

Title: Matrix Models and Noncritical Strings

Date: Jul 05, 2005 11:30 AM

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Abstract:

$$L_{inc} = \frac{1}{4}$$

$$\mathcal{L}_{\text{NS}} = \frac{1}{4\pi\alpha'} \left[(\partial\psi)^2 + (\partial\chi)^2 + 2\sqrt{\alpha'} \phi R \right]$$

$$\mathcal{L}_{\text{ms}} = \frac{1}{4\pi\alpha'} \left[(\partial\phi)^2 + (\partial X)^2 + 2\sqrt{\alpha'} \phi R \right]$$

$$T = -\frac{1}{\alpha'} \left[(\partial\phi)^2 + (\partial X)^2 + Q\partial^2\phi \right]$$

$$C=26 = 1 + 1 + \cancel{6\alpha'} Q^2 = 26 \quad Q = \frac{2}{\sqrt{\alpha'}}$$

$$\alpha' = \frac{1}{2}$$

$$\mathcal{L}_{\text{ms}} = \frac{1}{4\pi\alpha'} \left[(\partial\phi)^2 + (\partial X)^2 + 2\sqrt{\alpha'} \phi R \right]$$

$$T = -\frac{1}{\alpha'} \left[(\partial\phi)^2 + (\partial X)^2 + Q\partial^2\phi \right]$$

$$C = 26 = 1 + 1 + \frac{1}{6} Q^2 = 26 \quad Q = \frac{2}{\sqrt{\alpha'}}$$

$$\Delta(e^{i p \cdot X}) = \frac{1}{2} p^2 \quad \Delta(e^{\alpha\phi}) = -\frac{1}{2} \alpha(\alpha - Q)$$

$$\mathcal{L}_{\text{NLS}} = \frac{1}{4\pi\alpha'} \left[(\partial\phi)^2 + (\partial X)^2 + 2\sqrt{\alpha'} \phi R \right]$$

$$T = -\frac{1}{\alpha'} \left[(\partial\phi)^2 + (\partial X)^2 + Q\partial^2\phi \right]$$

Dilaton = $Q\phi$

$$C = 26 = 1 + 1 + \cancel{16} Q^2 = 26 \quad Q = \frac{2}{\sqrt{\alpha'}}$$

$$\Delta(e^{ipX}) = \frac{1}{2} p^2 \quad \Delta(e^{i\alpha\phi}) = -\frac{1}{2} \alpha(\alpha - Q)$$

Weak
Coupling

ϕ
Strong
Coupling

$$\mathcal{L}_{\text{NLS}} = \frac{1}{4\pi\alpha'} \left[(\partial\phi)^2 + (\partial X)^2 + 2\sqrt{\alpha'} \phi R \right]$$

$$T = -\frac{1}{2} \left[(\partial\phi)^2 + (\partial X)^2 + Q \partial^2 \phi \right]$$

$$\text{Dilaton} = Q\phi$$

$$g_s = e^{\frac{Q}{2}\phi}$$

$$26 = 1 + 1 + \cancel{6\alpha'} Q^2 = 26 \quad Q = \frac{2}{\sqrt{\alpha'}}$$

$$\alpha' = \frac{1}{2}$$

$$= \frac{1}{2} \text{Pl}^2$$

$$\Delta(e^{\alpha\phi}) = -\frac{1}{2}\alpha(\alpha - Q)$$

$$\mathcal{L}_{\text{NS}} = \frac{1}{4\pi\alpha'} \left[(\partial\phi)^2 + (\partial x)^2 + 2\sqrt{\alpha'} \phi R \right]$$

$$T = -\frac{1}{\alpha'} \left[(\partial\phi)^2 + (\partial x)^2 + Q \partial^2 \phi \right]$$

$$\text{Dilaton} = Q\phi$$

$$g_s = e^{\frac{Q}{2}\phi}$$

$$C = 26 = 1 + 1 + 6\alpha' Q^2 = 26 \quad Q = \frac{2}{\sqrt{\alpha'}}$$

$$\alpha' = \frac{1}{2} \quad \Delta(e^{i p x}) = \frac{1}{2} p^2 \quad \Delta(e^{\alpha\phi}) = -\frac{1}{2} \alpha(\alpha - Q)$$

Graviton, $B_{\mu\nu}$, D

Graviton, $B_{\mu\nu}$, D

$$\begin{array}{ccccccc} & -1 & & 0 & & 1 & = 0 \\ & \frac{d(d+1)}{2} & -2d & & & & \end{array}$$

Graviton, $B_{\mu\nu}$, D

$$\begin{array}{cccc} -1 & 0 & 1 & = 0 \\ \frac{d(d+1)}{2} - 2d & & & \end{array}$$

'Tachyon' massless scalar

ℓ

Graviton, $B_{\mu\nu}$, D

$$-1 \quad 0 \quad 1 = 0$$

$$\frac{d(d+1)}{2} - 2d$$

'Tachyon' massless scalar
 $z=1$

$$\left(\frac{1}{2} \right) \psi$$

$$\left(\frac{1}{2} \right) \psi$$

$$\left(\frac{1}{2} \right) \psi$$

Graviton, $B_{\mu\nu}$, D

$$\begin{array}{cccc} -1 & 0 & 1 & = 0 \\ \frac{d(d-1)}{2} - 2d & & & \end{array}$$

'Tachyon' massless scalar
 $z = ix$

$$e^{-iE(t-\varphi)} e^{\frac{\omega}{2}\psi}$$

$$E > 0$$



$$e^{-iE(t+\varphi)} e^{\frac{\omega}{2}\psi}$$

Graviton, $B_{\mu\nu}$, D

$$-1 \quad 0 \quad 1 = 0$$

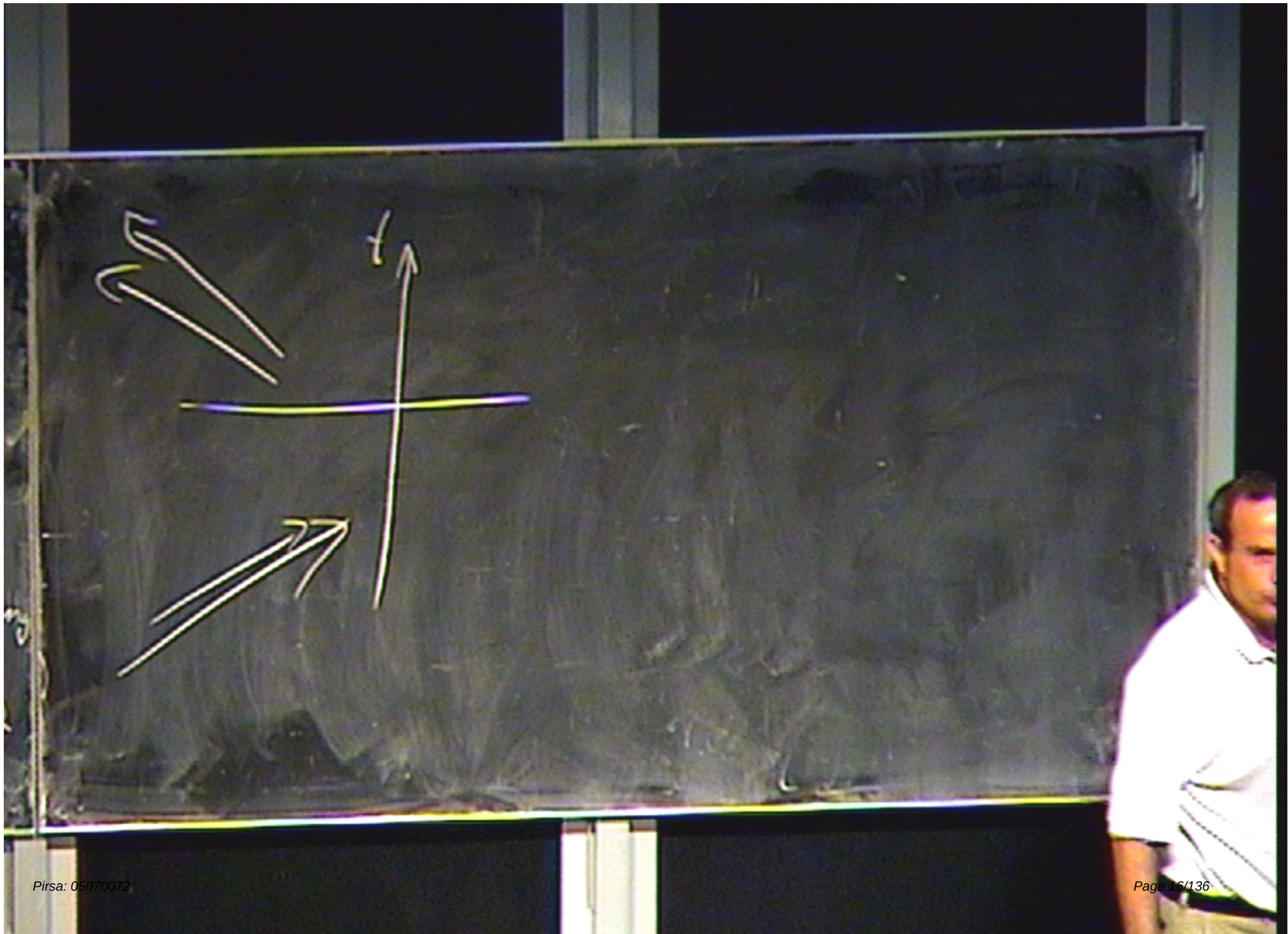
$$\frac{d(d-1)}{2} - 2d$$

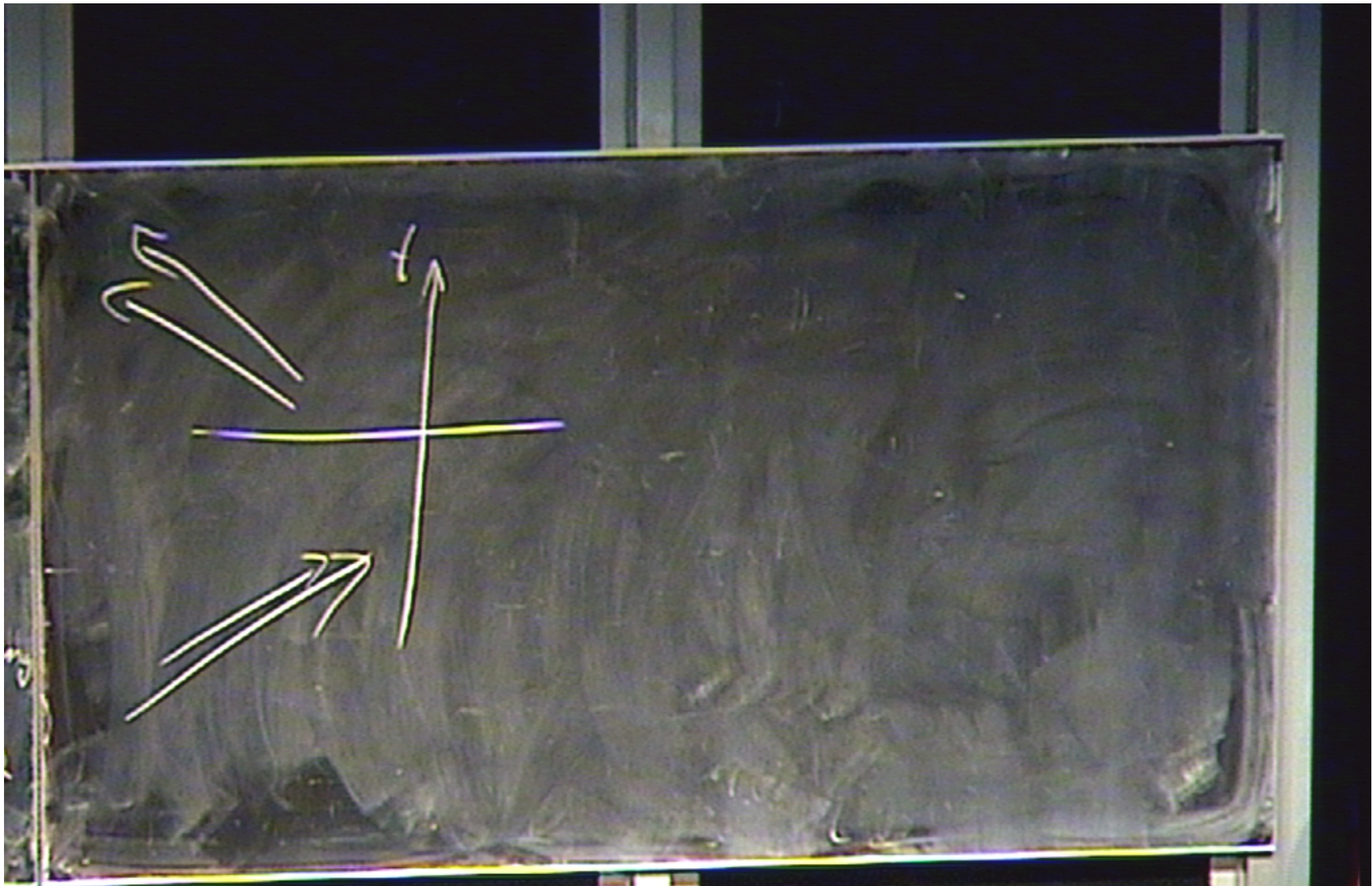
'Tachyon' massless scalar
 $z = ix$

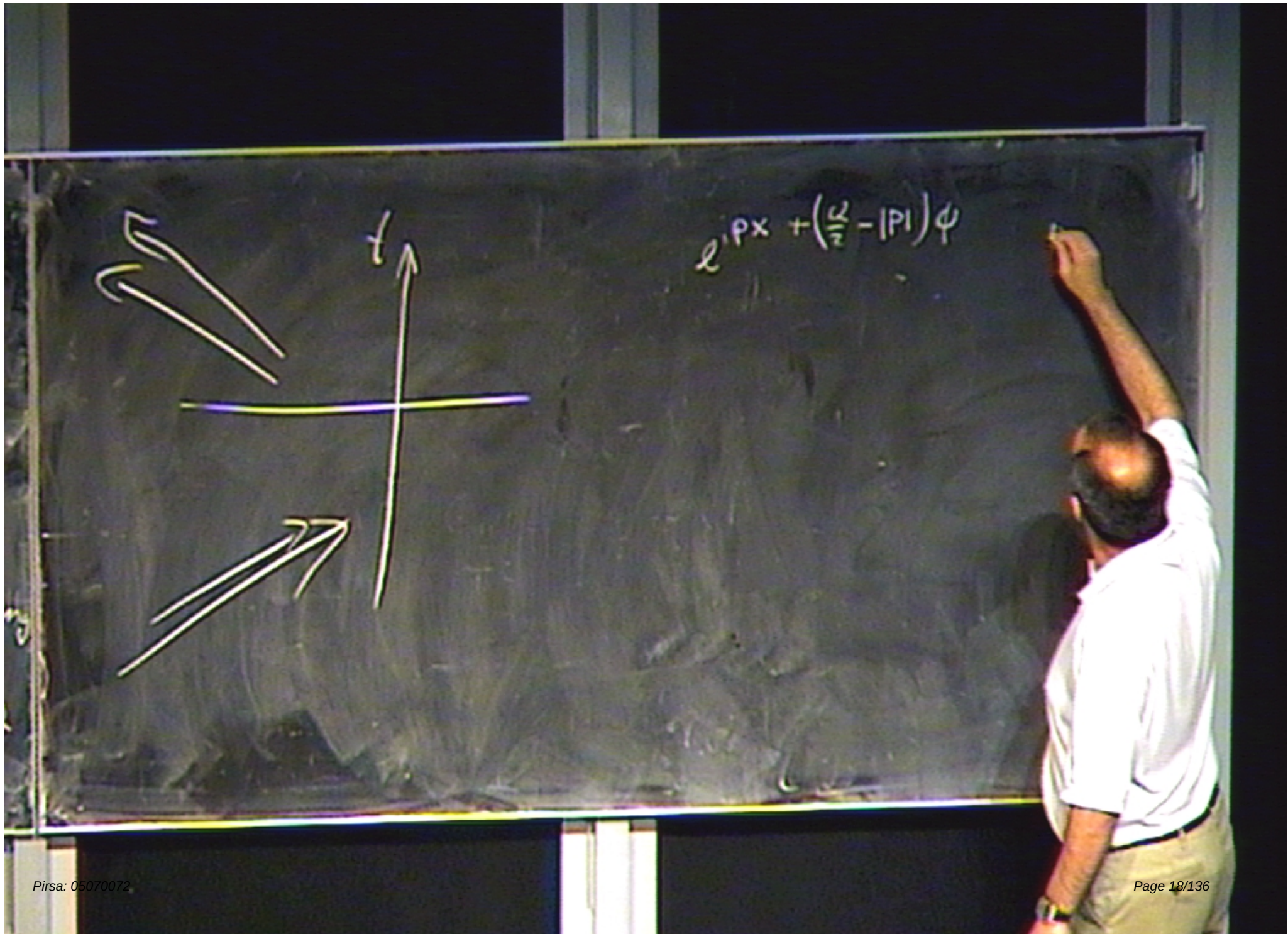
$$e^{-iE(t-\varphi)} e^{\frac{\omega}{2}\varphi}$$

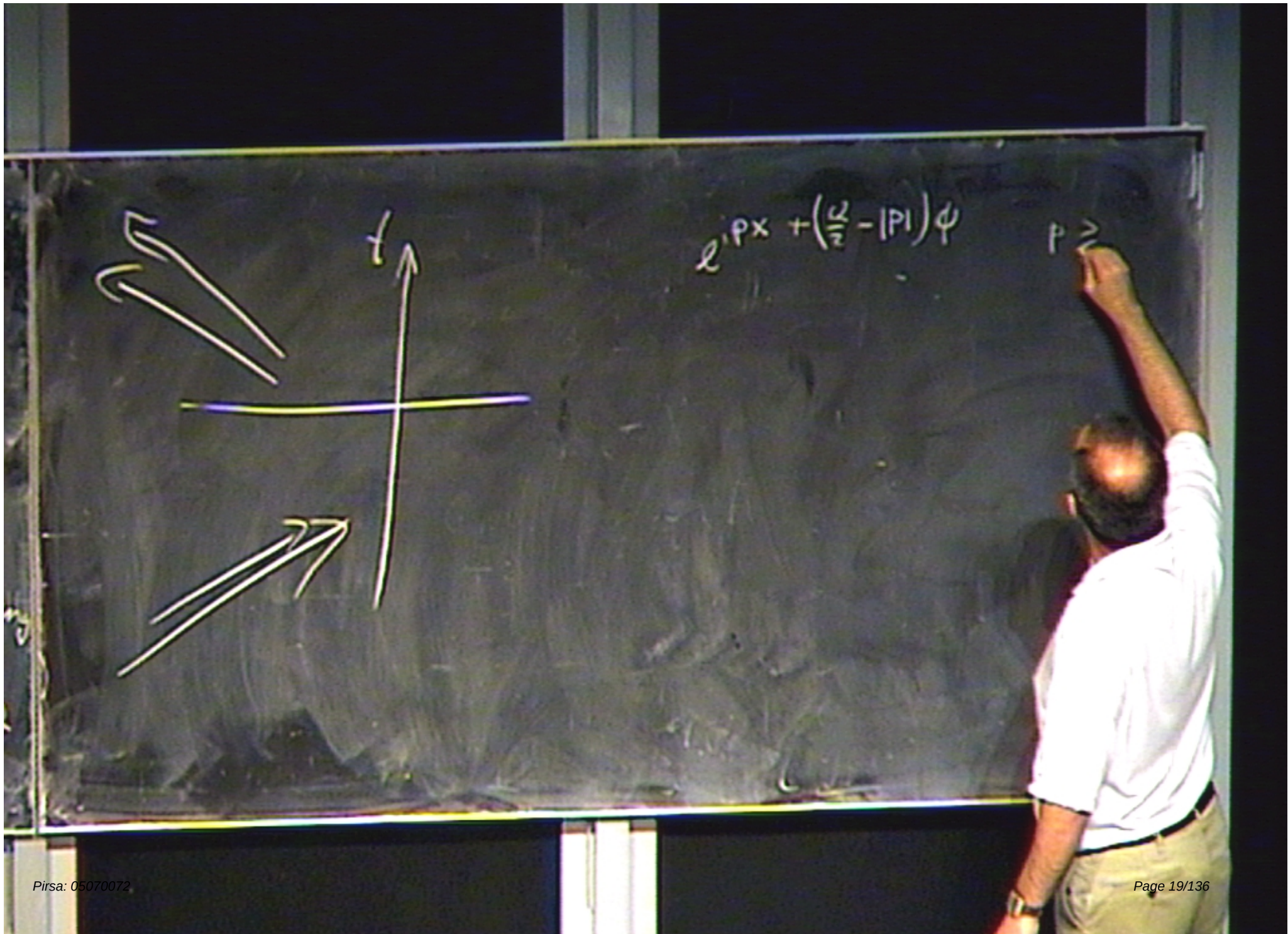
$E > 0 \leftarrow$ incoming / right moving
 outgoing / left moving

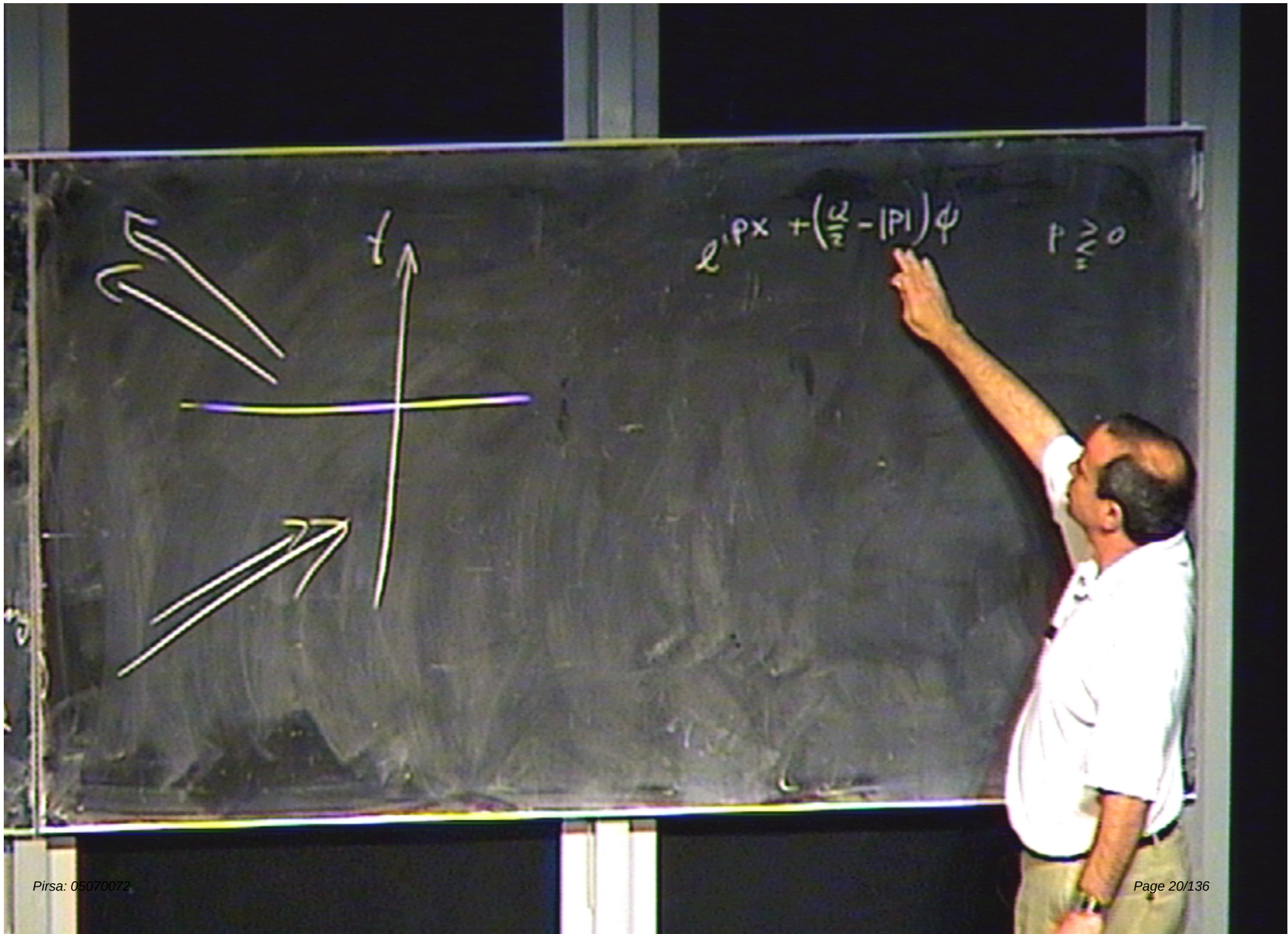
$$e^{-iE(t+\varphi)} e^{\frac{\omega}{2}\varphi}$$

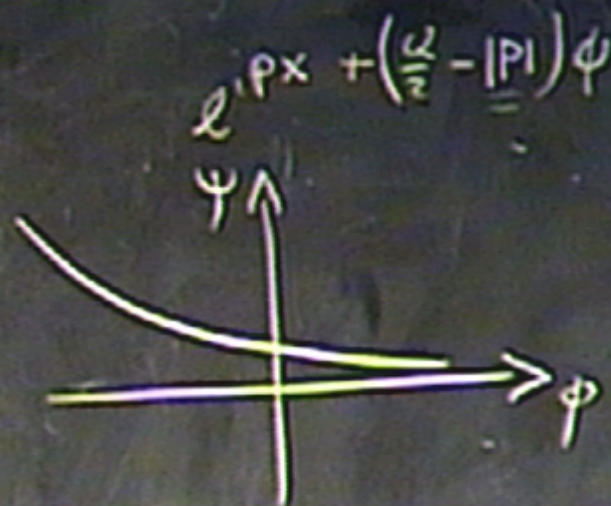
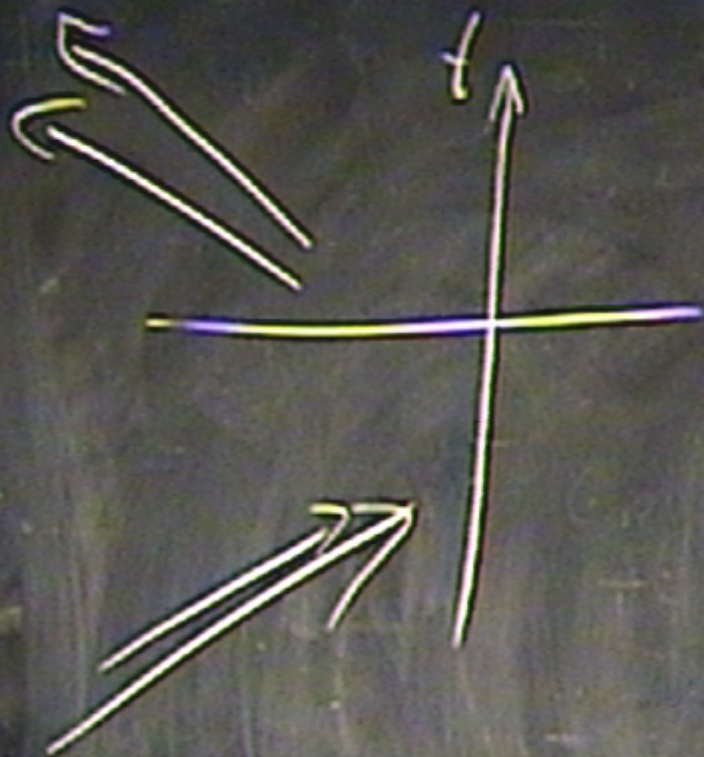






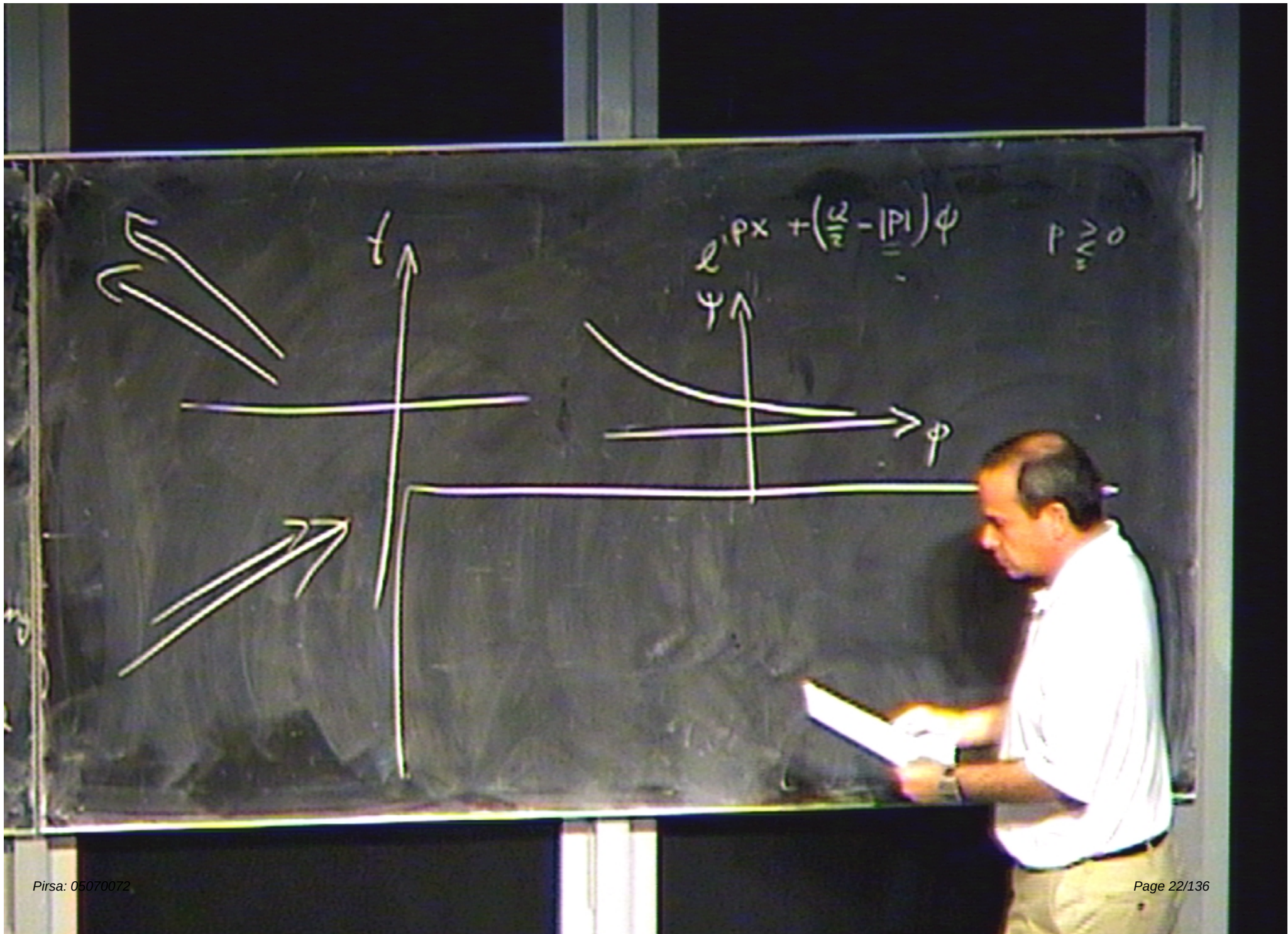


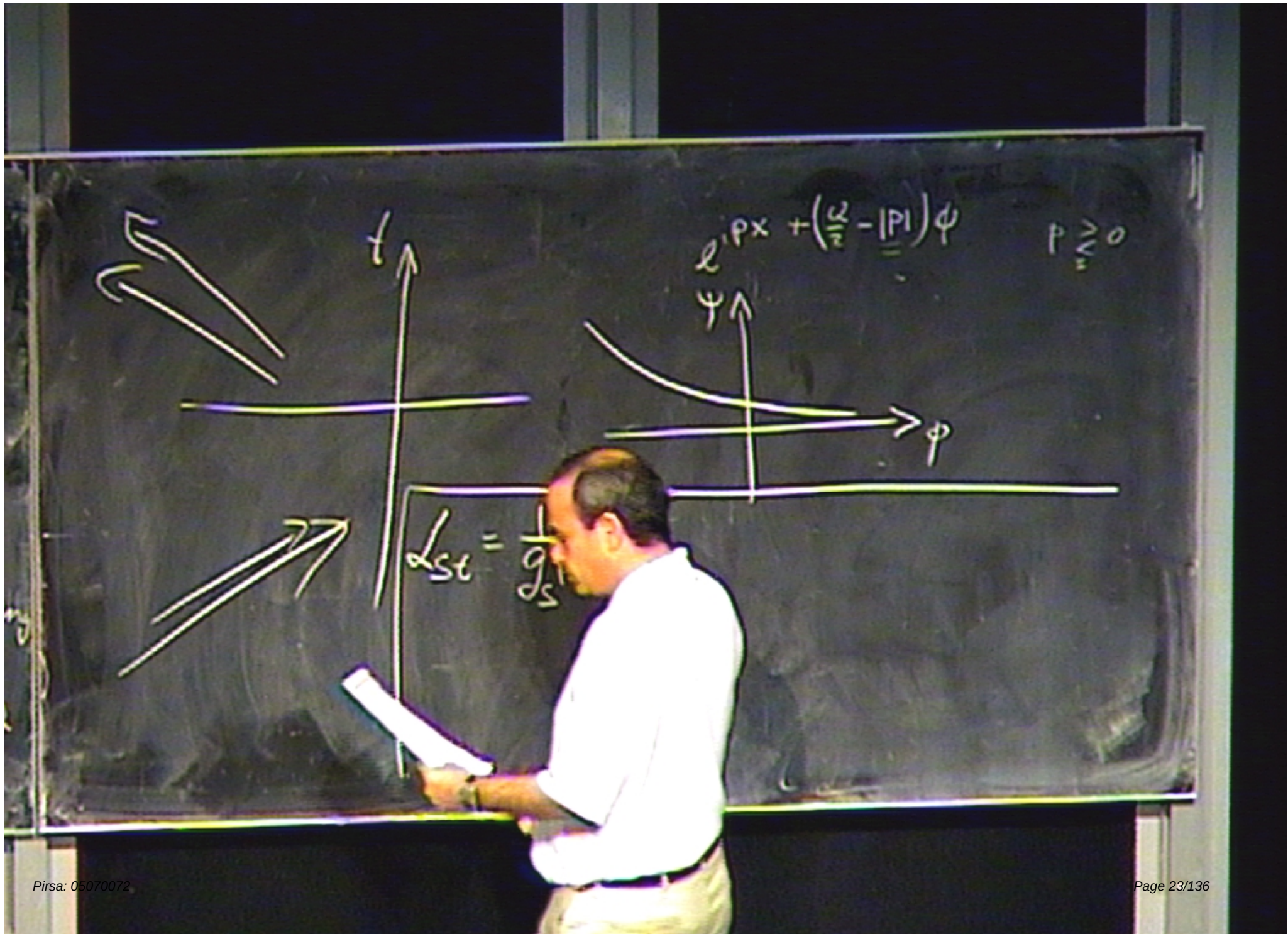


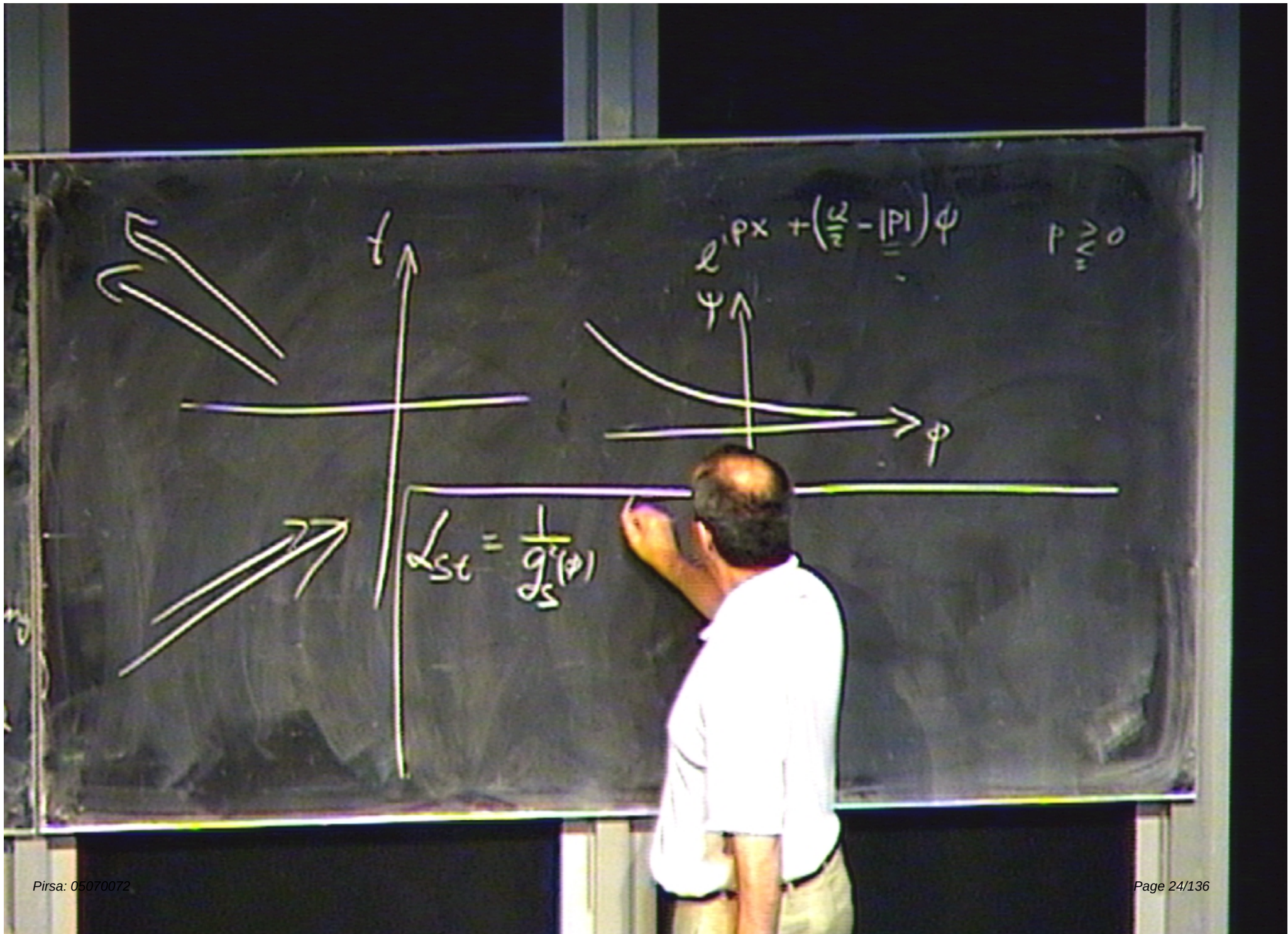


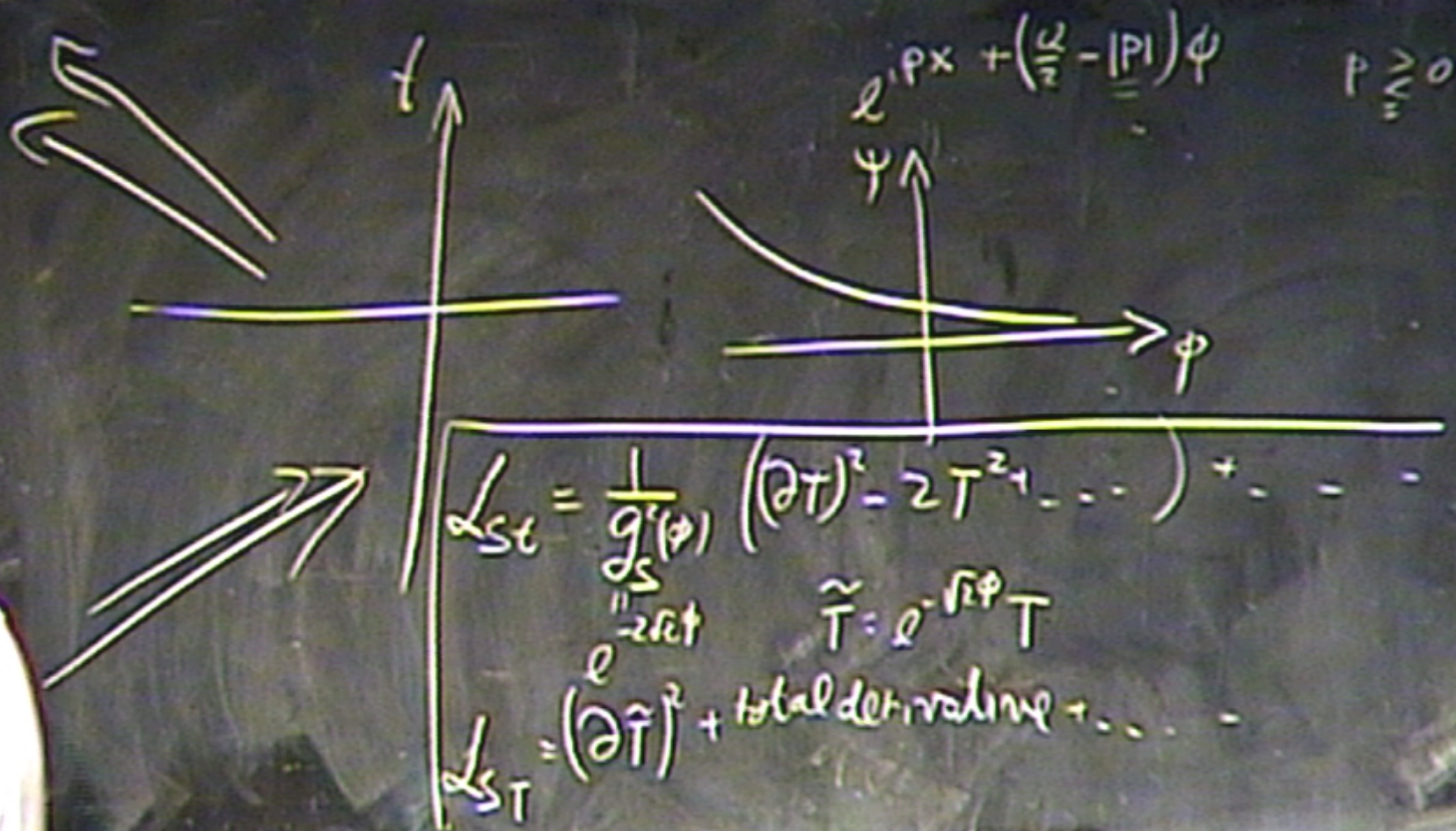
$$e^{ipx} + \left(\frac{\omega}{2} - |p|\right)\psi$$

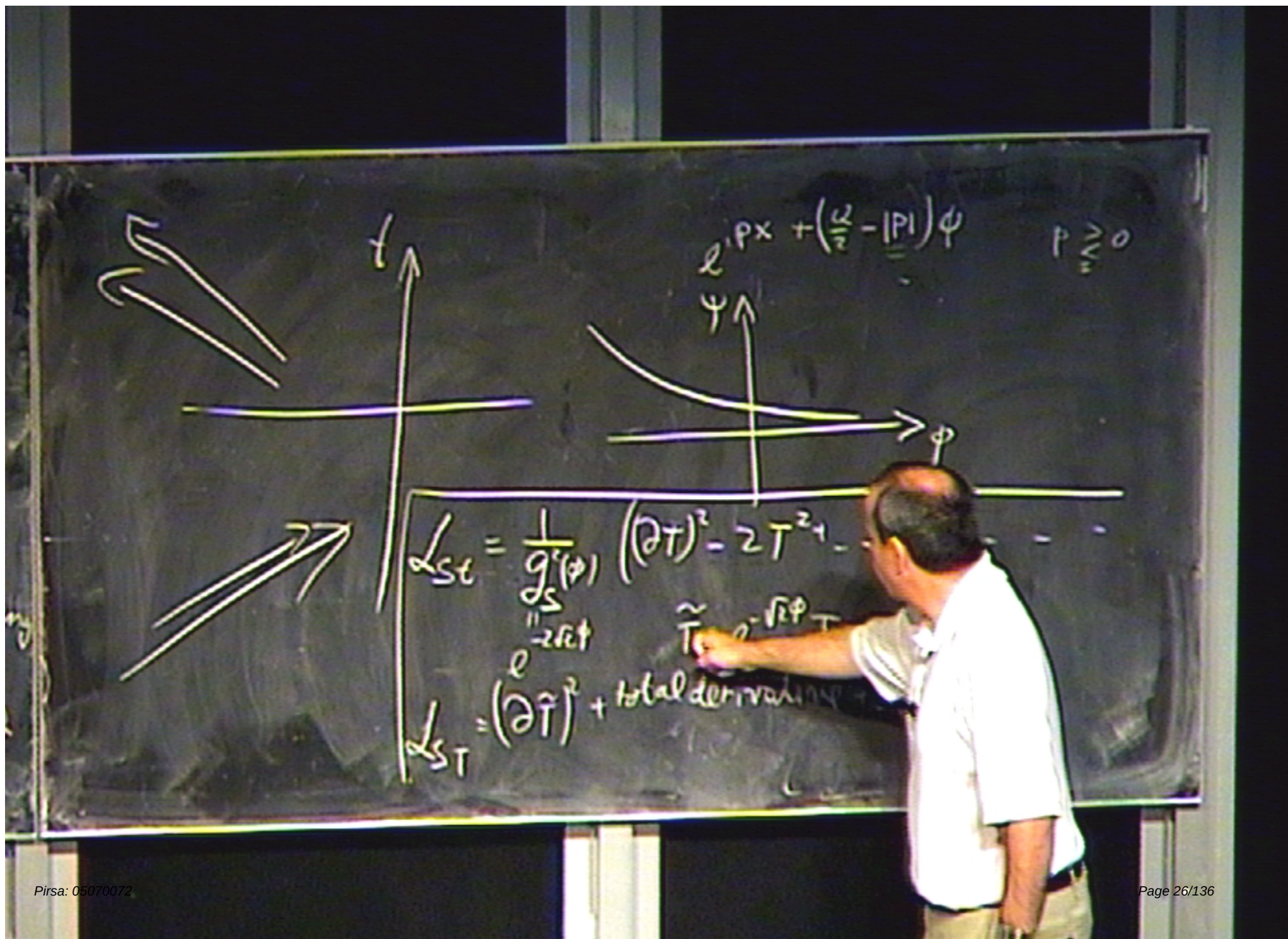
$$p \geq 0$$

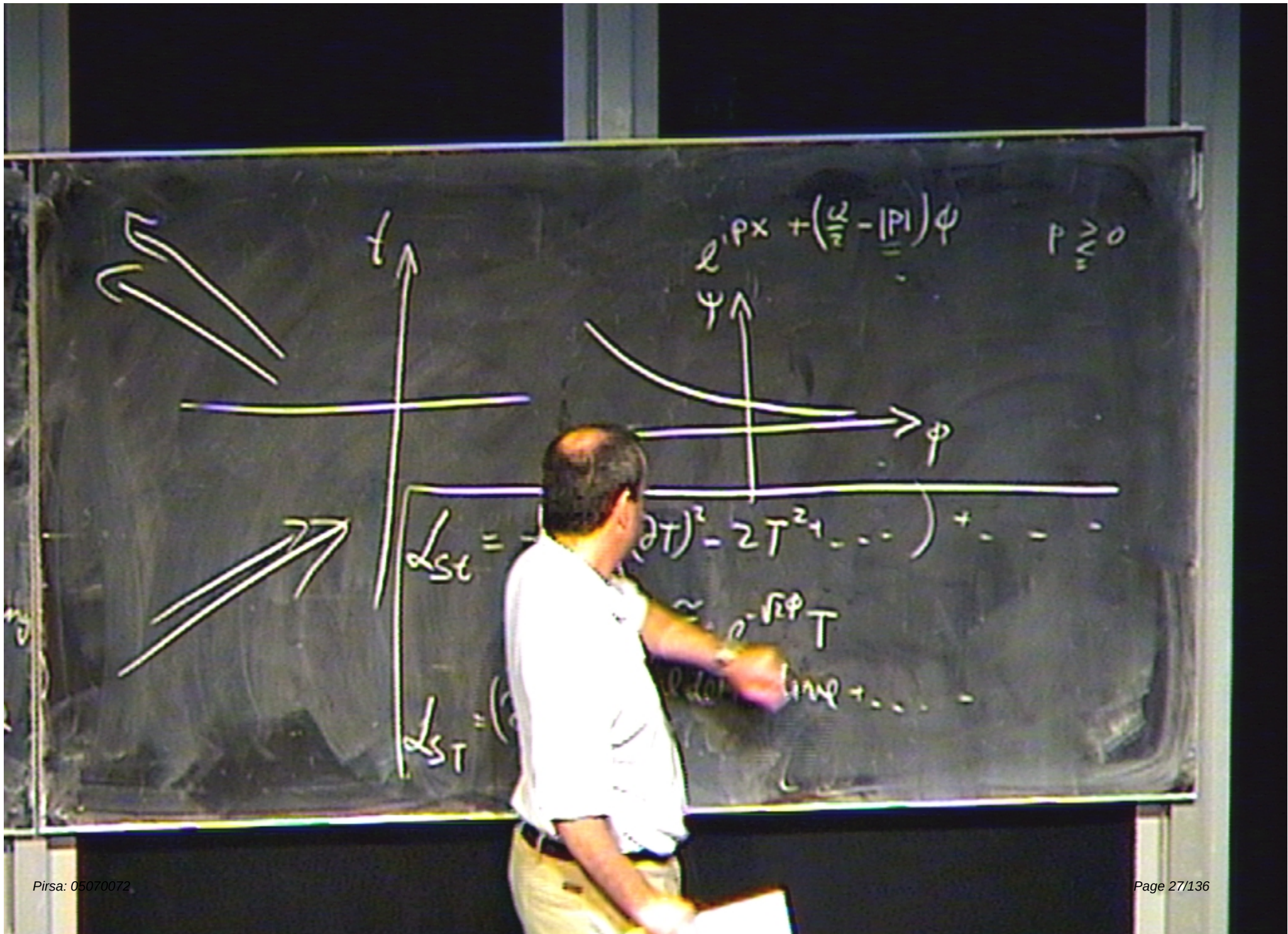












$$[(\partial\phi)^2 + (\partial x)^2 + 2\sqrt{\alpha'} \phi R]$$

$$[(\partial\phi)^2 + (\partial x)^2 + Q\partial^2\phi]$$

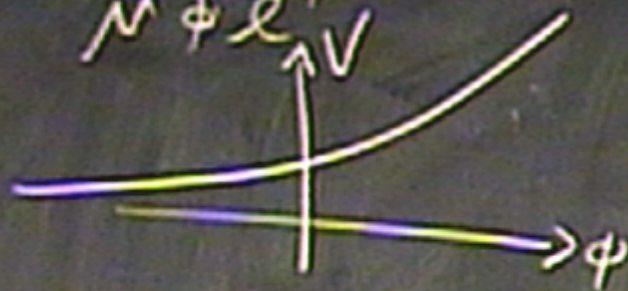
$$\text{Dilaton} = \alpha\phi$$

$$g_s = e^{\frac{\alpha}{2}\phi}$$

$$1 + \alpha' Q^2 = 26 \quad Q = \frac{2}{\sqrt{\alpha'}}$$

$$\Delta(e^{\alpha\phi}) = -\frac{1}{2}\alpha(\alpha - Q)$$

$$\langle T \rangle = \frac{N Q^{\phi}}{N \phi e^{\psi}} V$$



$$\langle T \rangle = \mu Q^\phi$$

$$N \phi e^{\psi}$$

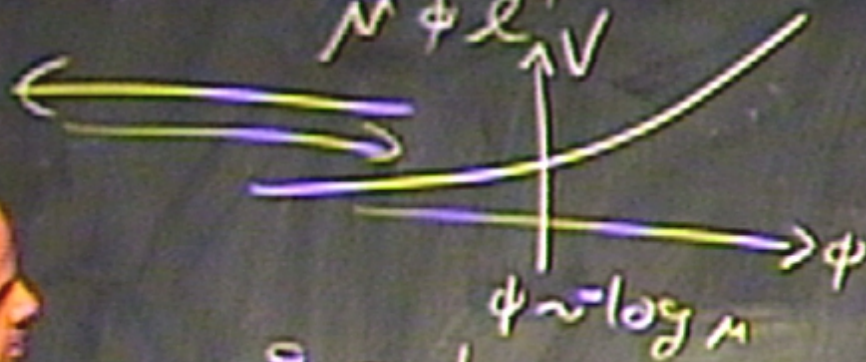
$$V$$

$$\phi \sim -\log \mu$$

$$g_s \sim \frac{1}{\mu}$$

$$\langle T \rangle = M Q^\phi$$

$$M \phi e^{\psi} V$$



$$g_s \sim \frac{1}{M}$$

$$\langle T \rangle = M Q^\phi$$

$$M \phi Q^\psi$$

$\uparrow V$

$$\phi \sim -\log M$$

$$g_S \sim \frac{1}{M}$$

Type 0

$$\langle T \rangle = M Q^\phi$$

$$M \phi e^{\psi}$$

$\psi \sim \log \mu$

$g_s \sim \frac{1}{\mu}$

Type 0

$\xi_L, (\xi_R)$ left, (right)

World sheet fermion #

NSNS $(-)^{\xi_L + \xi_R} = 1$

RR: $-1^{\xi_L + \xi_R} = \begin{cases} 1 & \text{OB} \\ -1 & \text{OA} \end{cases}$

OR

OR

NS "tachyon"

OR
NS "tachyon"

$$e^{-\frac{1}{2\alpha'} \int d\tau \left(\dot{\vec{\varphi}} - \vec{\varphi}' + i p X + (1 - |P|^2) \varphi \right)}$$

OR

NS "tachyon" T $e^{-\int d\tau \left(\dot{\phi} - \overline{\psi} + i p X + (1 - |p|) \phi \right)}$



RR scalar c :

OR

NS "tachyon" T $e^{-\frac{\phi}{2} - \bar{\phi} + i p X + (1 - |p|) \phi}$



RR scalar c : $e^{-\frac{\phi}{2} - \bar{\phi}} (H + \bar{H}) + i p X + (1 - |p|) \phi$

OR

NS "tachyon" T $e^{-\overset{\text{warphi}}{\downarrow} \dot{\phi} - \bar{\varphi} + i p X + \left(\frac{1}{2} - |P|\right) \phi}$



RR scalar c : $e^{-\frac{\phi}{2} - \frac{\bar{\phi}}{2} + \frac{i}{2}(H + \bar{H}) \cdot p X + (1 - |P|) \phi}$
 $e^{-\frac{\phi}{2} - \frac{\bar{\phi}}{2} - \frac{i}{2}(H + \bar{H}) \cdot p X + (1 - |P|) \phi}$

OR

NS "tachyon" T $\xrightarrow{\text{warp}} e^{-\frac{1}{2}\phi - \frac{1}{2}\bar{\phi} + i p X + (1-|p|)\phi}$



RR scalar c $\partial_+ c \sim e^{-\frac{1}{2}\phi - \frac{1}{2}\bar{\phi} + \frac{i}{2}(H+\bar{H}) + pX + (1-|p|)\phi}$
 $\partial_- c \sim e^{-\frac{1}{2}\phi - \frac{1}{2}\bar{\phi} - \frac{i}{2}(H+\bar{H}) + pX + (1-|p|)\phi}$

OR

transform

NS "tachyon" $T \quad e^{-\frac{1}{2}(\dot{\phi} - \bar{\phi} + i p_X + (1-|p|)\phi)}$



$R \quad C \quad \sim e^{-\frac{\phi}{2} - \frac{\bar{\phi}}{2} + \frac{i}{2}(H + \bar{H}) + p_X + (1-|p|)\phi}$

$\partial C \sim e^{-\frac{\phi}{2} - \frac{\bar{\phi}}{2} - \frac{i}{2}(H + \bar{H}) + p_X + (1-|H|)\phi}$

OR

NS "tachyon" T $\xrightarrow{\text{map to}} e^{-\frac{1}{2}(\phi - \bar{\phi}) + i p X + (1 - |p|)\phi}$



RR scalar c $\partial_+ c \sim e^{-\frac{1}{2}(\phi - \bar{\phi}) + \frac{i}{2}(H + \bar{H}) + pX + (1 - |p|)\phi}$
 $\partial_- c \sim e^{-\frac{1}{2}(\phi - \bar{\phi}) - \frac{i}{2}(H + \bar{H}) + pX + (1 - |p|)\phi}$

$$c = \mathcal{V}\phi + \tilde{\mathcal{V}}\tau$$

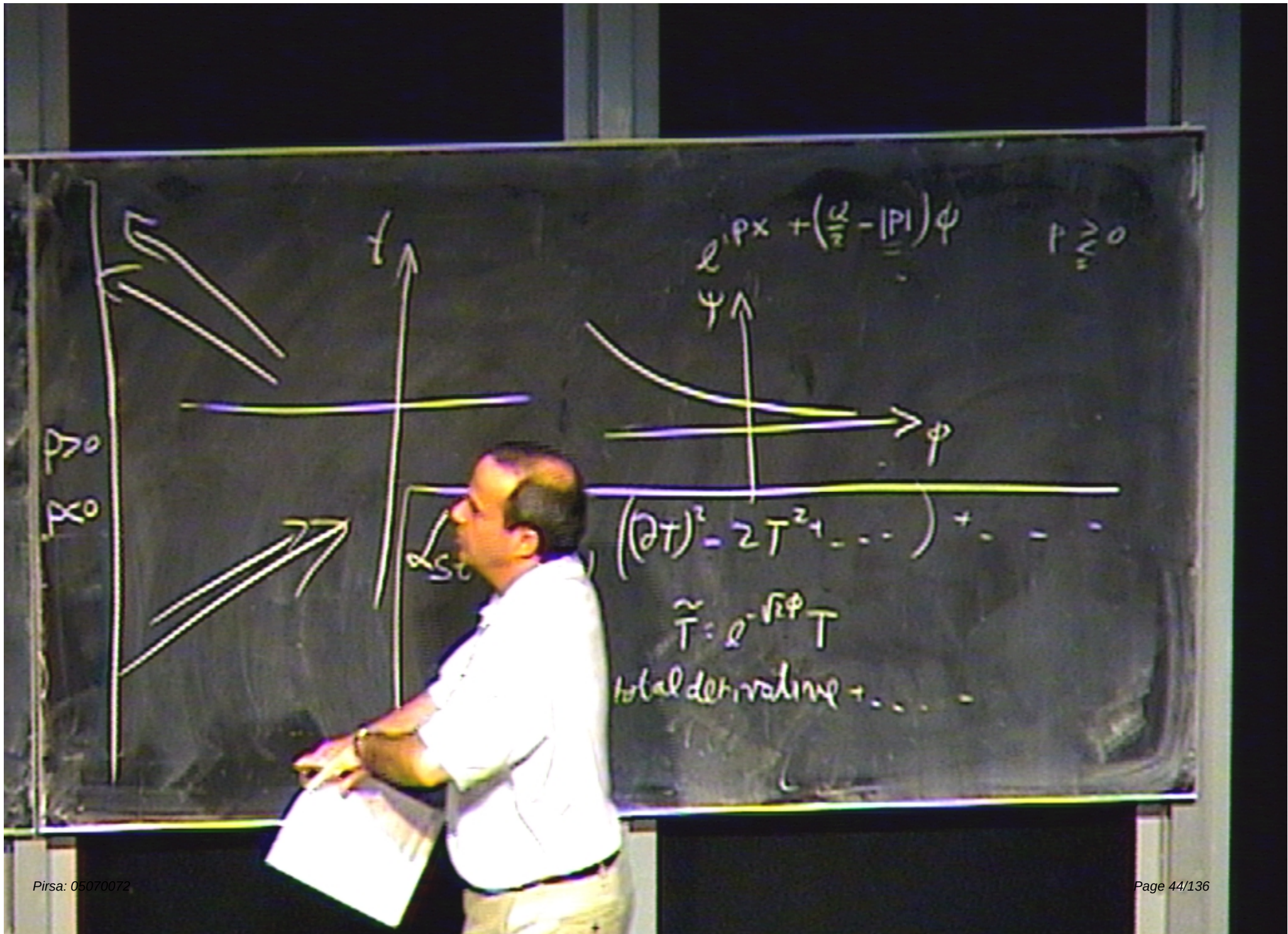
OR

NS "tachyon" T $\xrightarrow{\text{warp}} e^{-\frac{1}{2}(\phi - \bar{\phi}) + i p X + (1 - |p|)\phi}$



RR scalar C $\partial_+ C \sim e^{-\frac{1}{2}(\phi - \bar{\phi}) + \frac{i}{2}(H + \bar{H}) + p X + (1 - |p|)\phi}$
 $\partial_- C \sim e^{-\frac{1}{2}(\phi - \bar{\phi}) - \frac{i}{2}(H + \bar{H}) + p X + (1 - |p|)\phi}$

$$C = \mathcal{V}\phi + \tilde{\mathcal{V}}\tau = \mathcal{V}_{in}(\tau - \phi) + \mathcal{V}_{out}(\tau + \phi)$$



$p > 0$

$p < 0$

t

x_{st}

$$e^{ipx} + \left(\frac{\alpha}{2} - |p|\right)\psi$$

$$p \geq 0$$

ψ

ϕ

$$(\partial T)^2 - 2T^2 + \dots$$

$$\tilde{T} \sim e^{-\sqrt{2}\phi} T$$

total derivative + ...

OR

NS "tachyon" T $e^{-\phi - \bar{\phi} + i p X + (1 - |p|) \phi}$



RR scal $\partial_+ C \sim e^{-\frac{\phi}{2} - \frac{\bar{\phi}}{2} + \frac{i}{2}(H + \bar{H}) + p X + (1 - |p|) \phi}$

$\partial_- C \sim e^{-\frac{\phi}{2} - \frac{\bar{\phi}}{2} - \frac{i}{2}(H + \bar{H}) + p X + (1 - |p|) \phi}$

$C = \mathcal{V}_{in}(t - \phi) + \mathcal{V}_{out}(t + \phi)$

OR

NS "tachyon" T $e^{-\overset{\text{warp factor}}{\phi} - \bar{\phi} + i p X + \left(\frac{1}{2} - |P|\right) \phi}$



scalar c $\begin{aligned} \partial_+ c &\sim e^{-\frac{\phi}{2} - \frac{\bar{\phi}}{2} + \frac{i}{2}(H + \bar{H}) + pX + (1 - |P|)\phi} \\ \partial_- c &\sim e^{-\frac{\phi}{2} - \frac{\bar{\phi}}{2} - \frac{i}{2}(H + \bar{H}) + pX + (1 - |P|)\phi} \end{aligned}$

$= \underset{\substack{\uparrow \\ \text{RR flux}}}{\mathcal{V}} \phi + \tilde{\mathcal{V}} \tau = \mathcal{V}_{in}(\tau - \phi) + \mathcal{V}_{out}(\tau + \phi)$

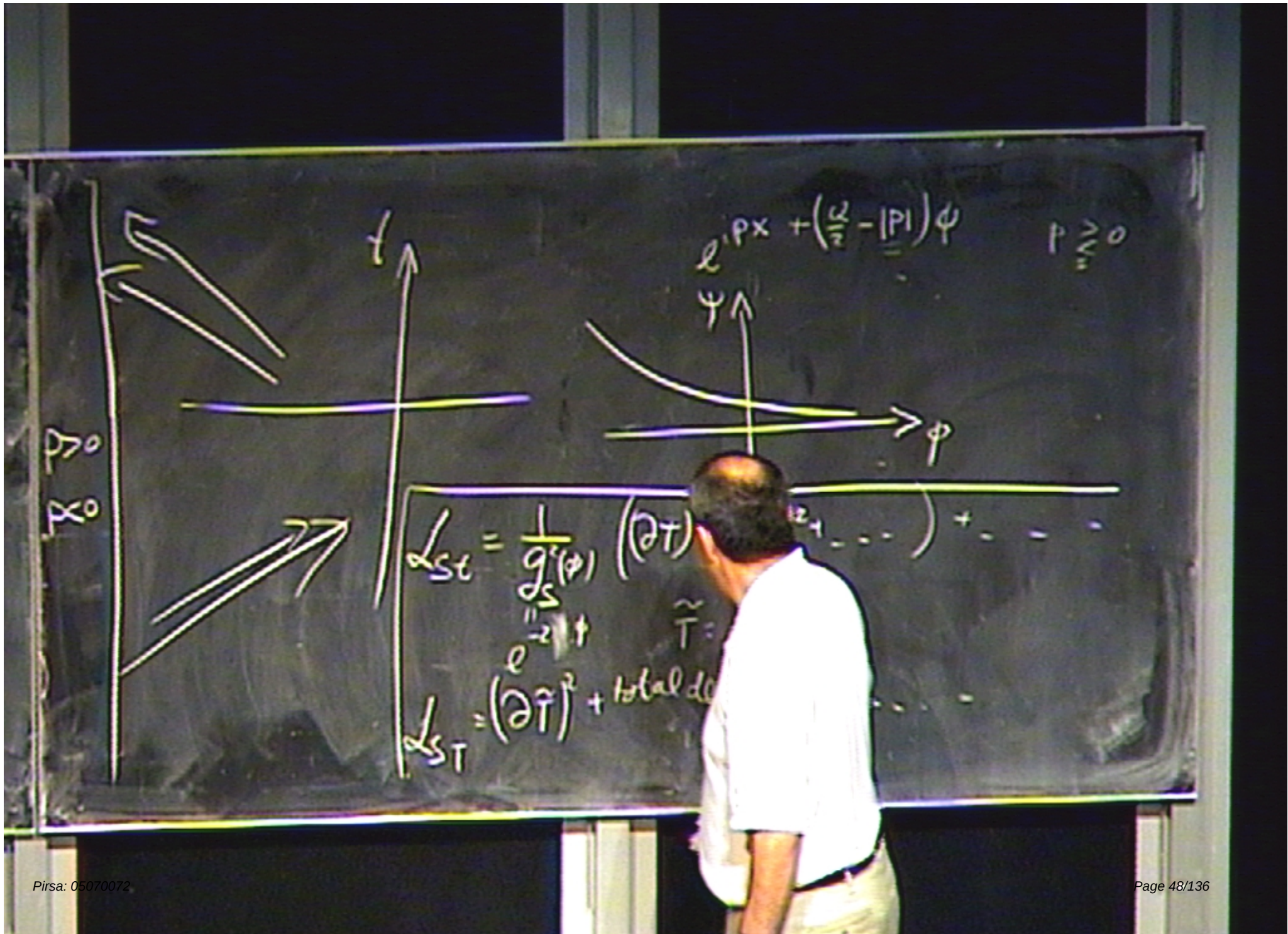
OR

NS "tachyon" T $\xrightarrow{\text{warp}} e^{-\frac{1}{2}(\phi - \bar{\phi}) + i p X + (1 - |p|)\phi}$



RR scalar C $\xrightarrow{\text{warp}} e^{-\frac{1}{2}(\phi - \bar{\phi}) + \frac{i}{2}(H + \bar{H}) + pX + (1 - |p|)\phi}$
 $\xrightarrow{\text{warp}} e^{-\frac{1}{2}(\phi - \bar{\phi}) - \frac{i}{2}(H + \bar{H}) + pX + (1 - |p|)\phi}$

$C = \underbrace{\gamma}_{\text{RR flux}} \phi + \tilde{\gamma} \tau = \gamma_{\text{in}}(\tau - \phi) + \gamma_{\text{out}}(\tau + \phi)$



$p > 0$
 x_0

t

$$e^{ipx} + \left(\frac{\omega^2}{2} - |p|^2\right)\psi$$

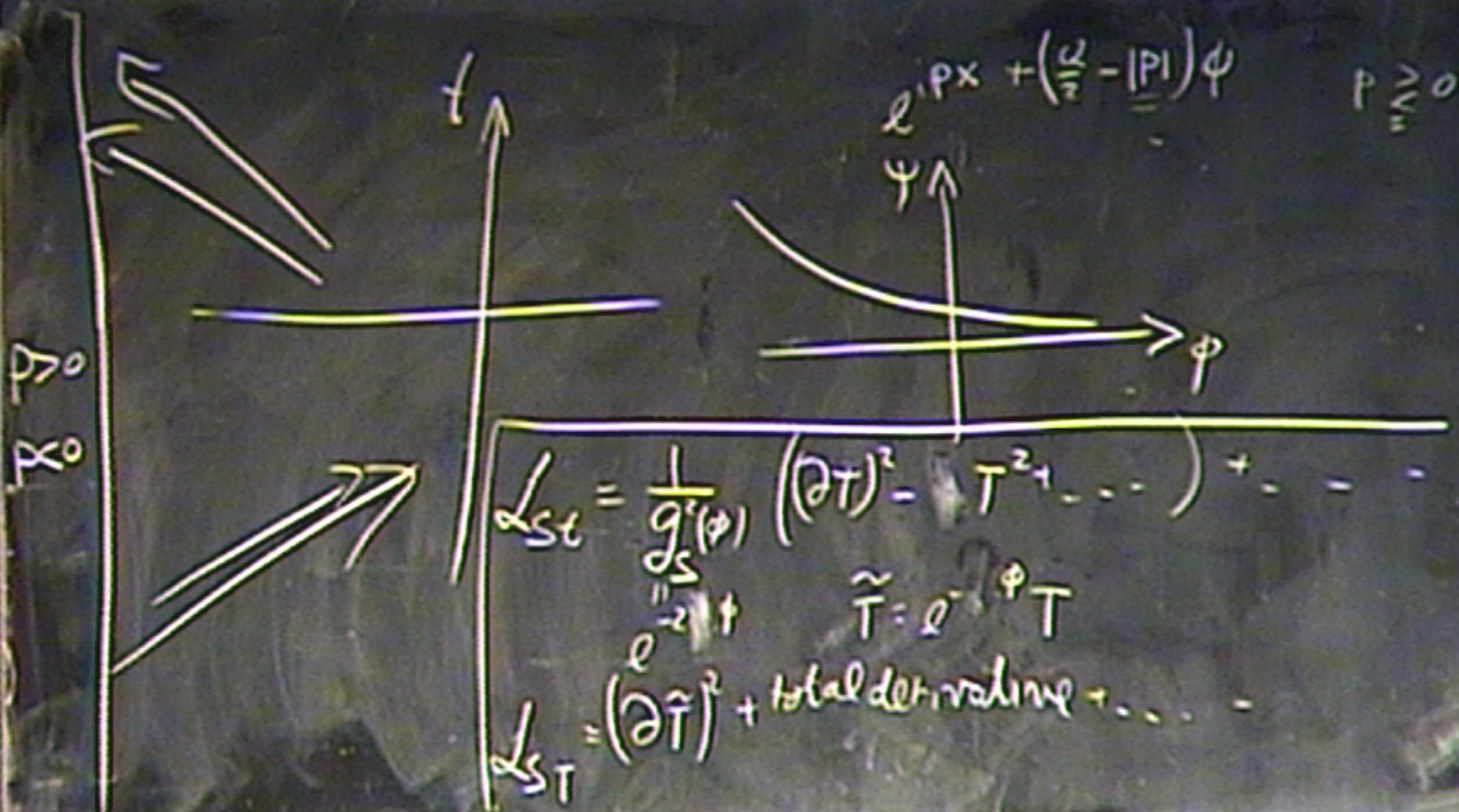
$$p \geq 0$$

ψ

ϕ

$$L_{St} = \frac{1}{g_s(\phi)} (\partial T)$$

$$L_{ST} = (\partial \tilde{T})^2 + \text{total de}$$



Trivial $\mathbb{Z}_2$ $(-)^{F_L}$

$F_L(F_R) = \text{left (right) moving}$

Trivial $\mathbb{Z}_2$ $(-)^{F_L}$

$F_L (F_R) =$ left (right) moving in w.s
spacetime fermion #.

Trivial $\mathbb{Z}_2$ $(-)^{F_L}$

$F_L (F_R) =$ left (right) moving in w.s
spacetime fermion #.

$$NS \rightarrow NS \rightarrow NS - NS$$

$$RR \rightarrow -RR$$

$$T \rightarrow T$$

$$C \rightarrow -C$$

Another z_2

$(-)^{f_2}$

$$T \rightarrow -T$$

$$C_+ \rightarrow C_+$$

$$C_- \rightarrow -C_-$$

Another z_2
 $(-)^{f_2}$

$$\bar{T} \rightarrow -T$$

$$c_+ \rightarrow c_+$$

$$c_- \rightarrow -c_-$$

) duality

flux

Another z_2
 $(-)^{f_2}$

$$\bar{T} \rightarrow -T$$

$$c_+ \rightarrow c_+$$

$$c_- \rightarrow -c_-$$

) duality

Another $\mathbb{Z}_2$
 $(-)^{F_L}$

$$T \rightarrow -T$$

$$c_+ \rightarrow c_+$$

$$c_- \rightarrow -c_-$$

) S-duality

Another z_2
 $(-)^{f_2}$

$$T \rightarrow -T$$

$$c_+ \rightarrow c_+$$

$$c_- \rightarrow -c_-$$

) S-duality

$$V \leftrightarrow \tilde{V}$$

$$\mathcal{L}_{st} = (\partial c)^2$$

$$c \rightarrow c + \text{const.}$$

flux

Another $\tilde{z}_2$
 $(-)^{f_z}$

$$T \rightarrow -T$$

$$c_+ \rightarrow c_+$$

$$c_- \rightarrow -c_-$$

) S-duality

$$V \leftrightarrow \tilde{V}$$

$$\mathcal{L}_{st} = (\partial c)^2$$

$$c \rightarrow c + \text{const.}$$

flux

γ -duality $\rightarrow$ c compact
 $c \sim c + 2\pi$

Another $\mathbb{Z}_2$
 $(-)^{F_L}$

$$T \rightarrow -T$$

$$c_+ \rightarrow c_+$$

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) S-duality

$$\mathcal{V} \leftrightarrow \tilde{\mathcal{V}}$$

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flux

γ -duality $\rightarrow$ c compact
 $c \sim c + 2\pi$

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 $(-)^{F_L}$

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$$V \leftrightarrow \tilde{V}$$

$$\mathcal{L}_{st} = (\partial c)^2$$

$$c \rightarrow c + \text{const.}$$

flux

γ -duality $\rightarrow$

c compact

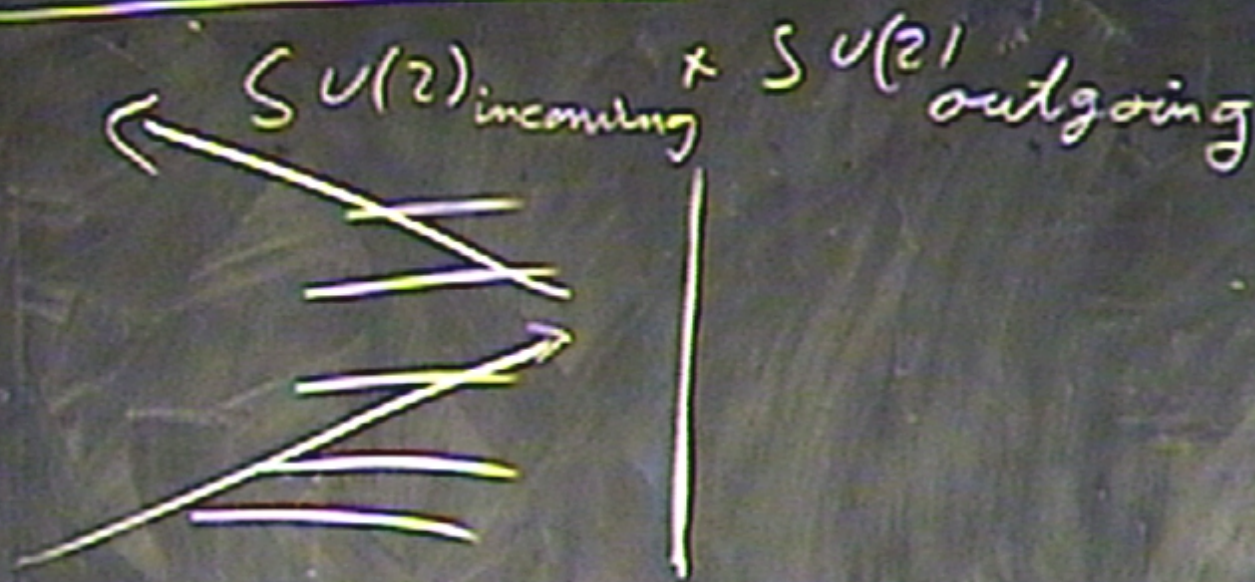
$$c \sim c + 2\pi n f_0$$

Self dual radius

At weak coupling

$SU(2)_{\text{incoming}} \times SU(2)_{\text{outgoing}}$

At weak coupling



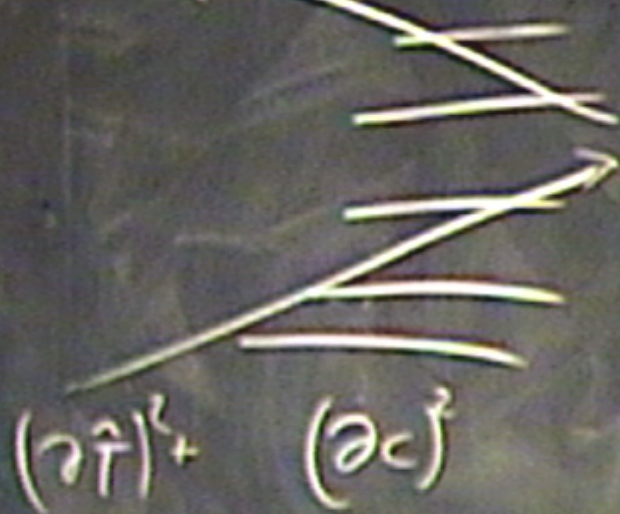
At weak coupling

$S U(2)_{\text{incoming}} \times S U(2)_{\text{outgoing}}$

$(\partial \hat{r})^2 + (\partial c)^2$

At weak coupling

$SU(2)_{\text{incoming}} \times SU(2)_{\text{outgoing}}$



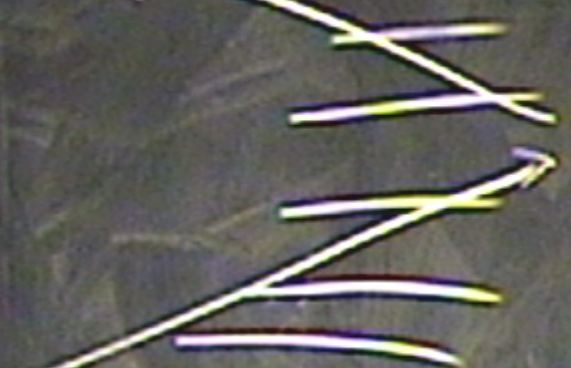
At weak coupling

$SU(2)_{\text{incoming}} \times SU(2)_{\text{outgoing}}$

$(\partial\bar{f})^2 + (\partial c)^2$

At weak coupling

$SU(2)_{\text{incoming}} \times SU(2)_{\text{outgoing}}$

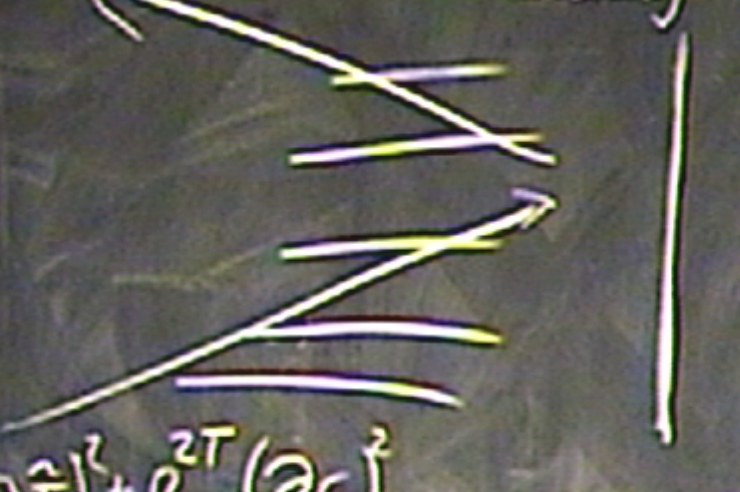


A Feynman diagram representing a four-point interaction at weak coupling. It consists of four external lines meeting at a central vertex. Two lines enter from the left, and two lines exit to the right. The lines are drawn with multiple parallel strokes to represent fermions. A vertical line is drawn to the right of the vertex, possibly indicating a separation or a specific gauge choice.

$$(\partial \hat{r})^2 + e^{2T} (\partial c)^2$$

At weak coupling

$SU(2)_{\text{incoming}} \times SU(2)_{\text{outgoing}}$



$$(\partial \tilde{r})^2 + e^{2T} (\partial c)^2$$

$$\hookrightarrow (\partial \tilde{r})^2 + e^{2T} (\partial \tilde{c})^2$$

At weak coupling

$SU(2)_{\text{incoming}} \times SU(2)_{\text{outgoing}}$

$$(\partial \hat{f})^2 + e^{2T} (\partial c)^2$$

At weak coupling

$SU(2)_{\text{incoming}} \times SU(2)_{\text{outgoing}}$

$$(\partial \tilde{r})^2 + e^{2T} (\partial c)^2$$



$$(\partial \tilde{r})^2 + e^{2T} (\partial \tilde{c})^2$$

Another $\tilde{z}_2$
 $(-)^{F_L}$

$$T \rightarrow -T$$

$$c_+ \rightarrow c_+$$

$$c_- \rightarrow -c_-$$

) S-duality

$$v \leftrightarrow \tilde{v}$$

$$\mathcal{L}_{st} \supset (\partial c)^2$$

γ-duality

$$c \rightarrow c + \text{const.} \frac{1}{v}$$

$$c = v\phi + \tilde{v}\tau = v_m(t -$$

α dual radius

Another $\tilde{z}_2$
 $(-)^{f_2}$

$$c = v\phi + \tilde{v}z = \underbrace{v_{in}(t-\phi)}_{c_-} + \underbrace{v_{out}(t+\phi)}_{c_+}$$

$$T \rightarrow -T$$

$$c_+ \rightarrow c_+$$

$$c_- \rightarrow -c_-$$

) S-duality

$$\frac{v \leftrightarrow \tilde{v}}{\mathcal{L}_{st} \propto (\partial c)^2}$$

$$c \rightarrow c + \text{const} \frac{1}{\sqrt{\lambda}}$$

T-duality $\rightarrow$ c compact
 $c \sim c + 2\pi R$

Self dual radius

At weak coupling

$S U(2)_{\text{incoming}} \times S U(2)_{\text{outgoing}}$



$$(\partial_c)^2 \left[(\partial_{\tilde{r}})^2 + e^{2\gamma} (\partial_{\tilde{c}})^2 \right]$$

At weak coupling

$SU(2)_{\text{incoming}} \times SU(2)_{\text{outgoing}}$

A Feynman diagram showing a Z boson exchange between two fermions. The incoming fermion on the left is represented by a solid line with an arrow pointing right. The outgoing fermion on the right is represented by a solid line with an arrow pointing left. A vertical dashed line represents the Z boson exchange. The vertices are represented by boxes containing the expression $(\partial \tilde{f})^2 + e^{2T} (\partial c)^2$. The diagram is labeled with $SU(2)_{\text{incoming}} \times SU(2)_{\text{outgoing}}$ at the top.

$$\langle T \rangle = \mu \epsilon^4$$

$$\langle T \rangle = \mu e^{\psi}$$

$$\partial(e^{\psi} \partial c) = 0$$

$$c = \nu$$

$$\langle T \rangle = \mu e^{\psi}$$

$$\partial(e^{2T} \partial c) = 0$$

$$c = \int_{\phi} e^{-2T(\phi)} d\phi + \tilde{v} t$$

$$\langle T \rangle = \mu e^{\psi}$$

$$\partial(\partial^2 \bar{\psi} c) = 0$$

$$c = \gamma^{\phi} \partial^2 T(\phi) d\psi + \tilde{\psi} t$$

$$\| \int c \|$$

$$\langle T \rangle = \mu e^{\psi}$$

$$\partial(e^{2T} \delta c) = 0$$

$$c = \nu \int_{\phi} e^{-2T(\phi')} d\psi + \tilde{\nu} t$$

$$\| \int c \|^2 = \int d\phi e^{2T(\phi')} \delta c^2$$

$$\langle T \rangle = \mu e^{\psi}$$

$$\partial(e^{2T} \delta c) = 0$$

$$c = \nu \int_{\phi} e^{-2T(\phi)} d\phi + \tilde{\nu} t$$

$$\| \int c \|^2 = \int d\phi e^{2T(\phi)} \delta c^2$$

$$\mu > 0$$

$$\tilde{\nu} \tau$$

$$C^2$$

Another

$$(-)^{f_z}$$

$$\overline{T} \rightarrow -$$

$$C_+ \rightarrow$$

$$C_- \rightarrow$$

$$\nu \leftrightarrow$$

$$\mathcal{L}_{st} \supset (\partial c$$

$$m > 0 \rightarrow \nu_j^0$$

$$\tilde{\nu}^t$$

$$c^2$$

Another

$$(-)^{f_z}$$

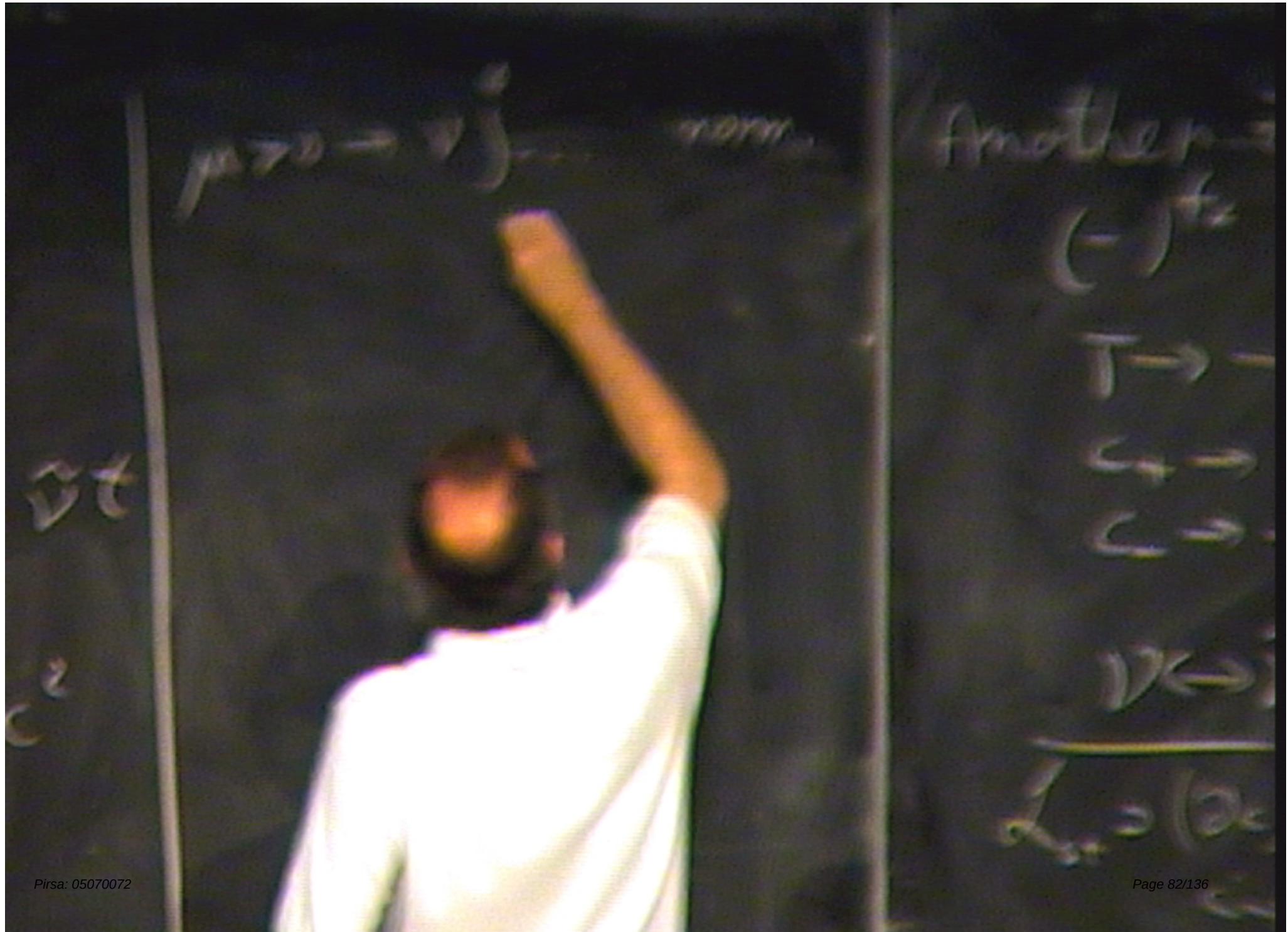
$$\overline{T} \rightarrow$$

$$c_+ \rightarrow$$

$$c_- \rightarrow$$

$$\nu \leftrightarrow$$

$$\mathcal{L}_{st} = (\partial c$$



$$\mu > 0 \rightarrow \gamma \int_{-\infty}^{\phi} \dots \text{ norm.} \\ \text{as } \phi \rightarrow +\infty$$

$$\vec{v} \tau$$

$$c^2$$

Another r-z

$$(-)^{f_z}$$

$$\overline{T} \rightarrow -$$

$$c_+ \rightarrow$$

$$c_- \rightarrow$$

$$v \leftrightarrow$$

$$\mathcal{L}_{st} \supset (\partial c$$

$$\mu > 0 \rightarrow \psi \int_{-\infty}^{\phi} \dots \text{norm.}$$

$$\text{as } \phi \rightarrow +\infty$$

$$\tilde{\psi}^T \propto \text{norm}$$

$$\text{as } \phi \rightarrow +\infty$$

$$\tilde{\psi}^T$$

$$c^2$$

Another

$$(-)^{f_L}$$

$$\bar{T} \rightarrow -$$

$$c_+ \rightarrow$$

$$c_- \rightarrow$$

$$V \leftrightarrow$$

$$\mathcal{L}_{st} \supset (\partial c$$

$$\mu > 0 \rightarrow \gamma \int_{-\infty}^{\phi} \dots \text{ norm.} \\ \text{as } \phi \rightarrow +\infty$$

$$\tilde{\gamma}^t \propto \text{norm}$$

$$\uparrow \text{ as } \phi \rightarrow +\infty$$

sourced by
an object at
 $\phi = +\infty$

Another $\tilde{\gamma}^t$
 $(-)^{t_z}$

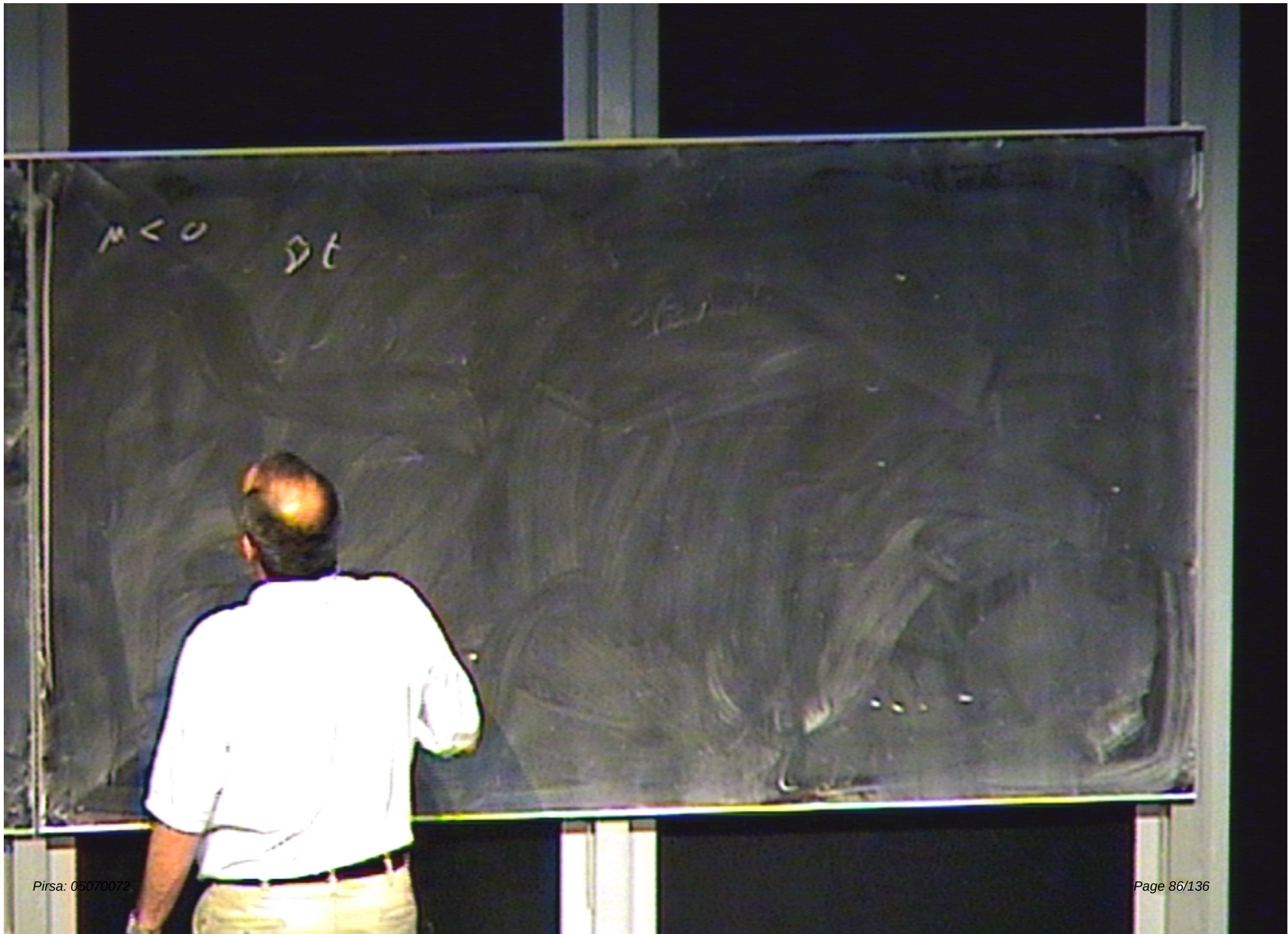
$$\tilde{T} \rightarrow -$$

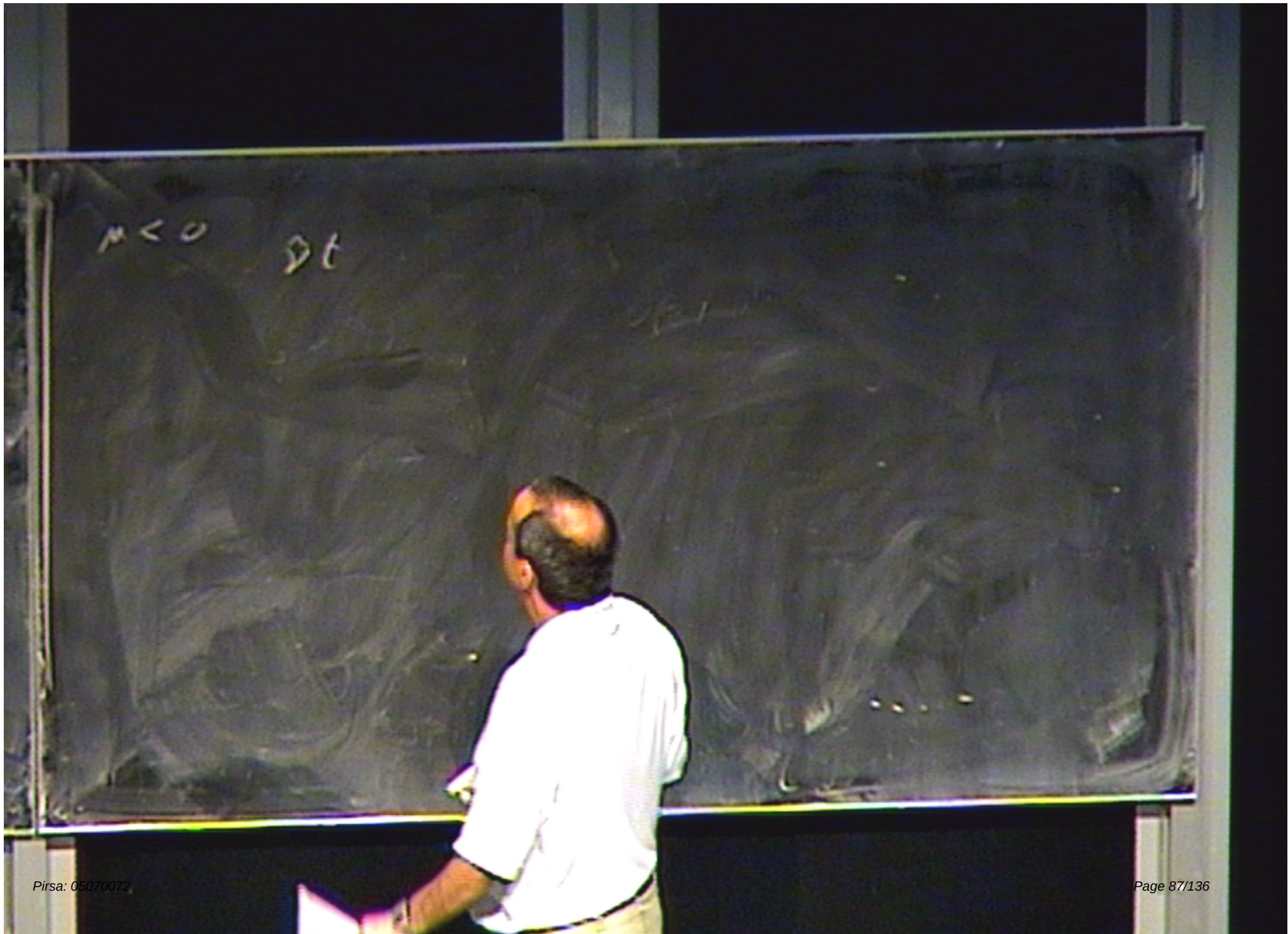
$$C_+ \rightarrow$$

$$C_- \rightarrow$$

$$V \leftrightarrow$$

$$\frac{1}{L_{st}} \propto (\partial_c$$





$$M < 0$$

$\exists t$

finite norm as $\phi \rightarrow +\infty$
not sourced by an object there

$$M < 0$$

$\forall t$

finite norm as $\phi \rightarrow +\infty$

not sourced by an object there

$$\forall \int_{\phi}^{\phi} \dots$$

∞ norm at $\phi \rightarrow +\infty$

$\rightarrow$ sourced by an object

$$m < 0$$

$\forall t$

finite norm as $\phi \rightarrow +\infty$

not sourced by an object there

$\forall \int_{\phi}^{\phi}$

∞ norm at $\phi \rightarrow +\infty$

$\rightarrow$ sourced by an object

$$M < 0$$

∂t

finite norm as $\phi \rightarrow +\infty$

not sourced by an object there

$$V \int_{\phi}^{\phi} \dots$$

∞ norm at $\phi \rightarrow +\infty$

$\rightarrow$ sourced by an object

$$\tilde{V} = 0 \quad V \neq 0$$

$$\Rightarrow (\partial_c)^2 \sim V^2$$

$$M < 0$$

∇t

finite norm as $\phi \rightarrow +\infty$

not sourced by an object there

$$\nabla \int_{\phi}^{\phi} \dots$$

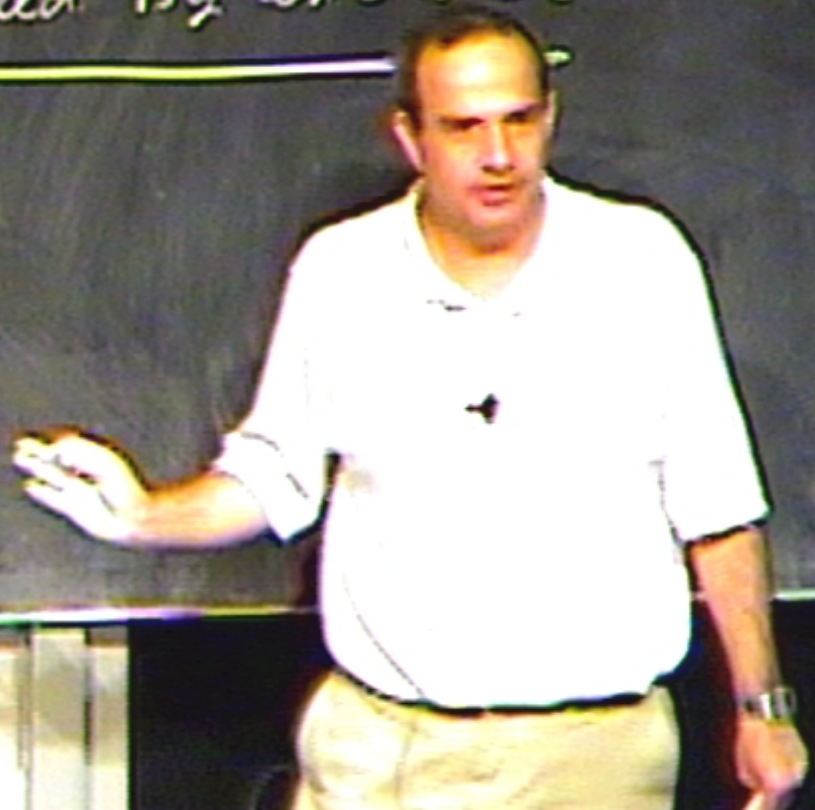
∞ norm at $\phi \rightarrow +\infty$

$\rightarrow$ sourced by an object

$$\tilde{V} = 0 \quad V \neq 0$$

$$\Delta (\partial_c)^2 \sim V^2$$

$$\text{Energy} = \underbrace{\log_1}_{V_{\text{space}}} V^2$$



$$M < 0$$

∇t

finite norm as $\phi \rightarrow +\infty$

not sourced by an object there

$\nabla \int_{\phi}^{\phi}$

∞ norm at $\phi \rightarrow +\infty$

$\rightarrow$ sourced by an object

$$\tilde{V} = 0 \quad V \neq 0$$

$$\Delta (\partial_c)^2 \sim V^2$$

$$\text{Energy} = \underbrace{\log_1 V^2}$$

$V_{\text{space}} = \text{Volume of } \phi$

$$M < 0$$

∇t

finite norm as $\phi \rightarrow +\infty$

not sourced by an object there

$\nabla \int^\phi$

∞ norm at $\phi \rightarrow +\infty$

$\rightarrow$ sourced by an object

$$\tilde{\nabla} = 0 \quad V \neq 0$$

$$\Delta (\partial_c)^2 \sim V^2$$

$$\text{Energy} = \log_1 V^2$$

$V_{\text{space}} = \text{Volume of } \phi$

$$\tilde{\nabla} \neq 0 \quad V = 0$$

$$\log_1 \tilde{V}^2$$

$$M < 0$$

∇t

finite norm as $\phi \rightarrow +\infty$

not sourced by an object there

$$\nabla \int \phi$$

∞ norm at $\phi \rightarrow +\infty$

$\rightarrow$ sourced by an object

$$\tilde{\nabla} = 0 \quad V \neq 0$$

$$\Delta (\partial_c)^2 \sim V^2$$

$$\text{Energy} = \log_1 V^2$$

$V = \text{Volume of } \phi$
space

$$\tilde{\nabla} \neq 0 \quad V = 0$$

$$\log_1 \tilde{\nabla}^2$$

both

indep

$$M < 0$$

∇t

finite norm as $\phi \rightarrow +\infty$

not sourced by an object there

$\nabla \int_{\phi} \dots$

∞ norm at $\phi \rightarrow +\infty$

$\rightarrow$ sourced by an object

$$\tilde{\nabla} = 0 \quad \nabla \neq 0$$

$$\Delta (\partial_c)^2 \sim \nabla^2$$

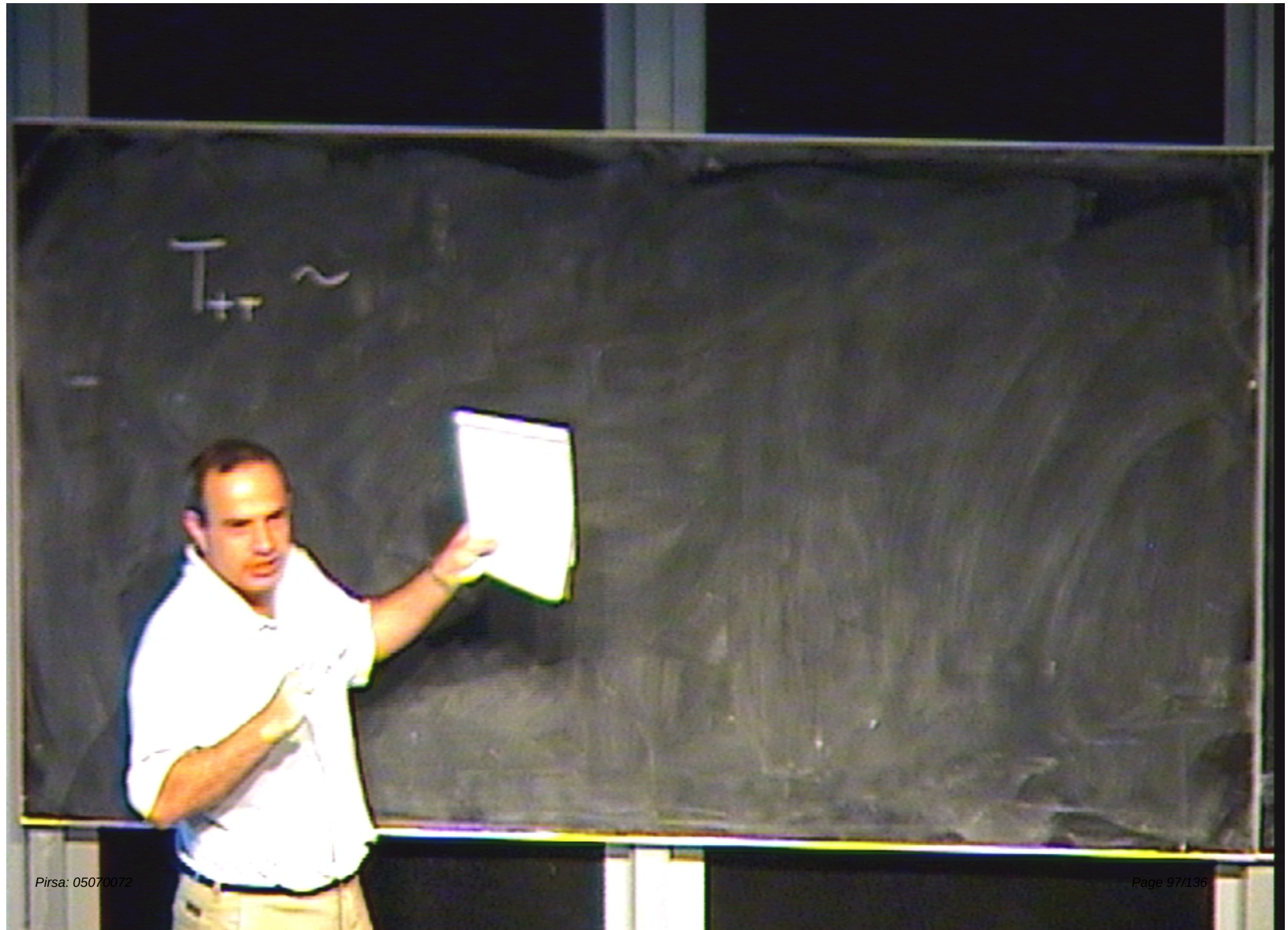
$$\text{Energy} = \log_1 \nabla^2$$

$\nabla = \text{Volume of } \phi$
space

$$\tilde{\nabla} \neq 0 \quad \nabla = 0$$

$$\log_1 \tilde{\nabla}^2$$

both time independent



$$T_{++} \sim v_{in}^2$$

$$T_{--} \sim v_{out}^2$$

$$T_{++} \sim v_{in}^2$$

$$T_{--} \sim v_{out}^2$$

$$\int T_{++} \neq \int T_{--}$$

$$T_{++} \sim v_{in}^2$$

$$T_{--} \sim v_{out}^2$$

$$\int T_{++} \neq \int T_{--}$$



$$T_{++} \sim v_{in}^2$$

$$T_{--} \sim v_{out}^2$$

$$\int T_{++} \neq \int T_{--}$$

Add more
quanta



$$T_{++} \sim v_{in}^2$$

$$T_{--} \sim v_{out}^2$$

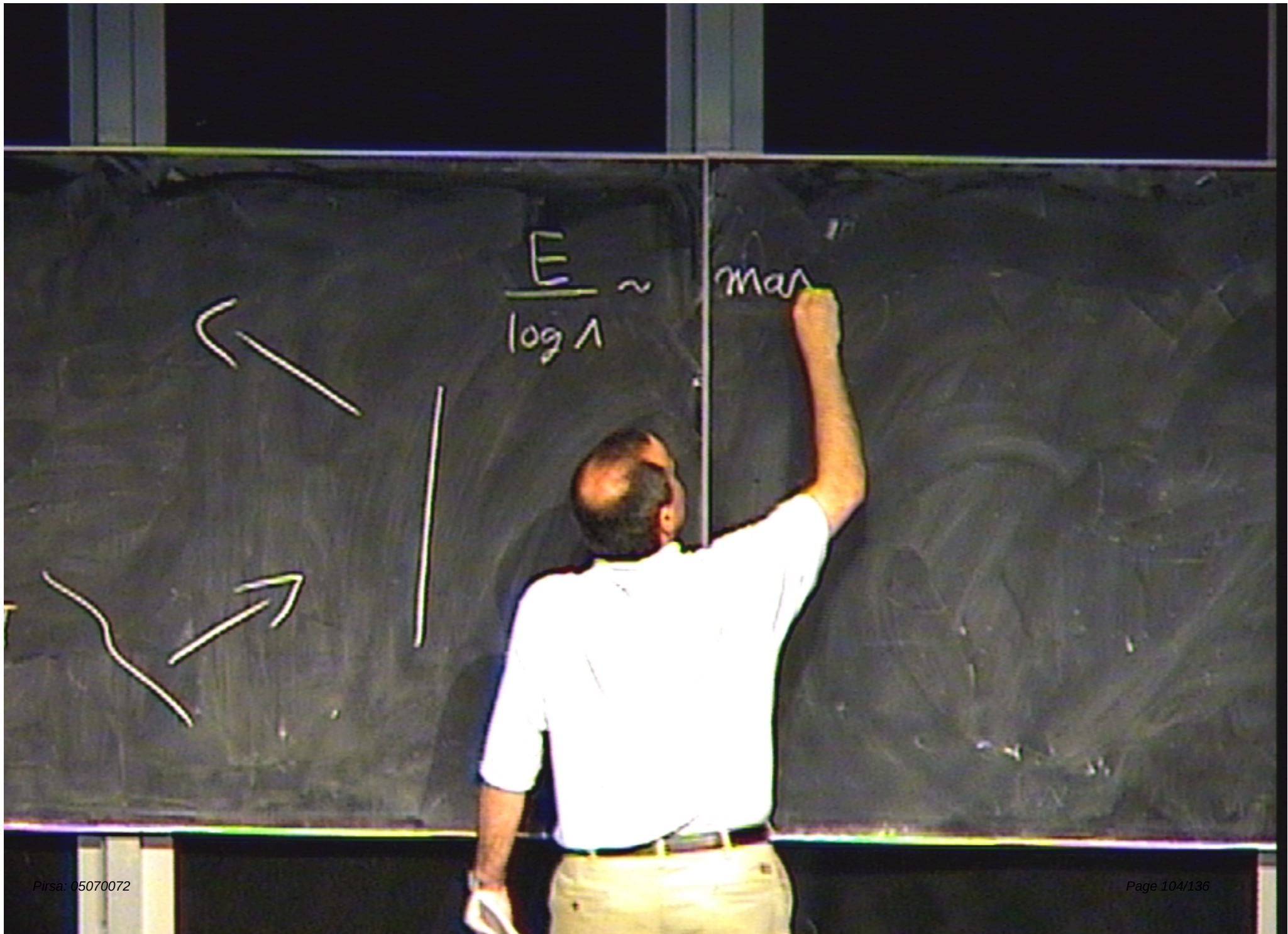
$$\int T_{++} \neq \int T_{--}$$

Add more
quanta

cor T

$$\Sigma Z = \underbrace{V_{in}(t-u)}_{C_-} + \underbrace{V_{out}(t+u)}_{C_+}$$

$\Rightarrow C$ compact
 $C \sim C + 2\pi R \mathbb{Z}$
 Self dual radius



$$\frac{E}{\log \lambda} \sim \max$$

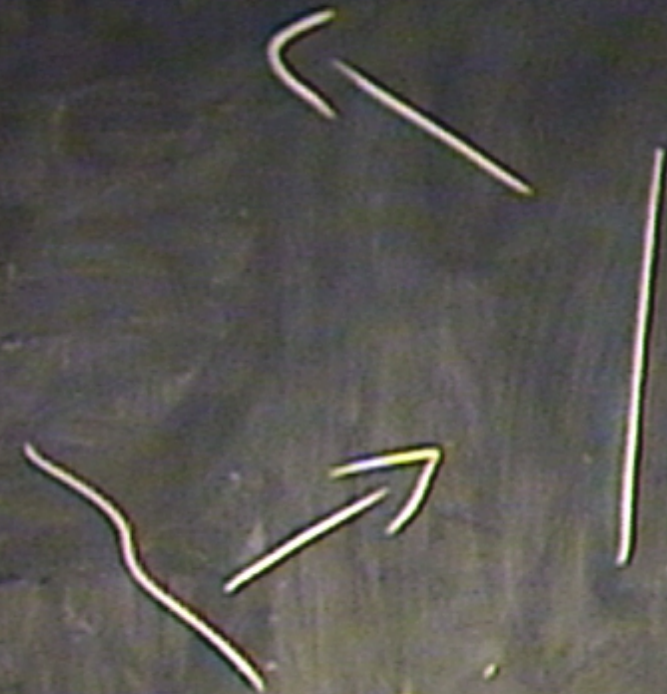
$$\frac{E}{\log \Lambda} \sim$$

$$\max(v_{in}^2, v_{out}^2)$$

$$\sim ||v||$$

$$\frac{E}{\log \Lambda} \sim$$

$$\max(v_{in}^2, v_{out}^2) \sim (|v| + |\tilde{v}|)^2$$



$$\frac{E}{\log \Lambda} \sim$$

$$\max(v_{in}^2, v_{out}^2) \\ \sim (|v| + |\tilde{v}|)^2$$

$$\frac{E}{\log \Lambda} \sim$$

$$\max(v_{in}^2, v_{out}^2)$$

$$\sim (|v| + |\tilde{v}|)^2$$

on



$$\frac{E}{\log \Lambda} \sim$$

$$\max(v_{in}^2, v_{out}^2) \\ \sim (|v| + |\tilde{v}|)^2$$

NS Tachyon

$$\frac{0A}{e^{-\varphi - \bar{\varphi}} + i p x + (1 - |A|) \phi}$$

$$\frac{E}{\log \Lambda} \sim$$

$$\max(V_{in}^2, V_{out}^2) \sim (|V| + |\tilde{V}|)^2$$

NS Tachyon

RR sector

0A

$$e^{-\varphi - \bar{\varphi} + i p X + (p \cdot \Pi) \phi}$$

$$e^{-\varphi/2 - \bar{\varphi}/2 + \frac{i}{2} (p \cdot \Pi) \phi}$$

$$\frac{E}{\log \Lambda} \sim$$

$$\max(v_{in}^2, v_{out}^2) \\ \sim (|v| + |\tilde{v}|)^2$$

NS Tachyon

$$p - \bar{\varphi} + i p x + (1 - |p|) \phi$$

RR sector

$$-\varphi/2 - \bar{\varphi}/2 \pm \frac{i}{2}(H - \bar{H}) + \psi$$

$$\frac{E}{\log \Lambda} \sim$$

$$\max(V_{in}^2, V_{out}^2) \\ \sim (|V| + |\tilde{V}|)^2$$

NS Tachyon

0A

$$e^{-\varphi - \bar{\varphi} + i p x + (1 - |p|) \phi}$$

RR sector

$$e^{-\varphi/2 - \bar{\varphi}/2 \pm \frac{i}{2}(H - \bar{H}) + \phi}$$

$$\frac{E}{\log \Lambda} \sim$$

$$\max(v_{in}^2, v_{out}^2) \\ \sim (|v| + |\tilde{v}|)^2$$

NS Tachyon

0A

$$e^{-\varphi - \bar{\varphi} + i p x + (1 - |p|) \phi}$$

RR sector

$$e^{-\varphi/2 - \bar{\varphi}/2 + \frac{i}{2}(H - \bar{H}) + \phi} = F, \bar{F}$$

$$\frac{E}{\log \Lambda} \sim$$

$$\max(V_{in}^2, V_{out}^2) \\ \sim (|V| + |\tilde{V}|)^2$$

NS Tachyon

0A

$$e^{-\varphi - \bar{\varphi} + i p x + (1 - |p|) \phi}$$

RR sector

$$e^{-\varphi/2 - \bar{\varphi}/2 \pm \frac{i}{2}(H - \bar{H}) \mp \phi} = F, \bar{F}$$

$$\frac{E}{\log \Lambda} \sim$$

$$\max(v_{in}^2, v_{out}^2) \\ \sim (|v| + |\tilde{v}|)^2$$

NS Tachyon

RR sector

0A

$$e^{-\varphi - \bar{\varphi}} + i p x + (1 - |p|) \phi$$

$$e^{-\varphi/2 - \bar{\varphi}/2} \pm \frac{i}{2} (H - \bar{H}) \mp \phi = F, \bar{F}$$

$$V_{st} = e^{2T} F^7$$

$$\frac{E}{\log \Lambda} \sim$$

$$\max(V_{in}^2, V_{out}^2) \\ \sim (|V| + |\tilde{V}|)^2$$

NS Tachyon

0A

$$e^{-\varphi - \bar{\varphi}} + i p x + (1 - |H|) \phi$$

RR sector

$$e^{-\varphi/2 - \bar{\varphi}/2} \pm \frac{i}{2} (H - \bar{H}) \psi = F, \tilde{F}$$

$$V_{st} = e^{2T} \tilde{F}^2$$

Sidus

$$T \rightarrow -T \\ F \rightarrow \tilde{F}$$

$$\frac{E}{\log \Lambda} \sim$$

$$\max(V_{in}^2, V_{out}^2) \\ \sim (|V| + |\tilde{V}|)^2$$

NS Tachyon

OA

$$e^{-\varphi - \bar{\varphi}} + i p x + (1 - |H|) \phi$$

RR sector

$$e^{-\varphi/2 - \bar{\varphi}/2} + \frac{i}{2} (H - \bar{H}) + \phi = F, \tilde{F}$$

$$V_{st} = e^{2T} F^2 + e^{-2T} \tilde{F}^2$$

S-duality

$$T \rightarrow -T \\ F \leftrightarrow \tilde{F}$$

$$\frac{E}{\log \Lambda}$$

$$\max(V_{\text{in}}^2, V_{\text{out}}^2) \sim (|V| + |\tilde{V}|)^2$$

NS Tachyon

OA

$$e^{-\frac{1}{2}(\tilde{V} + \tilde{\bar{V}})} + i(P\tilde{V} + (1-H)\phi)$$

RR scalar

$$e^{-\frac{1}{2}(\tilde{V} + \tilde{\bar{V}})} + \frac{i}{2}(H + \tilde{H})\phi = F, \tilde{F}$$

$$V_{\text{st}} = e^{2T} F^2 + e^{-2T} \tilde{F}^2$$

S-duality

$$T \rightarrow -T, F \rightarrow \tilde{F}$$

$$\frac{E}{\log \Lambda} \sim$$

$$\max(V_{in}^2, V_{out}^2) \sim (|V| + |\tilde{V}|)^2$$

NS Tachyon

RR sector

0A-

$$e^{-\varphi - \bar{\varphi}} + i p \chi + (1 - |p|)$$

$$e^{-\varphi/2 - \bar{\varphi}/2} + \frac{i}{2} (H \cdot \bar{H}) + \phi$$

$$(1 + 2T + \frac{1}{2}(2T)^2 + \dots) F^2$$

$$V_{st} = e^{2T} F^2 + e^{-2T} \tilde{F}^2$$

locality

$$T \rightarrow -T$$

$$F \leftrightarrow \tilde{F}$$

$$\frac{E}{\log \Lambda} \sim$$

$$\max(V_{in}^2, V_{out}^2) \\ \sim (|V| + |\tilde{V}|)^2$$

NS Tachyon

OA

$$e^{-\varphi - \bar{\varphi}} + i p x + (1 - |H|) \phi$$

RR sector

$$e^{-\varphi/2 - \bar{\varphi}/2} + \frac{i}{2} (H \cdot \bar{H}) + \phi = F, \tilde{F}$$

$$(1 + 2T + \frac{1}{2}(2T)^2 + \dots) F^2$$

$$V_{st} = e^{2T} F^2 + e^{-2T} \tilde{F}^2$$

S.duality

$$T \rightarrow -T \\ F \leftrightarrow \tilde{F}$$

$\mu > 0$ q is not sourced by a brane
 $\hat{q}$ is " " " "

$\mu < 0$ $q \leftrightarrow \hat{q}$

$\mu > 0$ q is not sourced by a brane
 $\hat{q}$ is " " " "

$\mu < 0$ $q \leftrightarrow \hat{q}$

$\mu > 0$ q is not sourced by a brane
 $\hat{q}$ is " " " "

$\mu < 0$ $q \mapsto \tilde{q}$

$q, \tilde{q} \in \mathbb{Z}$

$\mu > 0$ q is not sourced by a brane
 $\hat{q}$ is " " " "

$\mu < 0$ $q \leftrightarrow \hat{q}$

$q, \hat{q} \in \mathbb{Z}$

$\frac{E}{\log \Lambda} \sim q^2$ $\tilde{q} = 0$
 $\sim \hat{q}^2$ $q = 0$

$\mu > 0$ q is not sourced by a brane
 $\hat{q}$ is

$\mu < 0$ $q \leftrightarrow \hat{q}$

$q, \hat{q} \in \mathbb{Z}$

$$\begin{array}{ll}
 \frac{F}{\log \Lambda} \sim q^2 & \hat{q} = 0 \\
 \sim \hat{q}^2 & q = 0
 \end{array}$$

$\mu > 0$ q is not sourced by a brane
 $\hat{q}$ is

$\mu < 0$ $q \leftrightarrow \hat{q}$

$$q, \hat{q} \in \mathbb{Z}$$

$$\begin{array}{ll}
 \frac{E}{\log \Lambda} \sim q^2 & \hat{q} = 0 \\
 \sim \hat{q}^2 & q = 0
 \end{array}$$

when $q\hat{q} \neq 0$
 need to add $q\hat{q}$ fundamental
 strings str along

$\mu > 0$ q is not sourced by a brane
 $\tilde{q}$ is " " " "

when $q\tilde{q} \neq 0$
 need to add $q\tilde{q}$ fundamental
 strings str. along

$\mu < 0$ $q \leftrightarrow \tilde{q}$

$q, \tilde{q} \in \mathbb{Z}$

$$\begin{array}{ll}
 \frac{E}{\log \Lambda} \sim q^2 & \tilde{q} = 0 \\
 \sim \tilde{q}^2 & q = 0
 \end{array}$$

$$\frac{E}{\log \Lambda} \sim q^2 + \tilde{q}^2 + 2|q\tilde{q}| = (|q| + |\tilde{q}|)^2$$

$\mu > 0$ q is not sourced by a brane
 $\tilde{q}$ is " " " "

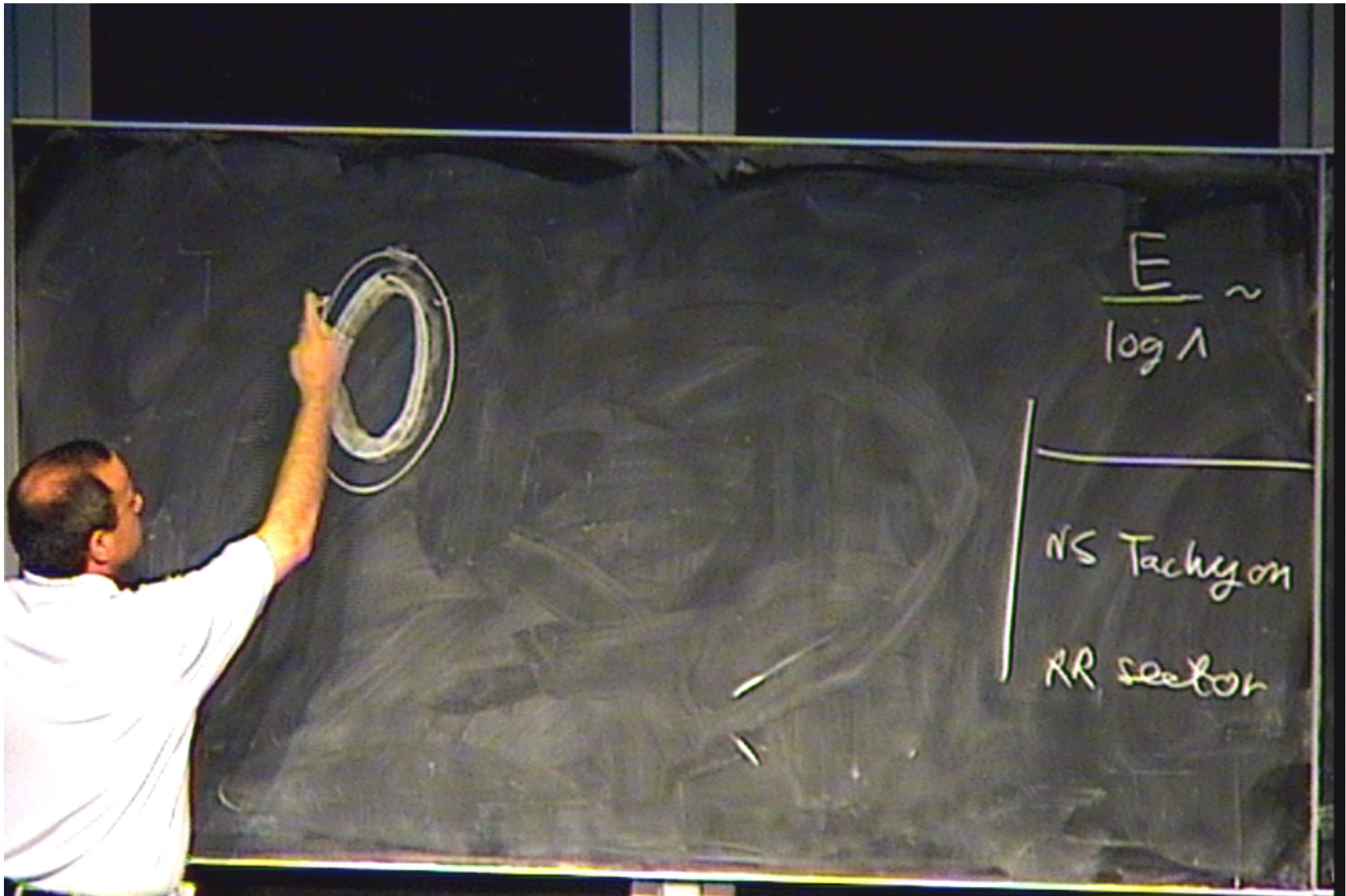
when $q\tilde{q} \neq 0$
 need to add
 $q\tilde{q}$ fundamental
 strings str. along ϕ

$\mu < 0$ $q \leftrightarrow \tilde{q}$

$q, \tilde{q} \in \mathbb{Z}$

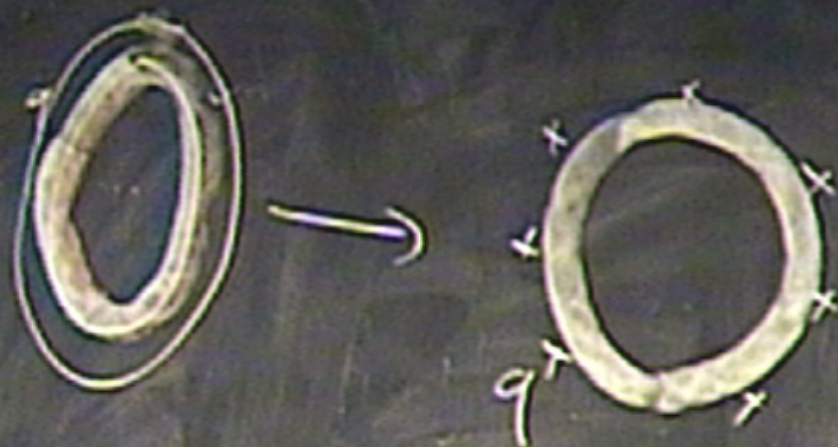
$$\begin{array}{ll}
 \frac{E}{\log \Lambda} \sim q^2 & \tilde{q} = 0 \\
 \sim \tilde{q}^2 & q = 0
 \end{array}$$

$$\frac{E}{\log \Lambda} \sim q^2 + \tilde{q}^2 + 2|q\tilde{q}| = (|q| + |\tilde{q}|)^2$$



$$\frac{E}{\log \Lambda} \sim$$

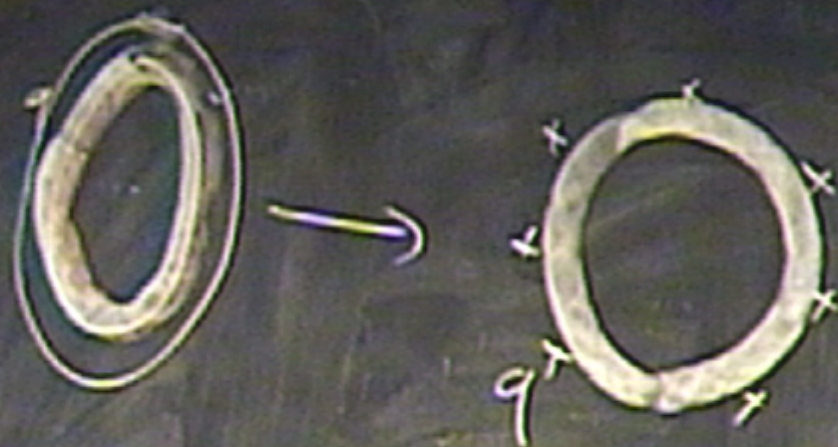
NS Tachyon
RR sector



$$\frac{E}{\log \Lambda} \sim$$

NS Tachyon

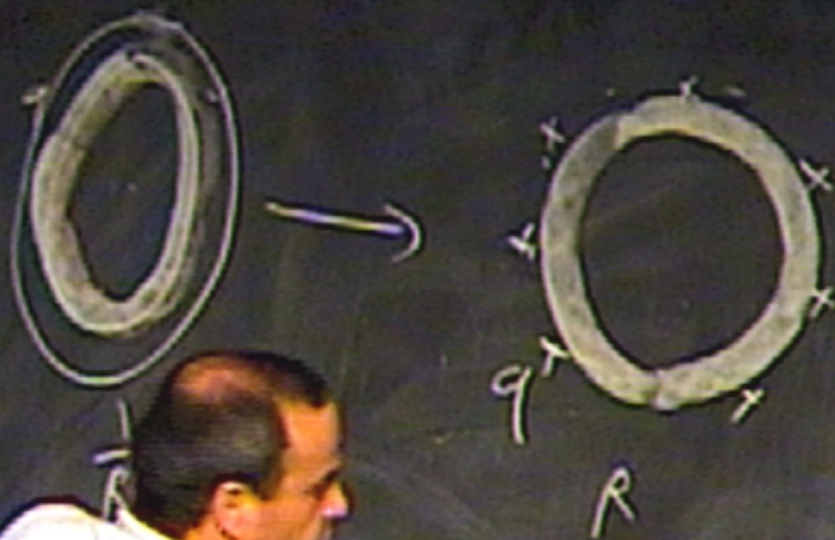
RR sector



$$\frac{E}{\log \Lambda} \sim$$

NS Tachyon

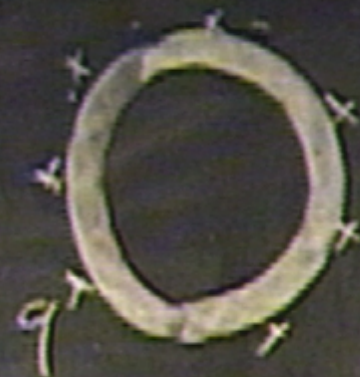
RR sector



$$\frac{E}{\log \Lambda} \sim$$

NS Tachyon

As $R \rightarrow \infty$ RR sector



R

$\frac{q}{R}$

As $R \rightarrow \infty$

$$\frac{E}{\log \Lambda} \sim$$

NS Tachyon

RR sector



$$\frac{E}{\log \Lambda} \sim$$

NS Tachyon

RR sector

$$\frac{q}{R}$$

As $R \rightarrow \infty$

$q \rightarrow \infty$

keeping $v = \frac{q}{R}$ fixed



$$\frac{E}{\log \Lambda} \sim$$

NS Tachyon

RR sector

$$\frac{q}{R}$$

As $R \rightarrow \infty$

keeping $q \rightarrow \infty$
keeping $v = \frac{q}{R}$ fixed



$$\frac{E}{\log \Lambda} \sim$$

NS Tachyon

RR sector

$$\frac{q}{R}$$

As $R \rightarrow \infty$

$q \rightarrow \infty$

keeping $v = \frac{q}{R}$ fixed