Title: Ashoke Sen (Harish Chandra Research Institute)

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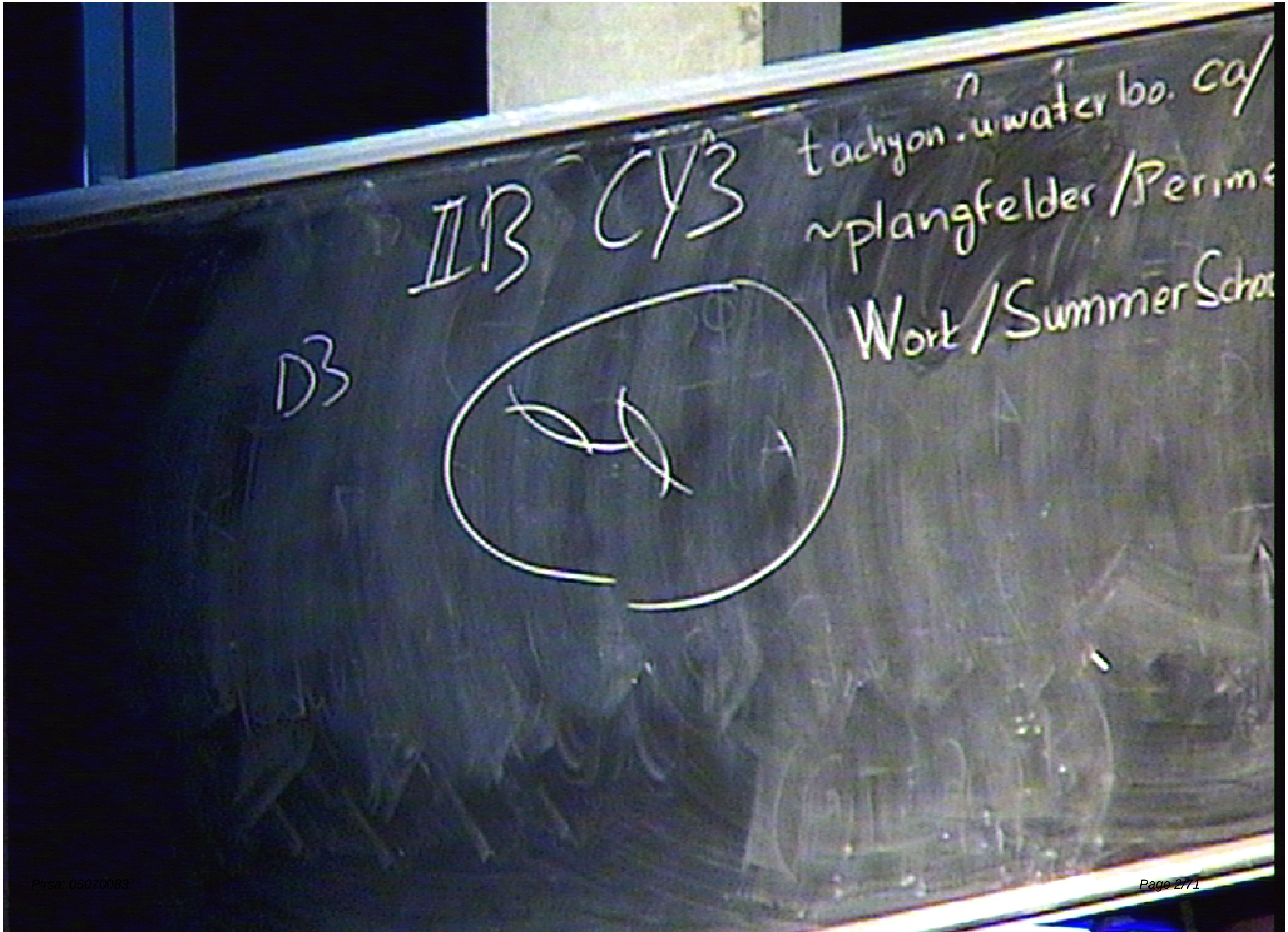
Abstract: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D3

$$T = \sqrt{G^{(10)}_{99}}$$

$$A^{(1)}_{\mu} = \frac{1}{2} B^{(10)}_{9\mu}$$

$$A^{(1)}_{\mu} = \frac{1}{2} \left(G^{(10)}_{99}\right)^{-1} G^{(10)}_{9\mu}$$

$$q_1 = \frac{1}{2} n , \quad q_2 = \frac{1}{2} \omega$$

$$g = 0 \, , \quad y_3 \frac{\partial g}{\partial y_3} = y_4 \frac{\partial g}{\partial y_4}$$

$$\frac{\partial g}{\partial y_1} = 0 \, , \quad \frac{\partial g}{\partial y_2} = 0$$

$$q_1 = u_5 u_7 \frac{\partial g}{\partial y_3}$$

$$q_2 = u_5 u_7 \frac{\partial g}{\partial y_4}$$

$$ds^2 = \vartheta_1\left(-r^2 dt^2 + \frac{dr^2}{r^2}\right) + \vartheta_2\, d\Omega^2_{D-2}$$

$$S = u_S, \quad T = u_T, \quad F^{(i)}_{rt} = e_i$$

$$y_1 = \vartheta_1, \quad y_2 = \vartheta_2, \quad y_3 = e_1 u_T, \quad y_4 = e_2 u_T^{-1}$$

$$f(u_S, u_T, \vartheta_1, \vartheta_2, e_1, e_2) = u_S\, g(y_1, y_2, y_3, y_4)$$

①

Summary of Results

D=4:

In a general higher derivative theory of gravity coupled to scalars ϕ_s and gauge fields $A_\mu^{(i)}$, the near horizon geometry of extremal black holes is:

$$ds^2 = v_1\left(-r^2dt^2 + \frac{dr^2}{r^2}\right) + v_2\left(d\theta^2 + \sin^2\theta d\phi^2\right)$$
$$\phi_s = u_s$$
$$F_{rt}^{(i)} = e_i, \qquad F_{\theta\phi}^{(i)} = \frac{p_i}{4\pi}\sin\theta\,.$$

For this background:

$$R_{\alpha\beta\gamma\delta} = -v_1^{-1}(g_{\alpha\gamma}g_{\beta\delta} - g_{\alpha\delta}g_{\beta\gamma}), \qquad \alpha,\beta,\gamma,\delta = r,t$$
$$R_{mnpq} = v_2^{-1}(g_{mp}g_{nq} - g_{mq}g_{np}), \qquad m,n,p,q = \theta,\phi$$

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②

Let $\sqrt{-\det g}\,\mathcal{L}$ be the Lagrangian density.

Define:

$$f(\vec{u},\vec{v},\vec{e},\vec{p}) \equiv \int d\theta\, d\phi \sqrt{-\det g}\,\mathcal{L}$$

$$F(\vec{u},\vec{v},\vec{e},\vec{q},\vec{p}) \equiv 2\pi(e_i\, q_i - f(\vec{u},\vec{v},\vec{e},\vec{p}))$$

For an extremal black hole of electric charge $\vec{q}$ and magnetic charge $\vec{p}$,

1. the values of $\{u_s\}$, $\{e_i\}$, v_1 and v_2 are obtained by solving

$$\frac{\partial F}{\partial u_s} = 0, \quad \frac{\partial F}{\partial v_1} = 0, \quad \frac{\partial F}{\partial v_2} = 0, \quad \frac{\partial F}{\partial e_i} = 0.$$

2. the black hole entropy S_{BH} is given by

$$S_{BH} = F(\vec{u},\vec{v},\vec{e},\vec{q},\vec{p})$$

at the extremum of F with respect to $\vec{u}$, $\vec{v}$, $\vec{e}$.

Similar results hold for higher dimensional black holes.

Generalization to $D > 4$:

Consider a theory of gravity coupled to scalars ϕ_s and anti-symmetric tensor gauge fields of different rank.

We define a spherically symmetric extremal black hole to have

1. near horizon geometry $AdS_2 \times S^{D-2}$
2. all background fields invariant under the $SO(2,1) \times SO(D-1)$ symmetry of $AdS_2 \times S^{D-2}$.

General near horizon field configuration:

$$ds^2 \equiv g_{\mu\nu}dx^\mu dx^\nu = v_1\left(-r^2dt^2 + \frac{dr^2}{r^2}\right) + v_2\, d\Omega_{D-2}^2$$

$$\phi_s = u_s\,, \qquad F^{(i)}_{rt} = e_i,$$

$$H^{(a)}_{l_1\cdots l_{D-2}} = p_a\, \epsilon_{l_1\cdots l_{D-2}} \sqrt{\det h^{(D-2)}}\,/\Omega_{D-2}\,.$$

$d\Omega_{D-2}^2 = h^{(D-2)}_{ll'}dx^l dx^{l'}$: the line element on the unit $(D-2)$-sphere

$H^{(a)}_{\mu_1\cdots\mu_{D-2}}$: field strength associated with the antisymmetric tensor $(D-3)$-form fields

Ω_{D-2} : the area of the unit $(D-2)$-sphere.

x^{l_i} with $2 \le l_i \le (D-1)$: coordinates along this sphere.

ϵ is the totally anti-symmetric symbol with

$$\epsilon_{2\ldots(D-1)} = 1$$

(24)

The analysis proceeds in a manner identical to the D=4 case.

Define $f(\vec{u}, \vec{v}, \vec{e}, \vec{p}) = \int_{S^{D-2}} \sqrt{-\det g}\, \mathcal{L}$.

$F(\vec{u}, \vec{v}, \vec{q}, \vec{p}) \equiv 2\pi \times$ the Legendre transform of f with respect to $\vec{e}$.

1. $\vec{u}$, $\vec{v}$ are obtained by extremizing F.

2. $S_{BH} = F$ at this extremum.

Assumption: The part of the lagrangian which is relevant for the construction of the black hole solution depends only on the field strengths and not on the gauge fields.

i.e. Chern-Simons terms, if present, do not affect the black hole solution.

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③

Application to computation of the entropy of elementary string excitations:

Consider heterotic string on $\mathcal{M} \times S^1$

$\mathcal{M}$: Some manifold of dimension ≤ 5, such that the resulting theory has $\mathcal{N} \geq 2$ supersymmetry.

Example of $\mathcal{M}$: T^n, $K3$, $K3 \times S^1$, SUSY preserving orbifolds of these, $\cdots$.

Consider a fundamental heterotic string wound multiple (w) times along S^1.

Excitations of the string: Left and right-moving oscillation modes circulating along the string.

24 independent left-moving modes and 8 (+8) independent right-moving modes.

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4

BPS states: involve excitation of left-moving modes only.

→ carry a net momentum along S^1 which is quantized in units of $1/R$.

R: radius of S^1.

Consider a BPS state that winds w times along S^1 and carries total momentm n/R along S^1.

Number of such states: The number of ways we can distribute the total momentum n/R among individual modes.

Result for large n and w:

$$d(n,w) \sim \exp(4\pi\sqrt{nw})$$

→ we associate a statistical entropy to this system:

$$S_{stat} = \ln d(n,w) \simeq 4\pi\sqrt{nw}$$

5

For large n and w this state has large mass.

→ might be described as a black hole.

(a solution to the equations of motion derived from the effective action of string theory)

→ we can associate a Bekenstein-Hawking entropy S_{BH} to this black hole.

Question: Is

$$S_{BH}=S_{stat}?$$

If true, this would provide a microscopic explanation of the black hole entropy.

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HORIZON IS SINGULAR.

This seems to be bad news!

However a look at the solution shows that

1. String coupling is small at the horizon.

2. Higher derivative correction terms are large at the horizon.

→ can ignore string loop corrections but cannot ignore the tree level higher derivative corrections to the effective action.

Goal:

1. Construct the effective field theory describing heterotic string on $\mathcal{M} \times S^1$, keeping tree level higher derivative terms.

2. Calculate the black hole entropy in this theory using the techniques described earlier.

Convention:

$$\hbar = 1, \qquad c = 1, \qquad \alpha' = 16$$

x^9: the coordinate along S^1

x^9 is periodic with period $2\pi\sqrt{\alpha'} = 8\pi$.

Define D-dimensional fields in terms of the ten dimensional fields $G^{(10)}_{MN}$, $B^{(10)}_{MN}$ and $\Phi^{(10)}$:

$$\Phi = \Phi^{(10)} - \frac{1}{4}\ln(G^{(10)}_{99}),$$

$$S = e^{-2\Phi}, \qquad T = \sqrt{G^{(10)}_{99}},$$

$$G_{\mu\nu} = G^{(10)}_{\mu\nu} - (G^{(10)}_{99})^{-1} G^{(10)}_{9\mu} G^{(10)}_{9\nu},$$

$$A^{(1)}_{\mu} = \frac{1}{2}(G^{(10)}_{99})^{-1} G^{(10)}_{9\mu},$$

$$A^{(2)}_{\mu} = \frac{1}{2} B^{(10)}_{9\mu},$$

$G_{\mu\nu}$: string metric

⑧

$$S = \frac{1}{32\pi}\int d^D x \sqrt{-\det G}\, S\, [R_G + S^{-2} G^{\mu\nu}\, \partial_\mu S \partial_\nu S$$
$$- T^{-2} G^{\mu\nu}\, \partial_\mu T \partial_\nu T - T^2 G^{\mu\nu} G^{\mu'\nu'} F^{(1)}_{\mu\mu'} F^{(1)}_{\nu\nu'}$$
$$- T^{-2} G^{\mu\nu} G^{\mu'\nu'} F^{(2)}_{\mu\mu'} F^{(2)}_{\nu\nu'}]$$

$+$ higher derivative terms

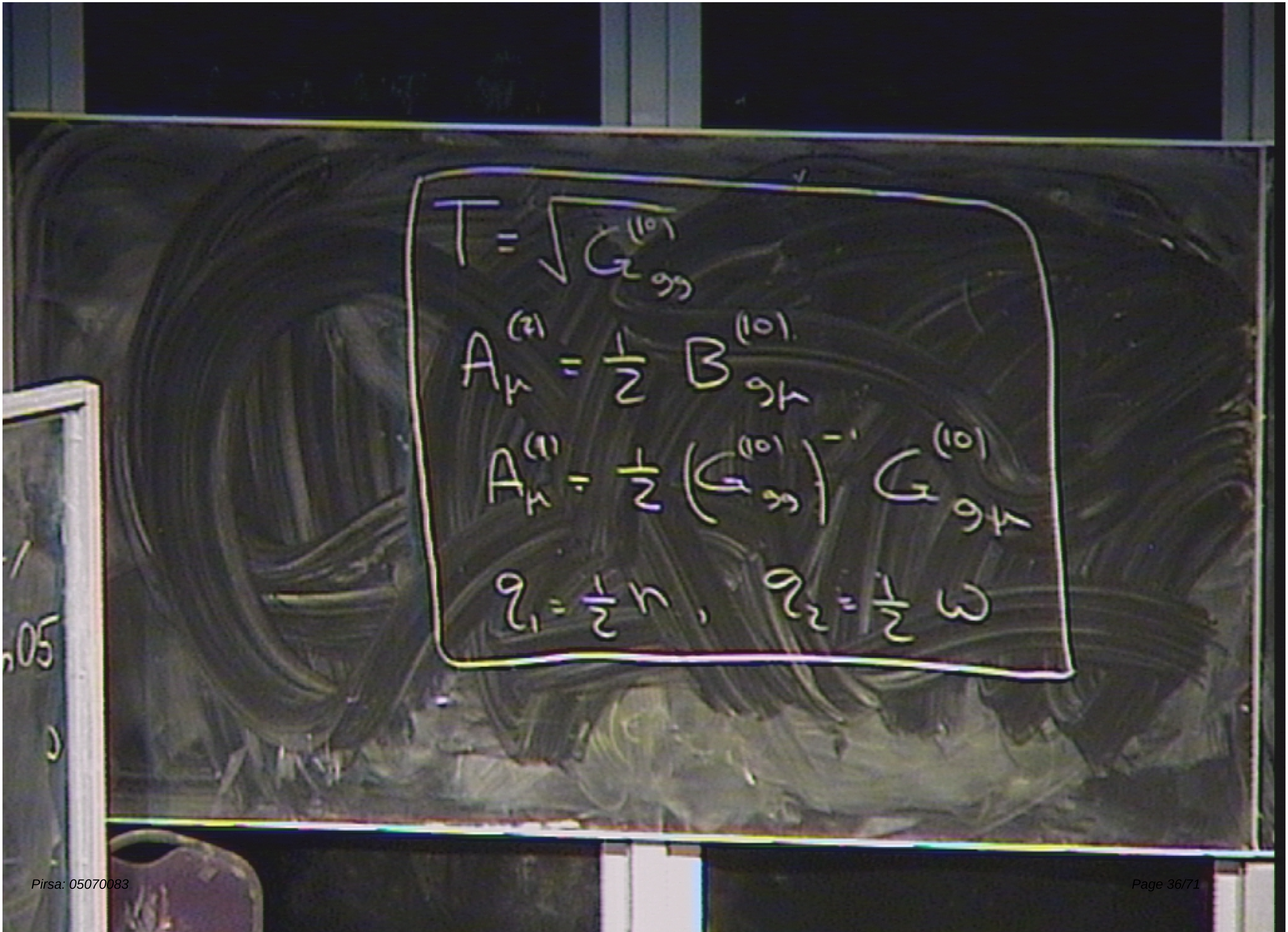
Consider a black hole with charge quantum numbers (n, w).

$A^{(1)}_\mu$ couples to n

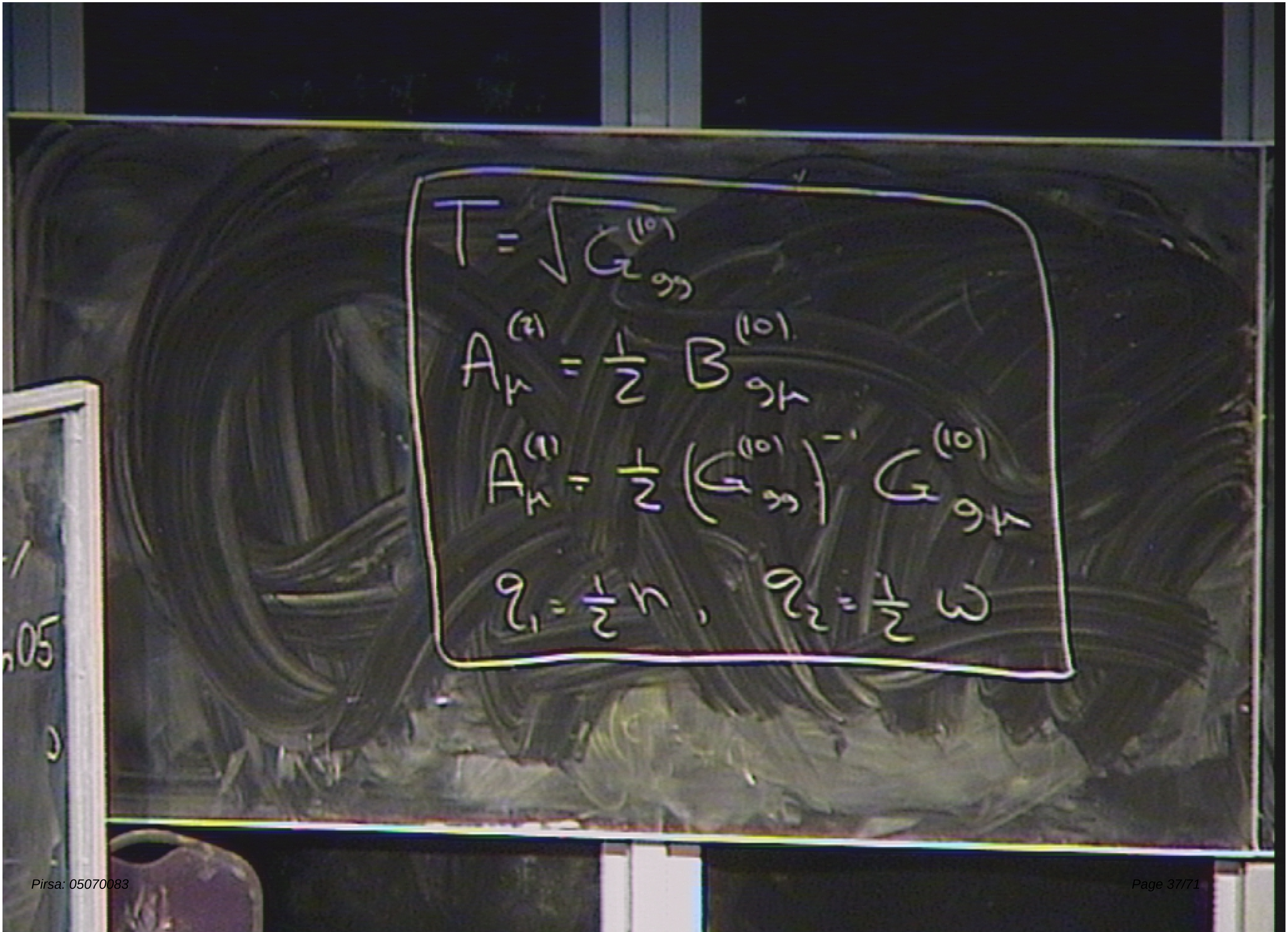
$A^{(2)}_\mu$ couples to w.

Precise normalization:

$$q_1 = \frac{1}{2} n, \qquad q_2 = \frac{1}{2} w.$$



$T = \sqrt{G^{(10)}_{99}}$
$A^{(7)}_{\mu} = \frac{1}{2} B^{(10)}_{9\mu}$
$A^{(7)}_{\mu} = \frac{1}{2} \left(G^{(10)}_{99}\right)^{-1} G^{(10)}_{9\mu}$
$q_1 = \frac{1}{2} n , \quad q_2 = \frac{1}{2} \omega$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$$S = u_s, \quad T = u_T, \quad F^{(A)}_{rt} = e_A$$

$$y_1 = v_1, \quad y_2 = v_2, \quad y_3 = e_1 u_T, \quad y_4 = e_2 u_T^{-1}$$

$$f(u_s, u_T, v_1, v_2, e_1, e_2) = u_s\, g(y_1, y_2, y_3, y_4)$$

8

$$S = \frac{1}{32\pi}\int d^Dx\sqrt{-\det G}\,S\,[R_G + S^{-2}G^{\mu\nu}\,\partial_\mu S\partial_\nu S$$
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Precise normalization:

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⑨

Look for a near horizon solution of the form:

$$ds^2 = v_1\left(-r^2dt^2 + \frac{dr^2}{r^2}\right) + v_2 d\Omega_{D-2}^2,$$

$$S = u_S, \qquad T = u_T$$

$$F_{rt}^{(i)} = e_i$$

Note: The solution we are looking for does not carry any magnetic charge.

Define:

$$f(u_S, u_T, v_1, v_2, e_1, e_2) = \int d^{D-2}\Omega\,\sqrt{-\det G}\,\mathcal{L}.$$

If we use string metric then $\mathcal{L}$ has an overall factor of S and hence f has the form:

$$f = u_S\,h(u_T, v_1, v_2, e_1, e_2)$$

for some function h.

Also $\mathcal{L}$ is invariant under:

$$T \to e^{\beta} T, \quad A_\mu^{(1)} \to e^{-\beta} A_\mu^{(1)}, \quad A_\mu^{(2)} \to e^{\beta} A_\mu^{(2)}$$

(A consequence of $x^9 \to e^{-\beta} x^9$ transformation:

$$G_{99}^{(10)} \to e^{2\beta} G_{99}^{(10)}, \quad G_{9\mu}^{(10)} \to e^{\beta} G_{9\mu}^{(10)},$$
$$B_{9\mu}^{(10)} \to e^{\beta} B_{9\mu}^{(10)}.)$$

$\to$

$$\begin{aligned} & f(u_S, e^{\beta} u_T, v_1, v_2, e^{-\beta} e_1, e^{\beta} e_2) \\ = \; & f(u_S, u_T, v_1, v_2, e_1, e_2) \end{aligned}$$

Combined effect:

$$f(u_S, u_T, v_1, v_2, e_1, e_2) = u_S\, g(v_1, v_2, u_T\, e_1, u_T^{-1}\, e_2)$$

for some function g.

(11)

$$f(u_S, u_T, v_1, v_2, e_1, e_2) = u_S\, g(y_1, y_2, y_3, y_4)$$

$$y_1 \equiv v_1, \quad y_2 = v_2, \quad y_3 = e_1\, u_T, \quad y_4 = e_2\, u_T^{-1}$$

Equations for $u_S, u_T, v_1, v_2, e_1, e_2$:

$$\frac{\partial f}{\partial u_S} = 0 \quad \rightarrow \quad g(y_1, y_2, y_3, y_4) = 0$$

$$\frac{\partial f}{\partial u_T} = 0 \quad \rightarrow \quad y_3 \frac{\partial g}{\partial y_3} = y_4 \frac{\partial g}{\partial y_4}$$

$$\frac{\partial f}{\partial v_1} = 0 \quad \rightarrow \quad \frac{\partial g}{\partial y_1} = 0$$

$$\frac{\partial f}{\partial v_2} = 0 \quad \rightarrow \quad \frac{\partial g}{\partial y_2} = 0$$

$$q_1 = \frac{\partial f}{\partial e_1} = u_S\, u_T\, \frac{\partial g}{\partial y_3}$$

$$q_2 = \frac{\partial f}{\partial e_2} = u_S\, u_T^{-1}\, \frac{\partial g}{\partial y_4}$$

$$g(y_1, y_2, y_3, y_4) = 0, \quad y_3 \frac{\partial g}{\partial y_3} = y_4 \frac{\partial g}{\partial y_4},$$

$$\frac{\partial g}{\partial y_1} = 0, \quad \frac{\partial g}{\partial y_2} = 0$$

Solving these equations we get universal (q_i independent) values of y_1, y_2, y_3, y_4.

The equations

$$q_1 = u_S\, u_T \frac{\partial g}{\partial y_3}, \quad q_2 = u_S\, u_T^{-1} \frac{\partial g}{\partial y_4}$$

now give:

$$u_S = \sqrt{q_1 q_2} \Big/ \sqrt{\frac{\partial g}{\partial y_3}\frac{\partial g}{\partial y_4}}$$

$$u_T = \sqrt{(q_1/q_2)\, \frac{\partial g}{\partial y_4} \Big/ \frac{\partial g}{\partial y_3}} = \sqrt{(q_1/q_2)\,(y_3/y_4)}$$

$y_3 = e_1\, u_T,$	$y_4 = e_2\, u_T^{-1}$
$g(y_1, y_2; y_3, y_4) = 0$	
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The entropy is given by:

$$\begin{aligned} S_{BH} &= 2\pi(e_1 q_1 + e_2 q_2 - u_S\, g) \\ &= 2\pi\left(y_3\, u_T^{-1}\, q_1 + y_4\, u_T\, q_2\right) \\ &= 4\pi\sqrt{q_1 q_2}\,\sqrt{y_3 y_4} = 2\pi\sqrt{nw}\,\sqrt{y_3 y_4}\,. \end{aligned}$$

Since y_i are universal,

$$S_{BH} = 4\pi\, K\, \sqrt{nw}$$

K: a universal number

Also note: Since $u_S \propto \sqrt{nw}$ is large for large n, w, string coupling at the horizon is small.

$y_3 = e_1 u_T,$	$y_4 = e_2 u_T^{-1}$
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(14)

$$S_{BH} = 4\pi K \sqrt{nw}$$

$$S_{stat} = 4\pi \sqrt{nw}$$

S_{BH} and S_{stat} have the same dependence on n and w.

A-S. ; PEET (1995)

Can we calculate K?

For this we need to know $\mathcal{L}$ to all orders in α'.

For $D = 4$, K has been calculated by taking into account a special class of higher derivative terms which come from the supersymmetrization of the curvature squared terms.

Cardoso, de Wit, Mohaupt

Result: $K = 1$

Dabholkar

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We shall leave this calculation as an exercise:

1. Calculate the function f after taking into account the correction terms arising out of supersymmetrization of curvature squared terms.

These correction terms can be found in hep-th/0007195

The translation from the variables used in hep-th/0007195 to the variables used here can be found in hep-th/0411255.

2. Using f, calculate the entropy function F, extremize it, and evaluate its value at the extremum.

Show that the result is $4\pi\sqrt{nw}$.

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We shall illustrate the method by considering a different truncation of higher derivative corrections for general D.

Begin with the supergravity approximation to the effective action.

$$S = \frac{1}{32\pi}\int d^D x \sqrt{-\det G}\, S\, [R_G + S^{-2} G^{\mu\nu}\, \partial_\mu S \partial_\nu S - T^{-2} G^{\mu\nu}\, \partial_\mu T \partial_\nu T - T^2 G^{\mu\nu} G^{\mu'\nu'} F^{(1)}_{\mu\mu'} F^{(1)}_{\nu\nu'} - T^{-2} G^{\mu\nu} G^{\mu'\nu'} F^{(2)}_{\mu\mu'} F^{(2)}_{\nu\nu'}],$$

This gives

$$f(v_1, v_2, u_S, u_T, e_1, e_2) = \frac{\Omega_{D-2}}{32\pi} v_1\, v_2^{(D-2)/2}\, u_S \left[-\frac{2}{v_1} + \frac{(D-2)(D-3)}{v_2} + \frac{2\, u_T^2 e_1^2}{v_1^2} + \frac{2\, e_2^2}{u_T^2 v_1^2}\right]$$

Ω_{D-2}: volume of unit $(D-2)$-sphere.

$\partial f/\partial u_S = 0$ and $\partial f/\partial v_2 = 0$ has no solution.

We shall illustrate the method by considering a different truncation of higher derivative corrections for general D.

Begin with the supergravity approximation to the effective action.

$$S = \frac{1}{32\pi}\int d^D x\sqrt{-\det G}\, S\,[R_G + S^{-2}G^{\mu\nu}\,\partial_\mu S\partial_\nu S - T^{-2}G^{\mu\nu}\,\partial_\mu T\partial_\nu T - T^2 G^{\mu\nu}G^{\mu'\nu'}F^{(1)}_{\mu\mu'}F^{(1)}_{\nu\nu'} - T^{-2}G^{\mu\nu}G^{\mu'\nu'}F^{(2)}_{\mu\mu'}F^{(2)}_{\nu\nu'}]\,,$$

This gives

$$f(v_1,v_2,u_S,u_T,e_1,e_2) = \frac{\Omega_{D-2}}{32\pi}v_1\,v_2^{(D-2)/2}\,u_S \Big[-\frac{2}{v_1}+\frac{(D-2)(D-3)}{v_2} + \frac{2\,u_T^2 e_1^2}{v_1^2}+\frac{2\,e_2^2}{u_T^2 v_1^2}\Big]$$

Ω_{D-2}: volume of unit $(D-2)$-sphere.

$\partial f/\partial u_S = 0$ and $\partial f/\partial v_2 = 0$ has no solution.

Add to $\mathcal{L}$ the Gauss-Bonnet combination:

$$\Delta\mathcal{L} = C\frac{S}{16\pi}\left\{R_{G\mu\nu\rho\sigma}R_G^{\mu\nu\rho\sigma} - 4R_{G\mu\nu}R_G^{\mu\nu} + R_G^2\right\}$$

For heterotic string theory $C = 1$.

→ additional contribution to f:

$$\Delta f = \frac{\Omega_{D-2}}{32\pi} v_1 v_2^{(D-2)/2} u_S \left[\frac{2C}{v_2^2}(D-2)(D-3)(D-4)(D-5) - \frac{8C}{v_1 v_2}(D-2)(D-3)\right]$$

(18)

Now solve

$$\frac{\partial f}{\partial v_1} = \frac{\partial f}{\partial v_2} = \frac{\partial f}{\partial u_S} = \frac{\partial f}{\partial u_T} = 0,$$

$$\frac{\partial f}{\partial e_1} = q_1 = \frac{n}{2}, \quad \frac{\partial f}{\partial e_2} = q_2 = \frac{w}{2}.$$

A solution exists.

$$\begin{aligned}
v_2 &= 4C\left[(D-2)(D-3)-(D-4)(D-5)\right], \\
v_1 &= \frac{2v_2}{(D-2)(D-3)}, \\
\tilde{u}_S &\equiv \frac{\Omega_{D-2}}{32\pi} v_1 v_2^{(D-2)/2} u_S \\
&= \left[\frac{16}{v_1^3 v_2}(v_2 + 4C(D-2)(D-3))\right]^{-1/2} \sqrt{nw}, \\
u_T &= \sqrt{\frac{n}{w}}, \\
e_1 &= \frac{v_1^2}{8\tilde{u}_S}\sqrt{\frac{w}{n}}, \qquad e_2 = \frac{v_1^2}{8\tilde{u}_S}\sqrt{\frac{n}{w}}.
\end{aligned}$$

$$
\begin{aligned}
S_{BH} &= 2\pi(q_1 e_1 + q_2 e_2 - f) \\
&= 4\pi\sqrt{nw}\sqrt{1 - \frac{(D-4)(D-5)}{2(D-2)(D-3)}}
\end{aligned}
$$

– has the correct dependence on n and w, but

– does not give the correct answer.

This is not surprising since we have included only part of the α' corrections.

However this analysis demonstrates that higher derivative corrections can make the entropy of these black holes finite even though it vanishes in the supergravity approximation.

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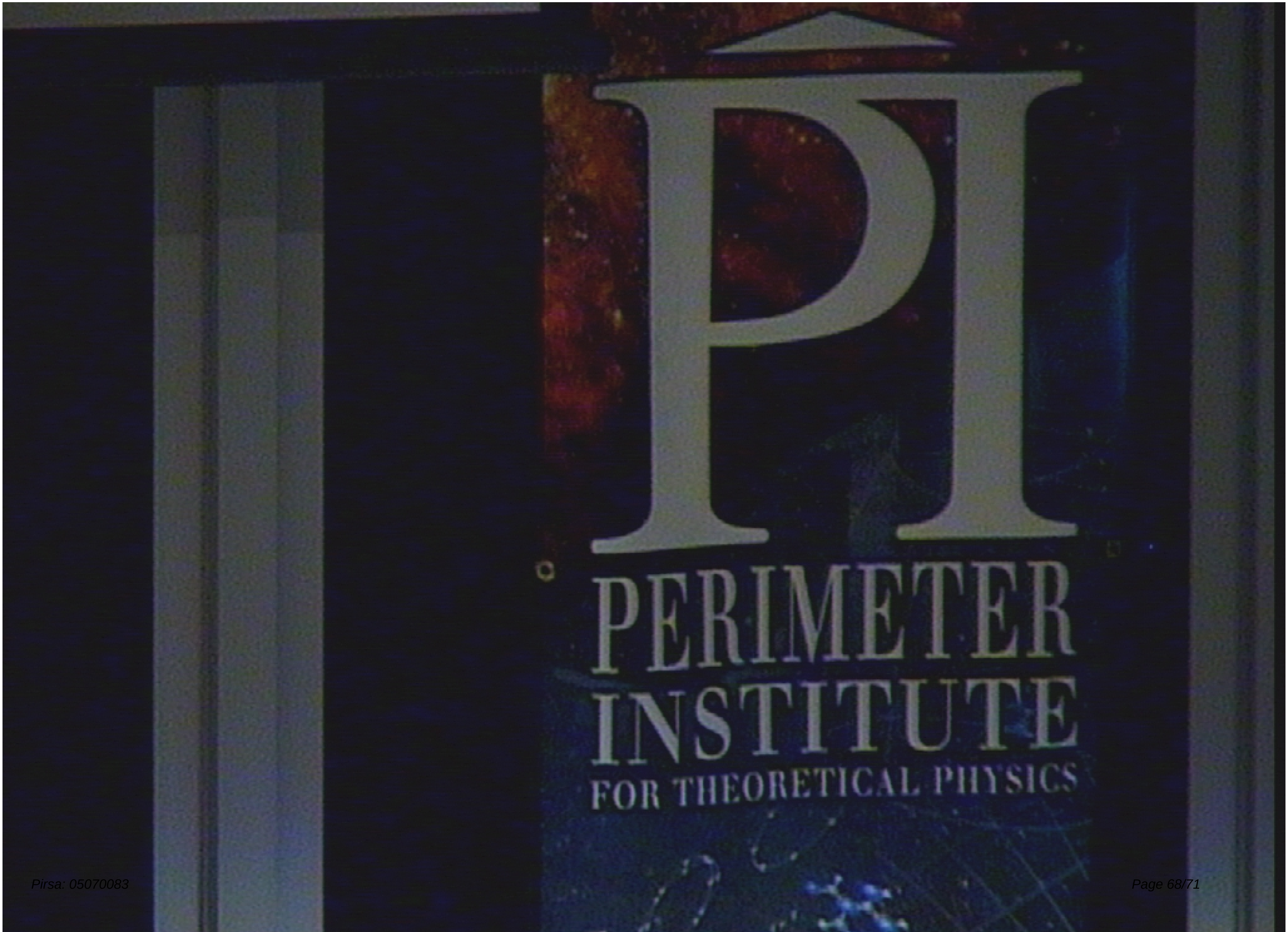
Note: For $D = 4$ (and $D = 5$) we get the correct answer.

Is this an accident?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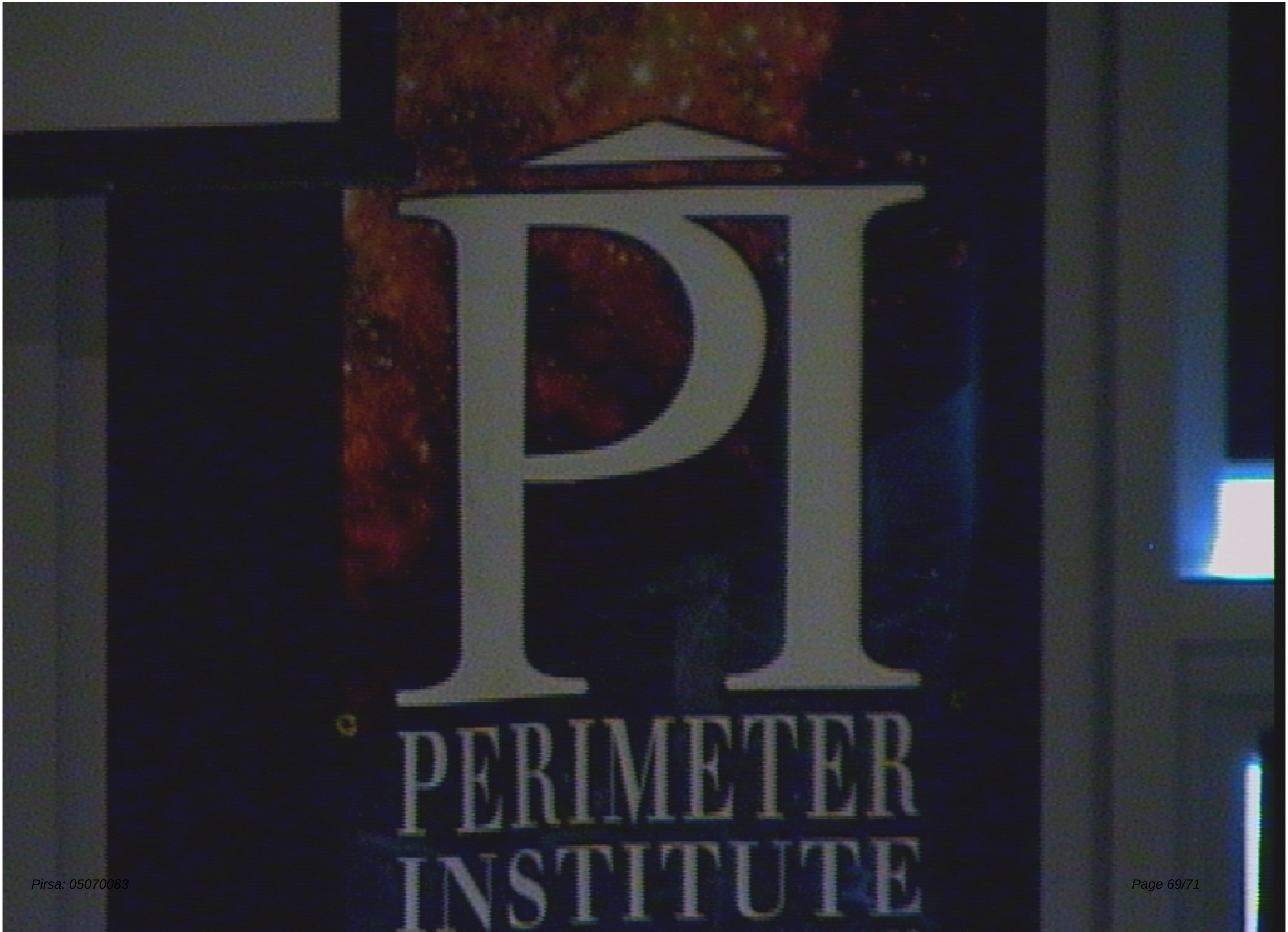
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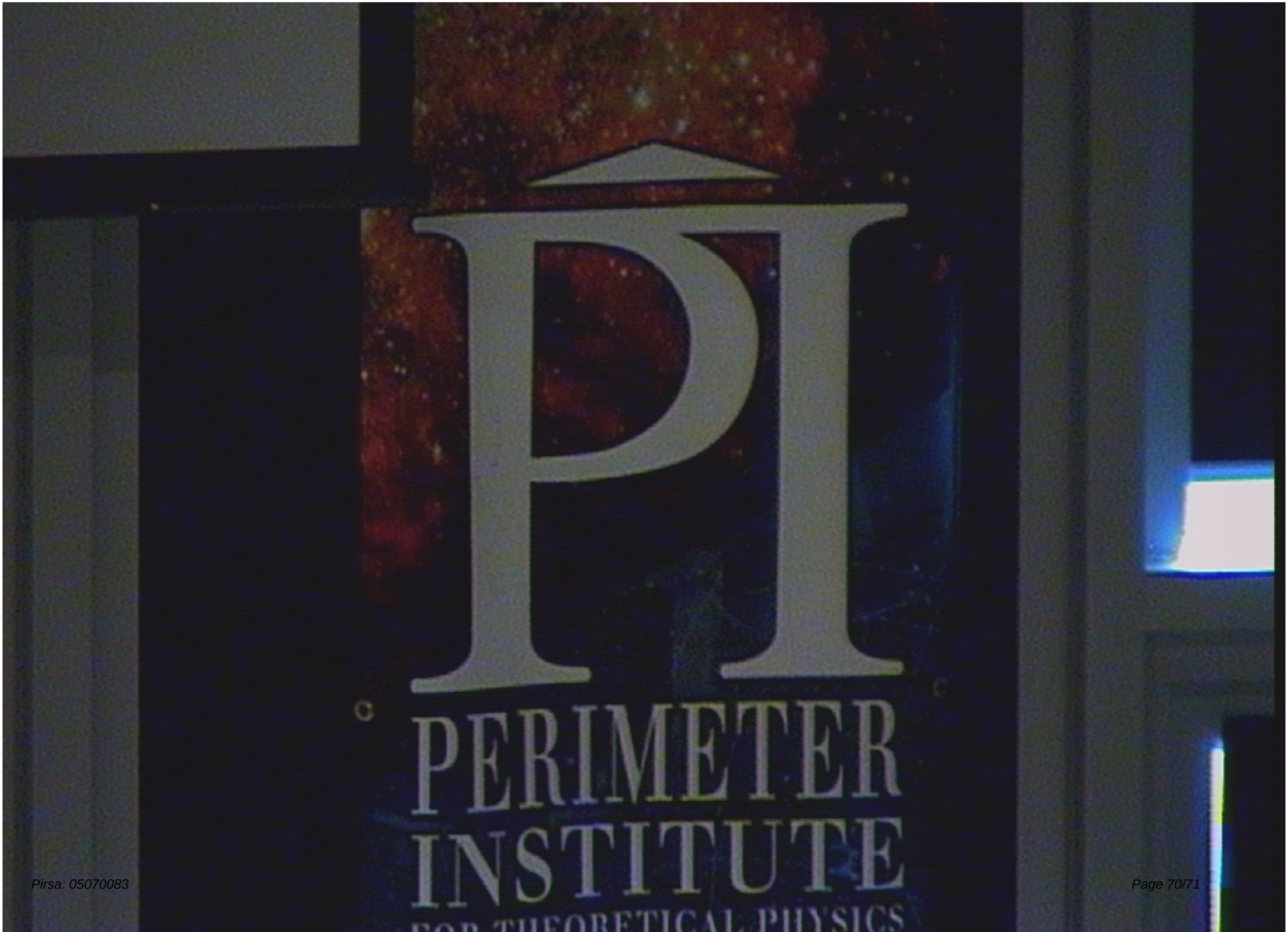
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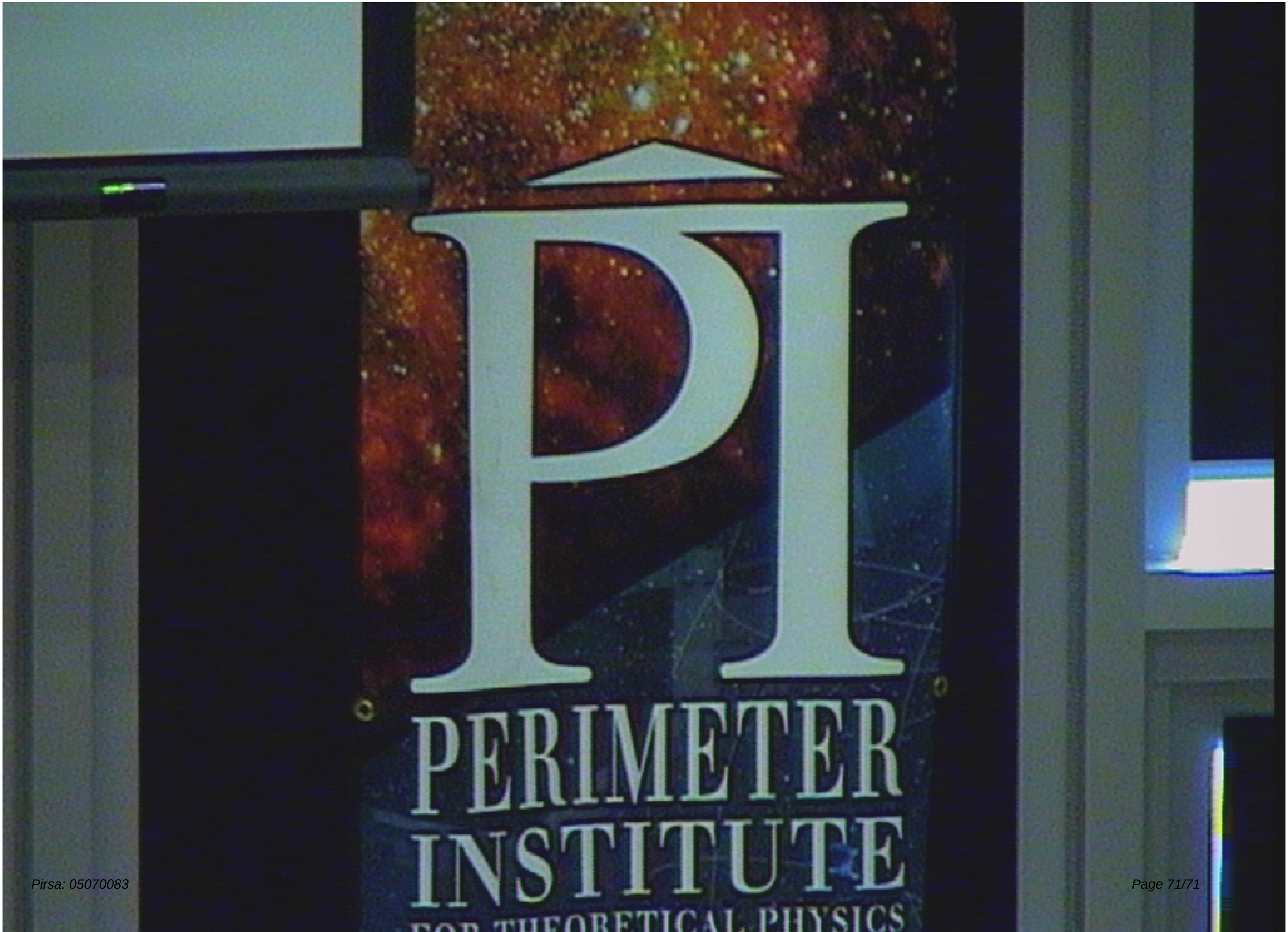
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