Title: Topological String Theory

Date: Jul 08, 2005 09:30 AM

URL: http://pirsa.org/05070084

Abstract:

references:

topological string theory

$g=0$: Witten / 9112056

$g \geq 1$: BCOV / 9309140

"Mirror Book" by K. Hori et al

black hole entropy

Mohaupt

eferences:

al string theory

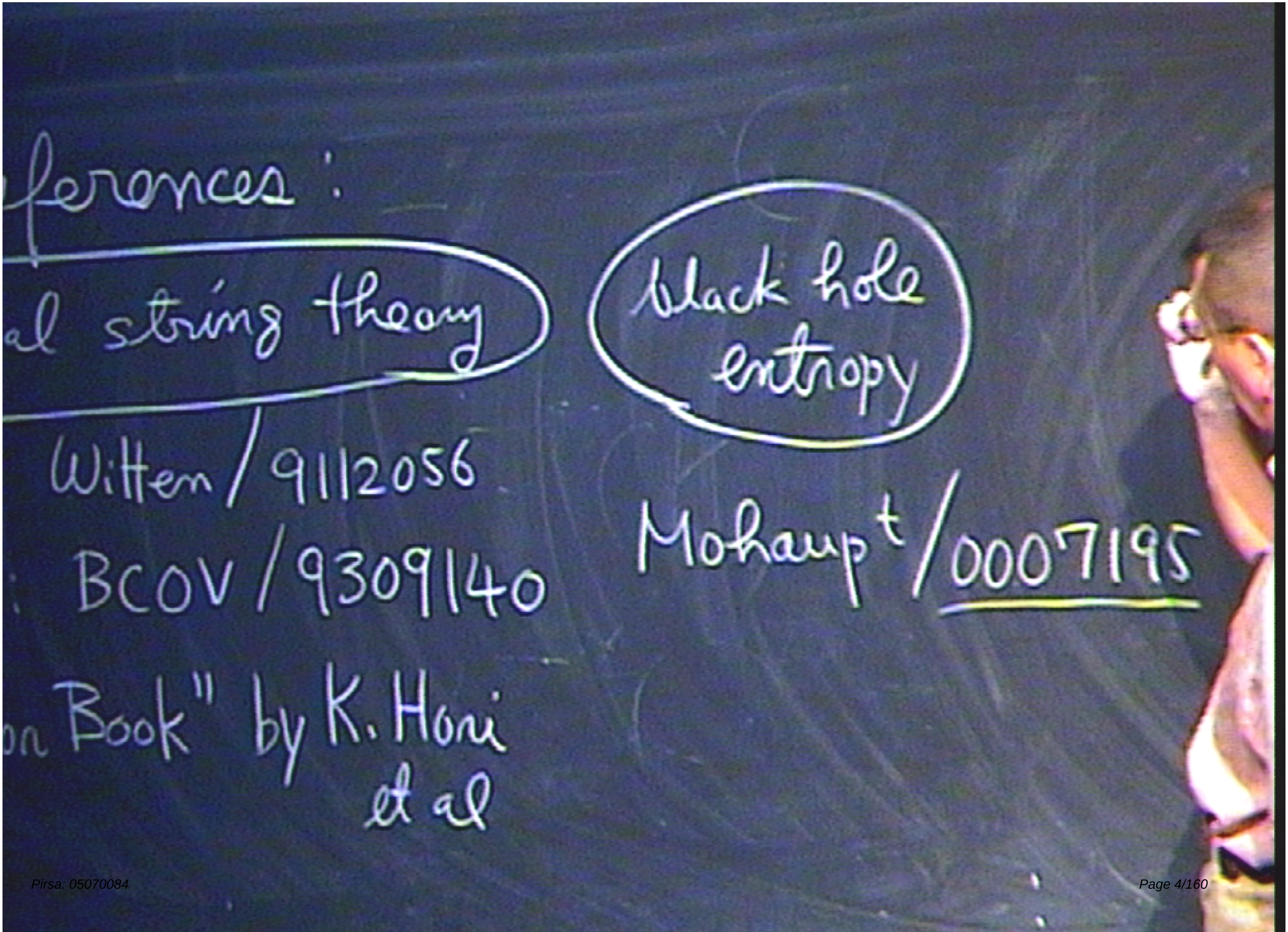
Witten / 9112056

: BCOV / 9309140

on Book" by K. Hori
et al

black hole
entropy

Mohaupt / 0007195


eferences :
al string theory
black hole entropy
Witten / 9112056
BCOV / 9309140
Mohaupt / 0007195
on Book" by K. Hori
et al

eferences:

al string theory

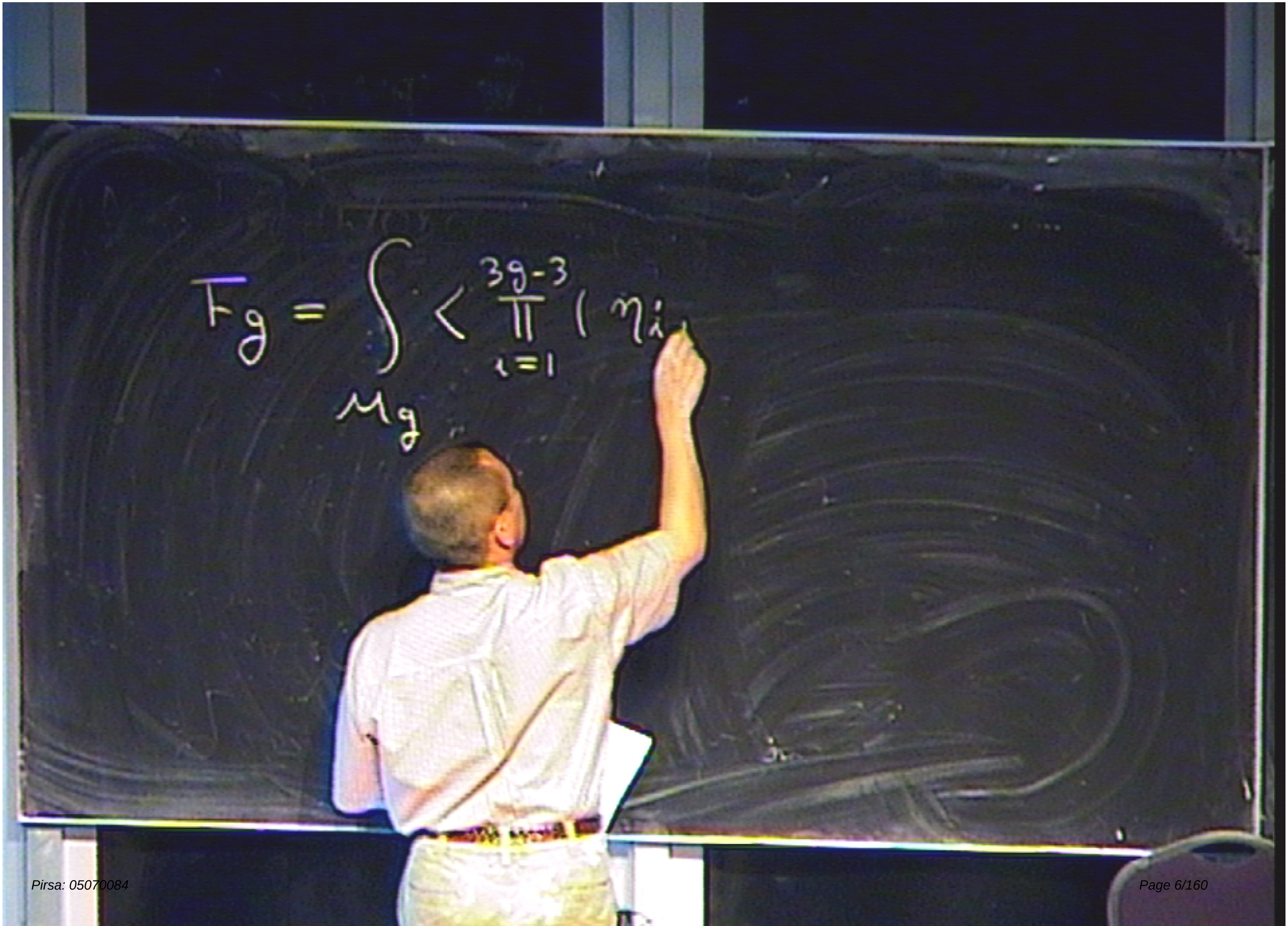
black hole entropy

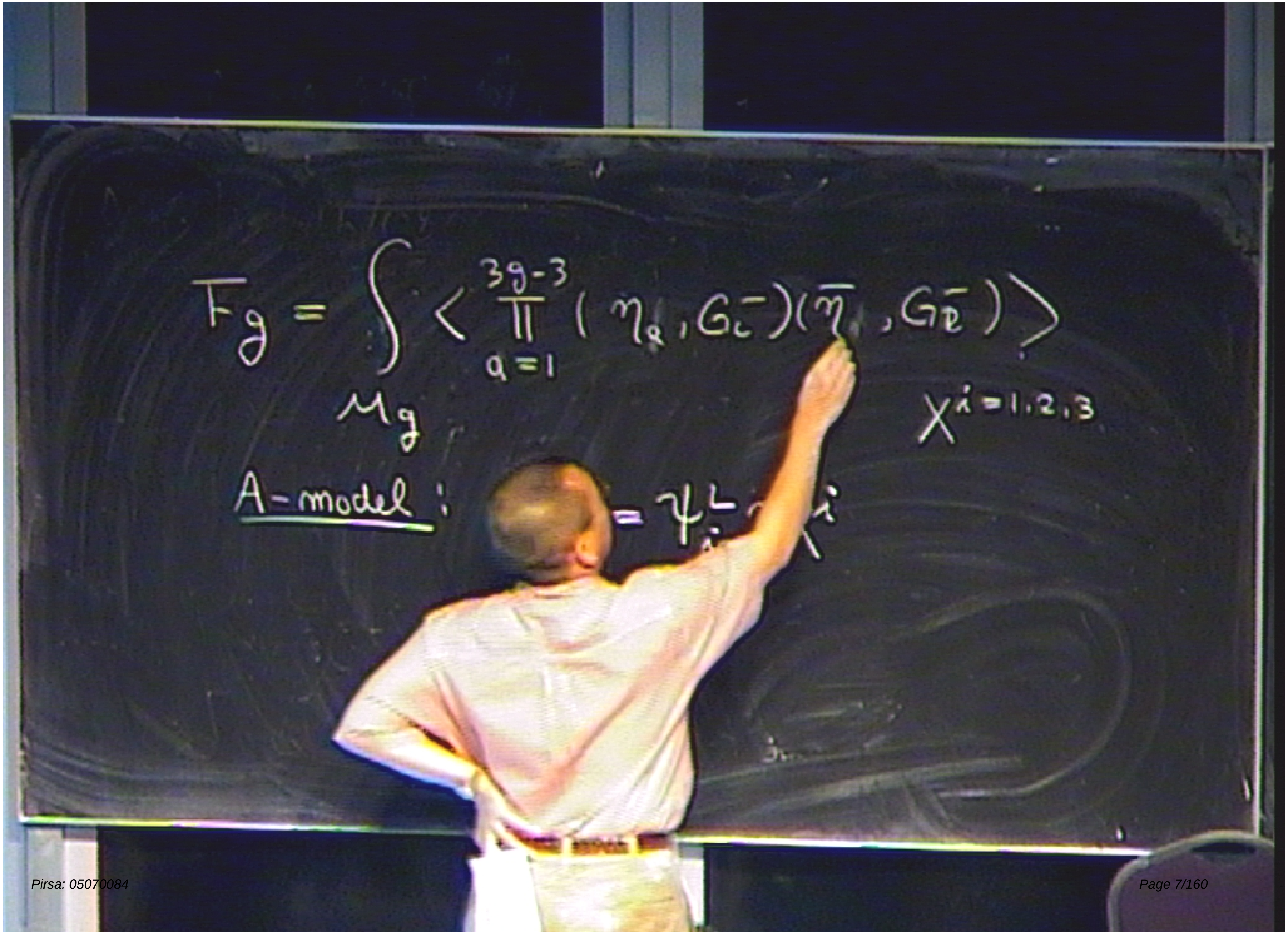
Witten / 9112056

Mohaupt / 0007195

: BCOV / 9309140

on Book" by K. Hori et al

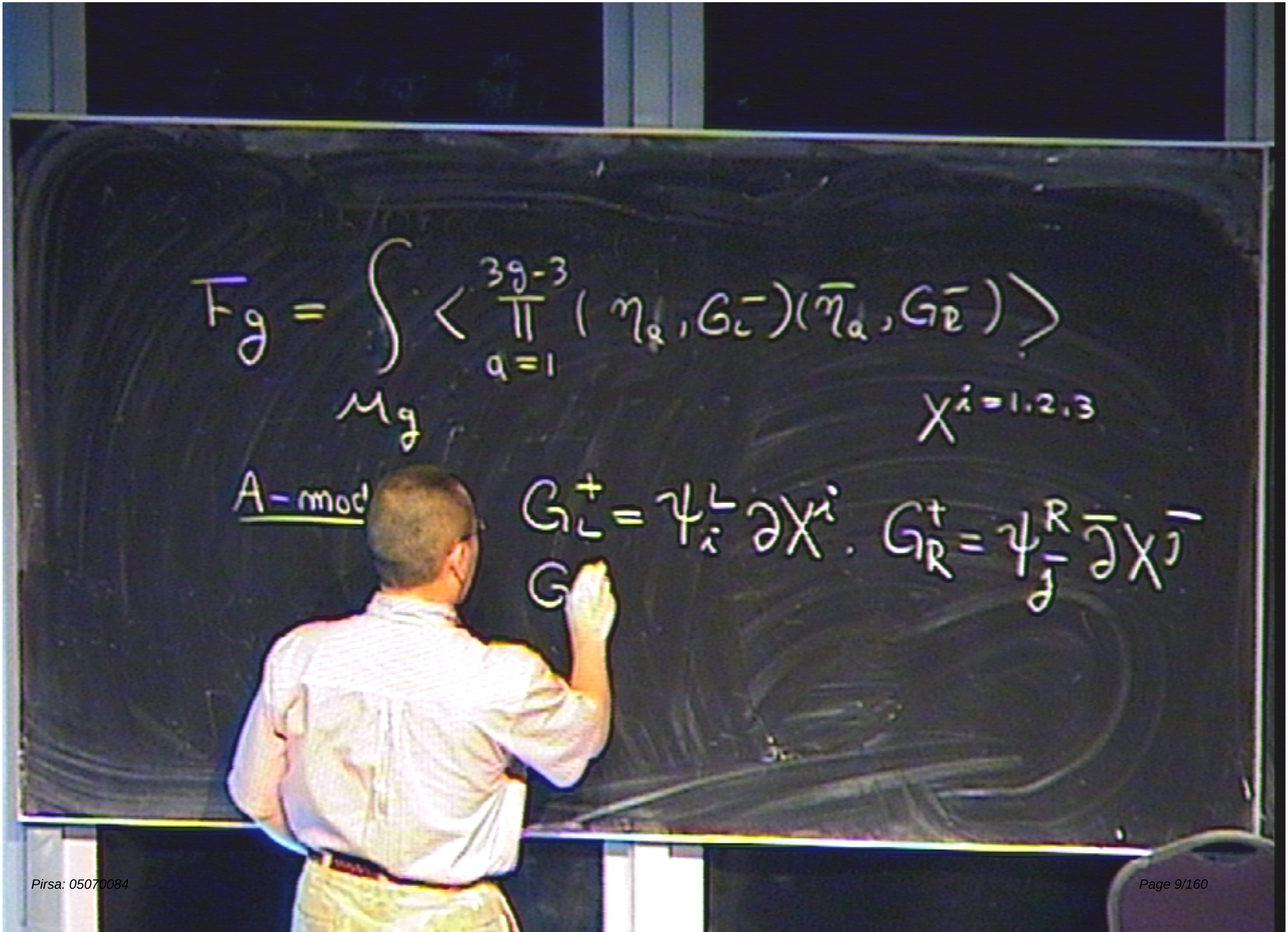

$F_g = \int_{\mathcal{M}_g} \langle \prod_{i=1}^{3g-3} (\eta_i$

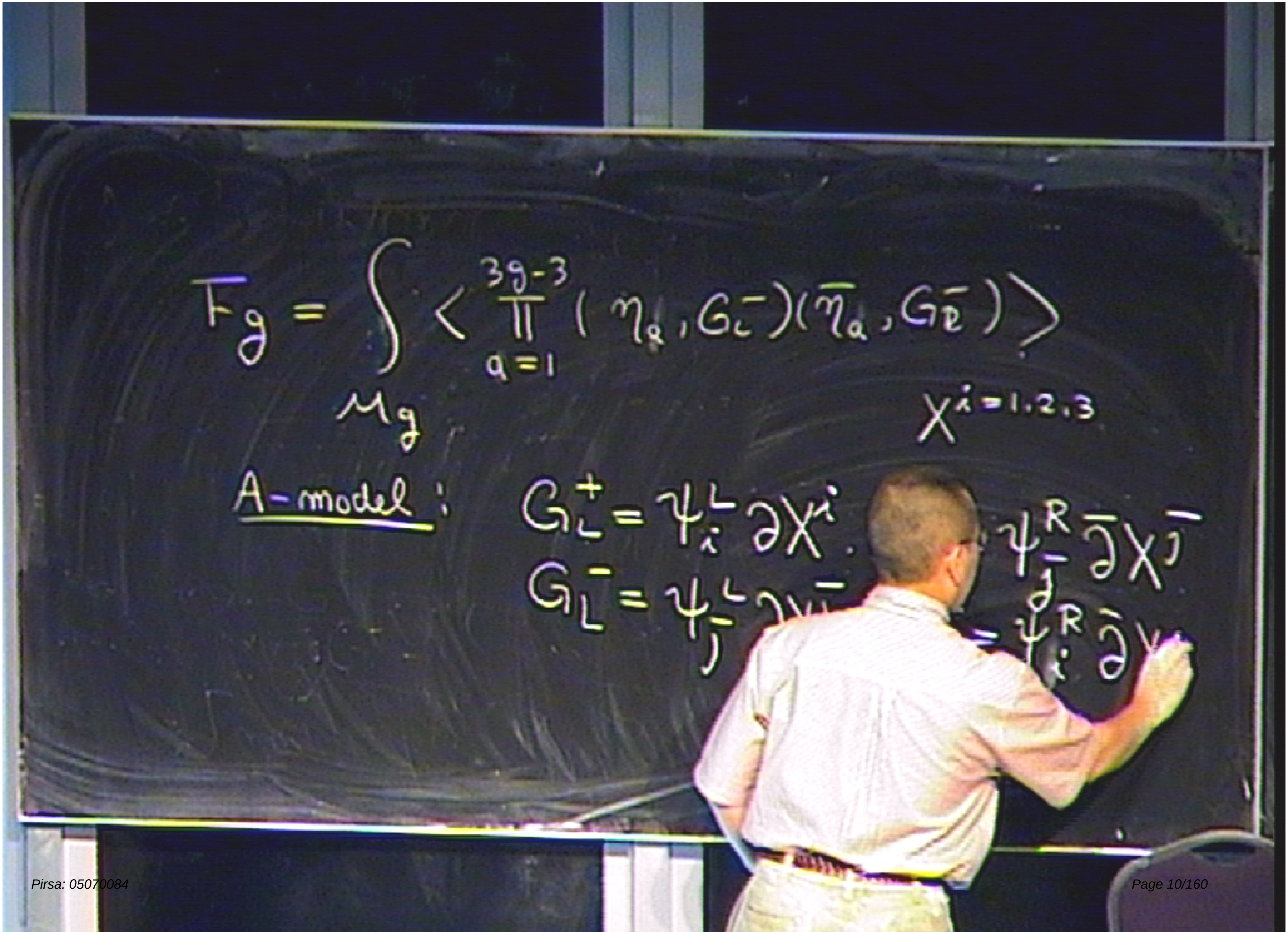

$F_g = \int_{\mathcal{M}_g} \langle \prod_{a=1}^{3g-3} (\eta_a, G_i^-)(\bar{\eta}_{\bar{a}}, G_{\bar{i}}^-) \rangle$
$X^{i=1,2,3}$
A-model :

$$F_g = \int_{\mathcal{M}_g} \left\langle \prod_{a=1}^{3g-3} (\eta_a, G_L^-)(\bar{\eta}_a, G_R^-) \right\rangle$$

$$X^{i=1,2,3}$$

A-model : $G_L^+ = \psi_i^L \partial X^i$


$F_g = \int_{\mathcal{M}_g} \langle \prod_{a=1}^{3g-3} (\eta_a, G_L^-)(\bar{\eta}_a, G_R^-) \rangle$
$X^{i=1,2,3}$
A-mod
$G_L^+ = \psi_i^L \partial X^i$, $G_R^+ = \psi_{\bar{j}}^R \bar{\partial} X^{\bar{j}}$
G


$F_g = \int_{\mathcal{M}_g} \langle \prod_{a=1}^{3g-3} (\eta_a, G_L^-)(\bar{\eta}_a, G_R^-) \rangle$
$X^{i=1,2,3}$
A-model:
$G_L^+ = \psi_i^L \partial X^i$
$\psi_{\bar{j}}^R \bar{\partial} X^{\bar{j}}$
$G_L^- = \psi_{\bar{j}}^L \partial X^{\bar{j}}$
$= \psi_i^R \bar{\partial} X$

$$F_g = \int_{\mathcal{M}_g} \left\langle \prod_{a=1}^{3g-3} (\eta_a, G_L^-)(\bar{\eta}_a, G_R^-) \right\rangle$$

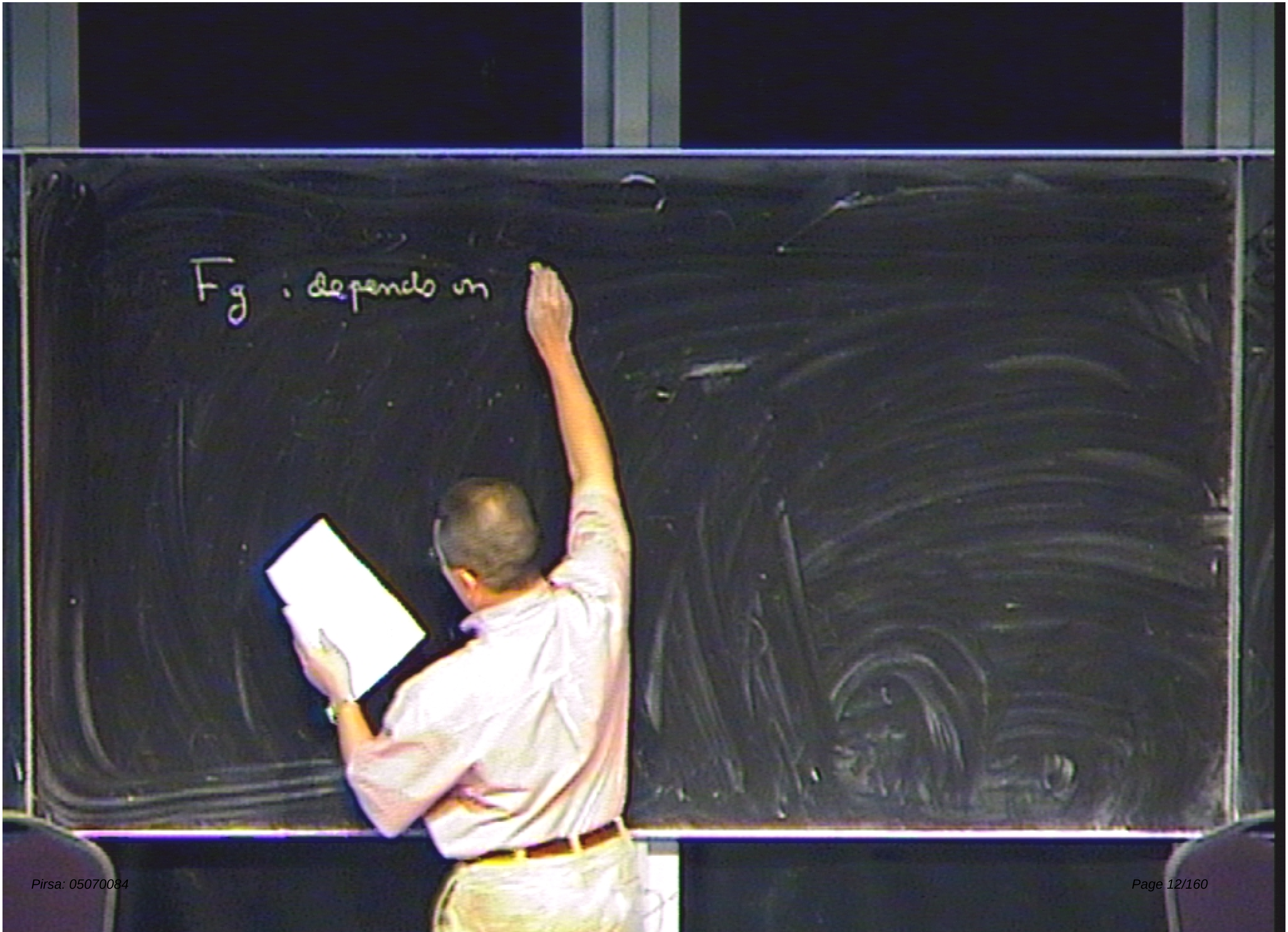
$$X^{i=1,2,3}$$

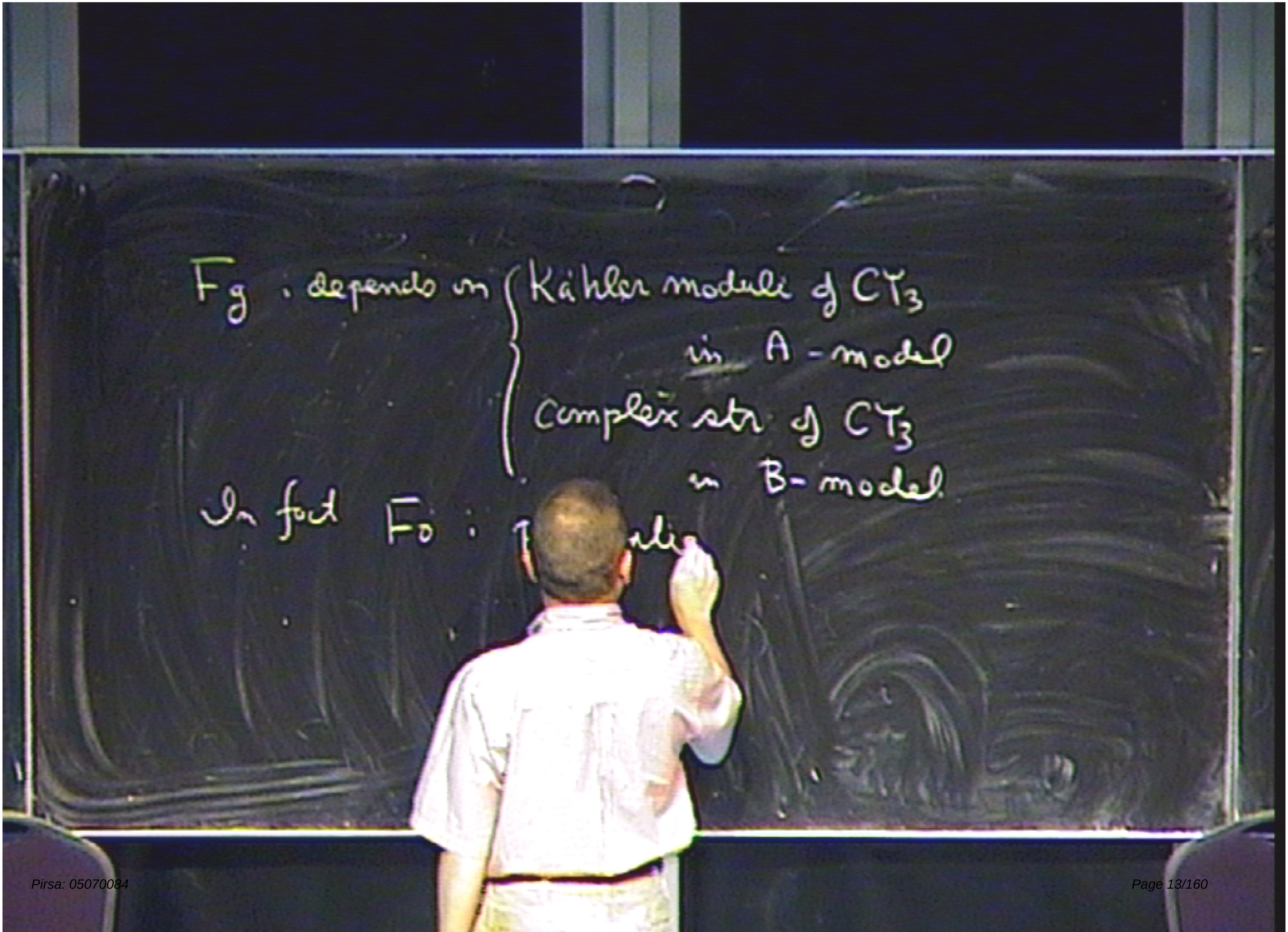
A-model: $G_L^+ = \psi_i^L \partial X^i$, $\quad G_R^+ = \psi_{\bar{j}}^R \bar{\partial} X^{\bar{j}}$

$G_L^- = \psi_{\bar{j}}^L \partial X^{\bar{j}}$ $\quad G_R^- = \psi_i^R \bar{\partial} X^i$

B-model

$$G_R^\pm \longrightarrow G_R^\mp$$


Fg : depends on


Fg : depends on
Kähler moduli of CY3
in A-model
complex str of CY3
in B-model
In fact F0 :

F_g : depends on $\begin{cases} \text{Kähler moduli of } CY_3 \\ \quad\quad \text{in A-model} \\ \text{complex str of } CY_3 \\ \quad\quad \text{in B-model} \end{cases}$

In fact F_0 : prepotential for type $\frac{\text{II A}}{\text{II B}}$ on CY_3

F_g : depends on $\begin{cases} \text{Kähler moduli of } CY_3 \\ \qquad \text{in A-model} \\ \text{complex str of } CY_3 \\ \qquad \text{in B-model} \end{cases}$

In fact F_0 : prepotential for type $\frac{\mathrm{II}A}{\mathrm{II}B}$ on CY_3

$\mathrm{II}A \quad M_V \leftarrow$ Kähler

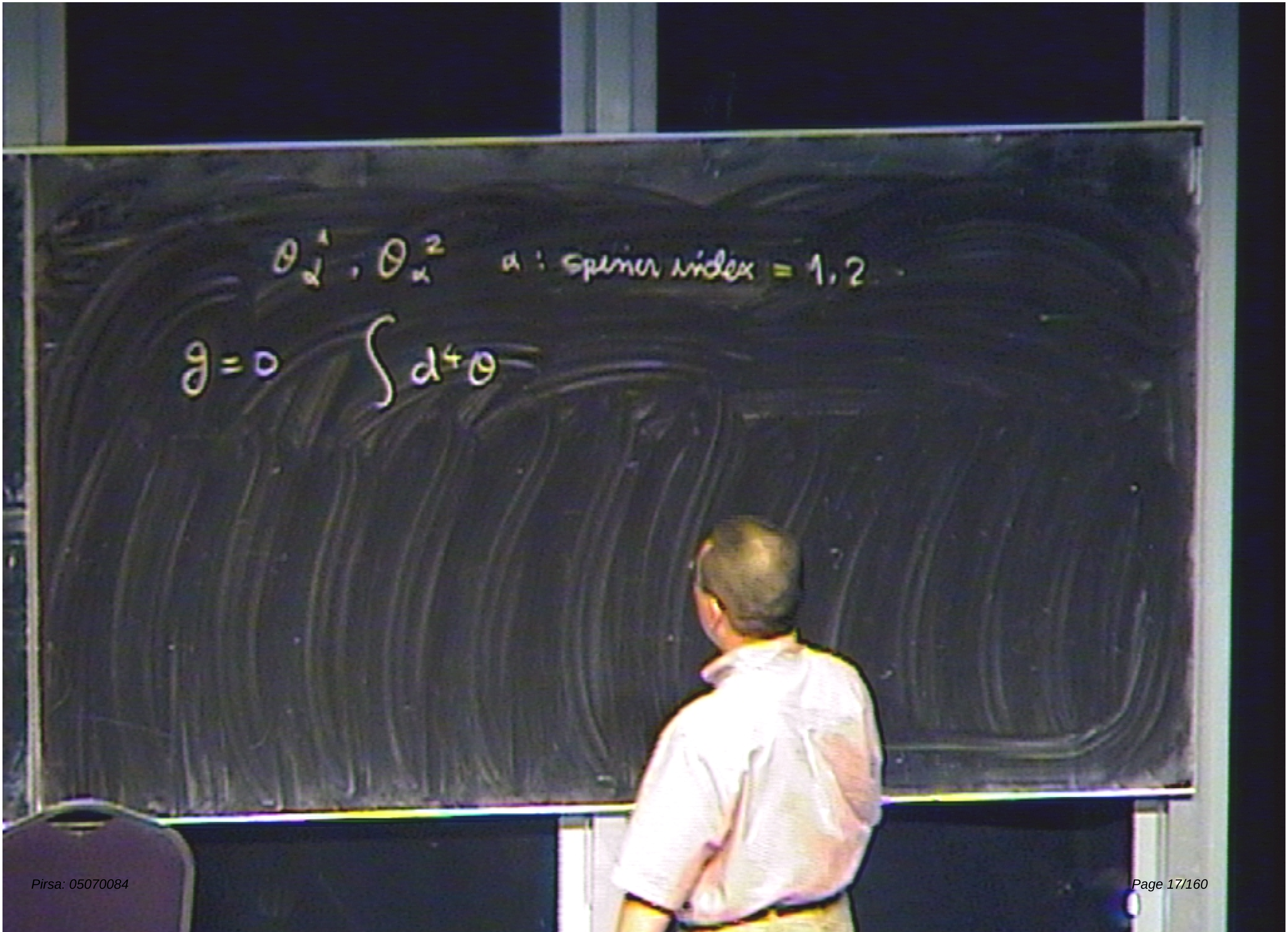
$\mathrm{II}B \quad M_V \leftarrow$ complex

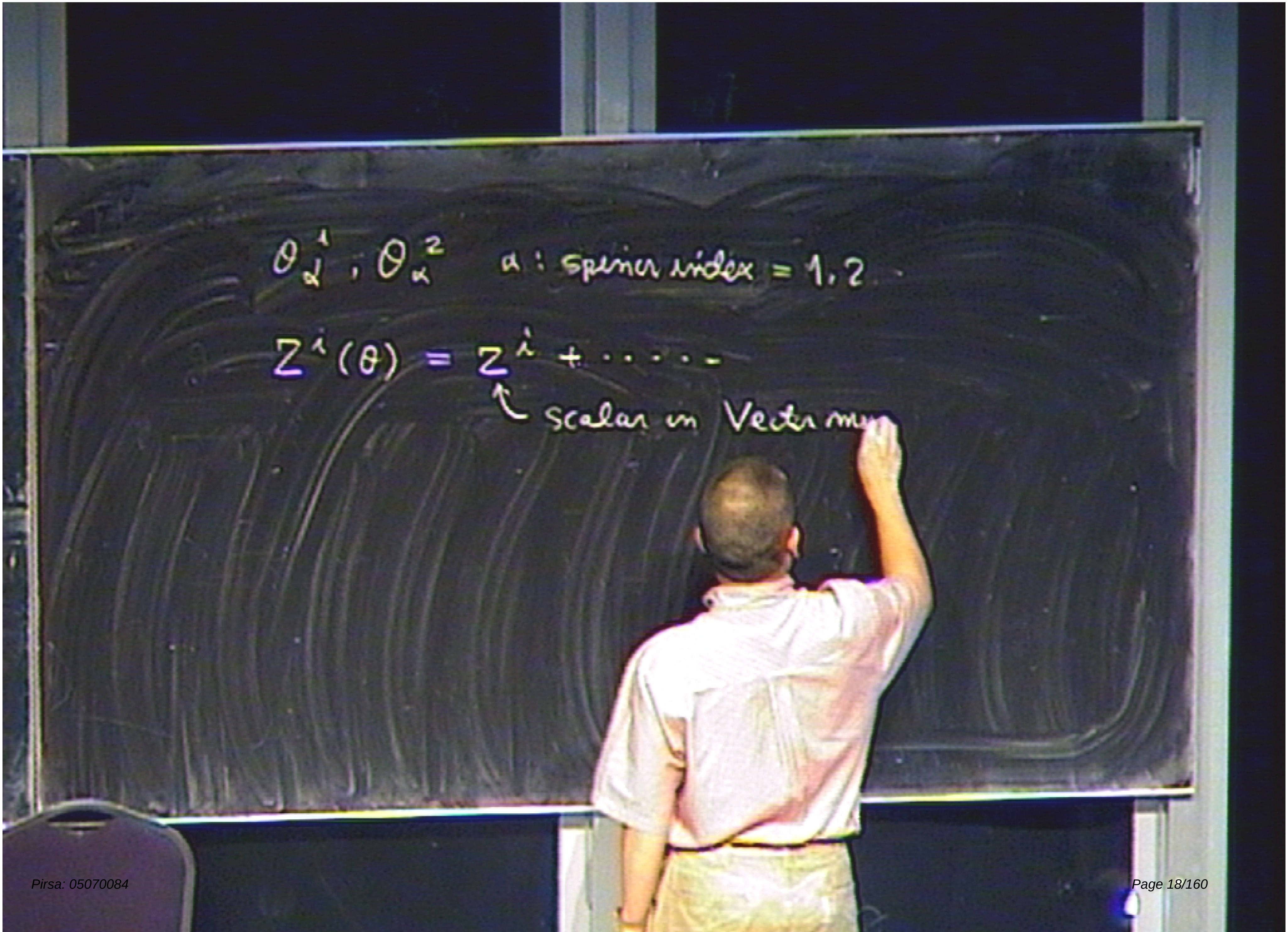
F_g : depends on $\begin{cases} \text{Kähler moduli of } CY_3 \\ \qquad \text{in A-model} \\ \text{complex str of } CY_3 \\ \qquad \text{in B-model} \end{cases}$

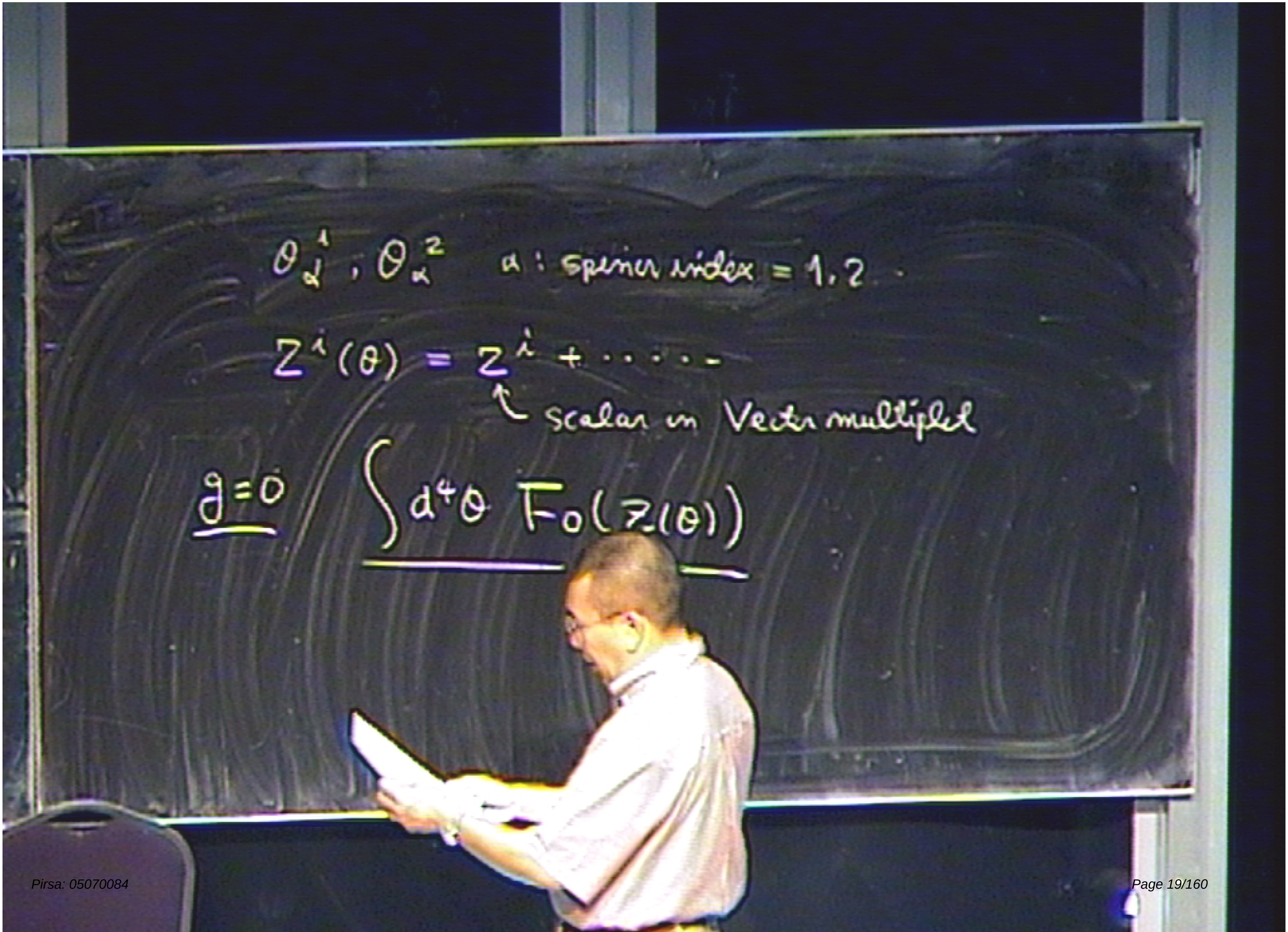
In fact F_0 : prepotential for type $\frac{\text{II A}}{\text{II B}}$ on CY_3

II A $\quad M_V \leftarrow$ Kähler

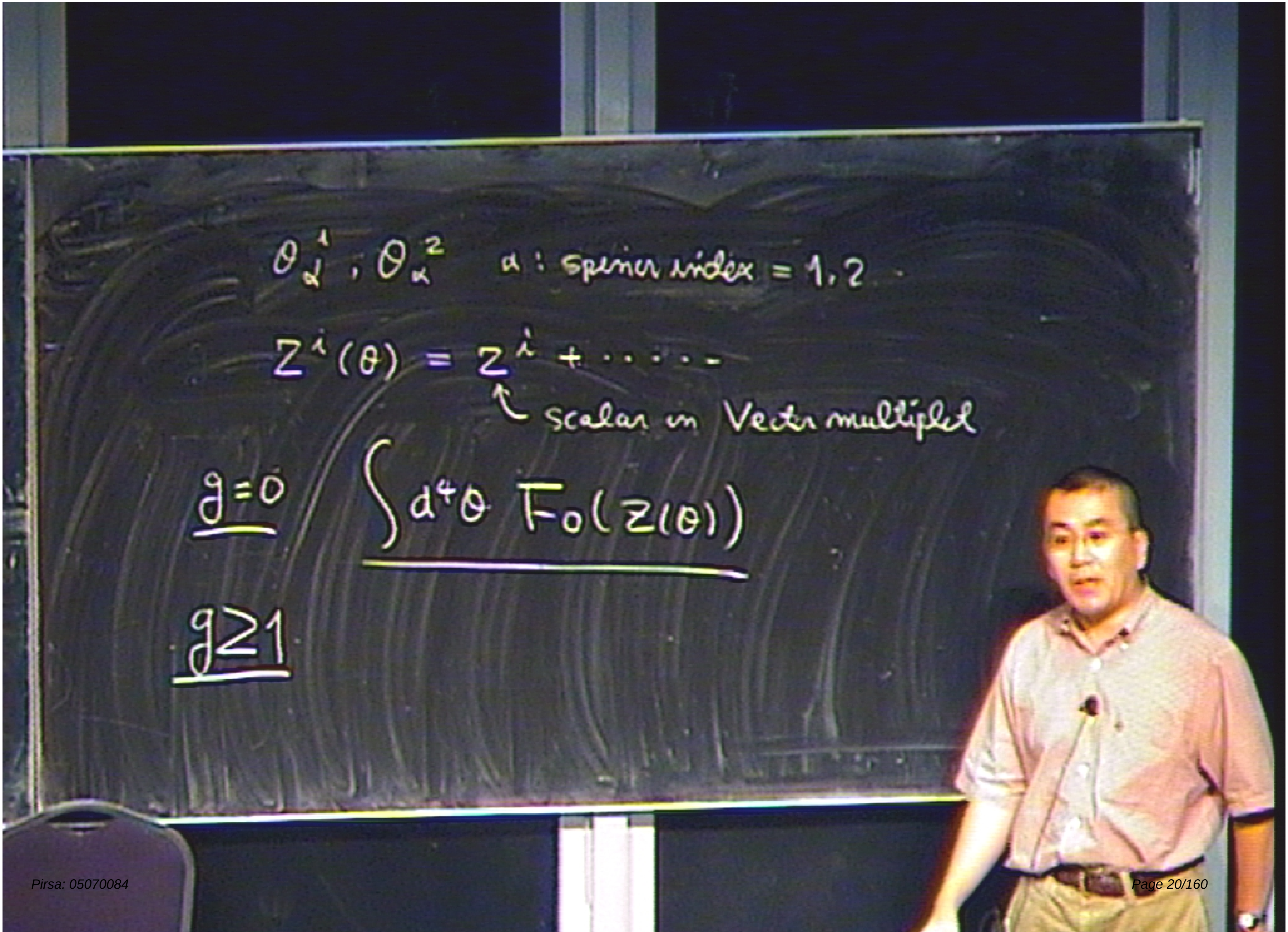
II B $\quad M_V \leftarrow$ complex

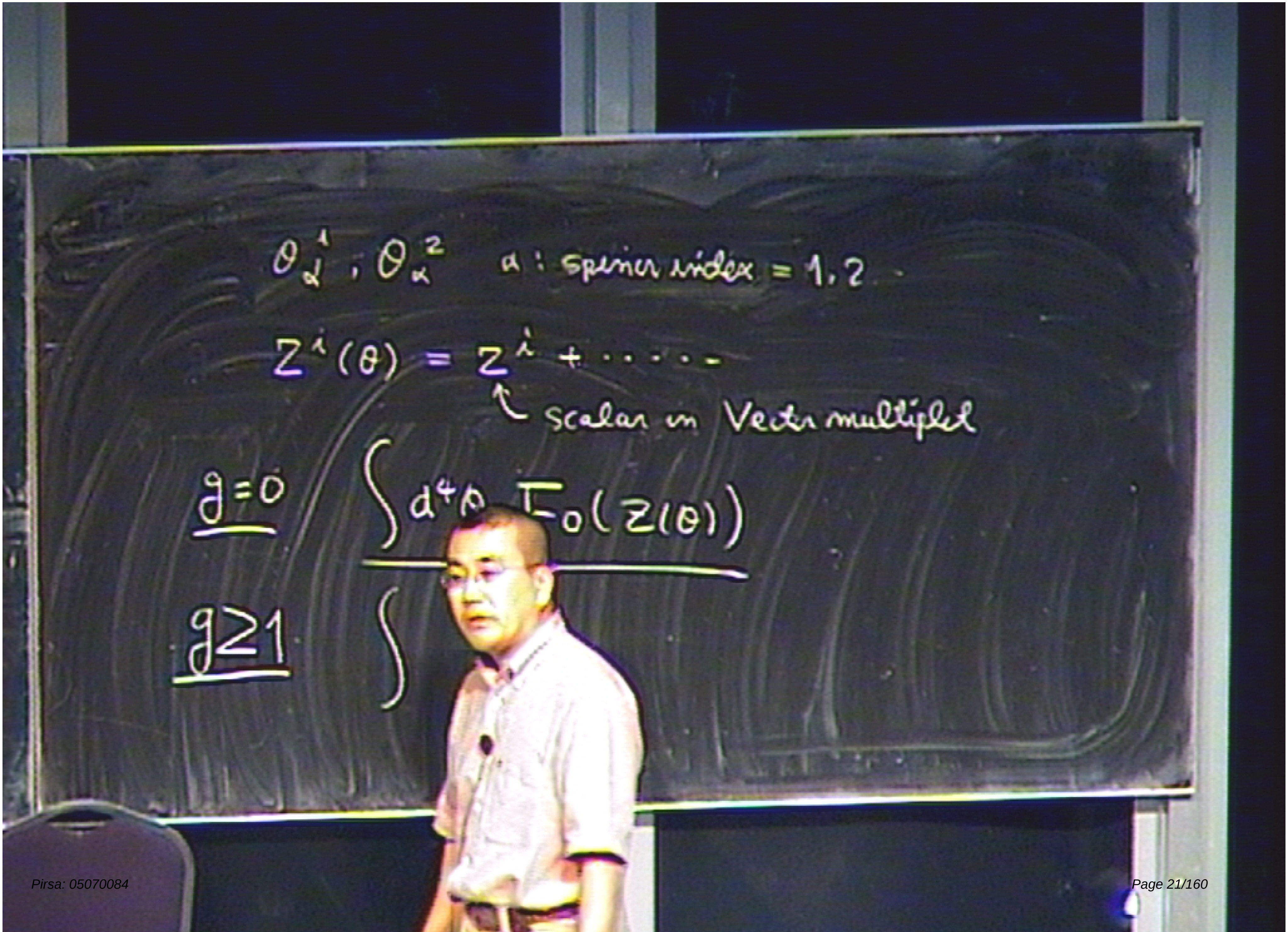

$\theta_\alpha^1 , \theta_\alpha^2$ α : spinor index = 1, 2
$g = 0$ $\int d^4\theta$


$\theta_\alpha^1 , \theta_\alpha^2$ α : spinor index = 1, 2
$Z^i(\theta) = z^i + \cdots$
scalar in Vector



$\theta_\alpha^1 , \theta_\alpha^2$ α : spinor index = 1, 2
$Z^i(\theta) = z^i + \cdots\cdots$
scalar in Vector multiplet
$g=0$
$\int d^4\theta \, F_0(Z(\theta))$


θ_α^1 , θ_α^2   α : spinor index = 1, 2
Z^i(θ) = Z^i + ...
scalar in Vector multiplet
g=0
∫d^4θ F_0(Z(θ))
g≥1


θ_α^1 ; θ_α^2 α : spinor index = 1, 2
$Z^i(\theta) = z^i + \cdots$
scalar in Vector multiplet
g=0 $\int d^4\theta\, F_0(Z(\theta))$
g≥1 $\int$

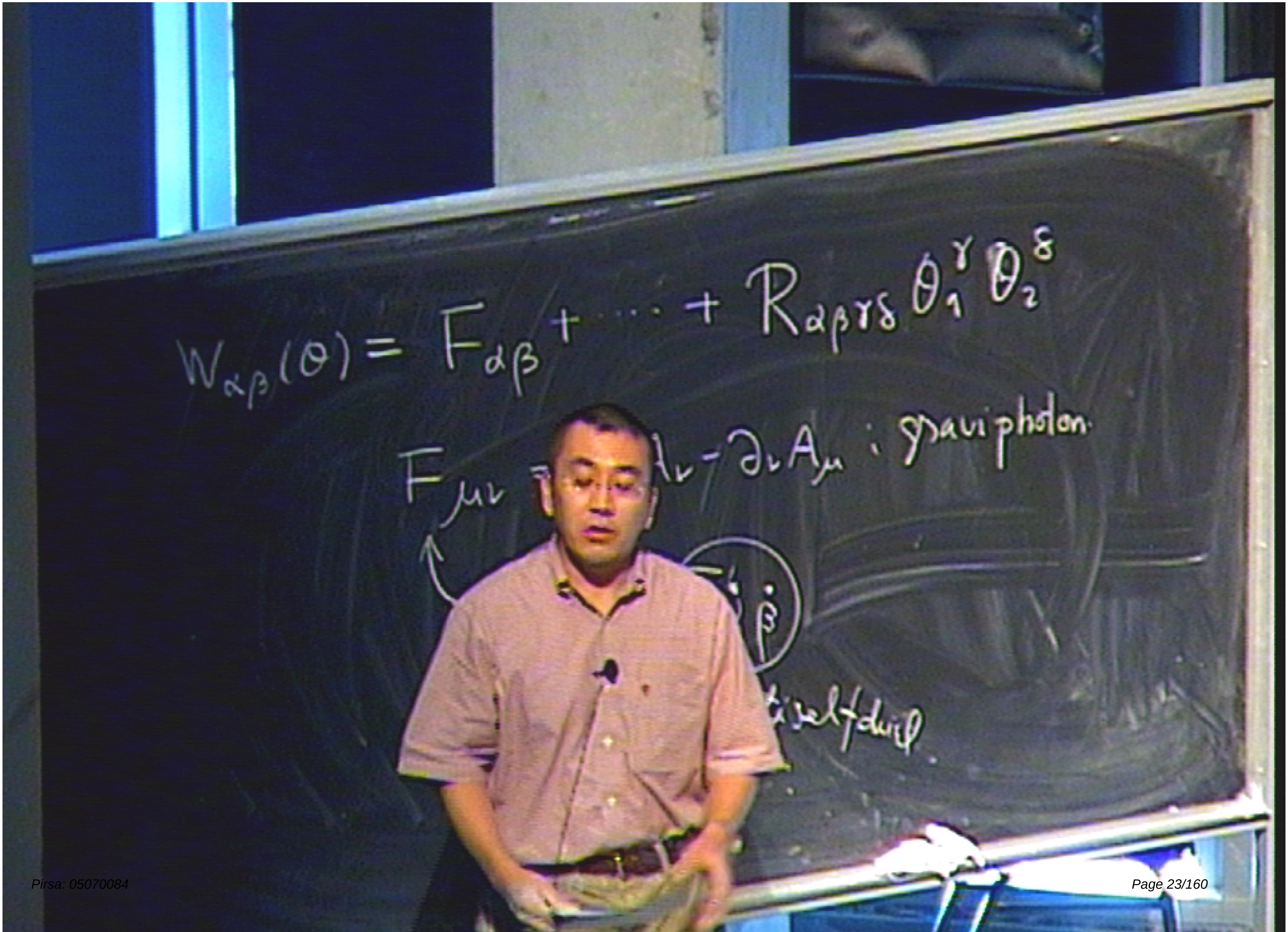
$\theta_\alpha^1 , \theta_\alpha^2$ α : spinor index $= 1, 2$.

$Z^i(\theta) = z^i + \cdots\cdots$

scalar in Vector multiplet

$\underline{g=0}$ $\int d^4\theta \, F_0(Z(\theta))$

$\underline{g\geq 1}$ $\int d^4 \cdots {}_g(Z(\theta)) \, (W^2)^{2g-2}$


$W_{\alpha\beta}(\theta) = F_{\alpha\beta} + \cdots + R_{\alpha\beta\gamma\delta}\,\theta_1^\gamma \theta_2^\delta$
$\partial_\nu A_\mu$: graviphoton

$$W_{\alpha\beta}(\theta) = F_{\alpha\beta} + \cdots + R_{\alpha\beta\gamma\delta}\,\theta_1^\gamma\,\theta_2^\delta$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$: graviphoton

$F_{\alpha\beta}$ selfdual

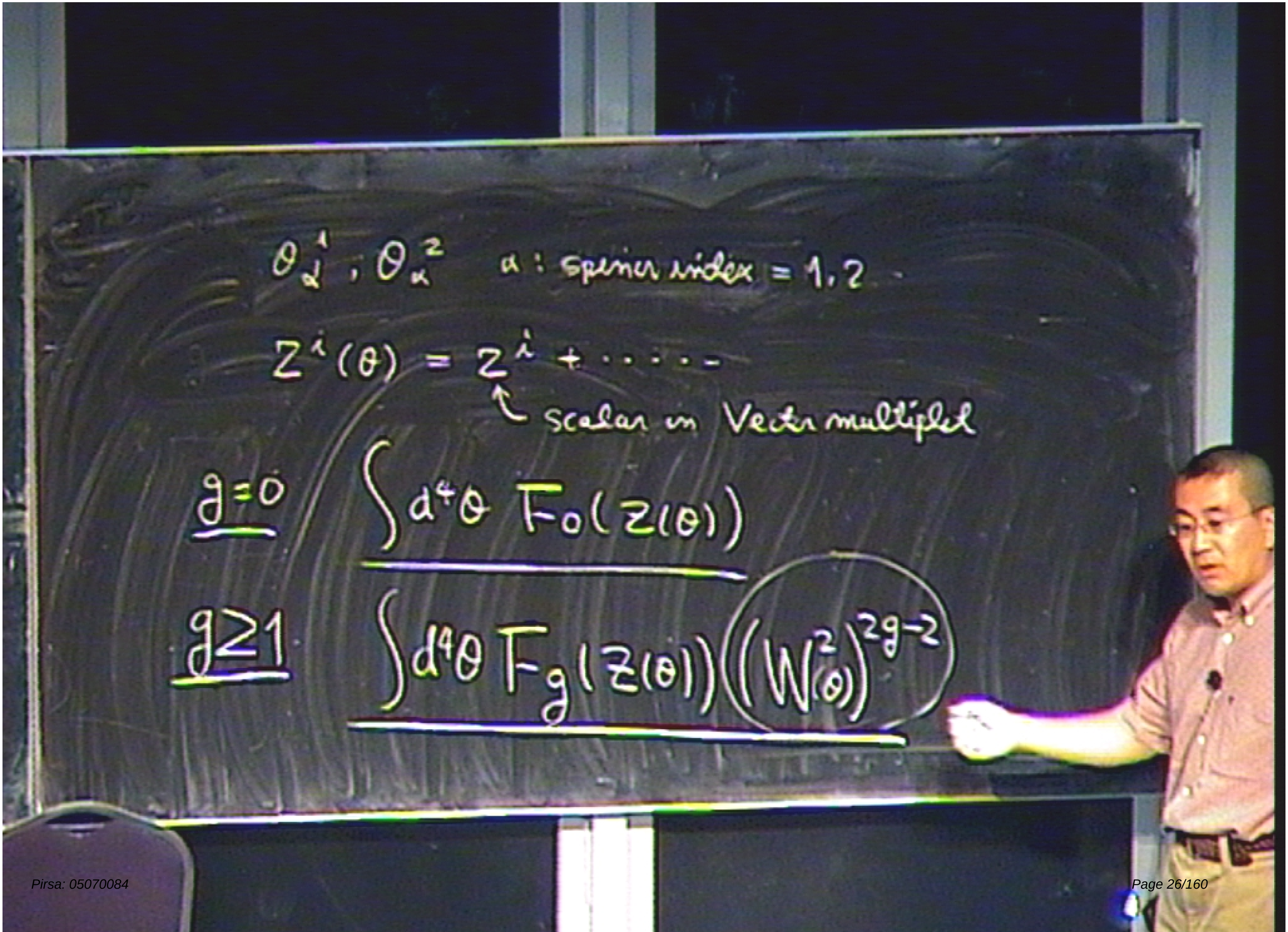
$F_{\dot\alpha\dot\beta}$ antiselfdual

$$W_{\alpha\beta}(\theta) = F_{\alpha\beta} + \cdots + \underline{R_{\alpha\beta\gamma\delta}}\,\theta_1^{\gamma}\,\theta_2^{\delta}$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$: graviphoton

$F_{\alpha\beta}$ $\qquad$ $F_{\dot\alpha\dot\beta}$

selfdual $\qquad$ antiselfdual



$\theta_\alpha^1 , \theta_\alpha^2$ α : spinor index = 1, 2
$Z^i(\theta) = z^i + \cdots$
scalar in Vector multiplet
g=0 $\int d^4\theta \, F_0(Z(\theta))$
g≥1 $\int d^4\theta \, F_g(Z(\theta)) \left(W^2_{(\theta)}\right)^{2g-2}$

$\underline{g=1}$

scalar

$$F_{g=1}(z)\cdot R^2 + \cdots$$

$\underline{g=1}$

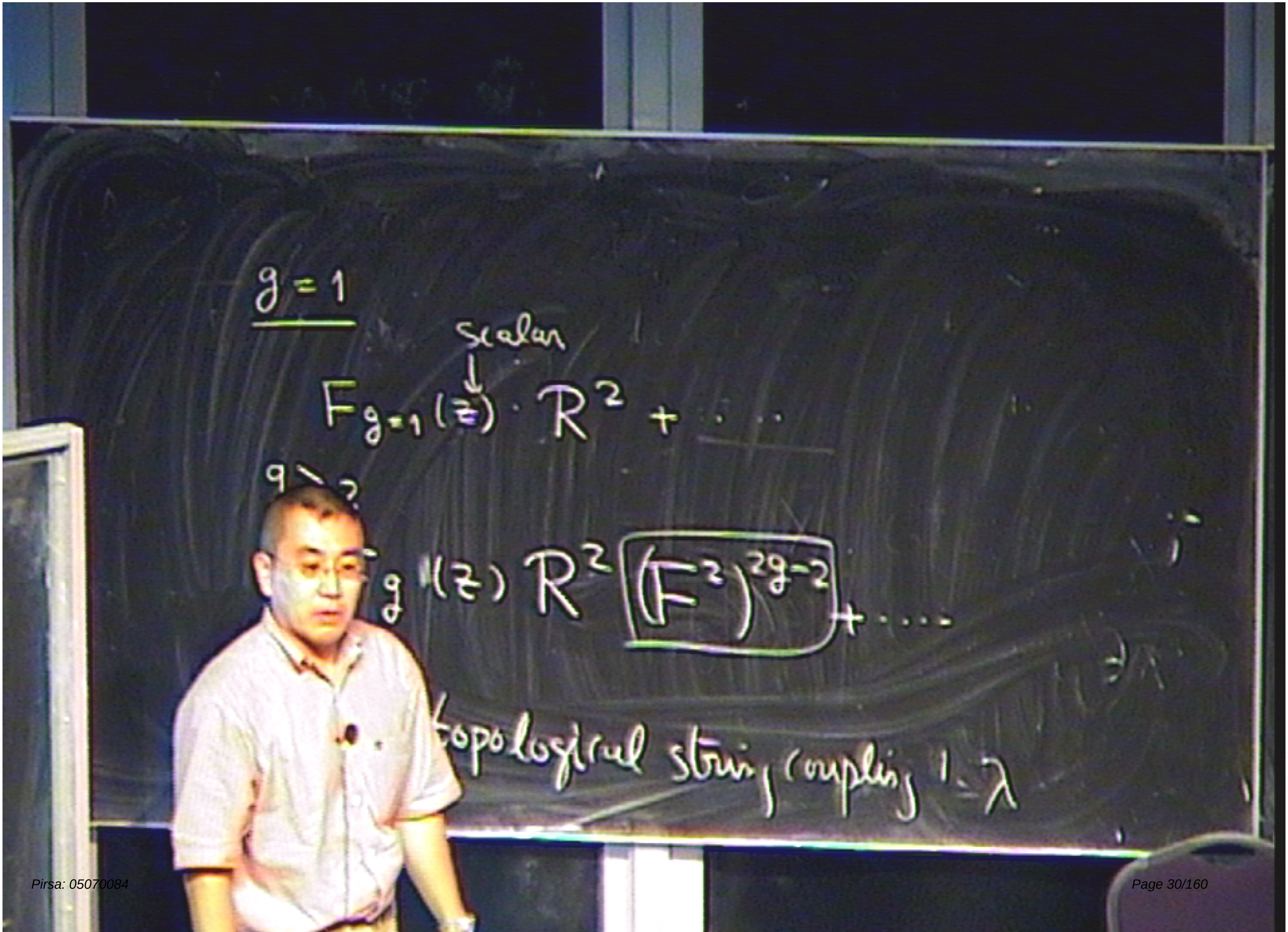
$$F_{g=1}(\overset{\text{scalar}}{\overset{\downarrow}{z}}) \cdot R^2 + \cdots$$

$g \geq 2$

$$F_g(z)\, R^2\, (F^2)^{2g-2} + \cdots$$

$F^2 \sim$ topological string coupling !

$$Z_{top} = \exp\left(\sum_g F_g(t) \cdot \lambda^{2g-2} \right)$$


g = 1
scalar
F_{g=1}(z) · R^2 + ...
g ≥ 2
F_g(z) R^2 (F^2)^{2g-2} + ...
topological string coupling

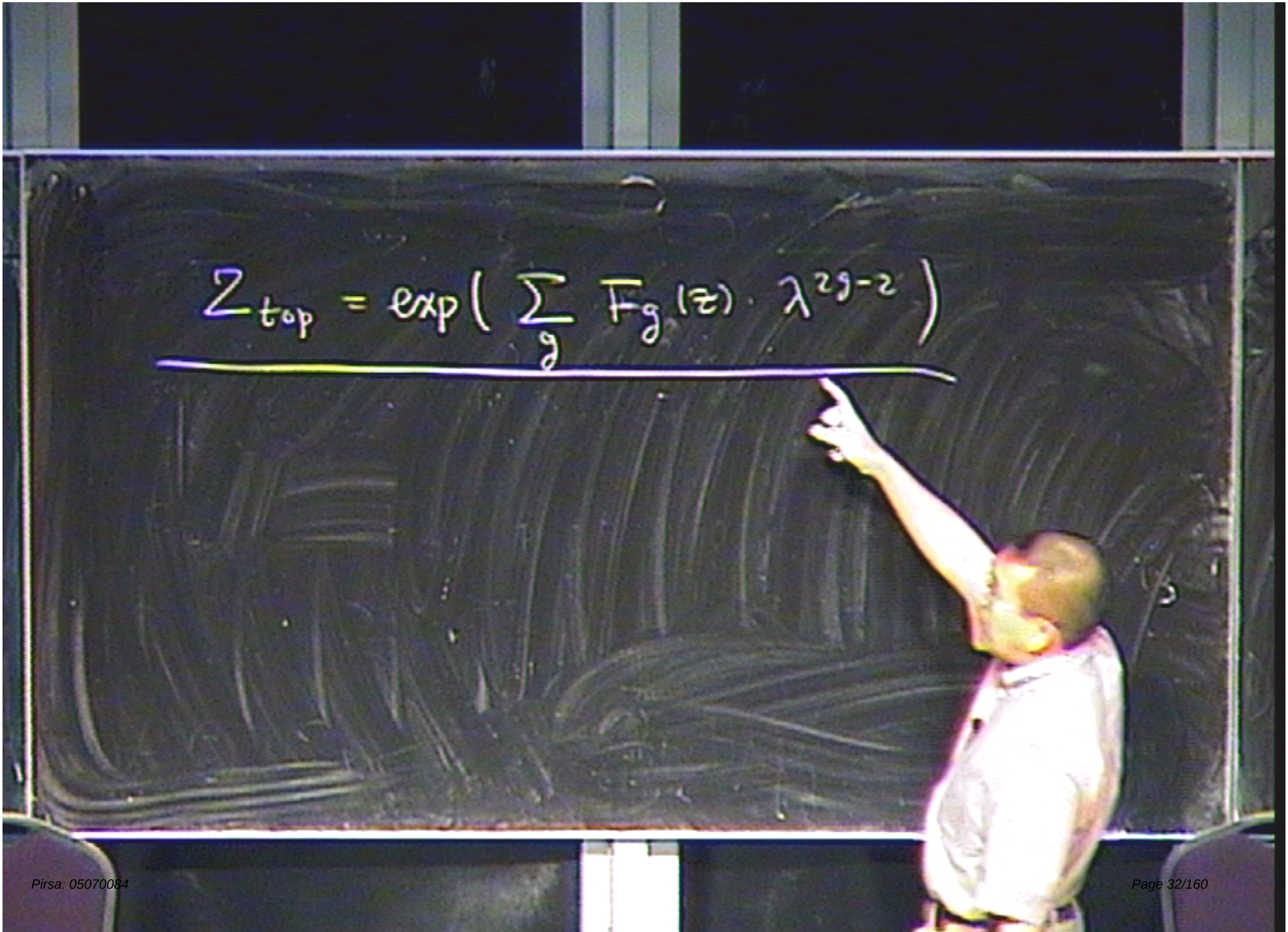
$\underline{g=1}$

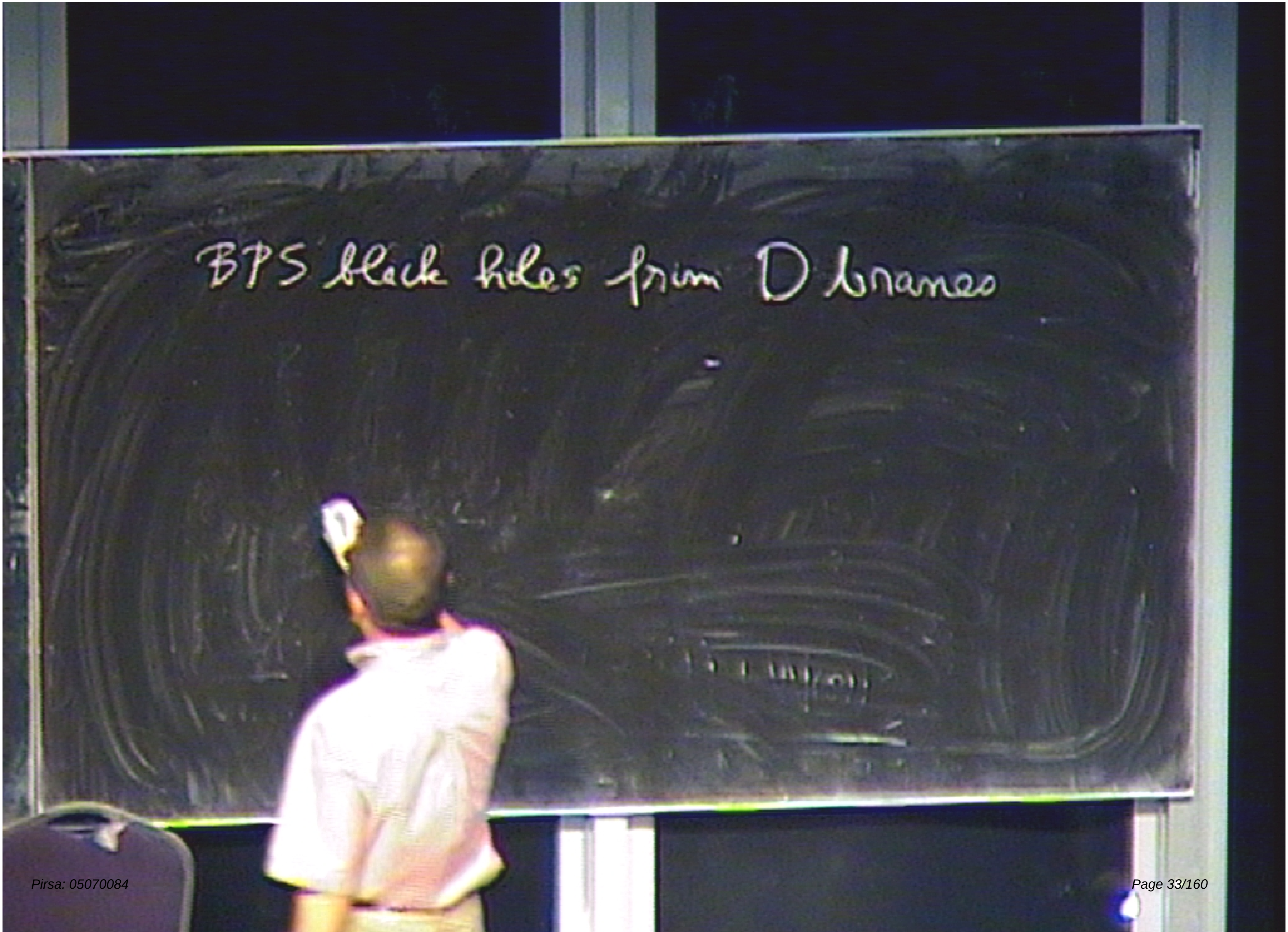
$$F_{g=1}(\overset{\text{scalar}}{\overset{\downarrow}{z}}) \cdot R^2 + \cdots$$

$g \geq 2$

$$F_g(z)\, R^2 \boxed{(F^2)^{2g-2}} + \cdots$$

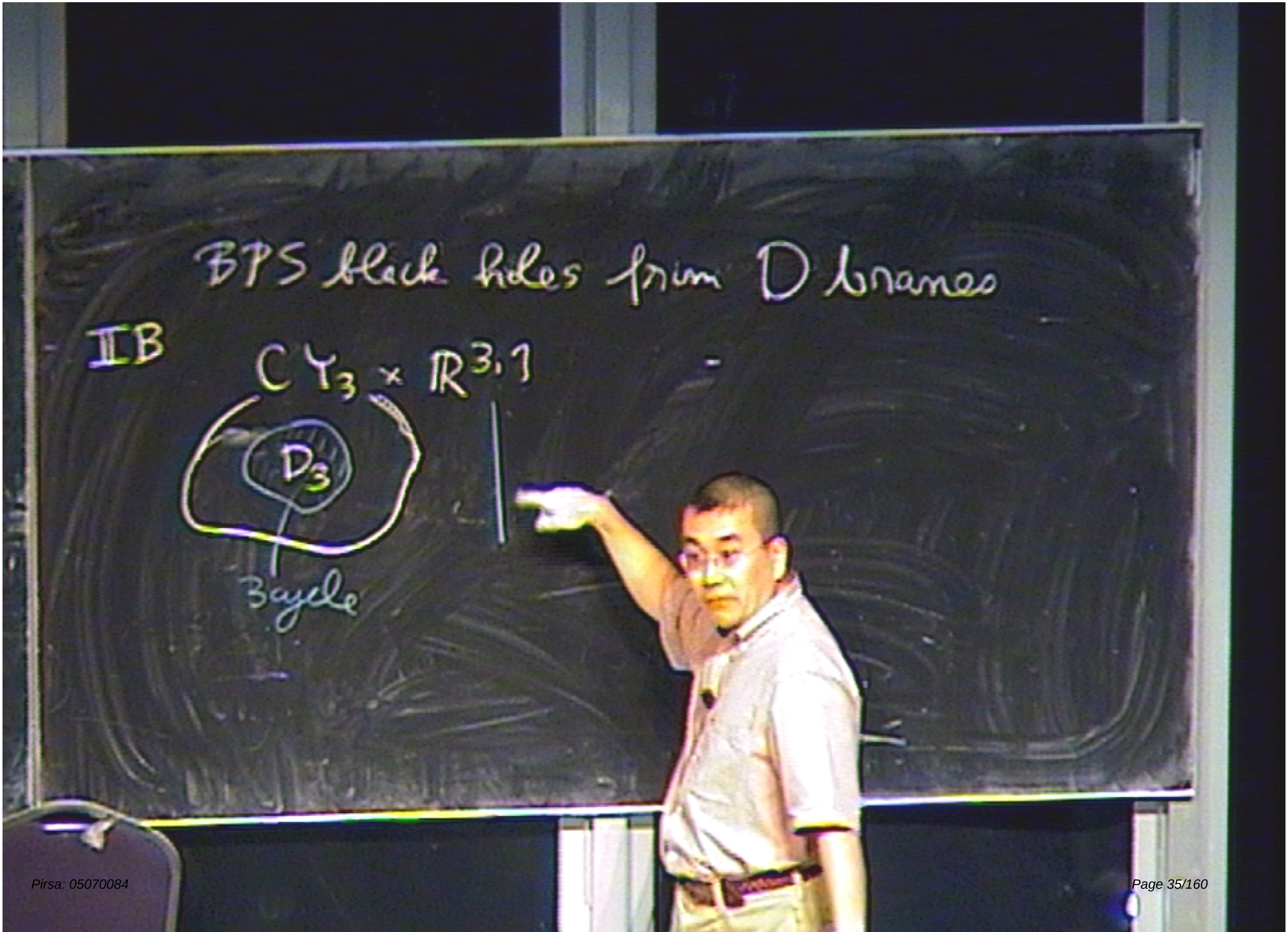
$F^2 \sim$ topological string coupling $1 = \lambda$


$Z_{top} = \exp\left(\sum_g F_g(z) \cdot \lambda^{2g-2} \right)$


BPS black holes from D branes

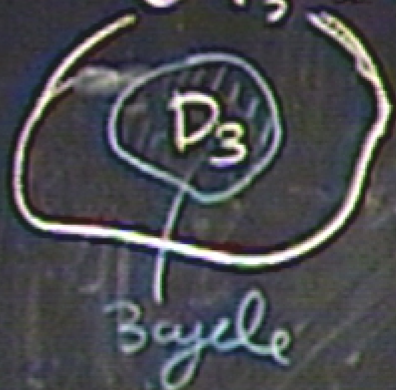
BPS black holes from D branes

IIB $CY_3 \times \mathbb{R}^{3,1}$


BPS black holes from D branes
IIB
$CY_3 \times R^{3,1}$
D_3
3cycle

BPS black holes from D branes

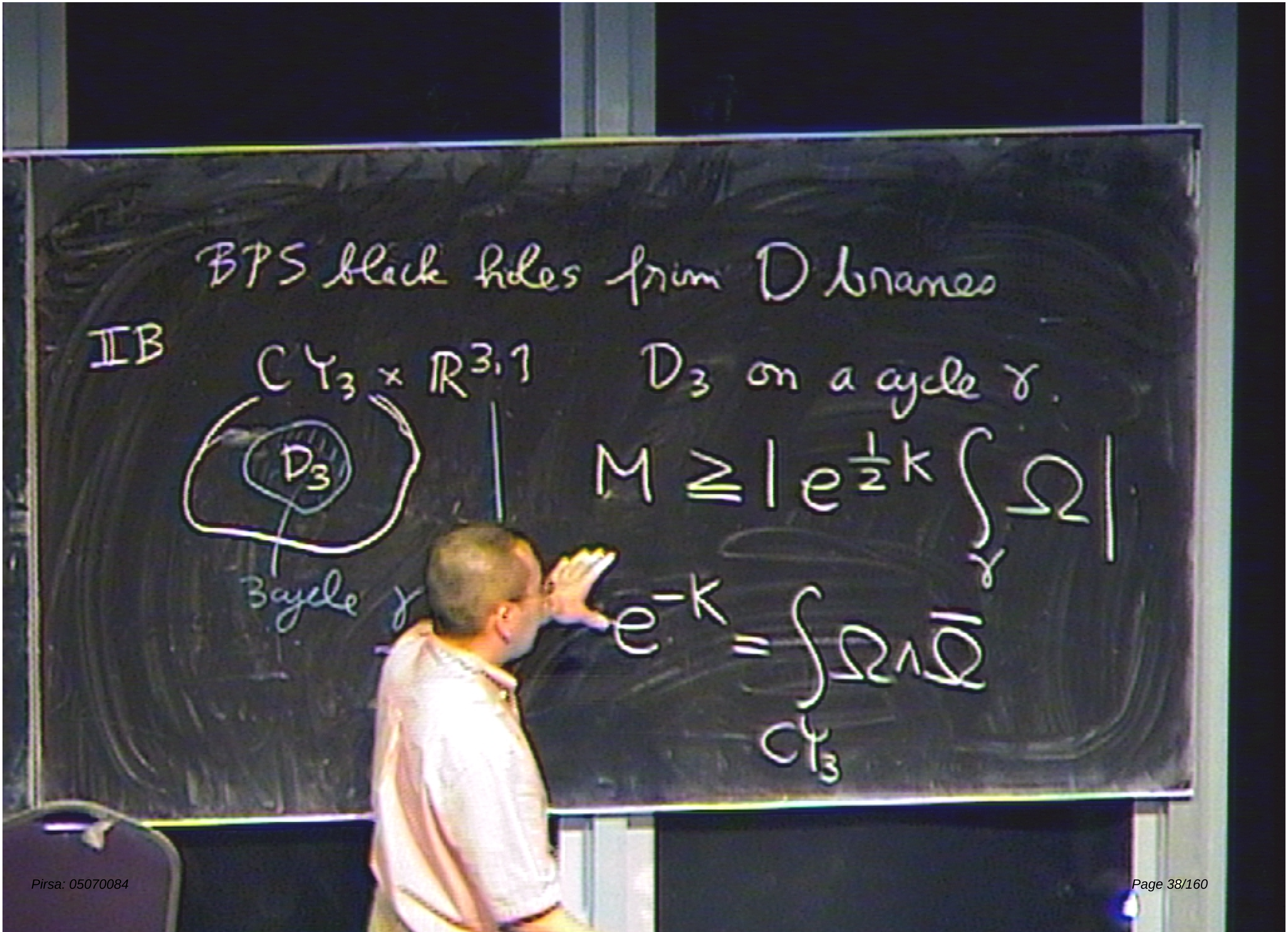
IIB $\quad CY_3 \times \mathbb{R}^{3,1}$



BPS black holes from D branes

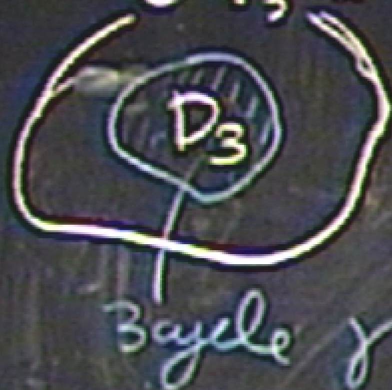
IIB $\quad CY_3 \times R^{3,1}$

3cycle γ


BPS black holes from D branes
IIB
$CY_3 \times \mathbb{R}^{3,1}$
D_3 on a cycle γ.
D_3
3cycle γ
$M \geq \left| e^{\frac{1}{2}K} \int_\gamma \Omega \right|$
$e^{-K} = \int_{CY_3} \Omega \wedge \bar{\Omega}$

BPS black holes from D branes

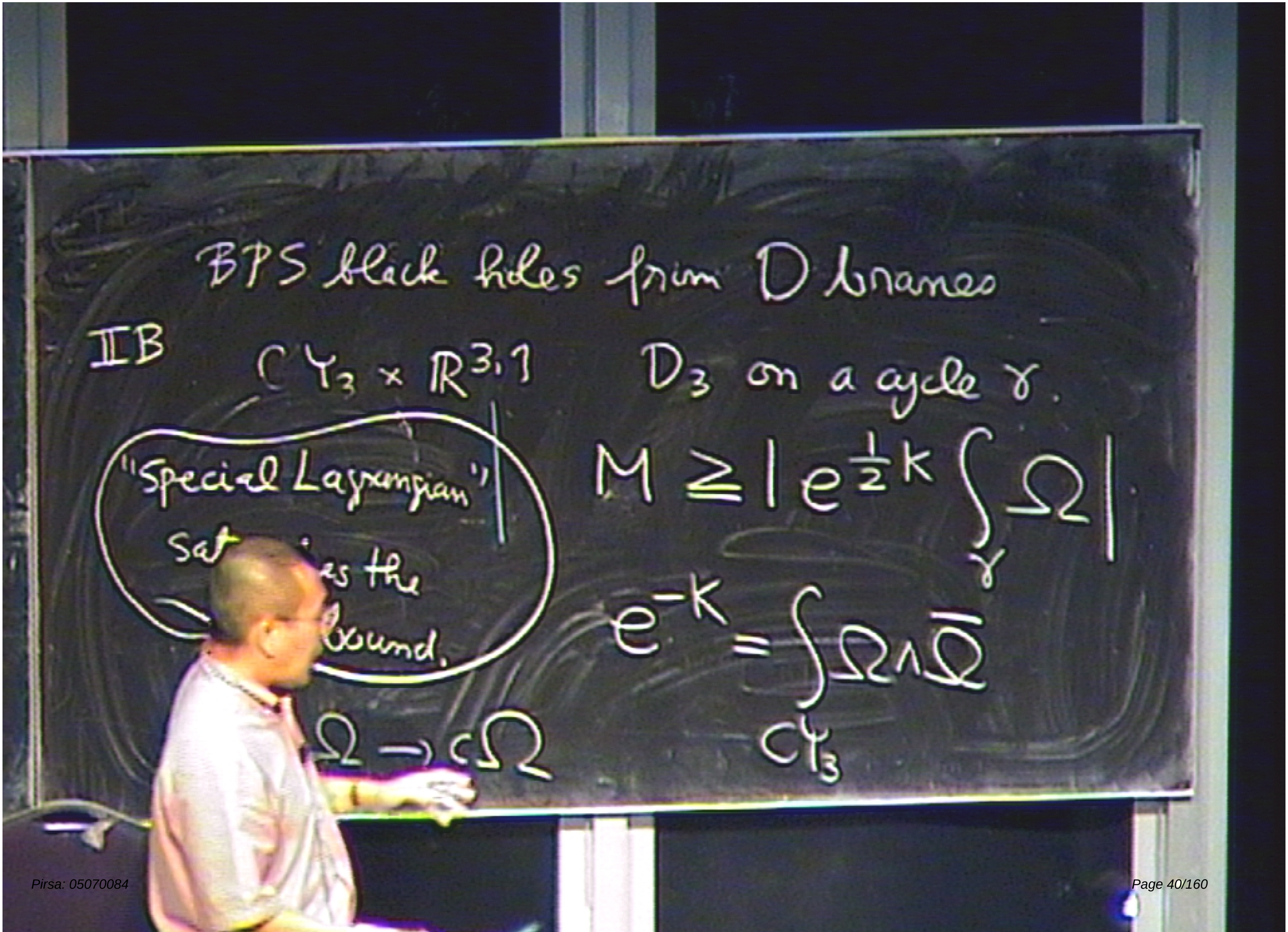
IIB $\quad CY_3 \times \mathbb{R}^{3,1} \qquad D_3$ on a cycle γ.



$$M \geq \left| e^{\frac{1}{2}K} \int_{\gamma} \Omega \right|$$

$$e^{-K} = \int_{CY_3} \Omega \wedge \bar{\Omega}$$

$$\Omega \to c\,\Omega$$


BPS black holes from D branes
IIB
$CY_3 \times \mathbb{R}^{3,1}$
D_3 on a cycle γ.
"Special Lagrangian"
$M \geq \left| e^{\frac{1}{2}K} \int_\gamma \Omega \right|$
$e^{-K} = \int_{CY_3} \Omega \wedge \bar{\Omega}$
$\Omega \to c\Omega$

BPS black holes from D branes

IIB $\quad CY_3 \times \mathbb{R}^{3,1} \quad D_3$ on a cycle γ.

"Special Lagrangian" saturates the bound.

$$M \geq \left| e^{\frac{1}{2}K} \int_\gamma \Omega \right|$$

$$e^{-K} = \int_{CY_3} \Omega \wedge \bar{\Omega}$$

$$\Omega \to c\,\Omega$$

BPS black holes from D branes

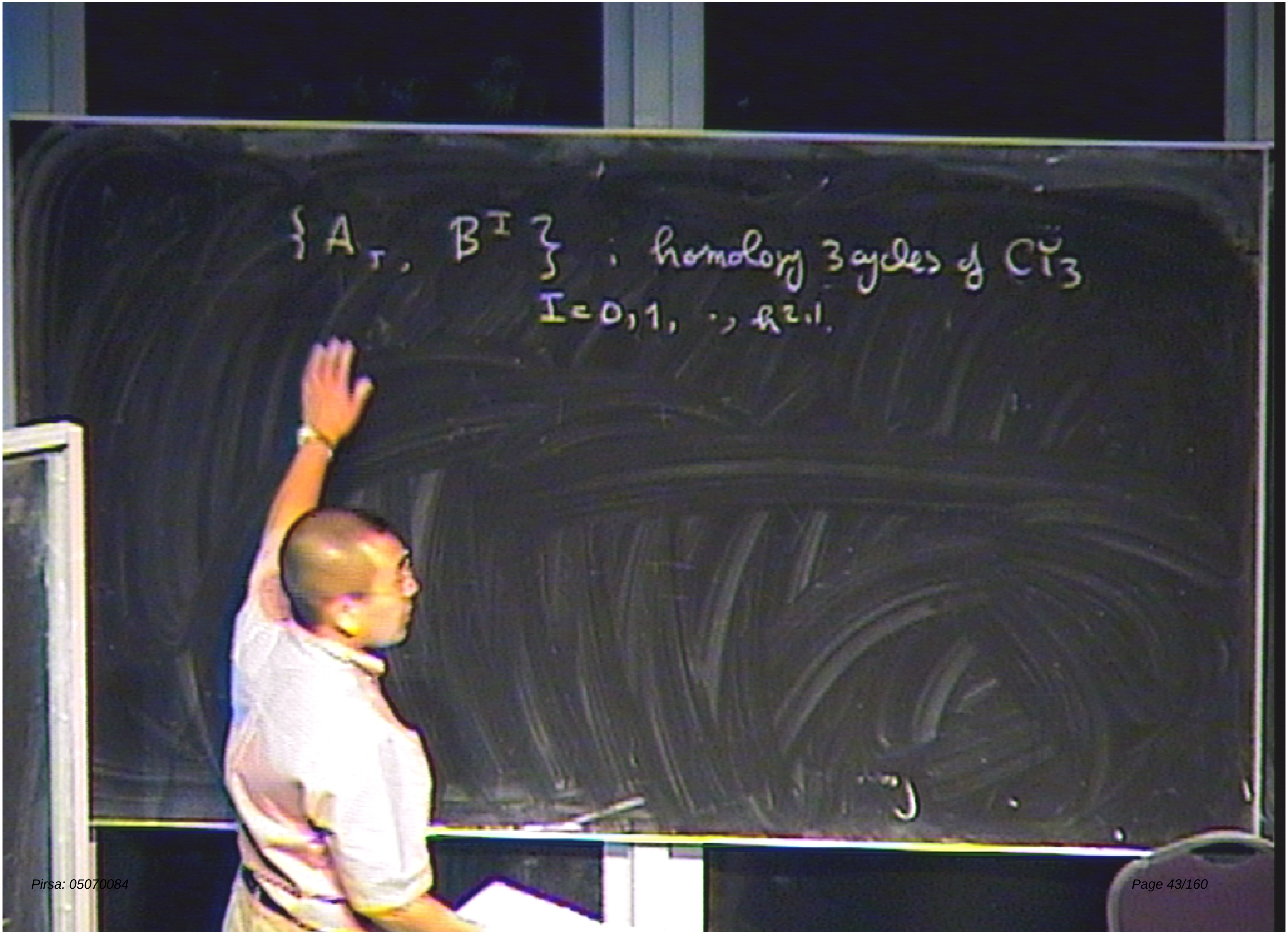
IIB $\quad CY_3 \times \mathbb{R}^{3,1}$ $\quad D_3$ on a cycle γ.

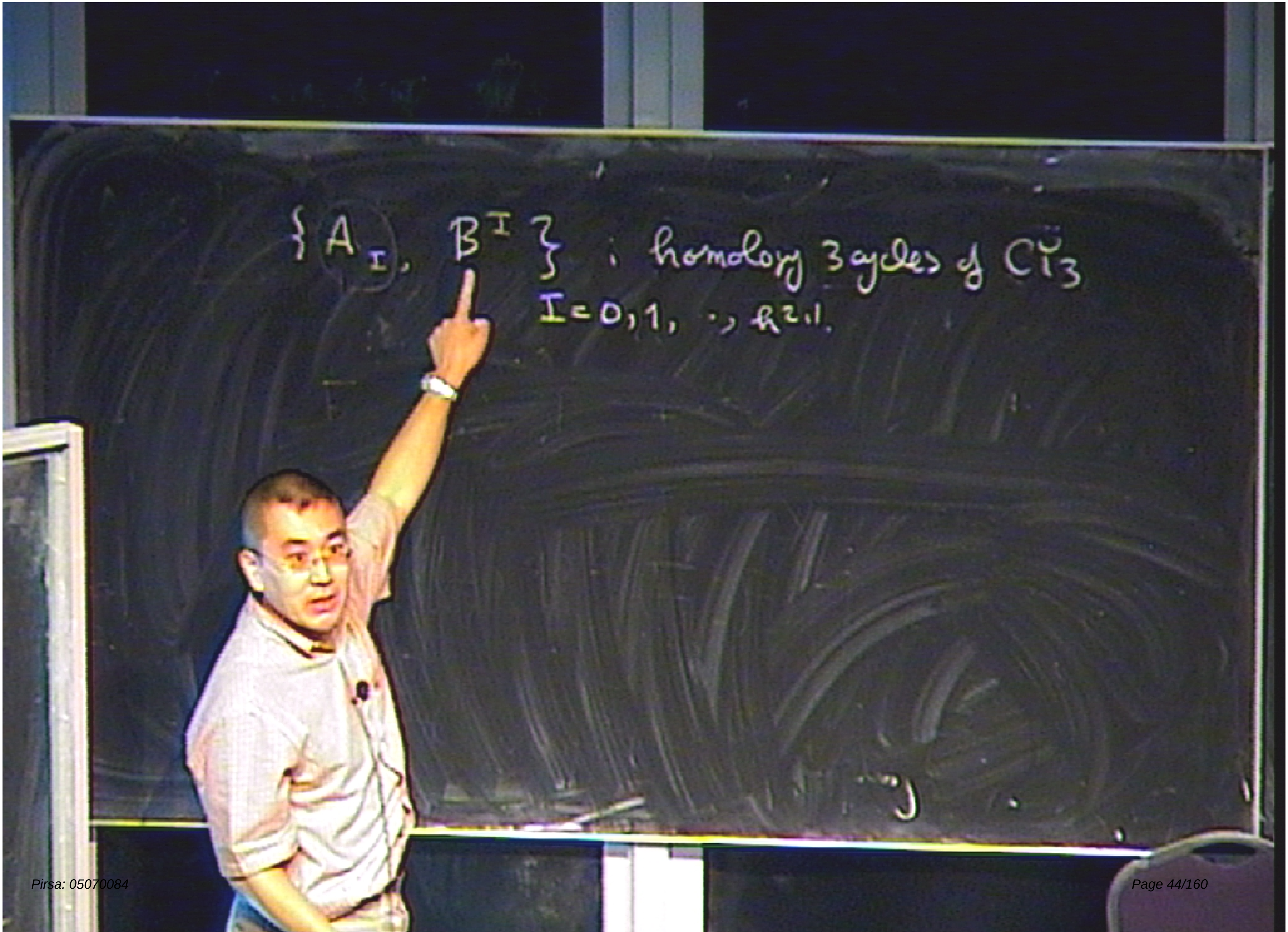
"Special Lagrangian" saturates the bound.

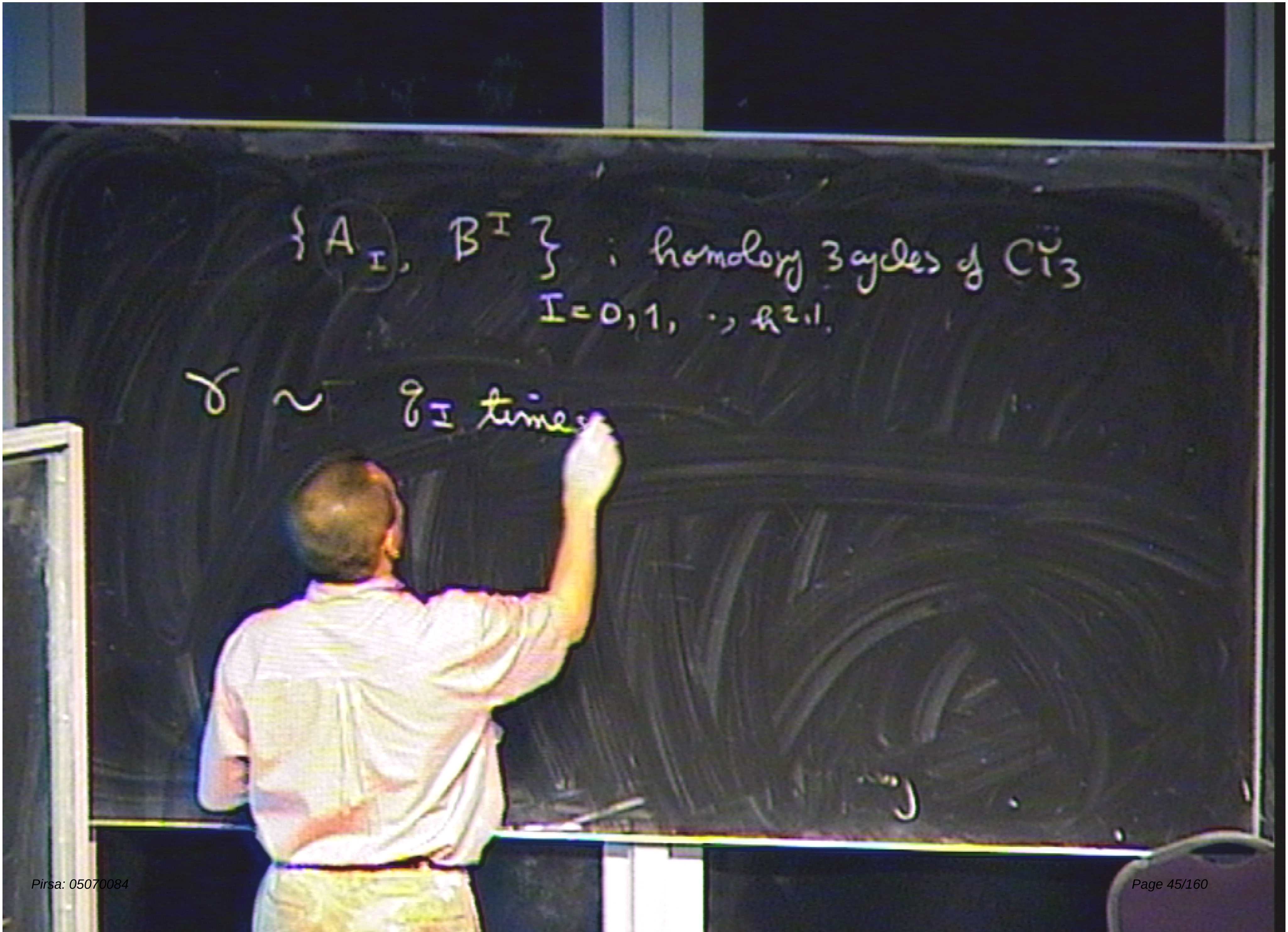
$$M \geq \left| e^{\frac{1}{2}K} \int_\gamma \Omega \right|$$

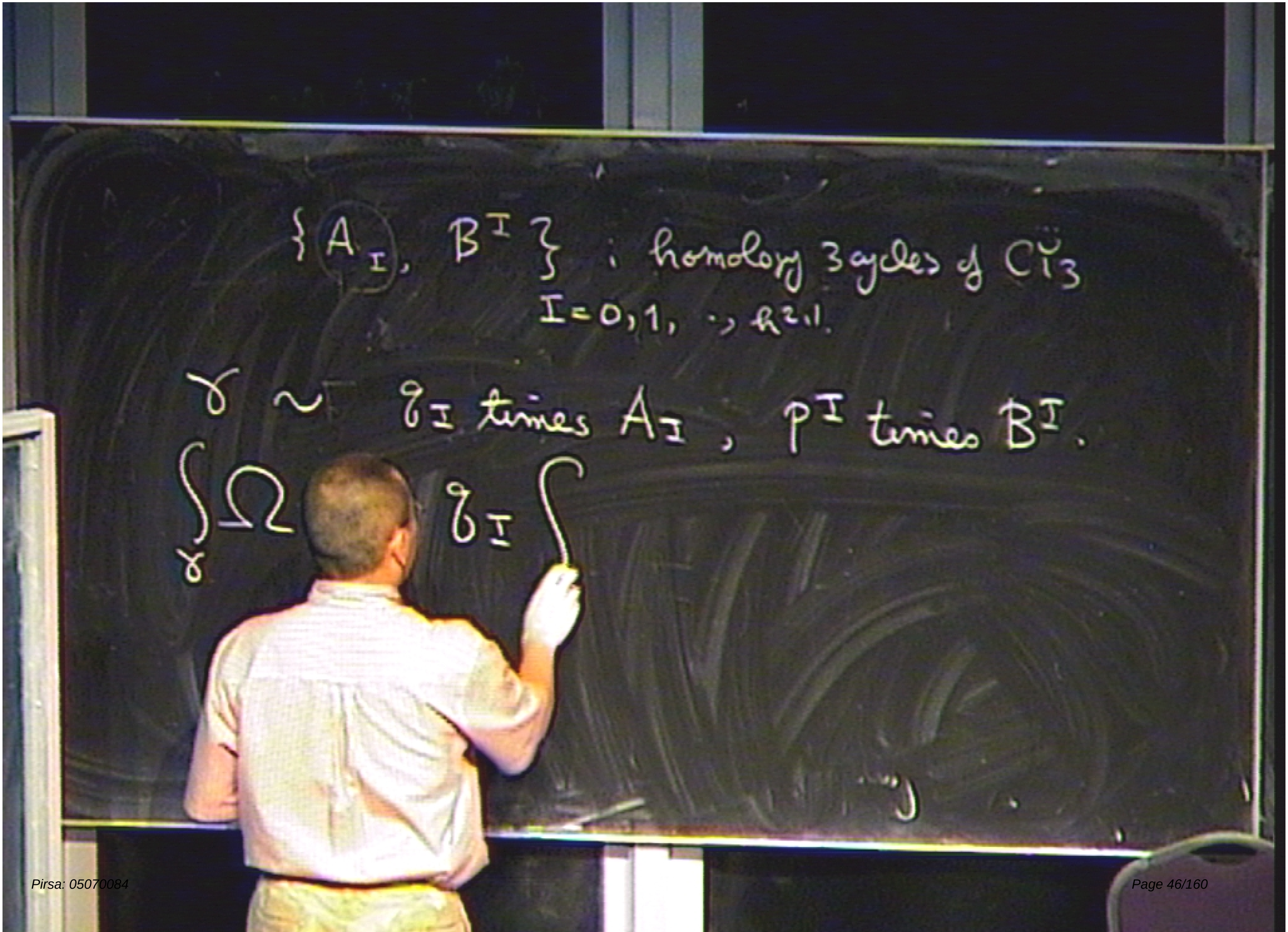
$$e^{-K} = \int_{CY_3} \Omega \wedge \bar{\Omega}$$

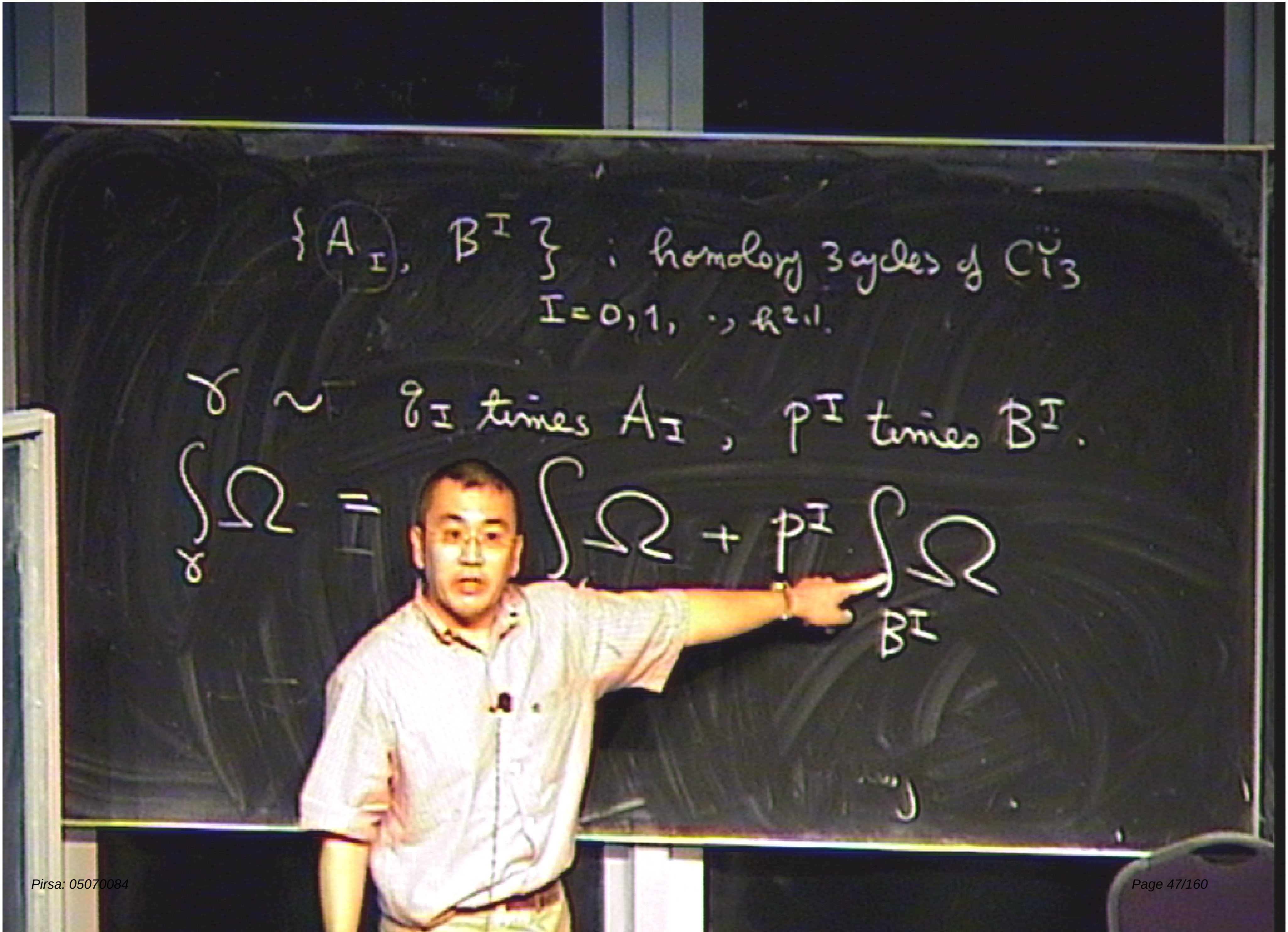
$$\Omega \to c\,\Omega$$

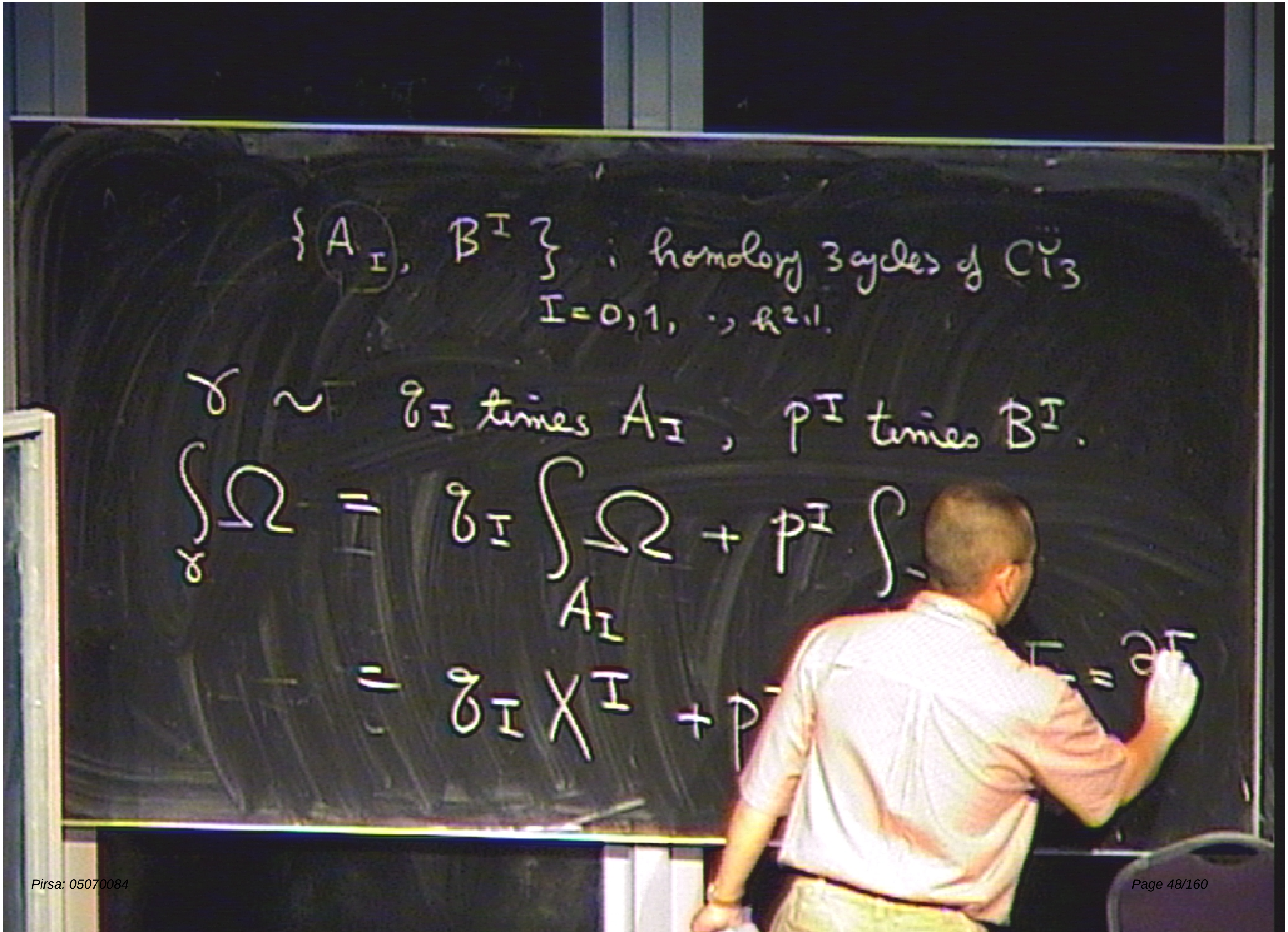

$\{A_I, B^I\}$: homology 3cycles of CY_3
$I = 0, 1, \cdots, h^{2,1}$


$\{A_I, B^I\}$: homology 3cycles of CY_3
$I = 0, 1, .., h^{2,1}$


{A_I, B^I} ; homology 3cycles of CY_3
I = 0, 1, .., h^{2,1}
γ ~ q_I times


$\{A_I, B^I\}$: homology 3cycles of CY_3
$I = 0, 1, \cdots, h^{2,1}$
$\gamma \sim q_I$ times A_I , p^I times B^I.
$\int_\gamma \Omega$ $q_I \int$


{A_I, B^I} : homology 3 cycles of CY_3
I = 0, 1, .., h^{2,1}
γ ~ q_I times A_I, p^I times B^I.
∫_γ Ω = ∫ Ω + p^I ∫_{B^I} Ω


{A_I, B^I} : homology 3cycles of CY_3
I = 0, 1, .., h^{2,1}
γ ~ q_I times A_I , p^I times B^I.
∫_γ Ω = q_I ∫_{A_I} Ω + p^I ∫
= q_I X^I + p
= ∂F

$\{A_I, B^I\}$: homology 3 cycles of CY_3

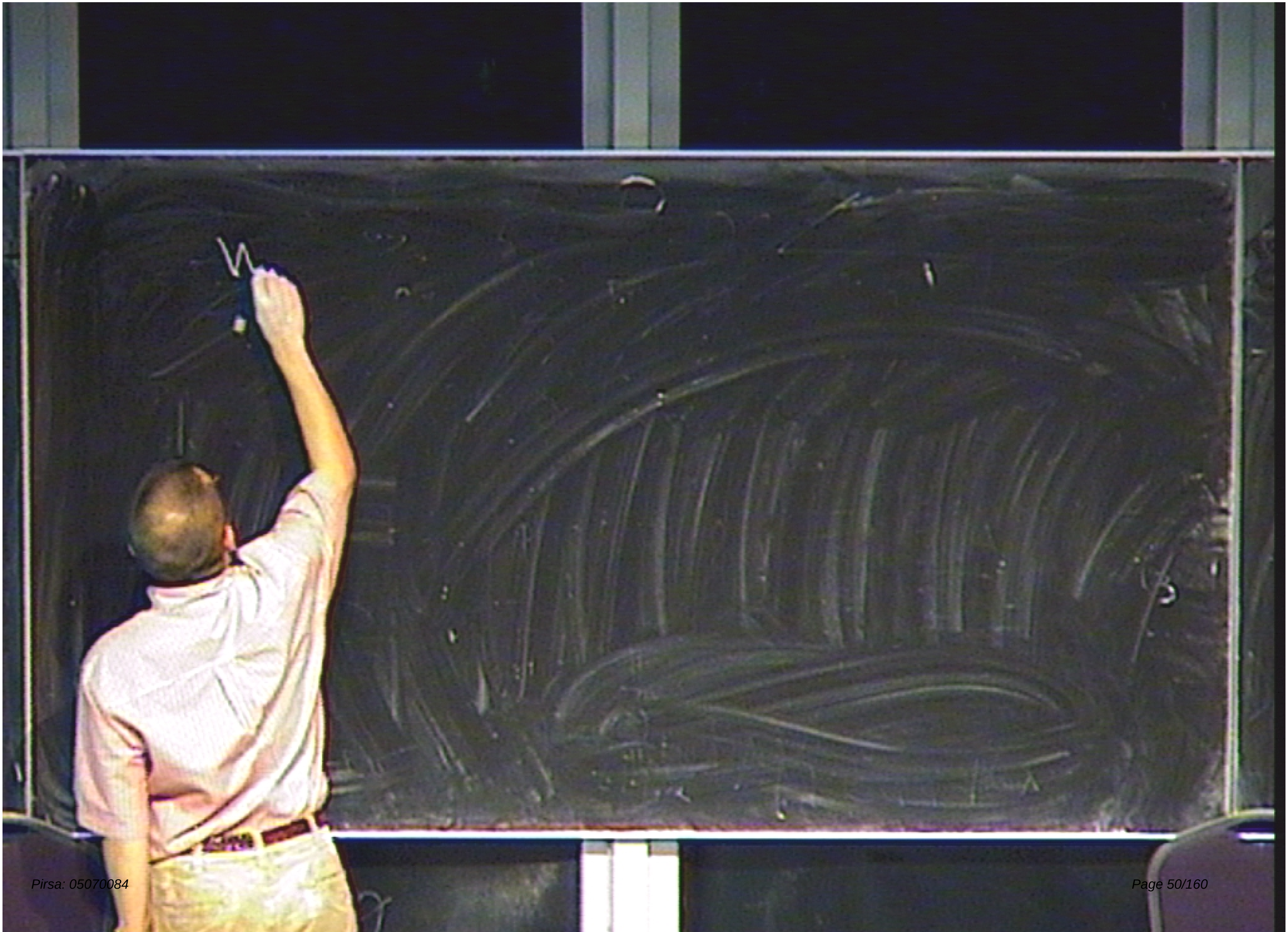
$I = 0, 1, \ldots, h^{2,1}$

$\gamma \sim q_I$ times A_I, p^I times B^I.

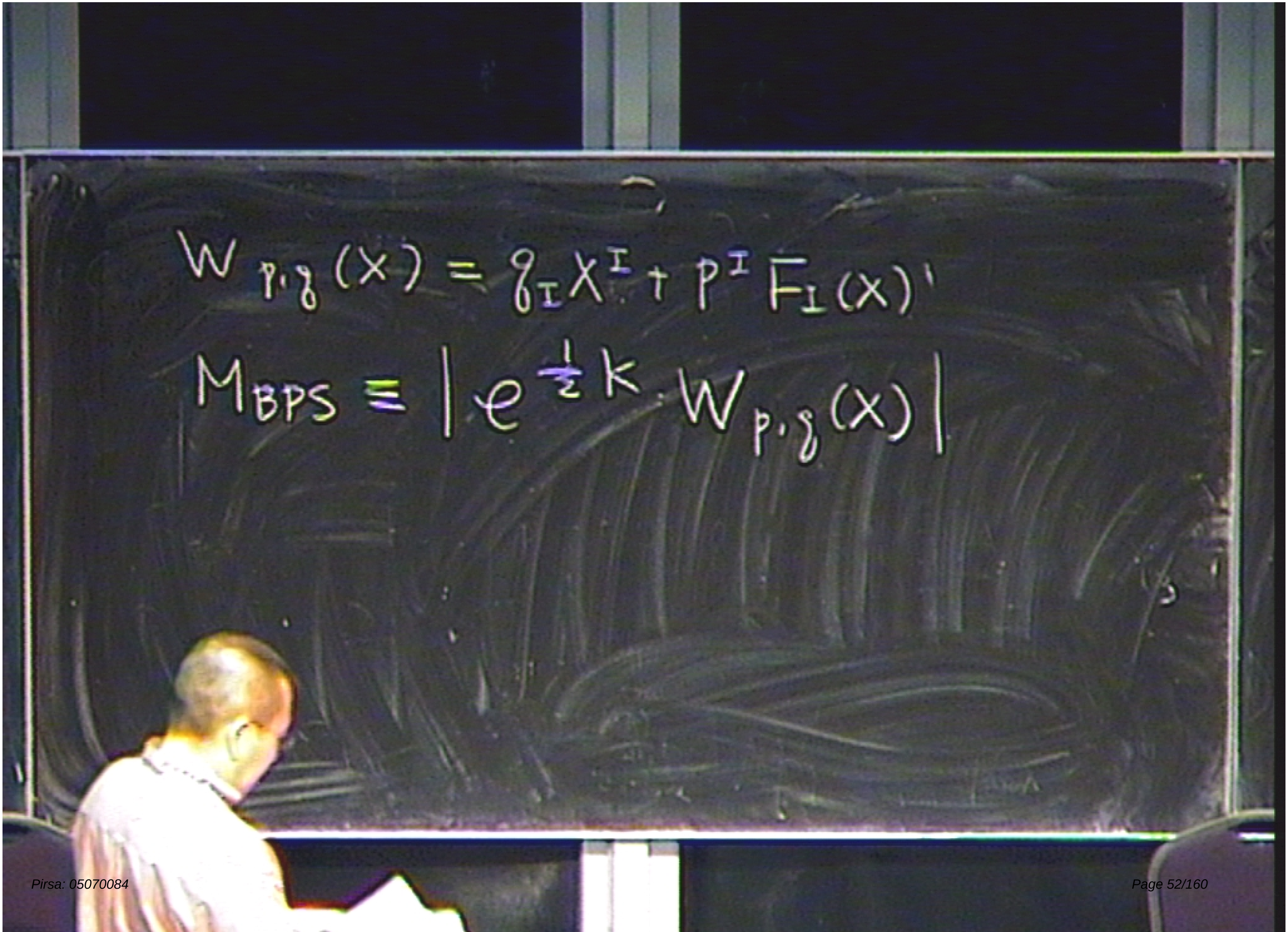
$$\int_\gamma \Omega = q_I \int_{A_I} \Omega + p^I \int_{B^I} \Omega$$

$$= q_I X^I + p^I F_I$$

$$F_I = \frac{\partial F}{\partial X^I}$$



$$W_{p,q}(X) = q_I X^I + p^I F_I(X)$$

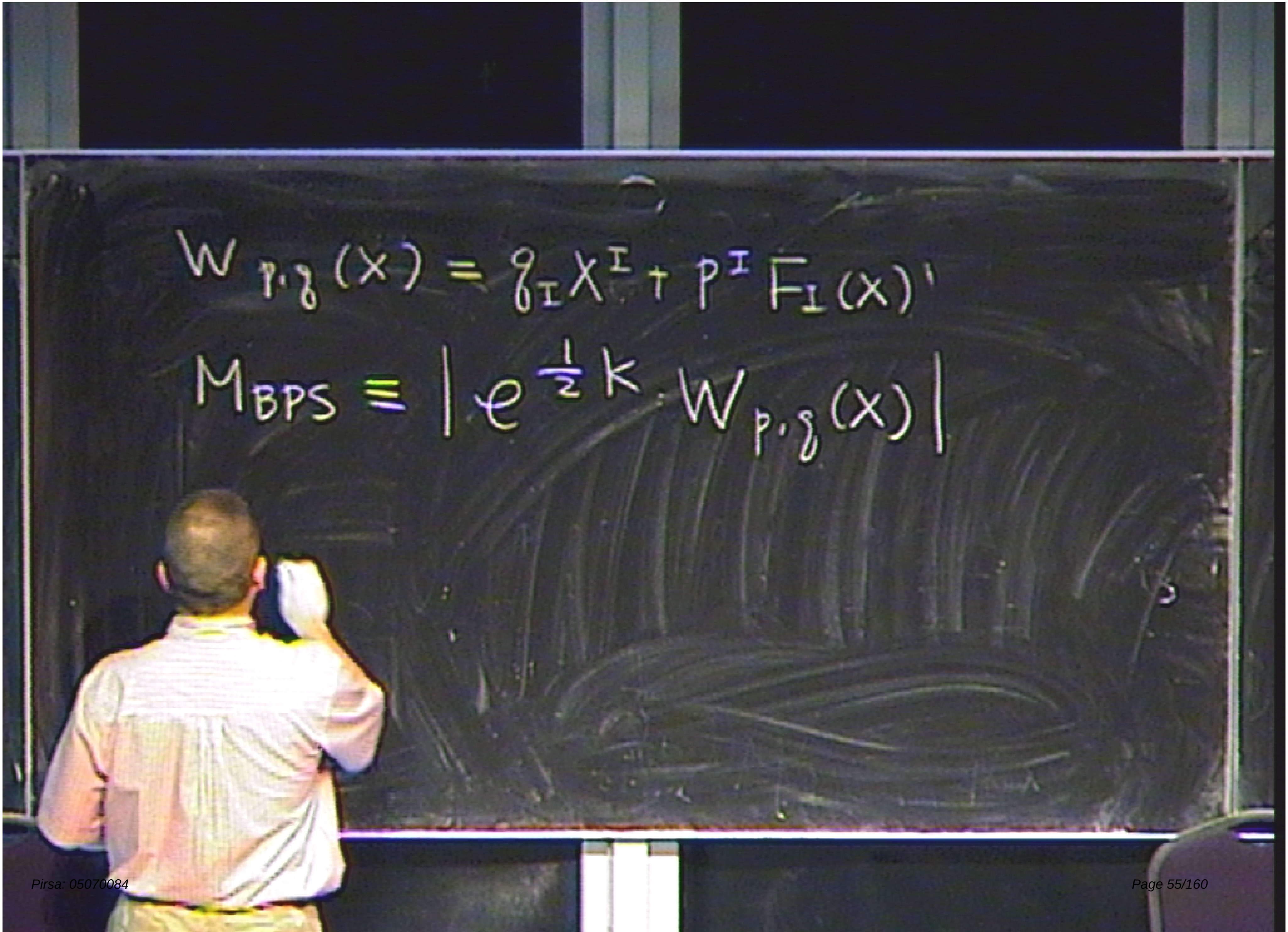

$W_{p,q}(X) = q_I X^I + p^I F_I(X)$
$M_{BPS} \equiv \left| e^{\frac{1}{2}K} W_{p,q}(X) \right|$

$$W_{p,q}(X) = q_I X^I + p^I F_I(X)$$

$$M_{BPS} \equiv \left| e^{\frac{1}{2}K} W_{p,q}(X) \right|$$

$$W_{p,q}(X) = q_I X^I + p^I F_I(X)$$

$$M_{BPS} \equiv \left| e^{\frac{1}{2}K} W_{p,q}(X) \right|$$


$W_{p,q}(X) = q_I X^I + p^I F_I(X)$
$M_{BPS} \equiv \left| e^{\frac{1}{2}K} W_{p,q}(X) \right|$

$$W_{p,q}(X) = q_I X^I + p^I F_I(X)$$

$$M_{BPS} \equiv \left| e^{\frac{1}{2}K} W_{p,q}(X) \right|$$

- M_{BPS} is a function on $\mathcal{M}_V$.
- "Attractor mechanism" minimizes M_{BPS}.

$$W_{p,q}(X) = q_I X^I + p^I F_I(X)$$

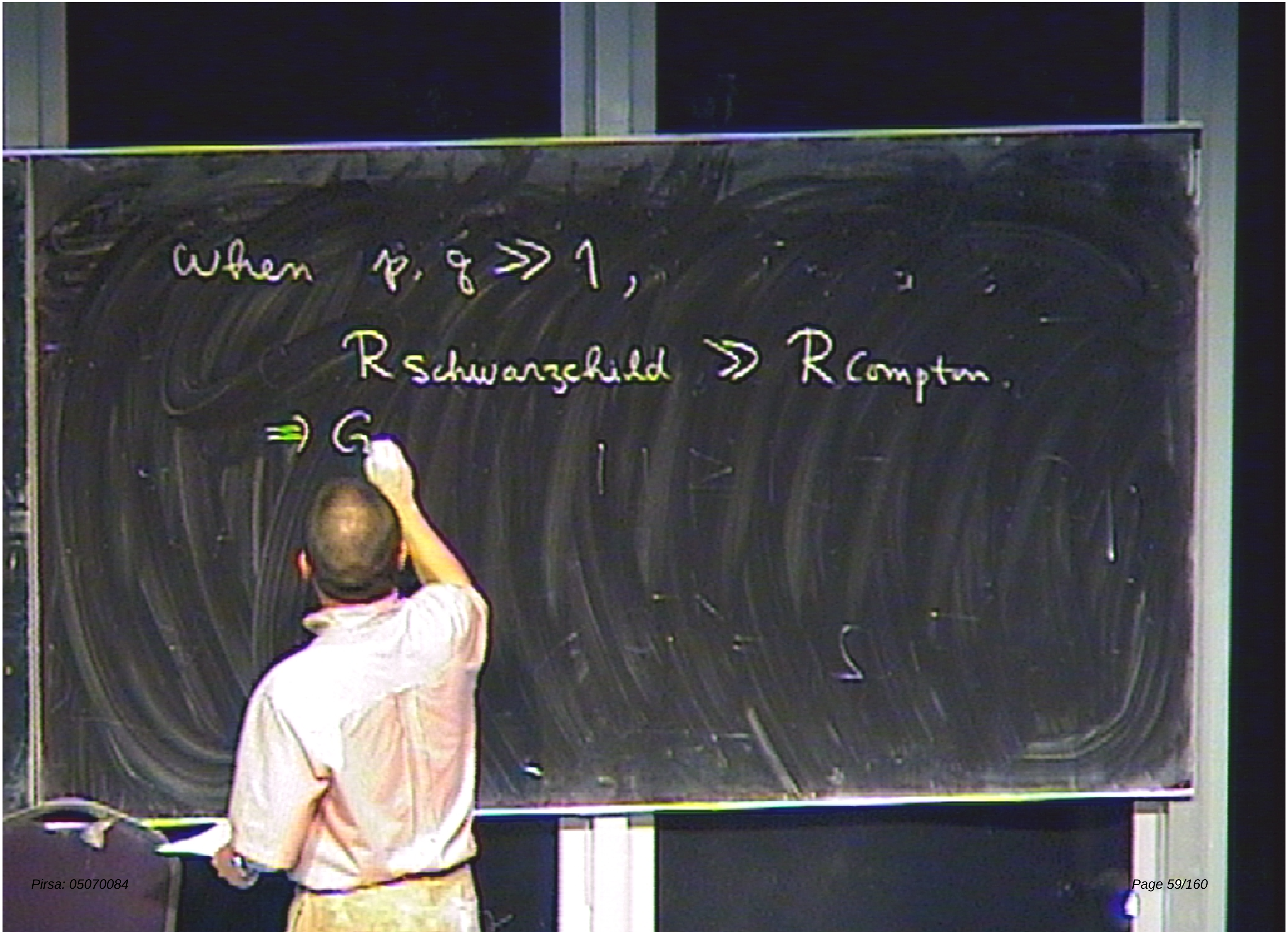
$$M_{BPS} \equiv \left| e^{\frac{1}{2}K} W_{p,q}(X) \right|$$

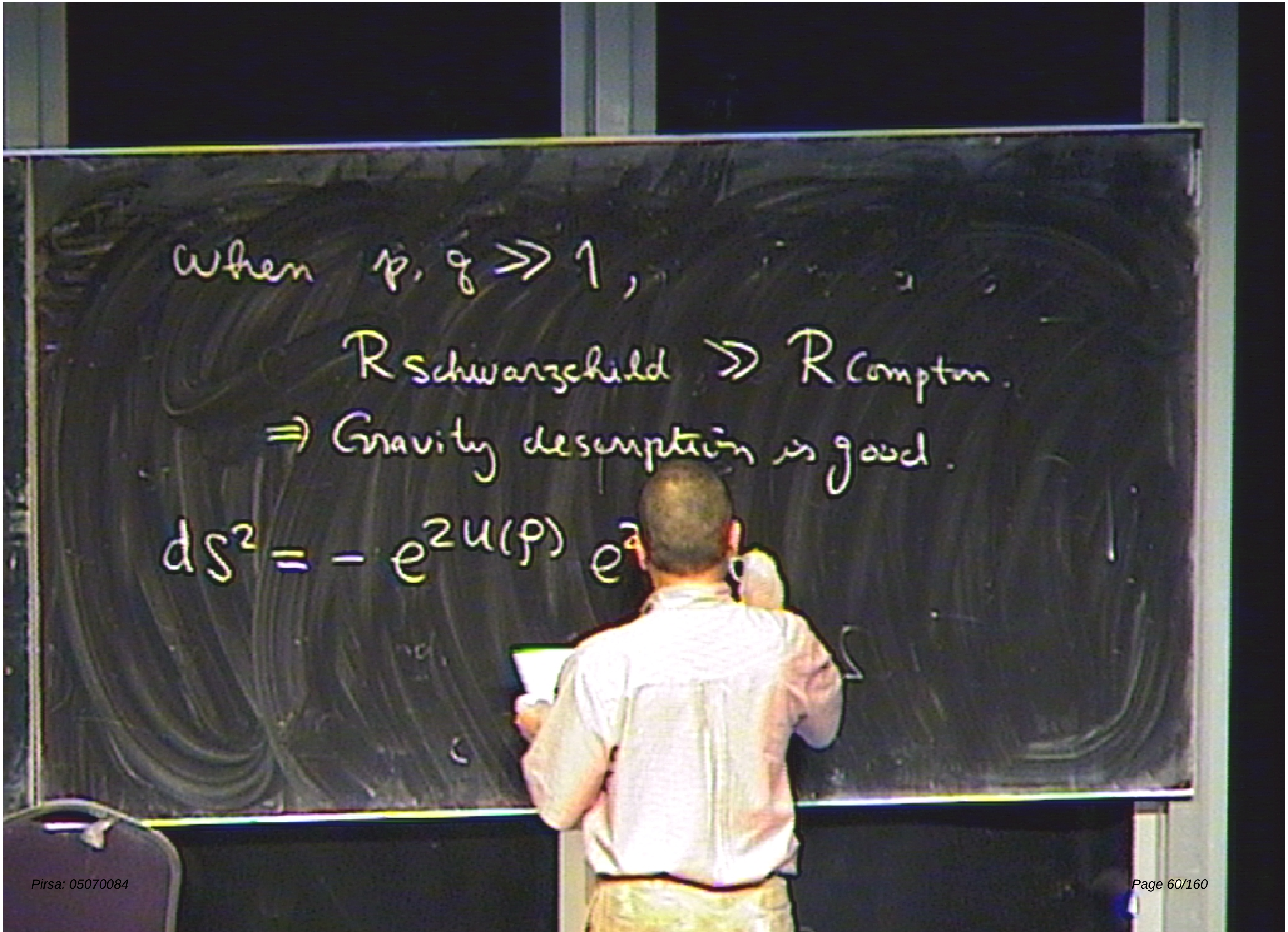
- M_{BPS} is a function on $\mathcal{M}_V$.
- "Attractor mechanism" minimizes M_{BPS}.

$$W_{p,q}(X) = q_I X^I + p^I F_I(X)$$

$$M_{BPS} \equiv \left| e^{\frac{1}{2}K} W_{p,q}(X) \right|$$

- M_{BPS} is a function on $\mathcal{M}_V$.
- "Attractor mechanism" minimizes M_{BPS}.


When p, q ≫ 1,
R Schwarzchild ≫ R Compton.
⇒ G


When p, q ≫ 1,
R Schwarzchild ≫ R Compton.
⇒ Gravity description is good.
ds² = − e^{2U(ρ)} e

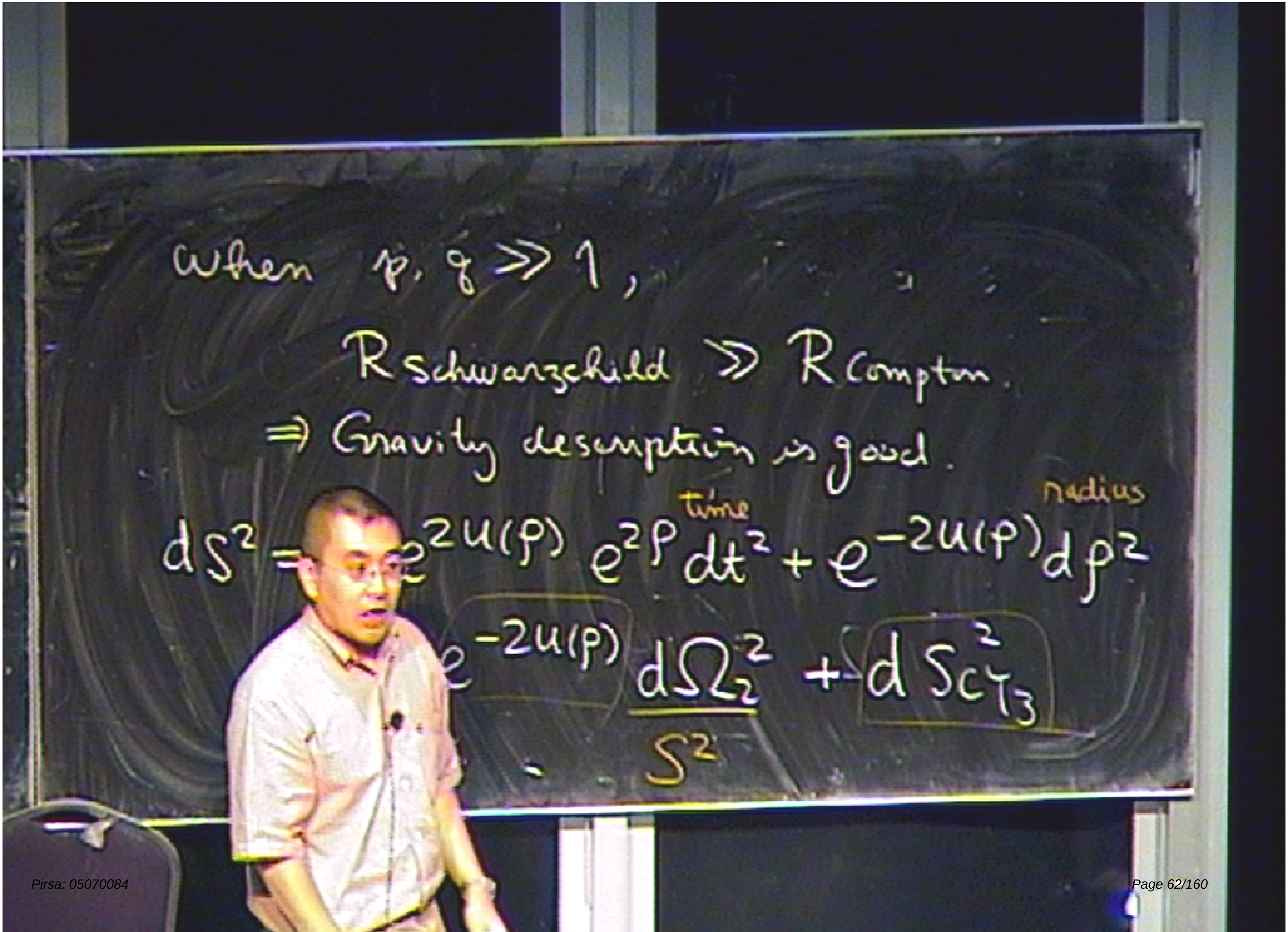
When $p, q \gg 1$,

$R_{\text{Schwarzschild}} \gg R_{\text{Compton}}$.

$\Rightarrow$ Gravity description is good.

$$ds^2 = -e^{2U(\rho)} e^{2\rho} \overset{\text{time}}{dt^2} + e^{-2U(\rho)} \overset{\text{radius}}{d\rho^2}$$

$$+ e^{-2U(\rho)} \frac{d\Omega_2^2}{S^2} + ds^2_{CY_3}$$


When p, q ≫ 1,
R Schwarzchild ≫ R Compton.
⇒ Gravity description is good.
time
radius
ds² = e^{2U(ρ)} e^{2ρ} dt² + e^{−2U(ρ)} dρ²
e^{−2U(ρ)} dΩ₂² + ds²_{CY₃}
S²

When $p, q \gg 1$,

$R_{\text{Schwarzschild}} \gg R_{\text{Compton}}$.

$\Rightarrow$ Gravity description is good.

$$ds^2 = -e^{2U(\rho)} e^{2\rho} \overset{\text{time}}{dt^2} + e^{-2U(\rho)} \overset{\text{radius}}{d\rho^2}$$

$$+ \boxed{e^{-2U(\rho)}} \underbrace{d\Omega_2^2}_{S^2} + \boxed{dS_{CY_3}^2}$$

BPS equation ($\Leftarrow \exists$ Killing spinor)

BPS equation ($\Leftarrow \exists$ Killing spinor)

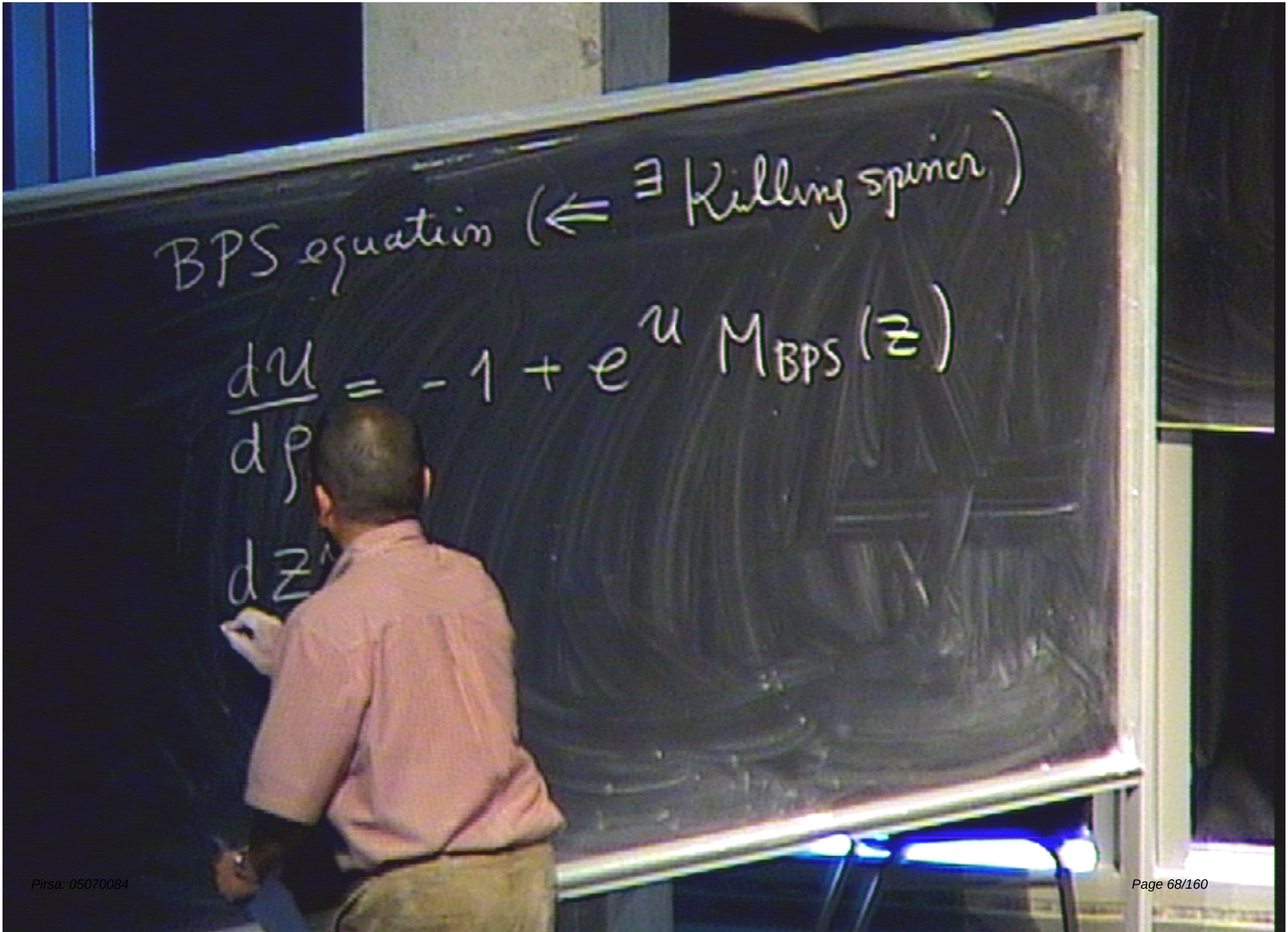
$$\frac{dU}{d\rho} = -1 + e^{U} M_{BPS}(z)$$

BPS equations ($\Leftarrow \exists$ Killing spinor)

$$\frac{dU}{d\rho} = -1 + e^{U} M_{BPS}(z)$$

BPS equation ($\Leftarrow \exists$ Killing spinor)

$$\frac{dU}{d\rho} = -1 + e^{U} M_{BPS}(z)$$

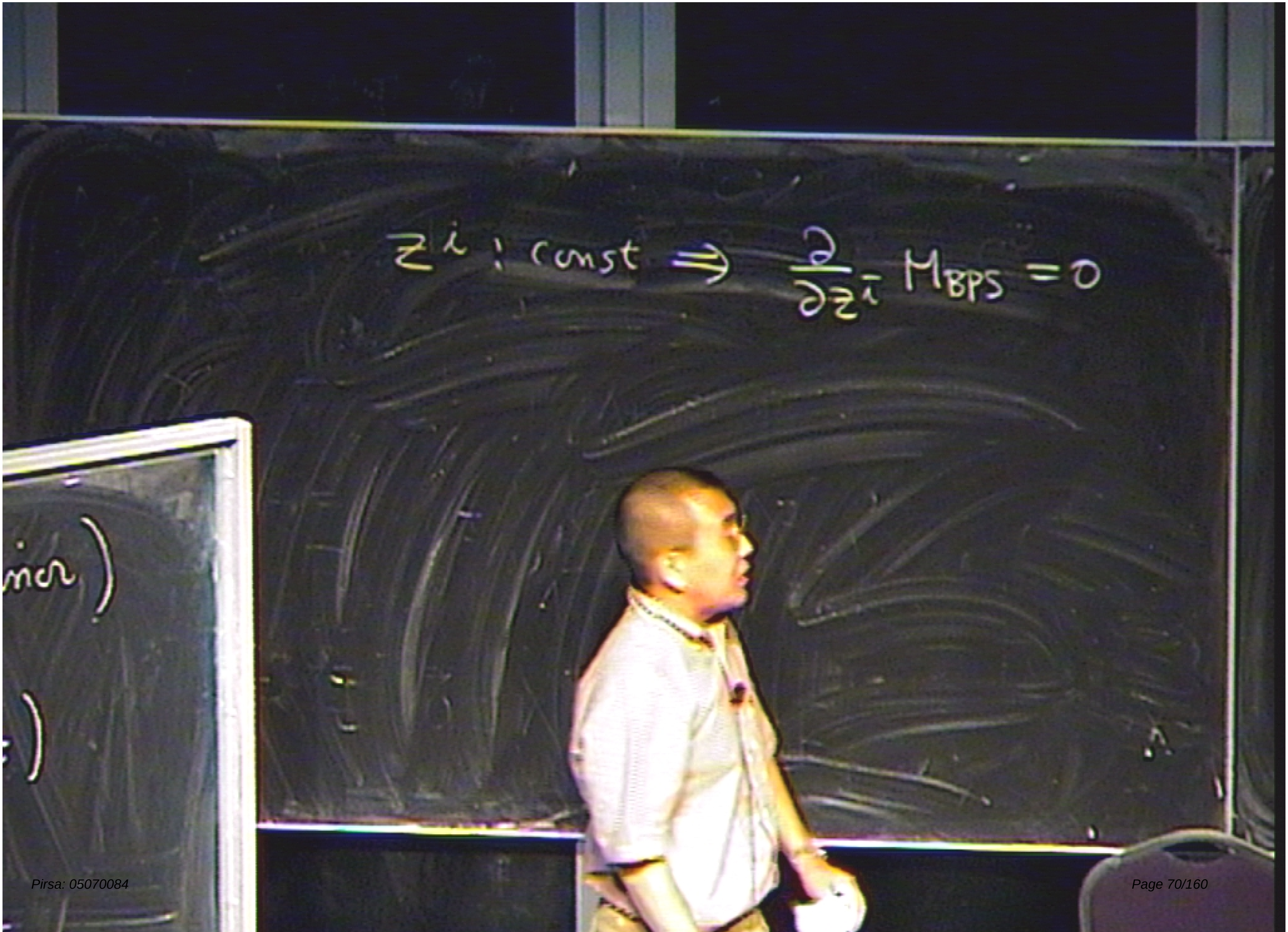

BPS equation (⇐ ∃ Killing spinor)
$\frac{dU}{d\rho} = -1 + e^{U} M_{BPS}(z)$
dz

BPS equation ($\Leftarrow \exists$ Killing spinor)

$$\frac{dU}{d\rho} = -1 + e^{U} M_{BPS}(z)$$

$$\frac{dz^i}{d\rho} = 2 e^{U} G^{i\bar{j}} \frac{\partial}{\partial \bar{z}^{j}} M_{BPS}$$

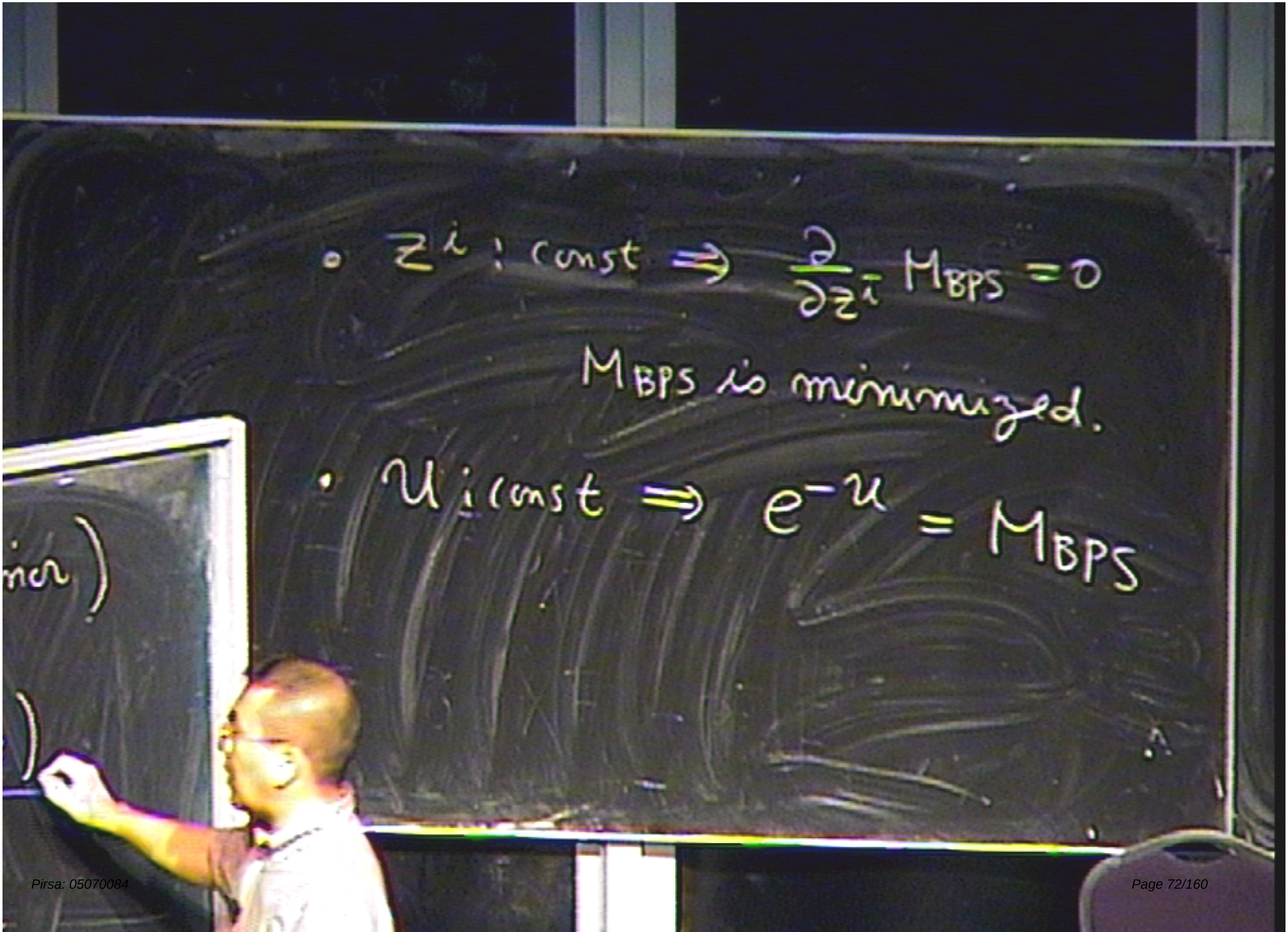
$$\left(G_{i\bar{j}} = \partial_i \partial_{\bar{j}} K \right)$$


z^i : const $\Rightarrow$ $\frac{\partial}{\partial \bar{z}^{\bar{i}}} M_{BPS} = 0$

- Z^i : const $\Rightarrow$ $\frac{\partial}{\partial \bar{Z}^{\bar{i}}} M_{BPS} = 0$

  M_{BPS} is minimized.

-


• z^i : const $\Rightarrow$ $\frac{\partial}{\partial \bar{z}^{\bar{i}}} M_{BPS} = 0$
M_{BPS} is minimized.
• U : const $\Rightarrow$ $e^{-U} = M_{BPS}$

BPS equation ($\Leftarrow \exists$ Killing spinor)

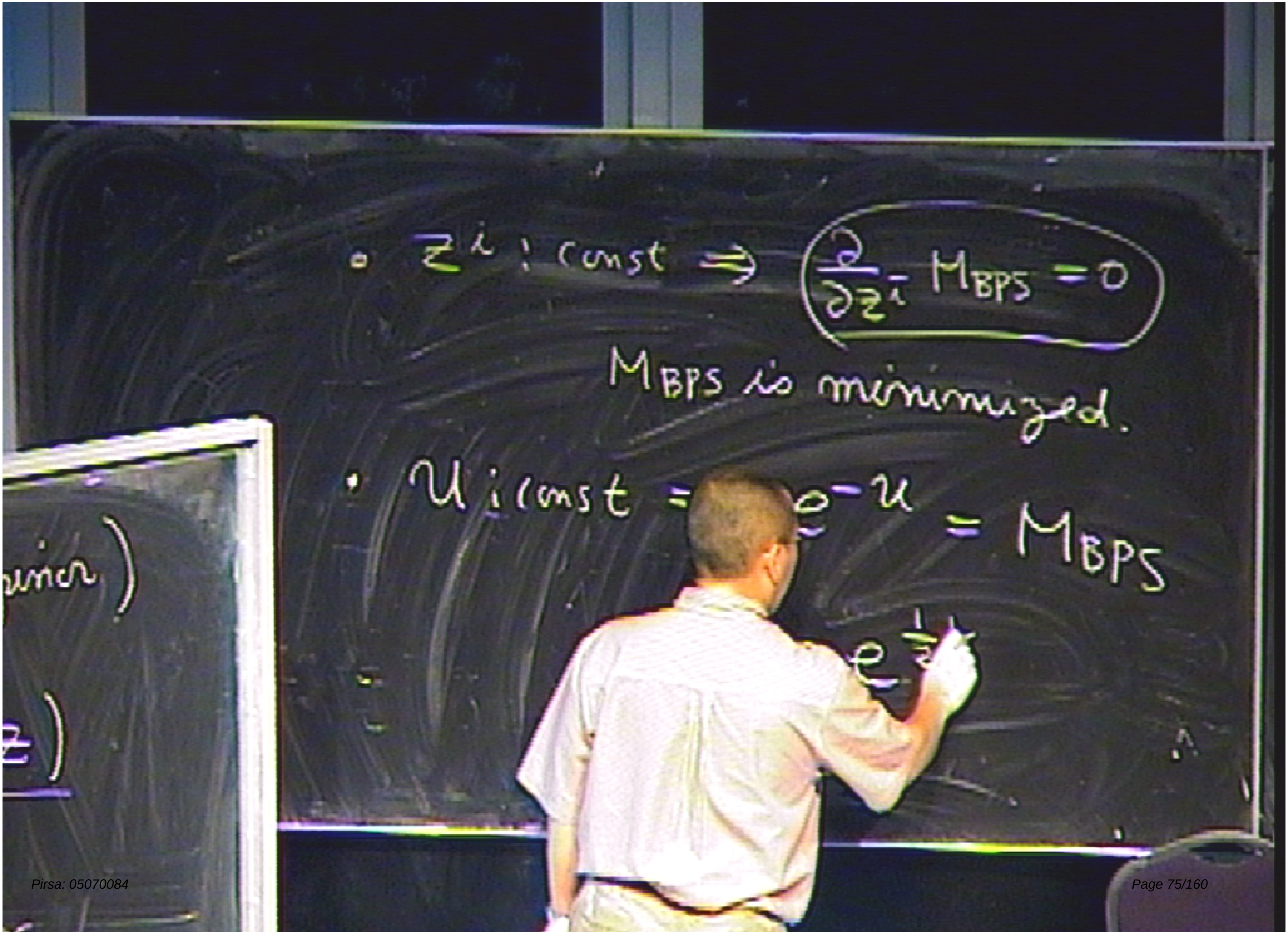
$$\frac{du}{d\rho} = -1 + \underline{e^{u} M_{BPS}(z)}$$

$$\frac{dz^i}{d\rho} = 2e^{u} \underline{G^{i\bar{j}}} \, \underline{\frac{\partial}{\partial \bar{z}^{j}}} \underline{M_{BPS}}$$

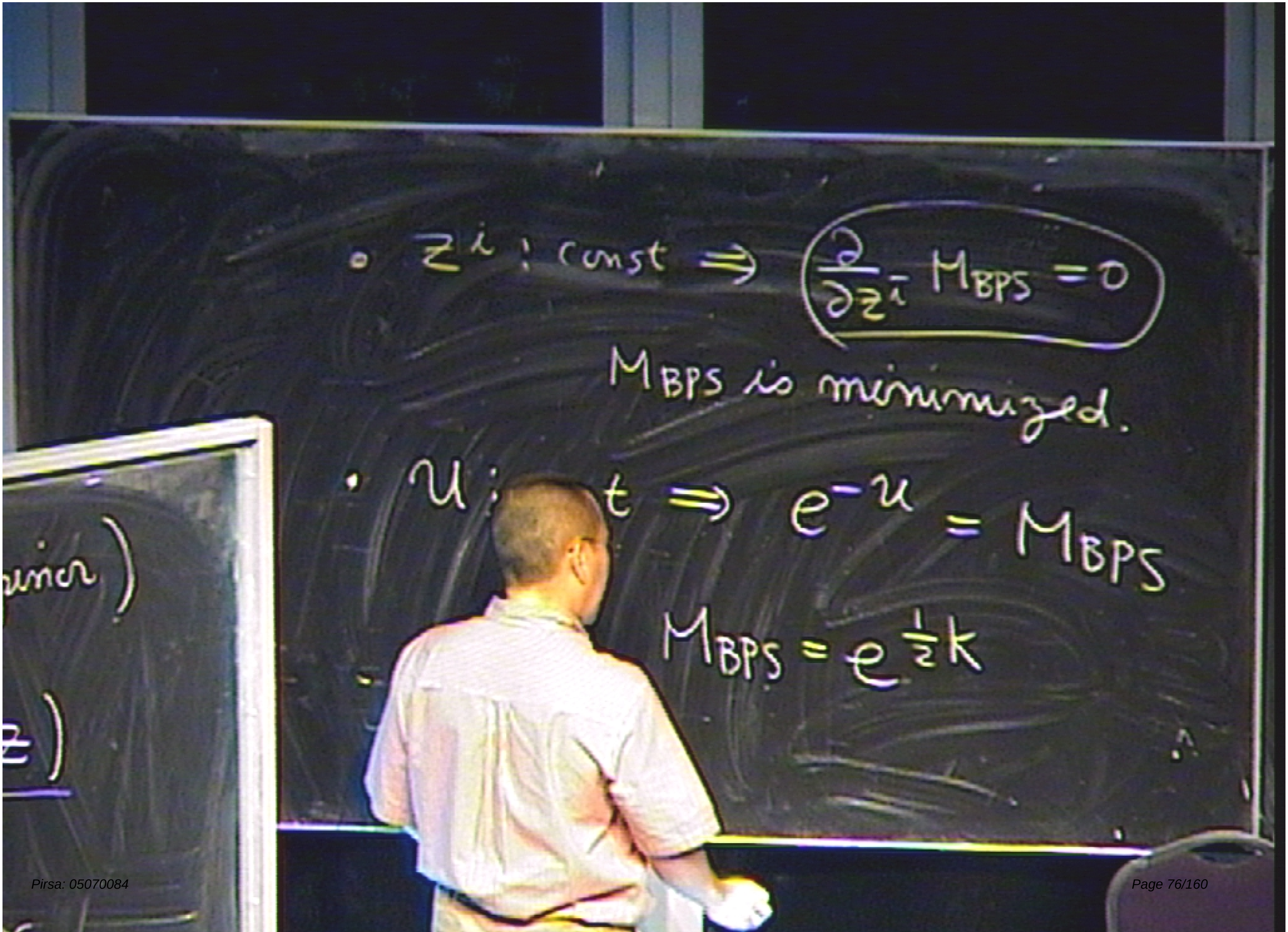
$$\left(G_{i\bar{j}} = \partial_i \partial_{\bar{j}} K \right)$$

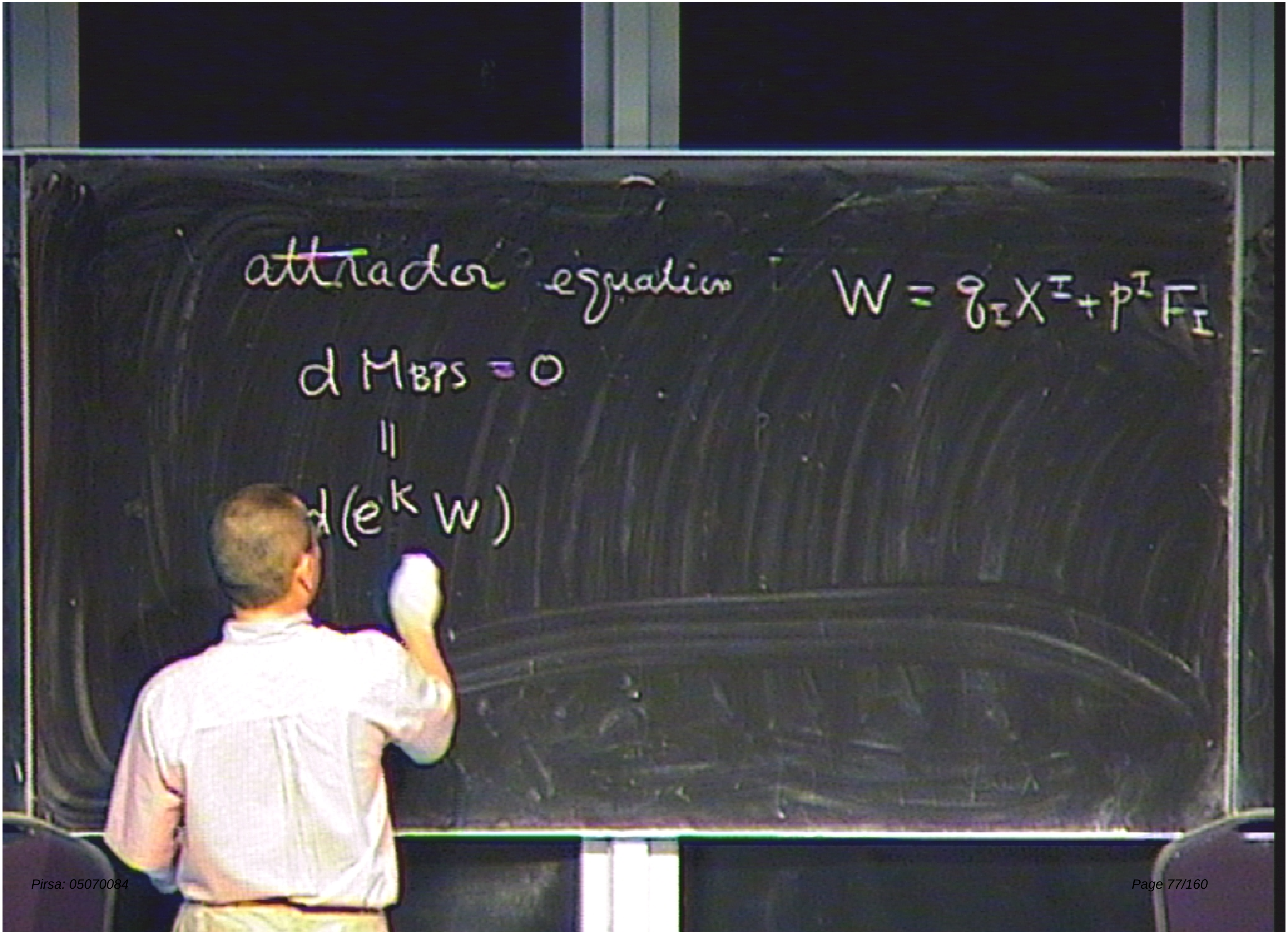
attractor equation

$$d\, M_{BPS} = 0$$



• z^i : const $\Rightarrow$ $\frac{\partial}{\partial z^i} M_{BPS} = 0$
M_{BPS} is minimized.
• U : const
$= M_{BPS}$


• z^i : const $\Rightarrow$ $\frac{\partial}{\partial \bar{z}^{\bar{i}}} M_{BPS} = 0$
M_{BPS} is minimized.
• u : t $\Rightarrow$ $e^{-u} = M_{BPS}$
$M_{BPS} = e^{\frac{1}{2}K}$

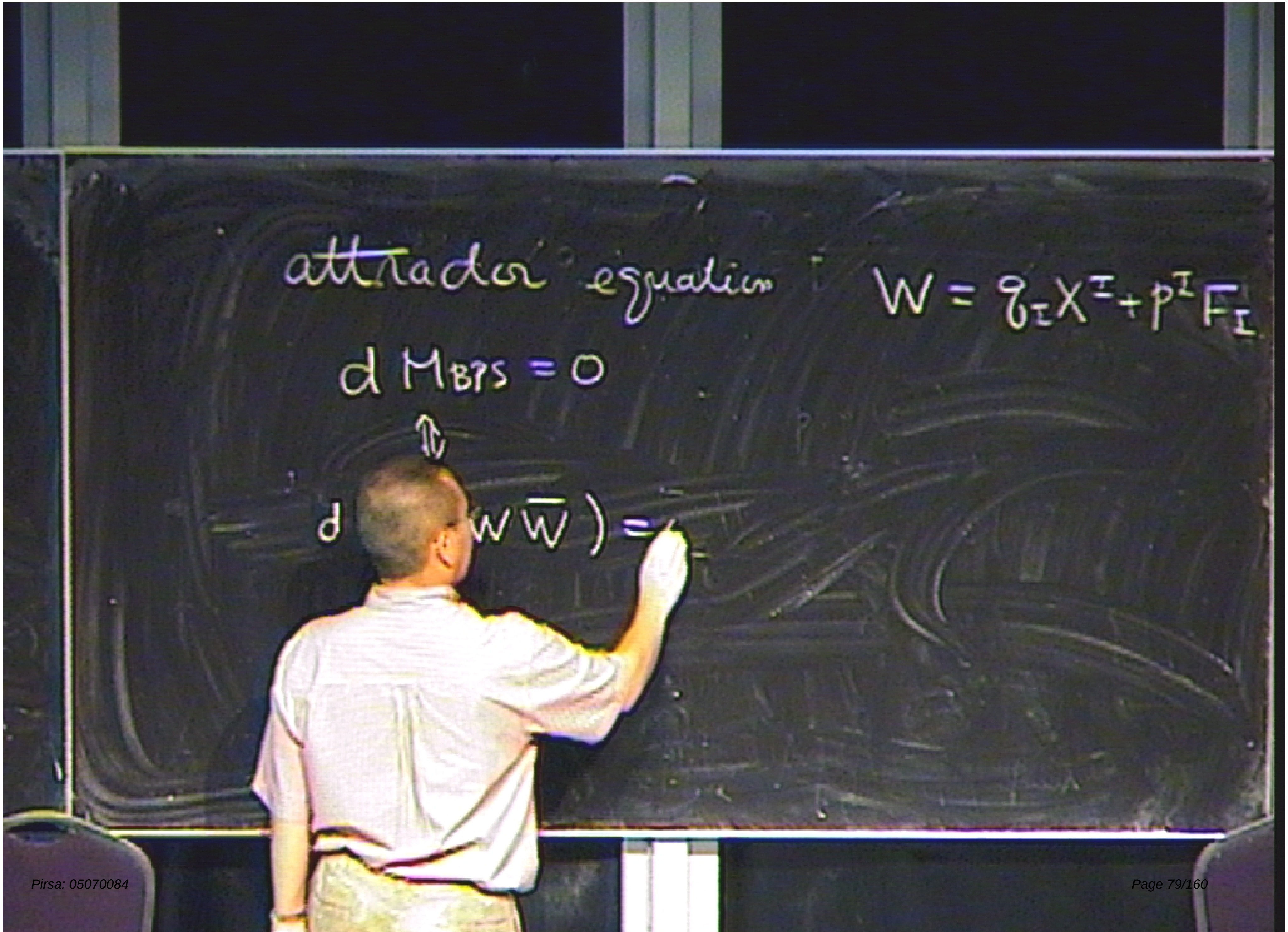

attractor equation
$W = q_I X^I + p^I F_I$
$d M_{BPS} = 0$
$\|$
$d(e^K W)$

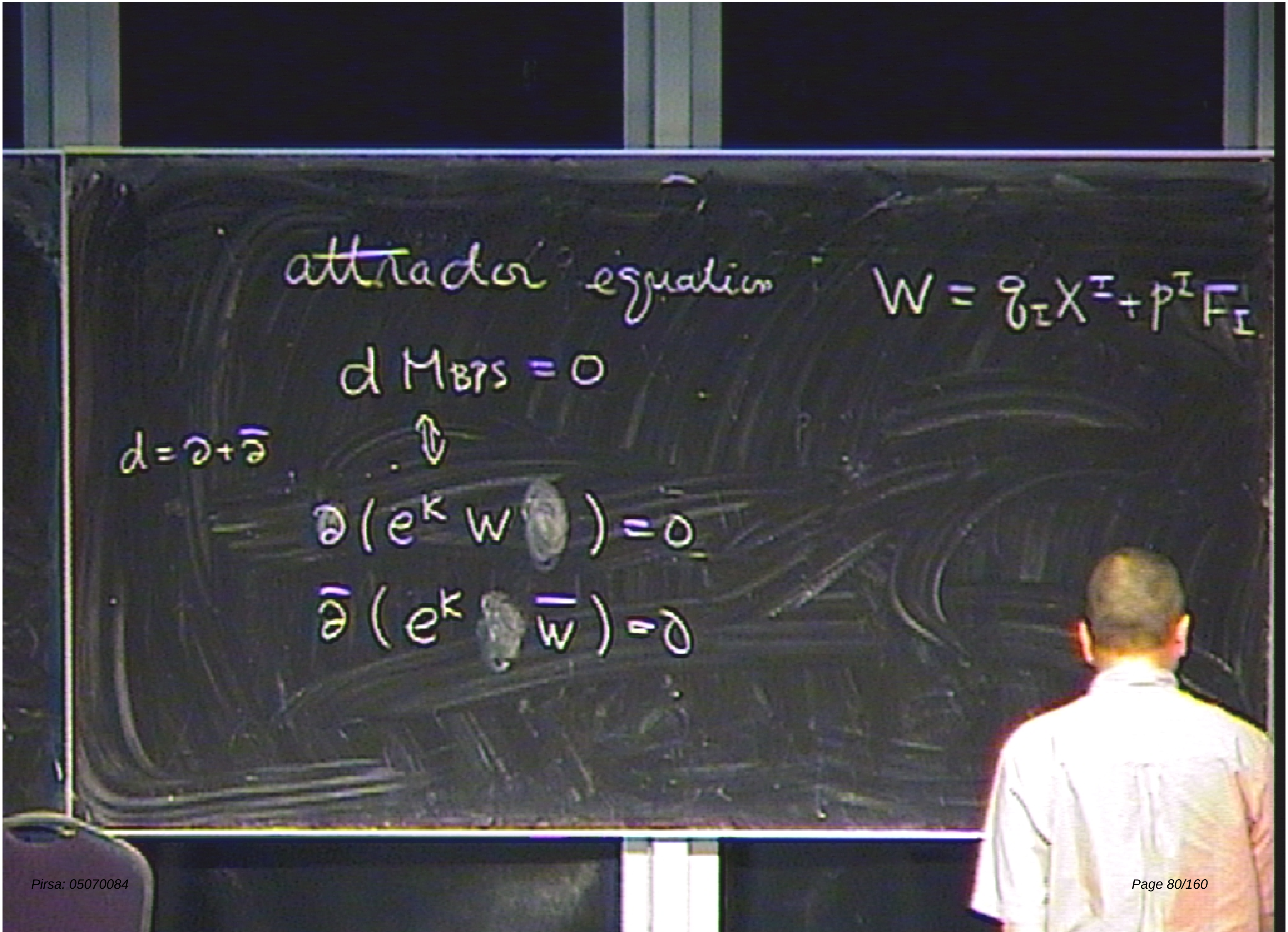
~~attractor~~ equation $\quad W = q_I X^I + p^I F_I$

$d M_{BPS} = 0$

$\parallel$

$e^{-K} d(e^K W) = \underline{DW} = 0$


attractor equation
$W = q_I X^I + p^I F_I$
$d M_{BPS} = 0$
$\Updownarrow$
$d \; (W \bar{W}) =$


attractor equation
$W = q_I X^I + p^I F_I$
$d M_{BPS} = 0$
$\Updownarrow$
$d = \partial + \bar{\partial}$
$\partial (e^K W) = 0$
$\bar{\partial} (e^K \bar{W}) = 0$

attractor equation

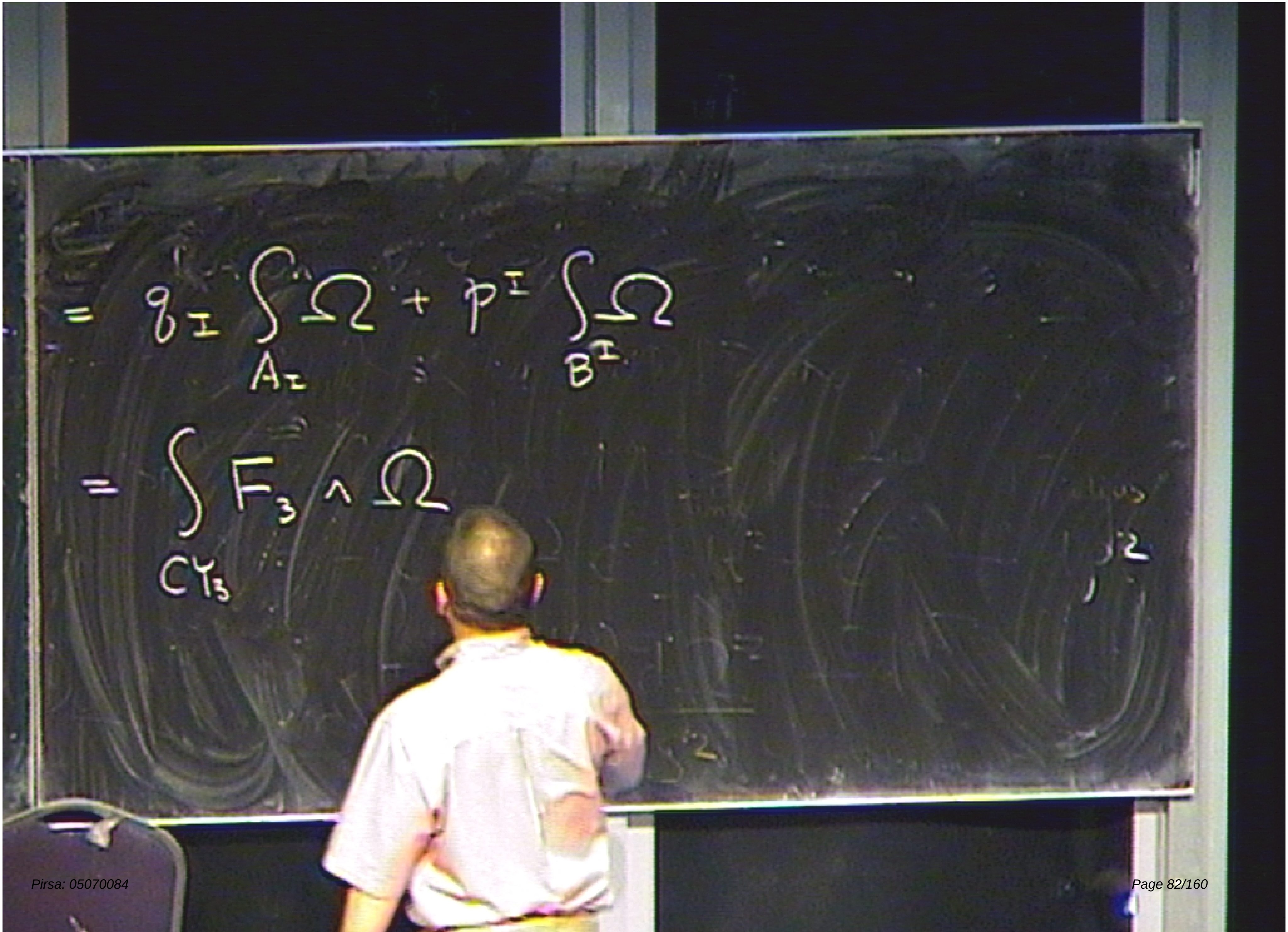
$$W = q_I X^I + p^I F_I$$

$$d M_{BPS} = 0$$

$$d = \partial + \bar{\partial} \qquad \Updownarrow$$

$$\partial (e^K W) = 0$$

$$\bar{\partial} (e^K \bar{W}) = 0$$


$= q_I \int_{A_I} \Omega + p^I \int_{B^I} \Omega$
$= \int_{CY_3} F_3 \wedge \Omega$

$$= q_I \int_{A_I} \Omega + p^I \int_{B^I} \Omega \qquad \int_{B^J} a^I = \delta^I_J$$

etc

$$= \int_{CY_3} F_3 \wedge \Omega \qquad F_3 = q_I a^I + p^I b_I$$

$$= q_I \int_{A_I} \Omega + p^I \int_{B^I} \Omega \qquad \int_{B^J} a^I = \delta^I_J$$

etc

$$= \int_{CY_3} F_3 \wedge \Omega \qquad F_3 = q_I a^I + p^I b_I$$

attractor equation

$$d\, M_{BPS} = 0$$

$$d = \partial + \bar{\partial} \qquad \Updownarrow$$

$$\partial (e^{K} W) = 0$$

$$\bar{\partial} (e^{K} \bar{W}) = 0$$

$$W = q_I X^I + p^I F_I$$

$$= \int_{CY_3} F_3 \wedge \Omega$$

attractor equation

$d M_{BPS} = 0$

$d = \partial + \bar{\partial}$

$\Updownarrow$

$\partial (e^{K} W) = 0$

$\bar{\partial} (e^{K} \bar{W}) = 0$

$W = q_I X^I + p^I F_I$

$= \int_{CY_3} F_3 \wedge \Omega$

$F_3 = q_I a^I + p^I$

$\{ a^I, b_I \}$:

attractor equation

$$d M_{BPS} = 0$$

$$d = \partial + \bar{\partial} \qquad \Updownarrow$$

$$\partial(e^{K} W) = 0$$

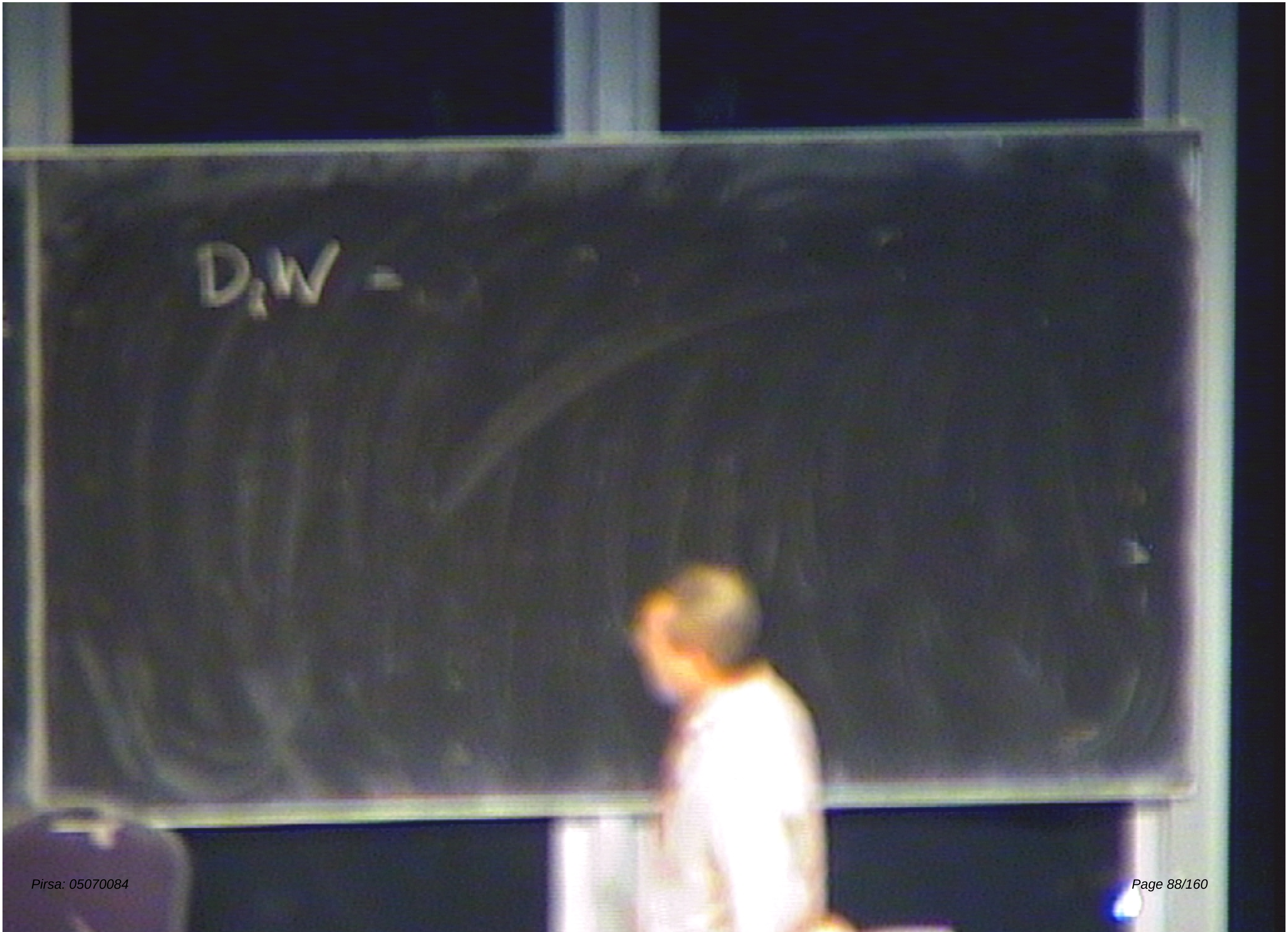
$$\bar{\partial}(e^{K} \bar{W}) = 0$$

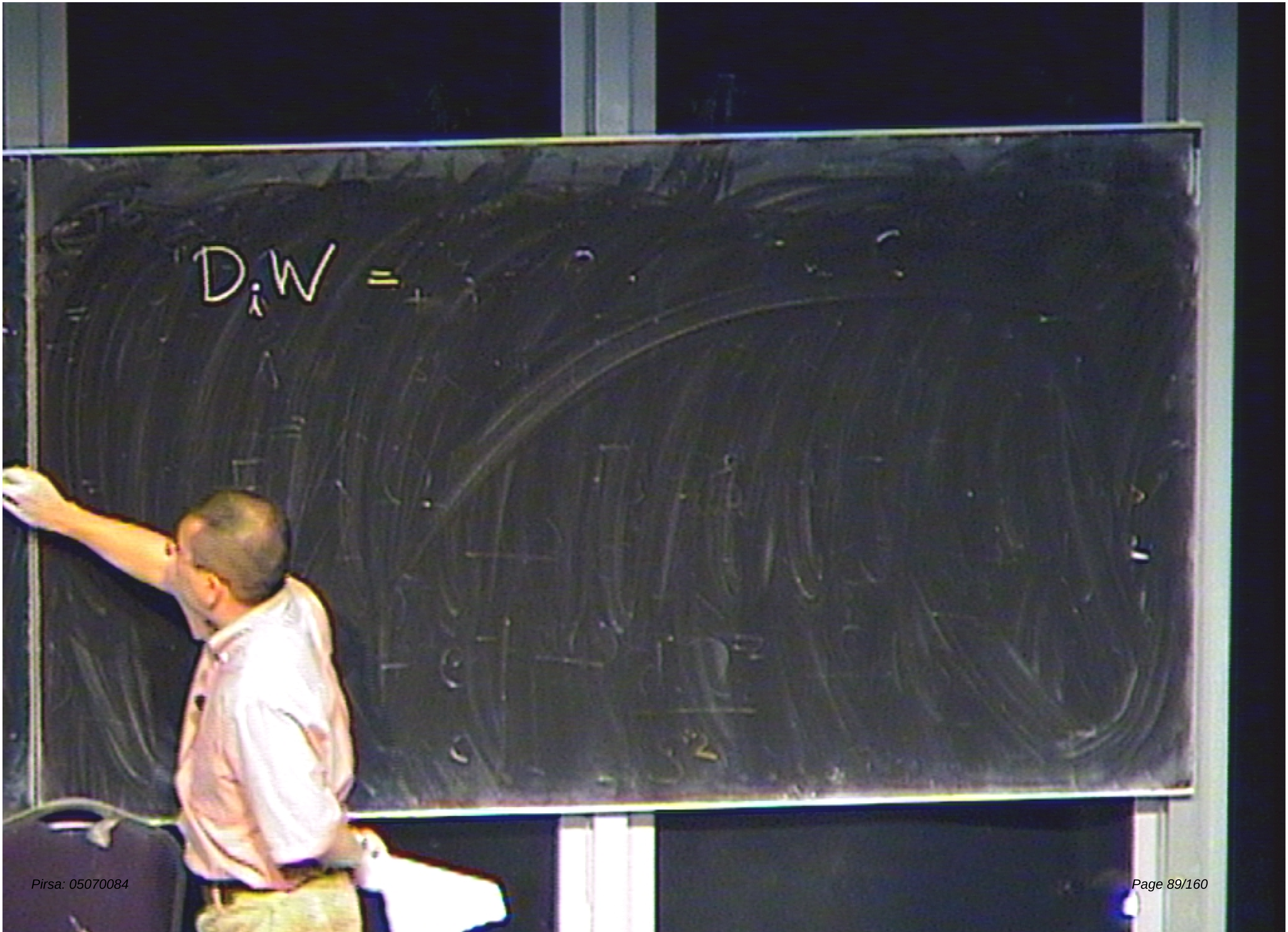
$$W = q_I X^I + p^I F_I$$

$$= \int_{CY_3} F_3 \wedge \Omega$$

$$F_3 = q_I a^I + p^I b_I$$

$$\{a^I, b_I\} : H^3$$


D_iW =


$D_i W =$

attractor equation:

$$d\, M_{BPS} = 0$$

$$\Updownarrow$$

$d = \partial + \bar{\partial}$

$$\partial\left(e^{K} W\right) = 0$$

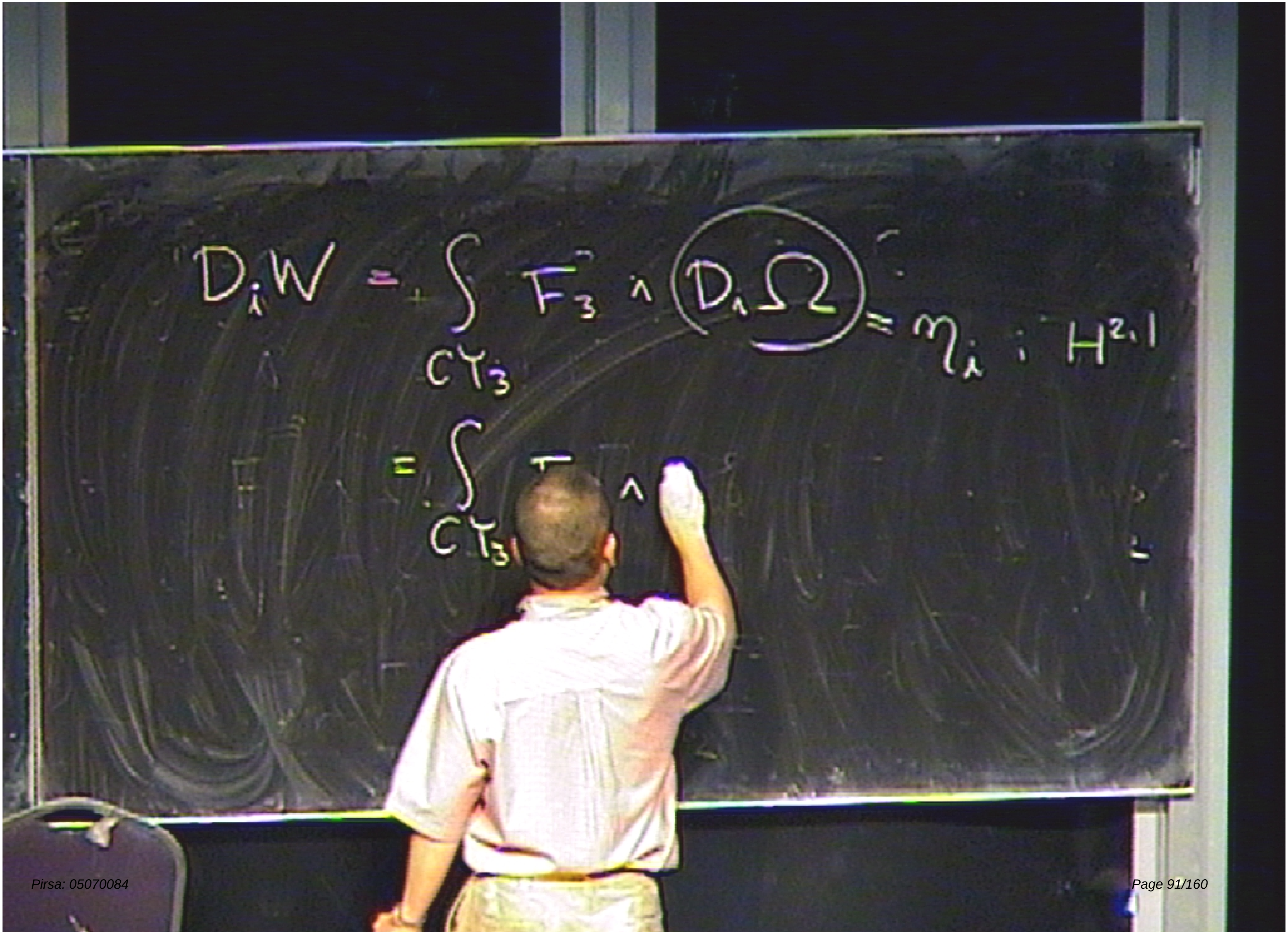
$$\bar{\partial}\left(e^{K} \overline{W}\right) = 0$$

$$W = q_I X^I + p^I F_I$$

$$= \int_{CY_3} F_3 \wedge \Omega$$

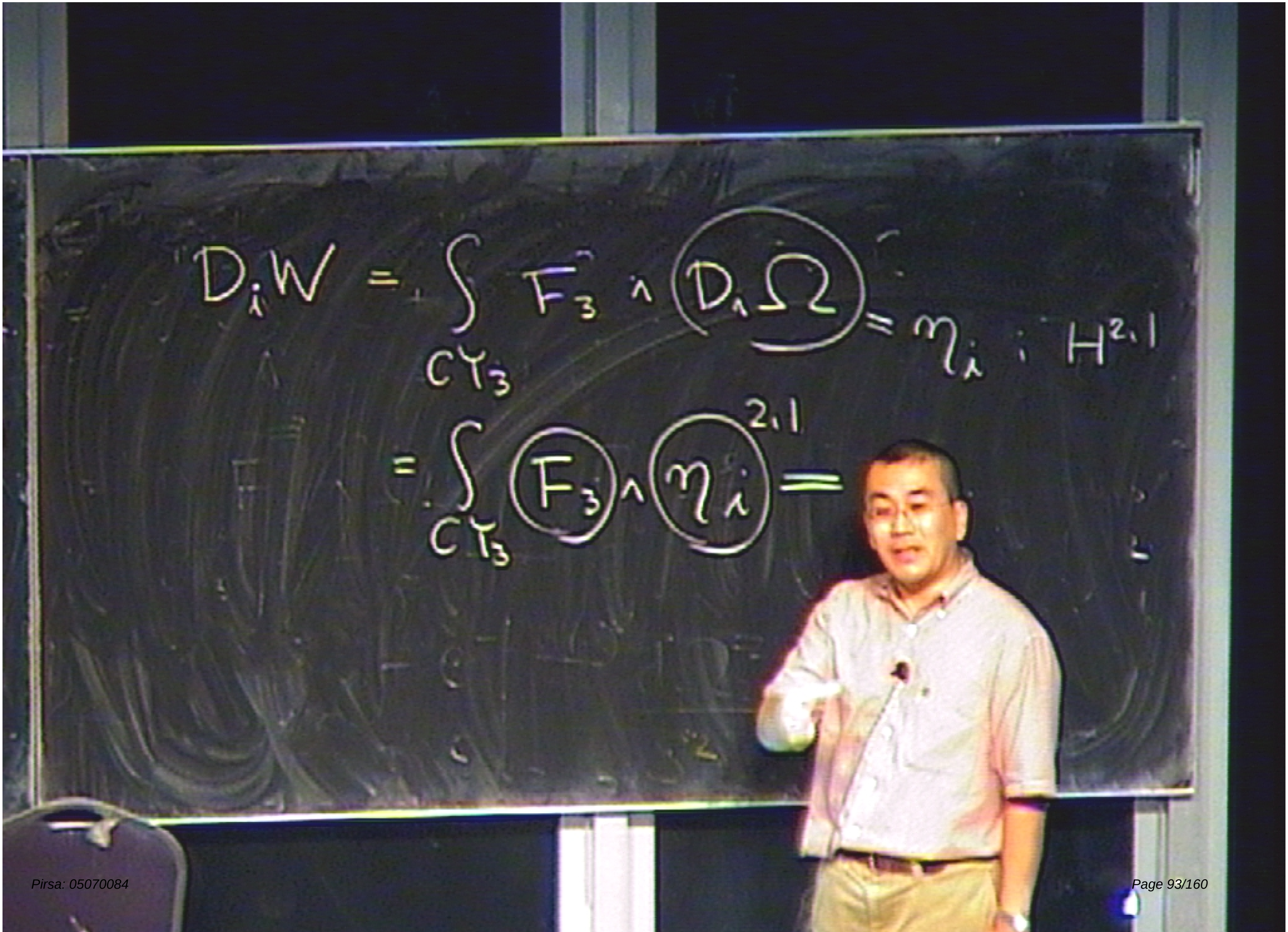
$$F_3 = q_I a^I + p^I b_I$$

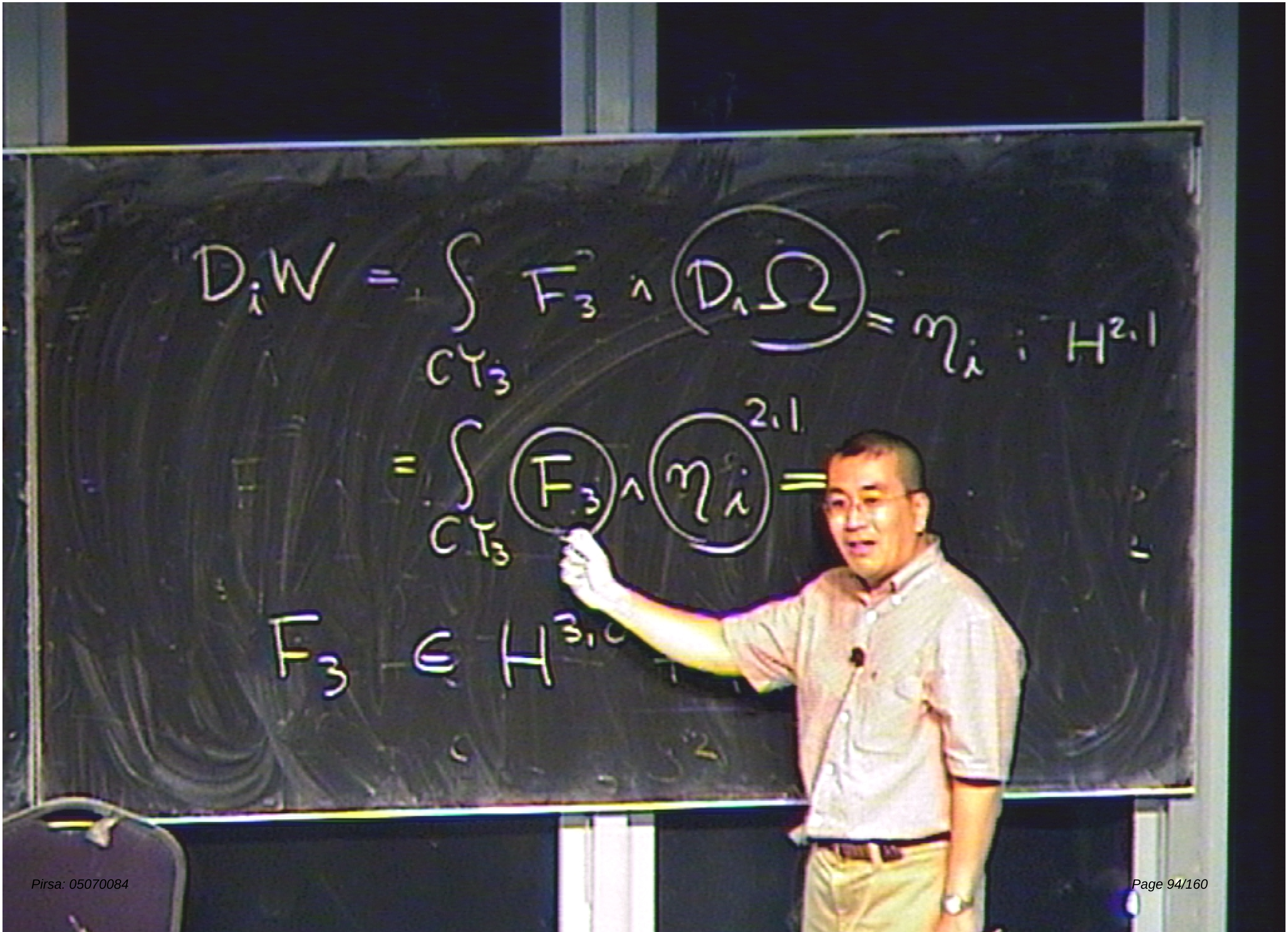
$\{a^I, b_I\} : H^3$


$D_i W = \int_{CY_3} F_3 \wedge D_i\Omega = \eta_i : H^{2,1}$
$= \int_{CY_3}$

$$D_i W = \int_{CY_3} F_3 \wedge D_i \Omega = \eta_i \; : \; H^{2,1}$$

$$= \int_{CY_3} F_3 \wedge \eta_i = 0$$

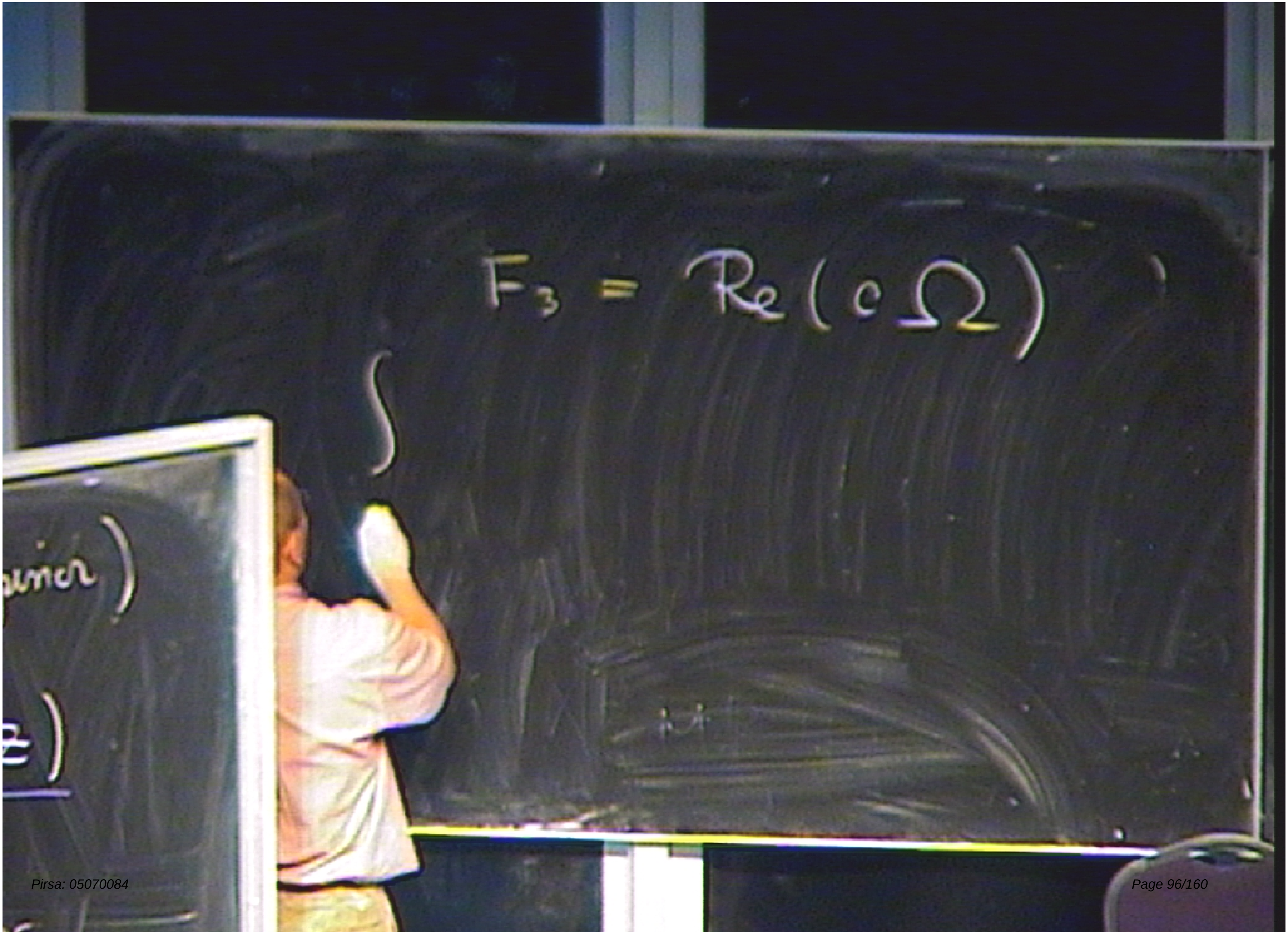

$D_i W = \int_{CY_3} F_3 \wedge D_i \Omega = \eta_i \; ; \; H^{2,1}$
$= \int_{CY_3} F_3 \wedge \eta_i^{2,1} =$

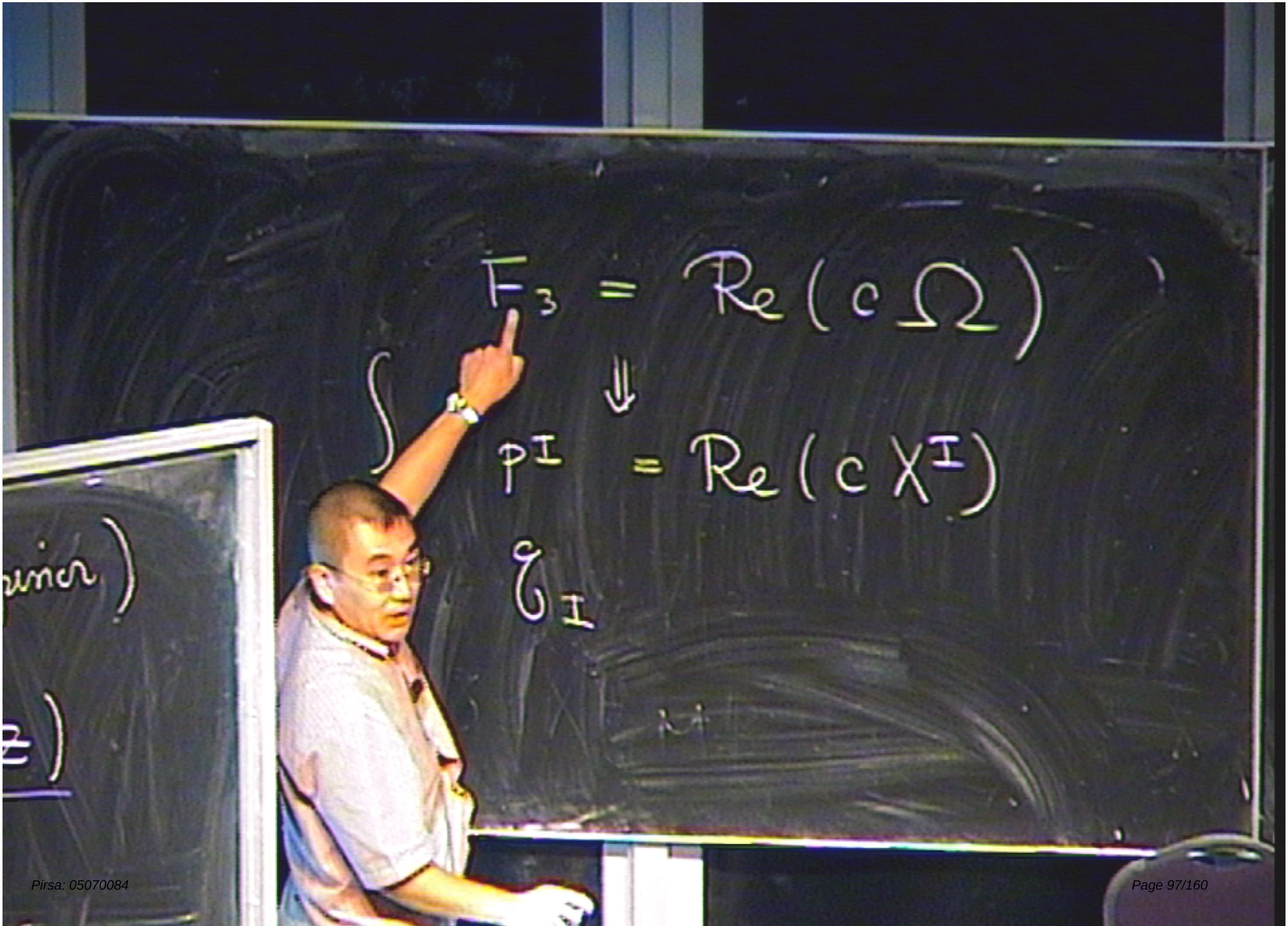

$D_i W = \int_{CY_3} F_3 \wedge D_i \Omega = \eta_i : H^{2,1}$
$= \int_{CY_3} F_3 \wedge \eta_i^{2,1} =$
$F_3 \in H^{3,}$

$$D_i W = \int_{CY_3} F_3 \wedge D_i \Omega = \eta_i \; : \; H^{2,1}$$

$$= \int_{CY_3} F_3 \wedge \eta_i^{2,1} = 0$$

$$F_3 \in \underline{H^{3,0} + H^{0,3}}$$


F3 = Re (c Ω)


$F_3 = Re(c\Omega)$
$\Downarrow$
$p^I = Re(cX^I)$
q_I

pinor)

z)

$$F_3 = \mathrm{Re}(C\,\Omega)$$

$$\Downarrow$$

$$\boxed{\begin{aligned} \int_{A_I}:\quad p^I &= \mathrm{Re}(C\,X^I) \\ \int_{B^I}:\quad q_I &= \mathrm{Re}(C\,F_I(X)) \end{aligned}}$$

attractor equations

$$D_i W = \int_{CY_3} F_3 \wedge D_i \Omega = \eta_i \; : \; H^{2,1}$$

$$= \int_{CY_3} F_3 \wedge \eta_i^{2,1} = 0$$

$$F_3 \in \underline{H^{3,0} \oplus H^{0,3}}$$

$$D_i W = \int_{CY_3} F_3 \wedge D_i \Omega = \eta_i \; : \; H^{2,1}$$

$$= \int_{CY_3} F_3 \wedge \eta_i^{2,1} = 0$$

$$F_3 \in \underline{H^{3,0} \oplus H^{0,3}}$$

attractor equation

$$\boxed{d M_{BPS} = 0}$$

$$\Updownarrow$$

$$d = \partial + \bar{\partial}$$

$$\partial\left(e^{K} W\right) = 0$$

$$\bar{\partial}\left(e^{K} \bar{W}\right) = 0$$

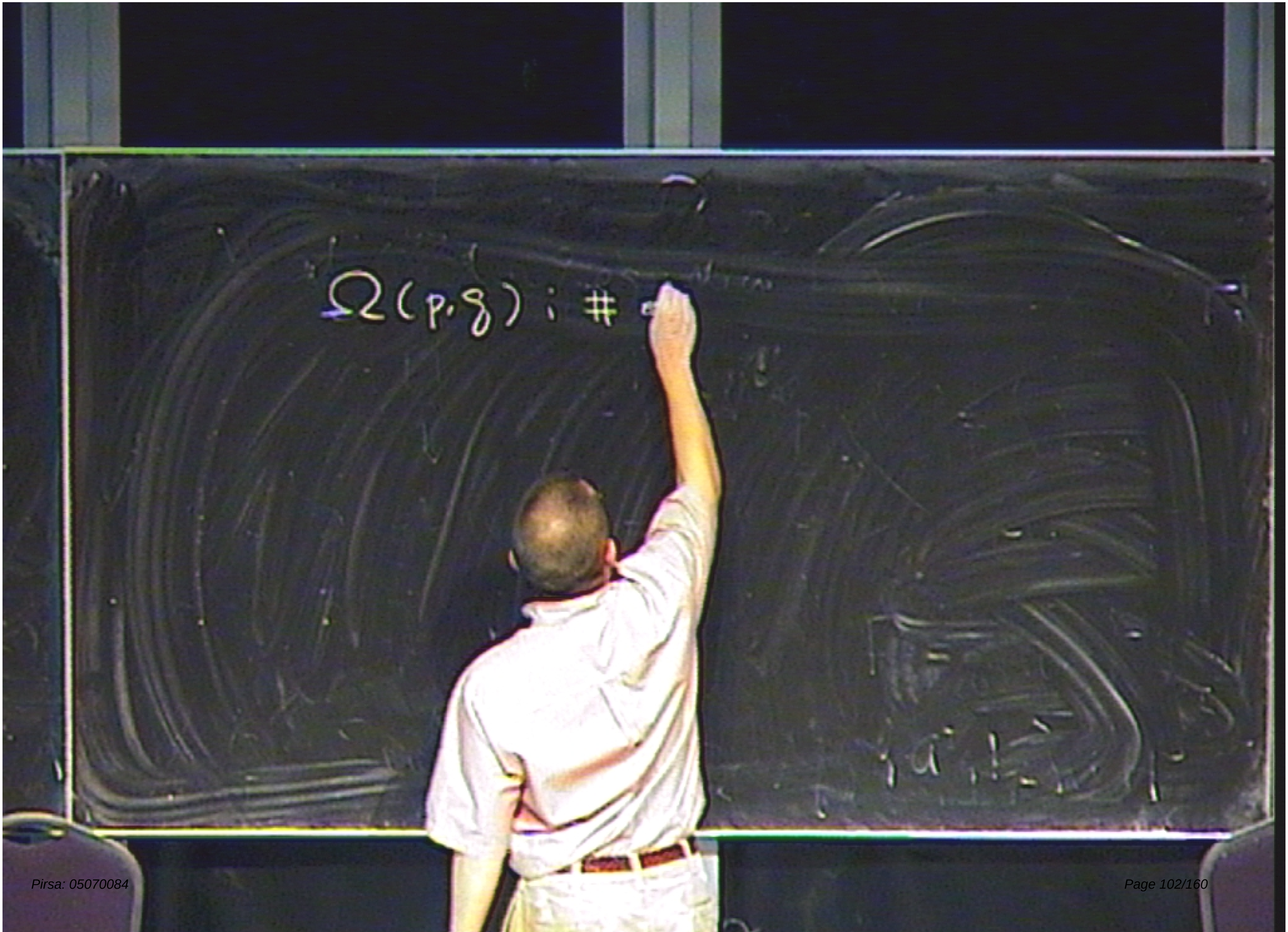
GVW

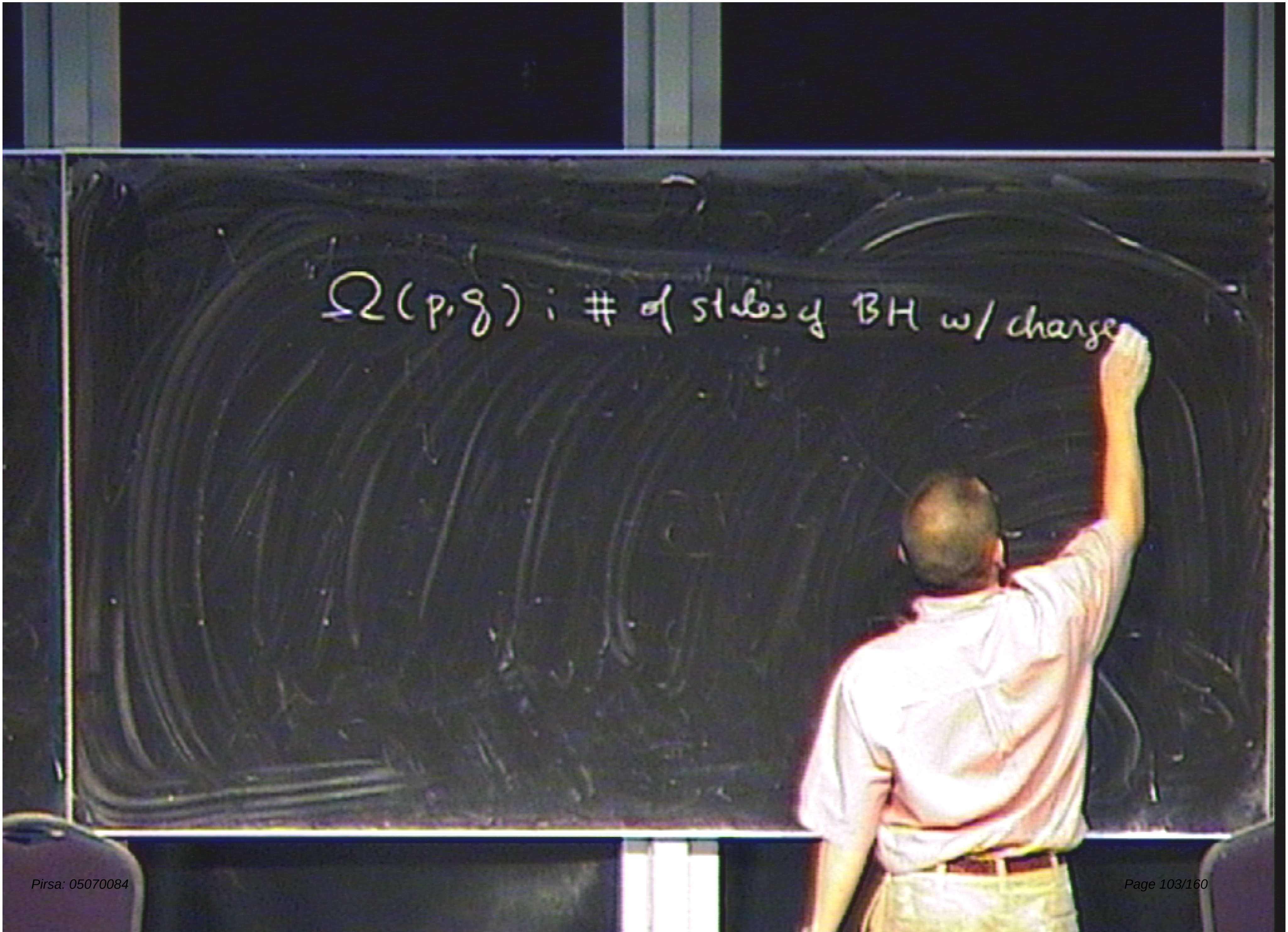
$$W = q_I X^I + p^I F_I$$

$$= \int_{CY_3} F_3 \wedge \Omega$$

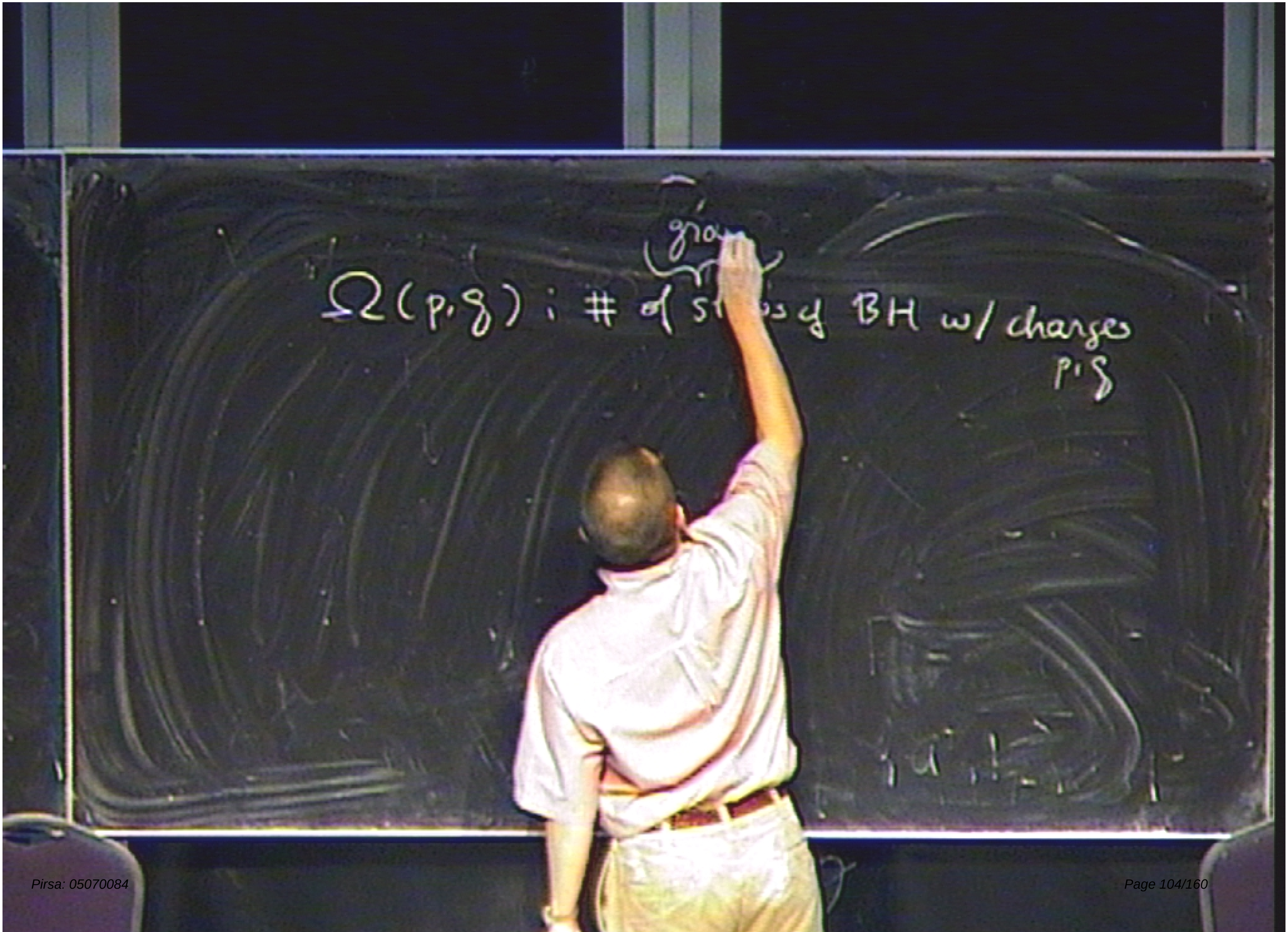
$$F_3 = q_I a^I + p^I b_I$$

$$\{a^I, b_I\} : H^3$$

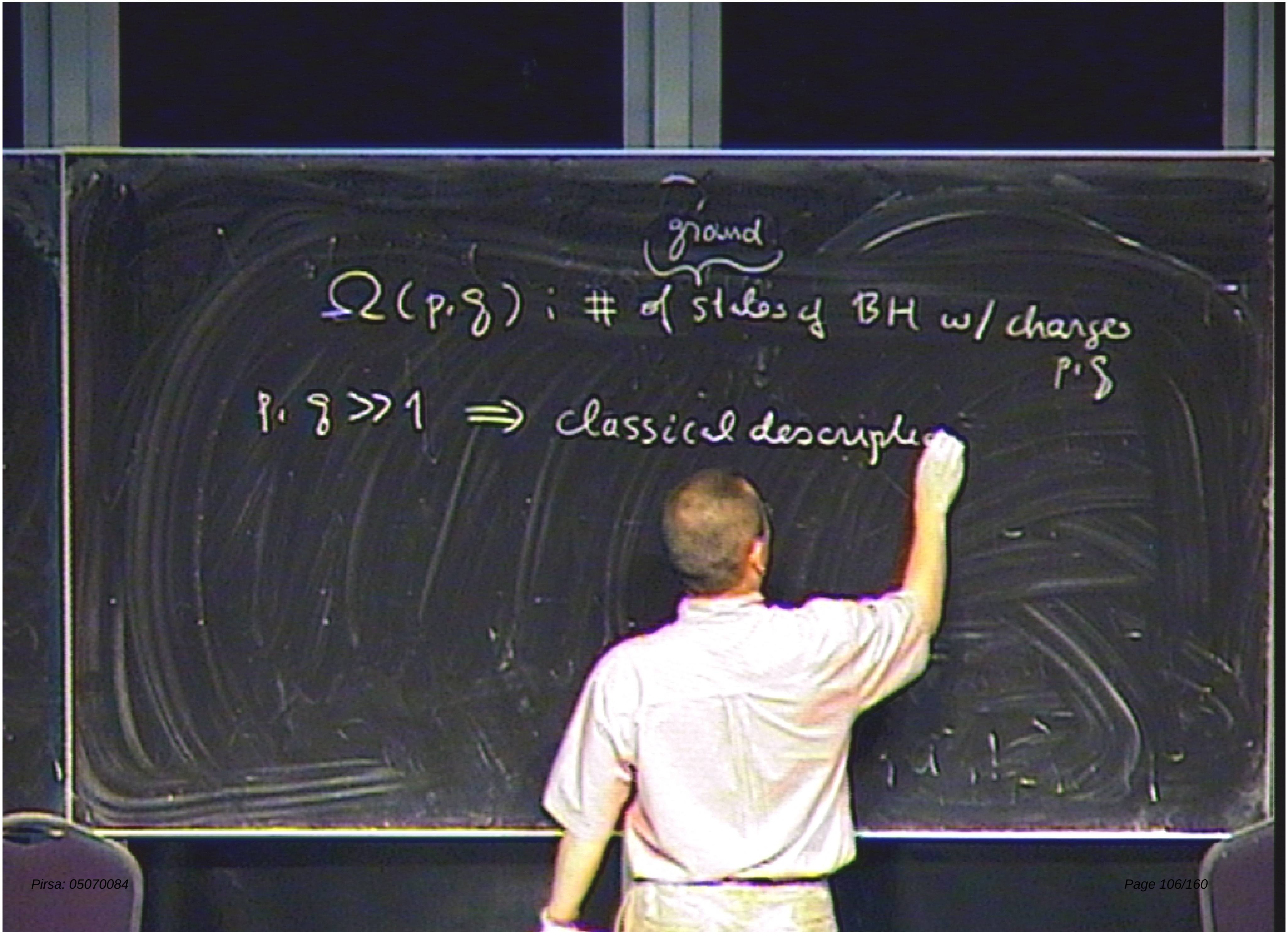

Ω(p,q) : # =

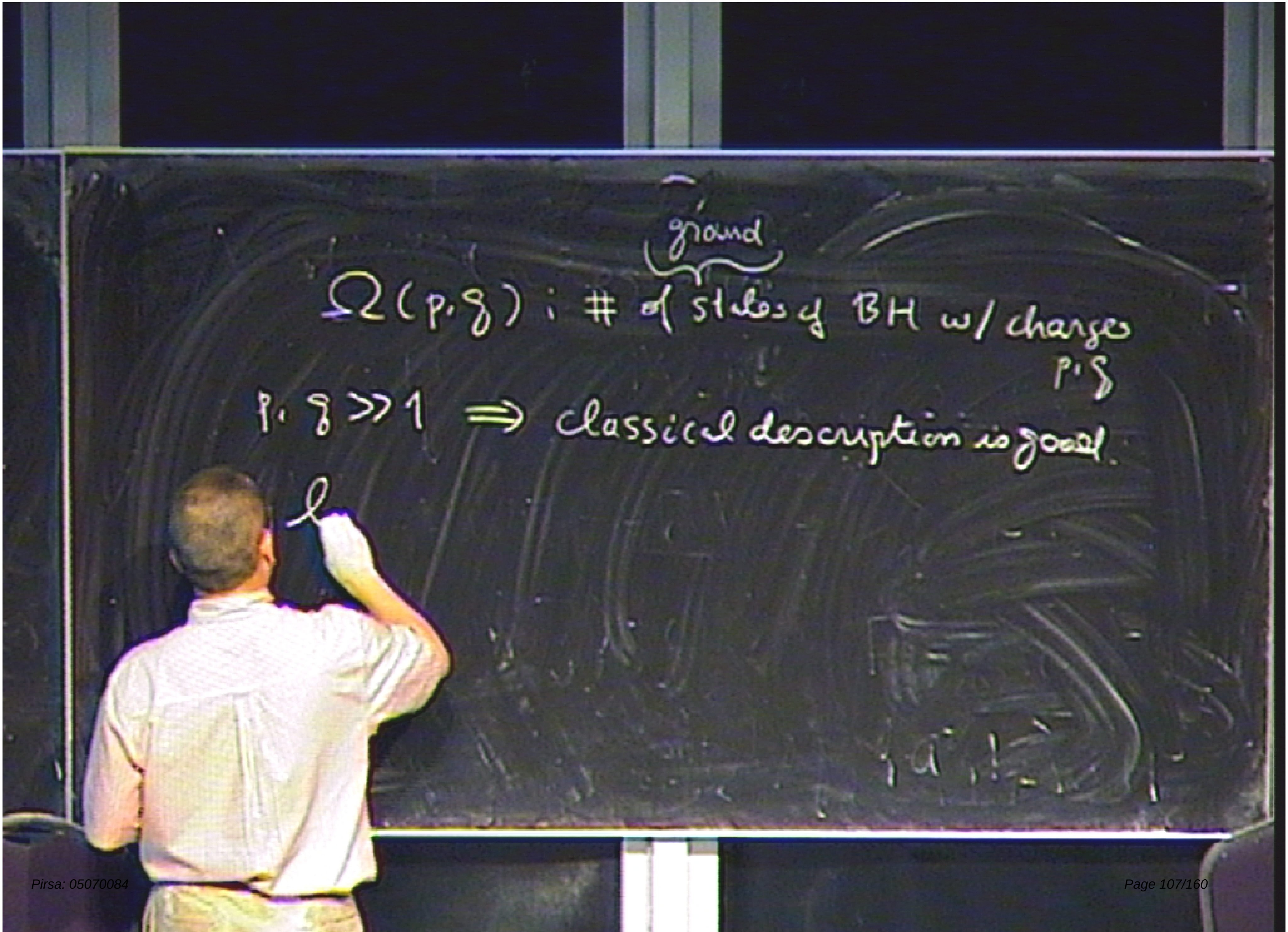


$\Omega(p,q)$: # of states of BH w/ charge


Ω(p,q) ; # of
of BH w/ charges
p,q

$\Omega(p,q)$: # of $\underbrace{\text{states}}_{\text{ground}}$ of BH w/ charges p,q


ground
Ω(p,q) : # of states of BH w/ charges p,q
p,q ≫ 1 ⇒ classical description


ground
Ω(p,q) : # of states of BH w/ charges
p,q
p,q >> 1 ⟹ classical description is good
l

$\Omega(p,q)$: # of $\overbrace{\text{states}}^{\text{ground}}$ of BH w/ charges p,q

$p,q \gg 1 \Rightarrow$ classical description is good.

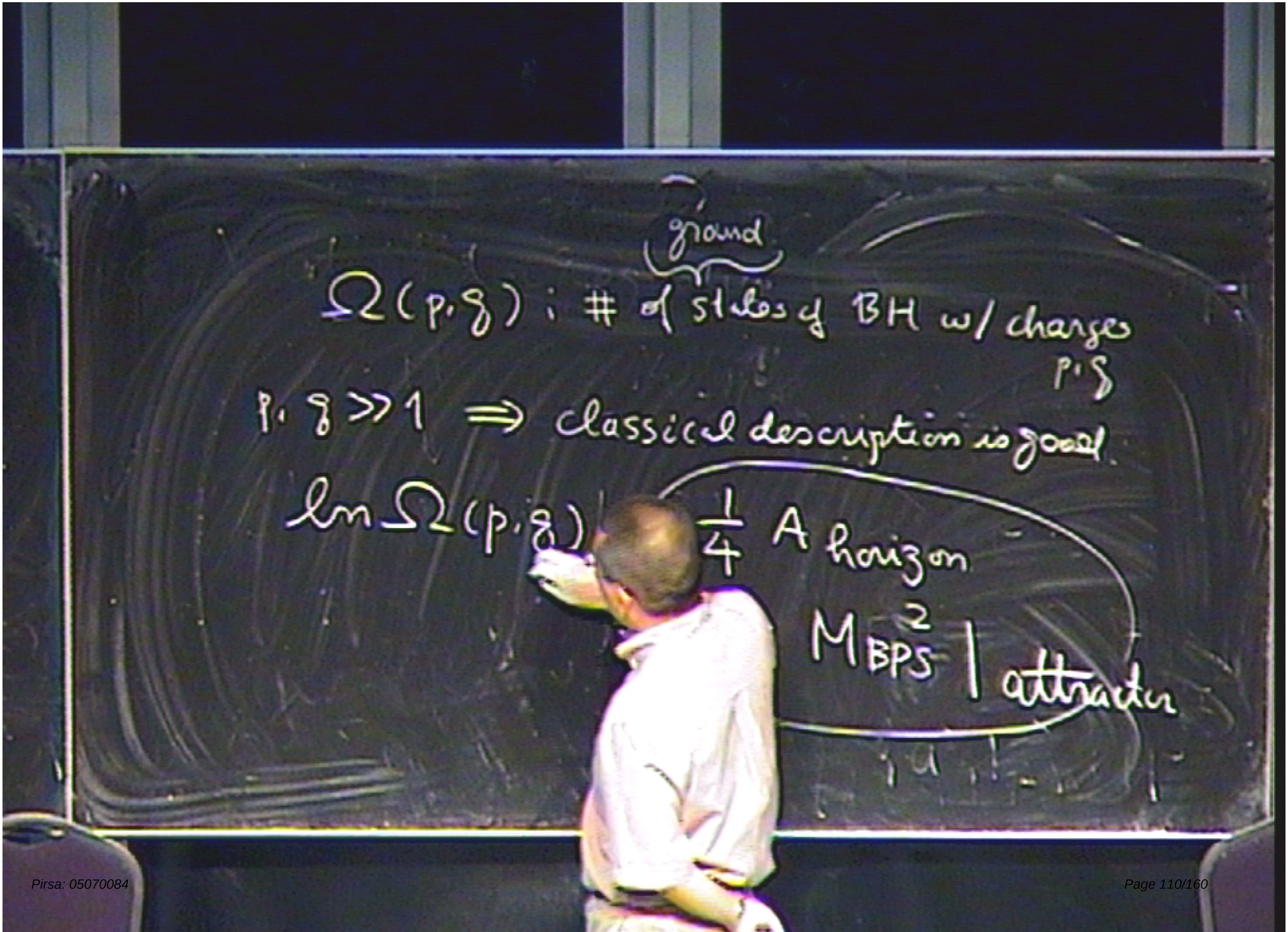
$\ln \Omega(p,q) \sim \frac{1}{4} A_{\text{horizon}}$

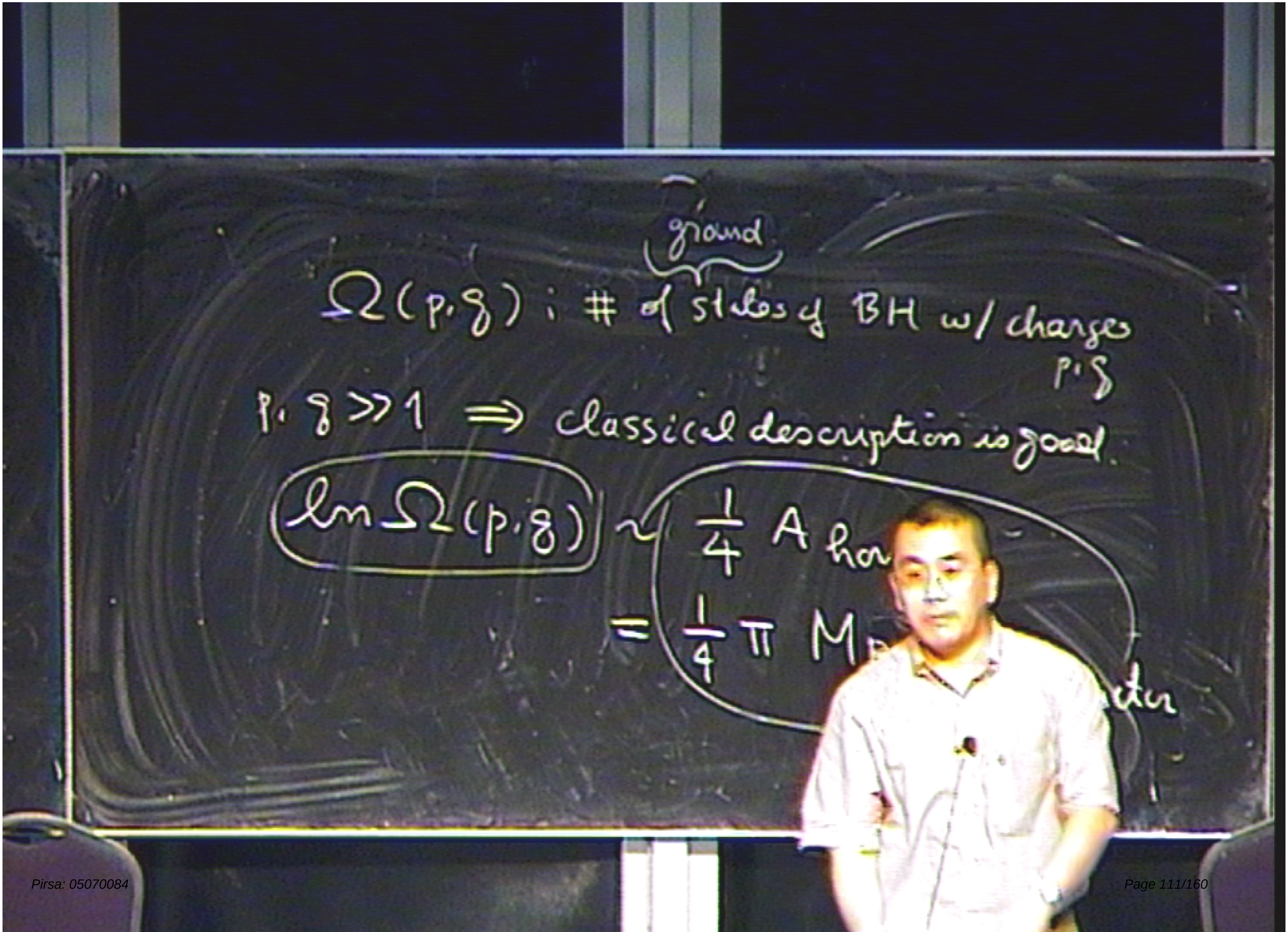
$$\Omega(p,q) : \# \text{ of } \overbrace{\text{states}}^{\text{ground}} \text{ of BH w/ charges } p, q$$

$$p, q \gg 1 \Longrightarrow \text{classical description is good}$$

$$\ln \Omega(p,q) \sim \frac{1}{2} \ldots \text{horizon}$$

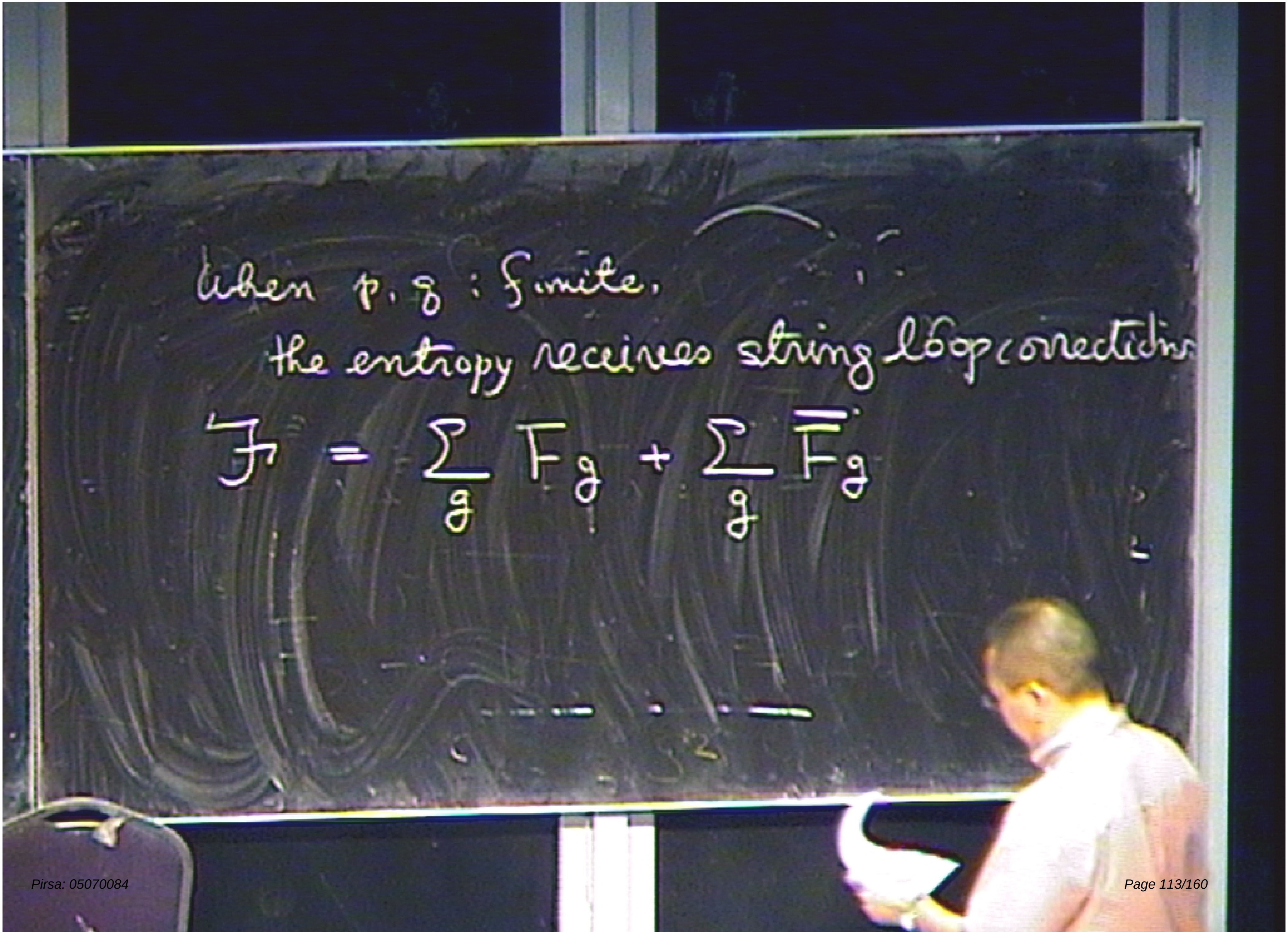
$$= \frac{1}{4} \ldots {}^{2} \ldots \Big|_{\text{attractor}}$$

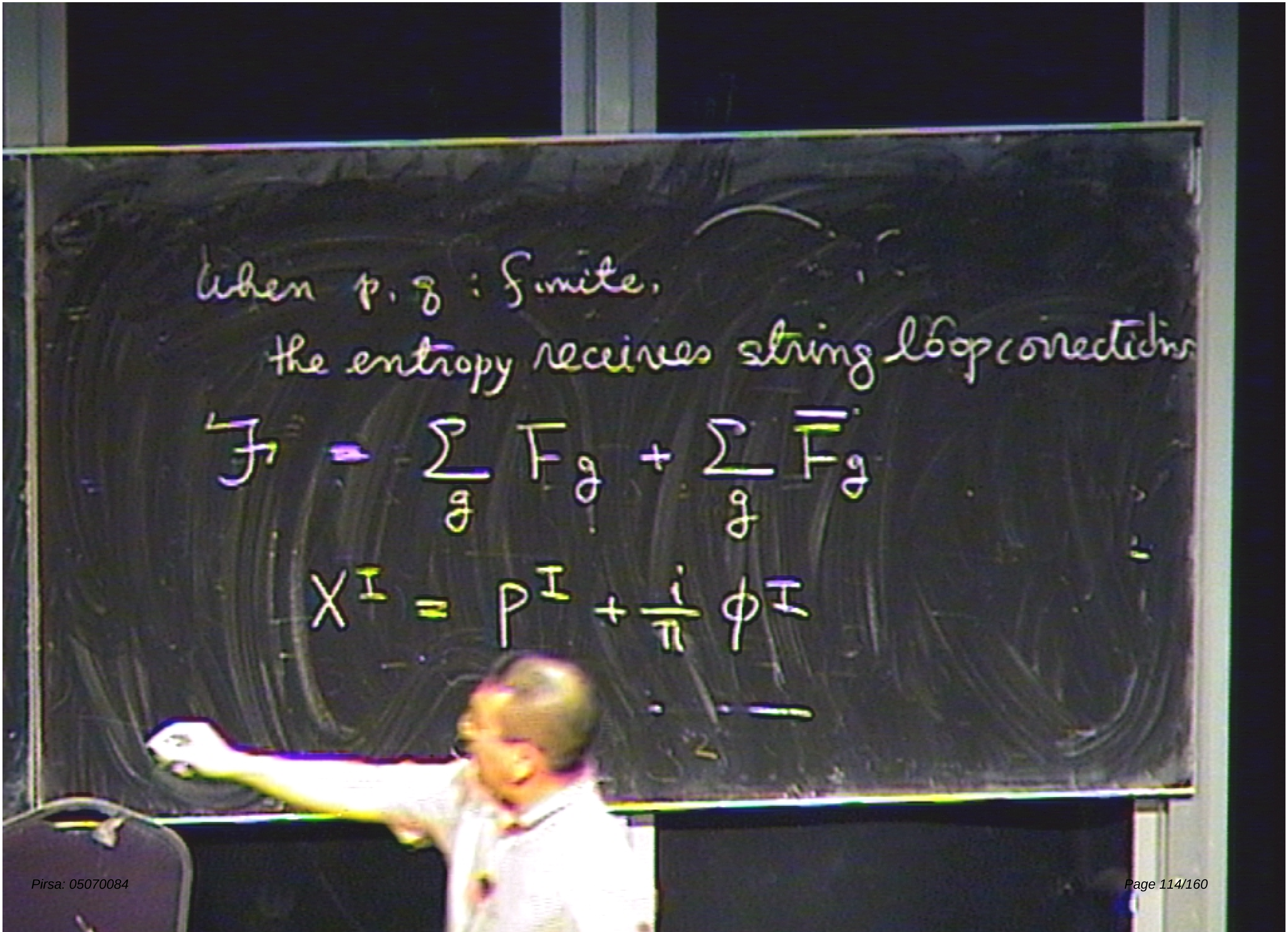

ground
Ω(p,q) : # of states of BH w/ charges p,q
p,q >> 1 ⇒ classical description is good
ln Ω(p,q) 1/4 A horizon
M BPS 2 | attractor

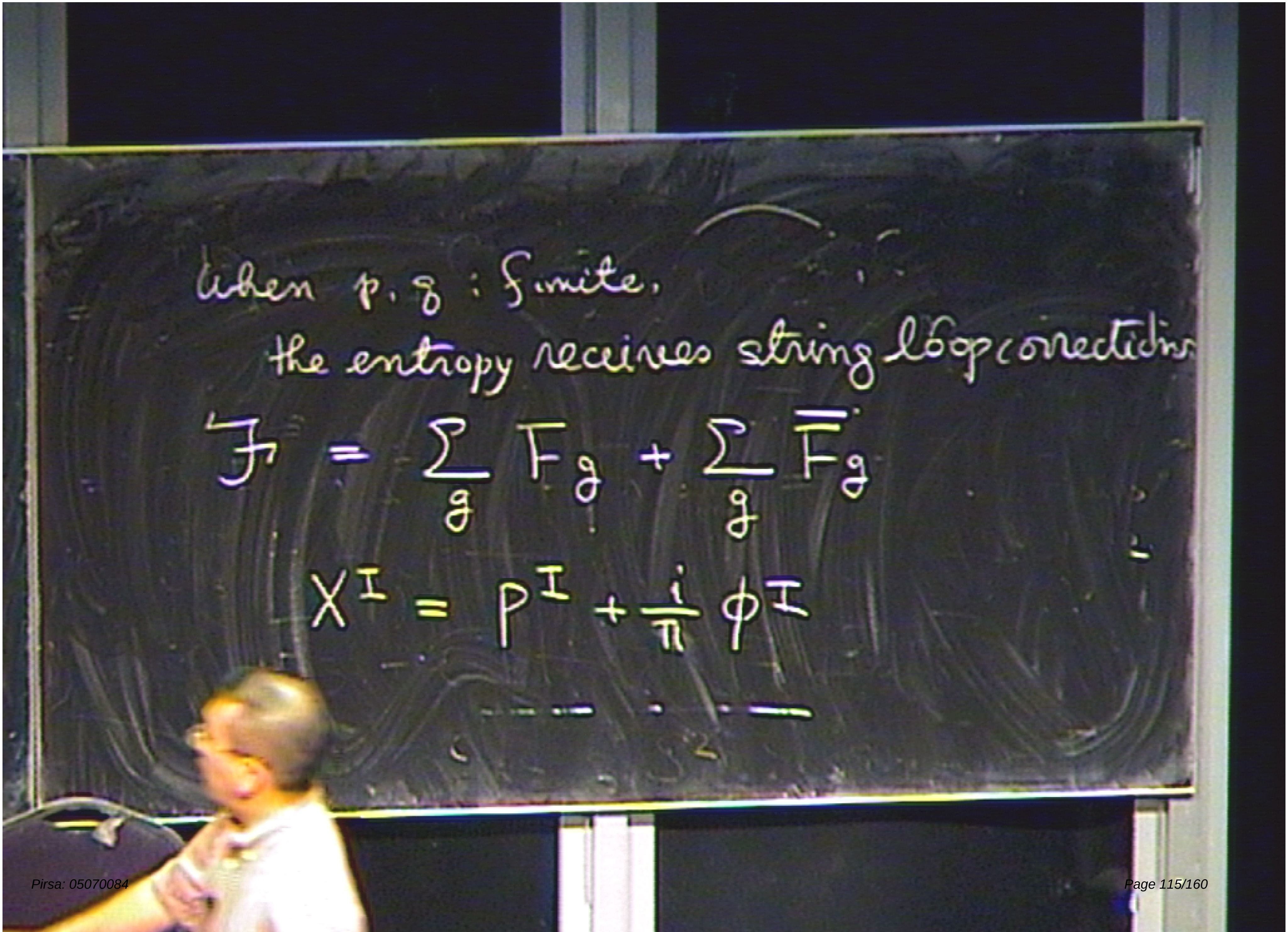

ground
Ω(p,q) : # of states of BH w/ charges p,q
p,q >> 1 ⇒ classical description is good
ln Ω(p,q) ~ 1/4 A hor
= 1/4 π M

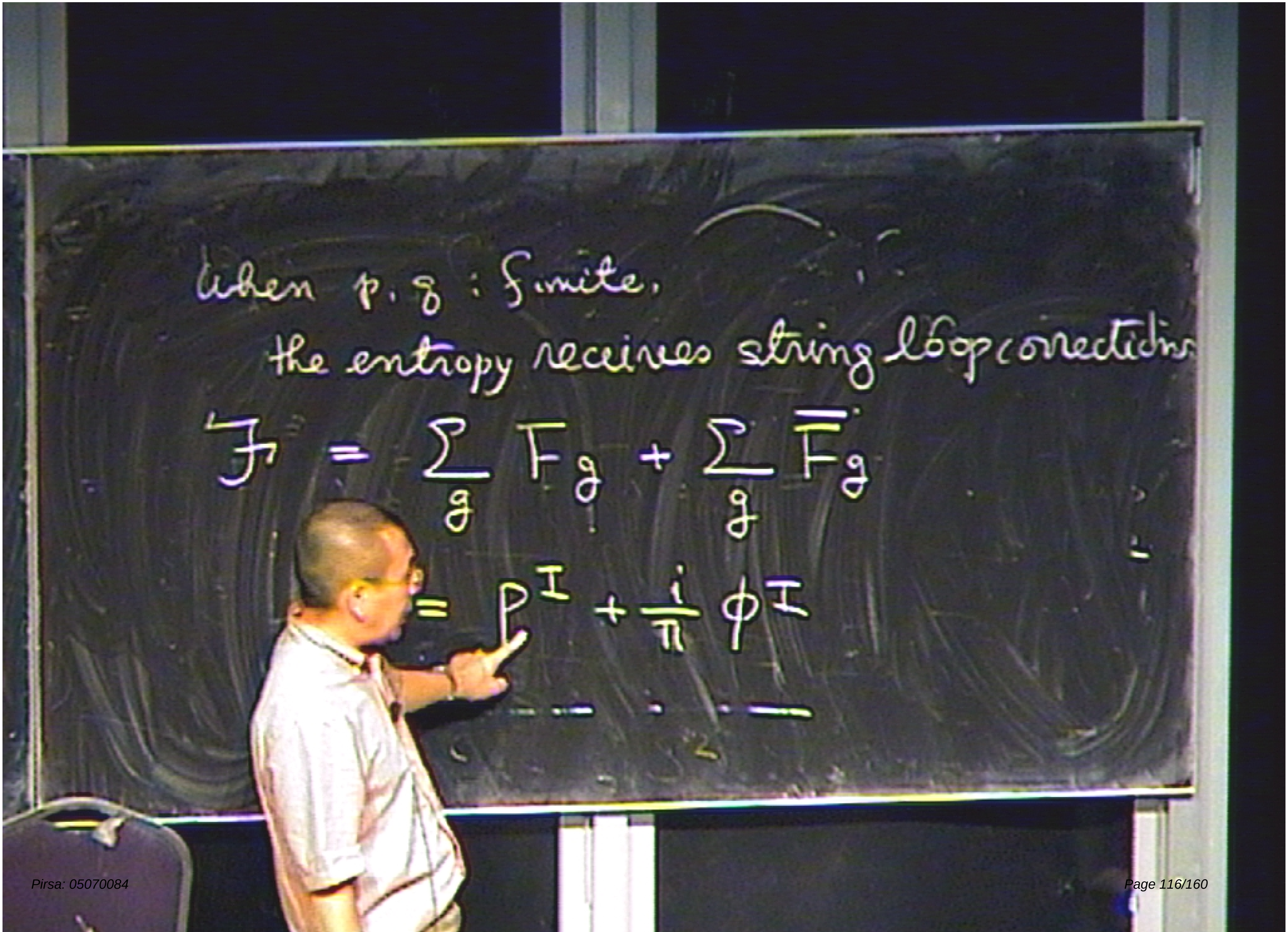
when p, q : finite,

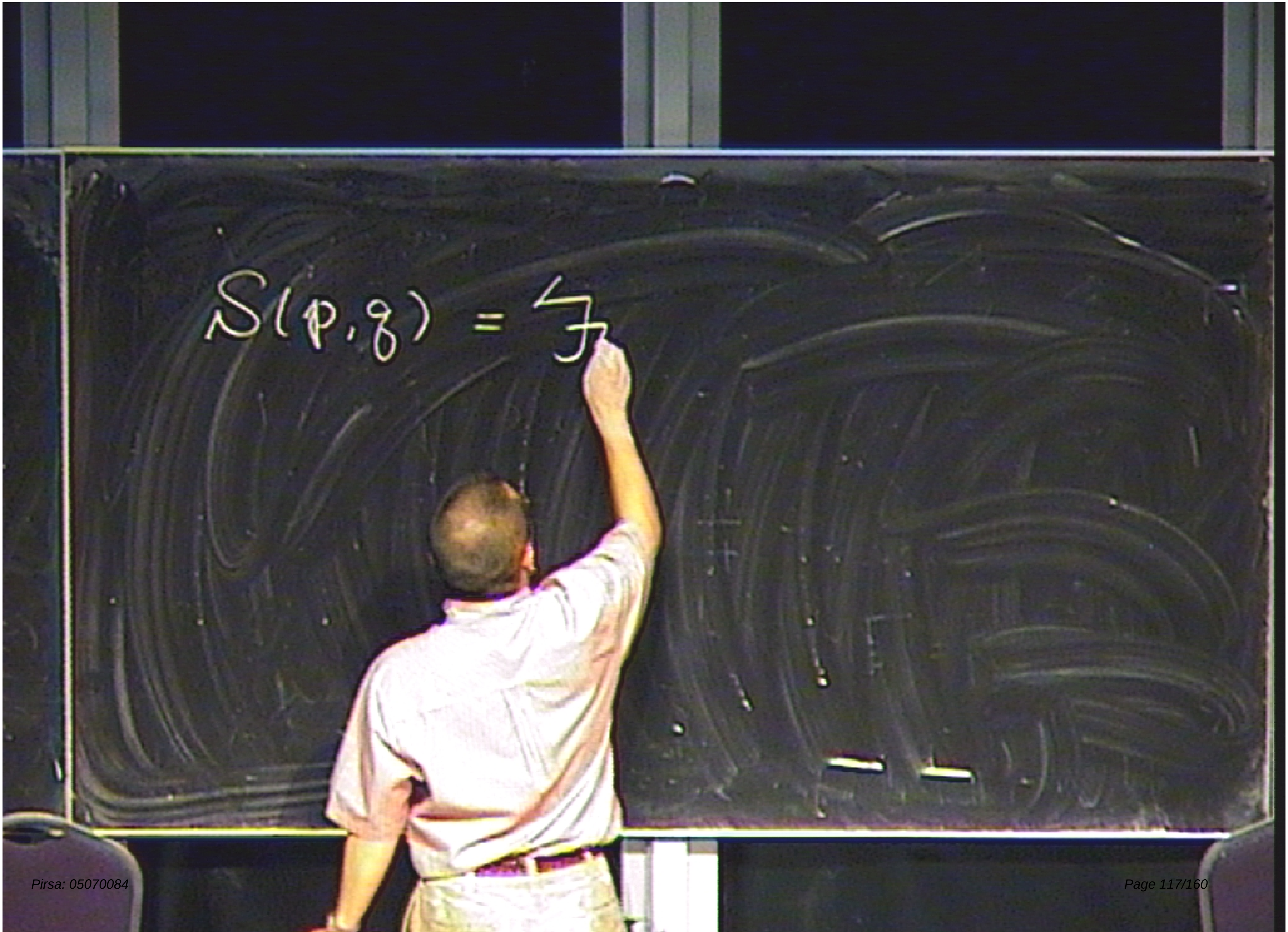
the entropy receives string loop correction


when p, q : finite,
the entropy receives string loop correction
$\mathcal{F} = \sum_g F_g + \sum_g \bar{F}_g$

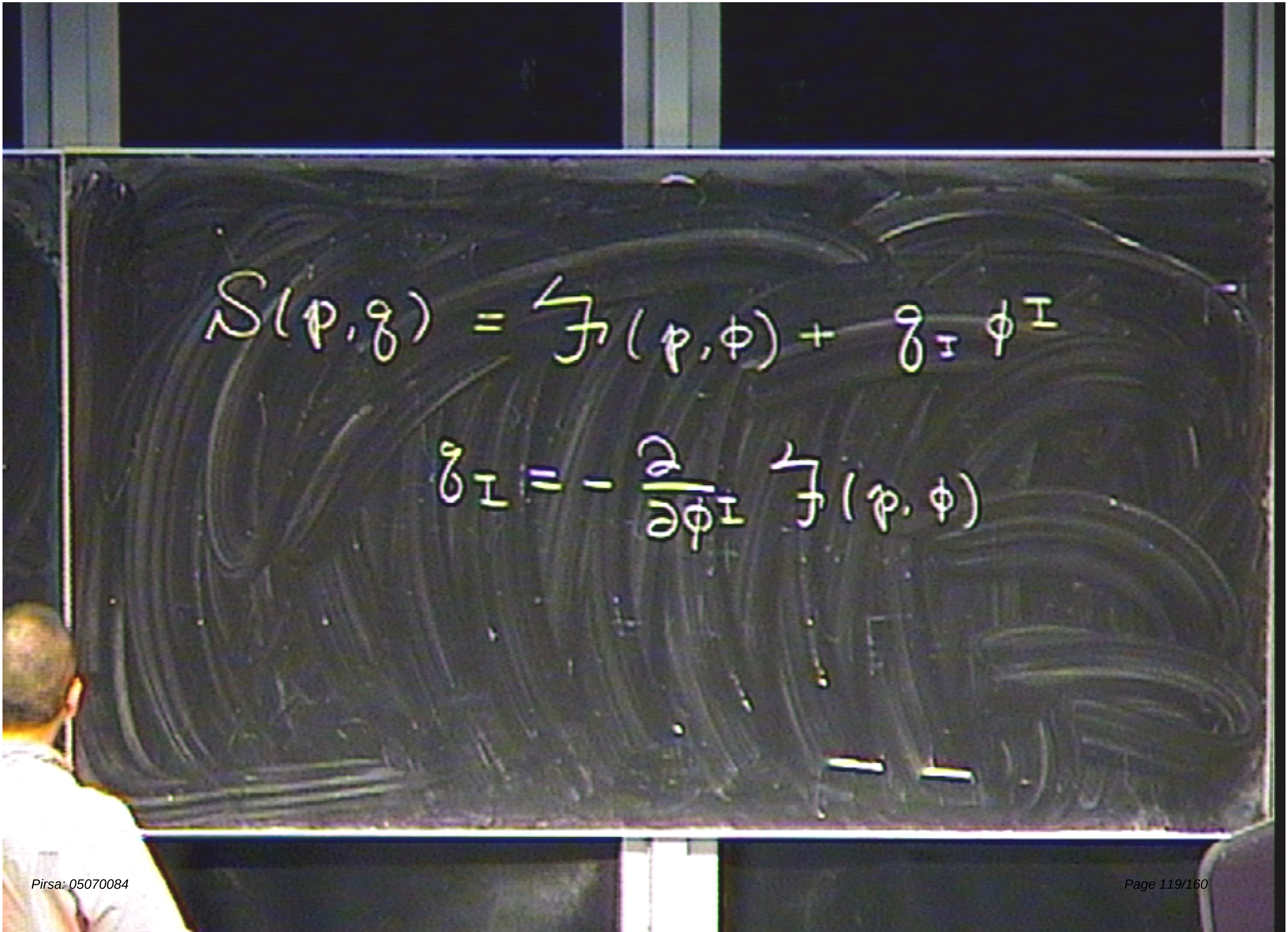

when p, q : finite,
the entropy receives string loop corrections
$\mathcal{F} = \sum_g F_g + \sum_g \bar{F}_g$
$X^I = p^I + \frac{i}{\pi} \phi^I$


when p, q : finite,
the entropy receives string loop correction
$\mathcal{F} = \sum_g F_g + \sum_g \bar{F}_g$
$X^I = p^I + \frac{i}{\pi}\phi^I$


when p, q : finite,
the entropy receives string loop corrections
$\mathcal{F} = \sum_g F_g + \sum_g \bar{F}_g$
$= p^I + \frac{i}{\pi}\phi^I$

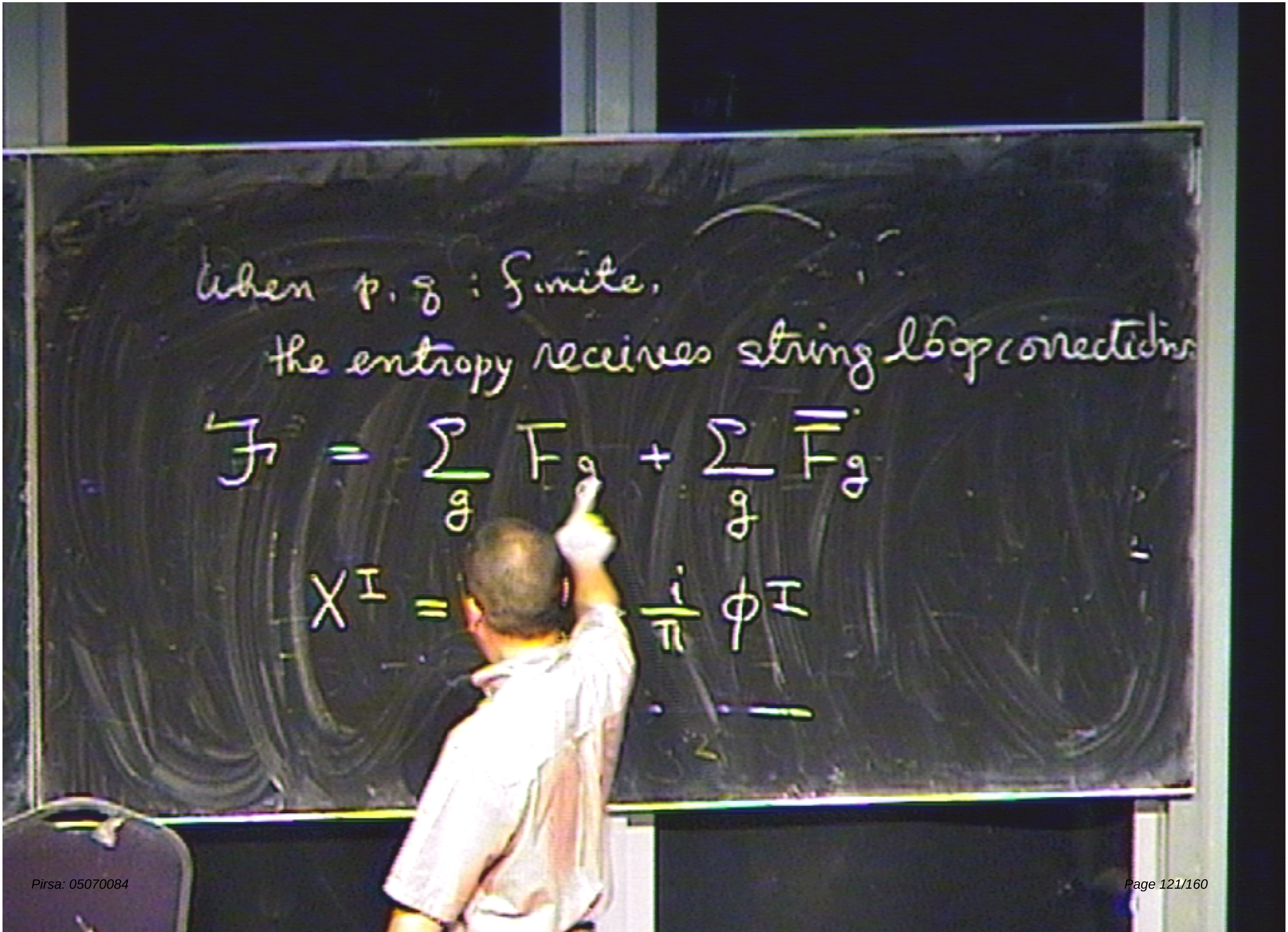

S(p,q) =

$$S(p,q) = \mathcal{F}(p,\phi)$$


$S(p,q) = \mathcal{F}(p,\phi) + q_I \phi^I$
$q_I = -\frac{\partial}{\partial \phi^I} \mathcal{F}(p,\phi)$

$$S(p,q) = \mathcal{F}(p,\phi) + q_I \phi^I$$

$$q_I = -\frac{\partial}{\partial \phi^I} \mathcal{F}(p,\phi)$$


when p, q : finite,
the entropy receives string loop corrections
$\mathcal{F} = \sum_g F_g + \sum_g \bar{F}_g$
$X^I =$
$\frac{i}{\pi} \phi^I$

$$S(p,q) = \mathcal{F}(p,\phi) + q_I \phi^I$$

$$q_I = -\frac{\partial}{\partial \phi^I} \mathcal{F}(p,\phi)$$

$$\sum_{g} \Omega(p,g)\, e^{-g_I \phi^I}$$

$$= \left| \exp\left(\sum_{g} F_g \right) \right|^2$$

$$\sum_{q} \Omega(p,q)\, e^{-q_I \phi^I}$$

$$= \left| \exp\left(\sum_{g} F_g \right) \right|^2$$

$$\sum_{g} \Omega(p,g)\, e^{-g_I \phi^I}$$

$$= \left| \exp\Big(\sum_{g} \ldots \Big) \right|^2$$

$$S(p,q) = \mathcal{F}(p,\phi) + q_I \phi^I$$

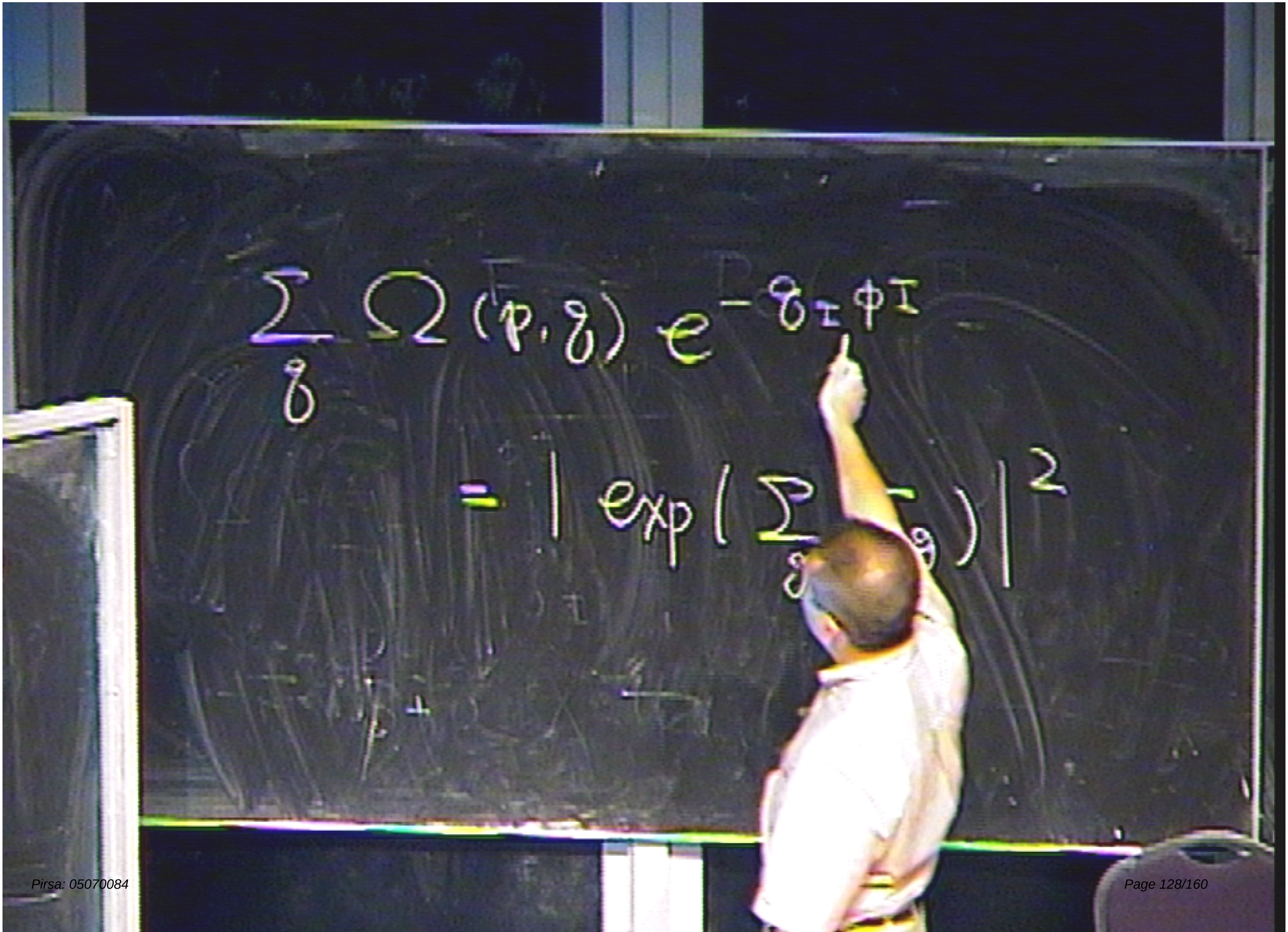
$$q_I = -\frac{\partial}{\partial \phi^I} \mathcal{F}(p,\phi)$$

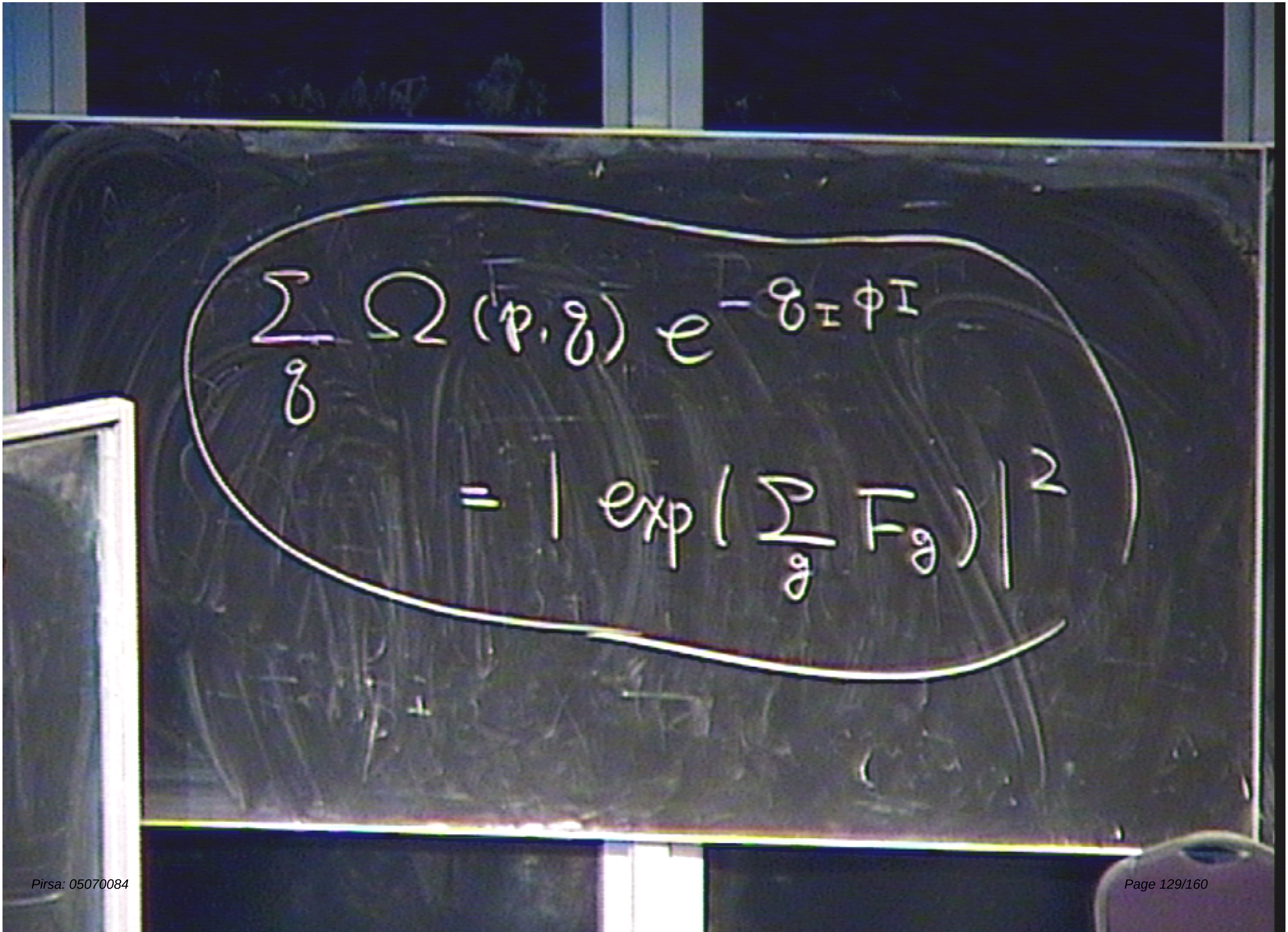
when p, q : finite,

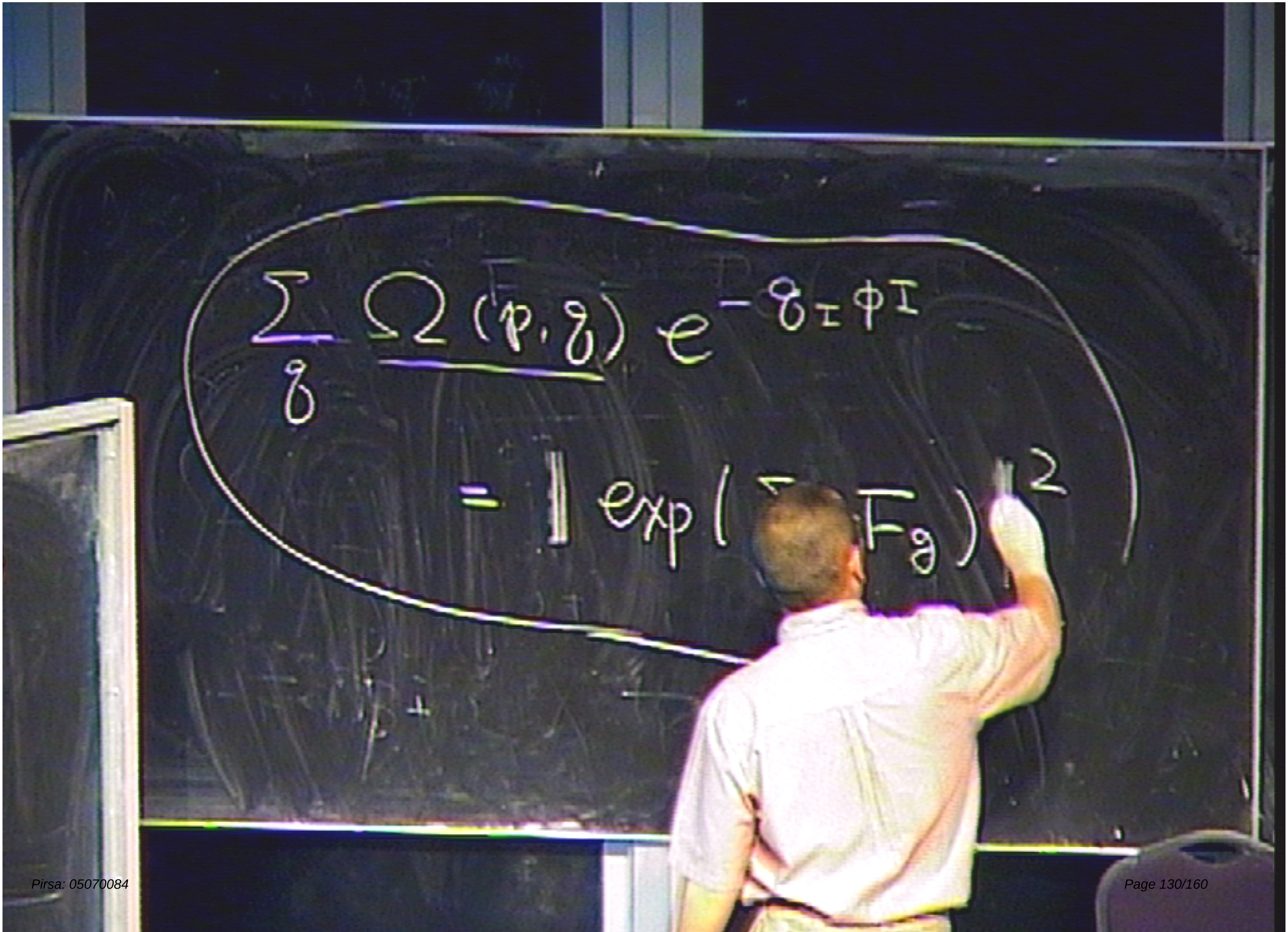
the entropy receives string loop corrections

$$\mathcal{F} = \sum_g F_g + \sum_g \overline{F}_g$$

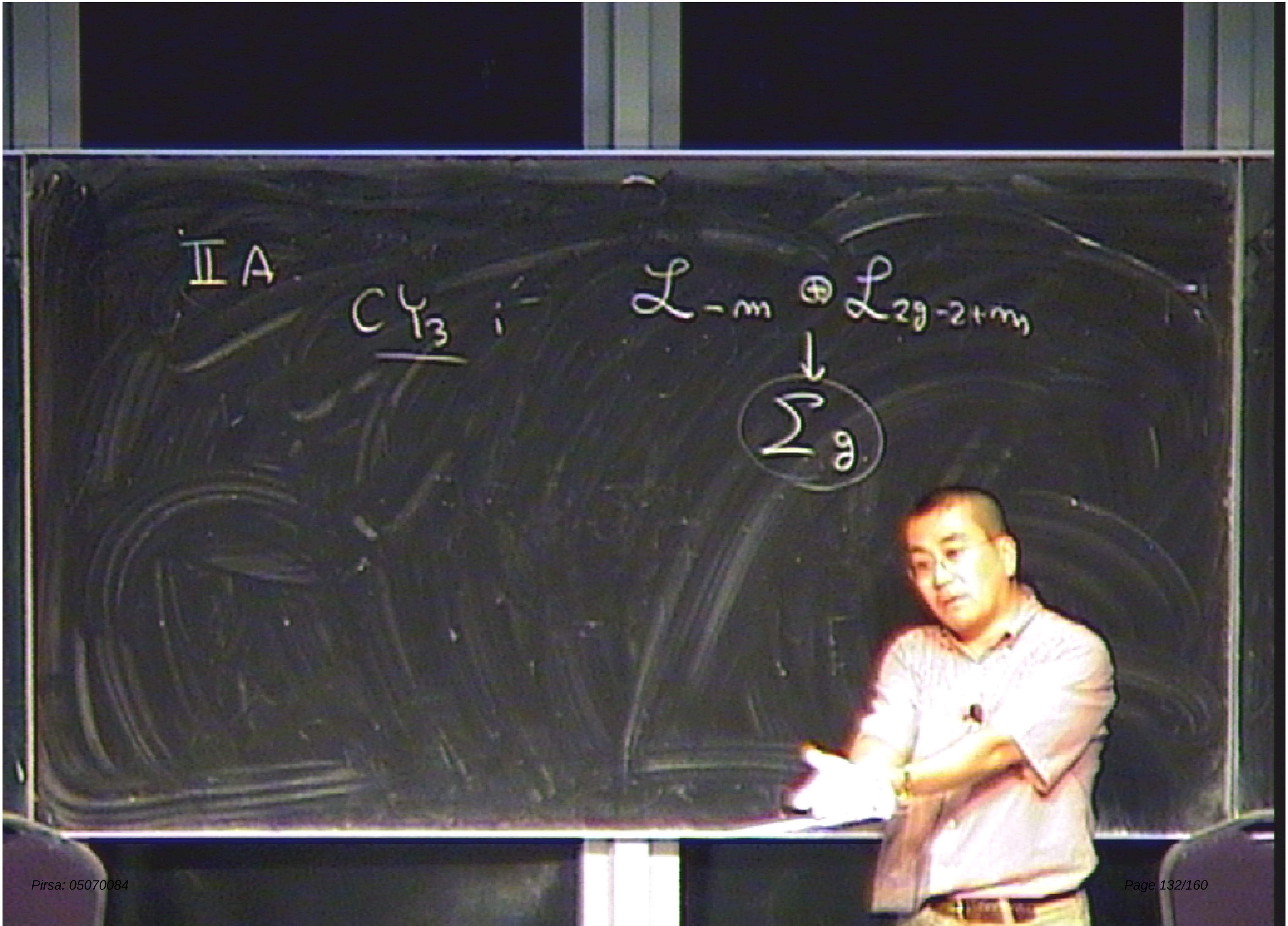
$$X^I = p^I + \frac{i}{\pi}\phi^I$$


$\sum_g \Omega(p,g)\, e^{-g_I \phi^I}$
$= \left| \exp\left(\sum_g \right) \right|^2$


$\sum_q \Omega(p,q)\, e^{-q_I \phi^I}$
$= \left| \exp\left(\sum_g F_g \right) \right|^2$


$\sum_{g} \Omega(p,g)\, e^{-g_I \phi^I}$
$= \| \exp($
$F_g) \|^2$

$$\sum_{g} \underline{\Omega(p,g)}\, e^{-g_I \phi^I} = \left\| \exp\left(\sum_g F_g\right) \right\|^2$$

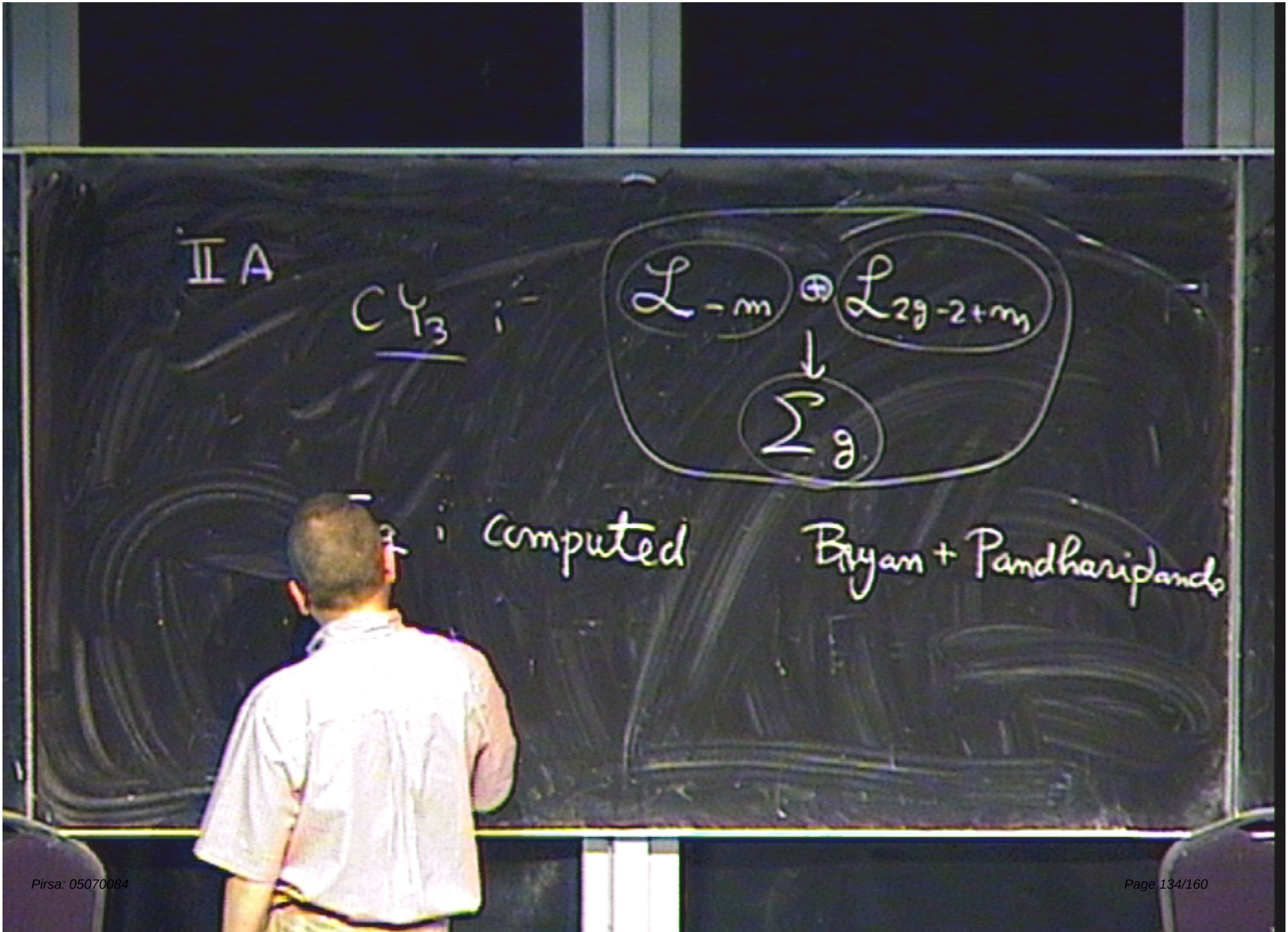


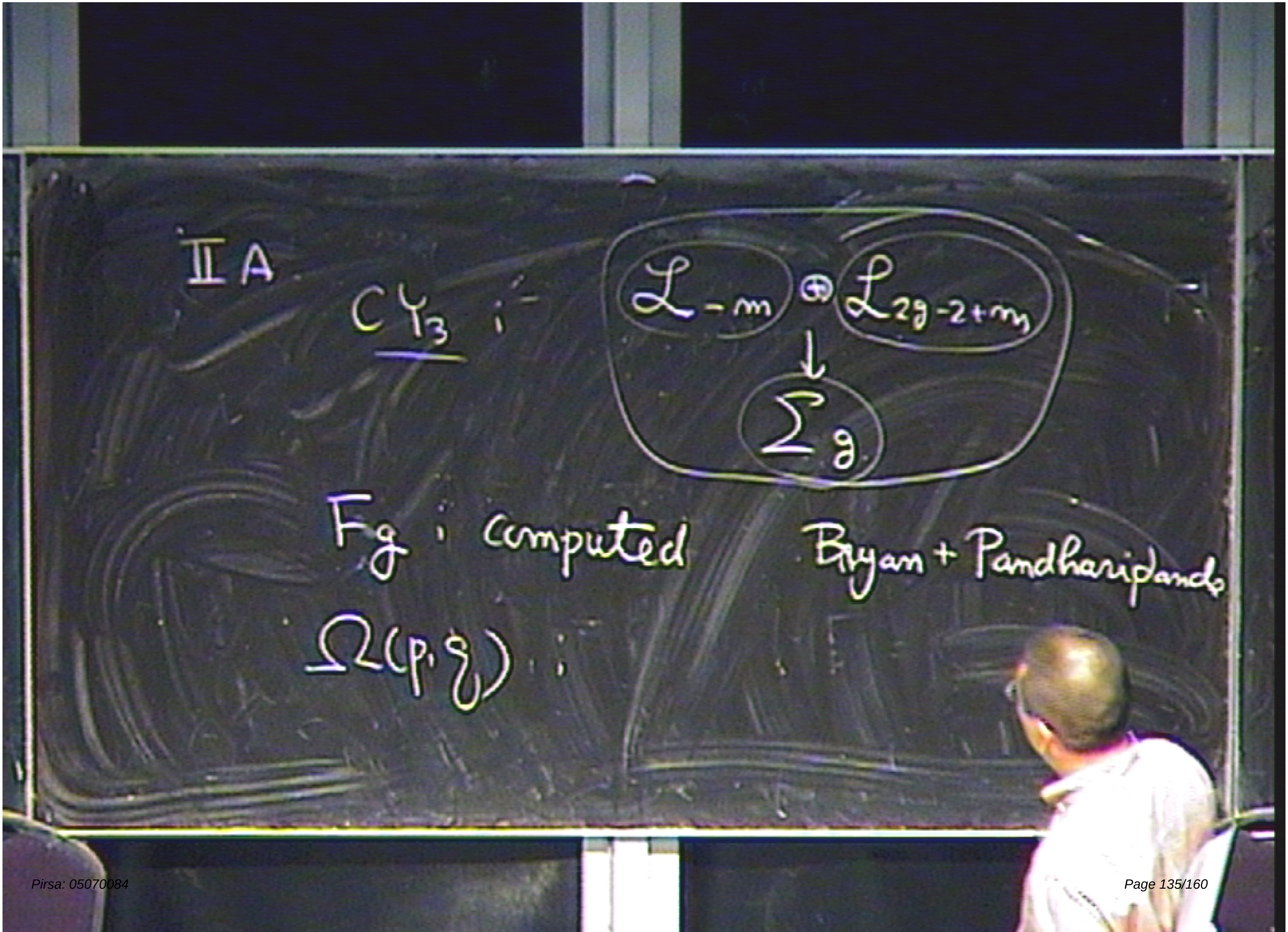
IIA
CY_3 :
$\mathcal{L}_{-m} \oplus \mathcal{L}_{2g-2+m}$
$\downarrow$
Σ_g

IIA

$\underline{CY_3}$:

$$\mathcal{L}_{-m} \oplus \mathcal{L}_{2g-2+m}$$
$$\downarrow$$
$$\Sigma_g$$


II A
CY3 :
L-m ⊕ L2g-2+m
Σg
: computed
Bryan + Pandharipande

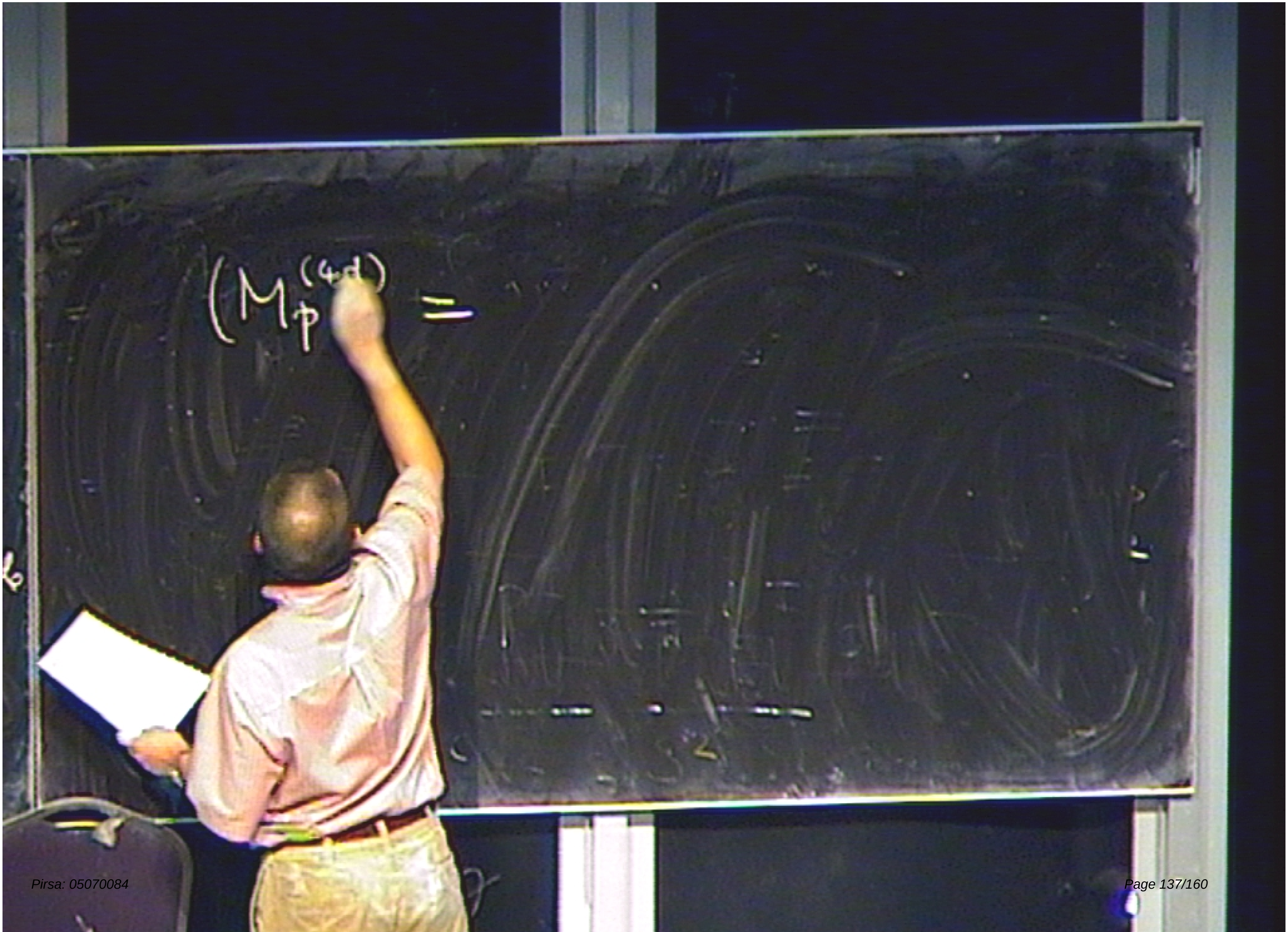

IIA
CY3 :
L−m ⊕ L2g−2+m
↓
Σg
Fg : computed
Bryan + Pandharipande
Ω(p,q) :

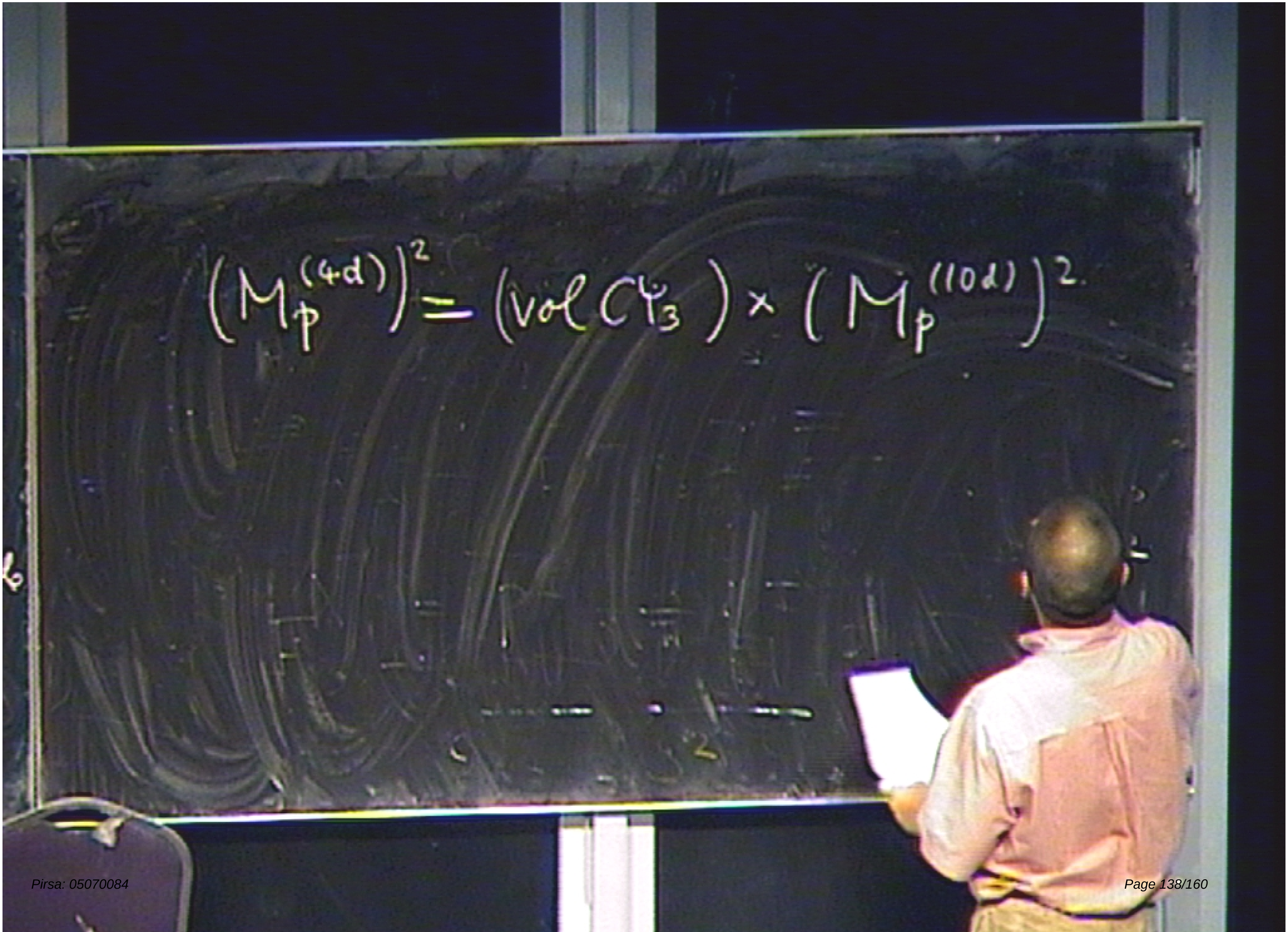
II A

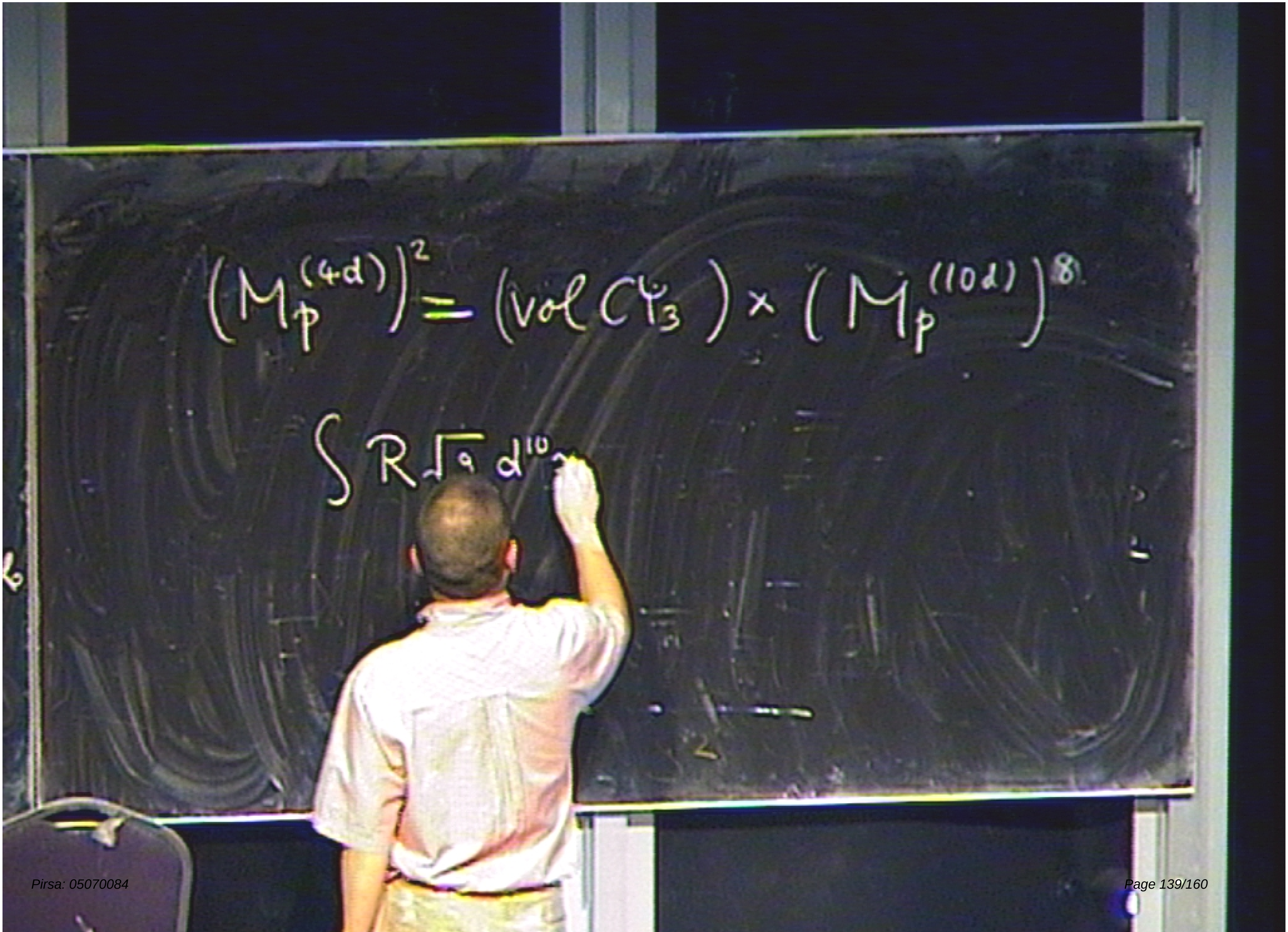
CY_3 : $\mathcal{L}_{-m} \oplus \mathcal{L}_{2g-2+m} \downarrow \Sigma_g$

F_g : computed   Bryan + Pandharipande

$\Omega(p,q)$: D4 on $\mathcal{L}_{-m} \to \Sigma_g$


$(M_p^{(4d)}) =$


$(M_p^{(4d)})^2 = (\mathrm{vol}\, CY_3) \times (M_p^{(10d)})^2$


$(M_p^{(4d)})^2 = (\mathrm{Vol}\, CY_3) \times (M_p^{(10d)})^8$
$\int R\sqrt{g}\, d^{10}$

$$\left(M_p^{(4d)}\right)^2 = \left(\mathrm{vol}\, CY_3\right) \times \left(M_p^{(10d)}\right)^8$$

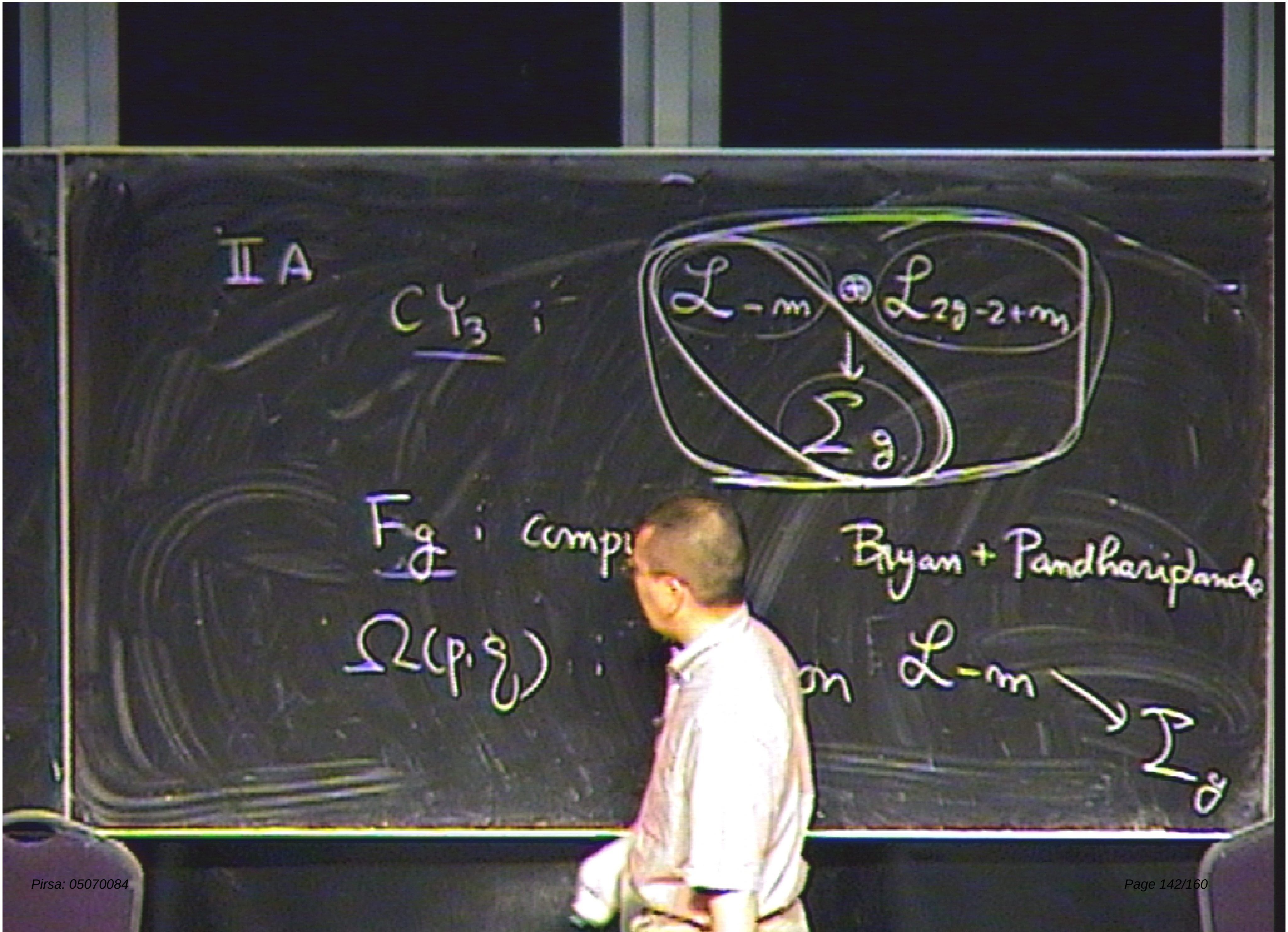
$$\int R\sqrt{g}\, d^{10}x = \left(\mathrm{vol}\, CY_3\right) \times \int R\sqrt{g}\, d^4x$$

IIA

$\underline{CY_3}$:
$$\begin{array}{c} \mathcal{L}_{-m} \oplus \mathcal{L}_{2g-2+m} \\ \downarrow \\ \Sigma_g \end{array}$$

F_g : computed   Bryan + Pandharipande

$\Omega(p,q)$: D4 on $\mathcal{L}_{-m} \to \Sigma_g$


II A
CY3 :
L-m ⊕ L2g-2+m
Σg
Fg : comp
Bryan + Pandharipande
Ω(p,q) :
on L-m → Σg

II A

$\underline{CY_3}$: $\mathcal{L}_{-m} \oplus \mathcal{L}_{2g-2+m}$

$\downarrow$

Σ_g

$\underline{F_g}$: computed Bryan + Pandharipande

$\underline{\Omega(p,q)}$: D4 on $\mathcal{L}_{-m} \to \Sigma_g$

IIA

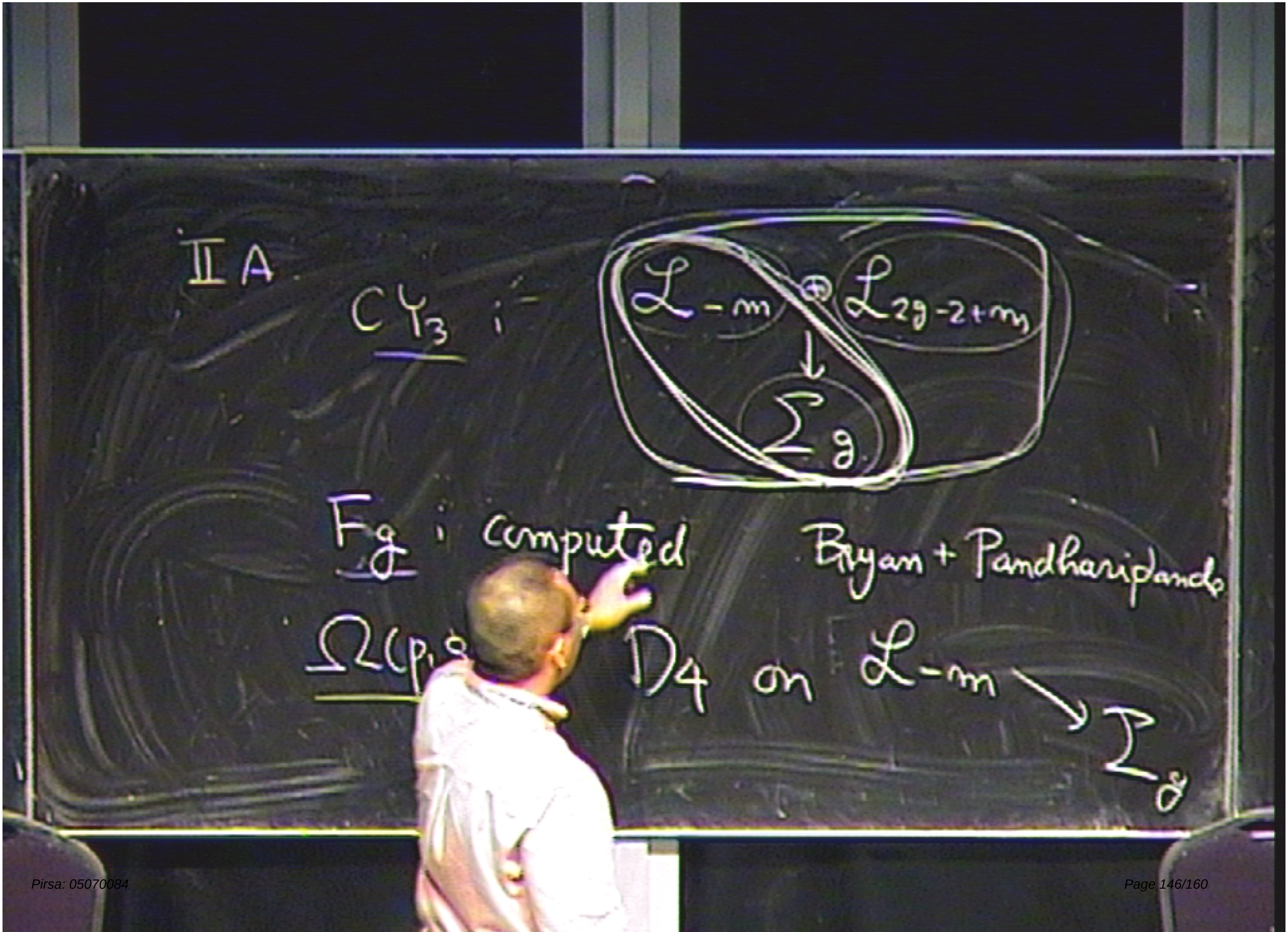
$\underline{CY_3}$: $\mathcal{L}_{-m} \oplus \mathcal{L}_{2g-2+m}$

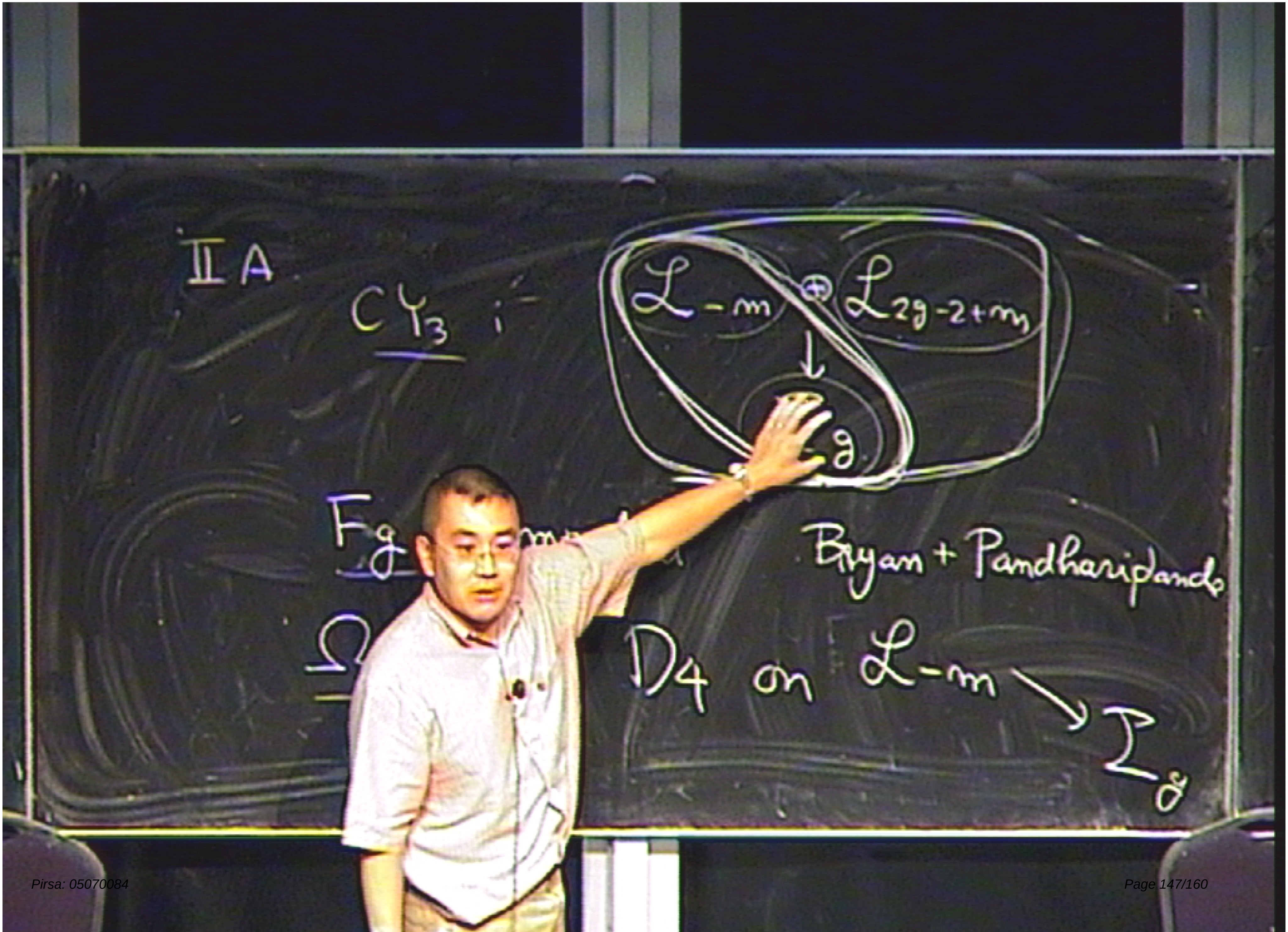
$\downarrow$

Σ_g

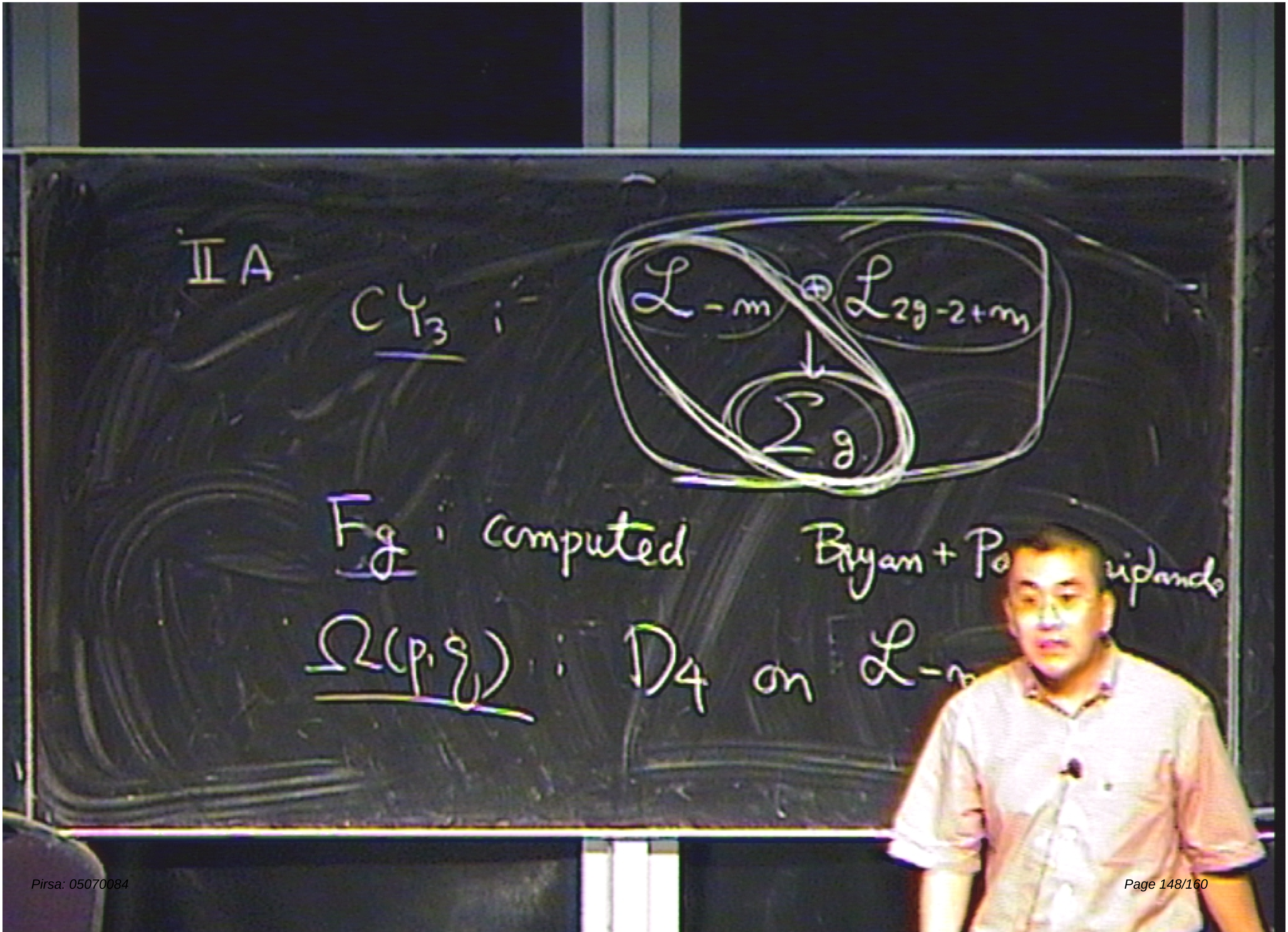
$\underline{F_g}$: computed Bryan + Pandharipande

$\underline{\Omega(p,q)}$: D4 on $\mathcal{L}_{-m} \rightarrow \Sigma_g$


II A
CY_3 : L_{-m} ⊕ L_{2g-2+m} → Σ_g
F_g : computed
Bryan + Pandharipande
D4 on L_{-m} → Σ_g



II A
CY3 :
L-m ⊕ L2g-2+m
Σg
Fg
Bryan + Pandharipande
D4 on L-m → Σg

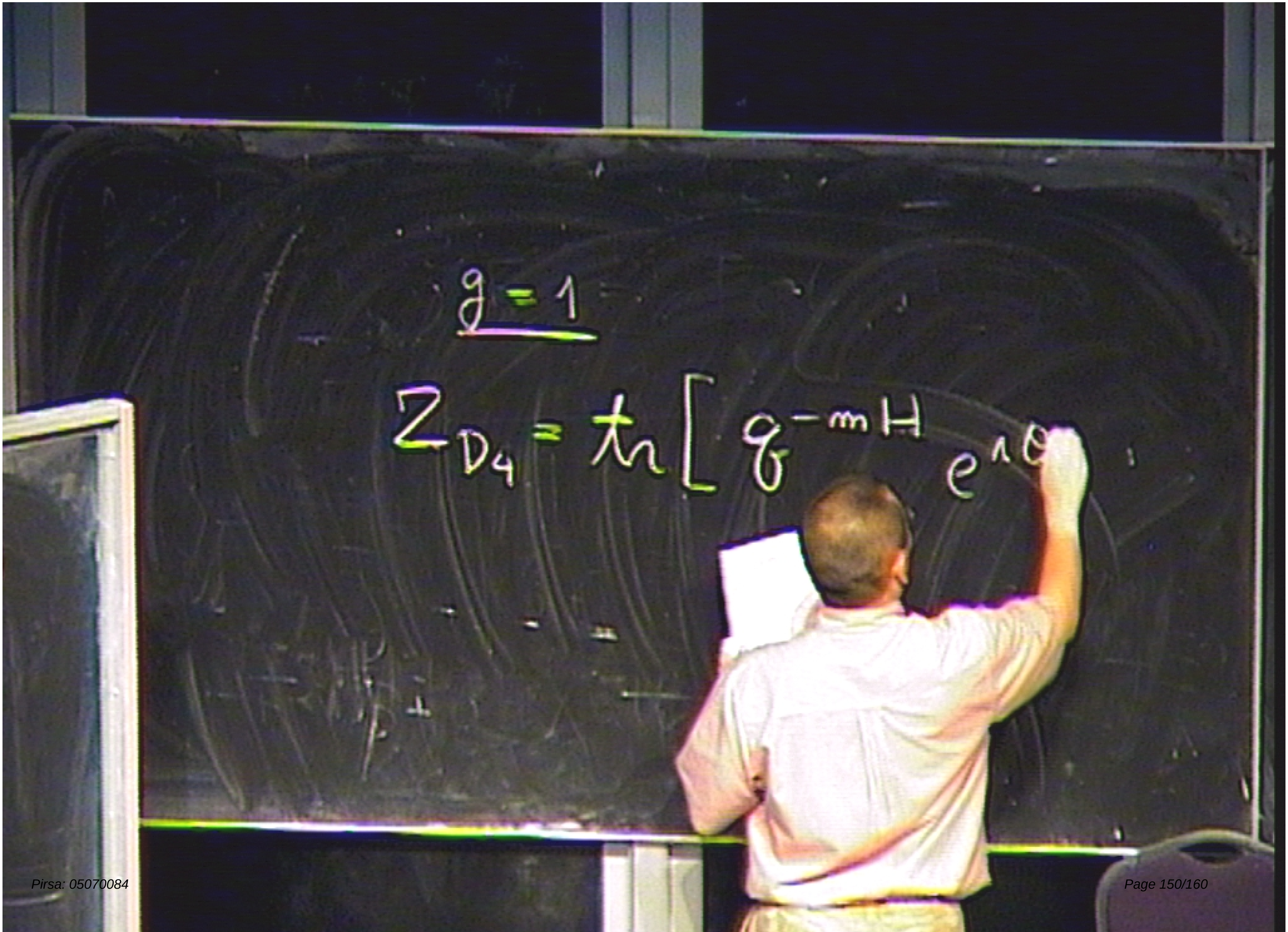

II A
CY3 : L-m ⊕ L2g-2+m
Σg
Fg : computed Bryan + Po
ipande
Ω(p,q) : D4 on L-

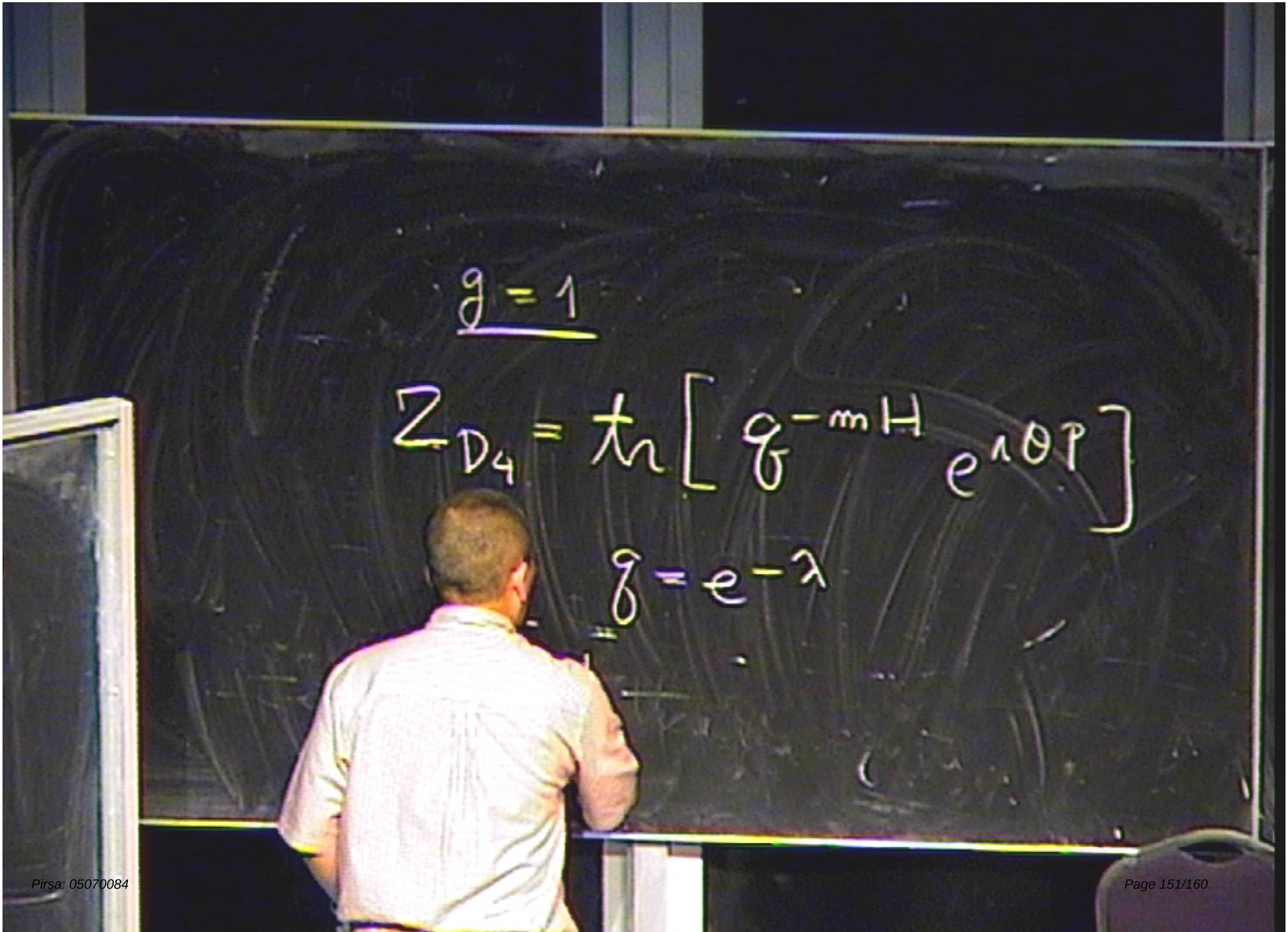
IIA

$$CY_3 : \quad \begin{array}{c} \mathcal{L}_{-m} \oplus \mathcal{L}_{2g-2+m} \\ \downarrow \\ \Sigma_g \end{array}$$

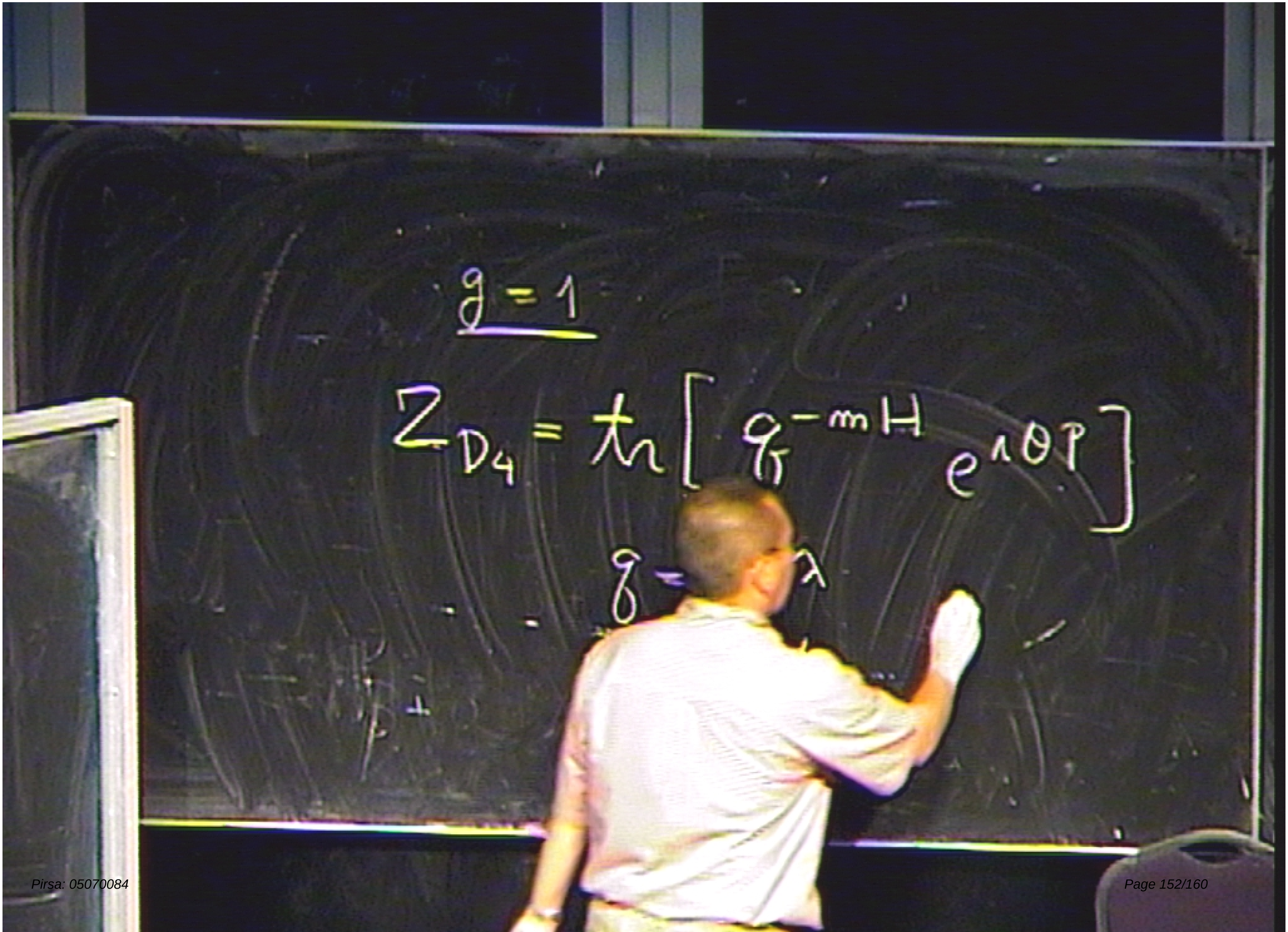
F_g : computed Bryan + Pandharipande

$\Omega(p,q)$: D4 on $\mathcal{L}_{-m} \to \Sigma_g$


g = 1
Z D4 = tr [q −mH e



$g = 1$
$Z_{D_4} = \mathrm{tr}\left[q^{-mH} e^{i\theta P} \right]$
$q = e^{-\lambda}$

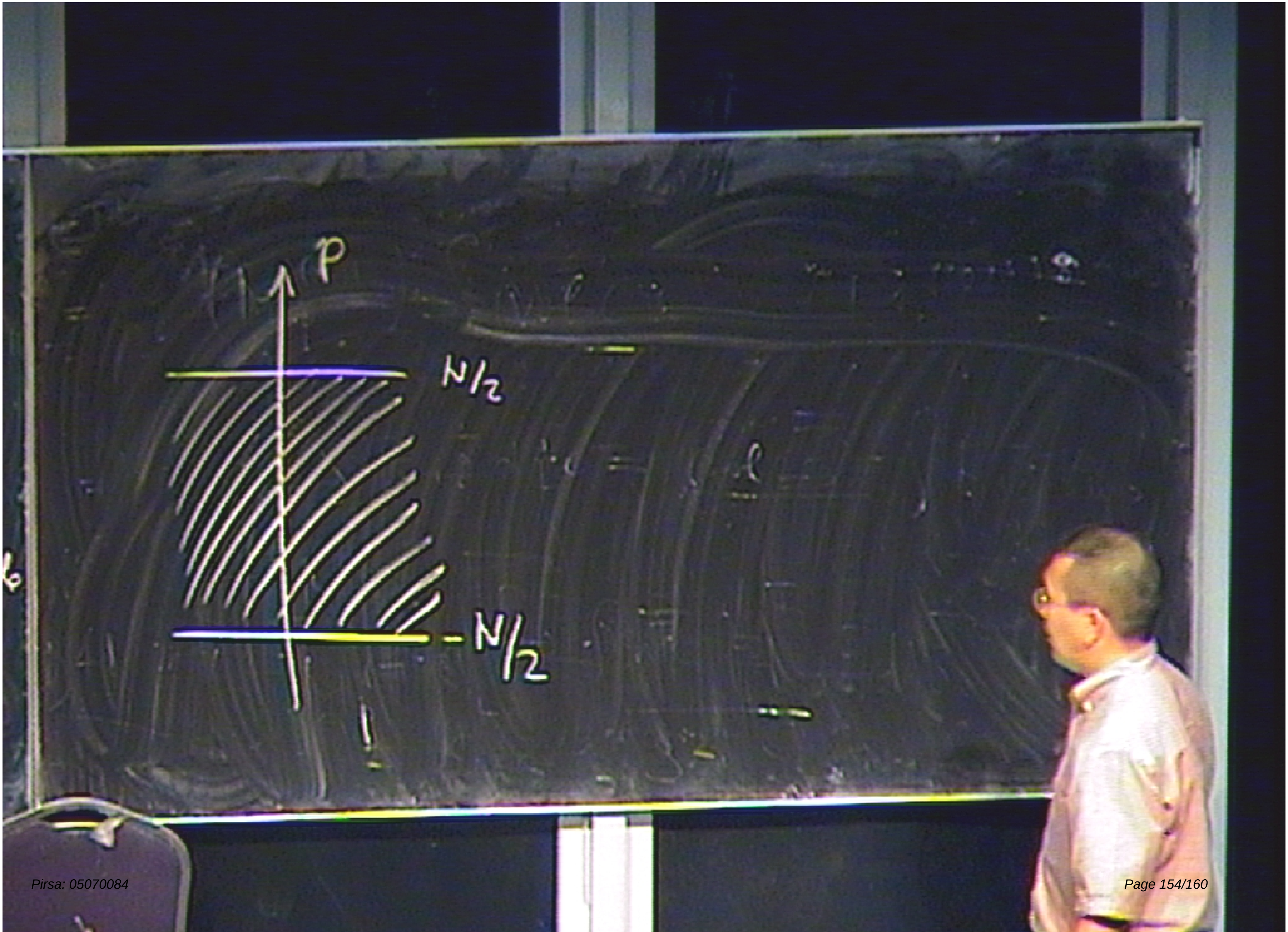

g = 1
Z_{D_4} = tr [q^{-mH} e^{iθP}]
q =

$\underline{g = 1}$

$$Z_{D_4} = \mathrm{tr}\left[q^{-mH}\, e^{i\theta P} \right]$$

$$q = e^{-\lambda} \qquad p_i = \pm\tfrac{1}{2}, \pm\tfrac{3}{2}, \dots$$

$$H = \frac{1}{2}\sum_{i=1}^{N} p_i \qquad N = \# D_4$$


P
N/2
-N/2

IIA

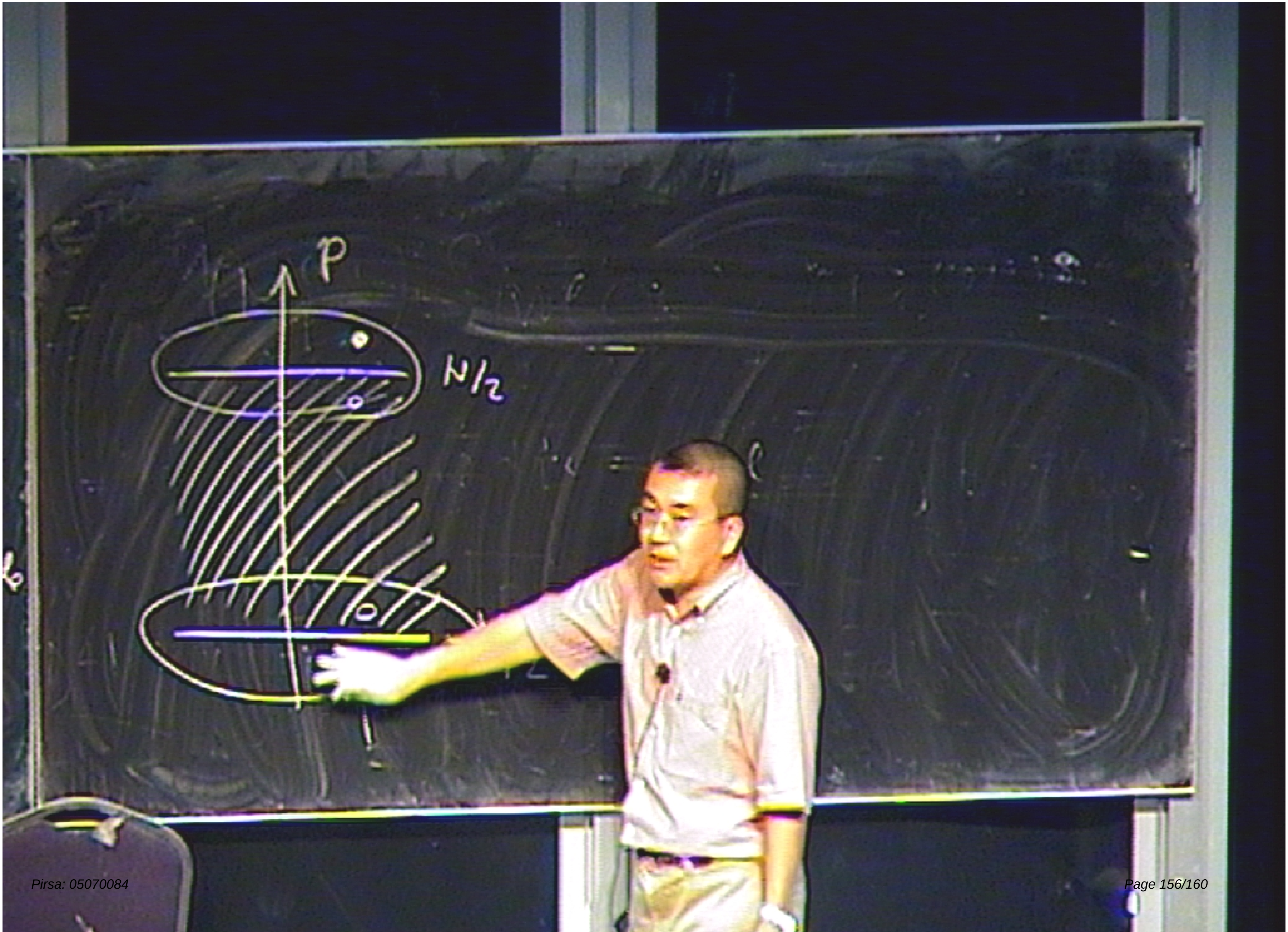
$\underline{CY_3}$: $\mathcal{L}_{-m} \oplus \mathcal{L}_{2g-2+m}$

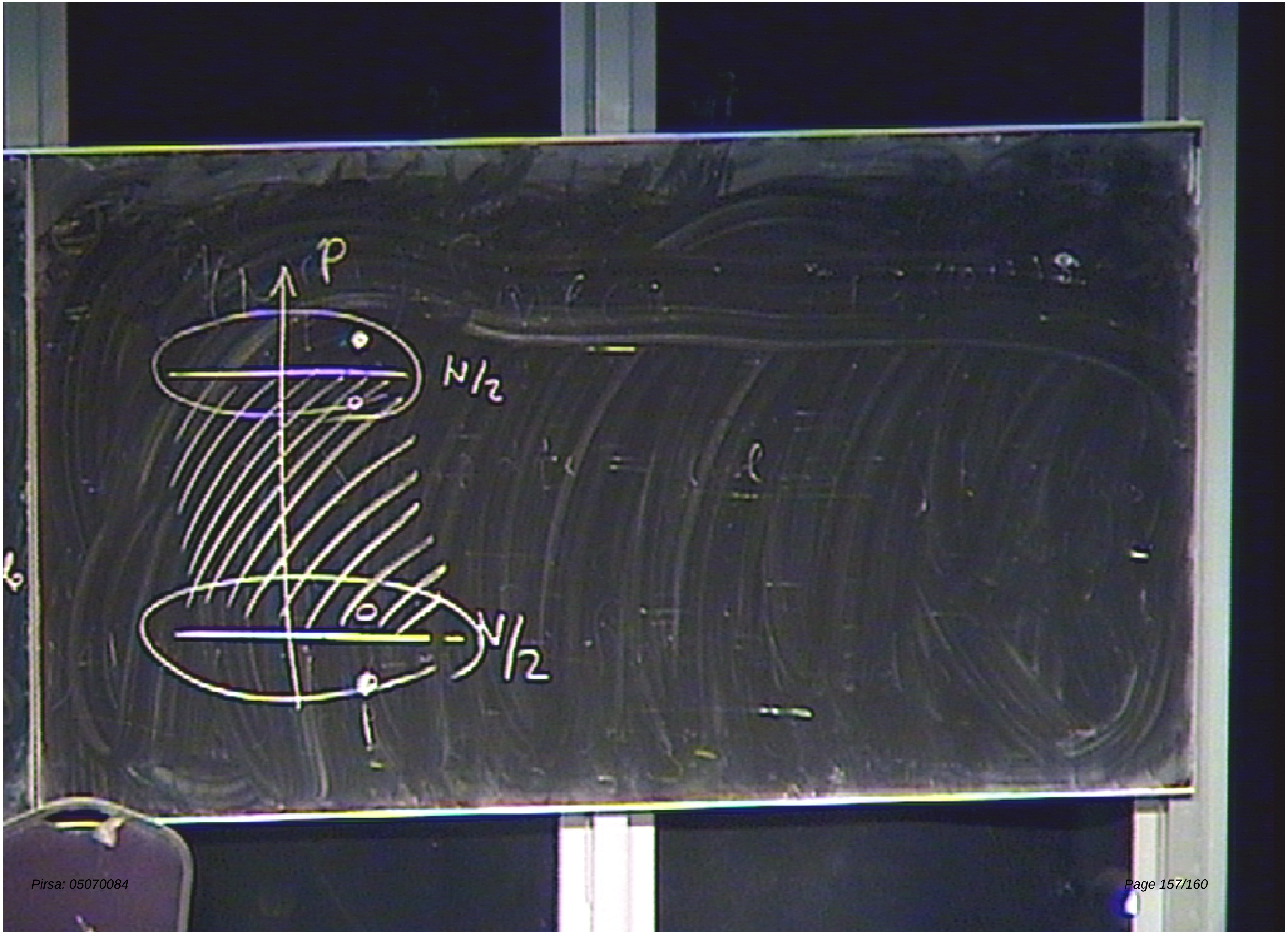
$\downarrow$

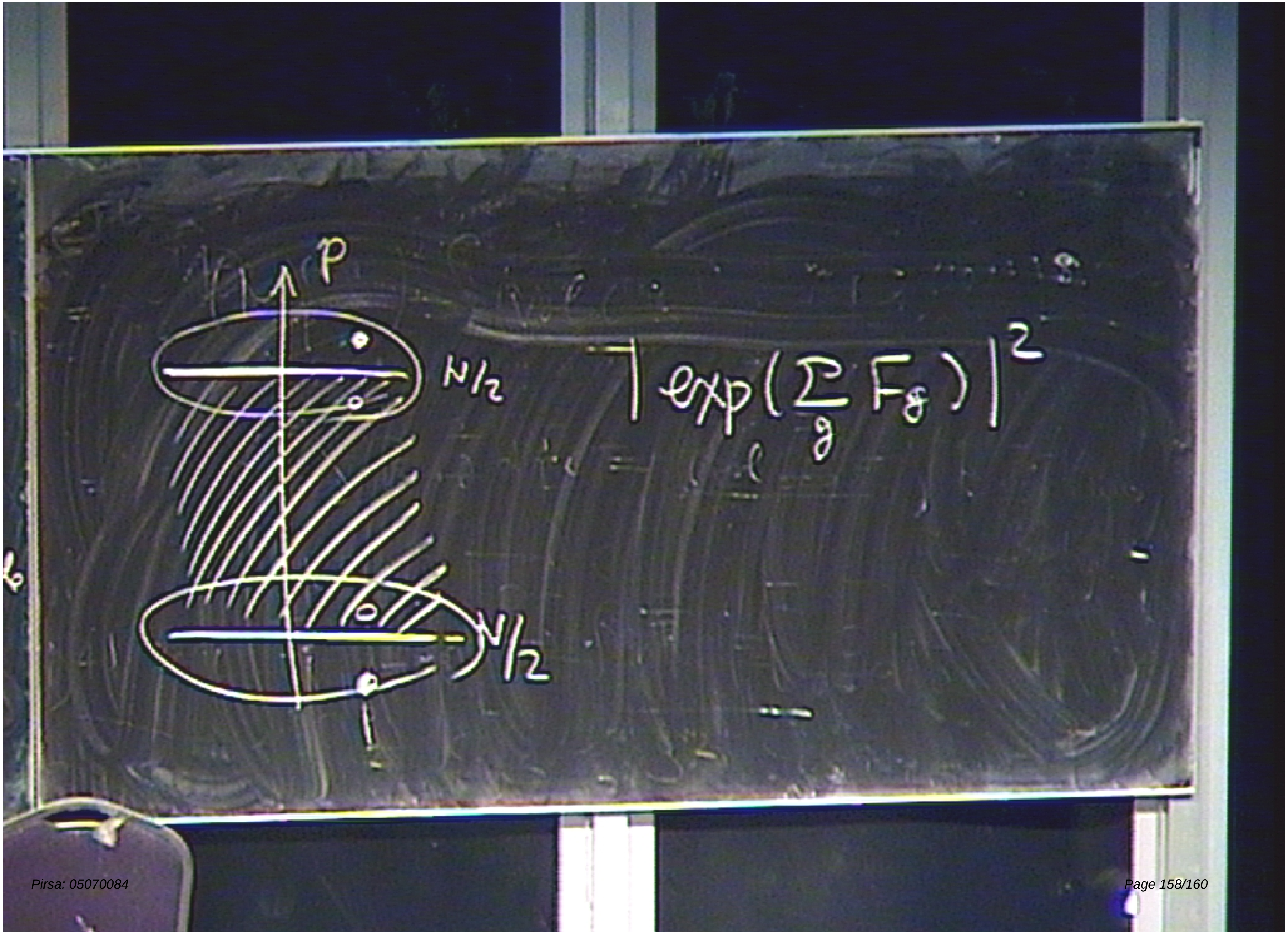
Σ_g

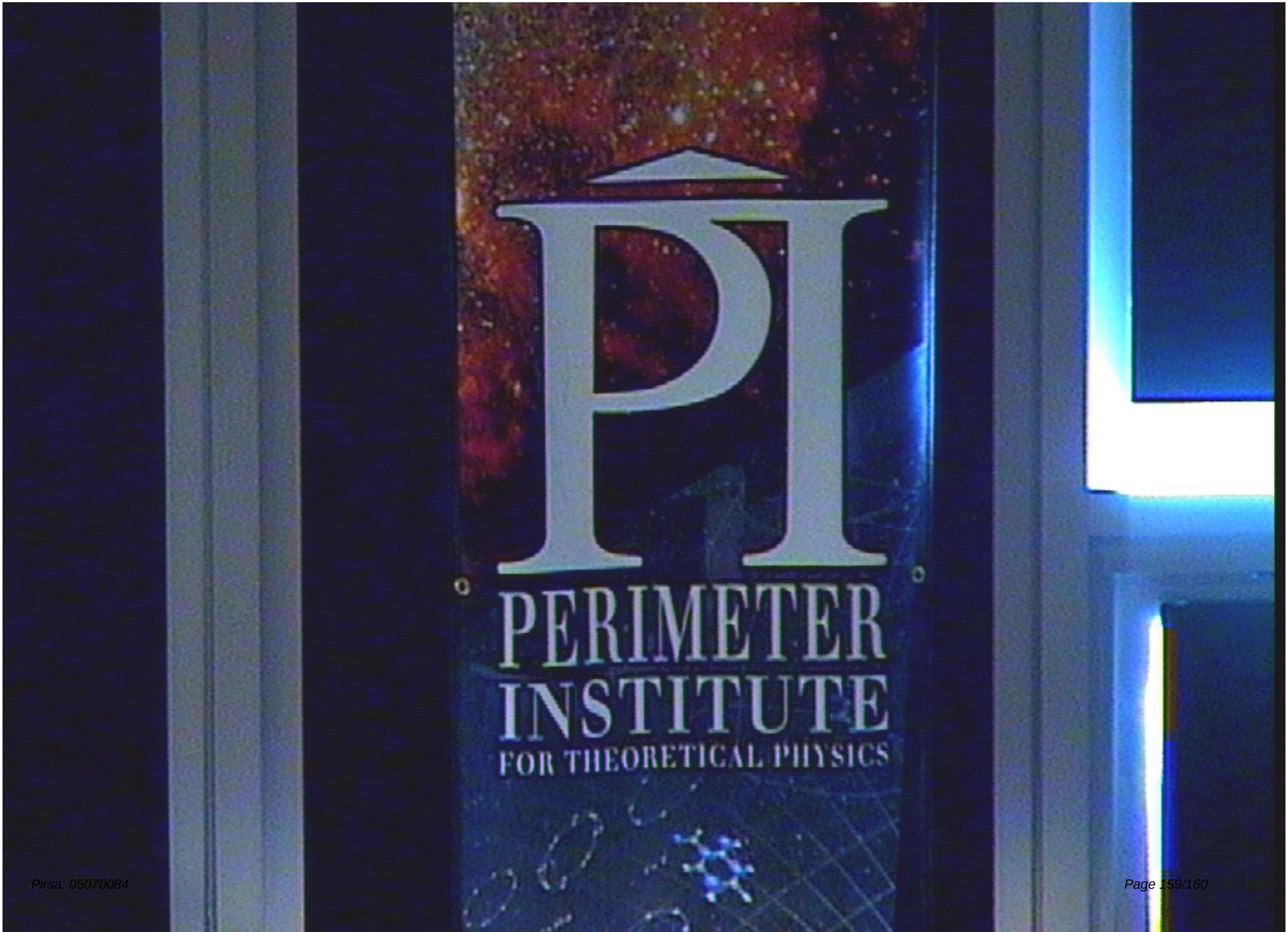
F_g : computed Bryan + Pandharipande

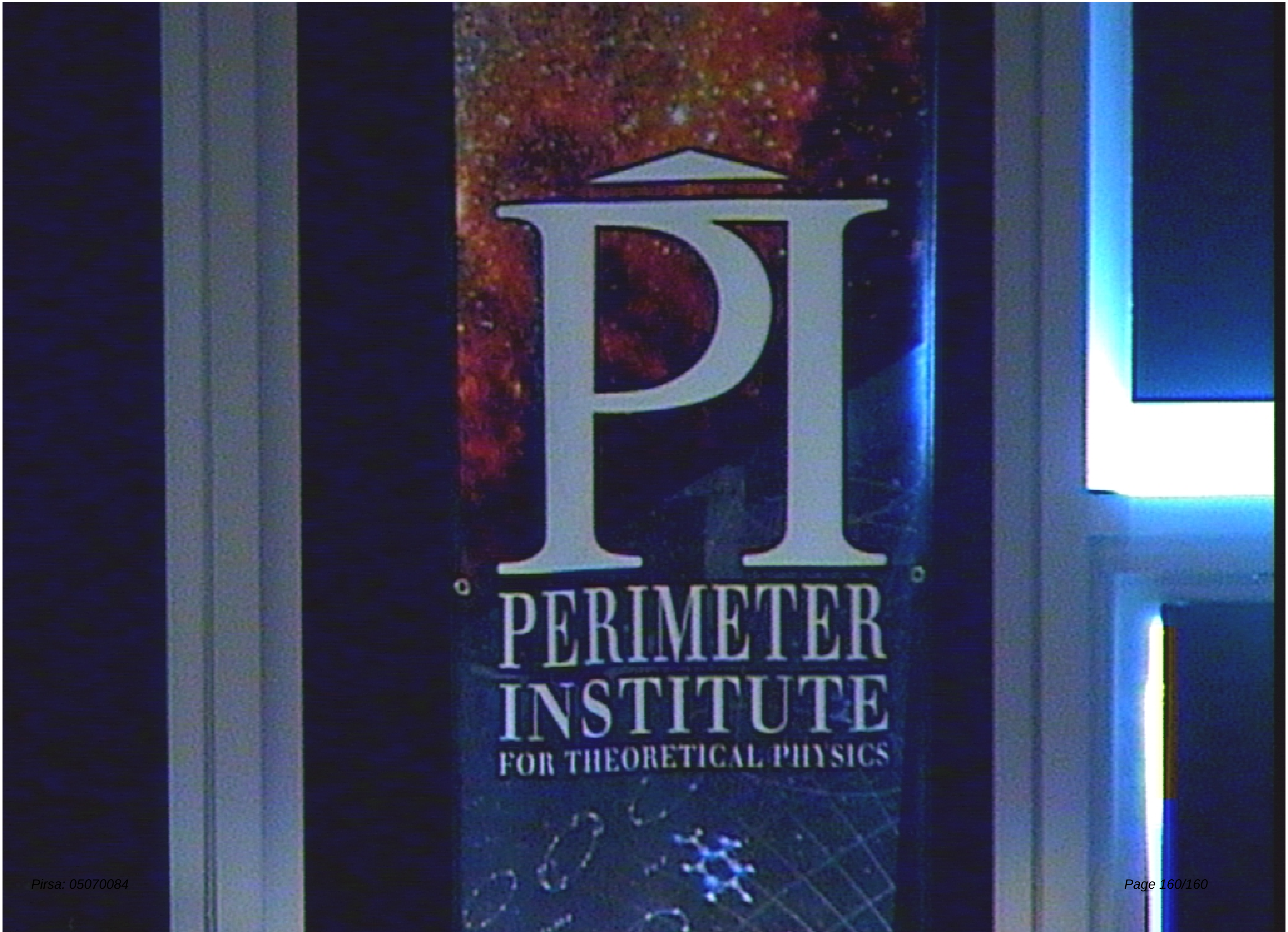
$\underline{\Omega(p,q)}$: $D4$ on $\mathcal{L}_{-m} \to \Sigma_g$


P
N/2


P
N/2
N/2


p
N/2
N/2
$|\exp(\sum_g F_g)|^2$


PI
PERIMETER
INSTITUTE
FOR THEORETICAL PHYSICS


PI
PERIMETER
INSTITUTE
FOR THEORETICAL PHYSICS