

Title: Matrix Models and Noncritical Strings

Date: Jul 06, 2005 11:30 AM

URL: <http://pirsa.org/05070076>

Abstract:

Id matrix mod

1d matrix model

$\Phi(t)$ $N \times N$ hermitian matrix

$U(N)$

~~size~~

S

1d matrix model

$\Phi(t)$ $N \times N$ hermitian matrix

$U(N)$ gauge symmetry

$$\Phi \rightarrow V^\dagger(t) \Phi(t) V(t)$$

Id matrix model

$\Phi(t)$ $N \times N$ hermitian matrix

$U(N)$ gauge symmetry

$$\bar{\Phi}(t) \rightarrow V(t)^\dagger \Phi(t) V(t)$$

1d matrix model

$\Phi(t)$ $N \times N$ hermitian matrix

$U(N)$ gauge symmetry

$$\bar{\Phi}(t) \rightarrow V(t)^\dagger \Phi(t) V(t)$$

$$\mathcal{L} = -\text{Tr}(\Phi^2)$$

1d matrix model

$\Phi(t)$ $N \times N$ hermitian matrix

$U(N)$ gauge symmetry

$$V(t) \Phi(t) V(t)^\dagger$$

$$\left[\partial_t \Phi + [A, \Phi] \right]^2 - U(\Phi)$$

Id matrix model

$\Phi(t)$ $N \times N$ hermitian matrix

$U(N)$ gauge symmetry

$$\bar{\Phi}(t) \rightarrow V^\dagger(t) \bar{\Phi}(t) V(t)$$

$$\mathcal{L} = \text{Tr} \left[\frac{1}{2} (\partial_t \bar{\Phi} + [A, \bar{\Phi}])^2 - V(\bar{\Phi}) \right]$$

$$A(t) \rightarrow V^\dagger \partial_t V + V^\dagger A V$$

Gauge choices:

1. $A=0$

Gauss law

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1. $A=0$

Gauss law

$$\left. \frac{\delta S}{\delta A(x)} \right|_{A=0} = 0 = (-\partial_i \Phi \bar{\Phi} + \bar{\Phi} \partial_i \Phi) = 0$$

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2.

Gauge choices:

1. $A=0$ —

Gauss law

$$\left. \frac{\delta S}{\delta A(x)} \right|_{A=0} = 0 = \left(-\nabla^2 \bar{\psi} + \frac{g}{f} \bar{\psi} \psi \right) = 0$$

2. Diagonal

Gauge choices:

1. $A=0$ — Axial gauge

Gauss law

$$\left. \frac{\delta S}{\delta A(x)} \right|_{A=0} = 0 = (-\partial_i \Phi \bar{\Phi} + \bar{\Phi} \partial_i \Phi) = 0$$

2. Diagonalize Φ — Unitary

Gauge choices:

1. $A=0$ — Axial gauge

Gauss law

$$\left. \frac{\delta S}{\delta A(t)} \right|_{A=0} = 0 = (-\partial_i \Phi \bar{\Phi} + \bar{\Phi} \partial_i \Phi) = 0$$

2. Diagonalize $\bar{\Phi}$ — unitary gauge

$$\bar{\Phi}(t) = \begin{pmatrix} \lambda_1(t) \\ \lambda_2(t) \\ \vdots \\ \lambda_N(t) \end{pmatrix}$$

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$$H = \sum_i \frac{1}{2} \dot{\lambda}_i^2 + V(\lambda_i)$$

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$$D\bar{\Phi} = D\Phi$$

Gauge choices:

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2. Diagonalization

$$\bar{\Phi}(t) = \begin{pmatrix} \lambda_1(t) \\ \lambda_2(t) \\ \vdots \\ \lambda_N(t) \end{pmatrix}$$

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$$D\bar{\Phi} = D\Omega(t) \prod_i d\lambda_i J(\lambda)$$

$$J = \Delta^2(\lambda)$$

$$\Delta(\lambda) = \prod_{i < j} (\lambda_i - \lambda_j)$$

Gauge choices:

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Gauss law

$$\left. \frac{\delta S}{\delta A(t)} \right|_{A=0} = 0 = (-\partial_t \Phi \bar{\Phi} + \bar{\Phi} \partial_t \Phi) = 0$$

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2. Diagonalize Φ — unitary gauge

$$\bar{\Phi}(t) = \begin{pmatrix} \lambda_1^{(t)} & & & \\ & \lambda_2^{(t)} & & \\ & & \ddots & \\ & & & \lambda_N^{(t)} \end{pmatrix}$$

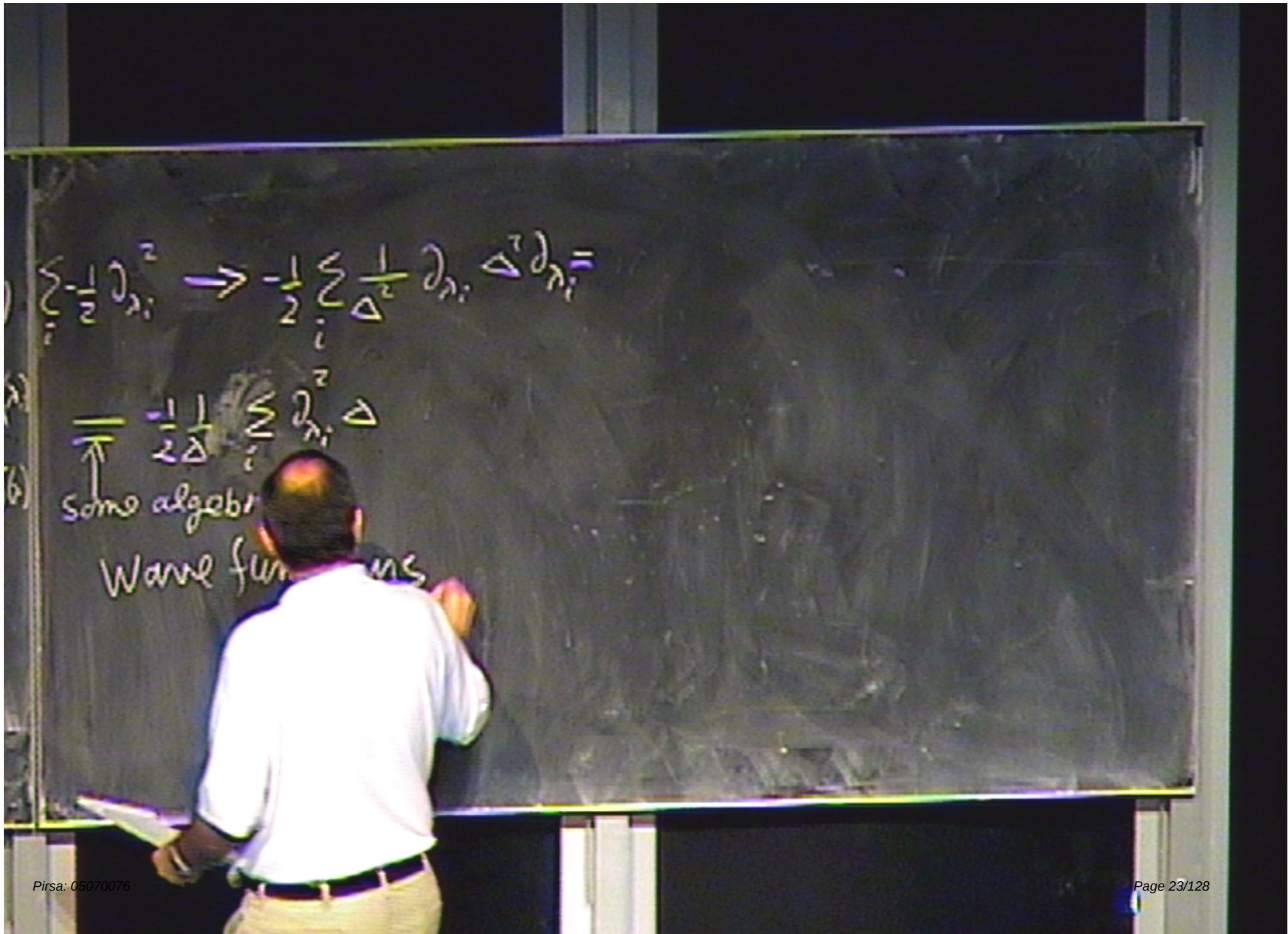
$$\mathcal{L}_{eff} = \sum_i \frac{1}{2} \dot{\lambda}_i^2 - U(\lambda_i)$$

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$$\Delta(\lambda) = \prod_{i < j} (\lambda_i - \lambda_j)$$



$$\sum_i -\frac{1}{2} \partial_{\lambda_i}^2 \rightarrow -\frac{1}{2} \sum_i \frac{1}{\Delta^2} \partial_{\lambda_i} \Delta^2 \partial_{\lambda_i} =$$

$$\frac{1}{2\Delta} \sum_i \partial_{\lambda_i}^2 \Delta$$

Some algebra

Wave functions

$$\sum_i -\frac{1}{2} \partial_{\lambda_i}^2 \rightarrow -\frac{1}{2} \sum_i \frac{1}{\Delta^2} \partial_{\lambda_i} \Delta^2 \partial_{\lambda_i} =$$

$$= -\frac{1}{2\Delta} \sum_i \partial_{\lambda_i}^2 \Delta$$

Some algebra

$$\text{Wave functions} = \frac{1}{\Delta} \psi(\lambda_i)$$

$$H\psi = E\psi$$

$$\psi(\lambda_i, \lambda_i) = -\psi(\lambda_i, \lambda_i)$$

$$\uparrow \quad \epsilon - \frac{1}{2} \partial_{\lambda_i}^2 + V(\lambda_i)$$

$$\sum_i -\frac{1}{2} \partial_{\lambda_i}^2 \rightarrow -\frac{1}{2} \sum_i \frac{1}{\Delta^2} \partial_{\lambda_i} \Delta^2 \partial_{\lambda_i} =$$

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Some algebra

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$$= -\frac{1}{2\Delta} \sum_i \partial_{\lambda_i}^2 \Delta$$

↑
some algebra

$$\text{Wave functions} = \frac{1}{\Delta} \Psi(\lambda_i)$$

$$H\Psi = E\Psi \quad \Psi(\lambda_i, \lambda_i) = -\Psi(\lambda_i, \lambda_i)$$

$$\uparrow \quad \left(\varepsilon - \frac{1}{2} \partial_{\lambda_i}^2 + V(\lambda_i) \right) \quad \Psi = \text{wave function of } N \text{ independent fermions}$$

$$\sum_i -\frac{1}{2} \partial_{\lambda_i}^2 \rightarrow -\frac{1}{2} \sum_i \frac{1}{\Delta^2} \partial_{\lambda_i} \Delta^2 \partial_{\lambda_i} =$$

$$= -\frac{1}{2\Delta} \sum_i \partial_{\lambda_i}^2 \Delta$$

Some algebra

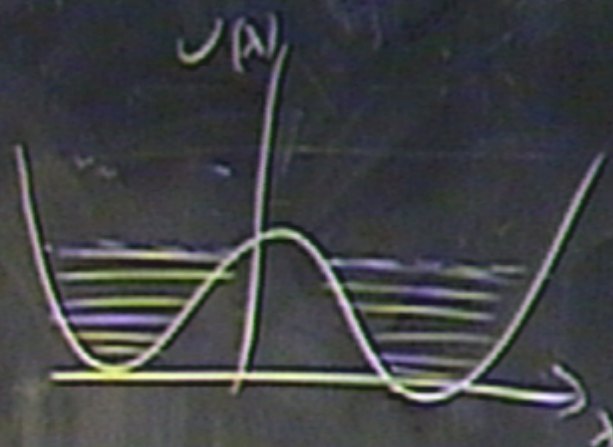
$$\text{Wave functions} = \frac{1}{\Delta} \Psi(\lambda_i)$$

$$H\Psi = E\Psi$$

$$\Psi(\lambda_i, \lambda_i) = -\Psi(\lambda_i, \lambda_i)$$

$$\left(\epsilon - \frac{1}{2} \partial_{\lambda_i}^2 + V(\lambda_i) \right)$$

$\Psi =$ wave function of N independent fermions

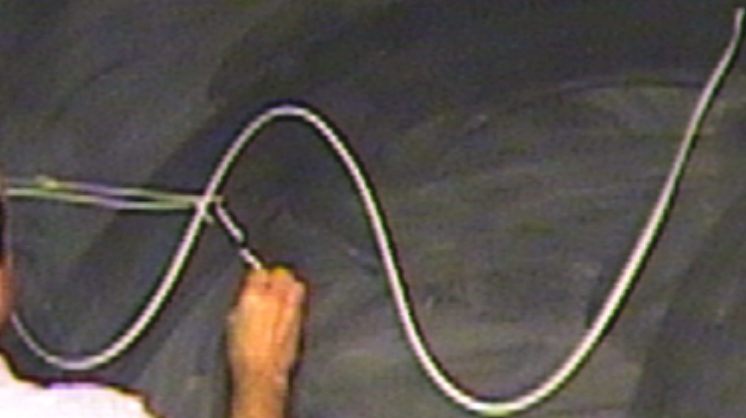


1d matrix model

utilian matrix

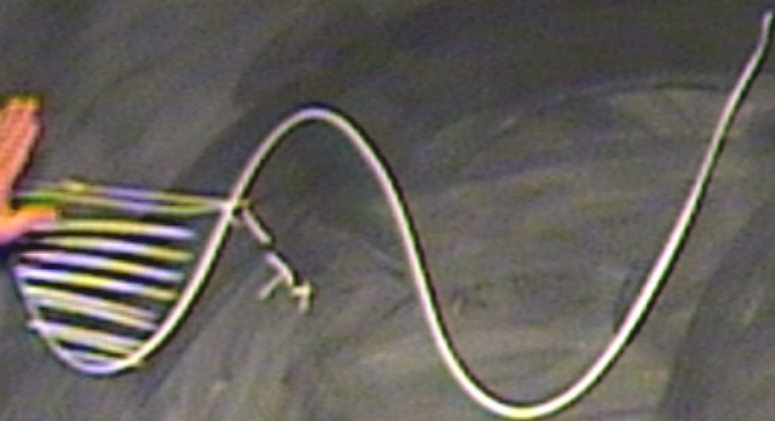
$$V(\phi)$$

1d matrix model



$$\oint p dx = 2\pi n$$

1d matrix model



$$\int_{\lambda(E)} p d\lambda = 2 \tilde{\Pi} \gamma$$

$$\int_{\lambda_-(E)}^{\lambda_+(E)} \sqrt{2(E - V(\lambda))} d\lambda = \tilde{\Pi} \gamma$$

$$P(E) = \sum_n J(E - E_n)$$

1d matrix model



$$\int_{\lambda_-(E)}^{\lambda_+(E)} p d\lambda = 2\pi \hbar n$$

$$\int_{\lambda_-(E)}^{\lambda_+(E)} \sqrt{2(E - U(\lambda))} d\lambda = \pi$$

$$\left[\begin{aligned} P(E) &= \sum_n J_n(E) \\ N &= \int_{E_0}^{\infty} P(E) dE \end{aligned} \right.$$

$$E = \int_{\mu_0}^{\lambda} E \rho(E) dE$$

$$\begin{aligned} \mathcal{L}_{\text{eff}} &= \sum_i \frac{1}{2} \dot{\lambda}_i^2 - U(\lambda_i) \\ H &= \sum_i \frac{1}{2} \dot{\lambda}_i^2 + U(\lambda_i) \\ D\bar{\Phi} &= D\Omega(t) \prod_i d\lambda_i J(\lambda_i) \\ J &= \Delta^2(\lambda) \\ \Delta(\lambda) &= \prod_{i \neq j} (\lambda_i - \lambda_j) \end{aligned}$$

1d matrix model



$$\int_{\lambda(E)} p d\lambda = 2\pi n$$

$$\int_{\lambda(E)} \sqrt{2(E - V(\lambda))} d\lambda = 2\pi n$$

$$f(E) = \sum_n \delta(E - E_n)$$

$$N \approx \int_{-\infty}^{\infty} f(E) dE$$

$$E = \int_{\mu_0}^{\lambda} E \rho(E) dE$$

$$\rho(E) = \dots$$

matrix model



$$\int_{\lambda(E)} p d\lambda = 2\pi n$$

$$\int_{\lambda(E)} \sqrt{2(E - V(\lambda))} d\lambda = \pi n$$

$$\left[\begin{aligned} \rho(E) &= \sum_n \delta(E - E_n) \\ N &= \int \rho(E) dE \end{aligned} \right.$$

$$E = \int_{\rho_0}^{\lambda} E \rho(E) dE$$

$$\rho(\lambda) = \frac{dN}{d\lambda} = \frac{1}{\pi} \int_{\lambda_-(E)}^{\lambda_+(E)} \frac{d\lambda}{\sqrt{2(\mu - V(\lambda))}}$$

$$\tilde{n}$$

$$\lambda = \tilde{n} \lambda_0$$

$$E_n)$$

$$E = \int_{\mu_0}^{\mu} E g(E) dE$$

$$g(\mu) = \frac{\Delta N}{\Delta \mu} = \frac{1}{\lambda_0(\mu)} \frac{d\lambda}{\sqrt{2(\mu - \mu_0)}} \quad \mu_0$$

$$\tilde{n}$$

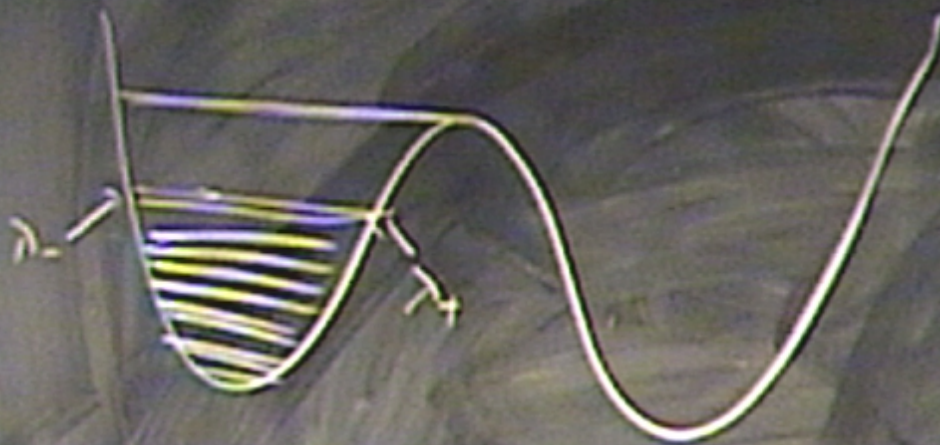
$$\lambda = \tilde{n} n$$

$$E_n)$$

$$E = \int_{\mu_0}^{\lambda} E g(E) dE$$

$$g(\mu) = \frac{\Delta N}{\Delta \mu} = \frac{1}{\tilde{n}} \int_{\lambda(\mu)}^{\lambda_0(\mu)} \frac{d\lambda}{\sqrt{2(\mu - \nu \lambda)}} \quad \mu \rightarrow \mu_0$$

1d matrix model



$$\oint_{\lambda(E)} p d\lambda = 2\pi n$$

$$\int_{\lambda_-(E)}^{\lambda_+(E)} \sqrt{2(E - U(\lambda))} d\lambda =$$

$$\left[\begin{aligned} \rho(E) &= \sum_n \delta(E - E_n) \\ N &= \int_{E_{\min}}^{E_{\max}} \rho(E) dE \end{aligned} \right.$$

$$E = \int_{\mu_0}^{\lambda} E g(E) dE$$

$$g(\mu) = \frac{\Delta N}{\Delta \mu} = \frac{1}{\pi} \int_{\lambda_-(\mu)}^{\lambda_+(\mu)} \frac{d\lambda}{\sqrt{2(\mu - V(\lambda))}} \stackrel{\mu \rightarrow 0}{\rightarrow} \frac{1}{\pi} \log \mu \dots$$

$$\mathcal{L}_{\text{eff}} = \sum_i \frac{1}{2}$$

$$H'' = \sum$$

$$D\bar{\Phi}H = D\Omega$$

$$J = \Delta^2(\lambda)$$

$$\Delta(\lambda) = \prod_{i=1}^n$$

$$E = \int_{\mu_0}^{\lambda} E g(E) dE$$

$$g(\mu) = \frac{\Delta N}{\Delta \mu} = \frac{1}{\pi} \int_{\lambda(\mu)}^{\lambda(\mu)} \frac{d\lambda}{v(\lambda)} = \frac{1}{\pi} \log \mu \dots$$

$$E = \frac{1}{\pi} \int_{\mu_0}^{\mu} \log E E dE \dots$$

$$L_{\text{eff}} = \sum \frac{1}{2}$$

$$H'' = \dots$$

$$D\bar{\Phi} = D\Omega$$

$$J = \Delta^2(\lambda)$$

$$\Delta(\lambda) = \dots$$

$$E = \int_{\mu_0}^{\Lambda} E \rho(E) dE$$

$$\rho(\mu) = \frac{\Delta N}{\Delta \mu} = \frac{1}{\pi} \int_{\lambda_-(\mu)}^{\lambda_+(\mu)} \frac{d\lambda}{\sqrt{2(\mu - V(\lambda))}} \stackrel{\mu \gg 0}{\approx} \frac{1}{\pi} \log \mu \dots$$

$$E = \frac{1}{\pi} \int_{\mu_0}^{\Lambda} \log E E dE = \frac{1}{2\pi} \mu^2 \log \mu$$

$$L_{eff} = \sum_i \frac{1}{2}$$

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$$E = \int_{\mu_0}^{\lambda} E g(E) dE$$

$$g(\mu) = \frac{\Delta N}{\Delta \mu} = \frac{1}{\pi} \frac{\lambda(\mu)}{\sqrt{2(\mu - \mu_0)}} \approx \frac{1}{\pi} \log \mu \dots$$

$\mu \approx 0$

$$E = \frac{1}{\pi} \int_{\mu_0}^{\lambda} \mu^2 \log \mu$$

A_P

$$L_{244} = \sum_i \frac{1}{2}$$

$$H'' = \sum$$

$$D\bar{\Phi} = D\Omega$$

$$J = \Delta^2(\lambda)$$

$$\Delta(\lambda) = \prod$$

$$E = \int_{\mu_0}^{\Lambda} E g(E) dE$$

$$g(\mu) = \frac{\Delta N}{\Delta \mu} = \frac{1}{\pi} \int_{\lambda_-(\mu)}^{\lambda_+(\mu)} \frac{d\lambda}{\sqrt{2(\mu - V(\lambda))}} \stackrel{\mu \gg 0}{\approx} \frac{1}{\pi} \log \mu \dots$$

$$E = \frac{1}{\pi} \int \log E E dE = \frac{1}{2\pi} \mu^2 \log \mu$$

Approximate $V(\lambda) \approx -\frac{1}{2}\lambda^2$

$$L_{244} = \sum_i \frac{1}{2}$$

$$H'' = \sum$$

$$D\bar{\Phi} = D\Omega$$

$$J = \Delta^2(\lambda)$$

$$\Delta(\lambda) = \prod_{i=1}^n$$

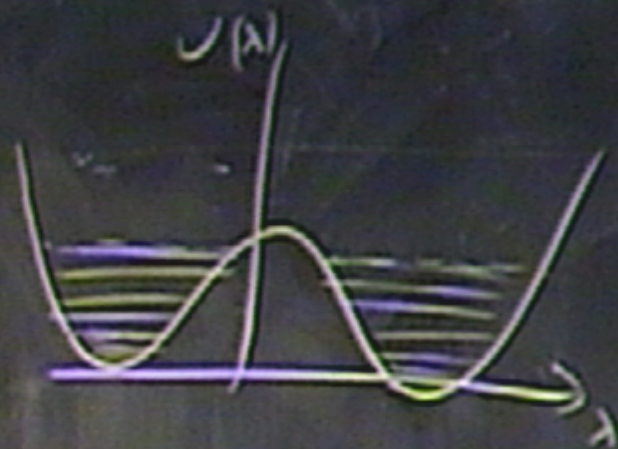
$$H = -\frac{1}{2} \partial_\lambda^2 - \frac{1}{2} \lambda^2$$

$$\frac{1}{\sqrt{\lambda}} \uparrow \frac{1}{\sqrt{\lambda}} \log \lambda \dots$$

$$u^2 \log \lambda$$

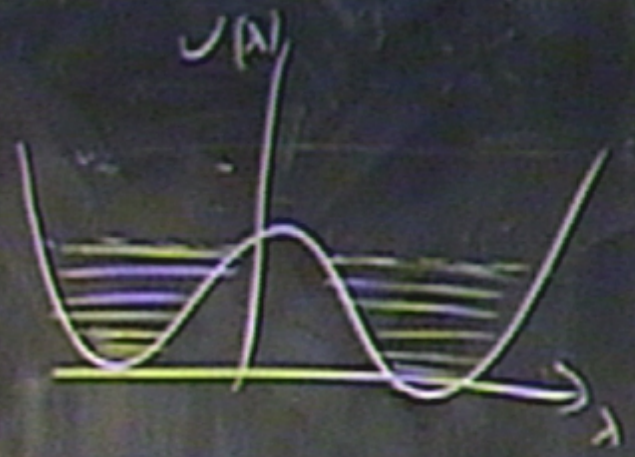


$$H\psi = E\psi$$



λ^2

$$S = \frac{1}{\sqrt{2}} (\lambda + P)$$
$$U = \frac{1}{\sqrt{2}} (\lambda - P)$$



λ^2

$$S = \frac{1}{\sqrt{2}} (\lambda + p)$$

$$U = \frac{1}{\sqrt{2}} (\lambda - p)$$

$$p = -i \frac{\partial}{\partial \lambda}$$

$$[U, S] = i$$

$$U = i \frac{\partial}{\partial S}$$

λ^2

$$S = \frac{1}{\sqrt{2}} (\lambda + p)$$

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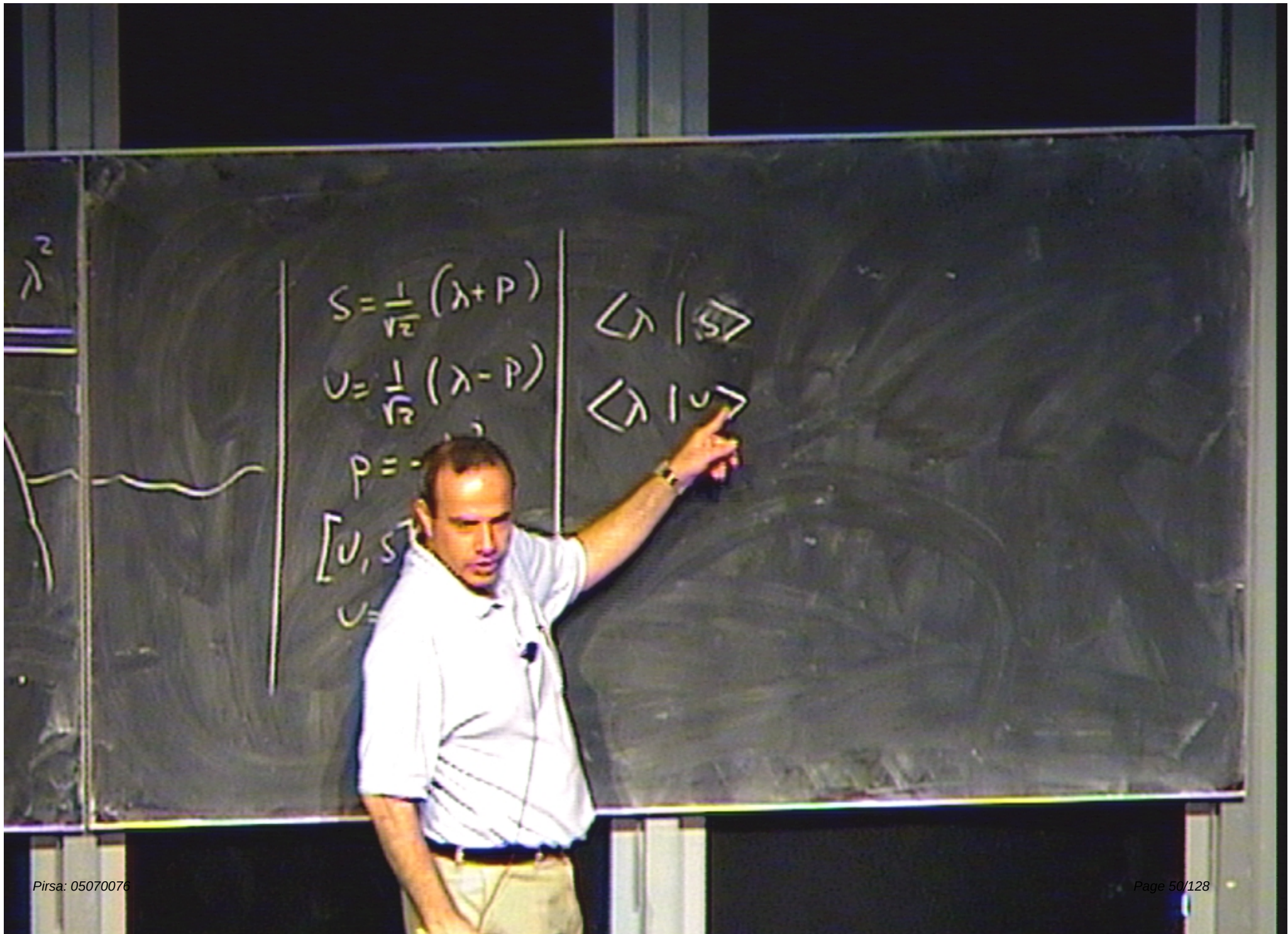
$$p = -i \frac{\partial}{\partial \lambda}$$

$$[U, S] = i$$

$$U = i \frac{\partial}{\partial S}$$

$$\langle \lambda | S \rangle$$

$$\langle \lambda | U \rangle$$



$$S = \frac{1}{\sqrt{2}}(\lambda + P)$$

$$U = \frac{1}{\sqrt{2}}(\lambda - P)$$

$$P = -$$

$$[U, S]$$

$$U =$$

$$\langle \lambda | S \rangle$$

$$\langle \lambda | \nu \rangle$$

$$S = \frac{1}{\sqrt{2}} (\lambda + p)$$

$$U = \frac{1}{\sqrt{2}} (\lambda - p)$$

$$p = -i \frac{\partial}{\partial \lambda}$$

$$[U, S] = i$$

$$U = i \frac{\partial}{\partial S}$$

$$\langle \lambda | S \rangle = \frac{2^{1/4}}{\sqrt{2\pi}} e^{i(-\frac{\lambda^2}{2} + \sqrt{2}\lambda S - \frac{S^2}{2})}$$

$$\langle \lambda | U \rangle = \frac{2^{1/4}}{\sqrt{2\pi}} e^{i(\frac{\lambda^2}{2} - \sqrt{2}\lambda U + \frac{U^2}{2})}$$

$$S = \frac{1}{\sqrt{2}} (\lambda + P)$$

$$U = \frac{1}{\sqrt{2}} (\lambda - P)$$

$$P = -i \frac{\partial}{\partial \lambda}$$

$$[U, S] = i$$

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$$\langle \lambda | U \rangle = \frac{2^{1/4}}{\sqrt{2\pi}} e^{i(\frac{\lambda^2}{2} - \sqrt{2}\lambda U + \frac{U^2}{2})}$$

$$\langle S | U \rangle = \sqrt{\frac{i}{2\pi}} e^{i S U}$$

$$S = \frac{1}{\sqrt{2}} (\lambda + P)$$

$$U = \frac{1}{\sqrt{2}} (\lambda - P)$$

$$P =$$

$$[U$$

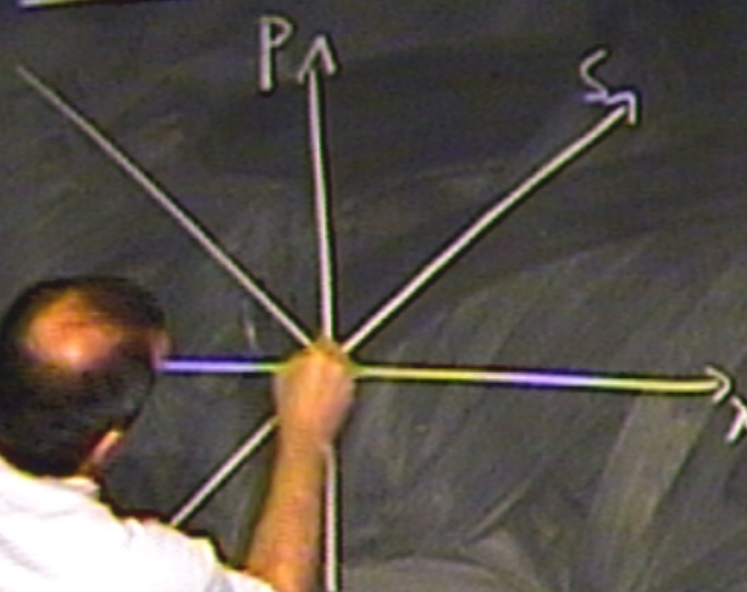
$$\langle \lambda | S \rangle = \frac{2^{1/4}}{\sqrt{2\pi}} e^{i(-\frac{\lambda^2}{2} + \sqrt{2}\lambda S - \frac{S^2}{2})}$$

$$\langle \lambda | U \rangle = \frac{2^{1/4}}{\sqrt{2\pi}} e^{i(\frac{\lambda^2}{2} - \sqrt{2}\lambda U + \frac{U^2}{2})}$$

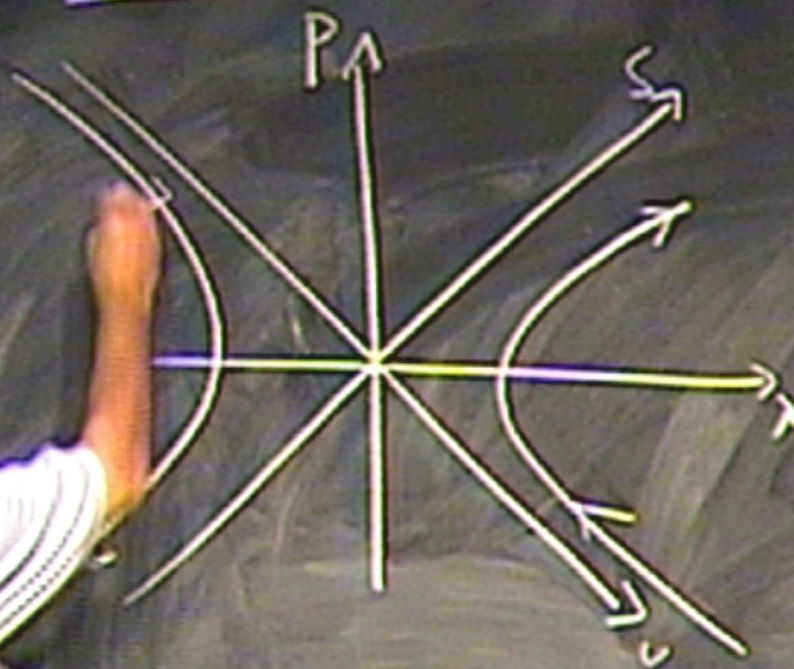
$$\langle S | U \rangle = \sqrt{\frac{i}{2\pi}} e^{iS U}$$

Id matrix model

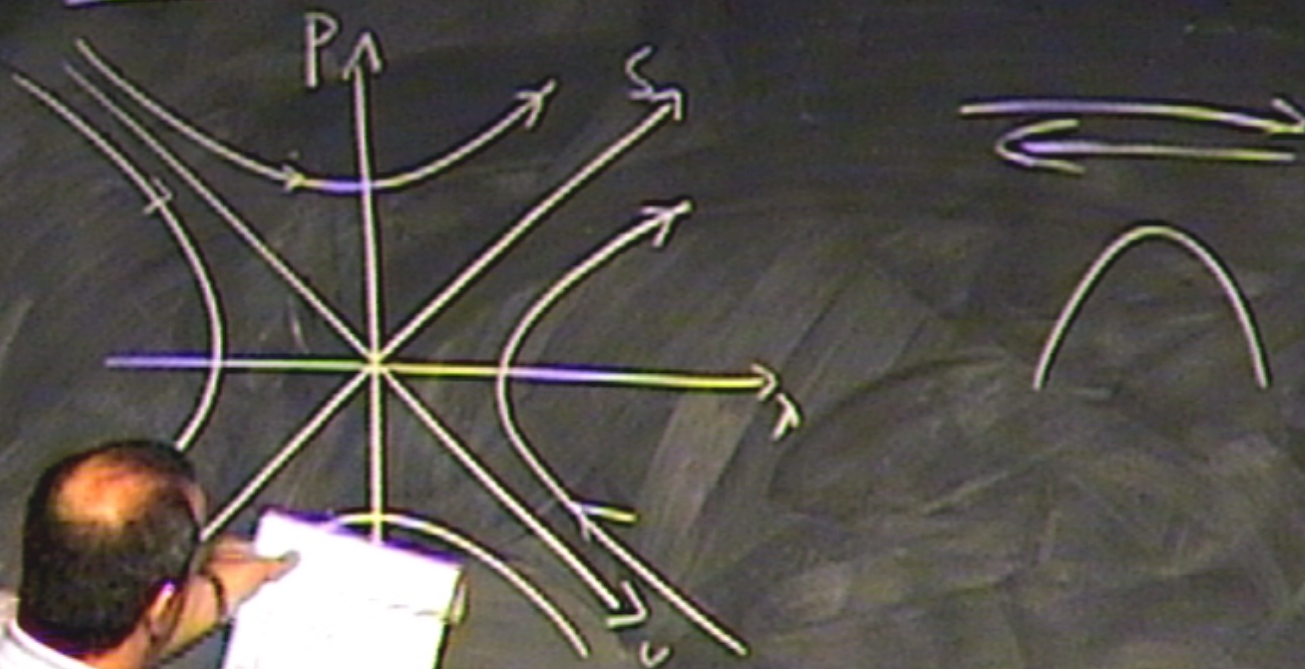
Id matrix model



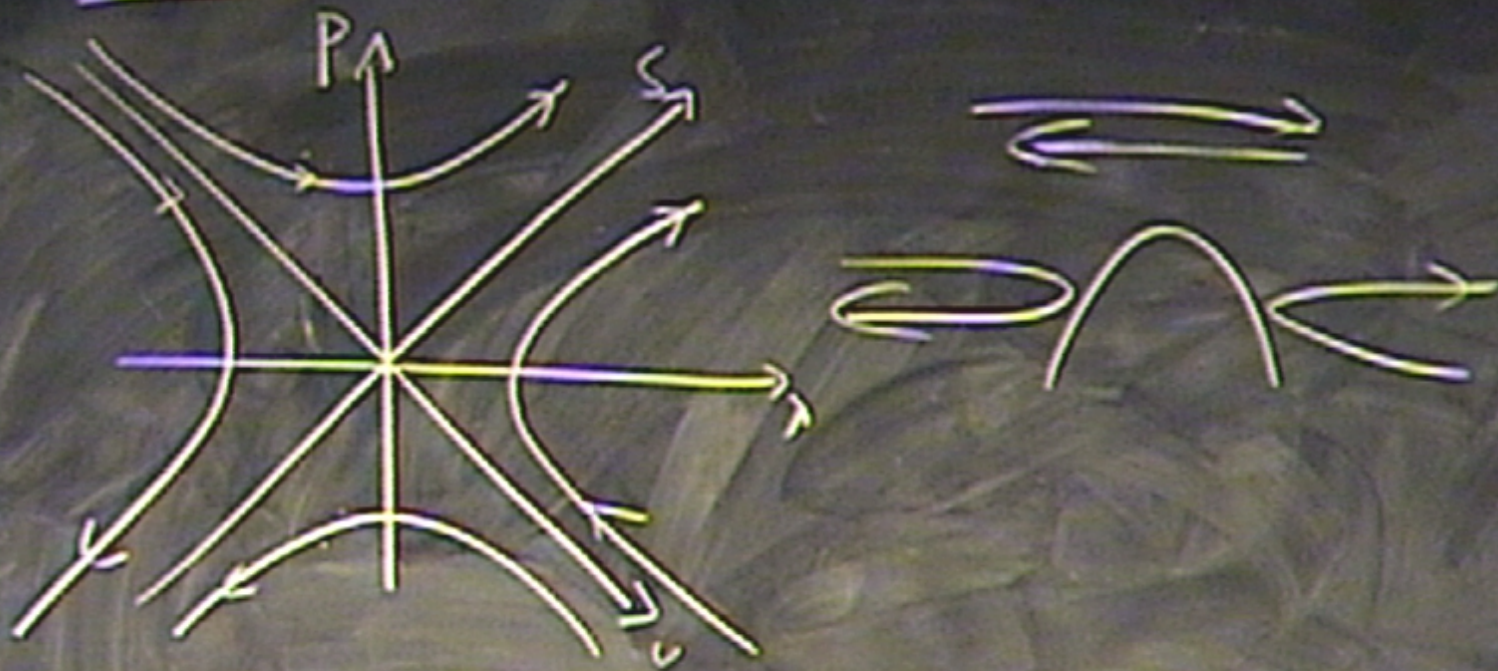
Id matrix model



Id matrix model



Id matrix model

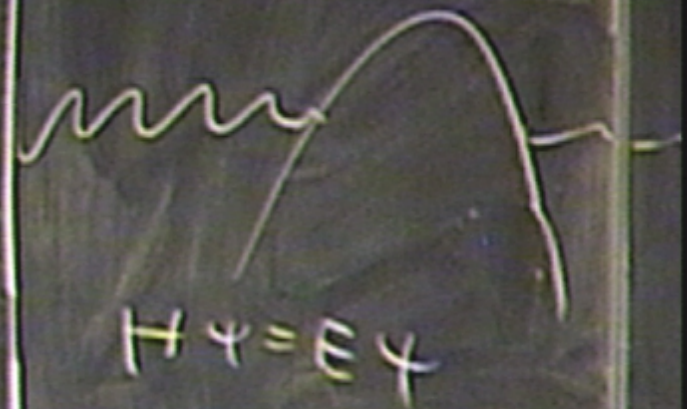


$$\mathcal{L} = \frac{1}{2}(\dot{\lambda}^2 + \lambda^2)$$

$$H = \frac{1}{2}(p^2 - \lambda^2) = -\frac{1}{2}(S\psi + \psi S) =$$

$$= -iS\frac{\partial}{\partial S} - \frac{i}{2}$$

$$\underline{\underline{H = -\frac{1}{2}\partial_\lambda^2 - \frac{1}{2}\lambda^2}}$$



$$H\psi = E\psi$$

$$\mathcal{L} = \frac{1}{2}(\dot{\lambda}^2 + \lambda^2)$$

$$H = \frac{1}{2}(P^2 - \lambda^2) = -\frac{1}{2}(S_0 + \psi S) =$$

$$= -iS \frac{\partial}{\partial S} - \frac{i}{2}$$

$$H = -\frac{1}{2} \partial_\lambda^2 - \frac{1}{2} \lambda^2$$



$$H\psi = E\psi$$

$$\mathcal{L} = \frac{1}{2}(\dot{\lambda}^2 + \lambda^2)$$

$$H = \frac{1}{2}(p^2 - \lambda^2) = -\frac{1}{2}(S_0 + \psi S) =$$

$$= -\frac{1}{2} - \frac{i}{2}$$

Sil eigenfunct

$$\underline{\underline{H = -\frac{1}{2} \partial_\lambda^2 - \frac{1}{2} \lambda^2}}$$



$$H\psi = E\psi$$

$$\mathcal{L} = \frac{1}{2}(\dot{\lambda}^2 + \lambda^2)$$

$$H = \frac{1}{2}(p^2 - \lambda^2) = -\frac{1}{2}(S_0 + \psi S) =$$

$$-i\frac{\partial}{\partial s} - \frac{i}{2}$$

$$S^2 = -\frac{1}{2}$$

eigenfunctions w/ energy E

$$H = -\frac{1}{2}\partial_\lambda^2 - \frac{1}{2}\lambda^2$$



$$H\psi = E\psi$$

$$\mathcal{L} = \frac{1}{2}(\dot{\lambda}^2 + \lambda^2)$$

$$H = \frac{1}{2}(P^2 - \lambda^2) = -\frac{1}{2}(S_0 + \psi S) =$$

$$= -iS \frac{\partial}{\partial S} - \frac{i}{2}$$

$S^{iE - \frac{1}{2}}$ eigenfunctions w/ energy E

E_{out}

$$\langle s | E, \text{out}, + \rangle =$$

$$\frac{1}{\sqrt{2\pi}} s^{iE - \frac{1}{2}} \quad s > 0$$

$$\langle s | E, \text{out}, - \rangle$$

$$\langle s | E, \text{out}, + \rangle = \begin{cases} \frac{1}{\sqrt{2\pi}} s^{iE - \frac{1}{2}} & s > 0 \\ 0 & s < 0 \end{cases}$$

$$\langle s | E, \text{out}, - \rangle = \begin{cases} 0 & s > 0 \\ \frac{1}{\sqrt{2}} & s < 0 \end{cases}$$

$$\langle s | E, \text{out}, + \rangle = \begin{cases} \frac{1}{\sqrt{2\pi}} s^{iE - \frac{1}{2}} & s > 0 \\ 0 & s < 0 \end{cases}$$

$$\langle s | E, \text{out}, - \rangle = \begin{cases} 0 & s > 0 \\ \frac{1}{\sqrt{2\pi}} |s|^{iE - \frac{1}{2}} & s < 0 \end{cases}$$

$$\langle \psi | E, im, + \rangle$$

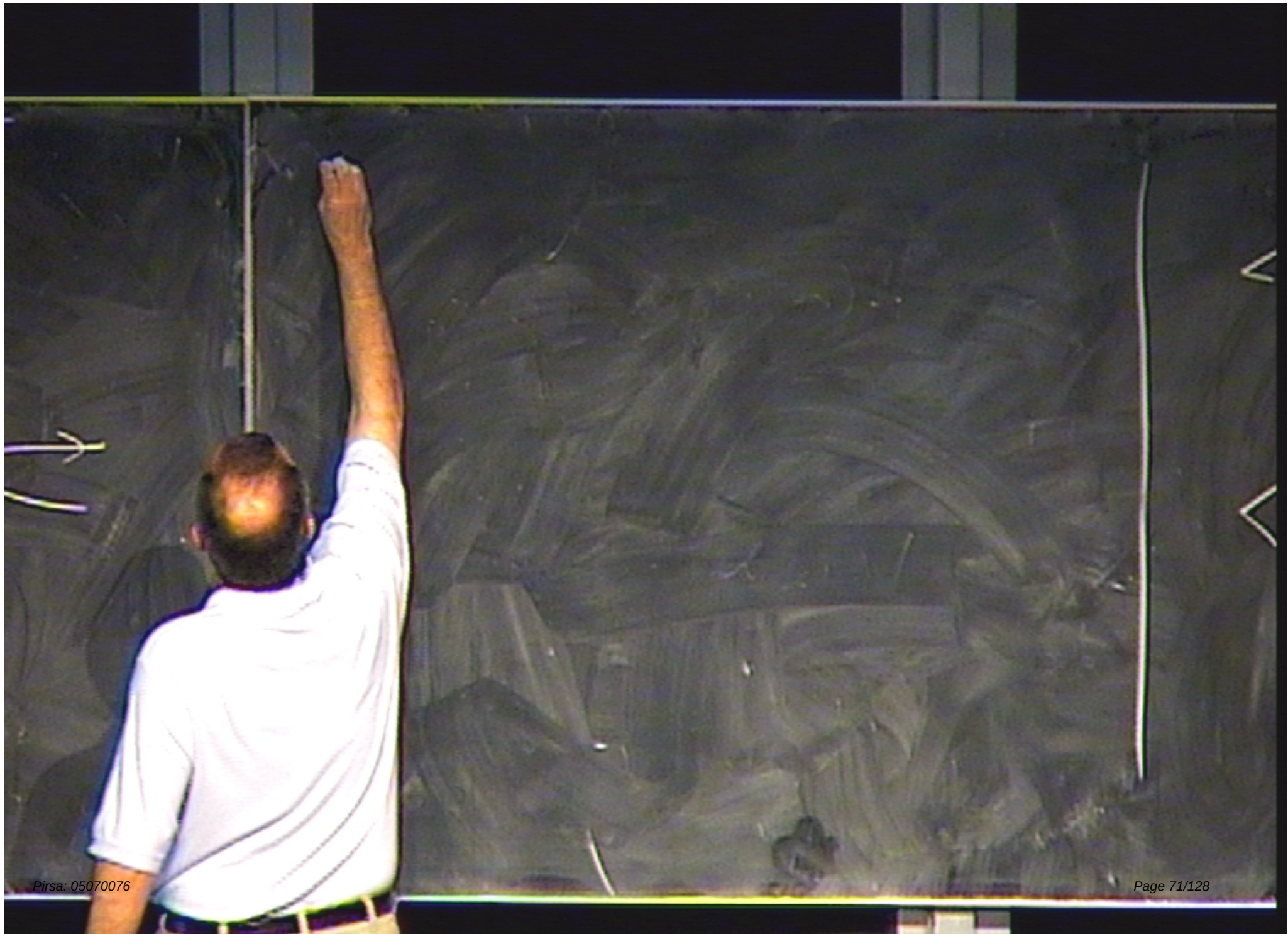
$$im \rightarrow =$$

$$\langle u | E, im, + \rangle = \begin{cases} \frac{1}{\sqrt{2\pi}} u^{-iE-\frac{1}{2}} & u > 0 \\ 0 & u < 0 \end{cases}$$

$$\langle u | E, im, - \rangle = \begin{cases} 0 & u > 0 \\ \dots & u < 0 \end{cases}$$

$$\langle u | E, im, + \rangle = \begin{cases} \frac{1}{\sqrt{2\pi}} u^{-iE-\frac{1}{2}} & u > 0 \\ 0 & u < 0 \end{cases}$$

$$\langle u | E, im, - \rangle = \begin{cases} 0 & u > 0 \\ \frac{1}{\sqrt{2\pi}} |u|^{-iE-\frac{1}{2}} & u < 0 \end{cases}$$



$$\langle s|u \rangle = \sqrt{\frac{L}{2\pi}} e^{-isu}$$

$$\langle s|u\rangle = \sqrt{\frac{i}{2\pi}} e^{-isu}$$

$in + \rangle$

$in - \rangle$

$$\langle s|u\rangle = \sqrt{\frac{i}{2\pi}} e^{-isu}$$

$$\begin{pmatrix} |E_{in+}\rangle \\ |E_{in-}\rangle \end{pmatrix}$$

$$\langle s|u\rangle = \sqrt{\frac{i}{2\pi}} e^{-isu}$$

$$\begin{pmatrix} |E_{in+}\rangle \\ |E_{in-}\rangle \end{pmatrix} = S \begin{pmatrix} |E_{out+}\rangle \\ |E_{out-}\rangle \end{pmatrix}$$

$$\langle s|u\rangle = \sqrt{\frac{i}{2\pi}} e^{-isu}$$

$$\begin{pmatrix} |E \text{ in } + \rangle \\ |E \text{ in } - \rangle \end{pmatrix} = \int \begin{pmatrix} |E \text{ out } + \rangle \\ |E \text{ out } - \rangle \end{pmatrix}$$

$$|E \text{ in } + \rangle = \frac{i}{\sqrt{2\pi}} \int_0^\infty ds \int_0^\infty du$$

$$\langle s|u\rangle = \sqrt{\frac{i}{2\pi}} e^{-isu}$$

$$\begin{pmatrix} |E_{in+}\rangle \\ |E_{in-}\rangle \end{pmatrix} = \int \begin{pmatrix} |E_{out+}\rangle \\ |E_{out-}\rangle \end{pmatrix}$$

$$\langle E_{out+} | E'_{in+} \rangle = \frac{i}{\sqrt{2\pi}} \int_0^\infty ds \int_0^\infty du \frac{e^{-isu}}{2\pi}$$

$|s\rangle \langle s| \rangle \langle u|$

$$= \int_0^\infty ds e^{-i(E-\frac{1}{2})s} \int_0^\infty du e^{-i(E'-\frac{1}{2})u} =$$

$$\langle s|u\rangle = \sqrt{\frac{i}{2\pi}} e^{-isu}$$

$$\begin{pmatrix} |E \text{ in } + \rangle \\ |E \text{ in } - \rangle \end{pmatrix} = S \begin{pmatrix} |E \text{ out } + \rangle \\ |E \text{ out } - \rangle \end{pmatrix}$$

$$\langle E \text{ out } + | E' \text{ in } + \rangle = \frac{i}{\sqrt{2\pi}} \int_0^\infty ds \int_0^\infty du \frac{e^{-isu}}{2\pi}$$

$|s\rangle \langle s| \rangle \langle u|$

$$e^{-i(E - \frac{1}{2})u} e^{-i(E' - \frac{1}{2})s} =$$

$$u = r e^{-i\varphi}$$

$$s = r e^{+i\varphi}$$

$$\langle s | u \rangle = \sqrt{\frac{i}{2\pi}} e^{-isu}$$

$$\begin{pmatrix} |E \text{ in } + \rangle \\ |E \text{ in } - \rangle \end{pmatrix} = S \begin{pmatrix} |E \text{ out } + \rangle \\ |E \text{ out } - \rangle \end{pmatrix}$$

$$\langle E \text{ out } + | E' \text{ in } + \rangle = \frac{i}{\sqrt{2\pi}} \int_0^\infty ds \int_0^\infty du \frac{e^{-isu}}{2\pi}$$

$1 \rightarrow \langle s | \rightarrow \langle u$

$$S^{-i(E-\frac{1}{2})u} - i(E'-\frac{1}{2})u = \int (E-E')$$

$$u = r e^{-i\varphi}$$

$$s = r e^{+i\varphi}$$

$$\int d\varphi$$

$$\langle s | u \rangle = \sqrt{\frac{i}{2\pi}} e^{-isu}$$

$$\begin{pmatrix} |E \text{ in } + \rangle \\ |E \text{ in } - \rangle \end{pmatrix} = S \begin{pmatrix} |E \text{ out } + \rangle \\ |E \text{ out } - \rangle \end{pmatrix}$$

$$\langle E \text{ out } + | E' \text{ in } + \rangle = \frac{i}{\sqrt{2\pi}} \int_0^\infty ds \int_0^\infty du \frac{e^{-isu}}{2\pi r}$$

$|s\rangle \langle s| |u\rangle \langle u|$

$$S^{-i(E-\frac{1}{2})u} - i(E'-\frac{1}{2})u = \int_0^\infty dt e^{-r^2(i+\frac{1}{2})t}$$

$\int d\varphi$

$u = r e^{-i\varphi}$
 $s = r e^{+i\varphi}$

$$\langle s | u \rangle = \sqrt{\frac{i}{2\pi}} e^{-isu}$$

$$\begin{pmatrix} |E \text{ in } + \rangle \\ |E \text{ in } - \rangle \end{pmatrix} = \int \begin{pmatrix} |E \text{ out } + \rangle \\ |E \text{ out } - \rangle \end{pmatrix}$$

$$\langle E \text{ out } + | E' \text{ in } + \rangle = \frac{i}{\sqrt{2\pi}} \int_0^\infty ds \int_0^\infty du \frac{e^{-isu}}{2\pi}$$

$$u = r e^{-i\varphi}$$

$$s = r e^{+i\varphi}$$

$$s^{-i(E-\frac{1}{2})} u^{-i(E'-\frac{1}{2})} = \int_0^\infty dt e^{-r^2(i+\frac{1}{2})t} e^{-2iEt}$$

$$\langle s | E, \text{out}, + \rangle = \left\{ \begin{array}{l} \sqrt{\frac{i}{2\pi}} e^{-isu} \\ \sqrt{\frac{i}{2\pi}} e^{-isu} \end{array} \right.$$

$$\langle s | E, \text{out}, - \rangle$$

$$S \begin{pmatrix} |E_{out} + \rangle \\ |E_{out} - \rangle \end{pmatrix}$$

$$in + \rangle = \frac{i}{\sqrt{2\pi}} \int_0^\infty ds \int_0^\infty dv \frac{e^{-isu}}{2\pi r}$$

$$S^{-iE-\frac{1}{2}}_U^{-iE'-\frac{1}{2}} = \int_0^\infty dE' \delta(E-E') \int_0^\infty dt \frac{e^{-r^2(i+0')}}{r} \frac{e^{-2iE}}{\sqrt{E}} = \frac{\Gamma(\frac{1}{2})}{\sqrt{2\pi}} \Gamma(\frac{1}{2}-iE) e^{-\pi E/2}$$

$$\langle s | E_{out}, + \rangle = \begin{cases} \frac{1}{\sqrt{2\pi}} S^{iE-\frac{1}{2}} & s > 0 \\ 0 & s < 0 \end{cases}$$

$$\langle s | E_{out}, - \rangle = \begin{cases} 0 & s > 0 \\ \frac{1}{\sqrt{2\pi}} |s|^{iE-\frac{1}{2}} & s < 0 \end{cases}$$

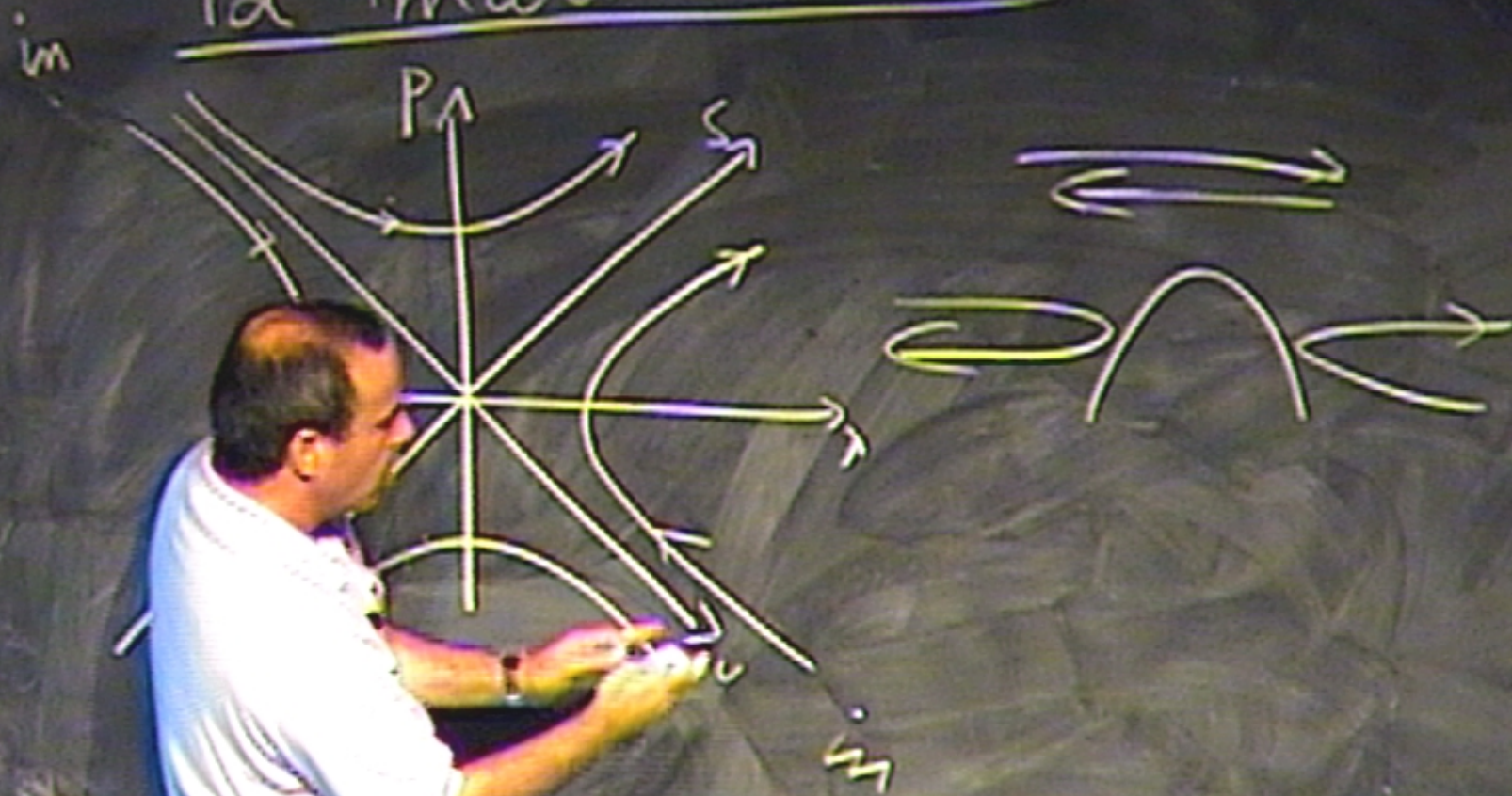
$$S = \frac{\Gamma(\frac{1}{2} - iE)}{\sqrt{2\pi}} \begin{pmatrix} e^{-\pi E/2} & i e^{\pi E/2} \\ i e^{\pi E/2} & e^{-\pi E/2} \end{pmatrix} = e^{i\Phi(E)}$$

$$S = \frac{\Gamma(\frac{1}{2} - iE)}{\sqrt{2\pi}} \begin{pmatrix} e^{-\pi E/2} & i e^{\pi E/2} \\ i e^{\pi E/2} & e^{-\pi E/2} \end{pmatrix} = e^{i\Phi(E)} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$S = \frac{\Gamma(\frac{1}{2} - iE)}{\sqrt{2\pi}} \begin{pmatrix} e^{-\pi E/2} & i e^{\pi E/2} \\ i e^{\pi E/2} & e^{-\pi E/2} \end{pmatrix} = e^{i\Phi(E)} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

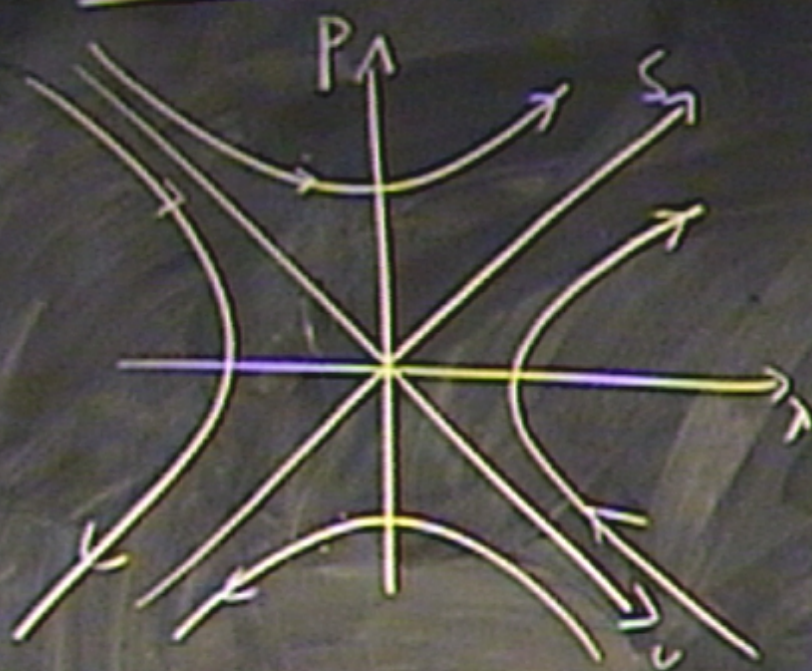
$$\begin{pmatrix} \frac{1}{\sqrt{1+e^{2\pi E}}} & \frac{i}{\sqrt{1+e^{-2\pi E}}} \\ \frac{i}{\sqrt{1+e^{-2\pi E}}} & \frac{1}{\sqrt{1+e^{2\pi E}}} \end{pmatrix}$$

Id matrix model



1d matrix model

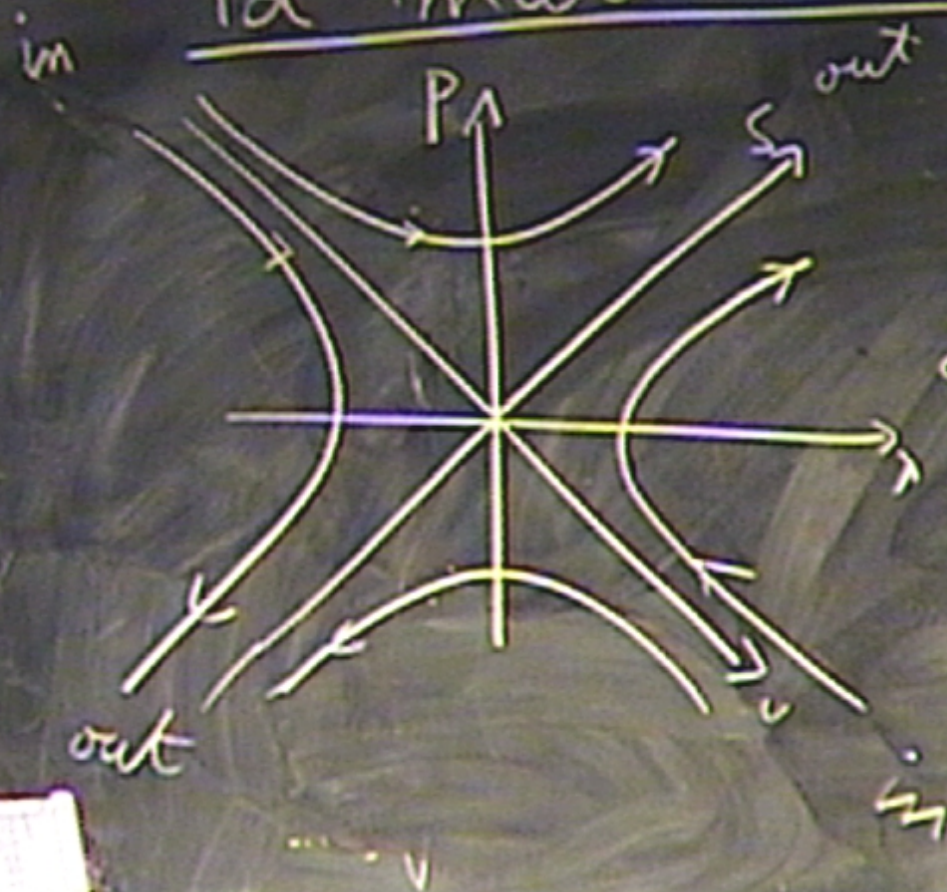
m



$$|v| \gg 1$$

$$s \approx 0$$

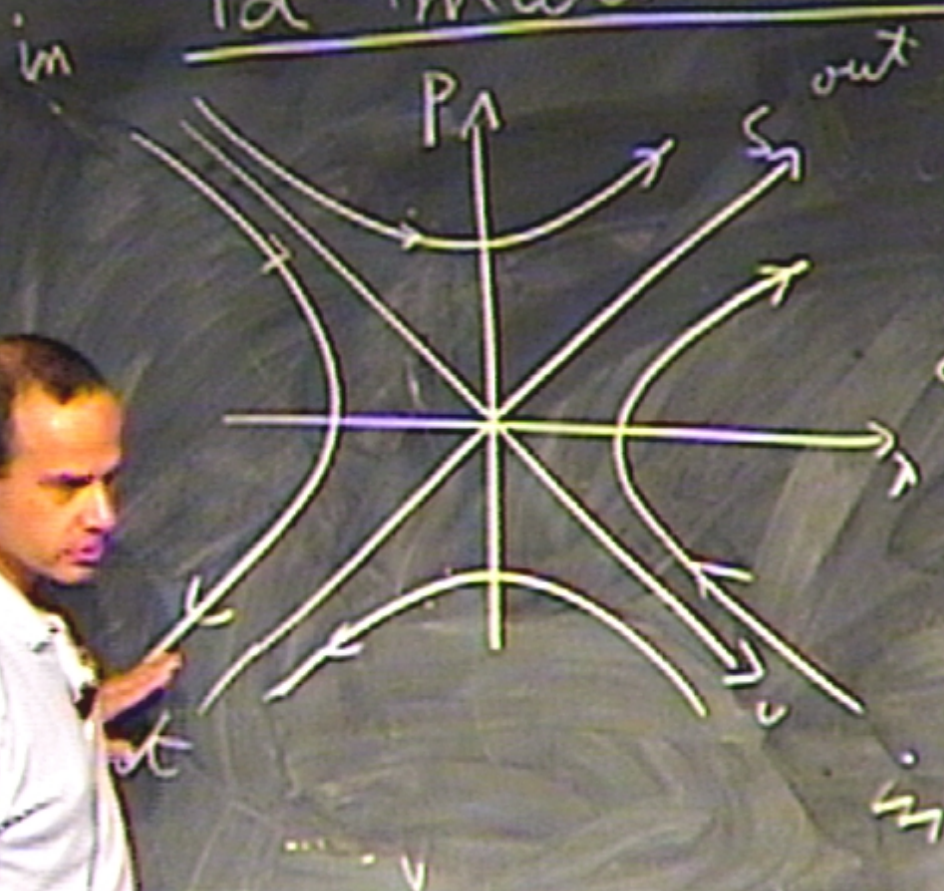
1d matrix model



incoming $|v| \gg 1$
 $s \approx 0$

outgoing $|s| \gg 1$
 $v \approx 0$

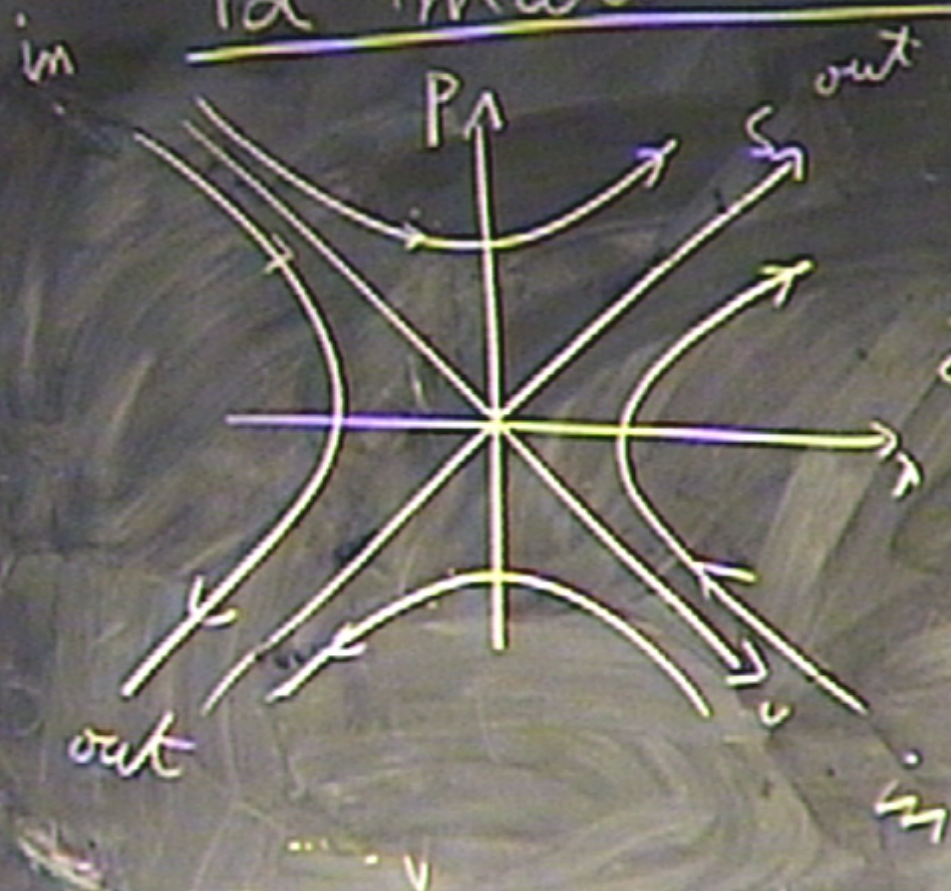
1d matrix model



incoming $|v| \gg 1$
 $s \approx 0$

outgoing $|s| \gg 1$
 $v \approx 0$

1d matrix model



incoming $|v| \gg 1$
 $s \approx 0$

outgoing $|s| \gg 1$
 $v \approx 0$

$$E \rightarrow +\infty$$

$$S = \frac{\Gamma(\frac{1}{2})}{\sqrt{2\pi}}$$

$$du \, e^{-isu} \\ \frac{1}{2\pi i}$$

$$\int_0^\infty dt \, e^{-r^2(i+0^+)} r^{-2iE} = \sqrt{\frac{i}{\pi}} \frac{\Gamma(\frac{1}{2})}{\sqrt{2\pi}} e^{-\pi E/2}$$

$$E \rightarrow +\infty$$

$$|E_{in}\rangle \approx |E_{out}\rangle \Rightarrow$$

$$S = \frac{\Gamma(\frac{1}{2})}{\sqrt{2\pi}}$$

$$du \frac{e^{-isu}}{2\pi}$$

$$\int_0^\infty dt \frac{e^{-r^2(i+0^+)} r^{-2iE}}{\sqrt{2\pi}} = \frac{\sqrt{\pi}}{\sqrt{2\pi}} \Gamma(\frac{1}{2}-iE) e^{-\pi E/r}$$

$$E \rightarrow +\infty$$

$$|E \text{ in} \rangle \propto |E \text{ out} \rangle \Rightarrow S = \frac{\Gamma(\frac{1}{2})}{\sqrt{2\pi}}$$

$$E \rightarrow -\infty$$

$$|E \text{ in}^\pm \rangle \propto |E \text{ out}^\pm \rangle$$

$$du \frac{e^{-isu}}{2\pi i}$$

$$\int_0^\infty dt \frac{e^{-r^2(i+0^+)} r^{-2iE}}{\sqrt{2\pi}} = \frac{\sqrt{2\pi}}{\sqrt{2\pi}} \Gamma(\frac{1}{2}-iE) e^{-\pi E/r}$$

$$E \rightarrow +\infty$$

$$|E \text{ in} \rangle \propto |E \text{ out} \rangle \quad S = \frac{\Gamma(\frac{1}{2})}{\sqrt{2\pi}}$$

$$E \rightarrow -\infty$$

$$|E \text{ in}^\pm \rangle \propto |E \text{ out}^\pm \rangle$$

$$du \, e^{-isu} \\ \frac{1}{2\pi i} \int_{-\infty}^{\infty} dr \, e^{-r^2(i+0^+)} r^{-2iE}$$

$$\delta(E-E') \int_{-\infty}^{\infty} dr \, e^{-r^2(i+0^+)} r^{-2iE} = \frac{\sqrt{\pi}}{\sqrt{2\pi}} \Gamma\left(\frac{1}{2}-iE\right) e^{-\pi E/r}$$

$$E \rightarrow +\infty$$

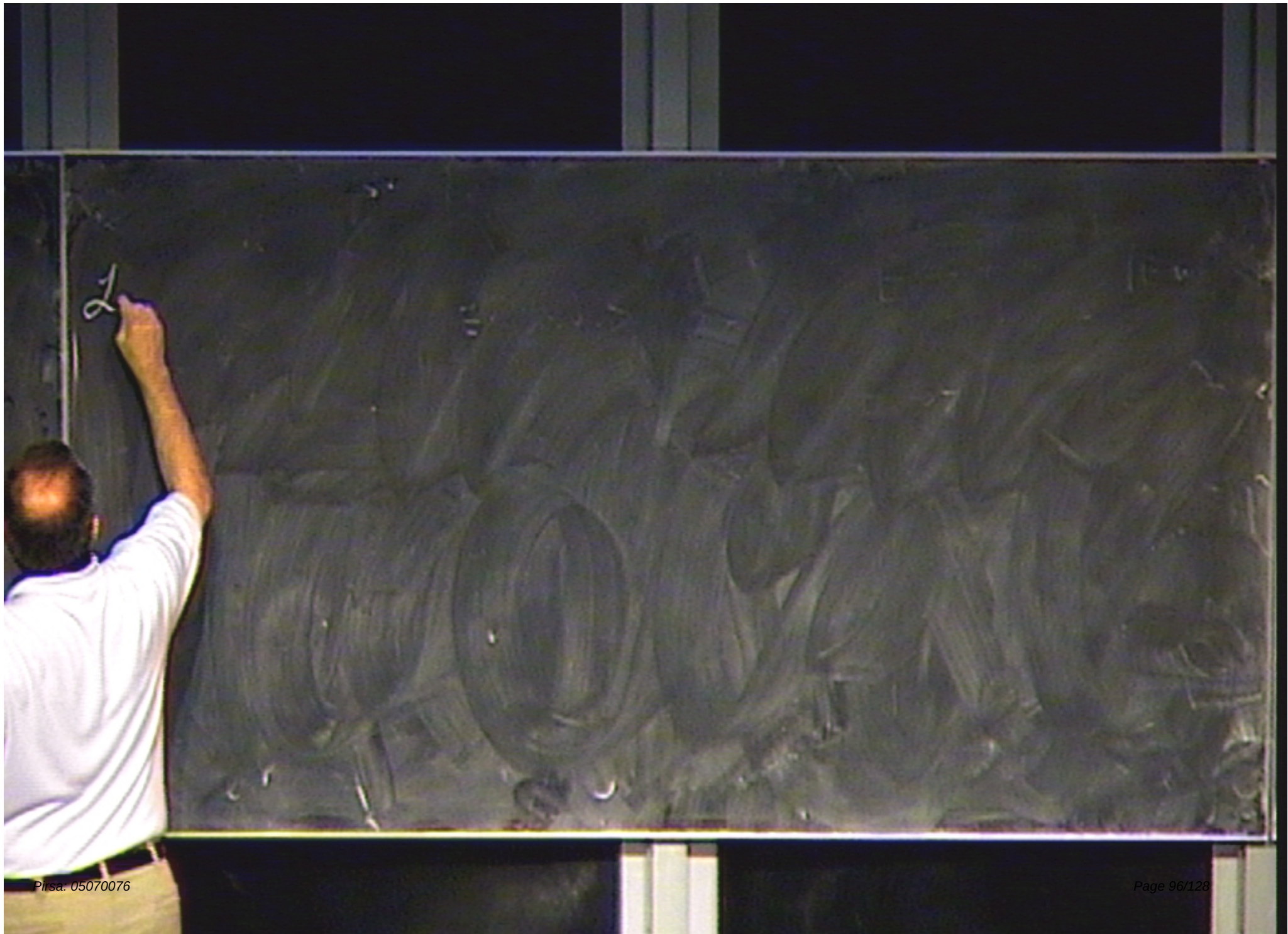
$$|E_{in}\rangle \propto |E_{out}\rangle \quad S = \frac{\Gamma(\frac{1}{2})}{\sqrt{2\pi}}$$

$$E \rightarrow -\infty$$

$$|E_{in}^{\pm}\rangle \propto |E_{out}^{\pm}\rangle$$

$$du \frac{e^{-isu}}{2\pi i}$$

$$\int_0^{\infty} dt \frac{e^{-r^2(i+0^+)}}{r} e^{-2iE} = \frac{\sqrt{\pi}}{\sqrt{2\pi}} \Gamma\left(\frac{1}{2}-iE\right) e^{-\pi E/r}$$



$$\mathcal{L} = \int dr \, \psi^\dagger(r,t) \left(i \partial_t + \frac{1}{2} \right)$$

$$\mathcal{L} = \int d\lambda \quad \psi^\dagger(\lambda, t) \left(i\partial_t + \frac{1}{2}\partial_\lambda^2 + \frac{1}{2}\lambda^2 - \mu \right) \psi(\lambda, t)$$

$$\mathcal{L} = \int d\lambda \quad \psi^\dagger(\lambda, t) \left(i\partial_t + \frac{1}{2}\partial_\lambda^2 + \frac{1}{2}\lambda^2 - \mu \right) \psi(\lambda, t)$$

$$\psi_s(s, t) = \frac{(2i)^{1/4}}{\sqrt{2\pi}} \int d\lambda \quad e^{i(\lambda^2 s)}$$

$$\mathcal{L} = \int d\lambda \quad \psi^\dagger(\lambda, \tau) \left(i\partial_\tau + \frac{1}{2}\partial_\lambda^2 + \frac{1}{2}\lambda^2 - \mu \right) \psi(\lambda, \tau)$$

$$\psi_s(s, \tau) = \frac{(2i)^{1/4}}{\sqrt{2\pi}} \int d\lambda \quad e^{i\left(\frac{\lambda^2}{2} - \sqrt{2}s\lambda + \frac{s^2}{2}\right)} \psi(\lambda, \tau)$$

$$\psi_v(v, \tau) = \frac{(-2i)^{1/4}}{\sqrt{2\pi}} \int d\lambda \quad e^{i\left(-\frac{\lambda^2}{2} + \sqrt{2}v\lambda - \frac{v^2}{2}\right)} \psi(\lambda, \tau)$$

$$\mathcal{L} = \int d\lambda \quad \psi^\dagger(\lambda, t) \left(i\partial_t + \frac{1}{2}\partial_\lambda^2 + \frac{1}{2}\lambda^2 - \mu \right) \psi(\lambda, t)$$

$$\psi_s(s, t) = \frac{(2i)^{1/4}}{\sqrt{2\pi}} \int d\lambda \quad e^{i\left(\frac{\lambda^2}{2} - \sqrt{2}s\lambda + \frac{s^2}{2}\right)} \psi(\lambda, t)$$

$$\psi_v(v, t) = \frac{(-2i)^{1/4}}{\sqrt{2\pi}} \int d\lambda \quad e^{i\left(-\frac{\lambda^2}{2} + \sqrt{2}v\lambda - \frac{v^2}{2}\right)} \psi(\lambda, t)$$

$$\psi_s(s, t) = \int \frac{dv}{\sqrt{2\pi}} \quad e^{-isv} \psi_v(v, t)$$

$$= \int_{-\infty}^{\infty} ds \, \psi_s^\dagger \left(i \partial_t + \underbrace{-\frac{1}{2}(s \partial_s + \partial_s s)} - \mu \right) \psi_s$$

$$= \int_{-\infty}^{\infty} ds \, \psi_s^\dagger \left(i \partial_t + \underbrace{\frac{i}{2} (s \partial_s + \partial_s s)} - \mu \right) \psi_s$$

$$= \int_{-\infty}^{\infty} ds \, \psi_s^\dagger \left(i\partial_t + \frac{i}{2}(s\partial_s + \partial_s s) - \mu \right) \psi_s$$

$$= \int_{-\infty}^{\infty} du \, \psi_u^\dagger \left(i\partial_t + \frac{i}{2}(u\partial_u + \partial_u u) - \mu \right) \psi_u$$

$$= \int_{-\infty}^{\infty} ds \, \psi_s^+ \left(i\partial_t + \frac{i}{2}(s\partial_s + \partial_s s) - \mu \right) \psi_s$$

$$= \int_{-\infty}^{\infty} du \, \psi_u^+ \left(i\partial_t + \frac{i}{2}(u\partial_u + \partial_u u) - \mu \right) \psi_u$$

$$\begin{aligned}
 &= \int_{-\infty}^{\infty} ds \, \psi_s^+ \left(i\partial_t + \frac{i}{2}(s\partial_s + \partial_s s) - \mu \right) \psi_s \\
 &= \int_{-\infty}^{\infty} du \, \psi_u^+ \left(i\partial_t + \frac{i}{2}(u\partial_u + \partial_u u) - \mu \right) \psi_u
 \end{aligned}$$

$$\begin{aligned}
 &= \int_{-\infty}^{\infty} ds \, \psi_s^\dagger \left(i\partial_t + \frac{i}{2}(\psi\partial_s + \partial_s\psi) - \mu \right) \psi_s \\
 &= \int_{-\infty}^{\infty} du \, \psi_u^\dagger \left(i\partial_t + \frac{i}{2}(u\partial_u + \partial_u u) - \mu \right) \psi_u
 \end{aligned}$$

$$= \int_{-\infty}^{\infty} ds \, \psi_s^+ \left(i \partial_t + \frac{1}{2} (s \partial_s + \partial_s s) - \mu \right) \psi_s$$

$$= \int_{-\infty}^{\infty} du \, \psi_u^+ \left(i \partial_t + \frac{1}{2} (u \partial_u + \partial_u u) - \mu \right) \psi_u$$

$$\psi_{\pm}^m(r,t) = e^{r/2} \psi_s$$

$$= \int_{-\infty}^{\infty} ds \psi_s^+ (i\partial_t + \frac{i}{2}(s\partial_s + \partial_s s) - \mu) \psi_s$$

$$= \int_{-\infty}^{\infty} dv \psi_v^+ (i\partial_t + \frac{i}{2}(v\partial_v + \partial_v v) - \mu) \psi_v$$

$$\psi_{\pm}^{\text{in}}(r,t) = e^{i\mu r} \psi_s(s = \pm e^r, t)$$

$$\psi_{\pm}^{\text{out}}(r,t) = e^{i\mu r} \psi_v(v = \pm e^r, t)$$

$$\begin{aligned}
 &= \int_{-\infty}^{\infty} ds \, \psi_s^+ \left(i\partial_t + \frac{i}{2}(s\partial_s + \partial_s s) - \mu \right) \psi_s = \int dr \sum_{i=\pm} \psi_i^+ (i\partial_t - i\partial_r - \mu) \psi_i \\
 &= \int_{-\infty}^{\infty} dv \, \psi_v^+ \left(i\partial_t + \frac{i}{2}(v\partial_v + \partial_v v) - \mu \right) \psi_v = \int dr \sum_{i=\pm} \psi_i^{\text{out}} (i\partial_t - i\partial_r - \mu) \psi_i
 \end{aligned}$$

$$\begin{aligned}
 \psi_{\pm}^{\text{in}}(r,t) &= e^{r/2} \psi_s(s=\pm e^r, t) \\
 \psi_{\pm}^{\text{out}}(r,t) &= e^{r/2} \psi_v(v=\pm e^r, t)
 \end{aligned}$$

$$\begin{aligned}
 &= \int_{-\infty}^{\infty} ds \, \psi_s^+ \left(i\partial_t + \frac{i}{2}(s\partial_s + \partial_s s) - \mu \right) \psi_s = \int dr \sum_{i=\pm} \psi_i^{\text{in}} (i\partial_t - i\partial_r - \mu) \psi_i^{\text{in}} \\
 &= \int_{-\infty}^{\infty} dv \, \psi_v^+ \left(i\partial_t + \frac{i}{2}(v\partial_v + \partial_v v) - \mu \right) \psi_v = \int dr \sum_{i=\pm} \psi_i^{\text{out}} (i\partial_t + i\partial_r - \mu) \psi_i^{\text{out}}
 \end{aligned}$$

$$\psi_{\pm}^{\text{in}}(r,t) = e^{r/2} \psi_s(s = \pm e^r, t)$$

$$\psi_{\pm}^{\text{out}}(r,t) = e^{r/2} \psi_v(v = \pm e^r, t)$$

$$\begin{aligned}
 &= \int_{-\infty}^{\infty} ds \, \psi_s^+ \left(i\partial_t - \frac{i}{2}(s\partial_s + \partial_s s) - \mu \right) \psi_s = \int dr \sum_{i=\pm} \psi_i^+ (i\partial_t - i\partial_r - \mu) \psi_i \\
 &= \int_{-\infty}^{\infty} dv \, \psi_v^+ \left(i\partial_t - \frac{i}{2}(v\partial_v + \partial_v v) - \mu \right) \psi_v = \int dr \sum_{i=\pm} \psi_i^+ (i\partial_t + i\partial_r - \mu) \psi_i
 \end{aligned}$$

$$\begin{aligned}
 \psi_{\pm}^{\text{in}}(r,t) &= e^{r/2} \psi_s(s=\pm e^r, t) \\
 \psi_{\pm}^{\text{out}}(r,t) &= e^{r/2} \psi_v(v=\pm e^r, t)
 \end{aligned}$$

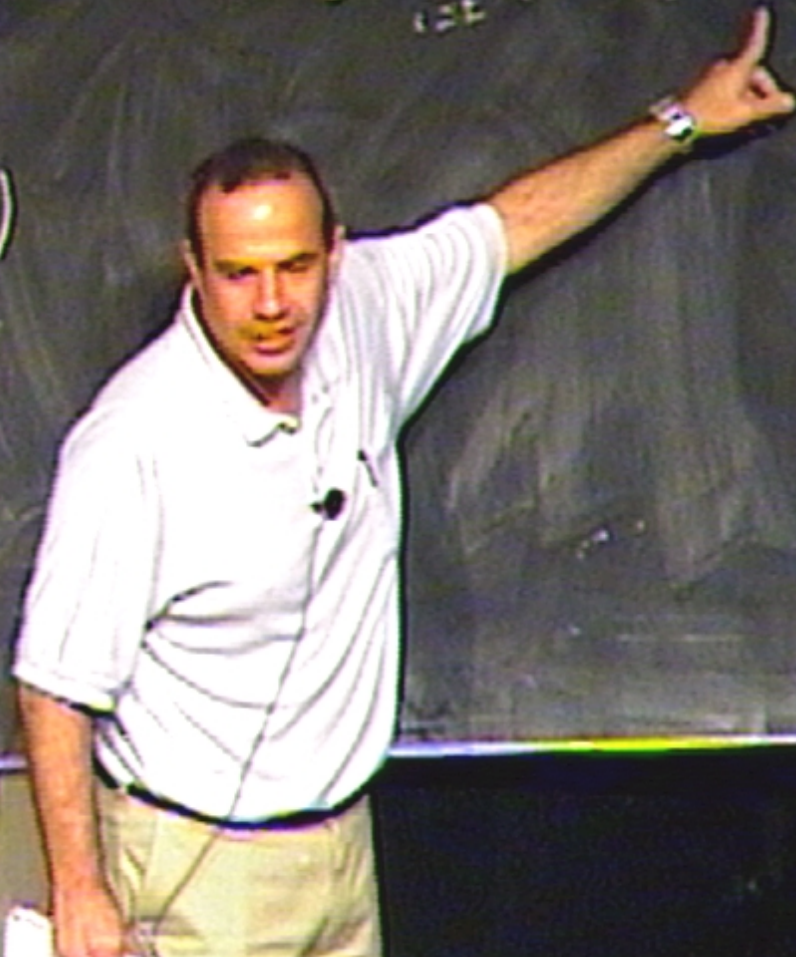
$$\begin{aligned}
 &= \int_{-\infty}^{\infty} ds \, \psi_s^+ \left(i\partial_t + \frac{i}{2}(s\partial_s + \partial_s s) - \mu \right) \psi_s = \int dr \sum_{i=\pm} \psi_i^{\text{in}} (i\partial_t - i\partial_r - \mu) \psi_i^{\text{in}} \\
 &= \int_{-\infty}^{\infty} dv \, \psi_v^+ \left(i\partial_t + \frac{i}{2}(v\partial_v + \partial_v v) - \mu \right) \psi_v = \int dr \sum_{i=\pm} \psi_i^{\text{out}} (i\partial_t + i\partial_r - \mu) \psi_i^{\text{out}}
 \end{aligned}$$

$$\psi_{\pm}^{\text{in}}(r,t) = e^{i\mu r} \psi_s(s=\pm e^r, t)$$

$$\psi_{\pm}^{\text{out}}(r,t) = e^{i\mu r} \psi_v(v=\pm e^r, t)$$

$$\begin{aligned}
 &= \int_{-\infty}^{\infty} ds \, \psi_s^+ (i\partial_t - \frac{i}{2}(s\partial_s + \partial_s s) - \mu) \psi_s = \int dr \sum_{i=\pm} \psi_i^{\text{in}} (i\partial_t - i\partial_r - \mu) \psi_i^{\text{in}} \\
 &= \int_{-\infty}^{\infty} dv \, \psi_v^+ (i\partial_t - \frac{i}{2}(v\partial_v + \partial_v v) - \mu) \psi_v = \int dr \sum_{i=\pm} \psi_i^{\text{out}} (i\partial_t + i\partial_r - \mu) \psi_i^{\text{out}}
 \end{aligned}$$

$$\begin{aligned}
 \psi_{\pm}^{\text{in}}(rt) &= e^{r/2} \psi_s(s=\pm e^r, t) \\
 \psi_{\pm}^{\text{out}}(rt) &= e^{r/2} \psi_v(v=\pm e^r, t)
 \end{aligned}$$



$$\begin{aligned}
 &= \int_{-\infty}^{\infty} ds \, \psi_s^+ (i\partial_t + \frac{i}{2}(s\partial_s + \partial_s s) - \mu) \psi_s = \int dr \sum_{i=\pm} \psi_i^{\text{in}} (i\partial_t + i\partial_r - \mu) \psi_i^{\text{in}} \\
 &= \int_{-\infty}^{\infty} dv \, \psi_v^+ (i\partial_t + \frac{i}{2}(v\partial_v + \partial_v v) - \mu) \psi_v = \int dr \sum_{i=\pm} \psi_i^{\text{out}} (i\partial_t + i\partial_r - \mu) \psi_i^{\text{out}}
 \end{aligned}$$

$$\psi_{\pm}^{\text{in}}(r,t) = e^{r/2} \psi_s(s=\pm e^r, t)$$

$$\psi_{\pm}^{\text{out}}(r,t) = e^{r/2} \psi_v(v=\pm e^r, t)$$

$$\mathcal{L} = \int d\lambda \psi^\dagger(\lambda, t) \left(i\partial_t + \frac{1}{2}\partial_\lambda^2 + \frac{1}{2}\lambda^2 - \mu \right) \psi(\lambda, t)$$

$$\psi_s(s, t) = \frac{(2i)^{1/4}}{\sqrt{2\pi}} \int d\lambda e^{i\left(\frac{\lambda^2}{2} - \sqrt{2}s\lambda + \frac{s^2}{2}\right)} \psi(\lambda, t)$$

$$\psi_v(v, t) = \frac{(-2i)^{1/4}}{\sqrt{2\pi}} \int d\lambda e^{i\left(-\frac{\lambda^2}{2} + \sqrt{2}v\lambda - \frac{v^2}{2}\right)} \psi(\lambda, t)$$

$$\psi_s(s, t) = \int \frac{dv}{\sqrt{2\pi}} e^{-isv} \psi_v(v, t)$$

$$\mathcal{L} = \int d\lambda \quad \psi^\dagger(\lambda, t) \left(i\partial_t + \frac{1}{2}\partial_\lambda^2 + \frac{1}{2}\lambda^2 - \mu \right) \psi(\lambda, t)$$

$$\psi_s(s, t) = \frac{(2i)^{1/4}}{\sqrt{2\pi}} \int d\lambda \quad e^{i\left(\frac{\lambda^2}{2} - \sqrt{2}s\lambda + \frac{s^2}{2}\right)} \psi(\lambda, t)$$

$$\psi_v(v, t) = \frac{(-2i)^{1/4}}{\sqrt{2\pi}} \int d\lambda \quad e^{i\left(-\frac{\lambda^2}{2} + \sqrt{2}v\lambda - \frac{v^2}{2}\right)} \psi(\lambda, t)$$

$$\psi_s(s, t) = \int \frac{dv}{\sqrt{2\pi}} e^{-isv} \psi_v(v, t)$$

$$\begin{aligned}
 &= \int_{-\infty}^{\infty} ds \, \psi_s^+ (i\partial_t + \frac{i}{2}(s\partial_s + \partial_s s) - \mu) \psi_s = \int dr \sum_{i=\pm} \psi_i^{\text{in}} (i\partial_t + i\partial_r - \mu) \psi_i^{\text{in}} \\
 &= \int_{-\infty}^{\infty} dv \, \psi_v^+ (i\partial_t + \frac{i}{2}(v\partial_v + \partial_v v) - \mu) \psi_v = \int dr \sum_{i=\pm} \psi_i^{\text{out}} (i\partial_t + i\partial_r - \mu) \psi_i^{\text{out}}
 \end{aligned}$$

$$\psi_{\pm}^{\text{in}}(r,t) = e^{i\mu r} \psi_s(s=\pm e^r, t)$$

$$\psi_{\pm}^{\text{out}}(r,t) = e^{i\mu r} \psi_v(v=\pm e^r, t)$$

$s, v \in \mathbb{R}$

$$\begin{aligned}
 &= \int_{-\infty}^{\infty} ds \, \psi_s^+ (i\partial_t + \frac{i}{2}(s\partial_s + \partial_s s) - \mu) \psi_s = \int dr \sum_{i=\pm} \psi_i^{\text{in}} (i\partial_t + i\partial_r - \mu) \psi_i^{\text{in}} \\
 &= \int_{-\infty}^{\infty} dv \, \psi_v^+ (i\partial_t + \frac{i}{2}(v\partial_v + \partial_v v) - \mu) \psi_v = \int dr \sum_{i=\pm} \psi_i^{\text{out}} (i\partial_t + i\partial_r - \mu) \psi_i^{\text{out}}
 \end{aligned}$$

$$\psi_{\pm}^{\text{in}}(r,t) = e^{i\omega_2} \psi_s(s=\pm e^r, t)$$

$$\psi_{\pm}^{\text{out}}(r,t) = e^{i\omega_2} \psi_v(v=\pm e^r, t)$$

$s dv$

$$\begin{aligned}
 &= \int_{-\infty}^{\infty} ds \, \psi_s^+ (i\partial_t + \frac{i}{2}(s\partial_s + \partial_s s) - \mu) \psi_s = \int dr \sum_{i=\pm} \psi_i^{\text{in}} (i\partial_t + i\partial_r - \mu) \psi_i^{\text{in}} \\
 &= \int_{-\infty}^{\infty} dv \, \psi_v^+ (i\partial_t + \frac{i}{2}(v\partial_v + \partial_v v) - \mu) \psi_v = \int dr \sum_{i=\pm} \psi_i^{\text{out}} (i\partial_t + i\partial_r - \mu) \psi_i^{\text{out}}
 \end{aligned}$$

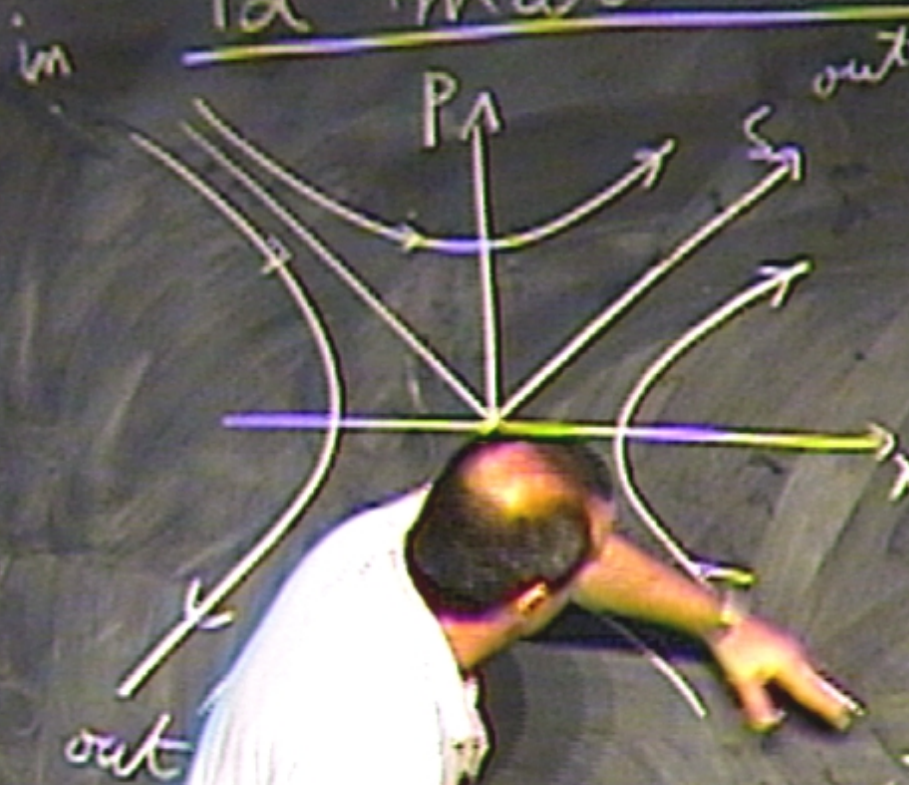
$$\psi_{\pm}^{\text{in}}(r,t) = e^{r/2} \psi_s(s=\pm e^r, t)$$

$$\psi_{\pm}^{\text{out}}(r,t) = e^{r/2} \psi_v(v=\pm e^r, t)$$

$s(z)_i$

$s(v)_i^{\text{out}}$

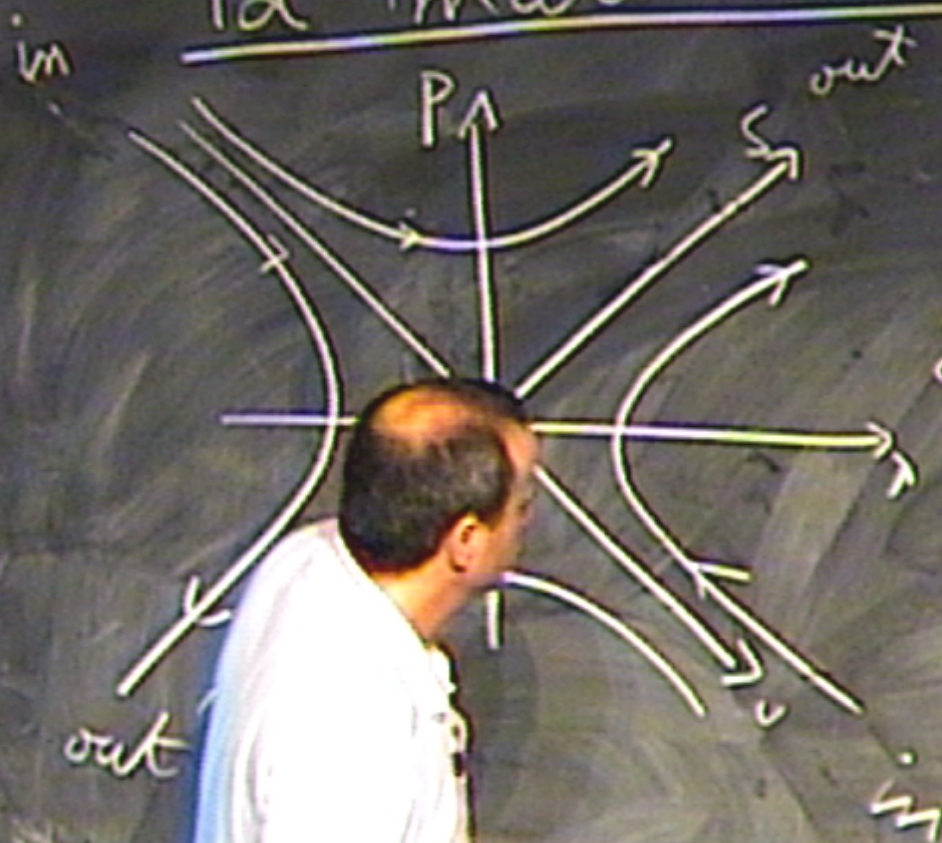
1d matrix model



incoming $|u| \gg 1$
 $s \approx 0$

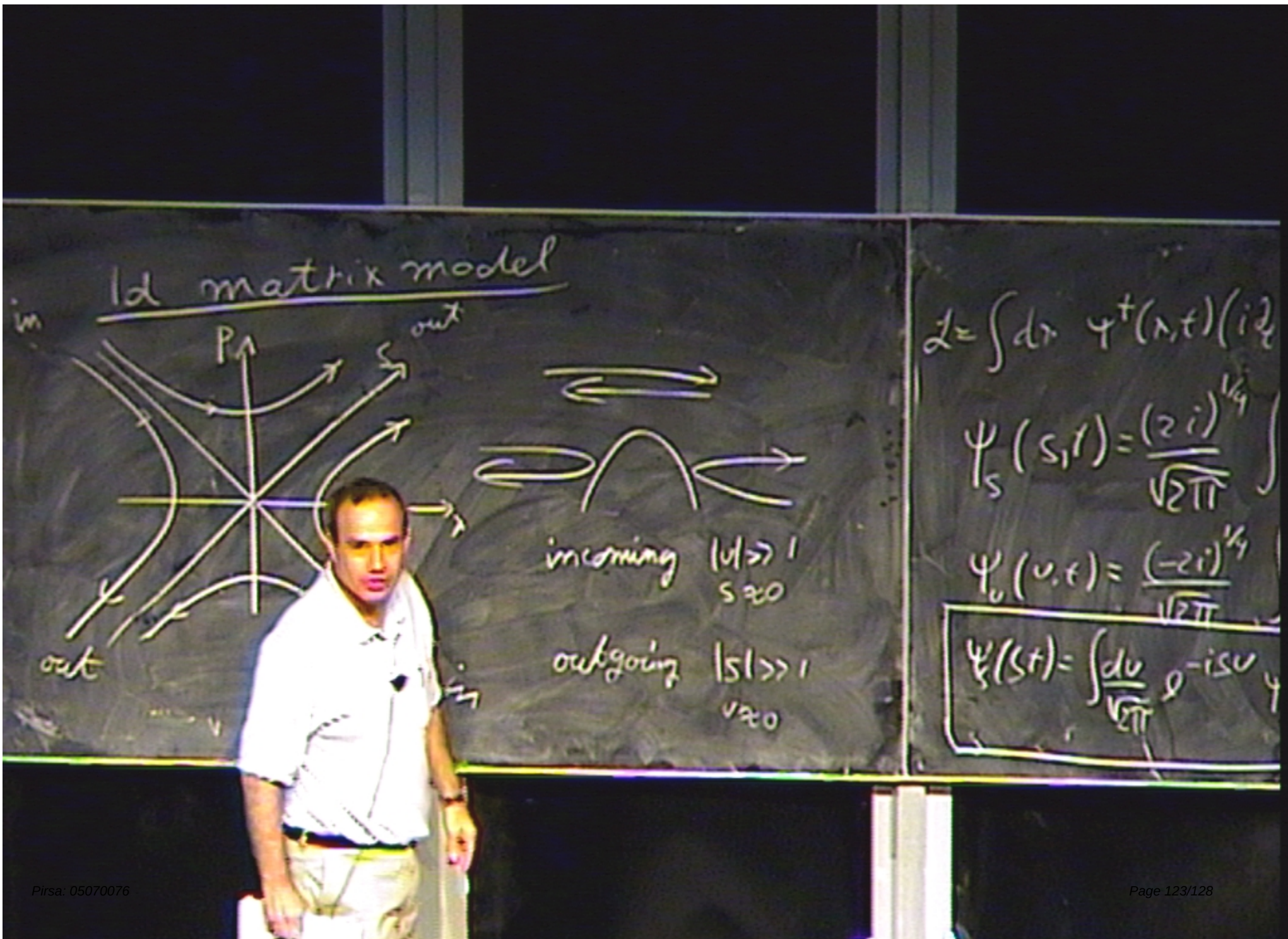
outgoing $|s| \gg 1$
 $v \approx 0$

1d matrix model

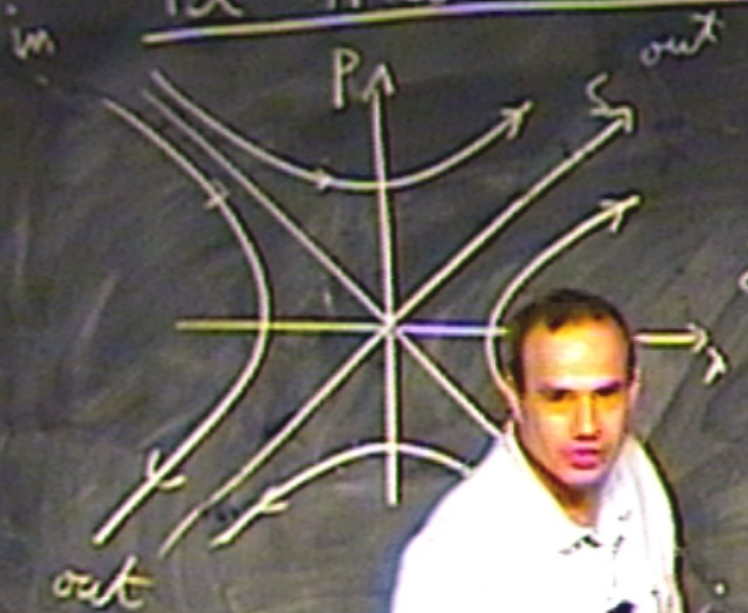


incoming $|v| \gg 1$
 $s \approx 0$

outgoing $|s| \gg 1$
 $v \approx 0$



1d matrix model



incoming $|v| \gg 1$
 $s \approx 0$

outgoing $|s| \gg 1$
 $v \approx 0$

$$\mathcal{L} = \int dt \psi^\dagger(x,t) (i \partial_t - H) \psi(x,t)$$

$$\psi_s(s,t) = \frac{(2i)^{1/4}}{\sqrt{2\pi}}$$

$$\psi_v(v,t) = \frac{(-2i)^{1/4}}{\sqrt{2\pi}}$$

$$\psi(s,t) = \int \frac{dv}{\sqrt{2\pi}} e^{-isv} \psi(v,t)$$

$$\begin{aligned}
 &= \int_{-\infty}^{\infty} ds \, \psi_s^\dagger \left(i\partial_t - \frac{i}{2}(\partial_s^2 + \partial_s^2) - \mu \right) \psi_s = \int dr \sum_{i=\pm} \psi_i^{\text{in}} (i\partial_t - i\partial_r - \mu) \psi_i^{\text{in}} \\
 &= \int_{-\infty}^{\infty} du \, \psi_u^\dagger \left(i\partial_t + \frac{i}{2}(\partial_u^2 + \partial_u^2) - \mu \right) \psi_u = \int dr \sum_{i=\pm} \psi_i^{\text{out}} (i\partial_t + i\partial_r - \mu) \psi_i^{\text{out}}
 \end{aligned}$$

$$\psi_{\pm}^{\text{in}}(r,t) = e^{r/2} \psi_s(s=\pm e^r, t)$$

$$\psi_{\pm}^{\text{out}}(r,t) = e^{r/2} \psi_u(u=\pm e^r, t)$$

$$\mathcal{J}^{\text{in}} \sim (\partial_t + \partial_r) \Phi$$

$s(u|z)_{\text{in}}$

$s(u|z)_{\text{out}}$

$$\begin{aligned}
 &= \int_{-\infty}^{\infty} ds \, \psi_s^{\dagger} \left(i\partial_t + \frac{i}{2}(s\partial_s + \partial_s s) - \mu \right) \psi_s = \int dr \sum_{i=\pm} \psi_i^{\dagger} (i\partial_t - i\partial_r - \mu) \psi_i^{\text{in}} \\
 &= \int_{-\infty}^{\infty} du \, \psi_u^{\dagger} \left(i\partial_t + \frac{i}{2}(u\partial_u + \partial_u u) - \mu \right) \psi_u = \int dr \sum_{i=\pm} \psi_i^{\text{out}} (i\partial_t + i\partial_r - \mu) \psi_i^{\text{out}}
 \end{aligned}$$

$$\psi_{\pm}^{\text{in}}(r,t) = e^{r/2} \psi_s(s=\pm e^r, t)$$

$$\psi_{\pm}^{\text{out}}(r,t) = e^{r/2} \psi_u(u=\pm e^r, t)$$

$$\begin{aligned}
 &S \psi(z)_{\text{in}} \quad S \psi(z)_{\text{out}} \\
 &J^{\text{in}} \sim (\partial_t + \partial_r) \Phi^{\text{in}} \sim \partial_r \psi \\
 &J^{\text{out}} \sim (\partial_t - \partial_r) \Phi^{\text{out}} \sim \partial_r \psi
 \end{aligned}$$

1d matrix model

$(-)^{F_c}$ charge conjugation



incoming $|u\rangle \gg 1$
 $s \gg 0$

outgoing $|s\rangle \gg 1$
 $v \gg 0$

1d matrix model

$(-)^{F_L}$ charge conjugation

$$(\lambda, \nu, s) \rightarrow (-\lambda, -\nu, -s)$$

$(-)^{F_L}$ S-duality

$$\psi_{\pm}^{\text{in}} \rightarrow \psi_{\mp}^{\text{in}}$$

$$\psi_{\pm}^{\text{out}} \rightarrow \psi_{\mp}^{\text{out}}$$



incoming $|\nu| \gg 1$
 $s \approx 0$

outgoing $|s| \gg 1$
 $\nu \approx 0$

1d matrix model

$(-)^{F_L}$ charge conjugation

$$(\lambda, \nu, S) \rightarrow (-\lambda, -\nu, -S)$$

$(-)^{F_L}$ S-duality

$$\begin{aligned} \psi_{\pm}^{\text{in}} &\rightarrow \psi_{\mp}^{\text{in}} \\ \psi_{\pm}^{\text{out}} &\rightarrow \psi_{\mp}^{\text{out}} \end{aligned}$$



incoming $|\nu| \gg 1$
 $S \approx 0$

outgoing $|S| \gg 1$
 $\nu \approx 0$