

Title: Black Holes in String Theory

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Abstract:

THE QUANTUM STRUCTURE OF BLACK HOLES

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Perimeter 2005

Puzzles arising from black holes:

- THE ENTROPY PUZZLE
- THE INFORMATION PUZZLE

Resolving these puzzles should
reveal important properties of
quantum gravity.

12
Details can be found in
review:

"The fuzzball proposal for
black holes: an elementary
review"

hep-th 0502050

Summarises work done in collaboration
with:

Oleg Lunin, Ashish Saxena,
Yogesh Srivastava, Stephano Giusto

THE ENTROPY PUZZLE

2.



$$\frac{dS}{dt} < 0 \quad ??$$

Black hole gets
BIGGER

$$S_{BH} = \frac{Ac^3}{4G\hbar} \rightarrow \frac{A}{4G}$$

$$[c=1, \hbar=1]$$

Then we get

$$\frac{dS_m}{dt} + \frac{dS_{BH}}{dt} \geq 0$$

So we save the second law
of thermodynamics

But by statistical mechanics:

3.

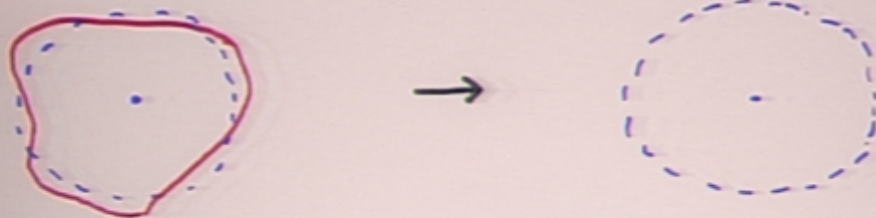
$$S_{BH} = \ln [\# \text{ states}]$$

Black hole should have $e^{S_{BH}}$ states for a given M, Q, J

Solar mass black hole: $10^{10^{77}}$ states

BUT:

"BLACK HOLES HAVE NO HAIR"



But by statistical mechanics:

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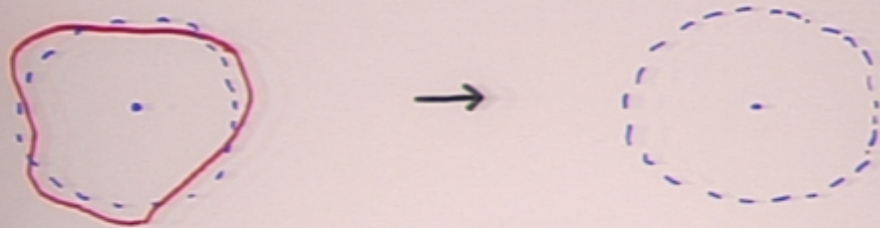
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Black hole should have $e^{S_{BH}}$
states for a given M, Q, J

Solar mass black hole: $10^{10^{77}}$ states

BUT:

"BLACK HOLES HAVE NO HAIR"



$$S = \ln [1] = 0 \quad ??$$

Puzzle: WHERE ARE THE STATES
OF THE BLACK HOLE ?

- AT THE HORIZON ?
- AT THE SINGULARITY ?
- EVERYWHERE IN THE HOLE ?

THE INFORMATION PUZZLE

Thermodynamics: $dS = TdE$

Schwarzschild hole in 3+1

$$S = \frac{A}{4G} = \frac{4\pi(2GM)^2}{4G}, \quad E = M$$

$$\frac{1}{T} = \frac{dS}{dM} = 8\pi GM$$

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THE INFORMATION PUZZLE

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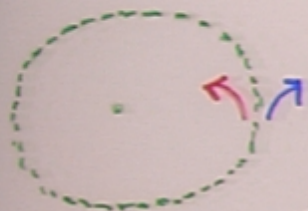
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Hawking:

Black holes radiate
with temperature T



Vacuum fluctuations
"pulled apart" by
"differential gravitational
field" of black hole



DUST CLOUD



BLACK
HOLE



RADIATION

BUT RADIATION HAS NO INFORMATION
OF INITIAL MATTER

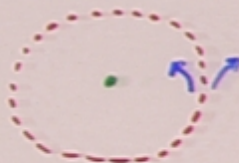
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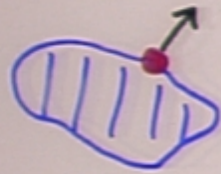
BUT RADIATION HAS NO INFORMATION
OF INITIAL MATTER

SO WE LOSE THE REVERSIBILITY
OF QUANTUM MECHANICS

$$|4\rangle_f = e^{-iHt} |4_i\rangle$$

$$|4_i\rangle = e^{iHt} |4_f\rangle$$

THE ENTROPY AND INFORMATION
PUZZLES ARE CLOSELY CONNECTED



BURN COAL

→ PHOTON

"SEES"

THE STATE

OF THE COAL



EVAPORATE BLACK HOLE

→ CANNOT "SEE"

THE STATE OF

THE HOLE

HAWKING'S DERIVATION IS VERY ROBUST

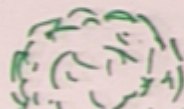
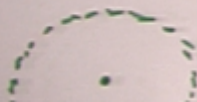
HAWKING "THEOREM" :

IF (A) All quantum gravity effects are confined to within a given distance $l < \ell_P, \ell_s$

(B) The vacuum is unique

Then black hole radiation will cause information loss.

STRING THEORY : SOME COMPUTATIONS SUGGEST THAT (A) WILL BE FALSE



??

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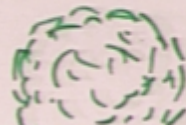
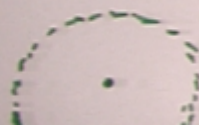
HAWKING "THEOREM":

IF (A) All quantum gravity
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??

HOW CAN THIS HAPPEN:

8.

QUANTUM GRAVITY: $c, \hbar, G$

$\rightarrow l_p$

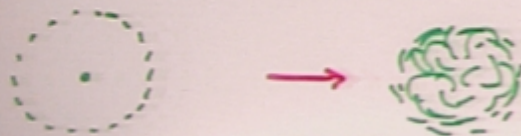
(Planck length
 $\sim 10^{-33}$ cm)



BUT TO MAKE A BLACK HOLE WE
NEED $N \gg 1$ QUANTA

QUANTUM GRAVITY EFFECTS : $\sim l_p$??
 $\sim N^{\alpha} l_p$??

PROPOSAL: $\alpha > 0$, such that
 $N^{\alpha} l_p \sim$ radius of horizon



BLACK HOLES IN STRING THEORY ⁹

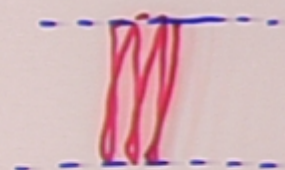
We have to make a concentrated mass using only the objects present in string theory

II A string theory

$$M_{9,1} \rightarrow M_{8,1} \times S^1$$



n_1 NS1
Branes



BOUND STATE
"LONG STRING"



$$\text{MASS} = (\text{WINDING})$$

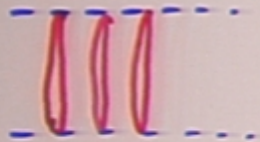
BLACK HOLES IN STRING THEORY

9

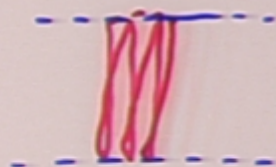
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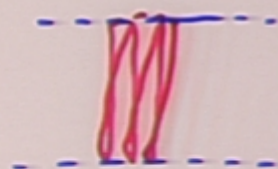
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BOUND STATE
"LONG STRING"



MASS = (WINDING)

$$ds_{\text{string}}^2 = H_1^{-1} [-dt^2 + dy^2] + \sum_{i=1}^8 dx_i dx_i$$

$$e^{2\phi} = H_1^{-1}$$

$$H_1 = 1 + \frac{Q_1}{r^6}$$

Entropy:

$$S_{\text{Bek}} = \frac{A_q}{4G_q} = \frac{A_{10}}{4G_{10}} = \frac{A_{11}}{4G_{11}}$$

All areas to be computed in the corresponding Einstein metrics



$$A_{10} = A_q L$$

$$G_{10} = G_q L$$

FIND: $|S_{21}| = 0$

$$ds_{\text{string}}^2 = H_1^{-1} [-dt^2 + dy^2] + \sum_{i=1}^8 dx_i dx_i$$

$$e^{2\phi} = H_1^{-1}$$

$$H_1 = 1 + \frac{Q_1}{r^6}$$

Entropy:

$$S_{\text{Bek}} = \frac{A_g}{4G_g} = \frac{A_{10}}{4G_{10}} = \frac{A_{11}}{4G_{11}}$$

All areas to be computed in the corresponding Einstein metric



$$A_{10} = A_g L$$

$$G_{10} = G_g L$$

FIND: $S_{g,11} = 0$

$$ds_{\text{string}}^2 = H_1^{-1} [-dt^2 + dy^2] + \sum_{i=1}^8 dx_i dx_i$$

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All areas to be computed in the
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$$A_{10} = A_q L$$

$$G_{10} = G_q L$$

FIND: $S_{\text{Bek}} = 0$



But $S_{\text{micro}} = \ln [256]$

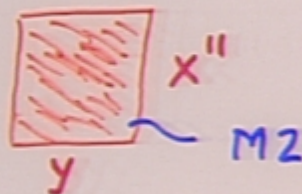
~ 0 also

[Does not grow with n_1]

$$S_{\text{Bek}} = S_{\text{micro}} = 0$$

Why was $A=0$?

11-D :



Branes :

- Shrink directions along brane
- Expand directions $\perp$ to brane

$$x'' \rightarrow 0 \Rightarrow \phi \rightarrow -\infty$$

$$x^9 \rightarrow 0$$

$$A_{11} \rightarrow 0$$



$$\Rightarrow S_{\text{micro}} \rightarrow 0$$

TO PREVENT x_{11} FROM SHRINKING,^{12.}
 PUT M5 BRANES $\perp$ TO x_{11}

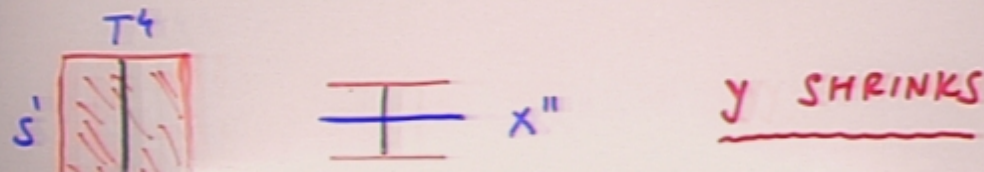
$\Rightarrow$ NS5 IN ΠA

$$M_{4,1} \rightarrow M_{4,1} \times T^4 \times \underbrace{S^1}_{\text{NS1}} \\ \text{NS5}$$

$$ds_{\text{string}}^2 = H_1^{-1} [-dt^2 + dy^2] + H_5 \sum_{i=1}^4 dx_i dx_i + \sum_{a=1}^4 dz_a dz_a$$

$$e^{2\phi} = \frac{H_5}{H_1}$$

$$H_1 = 1 + \frac{Q_1}{r^2}, \quad H_5 = 1 + \frac{Q_5}{r^2}$$



ADD MOMENTUM P ALONG y

13

$$E_p = \frac{n_p}{R_y} \quad : \quad R_y \text{ tends to EXPAND}$$

$$M_{q,1} \rightarrow M_{q,1} \times \underbrace{T^4 \times S^1}_{\substack{\text{NSI} \\ \text{NS5} \\ \text{P}}}$$

$$ds_{\text{String}}^2 = H_1^{-1} [-dt^2 + dy^2 + K(dt + dy)^2] + H_5 \sum_{i=1}^4 dx_i dx_i + \sum_{a=1}^4 dz_a dz_a$$

$$e^{2\phi} = \frac{H_5}{H_1}$$

$$H_1 = 1 + \frac{Q_1}{r^2}, \quad H_5 = 1 + \frac{Q_5}{r^2}, \quad K = \frac{Q_p}{r^2}$$

Near $r=0$, transverse space X_i

$$H_5 \sum_{i=1}^4 dx_i dx_i = H_5 [dr^2 + r^2 d\Omega_3^2] \\ \approx Q_5 \left[\frac{dr^2}{r^2} + d\Omega_3^2 \right]$$

$$[H_5 = 1 + \frac{Q_5}{r^2}]$$

$$\uparrow \\ A_{string}^{(S^3)} = 2\pi^2 Q_5^{3/2}$$

$$L_{string}^y = 2\pi R \left(\frac{K}{H_1} \right)^{1/2} \rightarrow 2\pi R \frac{Q_p^{1/2}}{Q_1^{1/2}}$$

$$V_{string}^{\tau^4} = (2\pi)^4 V$$

$$A_{10}^E = \left[\frac{g^E}{g^S} \right]^4 A_{10}^S = 2\pi^2 (2\pi R) (2\pi)^4 V \\ (Q_1 Q_5 Q_p)^{1/2}$$

Near $r=0$, transverse space X_i

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$$Q_1 = \frac{g^2 \alpha'^3}{V} n_1$$

15

$$Q_5 = \alpha' n_5$$

$$Q_{10} = 8\pi^6 g^2 \alpha'^4$$

$$Q_p = \frac{g^2 \alpha'^4}{V R^2} n_p$$

$$S_{\text{Bek}} = 2\pi \sqrt{n_1 n_5 n_p}$$

Note that all moduli V, R, g have cancelled out. This is an entropy of BPS holes [Mass = Charge]

So if the moduli did not cancel, then we could never explain S_{Bek} by a microscopic count

DUALITIES

16

$$\begin{array}{c}
 M_{9,1} \\
 \text{IIA}
 \end{array}
 \rightarrow
 \underbrace{M_{4,1}}_{x^0, x^1, x^2, x^3, x^4}
 \times
 \underbrace{T^4}_{x^6, x^7, x^8, x^9}
 \times
 \underbrace{S^1}_{x^5=y}$$

$$\begin{array}{c}
 \text{NSI} \\
 \hline
 \text{NS5} \\
 \hline
 \text{P}
 \end{array}$$

$$\begin{array}{c}
 \text{NSI-NS5-P} \\
 \text{IIA}
 \end{array}
 \xrightarrow{T_6}
 \begin{array}{c}
 \text{NSI-NS5-P} \\
 \text{IIB}
 \end{array}
 \xrightarrow{S}
 \begin{array}{c}
 \text{D1-D5-P} \\
 \text{IIB}
 \end{array}$$

Any permutation can be obtained:

$$\begin{array}{ccccc}
 \begin{array}{c} \text{D1-D5} \\ \text{IIB} \end{array} & \xrightarrow{S} & \begin{array}{c} \text{NSI-NS5} \\ \text{IIB} \end{array} & \xrightarrow{T_5} & \begin{array}{c} \text{P-NS5} \\ \text{IIA} \end{array} \\
 & & & & \downarrow T_6 \\
 & & & & \begin{array}{c} \text{P-NS5} \\ \text{IIB} \end{array} \\
 & & \begin{array}{c} \text{P-D5} \\ \text{IIB} \end{array} & \xleftarrow{S} & \begin{array}{c} \text{P-NS5} \\ \text{IIB} \end{array} \\
 & \begin{array}{c} \text{P-D1} \\ \text{IIB} \end{array} & \xleftarrow{T_{6789}} & \begin{array}{c} \text{P-D5} \\ \text{IIB} \end{array} & \\
 & \searrow S & & & \\
 & & \begin{array}{c} \text{P-NSI} \\ \text{IIA} \end{array} & &
 \end{array}$$

DUALITIES

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$$\begin{array}{c}
 \text{NSI} \\
 \hline
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 \text{NSI-NS5-P} & \xrightarrow{T_6} & \text{NSI-NS5-P} & \xrightarrow{S} & \text{D1-D5-P} \\
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 \text{IIB} & & \text{IIB} & & \text{IIA} \\
 & & & & \downarrow T_6 \\
 & & & & \text{P-NS5} \\
 & & & & \text{IIB} \\
 & & \xleftarrow{S} & \text{P-D5} & \xleftarrow{T_{6789}} \text{P-D1} \\
 & & & \text{IIB} & \text{IIB} \\
 & & & & \downarrow S \\
 & & & & \text{P-NSI} \\
 & & & & \text{IIB}
 \end{array}$$

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$$\begin{array}{c}
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 \end{array}$$

$$\begin{array}{c}
 \text{NSI} \\
 \hline
 \text{NSS} \\
 \hline
 \text{P}
 \end{array}$$

$$\begin{array}{c}
 \text{NSI-NSS-P} \xrightarrow{T_6} \text{NSI-NSS-P} \xrightarrow{S} \text{DI-DS-P} \\
 \text{IIA} \qquad \qquad \text{IIB} \qquad \qquad \text{IIB}
 \end{array}$$

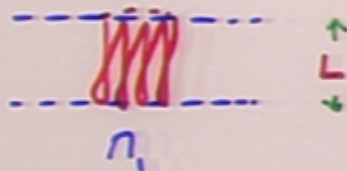
Any permutation can be obtained:

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 \text{IIB} & & \text{IIB} & & \text{IIA} \\
 & & & & \downarrow T_6 \\
 & & & & \text{P-NSS} \\
 & & & & \text{IIB} \\
 & \xleftarrow{T_{6789}} & \text{P-DS} & \xleftarrow{S} & \\
 & & \text{IIB} & & \\
 & \searrow S & & & \\
 & & \text{P-NSI} & & \\
 & & \text{IIB} & &
 \end{array}$$

THE MICROSCOPIC COUNT OF STATES

1-CHARGE

NSI



$$L_T = n_1 L \quad [\text{one long string}]$$

$$S_{\text{micro}} = 0$$

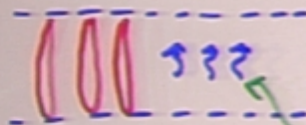
2-CHARGES

NSI - P



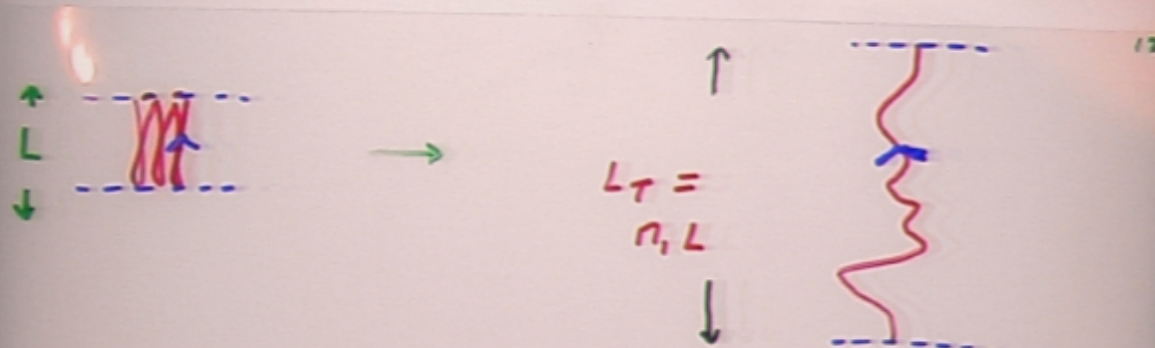
BOUND STATE

"TRAVELLING
WAVES ON
STRING"



massless modes

UNBOUND
STATE



Travelling wave carries momentum P up the string

Momentum is partitioned among different Fourier modes

1 quantum of Fourier mode K :

$$e_K = p_K = \frac{2\pi K}{L_T}$$

Total momentum

$$P = \frac{2\pi n_p}{L} = \frac{2\pi n_1 n_p}{L_T}$$

$$\sum_i m_i k_i = n_1 n_p$$

We must count partitions of

n, n_p

$$\rightarrow e^{2\pi\sqrt{\frac{n n_p}{6}}}$$

But. We have 8 transverse
directions of vibration

Susy $\Rightarrow$ 8 fermionic degrees
of freedom $\rightarrow \frac{8}{2} = 4$ bosons

$8 + 4 = 12$ Effective bosons

$\frac{n, n_p}{12}$ for each effective boson

$$\Rightarrow \left[e^{2\pi\sqrt{\frac{n n_p}{6 \times 12}}} \right]^{12} = e^{2\pi\sqrt{2} \sqrt{n n_p}}$$

$$S_{\text{micro}} = \ln [\#] = 2\pi\sqrt{2} \sqrt{n n_p}$$

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$$\Rightarrow \left[e^{2\pi\sqrt{\frac{n n_p}{6 \times 12}}} \right]^{12} = e^{2\pi\sqrt{2}\sqrt{n n_p}}$$

$$S_{\text{micro}} = \ln[\#] = 2\pi\sqrt{2}\sqrt{n n_p}$$