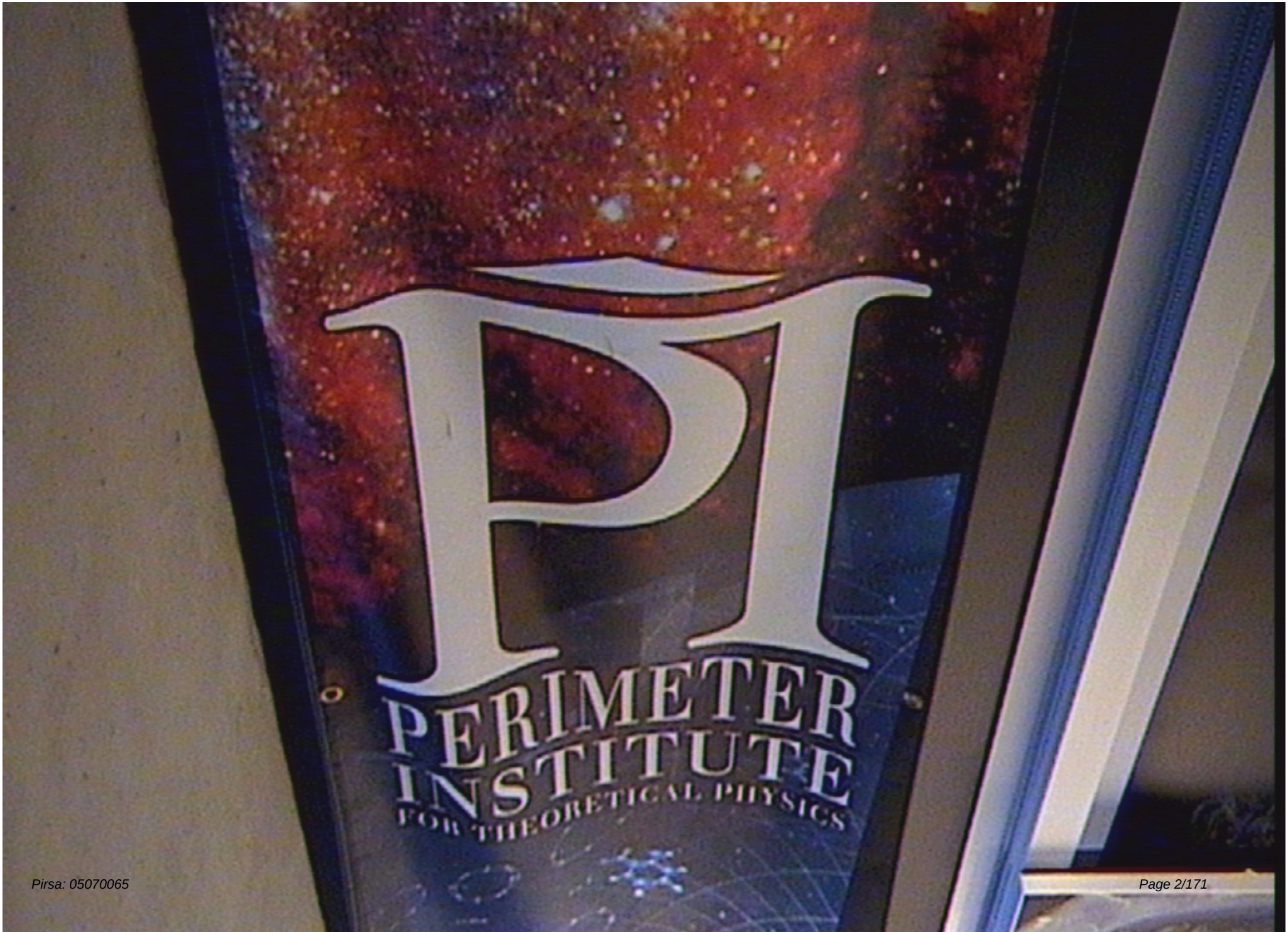
Title: Landscape Studies

Date: Jul 02, 2005 01:00 PM

URL: http://pirsa.org/05070065

Abstract:


PI
PERIMETER
INSTITUTE
FOR THEORETICAL PHYSICS

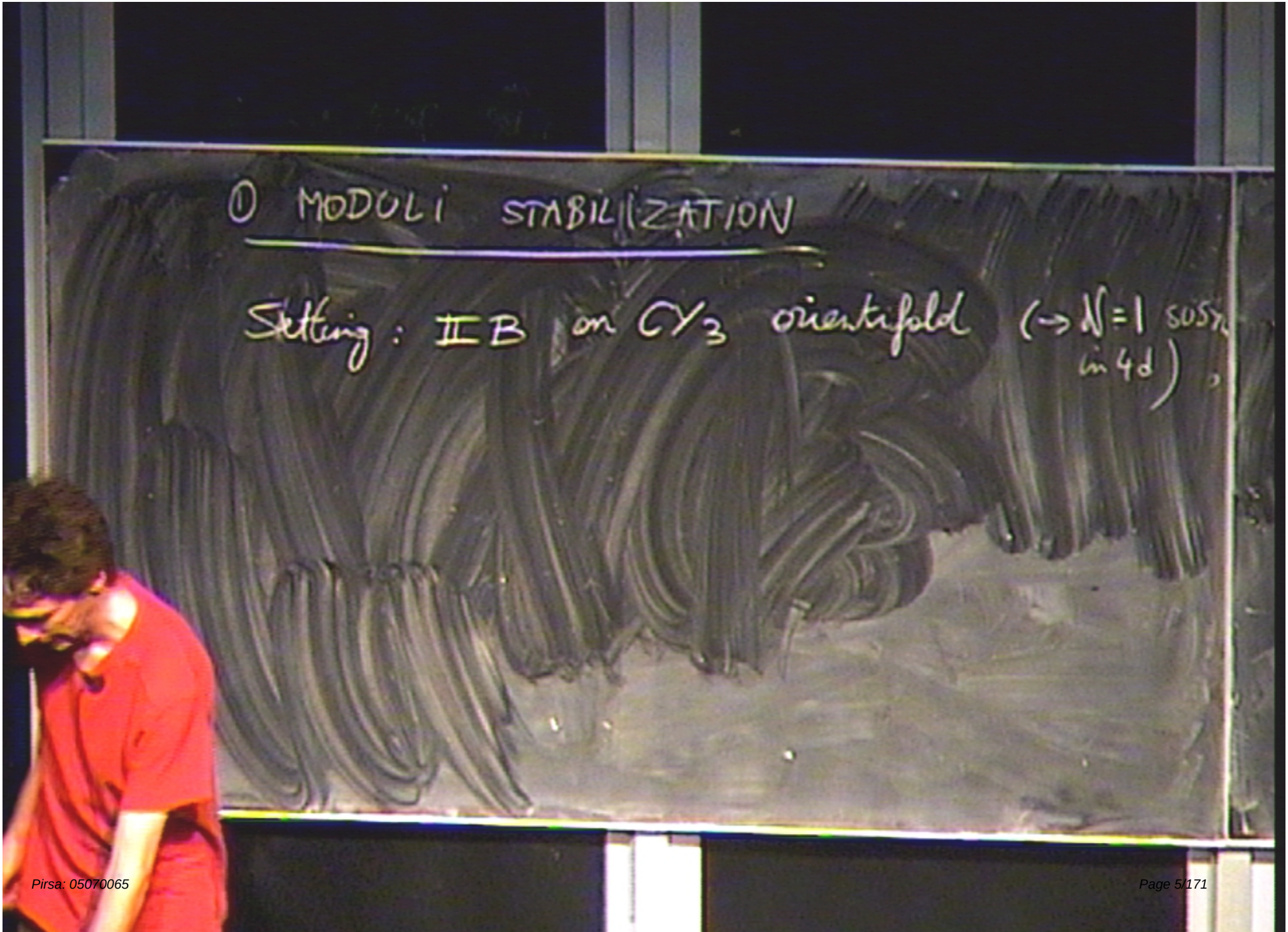
LANDSCAPE STUDIES - LECTURE 2

1) MODULI STABILIZATION (KKLT)

2) STATISTICS II

0 MODULI STABILIZATION

Setting : IIB on CY_3 orientifold (→ N


0 MODULI STABILIZATION
Setting : IIB on CY_3 orientifold (→ $\mathcal{N}=1$ SUSY in 4d),

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$\rightsquigarrow$ O3 and O7

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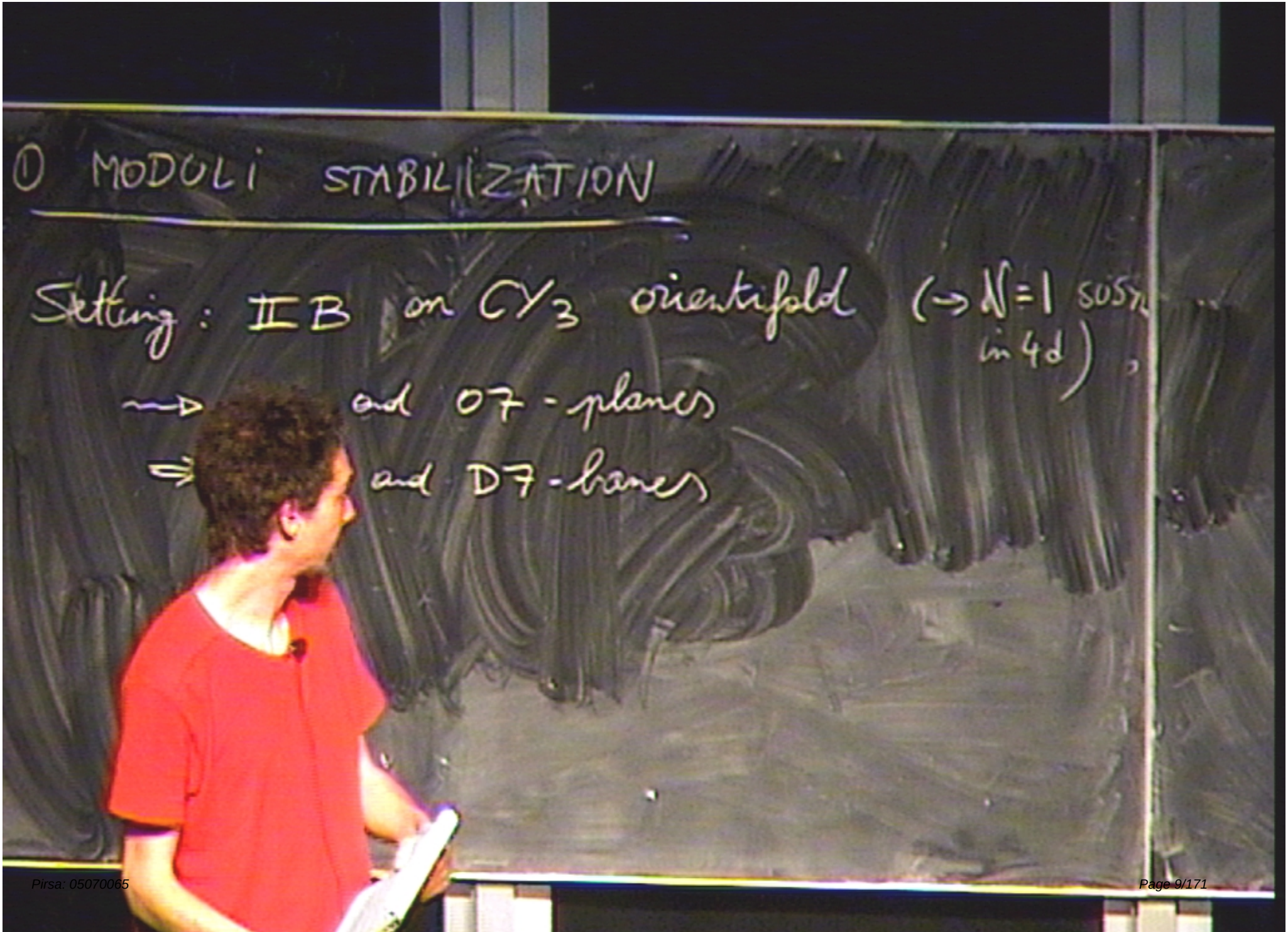
Setting: IIB on CY_3 orientifold ($\rightarrow \mathcal{N}=1$ SUSY in 4d).

$\rightsquigarrow$ O3 and O7-planes

$\Rightarrow$ D3 and D7-branes


0 MODULI STABILIZATION
Setting : IIB on CY_3 orientifold ($\to \mathcal{N}=1$ SUSY in 4d) ;
$\rightsquigarrow$ and O7-planes
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Setting: IIB on CY_3 orientifold X ($\to \mathcal{N}=1$ SUSY in 4d).

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Moduli: $h^{21}(X)$ $\mathbb{C}$-struct. mod.

$h^{11}(X)$ Kähler mod. $\to$ volumes of 4-cycles V_i

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Setting: IIB on CY_3 orientifold X ($\to \mathcal{N}=1$ SUSY in 4d).

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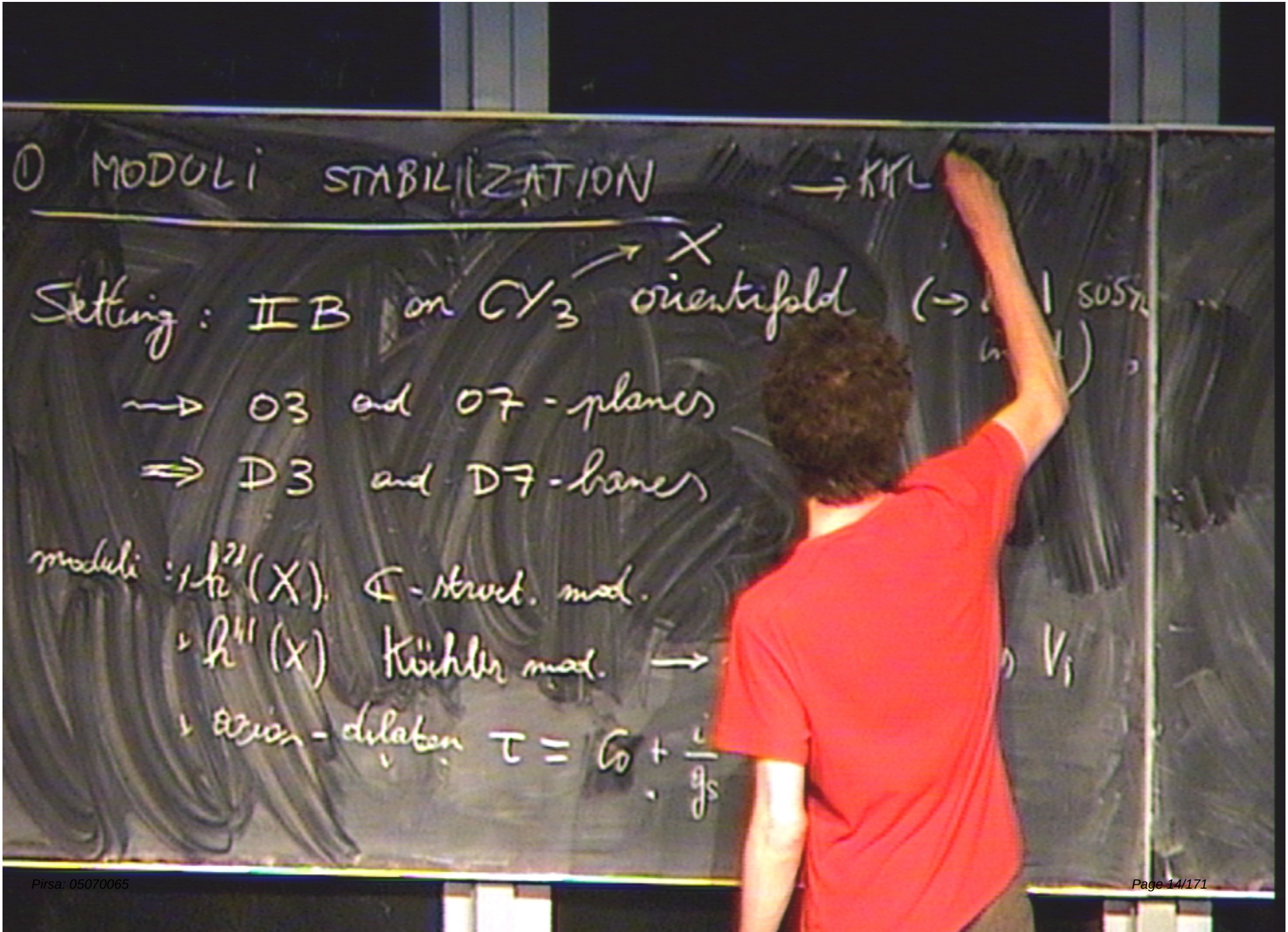
$h^{11}(X)$ Kähler mod. $\to$ volumes of 4-cycles V_i

axion-dilaton $\tau = C_0 + \frac{i}{g_s}$


0 MODULI STABILIZATION → KKL
Setting : IIB on CY3 → X orientifold (→
→ O3 and O7 - planes
⇒ D3 and D7 - branes
moduli : h21(X) ℂ - struct. mod.
h11(X) Kähler mod. →
axion - dilaton τ = C0 + i/gs

0 MODULI STABILIZATION → KKLT

Setting: IIB on CY_3 → X orientifold (→ $\mathcal{N}=1$ SUSY in 4d).

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⇒ D3 and D7-branes

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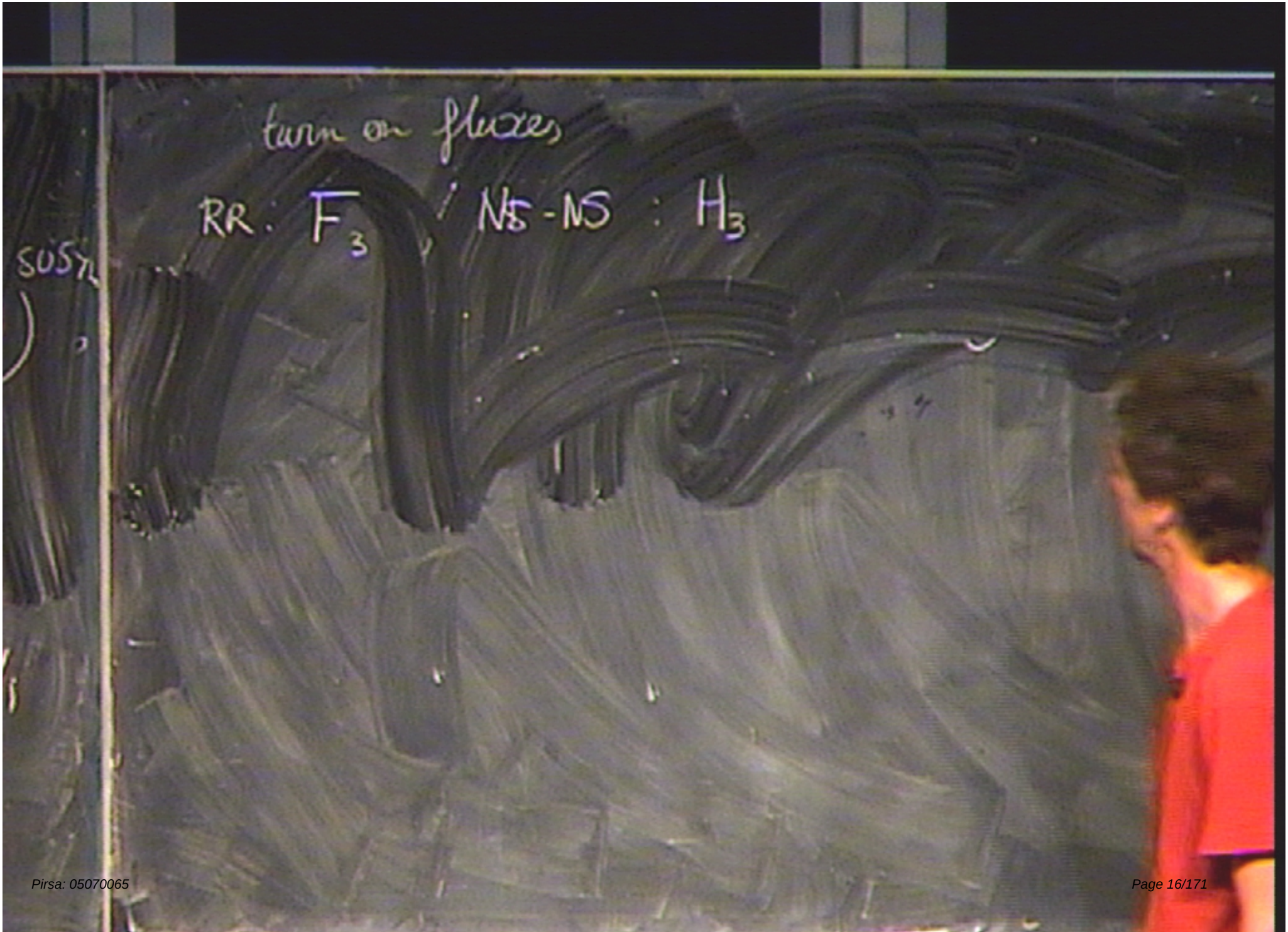
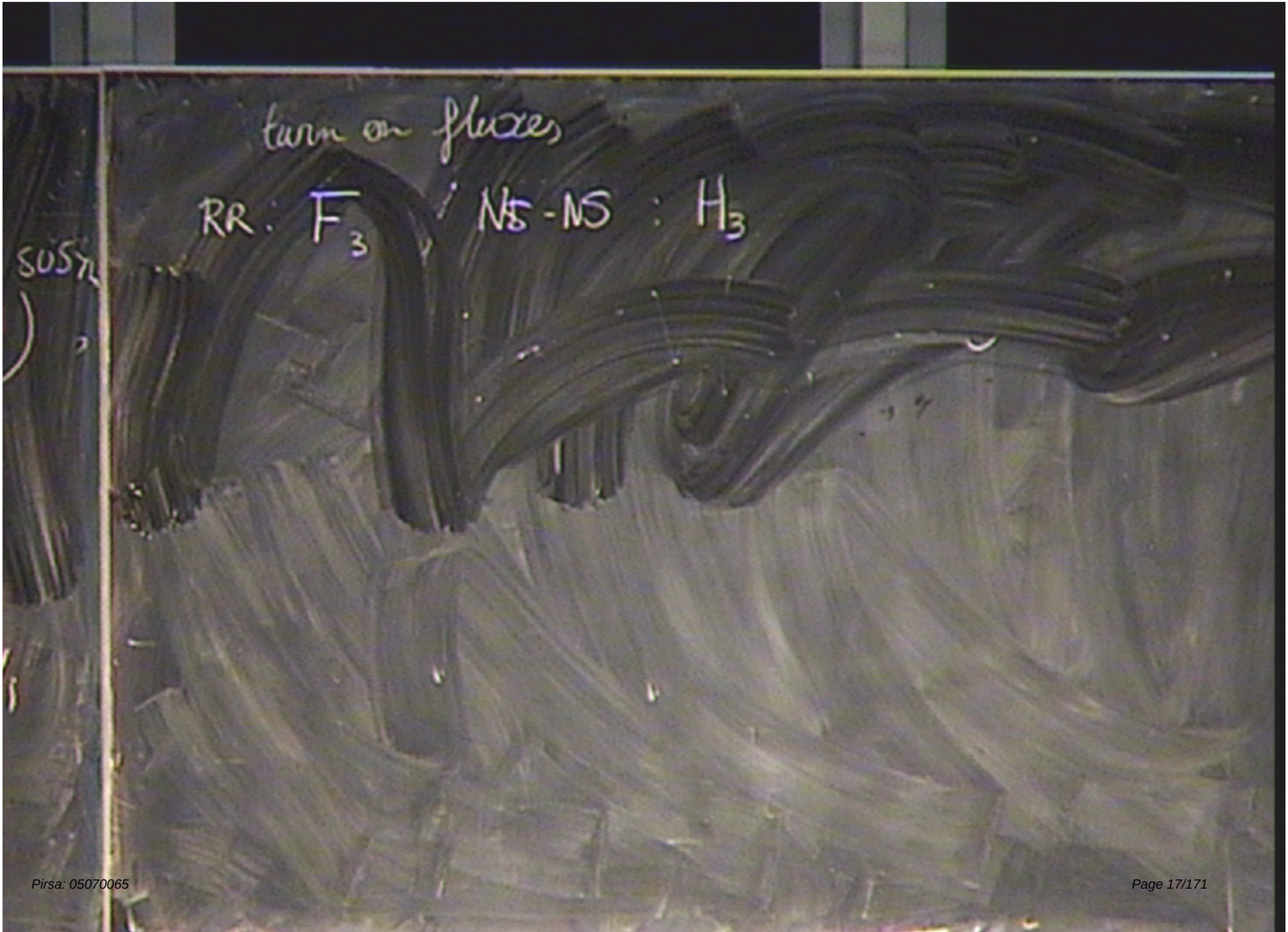

turn on fluxes
RR: F_3 / NS-NS : H_3
SUSY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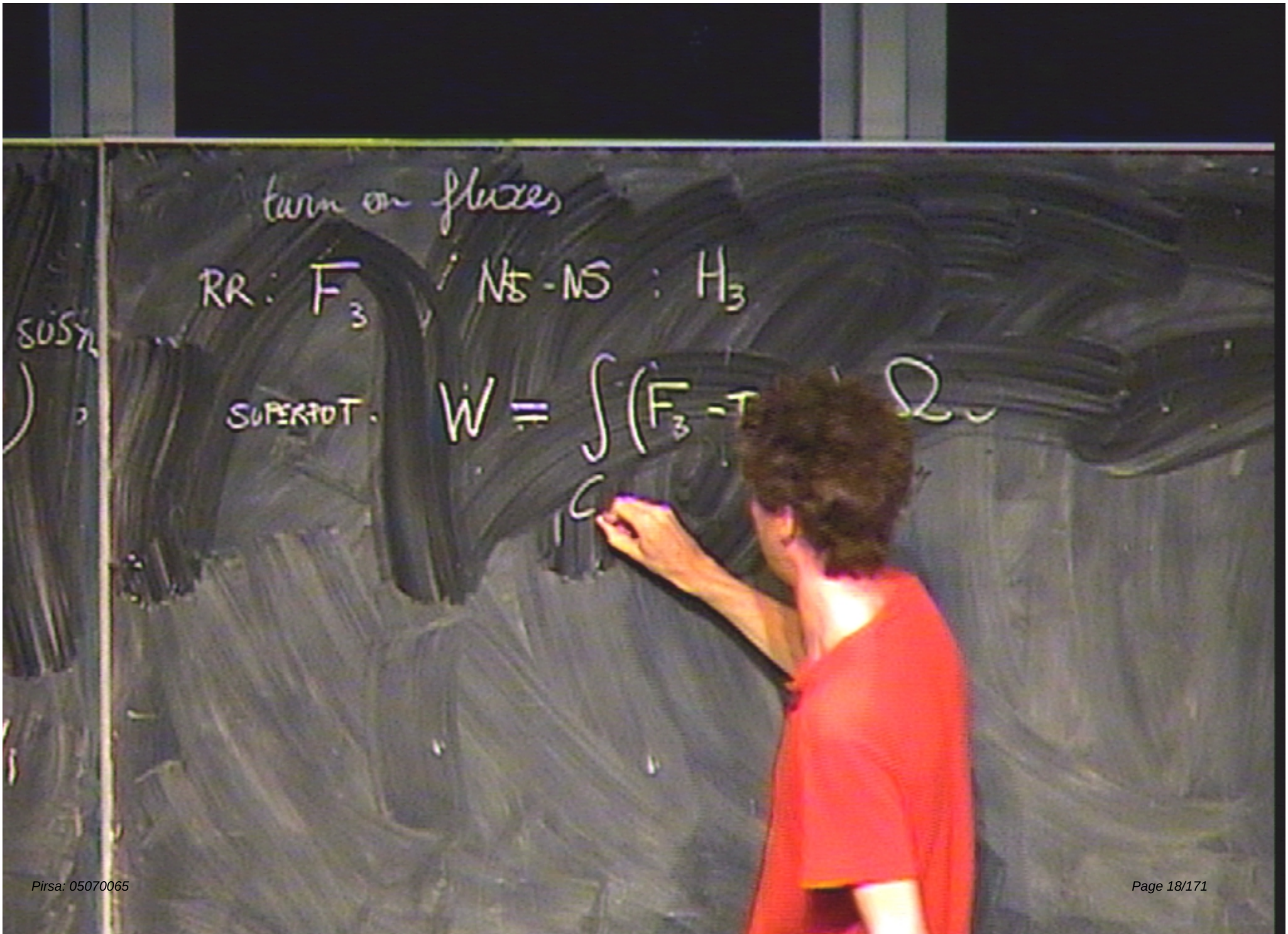
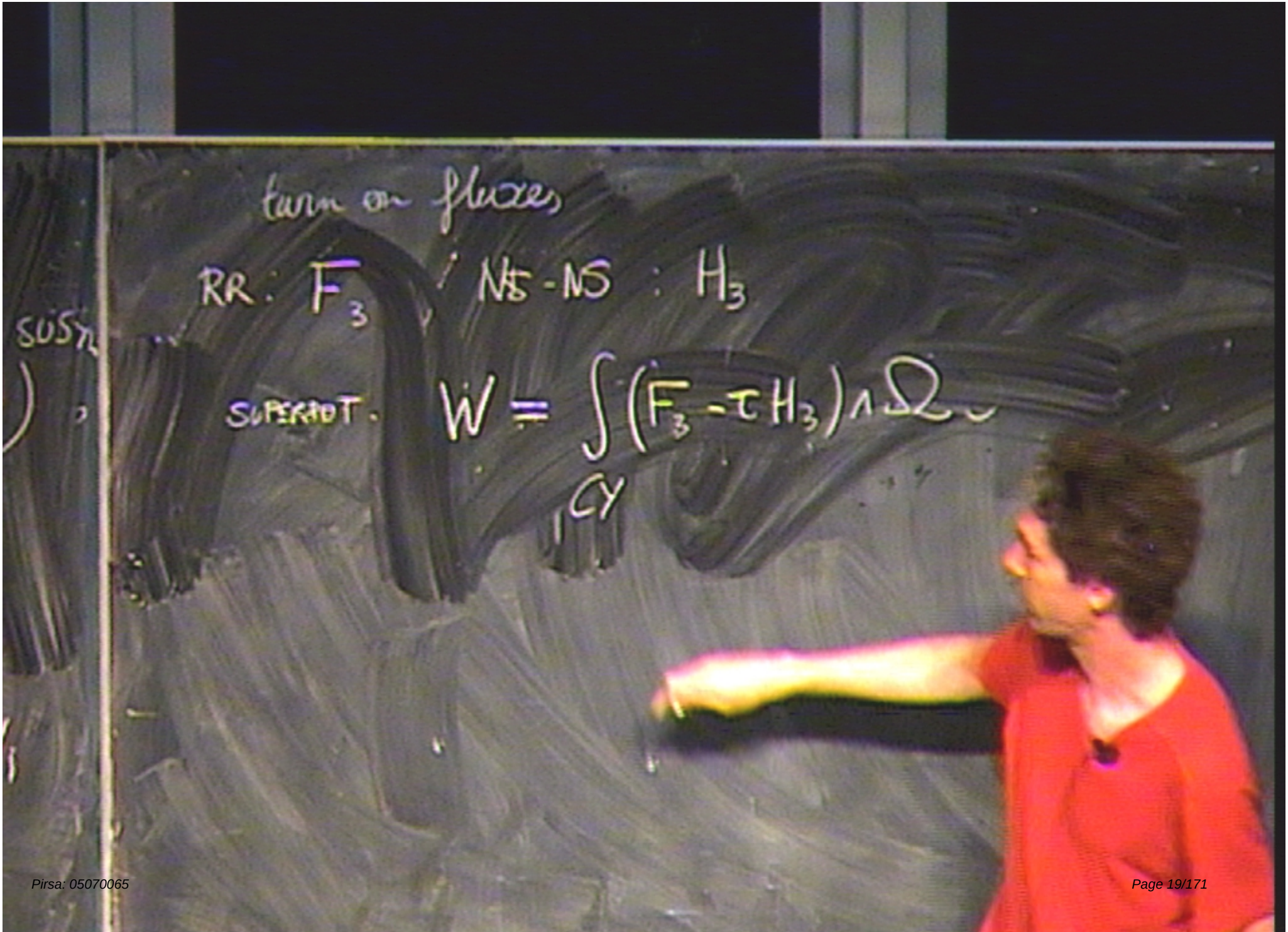
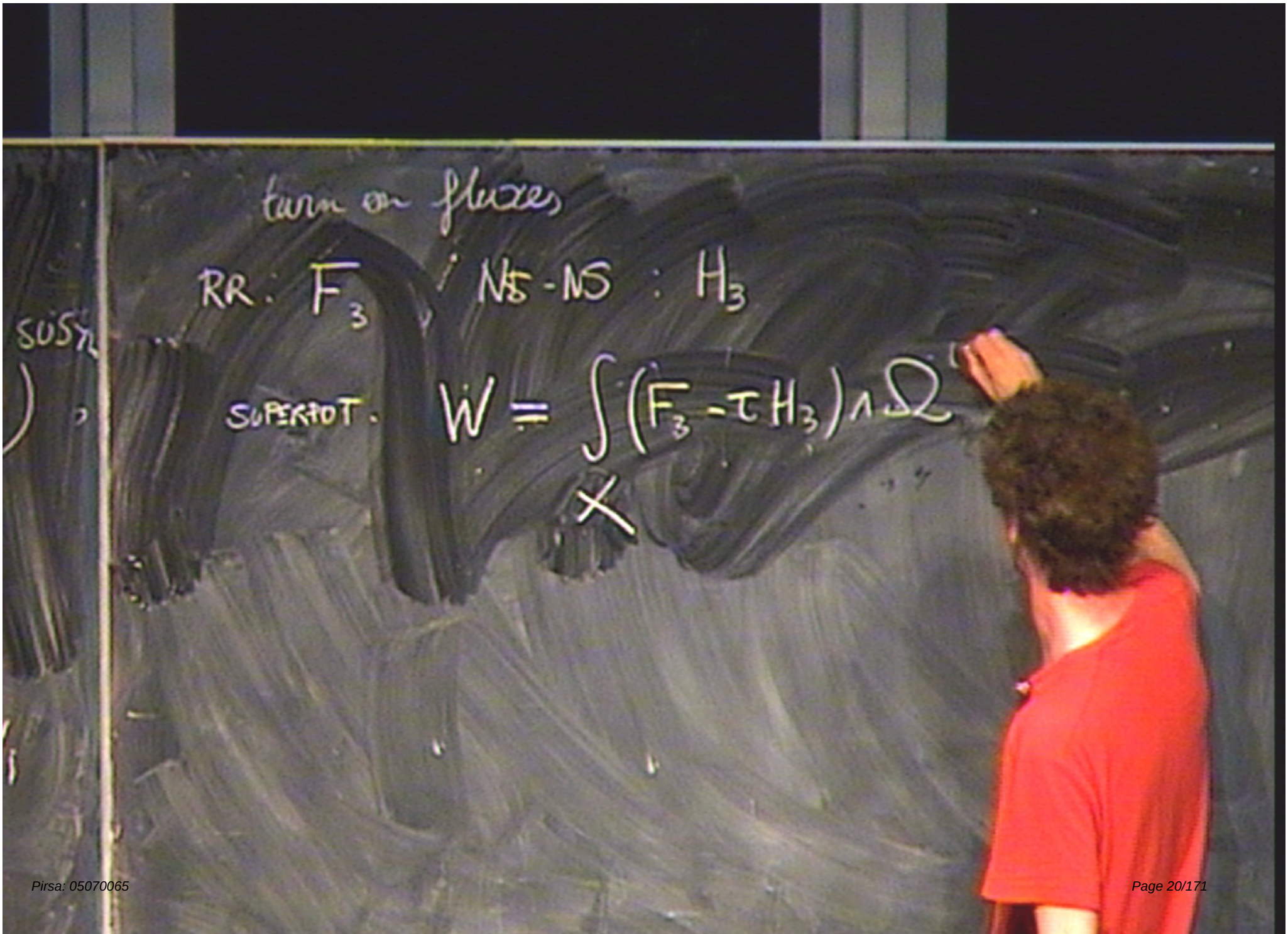
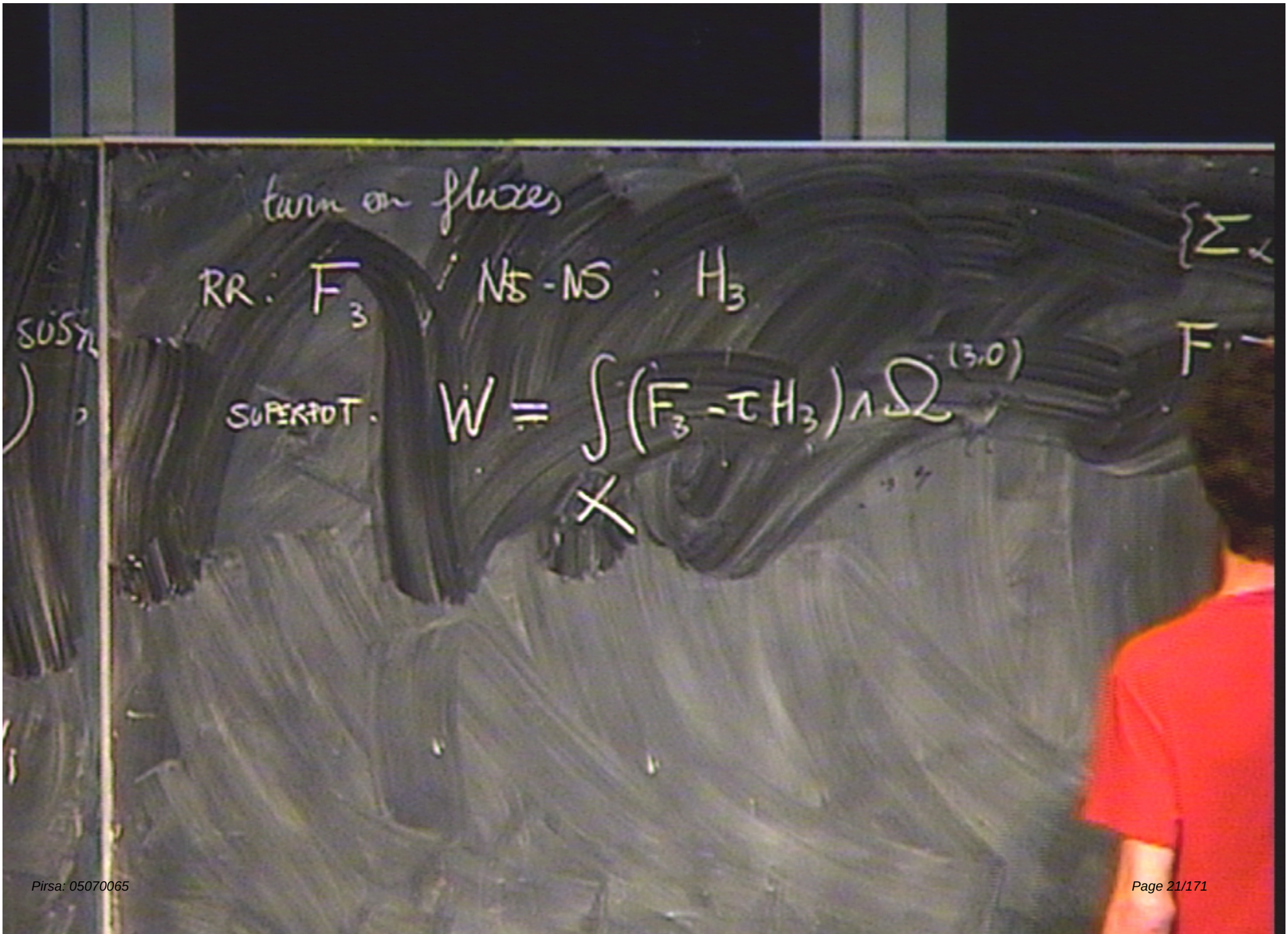

turn on fluxes
RR : F_3 / NS-NS : H_3
SUSY
SUPERPOT. $W = \int_C (F_3 - \tau$
Ω


turn on fluxes
RR: F_3 / NS-NS : H_3
SUSY
SUPERPOT. $W = \int_{CY} (F_3 - \tau H_3) \wedge \Omega$


turn on fluxes
RR : F_3 ; NS-NS : H_3
SUPERPOT. $W = \int_X (F_3 - \tau H_3) \wedge \Omega$


turn on fluxes
RR : F_3 / NS-NS : H_3
SUSY
SUPERPOT. $W = \int_X (F_3 - \tau H_3) \wedge \Omega^{(3,0)}$
$\{\Sigma_\alpha$
$F \cdot$

turn on fluxes

RR: F_3 / NS-NS: H_3

SUPERPOT. $W = \int_X (F_3 - \tau H_3) \wedge \Omega^{(3,0)}$

$\{\Sigma_\alpha\}$ basis of $H^2(X, \mathbb{Z})$

$F = N^\alpha \Sigma_\alpha$

$H = M^\alpha \Sigma_\alpha$

turn on fluxes

RR: F_3 ; NS-NS: H_3

SUPERPOT. $W = \int (F_3 - \tau H_3) \wedge \Omega^{(3,0)}$

$= (N^\alpha - \tau M^\alpha)\, \pi_\alpha(z)$

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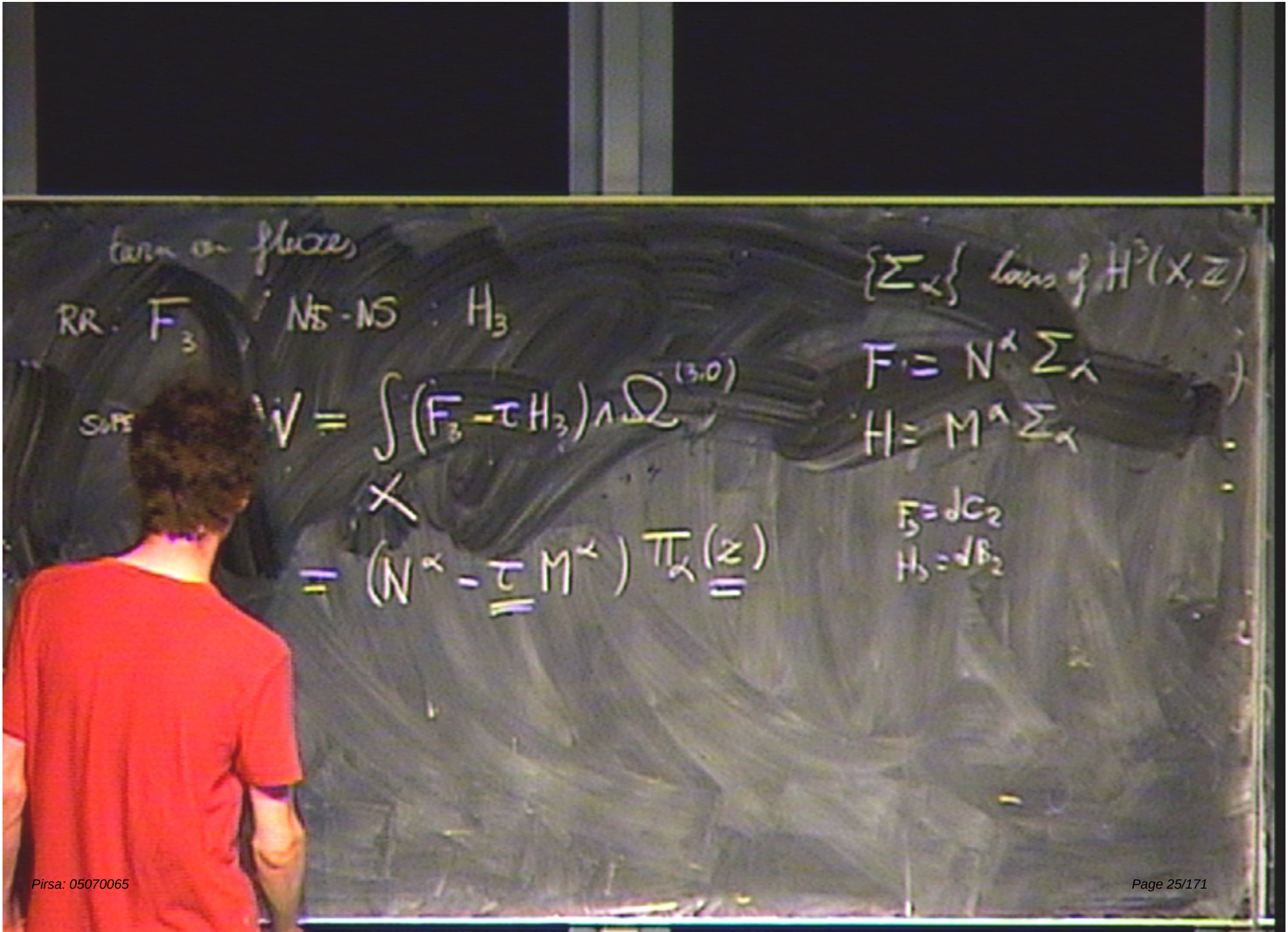
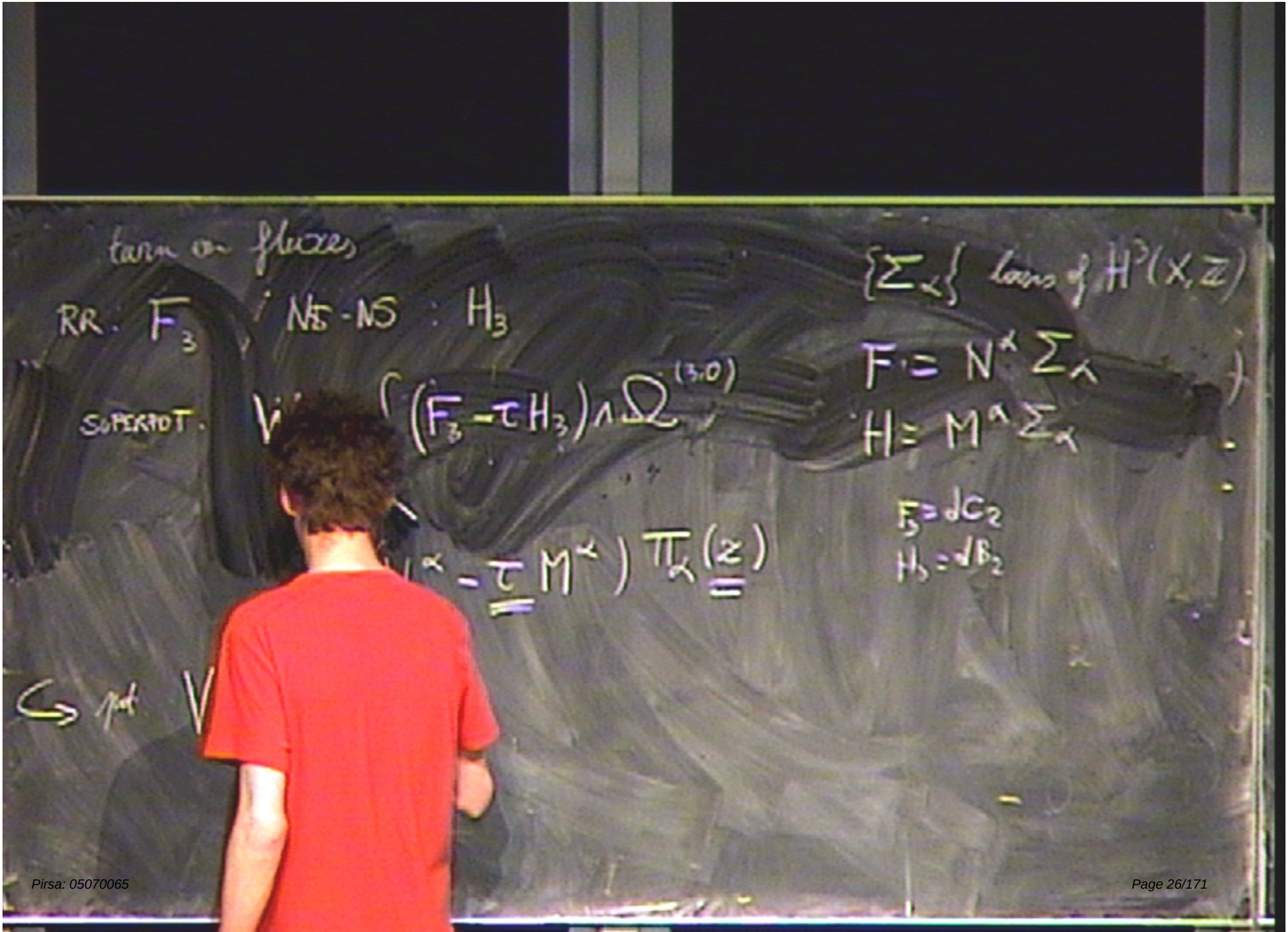

turn on fluxes
RR: F_3 ; NS-NS: H_3
$\{\Sigma_\alpha\}$ basis of $H^3(X,\mathbb{Z})$
$F = N^\alpha \Sigma_\alpha$
$H = M^\alpha \Sigma_\alpha$
$F_3 = dC_2$
$H_3 = dB_2$
$V = \int (F_3 - \tau H_3) \wedge \Omega^{(3,0)}$
$= (N^\alpha - \tau M^\alpha)\, \Pi_\alpha(z)$


turn on fluxes
RR: F_3 NS-NS : H_3
SUPERPOT. V $(F_3 - \tau H_3) \wedge \Omega^{(3,0)}$
$\{\Sigma_\alpha\}$ basis of $H^3(X, \mathbb{Z})$
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$\tau M^\alpha) \pi_\alpha(z)$
$\hookrightarrow$ pot V

turn on fluxes

RR: F_3 ; NS-NS: H_3

$\{\Sigma_\alpha\}$ basis of $H^2(X,\mathbb{Z})$

$F = N^\alpha \Sigma_\alpha$

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SUPERPOT. $W = \int_X (F_3 - \tau H_3) \wedge \Omega^{(3,0)}$

$= (N^\alpha - \tau M^\alpha)\, \Pi_\alpha(z)$

$\hookrightarrow$ pot $V = e^K (g^{a\bar{b}} D_a W \bar{D}_{\bar{b}} \bar{W} - 3|W|^2)$

Kähler pot

$D_a = (\partial_a + a_a k)$

turn on fluxes

RR: F_3 / NS-NS: H_3

SUPERPOT. $W = \int_X (F_3 - \tau H_3) \wedge \Omega^{(3,0)}$

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$\{\Sigma_\alpha\}$ basis of $H^2(X, \mathbb{Z})$

$F = N^\alpha \Sigma_\alpha$

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$F_3 = dC_2$

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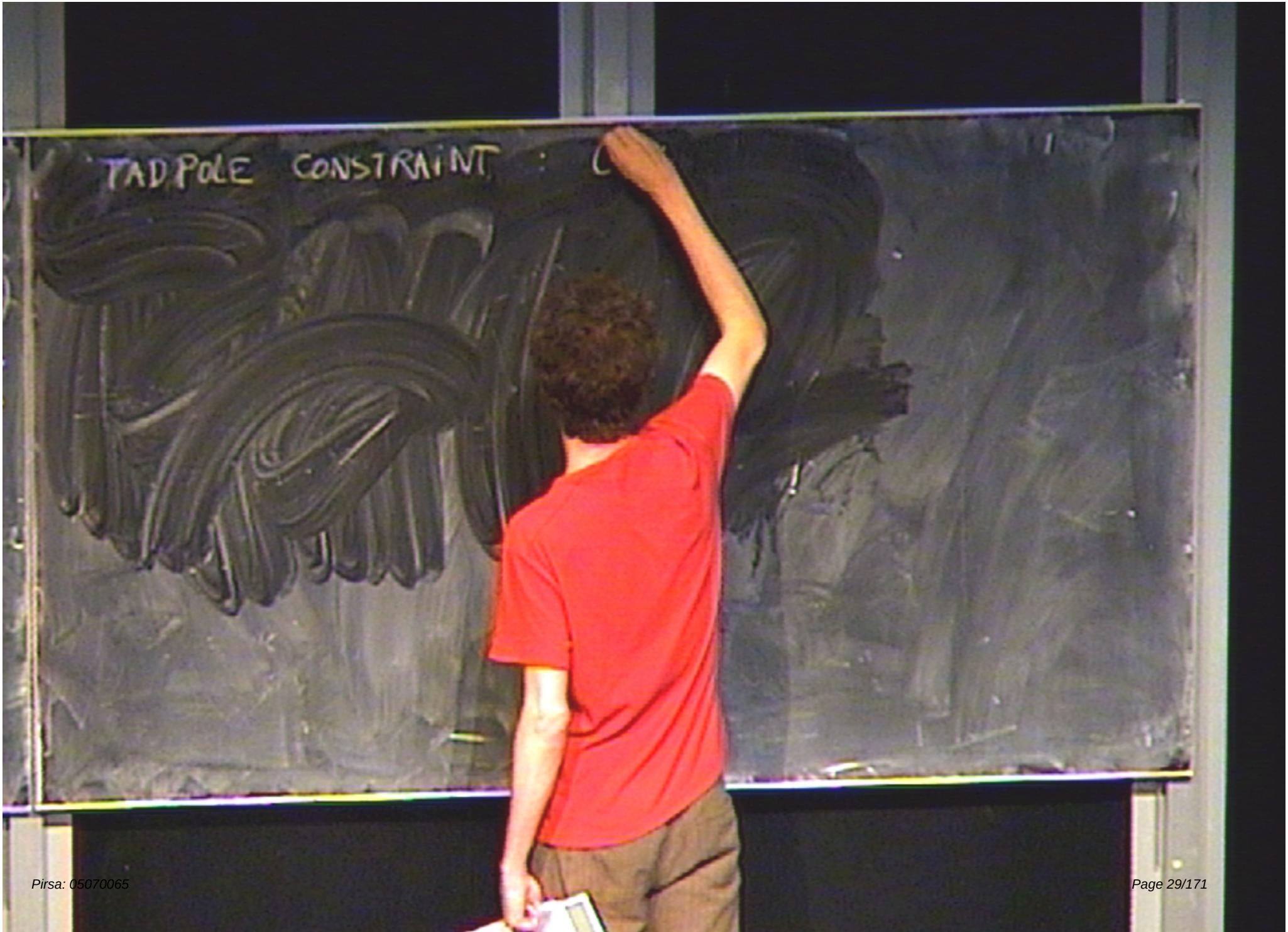
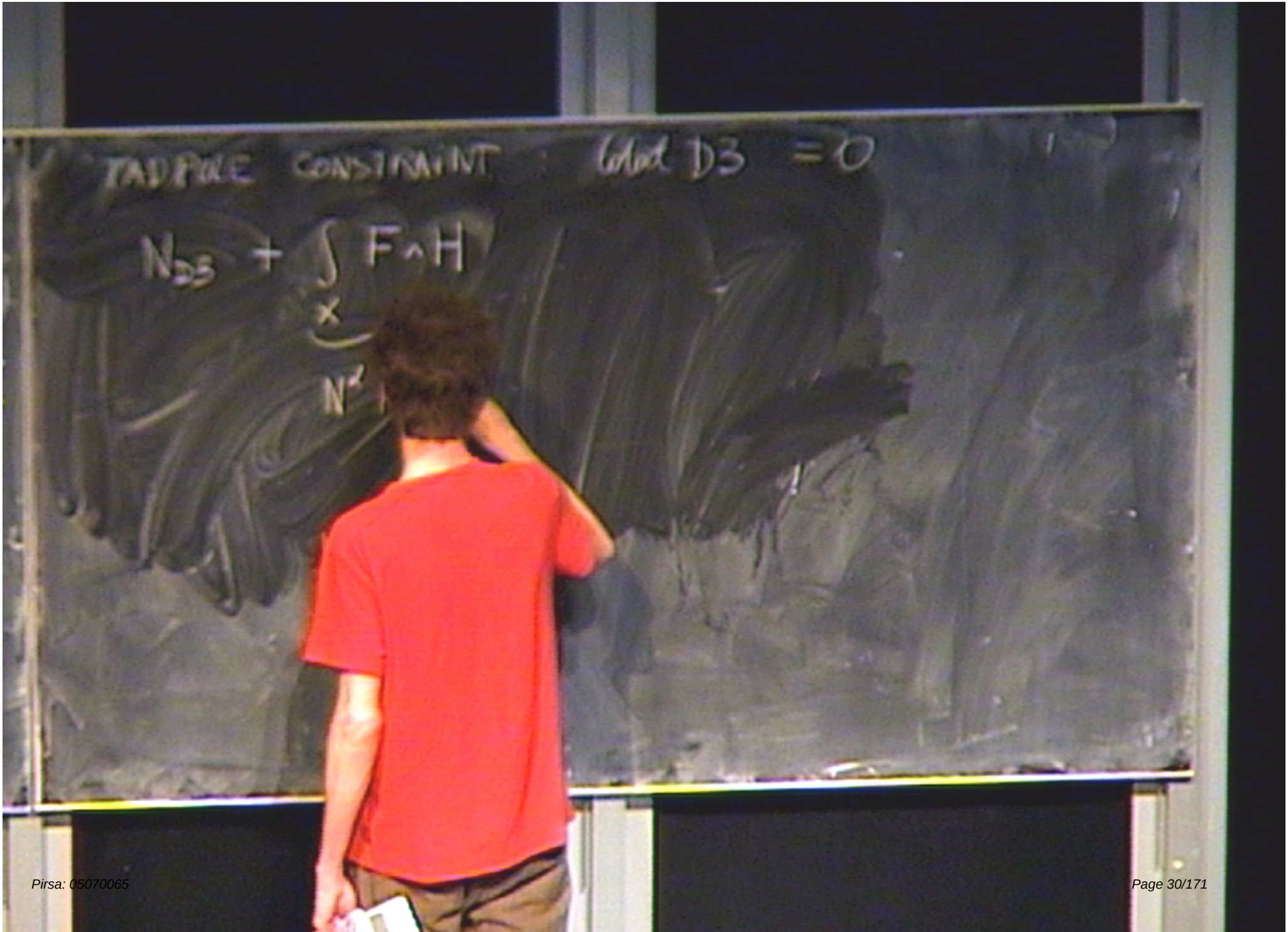
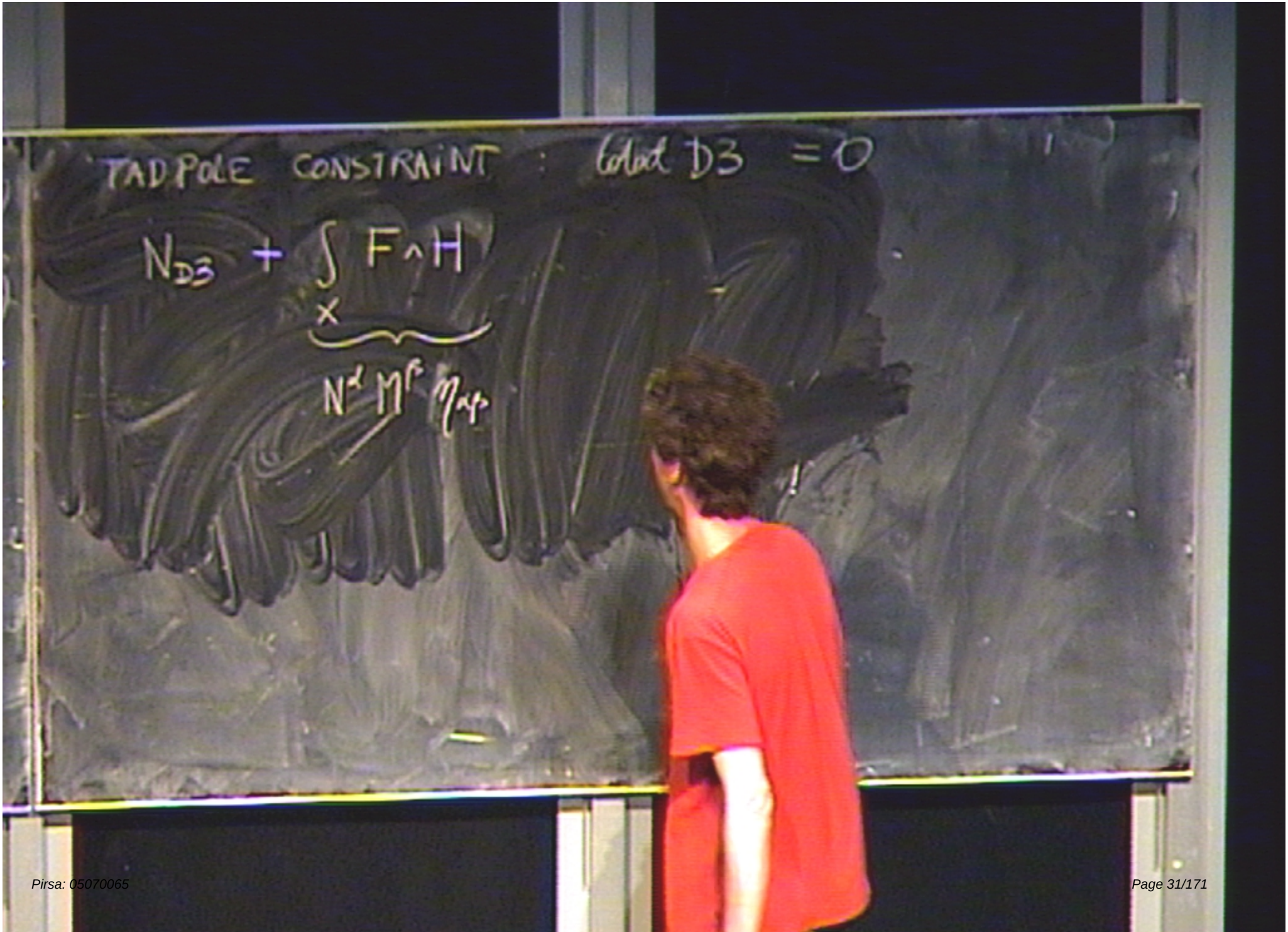
$\hookrightarrow$ 4d $V = e^{\mathcal{K}} (g^{a\bar{b}} D_a W \bar{D}_{\bar{b}} \bar{W} - 3|W|^2)$

Kähler 4d

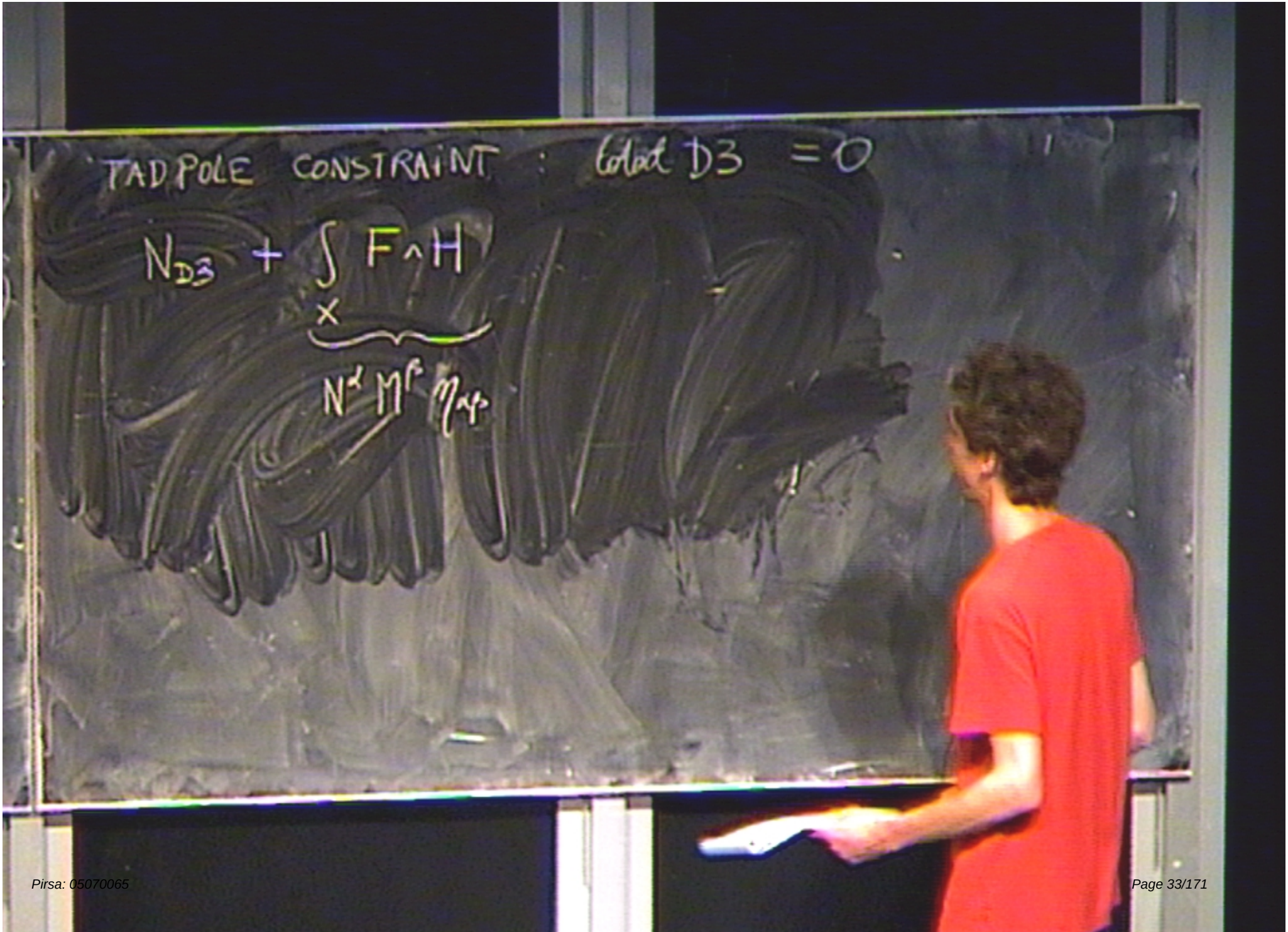
$\rightarrow D_a = (\partial_a + \partial_a K)$

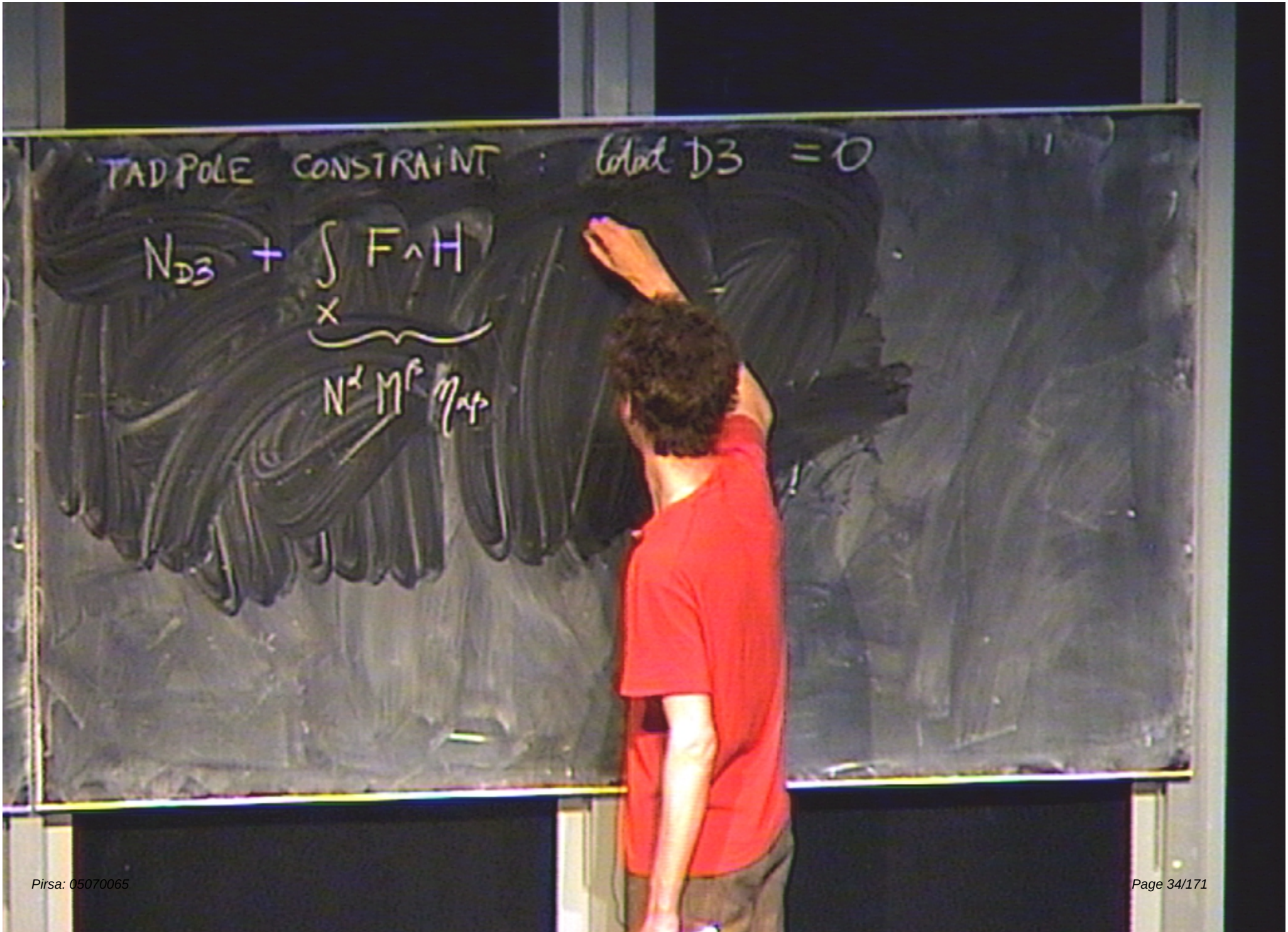

TADPOLE CONSTRAINT :

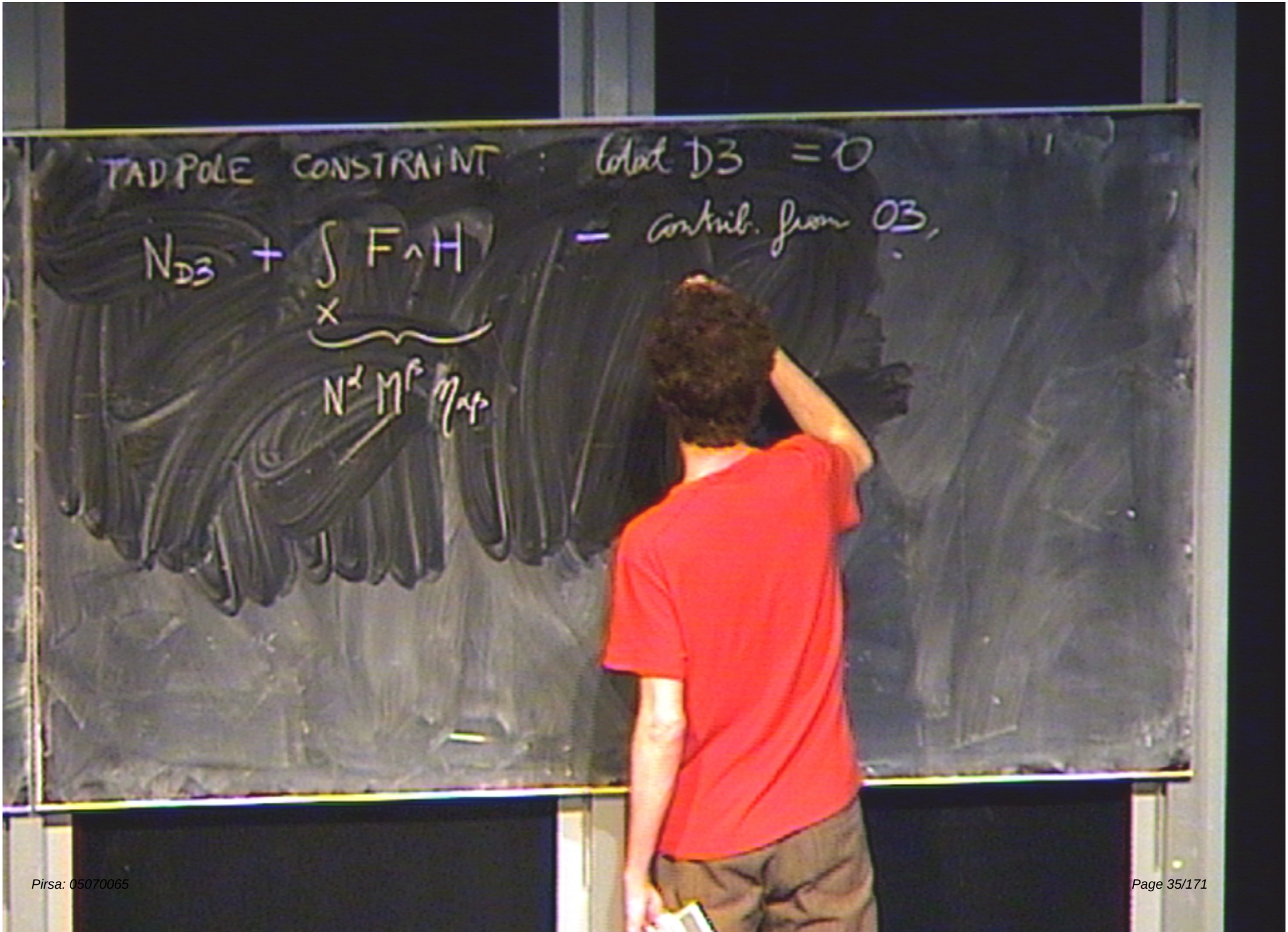

TADPOLE CONSTRAINT
Global D3 = 0
$N_{D3} + \int_X F \wedge H$
N^2


TADPOLE CONSTRAINT : Gauss D3 = 0
$N_{D3} + \int_{\times} F \wedge H$
$N^2 M^6 \eta_{AB}$


TADPOLE CONSTRAINT : Gauss $D3 = 0$
$N_{D3} + \int_X F \wedge H$
$N^2 M$


TADPOLE CONSTRAINT : Gauss D3 = 0
$N_{D3} + \int_{\times} F \wedge H$
$N^2 M^6 \eta_{4d}$

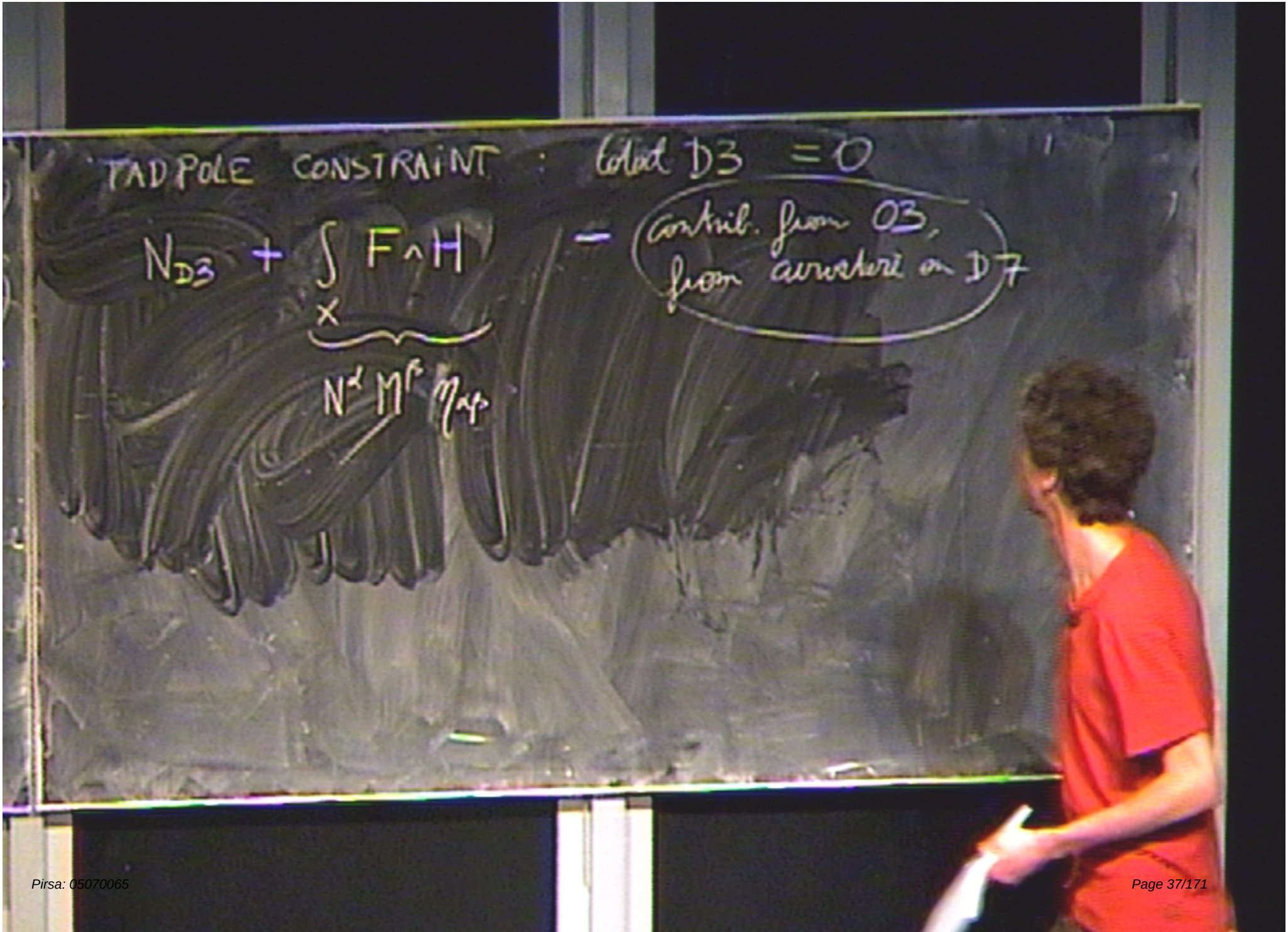

TADPOLE CONSTRAINT : total D3 = 0
$N_{D3} + \int_X F \wedge H$
$N^2 M^6 \eta_{AP}$

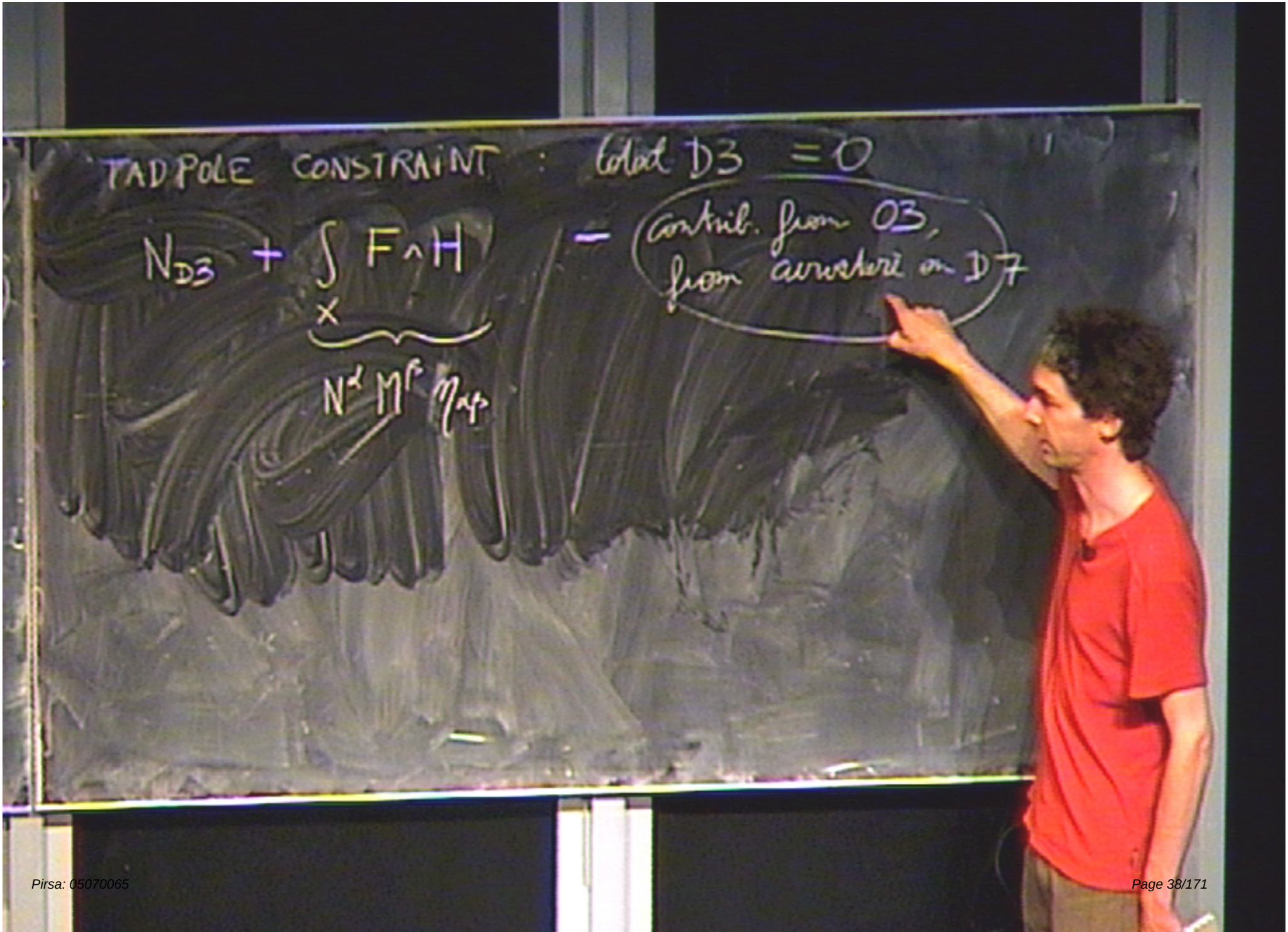

TADPOLE CONSTRAINT : Gauss D3 = 0
$N_{D3} + \int_{\times} F \wedge H$ — contrib. from O3,
$N^2 M^6 \eta_{AP}$

TADPOLE CONSTRAINT : total D3 = 0

$$N_{D3} + \underbrace{\int_X F \wedge H}_{N^\alpha M^\beta \eta_{\alpha\beta}}$$

— contrib. from O3, from curvature on D7


TADPOLE CONSTRAINT : Gauss D3 = 0
$N_{D3} + \int_{\times} F \wedge H$
$N^2 M^6$
− (contrib. from O3, from curvature on D7

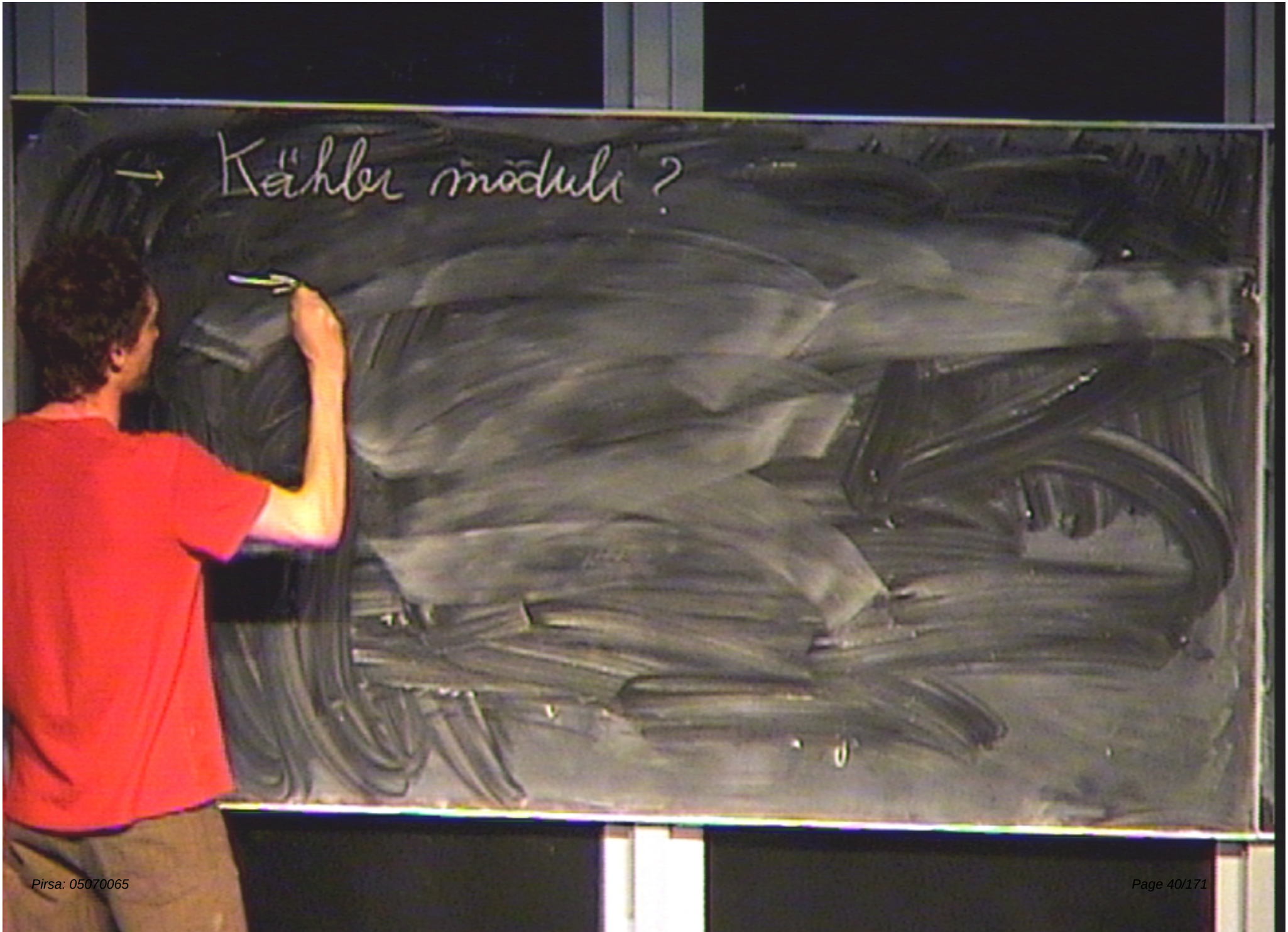

TADPOLE CONSTRAINT : Total D3 = 0
$N_{D3} + \int_{\times} F \wedge H$
$N^2 M^6 \eta_{AP}$
– (contrib. from O3, from anywhere on D7

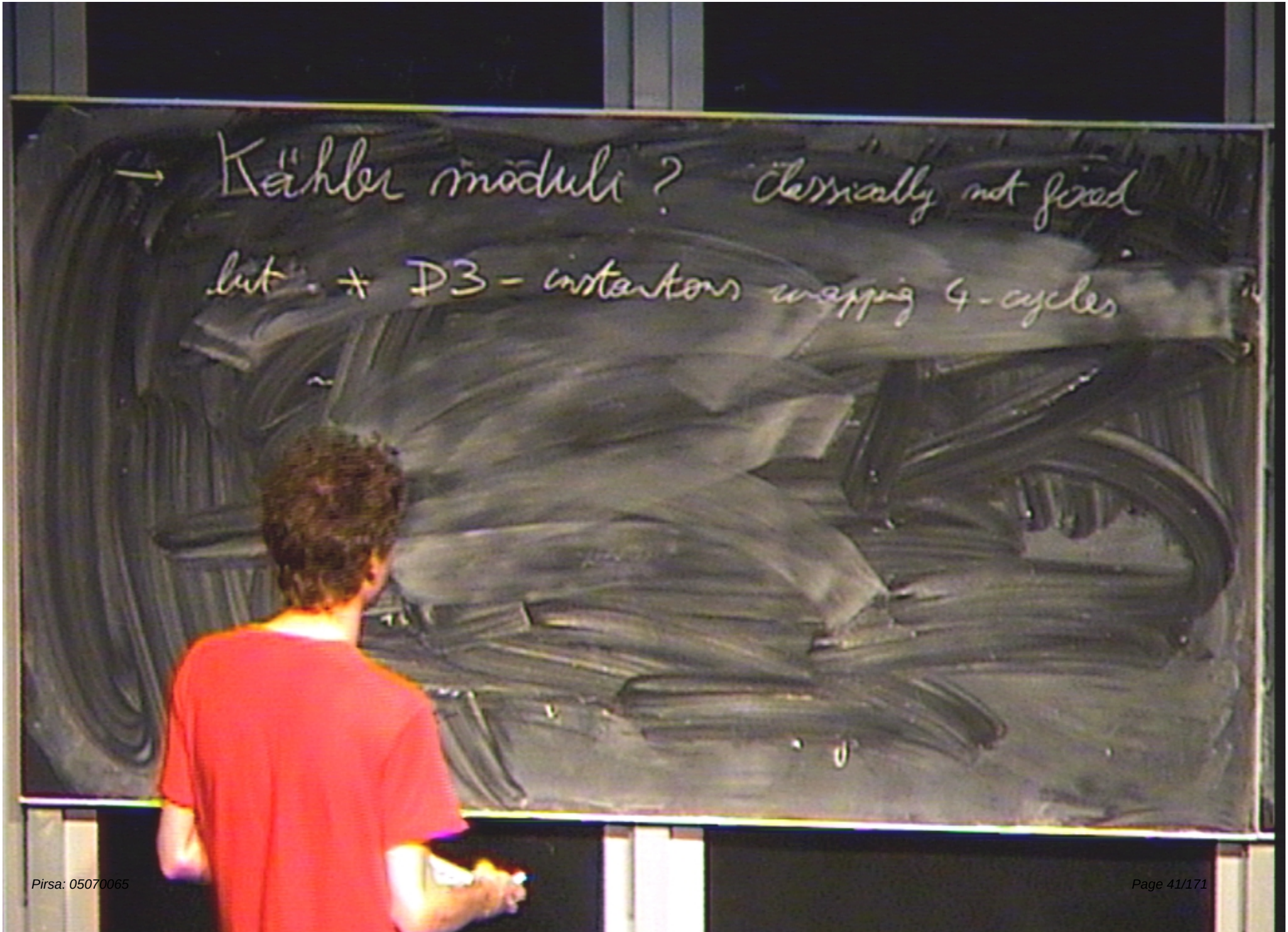
TADPOLE CONSTRAINT : Total D3 = 0

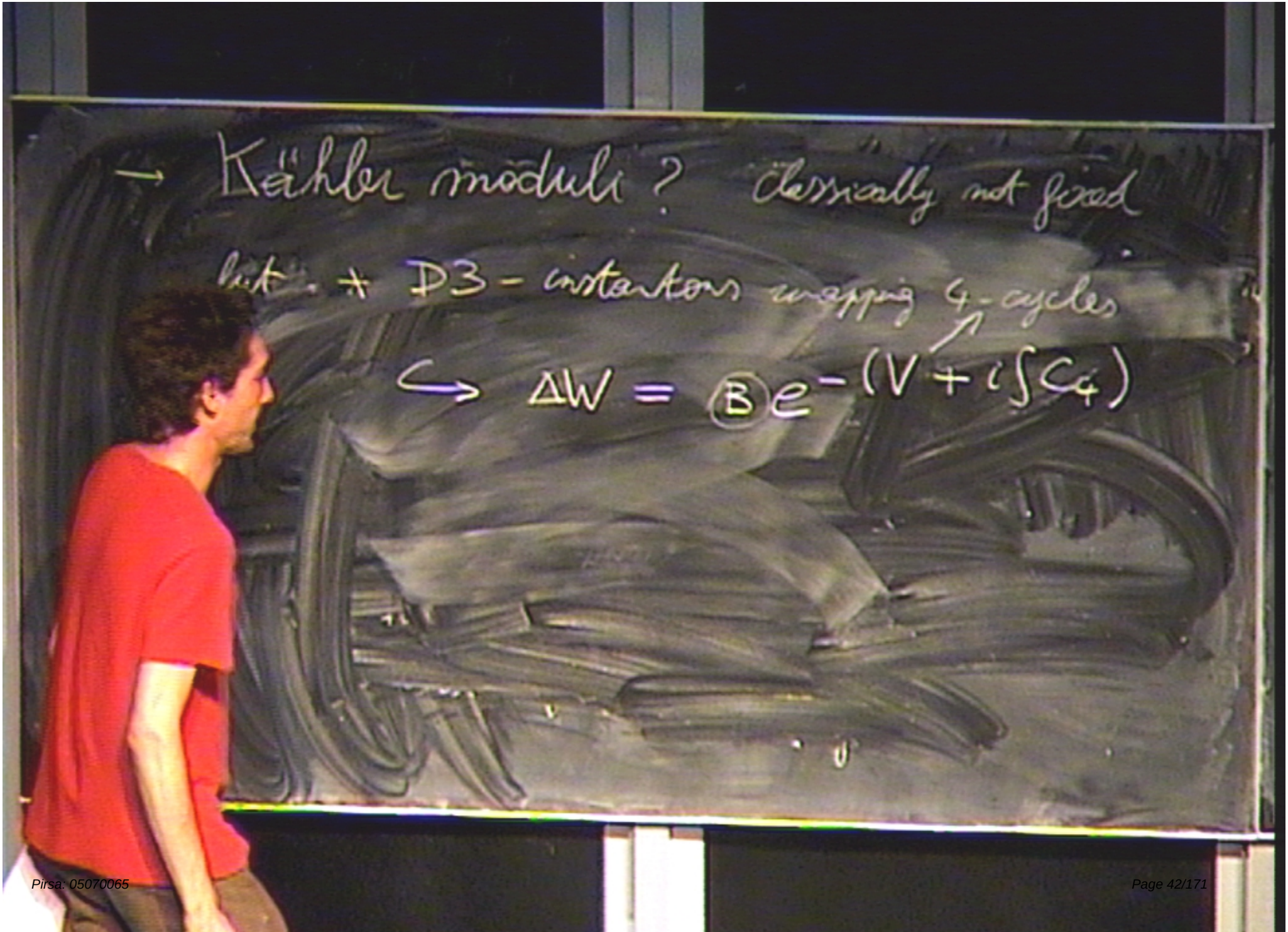
$N_{D3} + \underbrace{\int_{\times} F \wedge H}_{N^2 M^6 \eta^4} -$ (contrib. from O3, from curvature on D7)

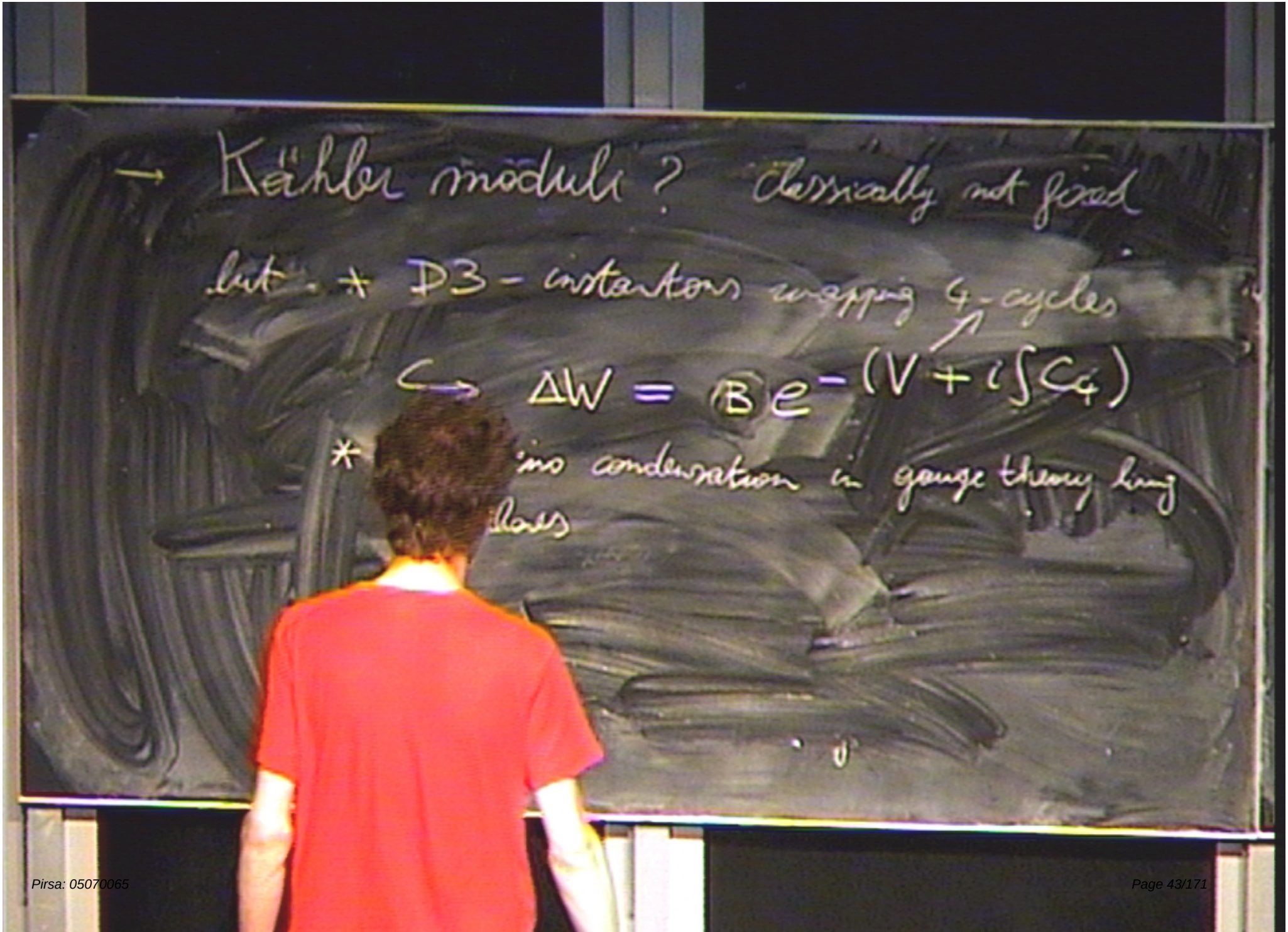
contrib. from O3 $\rightarrow -\frac{1}{4}$

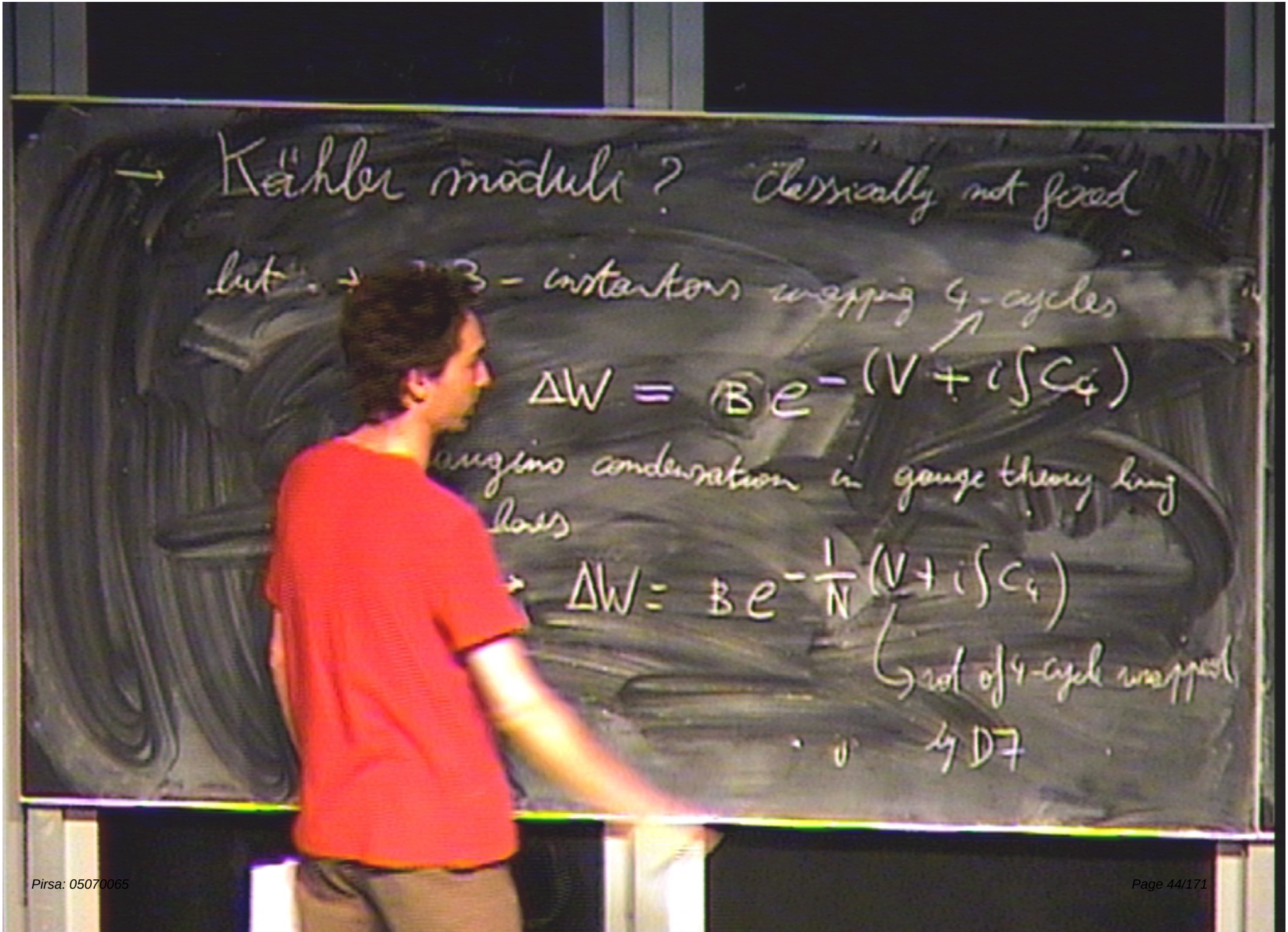
contrib. from curvature on D7 $\rightarrow -\frac{\chi(D7)}{24}$

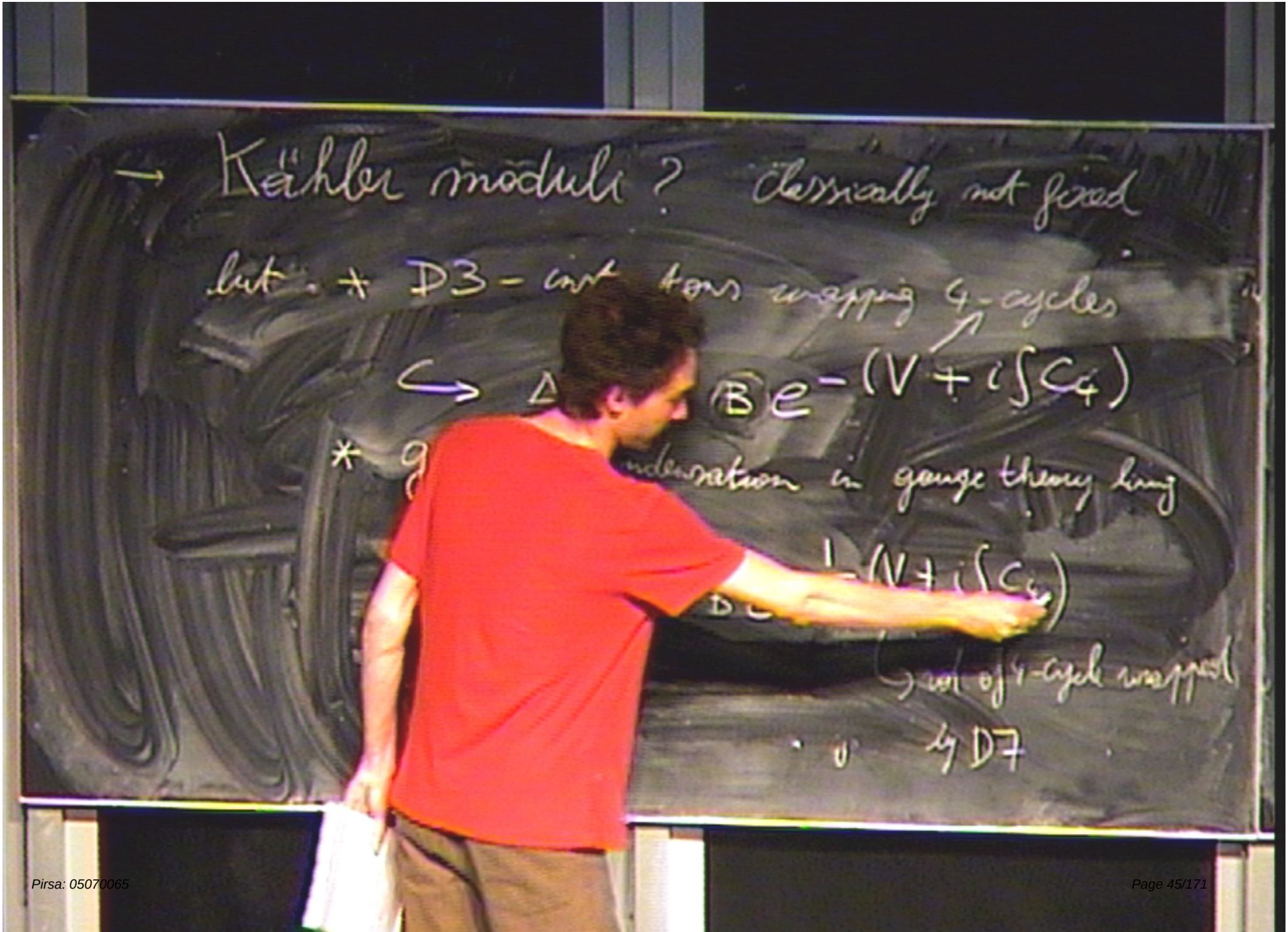

→ Kähler module ?


→ Kähler moduli ? classically not fixed
but + D3 - instantons wrapping 4-cycles


→ Kähler moduli ? classically not fixed
+ D3 - instantons wrapping 4-cycles
$\hookrightarrow \Delta W = B e^{-(V + i\int C_4)}$


→ Kähler moduli ? classically not fixed
but * D3 - instantons wrapping 4-cycles
↪ ΔW = B e^{-(V + i∫C4)}
* ino condensation in gauge theory lung
lows


→ Kähler moduli? classically not fixed
but → 3 - instantons wrapping 4-cycles
$\Delta W = B e^{-(V + i\int C_4)}$
gaugino condensation in gauge theory hung
loops
→ $\Delta W = B e^{-\frac{1}{N}(V + i\int C_4)}$
↳ vol of 4-cycle wrapped
by D7


→ Kähler moduli ? classically not fixed
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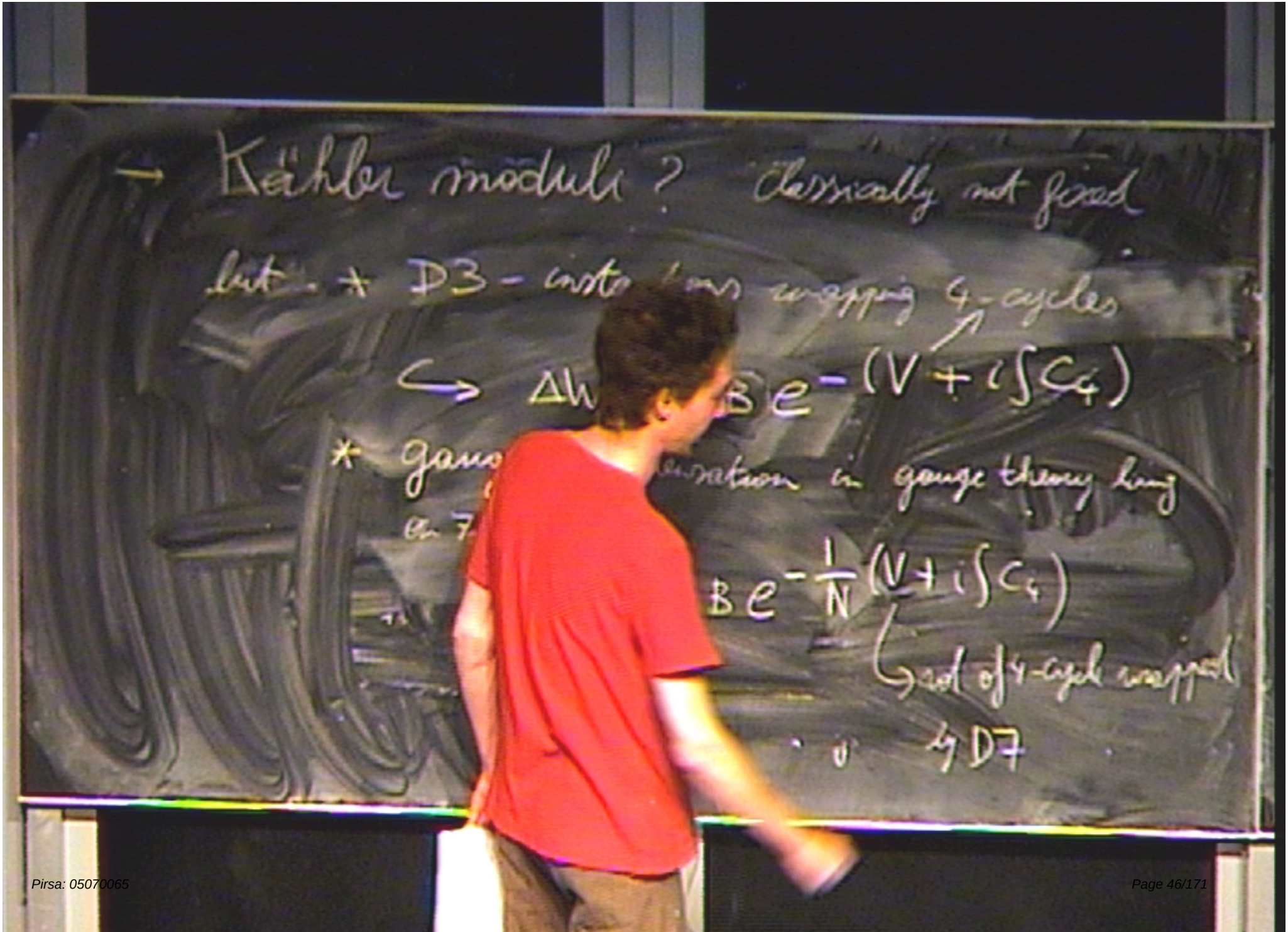
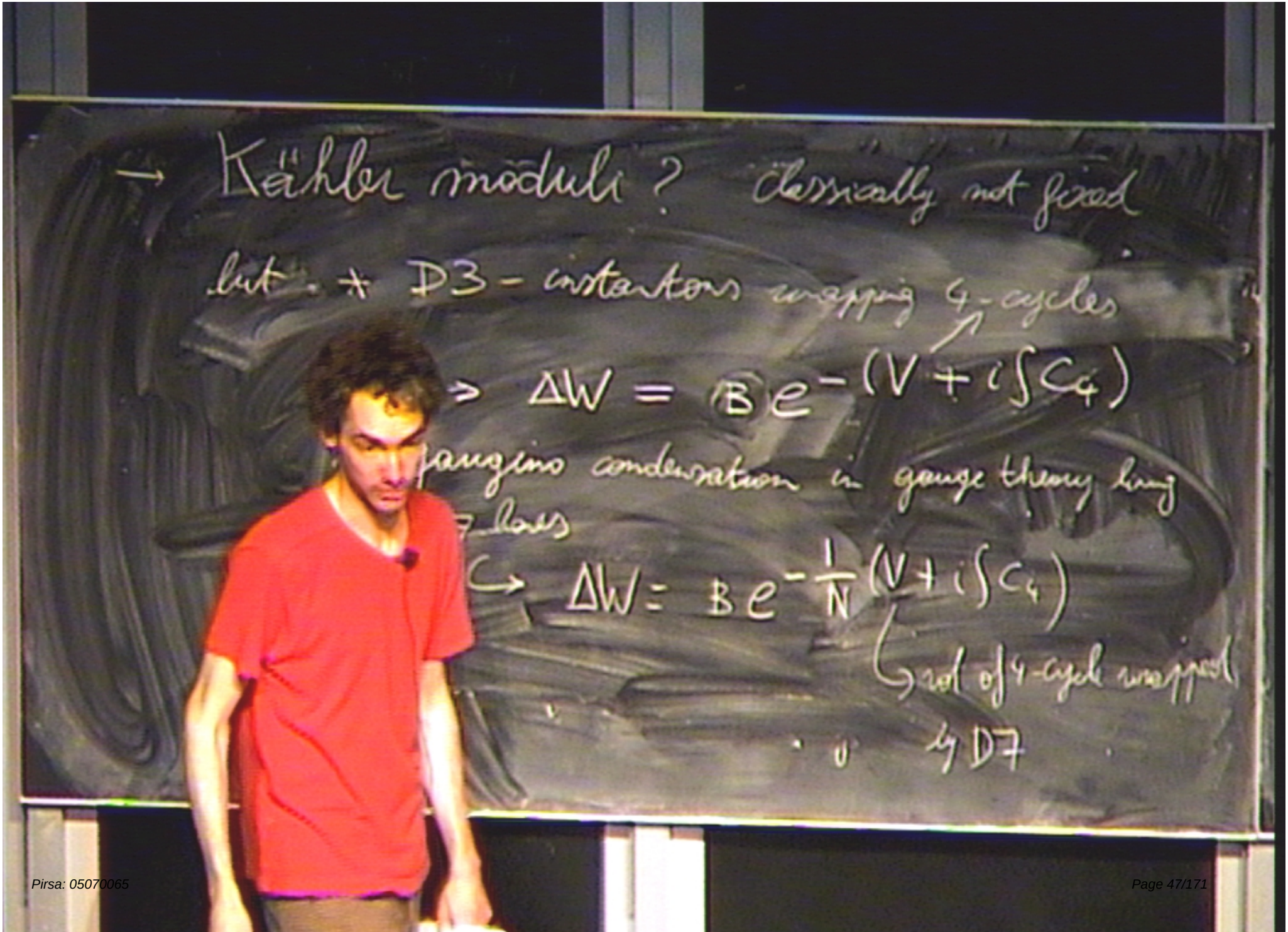
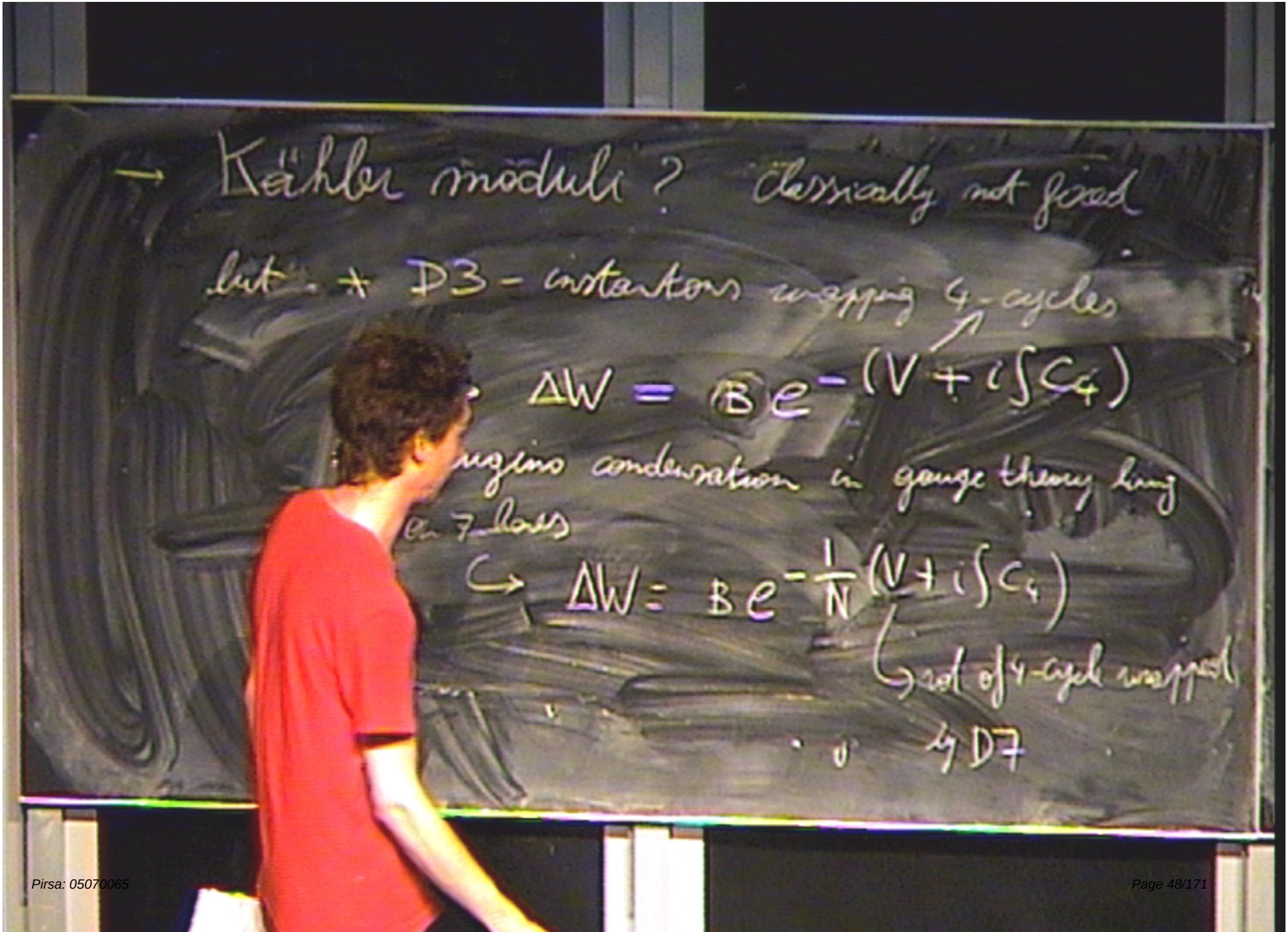

→ Kähler moduli ? Classically not fixed
but * D3 - instantons wrapping 4-cycles
↪ $\Delta W \sim B e^{-(V + i\int C_4)}$
* gaugino condensation in gauge theory living
on 7
$B e^{-\frac{1}{N}(V + i\int C_4)}$
↳ set of 4-cycle wrapped
by D7


→ Kähler moduli ? classically not fixed
but + D3 - instantons wrapping 4-cycles
$\Delta W = B e^{-(V + i\int C_4)}$
gaugino condensation in gauge theory hung
$\Delta W = B e^{-\frac{1}{N}(V + i\int C_4)}$
vol of 4-cycle wrapped
by D7


→ Kähler moduli? classically not fixed
but + D3-instantons wrapping 4-cycles
$\Delta W = B e^{-(V + i\int C_4)}$
gaugino condensation in gauge theory living
on 7-loss
$\hookrightarrow \Delta W = B e^{-\frac{1}{N}(V + i\int C_4)}$
↳ set of 4-cycle wrapped
by D7

→ Kähler moduli? Classically not fixed

but * D3-instantons wrapping 4-cycles

↪ $\Delta W = B e^{-(V + i\int C_4)}$

* gaugino condensation in gauge theory living on 7-branes

↪ $\Delta W = B' e^{-\frac{1}{N}(V + i\int C_4)}$

↑ ↪ vol of 4-cycle wrapped by D7

→ Kähler moduli? classically not fixed

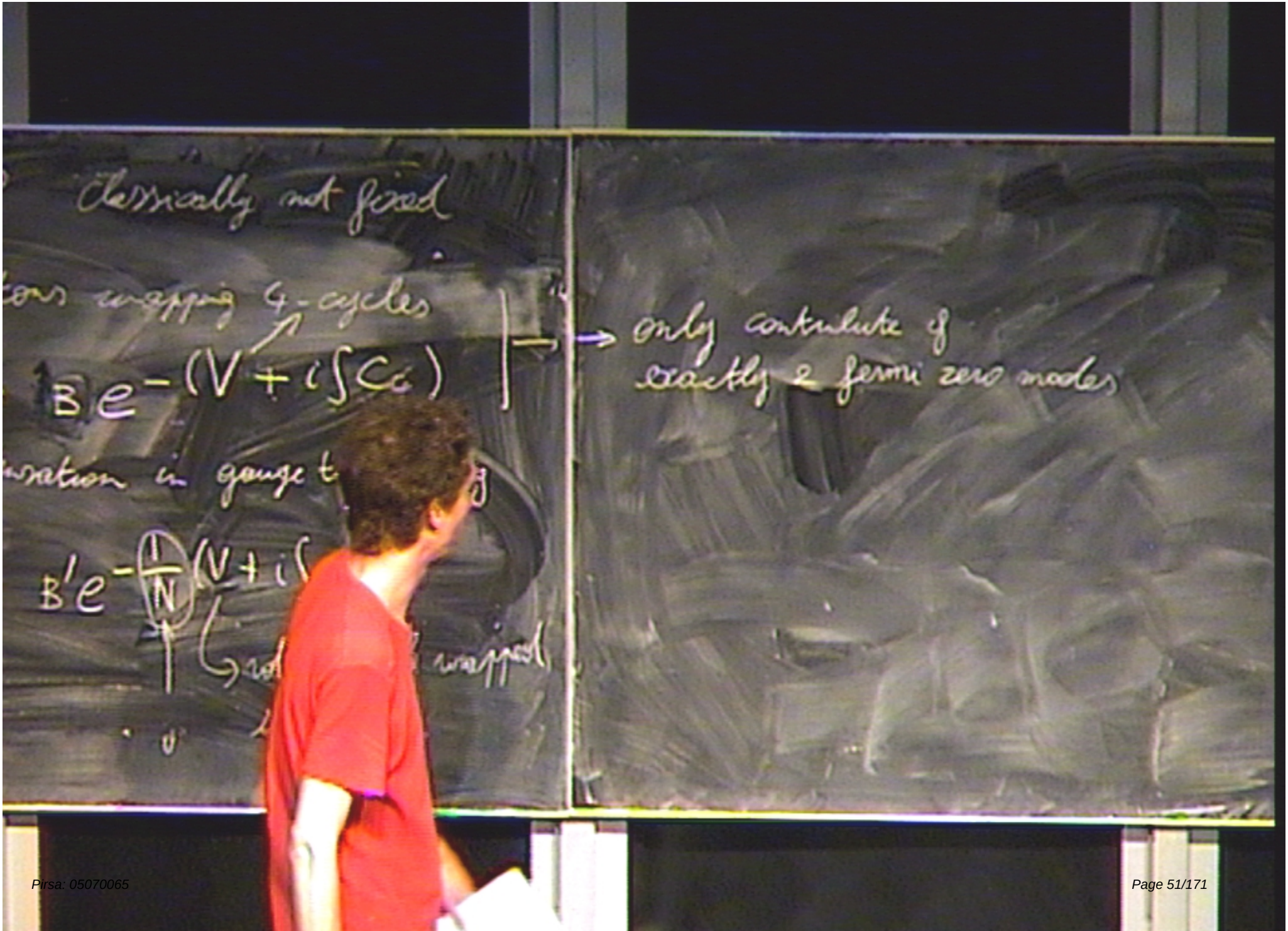
but: + D3-instantons wrapping 4-cycles

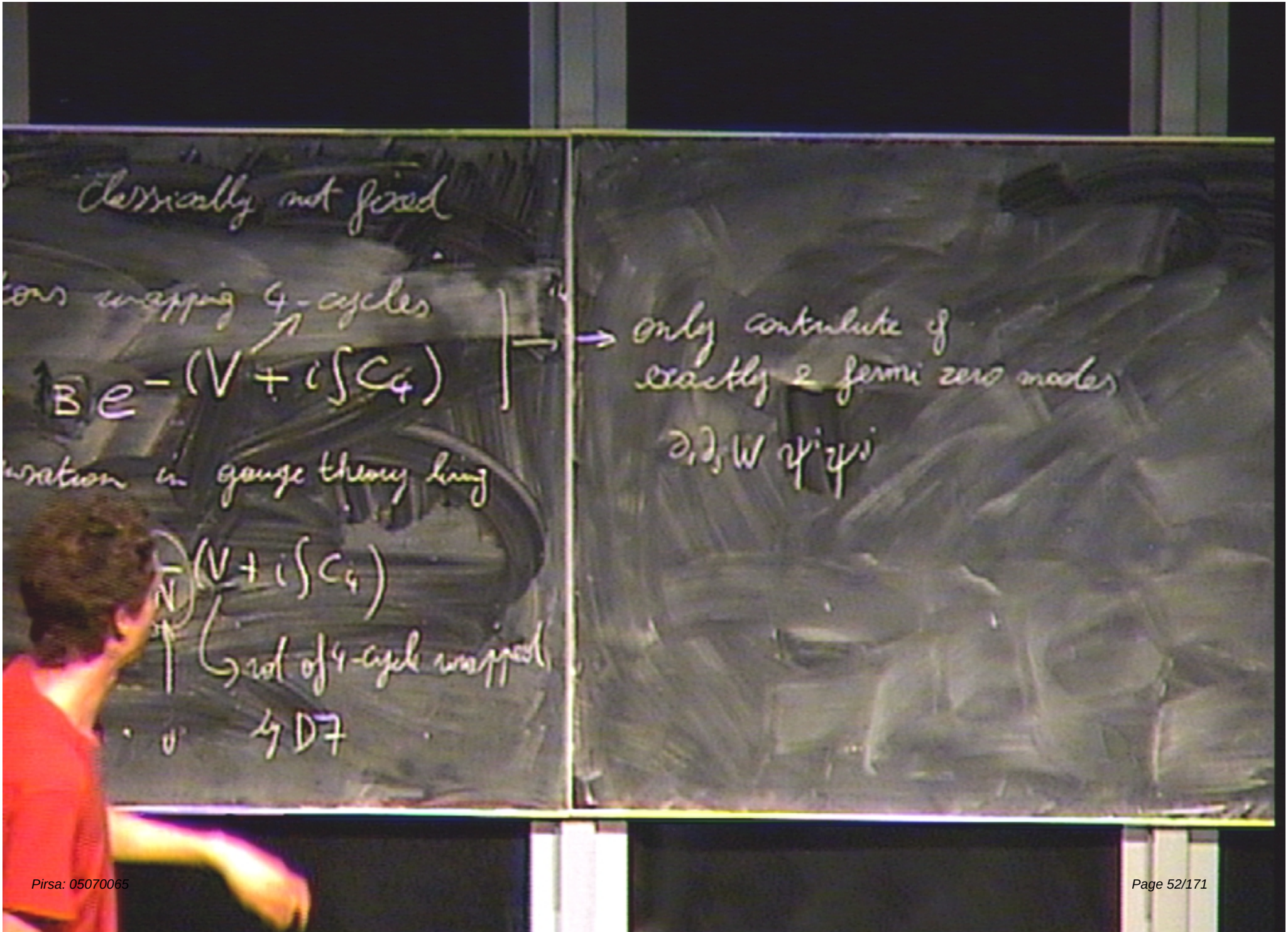
↪ $\Delta W = B e^{-(V + i\int C_4)}$

* gaugino condensation in gauge theory living on 7-branes

↪ $\Delta W = B' e^{-\frac{1}{N}(V + i\int C_4)}$

↑ ↪ rest of 4-cycle wrapped by D7


classically not fixed
4-cycles
$Be^{-(V+i\int C_4)}$
only contribute if
exactly 2 fermi zero modes
$B'e^{-\frac{1}{N}(V+i\int}$


Classically not fixed
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$\partial_i \partial_j W \psi^i \psi^j$
wsation in gauge theory living
$\frac{1}{N}(V + i\int C_4)$
rot of 4-cycle wrapped
by D7

classically not fixed

ons wrapping 4-cycles

$$B e^{-(V + i\int C_4)}$$

→ only contribute if exactly 2 fermi zero modes

$\partial_i \partial_j W \psi^i \psi^j$

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$$B' e^{-\frac{1}{N}(V + i\int C_4)}$$

↳ rest of 4-cycle wrapped by D7

Classically not fixed

...ons wrapping 4-cycles

$$B e^{-(V + i\int C_4)}$$

→ only contribute if exactly 2 fermi zero modes

$$\partial_i \partial_j W \, \psi^i \psi^j$$

...ation in gauge theory lung

$$B' e^{-\frac{1}{N}(V + i\int C_4)}$$

↳ rot of 4-cycle wrapped by D7

→ only contribute if exactly 2 fermi zero modes

$\partial_i \partial_j W \, \psi^i \psi^j$

→ in principle: sufficient to stabilize Kähler moduli

$$W = W_{flux}(z,\tau) + \sum B_i e^{-\left(V_i + i\int_D C_4\right)}$$

ρ_i

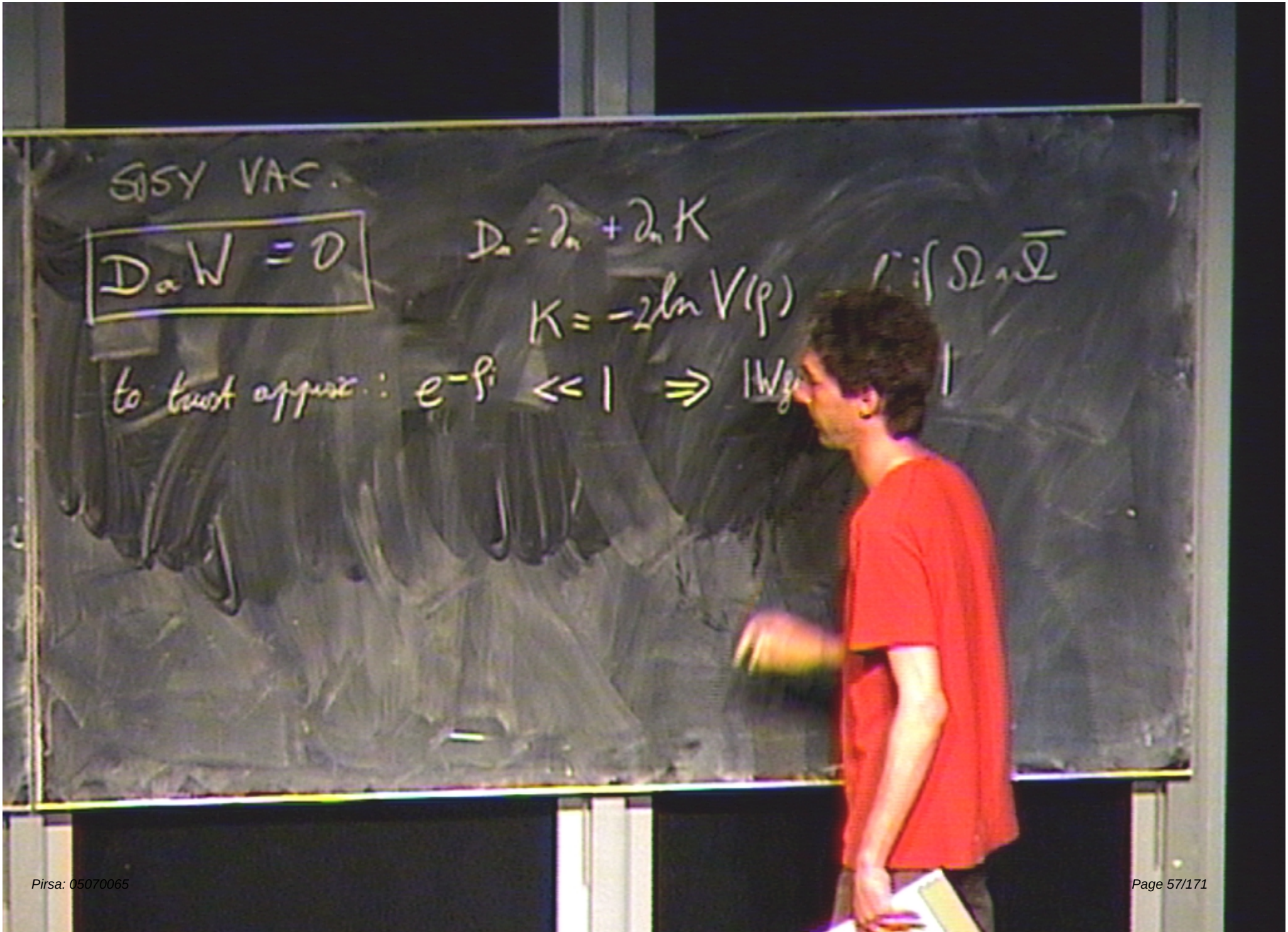
SUSY VAC.

$D_a W = 0$

$D_a = \partial_a + \partial_a K$

$K = -2 \ln V(\rho) - \ln i \int \Omega \wedge \bar{\Omega}$

to leading approx.: $e^{-\rho_i} \ll 1$


SUSY VAC.
$D_a W = 0$
$D_a = \partial_a + \partial_a K$
$K = -2 \ln V(\rho)$
to treat approx.: $e^{-\rho_i} \ll 1 \Rightarrow$

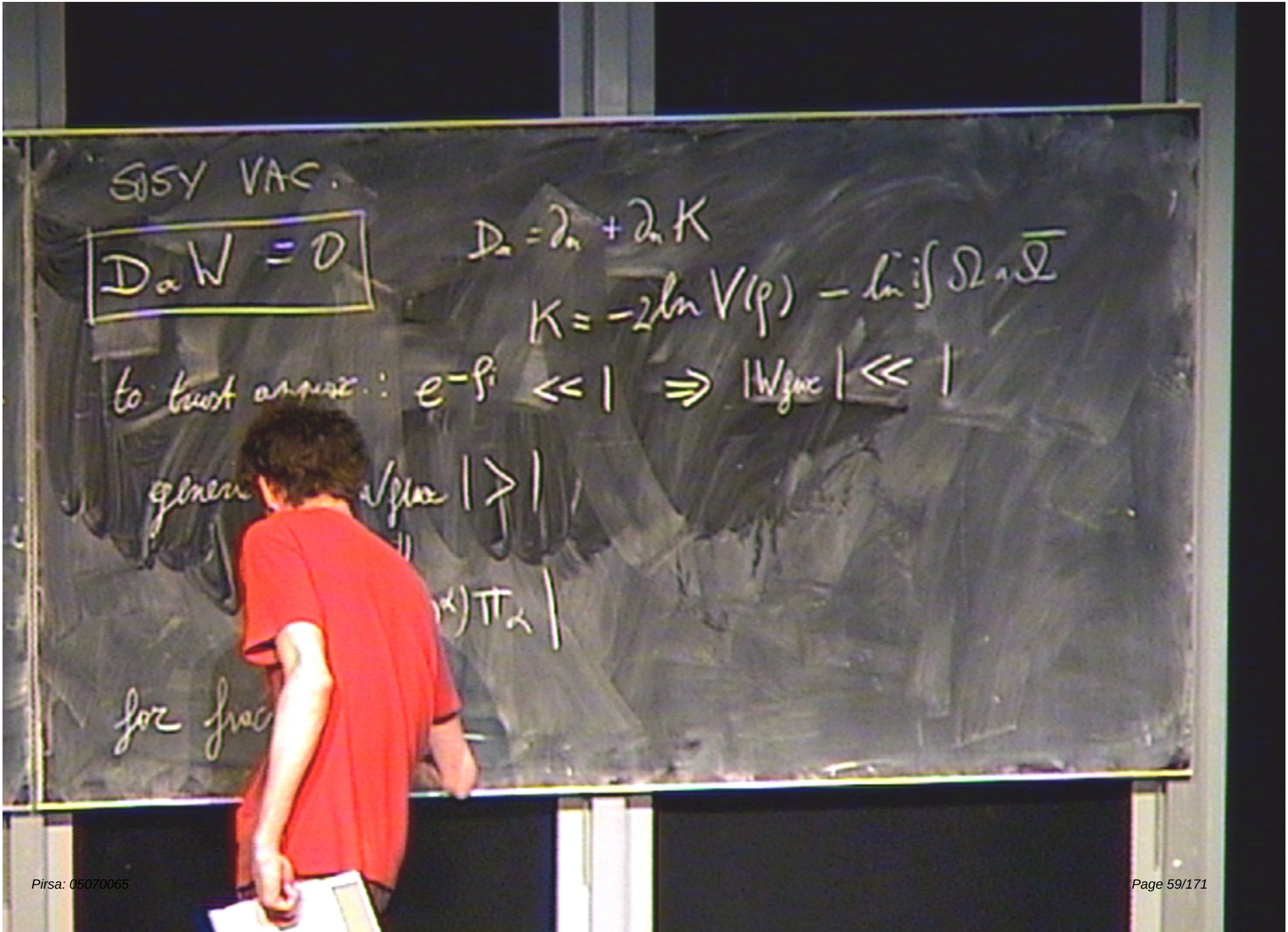
SUSY VAC.

$\boxed{D_\alpha W = 0}$

$D_\alpha = \partial_\alpha + \partial_\alpha K$

$K = -2 \ln V(\rho) - \ln i \int \Omega \wedge \bar{\Omega}$

to trust approx.: $e^{-\rho_i} \ll 1 \Rightarrow |W_{flux}| \ll 1$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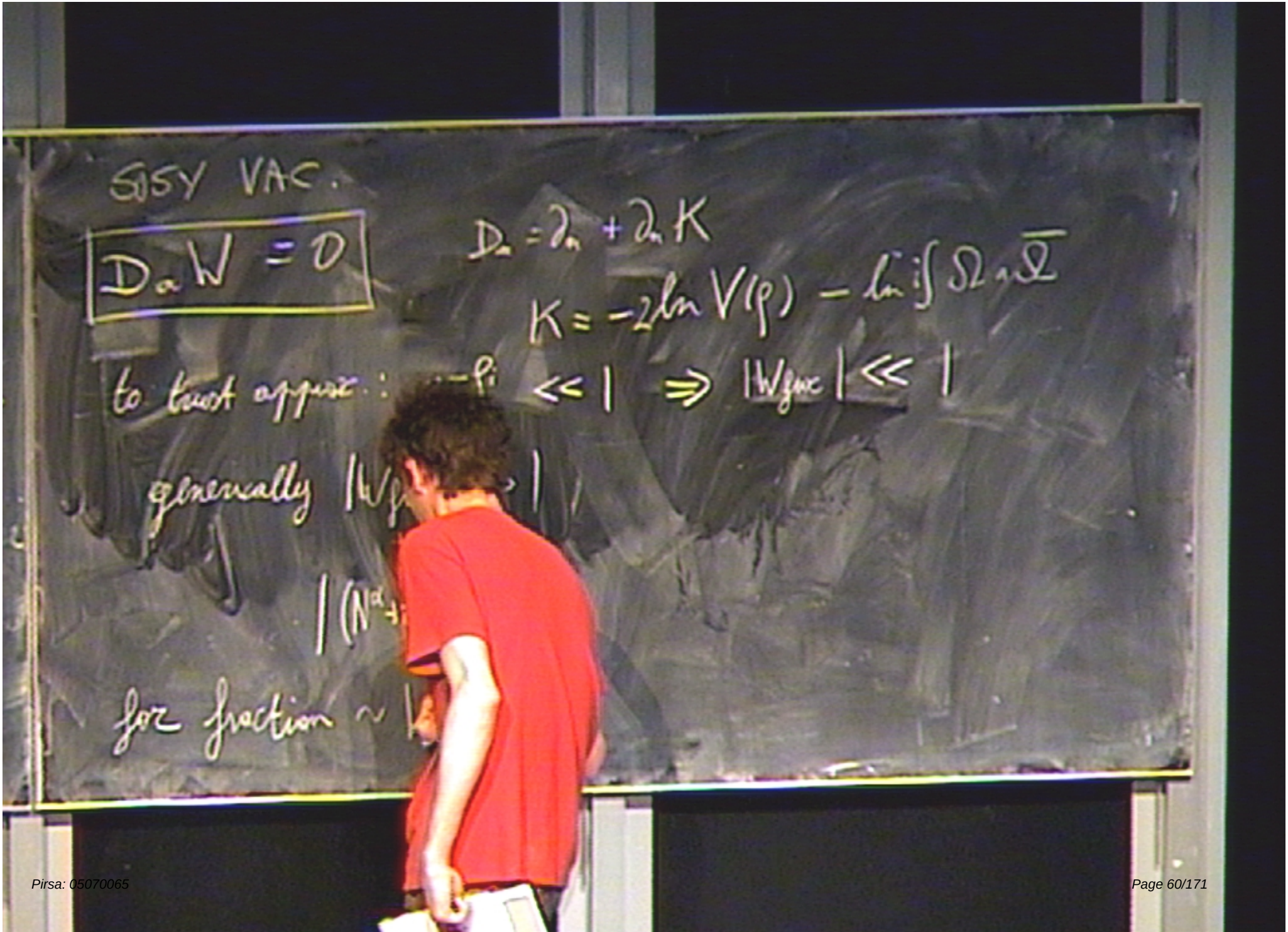
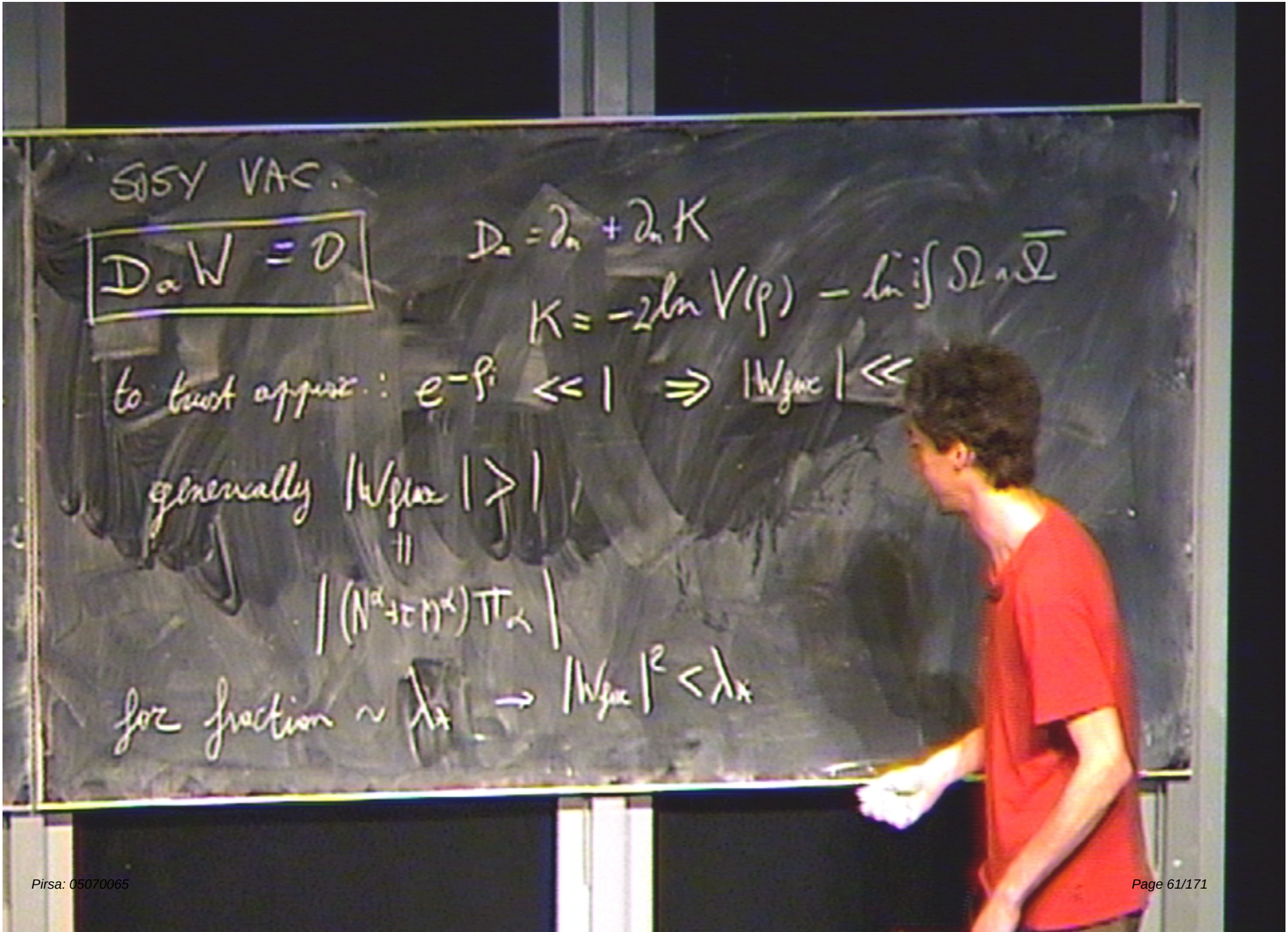
for frac


SUSY VAC.
$D_a W = 0$
$D_a = \partial_a + \partial_a K$
$K = -2 \ln V(\rho) - \ln i \int \Omega \wedge \bar{\Omega}$
to trust approx.: $-\rho_i \ll 1 \Rightarrow |W_{fluc}| \ll 1$
generically
for fraction


SUSY VAC.
$D_a W = 0$
$D_a = \partial_a + \partial_a K$
$K = -2 \ln V(\rho) - \ln i \int \Omega \wedge \bar{\Omega}$
to trust approx.: $e^{-\rho_i} \ll 1 \Rightarrow |W_{flux}| \ll$
generically $|W_{flux}| \geq 1$
$|(N^\alpha + \tau M^\alpha)\Pi_\alpha|$
for fraction $\sim \lambda_*$ $\rightarrow$ $|W_{flux}|^2 < \lambda_*$

SUSY VAC.

$D_a W = 0$

$D_a = \partial_a + \partial_a K$

$K = -2 \ln V(\rho) - \ln i \int \Omega \wedge \bar{\Omega}$

to trust approx.: $e^{-\rho_i} \ll 1 \Rightarrow |W_{flux}| \ll 1$

generically $|W_{flux}| \geq 1$

for many still

$= |(N^\alpha + \tau M^\alpha)\Pi_\alpha|$

for fraction $\sim \lambda_*$ $\rightarrow |W_{flux}|^2 < \lambda_*$

→ only contribute if
exactly 2 fermi zero modes

$\partial_i \partial_j W \neq 0$

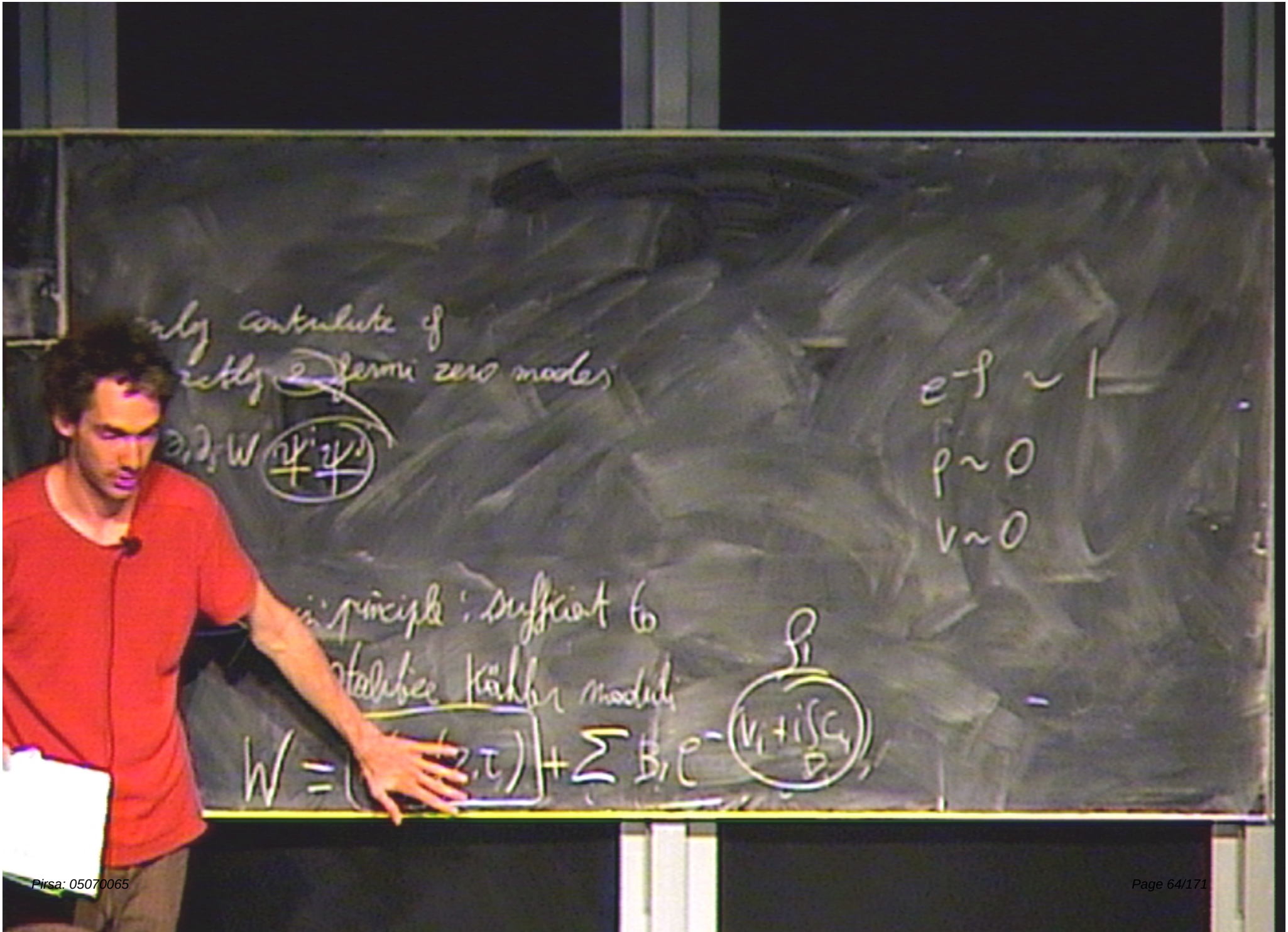
$e^{-\rho} \sim 1$

$\rho \sim 0$

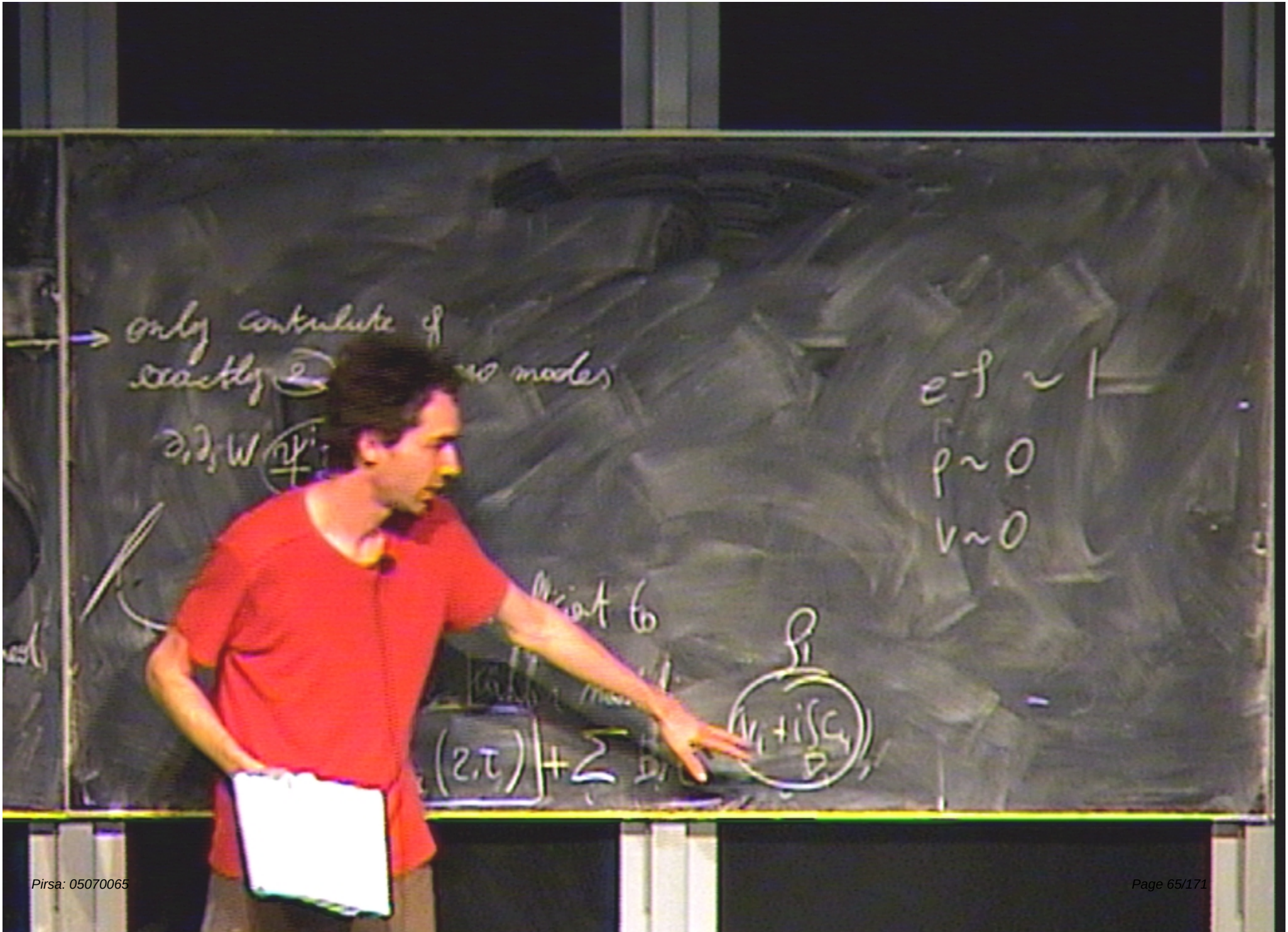
$V \sim 0$

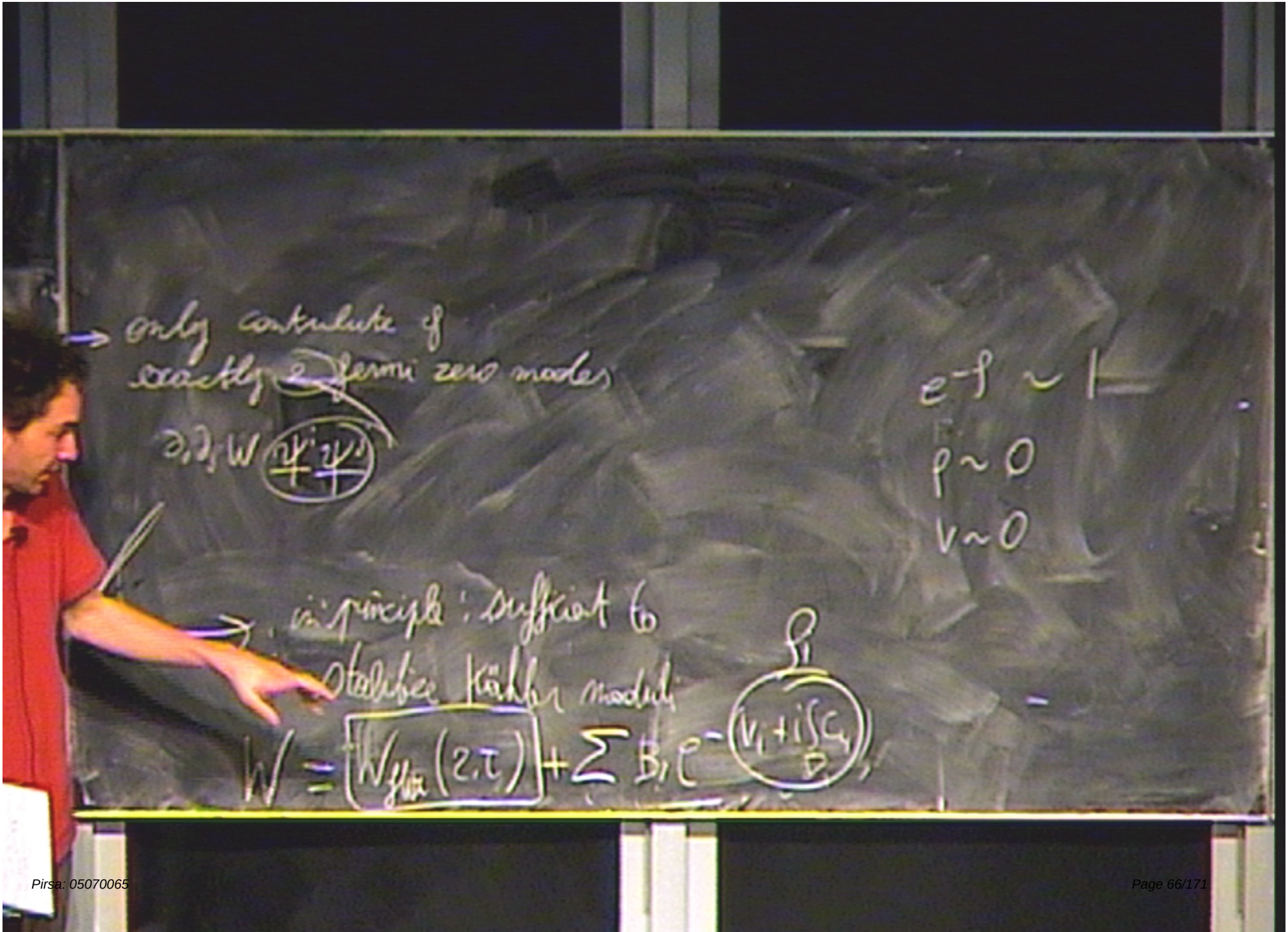
→ in principle: sufficient to
Stabilize Kähler moduli

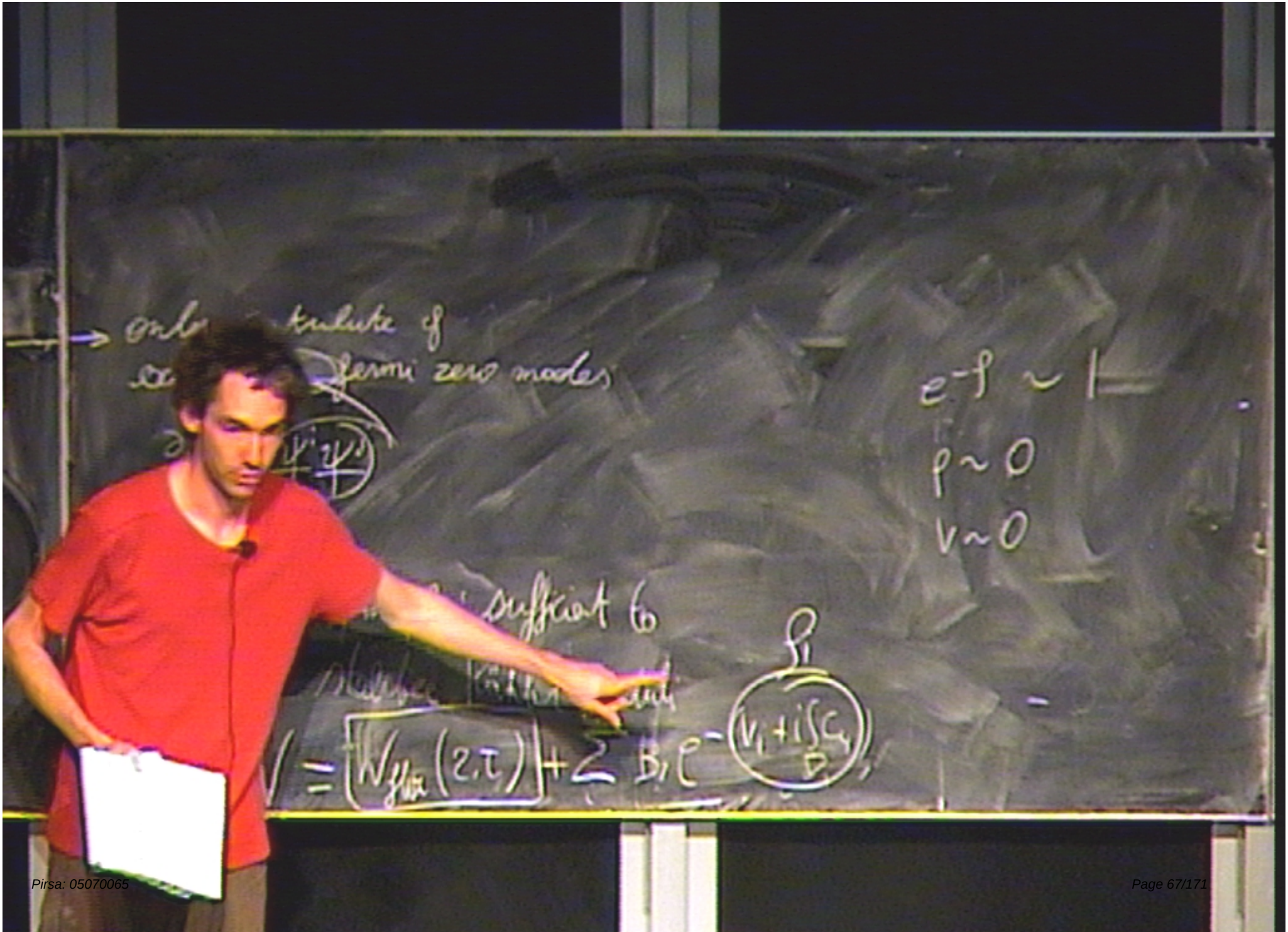
$$W = W_{flux}(z,\tau) + \sum B_i e^{-\left(\frac{T_i}{p} + i\frac{\zeta_i}{p}\right)}$$

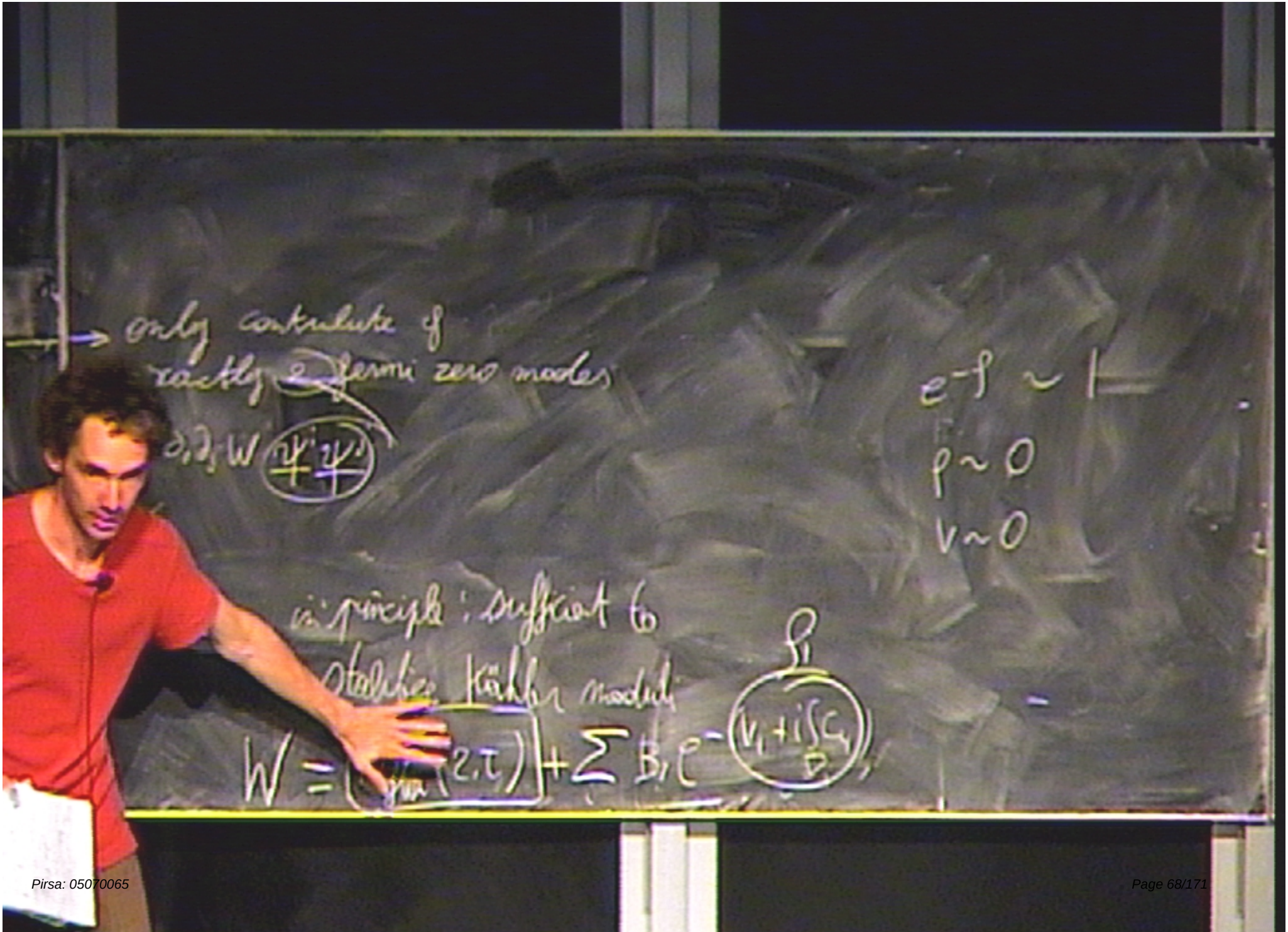


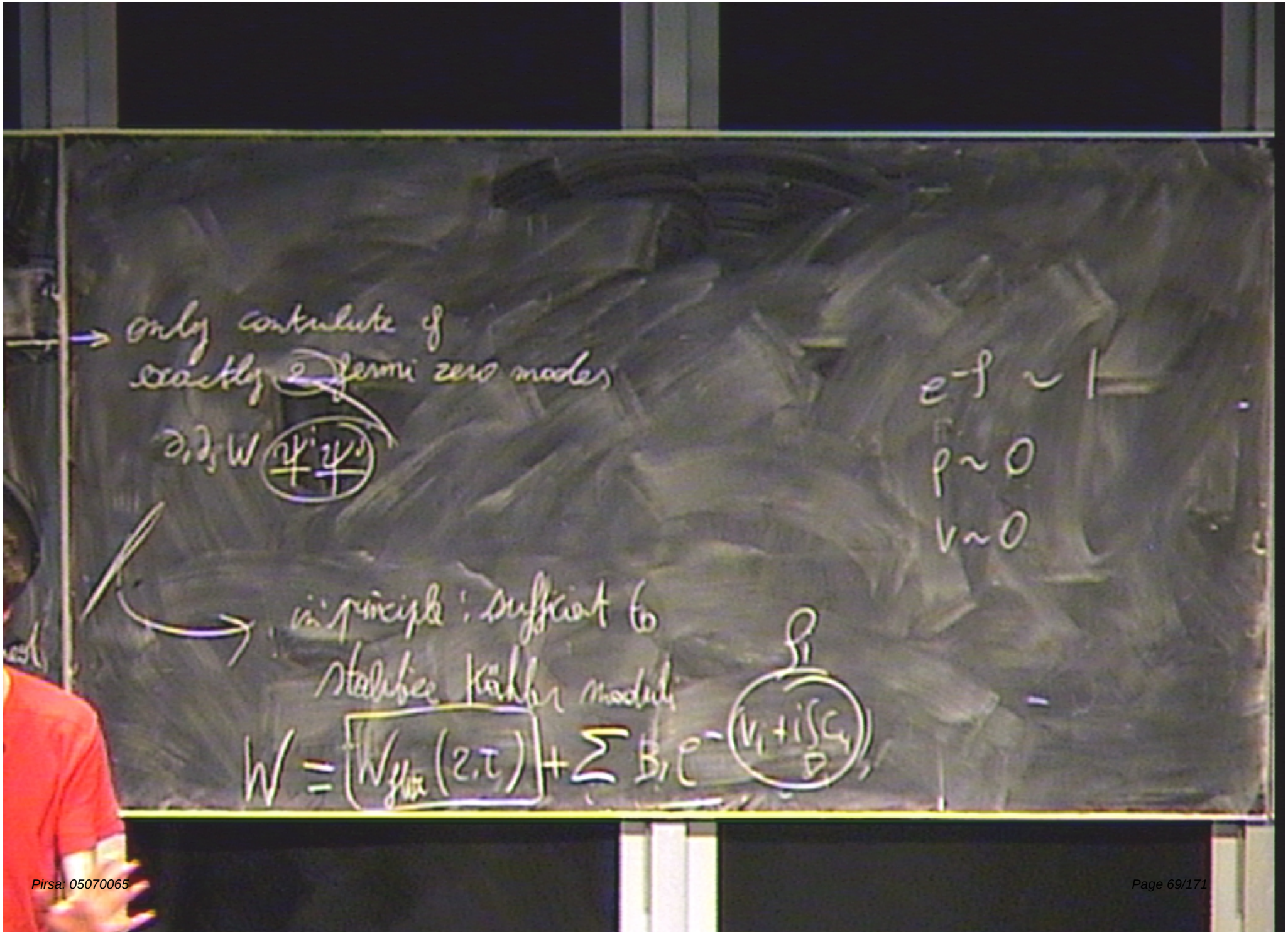
contribute of
fermi zero modes
principle : sufficient to
Kähler moduli
$e^{-\rho} \sim 1$
$\rho \sim 0$
$v \sim 0$

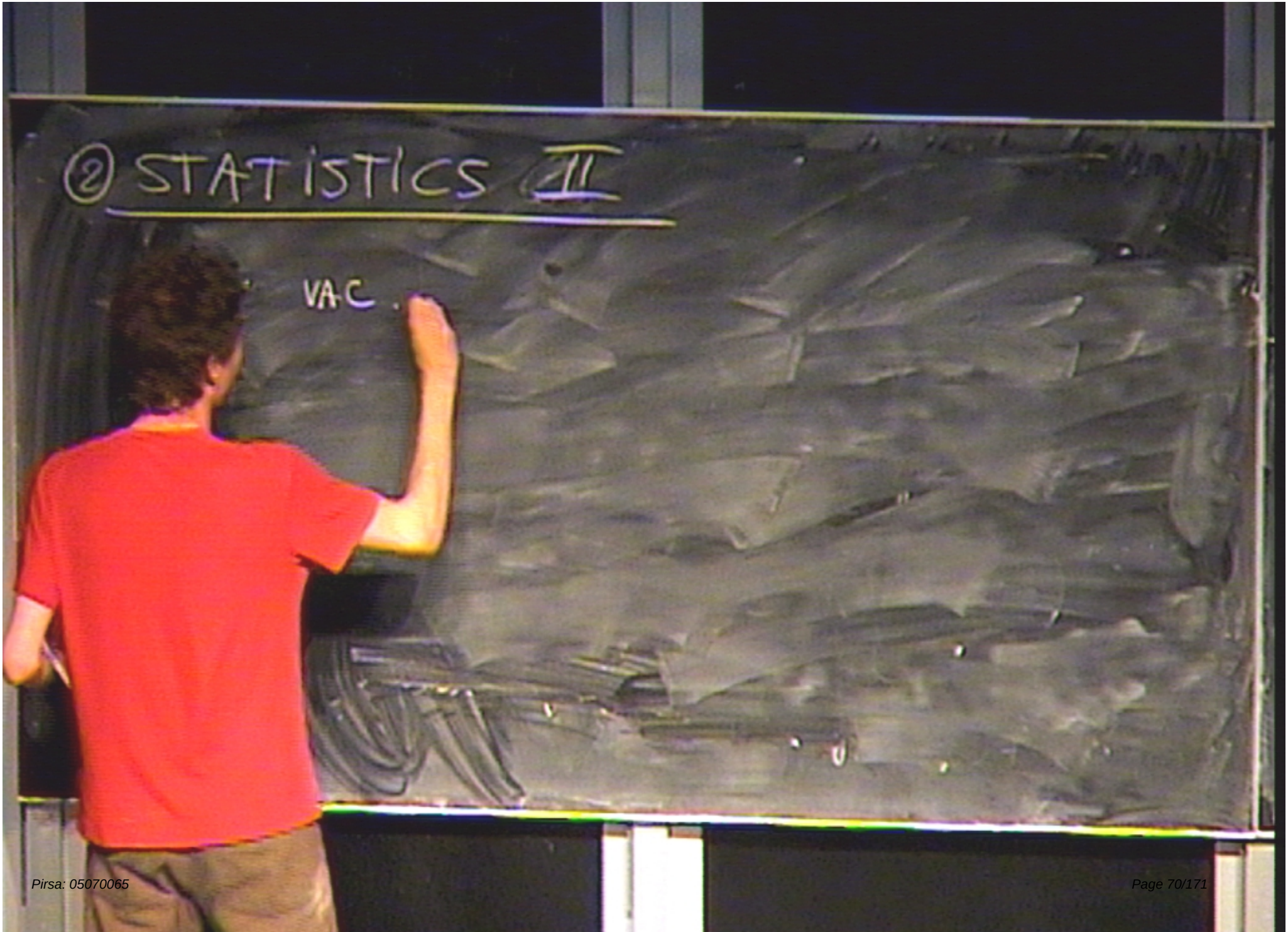

only contribute of
exactly
modes
$e^{-\rho} \sim 1$
$\rho \sim 0$
$v \sim 0$

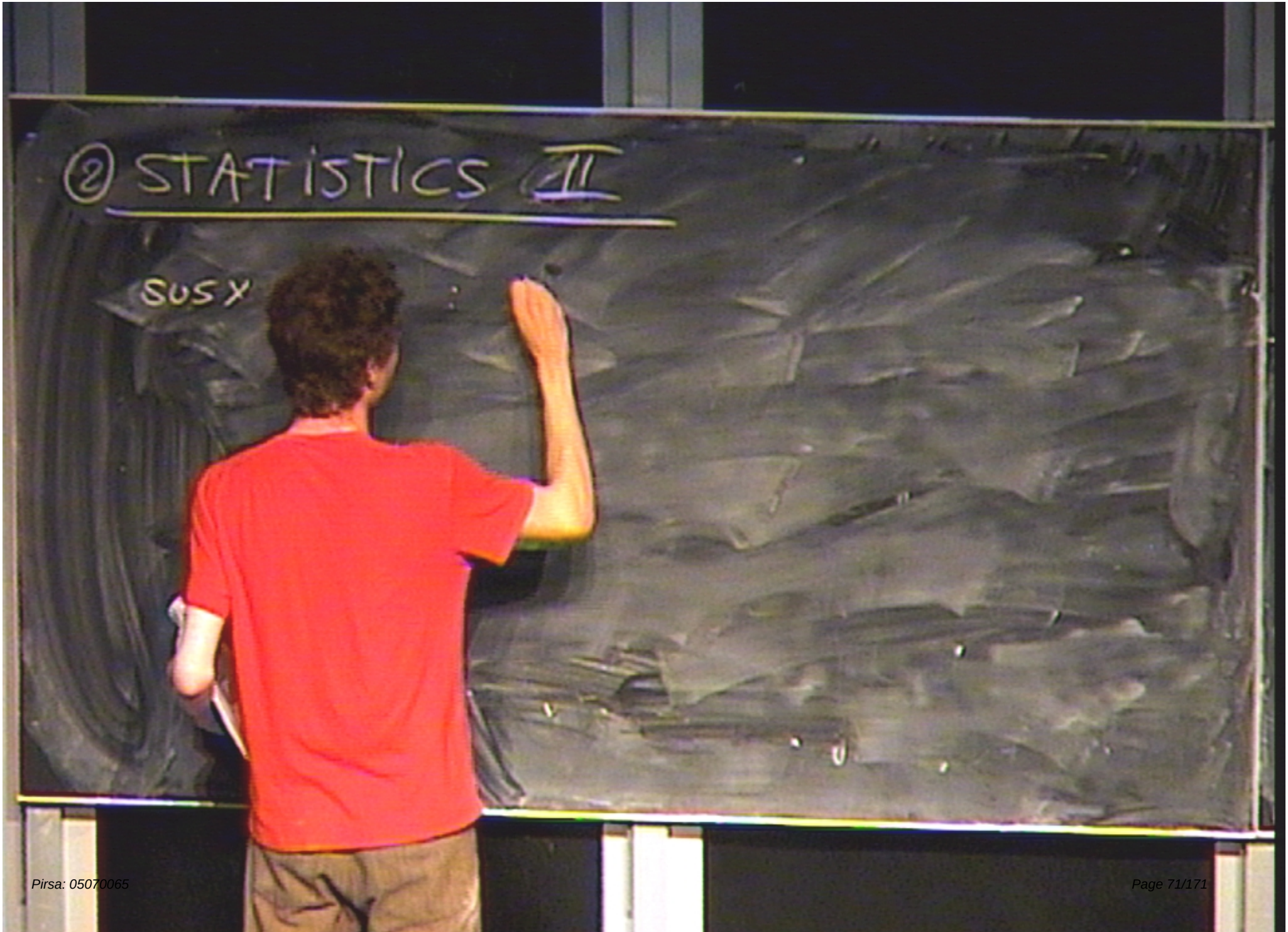

only contribute if
exactly 2 fermi zero modes
$\partial_i \partial_j W \psi_i \psi_j$
$e^{-f} \sim 1$
$\rho \sim 0$
$v \sim 0$
in principle: sufficient to
$W = W_{inst}(z,\tau) + \sum B_i e^{-(v_i + i\int_D C_4)}$

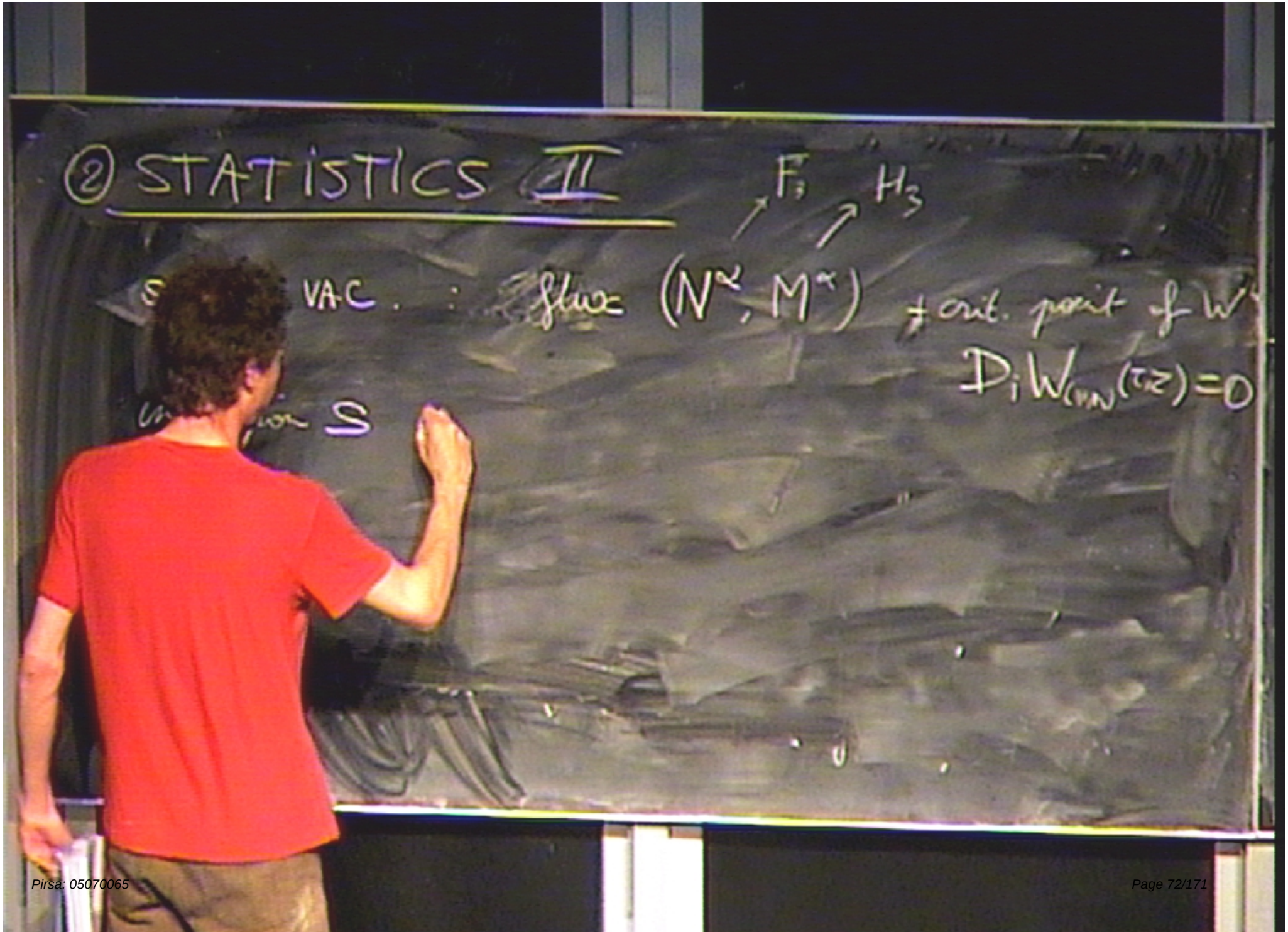

fermi zero modes
$e^{-S} \sim 1$
$\rho \sim 0$
$V \sim 0$
sufficient to
$V = [W_{inst}(z,\tau)] + \sum B_i e^{-(V_i + i\int_D C_4)}$

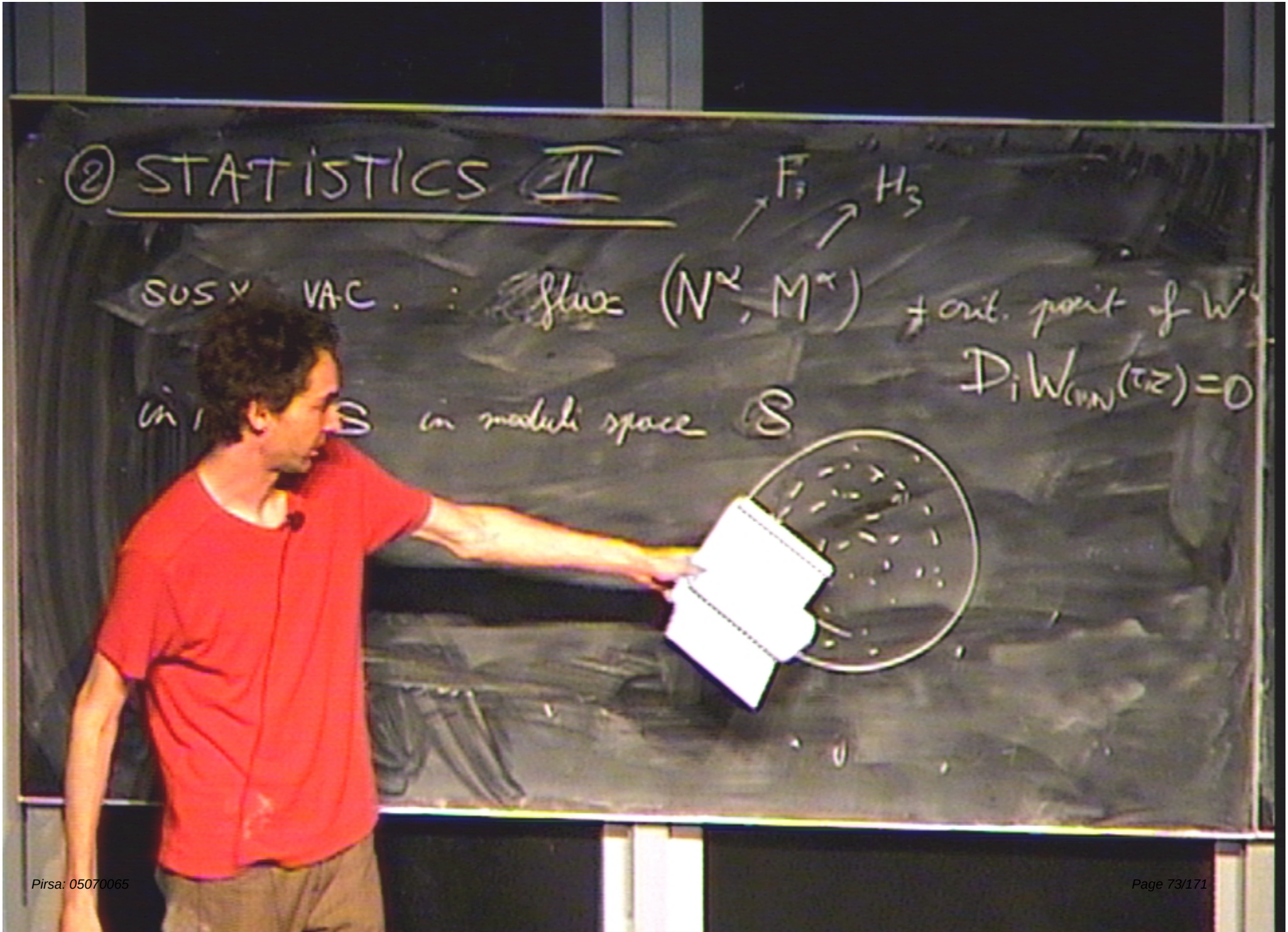

only contribute of
exactly 2 fermi zero modes
$\partial_i \partial_j W \psi^i \psi^j$
$e^{-\rho} \sim 1$
$\rho \sim 0$
$V \sim 0$
in principle: sufficient to
stabilize Kähler moduli
$W = \ldots (2,\tau) + \sum B_i e^{-(V_i + i\int_D C_4)}$

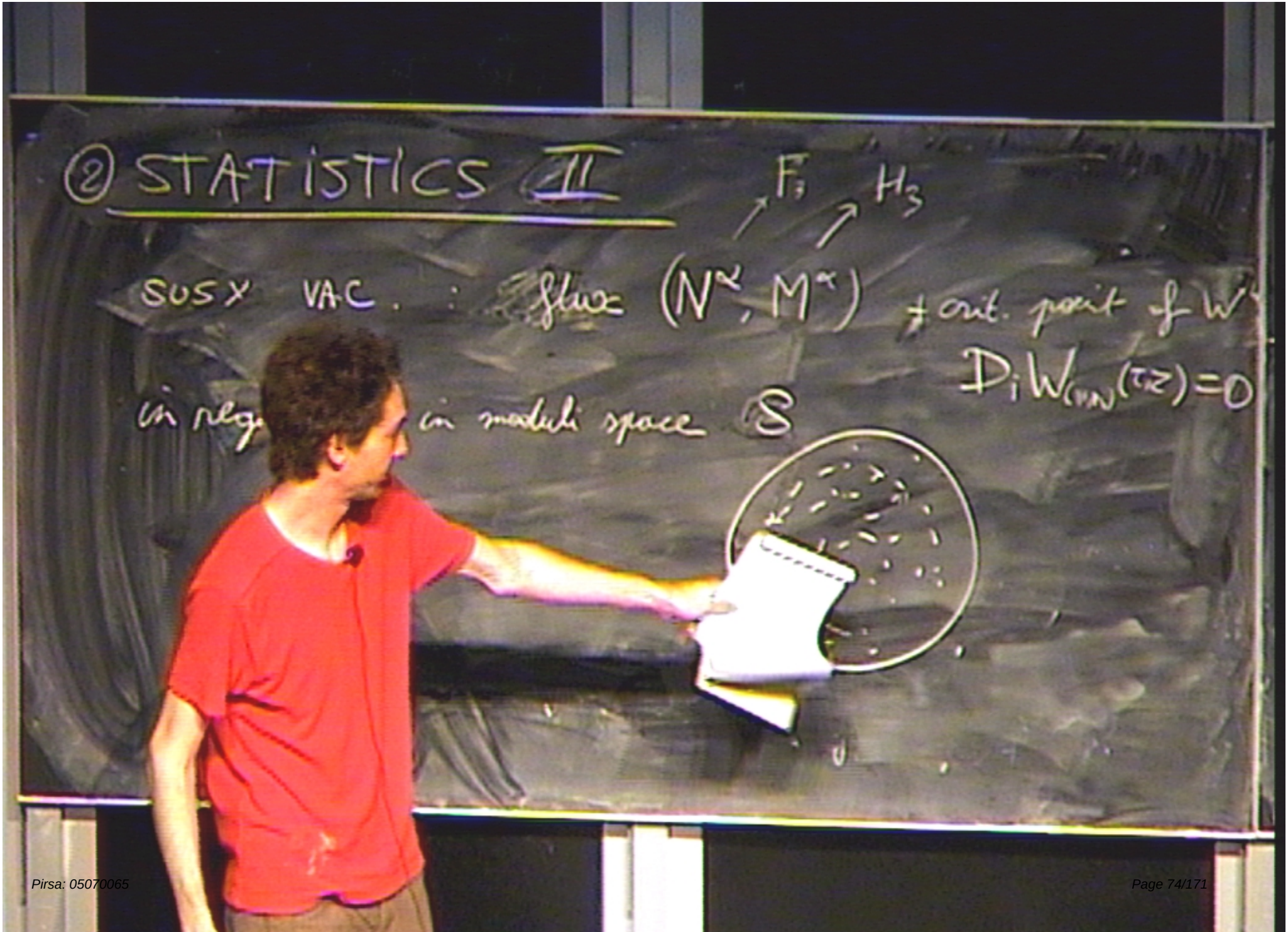

only contribute if
exactly 2 fermi zero modes
$\partial_i \partial_j W(\psi_i \psi_j)$
$e^{-S} \sim 1$
$\rho \sim 0$
$V \sim 0$
in principle: sufficient to
stabilize Kähler moduli
$W = W_{flux}(z,\tau) + \sum B_i e^{-(V_i + i\int_D C_4)}$

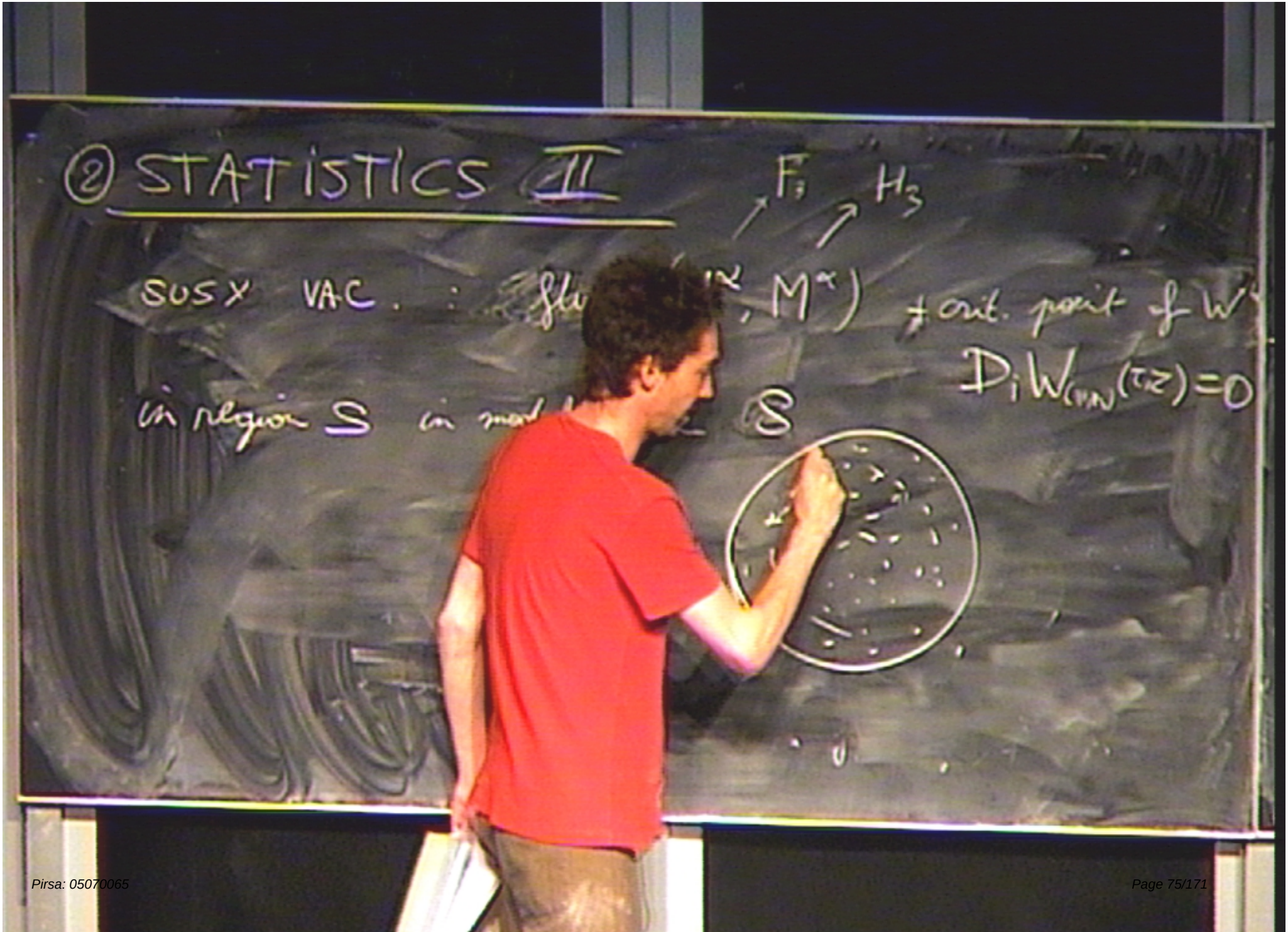

② STATISTICS II
VAC.

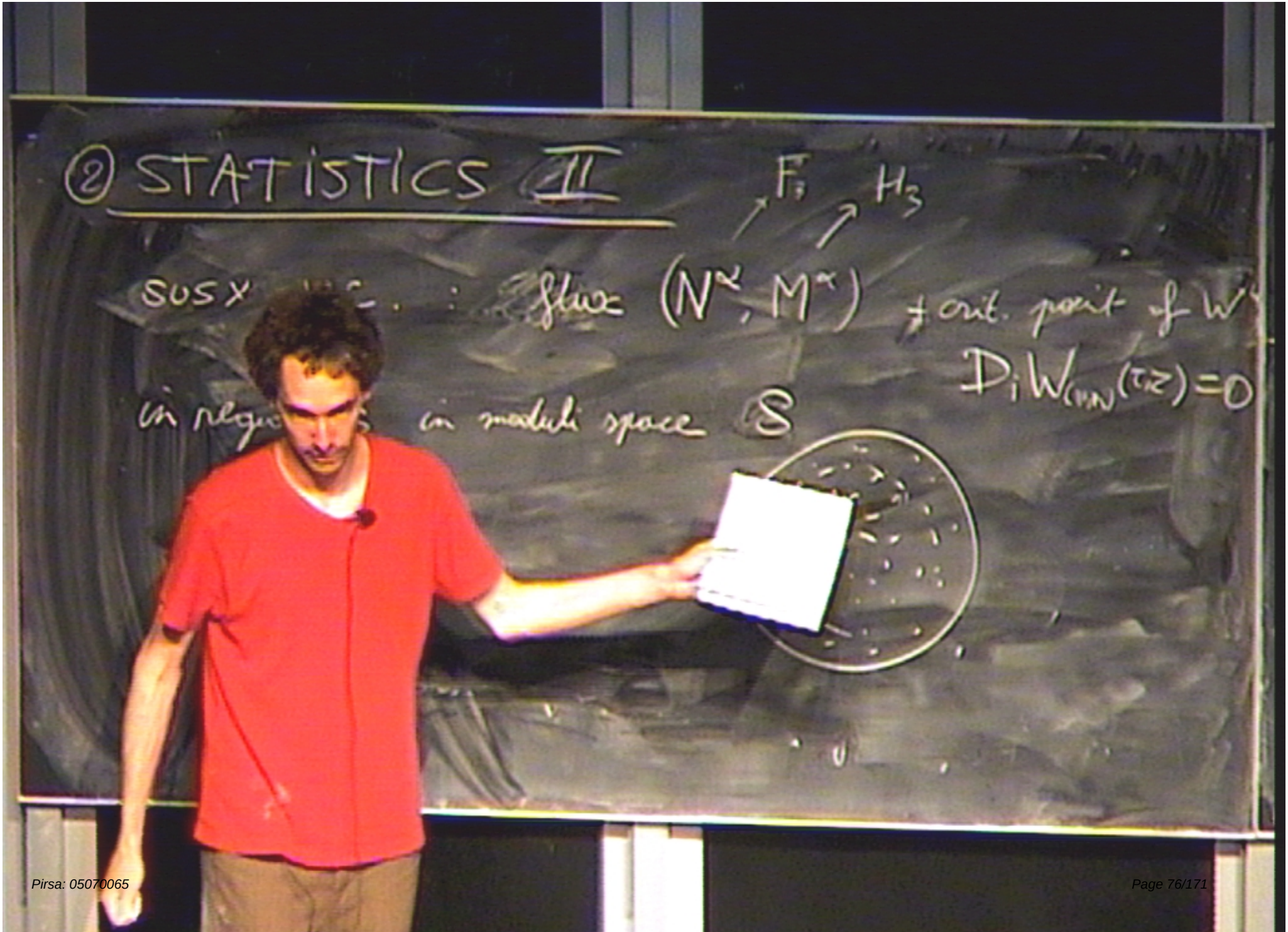

② STATISTICS II
SUSY

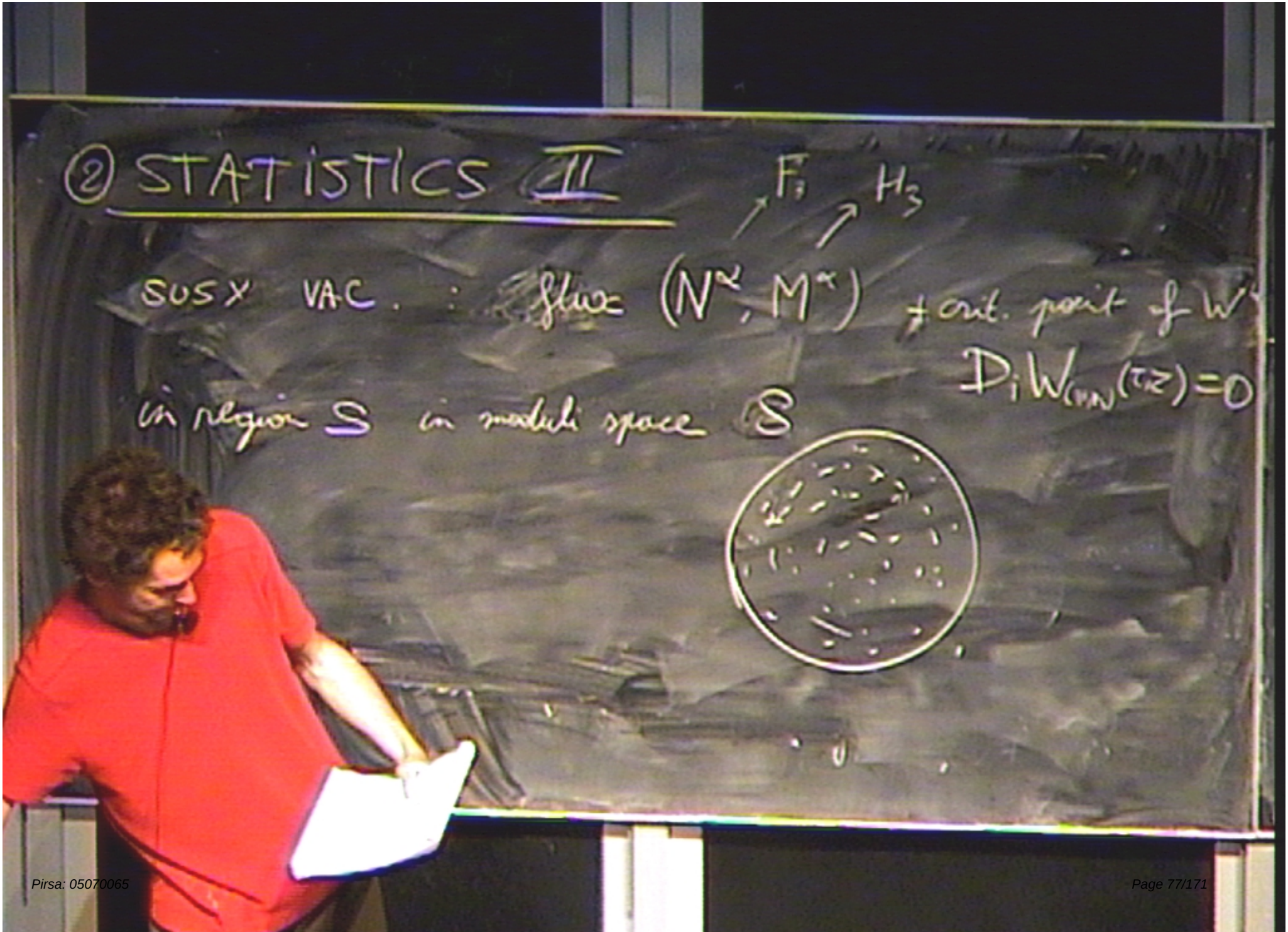

② STATISTICS II
F3
H3
VAC. : flux (N^α, M^α) + crit. point of W
D_i W_(N,N)(τ,z) = 0
S

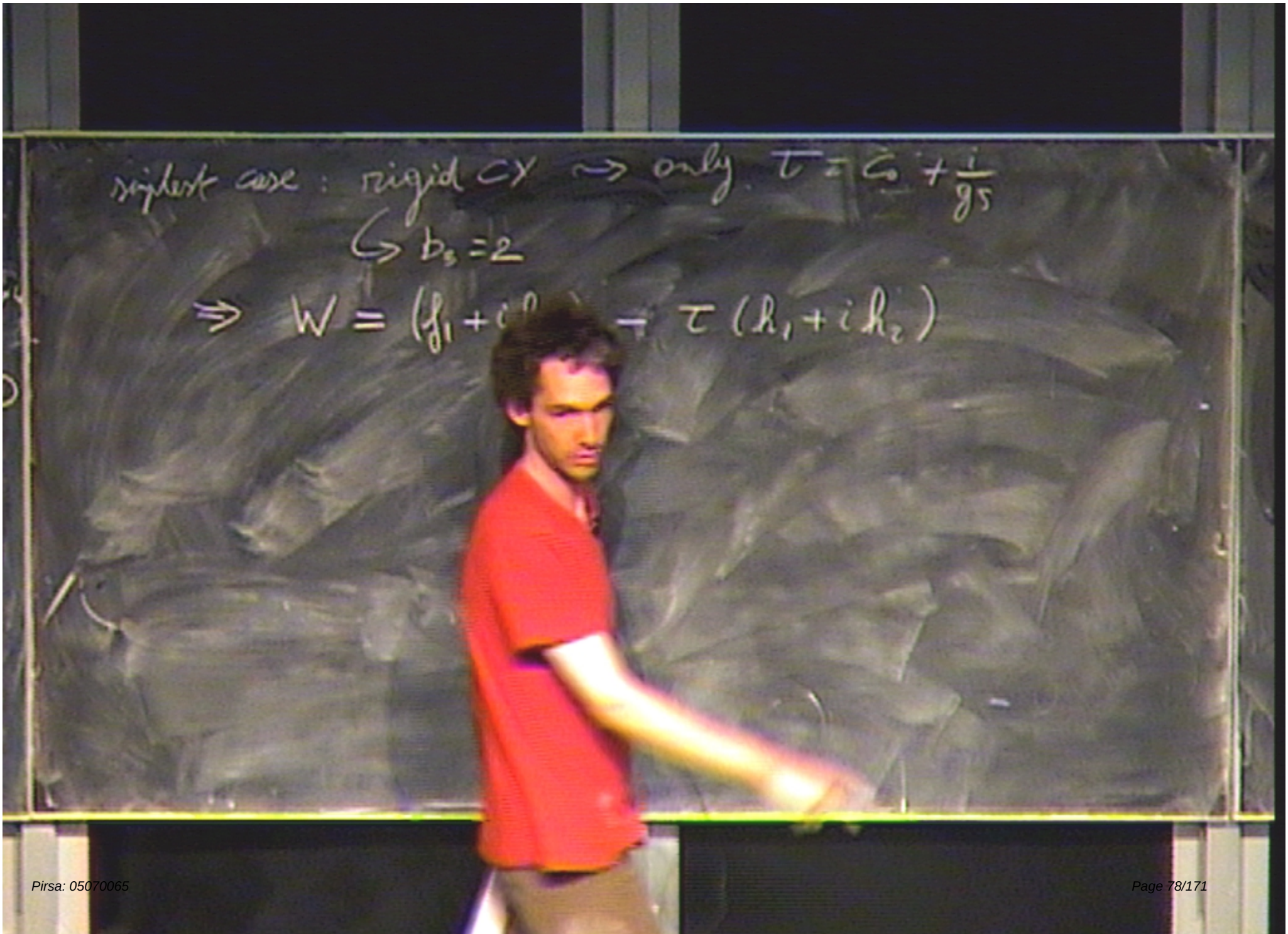

② STATISTICS II
F3
H3
SUSY VAC. : flux (N^α, M^α) + crit. point of W
D_i W_(N,M)(τ,z) = 0
in moduli space S


② STATISTICS II
F_i
H_3
SUSY VAC. : flux (N^α, M^α) ≠ crit. point of W
$D_i W_{(F,N)}(\tau z) = 0$
in moduli space $\mathcal{S}$


② STATISTICS II
F3
H3
SUSY VAC. :
+ crit. point of W
in region S
S


② STATISTICS II
F3
H3
SUSY
flux (N^α, M^α) + crit. point of W
D_i W_(N,M)(τ,z) = 0
in moduli space S

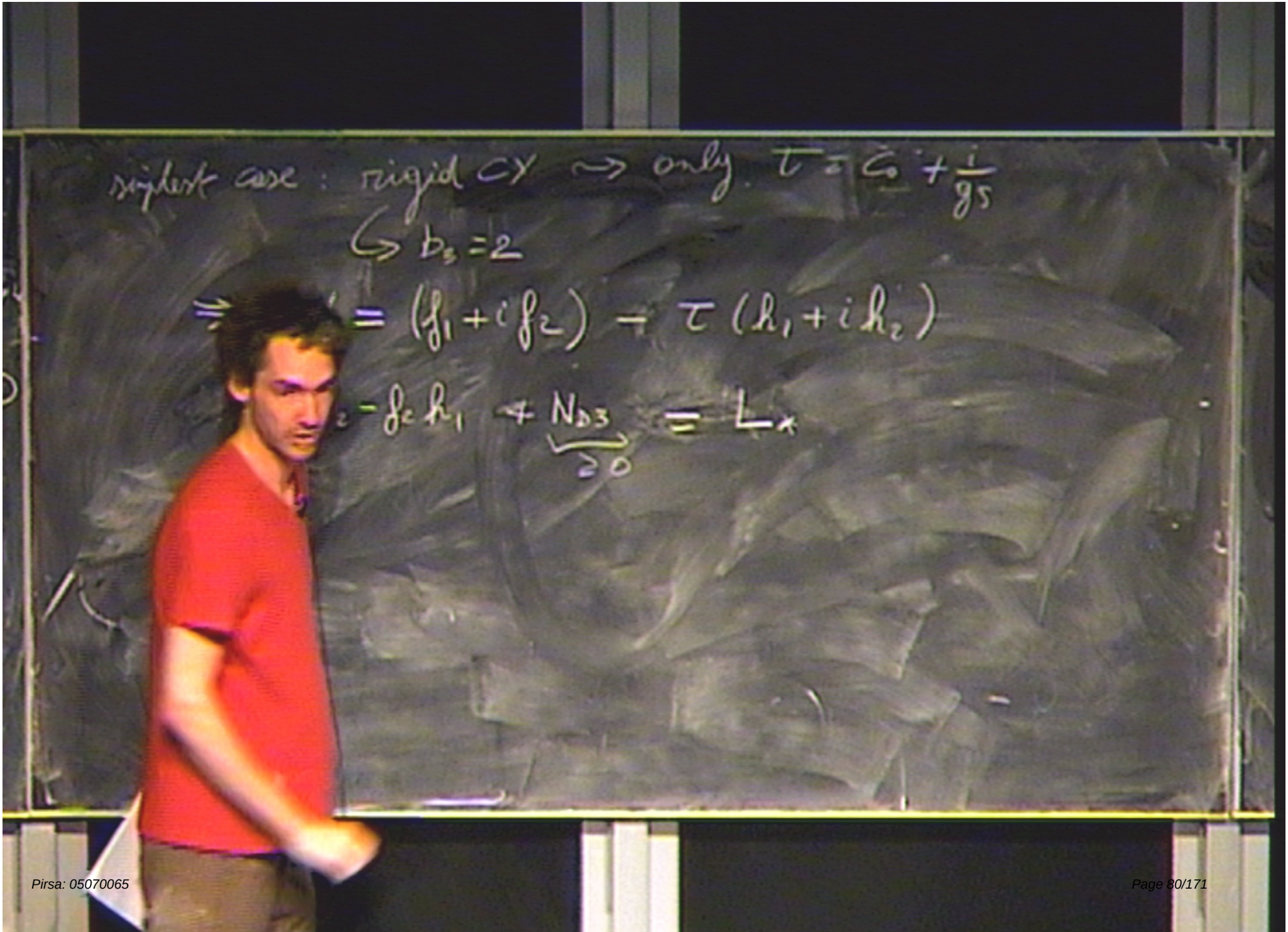

② STATISTICS II
F3
H3
SUSY VAC. : flux (N^α, M^α) + crit. point of W
D_i W_(N,M)(τ,z) = 0
in region S in moduli space S


highest case : rigid CY → only $\tau = C_0 + \frac{i}{g_s}$
↳ $b_3 = 2$
⇒ $W = (f_1 + i$ $\tau (h_1 + i h_2)$

slightest case : rigid CY $\rightsquigarrow$ only $\tau = C_0 + \frac{i}{g_s}$

$\hookrightarrow b_3 = 2$

$\Rightarrow W = (f_1 + i f_2) + \tau (h_1 + i h_2)$

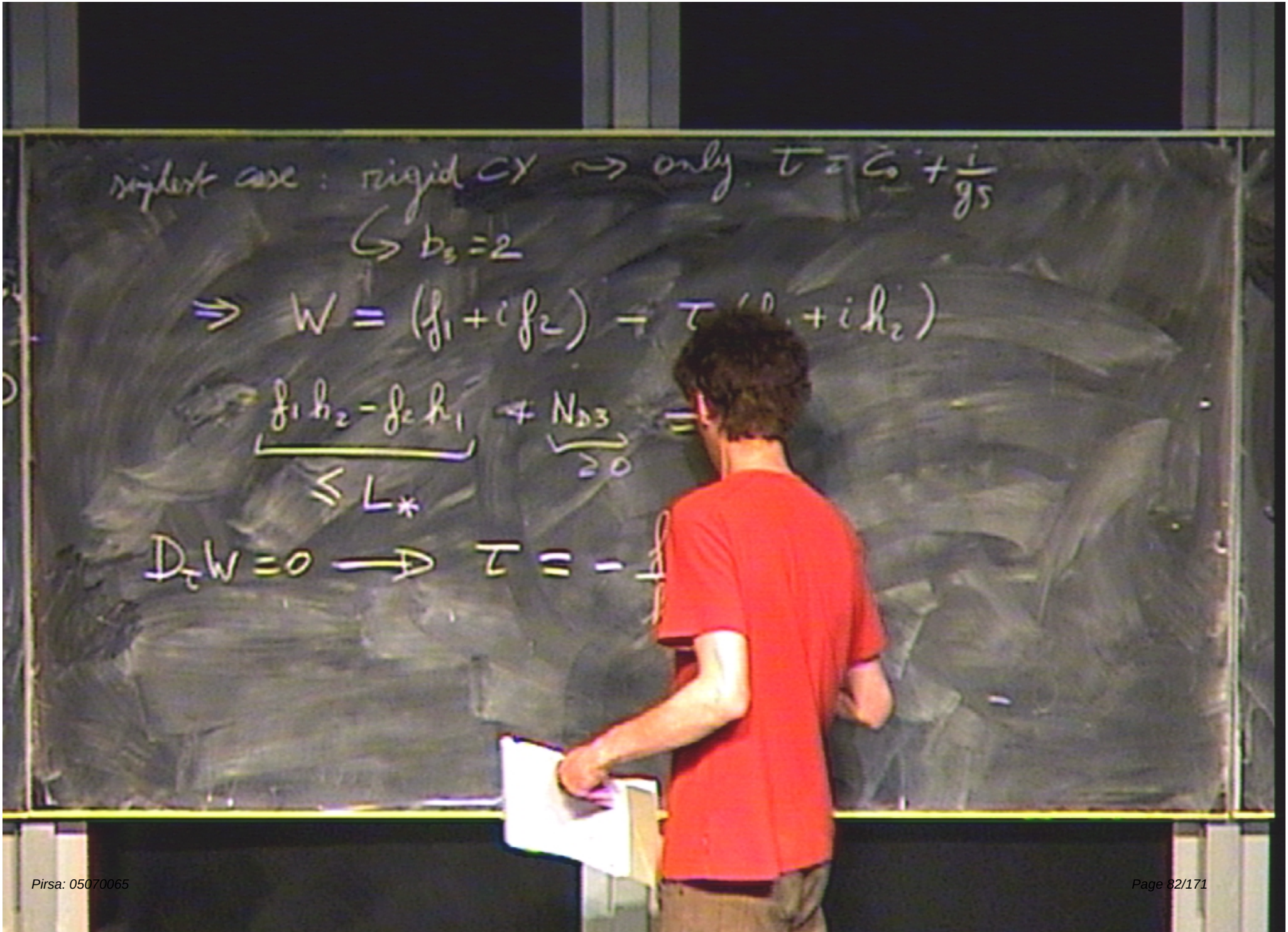

slightest case : rigid CY → only $\tau = C_0 + \frac{i}{g_s}$
↳ $b_3 = 2$
$= (f_1 + i f_2) + \tau (h_1 + i h_2)$
$- f_c h_1 + \underbrace{N_{D3}}_{\geq 0} = L$

highest case : rigid CY $\rightsquigarrow$ only $\tau = C_0 + \frac{i}{g_s}$

$\hookrightarrow b_3 = 2$

$$\Rightarrow W = (f_1 + i f_2) + \tau (h_1 + i h_2)$$

$$f_1 h_2 - f_2 h_1 + \underbrace{N_{D3}}_{\geq 0} = L_\star$$


slightest case : rigid CY → only $\tau = C_0 + \frac{i}{g_s}$
$\hookrightarrow b_3 = 2$
$\Rightarrow W = (f_1 + i f_2) + \tau (h_1 + i h_2)$
$\underbrace{f_1 h_2 - f_2 h_1}_{\leq L_*} + \underbrace{N_{D3}}_{\geq 0} =$
$D_\tau W = 0 \longrightarrow \tau =$

Simplest case: rigid CY $\rightsquigarrow$ only $\tau = C_0 + \frac{i}{g_s}$

$\hookrightarrow b_4 = 2$

$$\Rightarrow W = (f_1 + i f_2) + \tau (h_1 + i h_2)$$

$$\underbrace{f_1 h_2 - f_2 h_1}_{\leq L_*} + \underbrace{N_{D3}}_{\geq 0} = L_*$$

$$D_\tau W = 0 \longrightarrow \tau = -\frac{f_1 - i f_2}{h_1 - i h_2}$$

slightest case: rigid CY $\rightsquigarrow$ only $\tau = C_0 + \frac{i}{g_s}$

$\hookrightarrow b_2 = 2$

$$\Rightarrow W = (f_1 + i f_2) + \tau (h_1 + i h_2)$$

$$\underbrace{f_1 h_2 - f_2 h_1}_{\leq L_*} + \underbrace{N_{D3}}_{\geq 0} = L_*$$

$K = -\ln(-i(\tau - \bar{\tau}))$

$D_\tau = \partial_\tau + \partial_\tau K$

$$D_\tau W = 0 \longrightarrow \tau = -\frac{f_1 - i f_2}{h_1 - i h_2}$$

slightest case: rigid CY $\rightsquigarrow$ only $\tau = C_0 + \frac{i}{g_s}$

$\hookrightarrow b_2 = 2$

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$\underbrace{f_1 h_2 - f_2 h_1}_{\leq L_*} + \underbrace{N_{D3}}_{\geq 0} = L_*$ $\qquad K = -\ln(-i(\tau - \bar{\tau}))$

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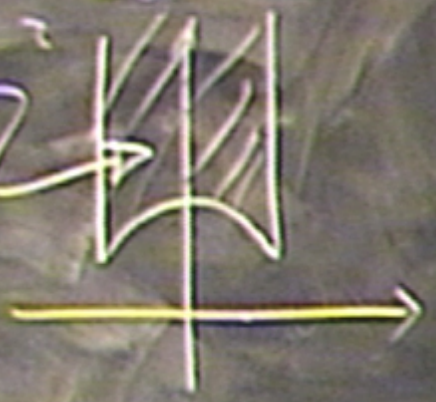


slightest case : rigid CY $\rightsquigarrow$ only $\tau = C_0 + \frac{i}{g_s}$

$\hookrightarrow b_3 = 2$

$$\Rightarrow W = (f_1 + i f_2) + \tau (h_1 + i h_2)$$

$$\underbrace{f_1 h_2 - f_2 h_1}_{\leq L_*} + \underbrace{N_{D3}}_{\geq 0} = L_* \qquad K = -\ln(-i(\tau - \bar{\tau}))$$

$$D_\tau = \partial_\tau + \partial_\tau K$$

$$D_\tau W = 0 \longrightarrow \tau = -\frac{f_1 - i f_2}{h_1 - i h_2}$$

ex. : $L_* = 150$

$$N_{TOT} = 37208$$



simplest case : rigid CY $\rightsquigarrow$ only $\tau = c_0 + \frac{i}{g_s}$

$\hookrightarrow b_3 = 2$

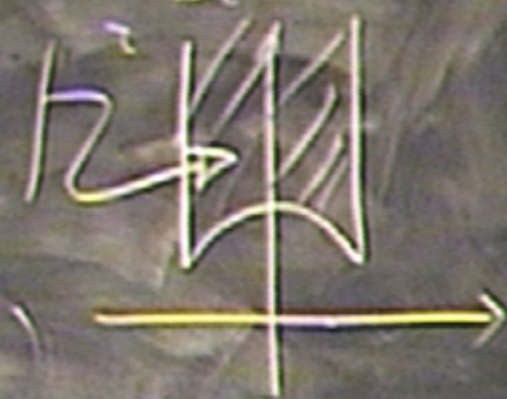
$$\Rightarrow W = (f_1 + i f_2) - \tau (h_1 + i h_2)$$

$$\underbrace{f_1 h_2 - f_2 h_1}_{\leq L_*} + \underbrace{N_{D3}}_{\geq 0} = L_*$$

$$K = -\ln(-i(\tau - \bar{\tau}))$$

$$D_\tau = \partial_\tau + \partial_\tau K$$

$$D_\tau W = 0 \longrightarrow \tau = -\frac{f_1 - i f_2}{h_1 - i h_2}$$



ex. : $L_* = 150$

$$N_{TOT} = 37208$$

SUSY VAC.

$D_a W = 0$

$D_a = \partial_a + \partial_a K$

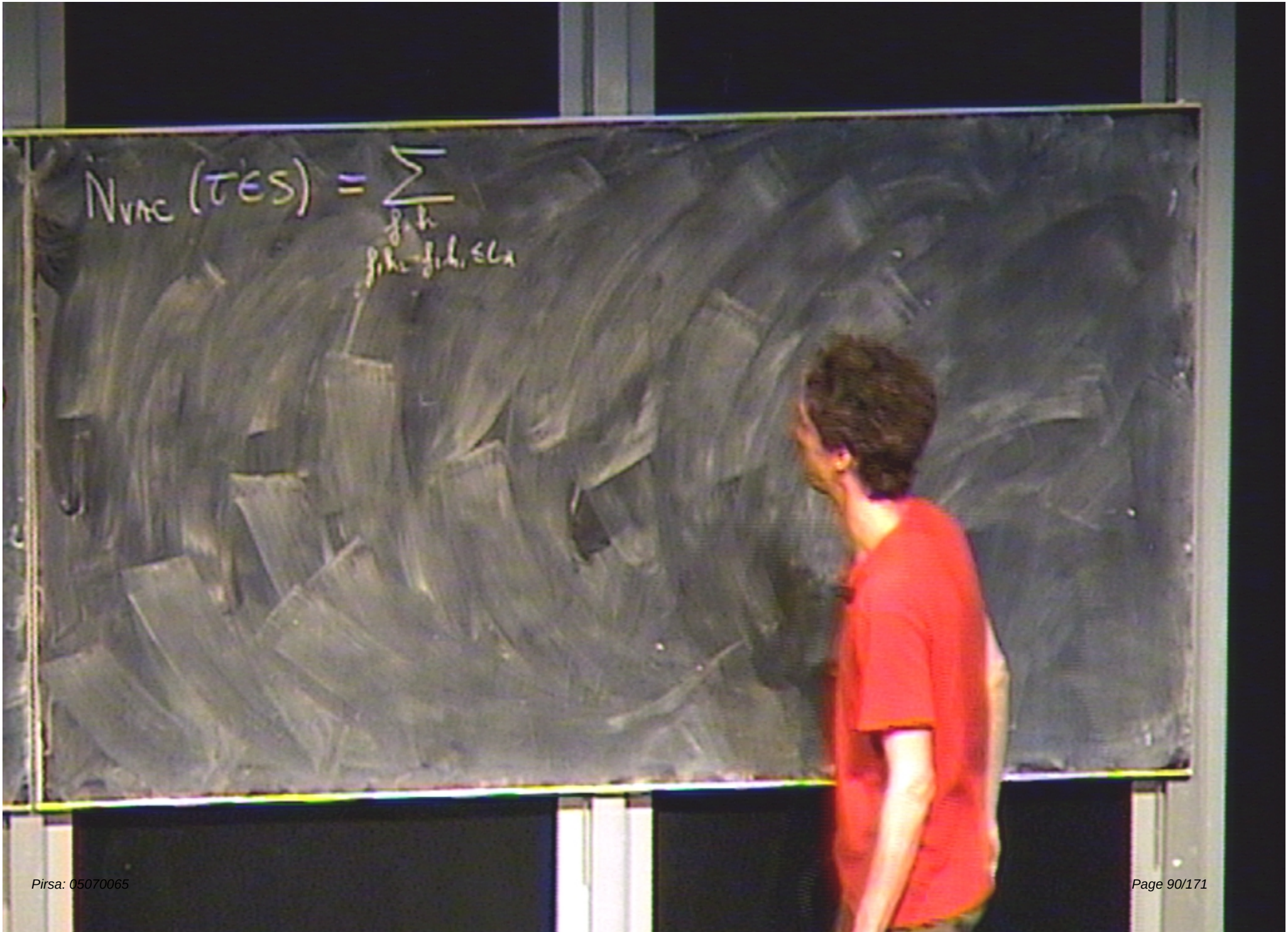
$K = -2 \ln V(\rho) - \ln i \int \Omega \wedge \bar{\Omega}$

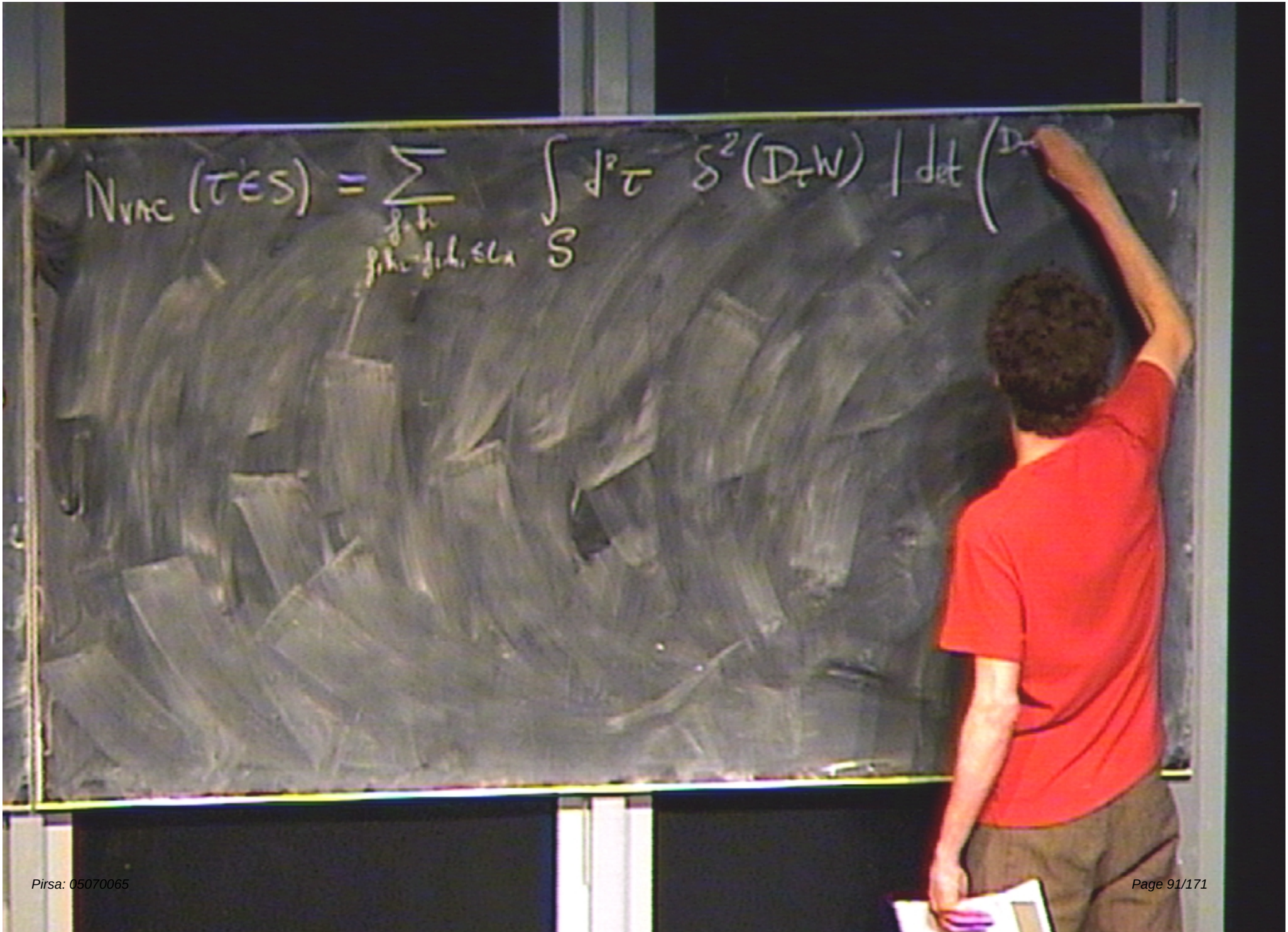
to trust approx.: $e^{-\rho_i} \ll 1 \Rightarrow |W_{flux}| \ll 1$

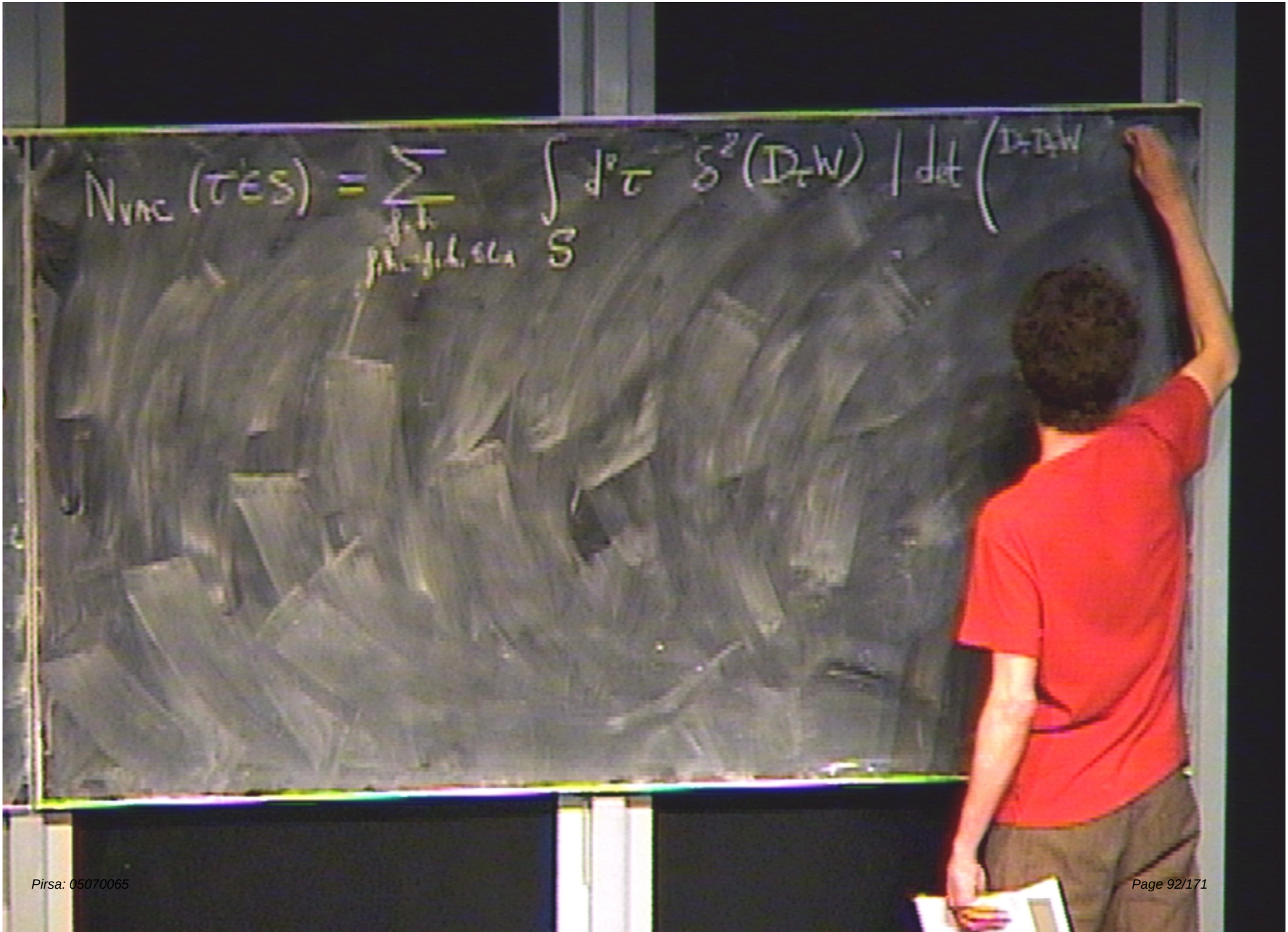
generically $|W_{flux}| > 1$ for many still.

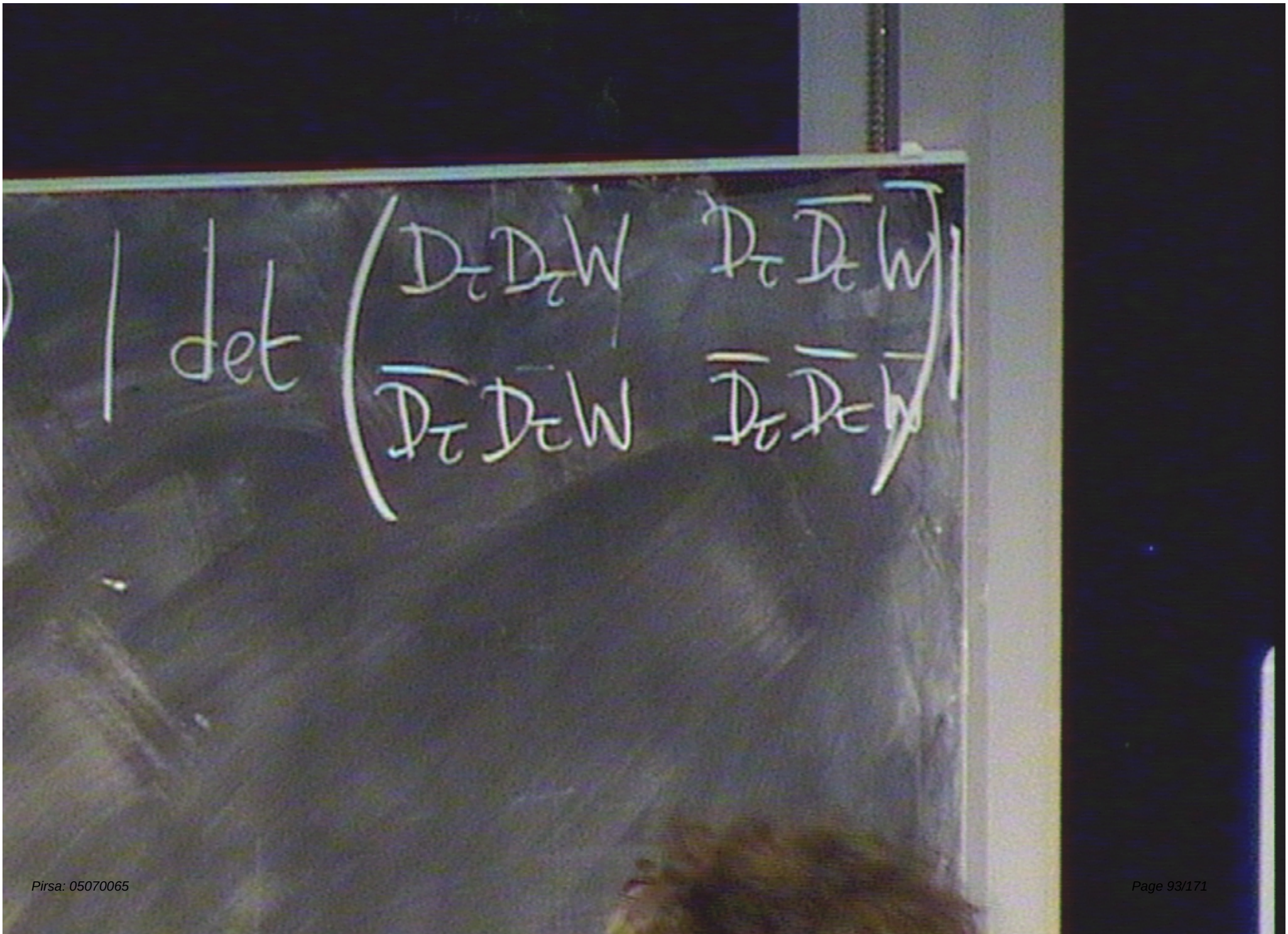
$= |(N^\alpha + \tau M^\alpha)\Pi_\alpha|$

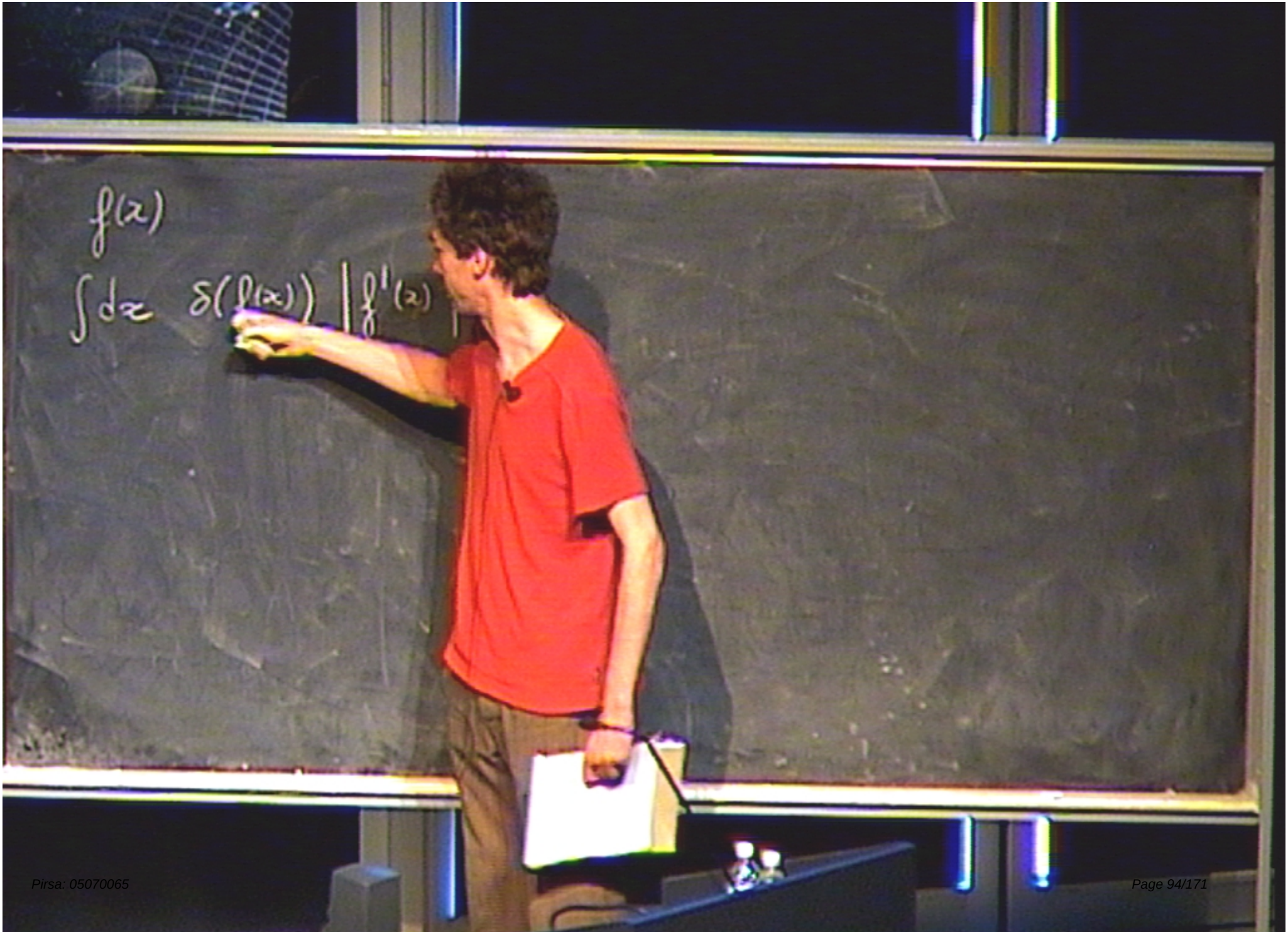
for fraction $\sim \lambda_*$ $\rightarrow |W_{flux}|^2 < \lambda_*$


$N_{VAC}(\tau \in S) = \sum_{f,h}$
$f_1 h_2 - f_2 h_1 \leq C_A$


$N_{vac}(\tau \in S) = \sum_{\substack{f,h \\ f_1 h_2 - f_2 h_1 \le c_k}} \int_S d^2\tau \; \delta^2(D_\tau W) \, |\det($

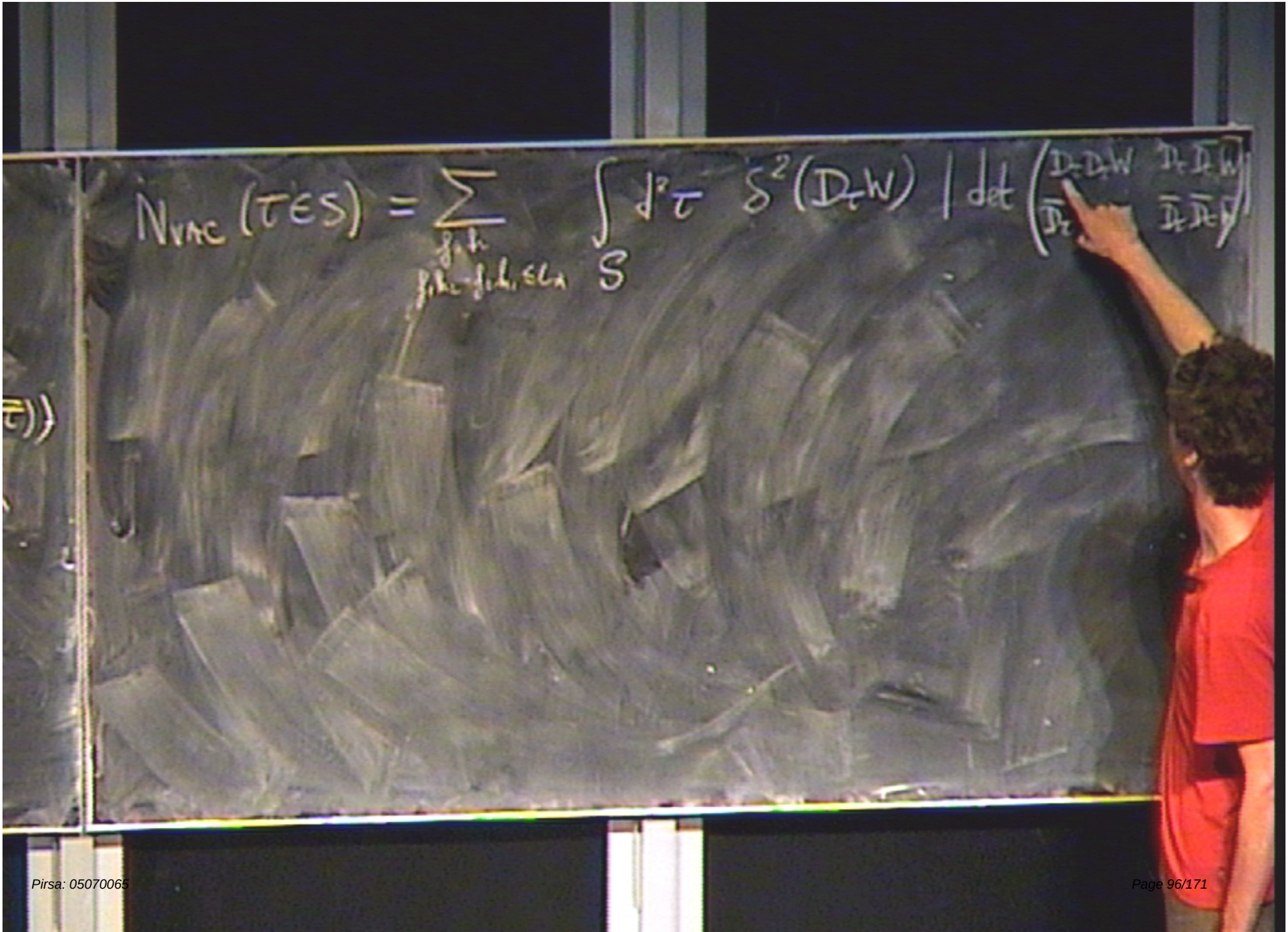

$N_{vac}(\tau \in S) = \sum_{\substack{f,h \\ f,h \to f,h \in L_a}} \int_S d^2\tau \; \delta^2(D_\tau W) \, |\det(D_\tau D_\tau W$

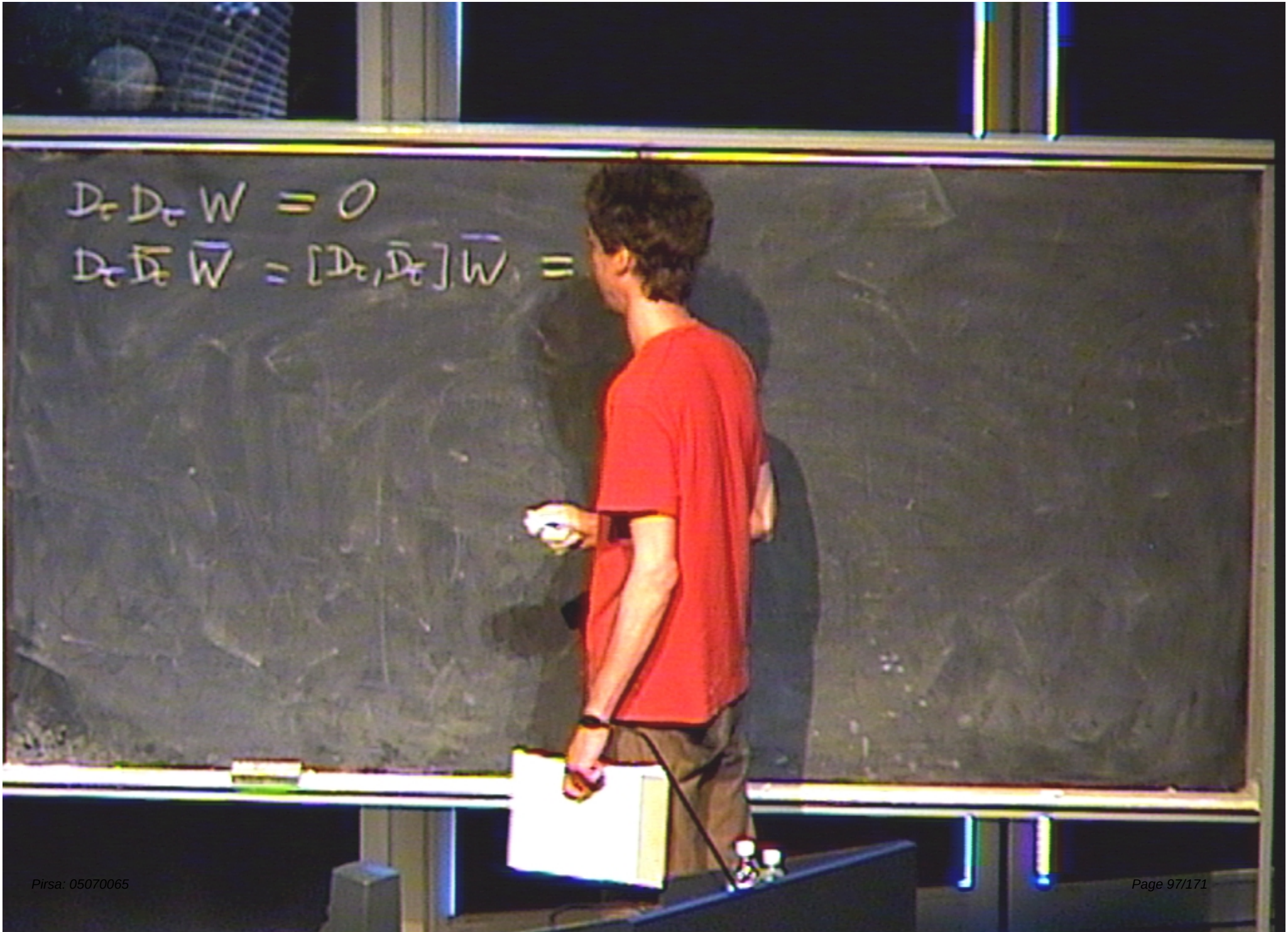

det

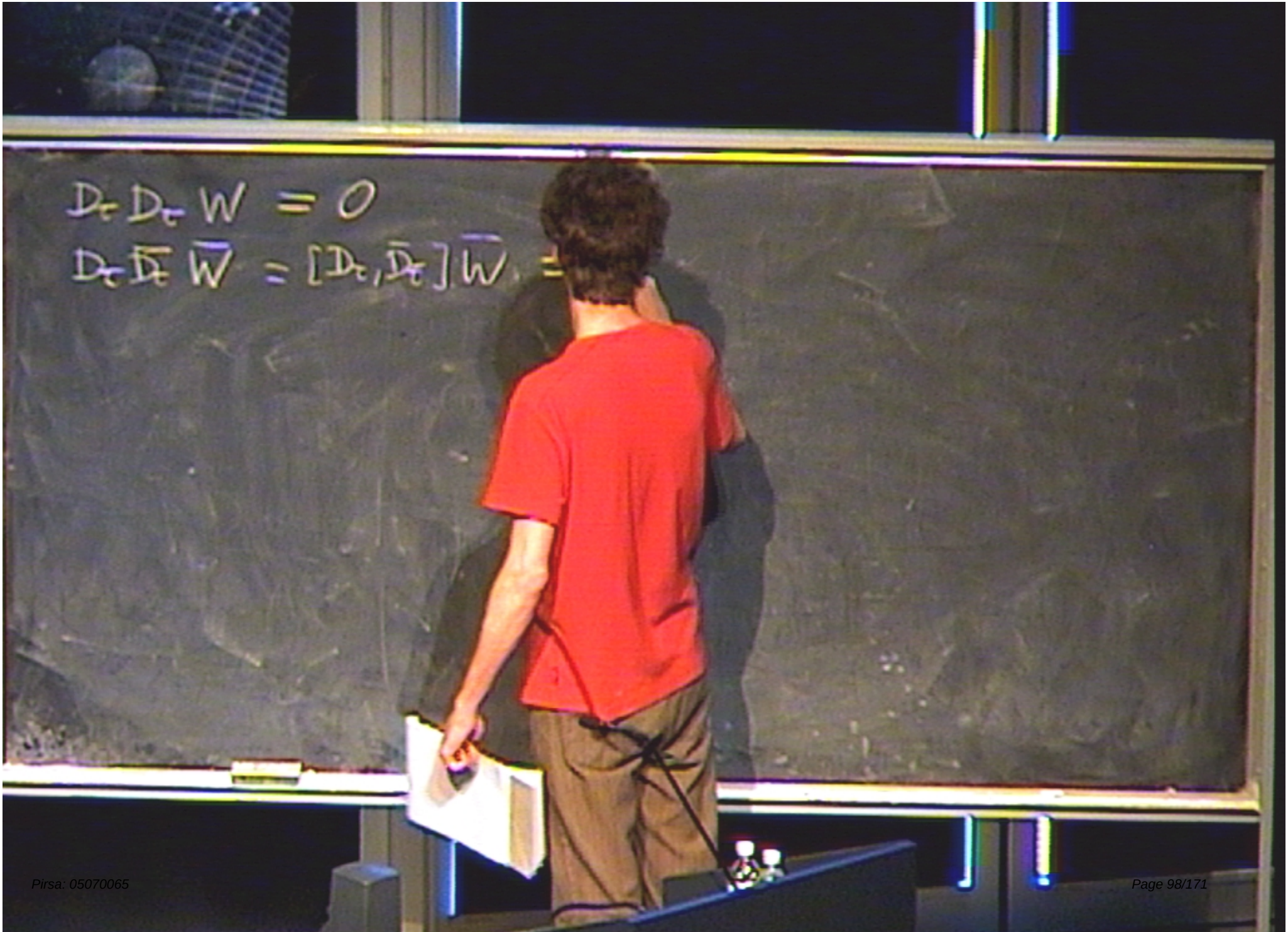

f(x)
∫dx δ(f(x)) |f'(x)|

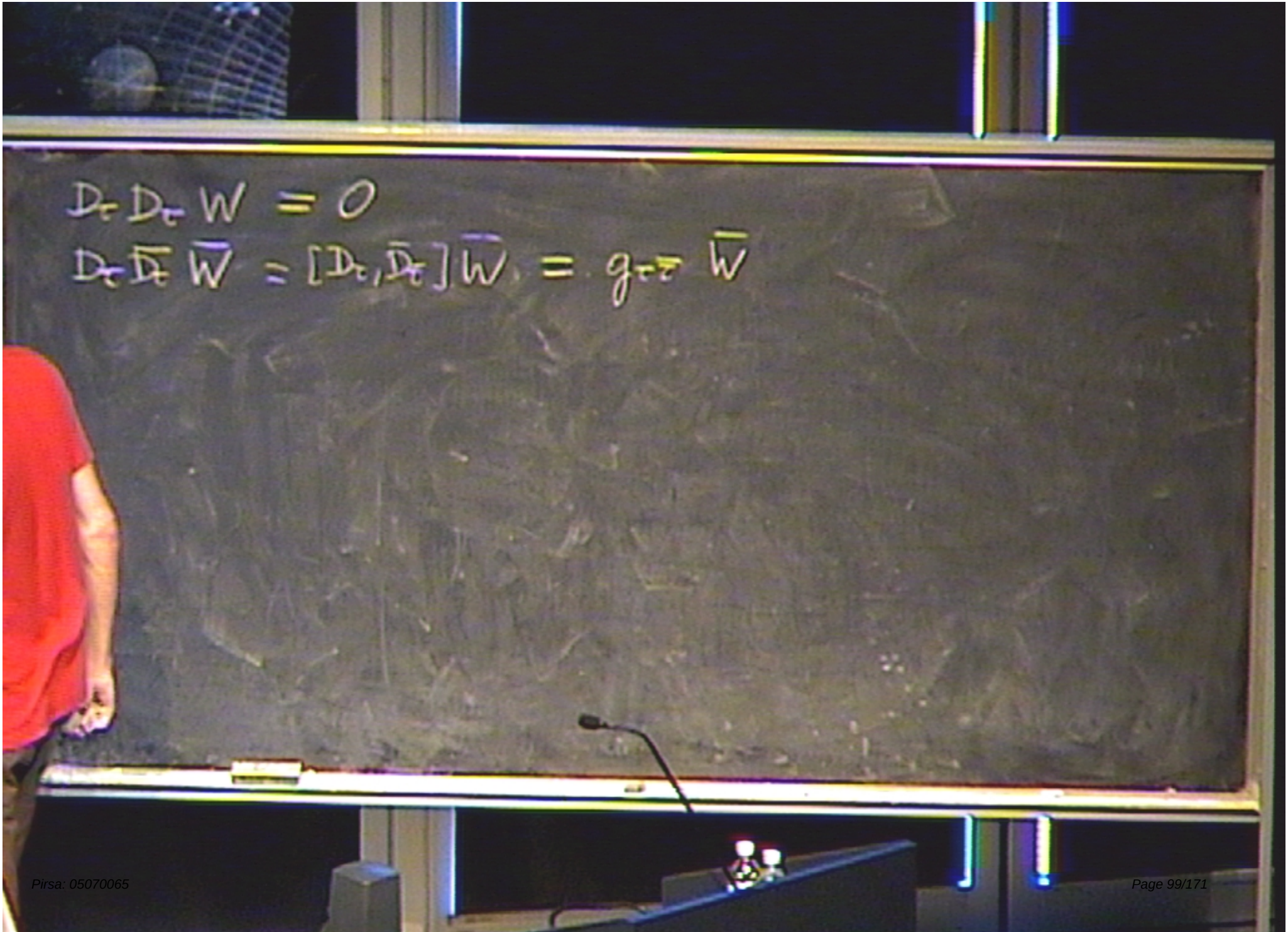
$f(x)$

$\int dx \; \delta(f(x)) \; |f'(x)|$


$N_{vac}(\tau \in S) = \sum_{\substack{f,h \\ f_1 h_2 - f_2 h_1 \le L_A}} \int_S d^2\tau \; \delta^2(D_\tau W) \left| \det \begin{pmatrix} D_\tau D_\tau W & D_\tau \bar{D}_\tau W \\ \bar{D}_\tau & \bar{D}_\tau \bar{D}_\tau W \end{pmatrix} \right|$

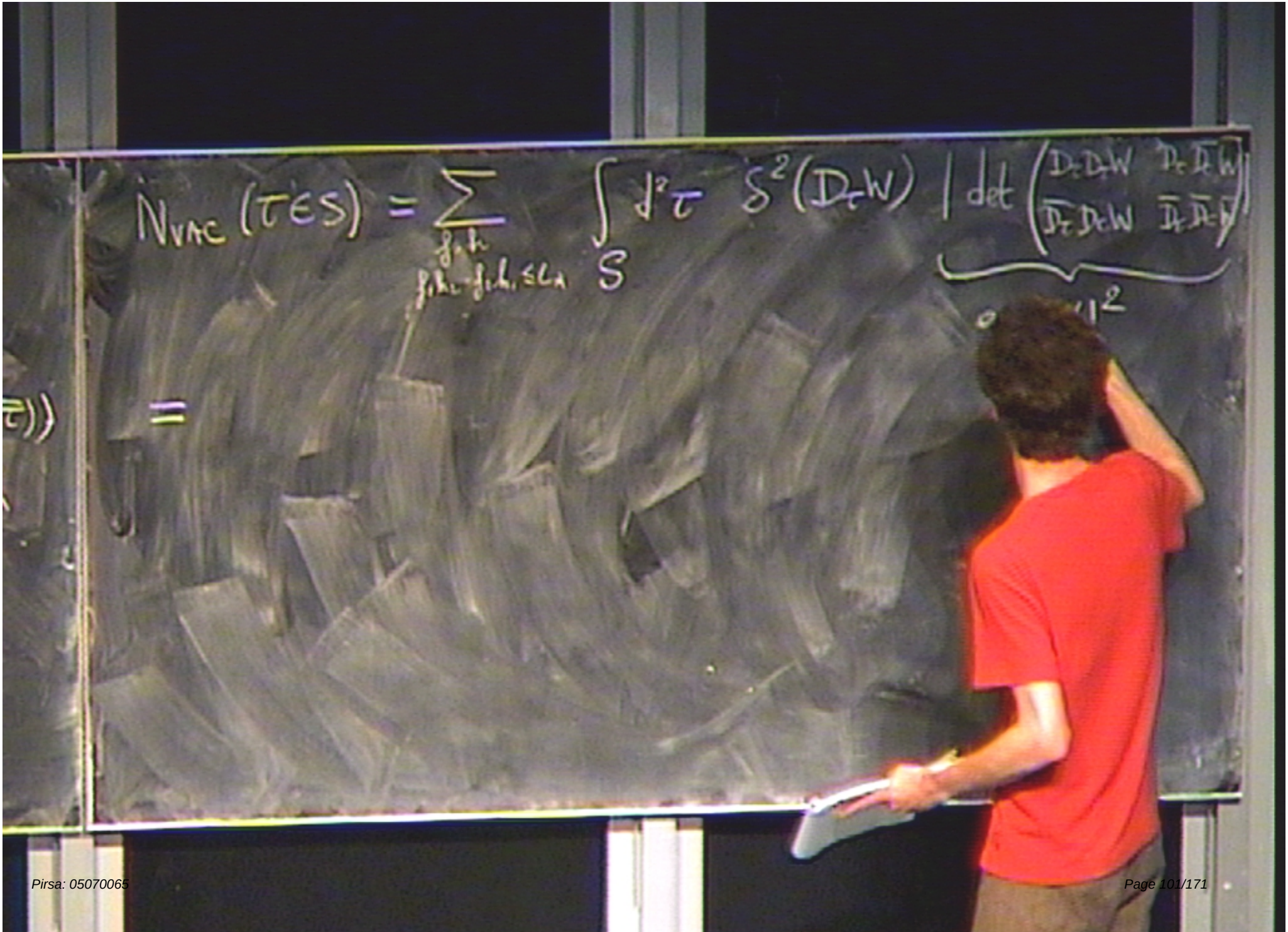

$D_+ D_+ W = 0$
$D_+ \bar{D}_+ \bar{W} = [D_+, \bar{D}_+] \bar{W} =$


$D_t D_t W = 0$
$D_t \bar{D}_t \bar{W} = [D_t, \bar{D}_t] \bar{W}$


$D_\tau D_\tau W = 0$
$D_\tau \bar{D}_\tau \bar{W} = [D_\tau, \bar{D}_\tau]\bar{W} = g_{\tau\bar{\tau}} \bar{W}$

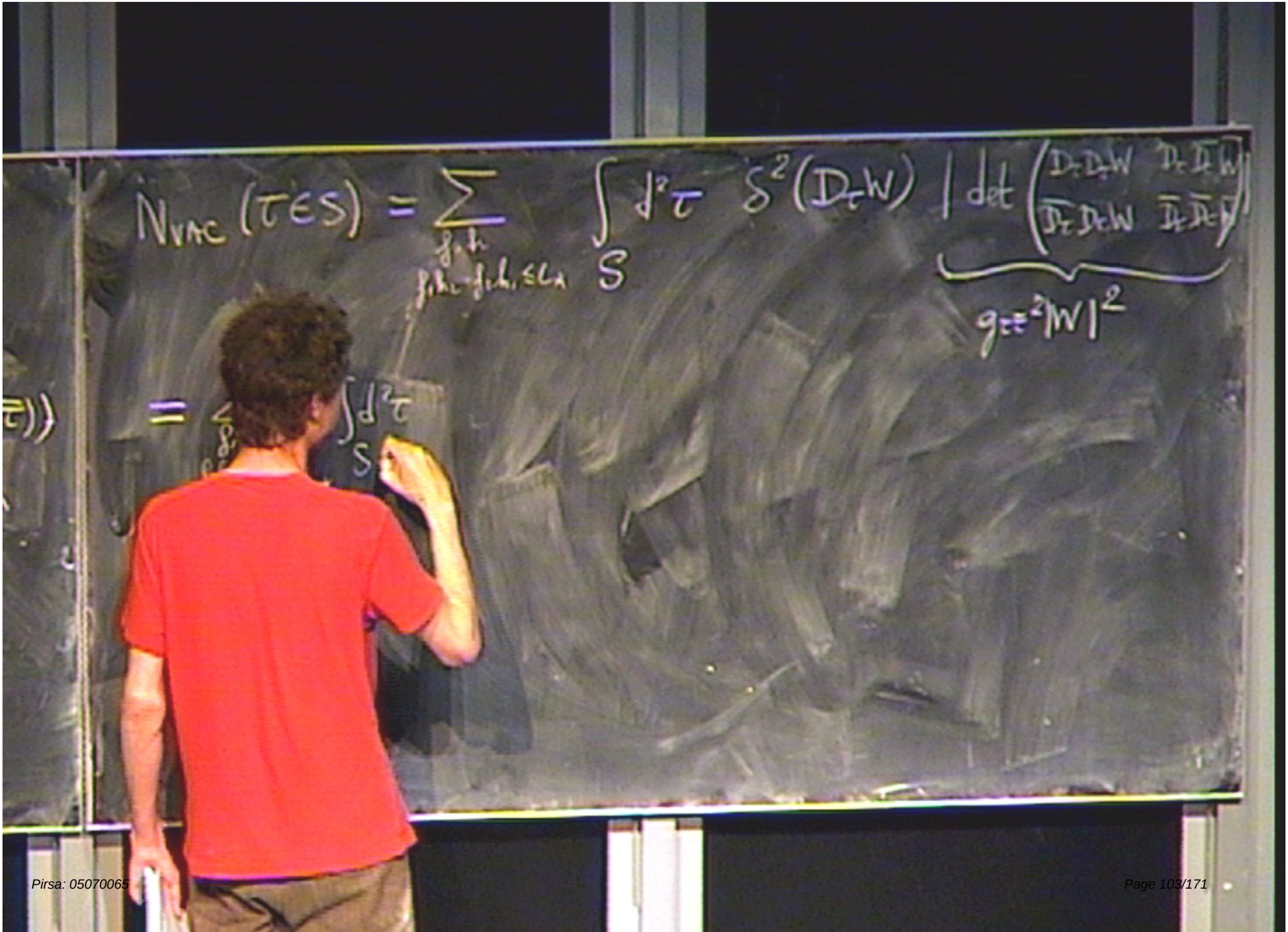
$D_\tau D_\tau W = 0$

$D_\tau \bar{D}_\tau \bar{W} = [D_\tau, \bar{D}_\tau]\bar{W} = g_{\tau\bar{\tau}} \bar{W}$


$N_{VAC}(\tau \in S) = \sum_{\substack{f_i, h_i \\ f_i h_i - f_i h_i \in \mathcal{L}_A}} \int_S d^2\tau \; \delta^2(D_{\bar{z}} W) \left| \det \begin{pmatrix} D_z D_z W & D_z \bar{D}_{\bar{z}} W \\ \bar{D}_{\bar{z}} D_z W & \bar{D}_{\bar{z}} \bar{D}_{\bar{z}} W \end{pmatrix} \right|$
$= \sum_{\substack{f_i, h_i \\ f_i h_i - f_i h_i \in \mathcal{L}_A}} \int_S d^2\tau \; \delta^2(D_{\bar{z}} W) \left| \det \begin{pmatrix} D_z D_z W & D_z \bar{D}_{\bar{z}} W \\ \bar{D}_{\bar{z}} D_z W & \bar{D}_{\bar{z}} \bar{D}_{\bar{z}} W \end{pmatrix} \right|^2$

$$N_{vac}(\tau \in S) = \sum_{\substack{f,h \\ f_1 h_1 - f_2 h_2 \le L_A}} \int_S d^2\tau \; \delta^2(D_\tau W) \underbrace{\left| \det \begin{pmatrix} D_\tau D_\tau W & D_\tau \bar{D}_\tau W \\ \bar{D}_\tau D_\tau W & \bar{D}_\tau \bar{D}_\tau W \end{pmatrix} \right|}_{g_{\tau\bar{\tau}}^2 |W|^2}$$

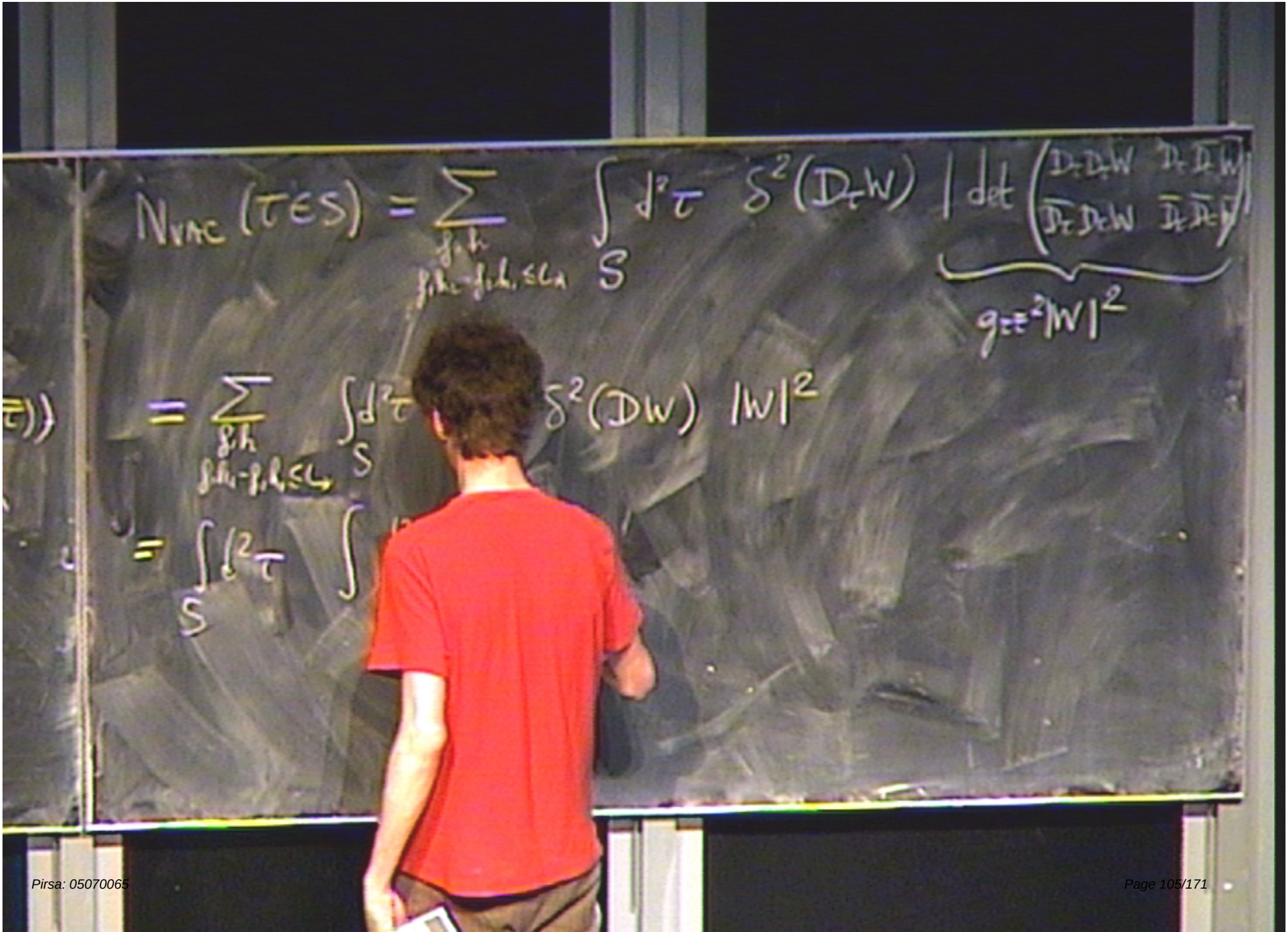
$$\bar{\tau})) \quad =$$

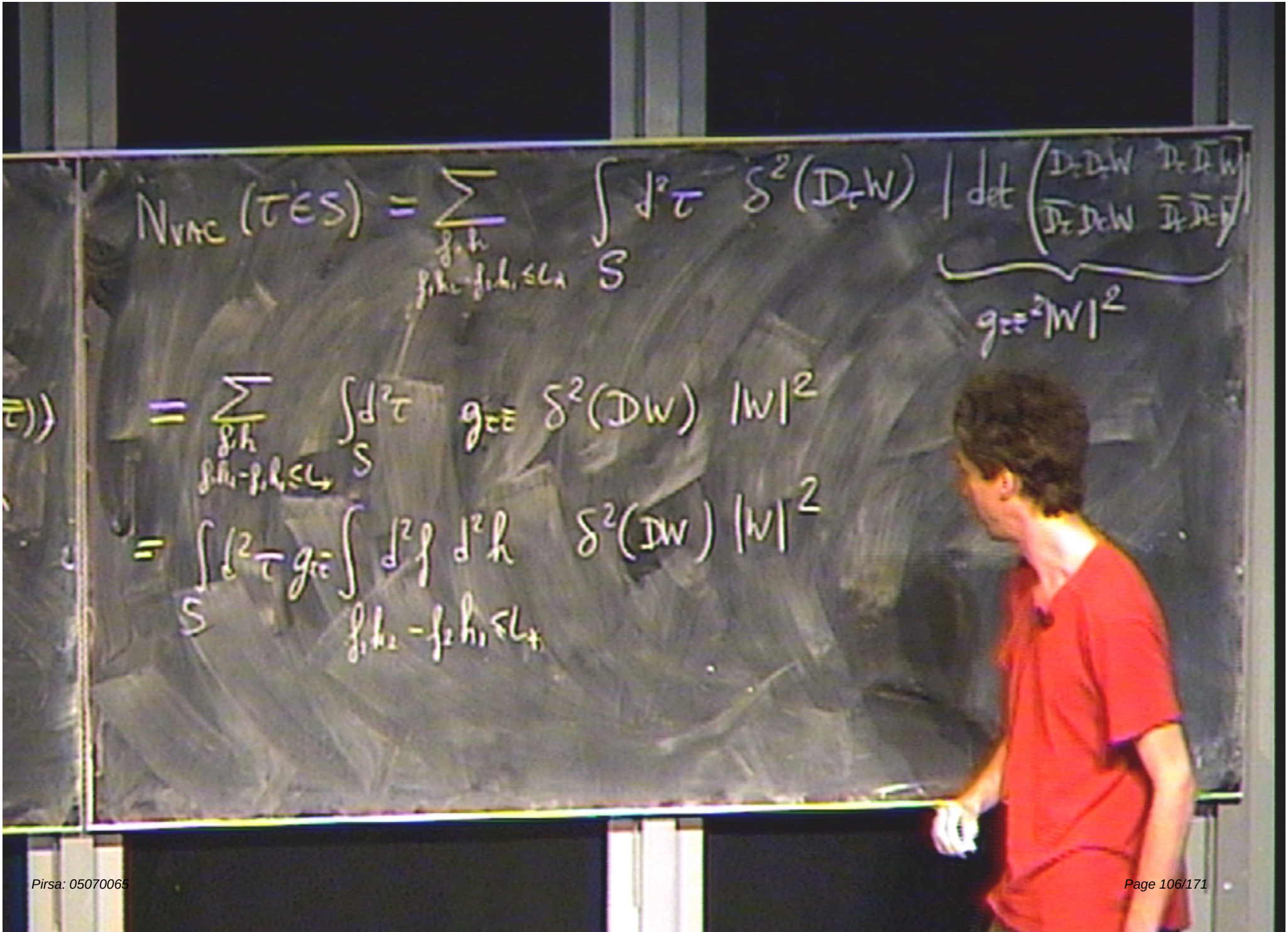

$N_{VAC}(\tau \in S) = \sum_{f,h} \int_S d^2\tau \; \delta^2(D_\tau W) \left| \det \begin{pmatrix} D_\tau D_\tau W & D_\tau \bar{D}_\tau W \\ \bar{D}_\tau D_\tau W & \bar{D}_\tau \bar{D}_\tau W \end{pmatrix} \right|$
$g_{\tau\bar{\tau}} = 2|W|^2$

$$N_{VAC}(\tau \in S) = \sum_{\substack{\beta,h \\ \beta_i h_i - \beta_j h_j \le L_A}} \int_S d^2\tau \; \delta^2(D_{\bar\tau} W) \underbrace{\left| \det \begin{pmatrix} D_\tau D_\tau W & D_\tau \bar D_\tau W \\ \bar D_\tau D_\tau W & \bar D_\tau \bar D_\tau W \end{pmatrix} \right|}_{g_{\tau\bar\tau}^2 |W|^2}$$

$$= \sum_{\substack{\beta,h \\ \beta_i h_i - \beta_j h_j \le L_A}} \int_S d^2\tau \; g_{\tau\bar\tau} \, \delta^2(DW) \, |W|^2$$

$=$

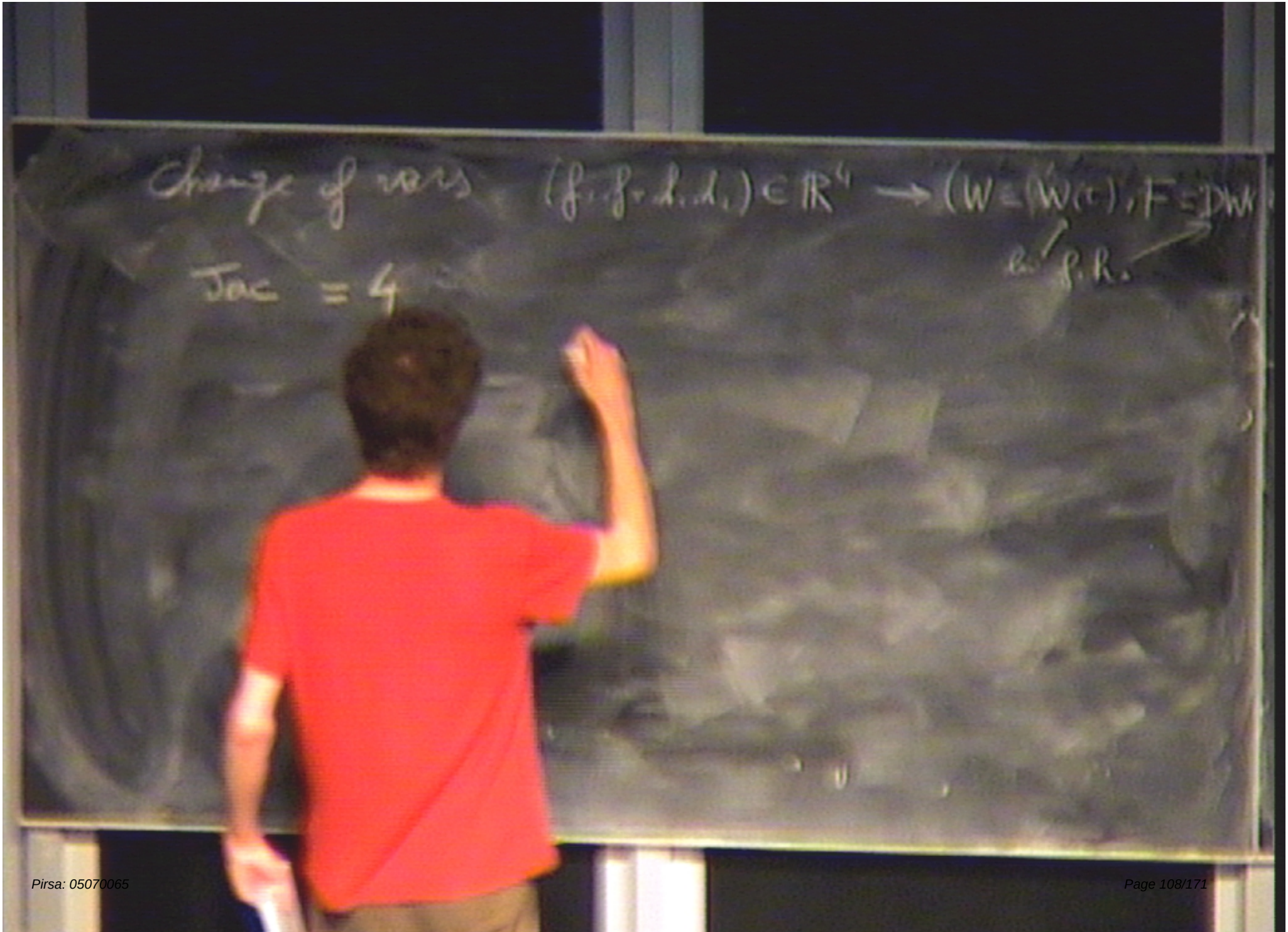

$N_{VAC}(\tau \in S) = \sum_{\substack{\beta,h \\ \beta_i h_i - \beta_j h_j \in C_A}} \int_S d^2\tau \; \delta^2(D_{\bar{z}} W) \left| \det \begin{pmatrix} D_z D_z W & D_z D_{\bar{z}} W \\ D_{\bar{z}} D_z W & D_{\bar{z}} D_{\bar{z}} W \end{pmatrix} \right|$
$g_{z\bar{z}} = e^2 |W|^2$
$= \sum_{\substack{\beta,h \\ \beta_i h_i - \beta_j h_j \in C_A}} \int_S d^2\tau \; \delta^2(DW) \; |W|^2$
$= \int_S d^2\tau$

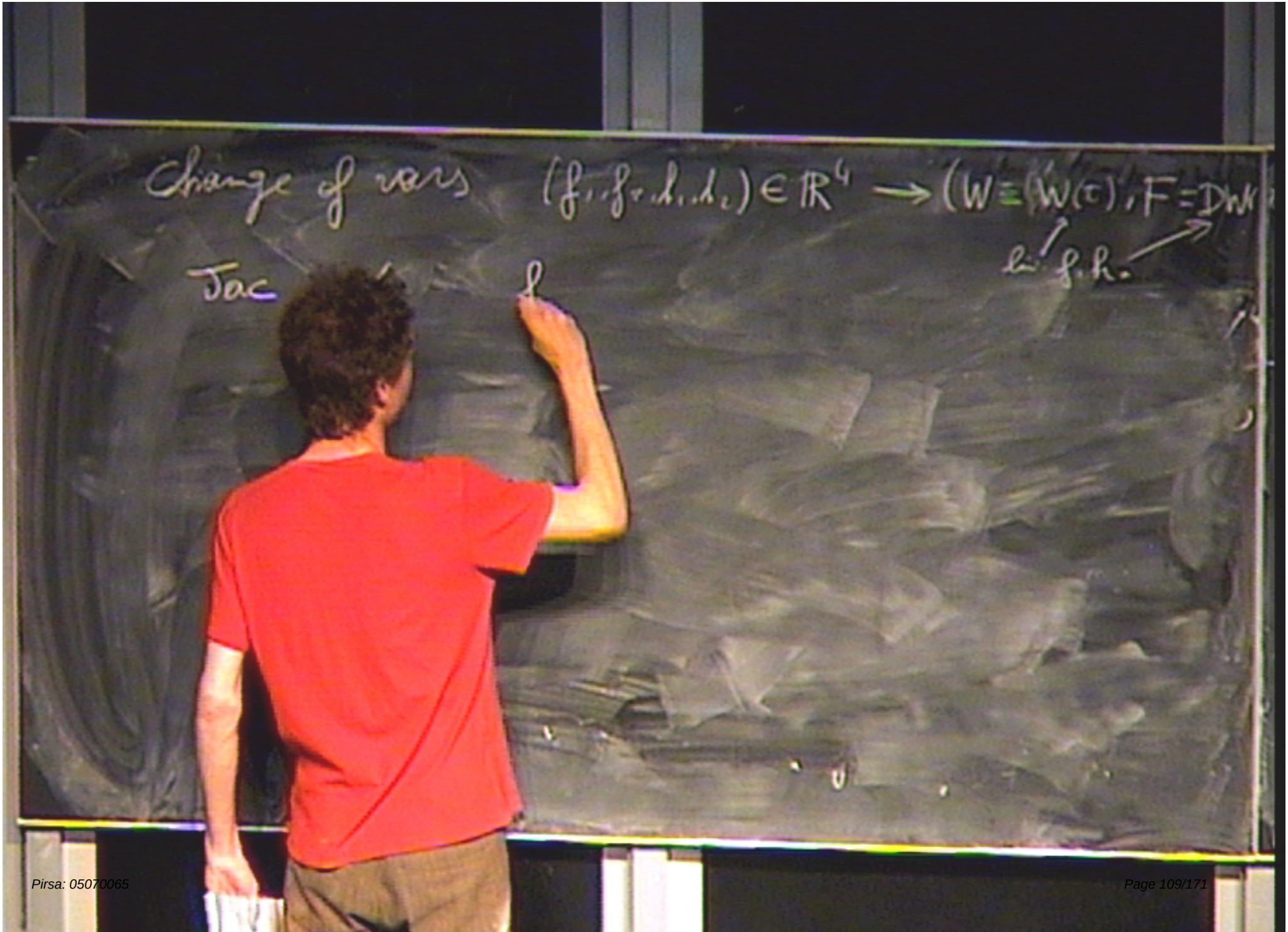

$N_{VAC}(\tau \in S) = \sum_{\substack{f,h \\ f_1h_2 - f_2h_1 \le L_4}} \int_S d^2\tau\ \delta^2(D_z W) \left| \det \begin{pmatrix} D_z D_z W & D_z \bar{D}_z W \\ \bar{D}_z D_z W & \bar{D}_z \bar{D}_z W \end{pmatrix} \right|$
$g_{zz}^2 |W|^2$
$= \sum_{\substack{f,h \\ f_1h_2 - f_2h_1 \le L_4}} \int_S d^2\tau\ g_{z\bar{z}}\, \delta^2(DW)\, |W|^2$
$= \int_S d^2\tau\, g_{z\bar{z}} \int_{f_1h_2 - f_2h_1 \le L_4} d^2f\, d^2h\ \delta^2(\mathcal{I}W)\, |W|^2$

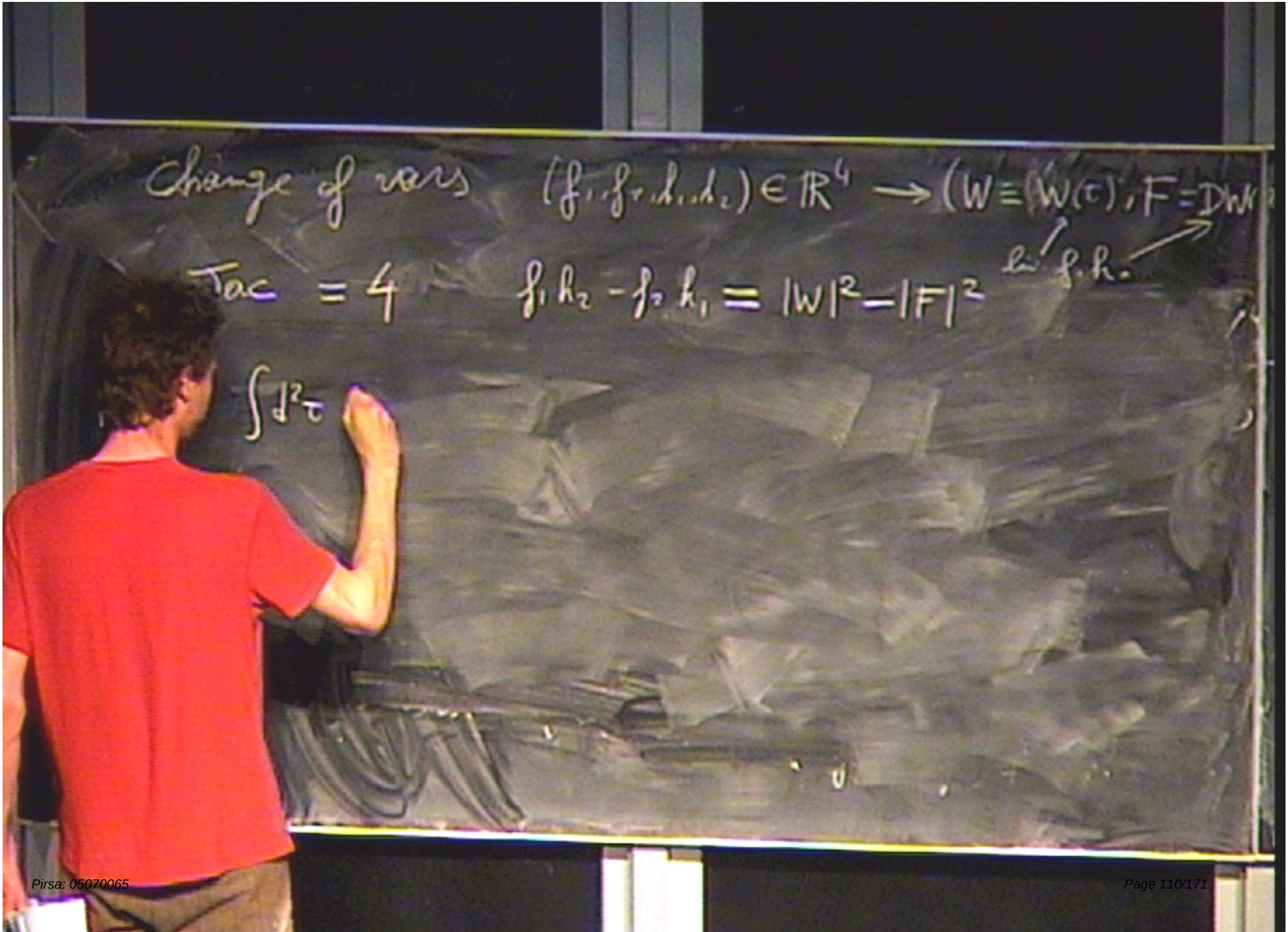
$$N_{VAC}(\tau \in S) = \sum_{\substack{f_1 h_2 - f_2 h_1 \le L_4}} \int_S d^2\tau \; \delta^2(D_{\bar z} W) \underbrace{\left| \det \begin{pmatrix} D_z D_{\bar z} W & D_z D_{\bar z} \bar W \\ \bar D_z D_{\bar z} W & \bar D_z D_{\bar z} \bar W \end{pmatrix} \right|}_{g_{z\bar z}^2 |W|^2}$$

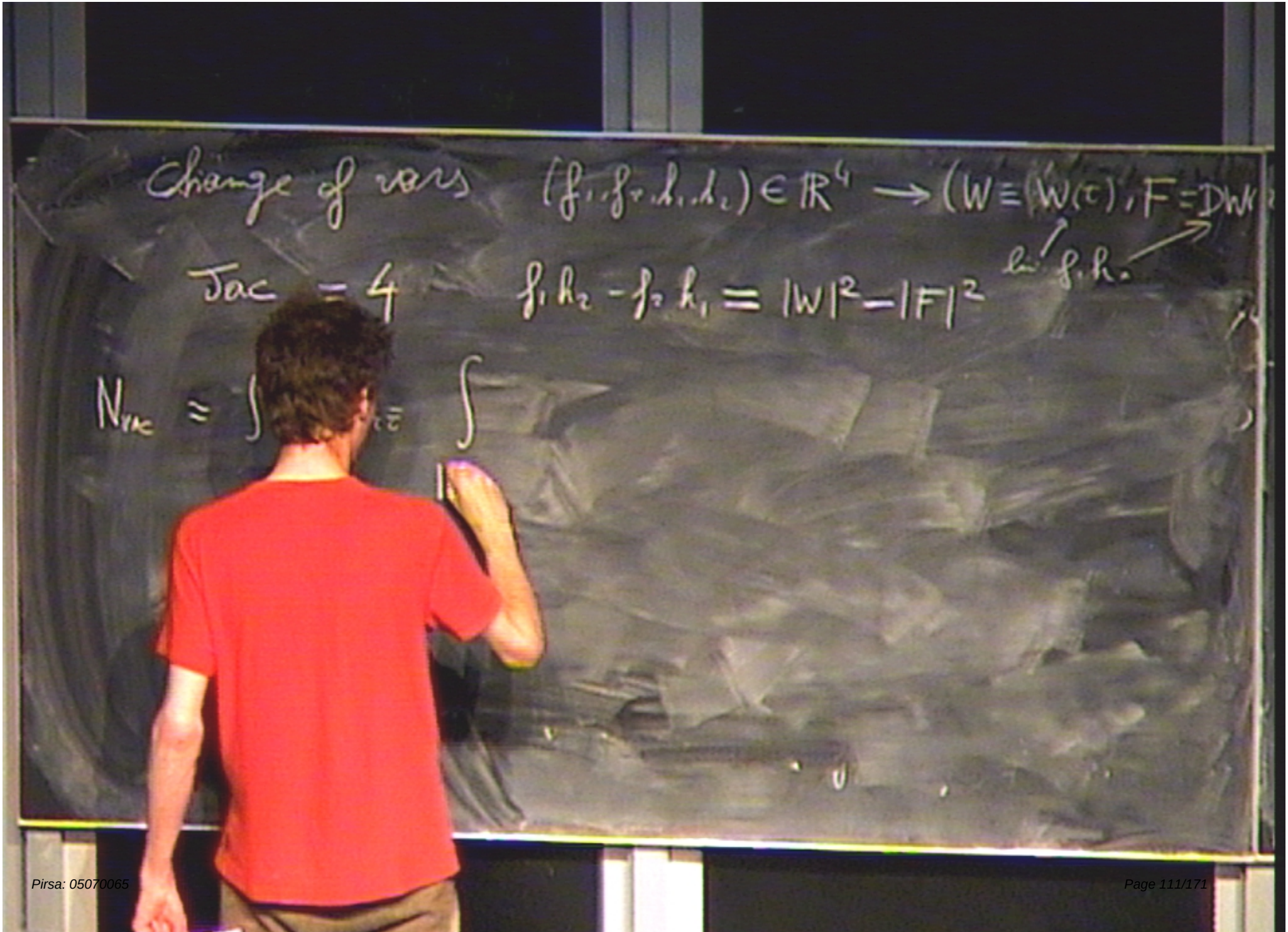
$$= \sum_{\substack{f,h \\ f_1 h_2 - f_2 h_1 \le L_4}} \int_S d^2\tau \; g_{z\bar z} \, \delta^2(DW) \, |W|^2$$

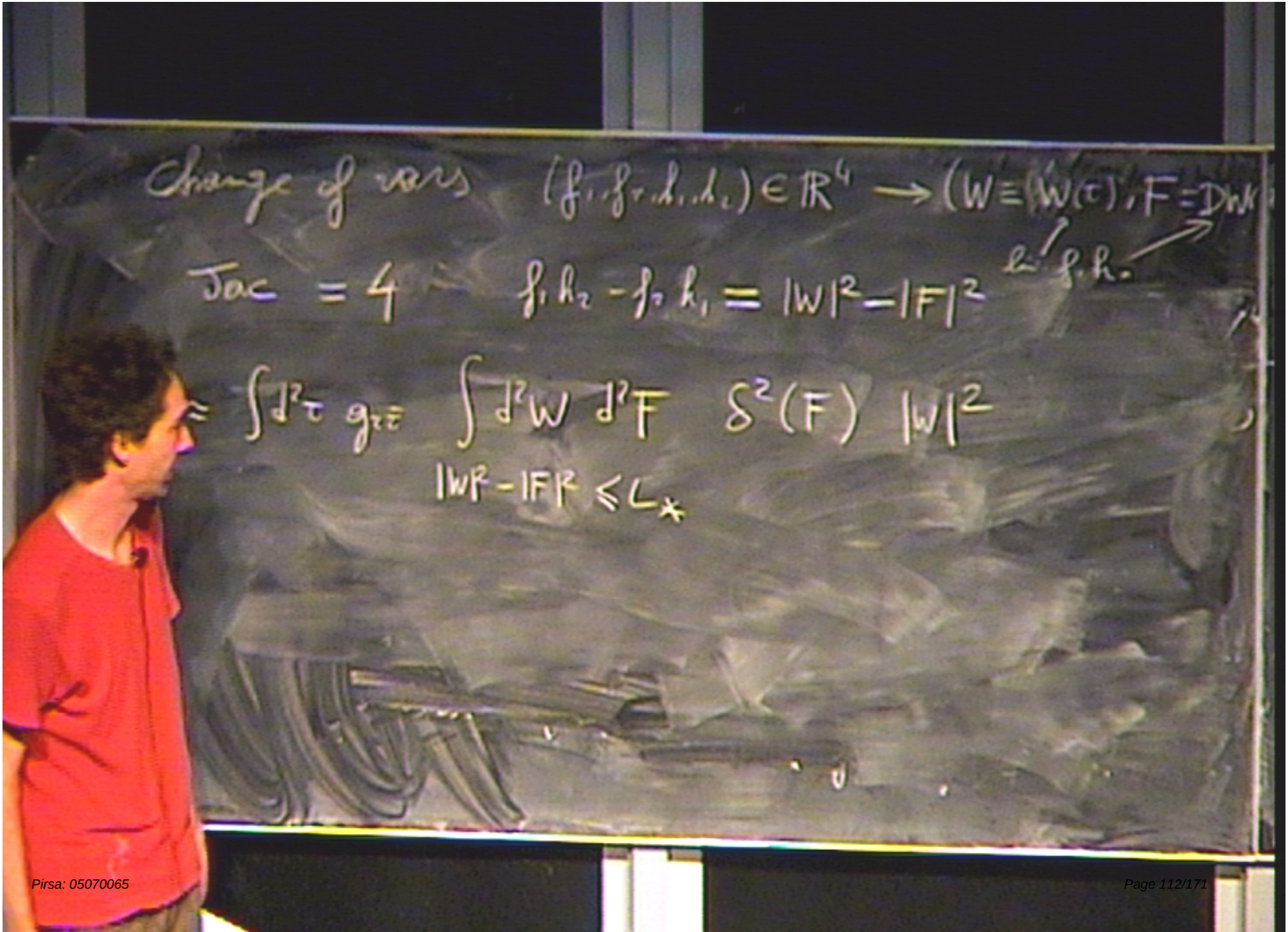
$$= \int_S d^2\tau \, g_{z\bar z} \int_{f_1 h_2 - f_2 h_1 \le L_4} d^2 f \, d^2 h \; \delta^2(DW(z)) \, |W|^2$$

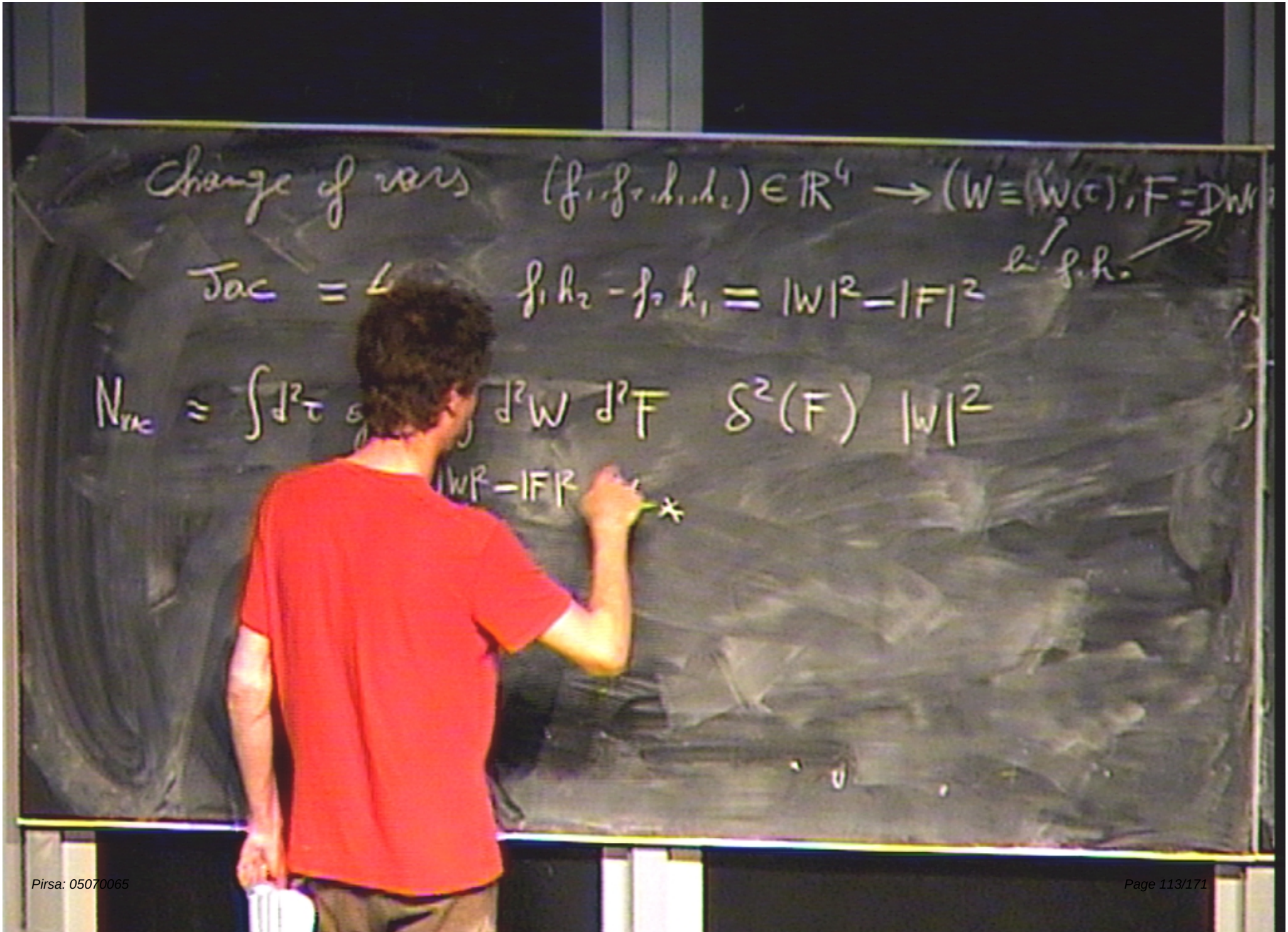

Change of vars
Jac = 4


Change of vars
Jac


Change of vars $(f_1, f_2, h_1, h_2) \in \mathbb{R}^4 \to (W \equiv W(\tau), F = DW$
lin f.h.
Tac $= 4$ $f_1 h_2 - f_2 h_1 = |W|^2 - |F|^2$
$\int d^2\tau$


Change of vars $(f_1, f_2, h_1, h_2) \in \mathbb{R}^4 \to (W \equiv W(\tau), F = DW$
$Jac = 4$
$f_1 h_2 - f_2 h_1 = |W|^2 - |F|^2$
$N_{vac} \approx \int$
$\int$


Change of vars $(f_1, f_2, h_1, h_2) \in \mathbb{R}^4 \to (W \equiv W(\tau), F = DW$
$\ln f_1 h_2$
Jac $= 4$ $f_1 h_2 - f_2 h_1 = |W|^2 - |F|^2$
$\approx \int d^2\tau \, g_{z\bar{z}} = \int d^2W \, d^2F \; \delta^2(F) \; |W|^2$
$|W|^2 - |F|^2 \leq L_*$


Change of vars $(f_1, f_2, h_1, h_2) \in \mathbb{R}^4 \to (W \equiv W(\tau), F = DW$
Jac $= 4$
$f_1 h_2 - f_2 h_1 = |W|^2 - |F|^2$
$N_{vac} \approx \int d^2\tau \int d^2W\, d^2F\, \delta^2(F)\, |W|^2$
$|W|^2 - |F|^2$

Change of vars $(f_1, f_2, h_1, h_2) \in \mathbb{R}^4 \to (W \equiv W(\tau), F = DW($

lin. $f_1 h_2$

$$\text{Jac} = 4 \qquad f_1 h_2 - f_2 h_1 = |W|^2 - |F|^2$$

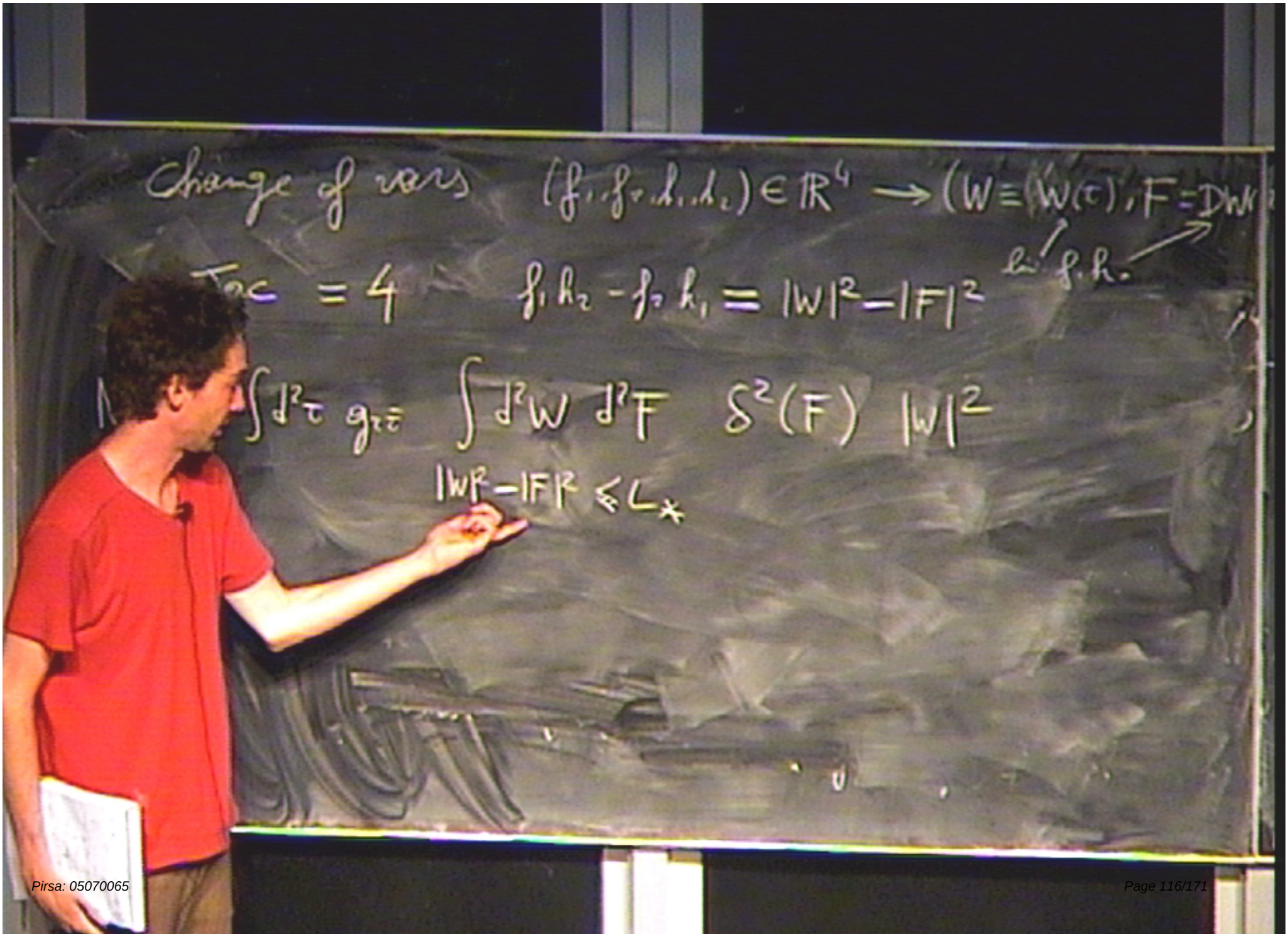
$$N_{vac} \simeq \int d^2\tau \, g_{\tau\bar{\tau}} \int_{|W|^2 - |F|^2 \leq L_*} d^2W \, d^2F \; \delta^2(F) \; |W|^2$$

Change of vars $(f_1, f_2, h_1, h_2) \in \mathbb{R}^4 \to (W \equiv W(\tau), F = DW($

lin. $f_1 h_2$

$$\text{Jac} = 4 \qquad f_1 h_2 - f_2 h_1 = |W|^2 - |F|^2$$

$$N_{vac} = \int d^2\tau \, g_{2\bar{\tau}} \int_{|W|^2 - |F|^2 \leq L_*} d^2W \, d^2F \; \delta^2(F) \; |W|^2$$


Change of vars $(f_1, f_2, h_1, h_2) \in \mathbb{R}^4 \to (W \equiv W(\tau), F = DW$
lin. f.h.
Jac $= 4$ $f_1 h_2 - f_2 h_1 = |W|^2 - |F|^2$
$\int d^2\tau \, g_{2\bar{\tau}} = \int d^2W \, d^2F \, \delta^2(F) \, |W|^2$
$|W|^2 - |F|^2 \leq L_*$

Change of vars $(f_1, f_2, h_1, h_2) \in \mathbb{R}^4 \to (W \equiv W(\tau), F = DW$

lin. $f_1 h_2$

$$\text{Jac} = 4 \qquad f_1 h_2 - f_2 h_1 = |W|^2 - |F|^2$$

$$N_{vac} = \int d^2\tau \, g_{\tau\bar\tau} \int_{|W|^2 - |F|^2 \leq L_*} d^2W \, d^2F \; \delta^2(F) \, |W|^2$$

$$= \int d^2\tau \, g_{\tau\bar\tau} \int_{|W|^2 \leq L_*} d^2W \, |W|^2 = 2\pi L_*^2 \underbrace{\int_S d^2\tau \, g_{\tau\bar\tau}}_{\mathrm{vol}(S)}$$

Change of vars $(f_1, f_2, h_1, h_2) \in \mathbb{R}^4 \longrightarrow (W \equiv W(\tau), F = D_n h)$

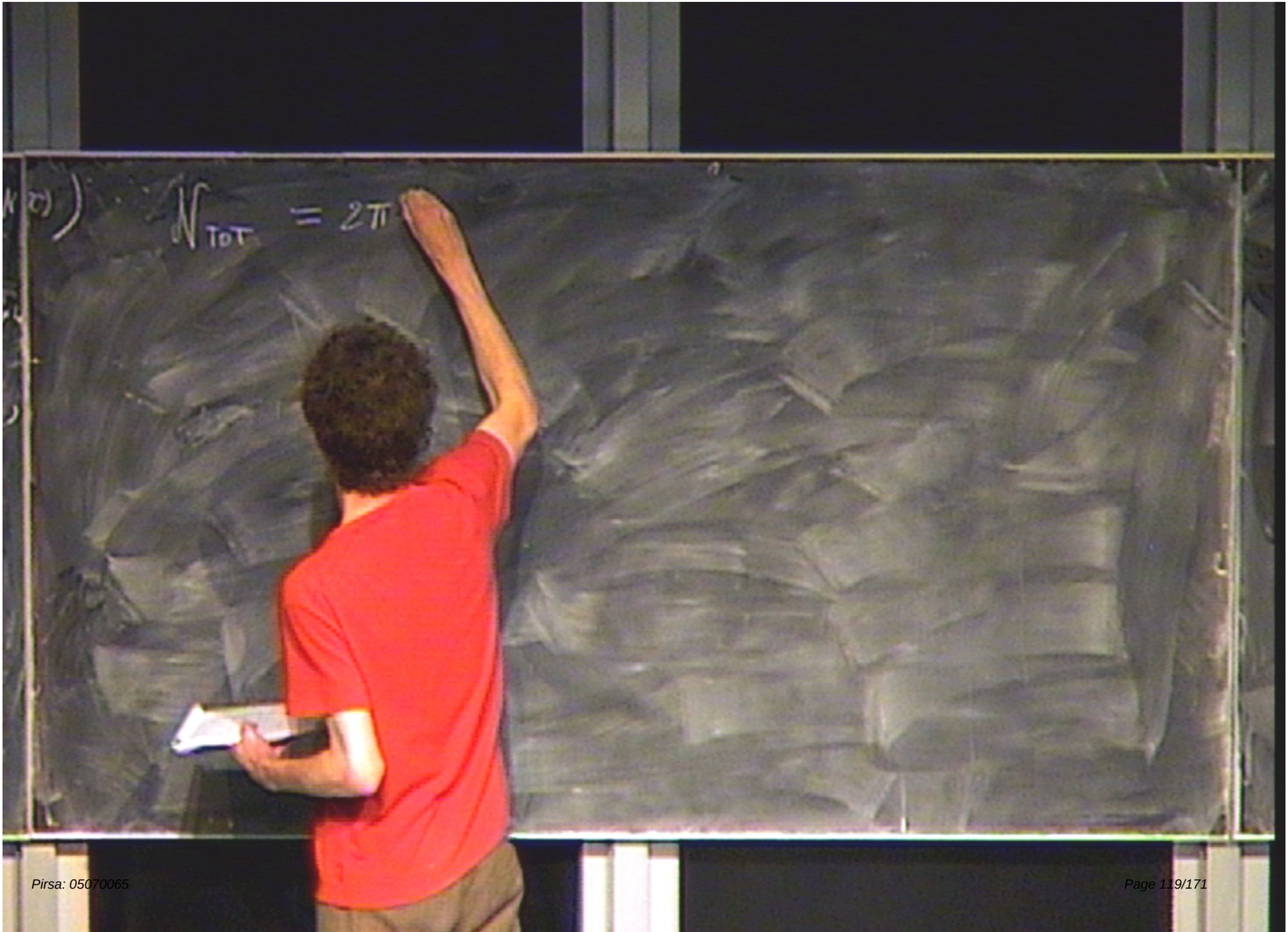
τ, f, h

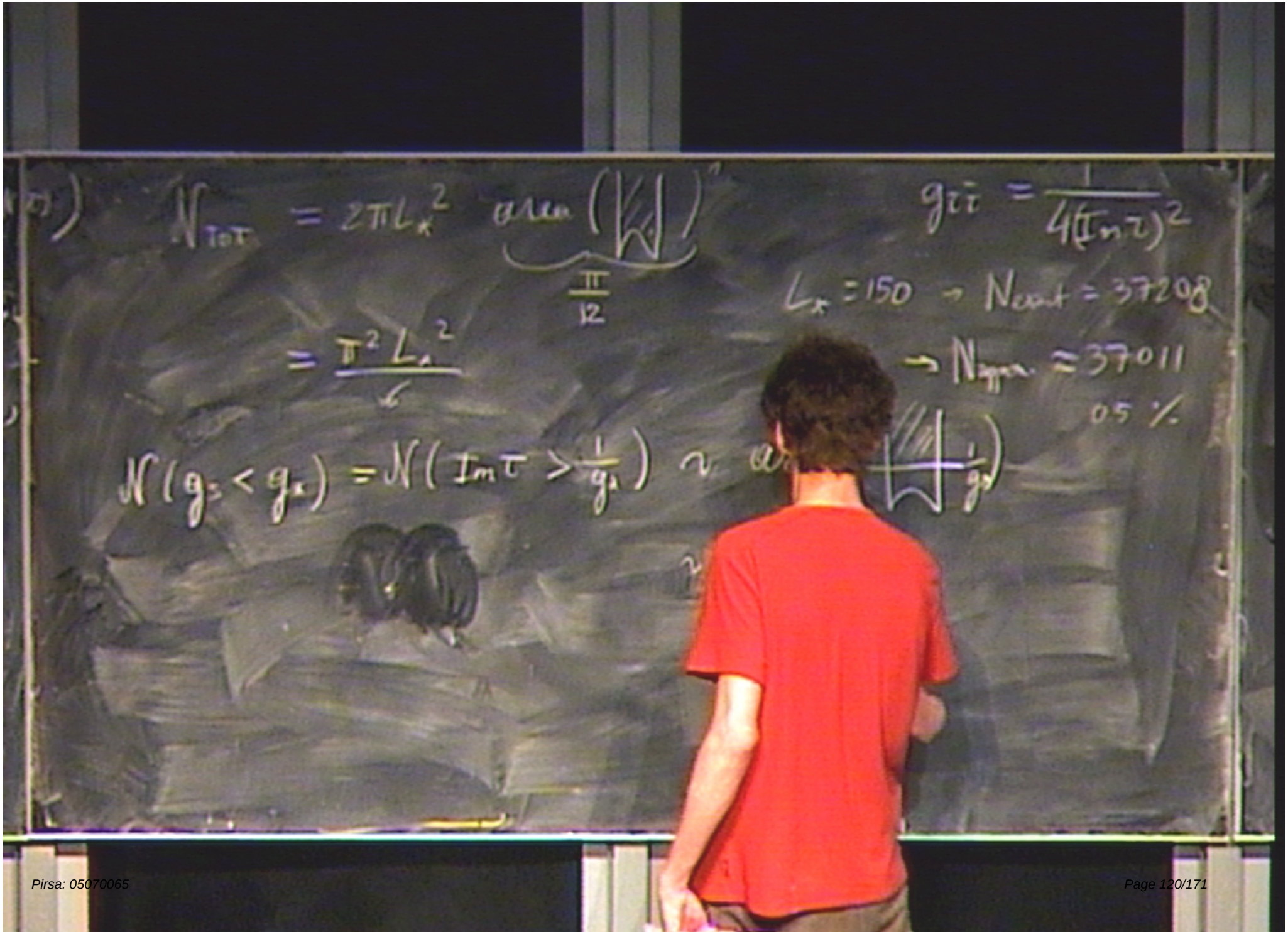
$\text{Jac} = 4 \qquad f_1 h_2 - f_2 h_1 = |W|^2 - |F|^2$

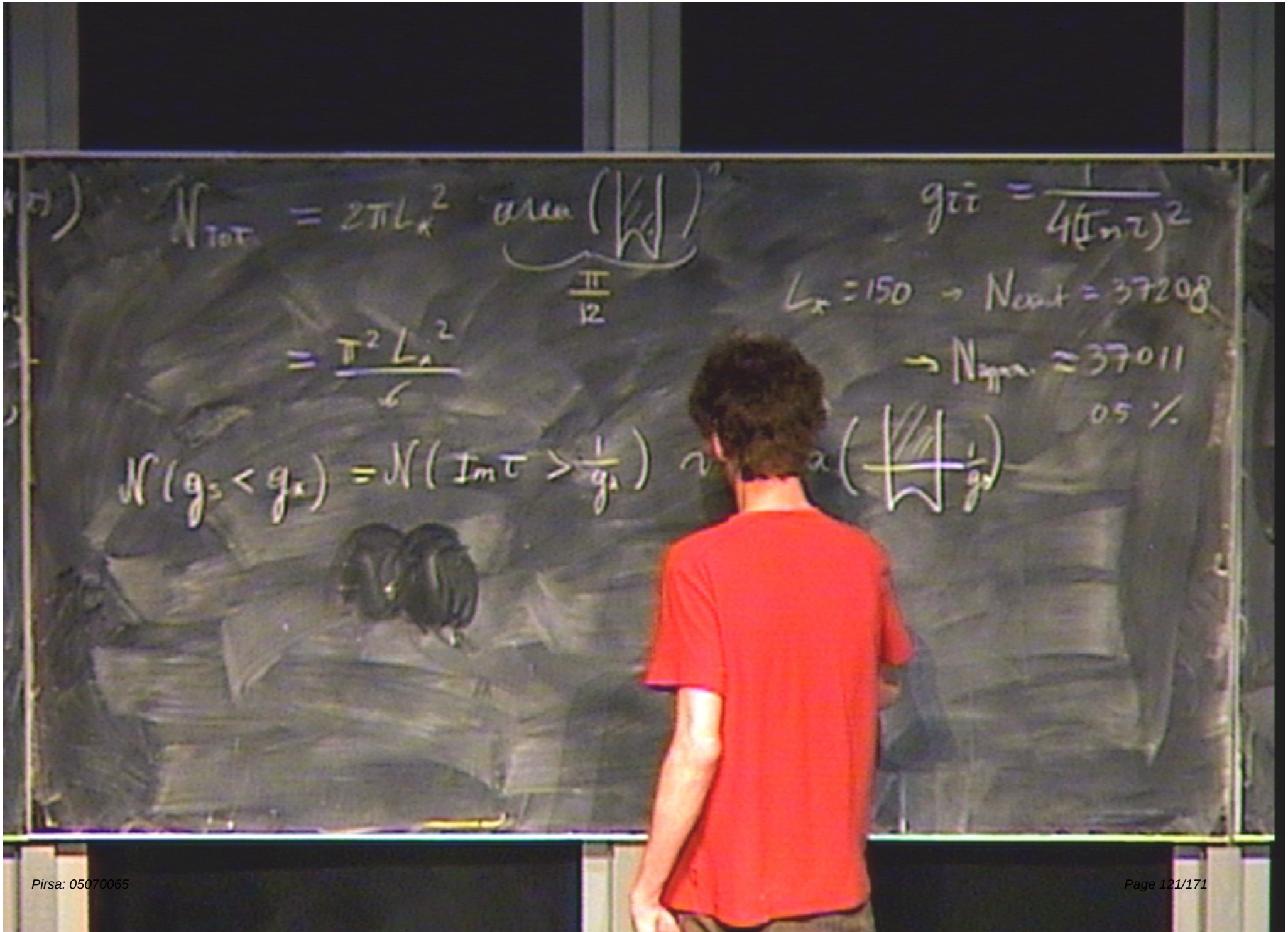
$$N_{inc} \approx \int d^2\tau \, g_{\tau\bar{\tau}} \int_{|W|^2 - |F|^2 \leq L_*} d^2W \, d^2F \; \delta^2(F) \; |W|^2$$

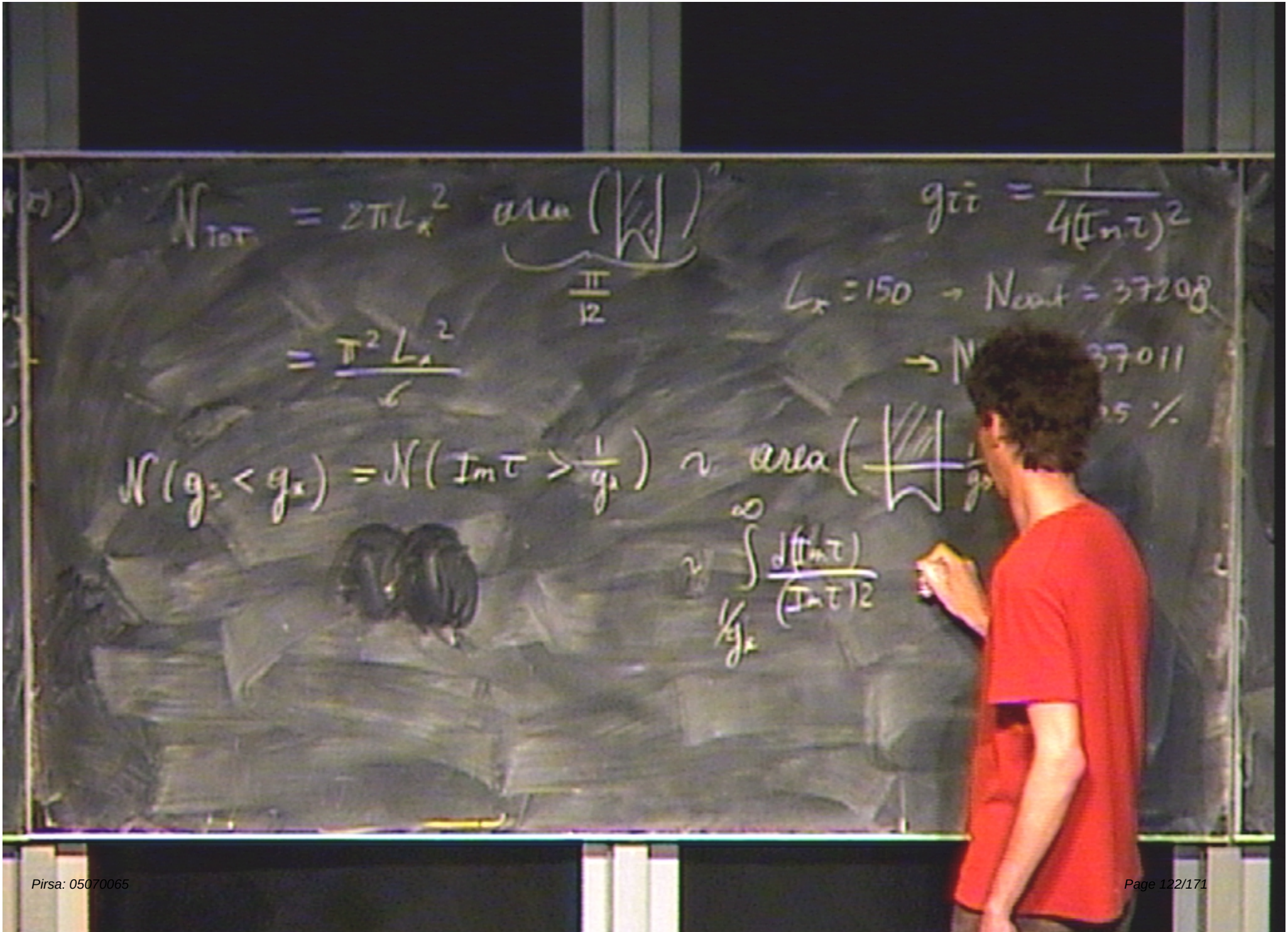
$$= \int d^2\tau \, g_{\tau\bar{\tau}} \int_{|W|^2 \leq L_*} d^2W \, |W|^2 = 2\pi L_*^2 \underbrace{\int d^2\tau \, g_{\tau\bar{\tau}}}_{S}$$

$\cdot \; \cdot \; \cdot \; \


$N_{TOT} = 2\pi$


$N_{tot} = 2\pi L_x^2 \cdot \frac{\pi}{12}$
$= \frac{\pi^2 L_x^2}{6}$
$g_{ii} = \frac{1}{4(\mathrm{Im}\,\tau)^2}$
$L_x = 150 \rightarrow N_{count} = 37208$
$\rightarrow N_{approx} \approx 37011$
0.5 %
$N(g_2 < g_x) = N(\mathrm{Im}\,\tau > \frac{1}{g_x})$


$N_{Tot} = 2\pi L_x^2 \underbrace{\text{area}(|K|)}_{\frac{\pi}{12}}$
$= \frac{\pi^2 L_x^2}{6}$
$g_{\tau\bar{\tau}} = \frac{1}{4(\text{Im}\tau)^2}$
$L_x = 150 \rightarrow N_{exact} = 37208$
$\rightarrow N_{approx} \approx 37011$
0.5 %
$N(g_s < g_x) = N(\text{Im}\tau > \frac{1}{g_s}) \sim \alpha(|K| \frac{1}{g_s})$


$N_{TOT} = 2\pi L_*^2$ area
$\frac{\pi}{12}$
$= \frac{\pi^2 L_*^2}{6}$
$g_{\tau\bar\tau} = \frac{1}{4(\mathrm{Im}\,\tau)^2}$
$L_* = 150 \to N_{cont} = 37208$
$\to N$ 37011
$N(g_s < g_*) = N(\mathrm{Im}\,\tau > \frac{1}{g_*}) \sim$ area
$\sim \int_{1/g_*}^{\infty} \frac{d(\mathrm{Im}\,\tau)}{(\mathrm{Im}\,\tau)^2}$

$N_{TOT} = 2\pi L_x^2 \cdot \underbrace{\text{area}(|K|)}_{\frac{\pi}{12}}$

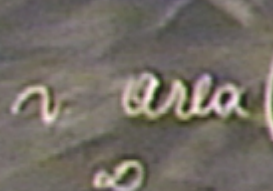
$= \frac{\pi^2 L_x^2}{6}$

$g_{\tau\bar{\tau}} = \frac{1}{4(\mathrm{Im}\,\tau)^2}$

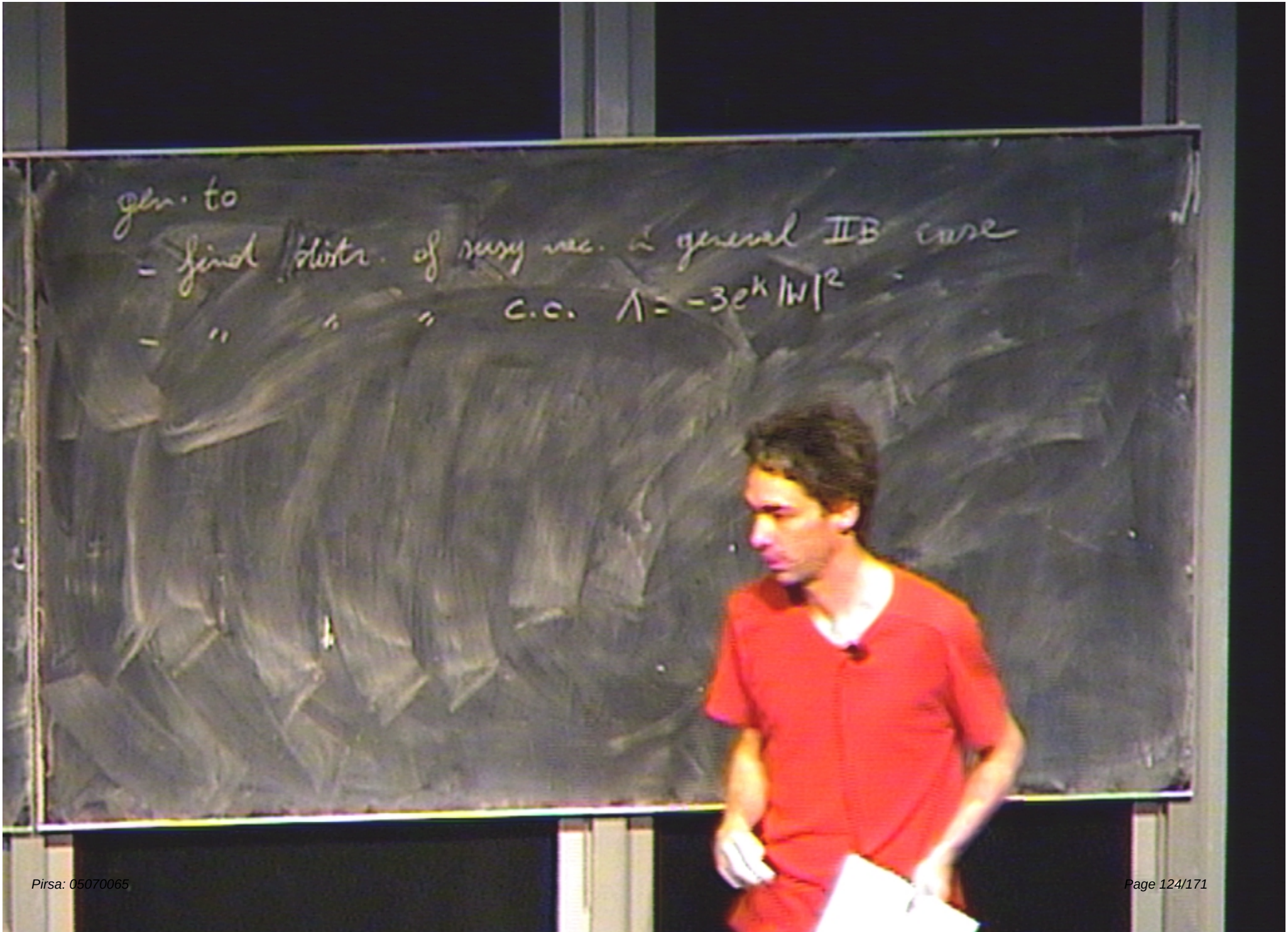
$L_x = 150 \rightarrow N_{exact} = 37208$

$\rightarrow N_{approx} \approx 37011$

0.5 %

$N(g_s < g_x) = N(\mathrm{Im}\,\tau > \frac{1}{g_x}) \sim \text{area}$ 

$\sim \int_{1/g_x}^{\infty} \frac{d(\mathrm{Im}\,\tau)}{(\mathrm{Im}\,\tau)^2} = g_x$

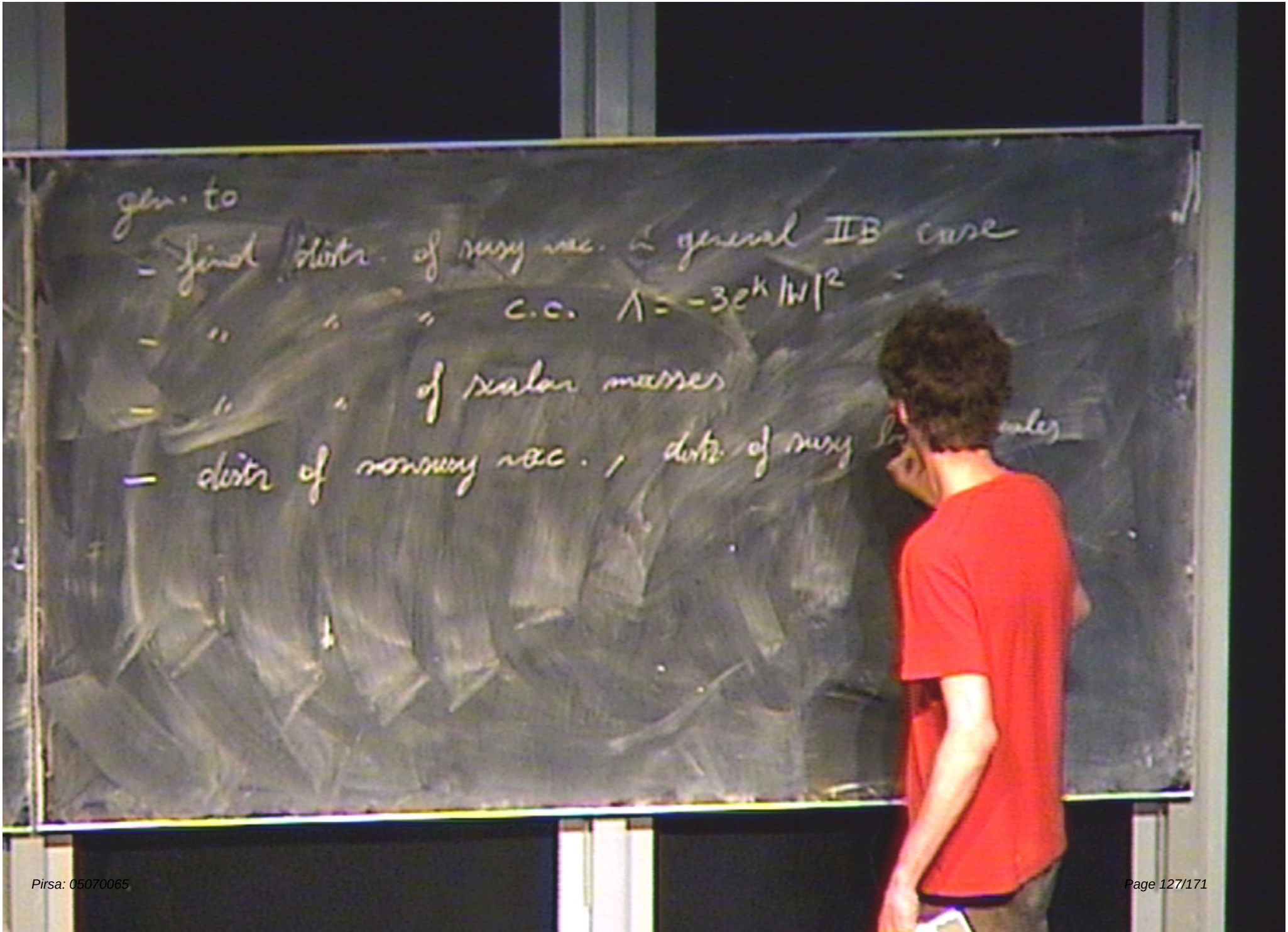

gen. to
- find / distr. of susy vac. in general IIB case
- " " " c.c. $\Lambda = -3e^{K}|W|^2$

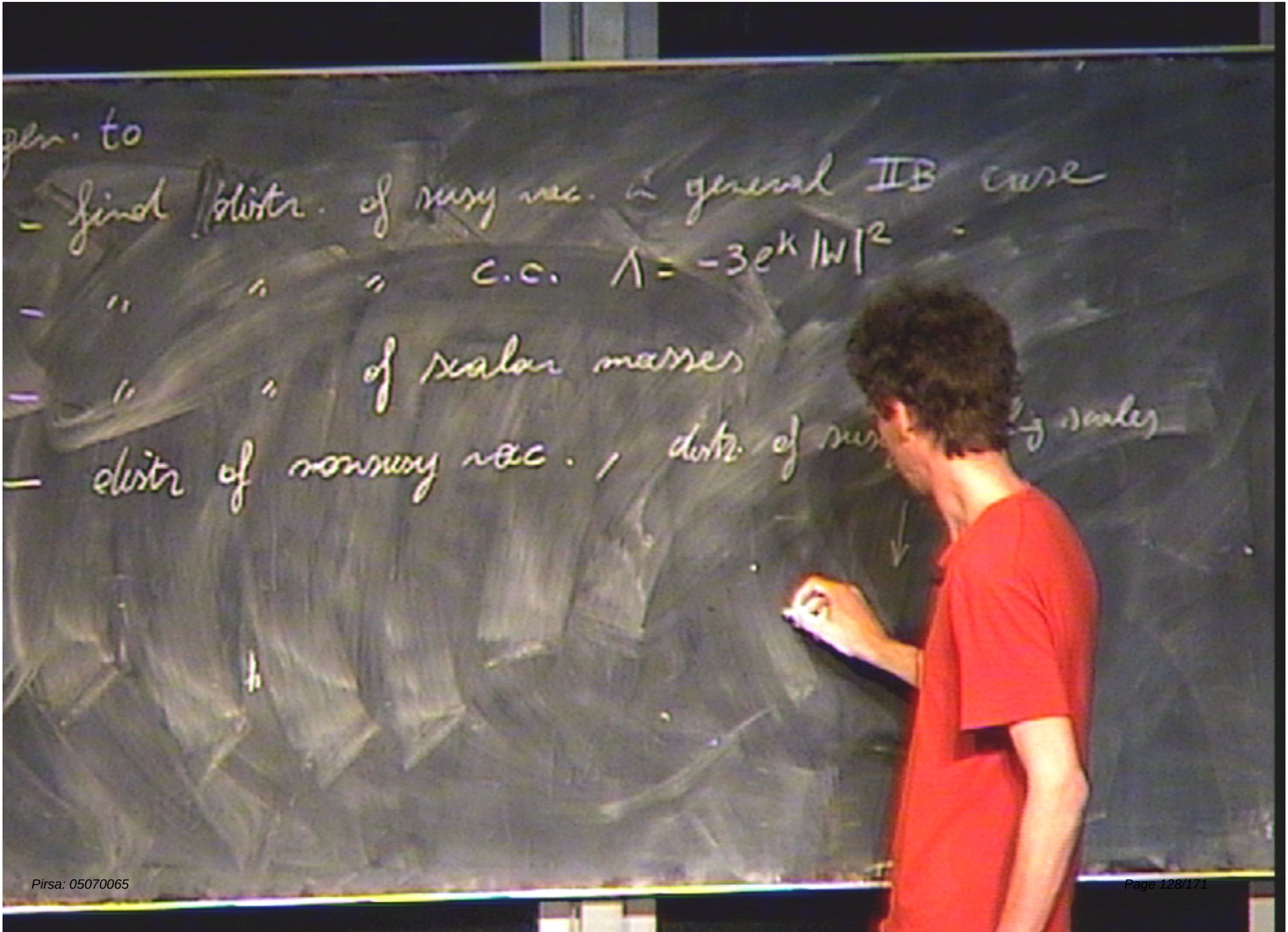
gen. to

- find / distr. of susy vac. in general IIB case
- " " " c.c. $\Lambda = -3e^{K}|W|^2$

gen. to

- find distr. of susy sec. in general IIB case
- " " " c.c. $\Lambda = -3e^{K}|W|^2$
- " " of scalar masses


gen. to
- find distr. of susy vac. ~ general IIB case
- " " " c.c. $\Lambda = -3e^K |W|^2$
- " " of scalar masses
- distr of nonsusy vac. , distr. of susy


gen. to
- find distr. of susy vac. in general IIB case
- " " " c.c. $\Lambda = -3e^K |W|^2$
- " " of scalar masses
- distr of nonsusy vac. , distr. of

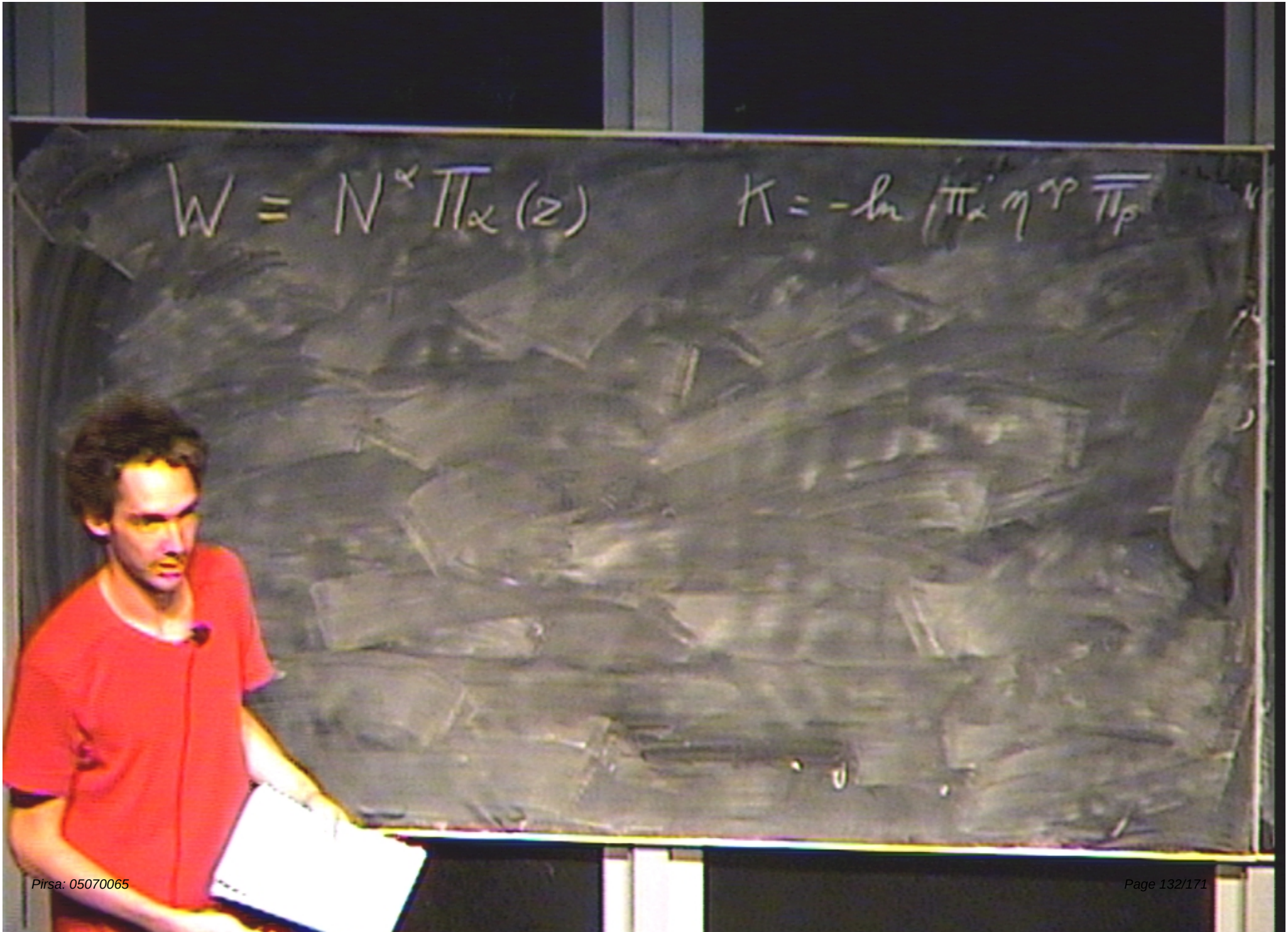
gen. to

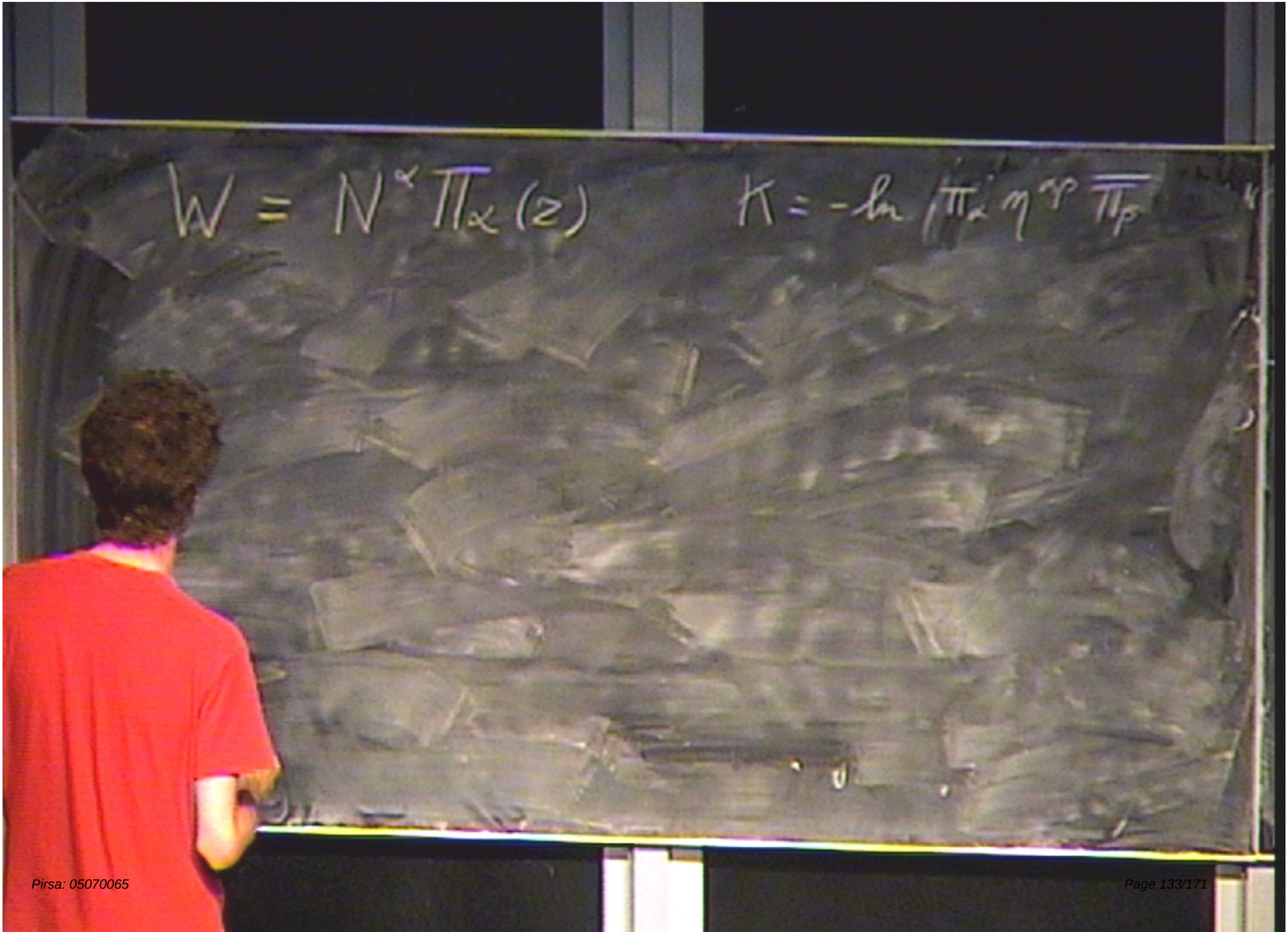
- find distr. of susy vac. in general IIB case
- " " " c.c. $\Lambda = -3e^K |W|^2$
- " " of scalar masses
- distr of nonsusy vac., distr. of susy breaking scales

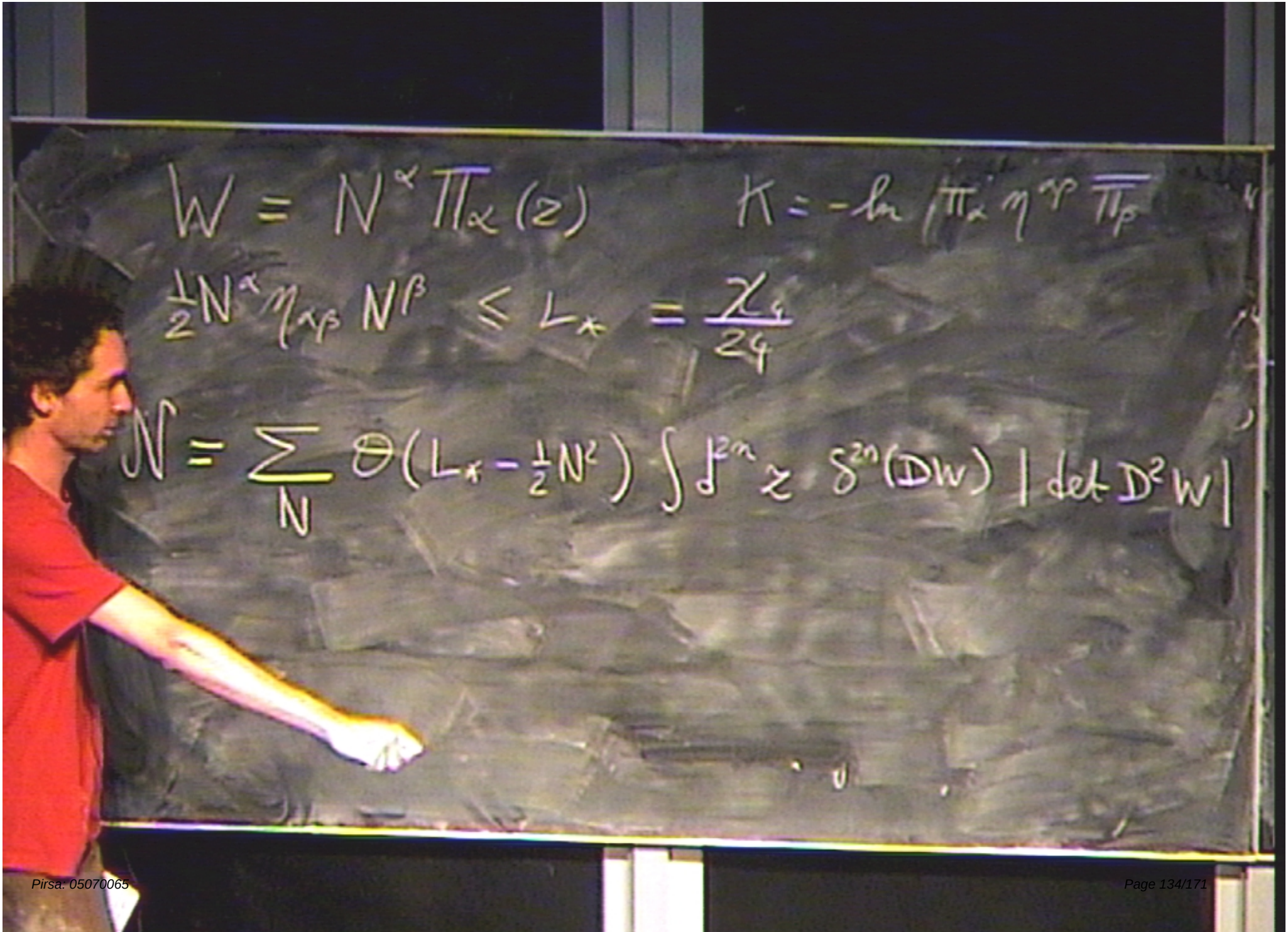
$$dN \sim M_{SUSY}^{10} \, d M_{SUSY}^2$$

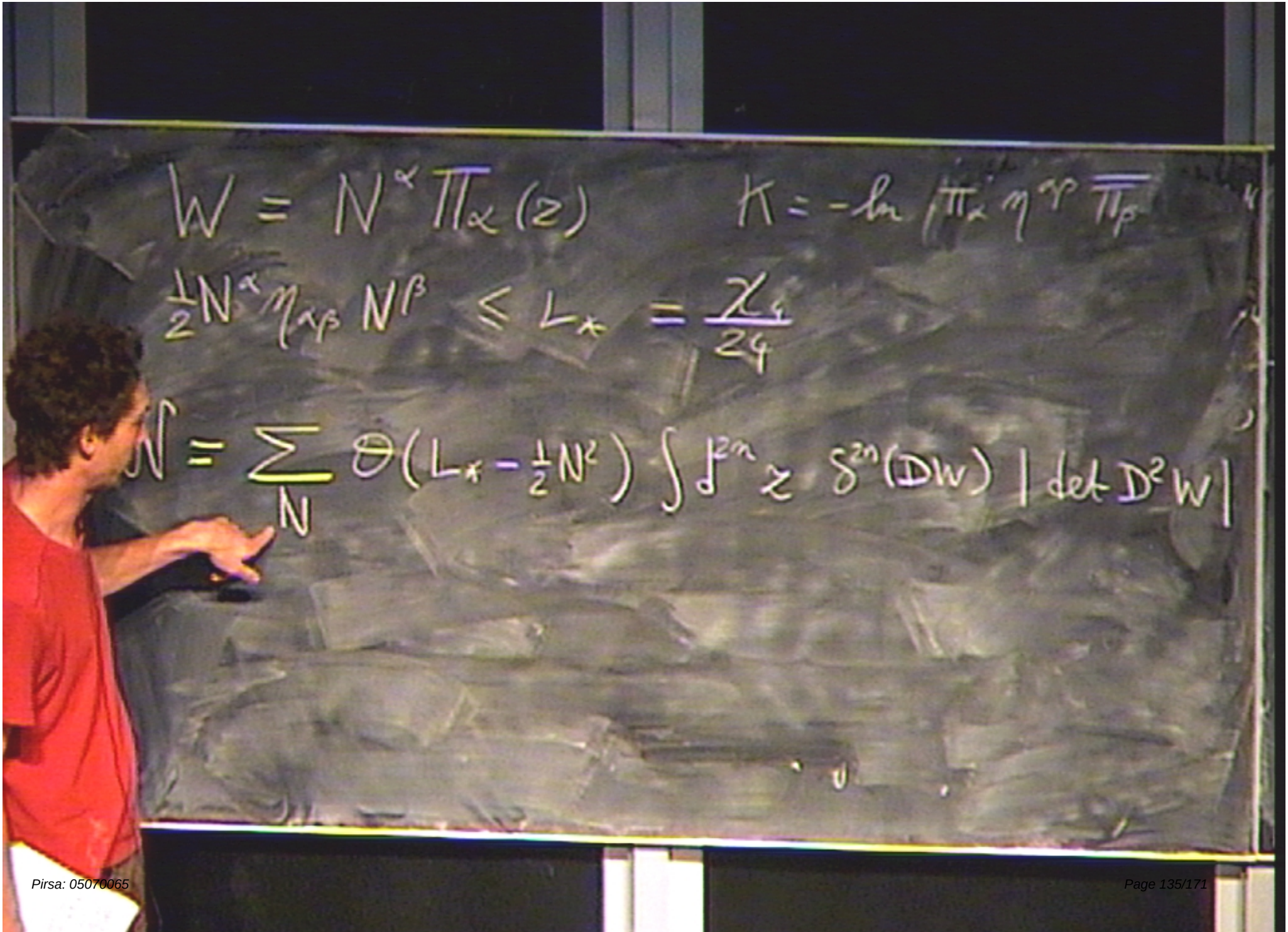


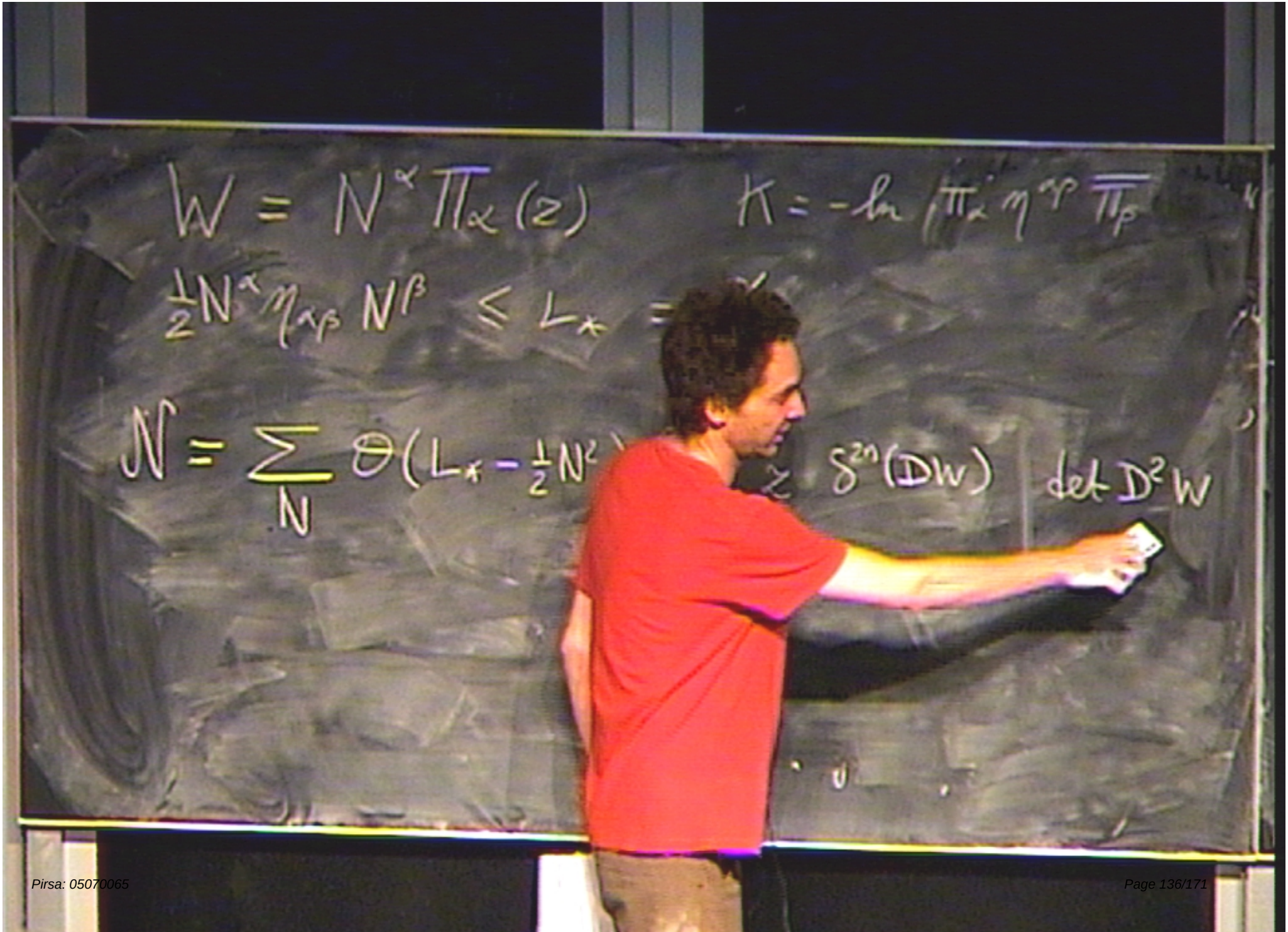


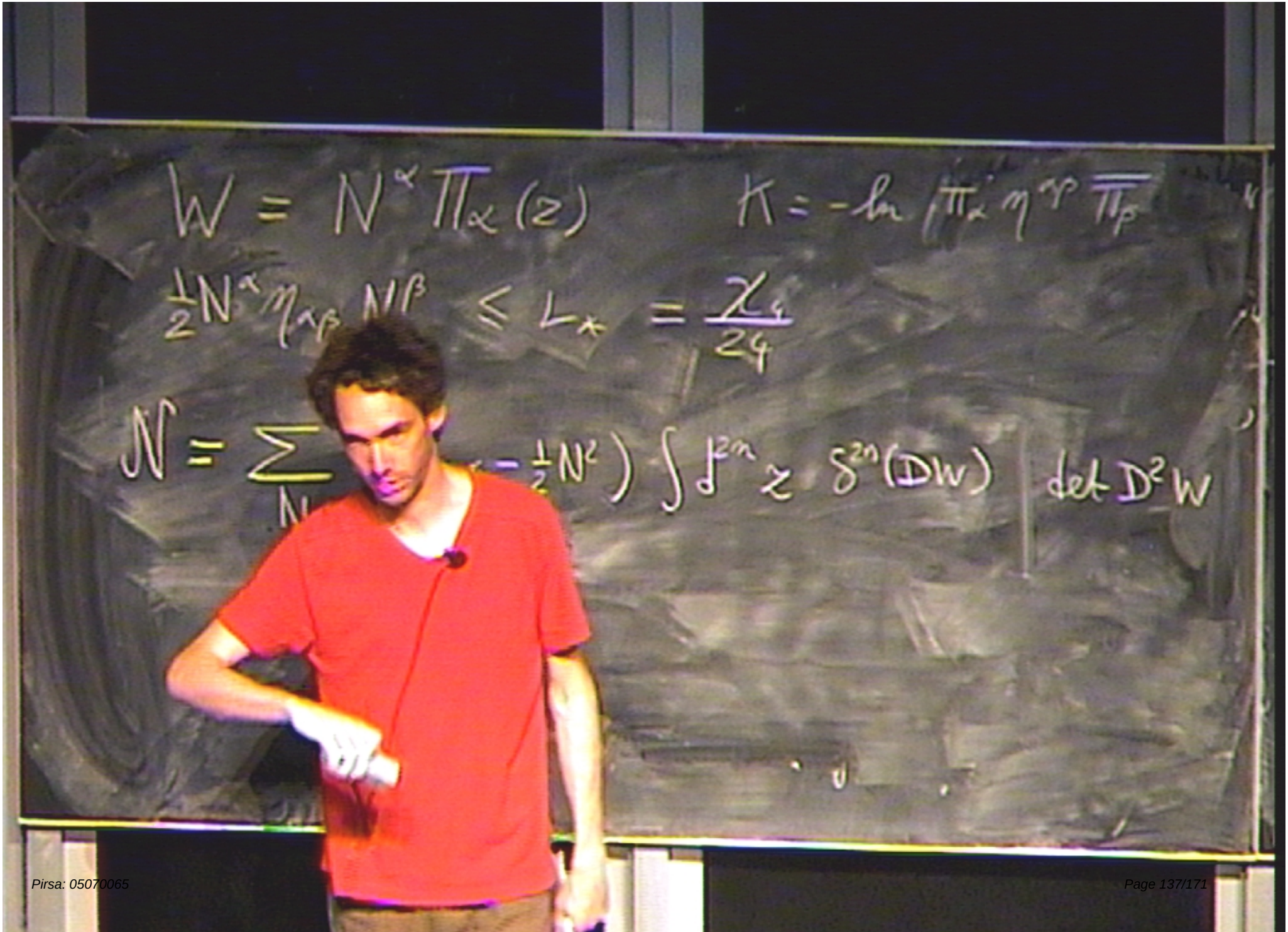

$W = N^{\alpha} \Pi_{\alpha}(z)$
$K = -\ln \langle \Pi_{\alpha}^{\dagger} \eta^{\alpha\beta} \Pi_{\beta} \rangle$

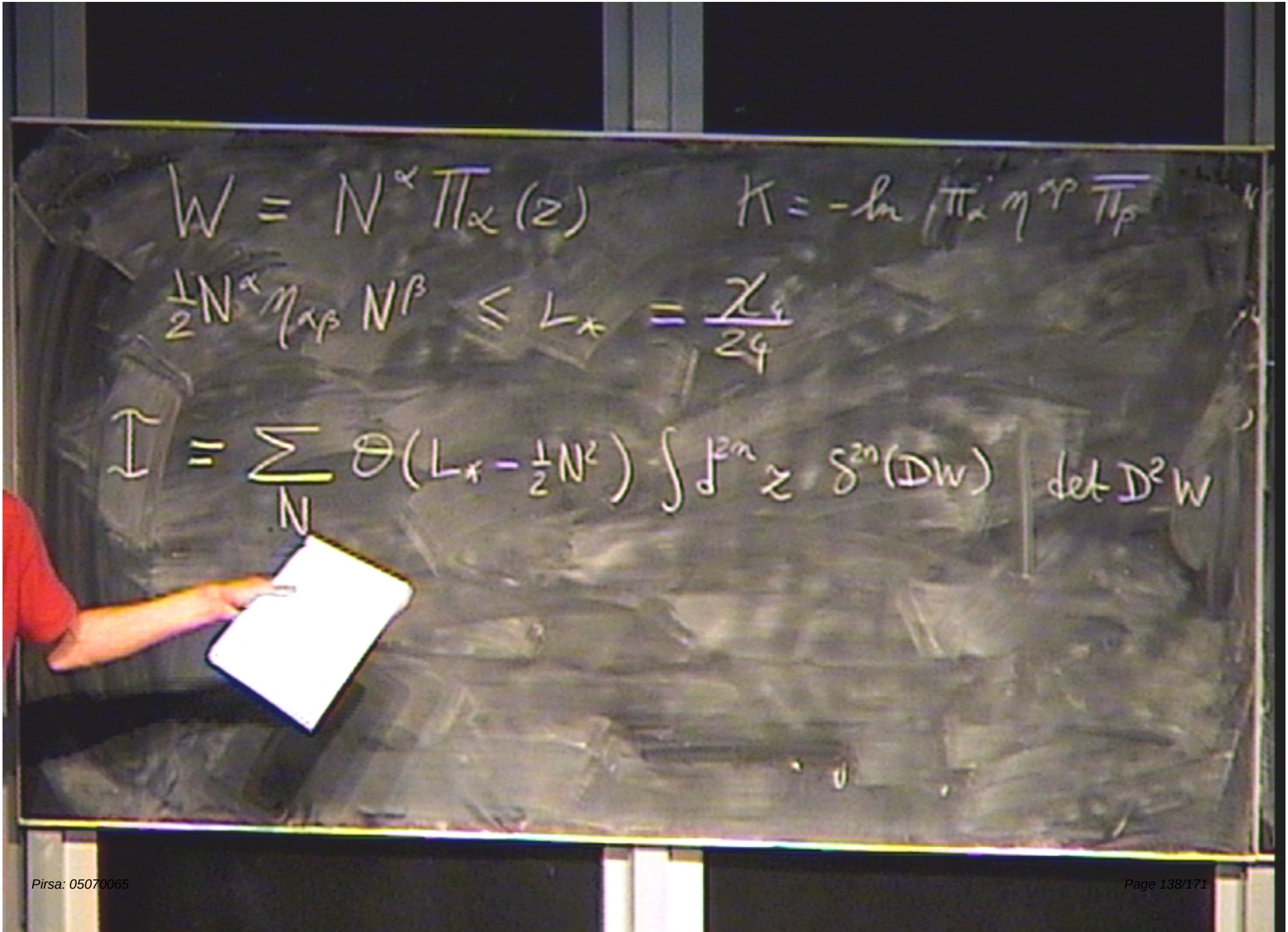

$W = N^{\alpha} \Pi_{\alpha}(z)$
$K = -\ln \Pi_{\alpha}^{\dagger} \eta^{\alpha\beta} \Pi_{\beta}$


$W = N^\alpha \Pi_\alpha(z)$
$K = -\ln |\Pi_\alpha \eta^{\alpha\beta} \bar{\Pi}_\beta$
$\frac{1}{2} N^\alpha \eta_{\alpha\beta} N^\beta \leq L_* = \frac{\chi_4}{24}$
$\mathcal{N} = \sum_N \Theta(L_* - \frac{1}{2}N^2) \int d^{2n} z \, \delta^{2n}(DW) \, |\det D^2 W|$


$W = N^\alpha \Pi_\alpha(z)$
$K = -\ln |\Pi_\alpha \eta^{\alpha\beta} \overline{\Pi_\beta}$
$\frac{1}{2} N^\alpha \eta_{\alpha\beta} N^\beta \leq L_* = \frac{\chi_4}{24}$
$\mathcal{N} = \sum_N \Theta(L_* - \frac{1}{2}N^2) \int d^{2n} z \, \delta^{2n}(DW) \, |\det D^2 W|$


$W = N^\alpha \Pi_\alpha(z)$
$K = -\ln \Pi_\alpha \eta^{\alpha\beta} \overline{\Pi_\beta}$
$\frac{1}{2} N^\alpha \eta_{\alpha\beta} N^\beta \leq L_* =$
$\mathcal{N} = \sum_N \theta(L_* - \frac{1}{2}N^2$
$\delta^{2n}(DW)$
$\det D^2 W$

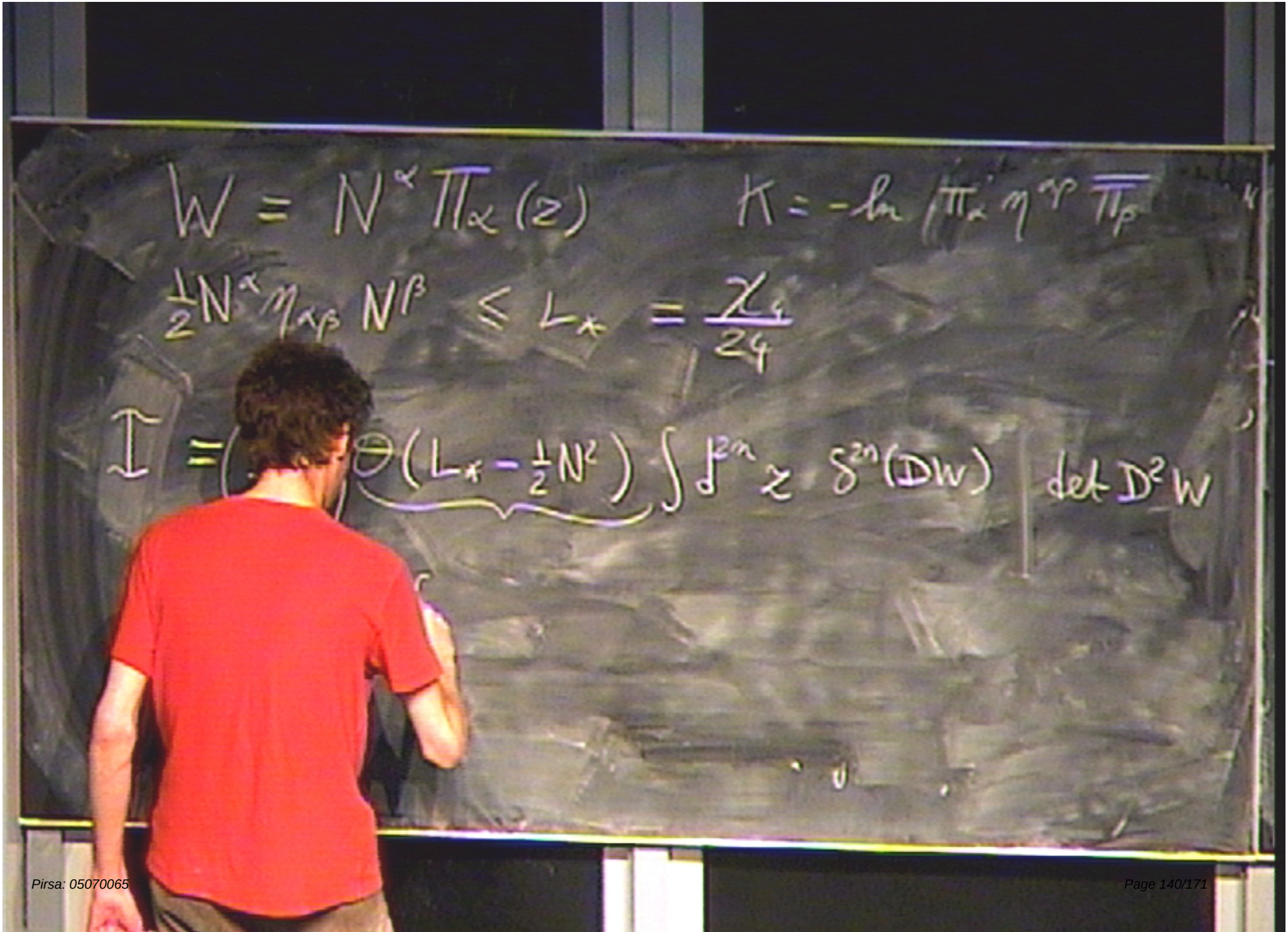

$W = N^\alpha \Pi_\alpha(z)$
$K = -\ln \Pi_\alpha \eta^{\alpha\beta} \overline{\Pi}_\beta$
$\frac{1}{2} N^\alpha \eta_{\alpha\beta} N^\beta \leq L_* = \frac{\chi_4}{24}$
$\mathcal{N} = \sum_N$
$-\frac{1}{2}N^2) \int d^{2n}z \; \delta^{2n}(DW) \; \det D^2 W$

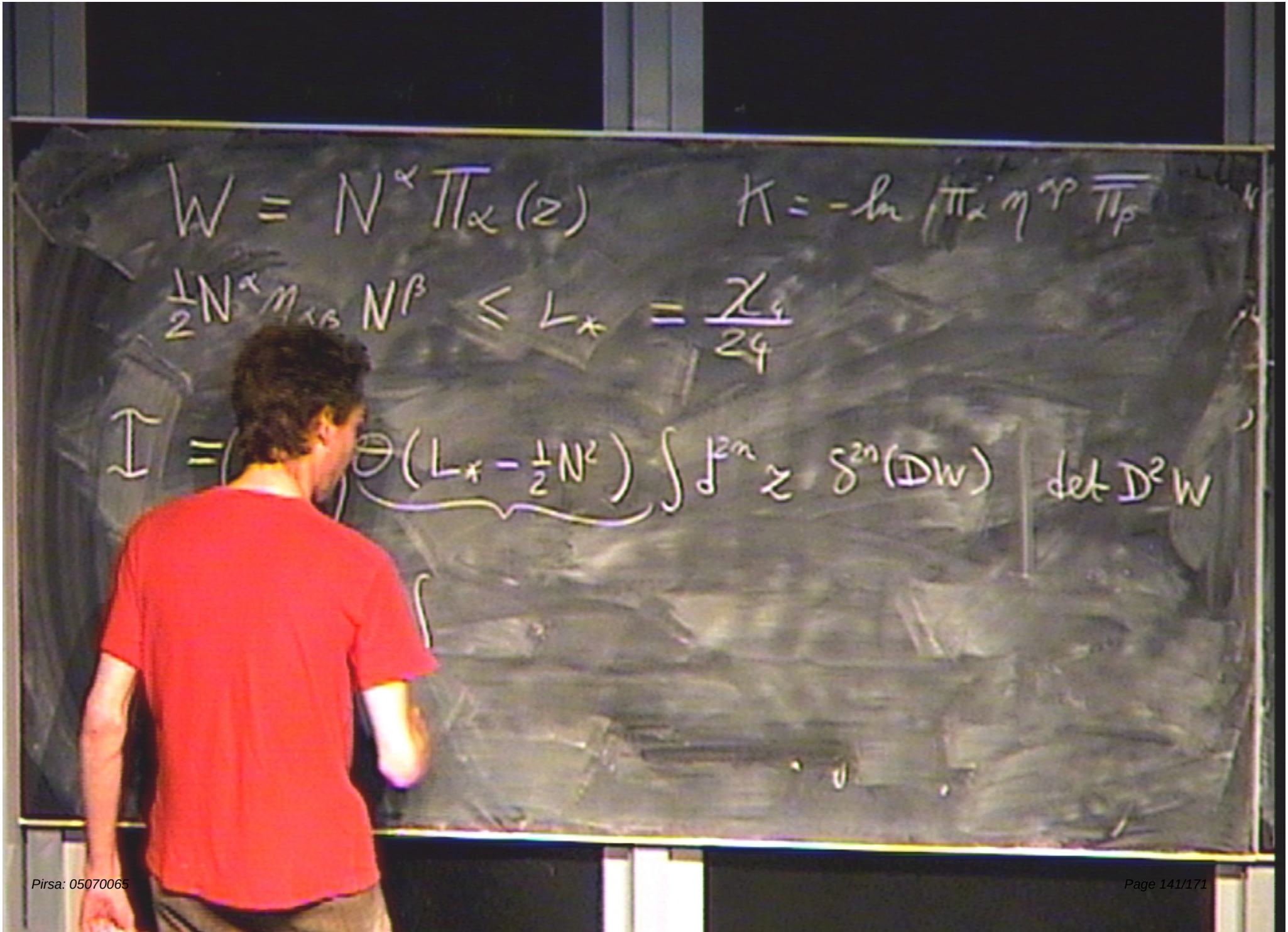

$W = N^\alpha \Pi_\alpha(z)$
$K = -\ln |\Pi_\alpha \eta^{\alpha\beta} \overline{\Pi_\beta}$
$\frac{1}{2} N^\alpha \eta_{\alpha\beta} N^\beta \leq L_* = \frac{\chi_4}{24}$
$I = \sum_N \Theta(L_* - \frac{1}{2}N^2) \int d^{2n} z \, \delta^{2n}(DW) \, |\det D^2 W|$

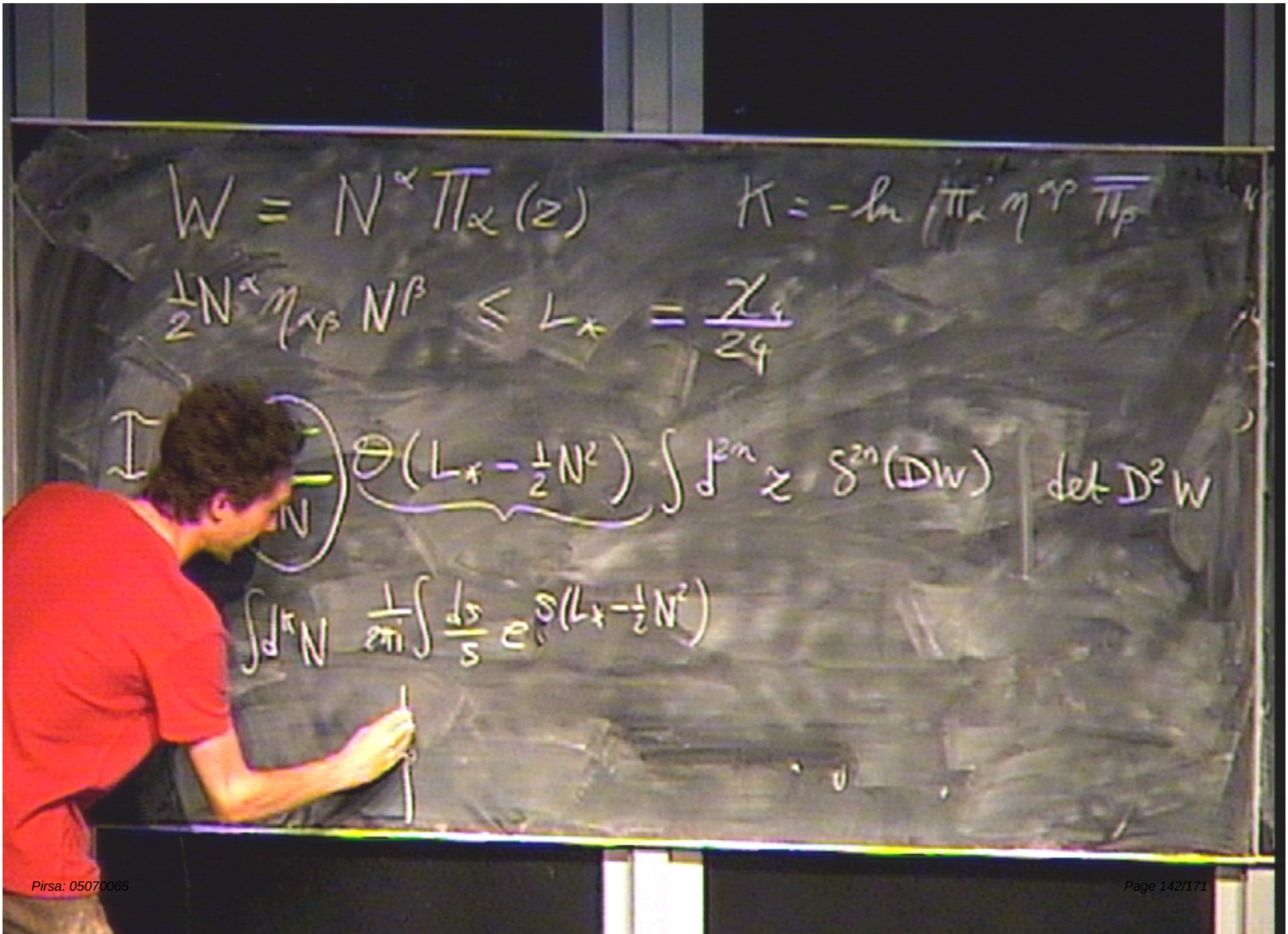
$$W = N^\alpha \Pi_\alpha(z) \qquad K = -\ln |\Pi_\alpha \eta^{\alpha\beta} \overline{\Pi_\beta}$$

$$\tfrac{1}{2} N^\alpha \eta_{\alpha\beta} N^\beta \leq L_* = \frac{\chi_4}{24}$$

$$\mathcal{I} = \sum_N \Theta\left(L_* - \tfrac{1}{2}N^2\right) \int d^{2n} z \; \delta^{2n}(DW) \; |\det D^2 W|$$


$W = N^\alpha \Pi_\alpha(z)$
$K = -\ln |\Pi_\alpha \eta^{\alpha\beta} \overline{\Pi_\beta}$
$\frac{1}{2} N^\alpha \eta_{\alpha\beta} N^\beta \leq L_+ = \frac{\chi_+}{24}$
$I = \theta(L_+ - \frac{1}{2}N^2) \int d^{2n} z \, \delta^{2n}(DW) \, |\det D^2 W|$


$W = N^\alpha \Pi_\alpha(z)$
$K = -\ln |\Pi_\alpha \eta^{\alpha\beta} \overline{\Pi_\beta}$
$\frac{1}{2} N^\alpha \eta_{\alpha\beta} N^\beta \leq L_* = \frac{\chi_4}{24}$
$I = \int \underbrace{\theta(L_* - \frac{1}{2}N^2)} \int d^{2n} z \, \delta^{2n}(DW) \, |\det D^2 W|$

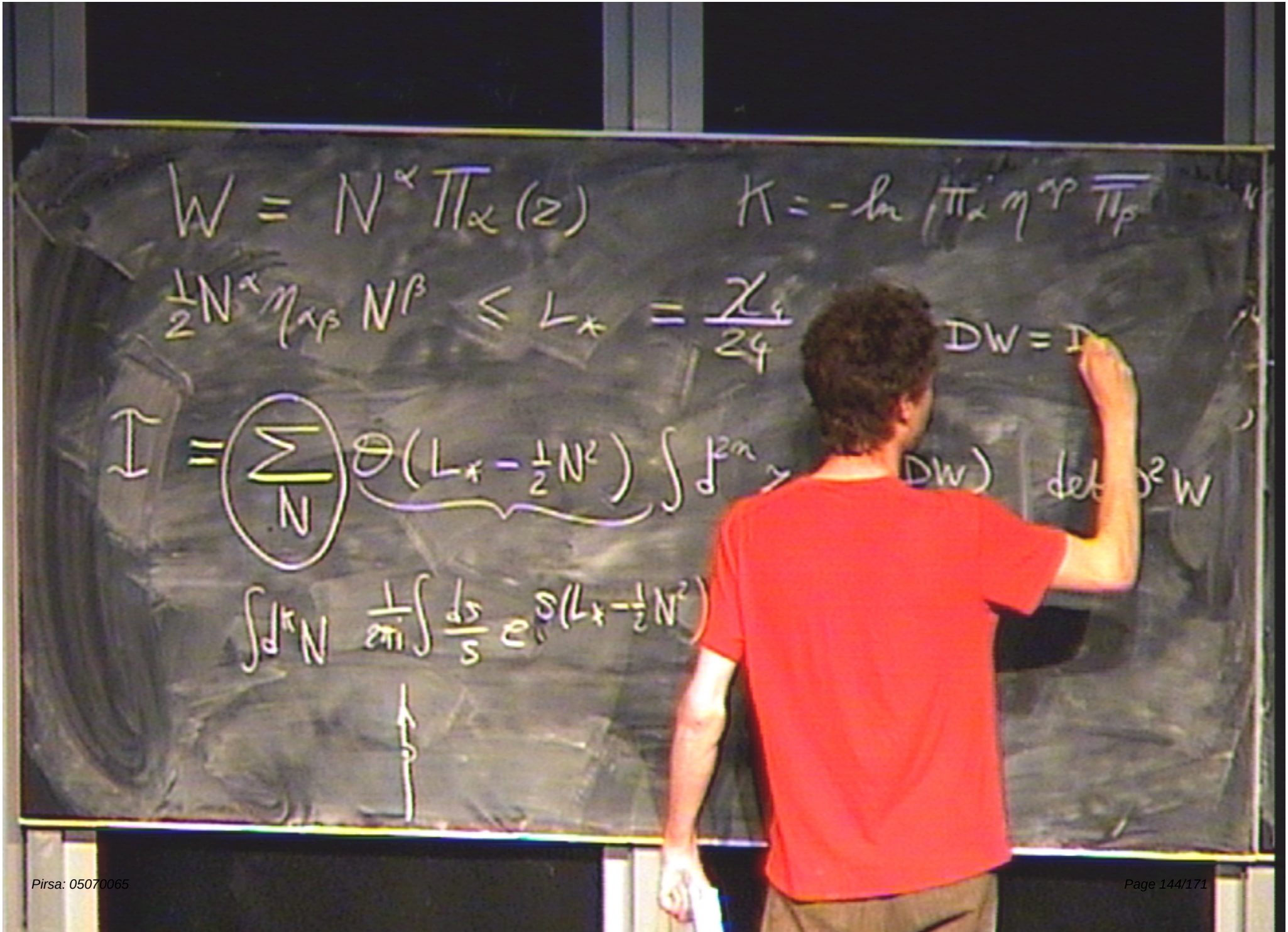

$W = N^\alpha \Pi_\alpha(z)$
$K = -\ln \Pi_\alpha \eta^{\alpha\beta} \overline{\Pi_\beta}$
$\frac{1}{2} N^\alpha \eta_{\alpha\beta} N^\beta \leq L_* = \frac{\chi_4}{24}$
$\Theta(L_* - \frac{1}{2}N^2) \int d^{2n}z \, \delta^{2n}(DW) \, \det D^2 W$
$\int d^k N \, \frac{1}{2\pi i} \int \frac{ds}{s} e^{s(L_* - \frac{1}{2}N^2)}$

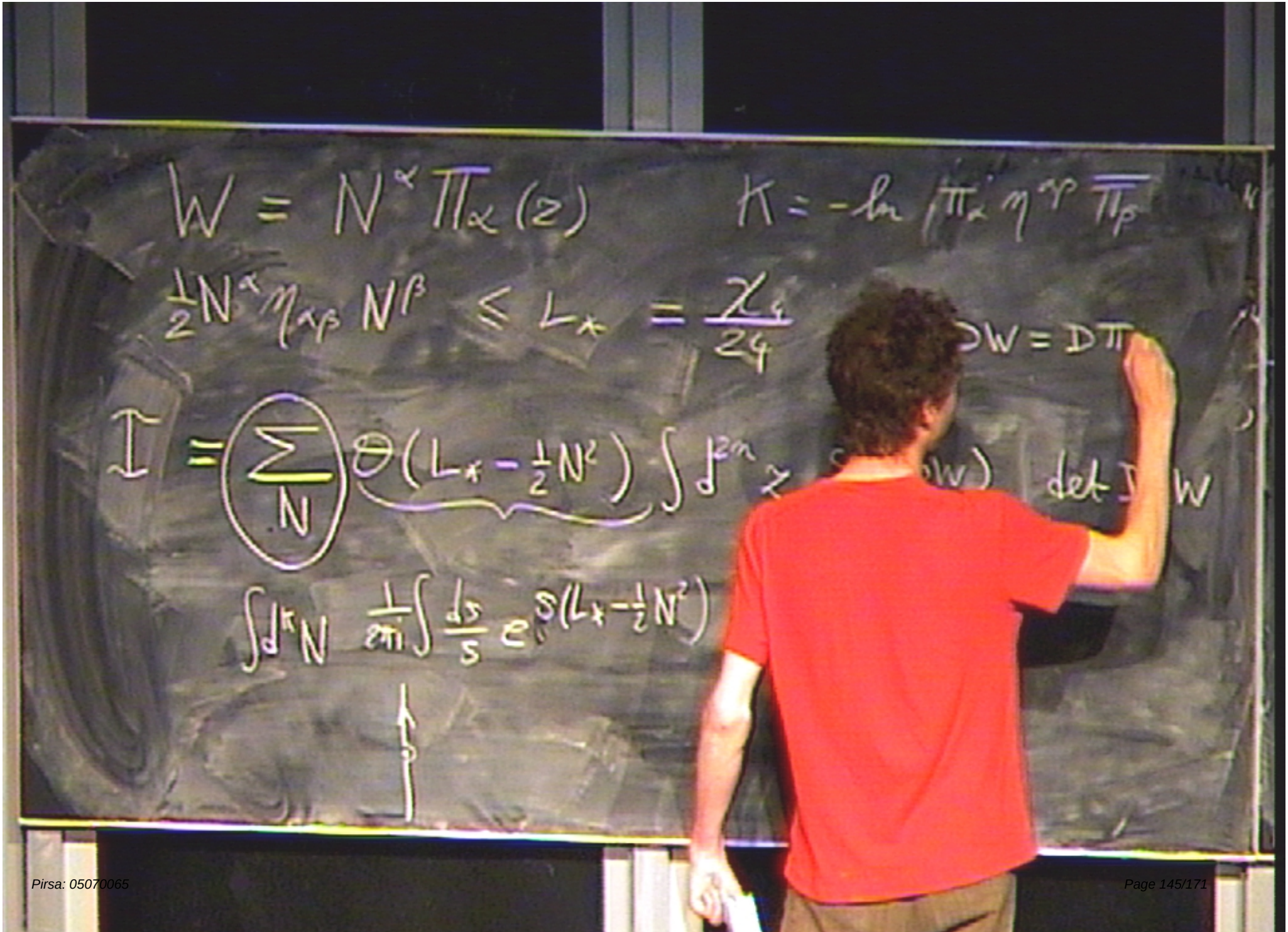
$$W = N^\alpha \Pi_\alpha(z) \qquad K = -\ln |\Pi_\alpha \eta^{\alpha\beta} \overline{\Pi_\beta}|^4$$

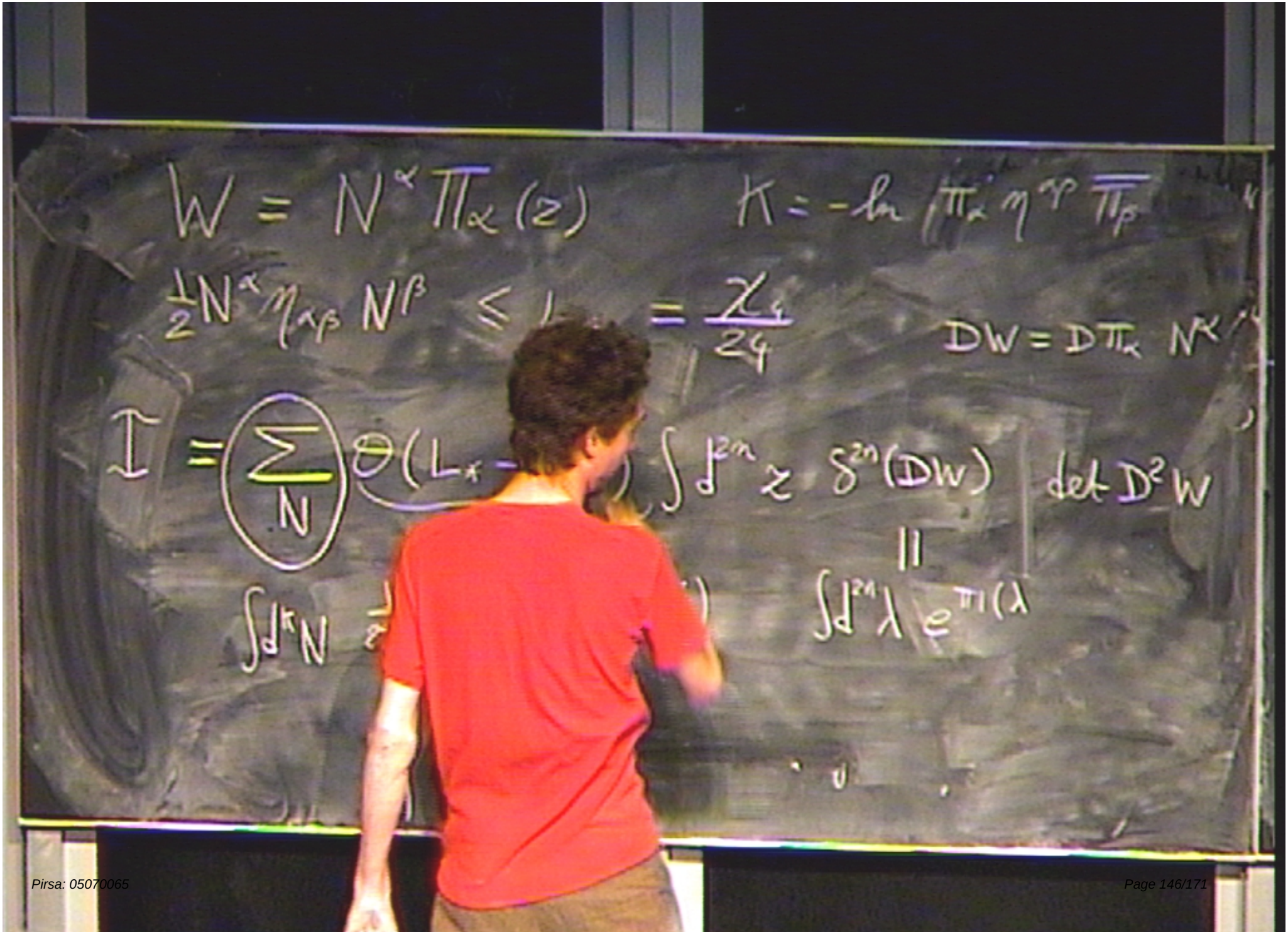
$$\frac{1}{2} N^\alpha \eta_{\alpha\beta} N^\beta \leq L_* = \frac{\chi_4}{24}$$

$$I = \sum_N \underbrace{\Theta\left(L_* - \tfrac{1}{2}N^2\right)} \int d^{2n} z \; \delta^{2n}(DW) \; |\det D^2 W|$$

$$N \; \frac{1}{2\pi i} \int \frac{ds}{s} e^{s\left(L_* - \frac{1}{2}N^2\right)}$$


W = N^α Π_α(z)
K = -ln ∫ Π_α η^αβ Π_β
½ N^α η_αβ N^β ≤ L_k = χ_k / 24
DW = D
I = Σ_N Θ(L_k - ½N²) ∫ d^2n z DW) det ∂²W
∫ d^k N 1/2πi ∫ ds/s e^s(L_k - ½N²)


$W = N^\alpha \Pi_\alpha(z)$
$K = -\ln \Pi_\alpha \eta^{\alpha\beta} \overline{\Pi_\beta}$
$\frac{1}{2} N^\alpha \eta_{\alpha\beta} N^\beta \leq L_* = \frac{\chi_4}{24}$
$\partial W = D\Pi$
$I = \sum_N \theta(L_* - \frac{1}{2}N^2) \int d^{2n} z$
$\det$
$\int d^k N \frac{1}{2\pi i} \int \frac{ds}{s} e^{s(L_* - \frac{1}{2}N^2)}$

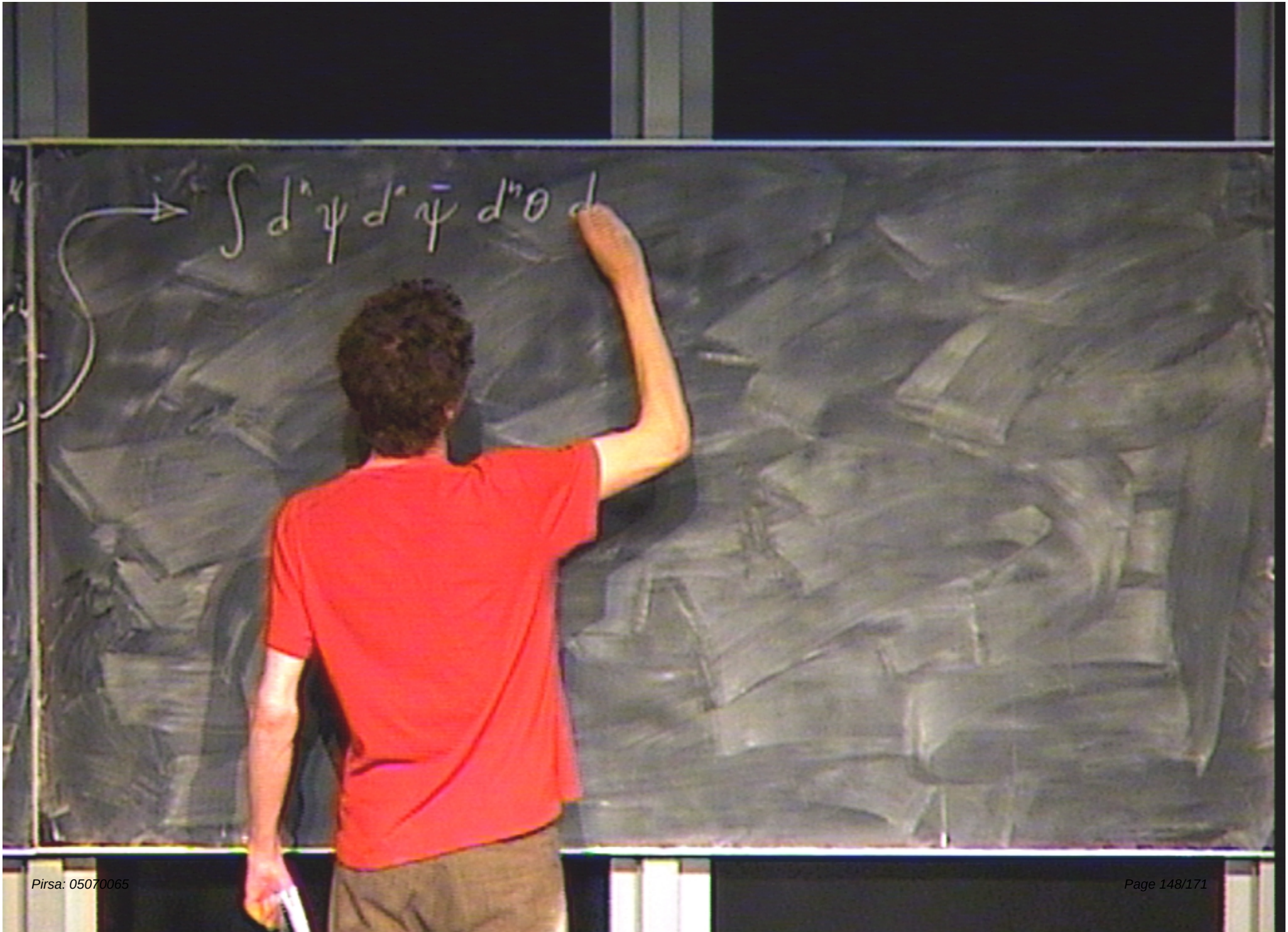

W = N^α Π_α(z)
K = -ln ∫ Π_α η^αβ Π_β
½ N^α η_αβ N^β ≤ 1
= χ_4 / 24
DW = DΠ_α N^α
I = Σ_N
θ(L_4 -
∫ d^2n z δ^2n(DW)
det D^2 W
∫ d^kN
∫ d^2n λ e^πi(λ

$$W = N^\alpha \Pi_\alpha(z) \qquad K = -\ln \int \Pi_\alpha \eta^{\alpha\beta} \overline{\Pi_\beta}$$

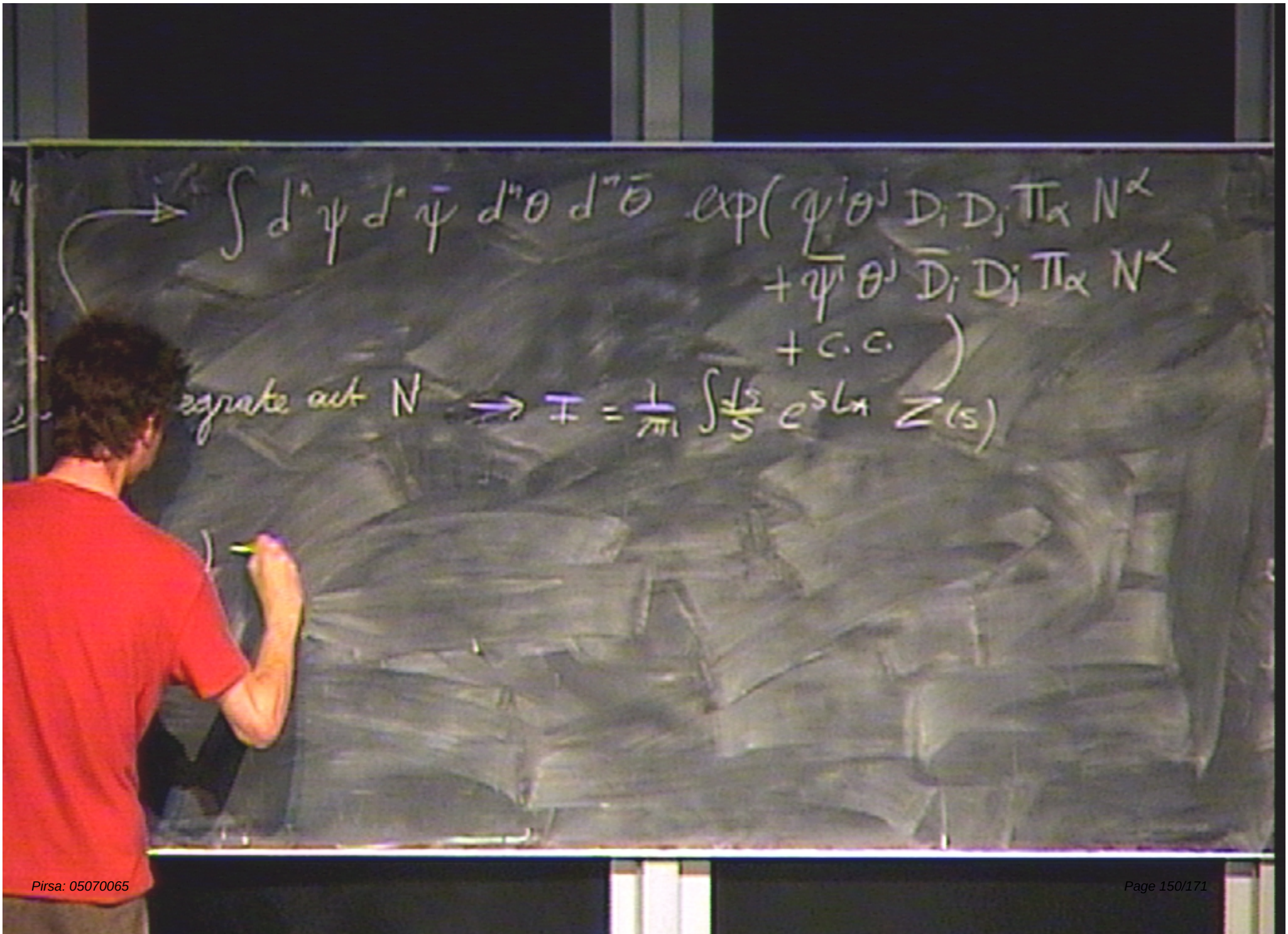
$$\tfrac{1}{2} N^\alpha \eta_{\alpha\beta} N^\beta \leq L_* = \frac{\chi_4}{24} \qquad DW = D\Pi_\alpha \, N^\alpha$$

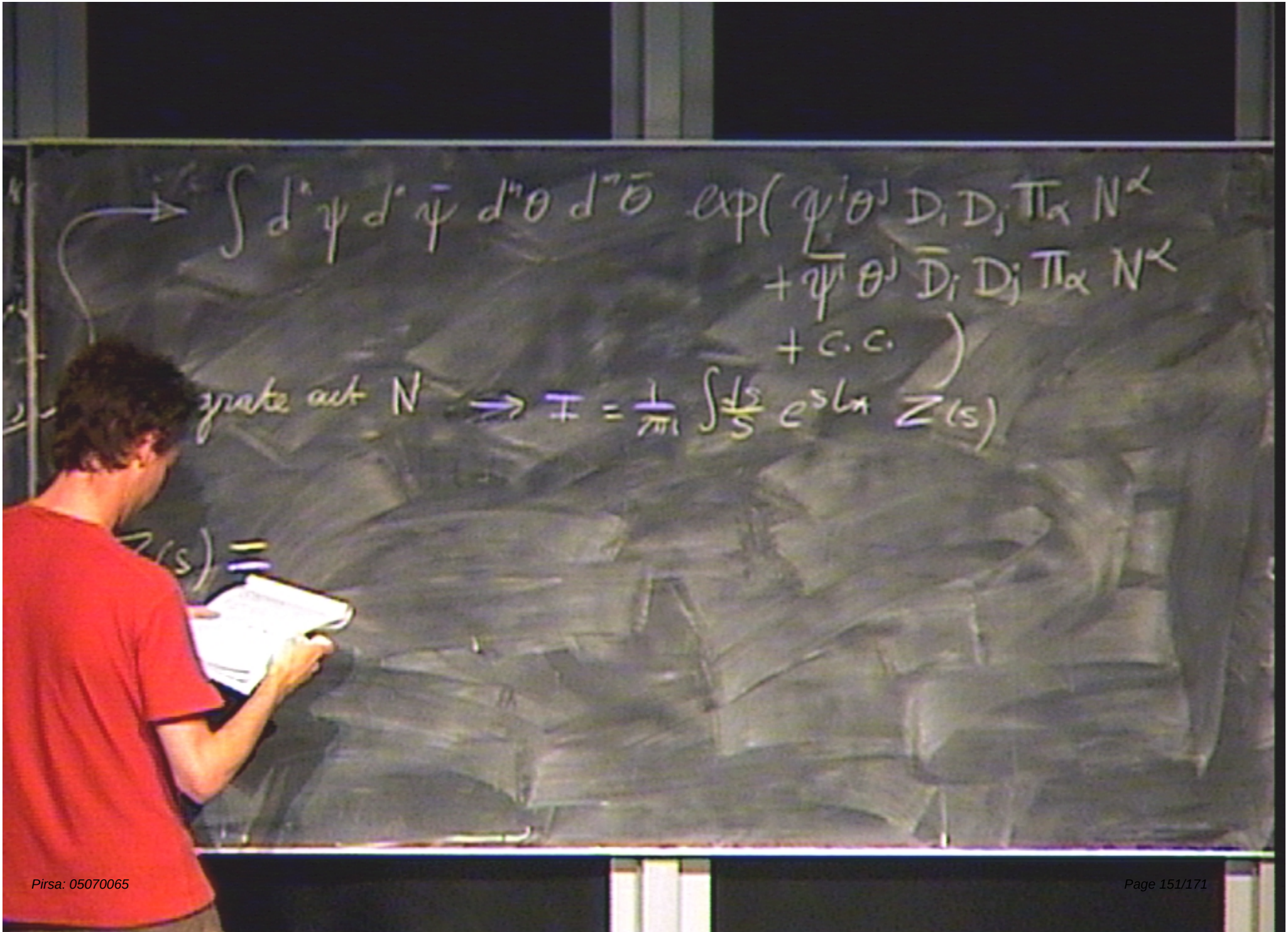
$$I = \sum_N \Theta\left(L_* - \tfrac{1}{2}N^2\right) \int d^{2n} z \; \delta^{2n}(DW) \; \det D^2 W$$

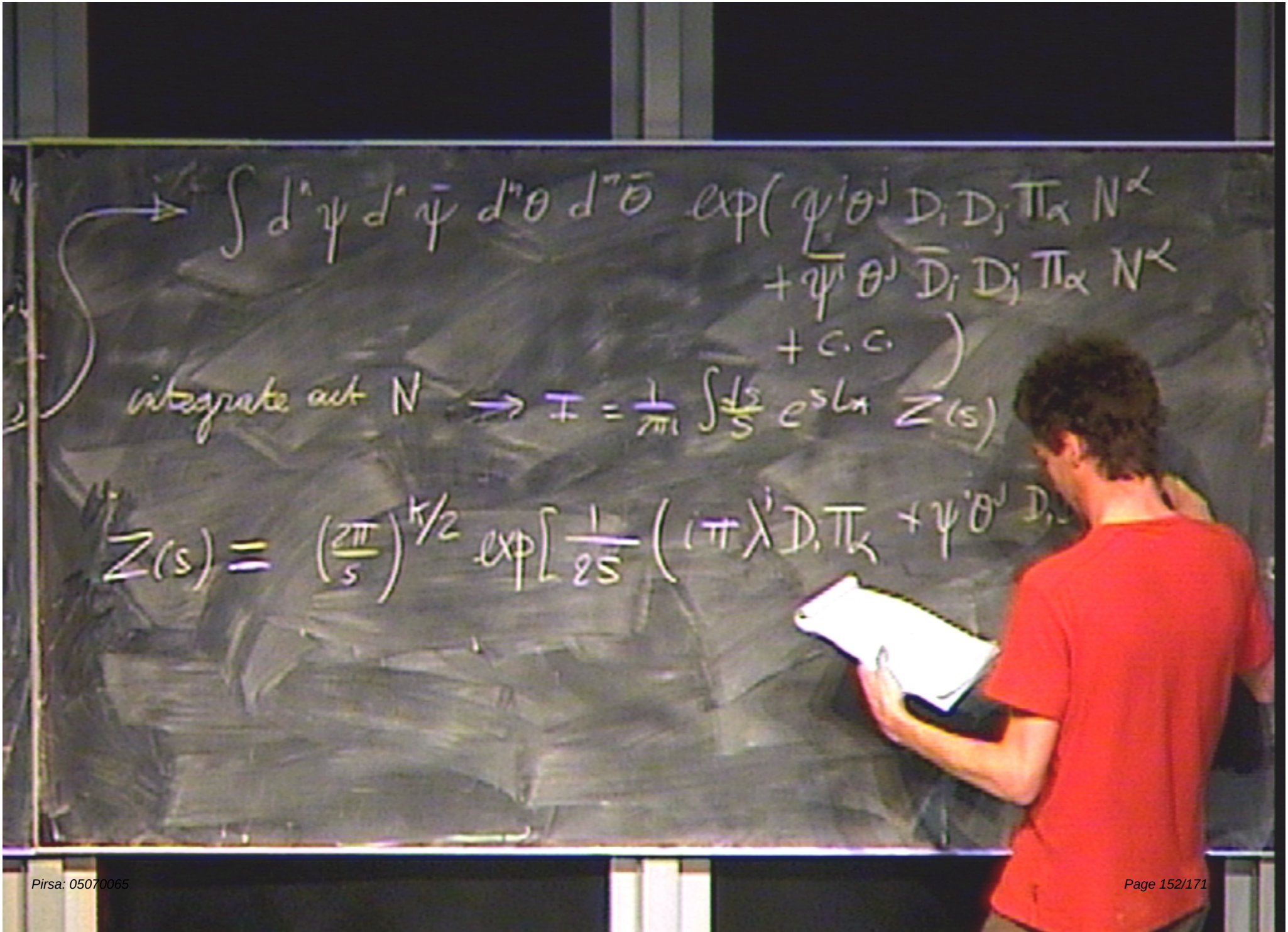
$$\int d^k N \; \frac{1}{2\pi i} \int \frac{ds}{s} e^{s\left(L_* - \frac{1}{2}N^2\right)} \qquad \int d^{2n}\lambda \, e^{\pi i \left(\lambda^i D_i \Pi_\alpha N^\alpha + \text{c.c.}\right)}$$


$\int d^n\psi\, d^n\bar{\psi}\, d^n\theta\, d$

$$\int d^4\psi\, d^4\bar{\psi}\, d^4\theta\, d^4\bar{\theta}\, \exp\Big(\psi^i \theta^j D_i D_j \Pi_\alpha N^\alpha + \bar{\psi}^i \theta^j \bar{D}_i D_j \Pi_\alpha N^\alpha + \text{c.c.} \Big)$$


$\int d^n\psi\, d^n\bar{\psi}\, d^n\theta\, d^n\bar{\theta}\, \exp(\psi^i\theta^j D_i D_j \Pi_\alpha N^\alpha$
$+\bar{\psi}^i\theta^j \bar{D}_i D_j \Pi_\alpha N^\alpha$
$+ c.c.)$
egrate out $N \Rightarrow Z = \frac{1}{2\pi i}\int \frac{ds}{s} e^{s\ln Z(s)}$


$\int d^n\psi\, d^n\bar{\psi}\, d^n\theta\, d^n\bar{\theta}\, \exp\big(\psi^i\theta^j D_i D_j \Pi_\alpha N^\alpha$
$+\bar{\psi}^i\theta^j \bar{D}_i D_j \Pi_\alpha N^\alpha$
$+ c.c.\big)$
grate out $N \Rightarrow \mathcal{I} = \frac{1}{\pi i}\int \frac{ds}{s} e^{s\ln} Z(s)$
$Z(s) =$


$\int d^n\psi\, d^n\bar{\psi}\, d^n\theta\, d^n\bar{\theta}\, \exp\big(\psi^i\theta^j D_i D_j \Pi_\alpha N^\alpha$
$+\bar{\psi}^i\theta^j \bar{D}_i D_j \Pi_\alpha N^\alpha$
$+ c.c.\big)$
integrate out $N \Rightarrow Z = \frac{1}{\pi i}\int \frac{ds}{s} e^{s\ln} Z(s)$
$Z(s) = \left(\frac{2\pi}{s}\right)^{k/2} \exp\left[\frac{1}{2s}\left(i\pi\lambda^i D_i \Pi_\alpha + \psi^i\theta^j D_i\right.\right.$

$$\int d^n\psi\, d^n\bar\psi\, d^n\theta\, d^n\bar\theta\, \exp\Big(\psi^i\theta^j D_i D_j \Pi_\alpha N^\alpha + \bar\psi^i\theta^j \bar D_i D_j \Pi_\alpha N^\alpha + c.c.\Big)$$

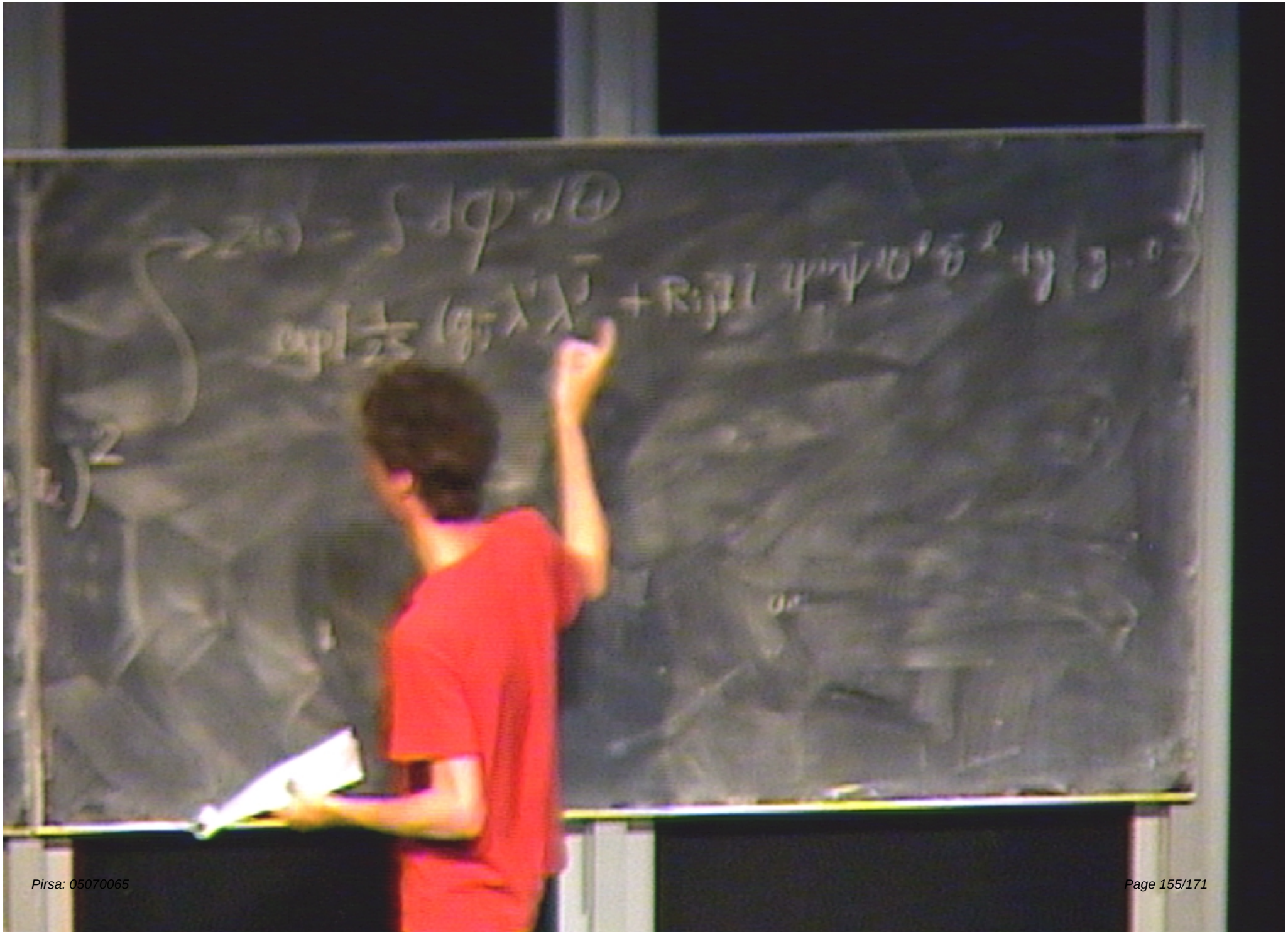
integrate out N $\Rightarrow I = \frac{1}{2\pi i}\int \frac{ds}{s} e^{s\ln} Z(s)$

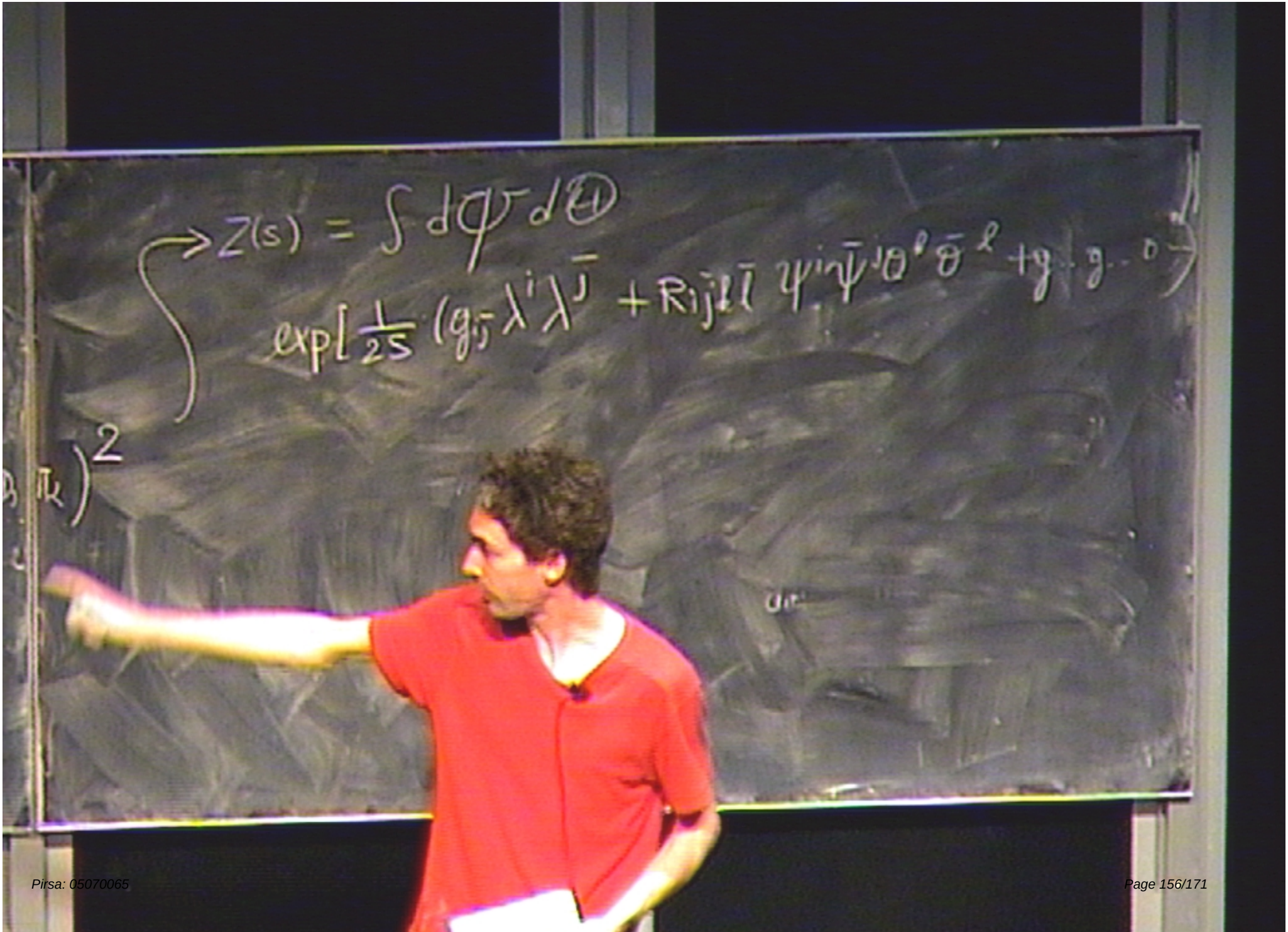
$$Z(s) = \left(\frac{2\pi}{s}\right)^{k/2} \exp\left[\frac{1}{2s}\Big(i\pi\lambda^i D_i \Pi_\alpha - c.c. + \psi^i\theta^j D_i D_j \Pi_\alpha + \bar\psi^i\theta^j \bar D_i D_j \Pi_\alpha + c.c.\Big)\right.$$

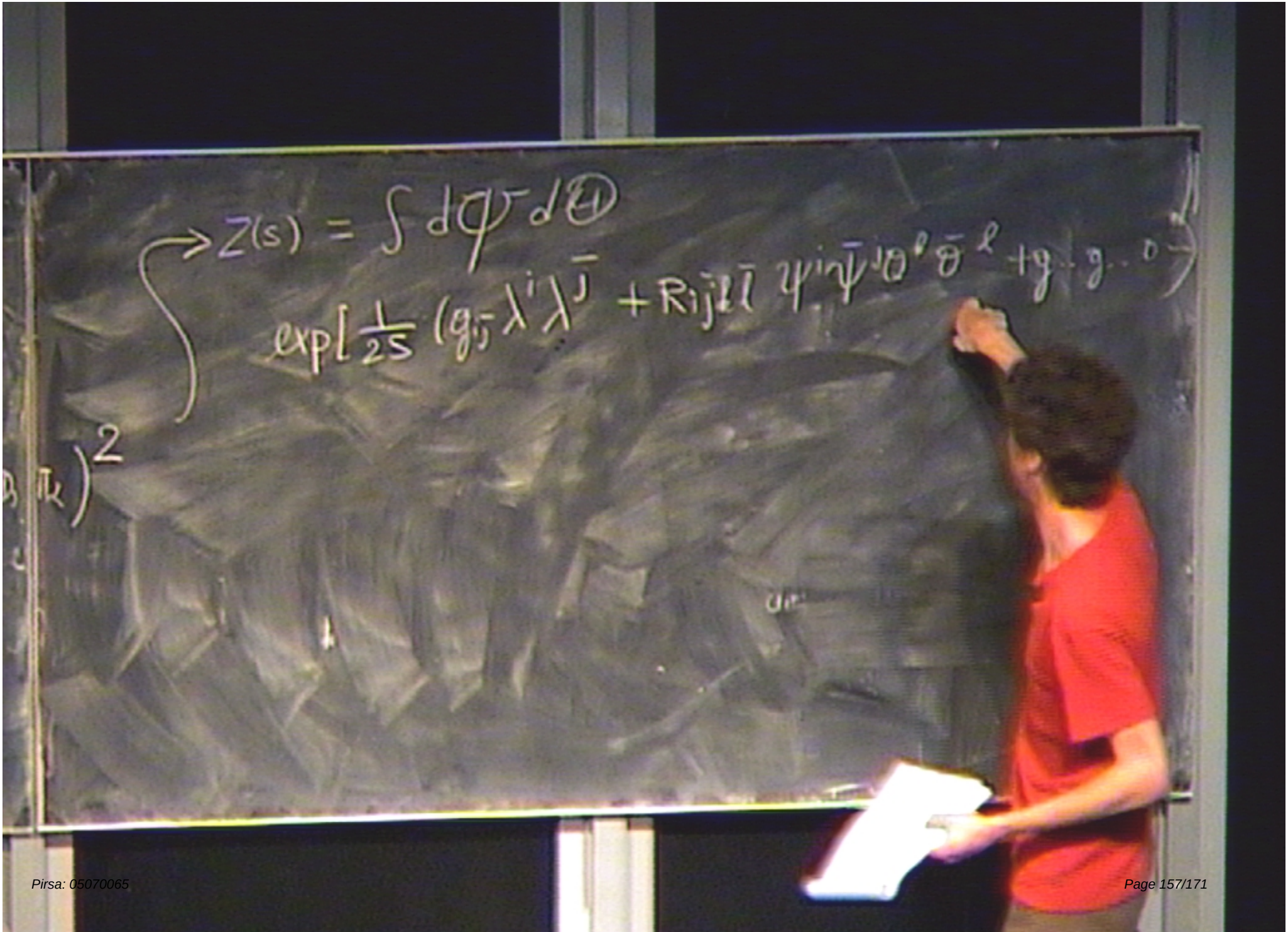
$$\int d^n\psi\, d^n\bar\psi\, d^n\theta\, d^n\bar\theta \exp\Big(\psi^i\theta^j D_i D_j \Pi_\alpha N^\alpha + \bar\psi^i \theta^j \bar D_i D_j \Pi_\alpha N^\alpha + \text{c.c.} \Big)$$

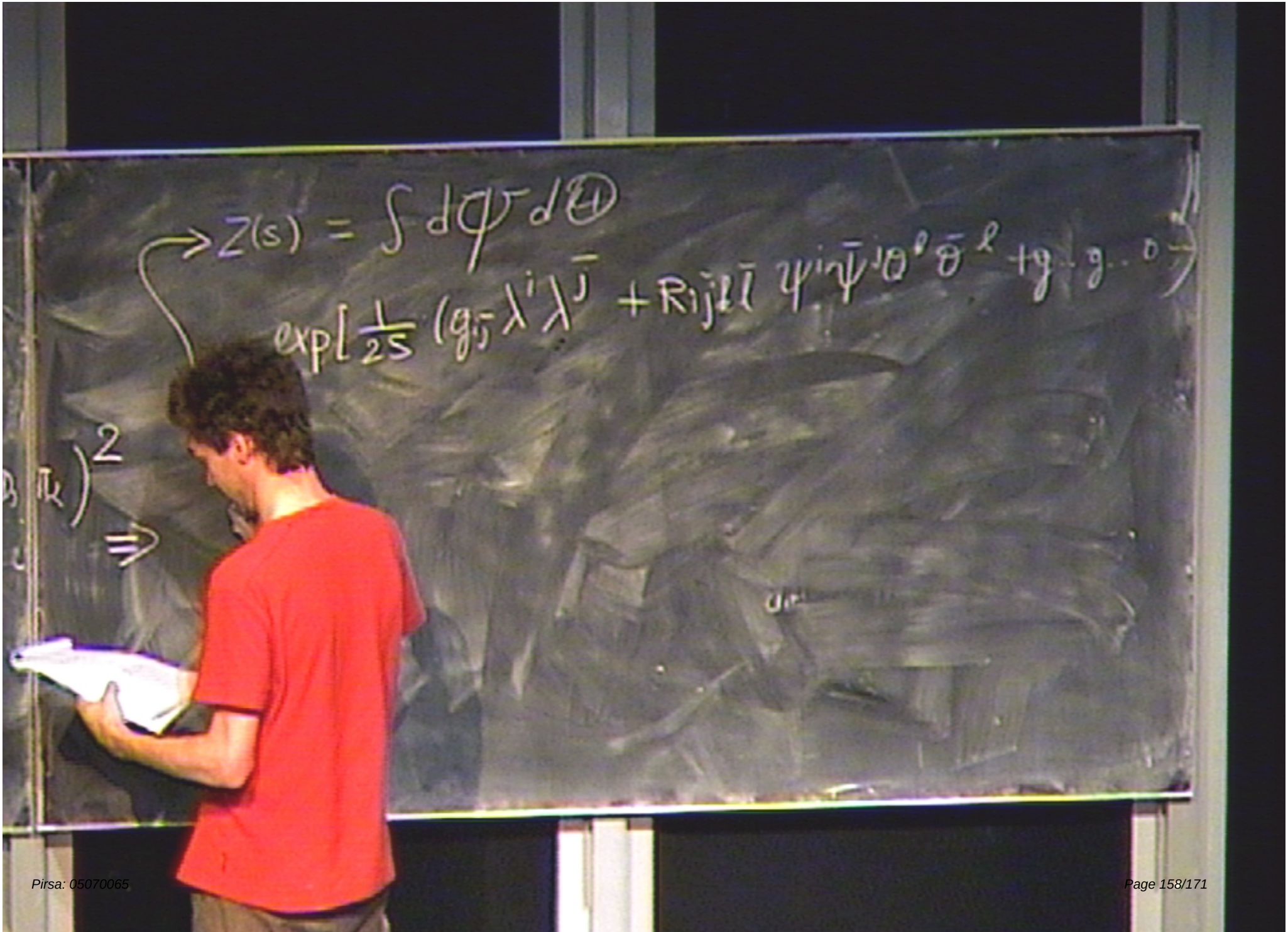
integrate out $N \Rightarrow \mathcal{I} = \frac{1}{\pi i}\int \frac{ds}{s} e^{s\ln} Z(s)$

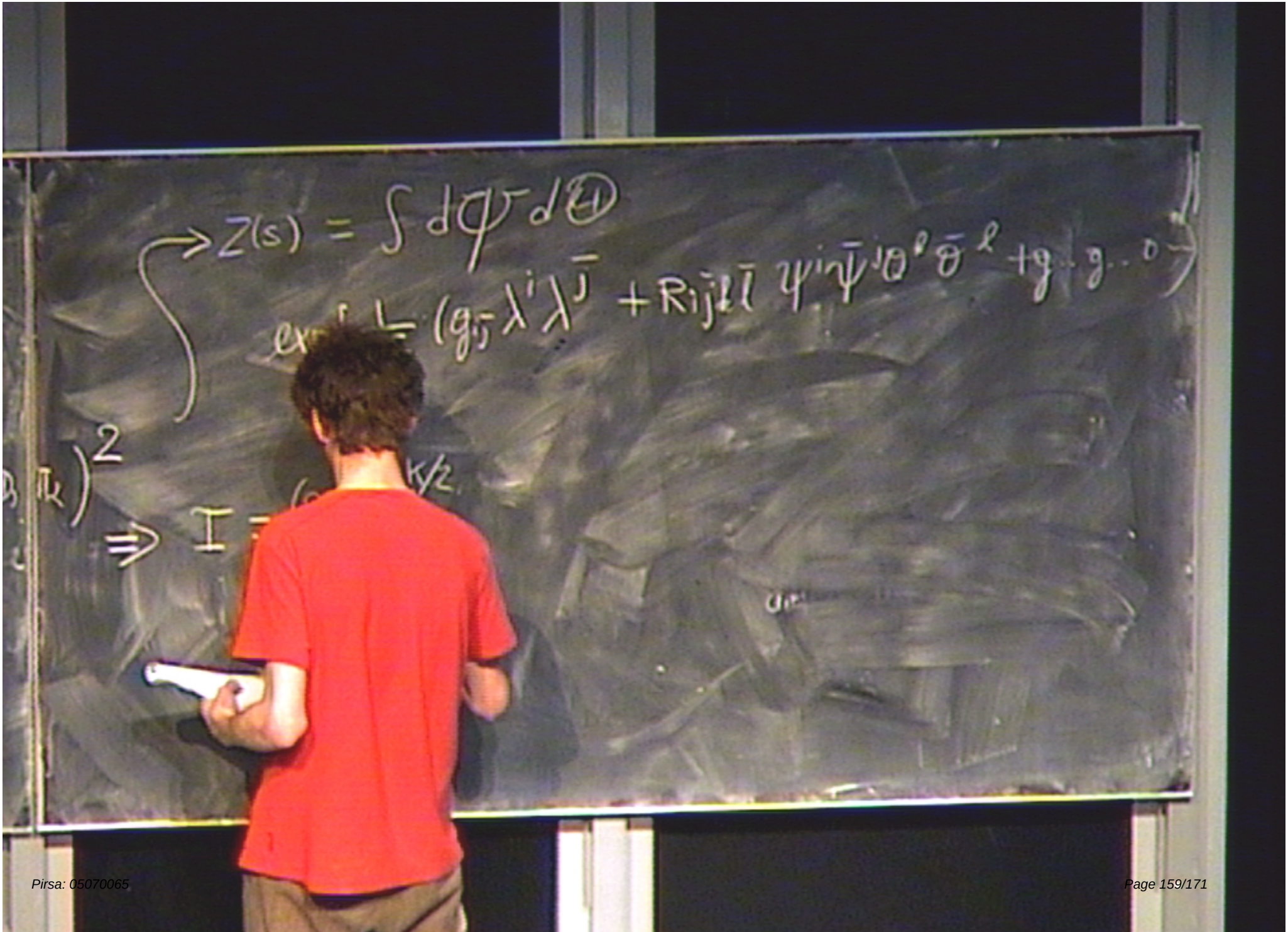
$$Z(s) = \int \prod \left(\frac{2\pi}{s}\right)^{K/2} \exp\Big[\frac{1}{2s} \Big((\pi \lambda^i D_i \Pi_k + \psi^i\theta^j D_i D_j \Pi_k + \bar\psi^{ij} \bar D_i D_j \Pi_k + \text{c.c.}) - \text{c.c.} \Big)^2$$

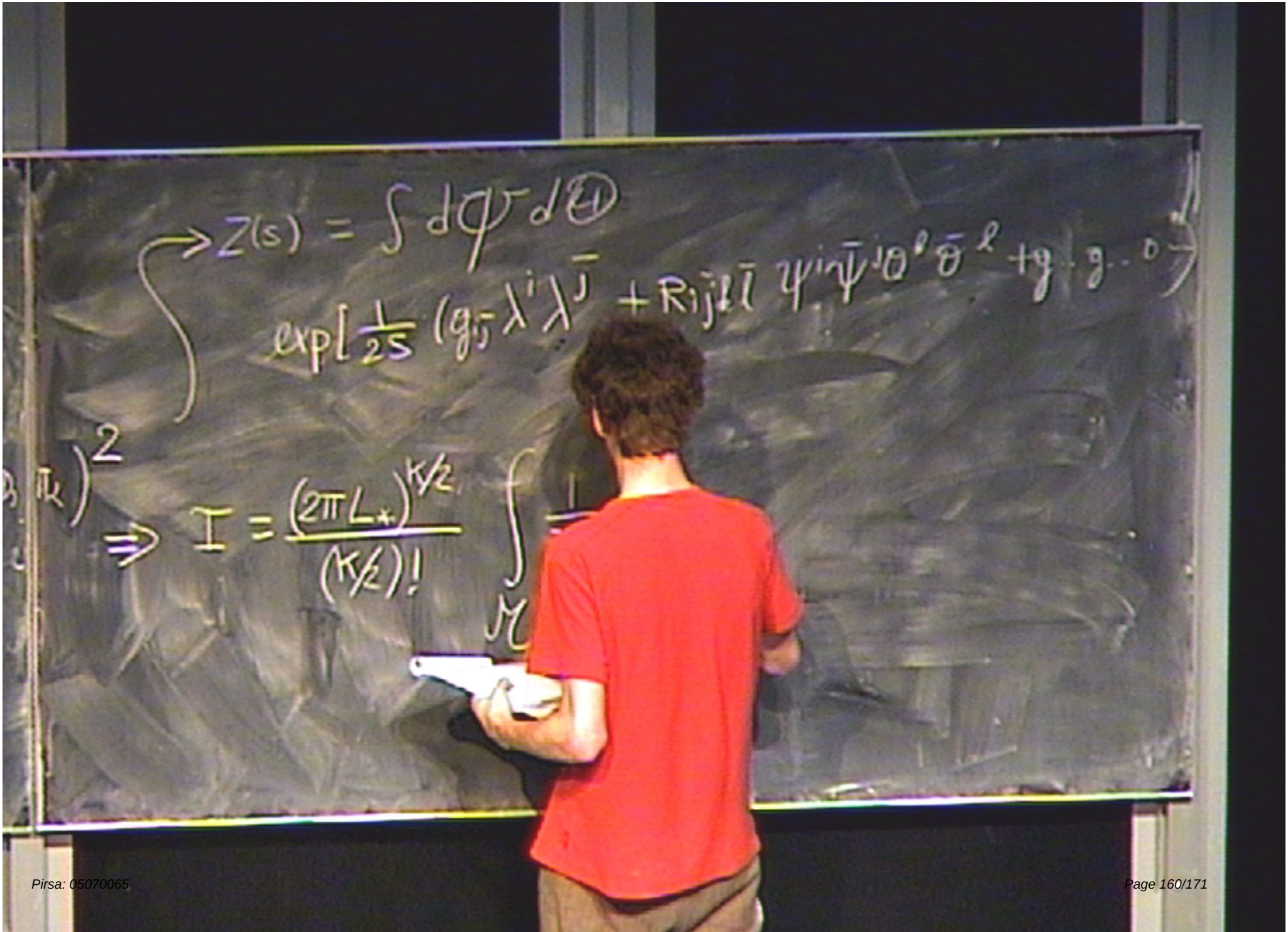


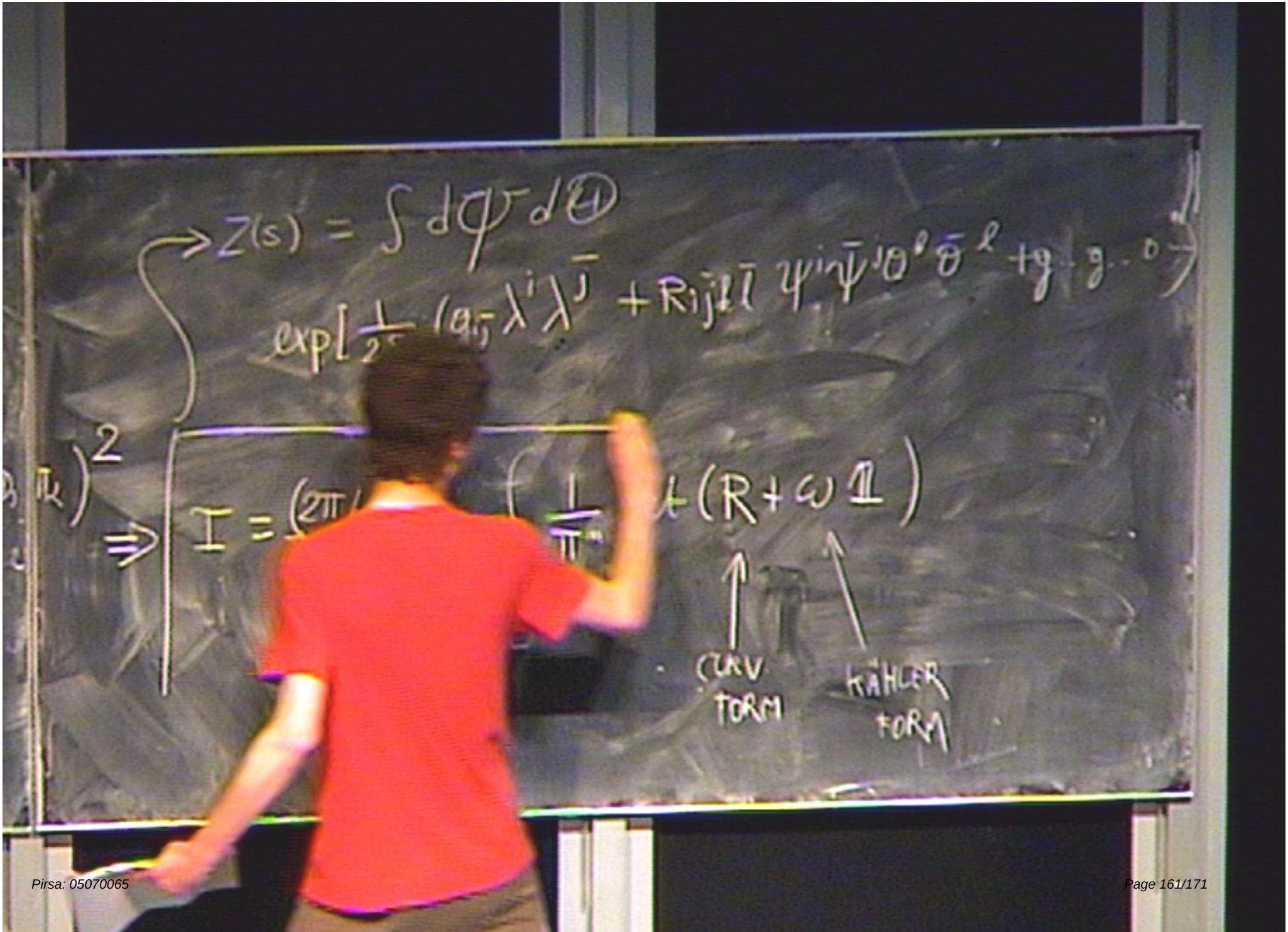

$\rightarrow Z(s) = \int dg\, d\Phi$
$\exp[\frac{1}{2s}(g_{ij}\lambda^i\lambda^{\bar{j}} + R_{i\bar{j}k\bar{l}}\,\psi^i\bar{\psi}^j\theta^k\bar{\theta}^{\bar{l}} + \ldots$
$\pi_k)^2$

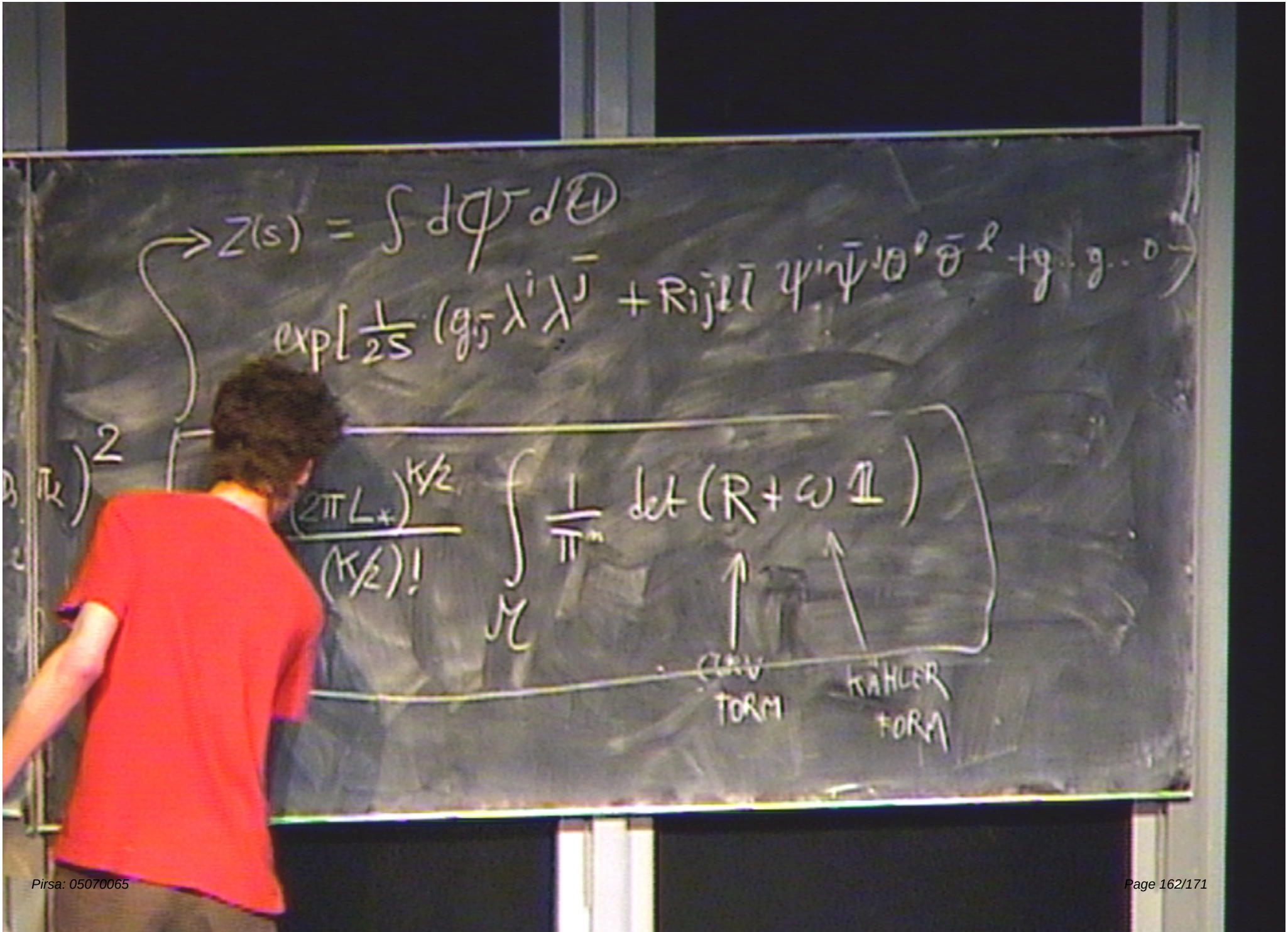

Z(s) = ∫ dΦ dΘ
exp[1/2s (g_ij λ^i λ^j + R_ijkl ψ^i ψ̄^j θ^l θ̄^l + g.. g.. 0)
)^2


Z(s) = ∫ dΦ dΘ
exp[1/2s (g_ij λ^i λ^j + R_ijkl ψ^i ψ̄^j θ^l θ̄^l + g ... g ... 0 ...
)^2
⇒


Z(s) = ∫ dΦ dΘ
ex
(g_ij λ^i λ^j̄ + R_ijkl̄ ψ^i ψ̄^j θ^l θ̄^l + g.. g.. 0
⇒ I =


Z(s) = ∫ dφ dΘ
exp[1/2s (g_ij λ^i λ^j̄ + R_ijkl̄ ψ^i ψ̄^j θ^l θ̄^l + g . g . . 0)
⇒ I = (2π L*)^{K/2} / (K/2)!


CURV FORM
KAHLER FORM


$Z(s) = \int d\varphi\, d\psi$
$\exp\left[\frac{1}{2s}\left(g_{ij}\lambda^i\lambda^{\bar{j}} + R_{ij\bar{k}\bar{l}}\,\psi^i\psi^j\theta^k\bar{\theta}^l + g\ldots g\ldots 0\right)\right]$
$\pi_k)^2$
$\frac{(2\pi L_+)^{k/2}}{(k/2)!}$
$\int_{\mathcal{M}} \frac{1}{\pi^m} \det(R + \omega\,\mathbb{1})$
CURV TORM
KAHLER FORM

$$Z(s) = \int d\varphi\, d\theta \exp\left[\frac{1}{2s}\left(g_{i\bar{j}}\lambda^i\lambda^{\bar{j}} + R_{ij\bar{k}\bar{l}}\,\psi^i\psi^j\theta^k\bar{\theta}^l + \right.\right.$$

$$I = \frac{(2\pi L_*)^{K/2}}{(K/2)!}\int_{\mathcal{M}} \frac{1}{\pi^n}\det(R+\omega\mathbb{1})$$

CURV FORM

KÄHLER FORM

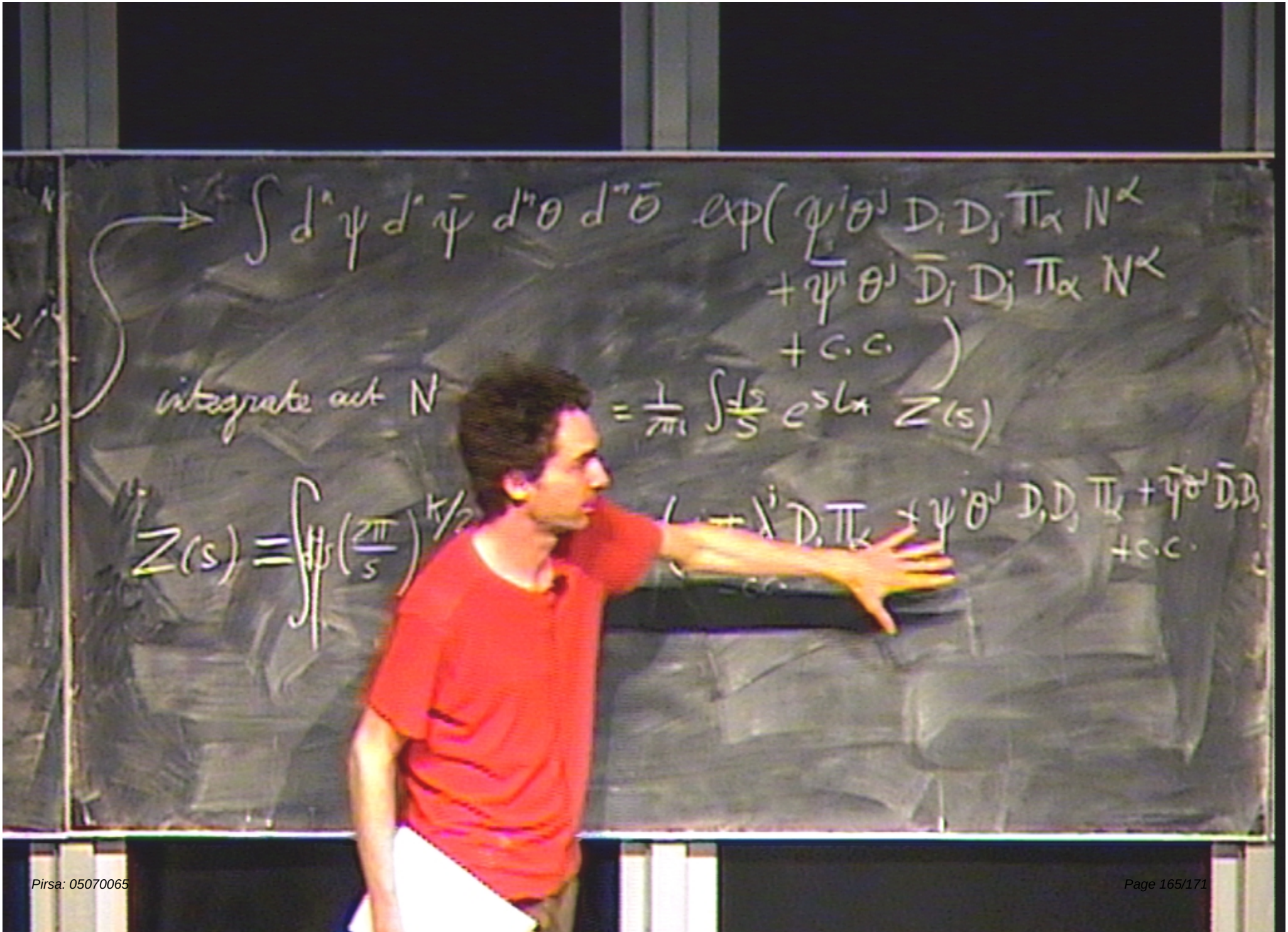
$$\rightarrow Z(s) = \int d\varphi\, d\theta$$

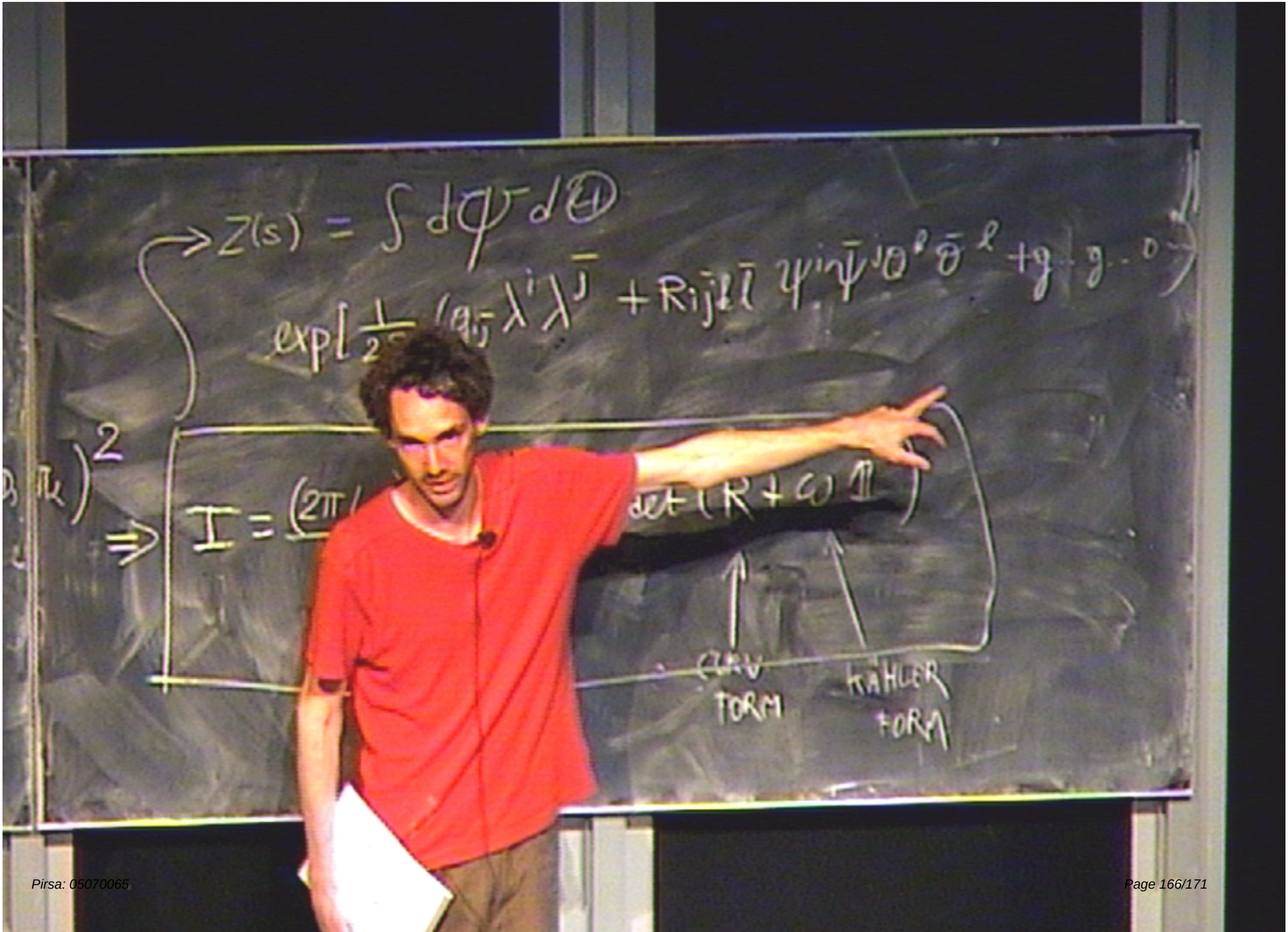
$$\exp\left[\frac{1}{2s}\left(g_{i\bar{j}}\lambda^i\lambda^{\bar{j}} + R_{ij\bar{k}\bar{l}}\,\psi^i\psi^j\theta^k\bar{\theta}^l + \right.\right.$$

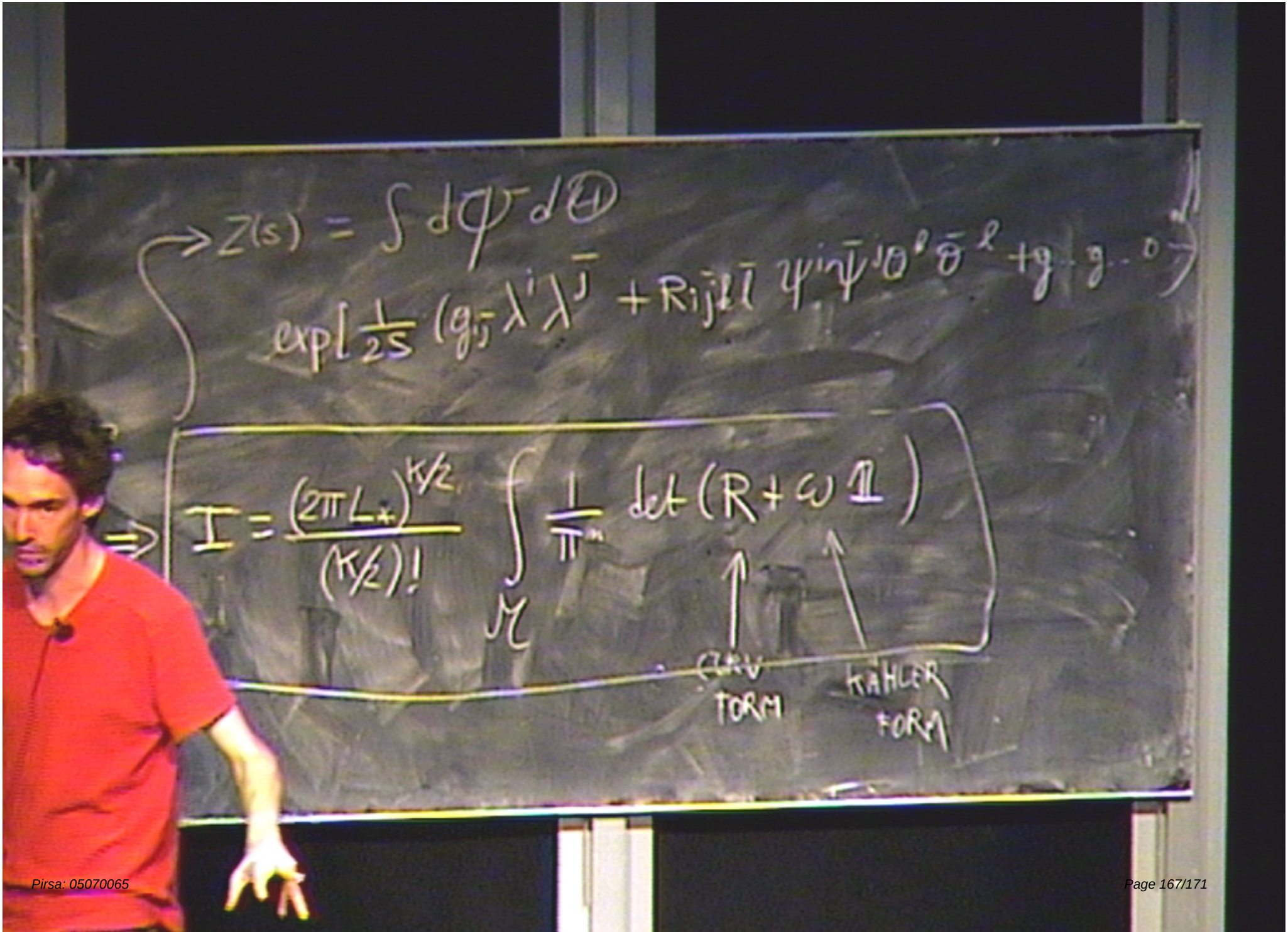
$$\Rightarrow \boxed{I = \frac{(2\pi L_+)^{K/2}}{(K/2)!}\int_{\mathcal{M}} \frac{1}{\pi^n}\det(R+\omega\mathbb{1})}$$

CURV FORM

KÄHLER FORM


integrate out N


Z(s) = ∫ dΦ dΘ
CURV
FORM
KAHLER
FORM


$\rightarrow Z(s) = \int d\varphi\, d\theta$
$\exp\left[\frac{1}{2s}\left(g_{ij}\lambda^i\lambda^{\bar{j}} + R_{ij\bar{k}\bar{l}}\,\psi^i\psi^j\theta^k\bar{\theta}^l + \ldots\right)\right]$
$\Rightarrow I = \frac{(2\pi L_+)^{K/2}}{(K/2)!}\int_{\mathcal{M}} \frac{1}{\pi^m}\det(R+\omega\mathbb{1})$
CURV TORM
KAHLER FORM

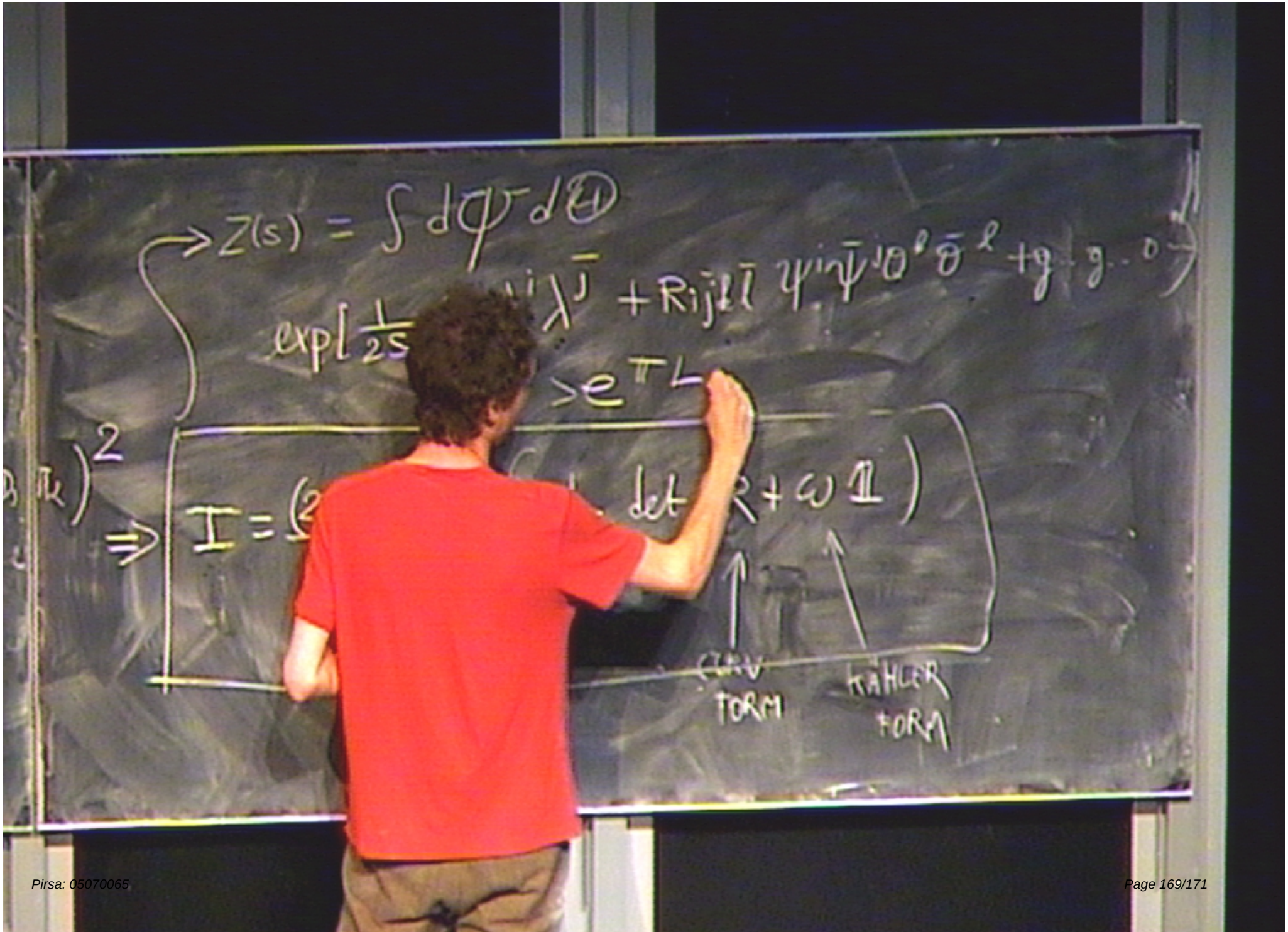
$$\rightarrow Z(s) = \int d\varphi\, d\theta\, d\bar{\theta}$$

$$\exp\left[\frac{1}{2s}\left(g_{i\bar{j}}\lambda^i\lambda^{\bar{j}} + R_{i\bar{j}k\bar{l}}\,\psi^i\psi^{\bar{j}}\theta^k\bar{\theta}^{\bar{l}} + g\cdot g\cdot 0\right)\right.$$

$$\Rightarrow \boxed{I = \frac{(2\pi L_+)^{K/2}}{(K/2)!}\int_{\mathcal{M}} \frac{1}{\pi^n}\det(R + \omega\,\mathbb{1})}$$

CURV FORM

KÄHLER FORM


Z(s) = ∫ dΦ dΘ
exp[1/2s
> e^{πL}
det R + ω 1
CURV
FORM
KAHLER
FORM

$$\rightarrow Z(s) = \int d\varphi\, d\Theta$$

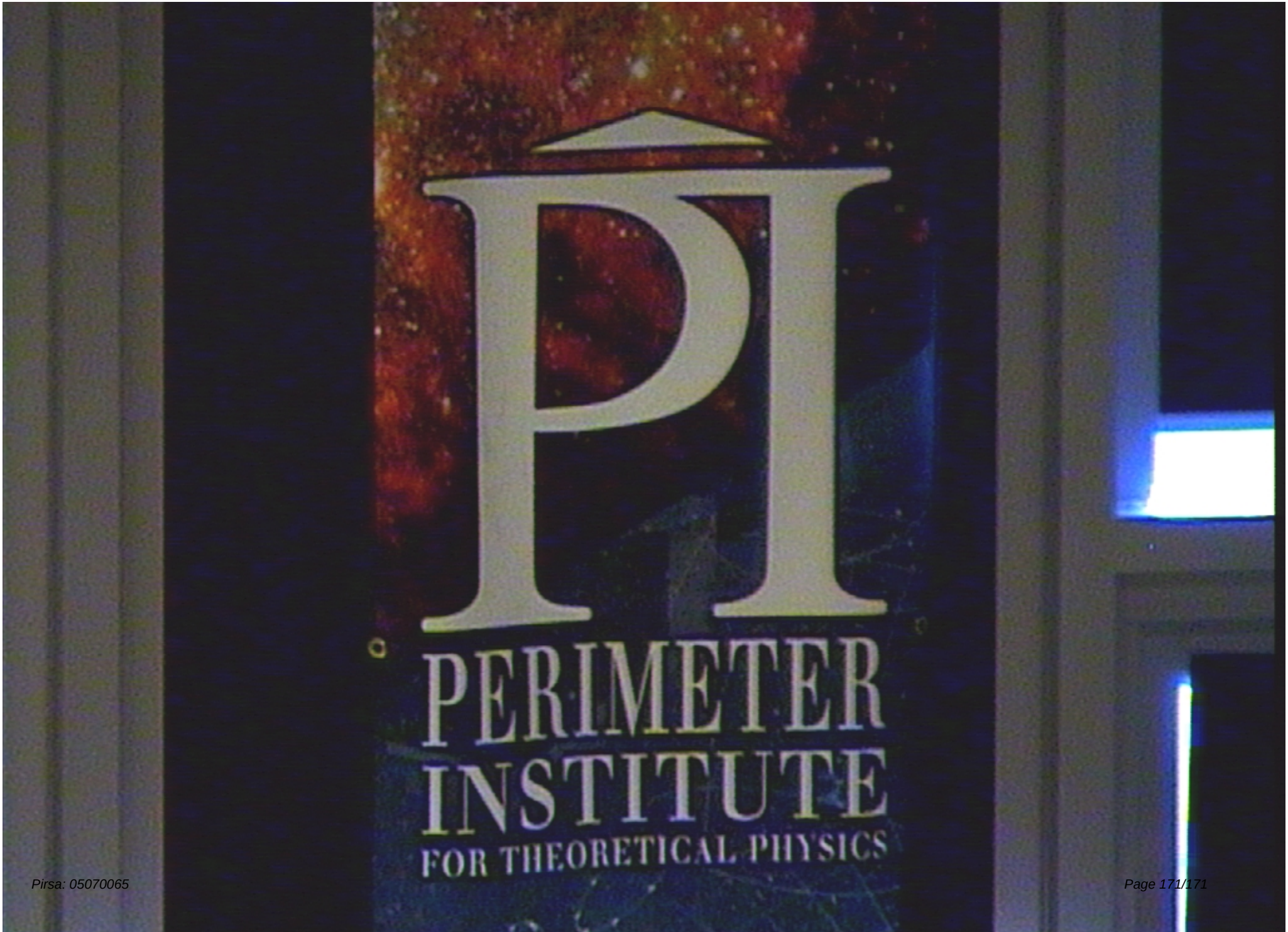
$$\exp\left[\frac{1}{2s}\left(g_{ij}\lambda^i\lambda^{\bar{j}} + R_{ij\bar{l}\bar{l}}\,\psi^i\psi^{\bar{j}}\theta^l\bar{\theta}^{\bar{l}} + g\ldots g\ldots 0\right)\right.$$

$$> e^{\pi L_*} \leftarrow \frac{\lambda}{2q}$$

$$\Rightarrow \boxed{I = \frac{(2\pi L_*)^{k/2}}{(k/2)!}\int_{\mathcal{M}} \frac{1}{\pi^n}\det(R + \omega\,\mathbb{1})}$$

CURV FORM

KÄHLER FORM


PI
PERIMETER
INSTITUTE
FOR THEORETICAL PHYSICS