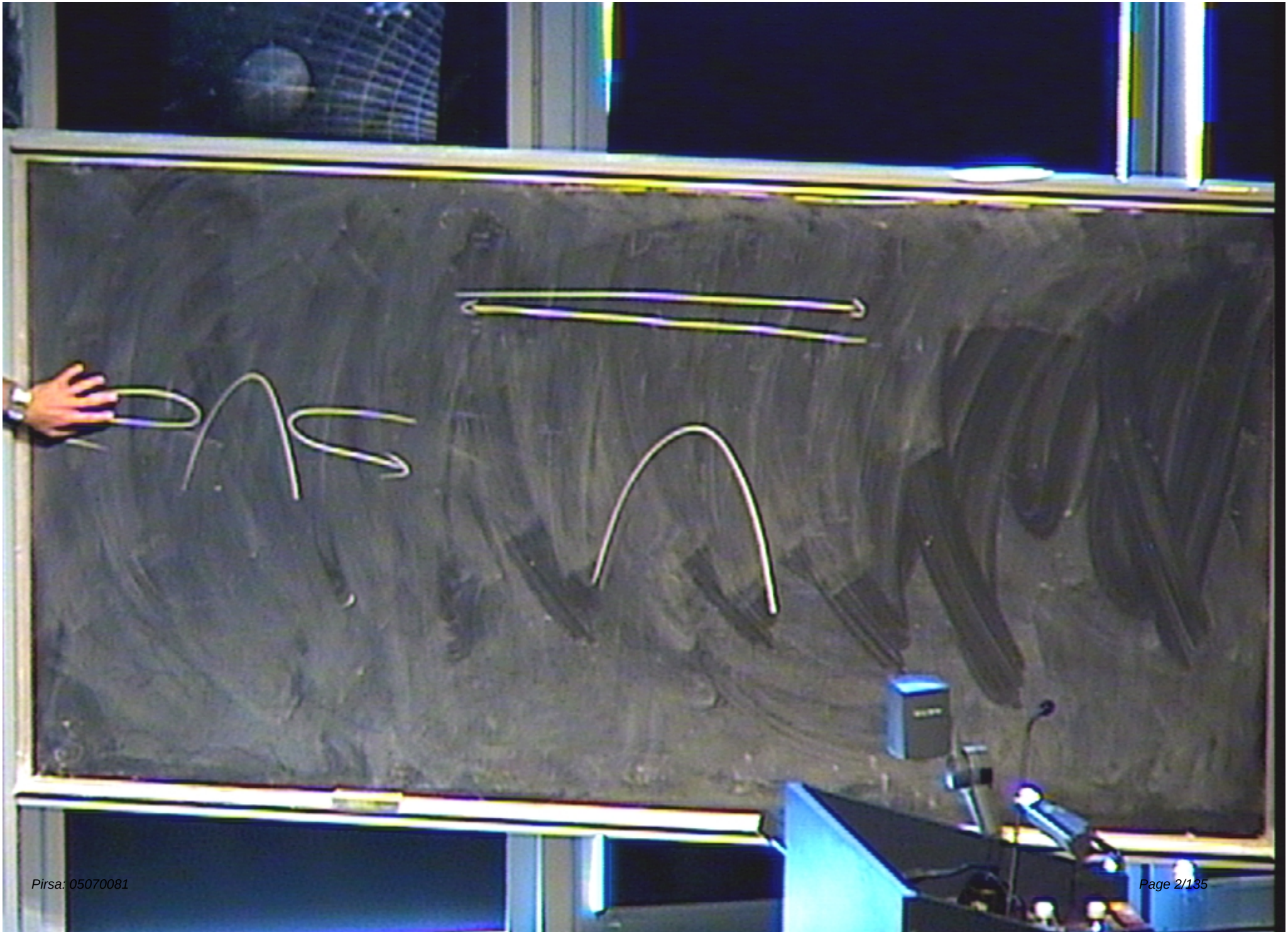
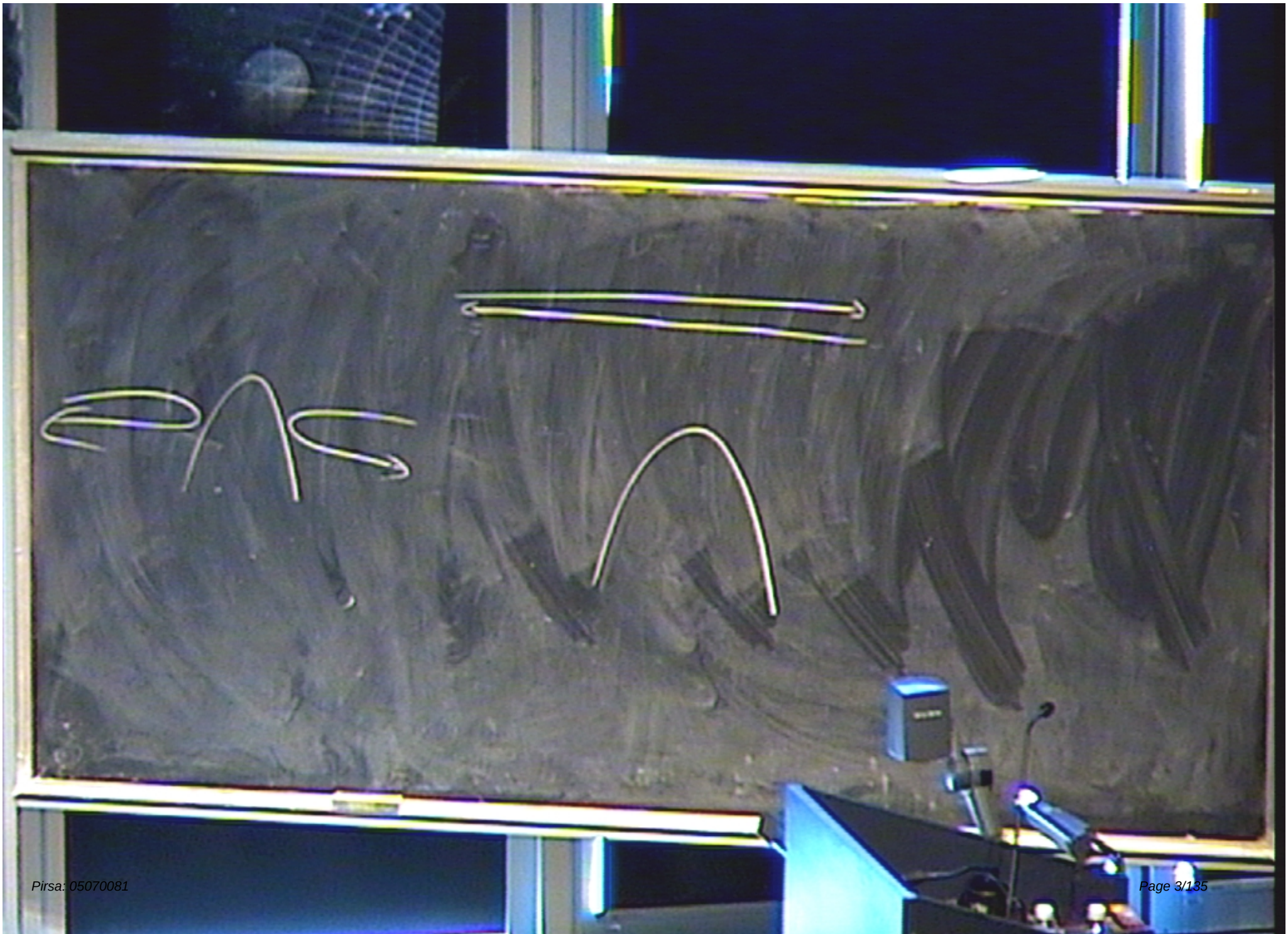
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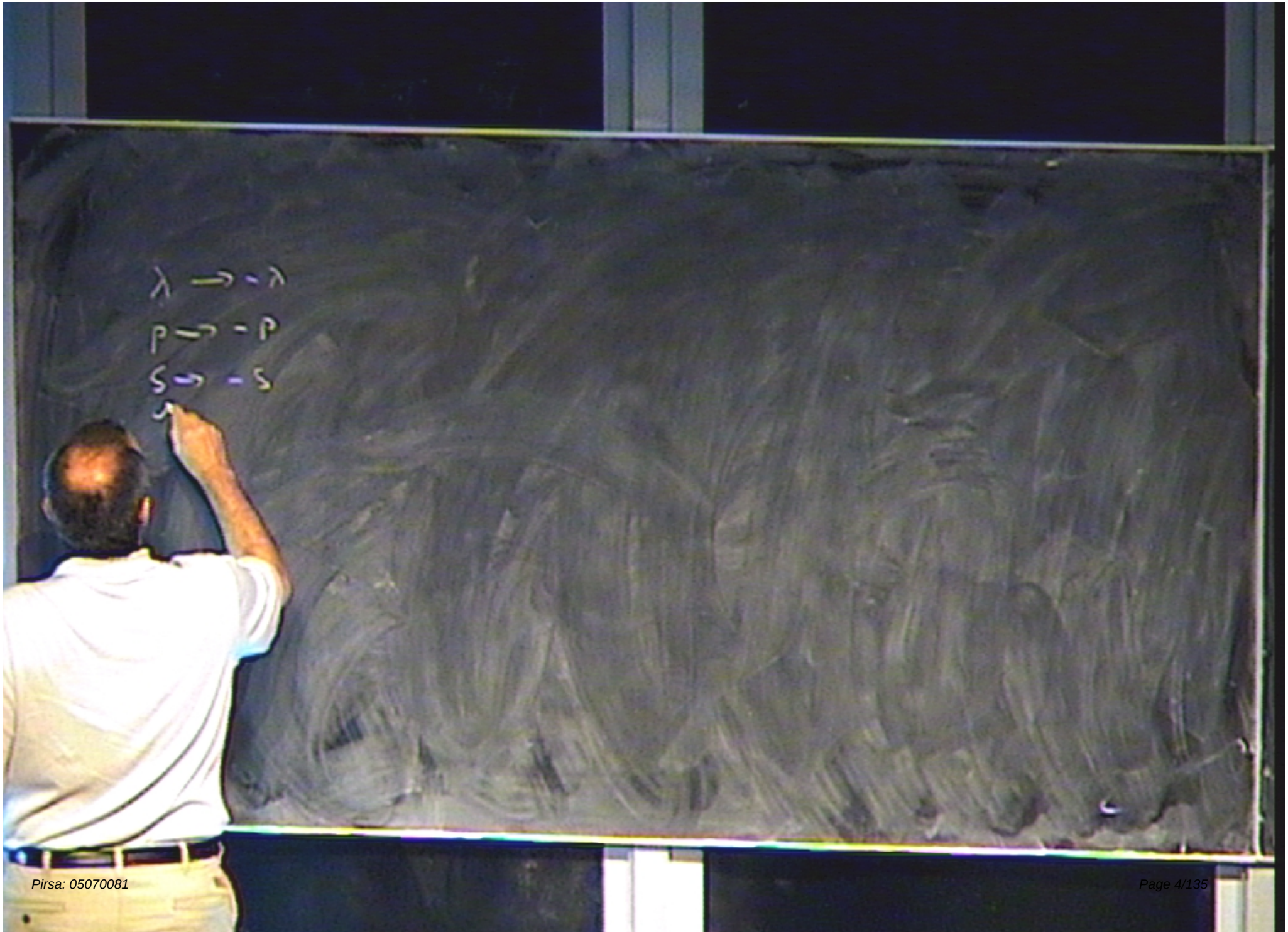
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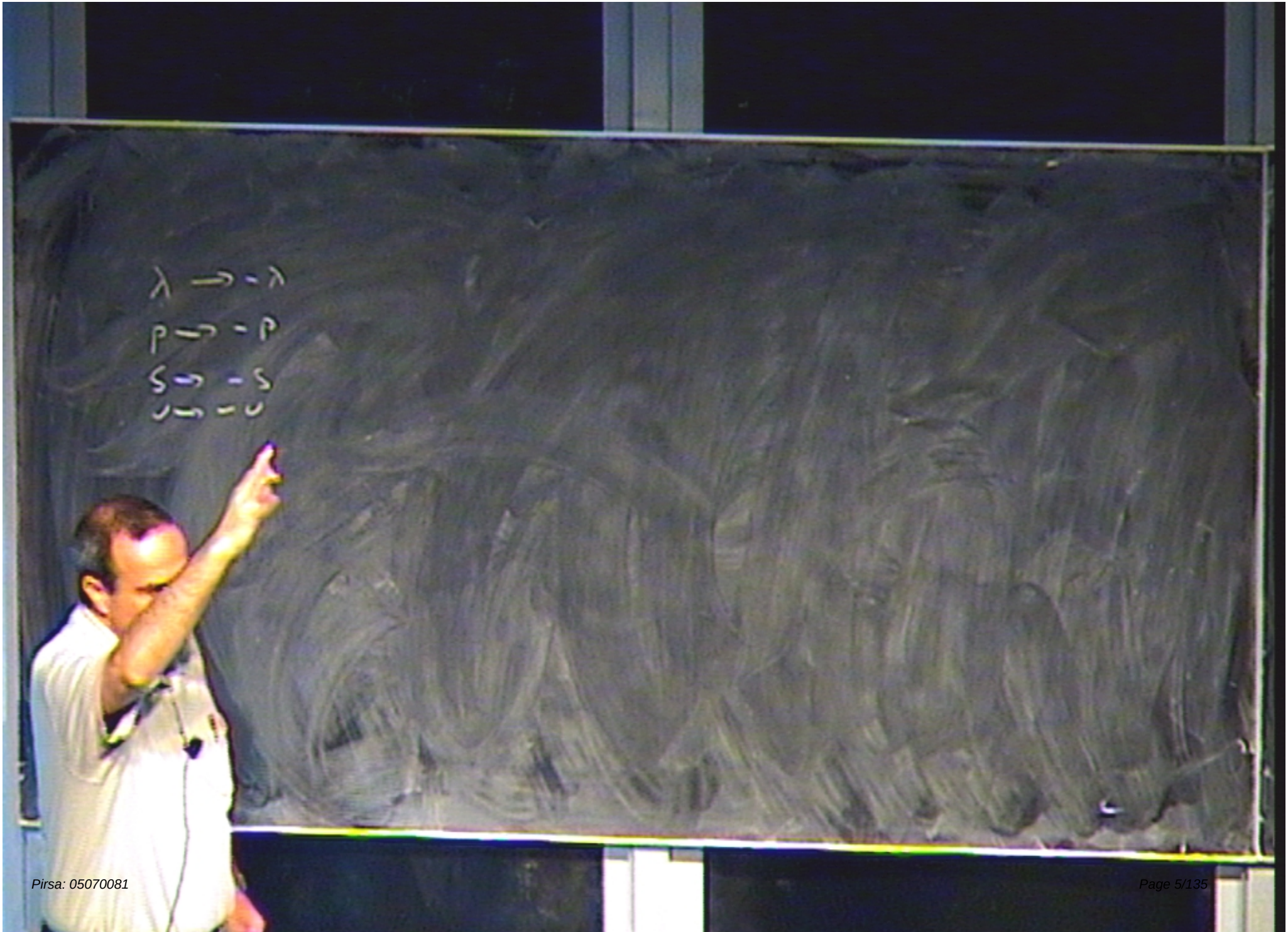
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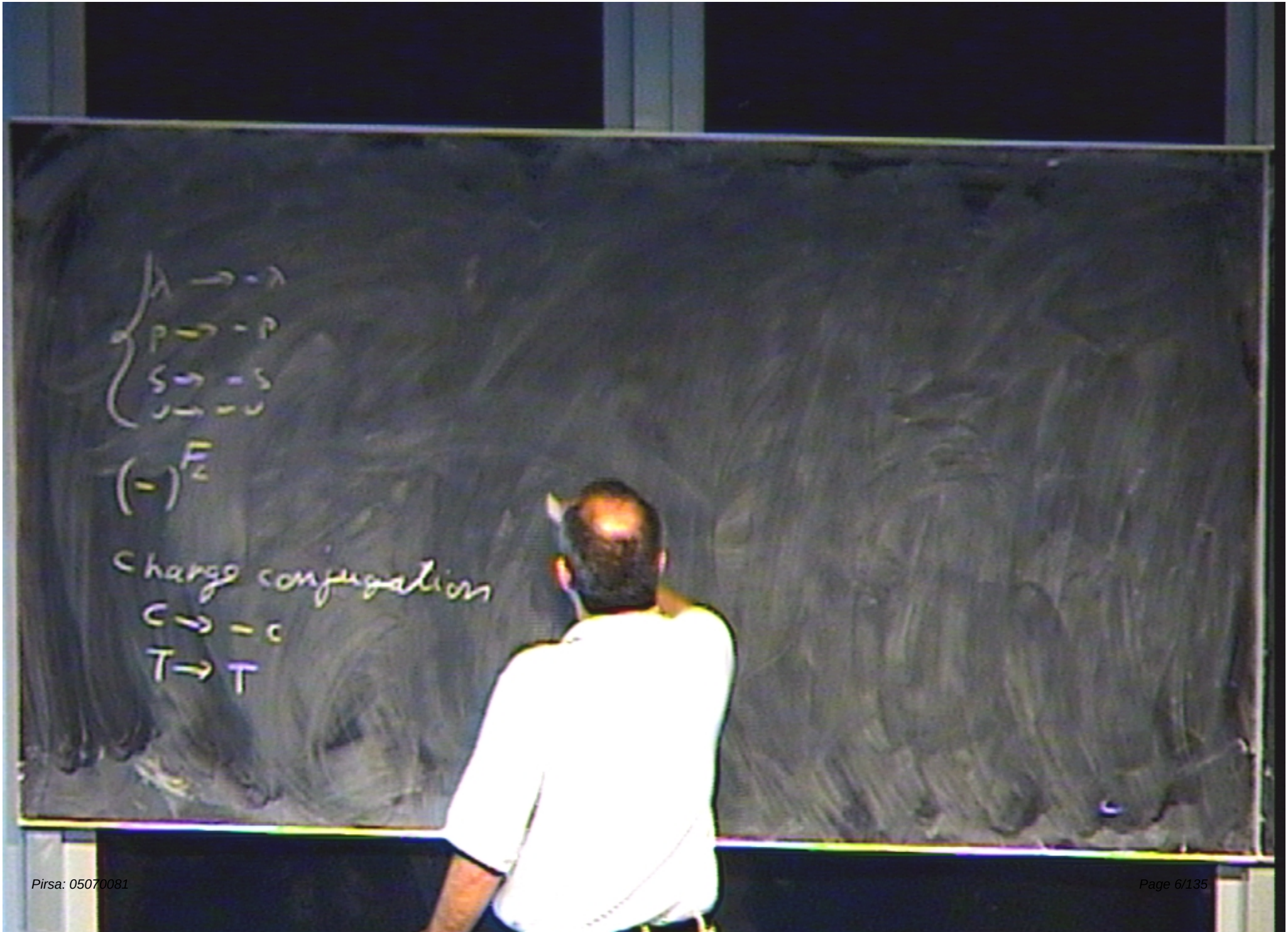
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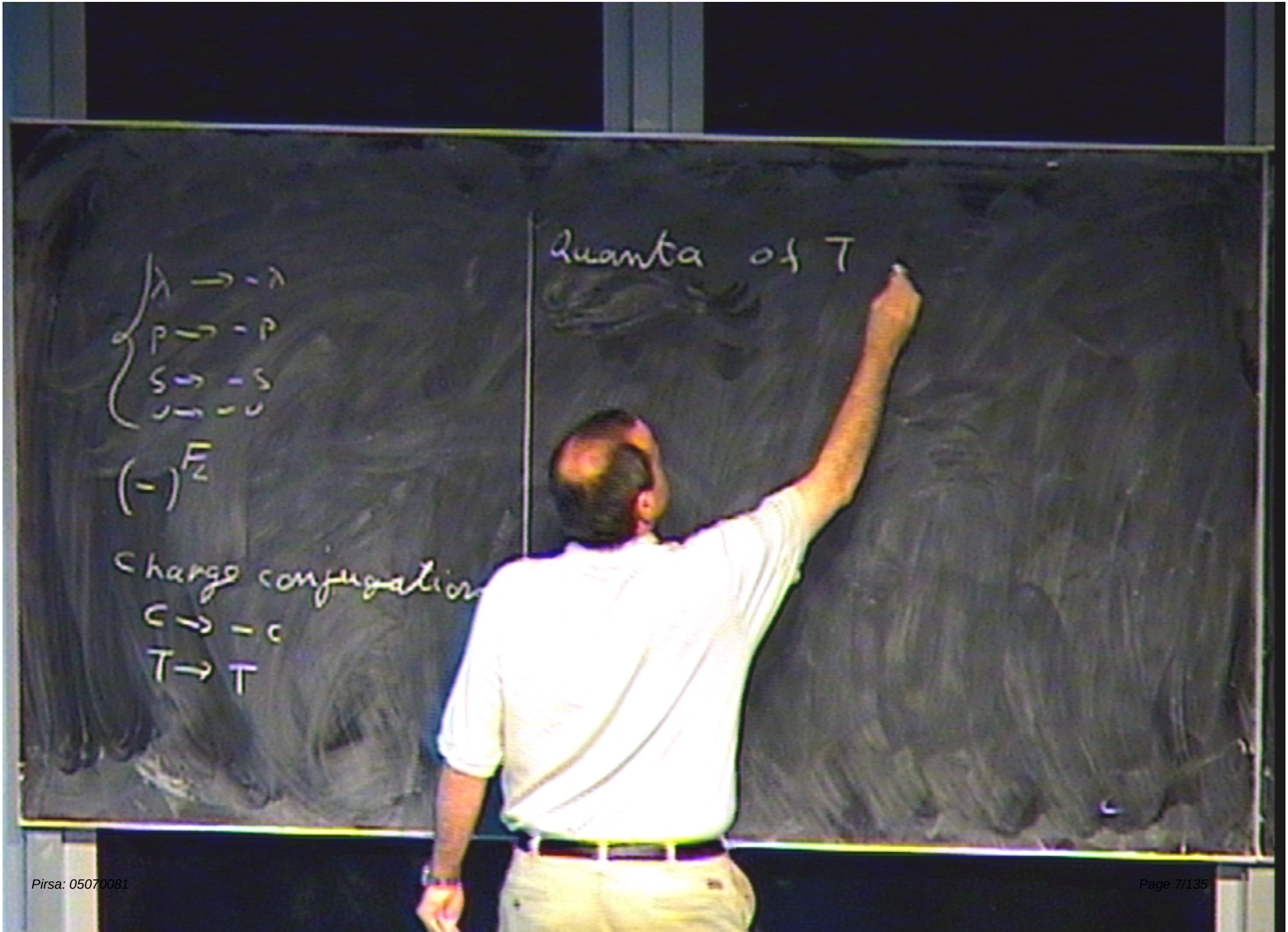


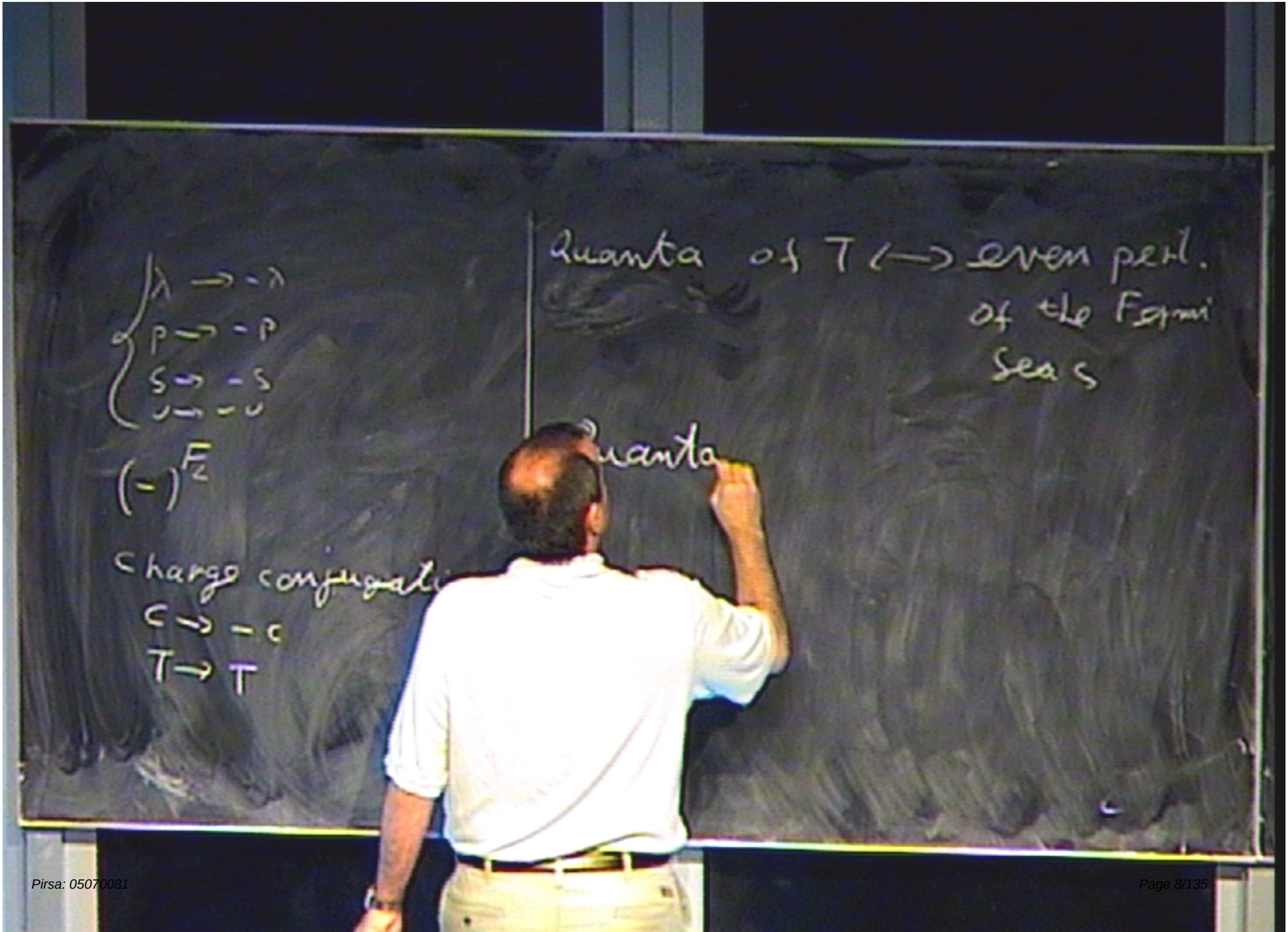



λ → -λ
ρ → -ρ
ς → -ς




λ → -λ
P → -P
S → -S
V → -V
(-)^F_L
Charge conjugation
C → -C
T → T


Quanta of T
$\lambda \to -\lambda$
$p \to -p$
$s \to -s$
$(-)^{F_z}$
Charge conjugation
$C \to -C$
$T \to T$


λ → -λ
p → -p
s → -s
v → -v
(-)^{F_L}
charge conjugati
C → -C
T → T
Quanta of T ⟷ even perl.
of the Fermi
seas
uanta

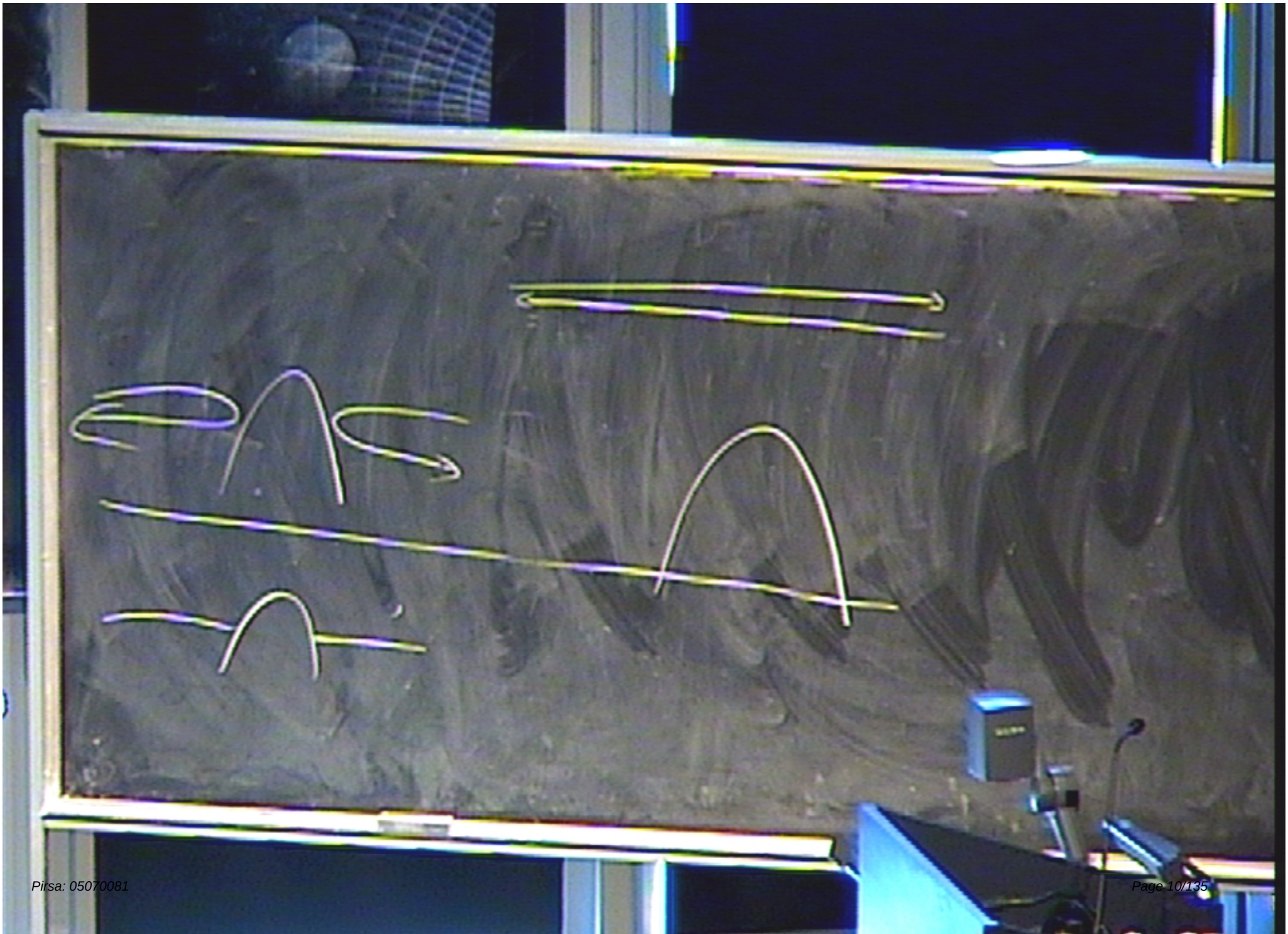
$\lambda \to -\lambda$
$p \to -p$
$S \to -S$
$v \to -v$

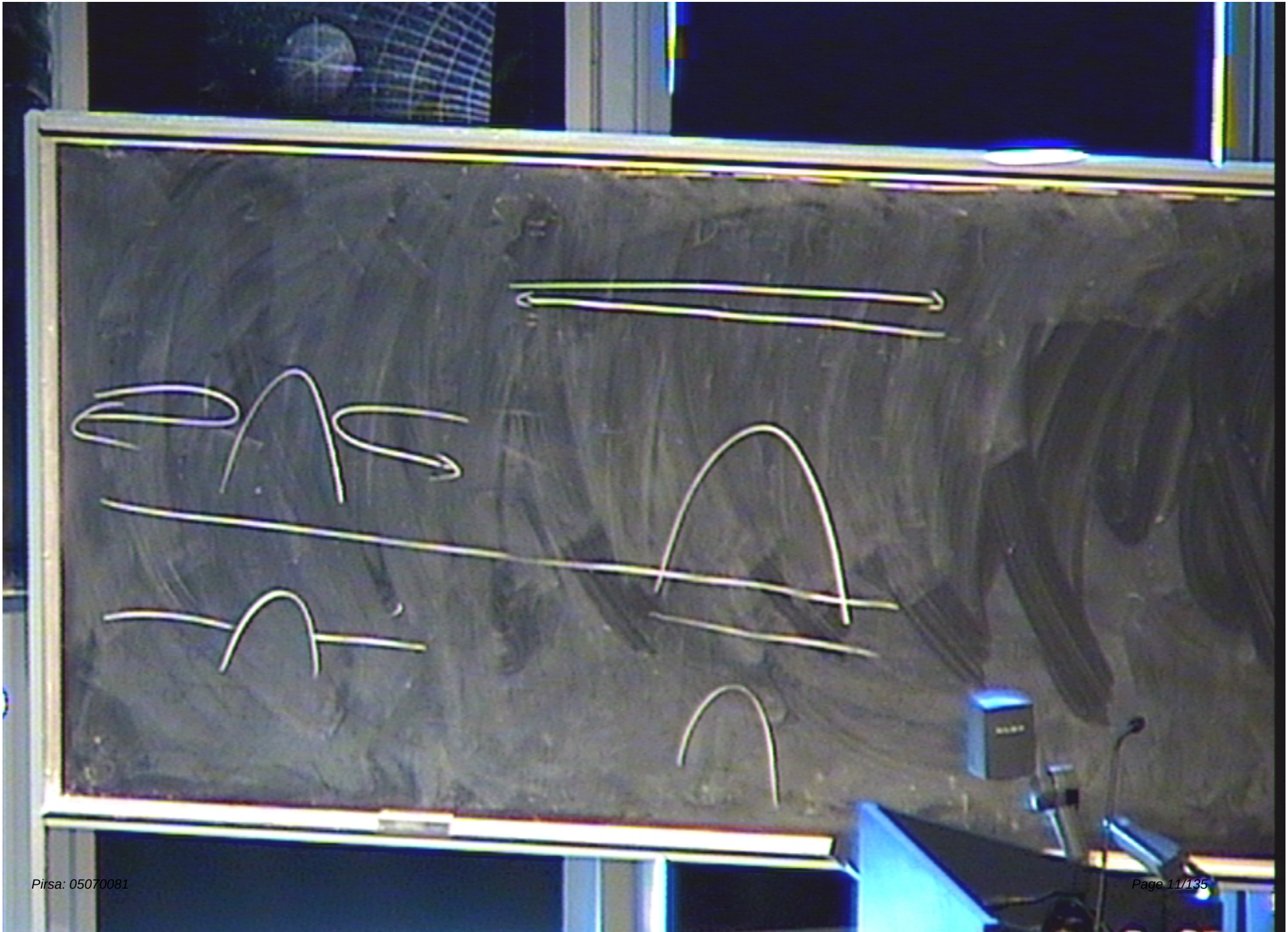
$(-)^{F_L}$

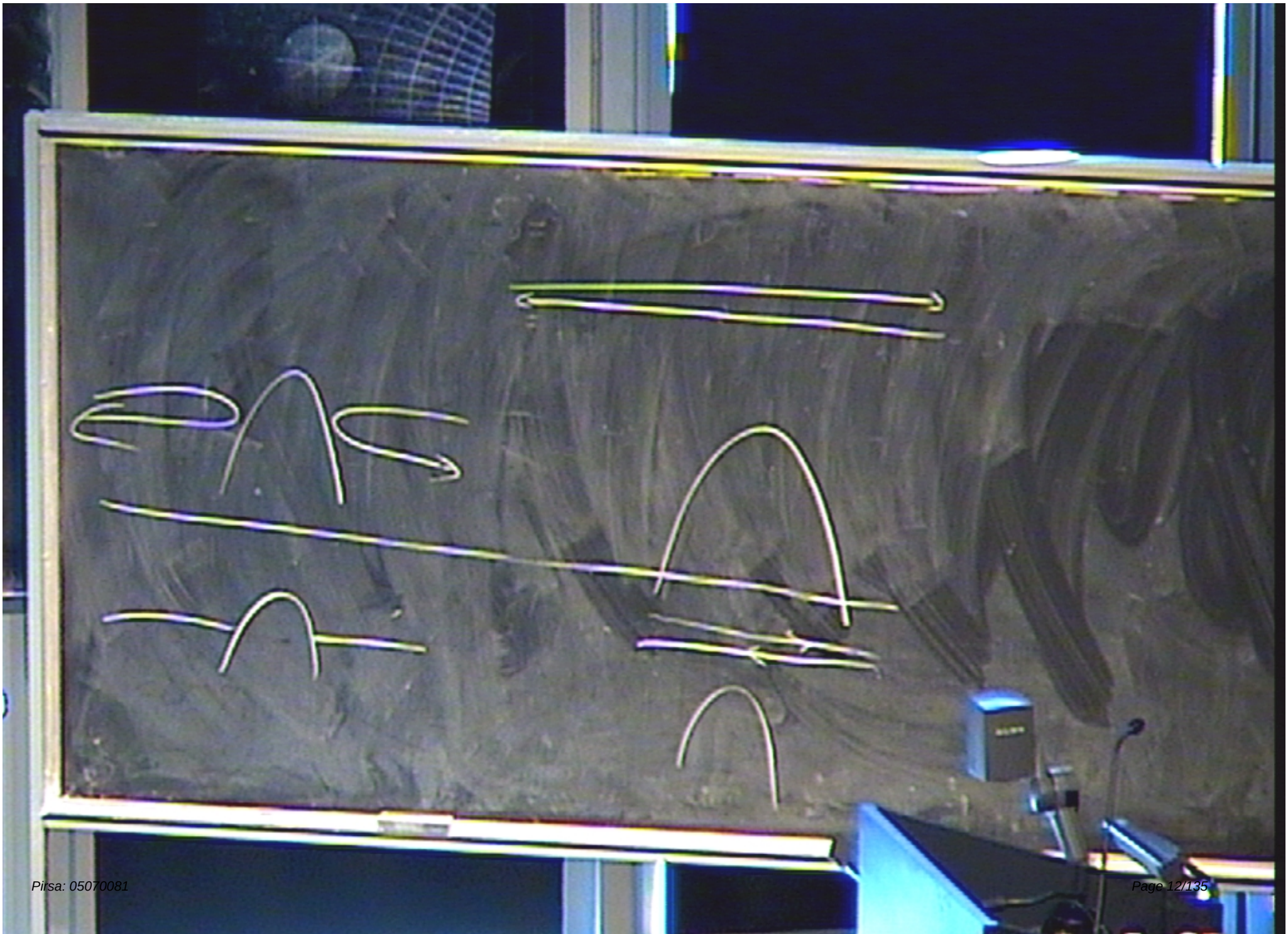
Charge conjugation
$C \to -C$
$T \to T$

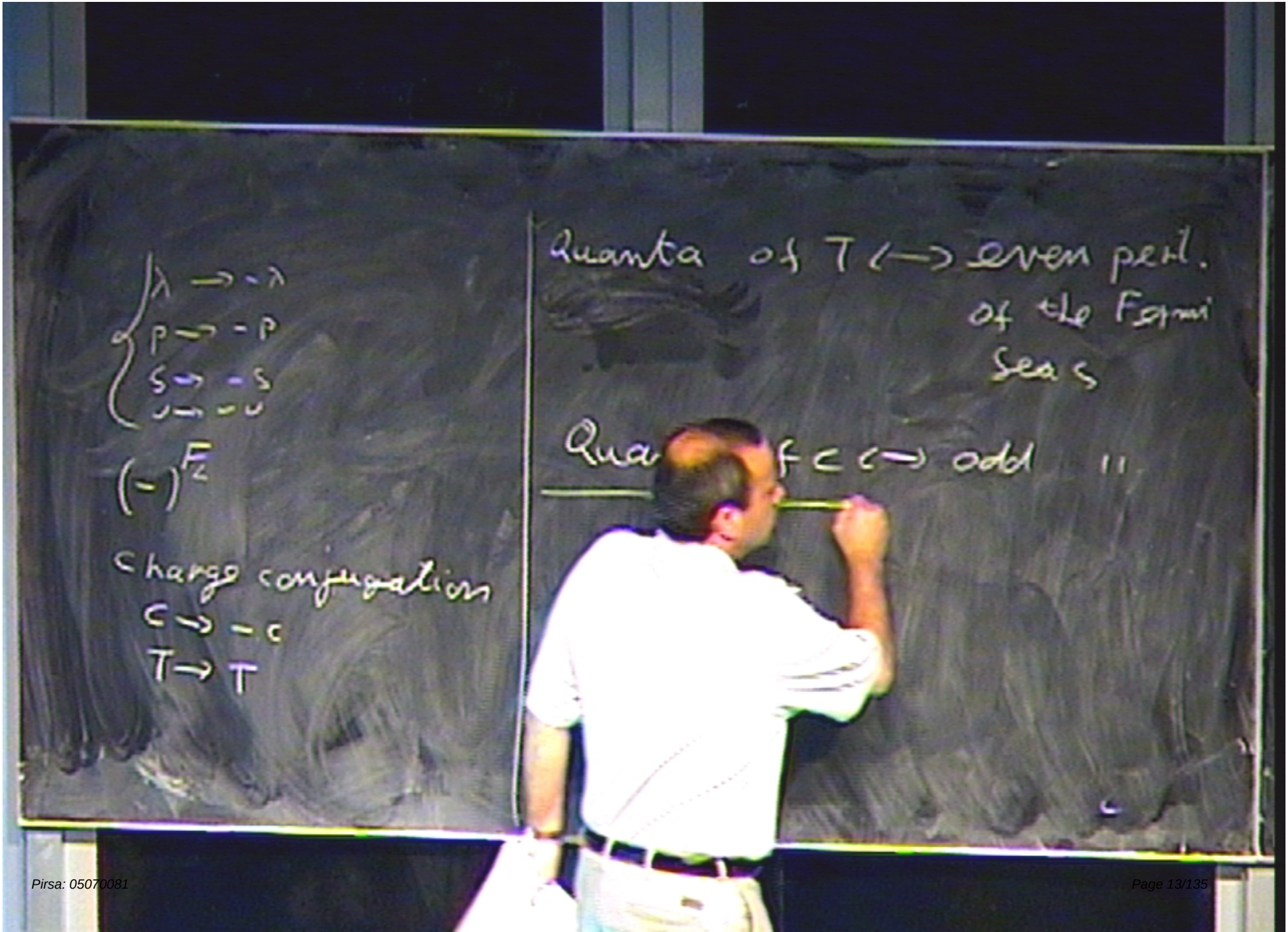
Quanta of T $\longleftrightarrow$ even pert. of the Fermi seas

Quanta of C $\longleftrightarrow$ odd "








λ → -λ
p → -p
s → -s
v → -v
$(-)^{F_L}$
charge conjugation
C → -C
T → T
Quanta of T ⟷ even perl. of the Fermi seas
Qua f ⟷ odd

$\begin{cases} \lambda \to -\lambda \\ p \to -p \\ s \to -s \\ v \to -v \end{cases}$

$(-)^{F_L}$

Charge conjugation

$C \to -C$

$T \to T$

Quanta of T $\longleftrightarrow$ even perl. of the Fermi seas

Quanta of C $\longleftrightarrow$ odd "

$\begin{cases} \lambda \to -\lambda \\ p \to -p \\ s \to -s \\ v \to -v \end{cases}$

$(-)^{F_L}$

Charge conjugation

$C \to -C$

$T \to T$

Quanta of T $\longleftrightarrow$ even perl. of the Fermi seas

Quanta of C $\longleftrightarrow$ odd "

$e^{\mp i\sqrt{2}\, C^{im}}$

$e^{2i\sqrt{2}\, C^{real}}$

$\begin{cases} \lambda \to -\lambda \\ p \to -p \\ s \to -s \\ v \to -v \end{cases}$

$(-)^{F_L}$

Charge conjugation

$C \to -C$

$T \to T$

Quanta of T $\longleftrightarrow$ even perl. of the Fermi seas

Quanta of C $\longleftrightarrow$ odd "

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$\lambda \to -\lambda$
$p \to -p$
$s \to -s$
$v \to -v$

$(-)^{F_L}$

charge conjugation
$C \to -C$
$T \to T$

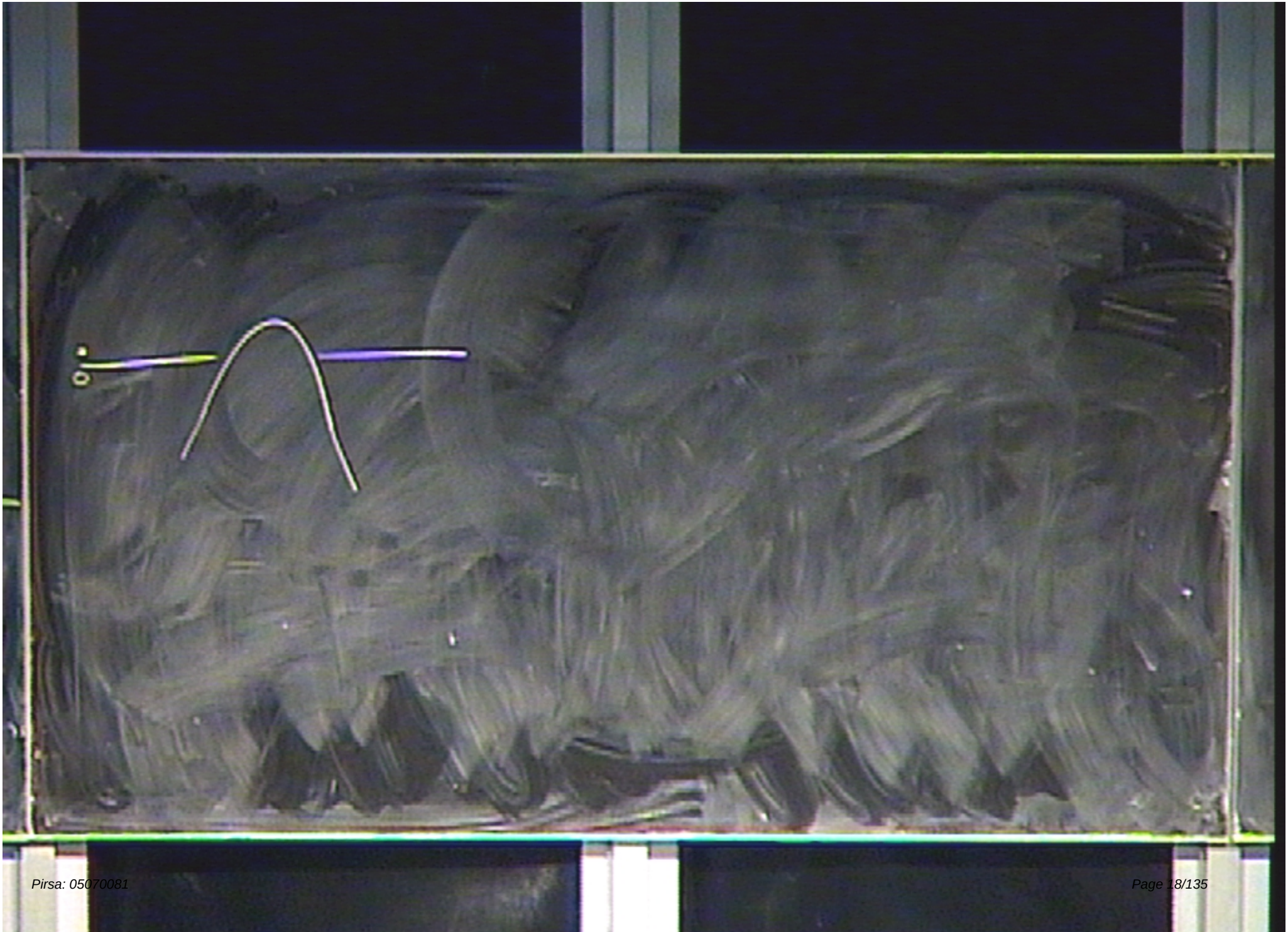
Quanta of T $\longleftrightarrow$ even perl. of the Fermi seas

Quanta of C $\longleftrightarrow$ odd "

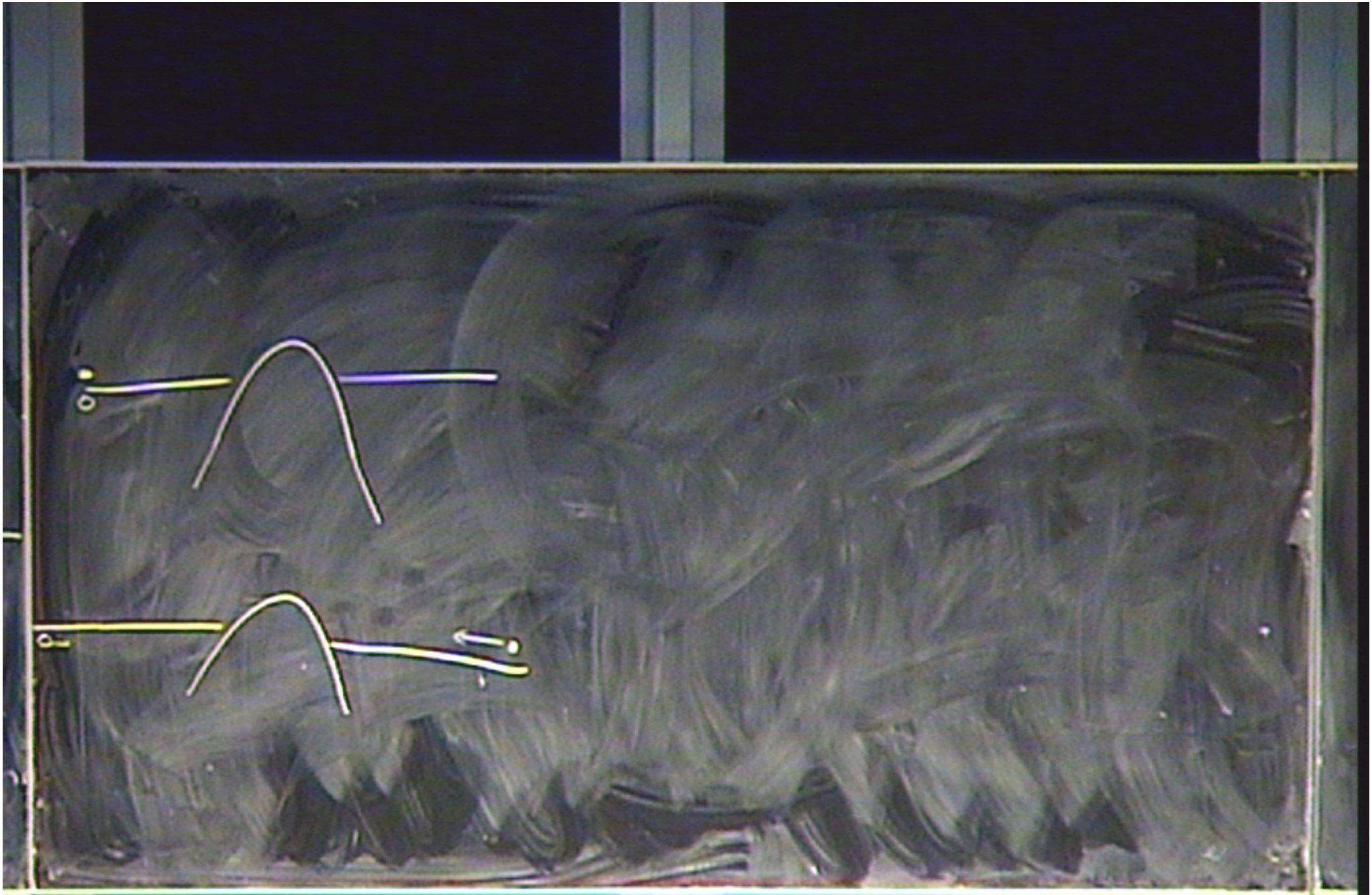
$e^{\mp i\sqrt{2}\,C}$

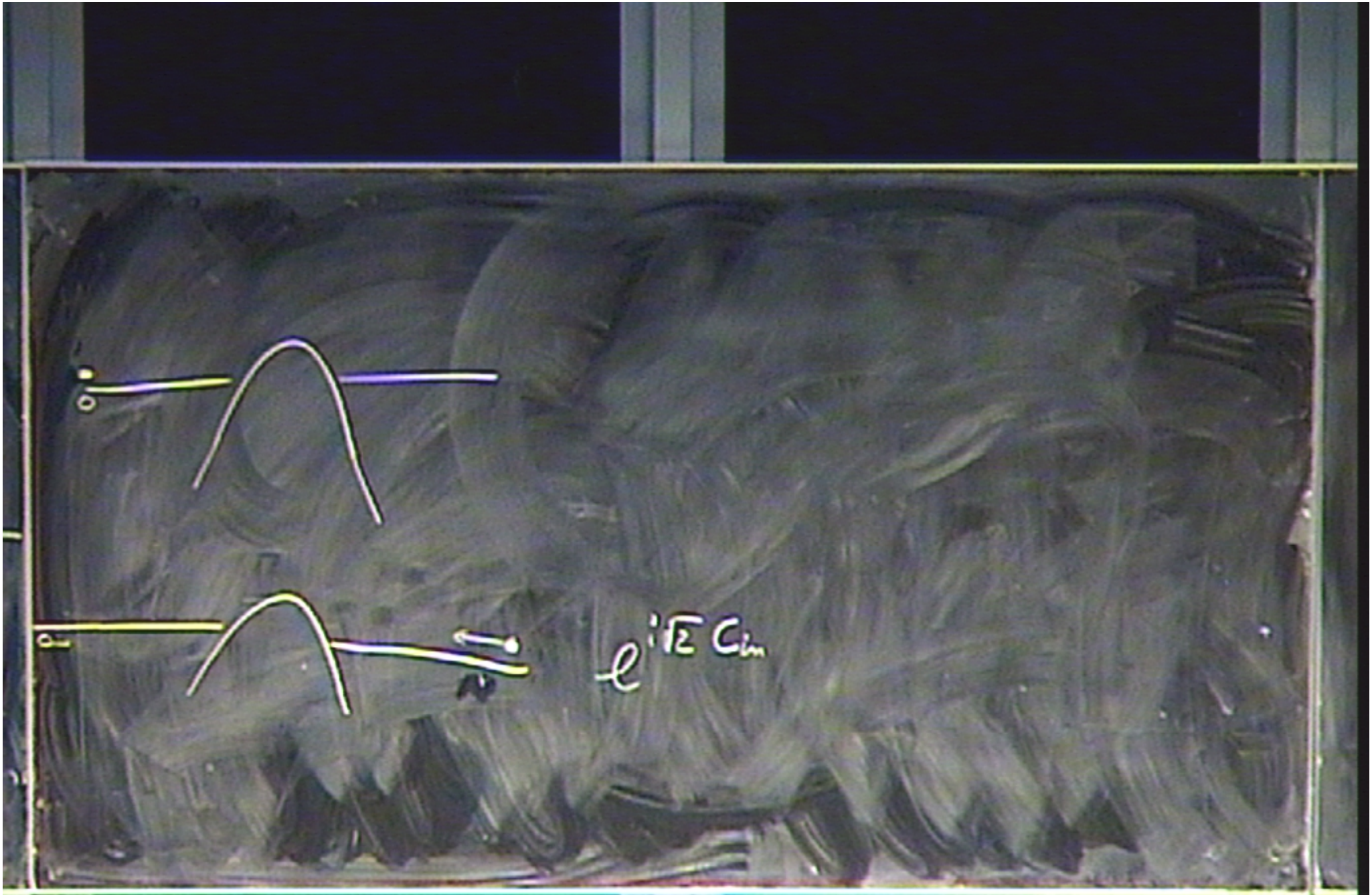
$e^{\pm i\sqrt{2}}$

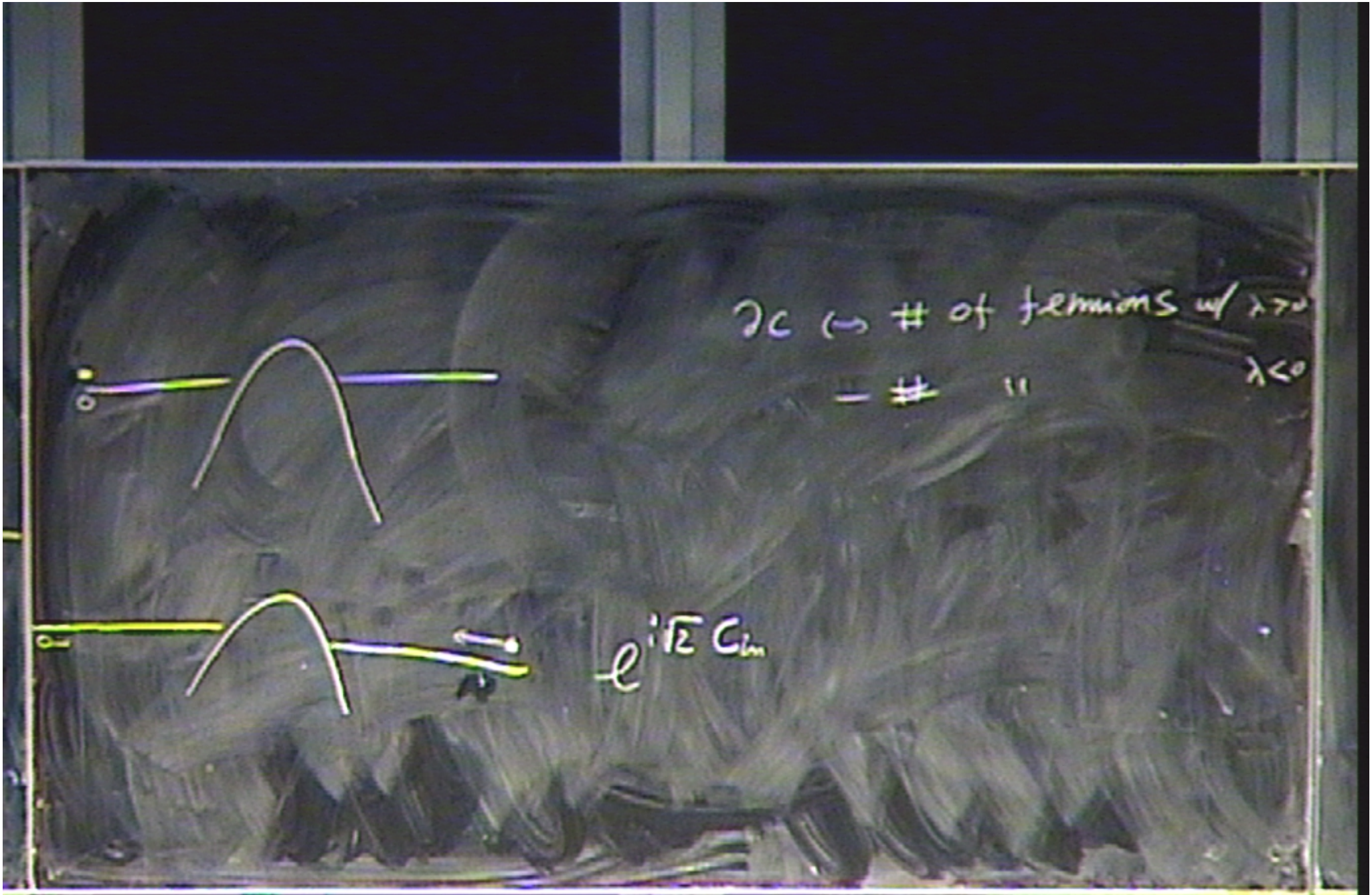


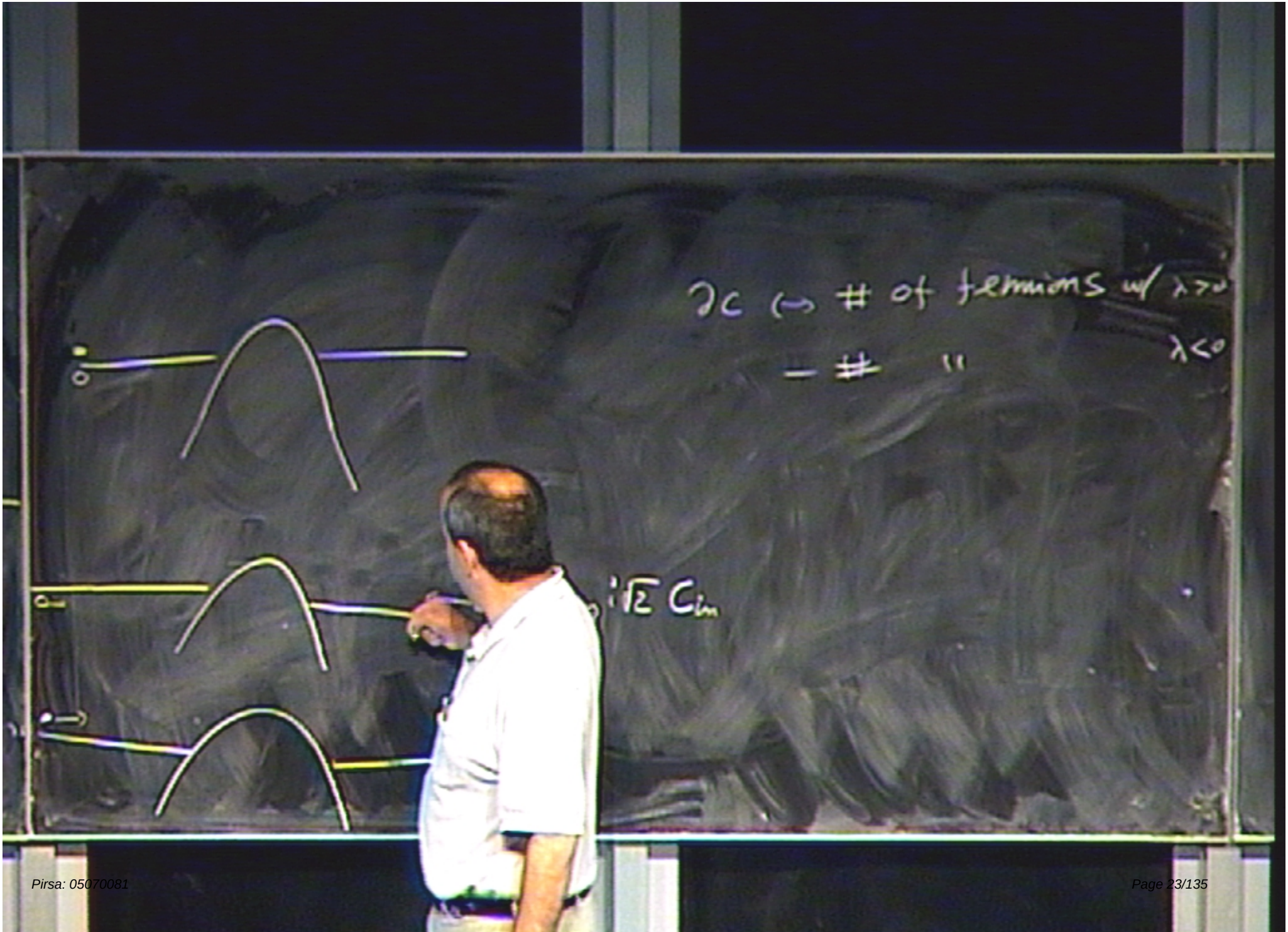






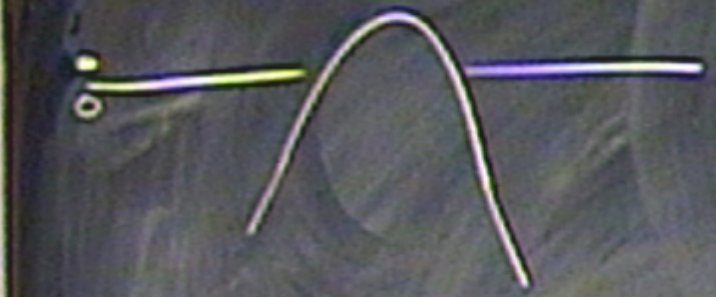

$e^{i\sum C_{lm}}$

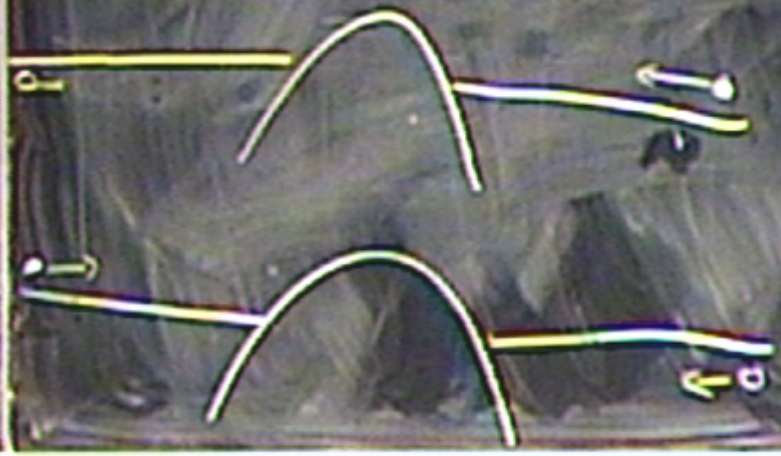

$\mathcal{I}c \hookrightarrow$ # of fermions w/ $\lambda > 0$
$-$ # " $\lambda < 0$
$e^{i\sum C_{km}}$


$\chi_c \leftrightarrow$ # of fermions w/ $\lambda > 0$
$\lambda < 0$
$- \#$ "
$\sum C_{lm}$

$\partial c \leftrightarrow$ # of fermions w/ $\lambda > 0$

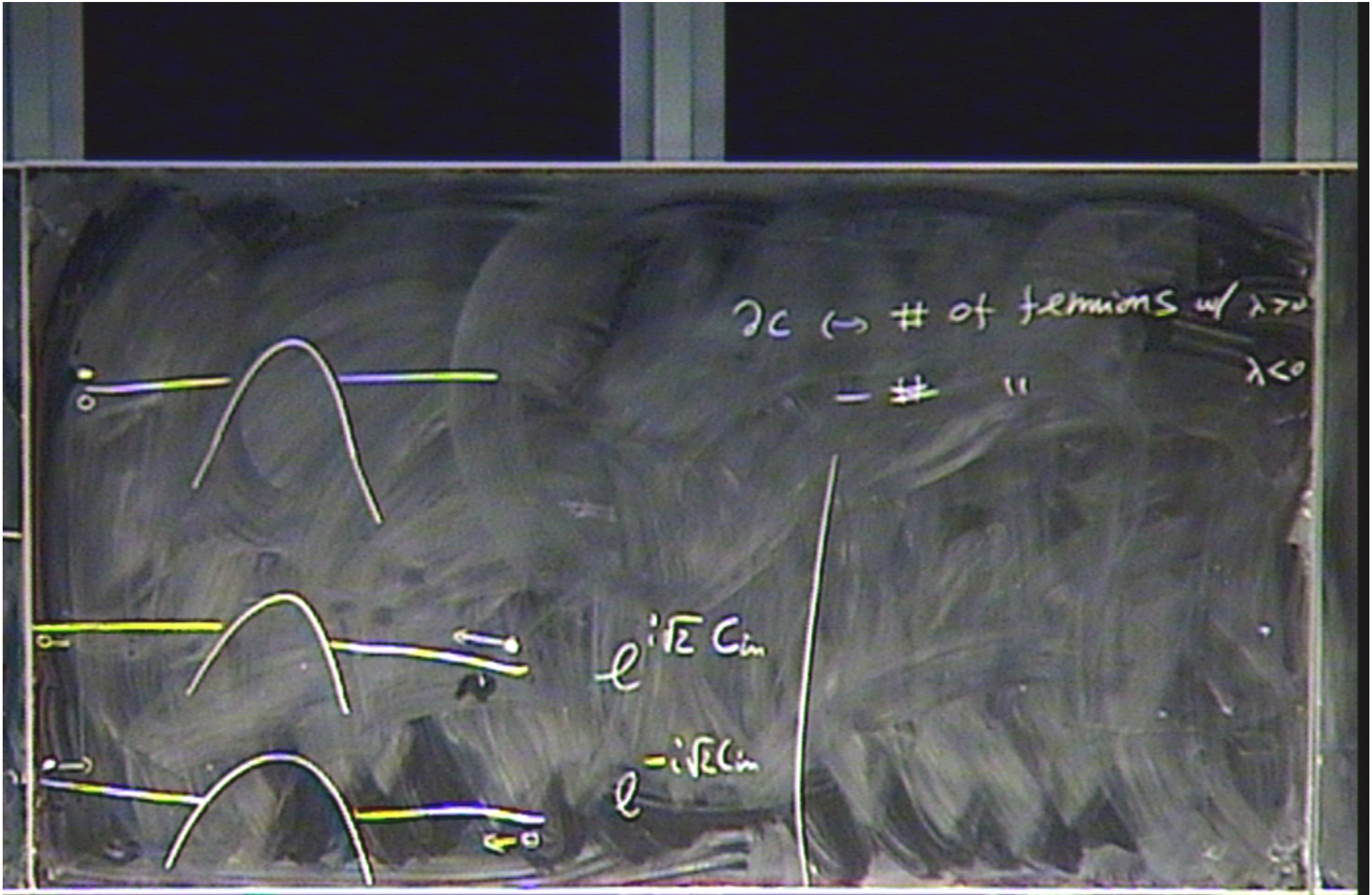
$-$ # " $\lambda < 0$

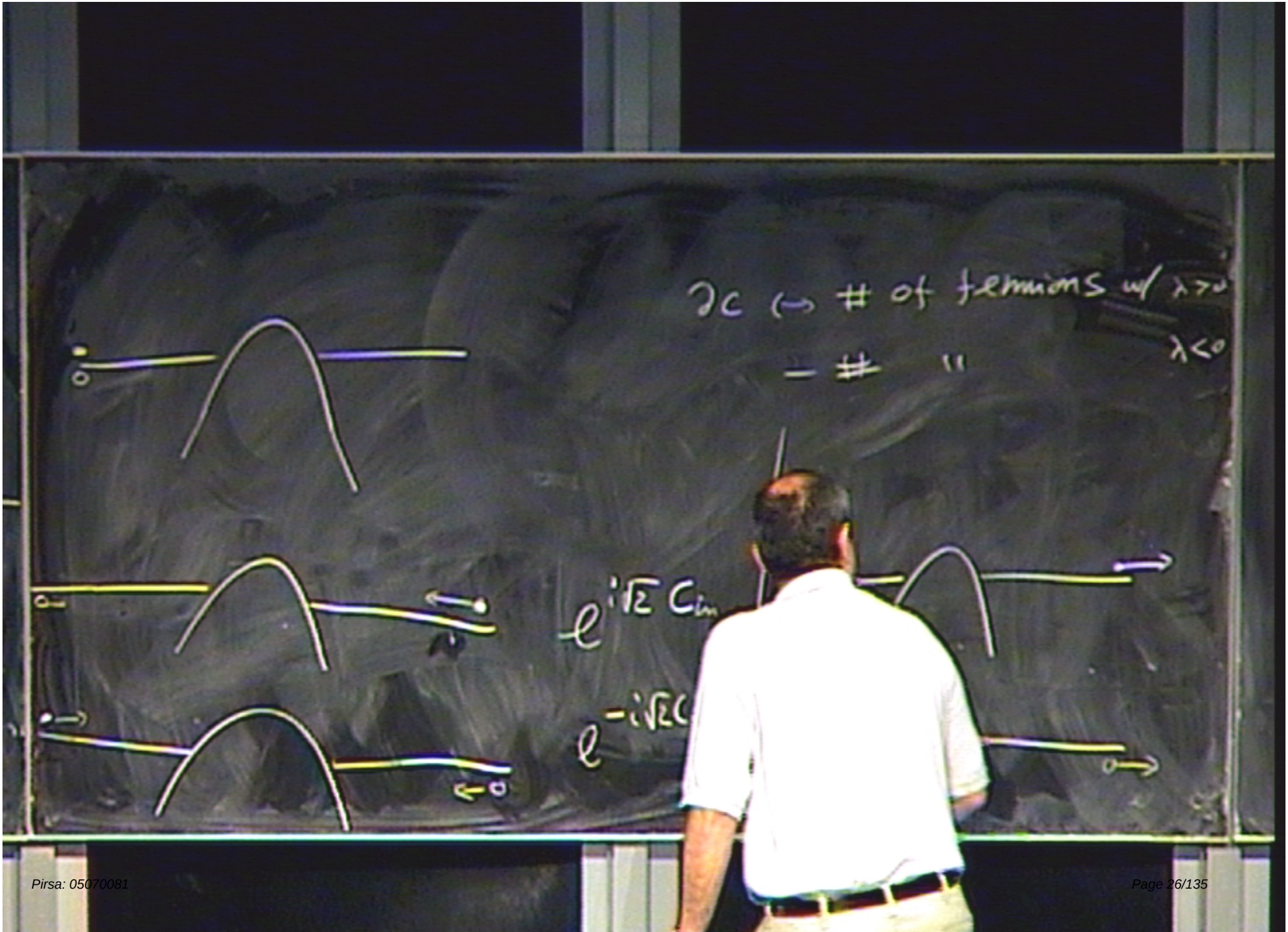


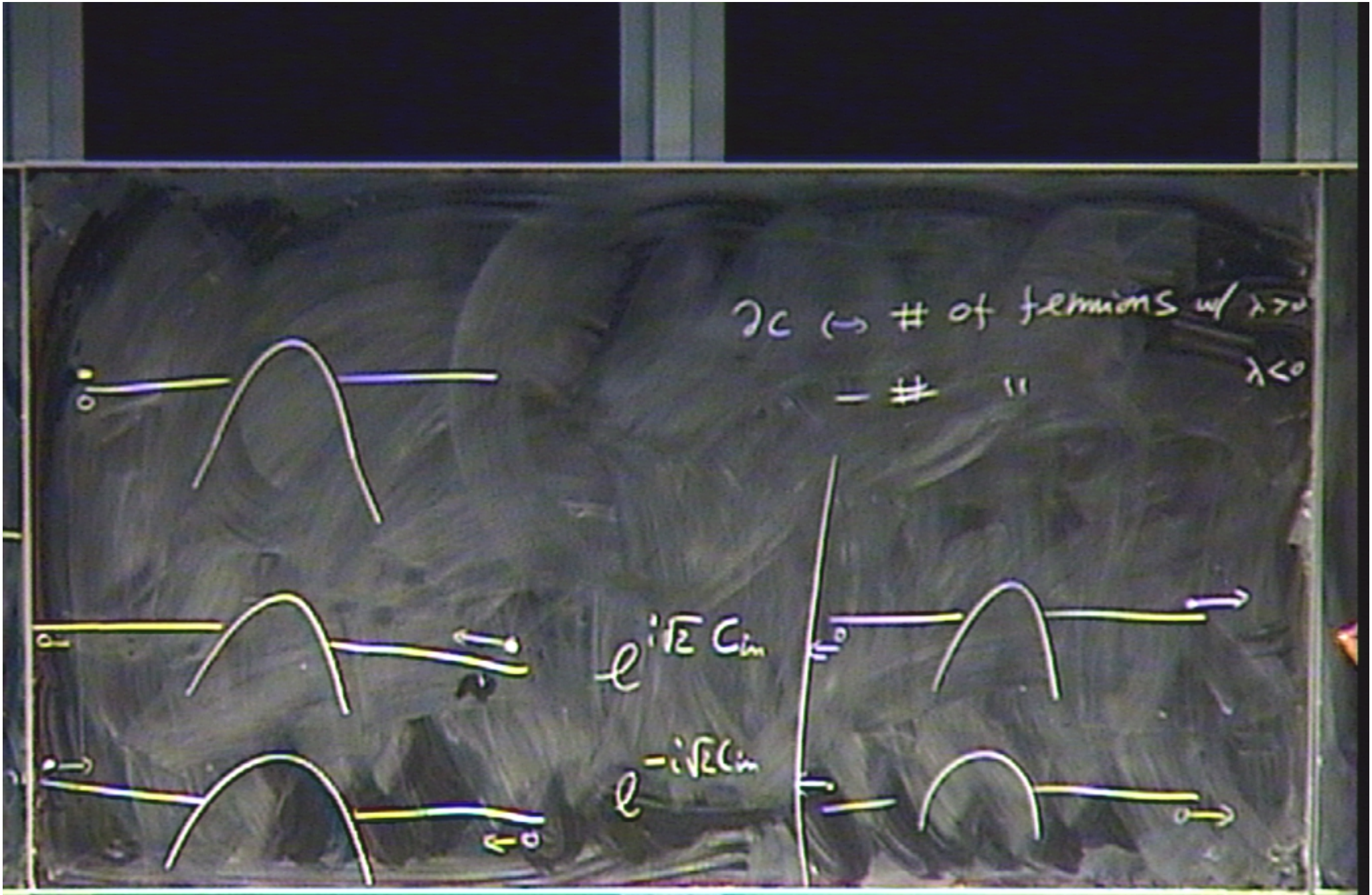


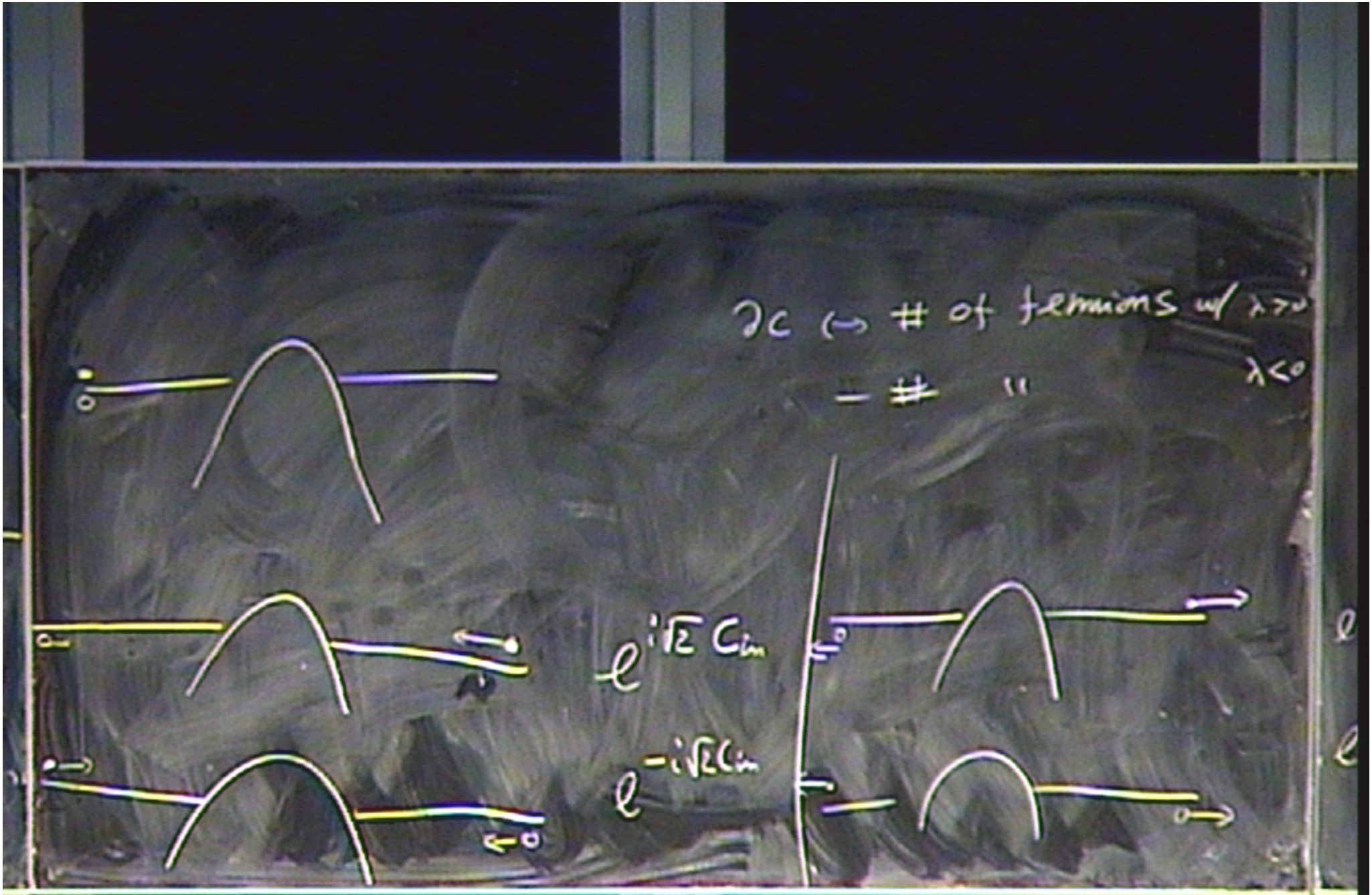
$e^{i\sqrt{2}\,C_{lm}}$

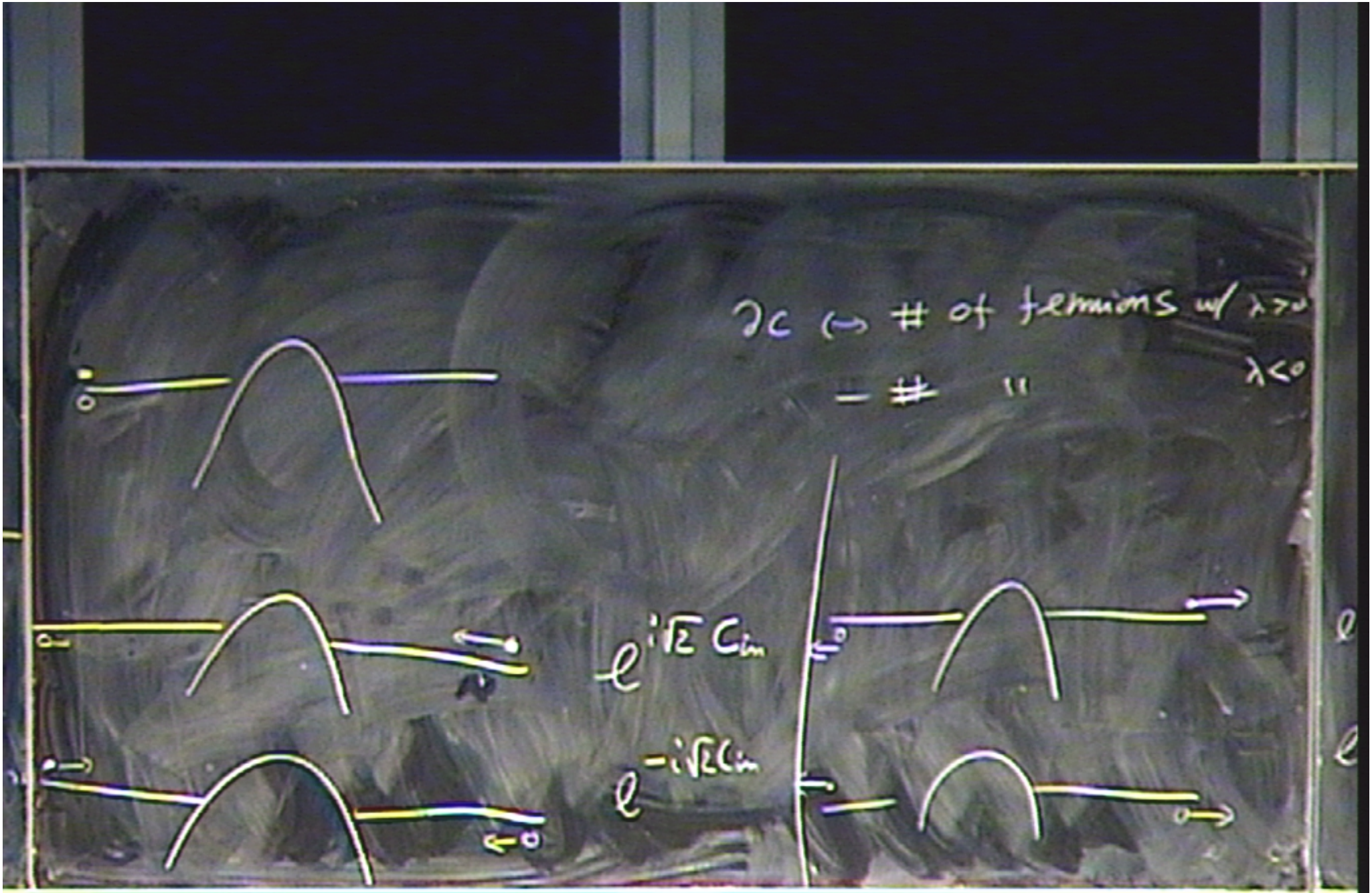
$e^{-i\sqrt{2}C_{lm}}$

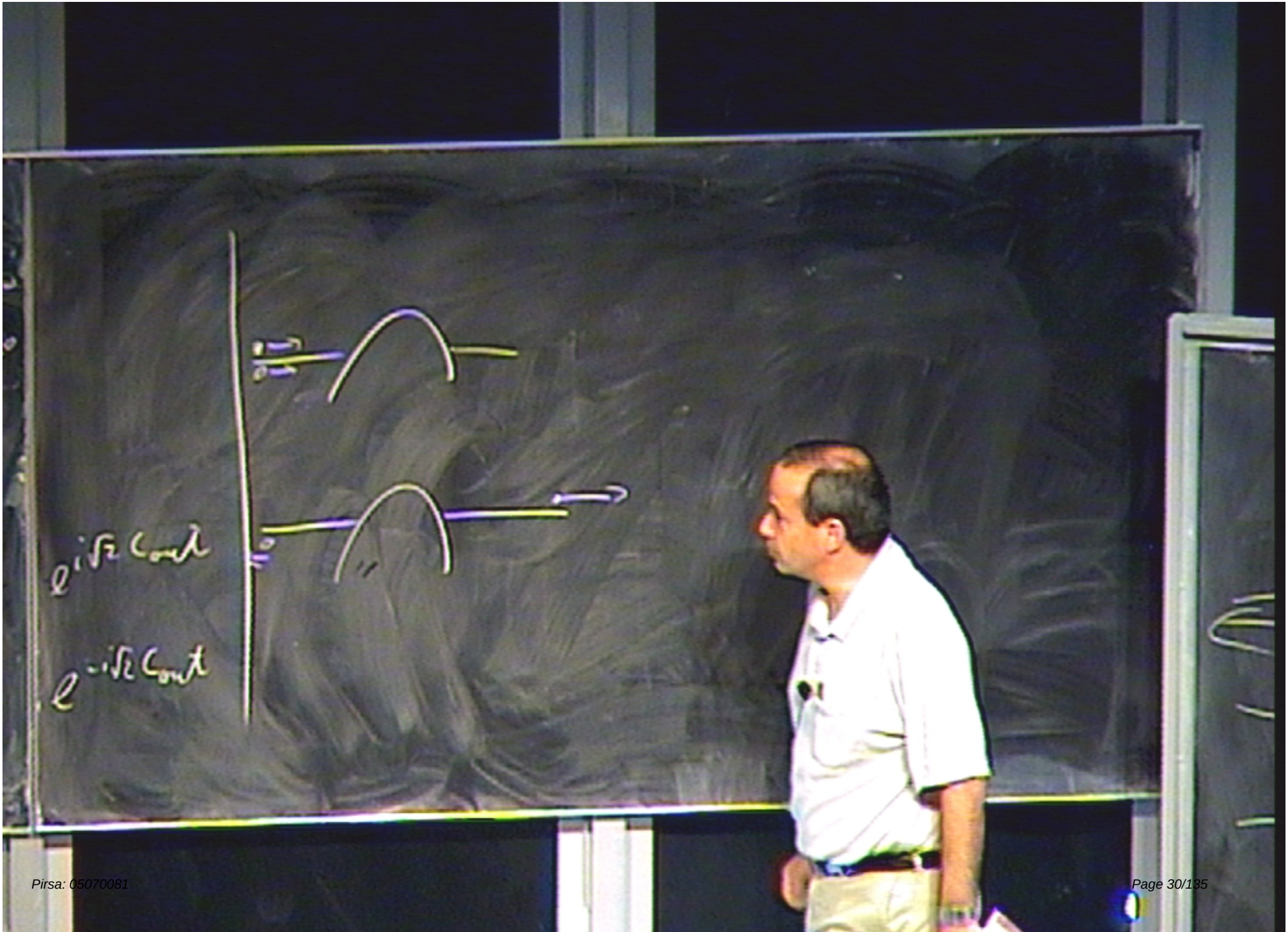

$\partial c \leftrightarrow$ # of fermions w/ $\lambda > 0$
$-$ # " $\lambda < 0$
$e^{i\sqrt{\pi} C_{in}}$
$e^{-i\sqrt{\pi} C_{in}}$

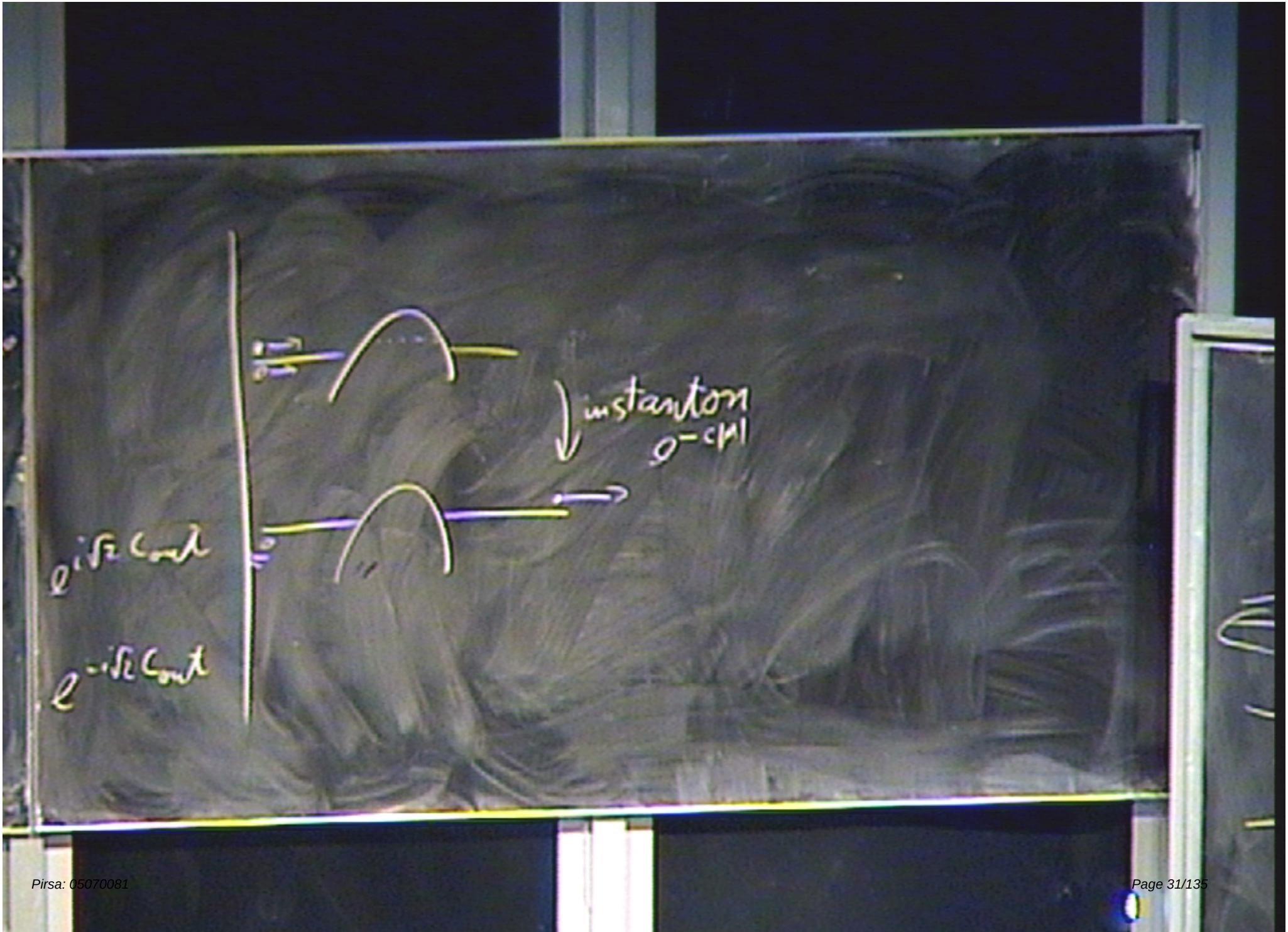

$\partial C \leftrightarrow$ # of fermions w/ $\lambda > 0$
$\lambda < 0$
$- \#$ "
$e^{i\sqrt{\Sigma} C_{lm}}$
$e^{-i\sqrt{\Sigma} C}$

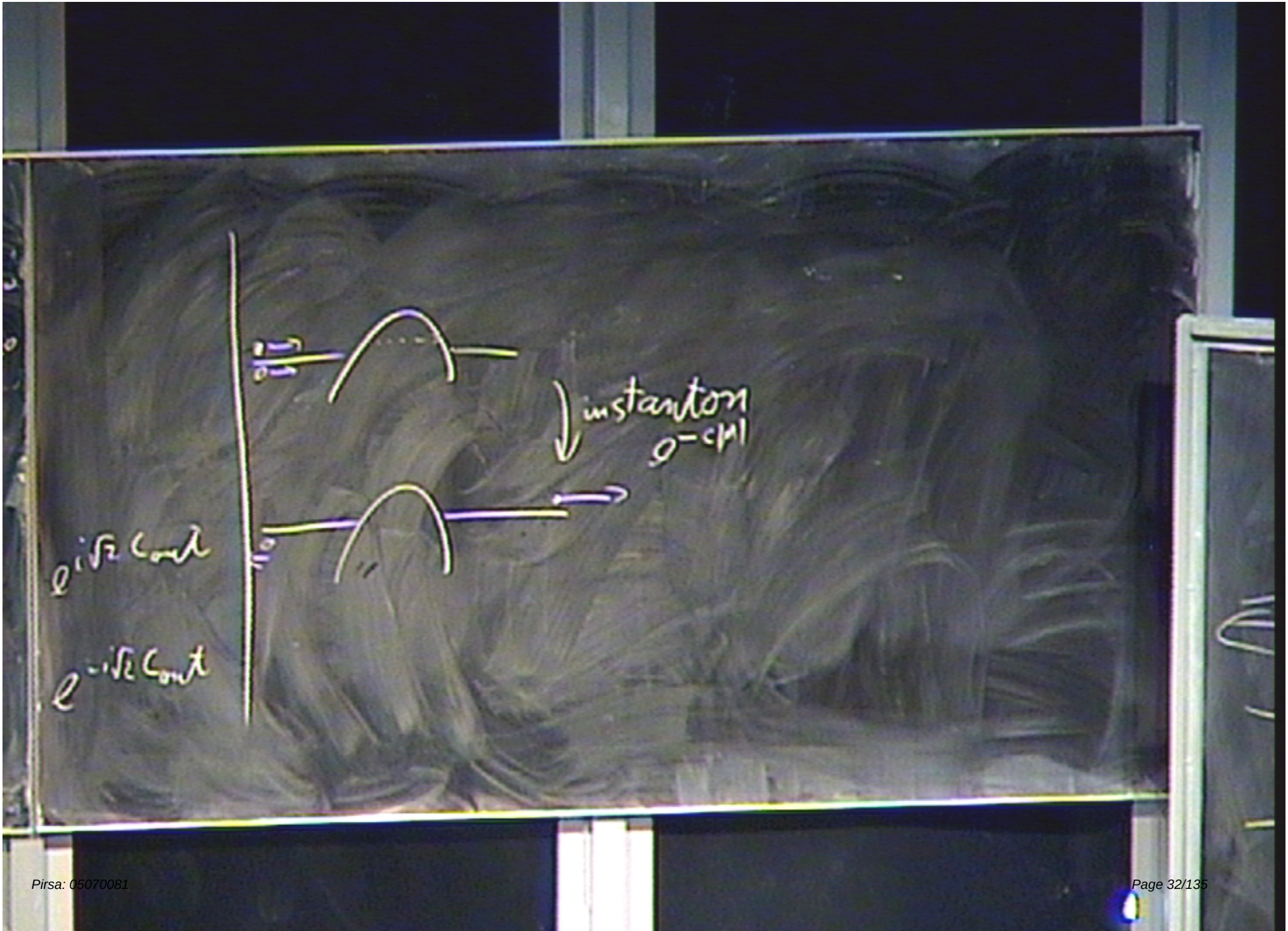

$\mathcal{Q}_C \leftrightarrow$ # of fermions w/ $\lambda > 0$
$-$ # " $\lambda < 0$
$e^{i\sqrt{2}C_{in}}$
$e^{-i\sqrt{2}C_{in}}$

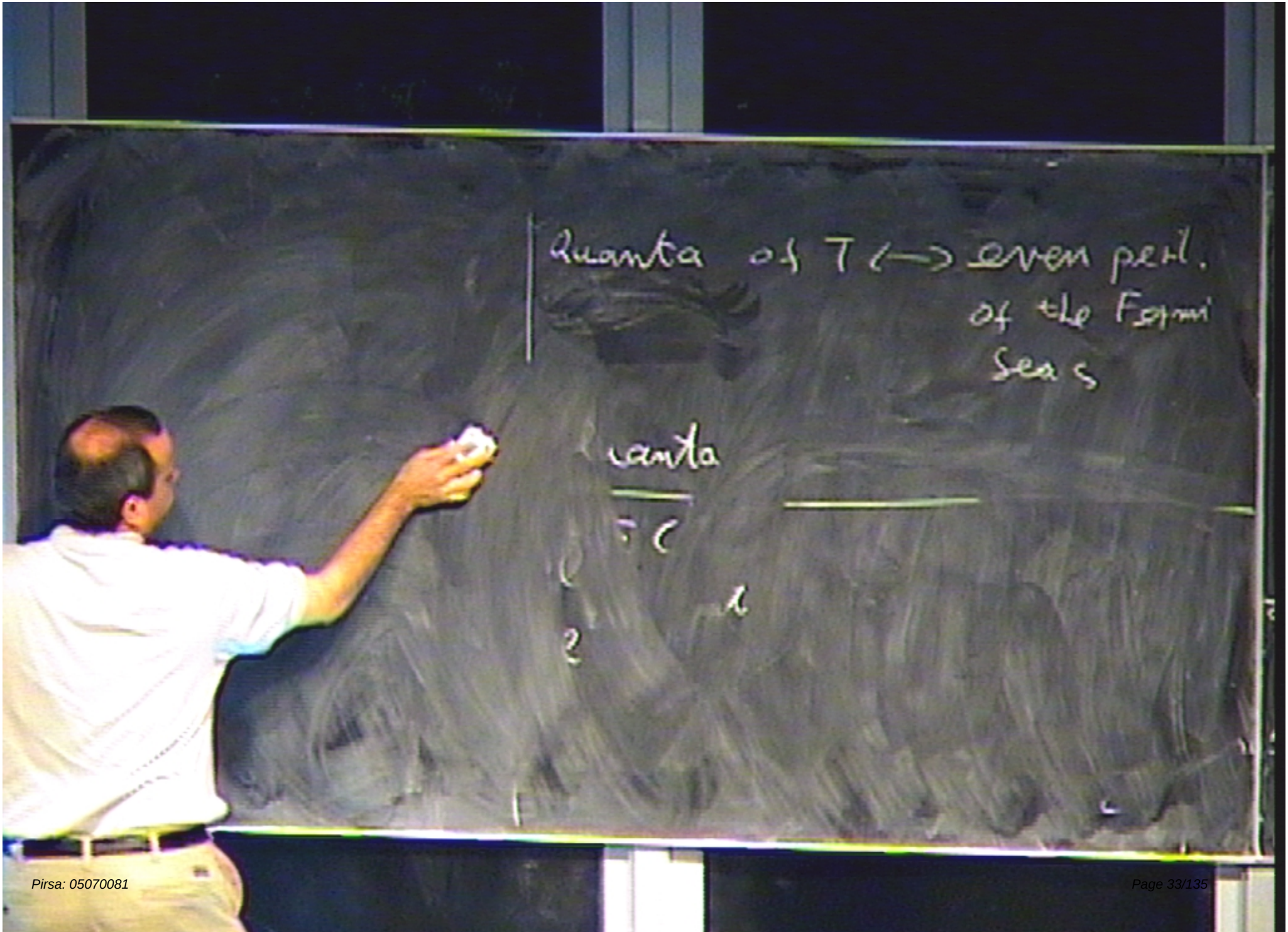

$\partial_C \hookrightarrow$ # of fermions w/ $\lambda > 0$
$-$ # " $\lambda < 0$
$e^{i\sqrt{2}C_{lm}}$
$e^{-i\sqrt{2}C_{lm}}$

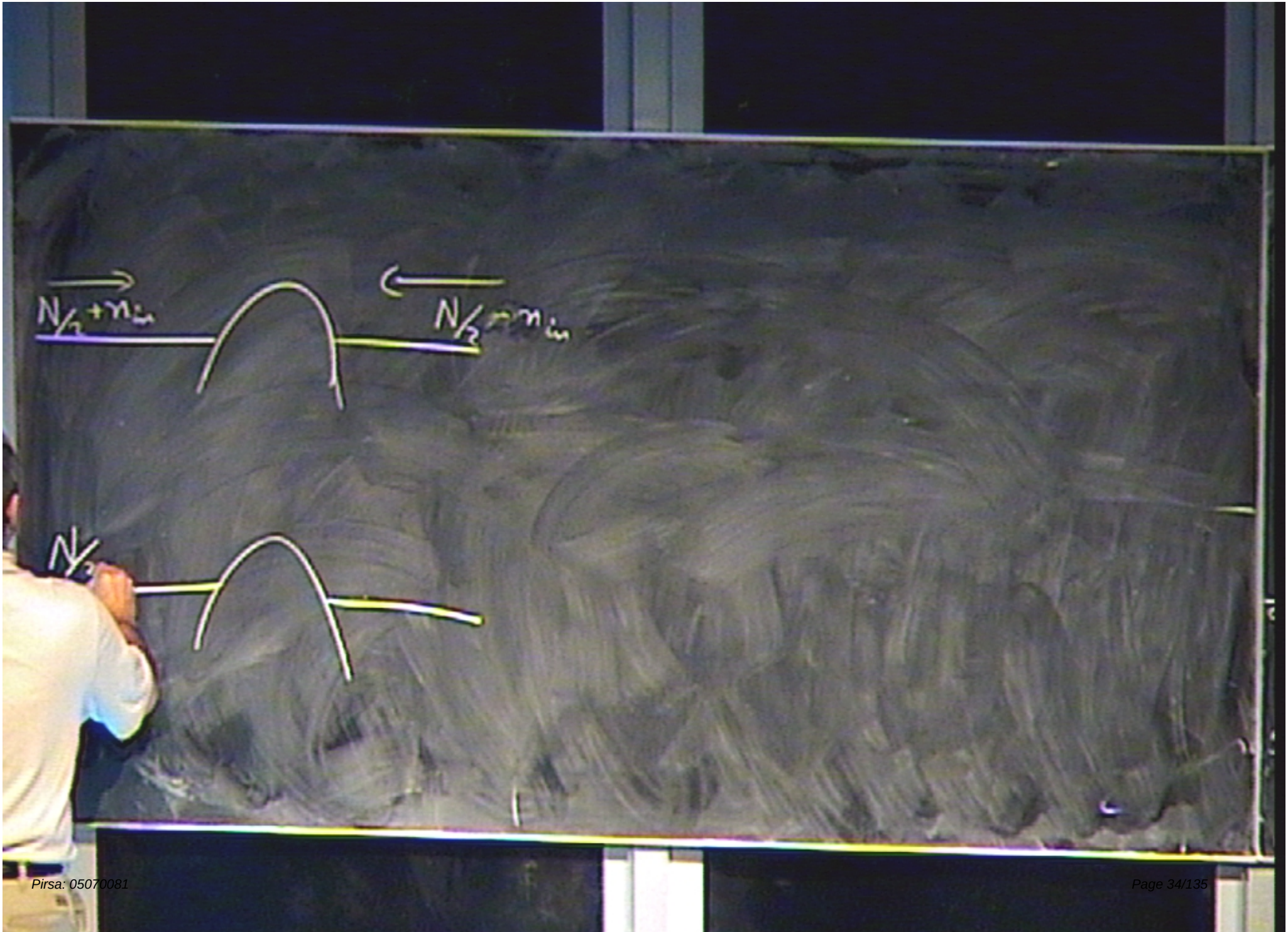

$\mathcal{J}C \hookrightarrow$ # of fermions w/ $\lambda > 0$
$-$ # " $\lambda < 0$
$e^{i\sqrt{2}C_{lm}}$
$e^{-i\sqrt{2}C_{lm}}$

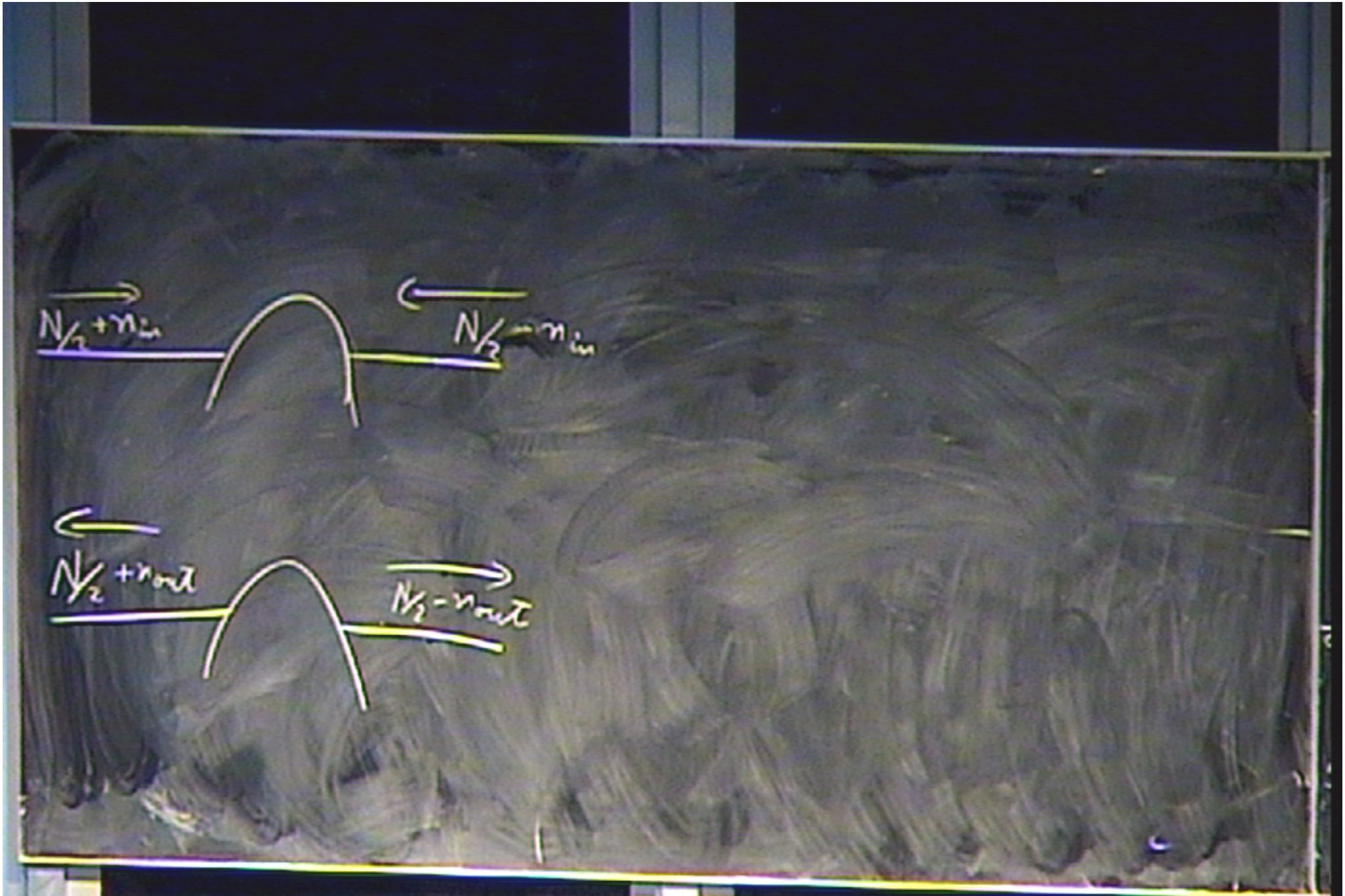

$e^{i\sqrt{2}\,C_{out}t}$
$e^{-i\sqrt{2}\,C_{out}t}$

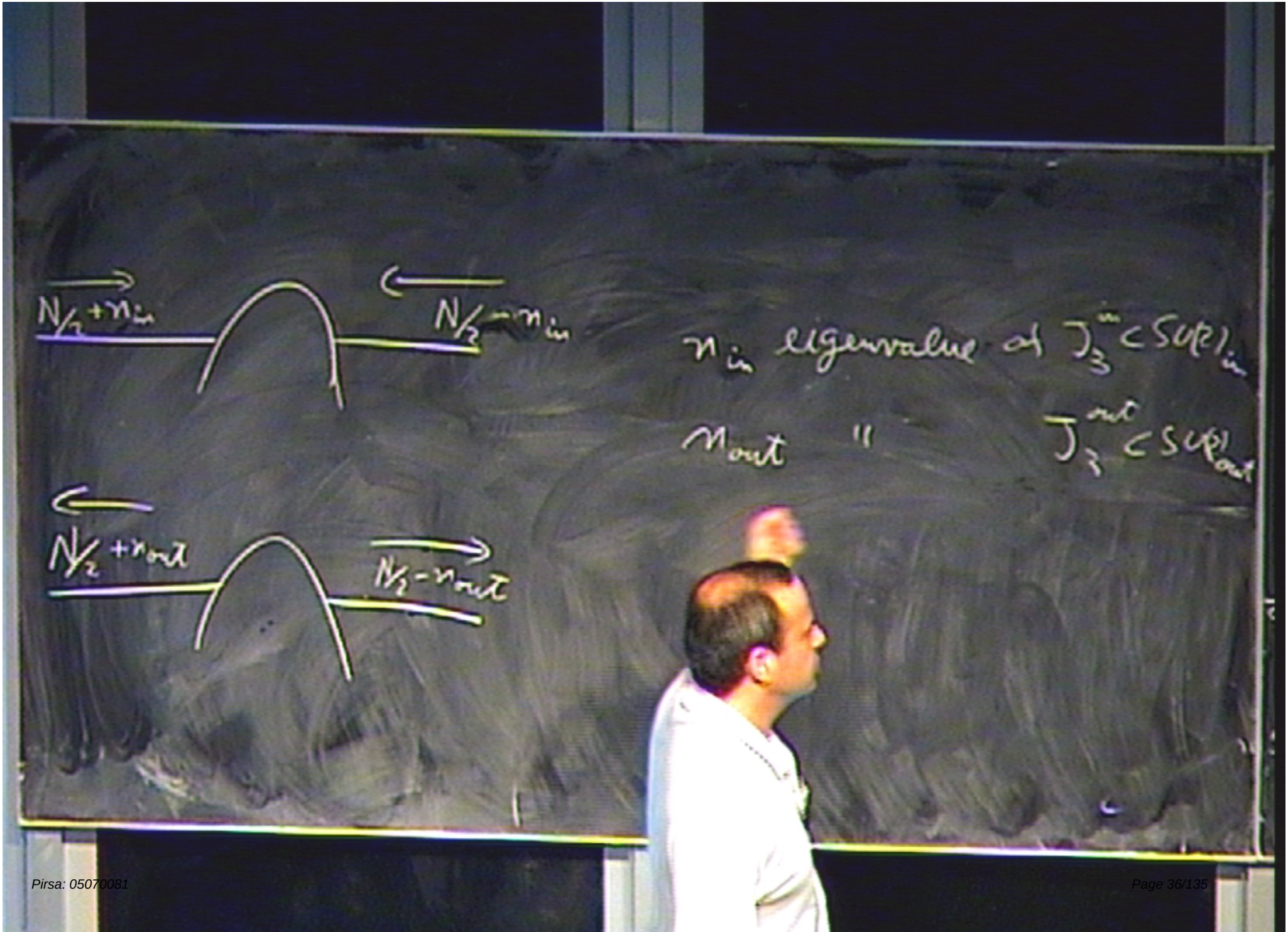

instanton
$e^{i\sqrt{2}c_{cm}t}$
$e^{-i\sqrt{2}c_{cm}t}$


instanton
$e^{i\sqrt{2}c_{cm}t}$
$e^{-i\sqrt{2}c_{cm}t}$


Quanta of T ⟷ even perl.
of the Fermi
Seas
Quanta
FC


N/2 + m_in
N/2 - m_in
N/2


$N/2 + n_{in}$
$N/2 - n_{in}$
$N/2 + n_{out}$
$N/2 - n_{out}$

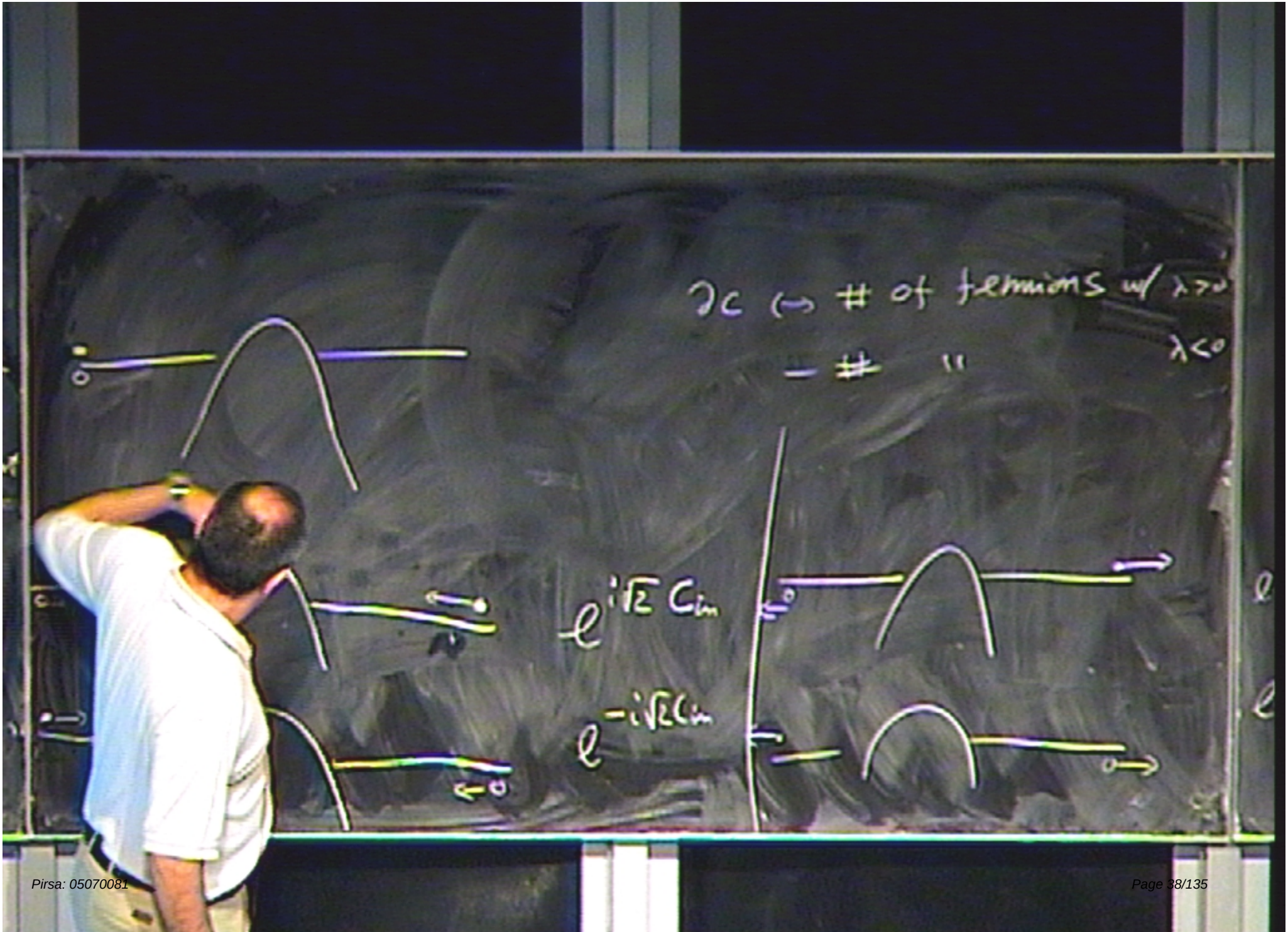

$N/2 + n_{in}$
$N/2 - n_{in}$
n_{in} eigenvalue of $J_3^{in} \subset SU(2)_{in}$
n_{out} " $J_3^{out} \subset SU(2)_{out}$
$N/2 + n_{out}$
$N/2 - n_{out}$

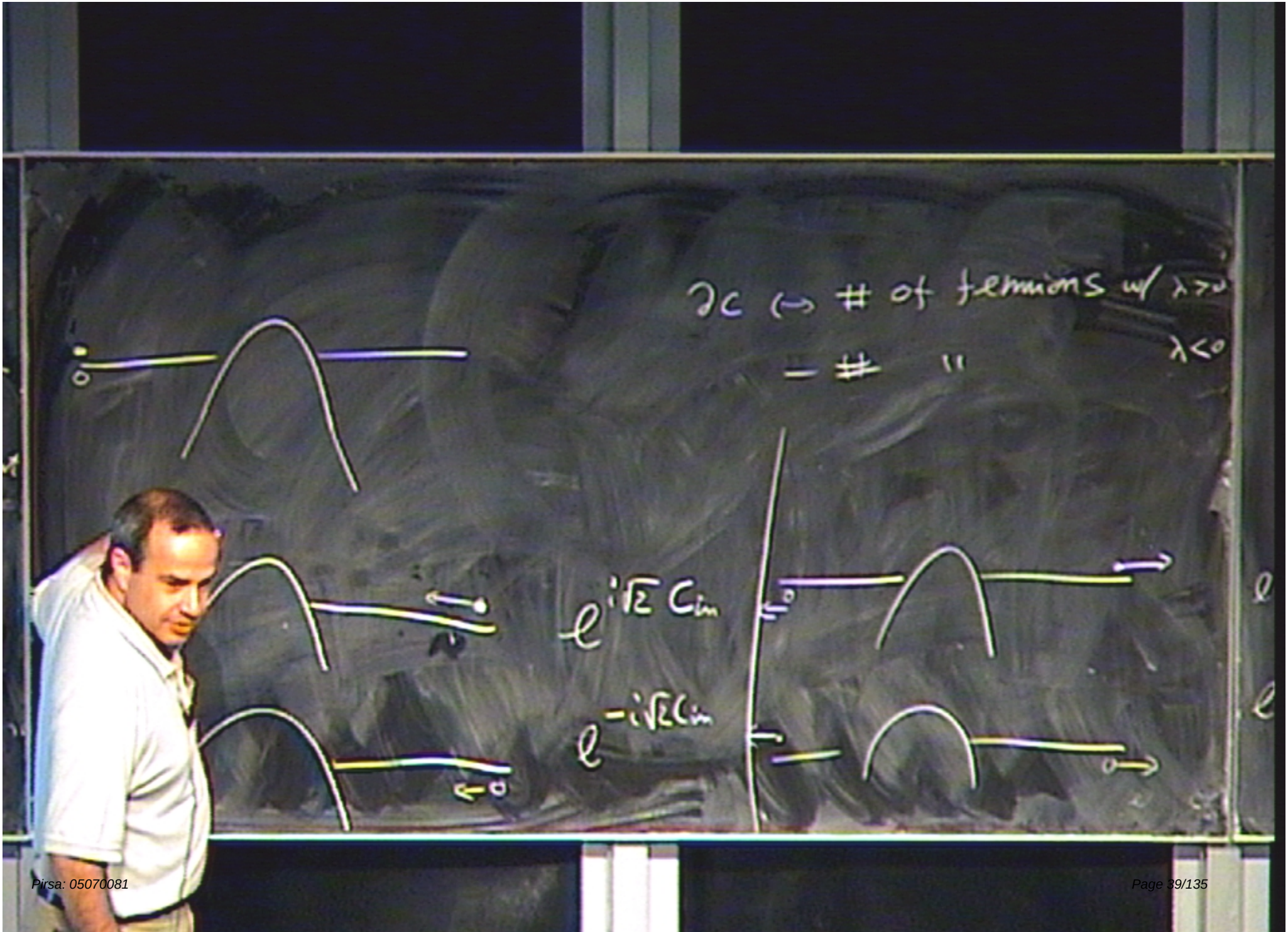


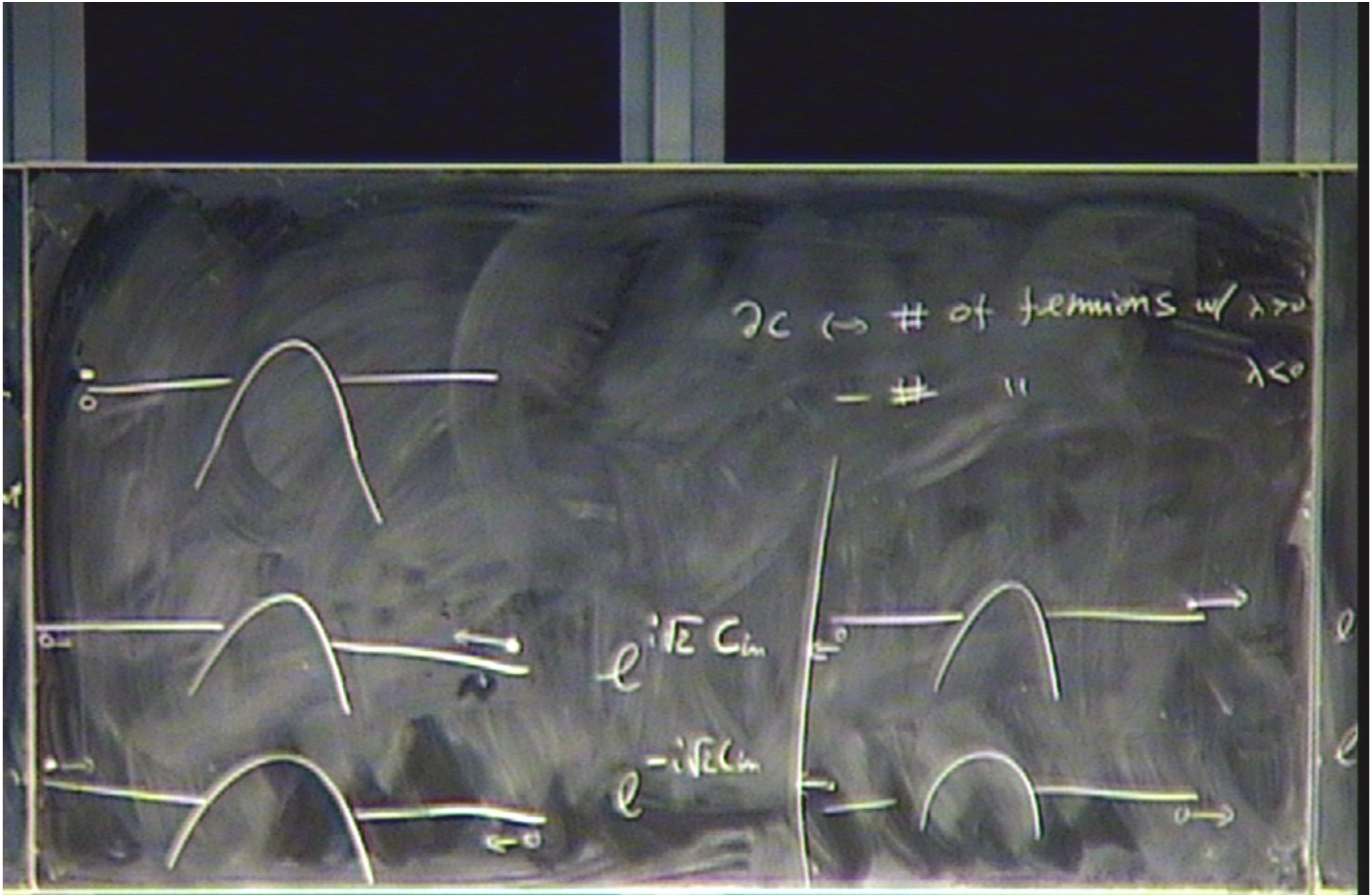
n_{in} eigenvalue of $J_3^{in} \subset SU(2)_{in}$

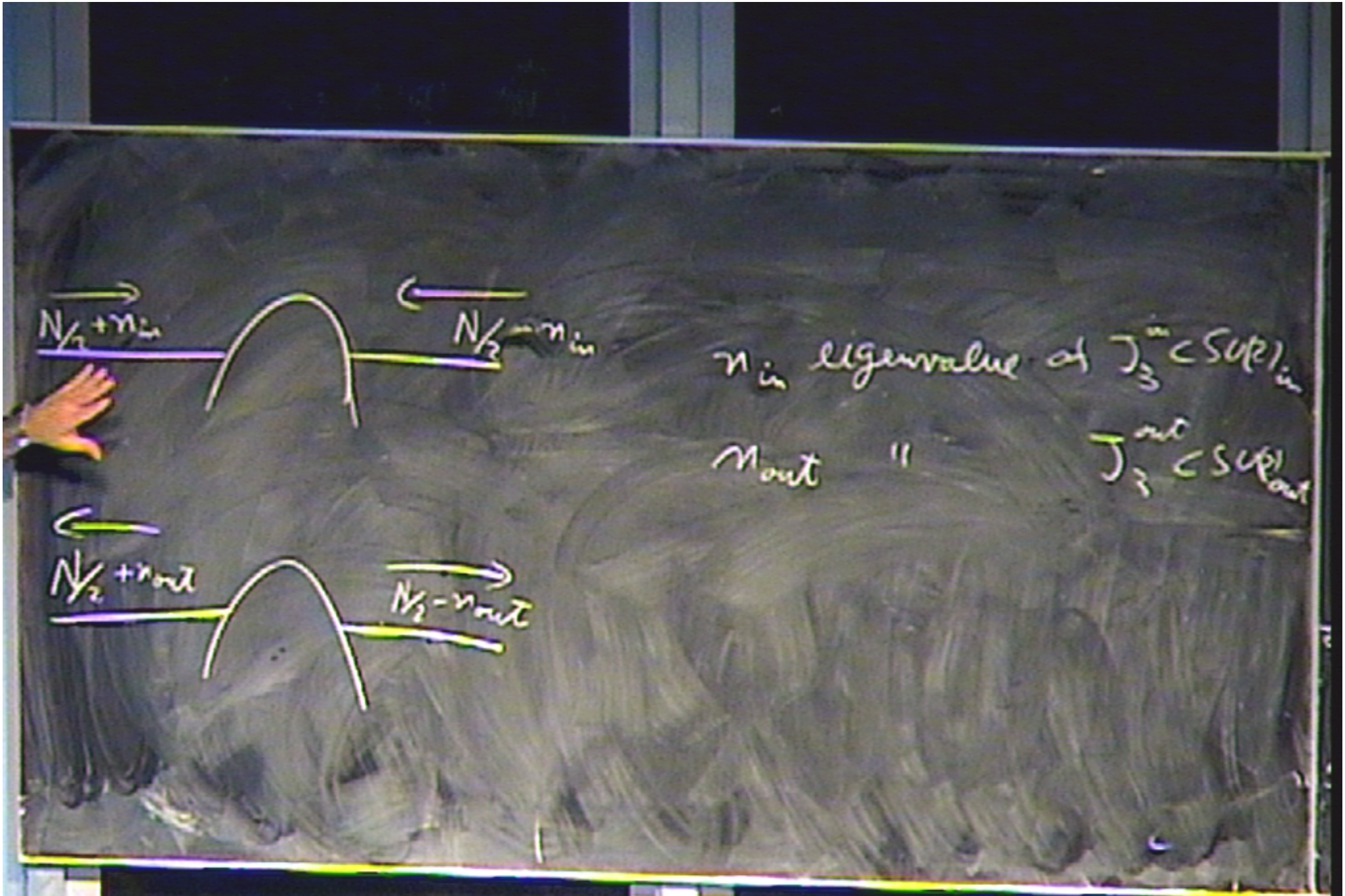
n_{out} " $J_3^{out} \subset SU(2)_{out}$

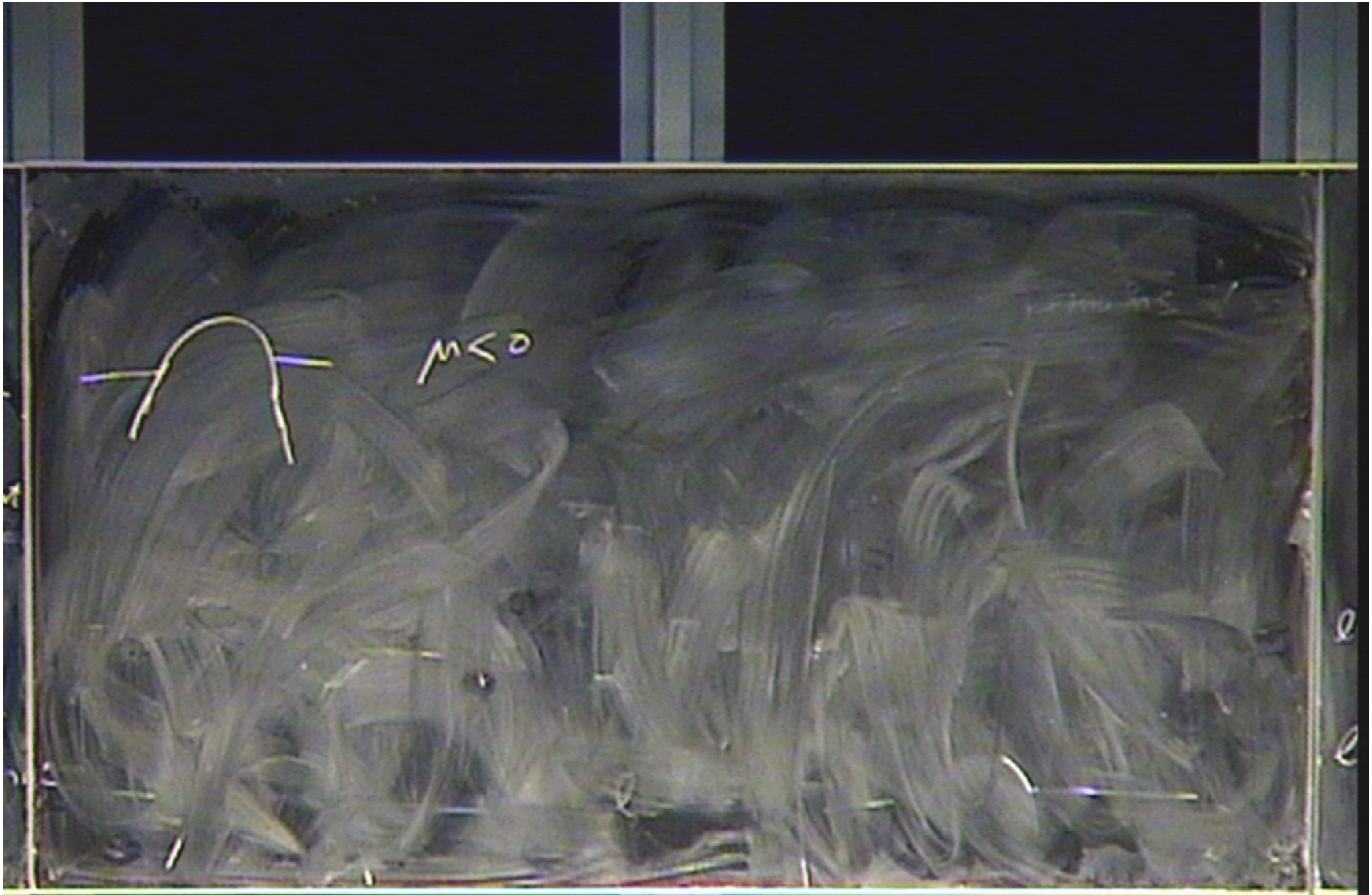


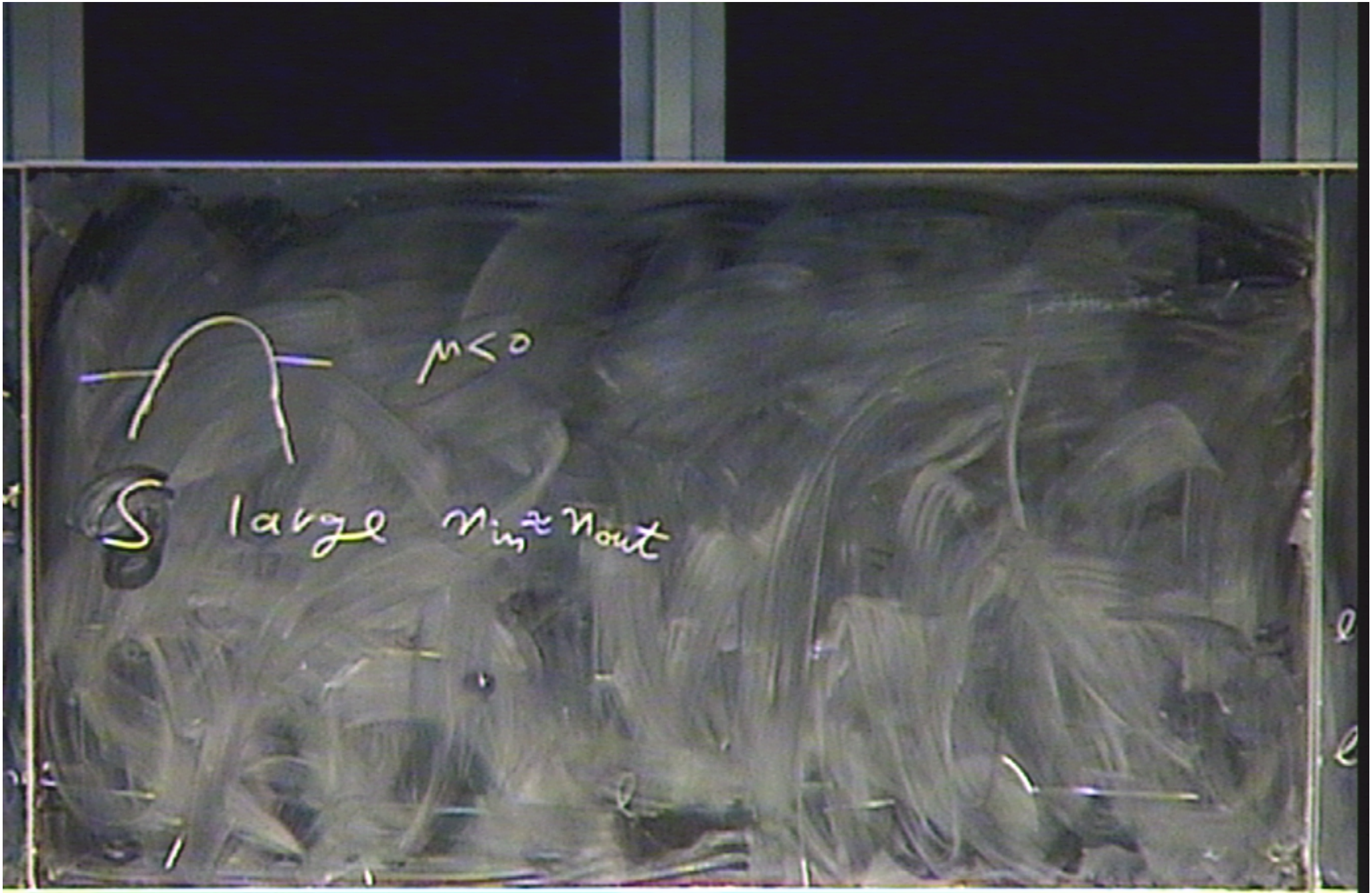

$\mathcal{H}_C \hookrightarrow$ # of fermions w/ $\lambda > 0$
$\lambda < 0$
$- \#$ "
$e^{i\sqrt{2}\, C_{in}}$
$e^{-i\sqrt{2} C_{in}}$

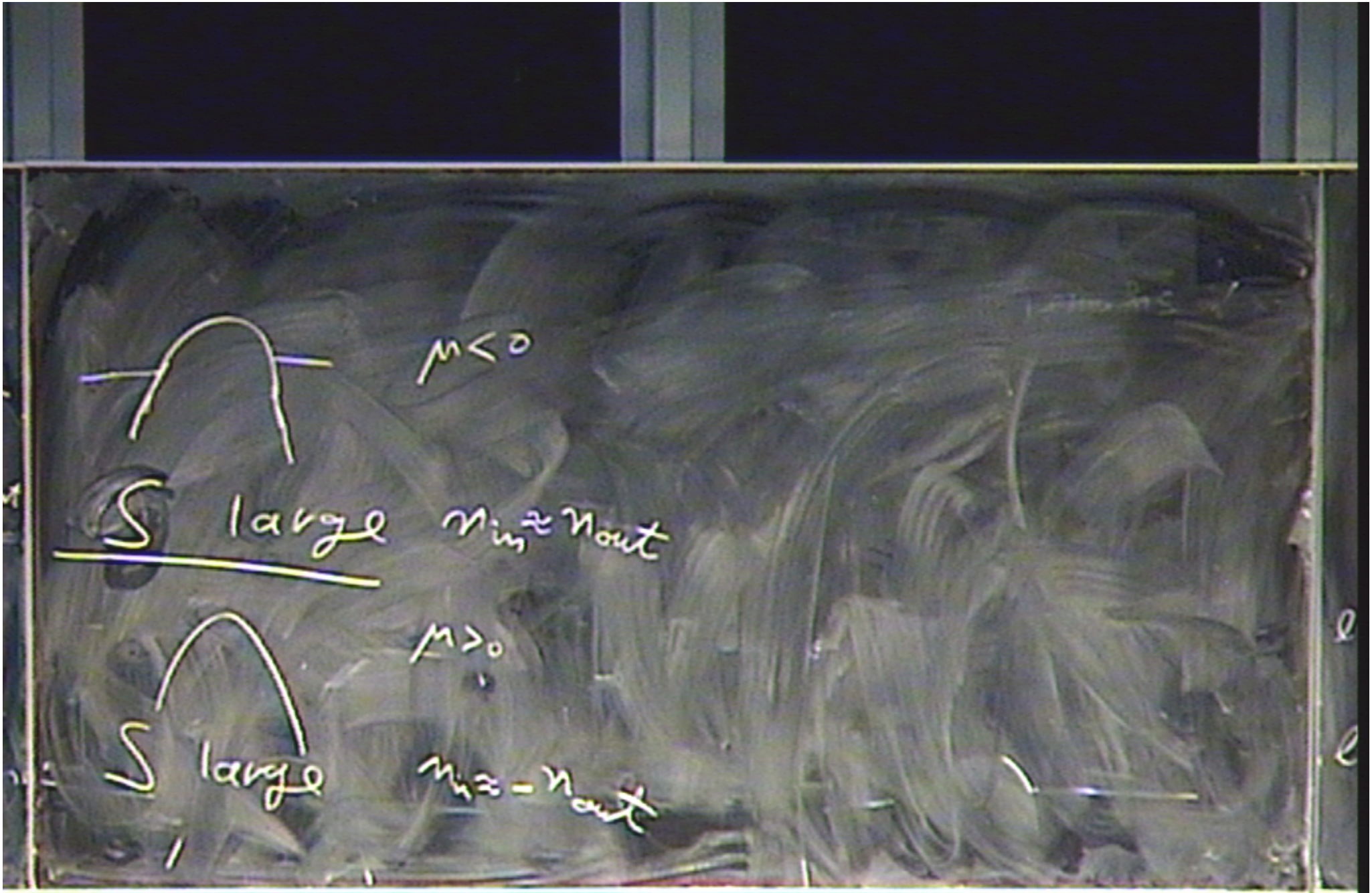

$\partial c \leftrightarrow$ # of fermions w/ $\lambda > 0$
$\lambda < 0$
$- \#$ "
$e^{i\sqrt{2}C_{lm}}$
$e^{-i\sqrt{2}C_{lm}}$

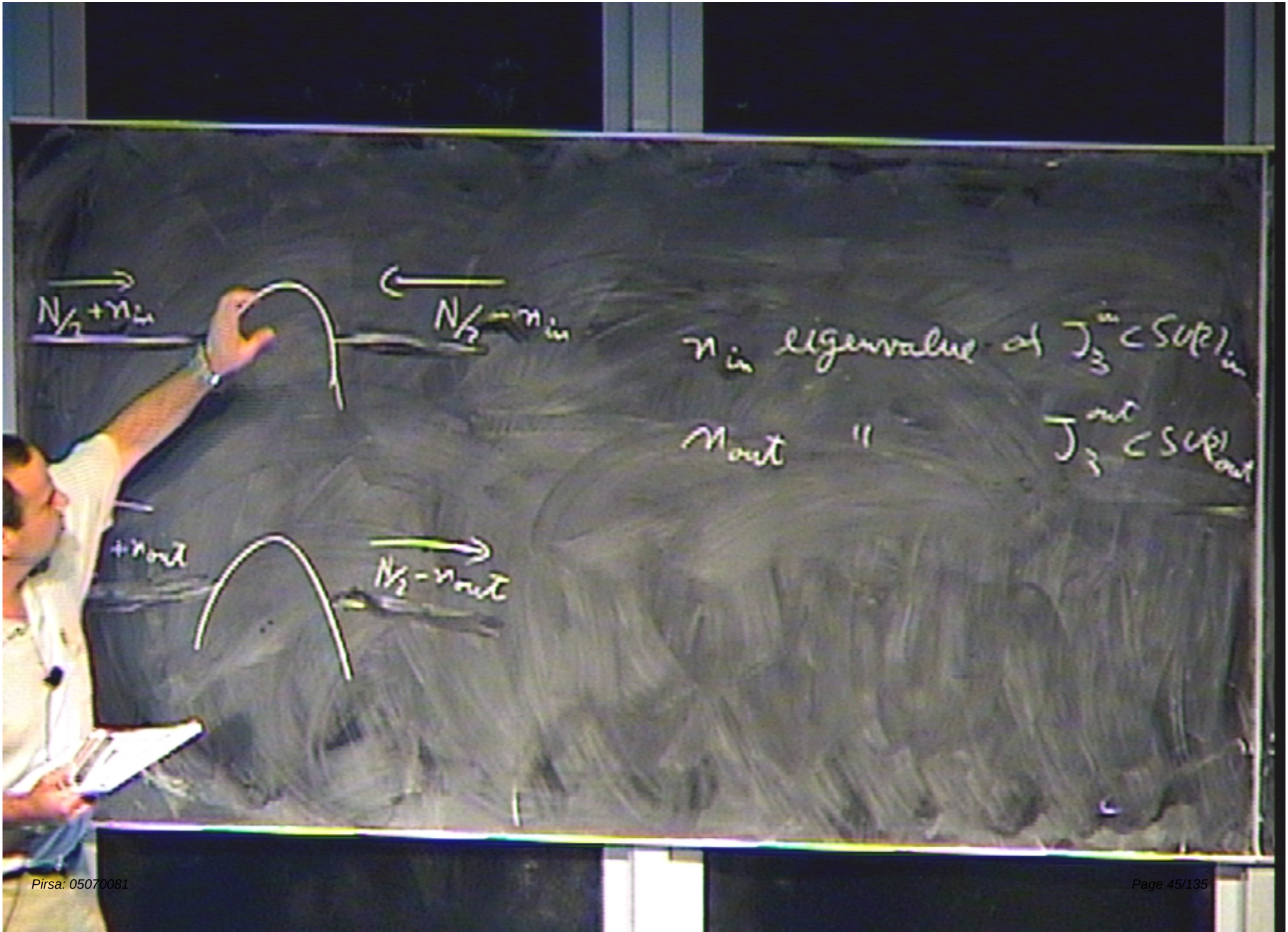

$\partial c \leftrightarrow$ # of fermions w/ $\lambda > 0$
$\lambda < 0$
$- \#$ "
$e^{i\sqrt{2}\, c_{in}}$
$e^{-i\sqrt{2} c_{in}}$

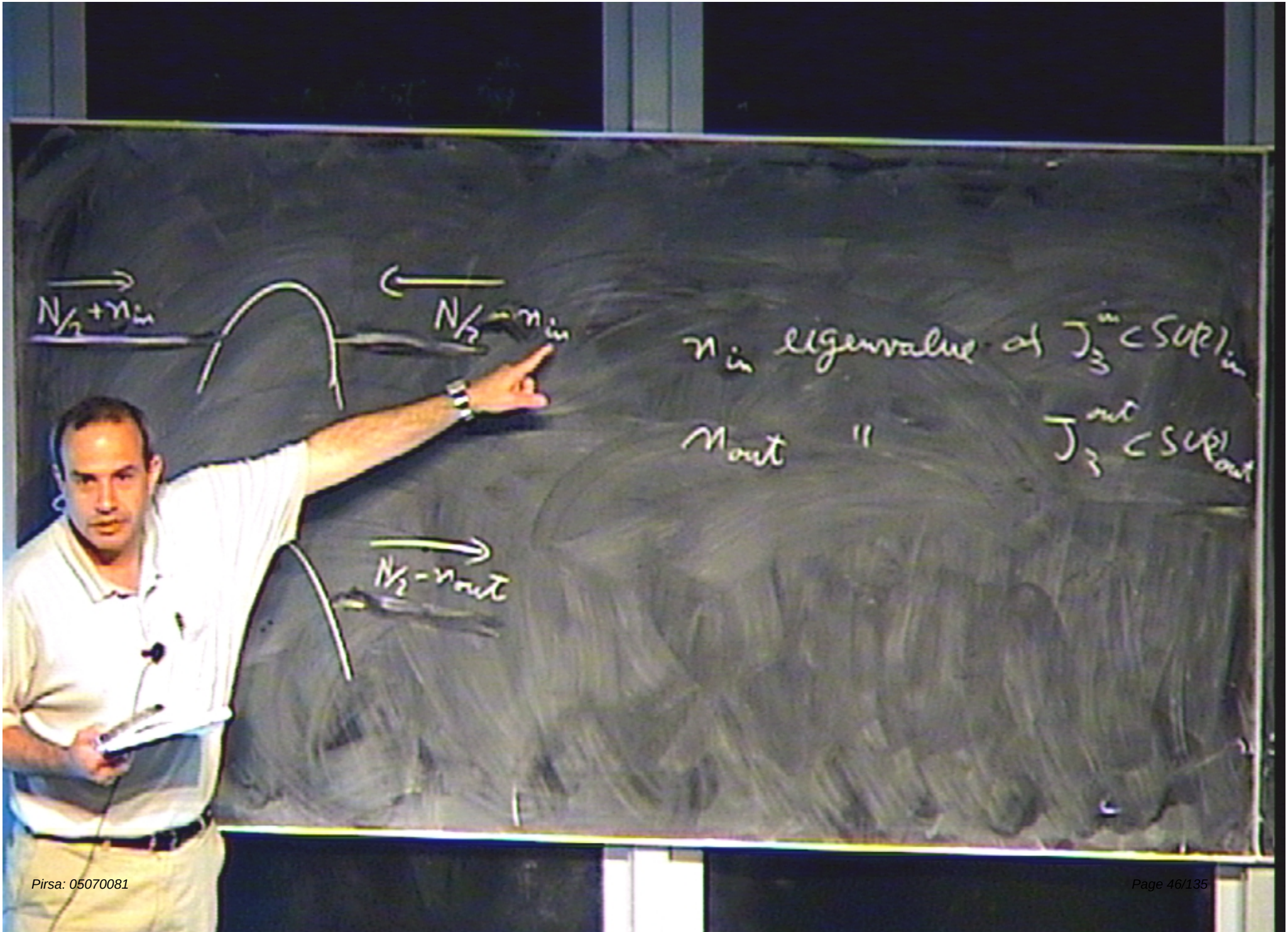

$N/2 + n_{in}$
$N/2 - n_{in}$
n_{in} eigenvalue of $J_3^{in} \subset SU(2)_{in}$
n_{out} " $J_3^{out} \subset SU(2)_{out}$
$N/2 + n_{out}$
$N/2 - n_{out}$

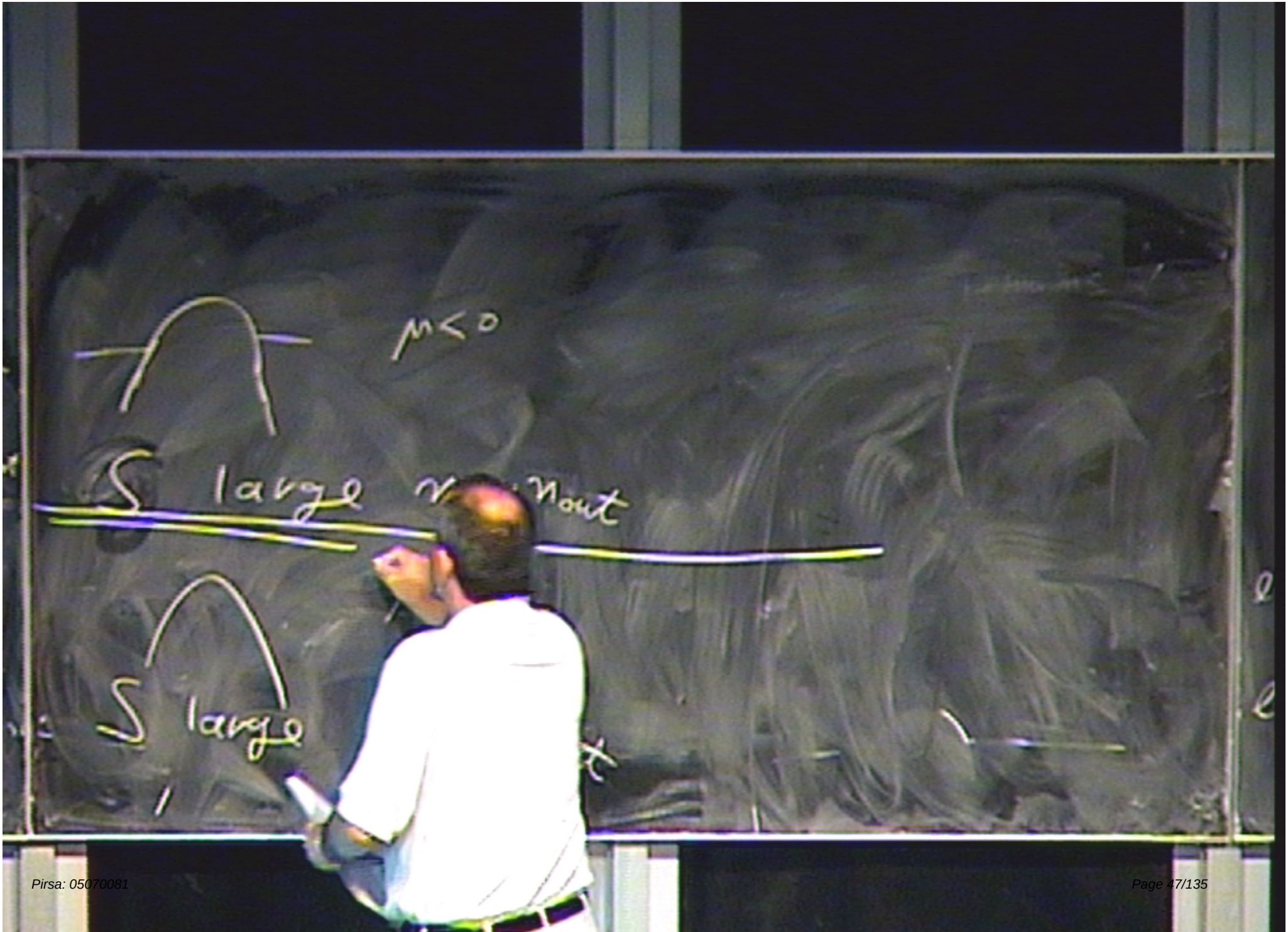

M<0

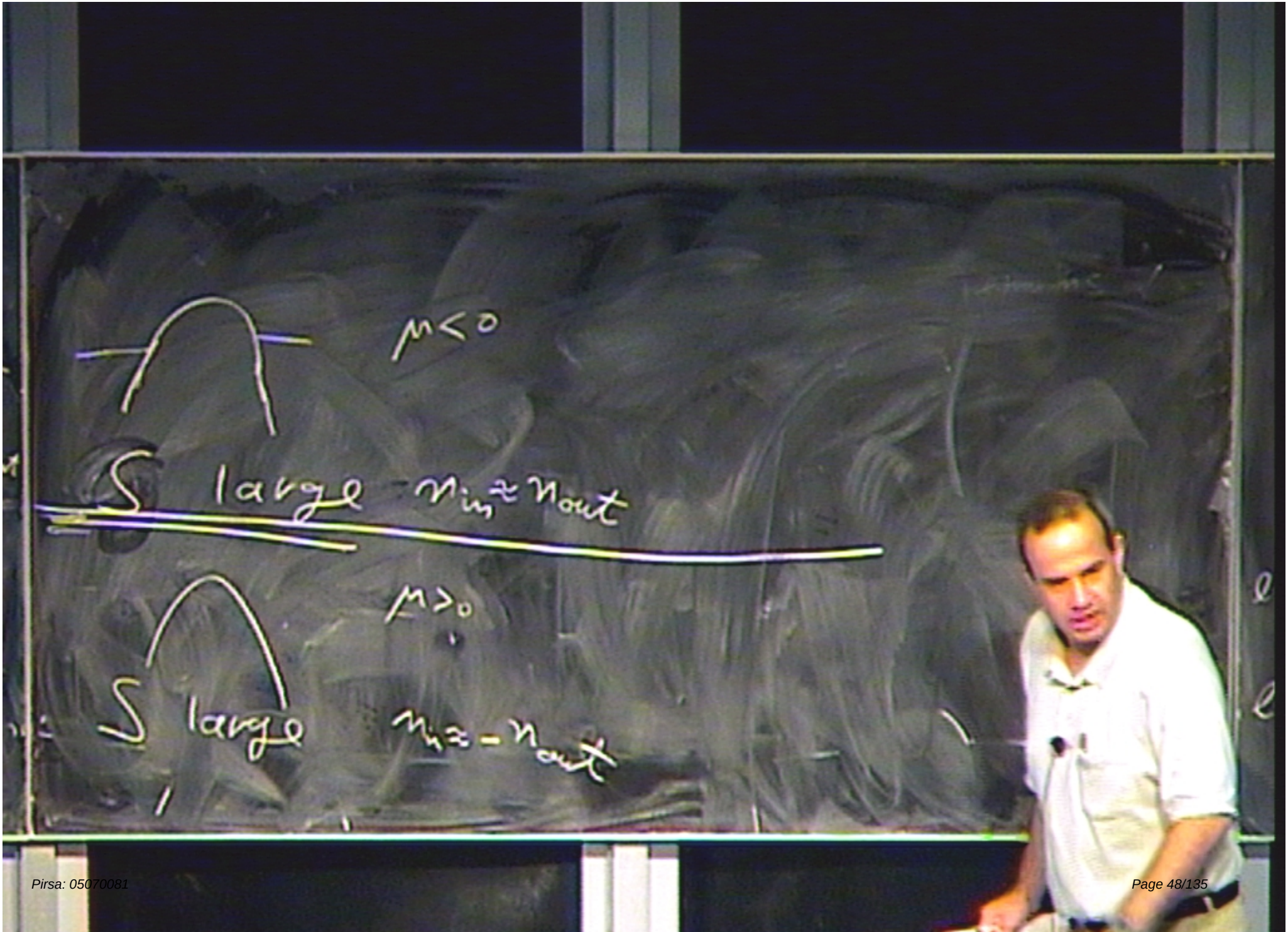

μ<0
S large $n_{in} \neq n_{out}$

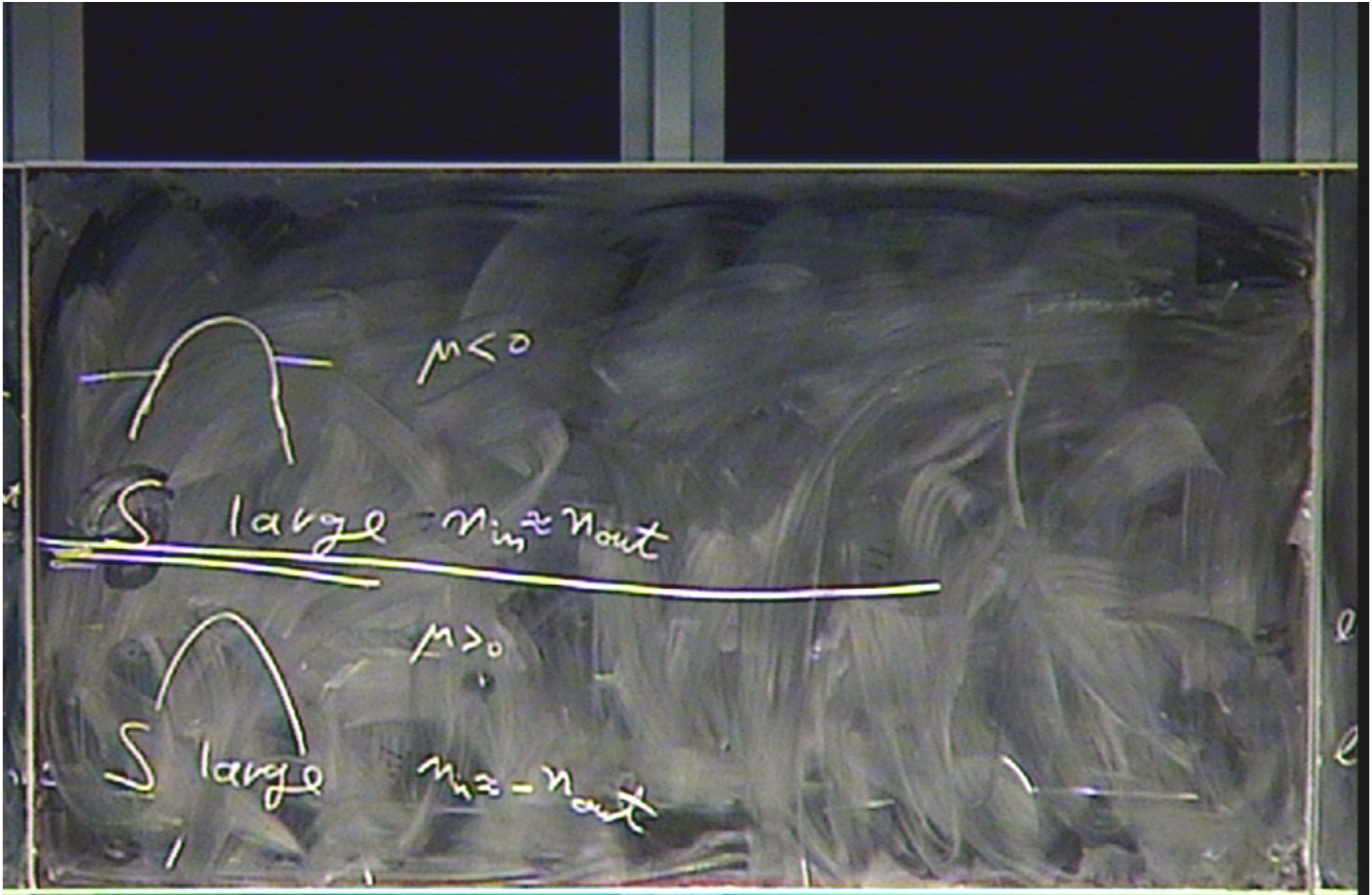

$\mu < 0$
S large $n_{in} \approx n_{out}$
$\mu > 0$
S large $n_{in} \approx -n_{out}$

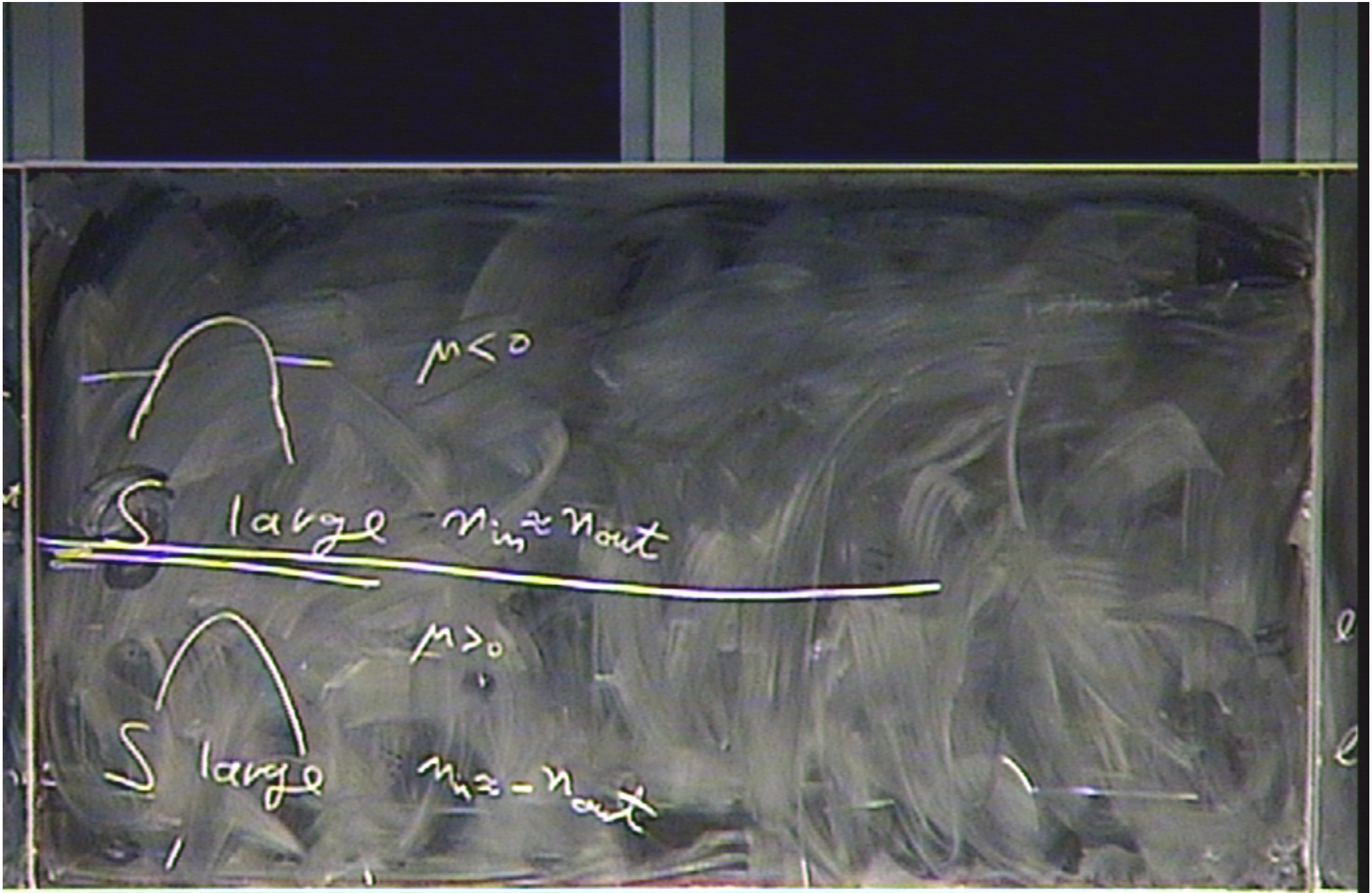

$N/2 + n_{in}$
$N/2 - n_{in}$
n_{in} eigenvalue of $J_3^{in} \subset SU(2)_{in}$
n_{out} " $J_3^{out} \subset SU(2)_{out}$
$+n_{out}$
$N/2 - n_{out}$

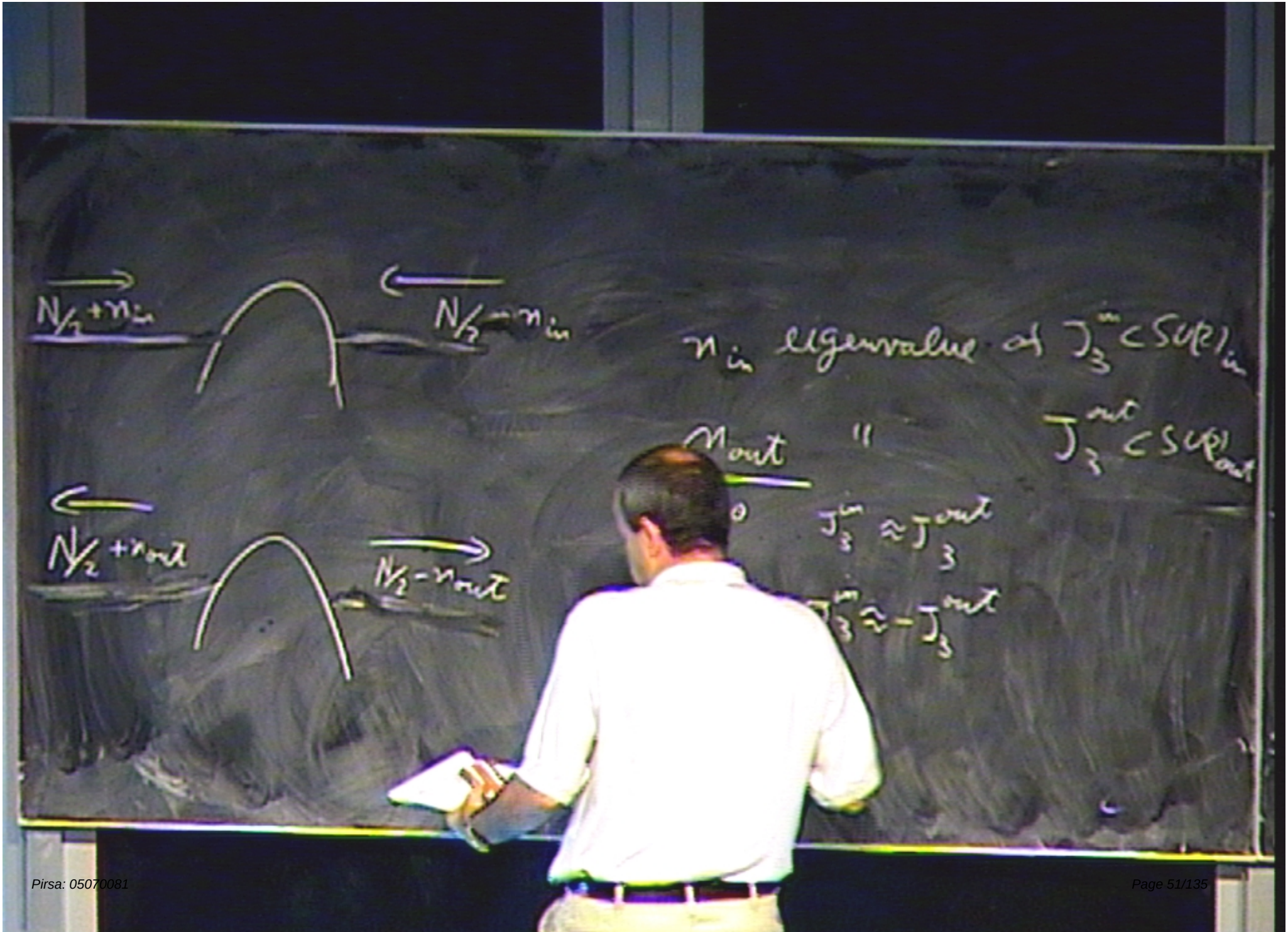

$N/2 + n_{in}$
$N/2 - n_{in}$
n_{in} eigenvalue of $J_3^{in} \subset SU(2)_{in}$
n_{out} " $J_3^{out} \subset SU(2)_{out}$
$N/2 - n_{out}$

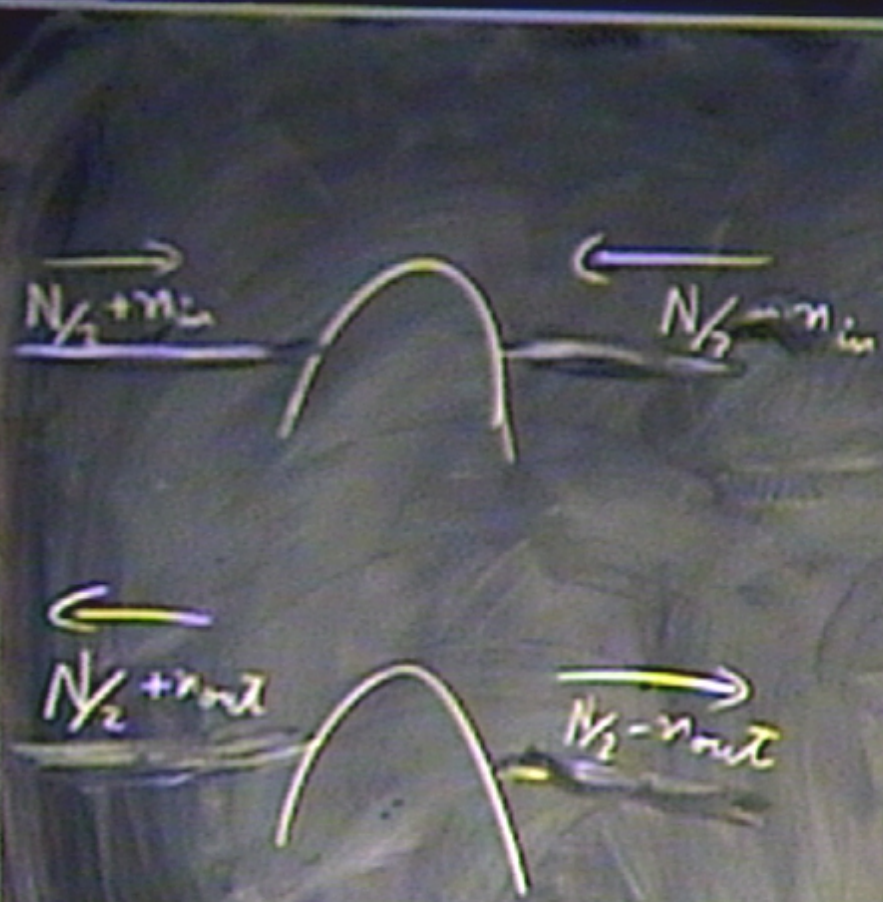

μ<0
S large
S large


$\mu < 0$
S large $n_{in} \approx n_{out}$
$\mu > 0$
S large $n_{in} \approx -n_{out}$


$\mu < 0$
S large $n_{in} \approx n_{out}$
$\mu > 0$
S large $n_{in} \approx -n_{out}$


$\mu < 0$
S large $n_{in} \approx n_{out}$
$\mu > 0$
S large $n_{in} \approx -n_{out}$


$N/2 + n_{in}$
$N/2 - n_{in}$
n_{in} eigenvalue of $J_3^{in} \subset SU(2)_{in}$
n_{out} = 0
$J_3^{out} \subset SU(2)_{out}$
$J_3^{in} \approx J_3^{out}$
$J_3^{in} \approx -J_3^{out}$
$N/2 + n_{out}$
$N/2 - n_{out}$

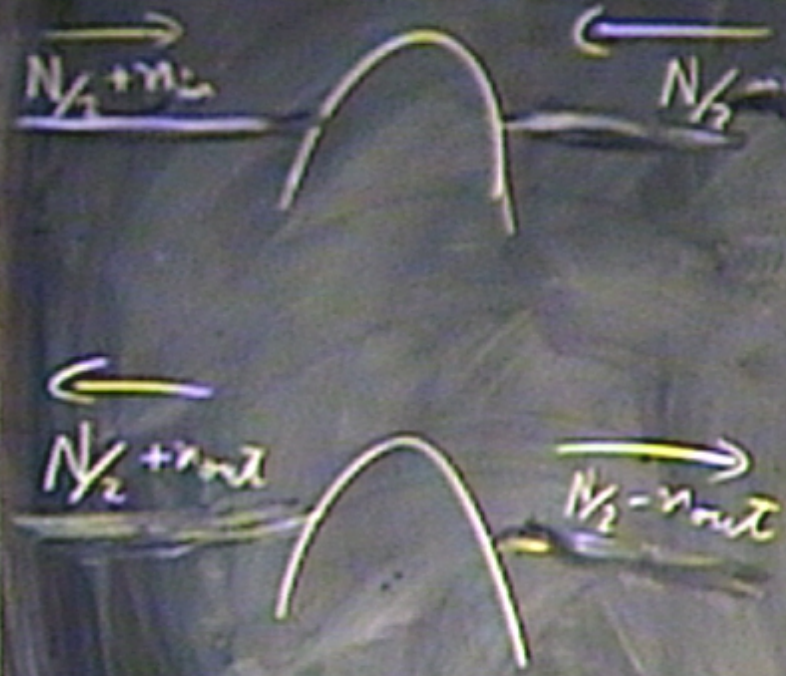


n_{in} eigenvalue of $J_3^{in} \subset SU(2)_{in}$

n_{out} " $J_3^{out} \subset SU(2)_{out}$

$\mu < 0$ $J_3^{in} \approx J_3^{out}$

$\mu > 0$ $J_3^{in} \approx -J_3^{out}$

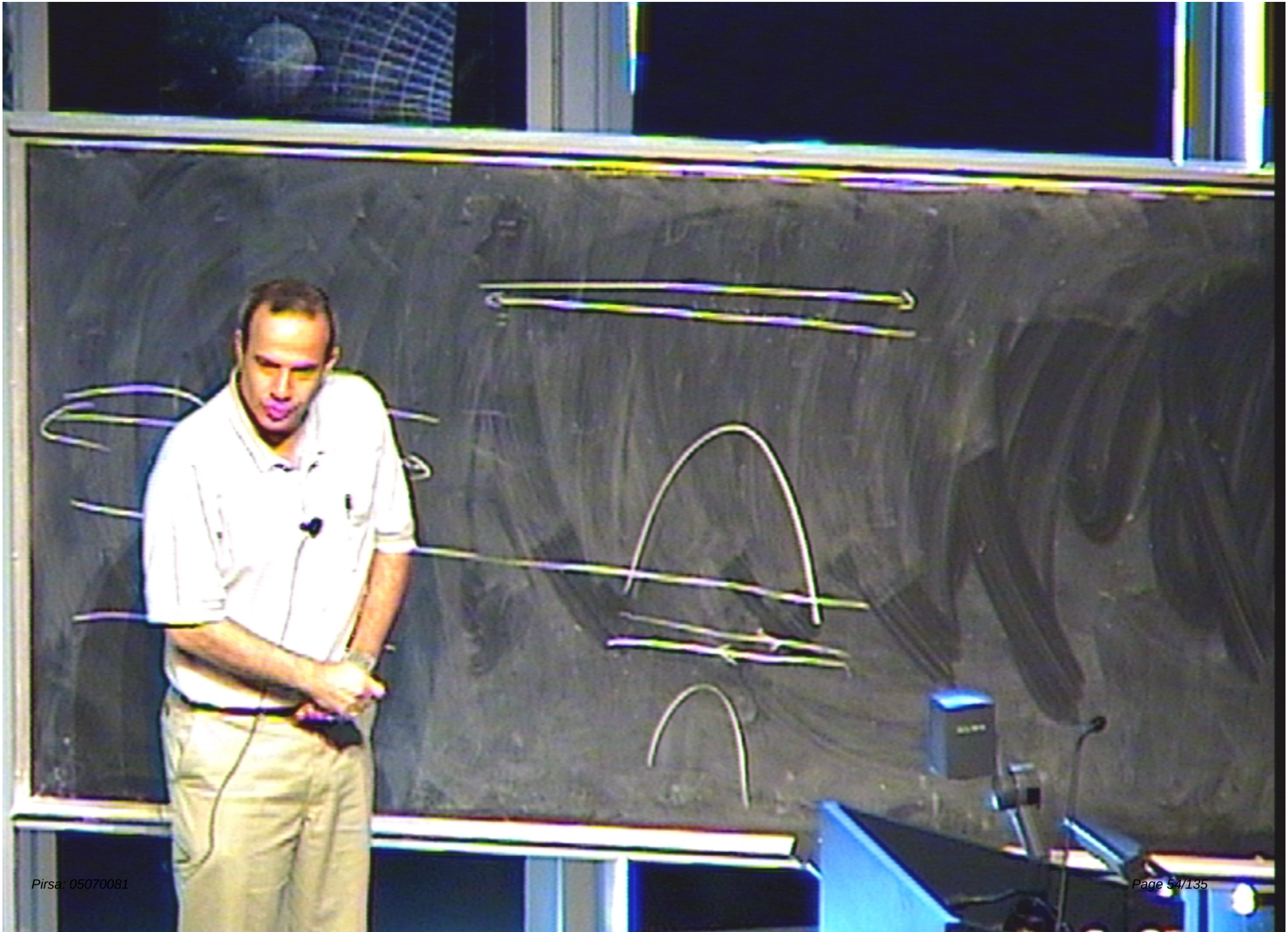


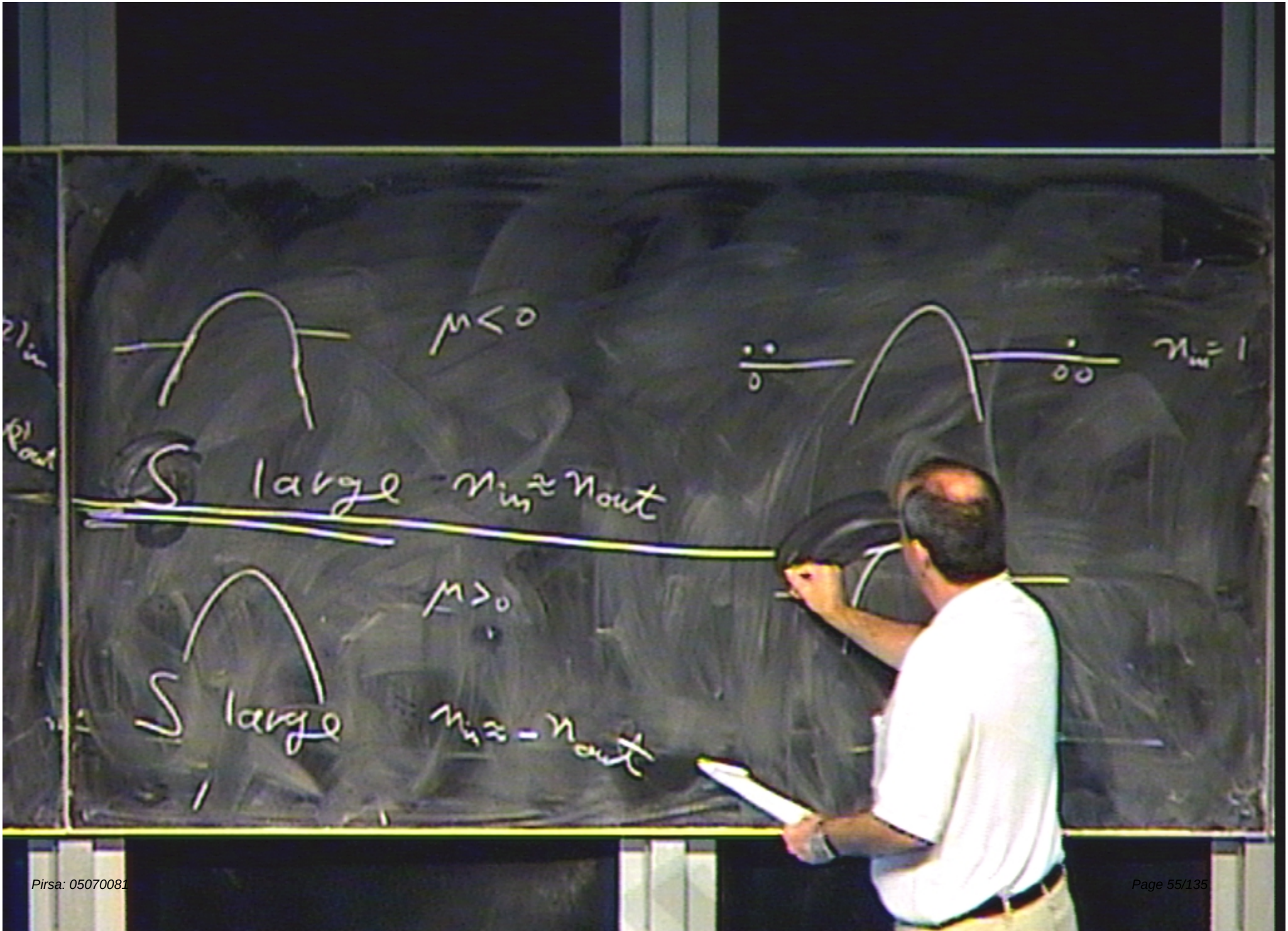
n_{in} eigenvalue of $J_3^{in} \subset SU(2)_{in}$

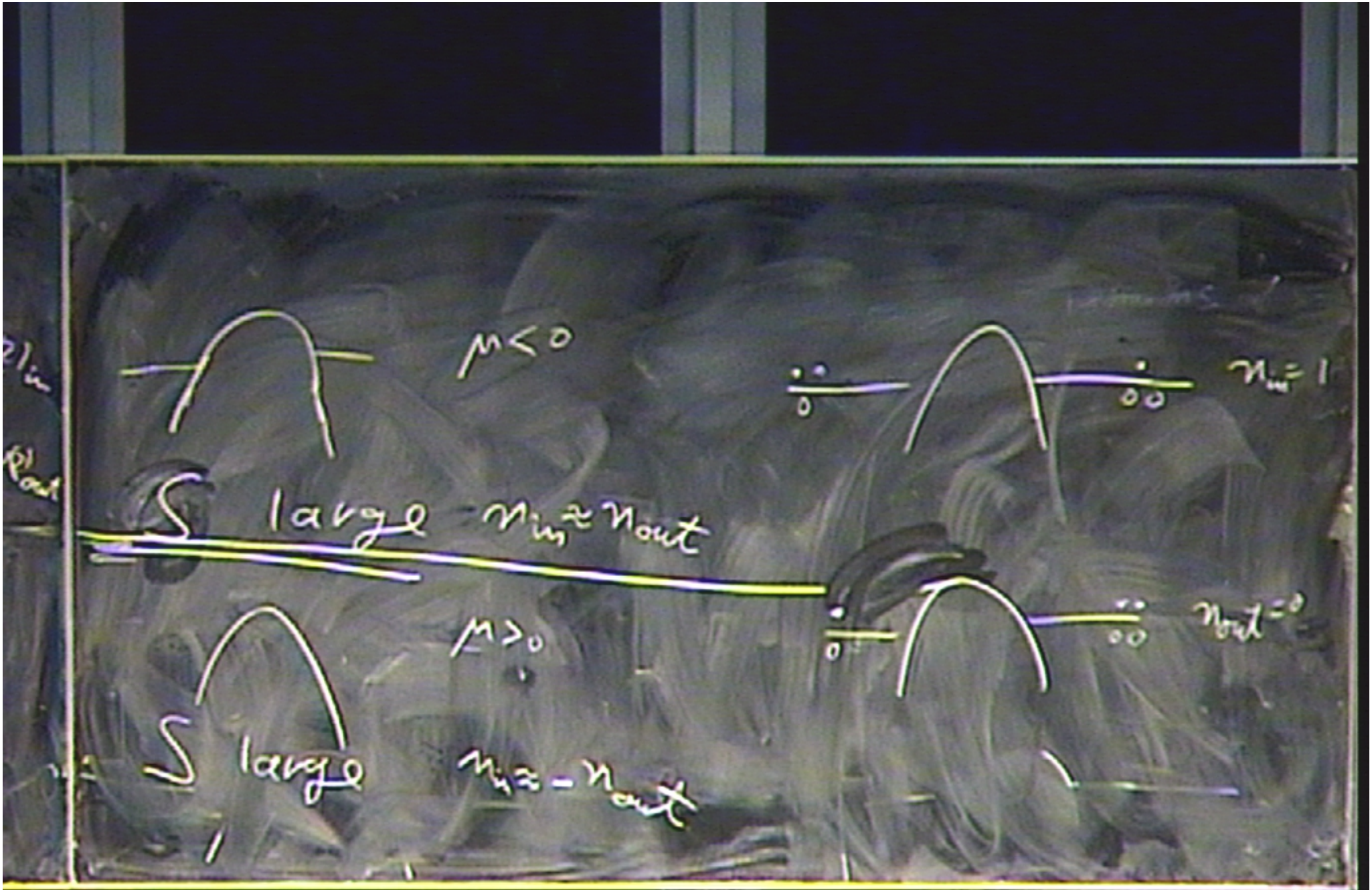
n_{out} " $J_3^{out} \subset SU(2)_{out}$

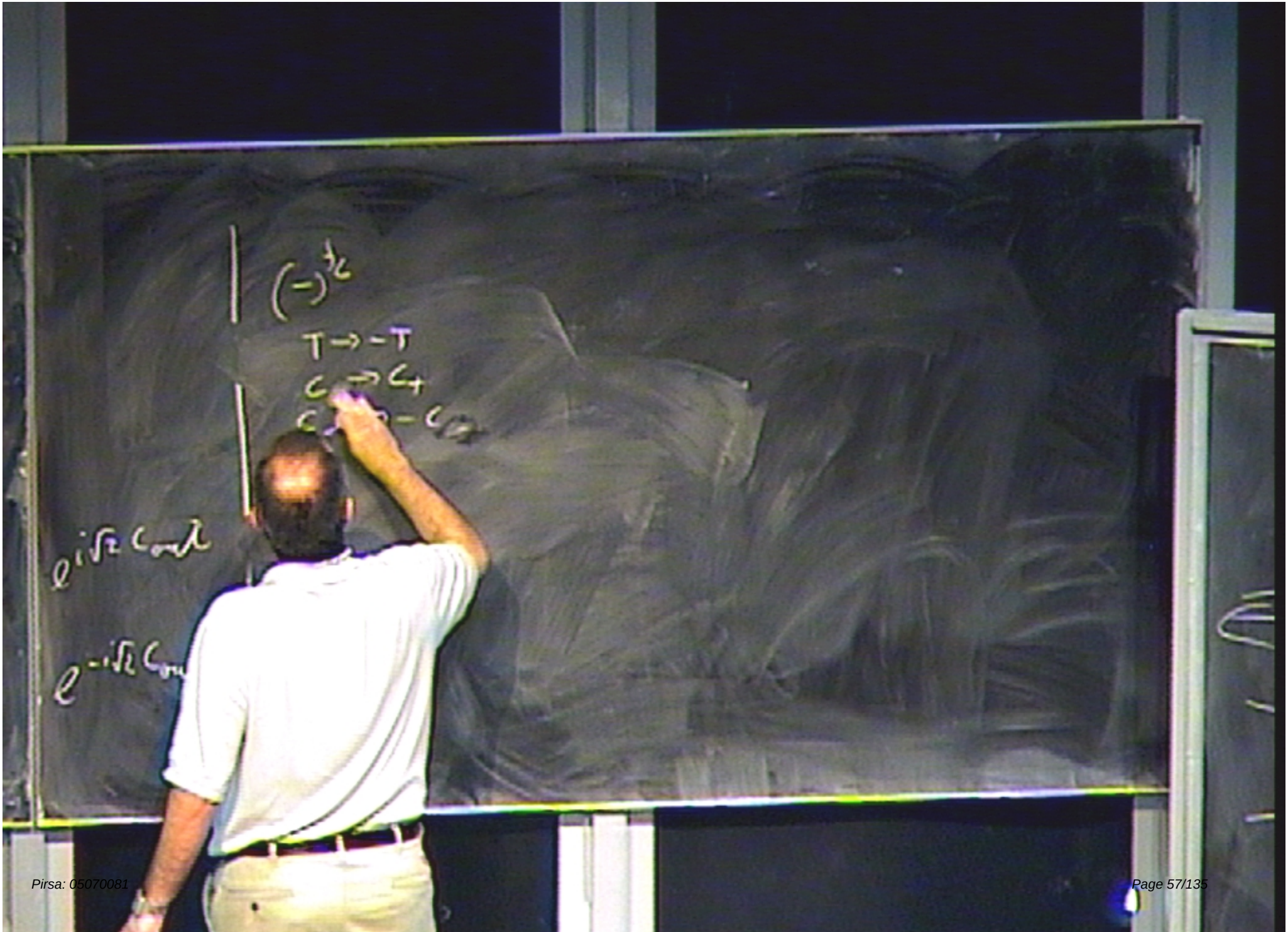
$\mu < 0$ $J_3^{in} \approx J_3^{out}$

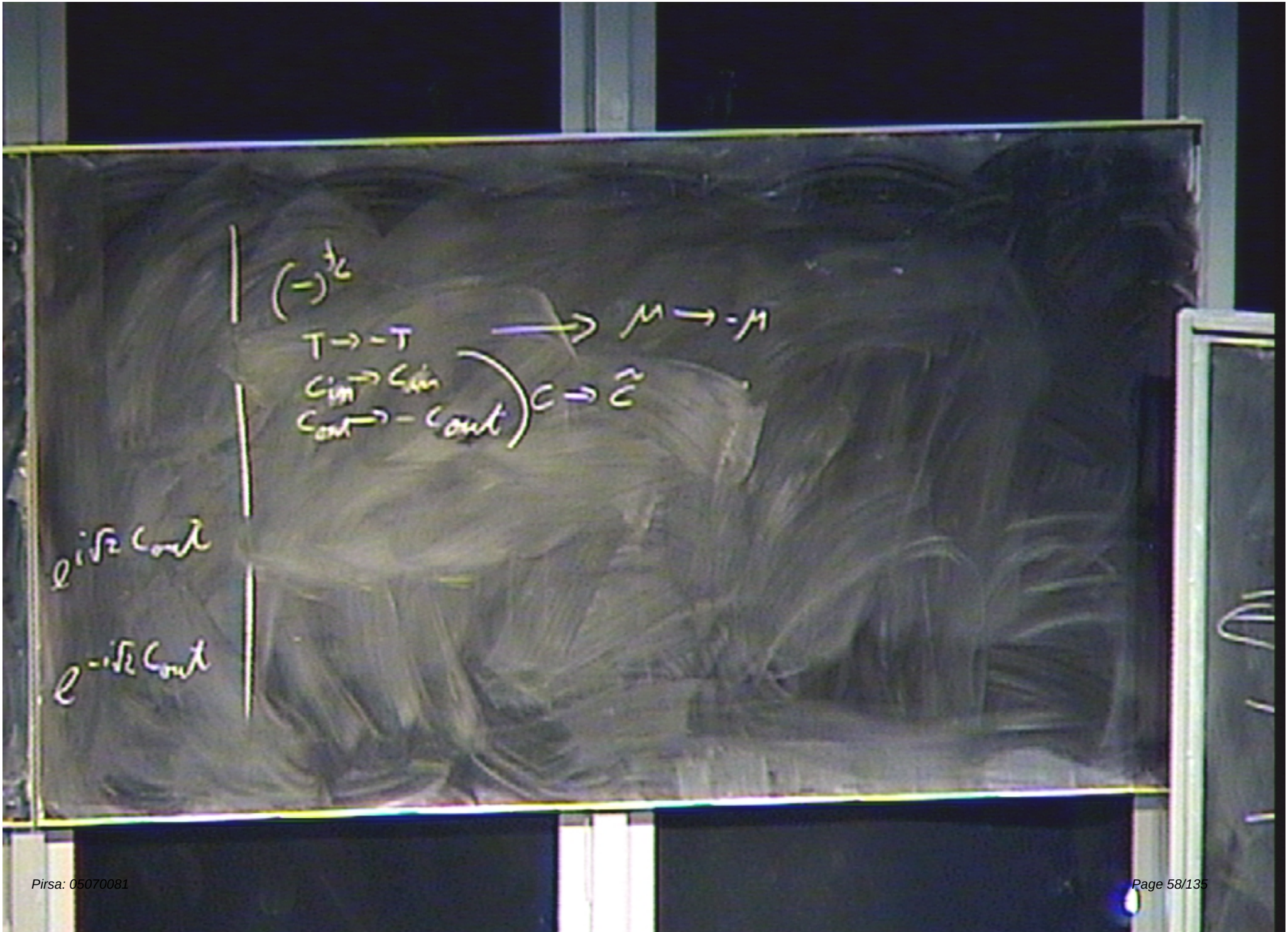
$\mu > 0$ $J_3^{in} \approx -J_3^{out}$




μ<0
$n_{in}=1$
S large $n_{in} \approx n_{out}$
μ>0
S large $n_{in} \approx -n_{out}$


μ<0
$n_{in}=1$
$n_{out}=0$
S large $n_{in} \approx n_{out}$
μ>0
S large $n_{in} \approx -n_{out}$


$(-)^{\frac{1}{2}}$
$T \to -T$
$c_+ \to c_+$
$e^{i\sqrt{2}\, c_{-1}}$
$e^{-i\sqrt{2}\, c_{-1}}$


$(-)^{F_L}$
$T \to -T$
$c_{in} \to c_{in}$
$c_{out} \to -c_{out}$
$c \to \tilde{c}$
$\mu \to -\mu$
$e^{i\sqrt{2}c_{out}t}$
$e^{-i\sqrt{2}c_{out}t}$

$(-)^{\frac{1}{2}}$

$T \to -T \longrightarrow \mu \to -\mu$

$c_{in} \to c_{out}$

$c_{out} \to -c_{out}$ $\Big) c \to \tilde{c}$

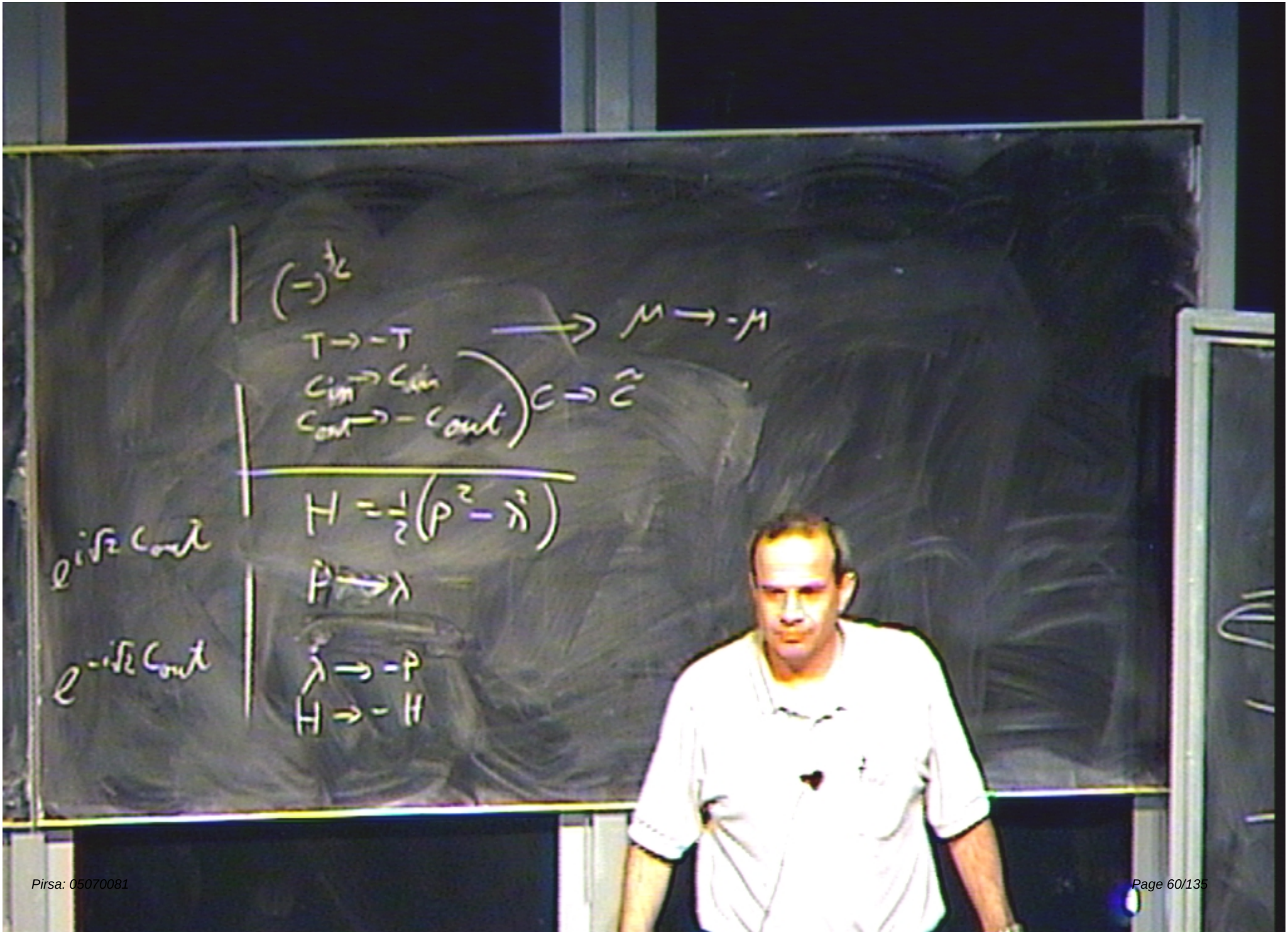
$H = -\frac{1}{2}(p^2 - \lambda^2)$

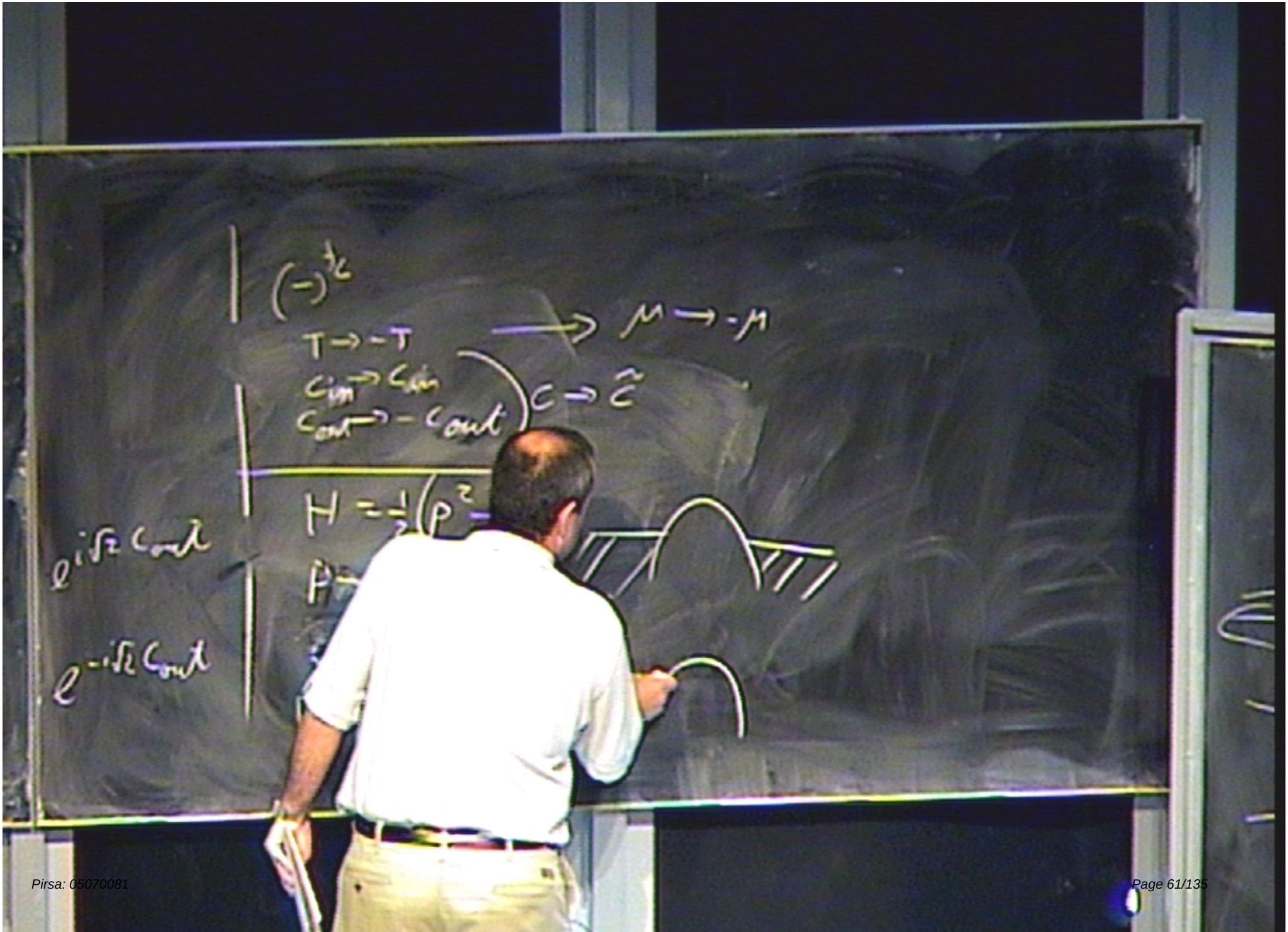
$e^{i\sqrt{2}\, c_{out} t}$

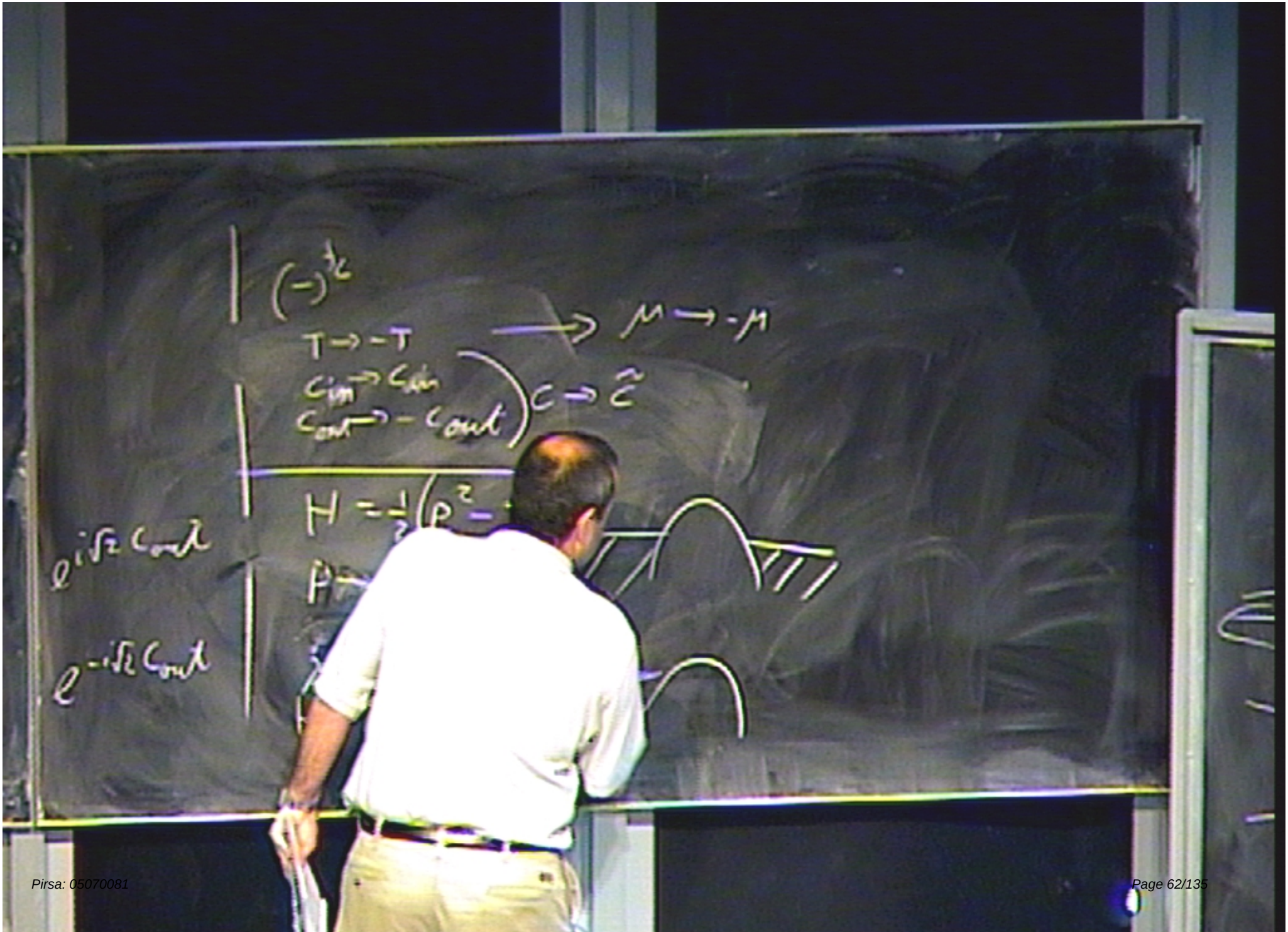
$p \leftrightarrow \lambda$

$H \to -H$

$e^{-i\sqrt{2}\, c_{out} t}$


$(-)^{\frac{1}{2}}$
$T \to -T$
$\mu \to -\mu$
$c_{in} \to c_{out}$
$c_{out} \to -c_{out}$
$c \to \tilde{c}$
$H = -\frac{1}{2}(p^2 - \lambda^2)$
$e^{i\sqrt{2}c_{out}t}$
$p \to \lambda$
$e^{-i\sqrt{2}c_{out}t}$
$\lambda \to -p$
$H \to -H$


$(-)^{\frac{1}{2}}$
$T \to -T$
$\mu \to -\mu$
$c_{in} \to c_{in}$
$c_{out} \to -c_{out}$
$c \to \tilde{c}$
$H = -\frac{1}{2}(p^2 -$
$e^{i\sqrt{2}\, c_{out} t}$
$e^{-i\sqrt{2}\, c_{out} t}$


$(-)^{\frac{1}{2}}$
$T \to -T$
$\mu \to -\mu$
$c_{in} \to c_{in}$
$c_{out} \to -c_{out}$
$c \to \tilde{c}$
$H = -\frac{1}{2}(p^2 -$
$e^{i\sqrt{2}\, c_{out} t}$
$e^{-i\sqrt{2}\, c_{out} t}$

$(-)^{\frac{1}{2}}$

$T \to -T \longrightarrow \mu \to -\mu$

$c_{in} \to c_{in}$
$c_{out} \to -c_{out}$ $\Big) c \to \tilde{c}$

$H = -\frac{1}{2}(p^2 - \lambda^2)$

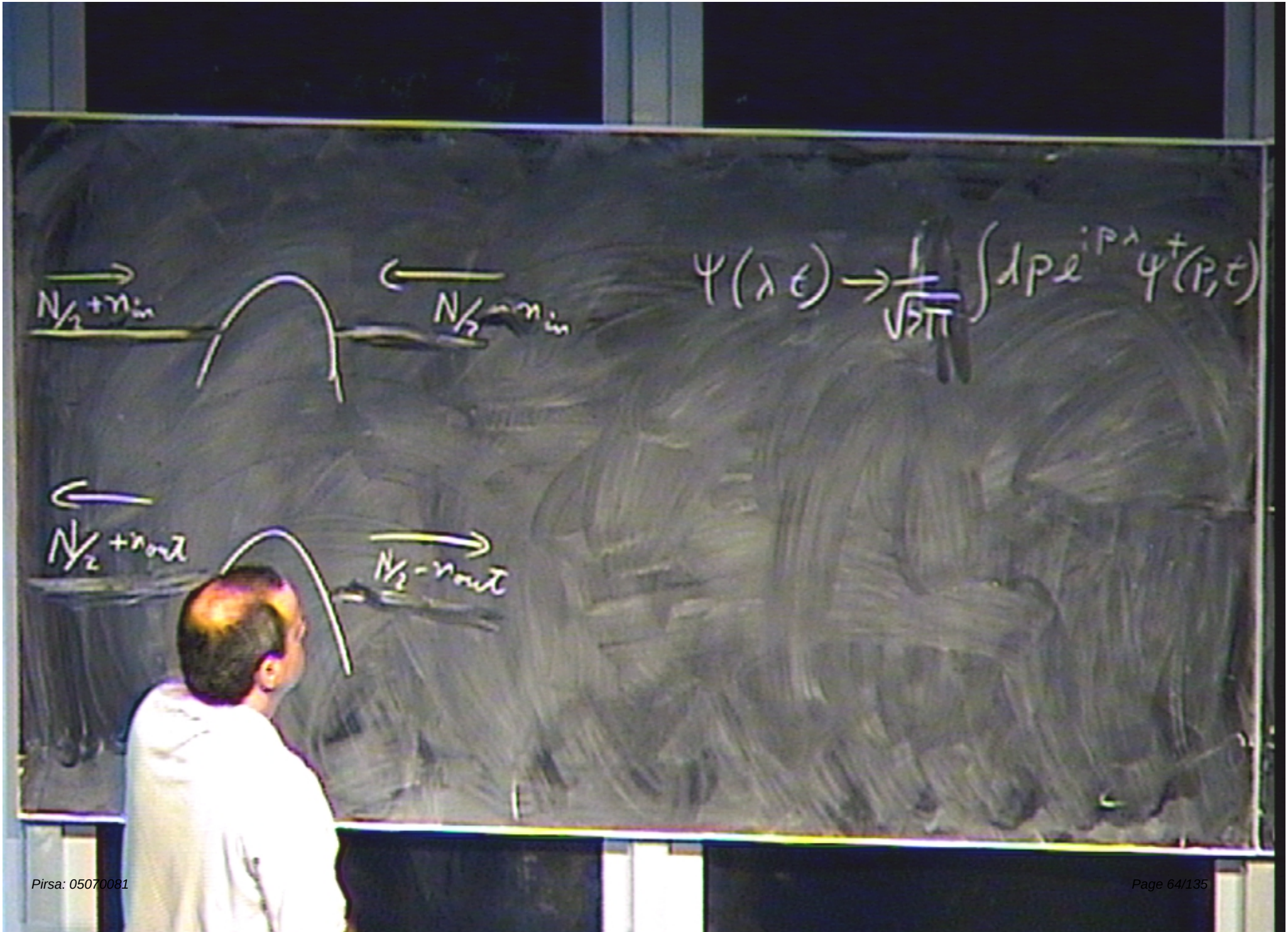
$e^{i\sqrt{2}\, c_{out} t}$ $\quad p \to \lambda$

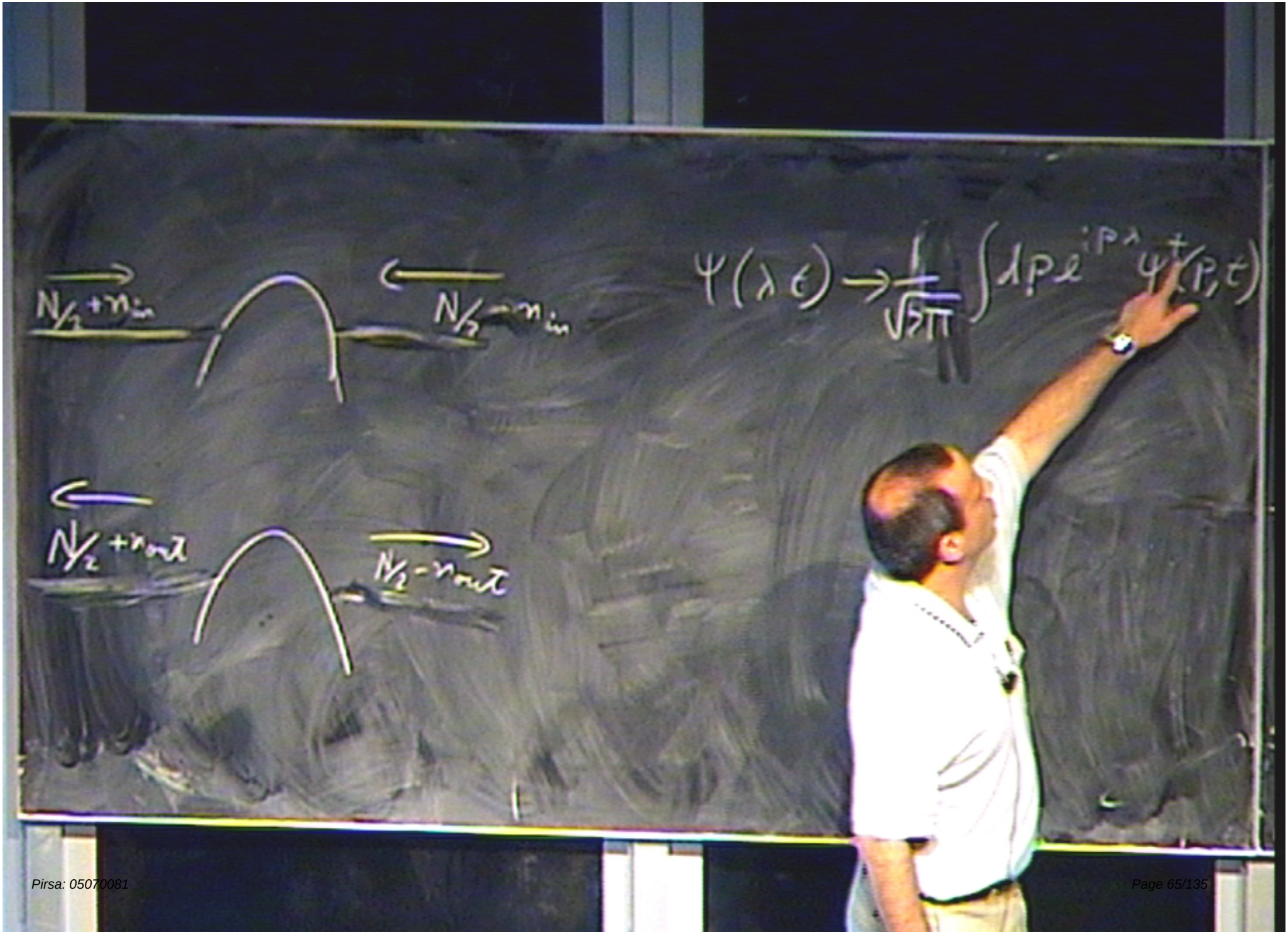
$e^{-i\sqrt{2}\, c_{out} t}$ $\quad \lambda \to -p$

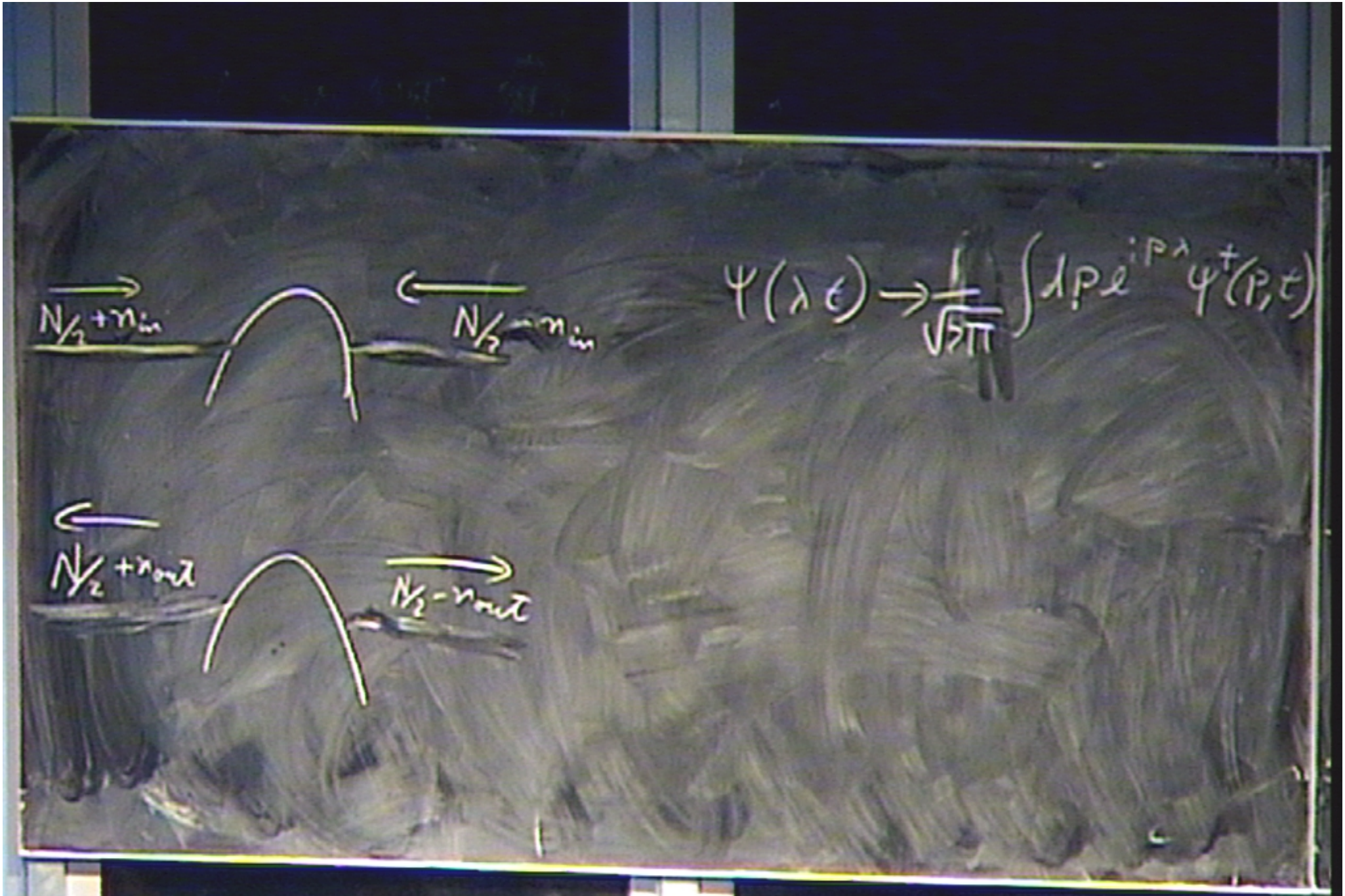
$H \to -H$

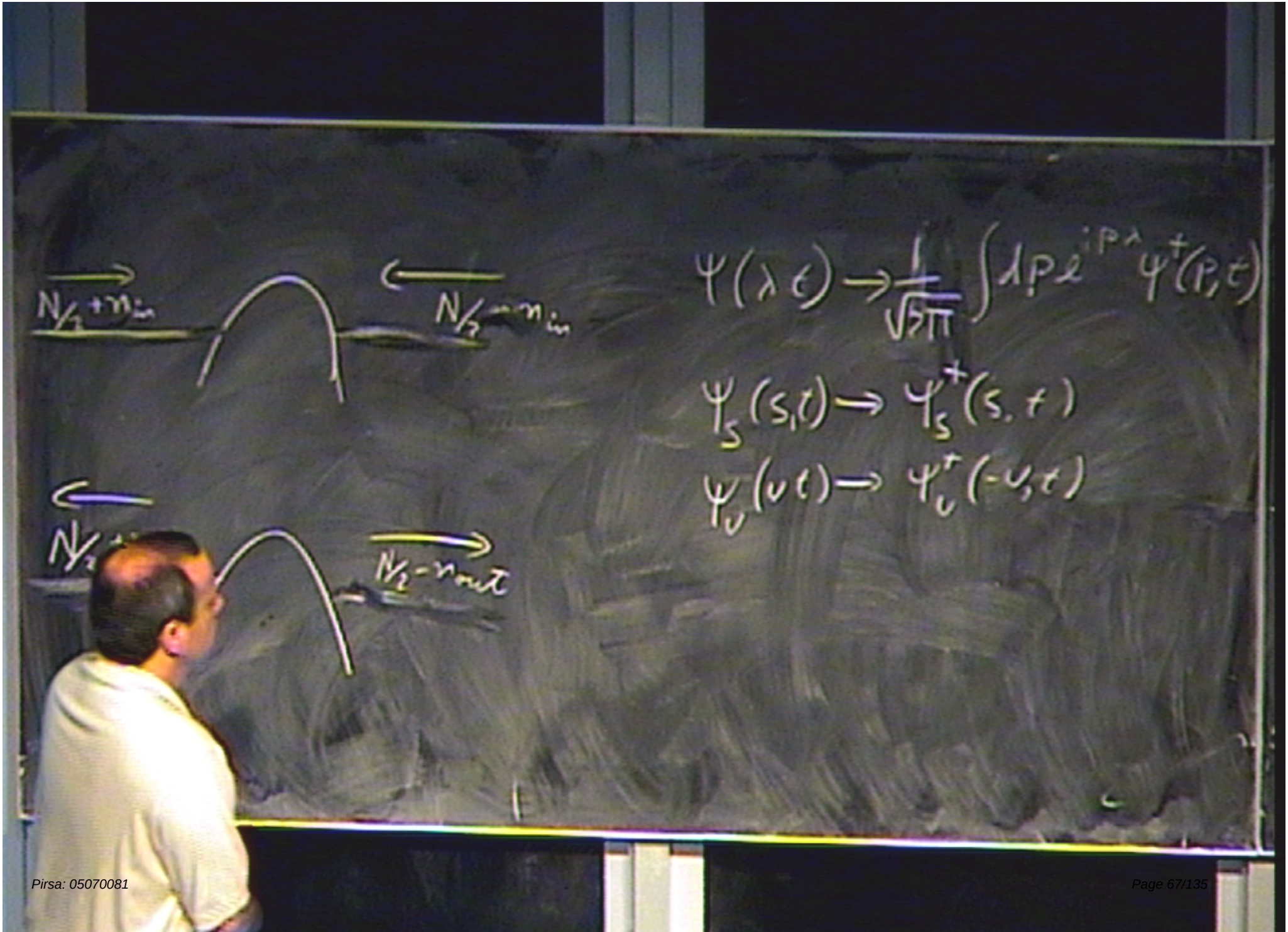
$\Psi \to \Psi^+$

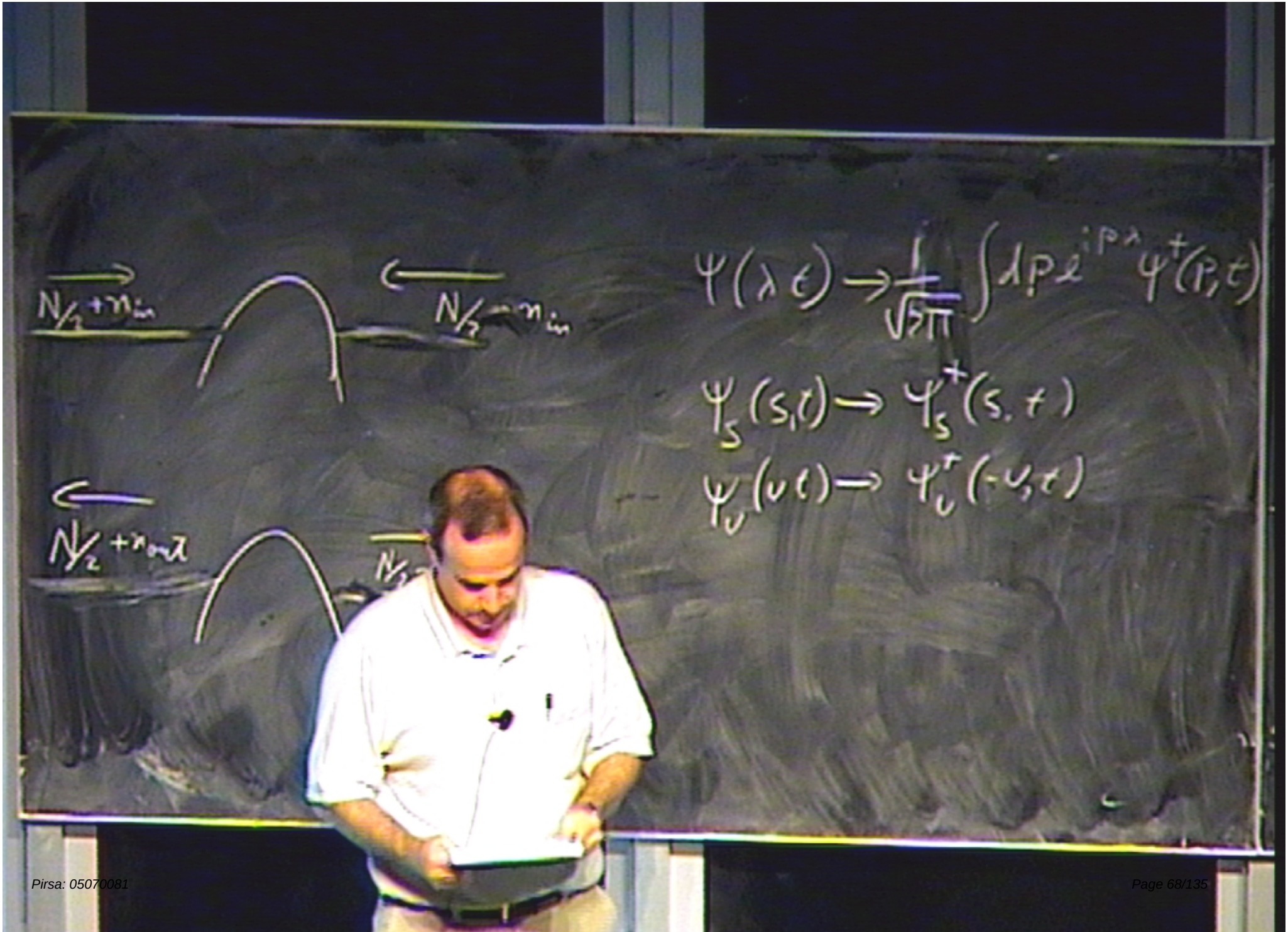


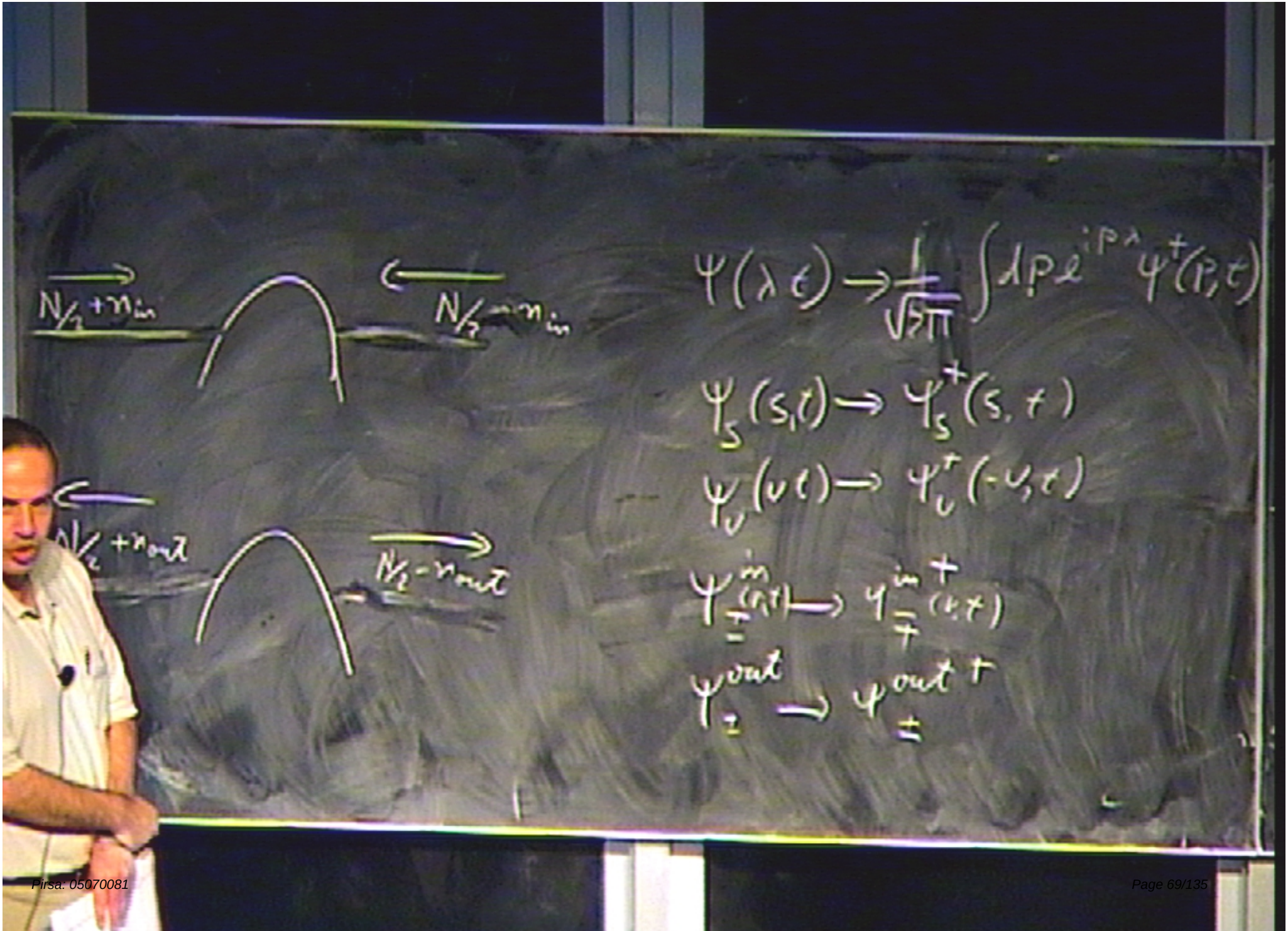

$N/2 + n_{in}$
$N/2 - n_{in}$
$N/2 + n_{out}$
$N/2 - n_{out}$
$\Psi(\lambda\epsilon) \rightarrow \frac{1}{\sqrt{2\pi}} \int dp\, e^{ip\lambda} \psi^{\dagger}(p,\epsilon)$

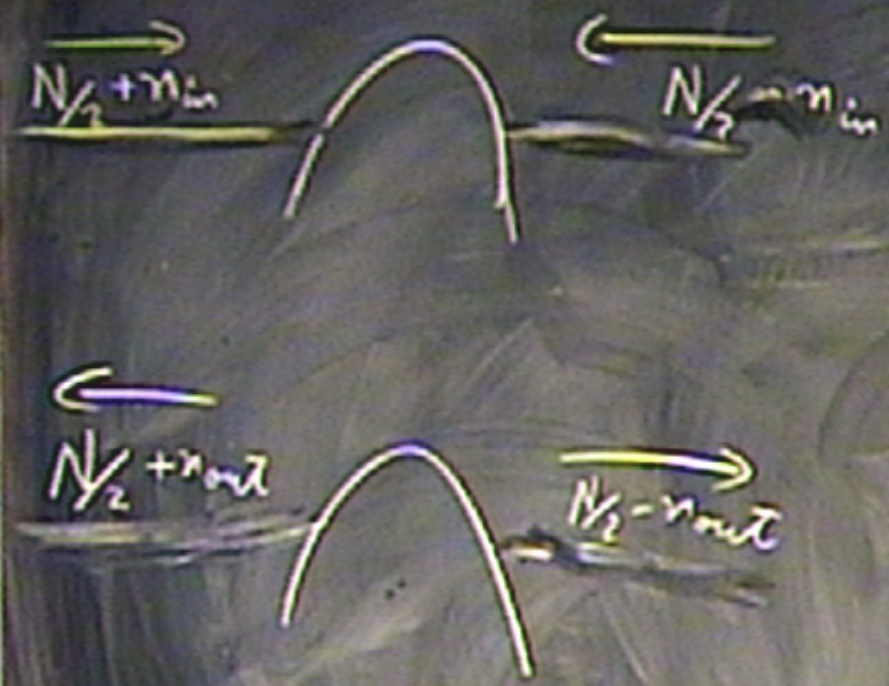

$N/2 + n_{in}$
$N/2 - n_{in}$
$N/2 + n_{out}$
$N/2 - n_{out}$
$\Psi(\lambda t) \rightarrow \frac{1}{\sqrt{2\pi}} \int dp\, e^{ip\lambda} \psi^{\dagger}(p,t)$


$N/2 + n_{in}$
$N/2 - n_{in}$
$N/2 + n_{out}$
$N/2 - n_{out}$
$\Psi(\lambda t) \to \frac{1}{\sqrt{2\pi}} \int dp\, e^{ip\lambda} \varphi^{\dagger}(p,t)$


$N/2 + n_{in}$
$N/2 - n_{in}$
$N/2 - n_{out}$
$\Psi(x,t) \rightarrow \frac{1}{\sqrt{2\pi}} \int dp\, e^{ipx} \psi^{\dagger}(p,t)$
$\Psi_s(s,t) \rightarrow \psi_s^{\dagger}(s,t)$
$\Psi_v(v,t) \rightarrow \psi_v^{\dagger}(-v,t)$


$N/2 + n_{in}$
$N/2 - n_{in}$
$N/2 + n_{out}$
$\Psi(\lambda t) \rightarrow \frac{1}{\sqrt{2\pi}} \int dp\, e^{ip\lambda} \psi^+(p,t)$
$\Psi_s(s,t) \rightarrow \psi_s^+(s,t)$
$\Psi_v(v,t) \rightarrow \psi_v^+(-v,t)$


$N/2 + n_{in}$
$N/2 - n_{in}$
$N/2 + n_{out}$
$N/2 - n_{out}$
$\Psi(\lambda t) \to \frac{1}{\sqrt{2\pi}} \int dp\, e^{ip\lambda} \psi^+(p,t)$
$\Psi_s(s,t) \to \psi_s^+(s,t)$
$\Psi_v(v,t) \to \psi_v^+(-v,t)$
$\Psi_{\pm}^{in}(n,t) \to \psi_{\pm}^{in\,+}(n,t)$
$\Psi_{\pm}^{out} \to \psi_{\pm}^{out\,+}$



$$\Psi(\lambda t) \to \frac{1}{\sqrt{2\pi}} \int \lambda p\, e^{ip\lambda}\, \psi^{+}(p,t)$$

$$\Psi_s(s,t) \to \psi_s^{+}(s,t)$$

$$\Psi_v(v,t) \to \psi_v^{+}(-v,t)$$

$$\Psi_{\pm}^{in}(r,t) \to \psi_{\pm}^{in\,+}(r,t)$$

$$\Psi_{\pm}^{out} \to \psi_{\pm}^{out\,+}$$

$(-)^{\frac{1}{2}}$

$T \to -T \longrightarrow \mu \to -\mu$

$c_{in} \to c_{in}$
$c_{out} \to -c_{out}$ $\Big) c \to \tilde{c}$

$H = \frac{1}{2}(p^2 - \lambda^2)$

$p \to \lambda$

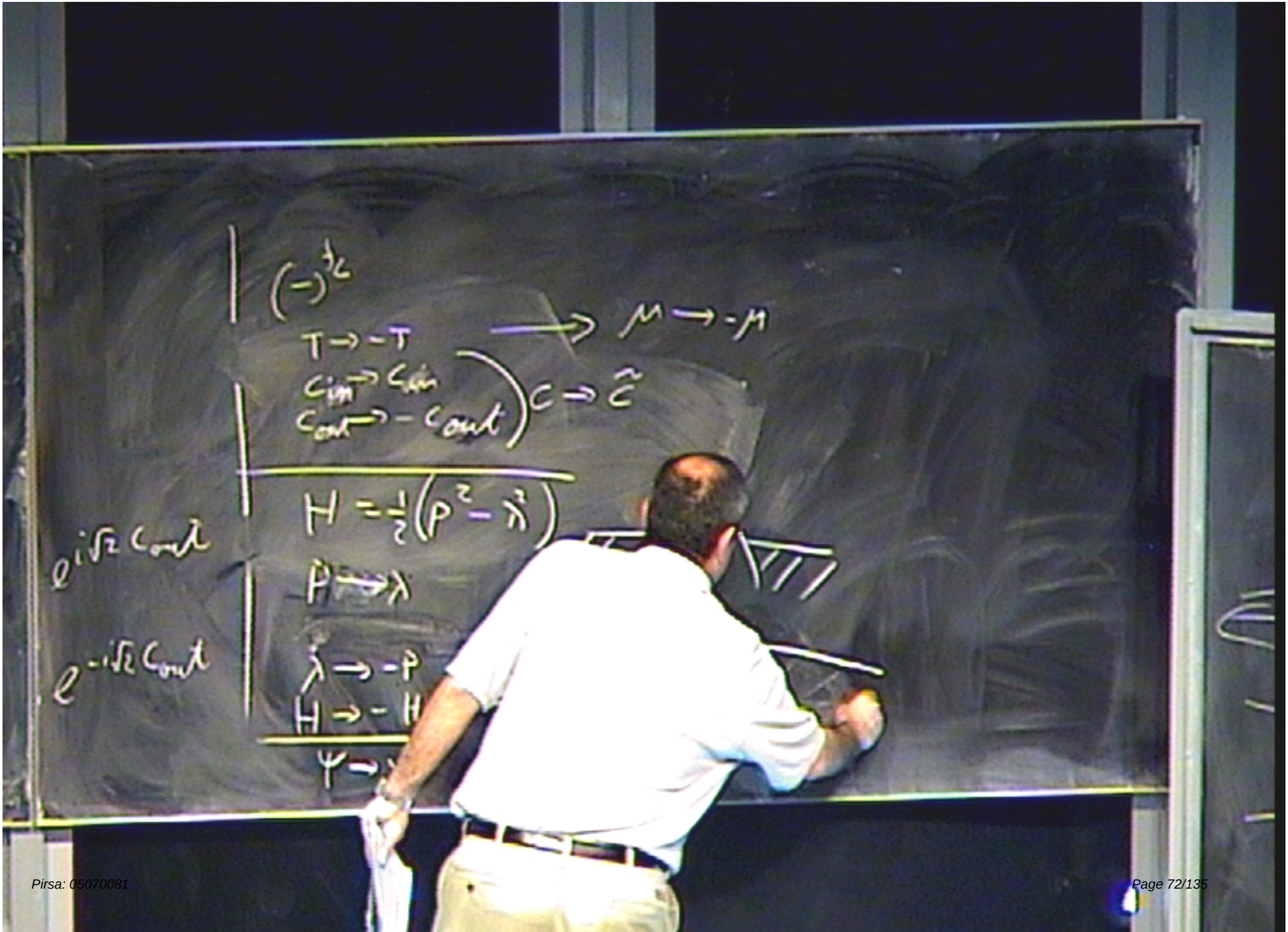
$\lambda \to -p$
$H \to -H$

$\Psi \to \Psi^+$

$e^{i\sqrt{2}\, c_{out} c}$

$e^{-i\sqrt{2}\, c_{out} c}$




$(-)^{1/2}$
$T \to -T$
$\mu \to -\mu$
$c_{in} \to c_{out}$
$c_{out} \to -c_{out}$
$c \to \tilde{c}$
$H = \frac{1}{2}(p^2 - \lambda^2)$
$e^{i\sqrt{2}c_{out}t}$
$p \to \lambda$
$e^{-i\sqrt{2}c_{out}t}$
$\dot{\lambda} \to -p$
$H \to -H$

$(-)^{\frac{1}{2}}$

$T \to -T \longrightarrow \mu \to -\mu$

$c_{in} \to c_{out}$

$c_{out} \to -c_{out}$ $\Big) c \to \tilde{c}$

$H = \frac{1}{2}(p^2 - \lambda^2)$

$e^{i\sqrt{2}\, c_{out}}$ $\quad p \to \lambda$

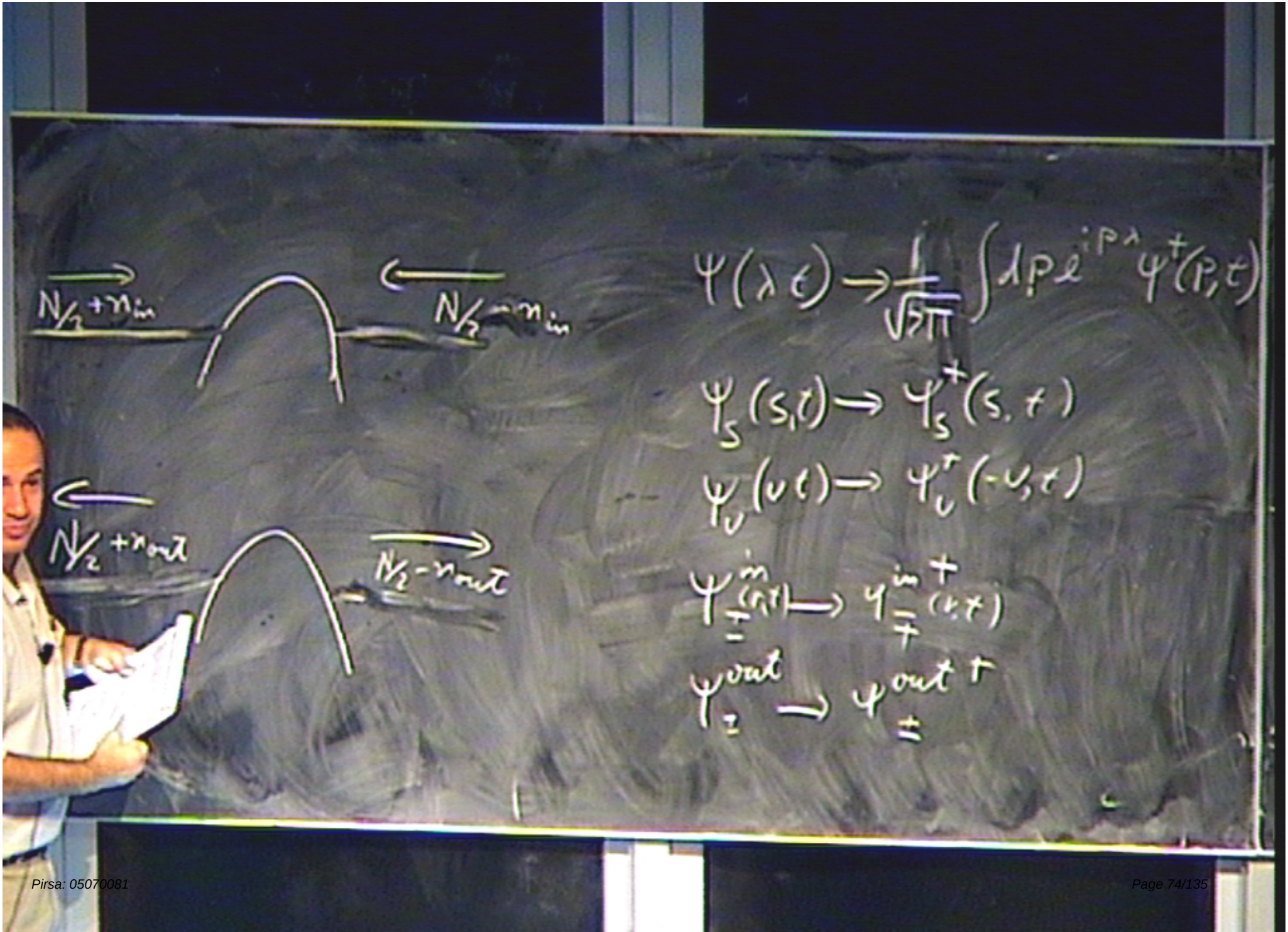
$e^{-i\sqrt{2}\, c_{out}}$ $\quad \lambda \to -p$

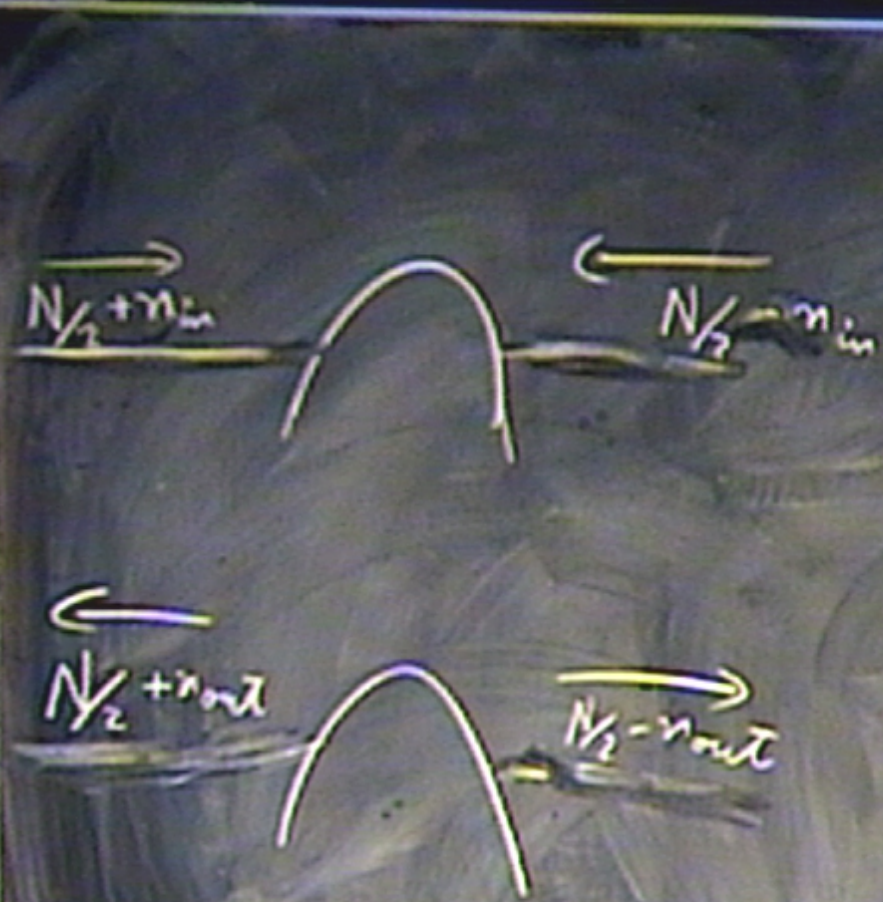
$H \to -H$

$\Psi \to \Psi^+$






$N/2 + n_{in}$
$N/2 - n_{in}$
$N/2 + n_{out}$
$N/2 - n_{out}$
$\psi(\lambda t) \to \frac{1}{\sqrt{2\pi}} \int dp\, e^{ip\lambda} \psi^{+}(p,t)$
$\psi_s(s,t) \to \psi_s^{+}(s,t)$
$\psi_v(v,t) \to \psi_v^{+}(-v,t)$
$\psi_{\pm}^{in}(n,t) \to \psi_{\pm}^{in\,+}(n,t)$
$\psi_{\pm}^{out} \to \psi_{\pm}^{out\,+}$



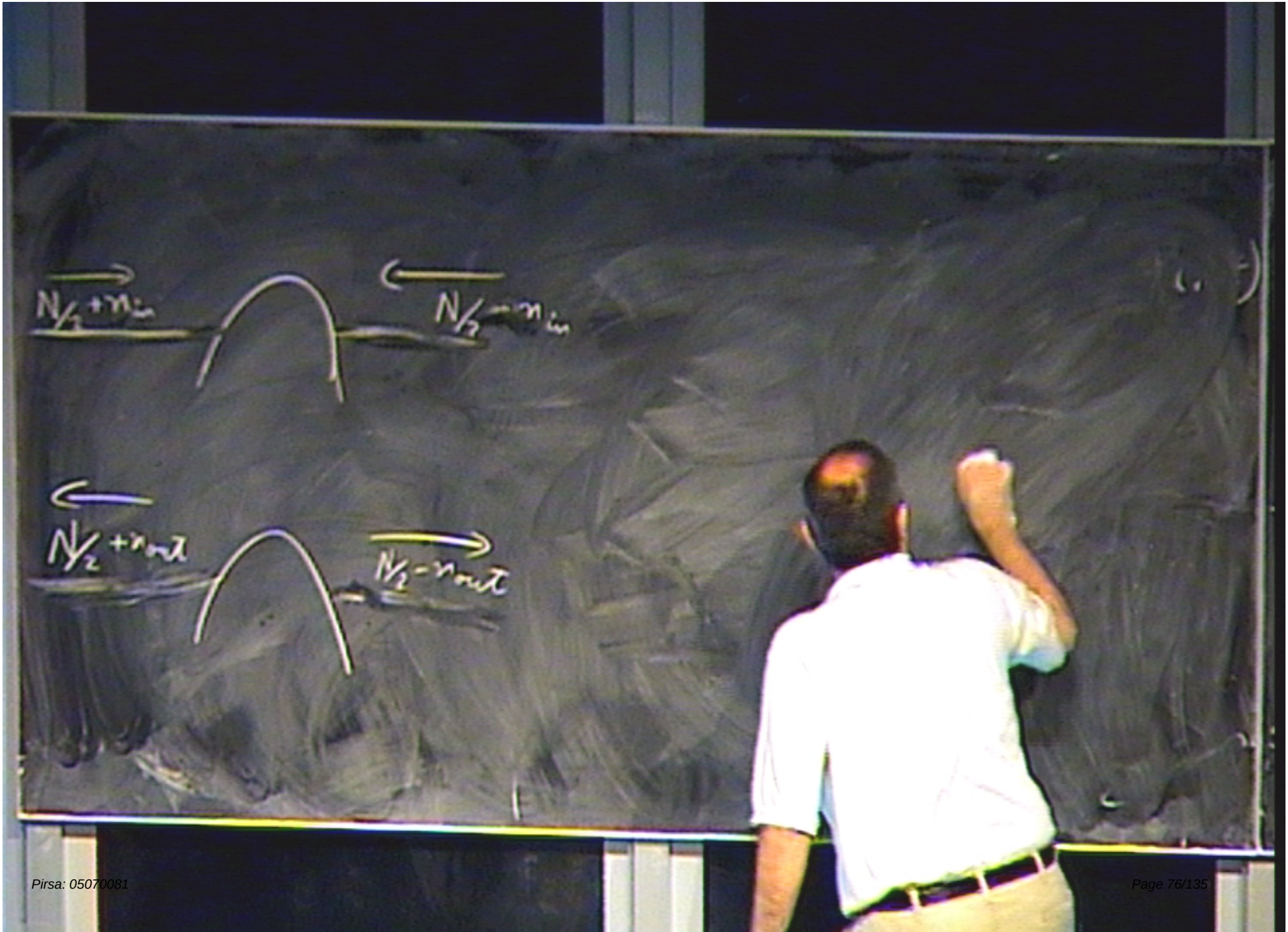
$$\Psi(\lambda, t) \to \frac{1}{\sqrt{2\pi}} \int dp\, e^{ip\lambda} \psi^{+}(p,t)$$

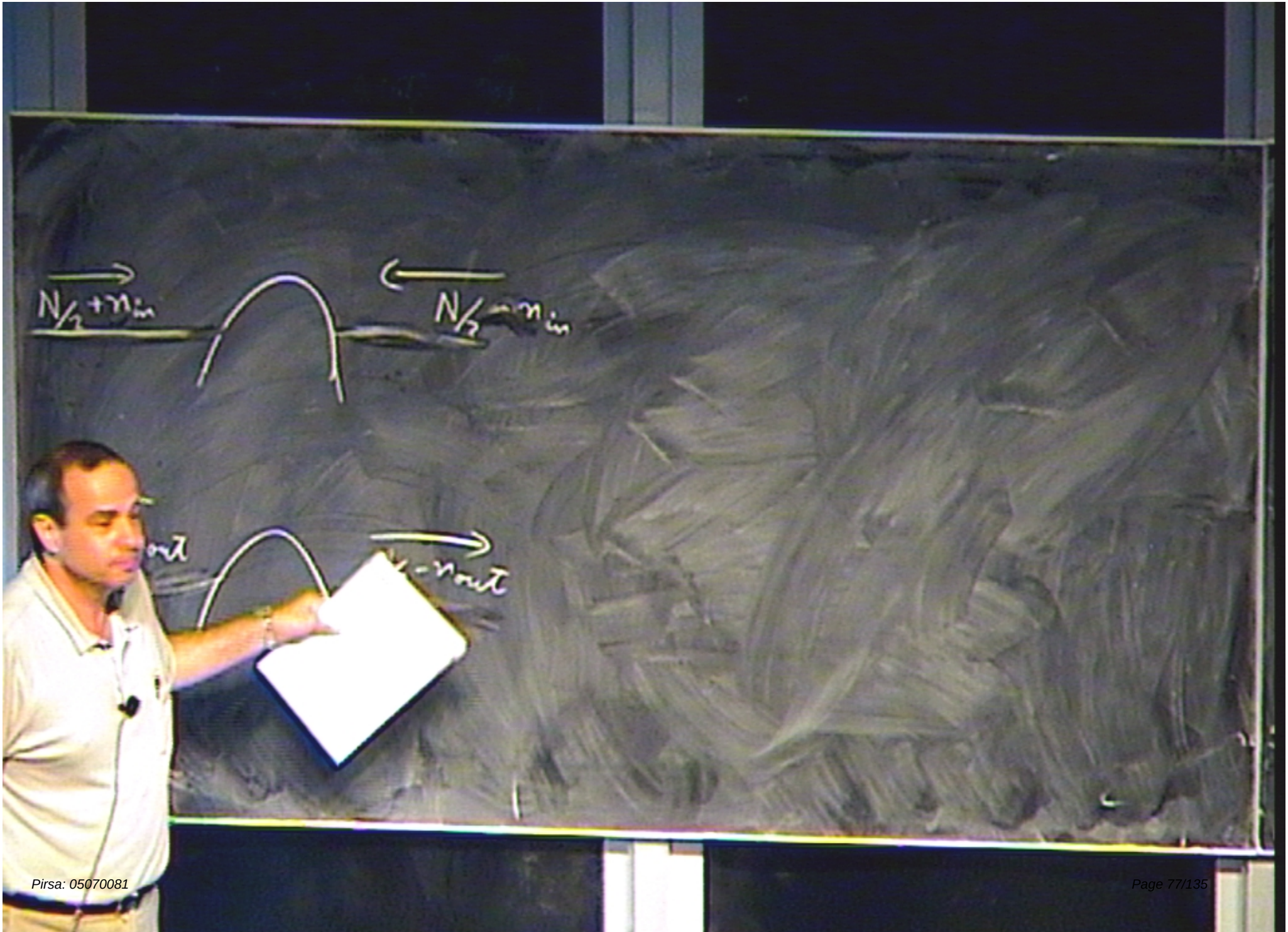
$$\Psi_s(s,t) \to \psi_s^{+}(s,t)$$

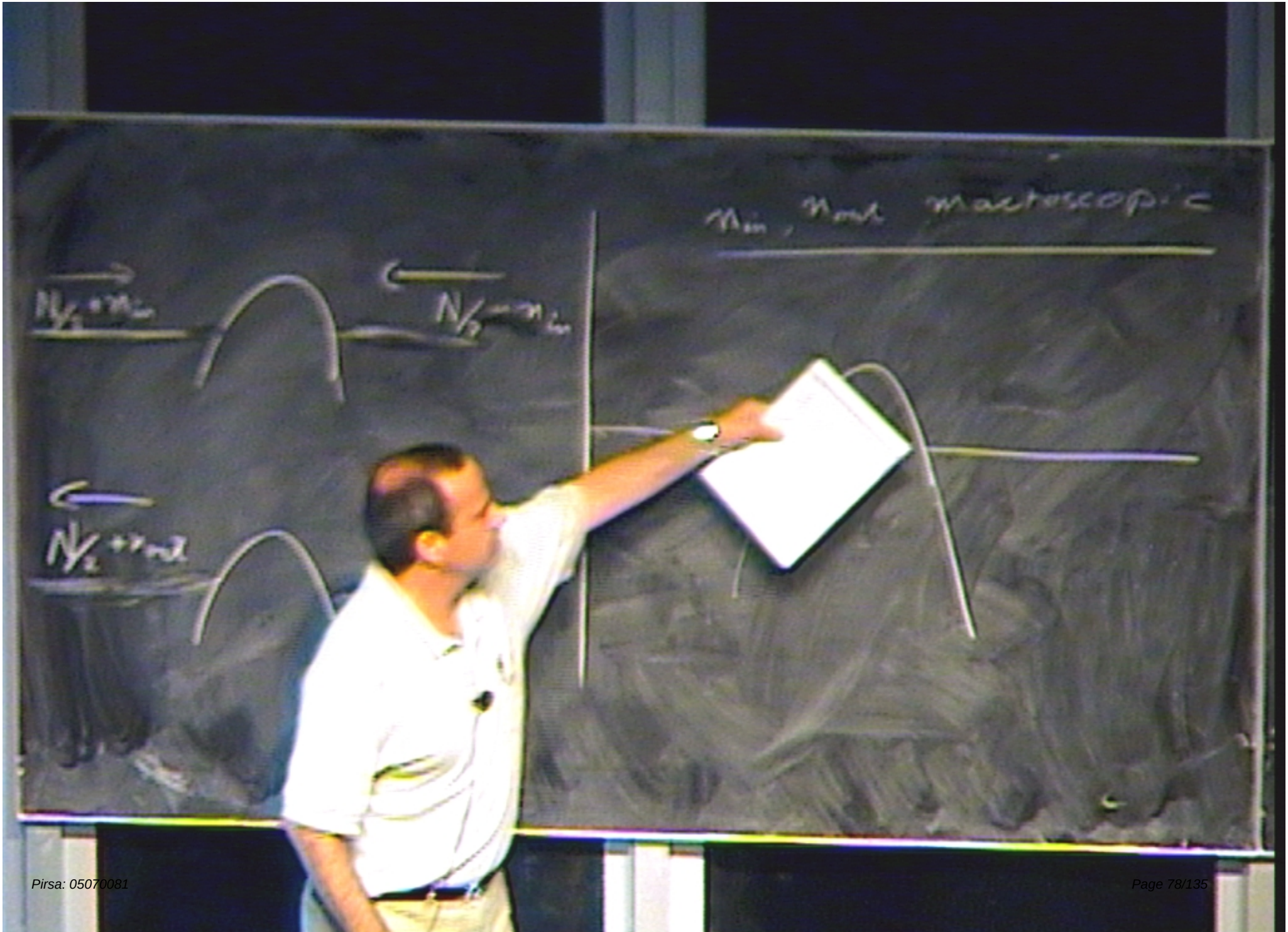
$$\Psi_v(v,t) \to \psi_v^{+}(-v,t)$$

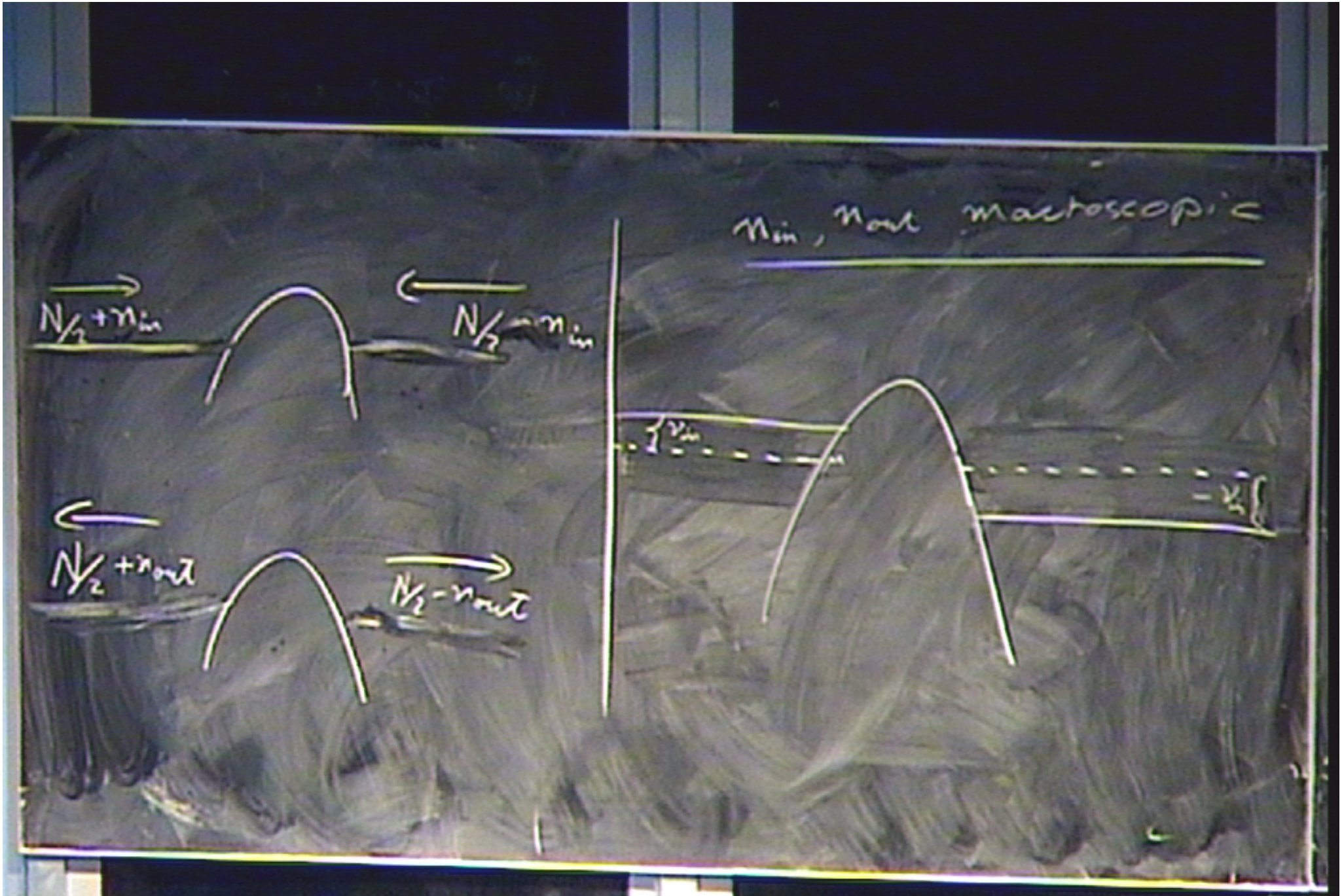
$$\Psi_{\pm}^{in}(n,t) \to \psi_{\pm}^{in\,+}(n,t)$$

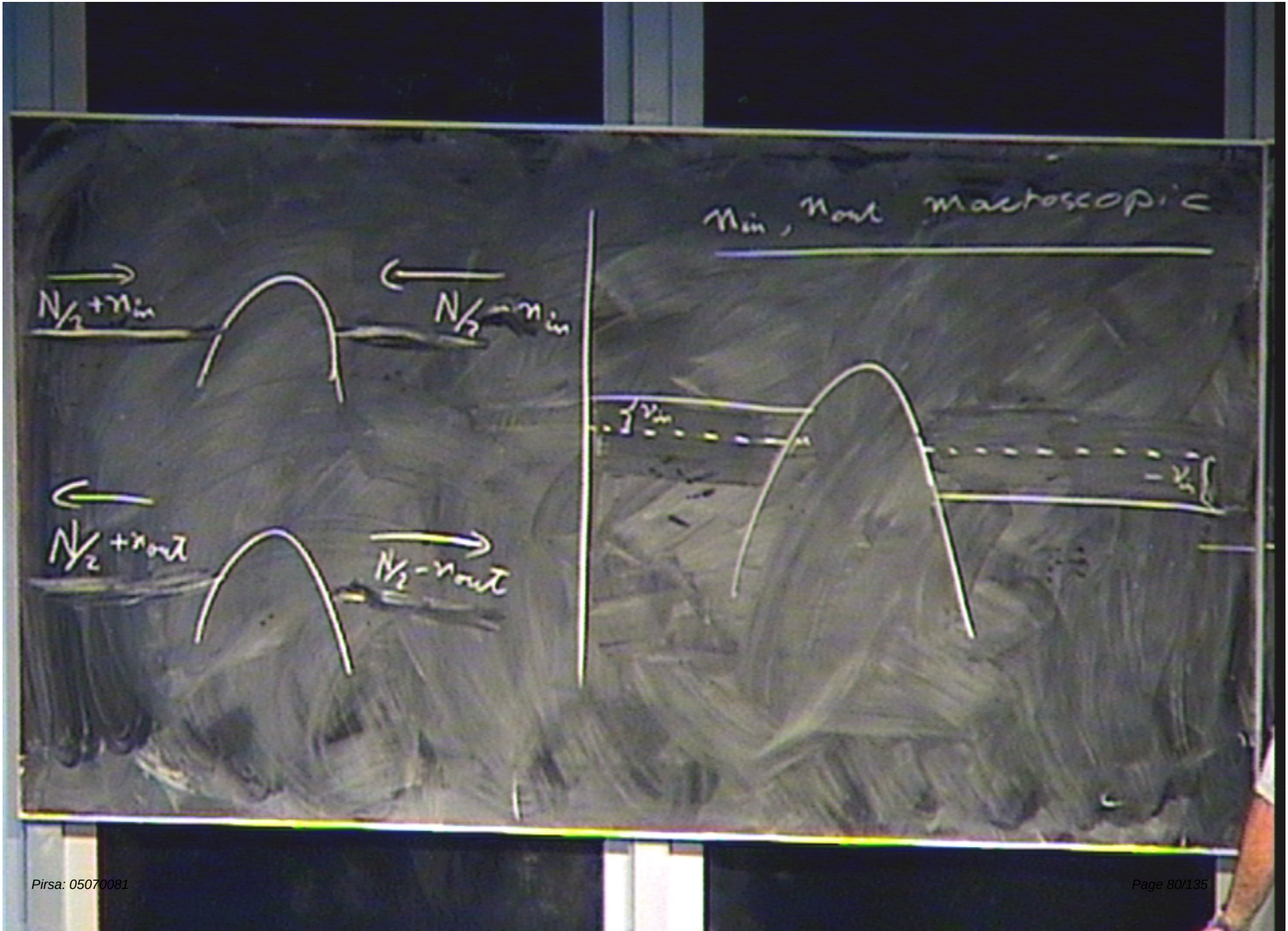
$$\Psi_{\pm}^{out} \to \psi_{\pm}^{out\,+}$$


$N/2 + n_{in}$
$N/2 - n_{in}$
$N/2 + n_{out}$
$N/2 - n_{out}$


$N/2 + n_{in}$
$N/2 - n_{in}$


macroscopic

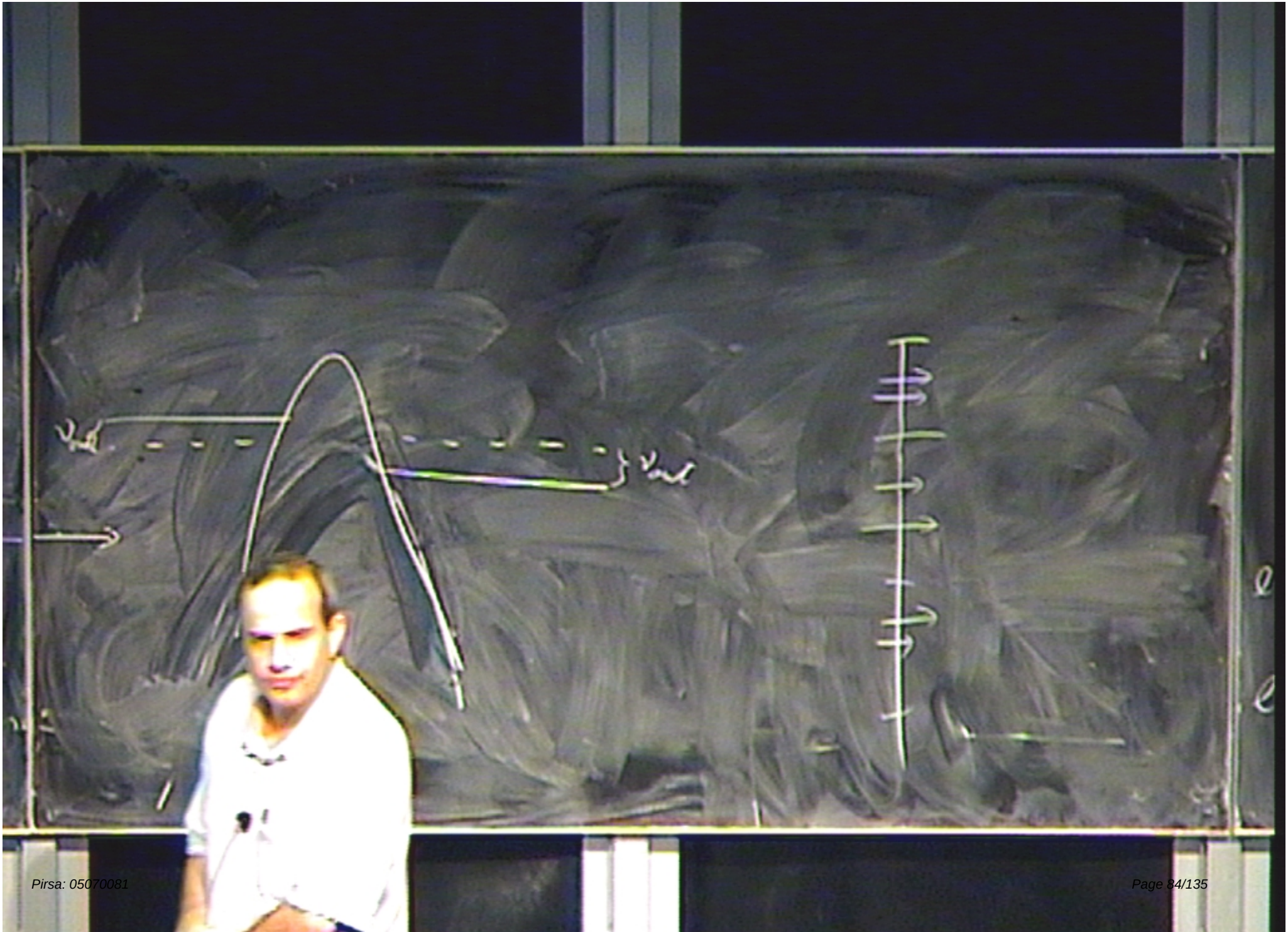

n_{in}, n_{out} macroscopic
$N/2 + n_{in}$
$N/2 - n_{in}$
$N/2 + n_{out}$
$N/2 - n_{out}$
n_{in}
$-n_{in}$

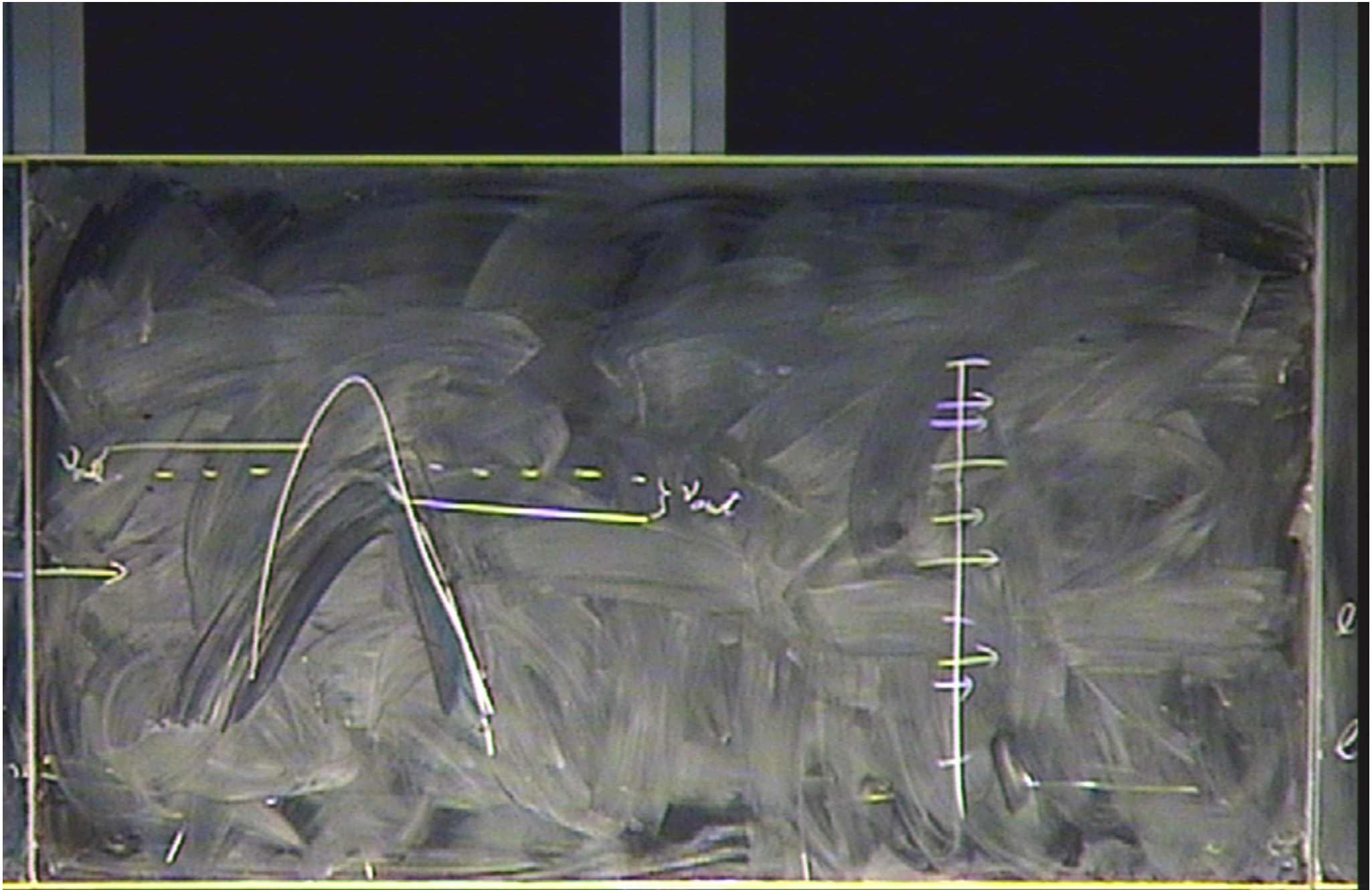

n_{in}, n_{out} macroscopic
$N/2 + n_{in}$
$N/2 - n_{in}$
$N/2 + n_{out}$
$N/2 - n_{out}$

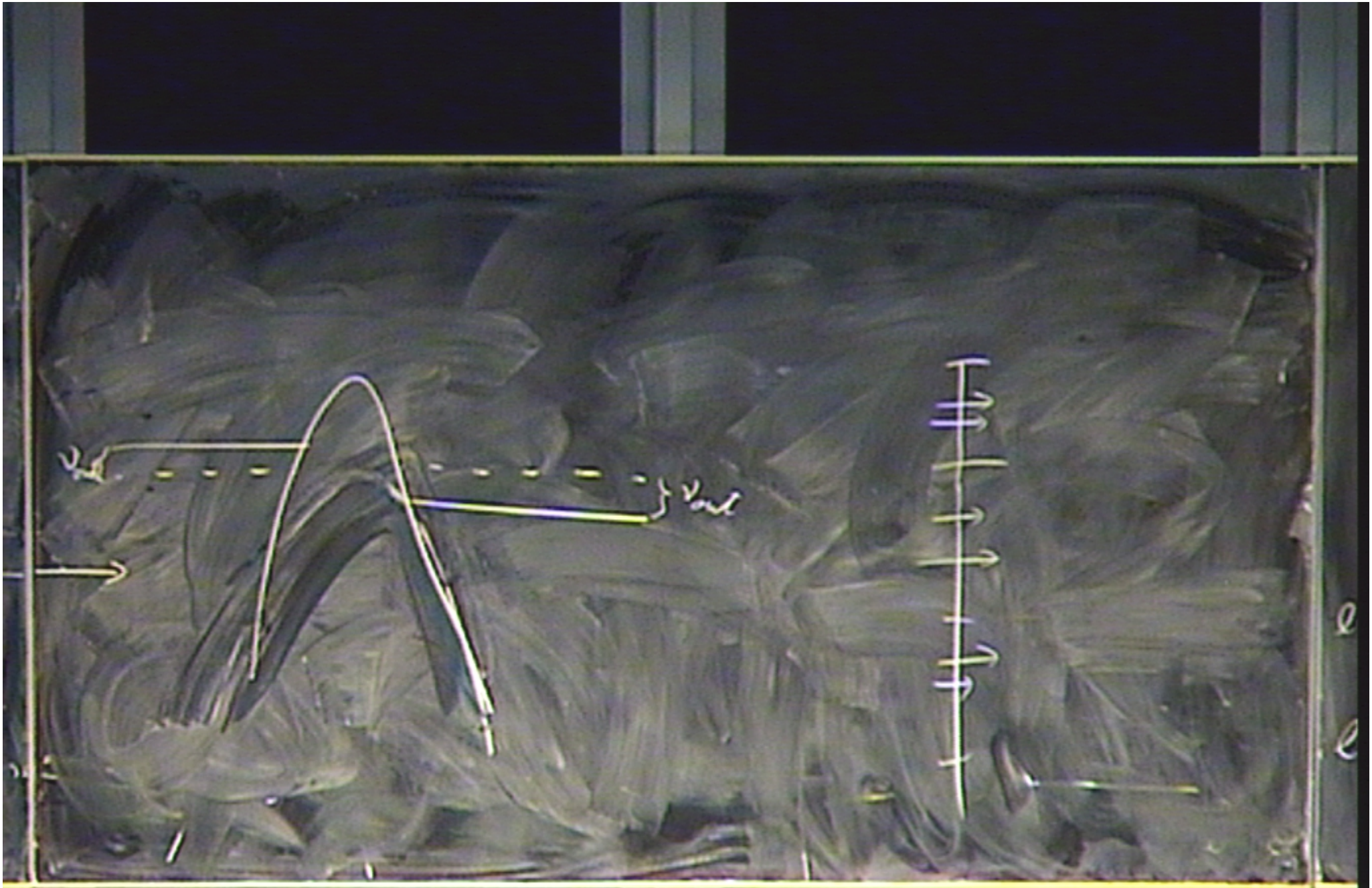


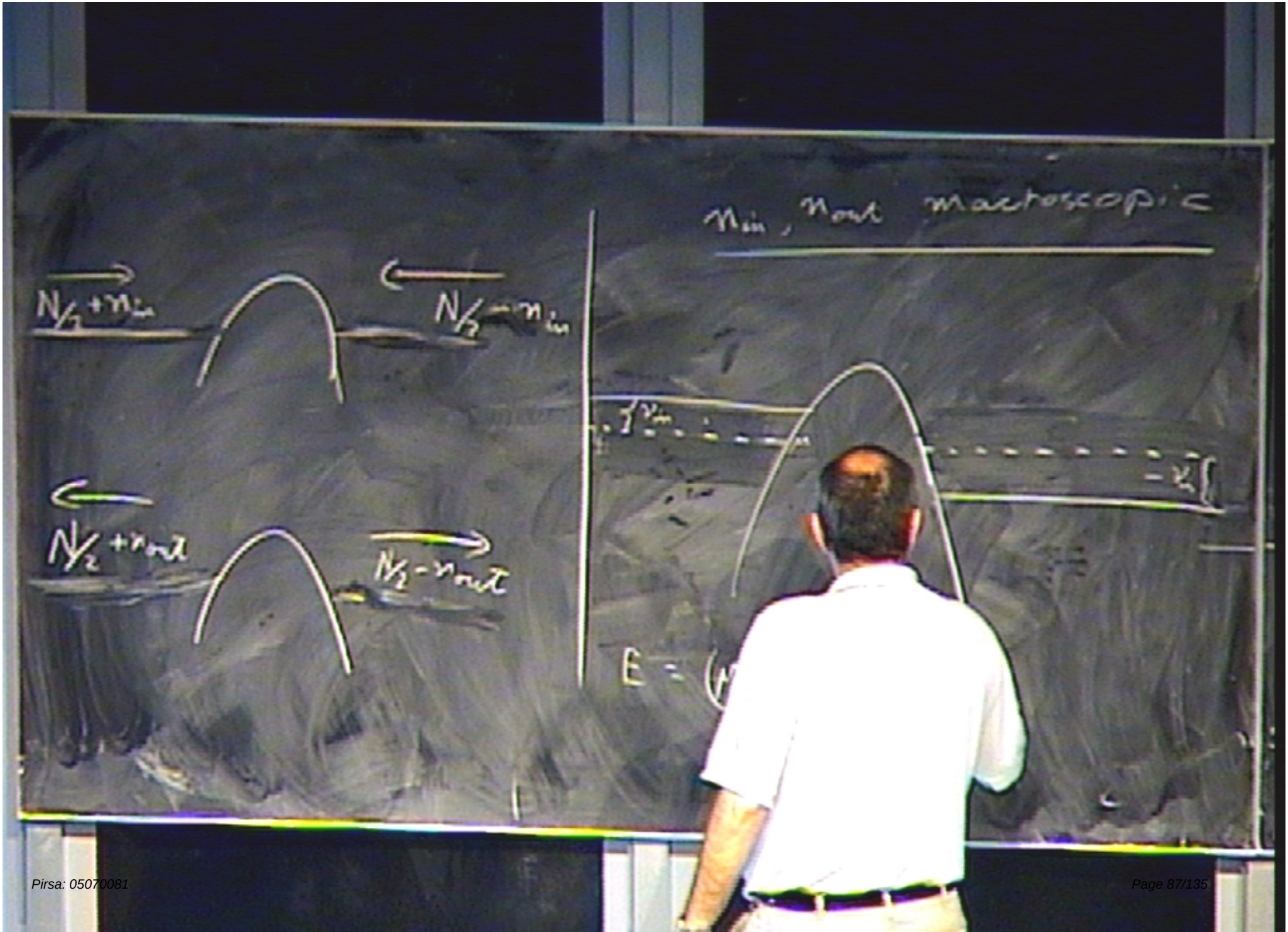


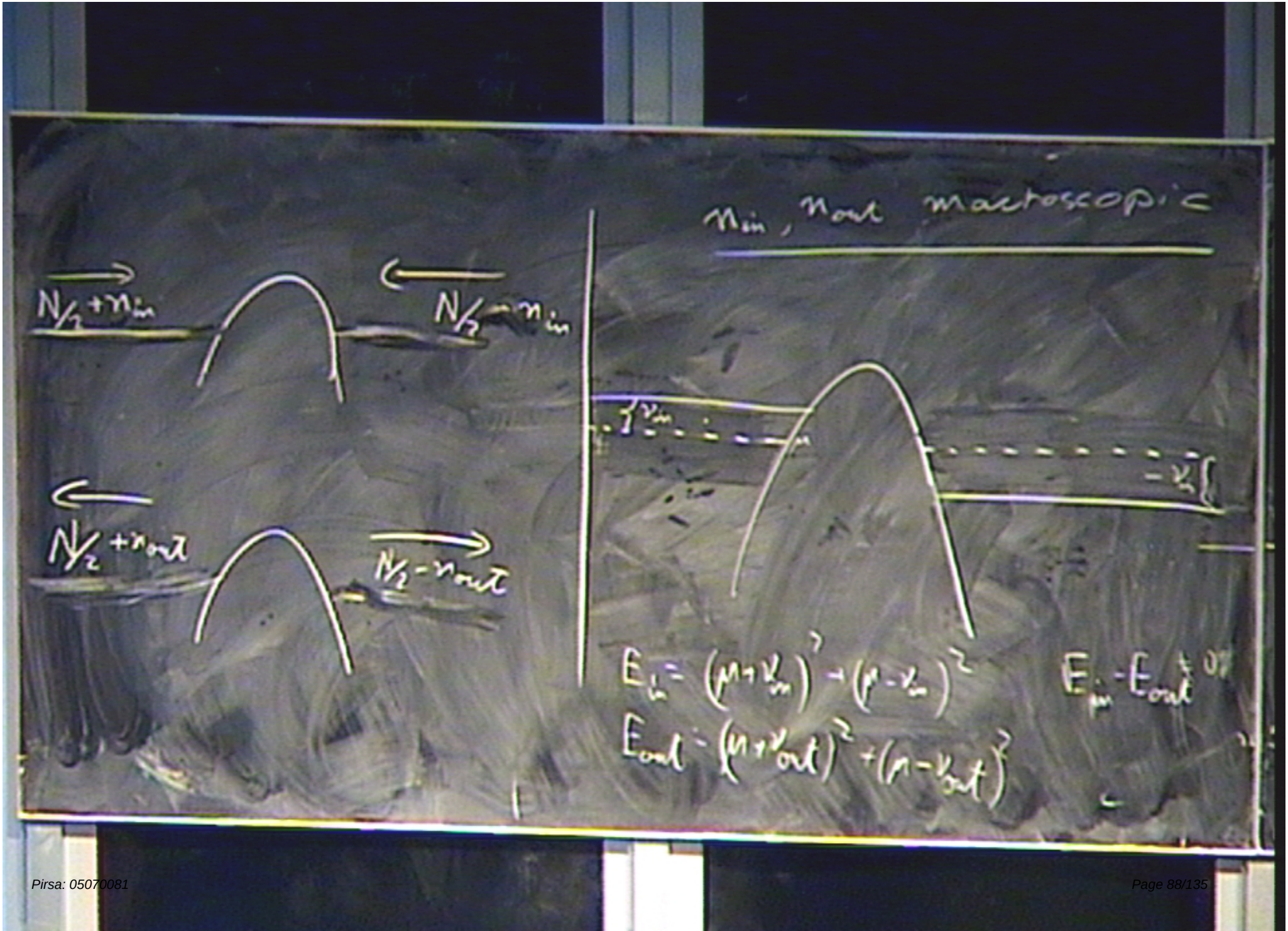


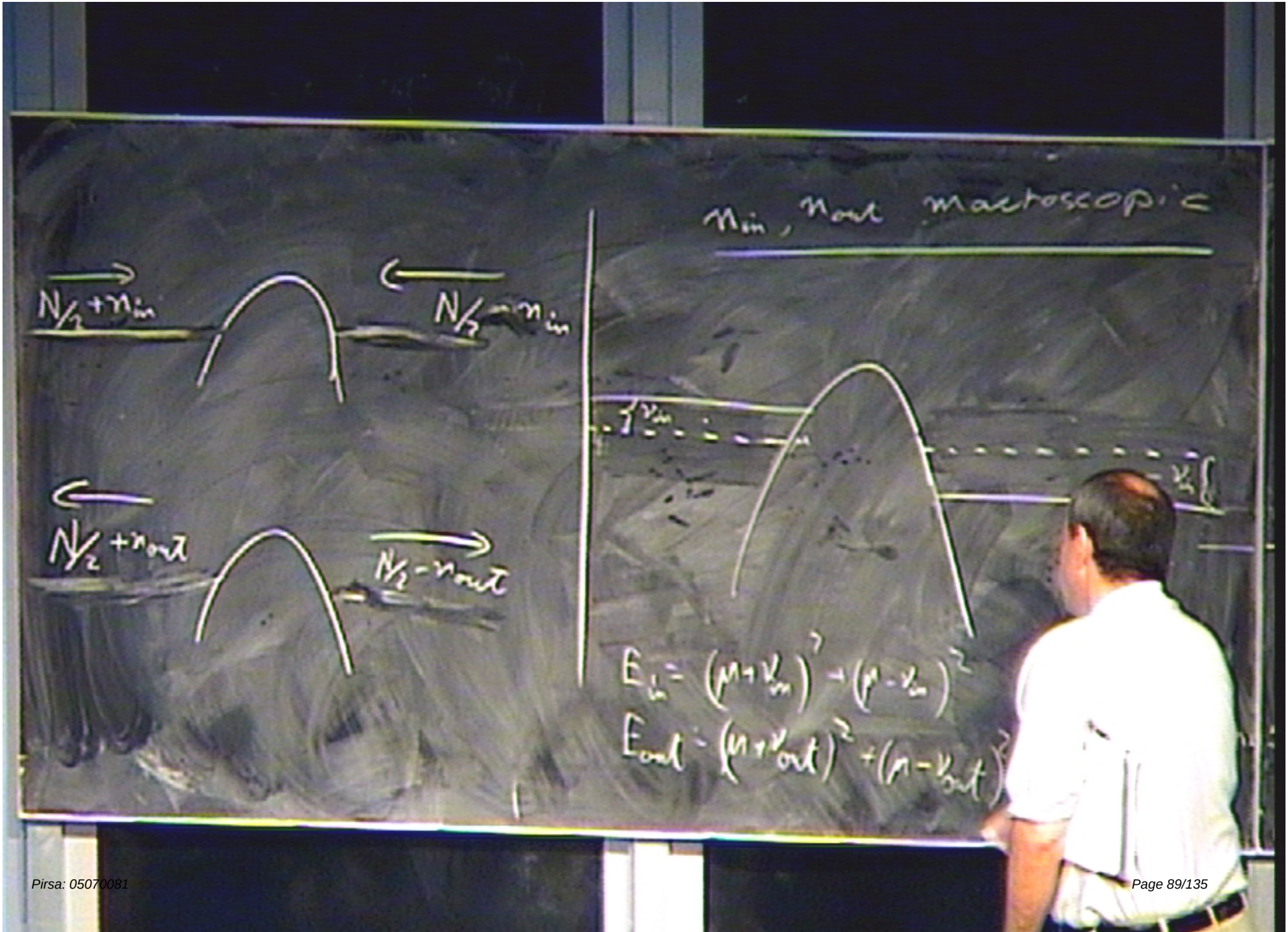


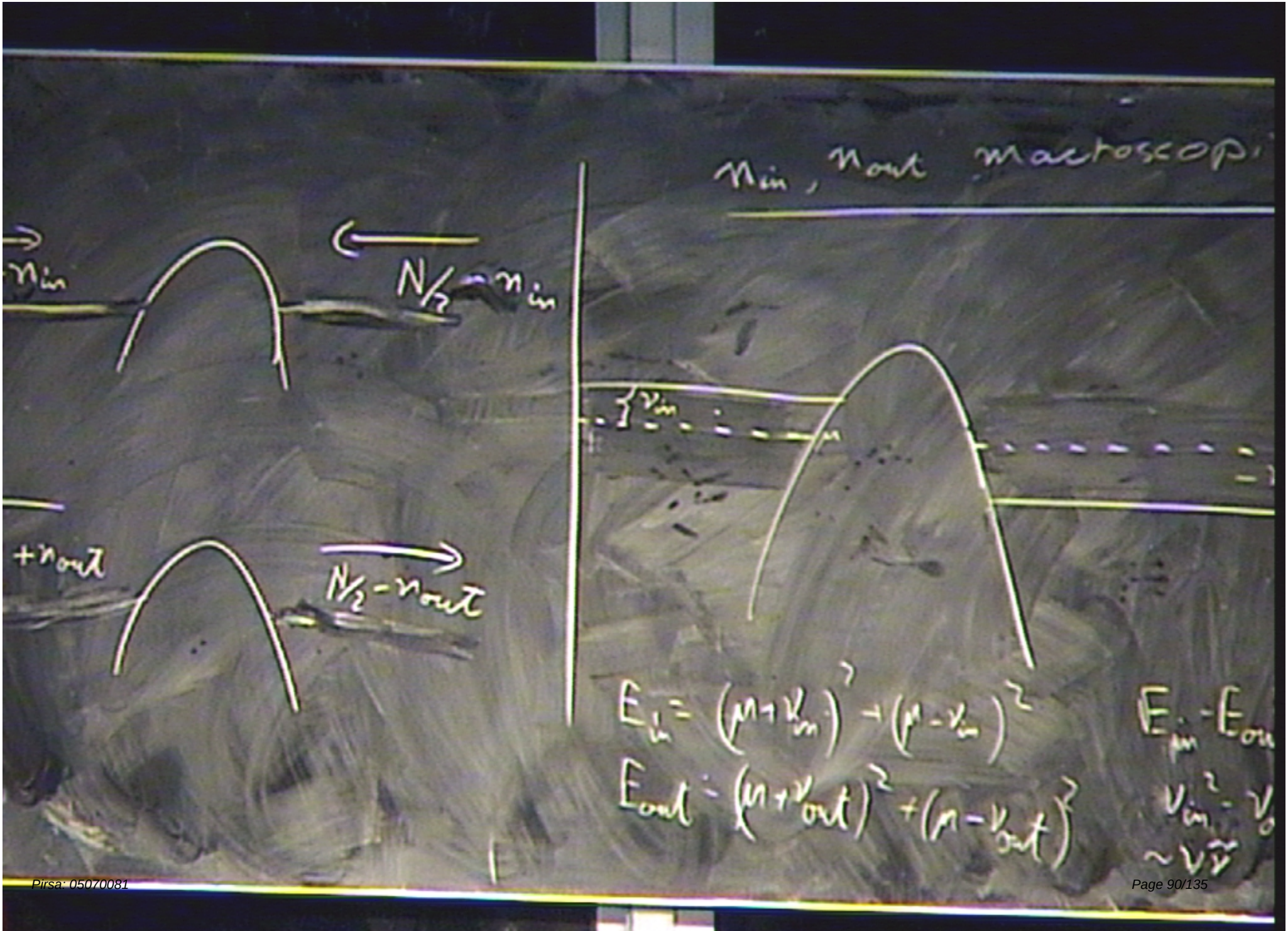


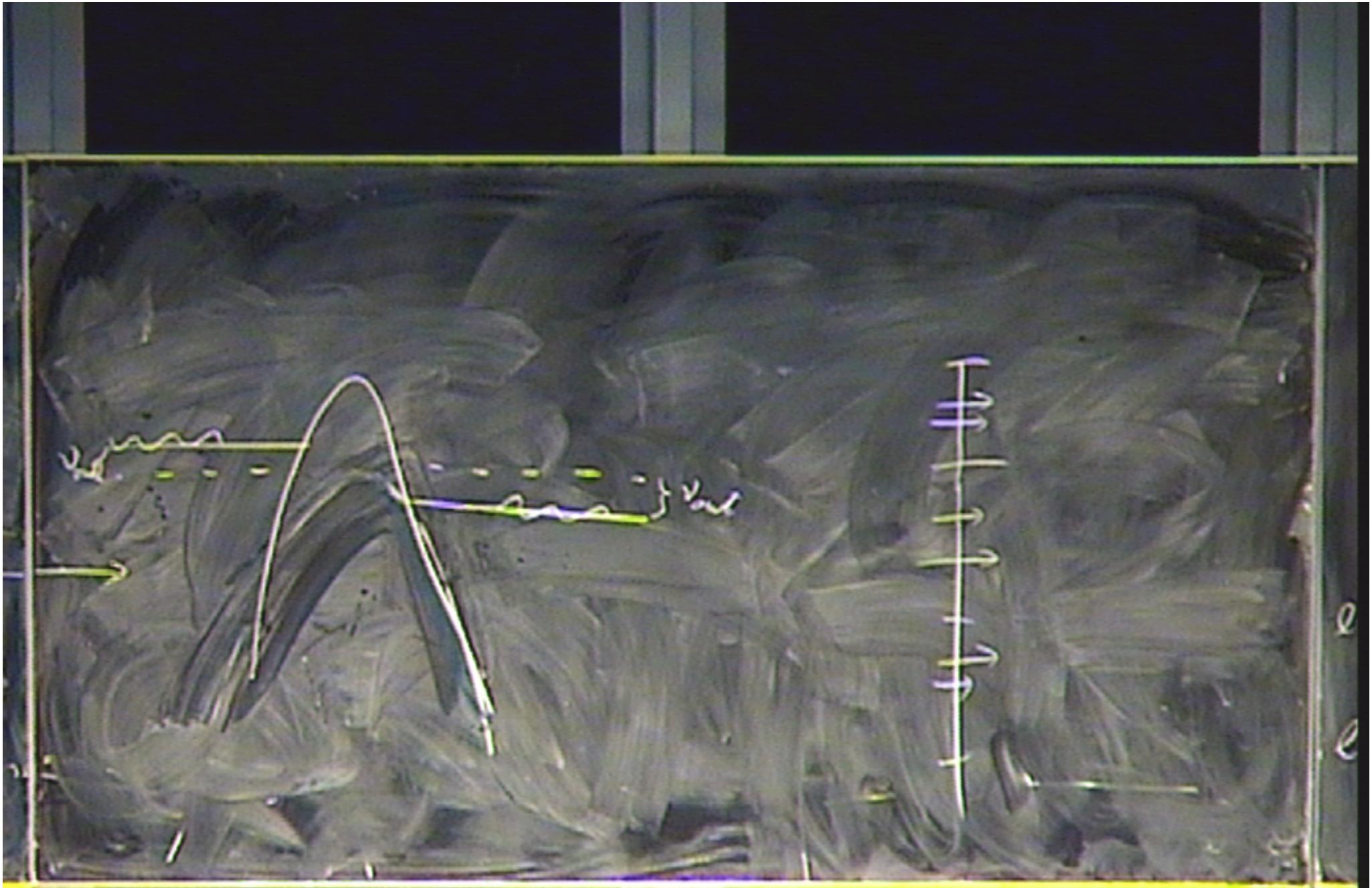


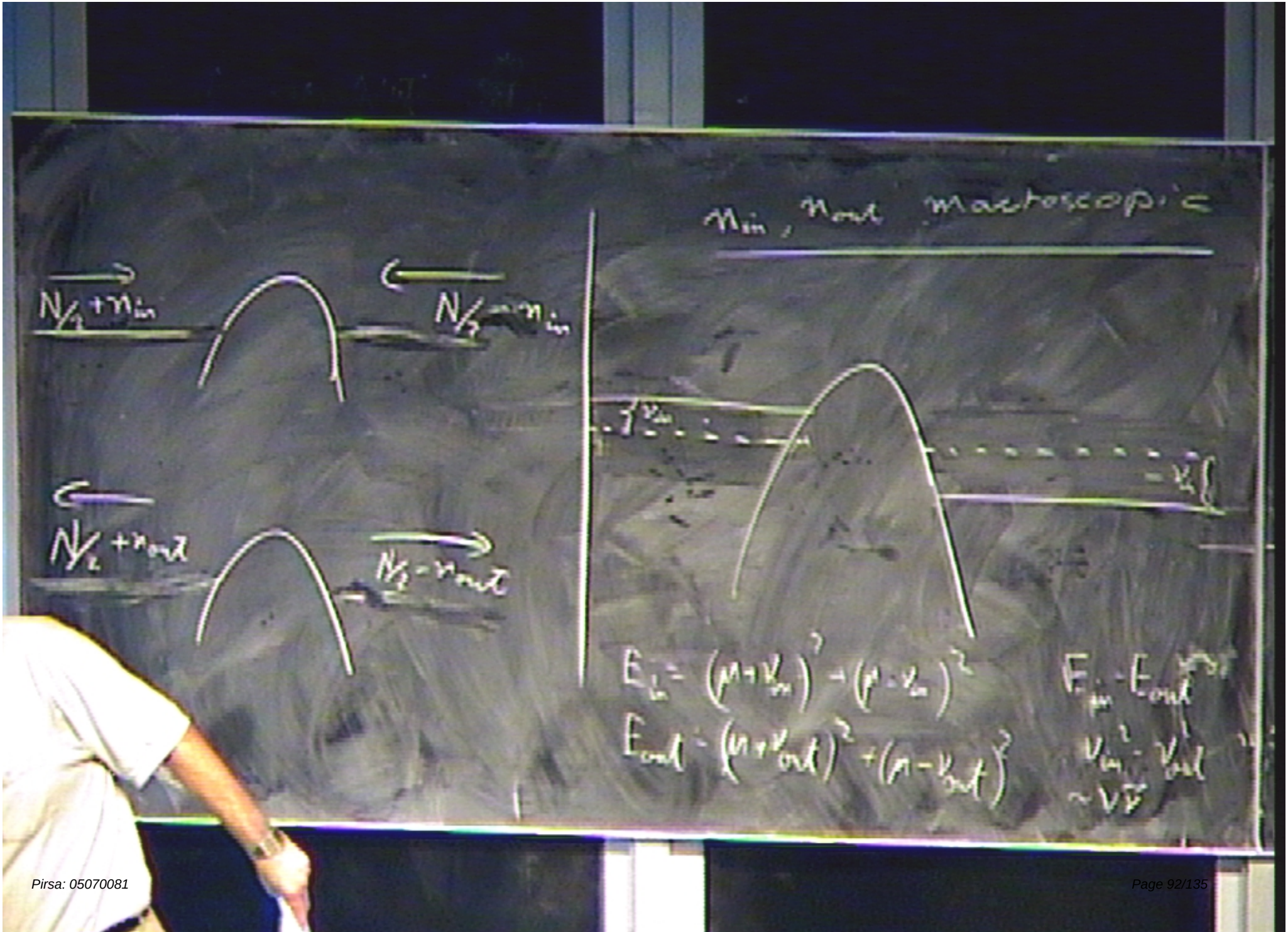

n_{in}, n_{out} macroscopic
$N/2 + n_{in}$
$N/2 - n_{in}$
$N/2 + n_{out}$
$N/2 - n_{out}$

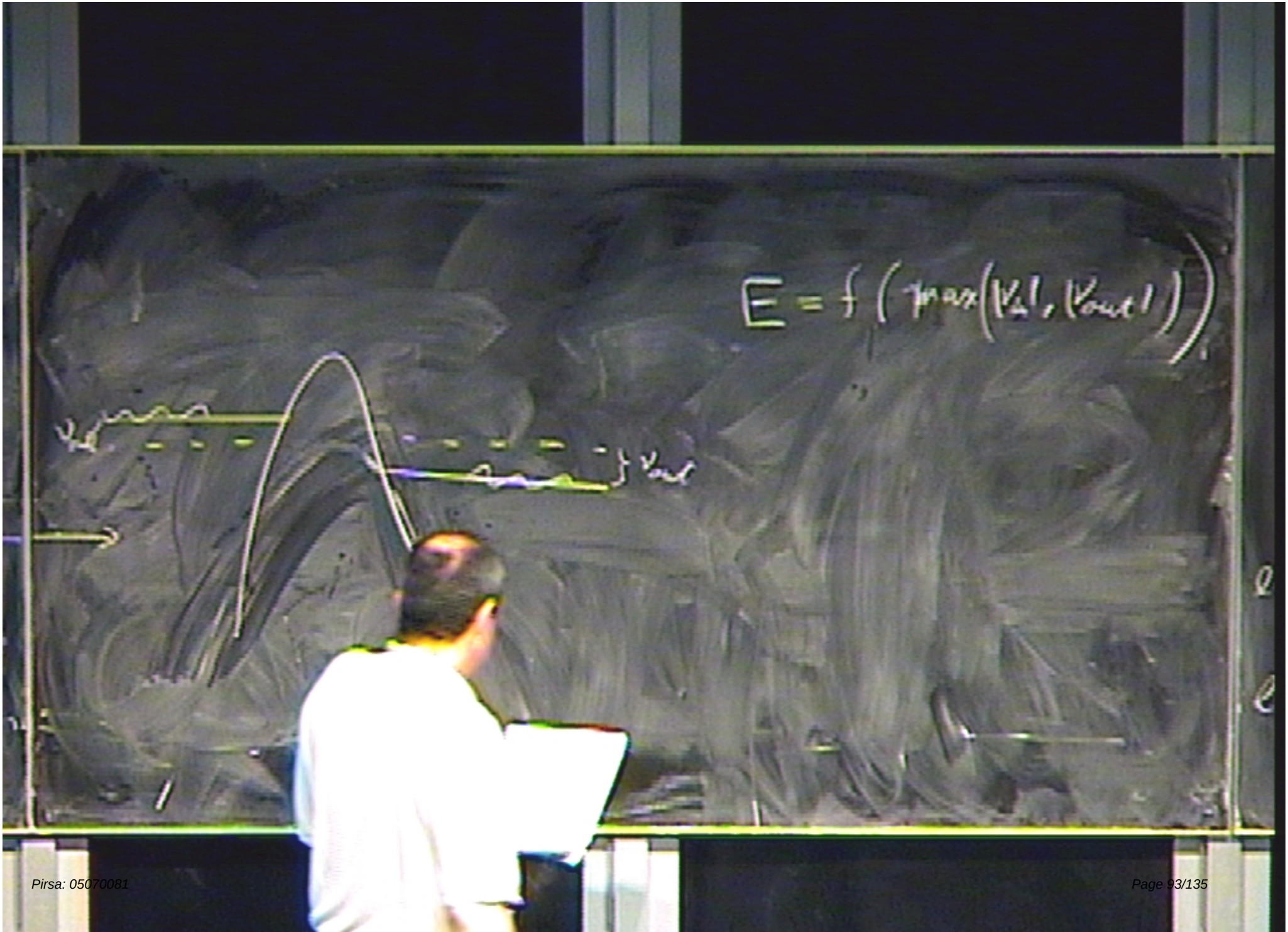

n_{in}, n_{out} macroscopic
$N/2 + n_{in}$
$N/2 - n_{in}$
$N/2 + n_{out}$
$N/2 - n_{out}$
$\mu + \nu_{in}$
$-\nu_{in}$
$E_{in} = (\mu + \nu_{in})^2 + (\mu - \nu_{in})^2$
$E_{out} = (\mu + \nu_{out})^2 + (\mu - \nu_{out})^2$
$E_{in} - E_{out} \neq 0$

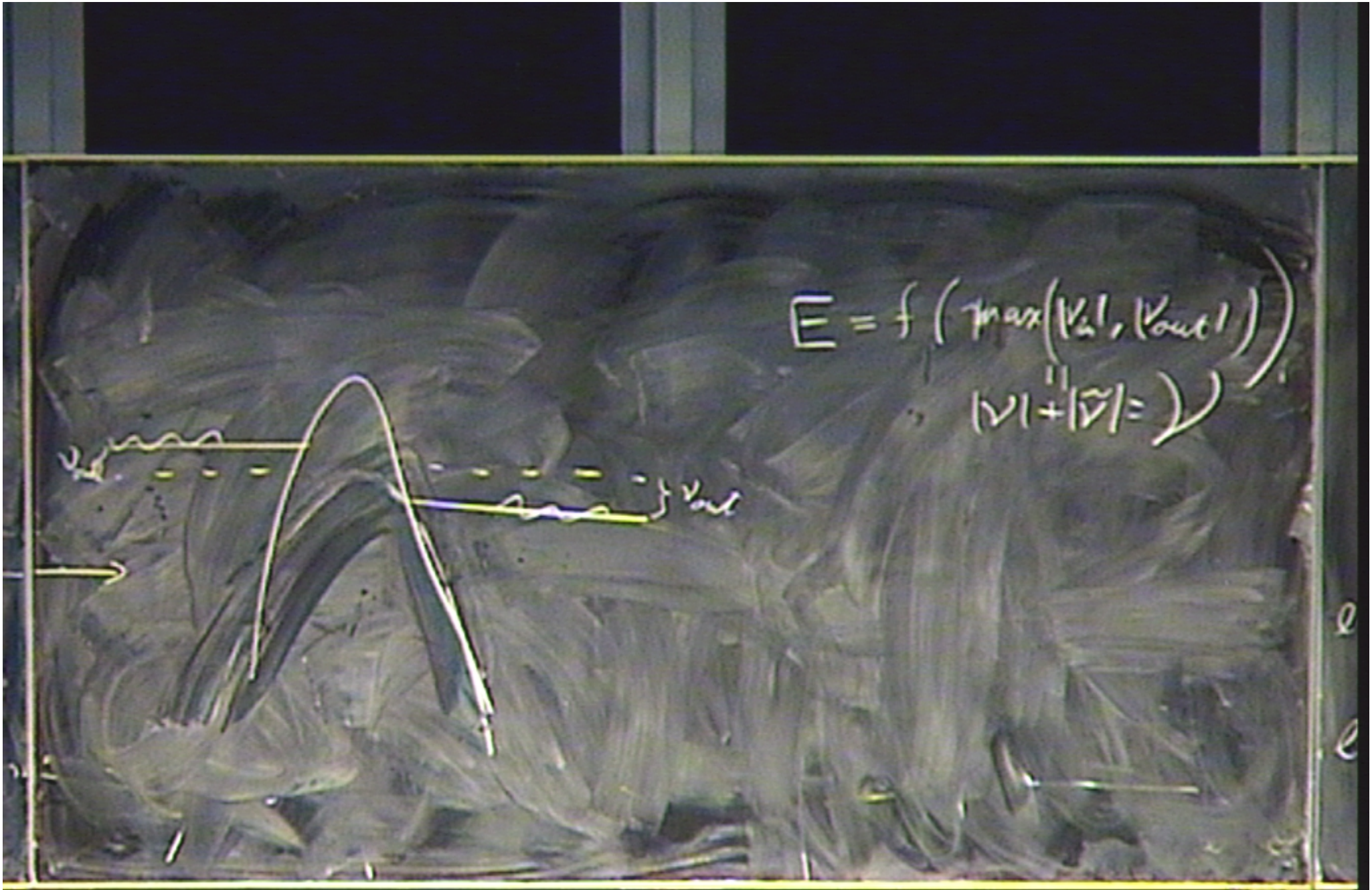

n_{in}, n_{out} macroscopic
$N/2 + n_{in}$
$N/2 - n_{in}$
$N/2 + n_{out}$
$N/2 - n_{out}$
$E_{in} = (\mu + \nu_{in})^2 + (\mu - \nu_{in})^2$
$E_{out} = (\mu + \nu_{out})^2 + (\mu - \nu_{out})^2$

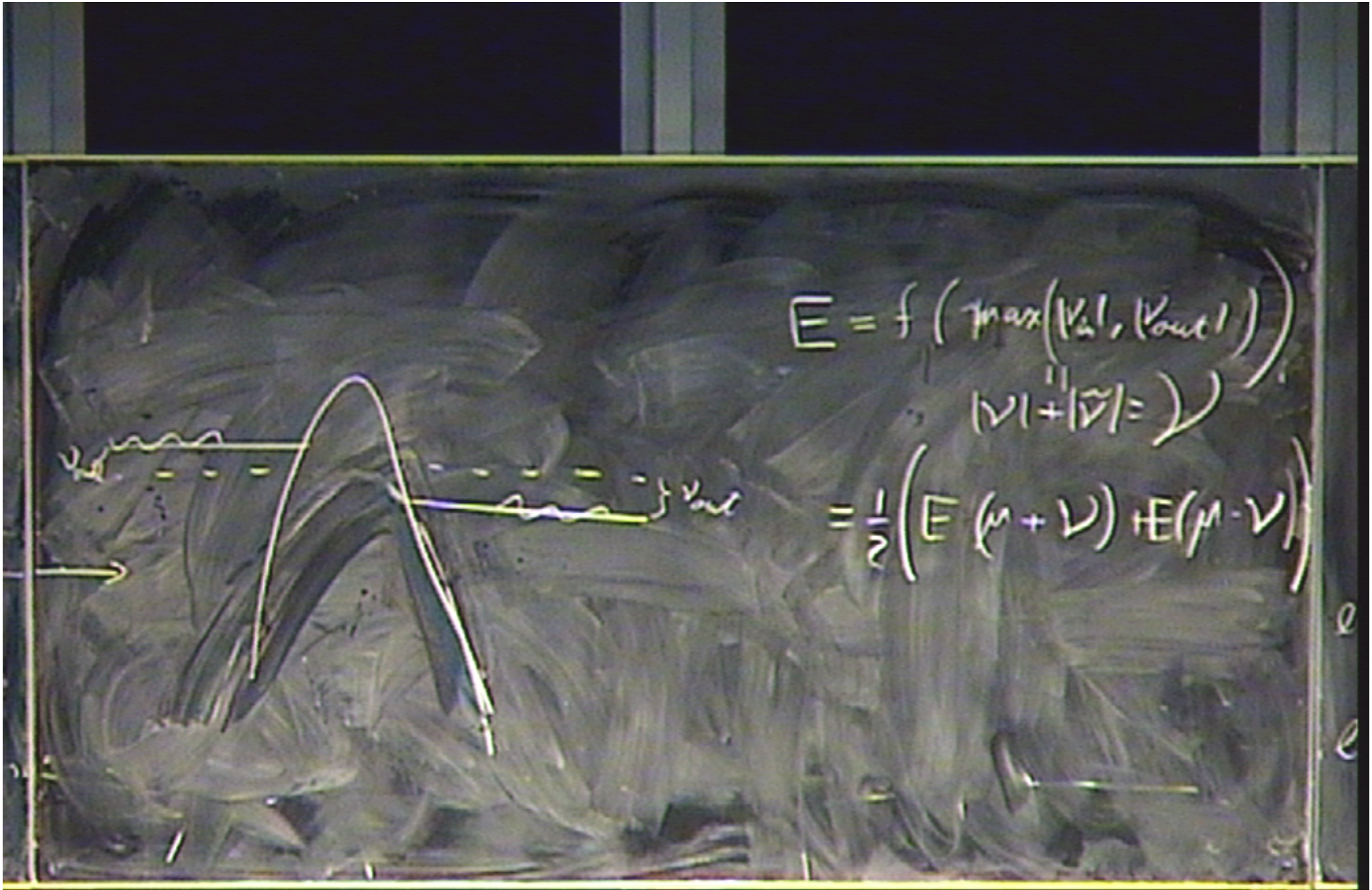

n_{in}, n_{out} macroscopi
n_{in}
$N/2 - n_{in}$
$+n_{out}$
$N/2 - n_{out}$
$E_{in} = (\mu + \nu_{in})^2 + (\mu - \nu_{in})^2$
$E_{out} = (\mu + \nu_{out})^2 + (\mu - \nu_{out})^2$
$E_{in} - E_{out}$
$\nu_{in}^2 - \nu_{out}^2$
$\sim \nu\tilde{\nu}$

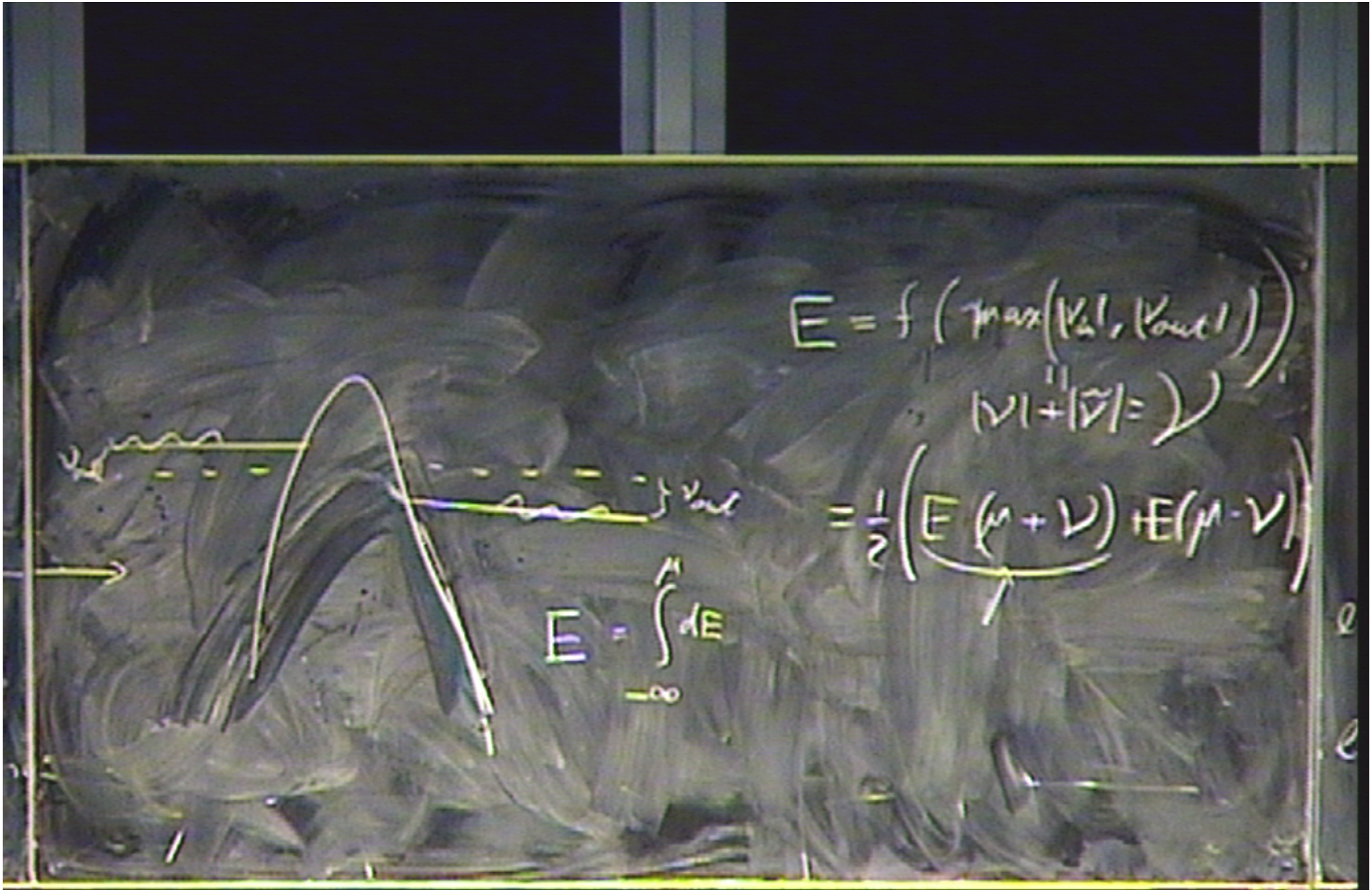


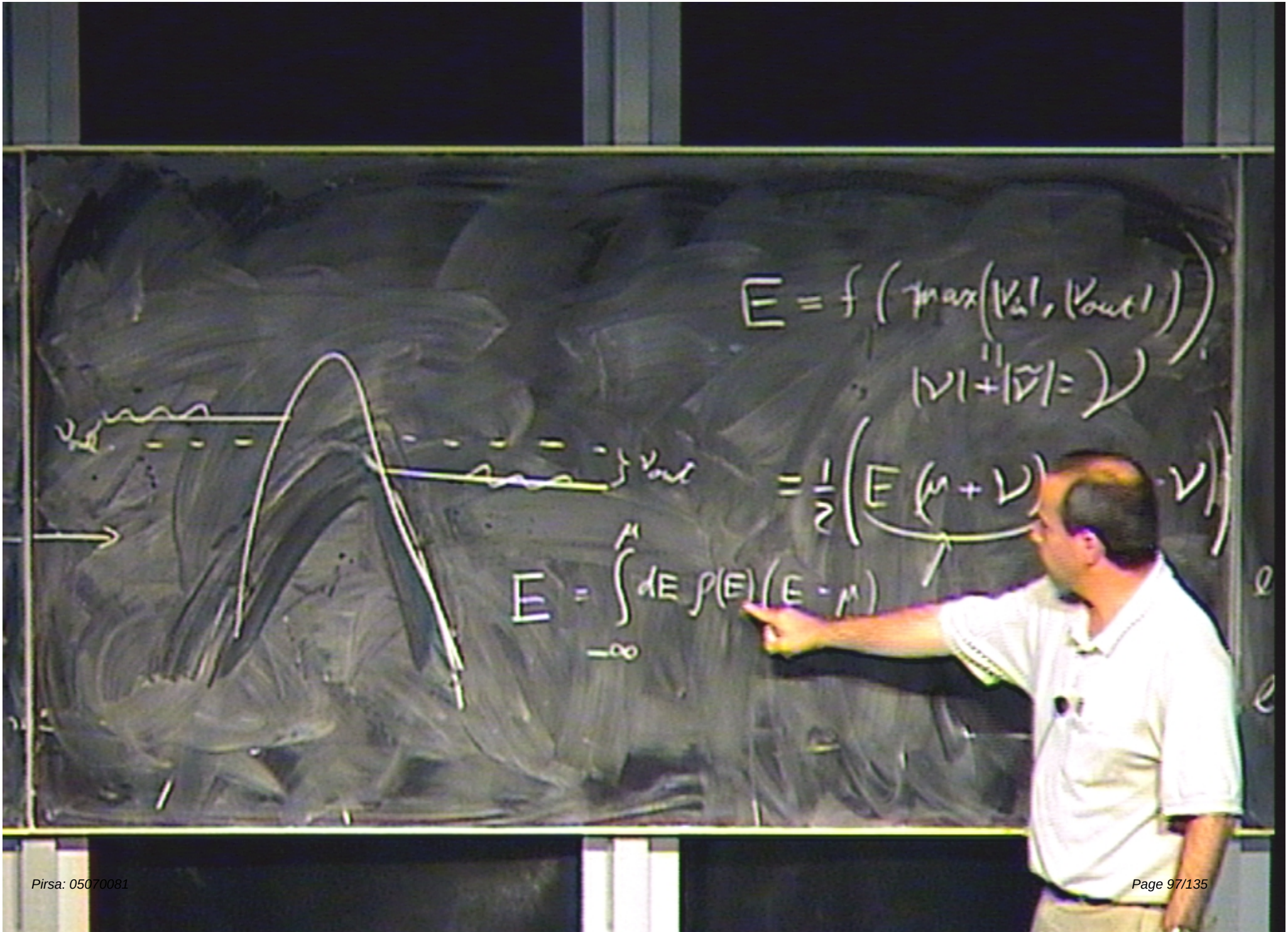

n_{in}, n_{out} macroscopic
$N/2 + n_{in}$
$N/2 - n_{in}$
$N/2 + n_{out}$
$N/2 - n_{out}$
$E_{in} = (\mu + v_{in})^2 + (\mu - v_{in})^2$
$E_{out} = (\mu + v_{out})^2 + (\mu - v_{out})^2$
$E_{in} - E_{out}$
$v_{in}^2 - v_{out}^2$
$\sim V\bar{V}$

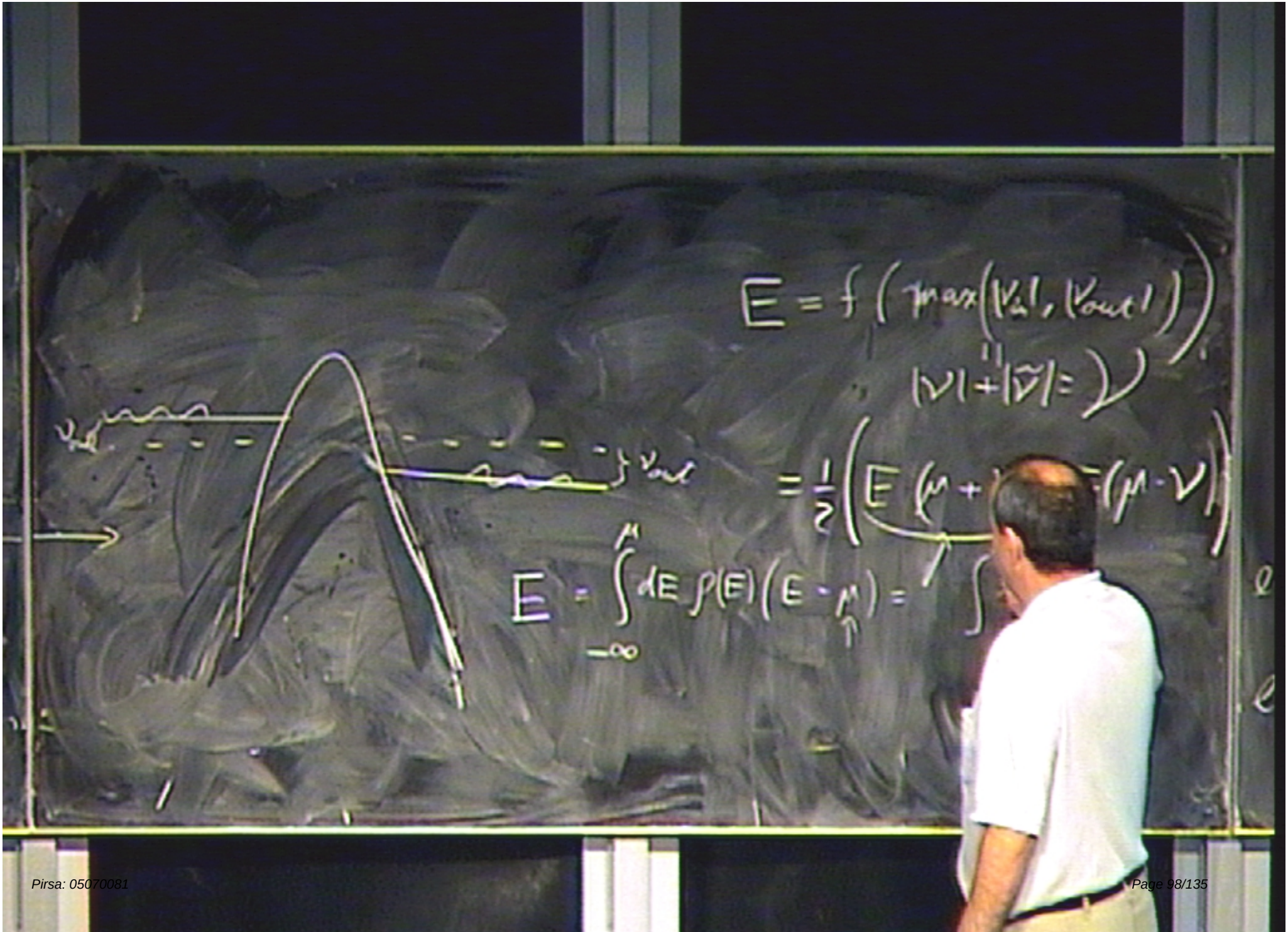

E = f (max(|V1|, |V2|))

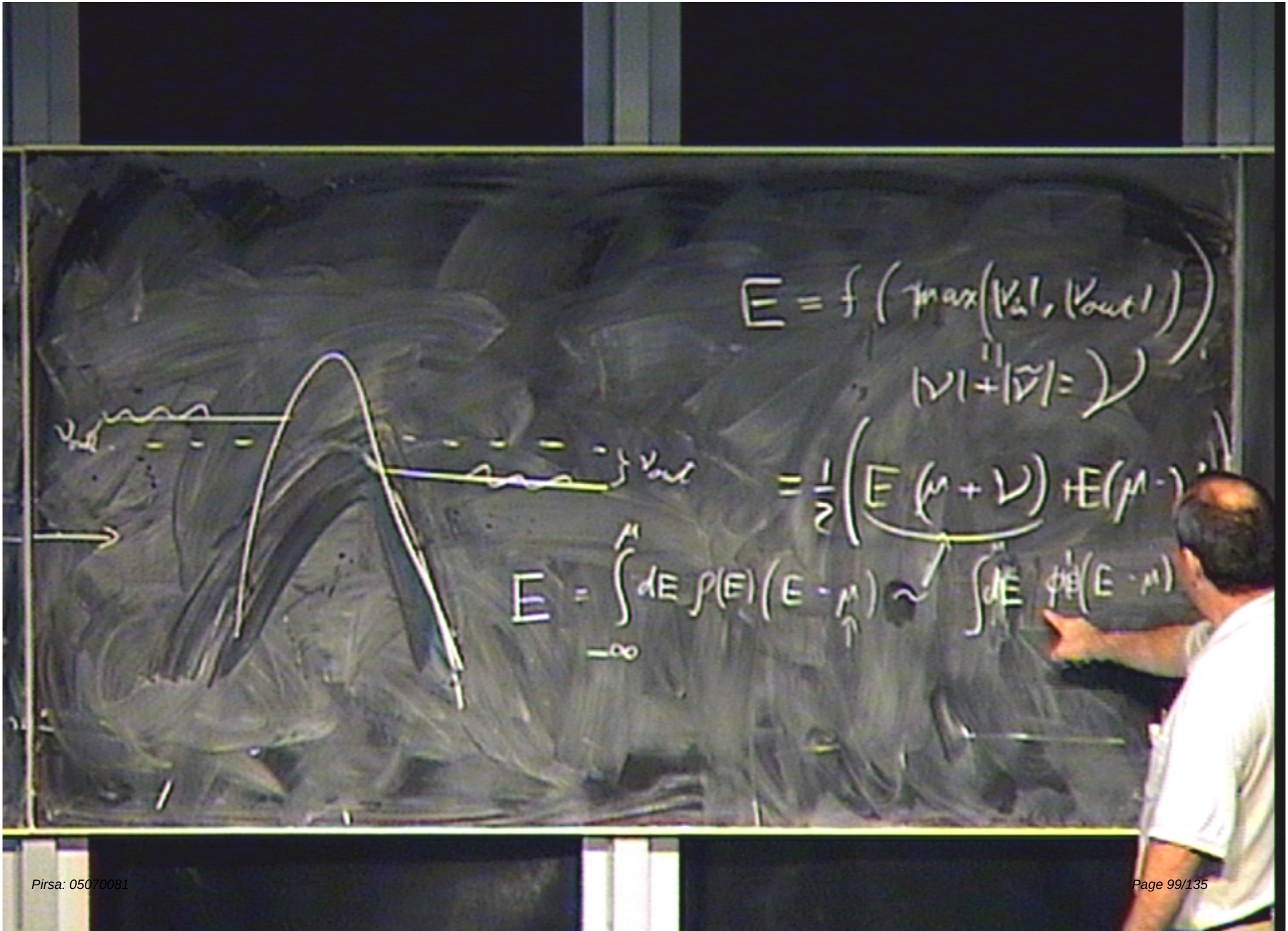

$E = f\left(\max\left(|V_{out}|, |V_{out}|\right)\right)$
$|V| + |\nabla| =$
V_{out}

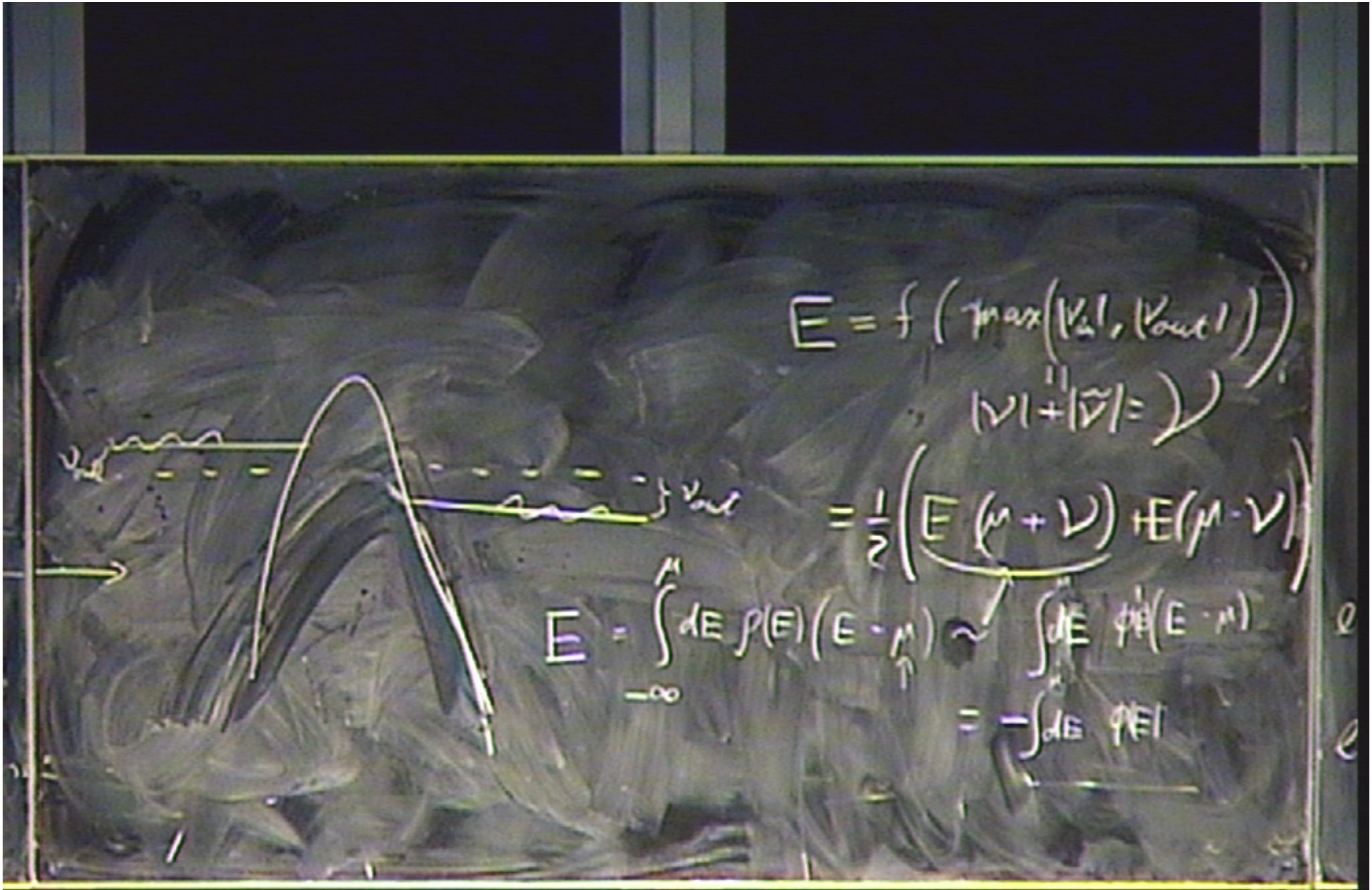

$E = f\left(\max(|V_{in}|, |V_{out}|)\right)$
$|V| + |\nabla| = V$
$= \frac{1}{2}\left(E(\mu + V) + E(\mu - V)\right)$
V_{in}
V_{out}

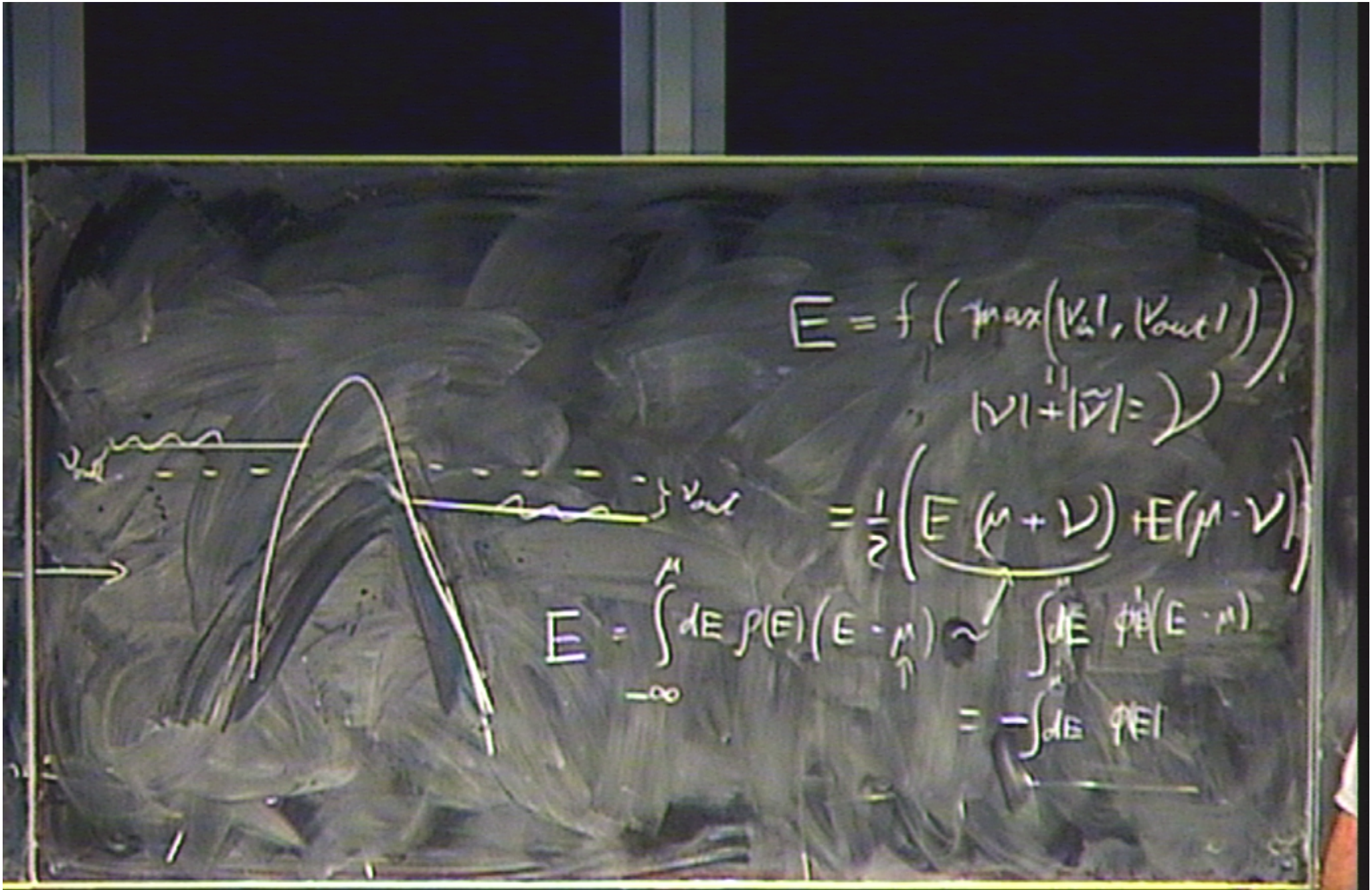

$E = f\left(\max\left(|V_L|, |V_{out}|\right)\right)$
$|V| + |\nabla| =$
V_{out}
$= \frac{1}{2}\left(E(\mu + V) + E(\mu - V)\right)$
$E = \int_{-\infty}^{\mu} dE$


E = f (max(|V_in|, |V_out|))
|V| + |∇V| =
= 1/2 (E (μ + V)
E = ∫_{-∞}^{μ} dE ρ(E) (E - μ)


E = f (max(|V_in|, |V_out|))
|V| + |∇V| =)
V_in
V_out
= 1/2 (E (μ +
(μ - V)
E = ∫_{-∞}^{μ} dE ρ(E) (E - μ) =

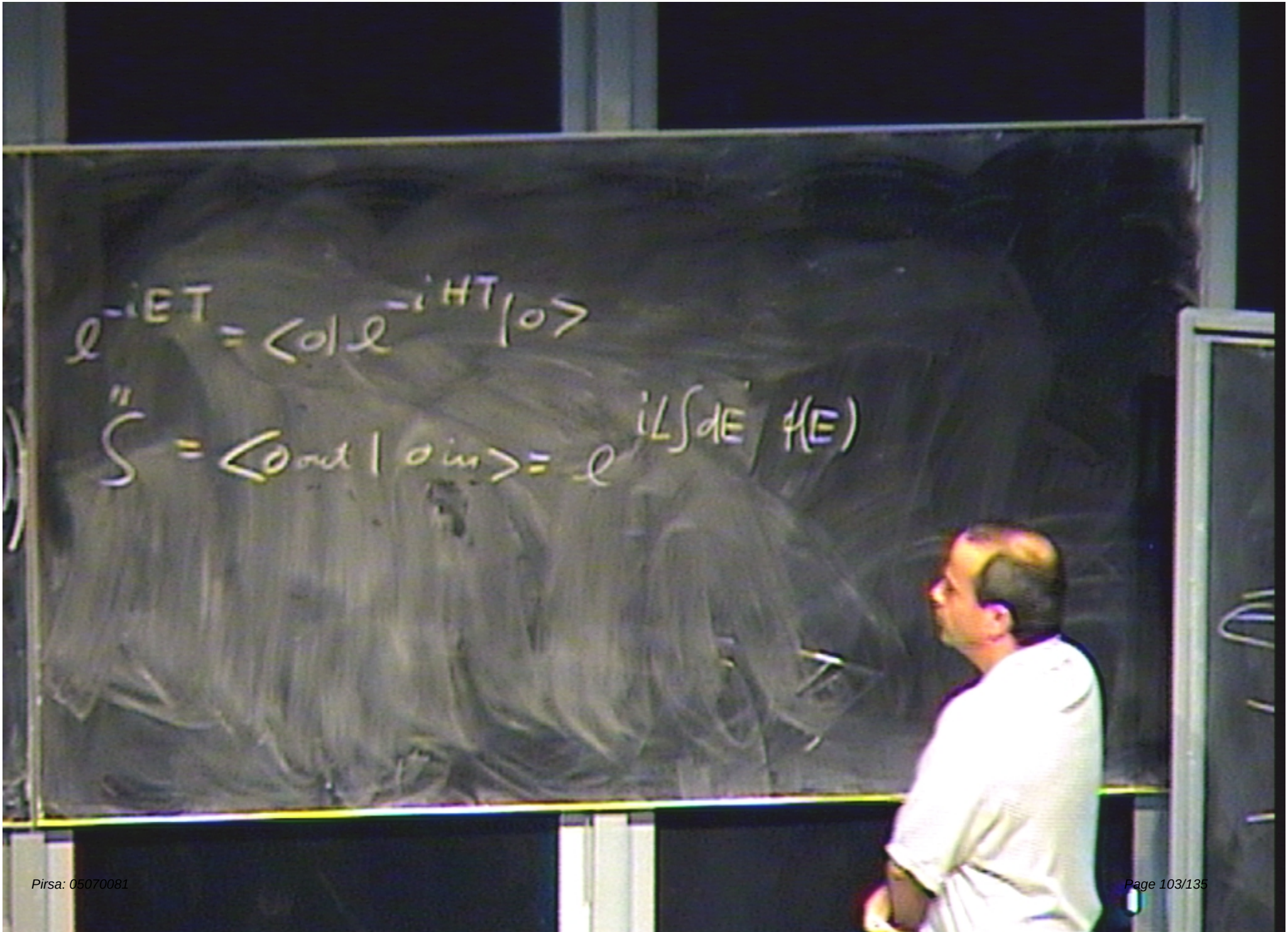

$E = f\left(\max\left(|V_{in}|, |V_{out}|\right)\right)$
$|\nabla| + |\nabla| =$
V_{in}
V_{out}
$= \frac{1}{2}\left(E(\mu + V) + E(\mu -\right.$
$E = \int_{-\infty}^{\mu} dE\, \rho(E)(E - \mu) \sim \int dE\, \rho(E)(E - \mu)$


$E = f\left(\max\left(|V_L|, |V_{out}|\right)\right)$
$|V| + |\nabla| =$
V_{out}
$= \frac{1}{2}\left(E(\mu+V) + E(\mu-V)\right)$
$E = \int_{-\infty}^{\mu} dE\, \rho(E)(E-\mu) \sim$
$= -\int dE\, \rho(E)$


$E = f\left(\max\left(|V_{in}|, |V_{out}|\right)\right)$
$|V| + |\nabla| =$
V_{out}
$= \frac{1}{2}\left(E(\mu+V) + E(\mu-V)\right)$
$E = \int_{-\infty}^{\mu} dE\, \rho(E)(E-\mu) \sim$
$= -\int dE\, \varphi(E)$

$$e^{-iET} = \langle 0 | e^{-iHT} | 0 \rangle$$

$$\text{"}S\text{"} = \langle 0_{out} | 0_{in} \rangle$$


$e^{-iET} = \langle o | e^{-iHT} | o \rangle$
$S = \langle o_{out} | o_{in} \rangle = e^{iL \int dE' \, \eta(E')}$

$$e^{-iET} = \langle 0 | e^{-iHT} | 0 \rangle$$

$$\text{"}S = \langle 0_{out} | 0_{in} \rangle = e^{i \int_{L=T} dE \; f(E)}$$

$$e^{-iET} = \langle 0 | e^{-iHT} | 0 \rangle$$

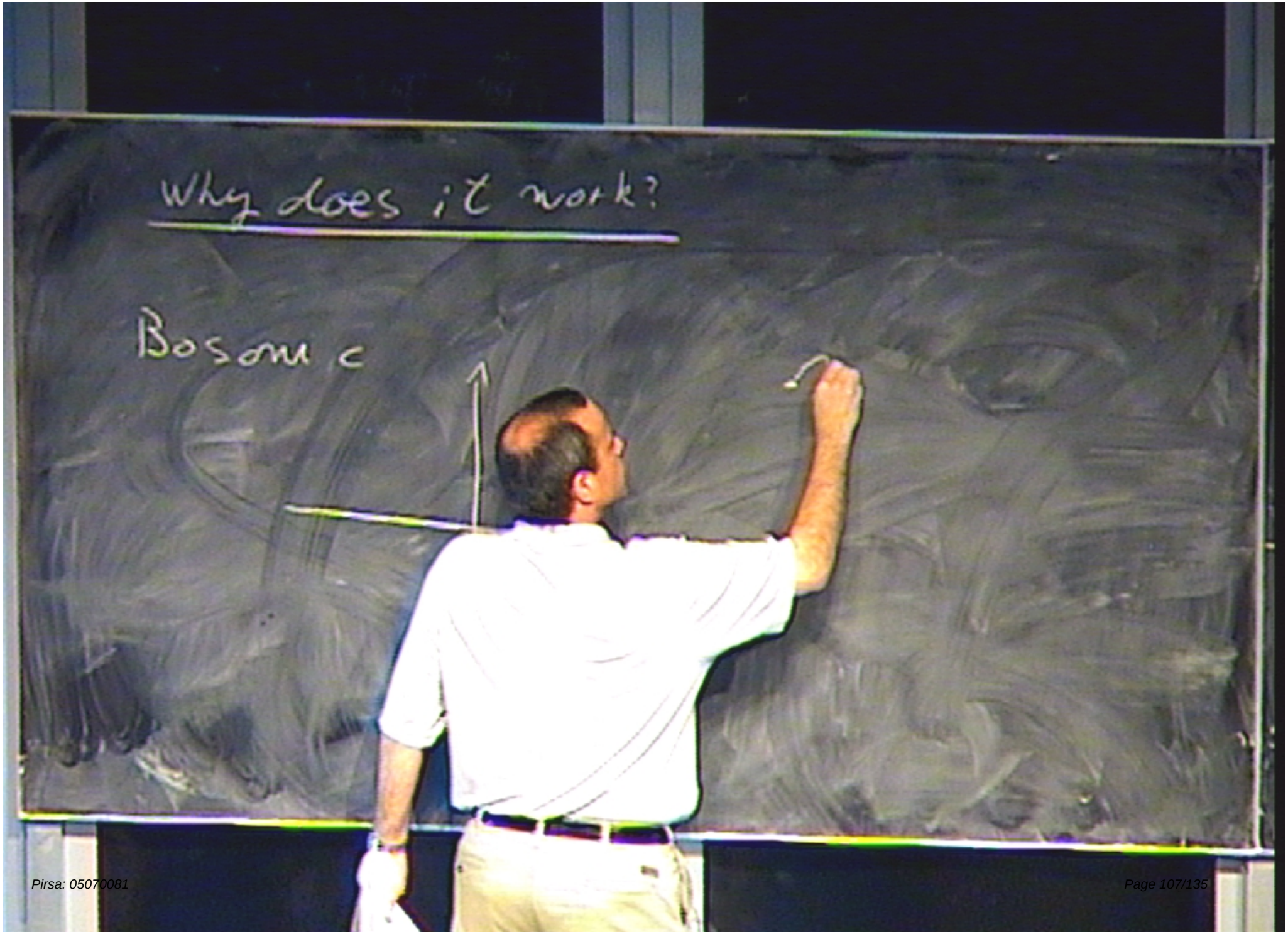
$$S = \langle 0_{out} | 0_{in} \rangle = e^{i \int dE \, f(E)}$$

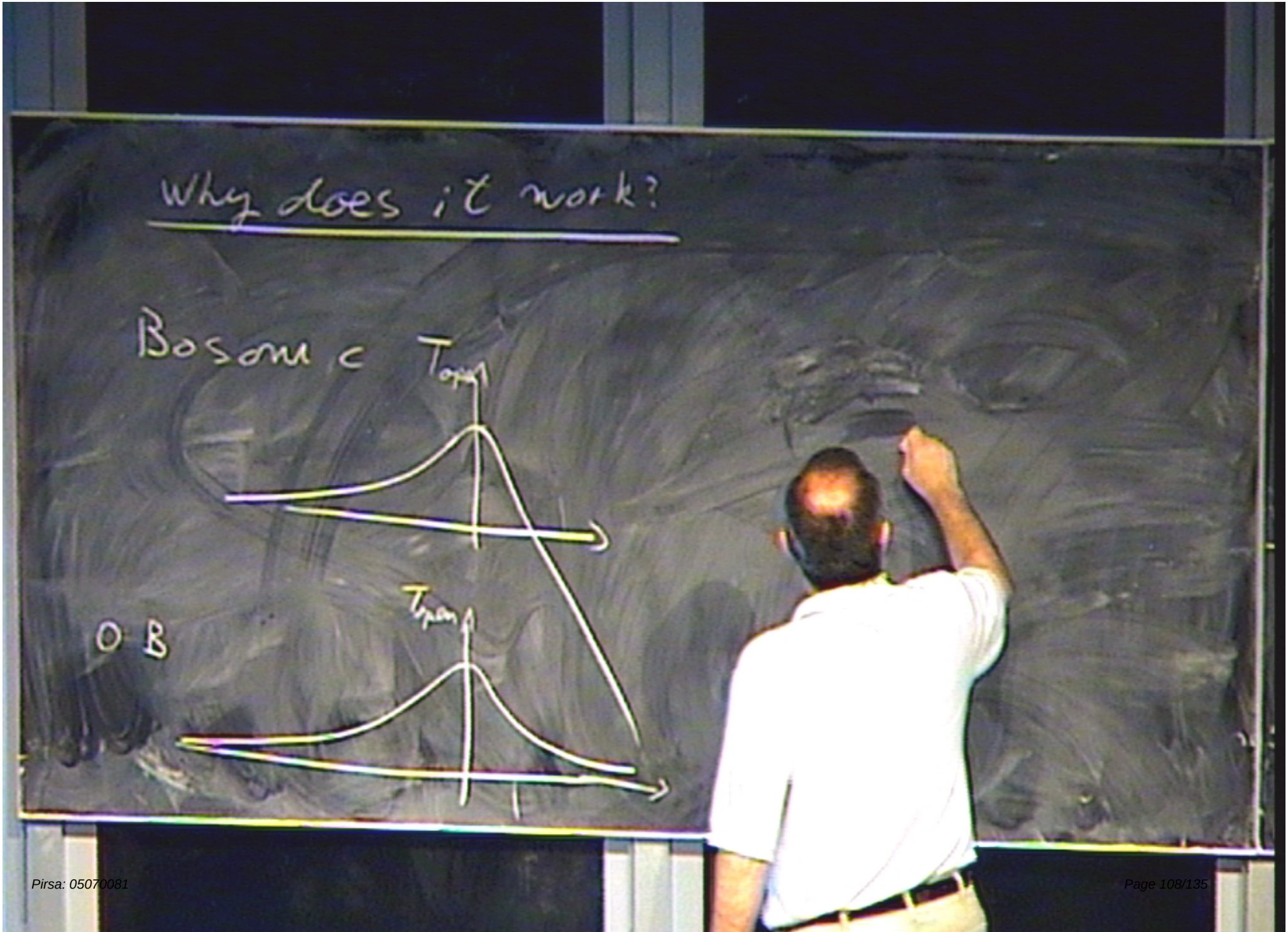
$$L = T$$

$$e^{-iET} = \langle 0 | e^{-iHT} | 0 \rangle$$

$$S = \langle 0_{out} | 0_{in} \rangle = e^{i \int dE \; A(E)}$$

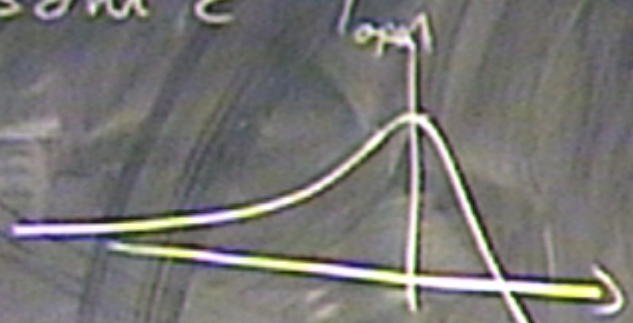
$$L = T$$


Why does it work?
Bosonic


Why does it work?
Bosons
OB

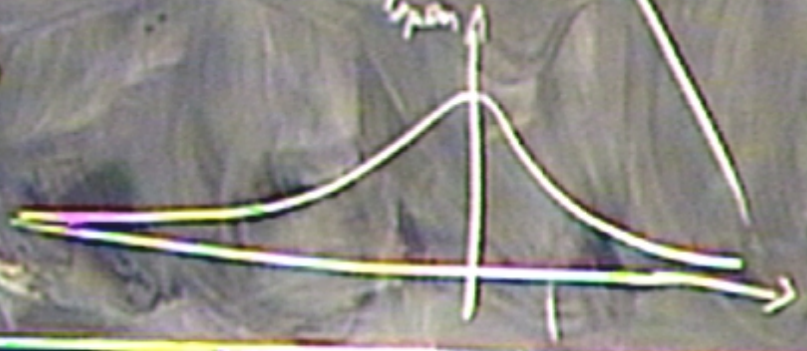
Why does it work?

Bosons: T_{opt}





FB: T_{opt}





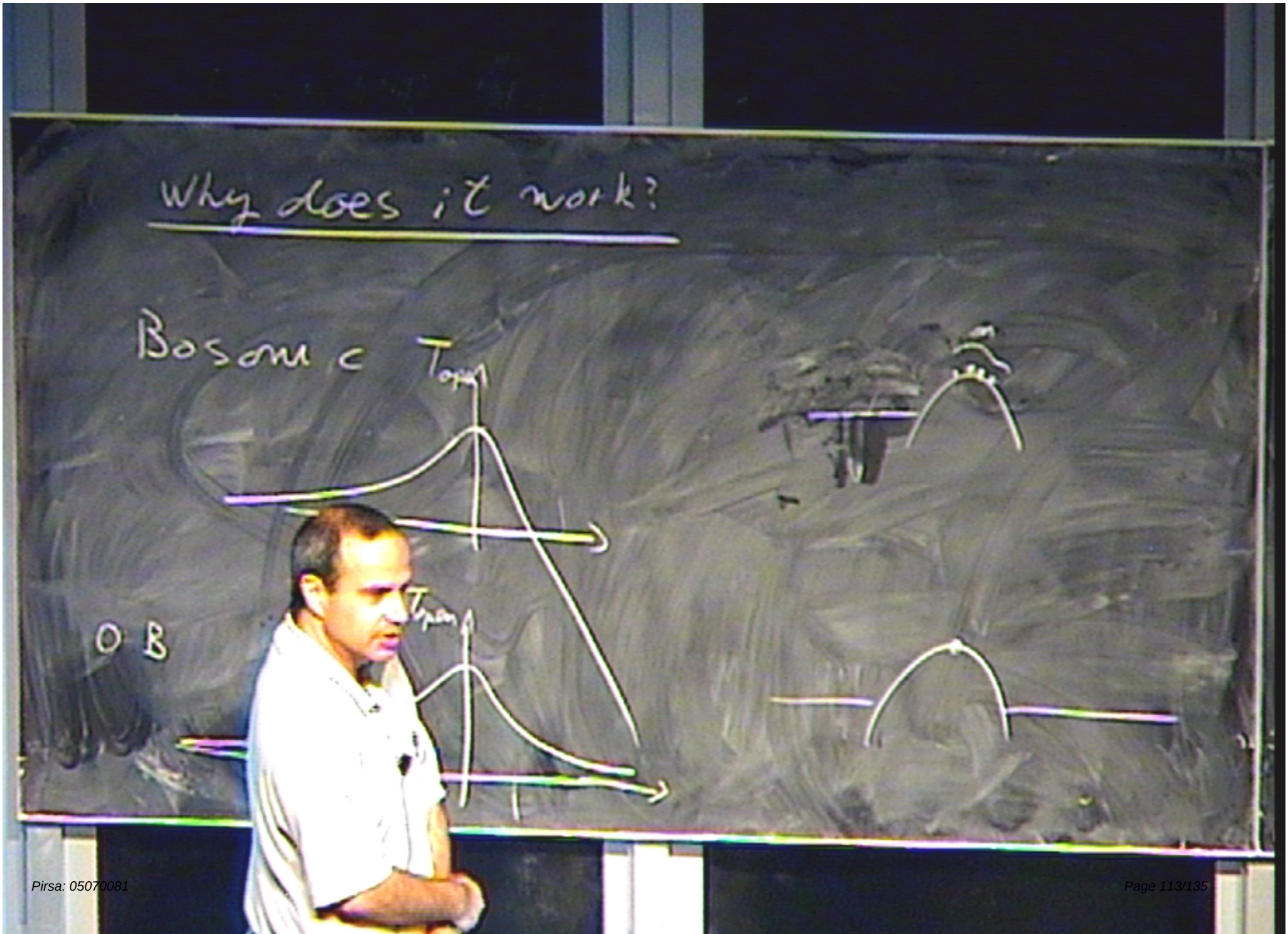


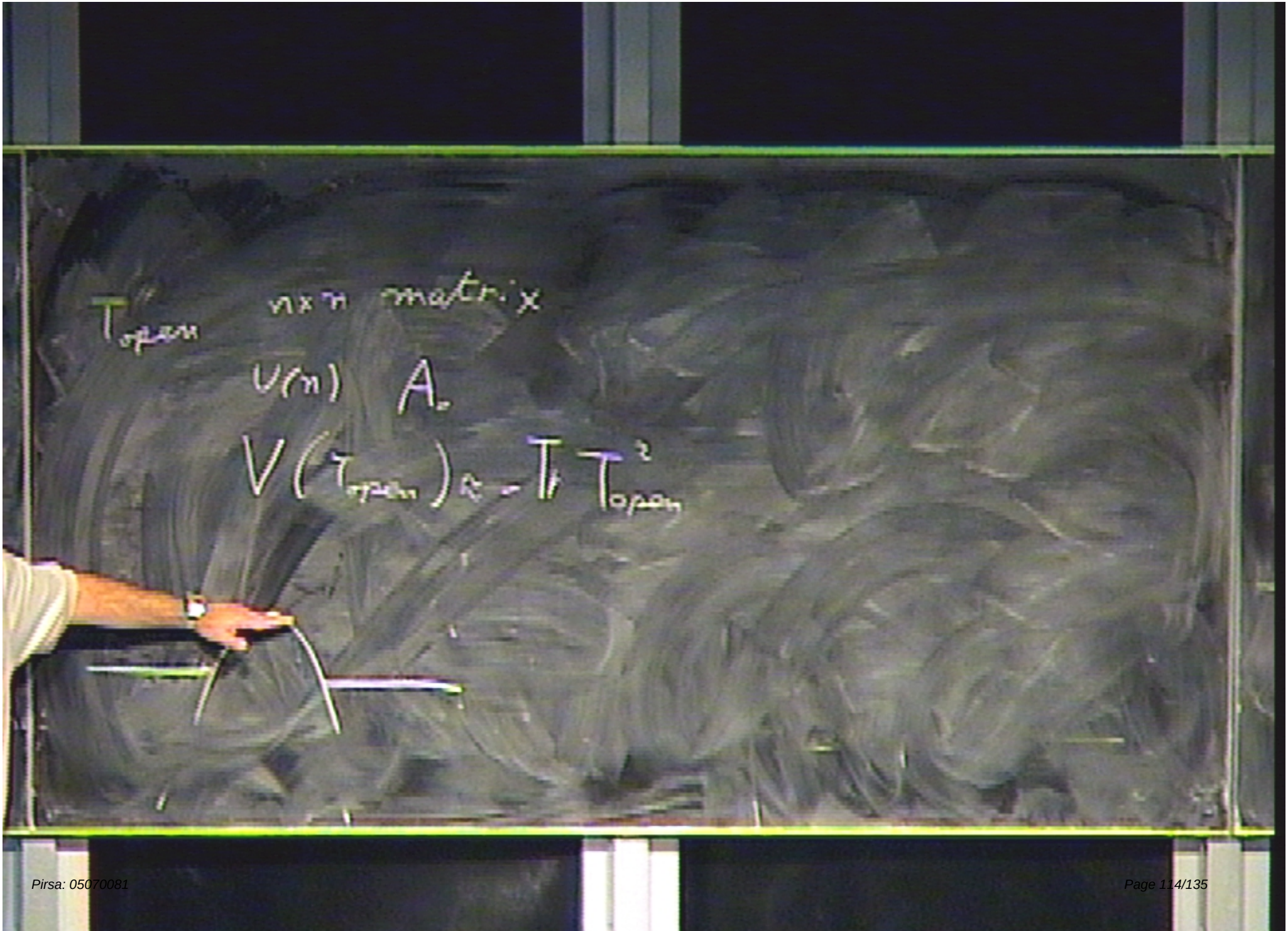

T open
n x n matrix
U(n)
A.

T_{open} $n \times n$ matrix

$U(n)$ A_0

$V(T_{open}) \approx - \mathrm{Tr}\, T_{open}^2$


Why does it work?
Bosons
OB

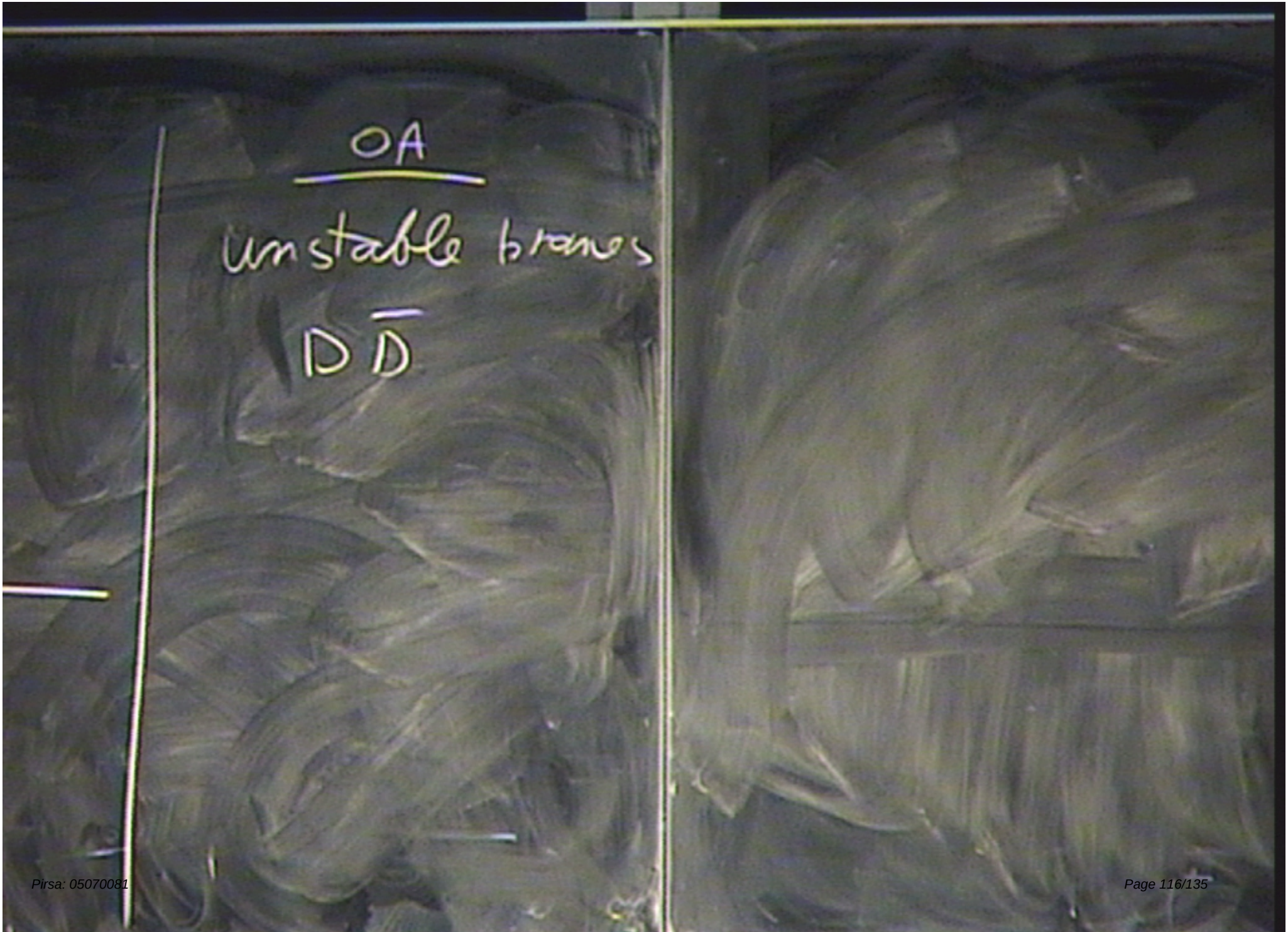

T_{open} n x n matrix
$U(n)$ A_α
$V(T_{open})_R = T_r\, T_{open}^2$

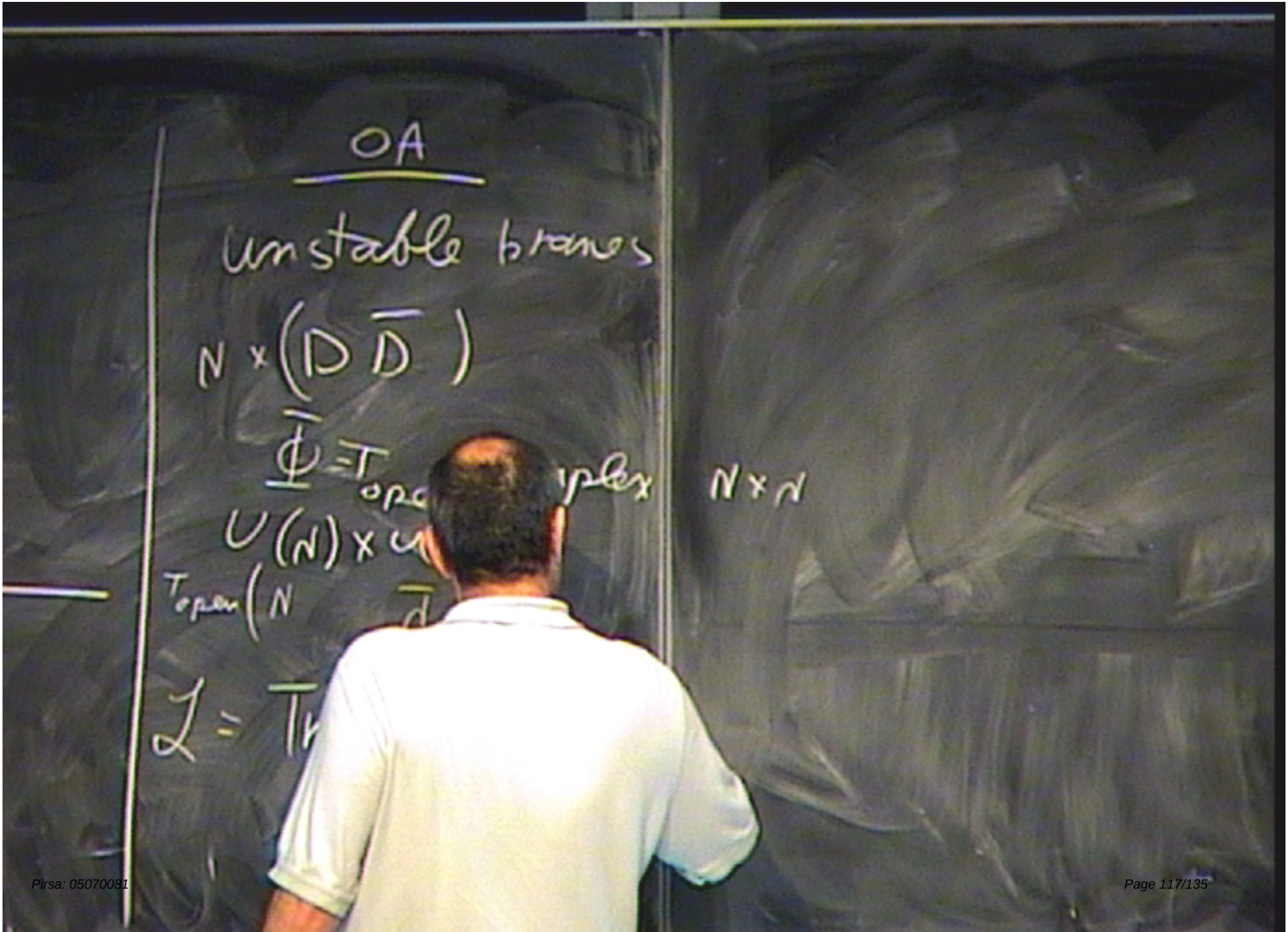
T_{open} $n \times n$ matrix

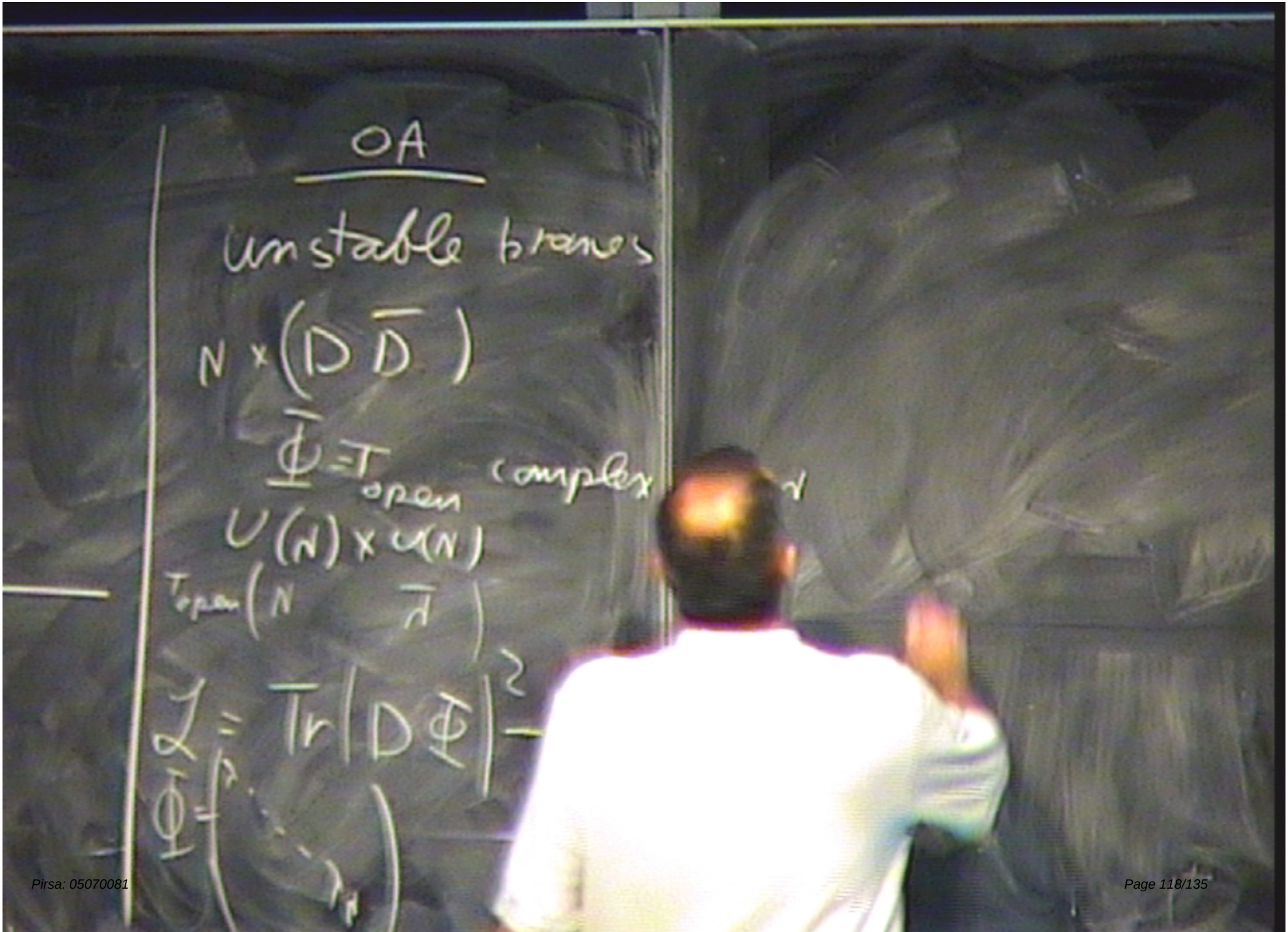
$U(n)$ A_0

$V(T_{open}) \approx - \mathrm{Tr}\, T_{open}^2$




OA
unstable branes
$D\bar{D}$


OA
unstable branes
$N \times (D\bar{D})$
$\bar{\Phi}$ = T
plex $N \times N$
$U(N) \times U$
$T_{open}(N$
$\mathcal{L} = \mathrm{Tr}$


OA
unstable branes
$N \times (D\bar{D})$
$\Phi = T_{open}$ (complex
$U(N) \times U(N)$
$T_{open}(N \quad \bar{N})$
$\mathcal{L} = \mathrm{Tr}|D\Phi|^2 -$
$\Phi = \begin{pmatrix} \cdot & & \\ & \cdot & \\ & & \cdot \end{pmatrix}$

OA

unstable branes

$N \times (D\bar{D})$

$\Phi = T_{open}$ (complex $N \times N$)

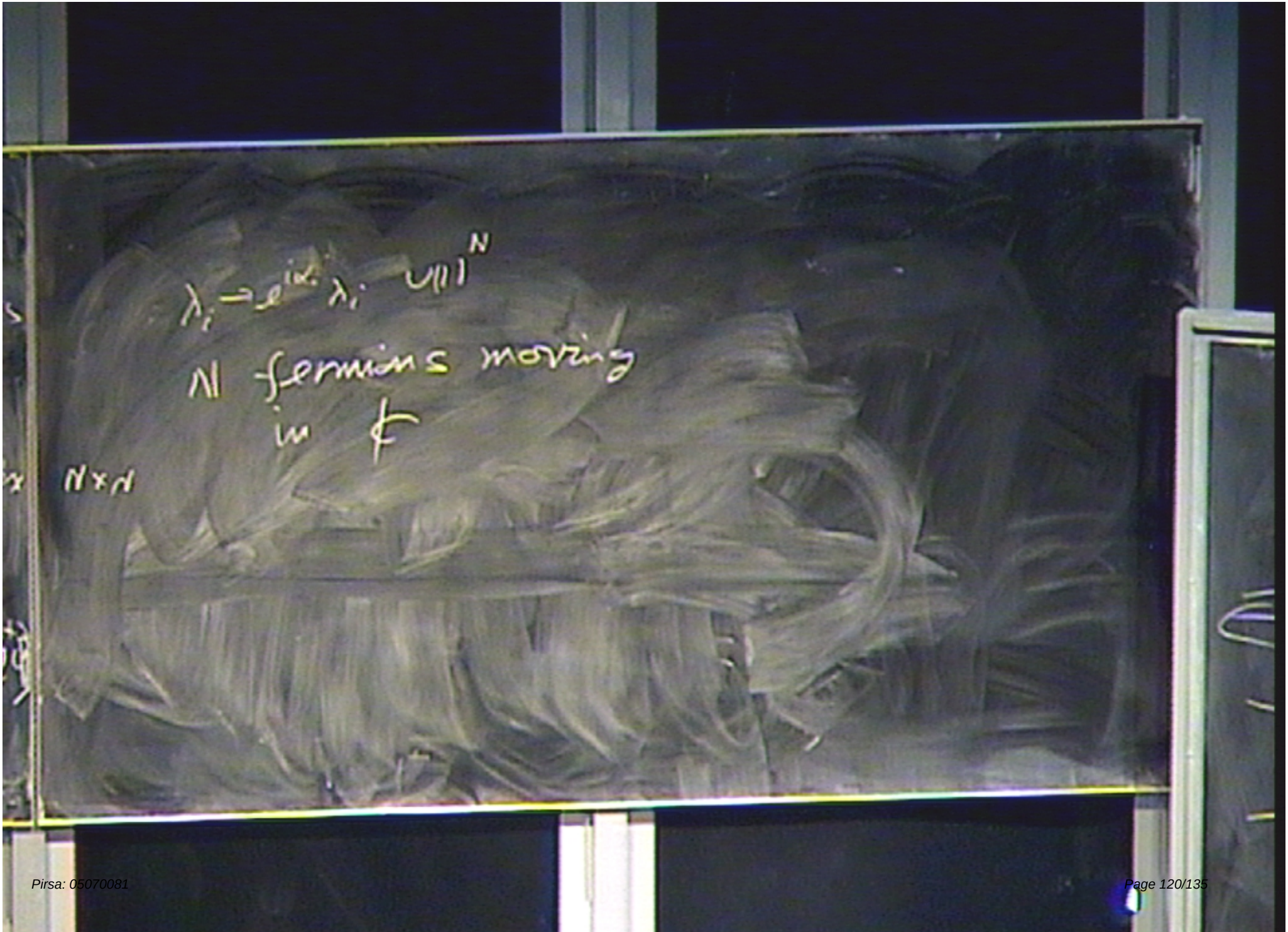
$U(N) \times U(N)$

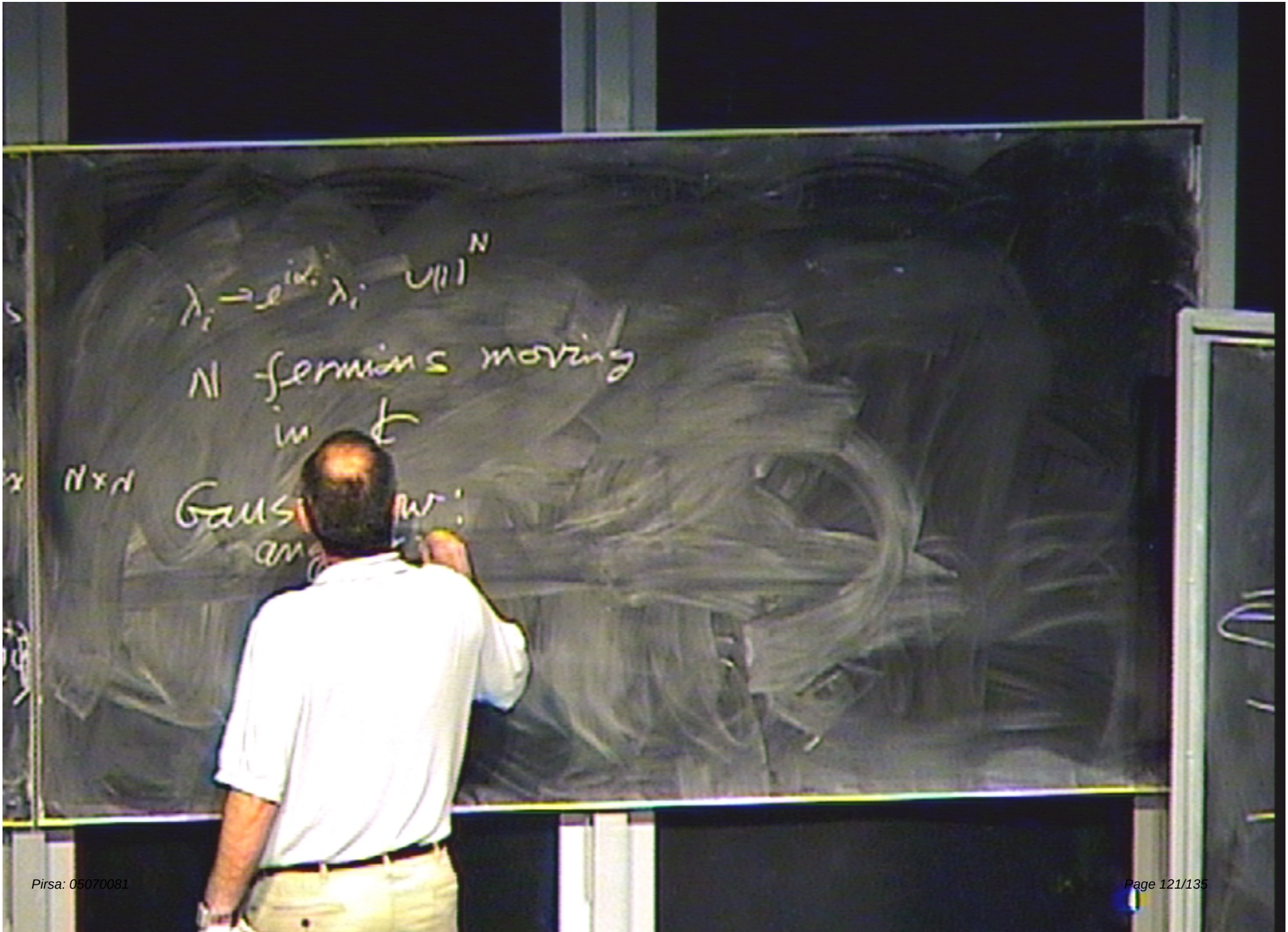
$T_{open}(N, \bar{N})$

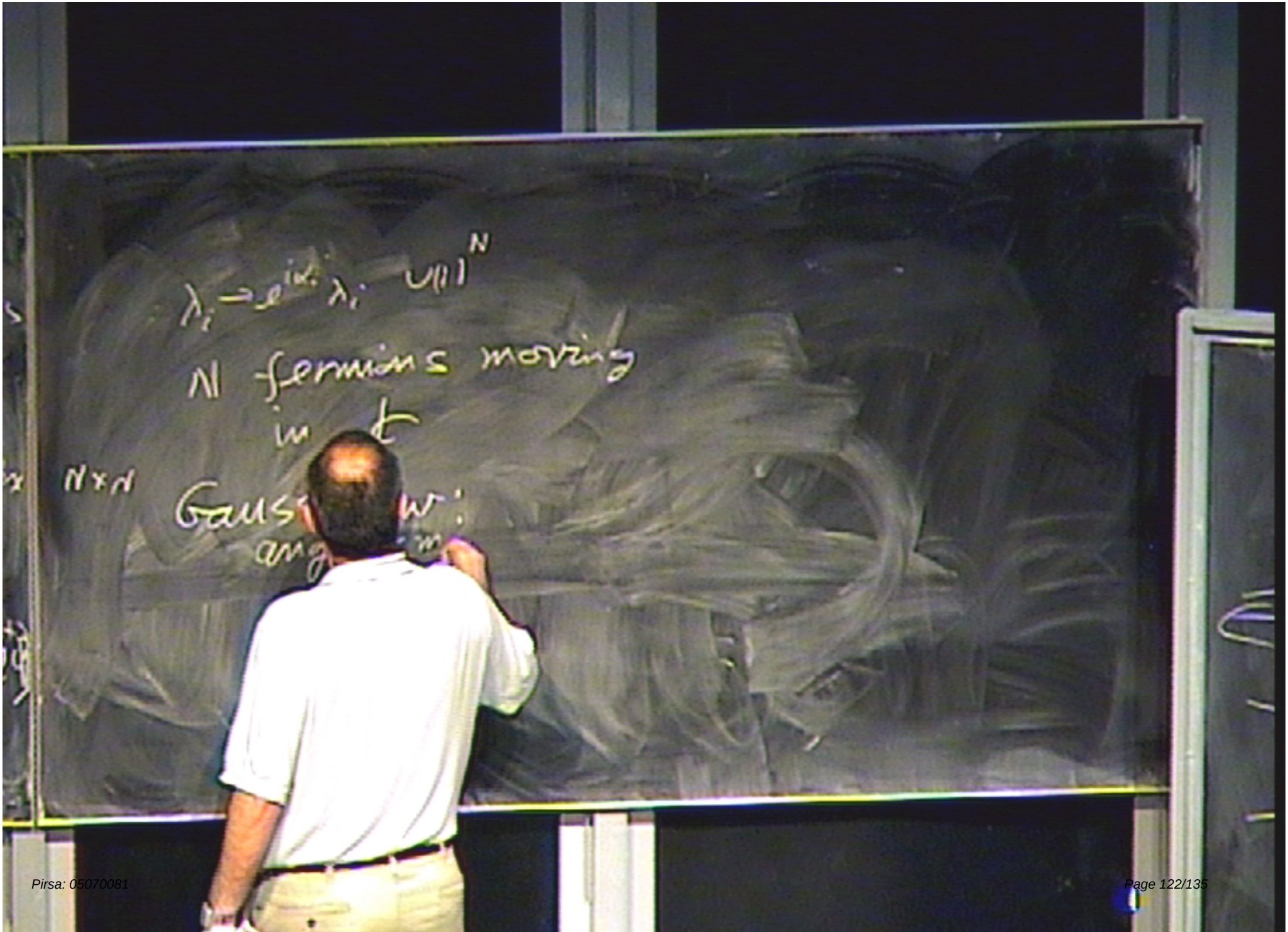
$\mathcal{L} = \mathrm{Tr}|D\Phi|^2 - V(\Phi^\dagger\Phi)$

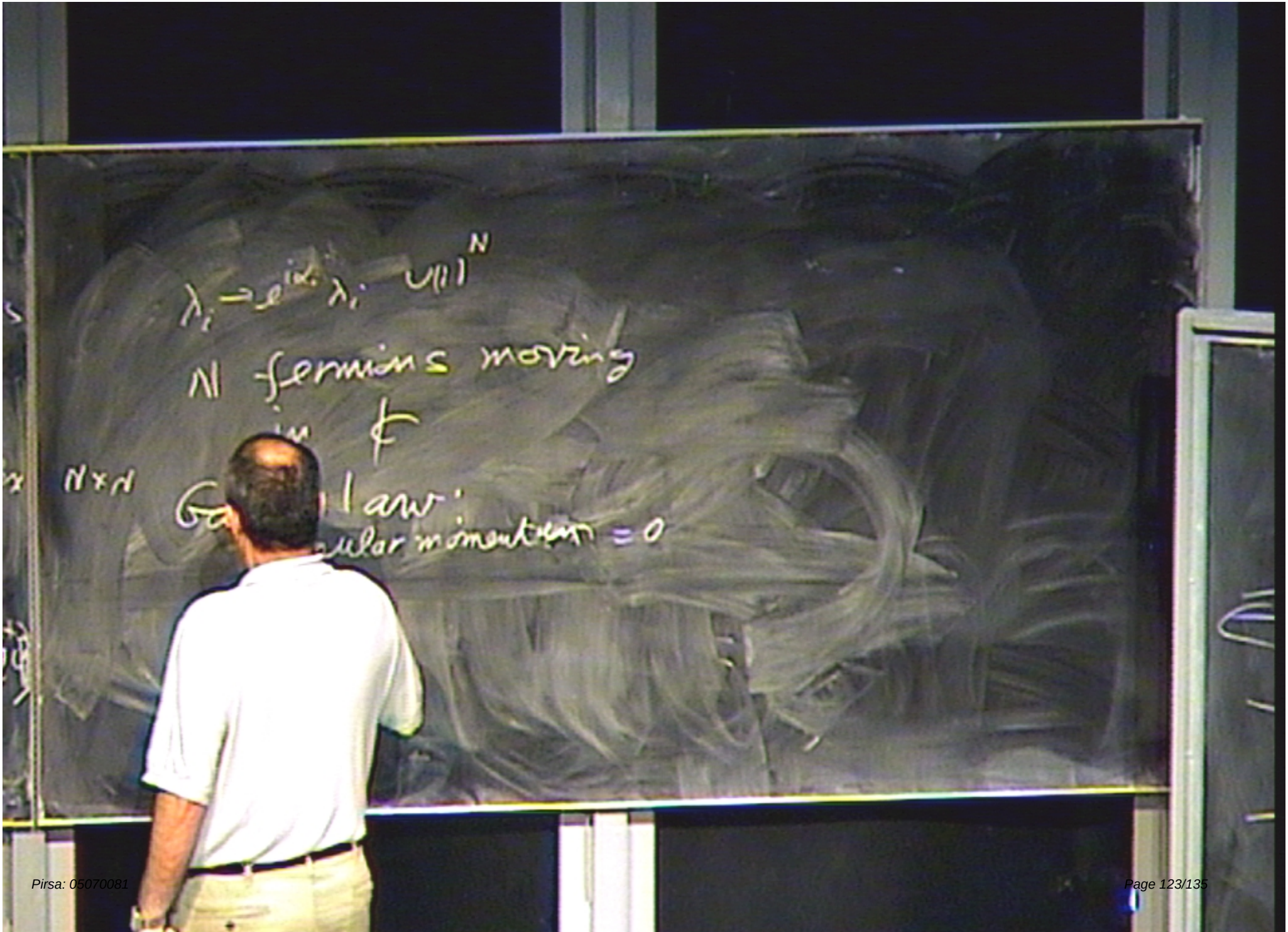
$\Phi = \begin{pmatrix} \lambda_1 & & \\ & \ddots & \\ & & \lambda_N \end{pmatrix}$

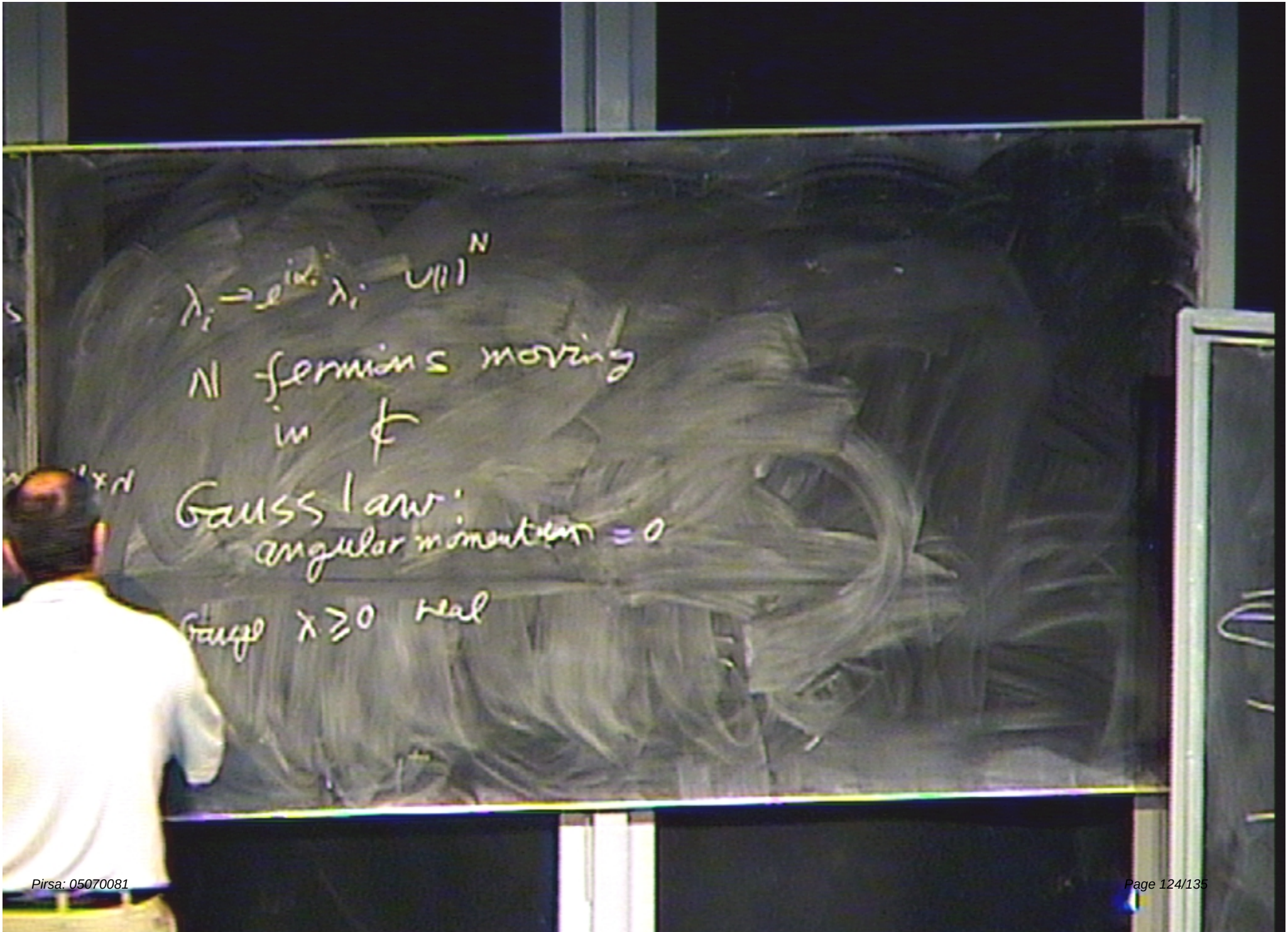
$\lambda_i \to e^{i\alpha_i}\lambda_i$ $U(1)$

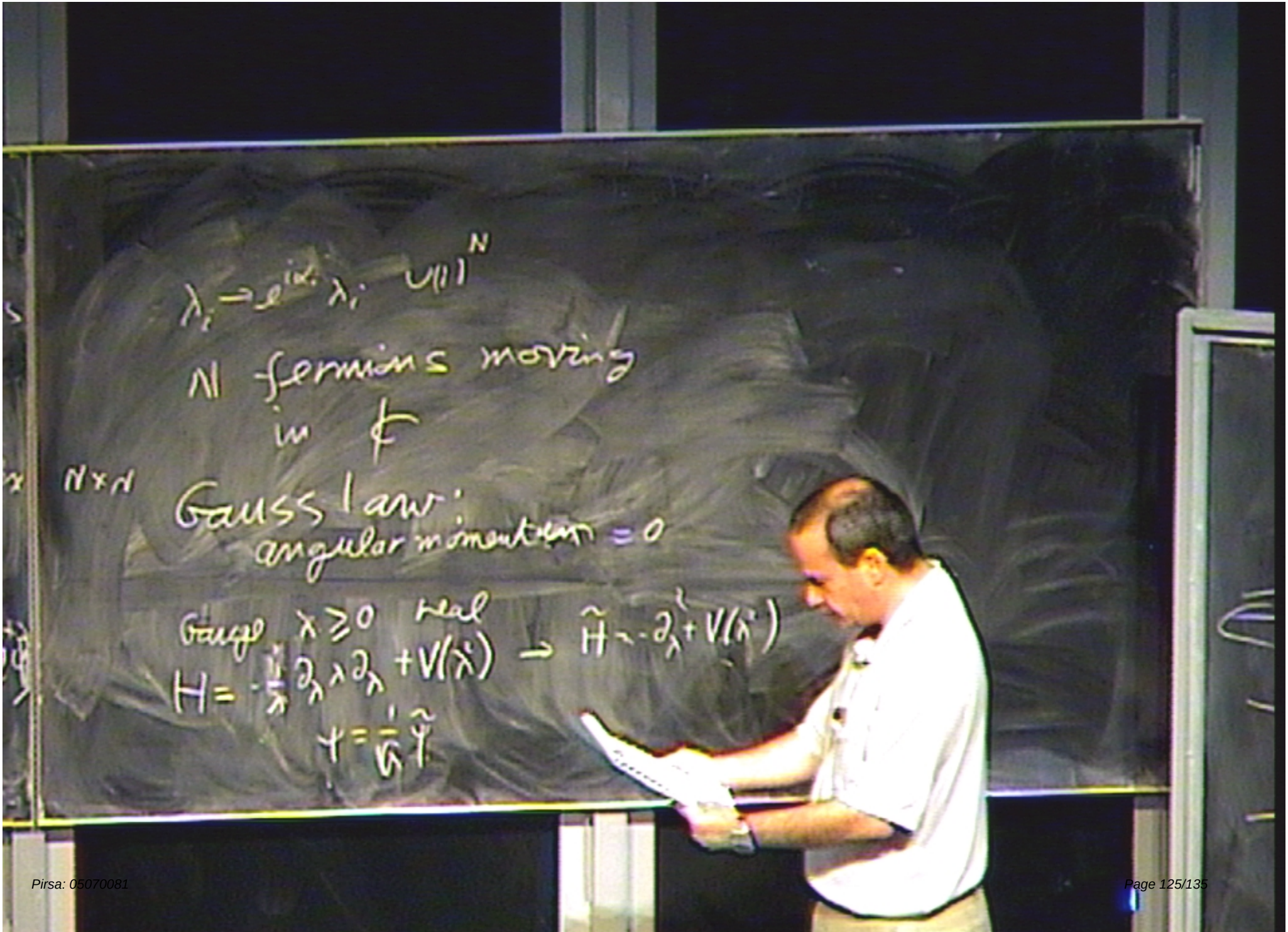

$\lambda_i \to e^{i\alpha} \lambda_i$ $U(1)^N$
N fermions moving
in
$N \times N$


$\lambda_i \to e^{i\alpha_i} \lambda_i$ $U(1)^N$
N fermions moving
in t
N×N
Gaus
an


$\lambda_i \to e^{i\alpha_i} \lambda_i$ $U(1)^N$
N fermions moving
in t
N×N
Gaus
ang


$\lambda_i \to e^{i\alpha_i}\lambda_i$ $U(1)^N$
N fermions moving
N×N
law:
angular momentum = 0


$\lambda_i \to e^{i\alpha_i} \lambda_i$ $U(1)^N$
N fermions moving
in
Gauss law:
angular momentum = 0
Gauge $\lambda \geq 0$ Real


$\lambda_i \to e^{i\alpha_i}\lambda_i$ $U(1)^N$
N fermions moving
in
$N \times N$
Gauss law:
angular momentum = 0
Gauge $\lambda \geq 0$ real
$H = -\frac{1}{\lambda}\partial_\lambda \lambda \partial_\lambda + V(\lambda) \to \tilde{H} = -\partial_\lambda^2 + V(\lambda)$
$\psi = \frac{1}{\sqrt{\lambda}}\tilde{\psi}$

$\lambda_i \to e^{i\alpha_i} \lambda_i$ $U(1)^N$

N fermions moving in $\mathbb{R}$

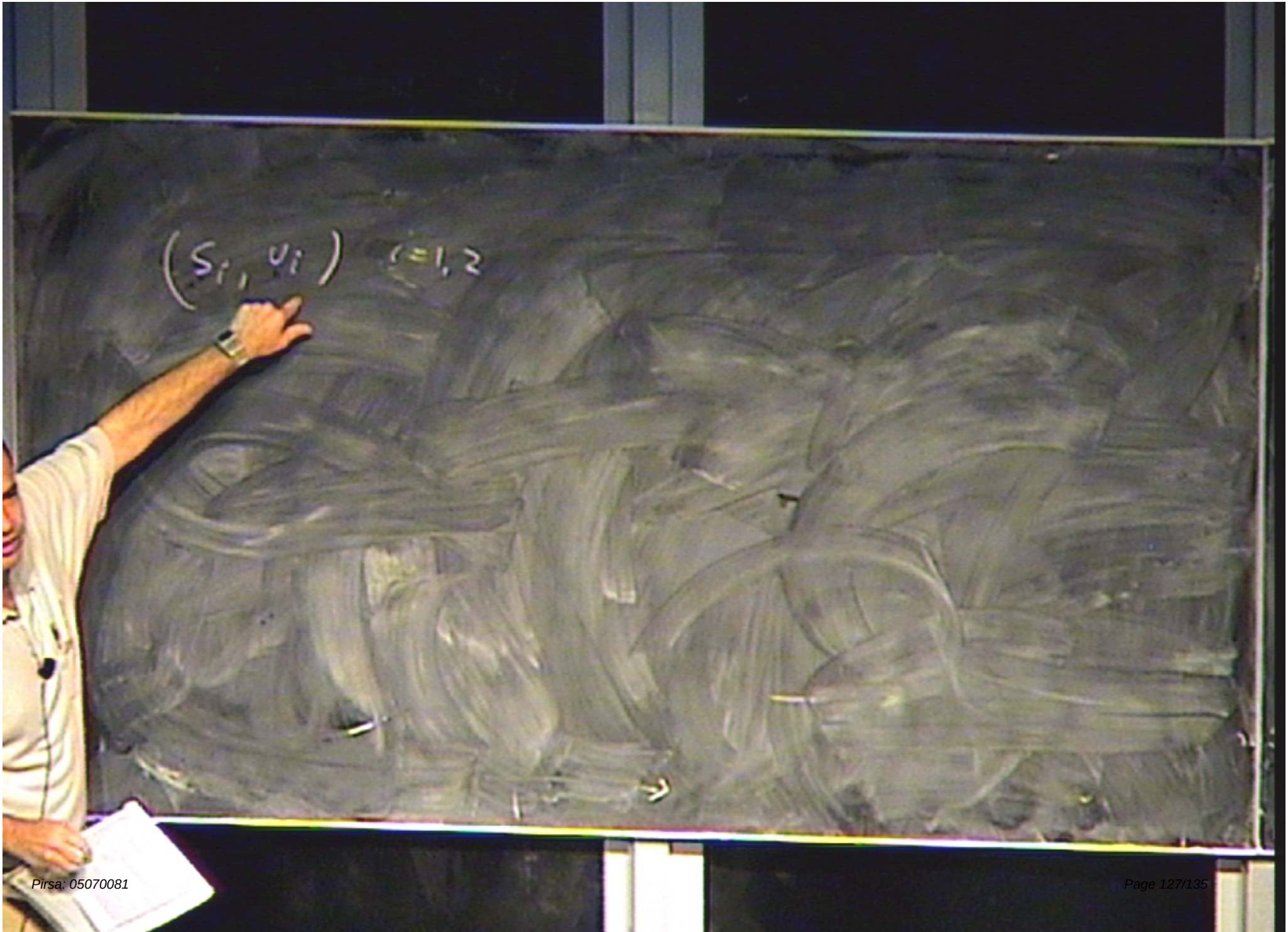
$N \times N$

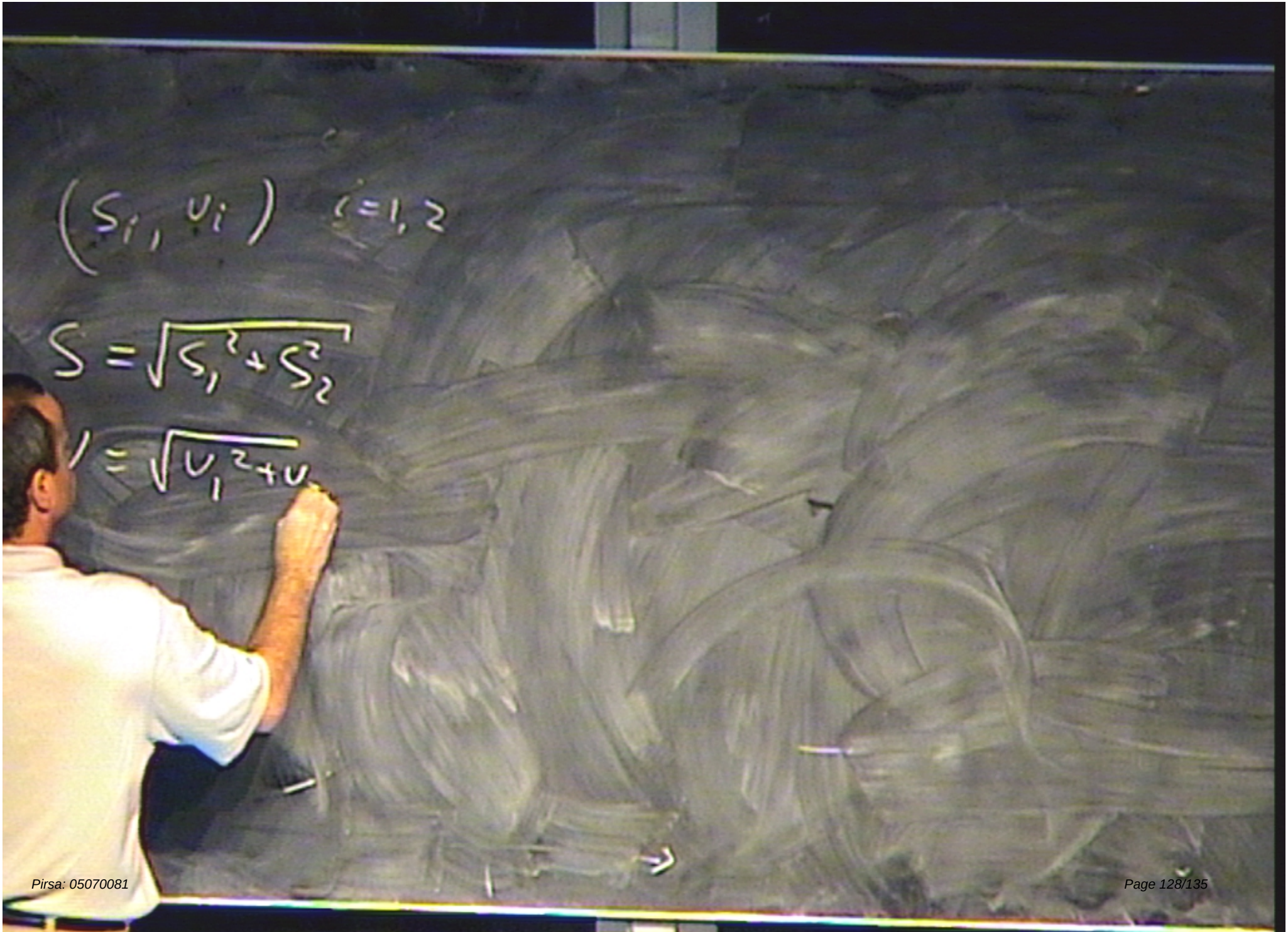
Gauss law: angular momentum $= 0$

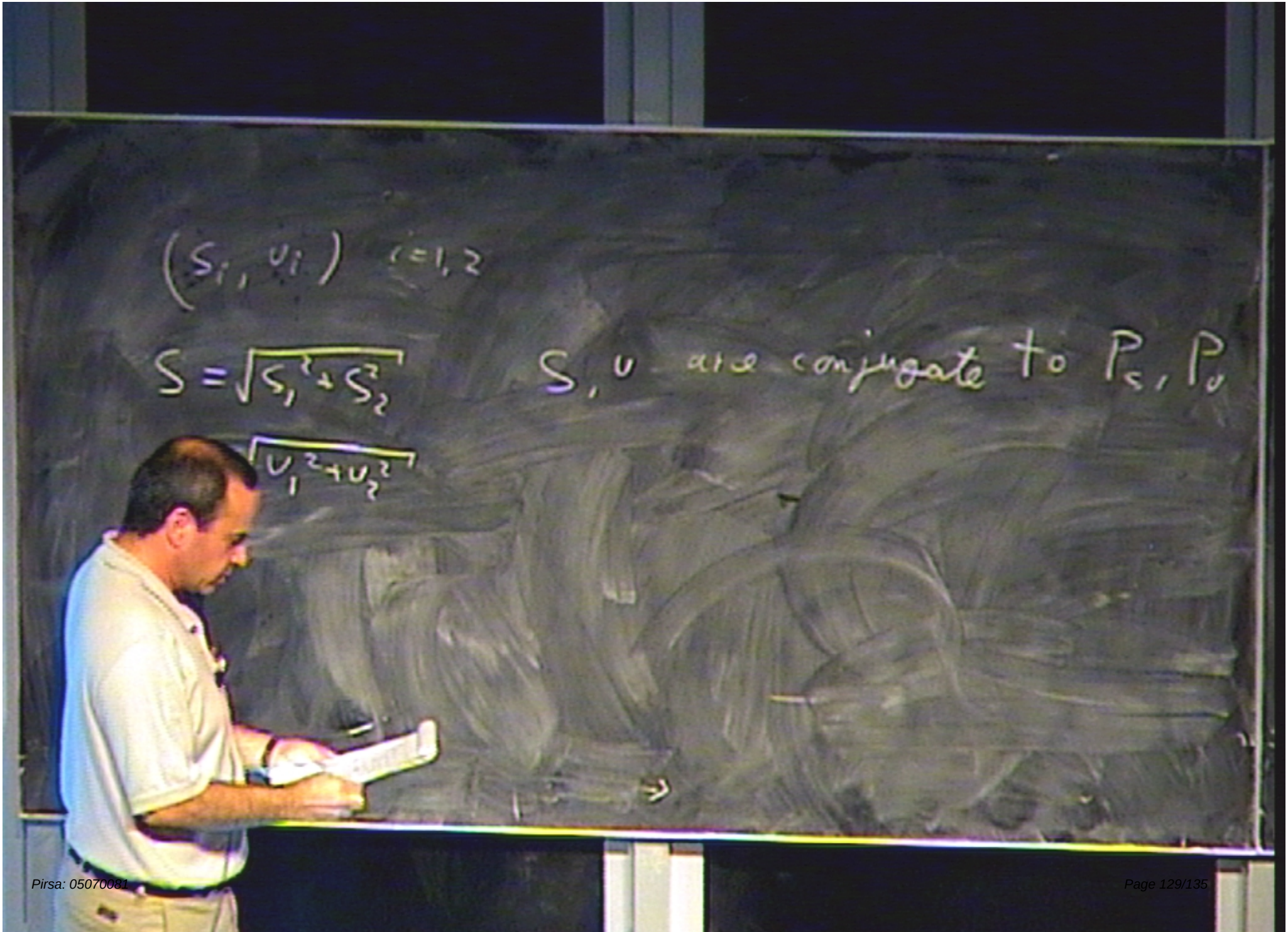
Gauge $\lambda \geq 0$ real

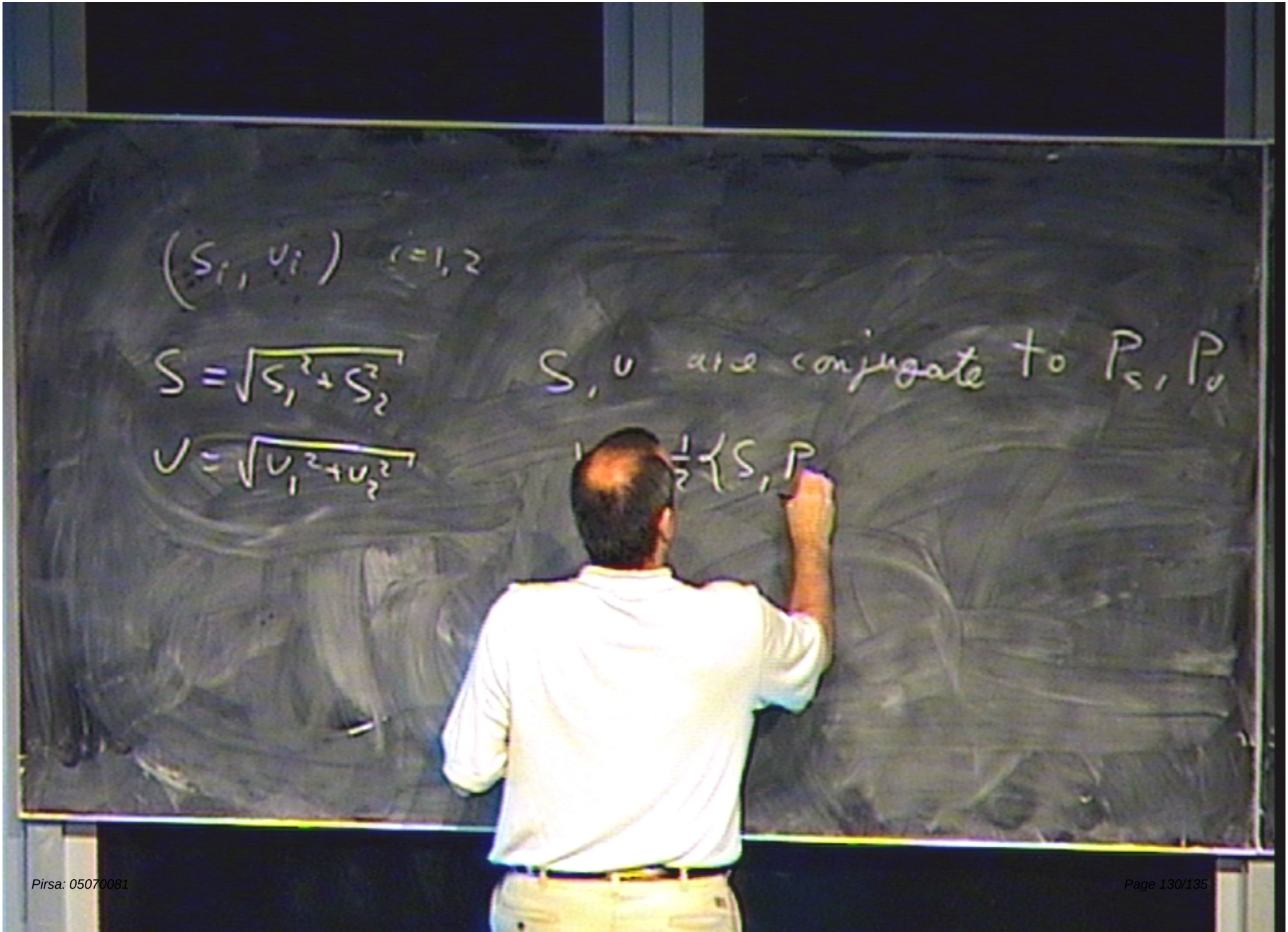
$H = -\frac{1}{2} \partial_\lambda \lambda \partial_\lambda + V(\lambda) \to \tilde{H} = -\partial_\lambda^2 + \frac{1}{4\lambda^2} + V(\lambda^2)$

$\Psi = \frac{1}{\sqrt{\lambda}} \tilde{\Psi}$


(s_i, v_i) $i=1,2$


(s_i, v_i) $i=1,2$
$s = \sqrt{s_1^2 + s_2^2}$
$v = \sqrt{v_1^2 + v$

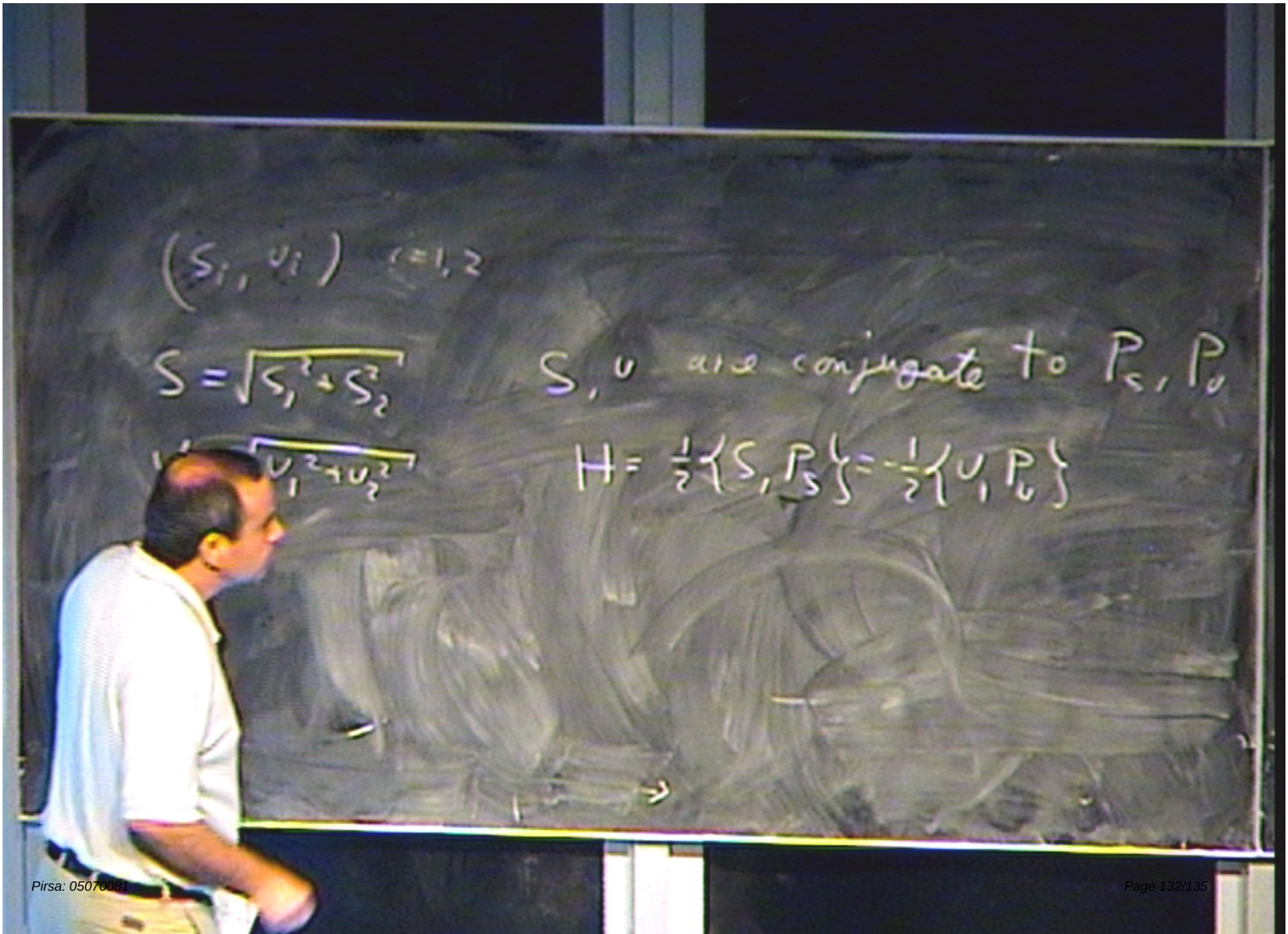

(S_i, v_i) $i=1,2$
$S=\sqrt{S_1^2+S_2^2}$
$\sqrt{v_1^2+v_2^2}$
S, v are conjugate to P_S, P_v


(S_i, v_i) $i=1,2$
$S = \sqrt{S_1^2 + S_2^2}$
$v = \sqrt{v_1^2 + v_2^2}$
S, v are conjugate to P_S, P_v

(S_i, v_i) $i=1,2$

$S = \sqrt{S_1^2 + S_2^2}$ $\quad$ S, v are conjugate to P_S, P_v

$V = \sqrt{v_1^2 + v_2^2}$ $\quad$ $H = \frac{1}{2}\{S, P_S\} = \frac{1}{2}\{V, P_v\}$


(s_i, v_i) $i=1,2$
$S = \sqrt{s_1^2 + s_2^2}$
$V = \sqrt{v_1^2 + v_2^2}$
S, v are conjugate to P_s, P_v
$H = \frac{1}{2}\{S, P_S\} = -\frac{1}{2}\{V, P_V\}$

(s_i, v_i) $i=1,2$

$S = \sqrt{s_1^2 + s_2^2}$ $\quad$ s, v are conjugate to P_s, P_v

$V = \sqrt{v_1^2 + v_2^2}$ $\quad$ $H = \frac{1}{2}\{s, P_s\} = \frac{1}{2}\{v, P_v\}$

$\langle s | E \text{ out} \rangle = \frac{1}{\sqrt{2\pi}} s^{iE - \frac{1}{2}}$

$\langle v | E \text{ in} \rangle = \frac{1}{\sqrt{2\pi}} v^{-iE - \frac{1}{2}}$

(s_i, v_i) $i=1,2$

$S = \sqrt{s_1^2 + s_2^2} \geq 0$ S, v are conjugate to P_s, P_v

$V = \sqrt{v_1^2 + v_2^2} \geq 0$ $H = \frac{1}{2}\{S, P_S\} = \frac{1}{2}\{V, P_V\}$

$\langle s | E \text{ out} \rangle = \frac{1}{\sqrt{2\pi}} s^{iE - \frac{1}{2}}$

$\langle v | E \text{ in} \rangle = \frac{1}{\sqrt{2\pi}} v^{-iE - \frac{1}{2}}$

(s_i, v_i) $i=1,2$

$S = \sqrt{s_1^2 + s_2^2} \geq 0$ S, v are conjugate to P_s, P_v

$V = \sqrt{v_1^2 + v_2^2} \geq 0$ $H = \frac{1}{2}\{S, P_S\} = -\frac{1}{2}\{V, P_V\}$

$\langle s | E \text{ out} \rangle = \frac{1}{\sqrt{2\pi}} s^{iE - \frac{1}{2}}$

$\langle v | E \text{ in} \rangle = \frac{1}{\sqrt{2\pi}} v^{-iE - \frac{1}{2}}$