

Title: Black holes in String Theory

Date: Jul 06, 2005 02:30 PM

URL: <http://pirsa.org/05070077>

Abstract:

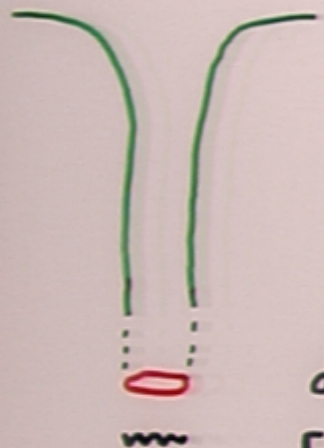
TOPICS FOR TODAY:

A) 3-CHARGE GEOMETRIES: Some results, what needs to be done

B) WHAT IS SPACETIME? CFT is VERY "DISCRETE", how does this manifest itself in dual gravity picture?

C) How do we understand the large size of states $\sim N^2 \ell_p$ by "fractionation"?

A THOUGHT ABOUT 3-CHARGE GEOMETRIES³¹



Naive geometry:

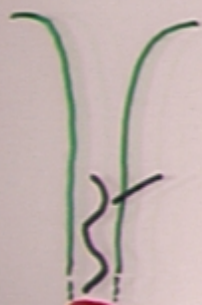
Horizon, Singularity
[Reissner-Nordstrom
in 4+1]

$$ds^2 \rightarrow \frac{dr^2}{r^2} :$$

$$\int_{\epsilon}^{r_1} \frac{dr}{r} = \log \frac{r_1}{\epsilon} \rightarrow \infty$$

Throat is infinitely long

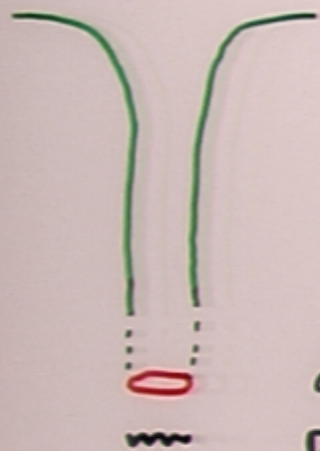
S^1 constant, S^3 constant, T^4 constant



Can put arbitrarily low
energy quanta in throat

$\rightarrow$ no energy gap

A THOUGHT ABOUT 3-CHARGE GEOMETRIES³¹



Naive geometry:

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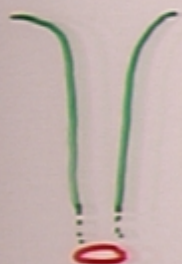
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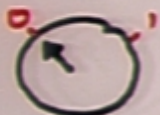
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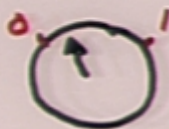
$$\begin{aligned} D1: \quad \pi_1 &= 10^6 \\ D5: \quad \pi_5 &= 10^6 \\ P: \quad \pi_P &= 10^6 \\ g &= -0.1 \end{aligned}$$

32.

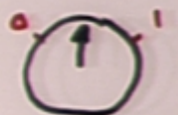
Is it reasonable that for FINITE values of the charges, compact volumes finite, we get an infinite throat, no gap in spectrum ??



$$g = 0$$

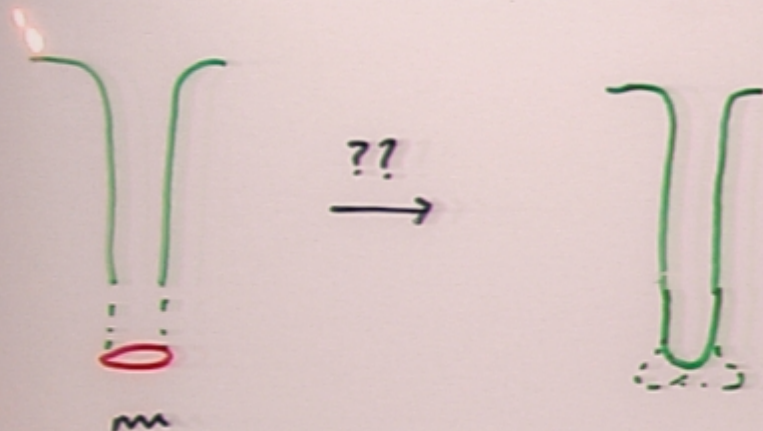


$$g = 10^{-6}$$



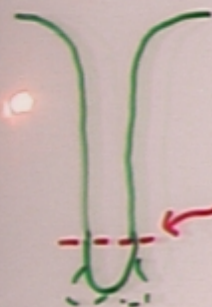
$$g = 10^{-2}$$

FINITE THROAT, NONZERO GAP,
LOOKS MORE REASONABLE.



Depth of throat $\sim (\eta_1 \eta_5 \eta_p)^\beta \sim \frac{1}{\epsilon^2}$

Throat looks infinite in classical limit



$$\frac{A}{4G} = 2\pi \sqrt{\eta_1 \eta_5 \eta_p} = S_{\text{micro}}$$

since in 3-charge case the S^3 , S^1 , T^4 all stabilise to constants down the throat.

(A) $D1-D5 + (1 \text{ UNIT OF } P) :$

9.



Solve wave equation for perturbations $[G_{\mu\nu}, \phi]$

$\rightarrow$ find normalisable solution by "matching"

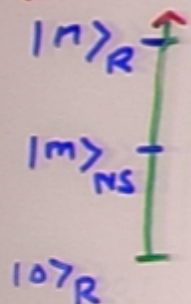
$$\sim e^{i \frac{2\pi}{L} (y-t)}$$

(B)



CFT IS IN THE
RAMOND - RAMOND SECTOR

Spectral flow:



$$|0>_R |0>_R \rightarrow |n>_R |\bar{0}>_R$$

$$P = L_0 - \bar{L}_0 = n$$

(A) D1-D5 + (1 UNIT OF P) :

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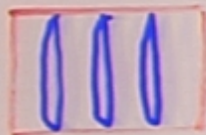


Solve wave equation for perturbations $[B_{\mu\nu}, \phi]$

→ find normalisable solution by "matching"

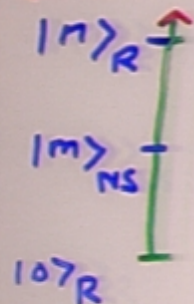
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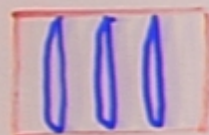


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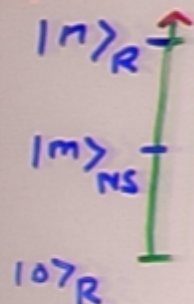
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(B)



CFT IS IN THE RAMOND - RAMOND SECTOR

Spectral flow:



$$|0>_R |0>_R \rightarrow |n>_R |\bar{0}>_R$$

$$p = L_0 - \bar{L}_0 = n$$

$$|4\rangle = J_{-(2n-2)}^+ J_{-(2n-4)}^+ \dots J_{-2}^+ J_0^+ |\phi\rangle_R$$

$$ds_{\text{string}}^2 = -\frac{1}{h} [dt^2 - dy^2] + \frac{\Phi_P}{hf} (dt - dy)^2$$

$$+ hf \left(\frac{dr^2}{r^2 + a^2 \eta} + d\theta^2 \right)$$

$$+ h \left[r^2 - na^2 \eta + \frac{(2n+1)a^2 \eta \Phi_1 \Phi_5 \cos^2 \theta}{h^2 f^2} \right] \cos^2 \theta d\psi^2$$

$$+ h \left[r^2 + (n+1)a^2 \eta - \frac{(2n+1)a^2 \eta \Phi_1 \Phi_5 \sin^2 \theta}{h^2 f^2} \right] \sin^2 \theta d\phi^2$$

$$+ \frac{a^2 \eta^2 \Phi_P}{hf} (\cos^2 \theta d\psi + \sin^2 \theta d\phi)^2$$

$$+ \frac{2a \sqrt{\Phi_1 \Phi_5}}{hf} [n \cos^2 \theta d\psi - (n+1) \sin^2 \theta d\phi] (dt - dy)$$

$$- \frac{2a \eta \sqrt{\Phi_1 \Phi_5}}{hf} [\cos^2 \theta d\psi + \sin^2 \theta d\phi] dy$$

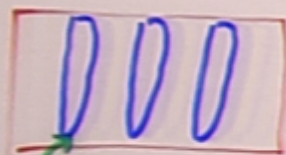
$$+ \sqrt{\frac{H_1}{H_5}} dz_0 dz_a$$

$$f = r^2 - a^2 \eta n \sin^2 \theta + a^2 \eta (n+1) \cos^2 \theta$$

$$h = \sqrt{H_1 H_5}, \quad H_1 = 1 + \frac{\Phi_1}{f}, \quad H_5 = 1 + \frac{\Phi_5}{h}, \quad \eta = \frac{\Phi_1 \Phi_5}{\Phi_1 \Phi_5 + \Phi_P (\Phi_1 + \Phi_5)}$$

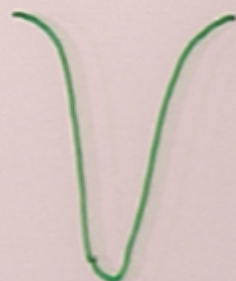
Is spacetime made of "bits"?

CFT



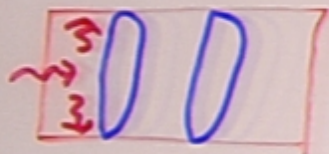
finite number of
"Component strings"

SUGRA



Where do we
see this finite
number?

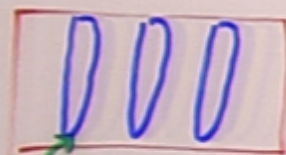
Recall the absorption process:



Let us be a little more
precise about this computation

Is spacetime made of "bits"?

CFT



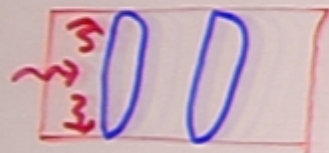
finite number of
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SUGRA



Where do we
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Recall the absorption process:



Let us be a little more
precise about this computation

What is the quantum that
fell in?

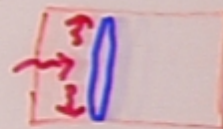
Massless scalar:

$$h_{ij}, \theta_{ij}^{RR} \quad [i,j=1,2,3,4]$$

$$\boxed{} \quad T^4 \quad x_1, x_2, x_3, x_4$$

$$h_{ij} [\partial x^i \bar{\partial} x^j + \partial x^j \bar{\partial} x^i]$$

$$\theta_{ij}^{RR} [\partial x^i \bar{\partial} x^j - \partial x^j \bar{\partial} x^i]$$



For convenience, define

$$S_{ij}^+ = h_{ij} + \theta_{ij}^{RR} \Rightarrow S_{ij}^+ \partial x^i \bar{\partial} x^j$$

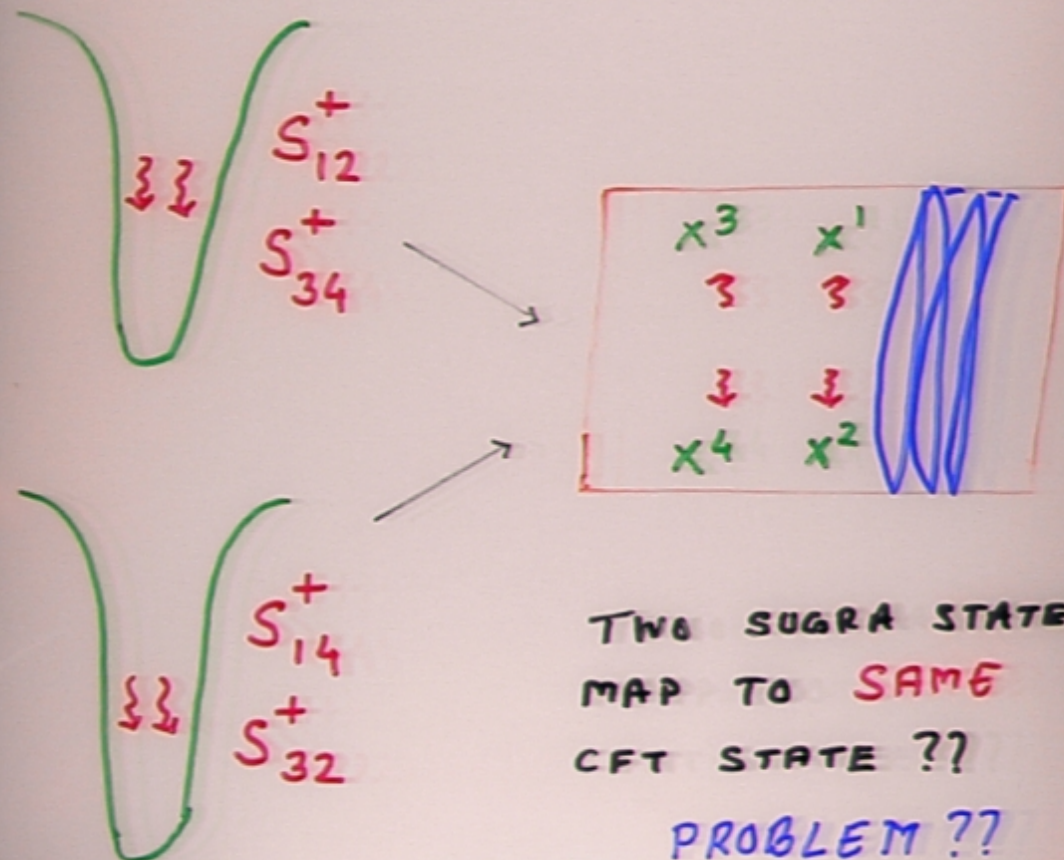
$$S_{ij}^- = h_{ij} - \theta_{ij}^{RR} \Rightarrow S_{ij}^- \bar{\partial} x^i \partial x^j$$

$$S_{12}^+ \quad \begin{array}{c} x^1 \\ \downarrow \\ \boxed{} \quad \downarrow \quad \downarrow \\ x^2 \end{array}$$

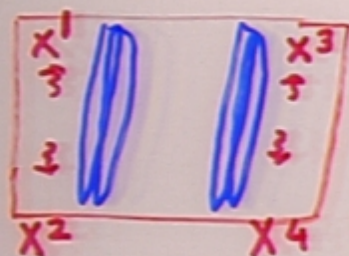
$$\begin{array}{c} \sim \\ \downarrow \\ S_{12}^+ \end{array}$$

Let the CFT state have just ¹³
ONE component string

Consider **TWO** quanta thrown in
the supergravity throat

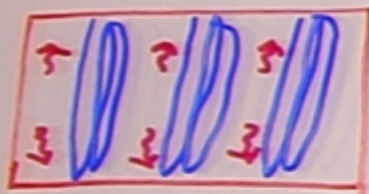


If we had **Two** component strings ¹⁴
it would be okay



But 3 quanta
in the throat
would be a problem

More generally



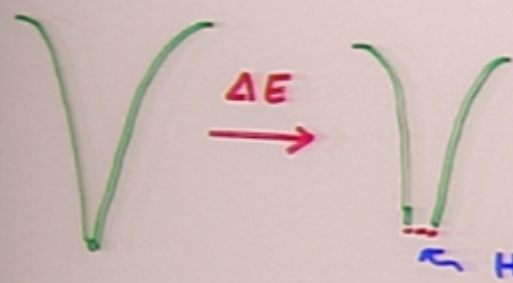
m component strings:
 m quanta okay,
 $m+1$: problem

$E_{\text{threshold}}^{\text{CFT}}$: energy to excite each
component string once
in lowest energy level

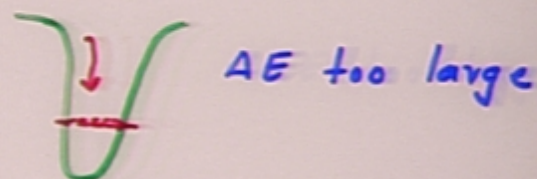
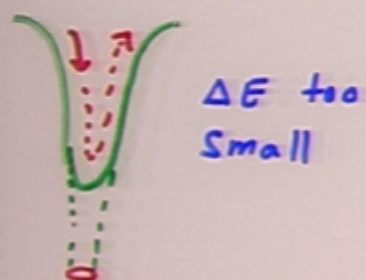
$$= m \times \frac{2\pi}{\lambda} \times 2$$

Supergravity : So far we have ignored backreaction

15



$$r_H = \left[\frac{4\pi g^2 \alpha'^4 \Delta E}{L v} \right]^{\frac{1}{2}}$$



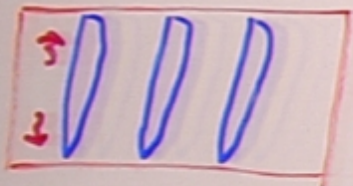
$\Delta E_{\text{threshold}}^{\text{SUGRA}}$: energy for which backreaction is $O(1)$
 $\rightarrow r_H = r$ at "cap"

FIND

$$\Delta t_{\text{threshold}}^{\text{CFT}} = \Delta t_{\text{threshold}}^{\text{SUGRA}}$$

WHAT DOES THIS MEAN?

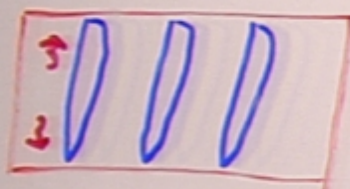
16



- Put one quantum in throat $\leftrightarrow$
excite one component string
- Put another quantum $\leftrightarrow$
excite a different component string
- Just when all component strings
get used up, there are enough
quanta in throat to create
backreaction $O(1) \rightarrow$ geometry deforms
- Prob. that two excitations are
on same component string $\sim \frac{1}{n}$
 $\leftrightarrow$ strength of gravitational interaction

WHAT DOES THIS MEAN?

16



- Put one quantum in throat $\leftrightarrow$ excite one component string
- Put another quantum $\leftrightarrow$ excite a different component string
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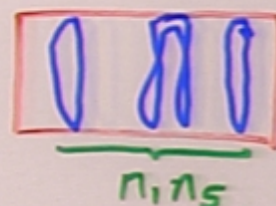
FRACTIONATION

F1

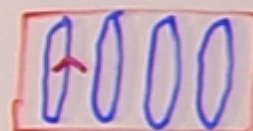
Near external entropy:

DI-DS external

$$S = 2\pi \sqrt{2} \sqrt{n_1 n_5}$$

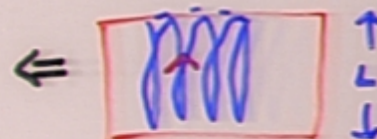


DI-DS + P



$$\frac{2\pi n_p}{L} \rightarrow \left\{ \begin{array}{l} \uparrow \\ L_T \\ \downarrow \end{array} \right.$$

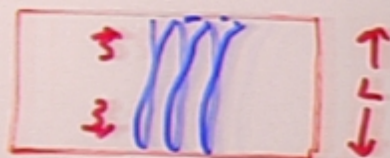
$$\frac{2\pi (n_1 n_5 n_p)}{L_T}$$



$$\Rightarrow S = 2\pi \sqrt{n_1 n_5 n_p}$$

$$NSI - P + \Delta E$$

$$\rightarrow NSI - P + \frac{P}{\bar{P}}$$



$\left\{ \frac{P}{\bar{P}} \right\}$ come in fractional units $\left(\frac{1}{n_1 n_5} \right) \left(\frac{2\pi}{L} \right)$

→ Near extremal entropy:

$$\begin{aligned} S_{\text{micro}} &= 2\pi \sqrt{n_1 n_5 n_p} + 2\pi \sqrt{n_1 n_5 \bar{n}_p} \\ &= S_{\text{Bek}} \end{aligned}$$

Question:

$$NSI - P + \Delta E = ??$$



$$NSI - P + \frac{P}{\bar{P}} : \Delta E_{\text{min}} = \frac{2\pi}{n_1} + \frac{2\pi}{n_2} = \frac{4\pi}{n_1}$$

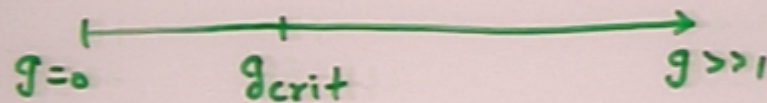
BUT:

F3

$$\begin{aligned} \mathcal{D}1 - \mathcal{D}5 + \Delta E &\rightarrow \mathcal{D}1 - \mathcal{D}5 + \frac{P}{\bar{P}} \left(\frac{1}{n, n_S} \right) \\ P - NS1 + \Delta E &\rightarrow P - NS1 + \frac{NS5}{\overline{NS5}} \left(\frac{1}{n, n_P} \right) \end{aligned}$$

$$\Delta E_{\min} = 2 \left(\frac{LV}{d^{1/3}} \frac{1}{g^2} \right) \left(\frac{1}{n, n_P} \right)$$

Recall: $P - NS1 + \frac{P}{\bar{P}} : \Delta E_{\min} = \frac{4\pi}{(n, L)}$



$$P - NS1 + \frac{P}{\bar{P}}$$

$$P - NS1 + \frac{NS5}{\overline{NS5}}$$



!



4-D BLACK HOLES

F4

$$M_{9,1} \rightarrow M_{3,1} \times \underbrace{T^4 \times \underbrace{S^1}_{D1}}_{D5} \times \underbrace{\tilde{S}^1}_{P}$$

KK monopoles

$$S_{\text{external}} = 2\pi \sqrt{n_1 n_2 n_3 n_4}$$

$$S_{\text{near-external}} = 2\pi \sqrt{n_1 n_2 n_3 n_4} + 2\pi \sqrt{n_1 n_2 n_3 \bar{n}_4}$$

$\overbrace{D1-D5-P-KK}^{\curvearrowright}$: All permutations possible by dualities

FS

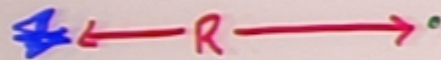
HOW BIG IS THE D1-D5-P
EXTREMAL BOUND STATE ?



SMALL $\Rightarrow$
HORIZON



BIG $\Rightarrow$ PERHAPS
NO HORIZON



DO BRANES REACH OUT AND TOUCH
THE TEST QUANTUM ?

FS

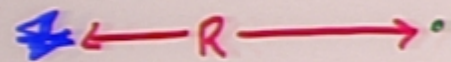
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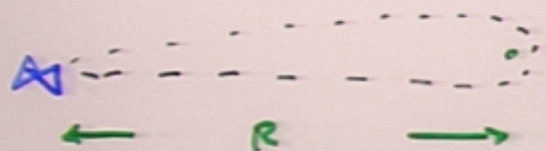


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F6.

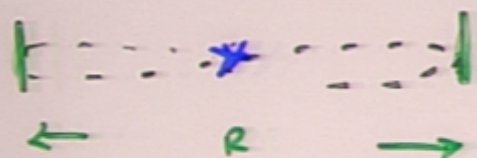


- THIS NEEDS ENERGY

ALL WE HAVE IS $\Delta E \sim \frac{1}{R}$

- WE ALSO WANT THE PROCESS TO BE PROBABLE, NOT JUST POSSIBLE

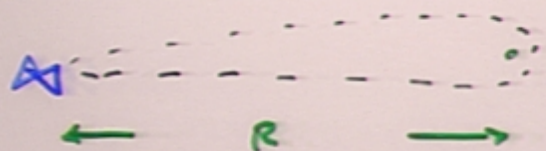
$$\Delta S \geq 1 : \frac{\text{Vol of phase space with pair}}{\text{Vol of ph. sp without pair}} \geq e$$



$$\Delta E = \frac{1}{R}$$

$$\Delta S = 1$$

F6.

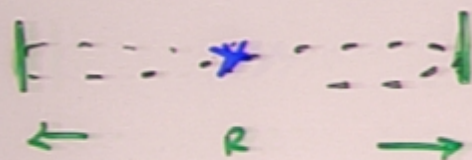


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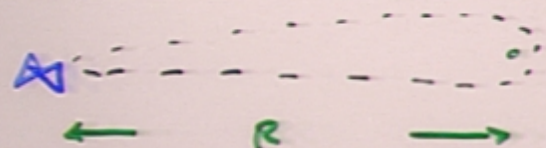
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F6.

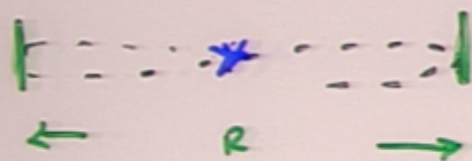


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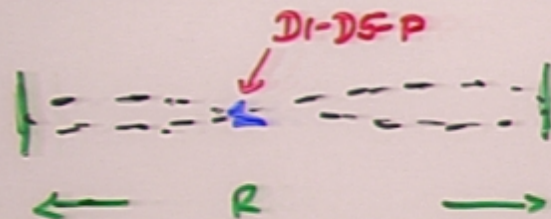


$$\Delta E = \frac{1}{R}$$

$$\Delta S = 1$$

$$R = ?$$

NOW THE COMPUTATION IS CLEAR F7



$$\text{DI-DS-P} \xrightarrow{\Delta E = 1/R} \text{DI-DS-P} + \frac{KK}{K\bar{K}}$$

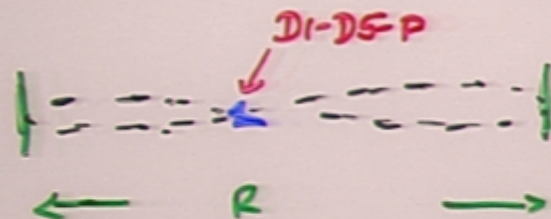
$$S = 2\pi \sqrt{n_i n_s n_p} \rightarrow 2\pi \sqrt{n_i n_s n_p n_{KK}} + 2\pi \sqrt{n_i n_s n_p \bar{n}_{KK}}$$

$$\boxed{\Delta S = 1} \Rightarrow \text{HAPPENS FOR}$$

$$R = \left[\frac{g^2 d'^4}{L V} \sqrt{n_i n_s n_p} \right]^{\frac{1}{3}}$$

$\sim R_H$ for DI-DS-P BLACK
11

NOW THE COMPUTATION IS CLEAR F7



$$\text{DI-DS-P} \xrightarrow{\Delta E = 1/R} \text{DI-DS-P} + \begin{matrix} KK \\ K\bar{K} \end{matrix}$$

$$S = 2\pi \sqrt{n_i n_s n_p} \rightarrow 2\pi \sqrt{n_i n_s n_p n_{KK}} + 2\pi \sqrt{n_i n_s n_p n_{K\bar{K}}}$$

$$\boxed{\Delta S = 1} \Rightarrow \text{HAPPENS FOR}$$

$$R = \left[\frac{g^2 d'^4}{L V} \sqrt{n_i n_s n_p} \right]^{\frac{1}{3}}$$

$\sim R_H$ for DI-DS-P BLACK

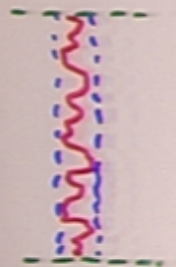
CONJECTURE

Bound states in a consistent
theory of quantum gravity
"swell up" to a size depending
on their degeneracy

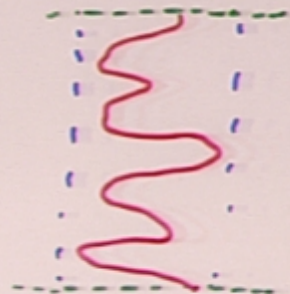


$$\frac{A}{4G} \sim S = \ln[\# \text{ states}]$$

May be just a question of
PHASE SPACE



FEW STATES



ALL STATES

CONJECTURE

Bound states in a consistent
theory of quantum gravity
"swell up" to a size depending
on their degeneracy

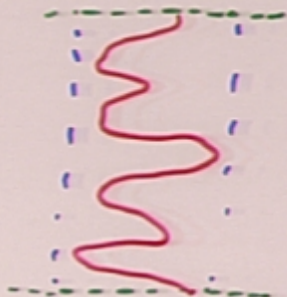


$$\frac{A}{4G} \sim S = \ln[\text{\# states}]$$

May be just a question of
PHASE SPACE



FEW STATES



ALL STATES

