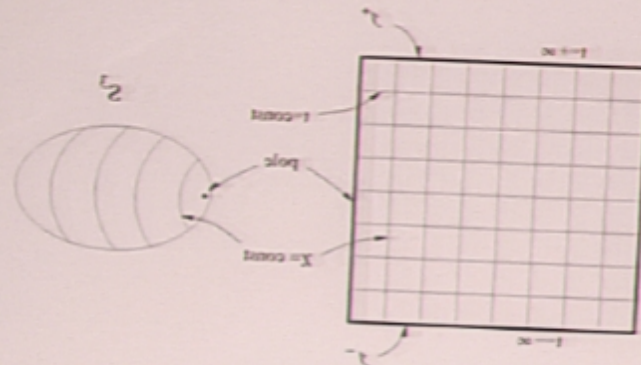
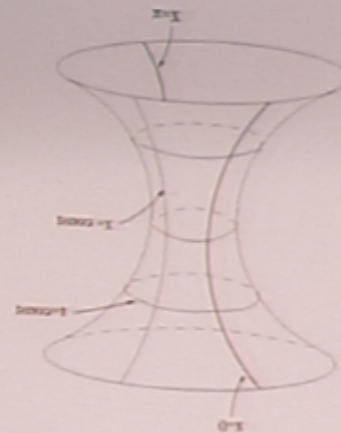


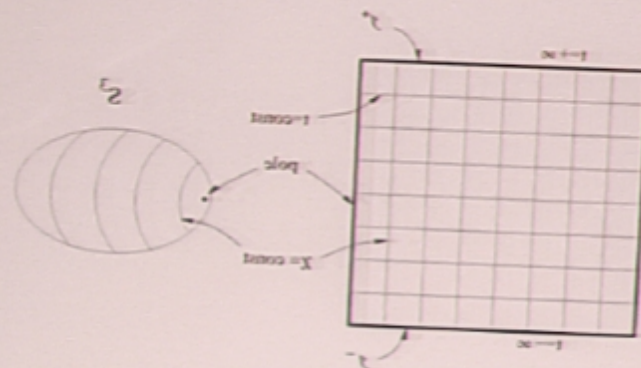
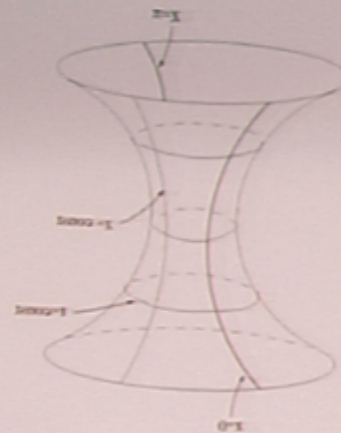
Title: Early Universe Cosmology 1

Date: Jul 01, 2005 11:30 AM

URL: <http://pirsa.org/05070064>

Abstract:





Geometry

$$ds^2 = a^2(\tau) \left[(\eta_{\mu\nu} + \delta g_{\mu\nu}) dx^\mu dx^\nu \right] + ds_c^2$$

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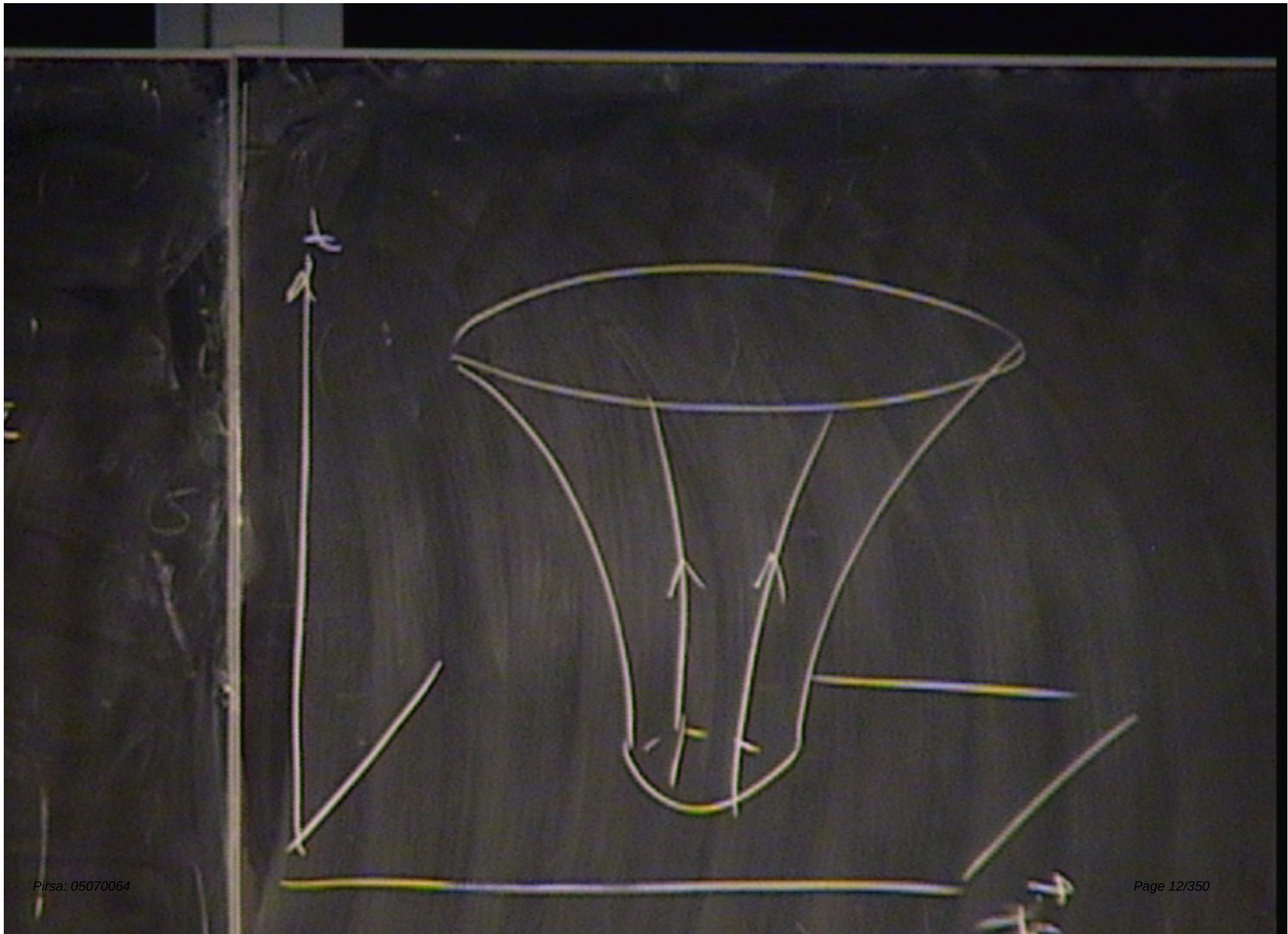
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$$\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$$

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$$\begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

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$$P = P(\epsilon)$$

$$\begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

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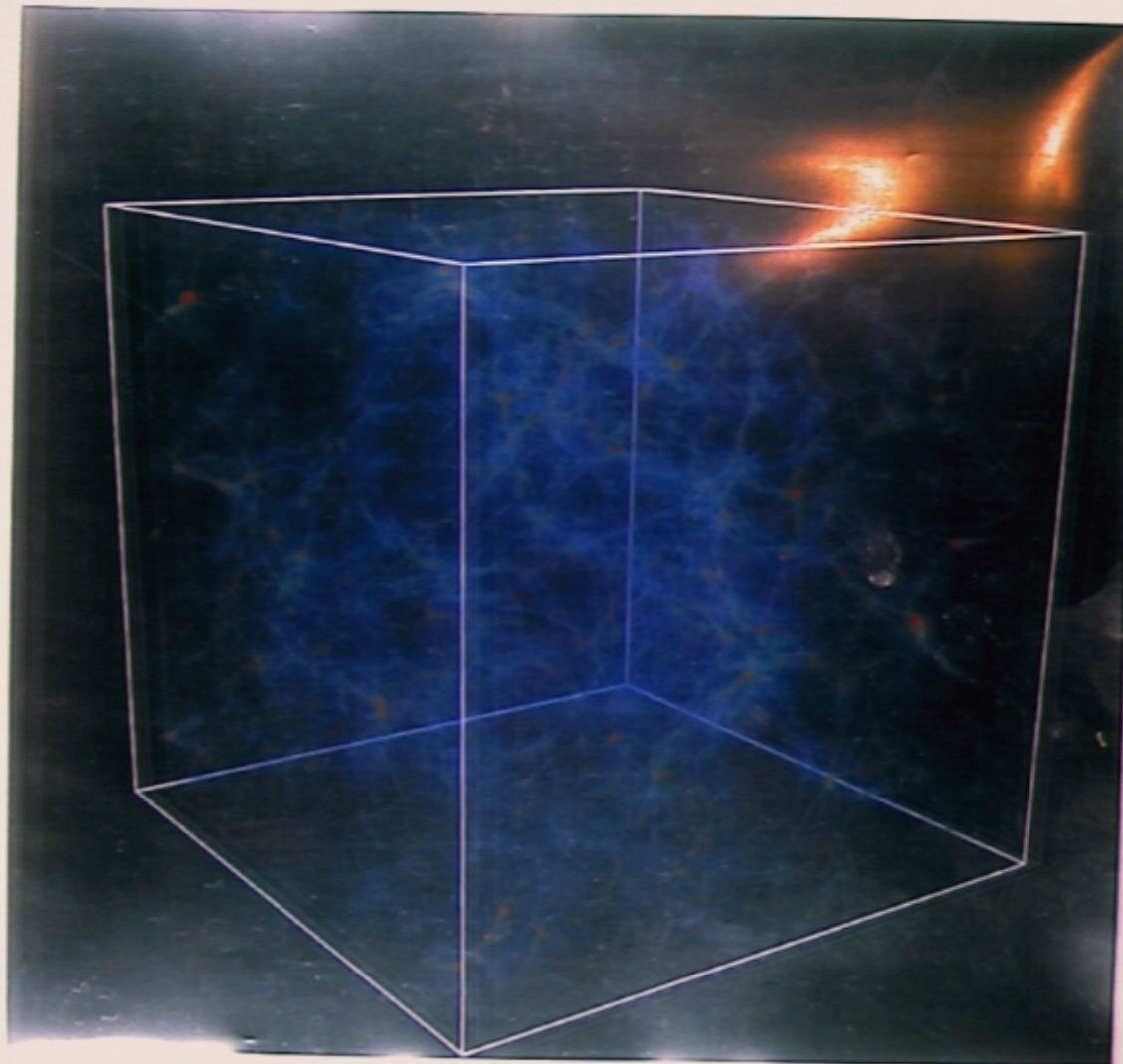
(0)

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3} \sum \epsilon_i$$

$$P = P(\epsilon)$$

(1)

$$\frac{\dot{a}}{a} + \frac{\ddot{a}}{a} = \frac{4\pi G}{3} \sum (\epsilon - 3P)_i$$



$$\epsilon_m = \frac{\epsilon_{m0}}{a^3}, p_m = 0$$

$$\epsilon_r = \frac{\epsilon_{r0}}{a^4}, p_r = \frac{1}{3}\epsilon_r$$

(3)

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$$\mathcal{E}_m = \frac{\mathcal{E}_{m0}}{a^3}, \quad p_m = 0$$

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$$\frac{d}{dt} = \frac{1}{a} \frac{d}{d\tau}$$

$$\boxed{a'' = 4\pi G \mathcal{E}_{m0}}$$

$$a(\tau) = a_0 \tau (\tau_0 + \tau)$$

$$\tau \ll \tau_0, \quad a \sim \tau$$

$$t = \int d\tau a(\tau)$$

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$$\tau \ll \tau_0, a \sim \tau$$

$$t = \int d\tau a(\tau) \sim \tau^2$$

$$a \sim \sqrt{t}$$

$$\tau \gg \tau_0$$

$$a \sim \tau^2$$

$$t \sim \tau^3$$

$$a \sim t^{2/3}$$

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$$\tau$$

$$a \propto \tau^2$$

$$t \propto \tau^3$$

$$\sim \tau^{2/3}$$

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$$a'' =$$

$$a(\tau) =$$

$a(t)$

$$\tau \gg \tau_0$$

$$a \sim \sqrt{t}$$

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$$t \sim \tau^3$$

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$$\epsilon_m = \frac{\epsilon_{m0}}{a^3}, P_m = 0$$

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$$\frac{d}{dt} = \frac{1}{a} \frac{d}{d\tau}$$

$$a'' = 4$$

$$a(\tau) = a$$

$a(t)$

$$a \sim \sqrt{t}$$

$$a \sim \tau^2$$

$$t \sim \tau^3$$

$$a \sim t^{2/3}$$

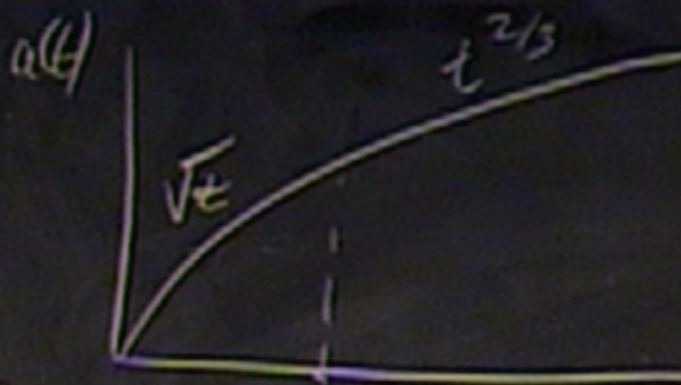
$$\epsilon_m = \frac{\epsilon_{m0}}{a^3}, P_m = 0$$

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$$\frac{d}{dt} = \frac{1}{a} \frac{d}{d\tau}$$

$$a'' =$$

$$a(\tau) =$$



$t \rightarrow \tau_0$

$$a \sim \sqrt{t}$$

$$a \sim \tau^2$$

$$t \sim \tau^3$$

$$a \sim t^{2/3}$$

$a(t)$

$t^{2/3}$

$\sqrt{t}$

$\frac{1}{3}\epsilon_r$

t_{eq}

$t a \sim \sqrt{t}$

$$\tau \gg \tau_0$$

$$a \sim \tau^2$$

$$t \sim \tau^3$$

$$a \sim t$$

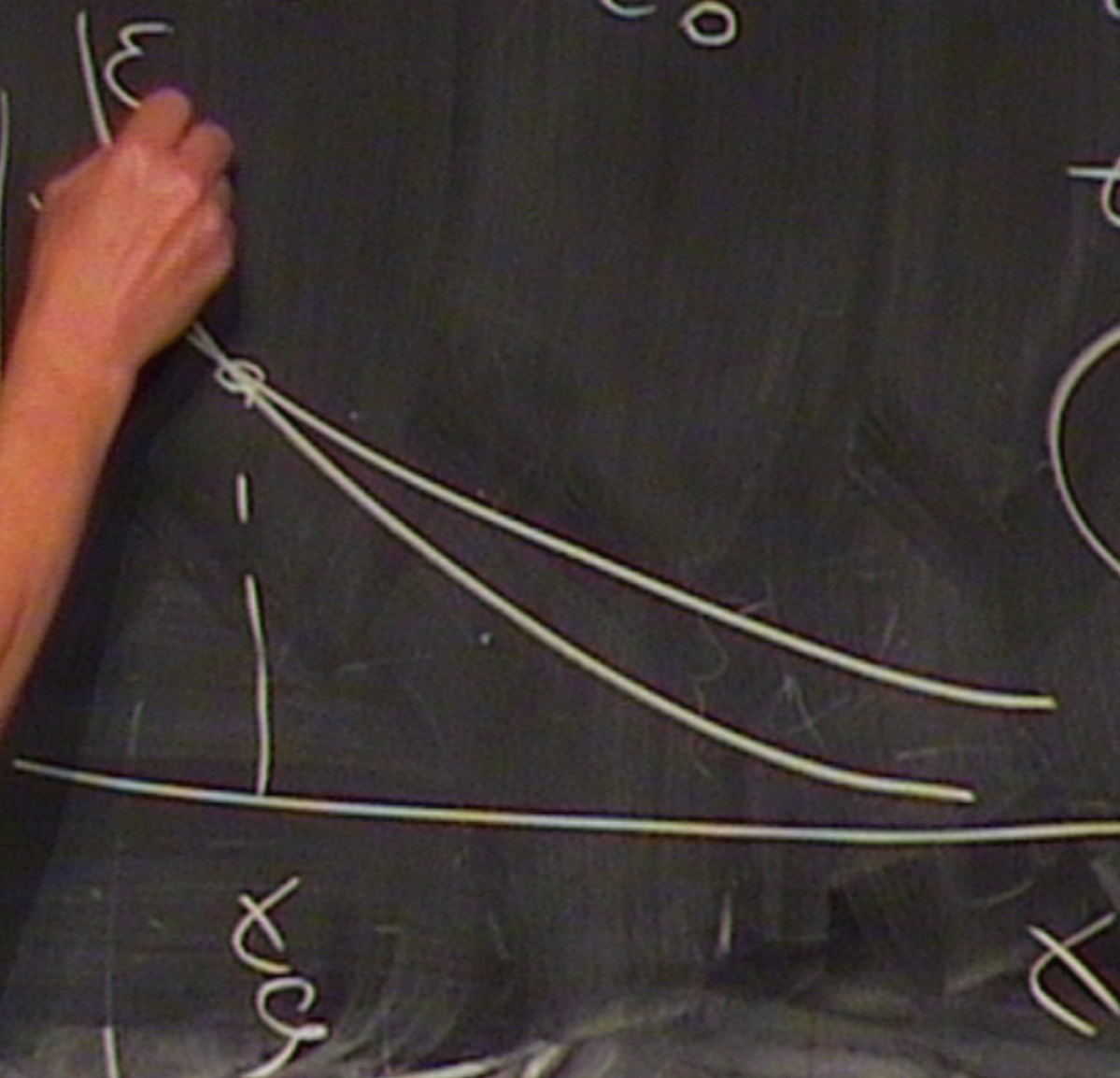


$$\tau \gg \tau_0$$

$$a \sim \tau^2$$

$$t \sim \tau^3$$

$$a \sim t$$

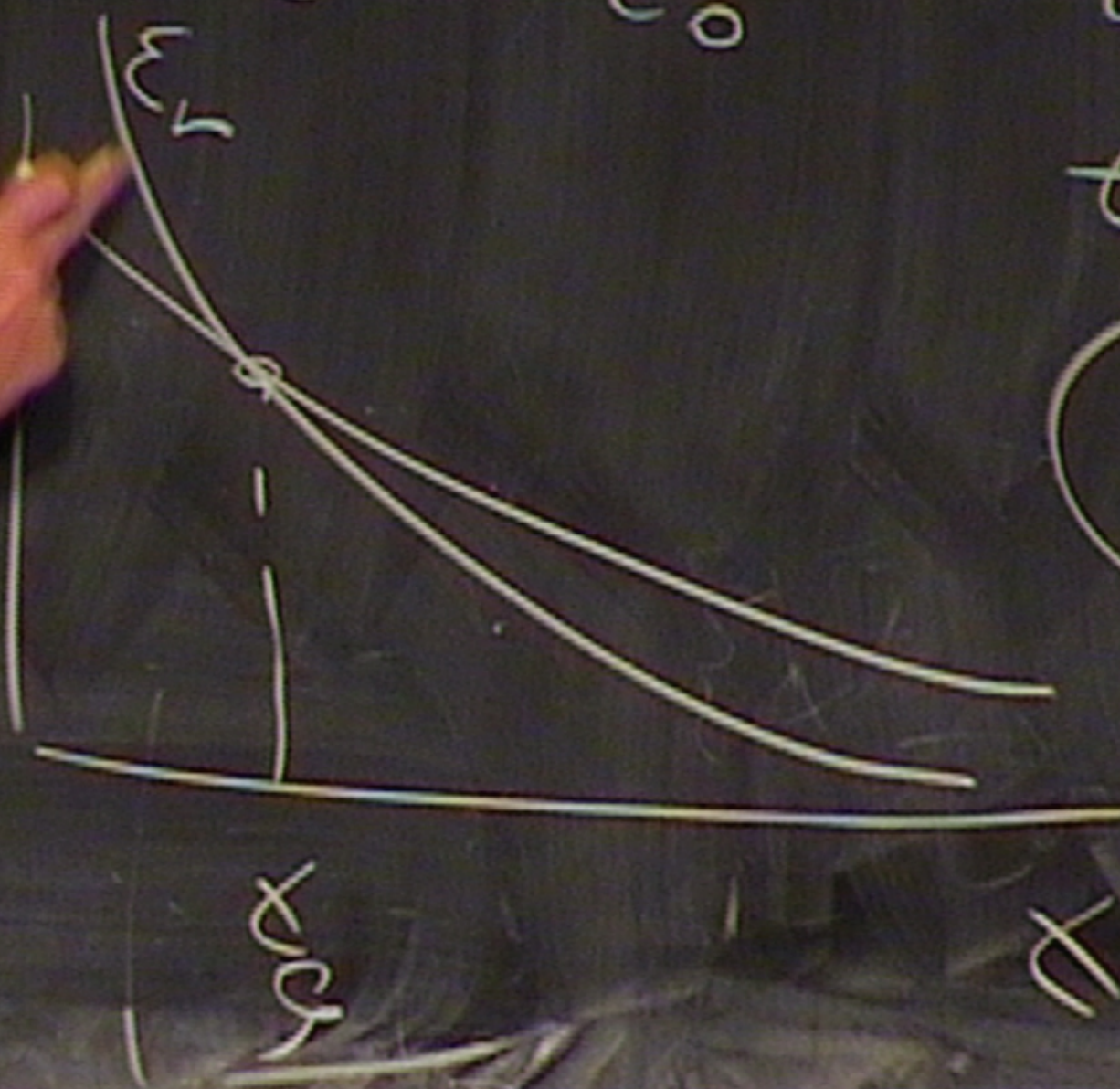


$$\tau \gg \tau_0$$

$$a \sim \tau^2$$

$$t \sim \tau^3$$

$$a \sim t$$

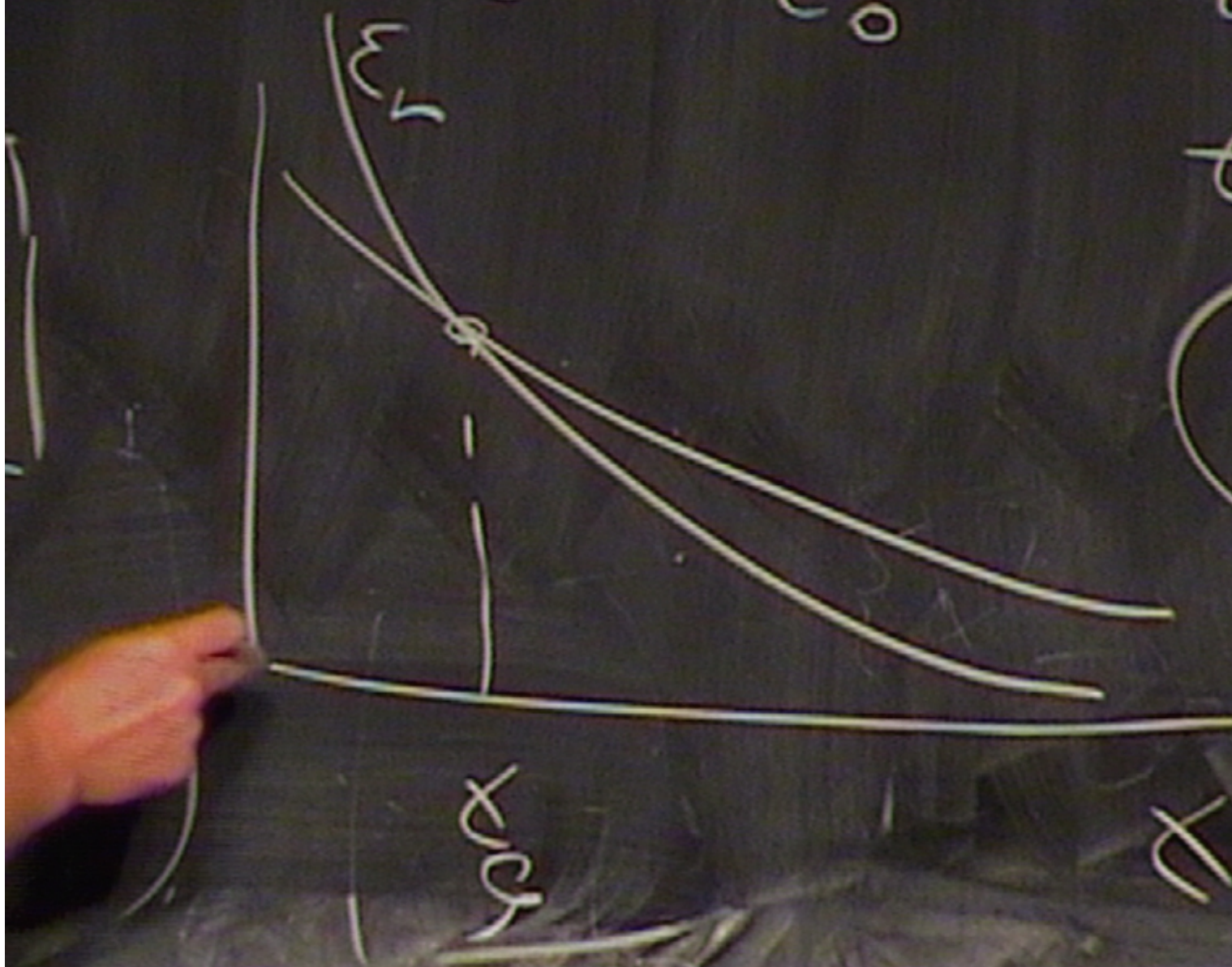


$$\tau \gg \tau_0$$

$$a \sim \tau^2$$

$$t \sim \tau^3$$

$$a \sim t$$



$$\tau \gg \tau_0$$

$$a \sim \tau^2$$

$$t \sim \tau^3$$

$$a \sim t$$



$$\tau \gg \tau_0$$

$$a \sim \tau^2$$

$$t \sim \tau^3$$

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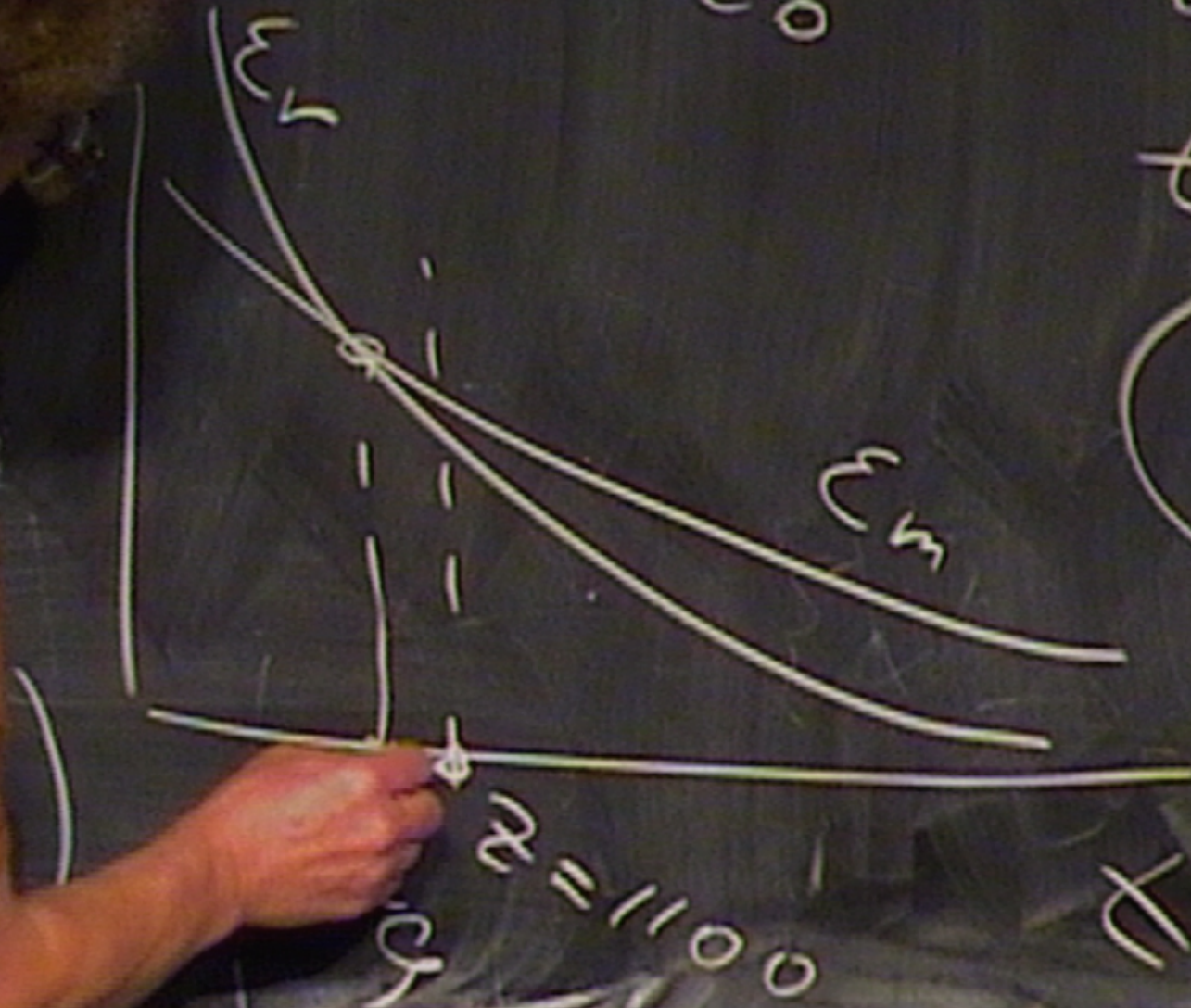


$$\tau \gg \tau_0$$

$$a \sim \tau^2$$

$$t \sim \tau^3$$

$$a \sim t^2$$

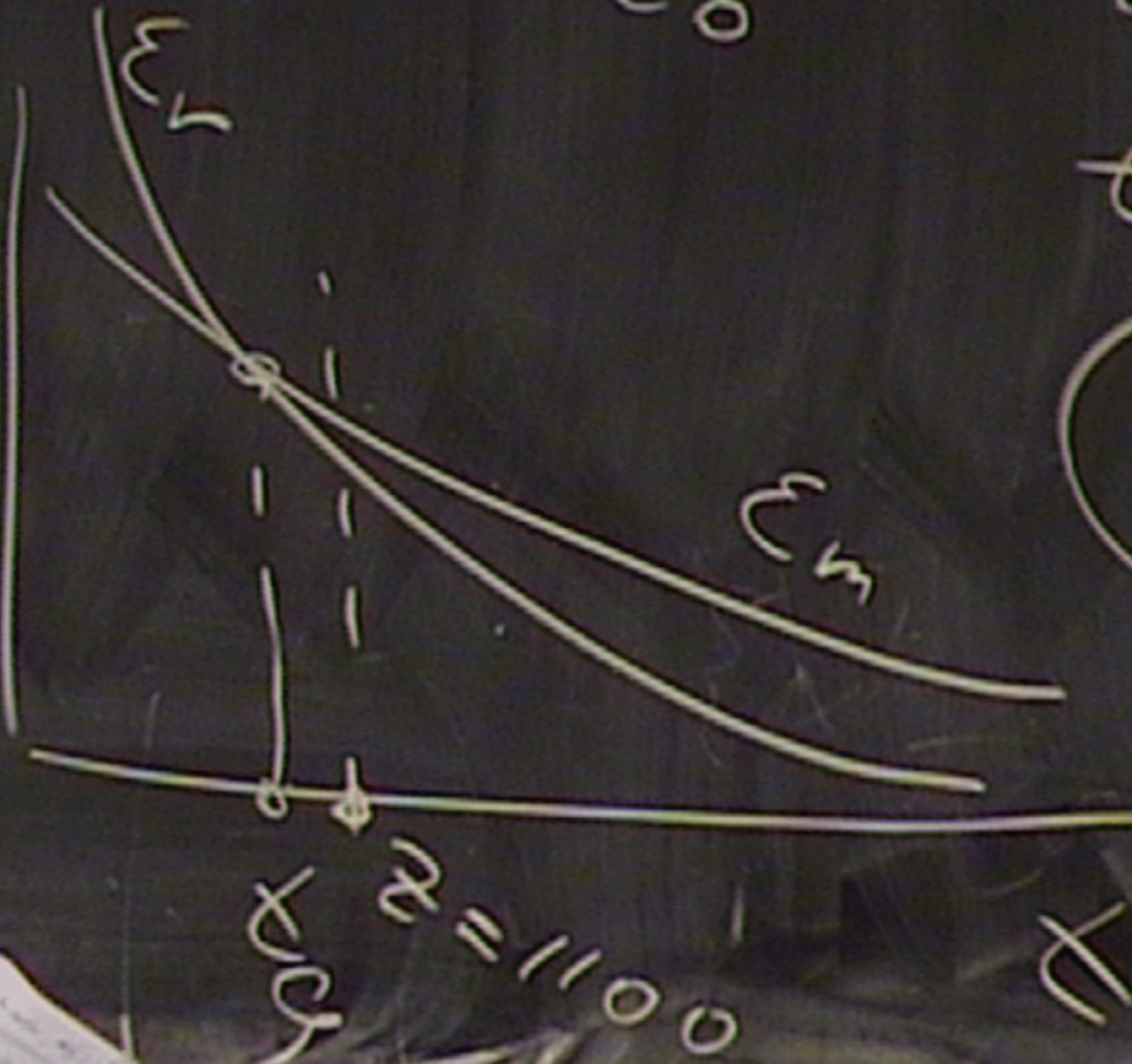


$$\tau \gg \tau_0$$

$$a \sim \tau^2$$

$$t \sim \tau^3$$

$$a \sim t^2$$

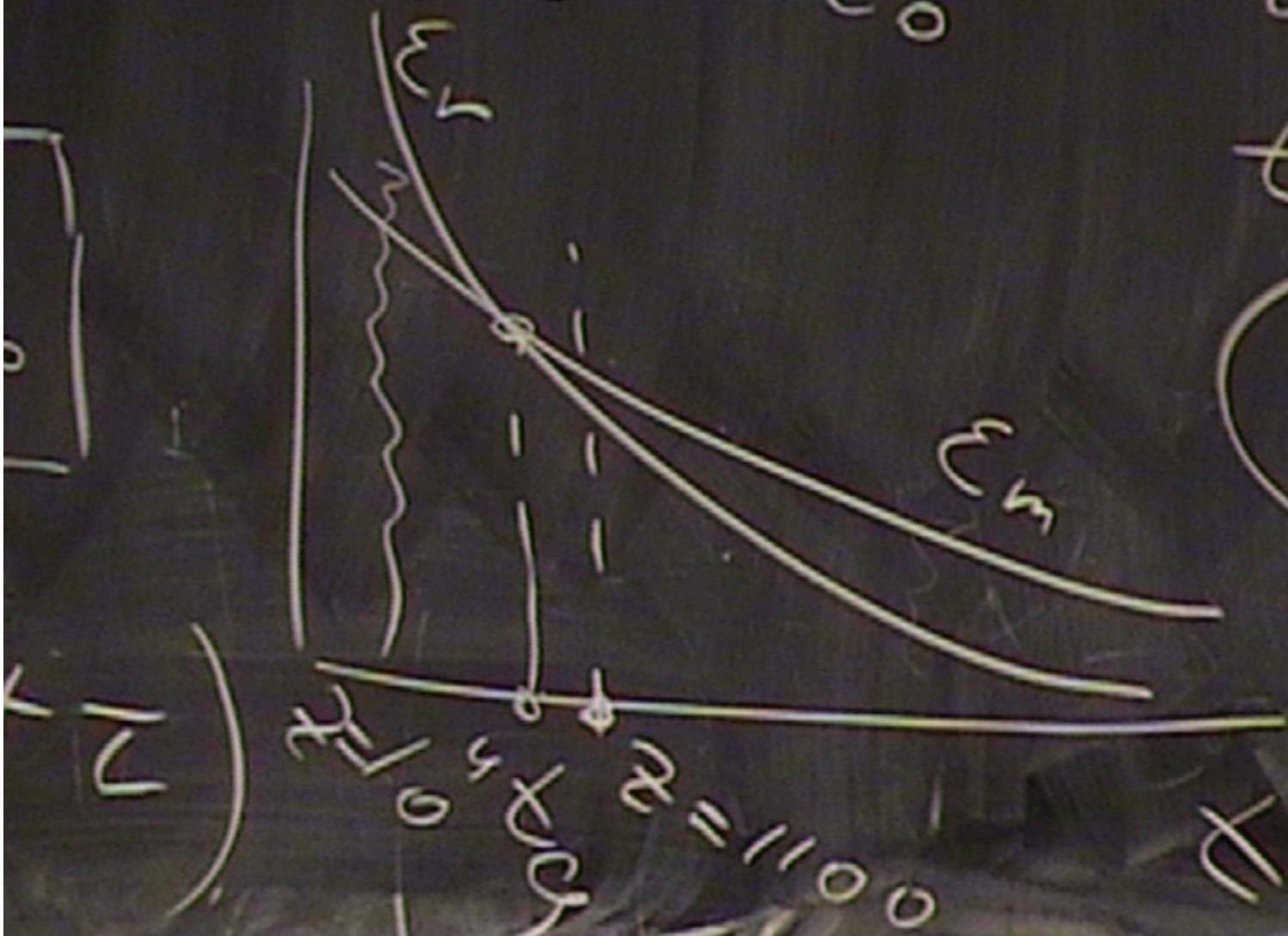


$$\tau \gg \tau_0$$

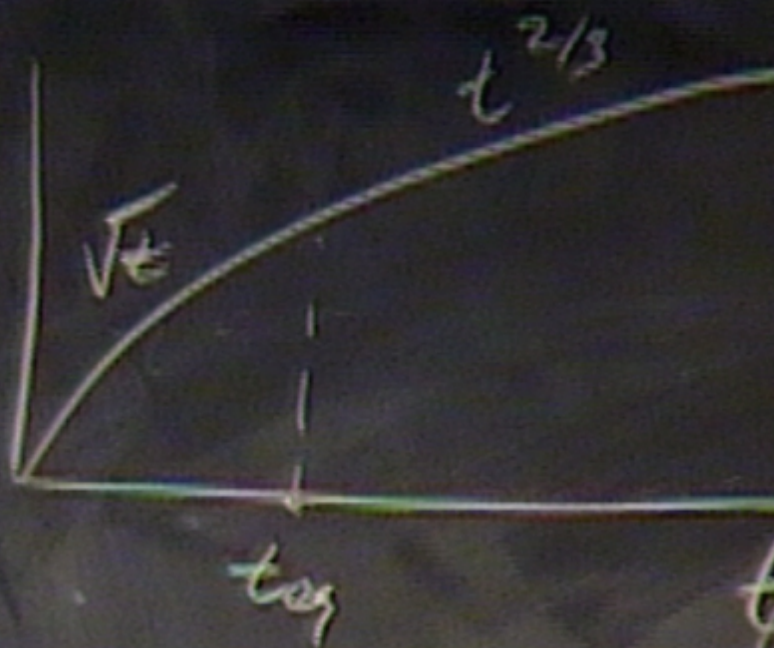
$$a \sim \tau^2$$

$$t \sim \tau^3$$

$$a \sim t^2$$



$a(t)$



$$\frac{\epsilon_{rad}}{a^3}, P_r = 0$$

$$\frac{\epsilon_{rad}}{a^4}, P_r = \frac{1}{3} \epsilon_r$$

$$\frac{d}{dt}$$

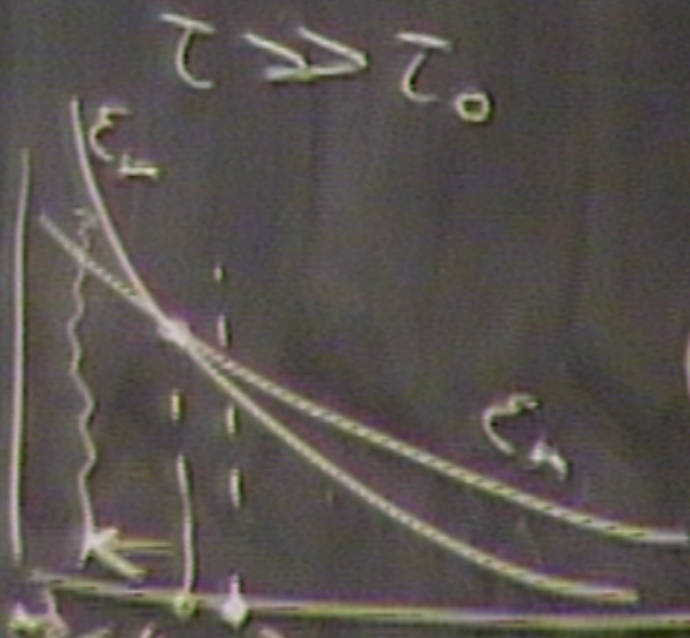
$$a'' = 4\pi G \epsilon_{rad}$$

$$a \sim \sqrt{t}$$

$$a \sim t^2$$

$$t \sim \tau^3$$

$$a \sim t^4$$



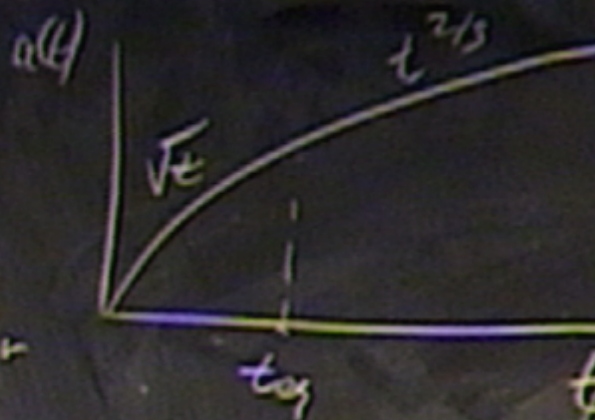
$$\epsilon_m = \frac{\epsilon_{m0}}{a^3}, P_m = 0$$

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$$a'' = 4\pi G \epsilon_{m0}$$

$$a(\tau) = a_0 \tau (\tau_0 + \tau)$$

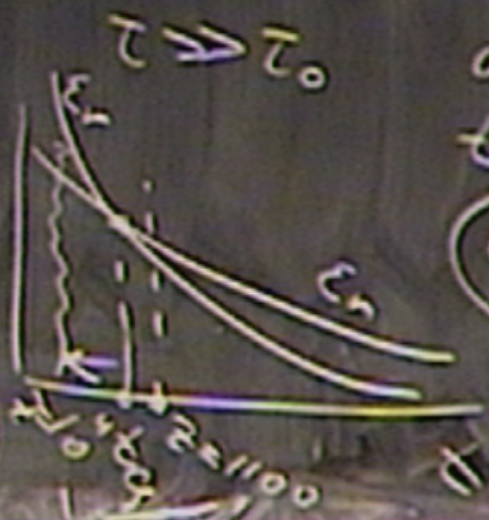


$$a \sim \sqrt{t}$$

$$a \sim \tau^2$$

$$t \sim \tau^3$$

$$a \sim t^{2/3}$$



$$p_1 \approx -\varepsilon_1$$

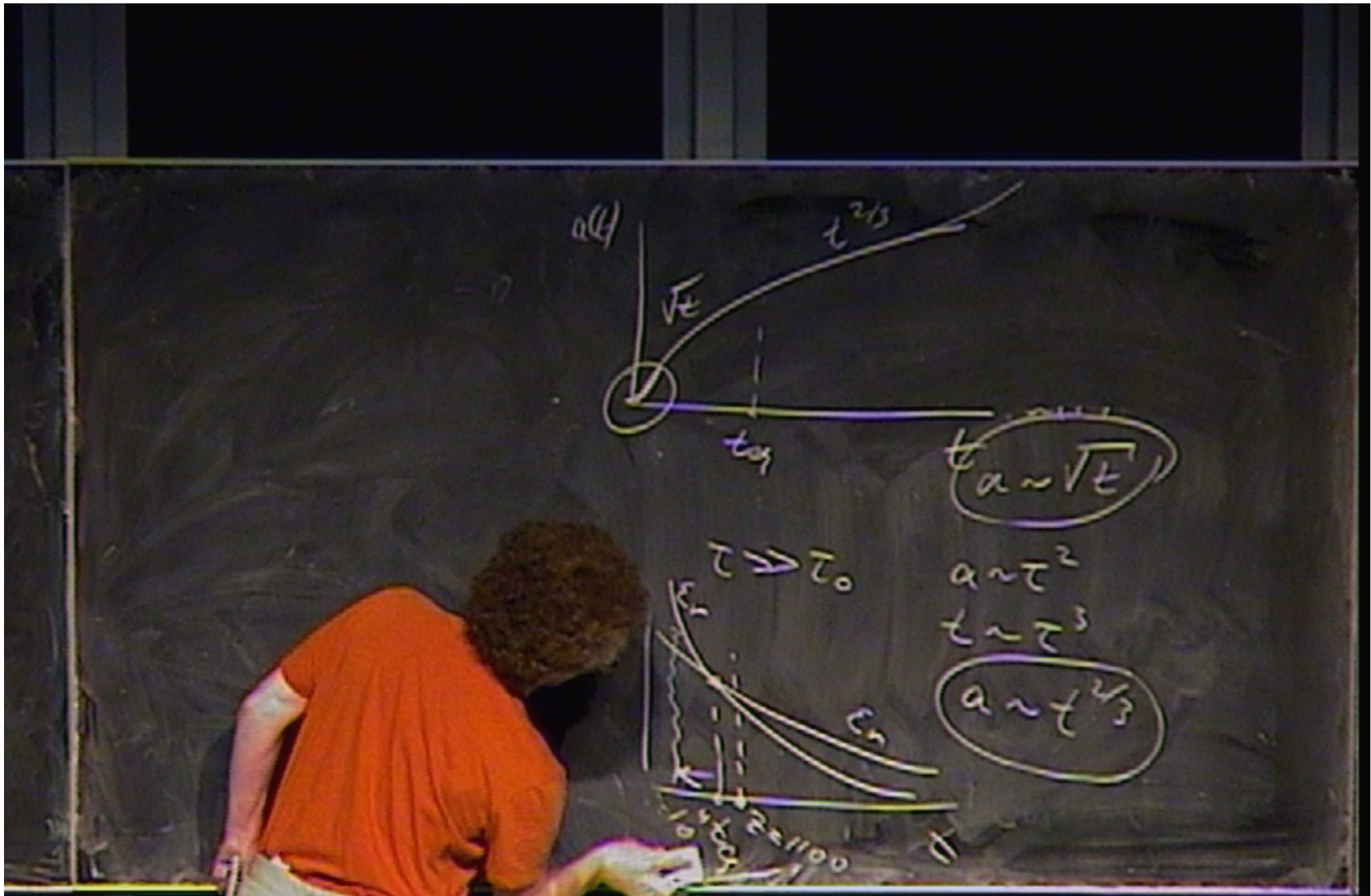
$$p_1 \approx -\epsilon_1$$

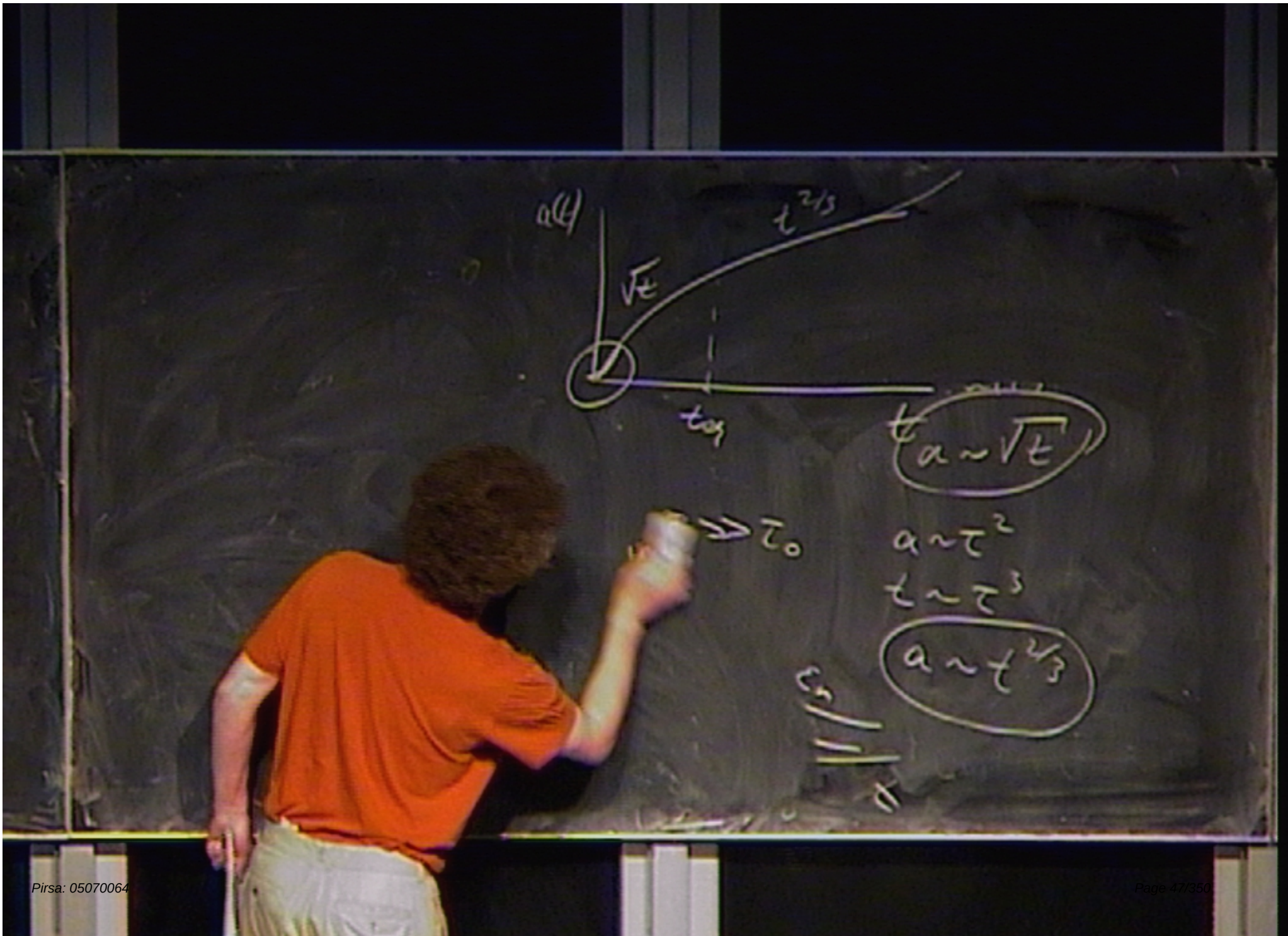
$$a = a_0 e^{Ht}$$

$$-\infty < t < +\infty$$

$$\alpha(\tau) = \frac{1}{H(-\tau)}$$

$$-\infty < \tau < 0$$









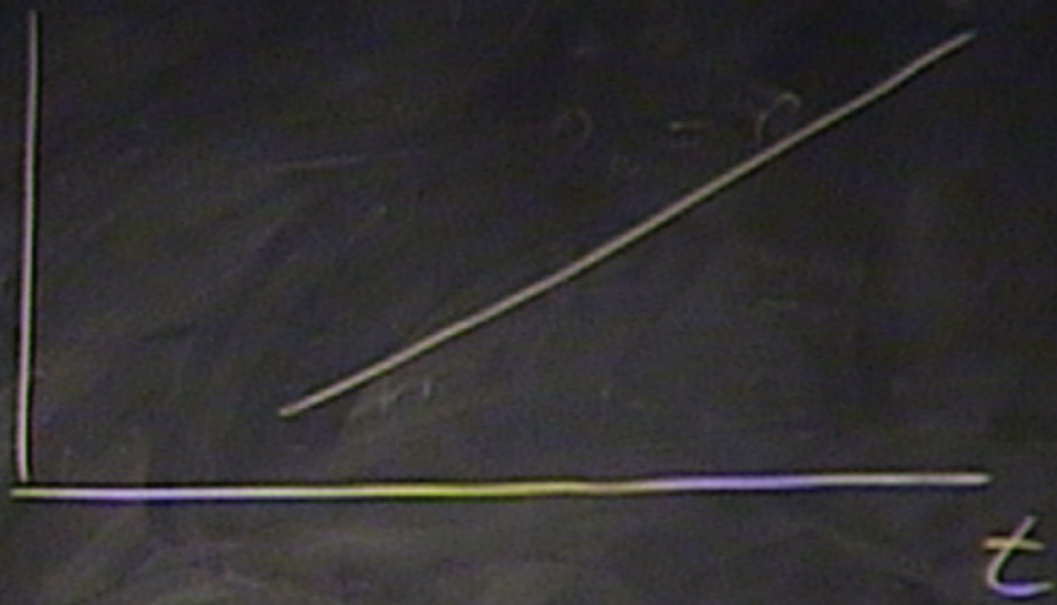
$$\frac{1}{H} \quad a \sim t^2$$

$$H = \frac{\dot{a}}{a} \sim \frac{1}{t}$$



$$\frac{1}{H} \sim t$$
$$a \sim t^2$$

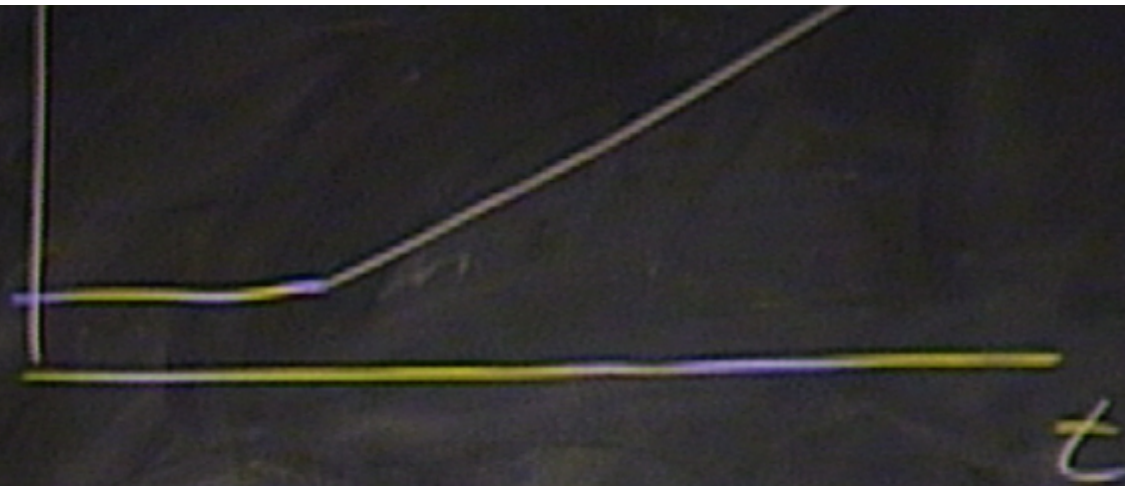
$$H = \frac{\dot{a}}{a} \sim \frac{1}{t}$$



$$\frac{1}{H} \sim t$$

$$a \sim t^2$$

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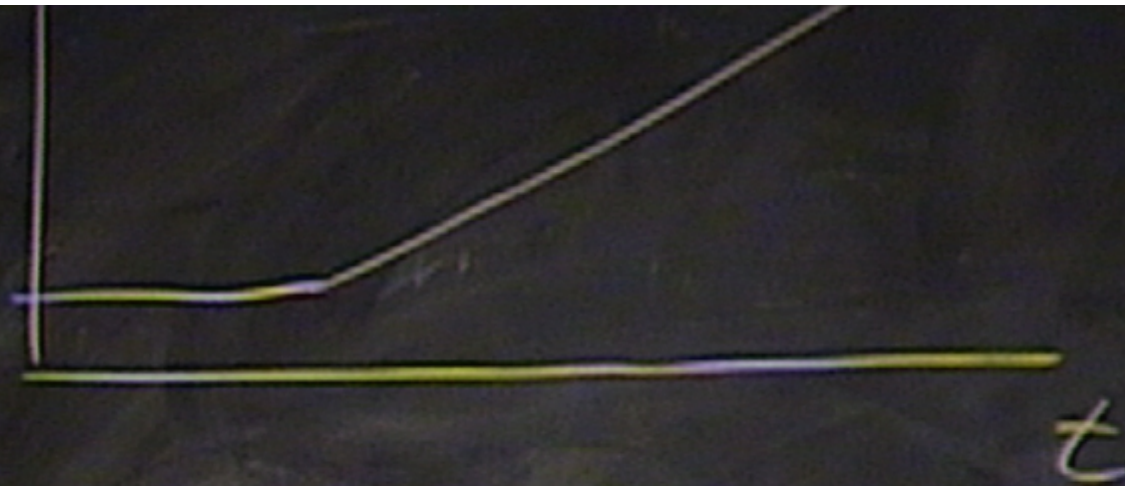


$$\frac{1}{H} \sim t$$

$$a \sim t^2$$

$$H = \frac{\dot{a}}{a} \sim \frac{1}{t}$$

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$$\frac{1}{H} \sim t$$

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$$H = \frac{\dot{a}}{a} \sim \frac{1}{t}$$

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$$\lambda_0 \quad a(t_0)$$

$$-\lambda$$

$$\lambda_0 \quad a(t_0)$$

$$-\lambda = \lambda_0 \quad \frac{a(t)}{a(t_0)}$$

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$$\lambda_0 \quad a(t_0)$$

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$$\lambda_0 \quad a(t_0)$$

$$-\lambda = \lambda_0 \frac{a(t)}{a(t_0)}$$

$$\lambda(t) = \lambda(t + \Delta t)$$

$$\lambda_0 \quad a(t_0)$$

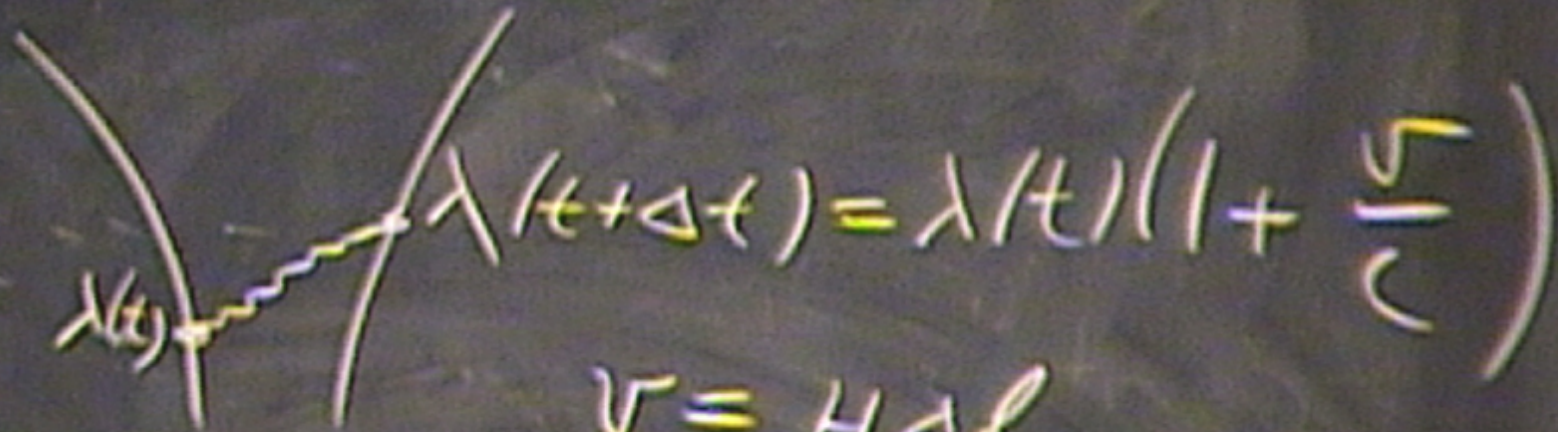
$$-\lambda = \lambda_0 \frac{a(t)}{a(t_0)}$$

$$\lambda(t + \Delta t) = \lambda(t) \left(1 + \frac{v}{c} \right)$$

$$v = H \Delta \ell$$

$$\lambda_0 \quad a(t_0)$$

$$-\lambda = \lambda_0 \frac{a(t)}{a(t_0)}$$



$$\lambda(t+\Delta t) = \lambda(t) \left(1 + \frac{H\Delta t}{c} \right)$$

$$v = H\Delta r$$

$$\lambda_0 \quad a(t_0)$$

$$-\lambda = \lambda_0 \frac{a(t)}{a(t_0)}$$

$$\lambda(t) = \lambda(t) \left(1 + \frac{v}{c} \right) = H \Delta r = H \Delta t = \lambda$$

$$\lambda_0 \quad a(t_0)$$

$$-\lambda = \lambda_0 \frac{a(t)}{a(t_0)}$$



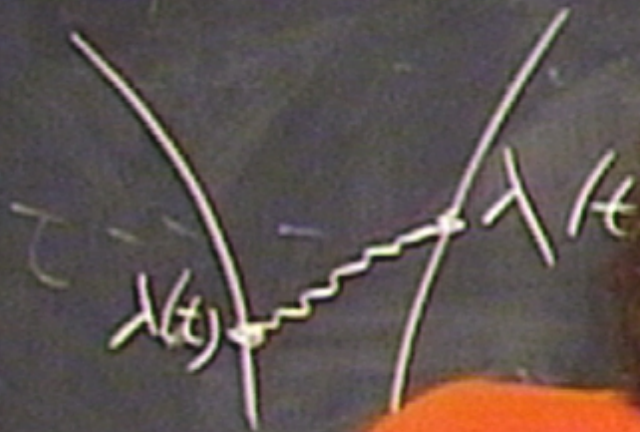
$$\lambda(t) \left(1 + \frac{v}{c}\right) =$$

$$H \Delta r = H \Delta t$$

$$= \lambda(t) \left(1 + \frac{v}{c}\right)$$

$$\lambda_0 \quad a(t_0)$$

$$-\lambda = \lambda_0 \frac{a(t)}{a(t_0)}$$



$$\lambda(t) \left(1 + \frac{v}{c} \right) =$$

$$\Delta \ell = H \Delta t$$

$$\lambda(t) \left(1 + \frac{v}{c} \right) =$$

$$\lambda_0 \quad a(t_0)$$

$$-\lambda = \lambda_0 \frac{a(t)}{a(t_0)}$$



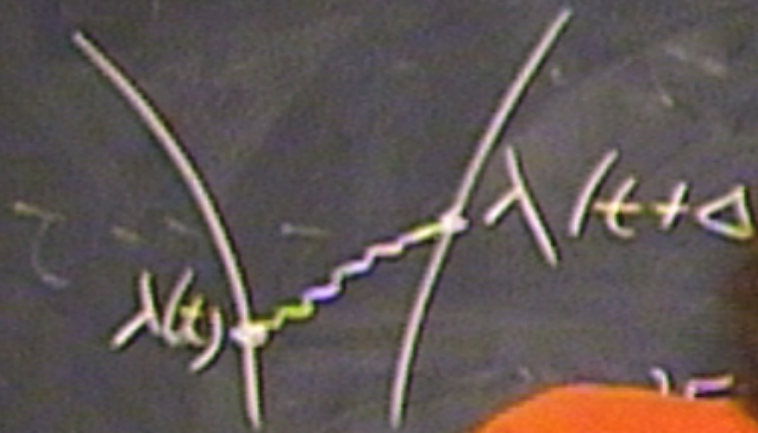
$$\left(1 + \frac{\delta}{\epsilon}\right) =$$

$$= H \Delta t$$

$$(t) (1 + H$$

$$\lambda_0 \quad a(t_0)$$

$$-\lambda = \lambda_0 \frac{a(t)}{a(t_0)}$$



$$\left(1 + \frac{\Delta \lambda}{\lambda}\right) =$$

$$= H \Delta t$$

$$\lambda(t) \left(1 + H \Delta t\right)$$

$$\lambda_0 \quad a(t_0)$$

$$-\lambda = \lambda_0 \frac{a(t)}{a(t_0)}$$

~~$$\lambda(t + \Delta t) = \lambda(t) \left(1 + \frac{v}{c}\right)$$~~

$$v = H \Delta r = H \Delta t$$

$$= \lambda(t) (1 + H \Delta t)$$

$$\lambda_0 \quad a(t_0)$$

$$-\lambda = \lambda_0 \frac{a(t)}{a(t_0)}$$

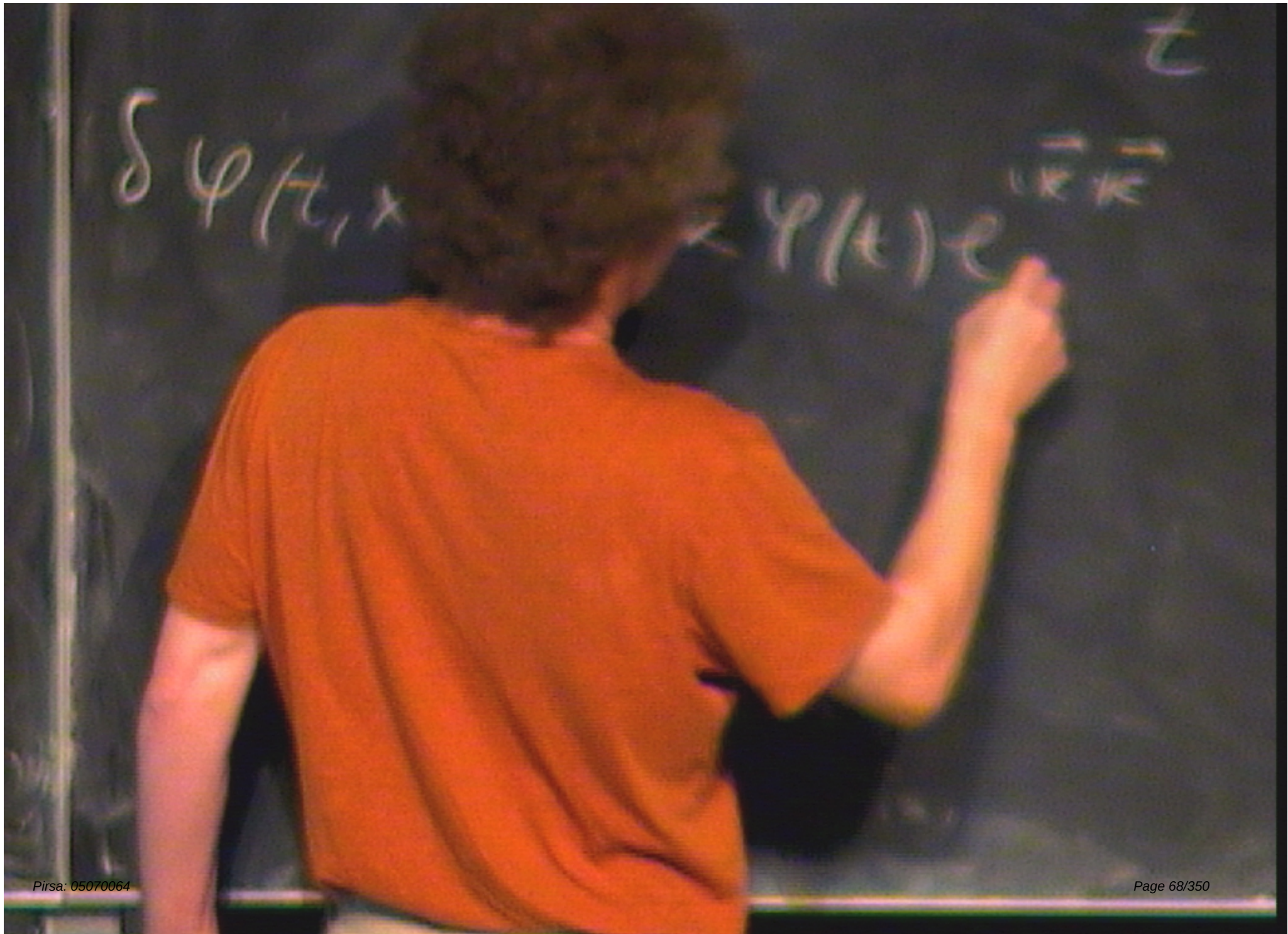
$$\lambda(t + \Delta t) = \lambda(t) \left(1 + \frac{v}{c} \right) =$$

$$v = H \Delta r = H \Delta t$$

$$\lambda(t) + \dot{\lambda} \Delta t = \lambda(t) \left(1 + H \Delta t \right)$$

$$= \lambda(t) \left(1 + \frac{\dot{\lambda}}{\lambda} \Delta t \right)$$

$$\frac{\dot{\lambda}}{\lambda} = \frac{\dot{a}}{a}$$



t

$$\delta \varphi(t, x) = \int d^3 k \varphi(t) e^{i \vec{k} \cdot \vec{x}}$$

t

$$\delta \varphi(t, x) = \int d^3 k \varphi(t) e^{i \vec{k} \cdot \vec{x}}$$

$$\psi(t, \mathbf{x}) = \int d^3k \, \varphi(k) e^{i\vec{k} \cdot \vec{x}}$$

$$\lambda = \frac{2\pi}{k} a(t)$$

t

$$\delta \varphi(t, x) = \int d^3 k \varphi(t) e^{i \vec{k} \cdot \vec{x}}$$

$$\lambda = \frac{2\pi}{k} a(t)$$

t

$$\delta \varphi(t, x) = \int d^3 k \varphi(t) e^{i \vec{k} \cdot \vec{x}}$$

$$\lambda = \frac{2\pi}{k} a(t)$$

t

$$\delta \psi(t, x) = \int d^3 k \psi(t) e^{i \vec{k} \cdot \vec{x}}$$

$$\lambda = \frac{2\pi}{k} a(t)$$

$$a \sim t^{1/2}$$

$$t^{2/3}$$

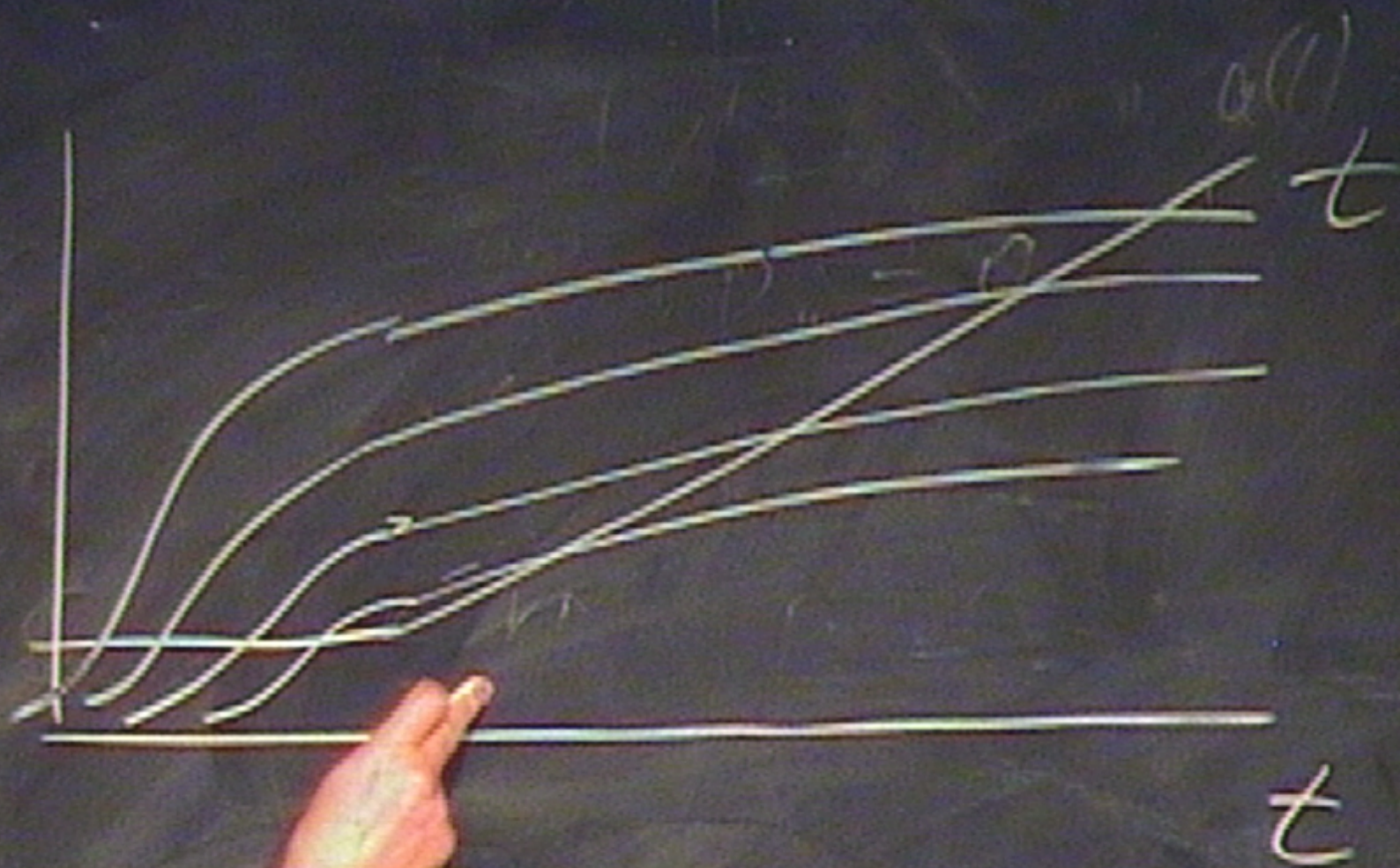
t

$$\delta \varphi(t, x) = \int d^3 k \varphi(t) e^{i \vec{k} \cdot \vec{x}}$$

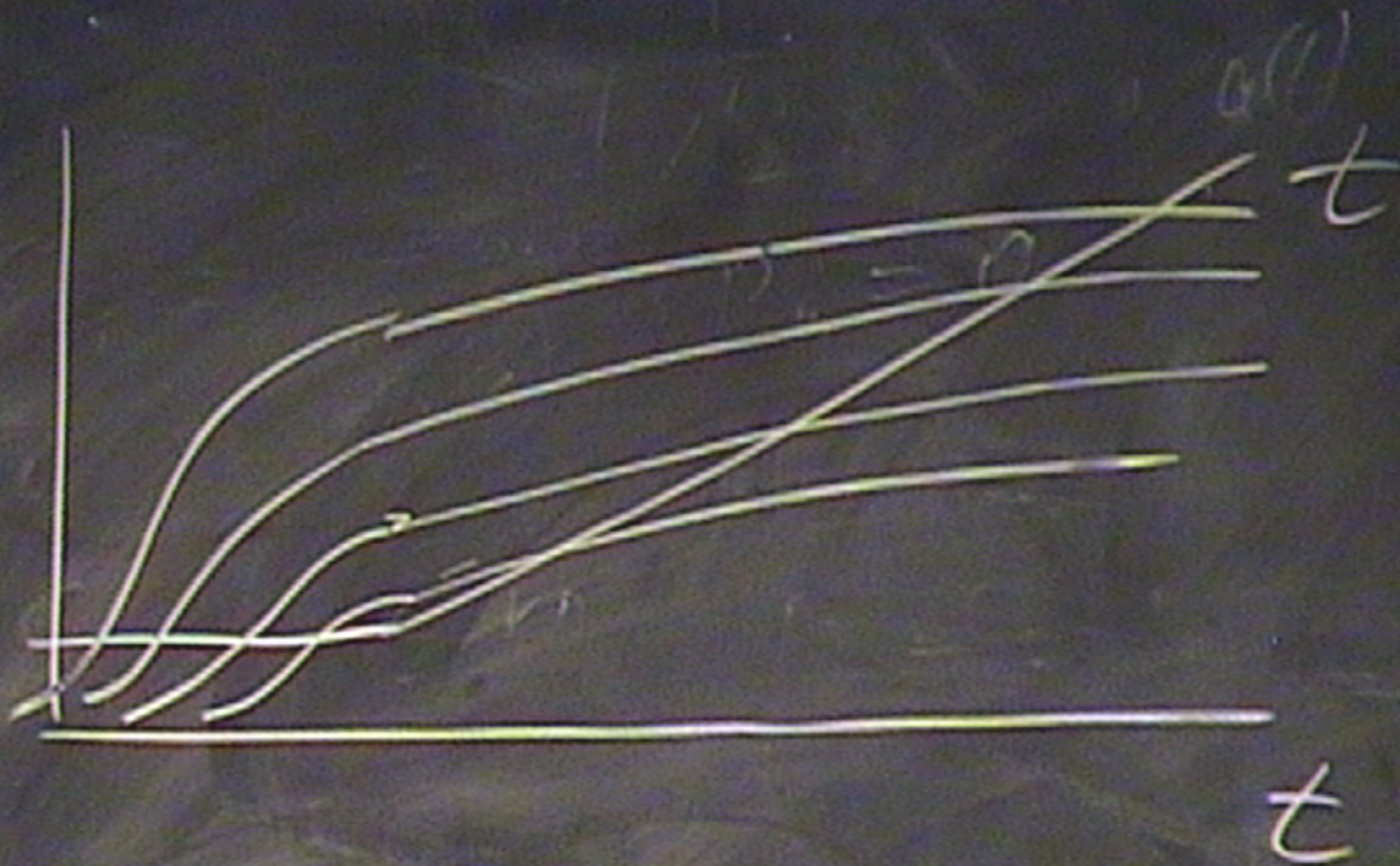
$$\lambda = \frac{2\pi}{k} a(t)$$

$$a \sim t^{1/2}$$

$$t^{2/3}$$



$$\psi(t) = \int d^3k \psi(t) e^{i\vec{k} \cdot \vec{k}}$$



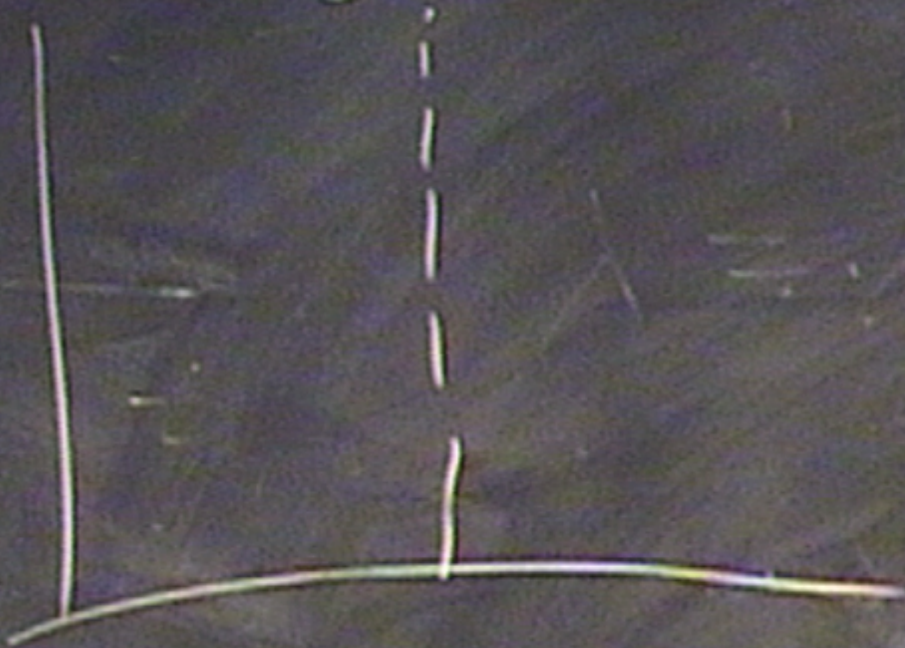
$$\delta \varphi(t, x) = \int d^3 k \varphi(k) e^{i \vec{k} \cdot \vec{x}}$$

3/3
Particle horizon

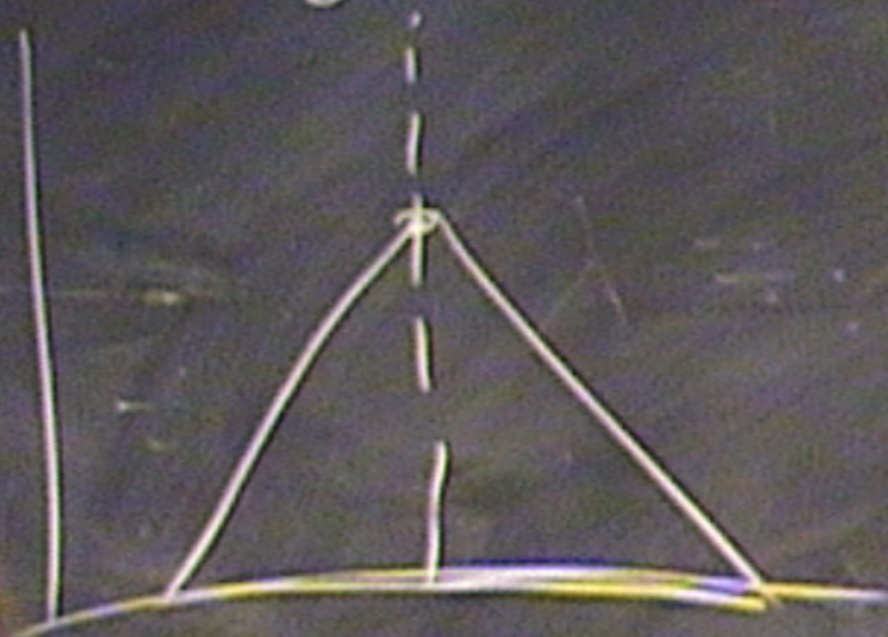
Particle horizon

Particle horizon

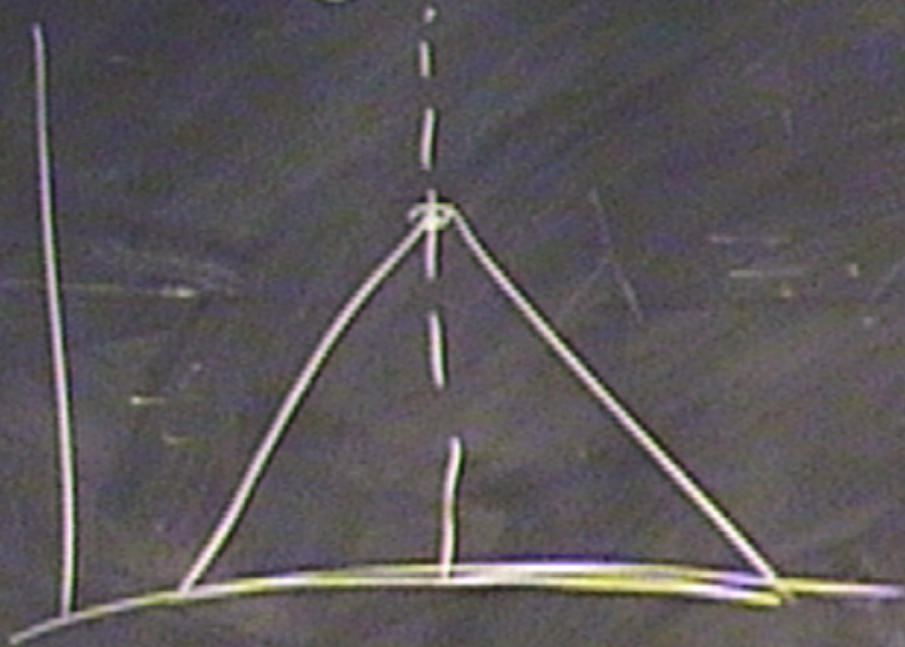
t
 t 0 Particle horizon



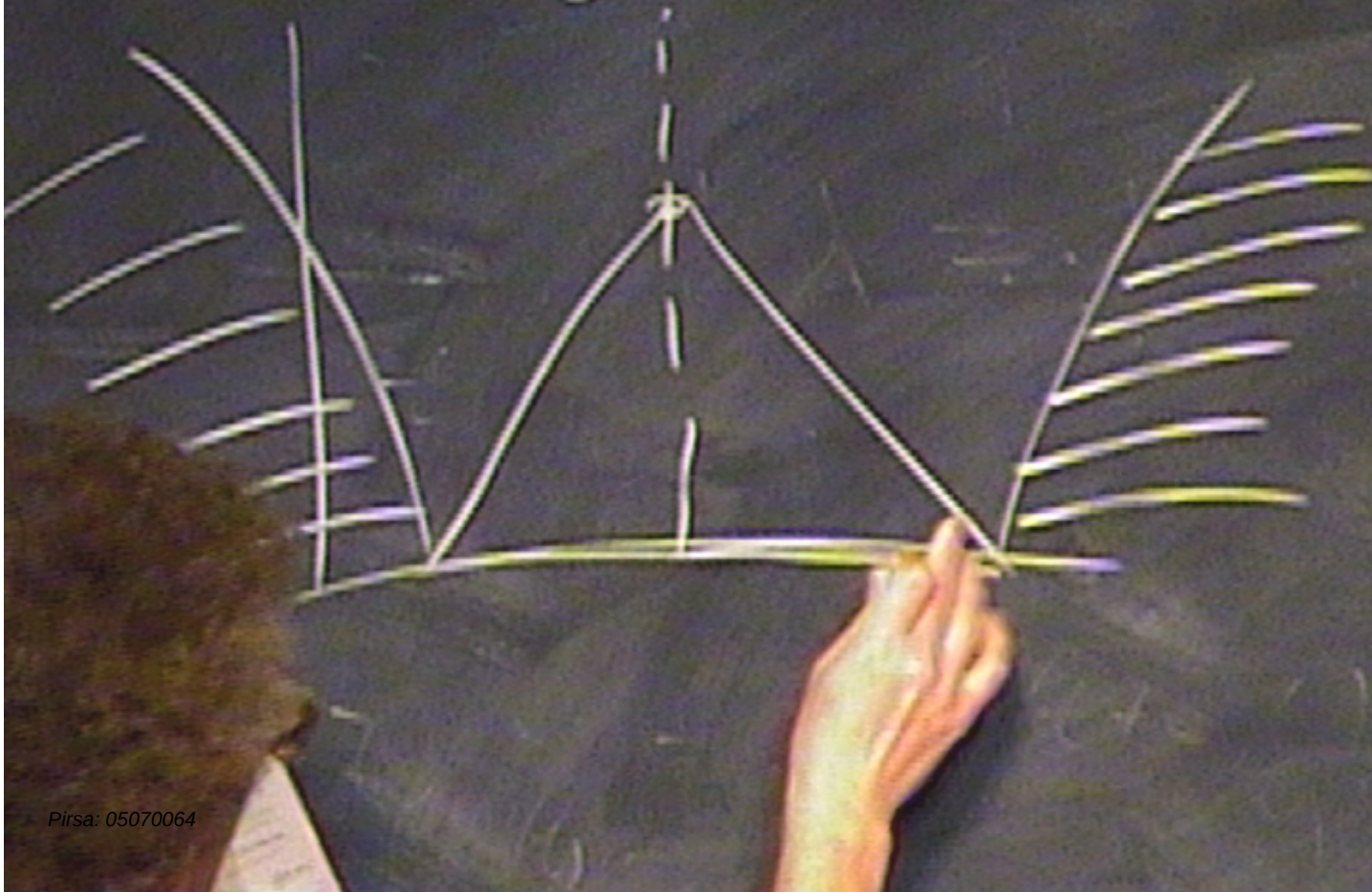
t 0 Particle horizon

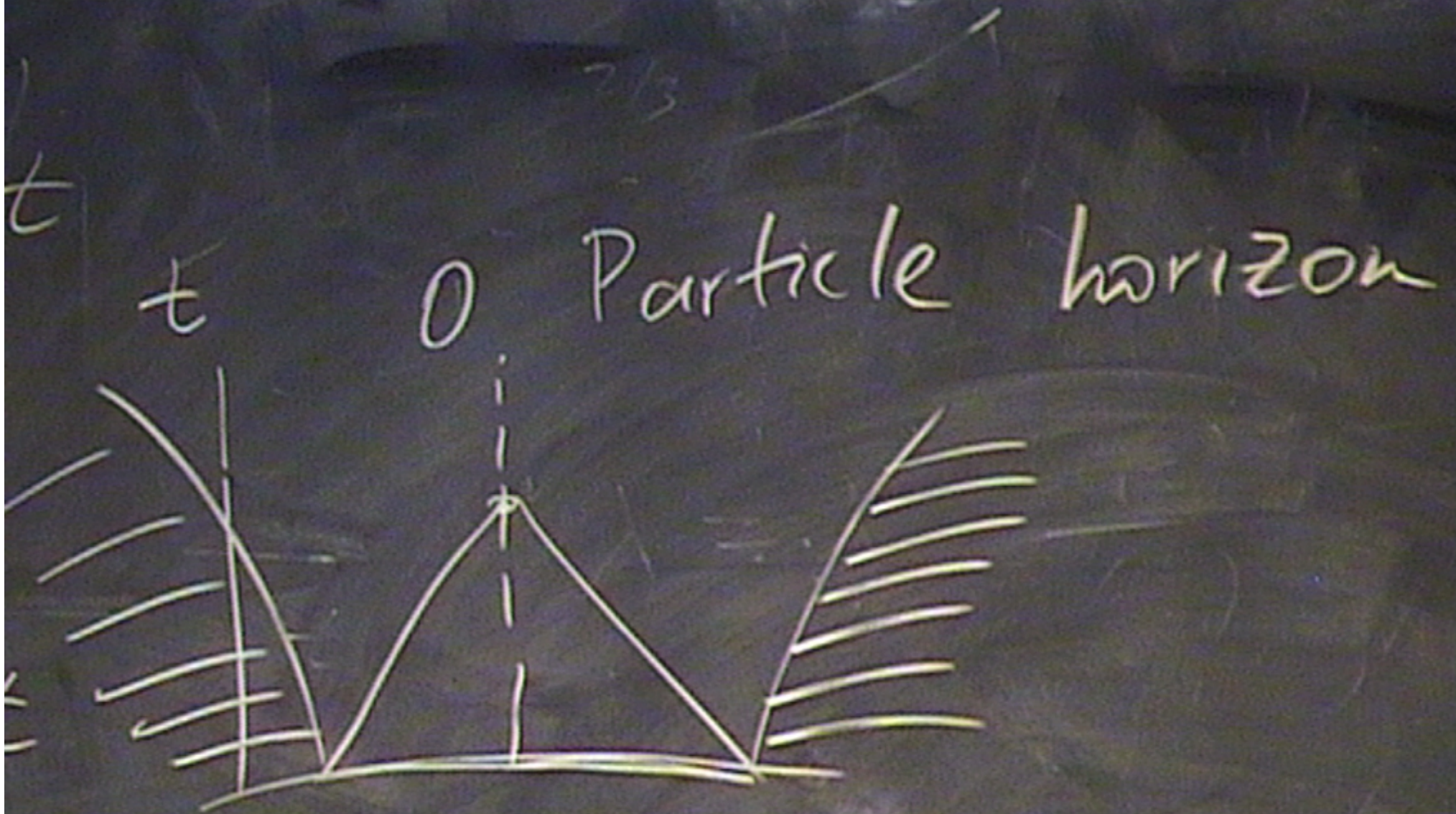


t 0 Particle horizon

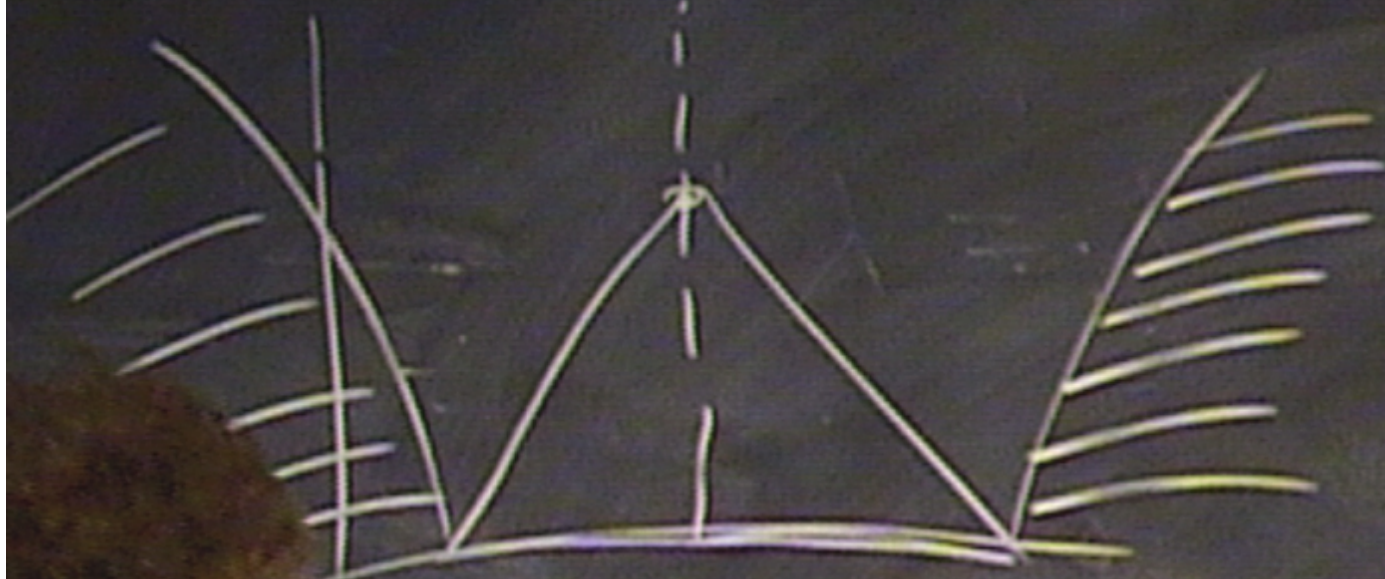


t
 t 0 Particle horizon





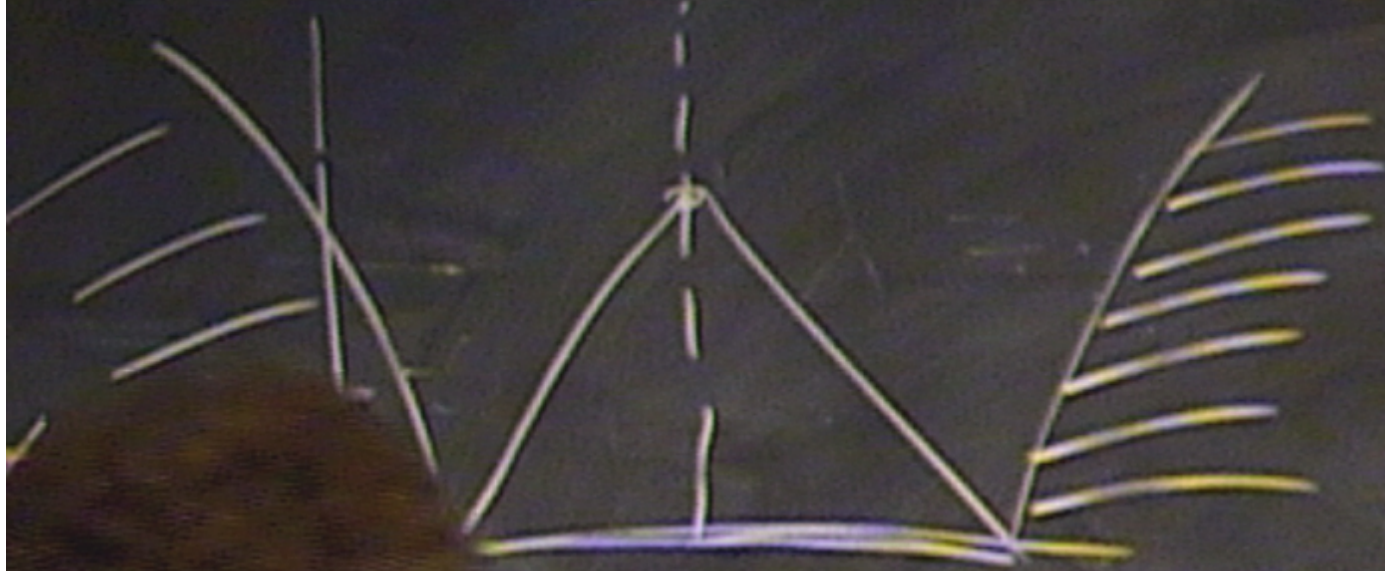
τ 0 parameter world



$$ds^2 = a^2(\tau)(d\tau^2 - d\vec{x}^2) = 0$$

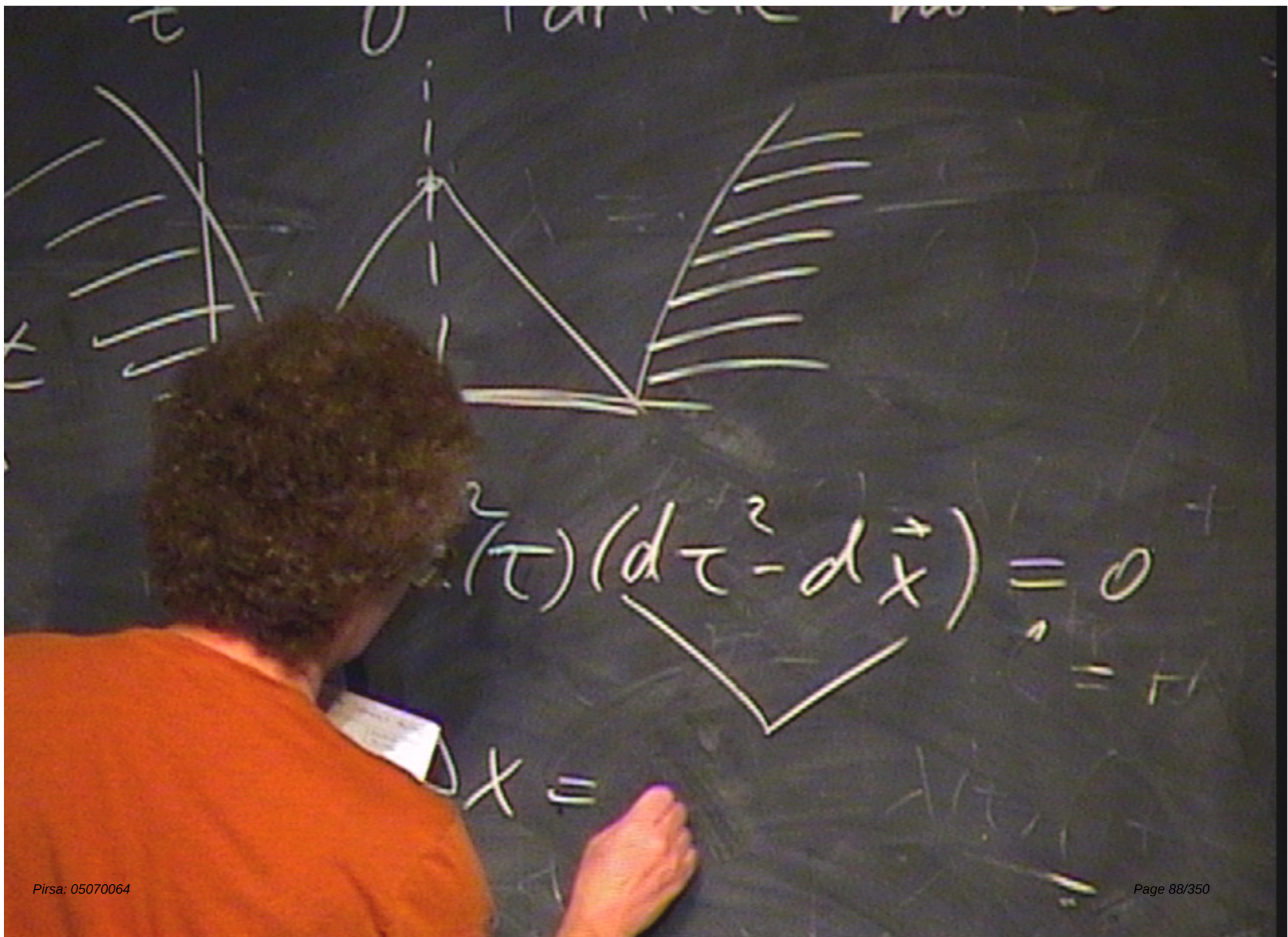
Δx

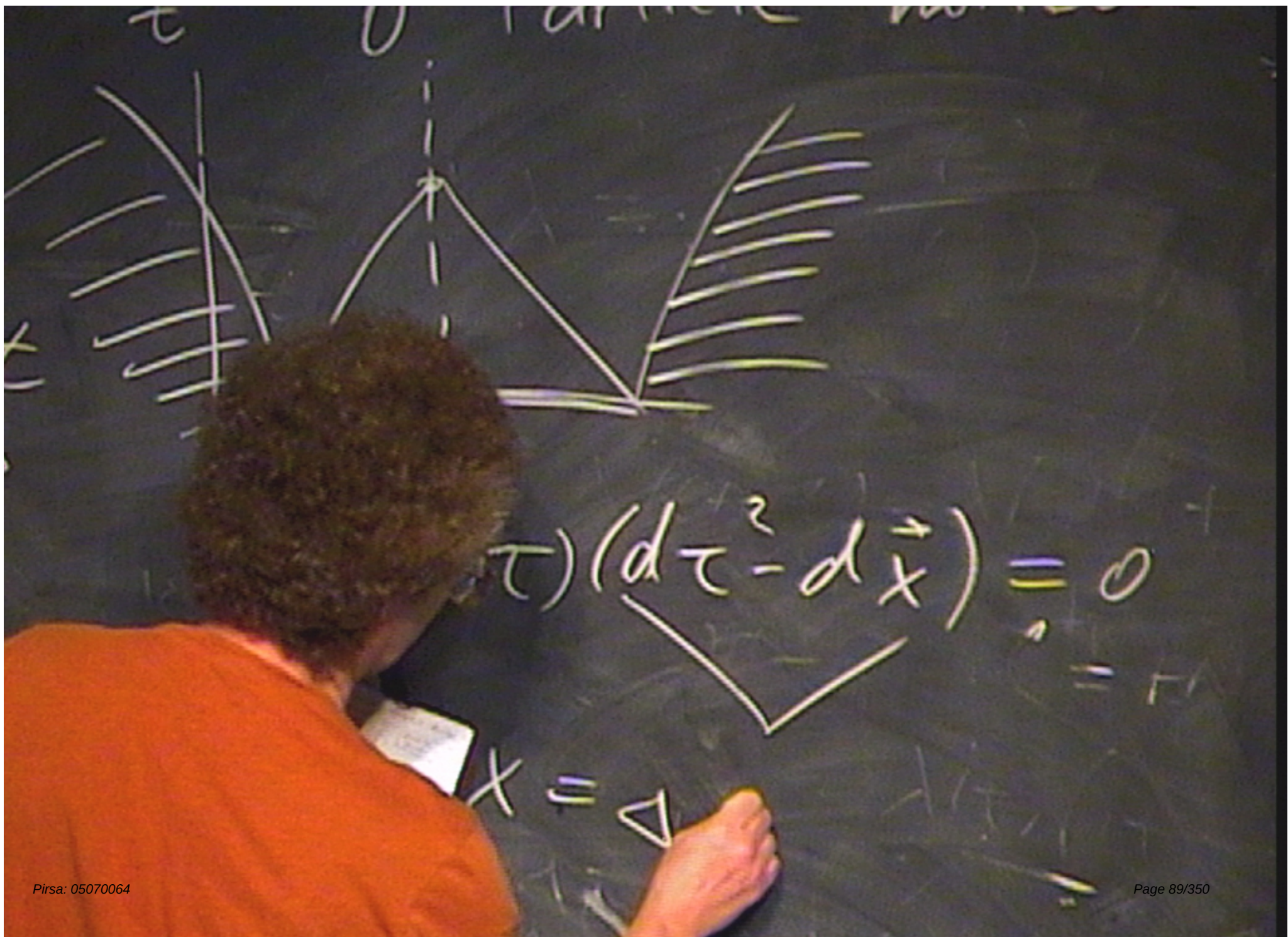
τ 0 parameter world



$$ds^2 = a^2(\tau)(d\tau^2 - d\vec{x}^2) = 0$$

$d\vec{x}$





$$ds^2 = a^2(\tau)(d\tau^2 - d\vec{x}^2) = 0$$

$$\Delta x = \Delta \tau$$

$t = 0$ Particle horizon



$$ds^2 = a^2$$

$$d\vec{x} = 0$$

$$t = \frac{t_i}{a(t_i)}$$

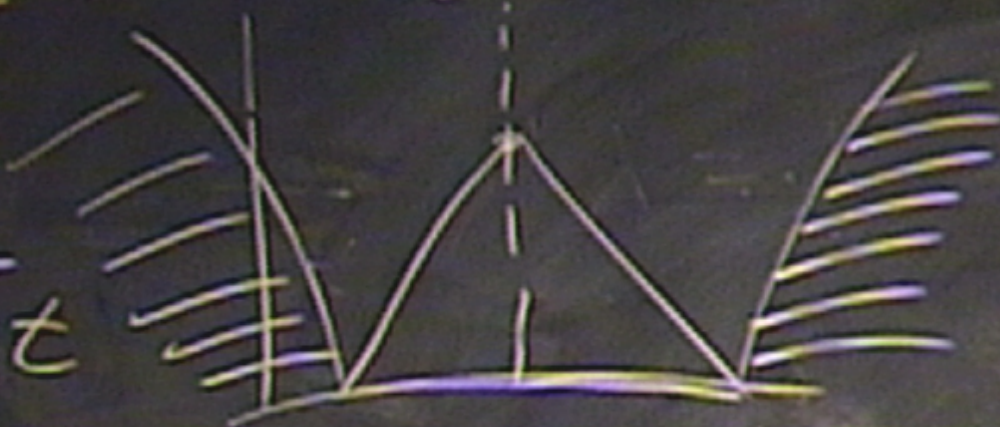
t 0 Particle horizon



$$ds^2 = a^2(\tau)(d\tau^2 - d\vec{x}^2) = 0$$

$$\Delta x = \Delta \tau = \int_0^t \frac{d\tau}{a(\tau)}$$

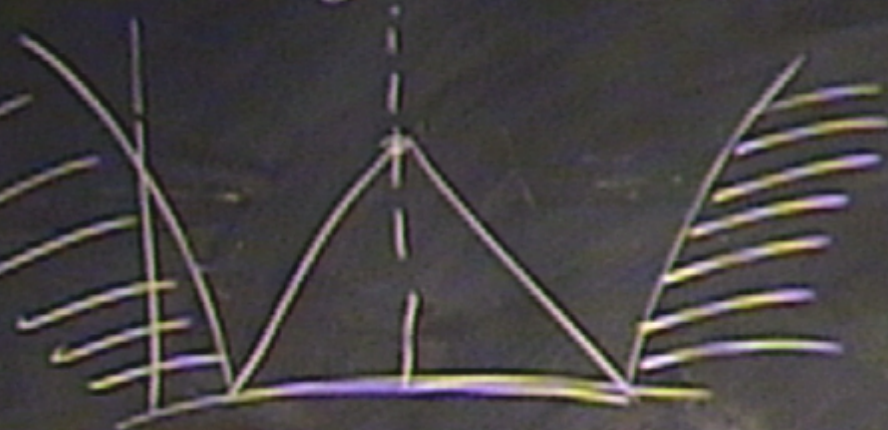
$t = 0$ Particle horizon



$$ds^2 = a^2(\tau)(d\tau^2 - d\vec{x}^2) = 0$$

$$\Delta x = \Delta \tau = \int_0^t \frac{dt}{a(t)}$$

$t = 0$ Particle horizon



$$a^2(\tau)(d\tau^2 - d\vec{x}^2) = 0$$

$$\Delta x = \Delta \tau = \int_0^t \frac{dt}{a(t)}$$

$$r = 0$$

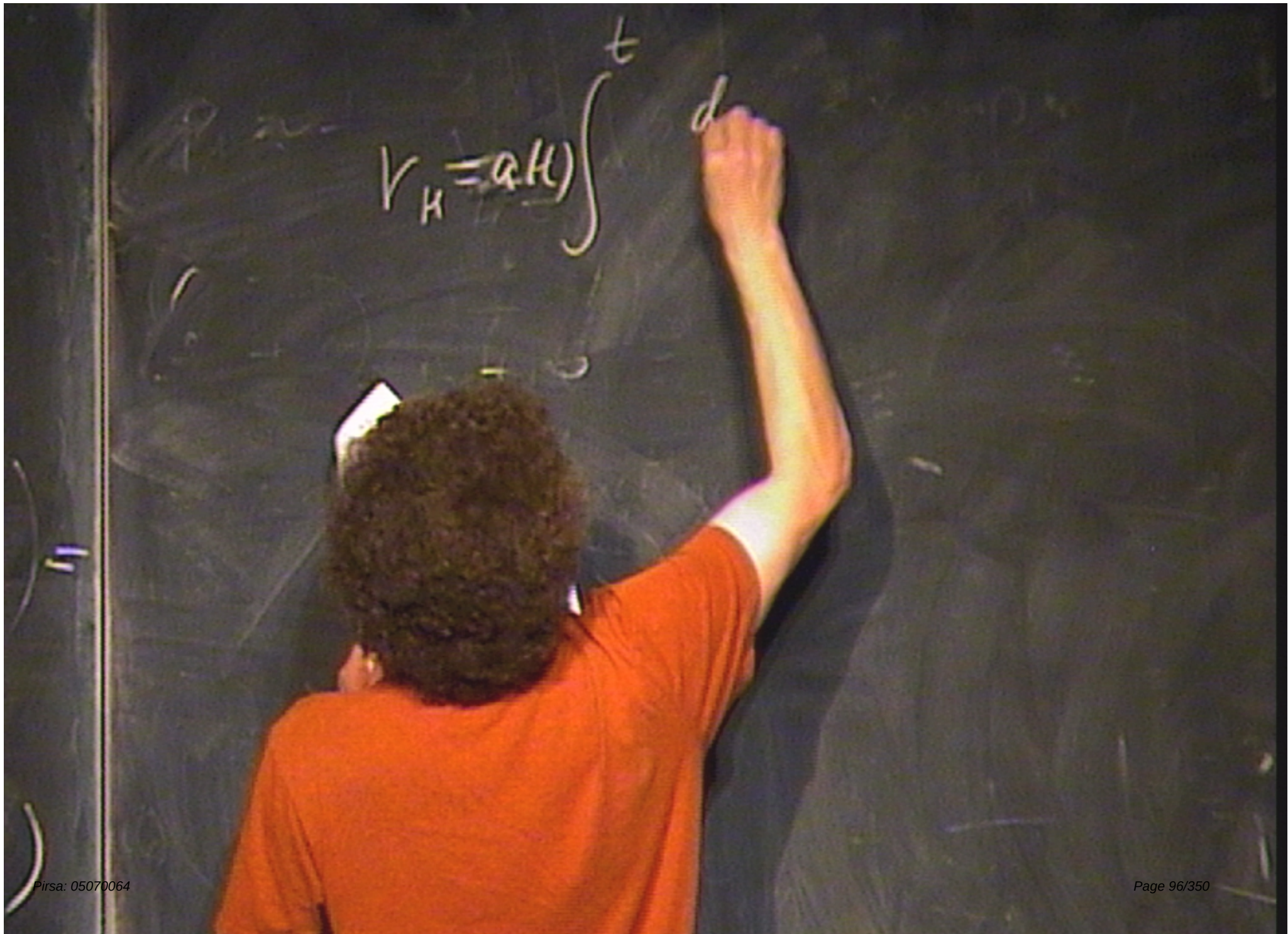
t 0 Particle horizon



$$ds^2 = a^2(\tau)(d\tau^2 - d\vec{x}^2) = 0$$

$$\Delta x = \Delta \tau = \int_0^t \frac{dt}{a(t)}$$

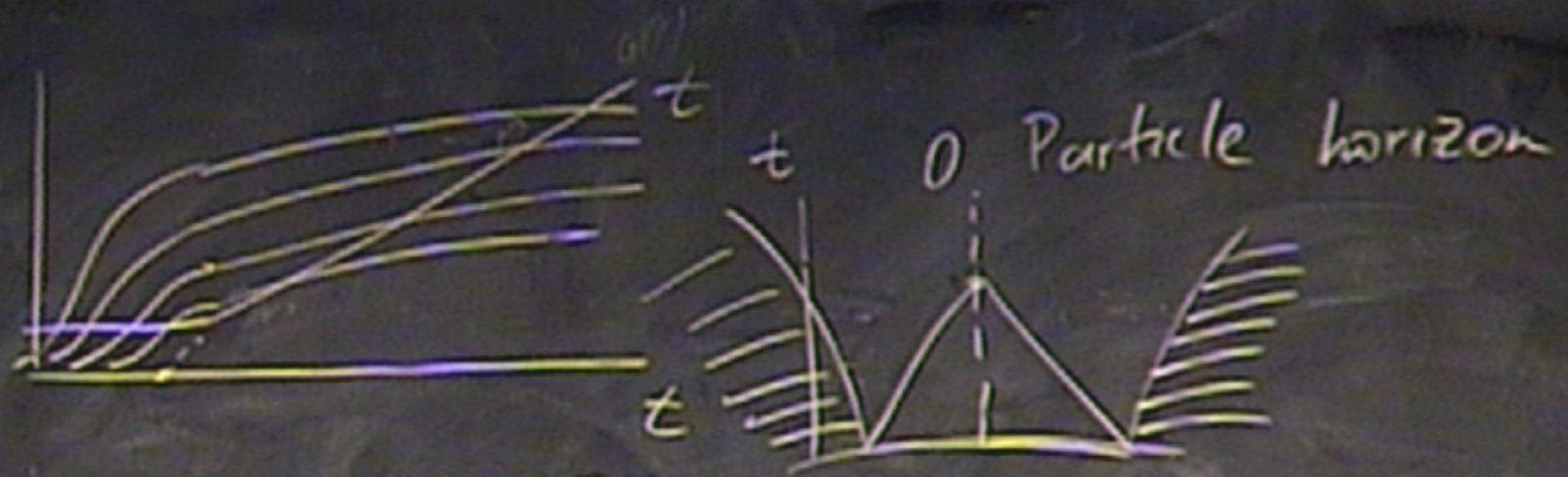
$$\Delta r = a(t) \Delta x$$



$$V_H = aH) \int^t d$$

$$r_H = a(t) \int_0^t \frac{dt'}{a(t')}$$

$$V_H = a(t) \int_0^t \frac{dt'}{a(t')}$$



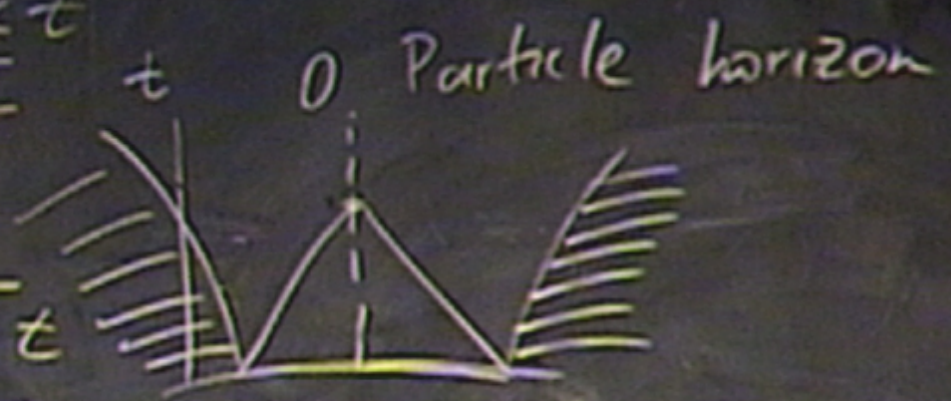
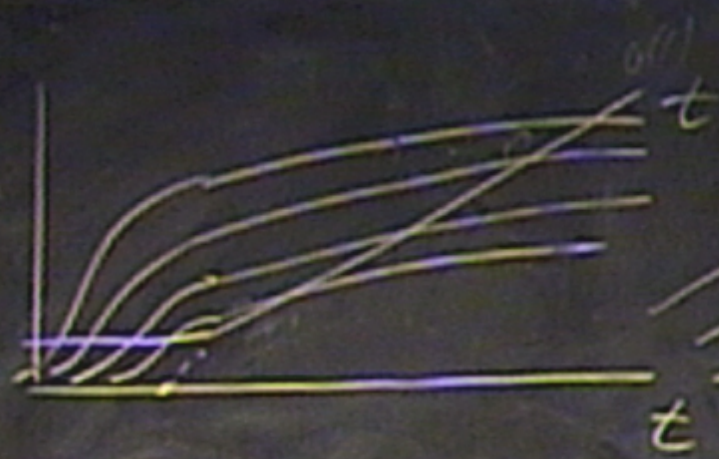
$$\delta \varphi(t, x) = \int d^3k \varphi(k)$$

$$\lambda = \frac{2\pi}{k} a(t)$$

$$ds^2 = a^2(t)(d\tau^2 - d\vec{x}^2) = 0$$

$$\Delta x = \Delta \tau = \int \frac{d\tau}{a(t)}$$

$$\Delta r = a(t) \Delta x$$



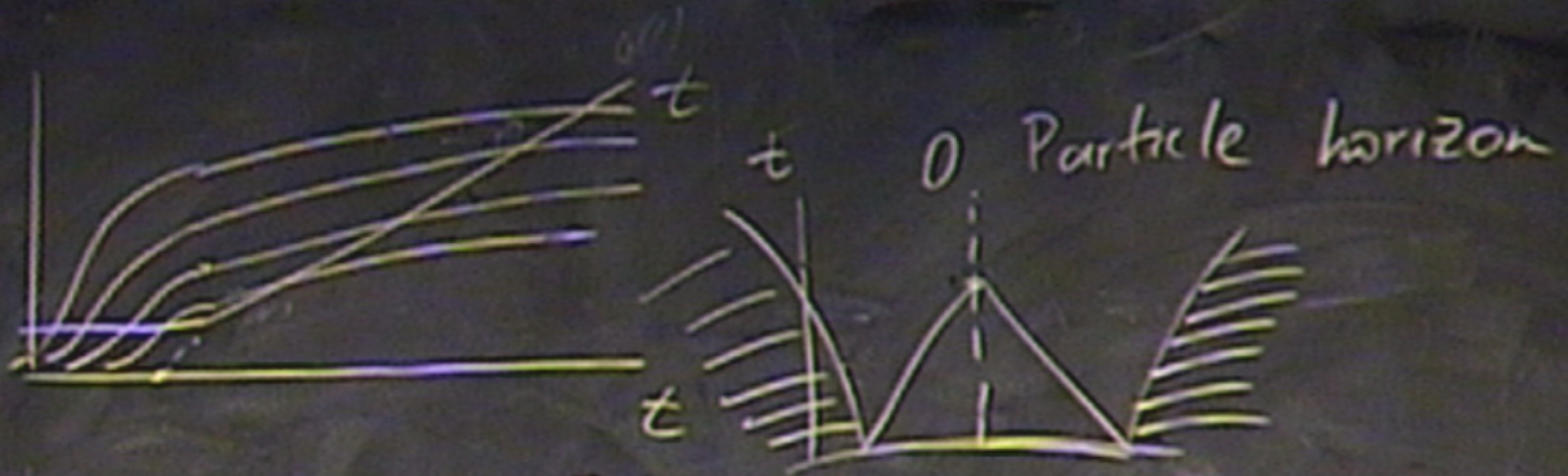
$$\delta \varphi(t) = \int d^3k \varphi(t) e^{i \vec{k} \cdot \vec{x}}$$

$a(t)$
 $t^{1/2}$
 $t^{2/3}$
 x

$$ds^2 = a^2(t) (d\tau^2 - d\vec{x}^2) = 0$$

$$\Delta x = \Delta \tau = \int \frac{d\tau}{a(t)}$$

$$\Delta r = a(t) \Delta x$$



$$\psi(\vec{x}) = \int d^3k \psi(k) e^{i\vec{x} \cdot \vec{k}}$$

$$= \frac{2\pi}{k} a(t) \left[\begin{array}{l} a \sim t^{1/2} \\ a \sim t^{2/3} \\ a \sim e^{Ht} \end{array} \right]$$

$$ds^2 = a^2(t) (d\tau^2 - d\vec{x}^2) = 0$$

$$\Delta x = \Delta \tau = \int \frac{d\tau}{a(t)}$$

$$\Delta r = a(t) \Delta x$$

$$r_H = a(t) \int_0^t \frac{dt'}{a(t')} = 1,456 \text{ Mpc}$$

$$\left| \int_0^t \frac{dr_H}{a(t')} \right| = 1,456 \text{ Mpc}$$

$$\left| \int_0^t \frac{dr_H}{a(t')} \right| = 1,456 \text{ Mpc}$$

$$\boxed{r_H = a(t) \int_0^t \frac{dt'}{a(t')} = 1,456 \text{ Mpc}}$$

$$1 \text{ Mpc} = 3.09 \times 10^{24} \text{ cm}$$

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$$1 \text{ Mpc} = 3.09 \times 10^{24} \text{ cm}$$

v_H

$$\left| \int_0^t \frac{dt'}{a(t')} \right| = 1,456 \text{ Mpc}$$

$$1 \text{ Mpc} = 3.09 \times 10^{24} \text{ cm}$$

v_H

$$\left| \frac{r_H = a(t) \int_0^t \frac{dt'}{a(t')}}{a(t)} \right| = 1,456 \text{ Mpc}$$

$$1 \text{ Mpc} = 3.09 \times 10^{24} \text{ cm}$$

$$r_H = a(t) \int_0^t \frac{dt'}{a(t')} = 1,456 \text{ Mpc}$$

$$1 \text{ Mpc} = 3.09 \times 10^{24} \text{ cm}$$

$$\approx \frac{1}{H}$$

$$\left| \int_0^t \frac{dr}{a(t')} \right| = 1,456 \text{ Mpc}$$

$$1 \text{ Mpc} = 3.09 \times 10^{24} \text{ cm}$$

$$\frac{1}{H}$$

$$\left| \int_0^t \frac{dt'}{a(t')} \right| = 1,456 \text{ Mpc}$$

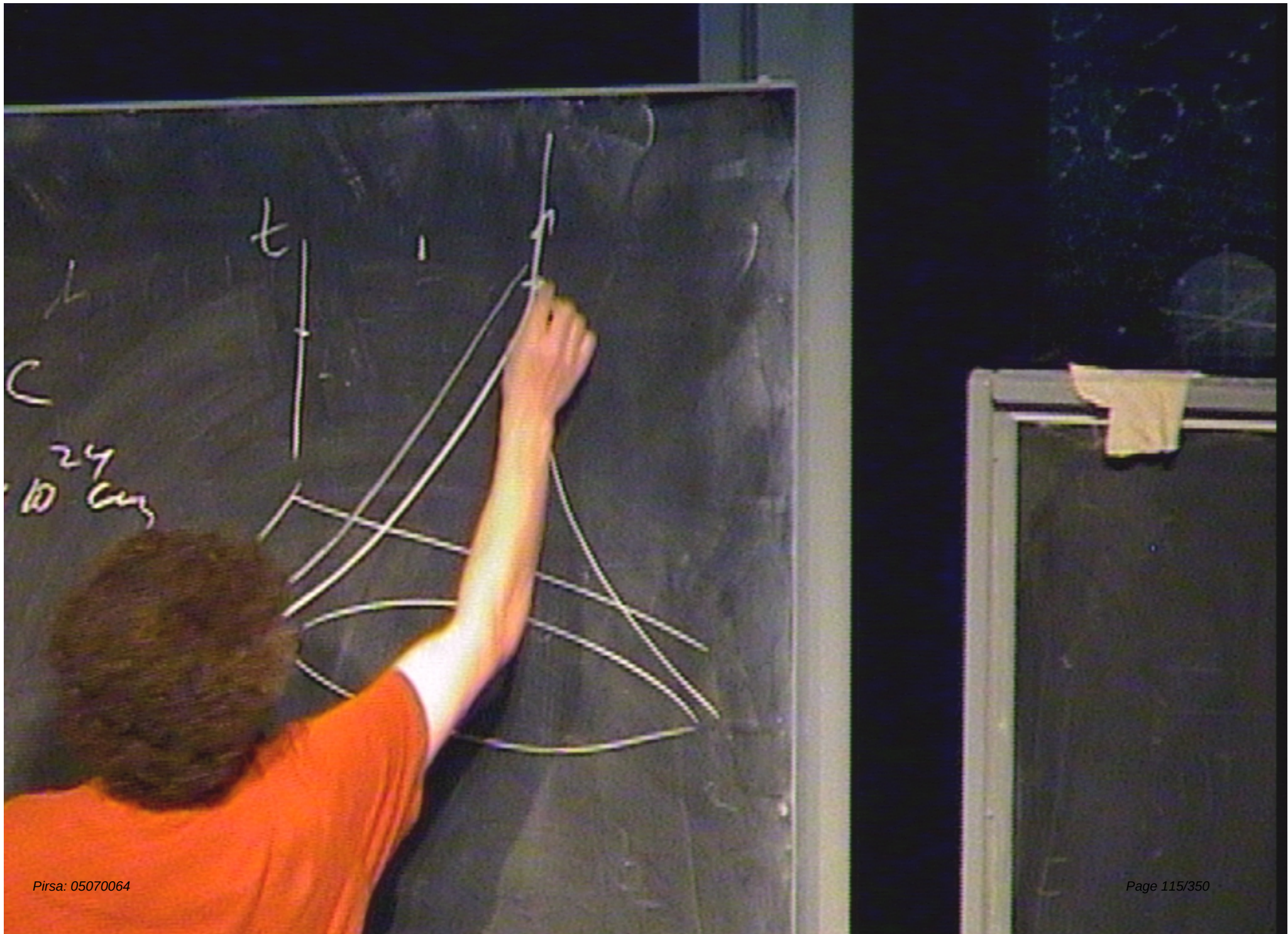
$$1 \text{ Mpc} = 3.09 \times 10^{24} \text{ cm}$$

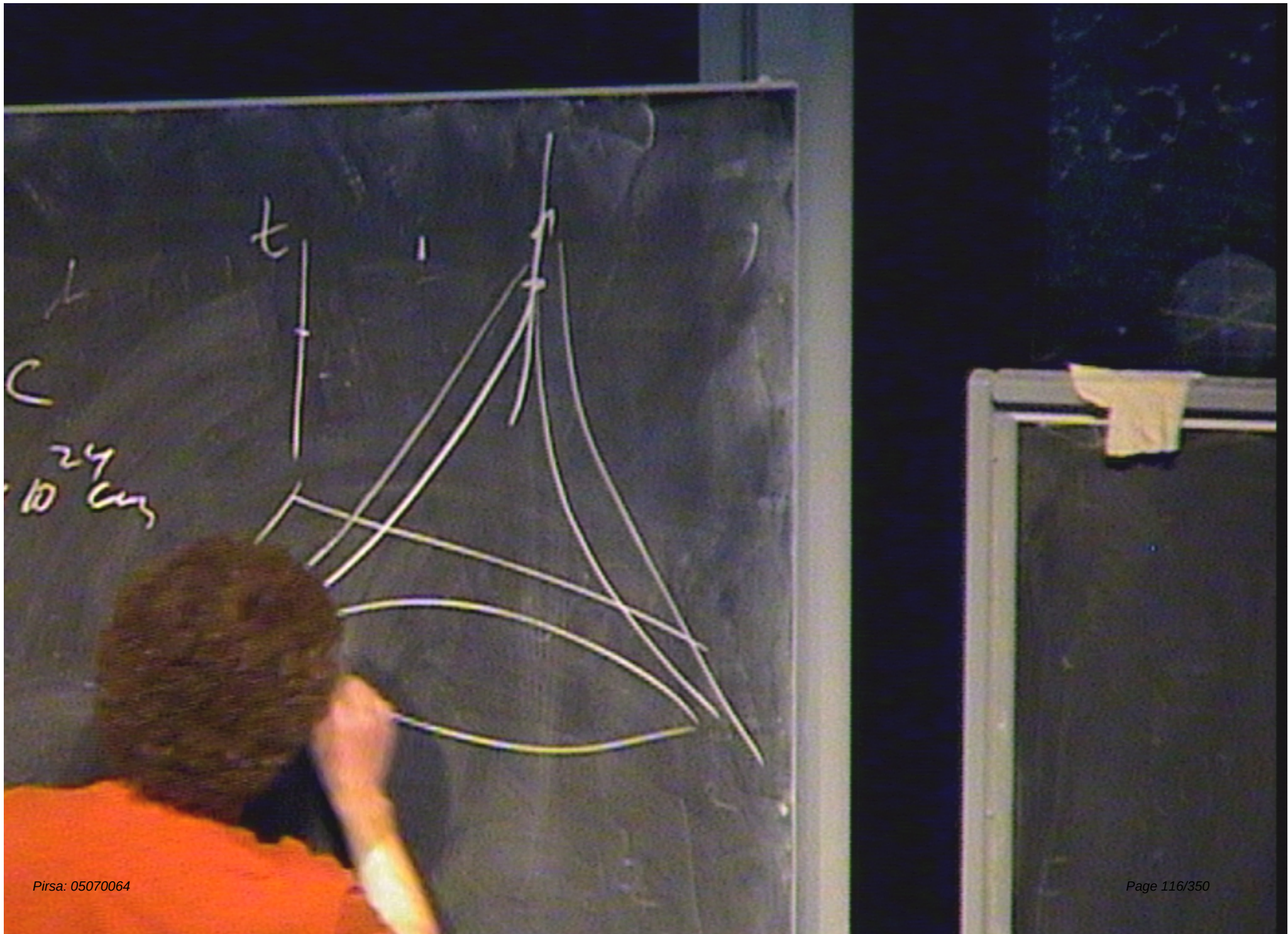
$$\approx \frac{1}{H} (e^{Ht} - 1)$$

$$\boxed{r_H = a(t) \int_0^t \frac{dt'}{a(t')}} = 1,456 \text{ Mpc}$$

$$1 \text{ Mpc} = 3.09 \times 10^{24} \text{ cm}$$

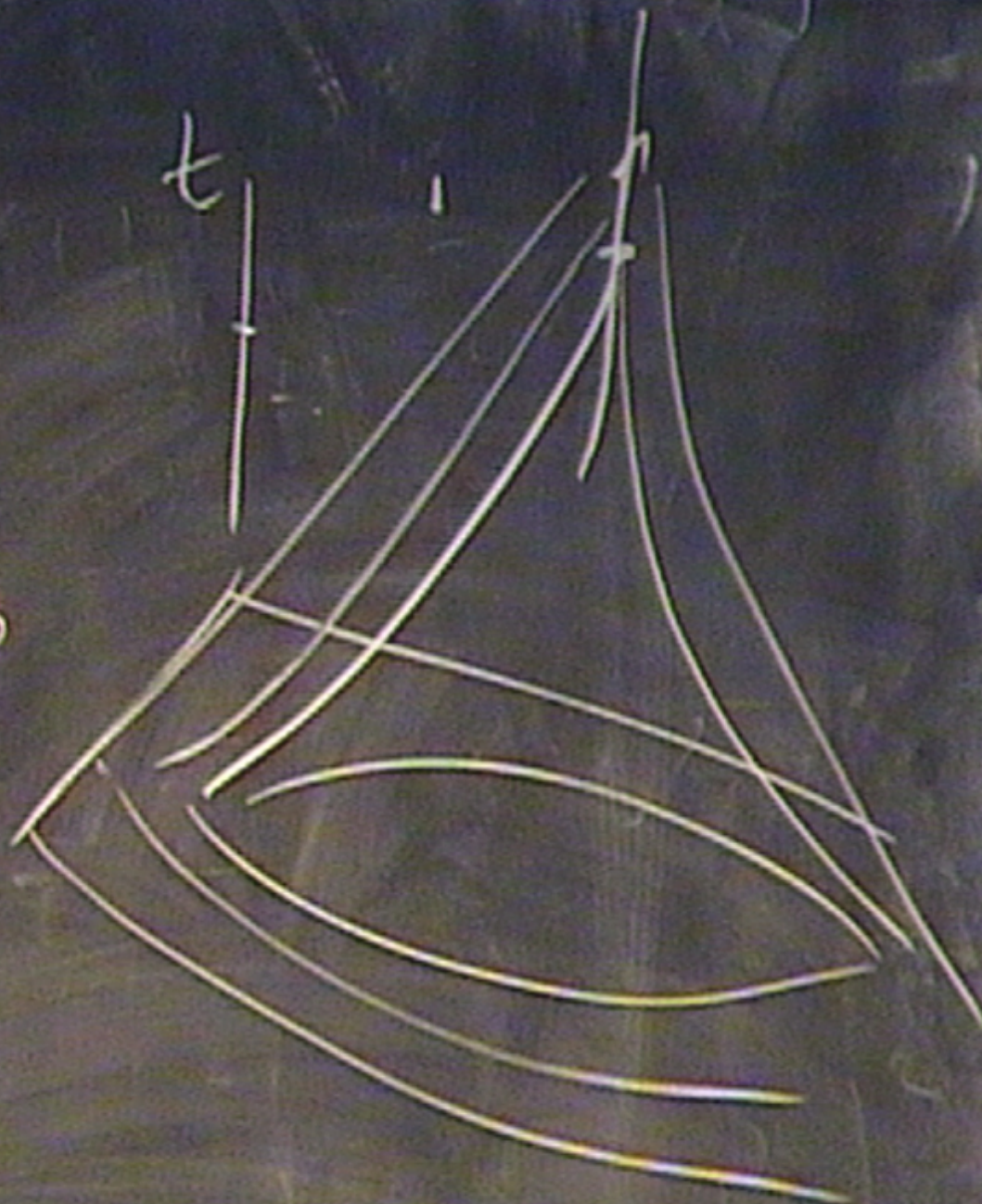
$$r_H \approx \frac{1}{H} (e^{Ht} - 1)$$





h₀ t
MPC

$9 \times 10^{24} \text{ cm}$

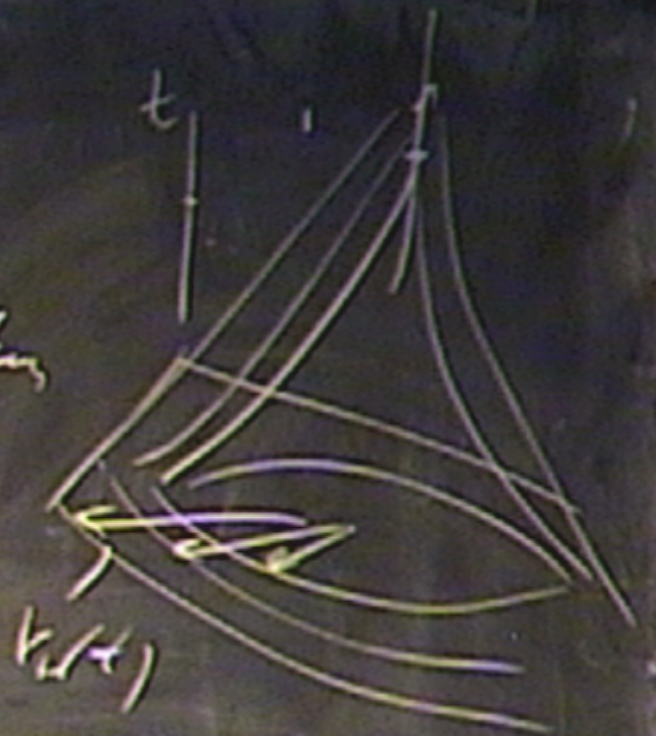


$$\boxed{v_H = aH \int_0^t \frac{dt'}{a(t')} = 1,456 \text{ Mpc}}$$

$$1 \text{ Mpc} = 3.09 \times 10^{24} \text{ cm}$$

$$v_H \approx \frac{1}{H} (e^{Ht} - 1)$$

$$v_c =$$

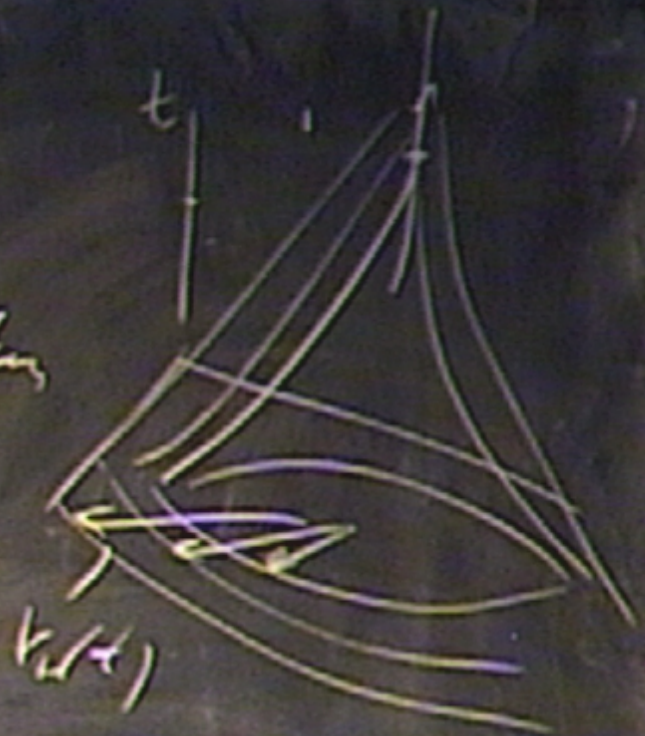


$$r_H = a(t) \int_0^t \frac{dt'}{a(t')} = 1,456 \text{ Mpc}$$

$$1 \text{ Mpc} = 3.09 \times 10^{24} \text{ cm}$$

$$= \frac{1}{H} (e^{Ht} - 1)$$

$$= a(t)$$



$$\left| \int_0^t \frac{dt'}{a(t')} \right| = 1,456 \text{ Mpc}$$

$$1 \text{ Mpc} = 3.09 \times 10^{24} \text{ cm}$$

$$v_H \approx \frac{1}{H} (e^{Ht} - 1)$$

$$v_e = a(t) \int_t^{t_{\text{max}}} \frac{dt'}{a(t')}$$

izon

$$\left| \frac{r_H = a(t) \int_0^t \frac{dt'}{a(t')}}{a(t)} \right| = 1,456 \text{ Mpc}$$

$$1 \text{ Mpc} = 3.09 \times 10^{24} \text{ cm}$$

$$r_H \approx \frac{1}{H} (e^{Ht} - 1)$$

$$\left| \frac{r_e = a(t) \int_t^{t_{\text{max}}} \frac{dt'}{a(t')}}{a(t)} \right|$$

izon

$$r_H = a(t) \int_0^t \frac{dt'}{a(t')} = 1,456 \text{ Mpc}$$

$$1 \text{ Mpc} = 3.09 \times 10^{24} \text{ cm}$$

$$= \frac{1}{H} (e^{Ht} - 1)$$

$$= a(t) \int_0^t \frac{dt'}{a(t')}$$

izon

$$r_H = a(t) \int_0^t \frac{dt'}{a(t')} = 1,456 \text{ Mpc}$$

$$1 \text{ Mpc} = 3.09 \times 10^{24} \text{ cm}$$

$$r_H \approx \frac{1}{H} (e^{Ht} - 1)$$

$$r_c = a(t) \int_t^{t_{\text{max}}} \frac{dt'}{a(t')}$$

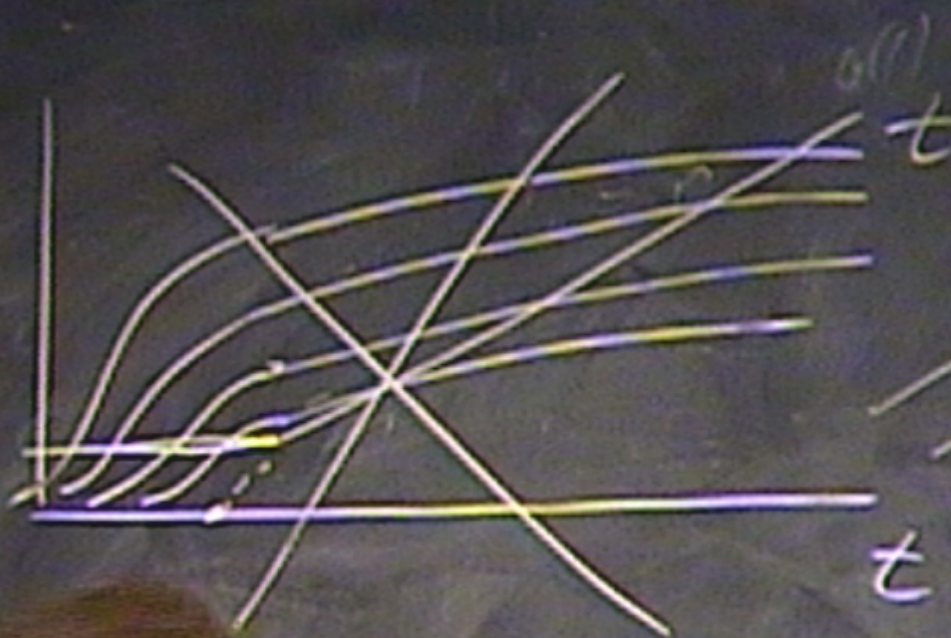
izon

$$\left| \int_0^t \frac{v_H = a(t)}{a(t)} dt \right| = 1,456 \text{ Mpc}$$

$$1 \text{ Mpc} = 3.09 \times 10^{24} \text{ cm}$$

$$v_H \approx \frac{1}{H} (e^{Ht} - 1)$$

$$\left| \int_t^{t_{\text{max}}} \frac{v_c = a(t)}{a(t)} dt \right| = \frac{1}{H}$$



Particle



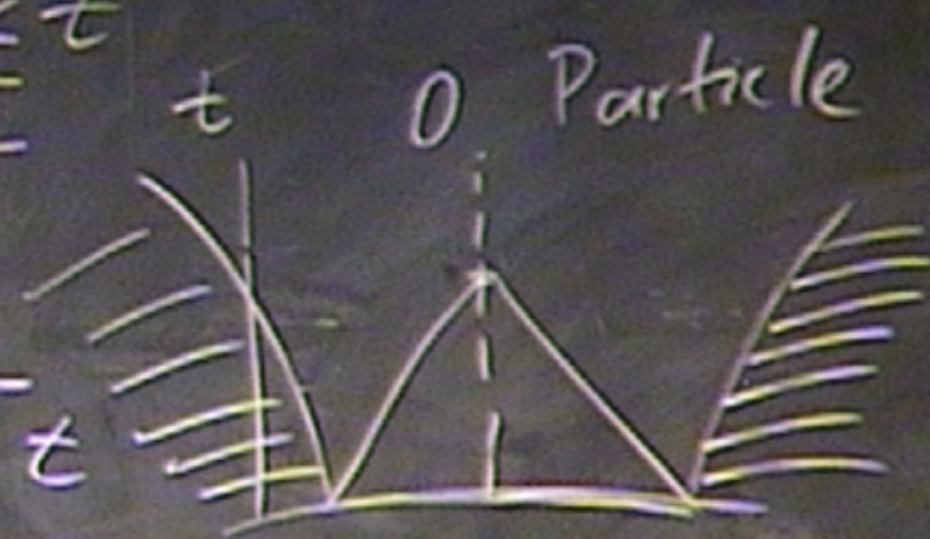
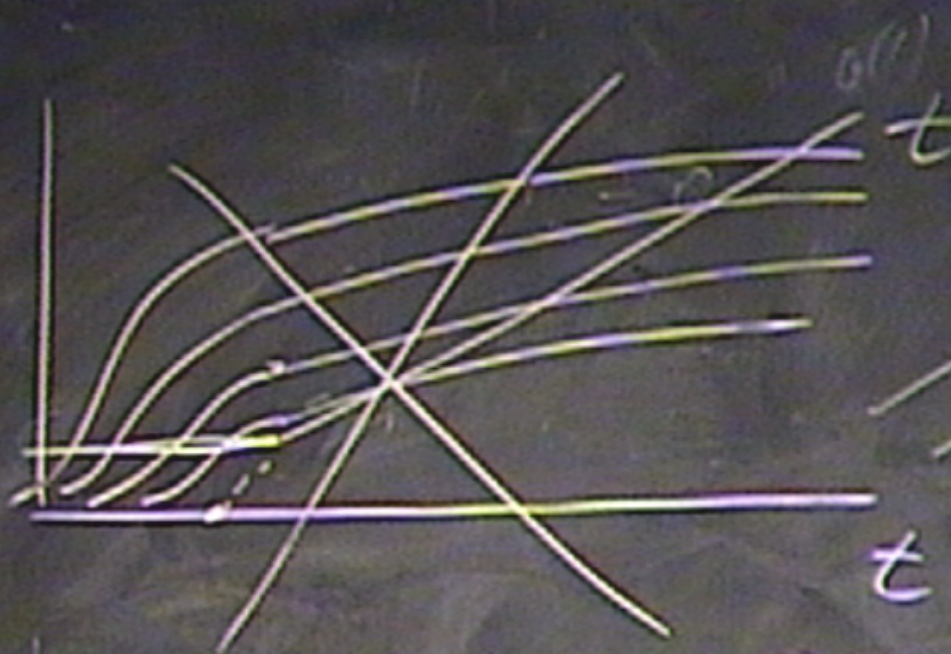
δ

$$\int d^3k \psi(t) e^{i\vec{x}\cdot\vec{k}}$$

$$ds^2 = a^2(t)(d\tau^2 - d\vec{x}^2)$$

$$= \frac{2\pi}{k} a(t) \quad a \sim t^{1/2}$$

$$\Delta x = \Delta \tau =$$

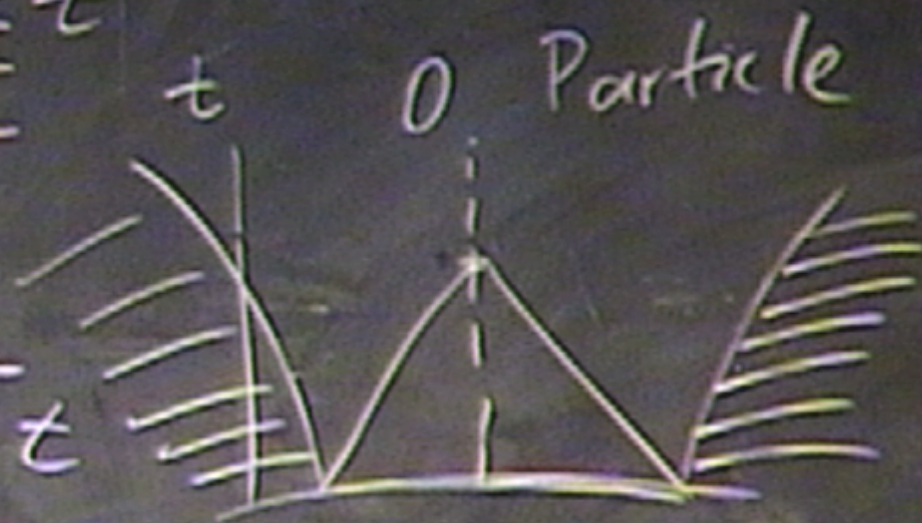
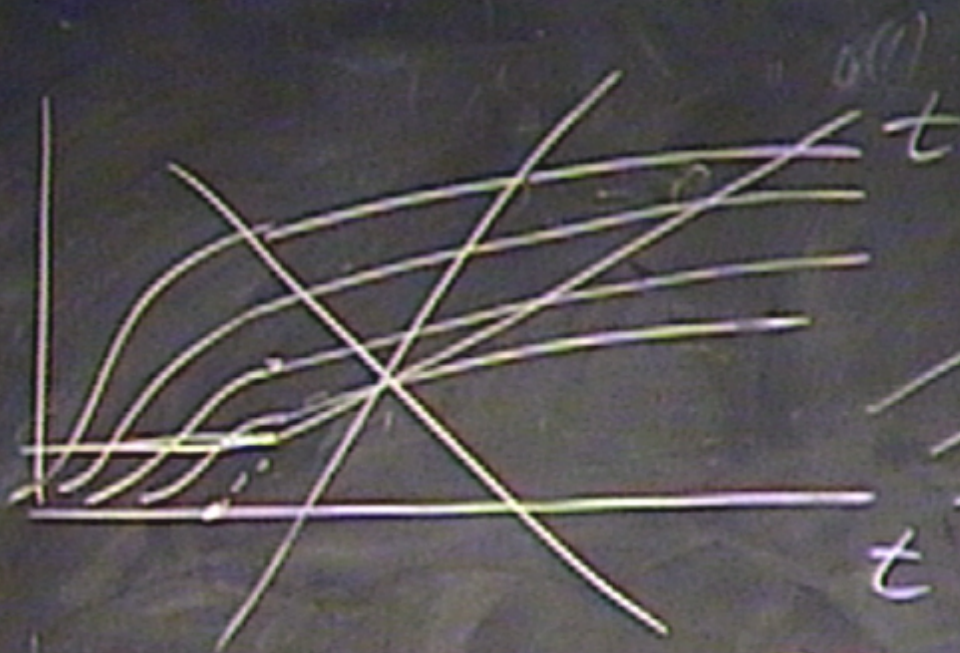


$$\delta \varphi(t, x) = \int d^3 k \varphi(k) e^{i \vec{x} \cdot \vec{k}}$$

$$ds^2 = a^2(\tau) (d\tau^2 - d\vec{x}^2)$$

$$\lambda = \frac{2\pi}{k} a(t) \quad a \sim t^{1/2}$$

$$\Delta x = \Delta \tau =$$



$$\delta \varphi(t, x) = \int d^3 k \varphi(k) e^{i \vec{x} \cdot \vec{k}}$$

$$ds^2 = a^2(\tau) (d\tau^2 - d\vec{x}^2)$$

$$\lambda = \frac{2\pi}{k} a(t) \quad a \sim t^{1/2}$$

$$\Delta x = \Delta \tau =$$

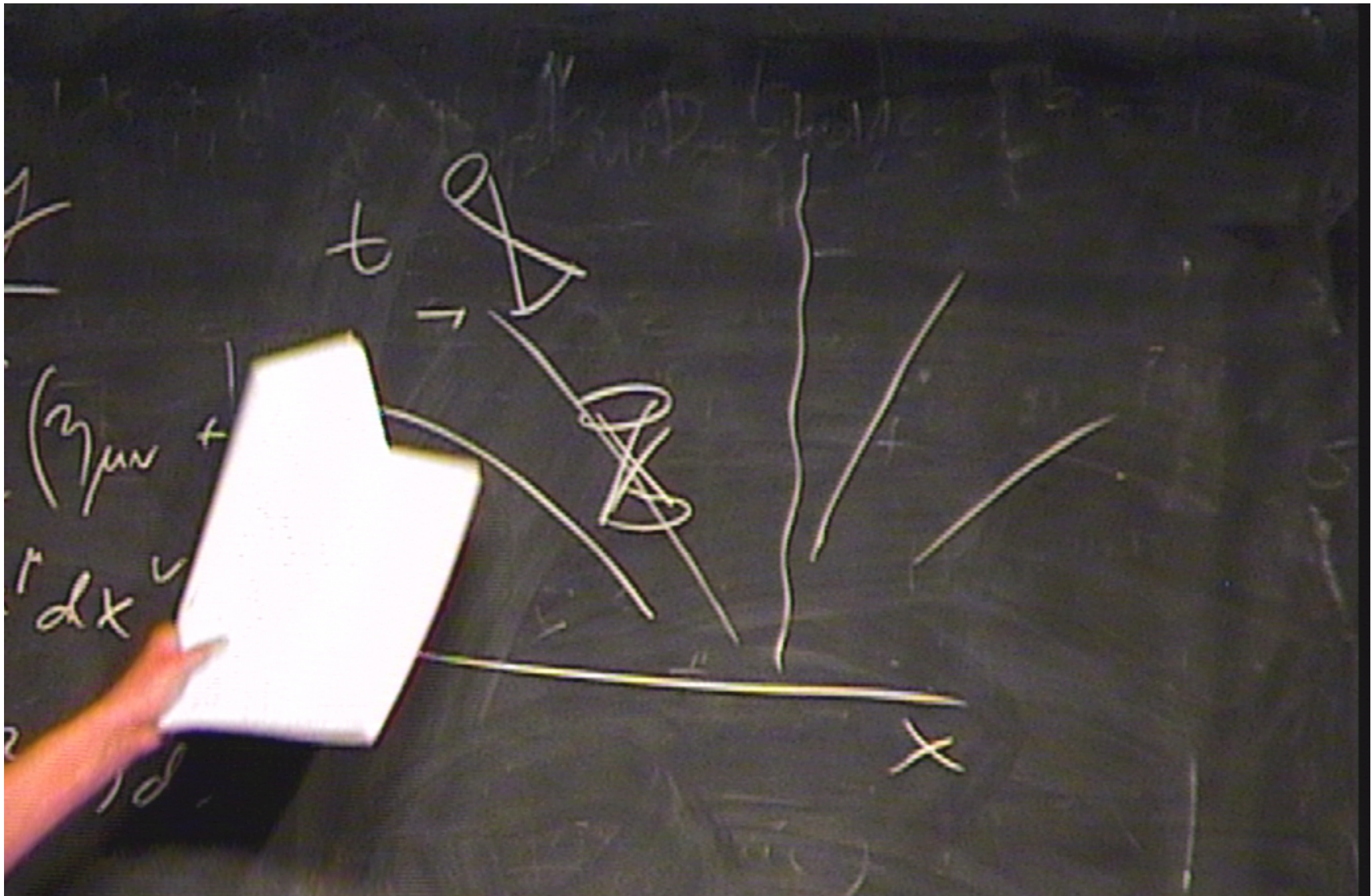
$$a(t) \int_0^t \frac{dt'}{a(t')} = 1,456 \text{ Mpc}$$

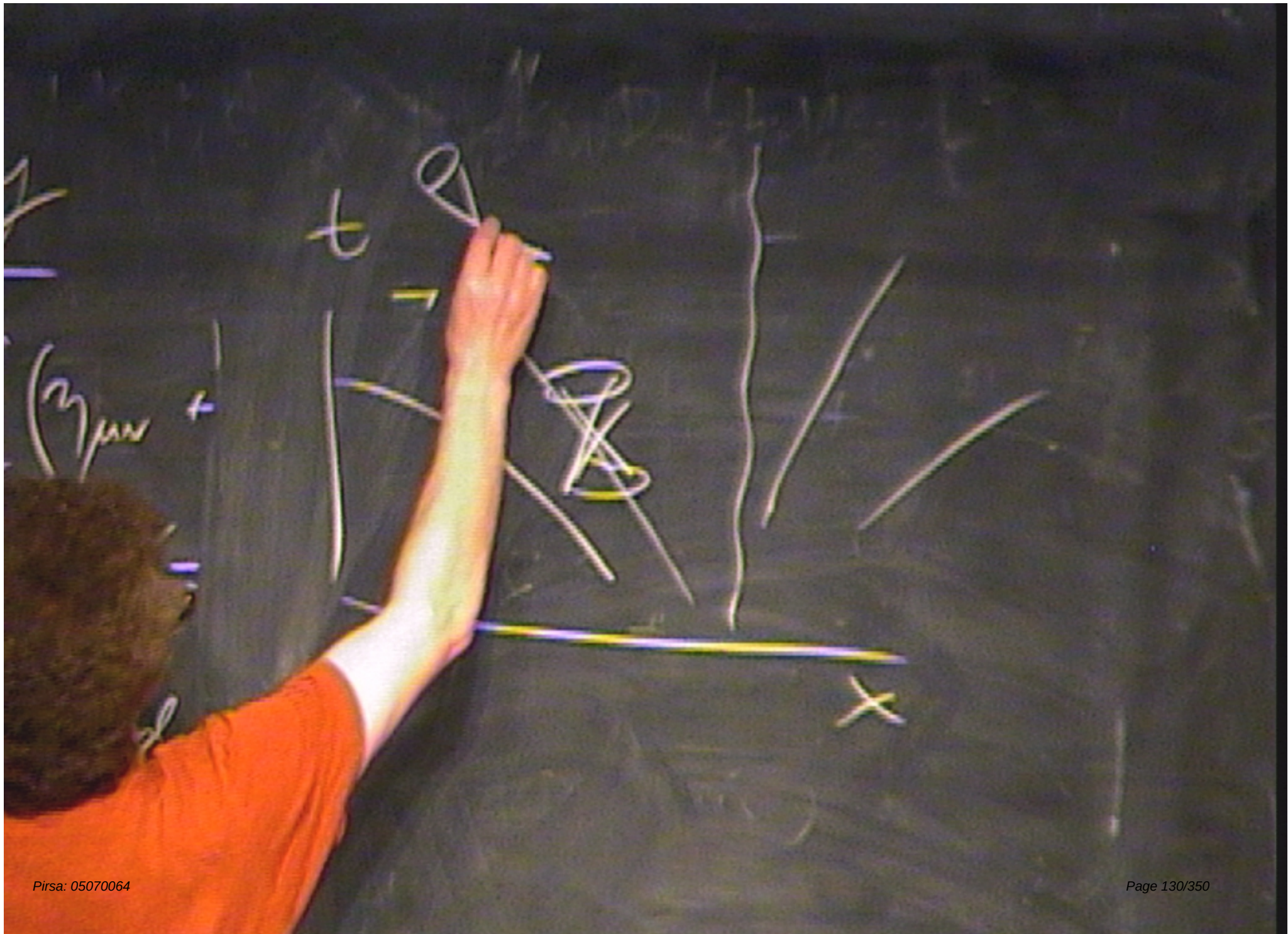
$$1 \text{ Mpc} = 3.09 \times 10^{24} \text{ cm}$$

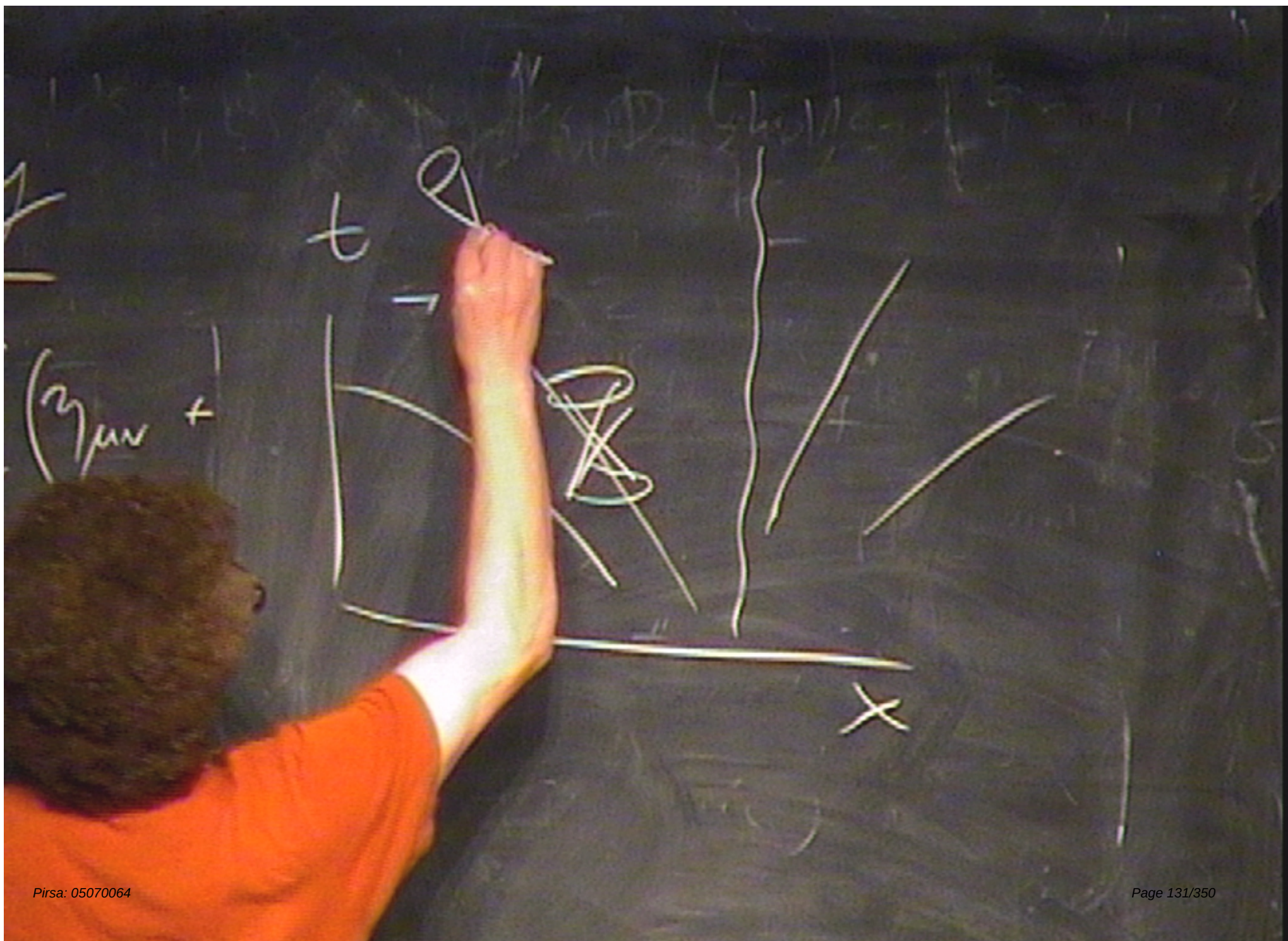
$$\frac{1}{H} \left(e^{Ht} - 1 \right)$$

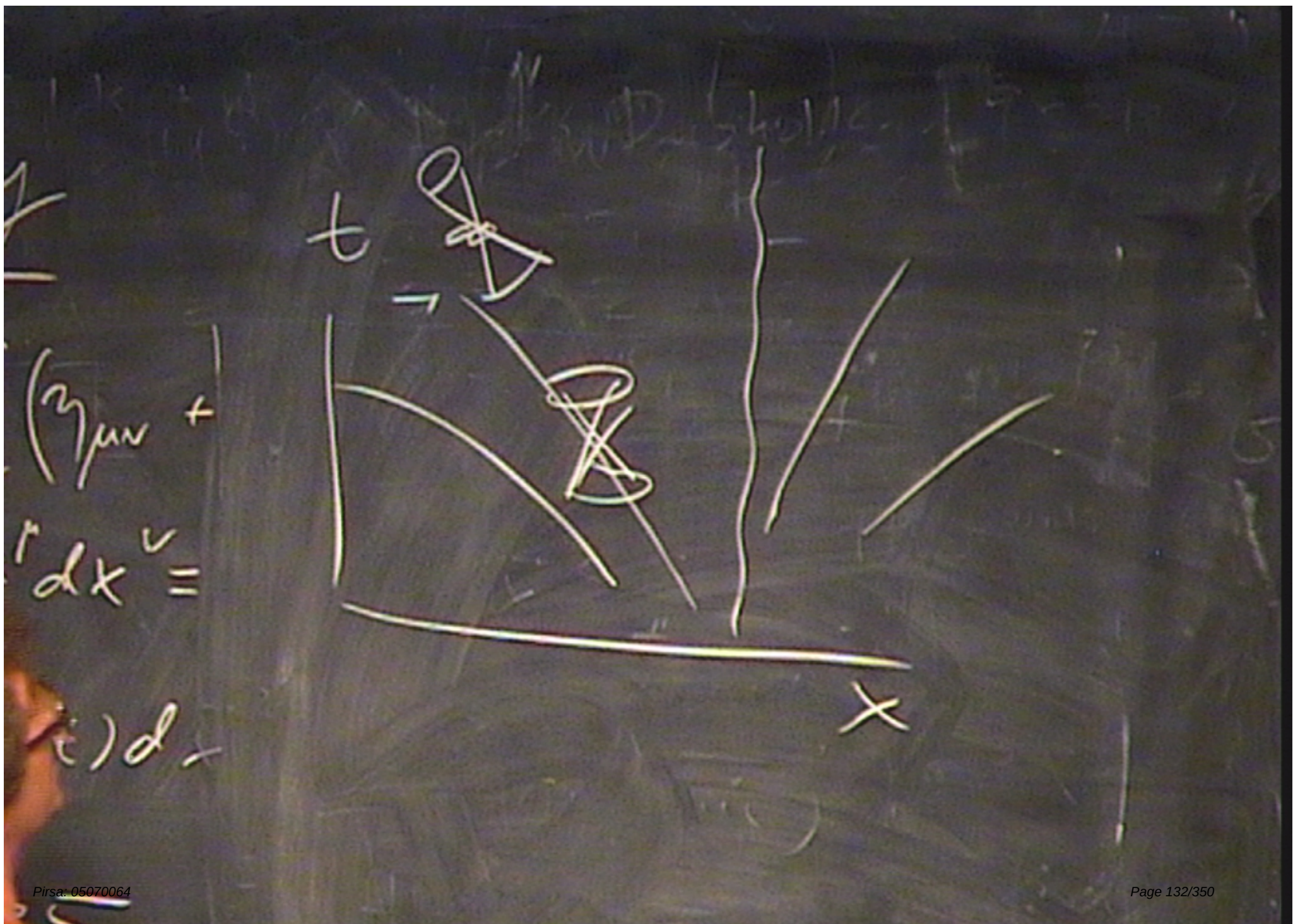
$$a(t) \int_t^{t_{\text{max}}} \frac{dt'}{a(t')} = \frac{1}{H}$$







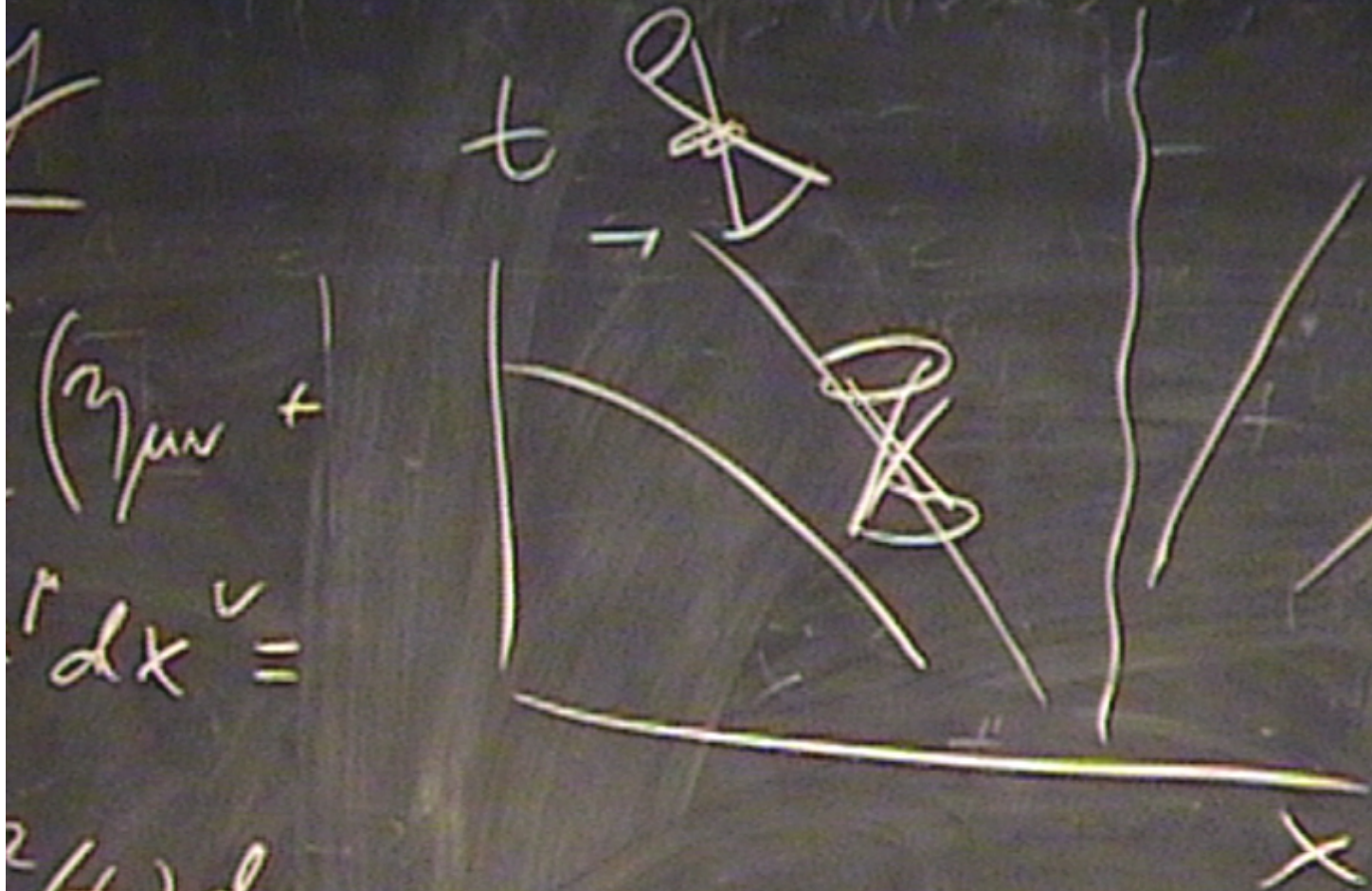




$$(g_{\mu\nu} +$$

$$dx^\nu =$$

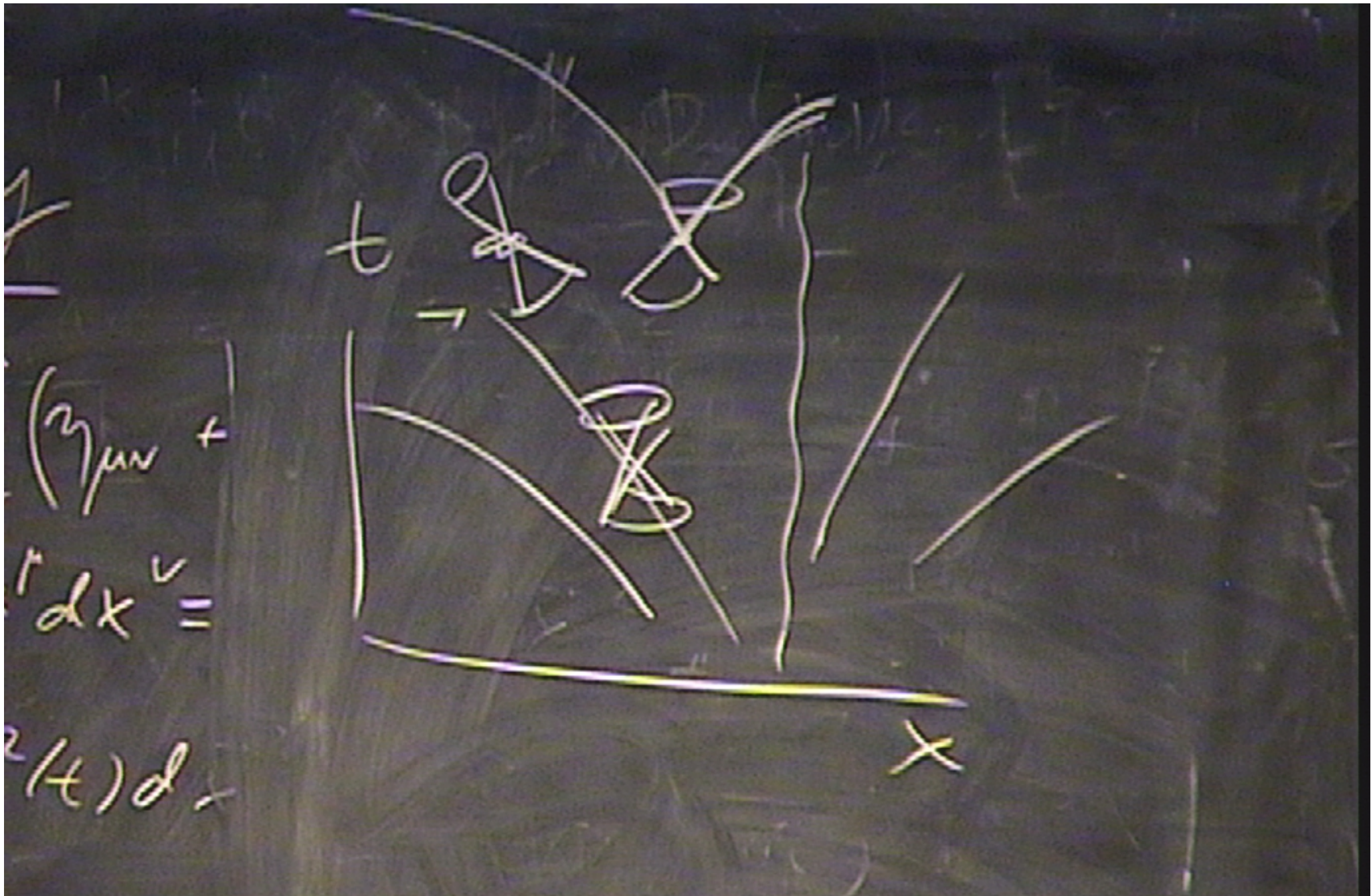
$$d\tau$$



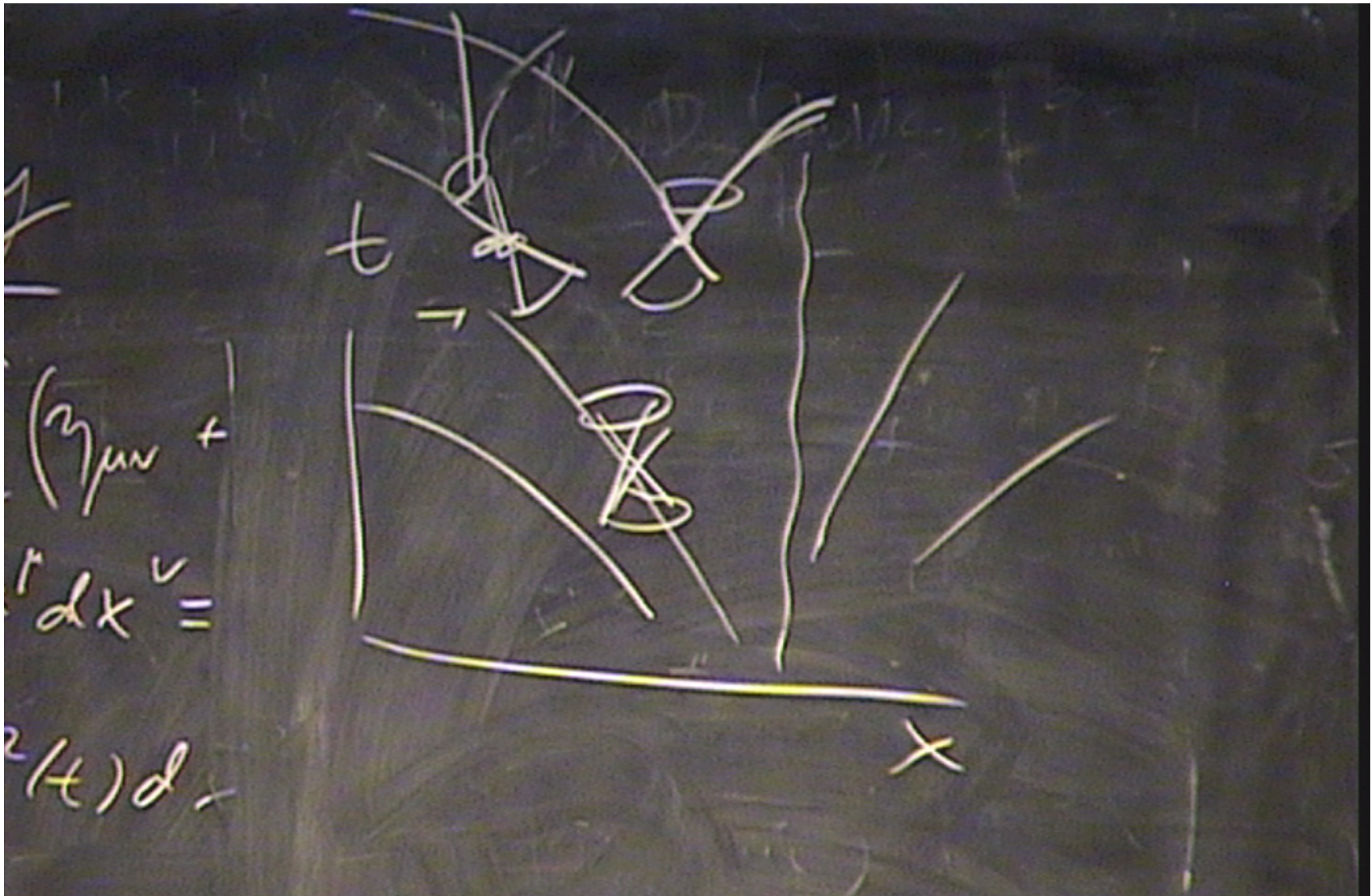
$$(\eta_{\mu\nu} +$$

$$dx^\nu =$$

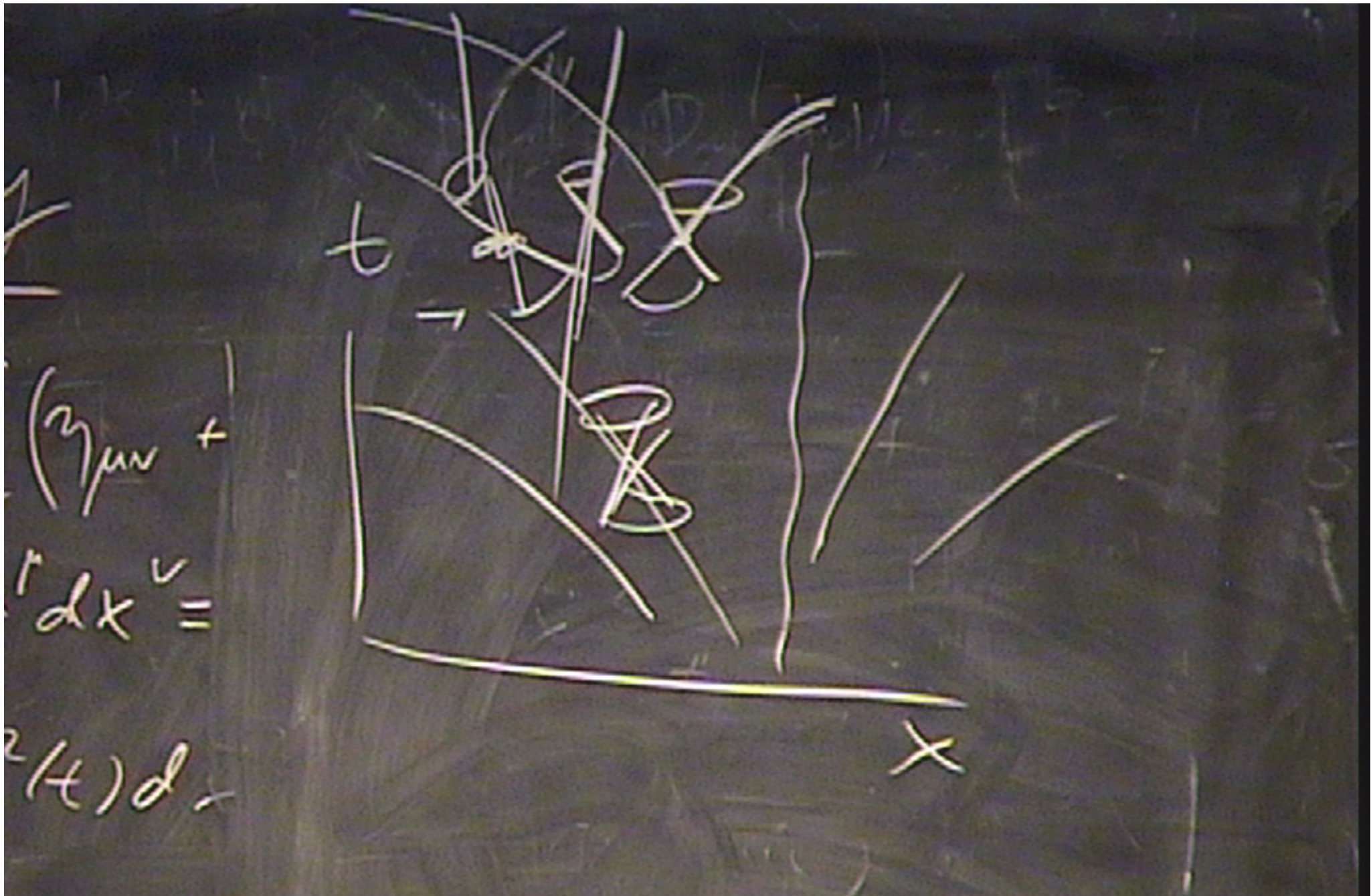
$$r(t) d\tau$$



$(\eta_{\mu\nu} +$
 $dx^\nu =$
 $z(t)dt$



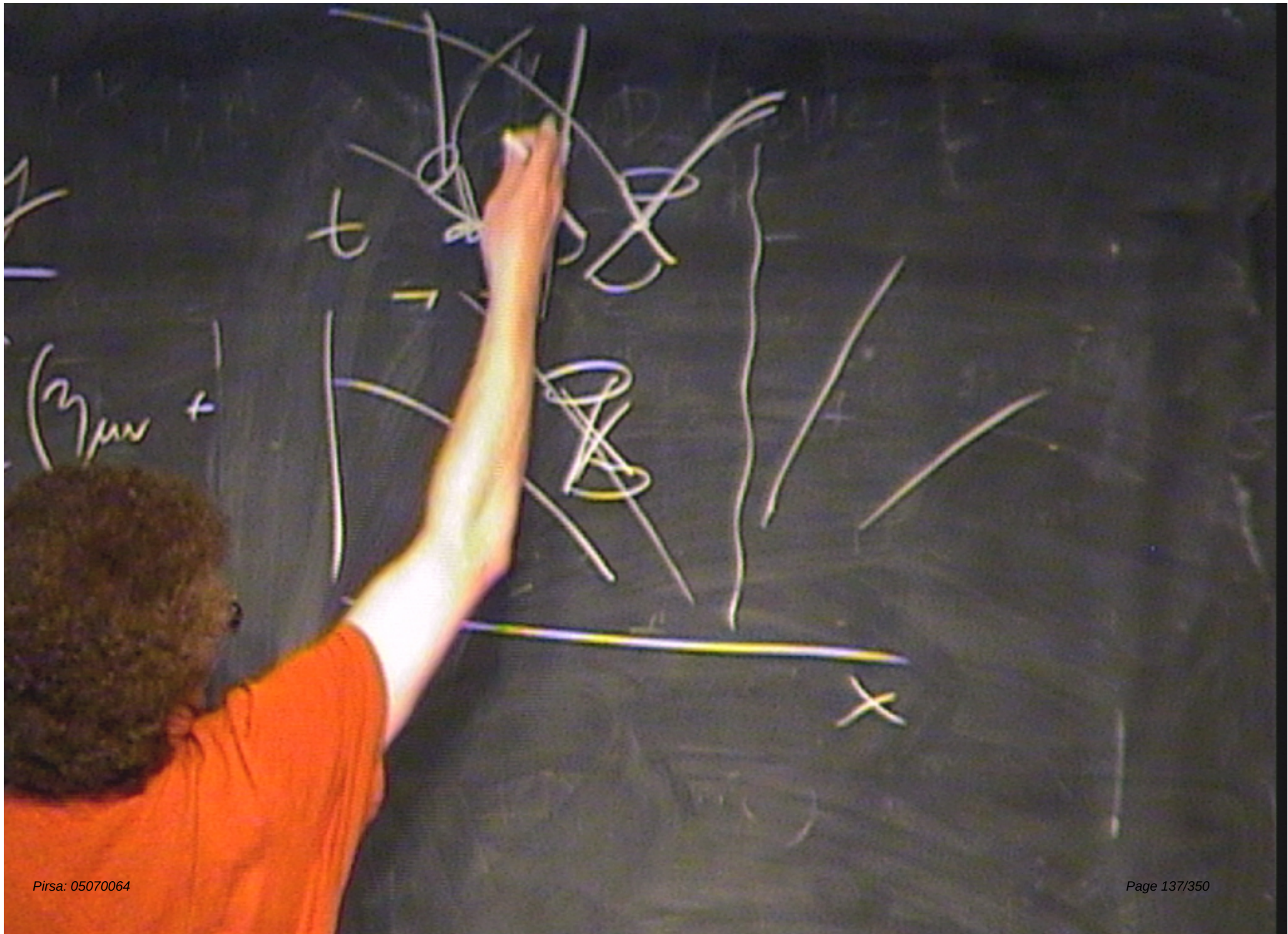
$$\begin{aligned} &(\gamma_{\mu\nu} + \\ &{}^{\mu}dx^{\nu} = \\ &{}^{\mu}(t)d. \end{aligned}$$

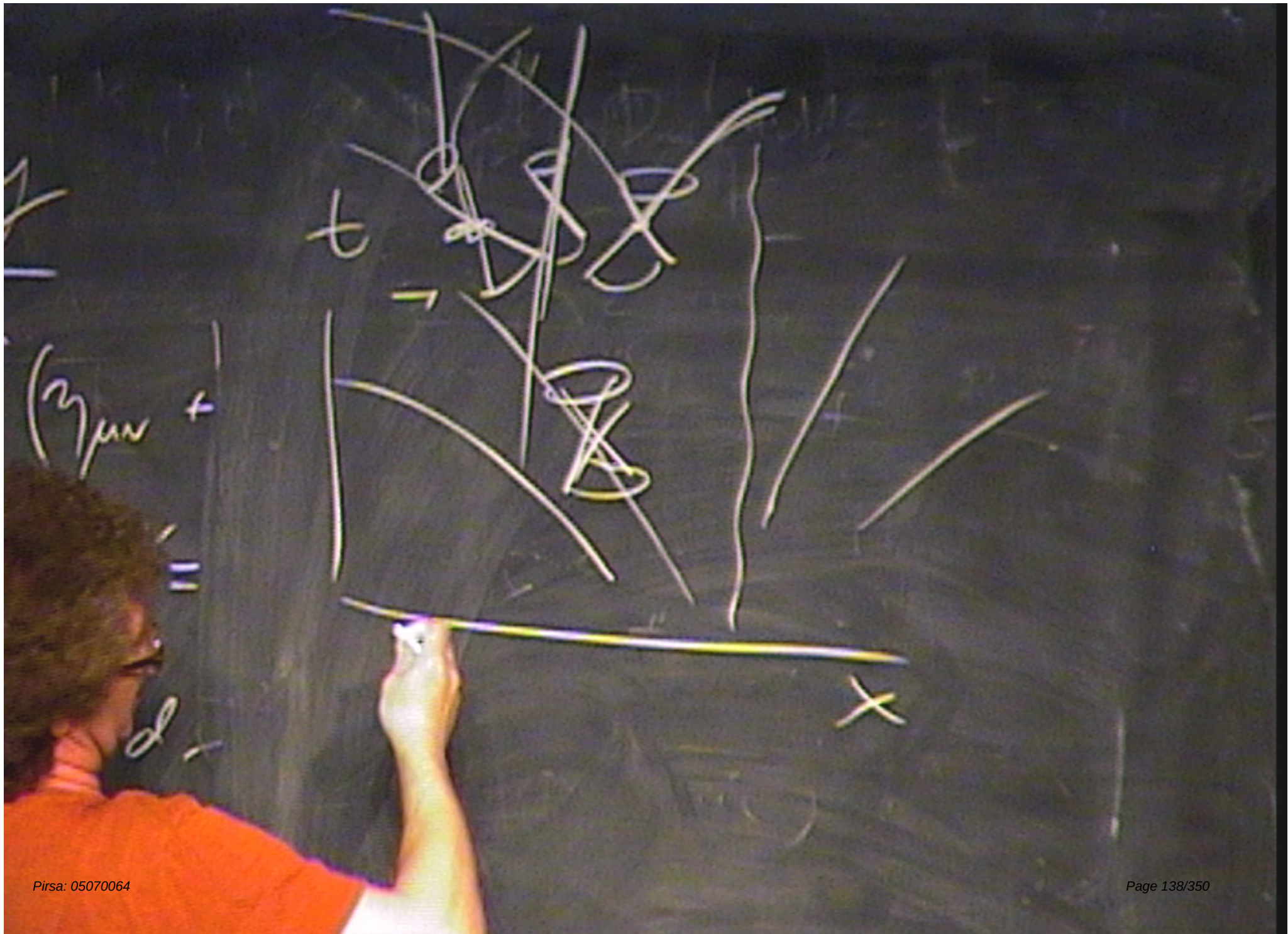


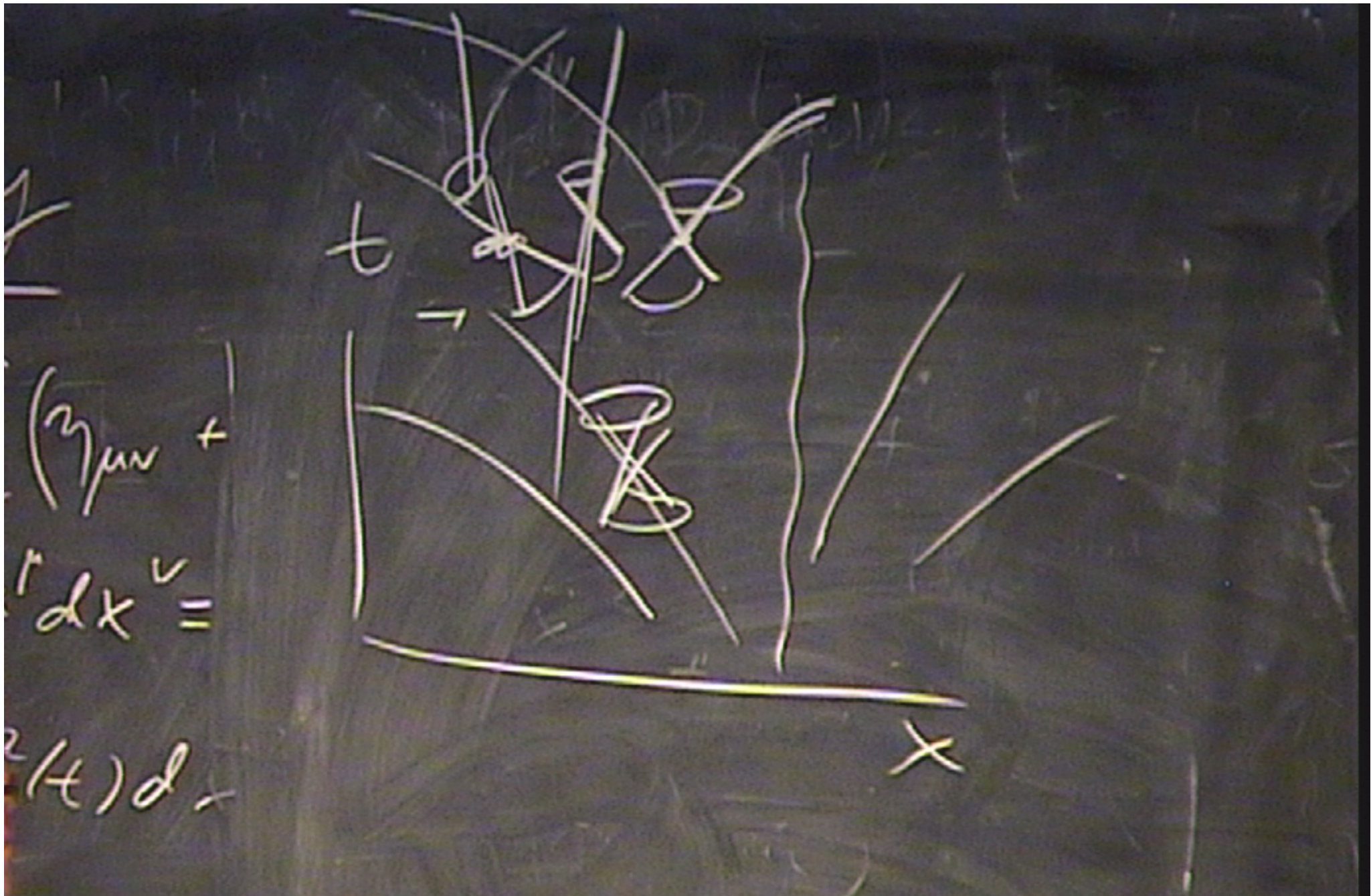
$$(g_{\mu\nu} +$$

$$dx^\nu =$$

$$r(t)dx$$



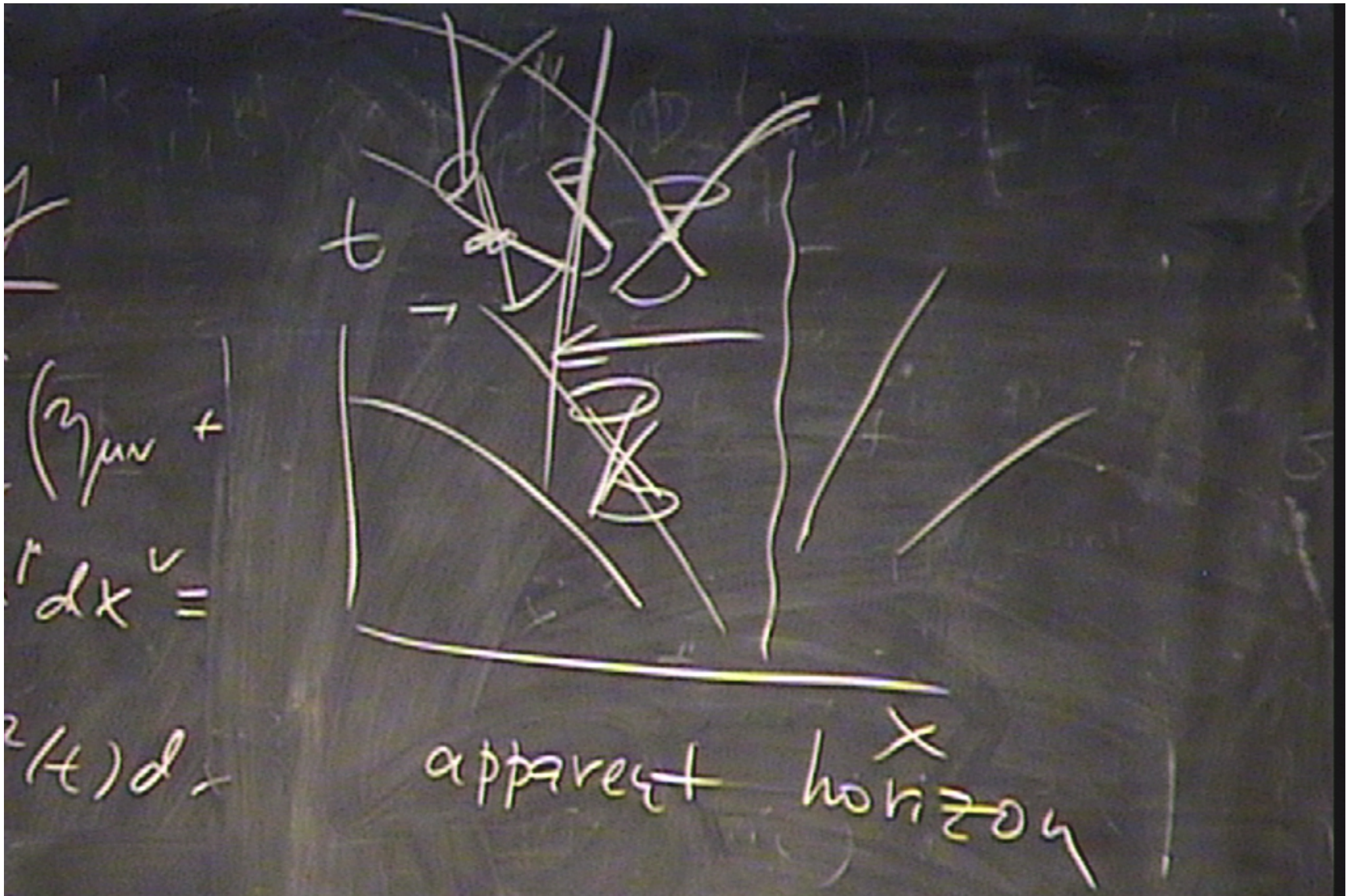




$$(\gamma_{\mu\nu} +$$

$$dx^\nu =$$

$$^2(t) d$$

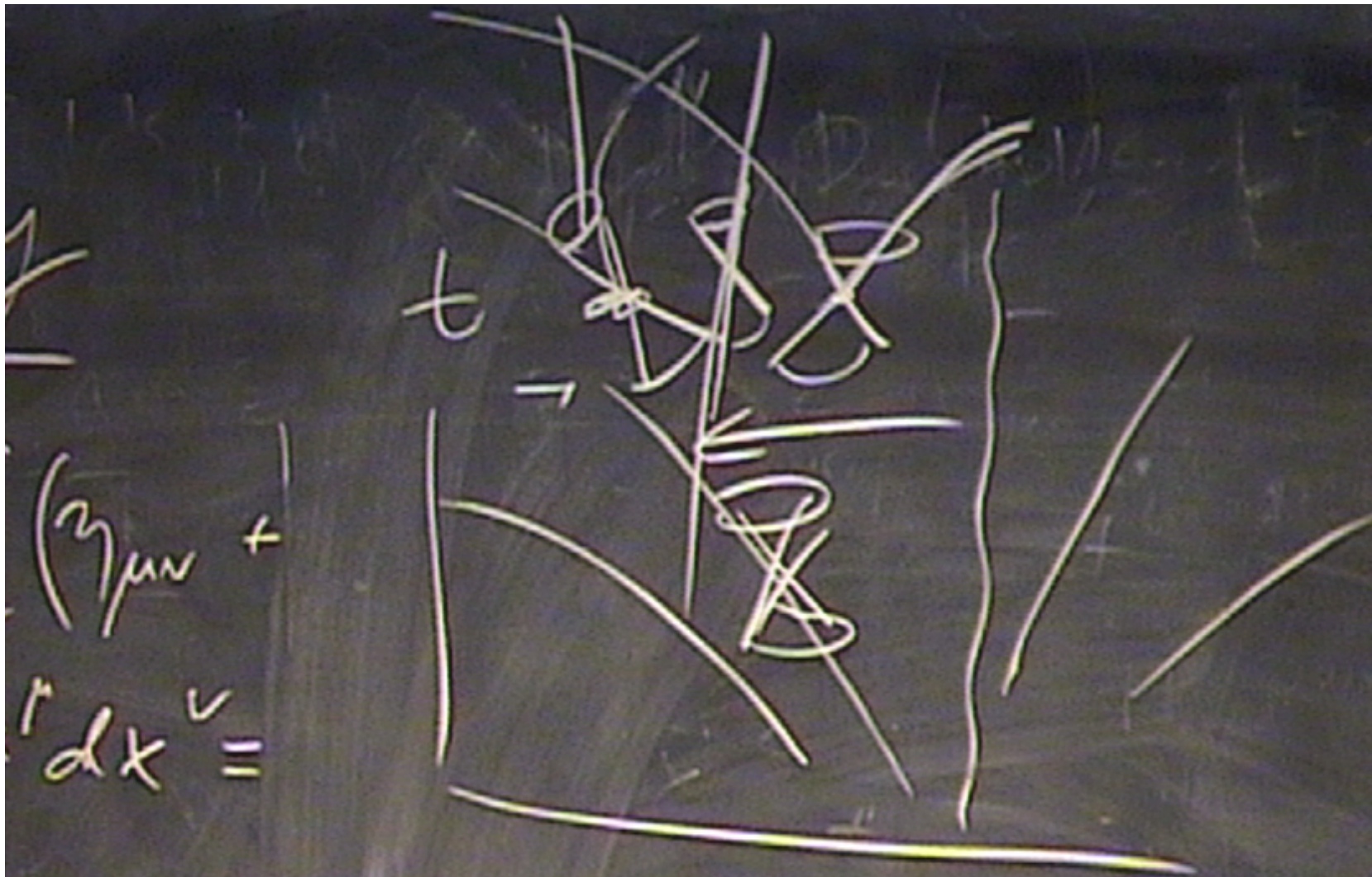


$$(\gamma_{\mu\nu} +$$

$$r dx^\nu =$$

$$^2(t) d$$

apparent horizon



$$(\gamma_{\mu\nu} +$$

$$r dx^\nu =$$

$$^2(t) d,$$

apparent horizon

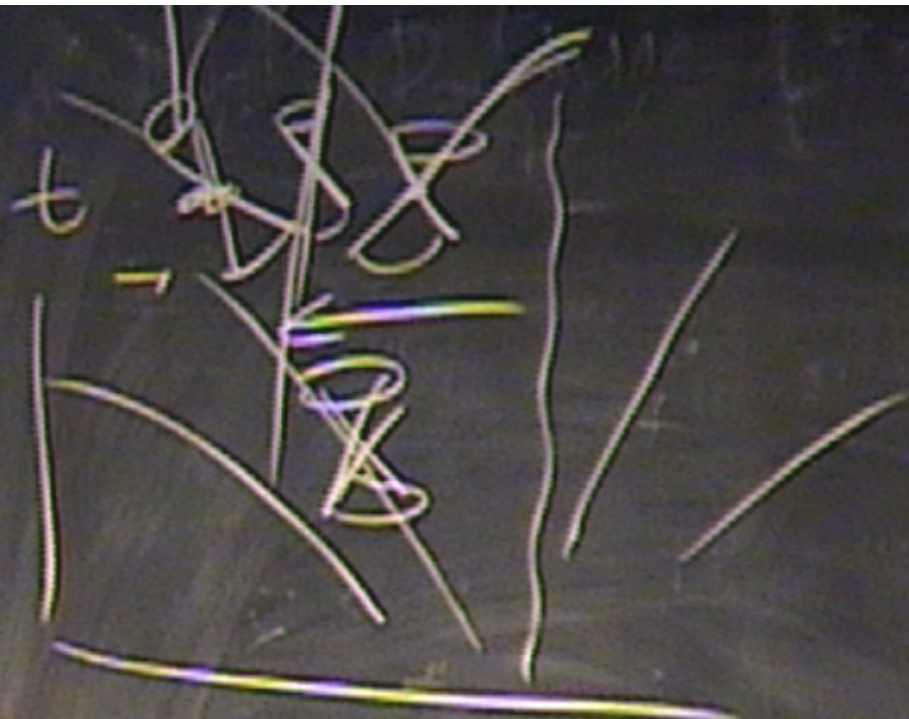
$$\frac{dL}{dt}$$

metry

$$\frac{1}{2}(\tau) \left[(\gamma_{\mu\nu} + \right.$$

$$\gamma_{\mu\nu} d$$

$$t^2$$



apparent horizon^x

$$\frac{dL}{dt} = H L$$

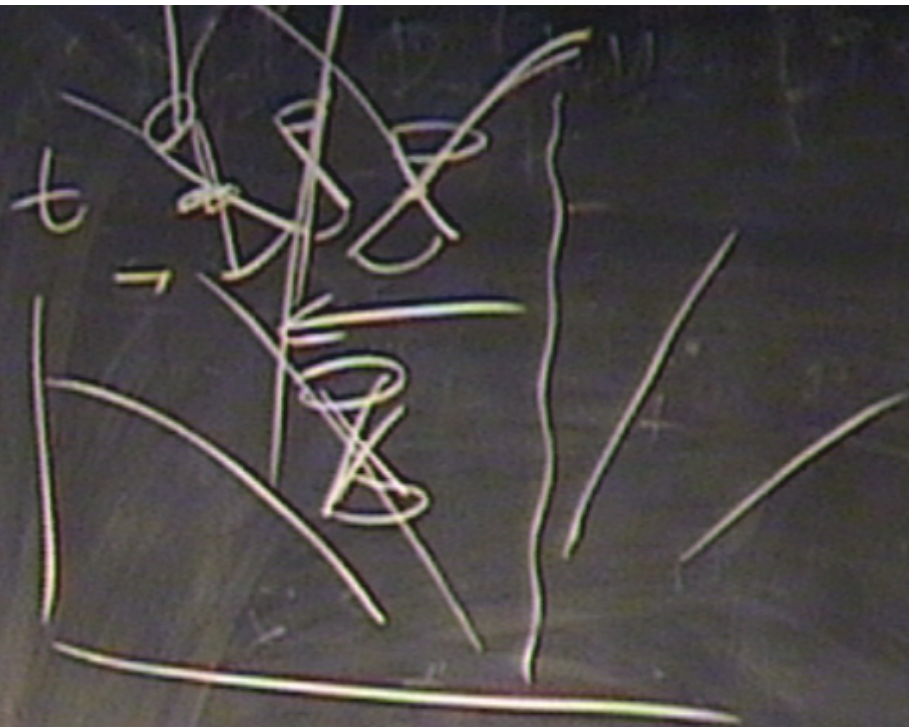
$\delta\varphi$

metry

$$^2/\tau) [(\gamma_{\mu\nu} +$$

$$\gamma_{\mu\nu} dx^\mu dx^\nu =$$

$$= a^2(t) d\tau^2$$



apparent horizon

$$\frac{8\pi G}{3} \sum \epsilon$$

$$\frac{dL}{dt} = H L$$

$$\frac{\dot{a}^2}{a^2} = 4\pi G \sum \rho$$

$\delta\varphi$

metry

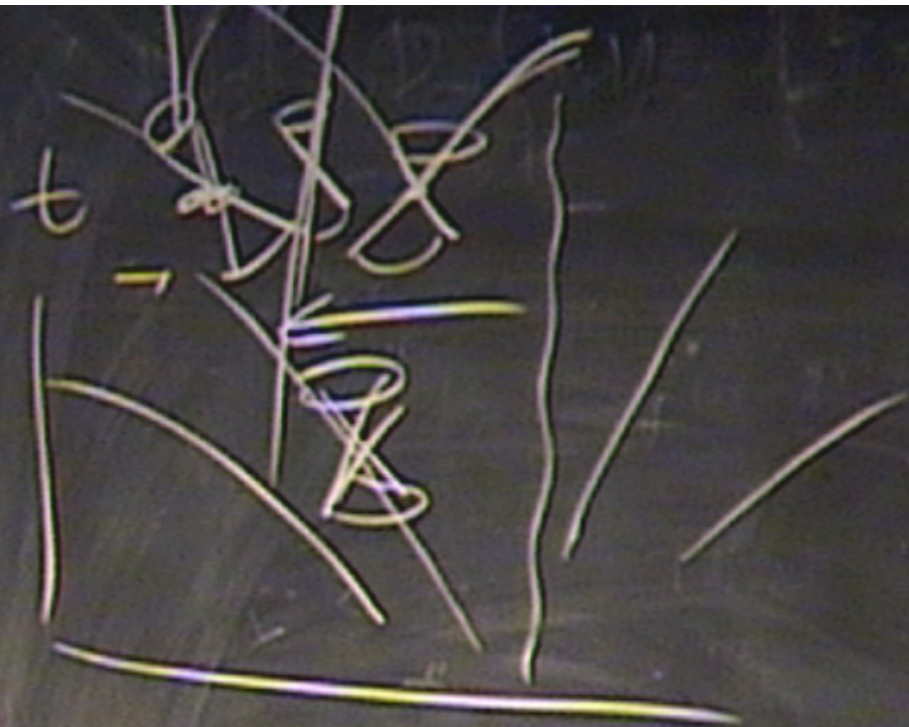
$$^2/\tau) [(\eta_{\mu\nu} +$$

$$\eta_{\mu\nu} dx^\mu dx^\nu =$$

$$(t^2 - a^2(t)) dt$$

$$= \frac{8\pi G}{3} \sum \epsilon$$

$$\frac{\dot{a}^2}{a^2} = \frac{4\pi G}{3} \sum \rho$$



apparent horizon^x

$$\frac{dL}{dt} = H L$$

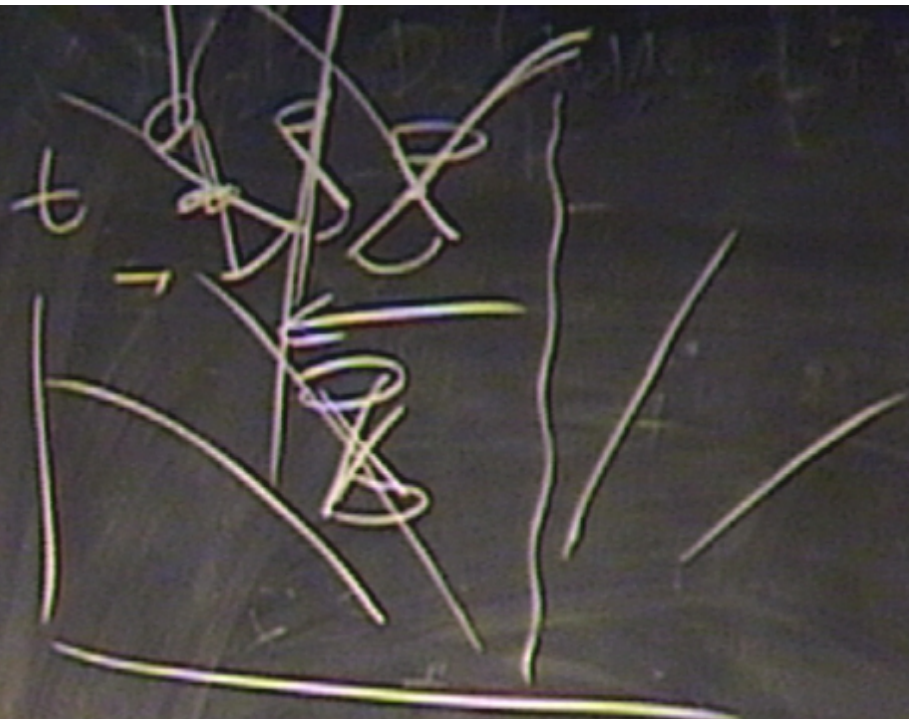
$\delta\varphi$

metry

$$\frac{1}{2} \left[(\eta_{\mu\nu} + \right.$$

$$\eta_{\mu\nu} dx^\mu dx^\nu =$$

$$(1 - a^2(t)) dt^2 -$$



apparent horizon

$$\frac{8\pi G}{3} \sum \epsilon$$

$$\frac{dr}{dt} = H r > 0$$

$$\frac{1}{a^2} = \frac{4\pi G}{3} \sum \rho$$

$$r > \frac{1}{H}$$

$\delta\varphi$

metry

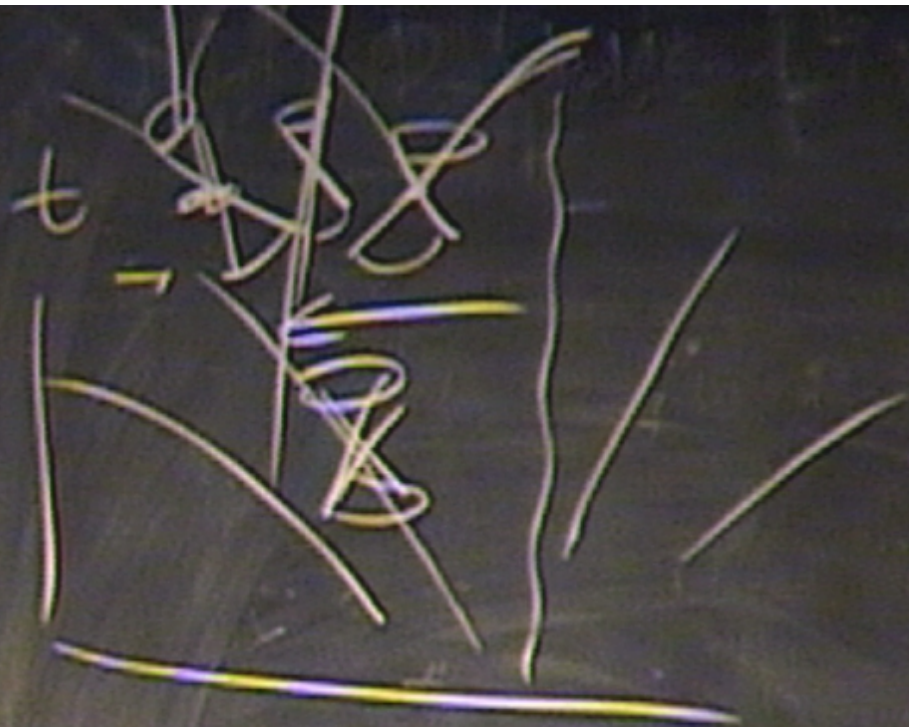
$$\frac{1}{2} \left[(\eta_{\mu\nu} + \right.$$

$$\eta_{\mu\nu} dx^\mu dx^\nu =$$

$$(1 - a^2(t)) dt^2 -$$

$$= \frac{8\pi G}{3} \sum \epsilon$$

$$\frac{\dot{a}^2}{a^2} = \frac{4\pi G}{3} \sum \rho$$



apparent horizon

$$\frac{dr}{dt} = H r > 0$$

$$r > \frac{1}{H}$$

$\delta\varphi$

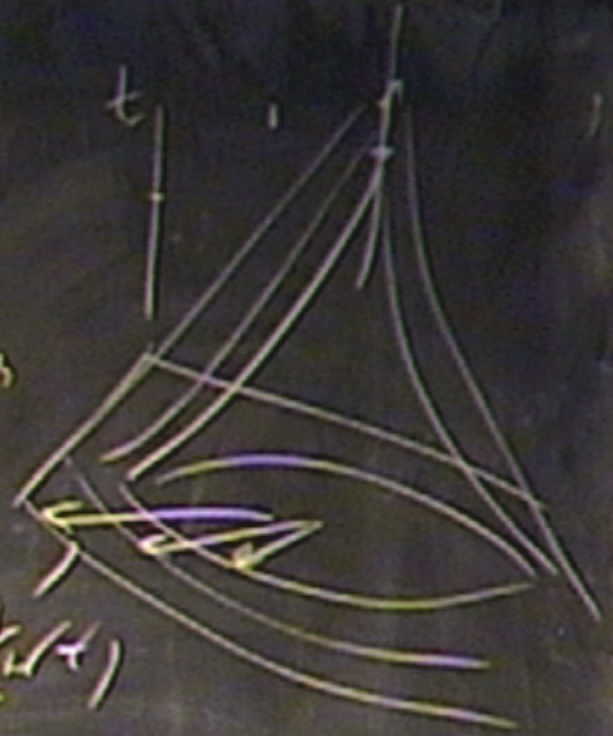
$$| \left. v_H = a(t) \int_0^t \frac{dt'}{a(t')} \right| = 1,456 \text{ Mpc}$$

$$1 \text{ Mpc} = 3,09 \times 10^{24} \text{ cm}$$

$$v_H \approx \frac{1}{H} (e^{Ht} - 1)$$

$$| \left. v_c = a(t) \int_t^{t_{\text{max}}} \frac{dt'}{a(t')} \right| = \frac{1}{H}$$

$$H \approx 1/3$$



Penrose diagrams

$$t \quad ds^2 = dt^2 - dr^2 - r^2 d\Omega_2^2$$

$$t = \pm r$$



Penrose diagrams

$$t \quad ds^2 = dt^2 - dr^2 - r^2 d\Omega_2^2$$

$$t = \pm r$$



Penrose diagrams

$$t \quad ds^2 = dt^2 - dr^2 - r^2 d\Omega_2^2$$

$$t = \pm r$$



Penrose diagrams

$$t \quad ds^2 = dt^2 - dr^2 - r^2 d\Omega_2^2$$

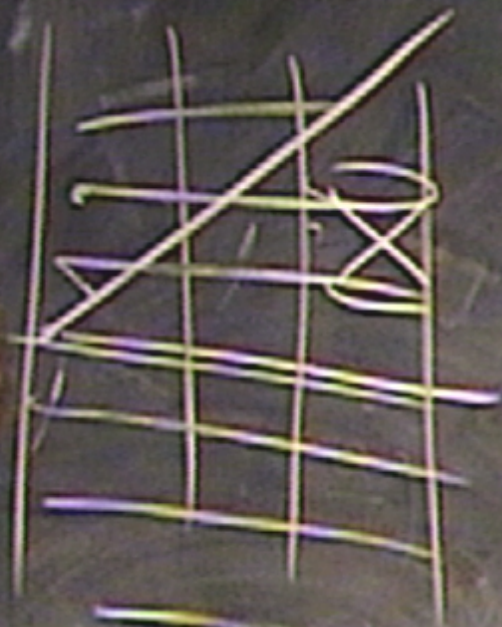
$$t = \pm r$$



Penrose diagrams

$$t \quad ds^2 = dt^2 - dr^2 - r^2 d\Omega_2^2$$

$$t = \pm r$$



Penrose diagrams

$$t \quad ds^2 = dt^2 - dr^2 - r^2 d\Omega_2^2$$

$$t = \pm r$$

Penrose diagrams

$$ds^2 = \underbrace{dt^2 - dr^2}_{t = \pm r} + r^2 d\Omega_2^2$$



Penrose diagrams

$$ds^2 = \underbrace{|dt^2 - dr^2|}_{t = \pm r} + r^2 d\Omega_2^2$$

$$t = \pm r$$

t

se diagrams

$$-dr^2 + r^2 d\Omega_2^2$$

$$t = \pm r$$

$$t + r = f(t' + r')$$

t

H

se diagrams

$$-dr^2 + r^2 d\Omega_2^2$$

$$t = \pm r$$

$$t + r = f(t' + r')$$

$$t - r = g(t' - r')$$

se diagrams

$$-dr^2 + r^2 d\Omega_2^2$$

$$t = \pm r$$

$$t + r = f(t' + r')$$

$$t - r = g(t' - r')$$

rose diagrams

$$t^2 - dr^2 + r^2 d\Omega_2^2$$

$$t = \pm r$$

$$t + r = f(t' + r')$$

$$t - r = g(t' - r')$$

$$dt^2 - dr^2 = f'g'(dt'^2 - dr'^2)$$

rose diagrams

$$\sqrt{t^2 - dr^2 + r^2 d\Omega_2^2}$$
$$t = \pm r$$

$$t + r = f(t' + r')$$

$$t - r = g(t' - r')$$

$$dt^2 - dr^2 = f'g'(dt'^2 - dr'^2)$$

rose diagrams

$$t^2 - dr^2 + r^2 d\Omega_2^2$$

$$t = \pm r$$

$$\begin{cases} t + r = f(t' + r') \\ t - r = g(t' - r') \end{cases}$$

$$dt^2 - dr^2 = f'g'(dt'^2 - dr'^2)$$

rose diagrams

$$\sqrt{t^2 - dr^2 + r^2 d\Omega_2^2}$$

$$t = \pm r$$

$$r = f(t' + r') = \tan\left(\frac{t' + r'}{2}\right)$$

$$= g(t' - r') = \tan\left(\frac{t' - r'}{2}\right)$$

$$-dr^2 = f'g'(dt'^2 - dr'^2)$$

Penrose diagrams

$$\sqrt{dt^2 - dr^2 + r^2 d\Omega_2^2}$$
$$t = \pm r$$

$$\begin{cases} t+r = f(t'+r') = \tan\left(\frac{t'+r'}{2}\right) \\ t-r = g(t'-r') = \tan\left(\frac{t'-r'}{2}\right) \end{cases}$$

$$ds^2 = dt^2 - dr^2 = f'g'(dt'^2 - dr'^2)$$

Penrose diagrams



$$dt^2 - dr^2 + r^2 d\Omega_2^2$$

$$t = \pm r$$

$$t + r = f(t' + r') = \tan\left(\frac{t' + r'}{2}\right)$$

$$t - r = g(t' - r') = \tan\left(\frac{t' - r'}{2}\right)$$

$$dt^2 - dr^2 = f'g'(dt'^2 - dr'^2)$$

Penrose diagrams



$$\sqrt{dt^2 - dr^2} + r^2 d\Omega_2^2$$

$t = \pm r$

$$\begin{cases} t + r = f(t' + r') = \tan\left(\frac{t' + r'}{2}\right) \\ t - r = g(t' - r') = \tan\left(\frac{t' - r'}{2}\right) \end{cases}$$

$$ds^2 = dt^2 - dr^2 = f'g'(dt'^2 - dr'^2)$$

Penrose diagrams

$$\sqrt{dt^2 - dr^2} + r^2 d\Omega_2^2$$

$$t = \pm r$$

$$\begin{cases} t + \\ t - \end{cases}$$

$$ds^2 = dt^2$$

$$t' + r' = \tan\left(\frac{t' + r'}{2}\right)$$

$$t' - r' = \tan\left(\frac{t' - r'}{2}\right)$$

$$(t'^2 - dr'^2)$$

Penrose diagrams

$$dt^2 - dr^2 + r^2 d\Omega^2$$

$$t = \pm r$$

$$\begin{cases} t + r = \text{const} \\ t - r = \text{const} \end{cases}$$

$$ds^2 = dt^2 - dr^2$$



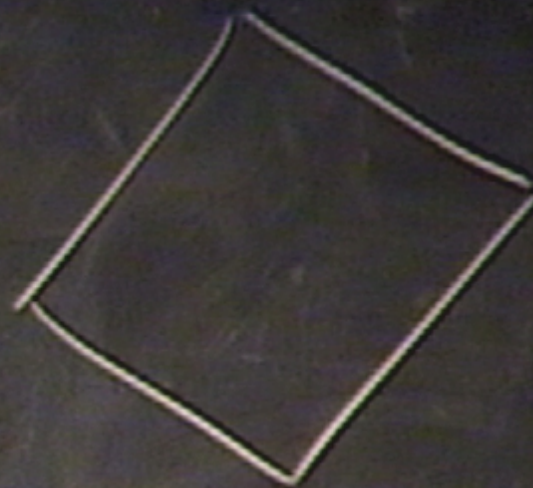
$$= \tan\left(\frac{r'}{2}\right)$$

$$\left(\frac{t' - r'}{2}\right)$$

Penrose diagrams

$$dt^2 - dr^2 + r^2 d\Omega_2^2$$

$$t = \pm r$$



$$t + r = f(t' + r') = \tan\left(\frac{t' + r'}{2}\right)$$

$$t - r = g(t' - r') = \tan\left(\frac{t' - r'}{2}\right)$$

$$dt^2 - dr^2 = f'g'(dt'^2 - dr'^2)$$

Penrose diagrams

$$dt^2 - dr^2 + r^2 d\Omega_2^2$$

$$t = \pm r$$

$$\begin{cases} t+r = r' \\ t-r = r' \end{cases} \Rightarrow \begin{cases} t' = \frac{t+r}{2} \\ r' = \frac{t-r}{2} \end{cases}$$

$$ds^2 = dt^2 - dr^2$$

$$dr'^2$$

Penrose diagrams

$$dt^2 - dr^2 + r^2 d\Omega^2$$

$$t = \pm r$$

$$\begin{aligned} \begin{cases} t+r \\ t-r \end{cases} &= \tan\left(\frac{t'+r'}{2}\right) \\ &= \tan\left(\frac{t'-r'}{2}\right) \end{aligned}$$

$$ds^2 = dt^2 - dr^2$$



Penrose diagrams

$$dt^2 - dr^2 + r^2 d\Omega^2$$

$$t = \pm r$$

$$\begin{cases} t+r \\ t-r \end{cases}$$

$$ds^2 = dt^2 - dr^2$$



$$\begin{aligned} & \tan\left(\frac{t+r'}{2}\right) \\ & = \tan\left(\frac{t'-r'}{2}\right) \\ & \quad - dr'^2 \end{aligned}$$

Penrose diagrams

$$dt^2 - dr^2 + r^2 d\Omega^2$$

$$t = \pm r$$

$$\begin{pmatrix} t+r \\ t-r \end{pmatrix}$$

$$ds^2 = dt^2 - dr^2$$



$$\begin{pmatrix} t' \\ r' \end{pmatrix} = \begin{pmatrix} \frac{t+r}{2} \\ \frac{t-r}{2} \end{pmatrix}$$

Penrose diagrams

$$dt^2 - dr^2 + r^2 d\Omega_2^2$$

$$t = \pm r$$

$$\begin{cases} t+r = \\ t-r = \end{cases}$$

$$ds^2 = dt^2 - dr^2$$

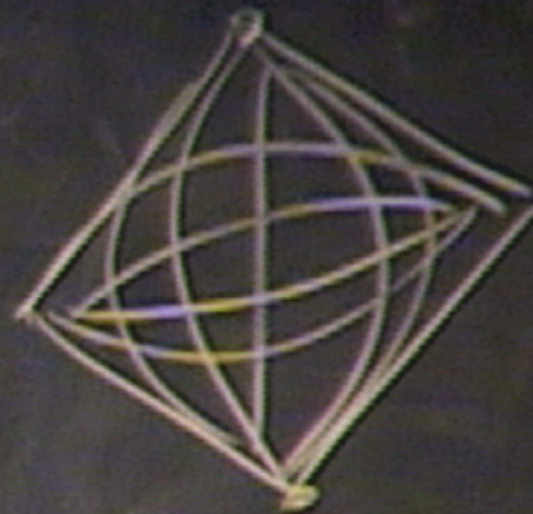


$$\begin{aligned} r' &= \sqrt{\left(\frac{t'+r'}{2}\right)} \\ &\quad \cosh\left(\frac{t'-r'}{2}\right) \\ &\quad dr'^2 \end{aligned}$$

Penrose diagrams

$$\sqrt{dt^2 - dr^2} + r^2 d\Omega_2^2$$

$$t = \pm 1$$

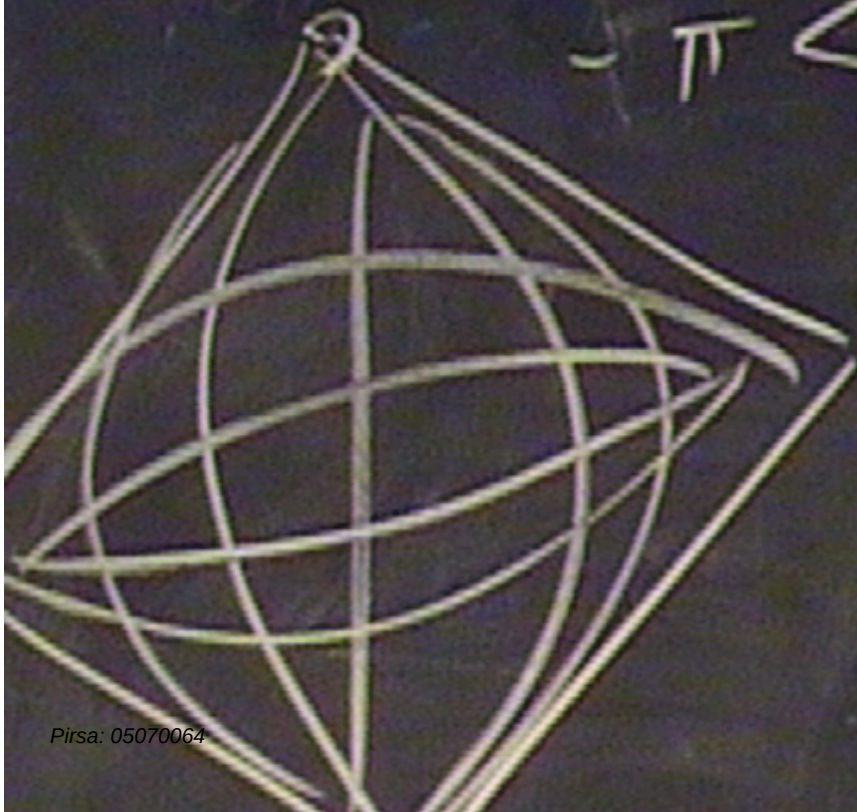


$$= f(t' + r') = \tan\left(\frac{t' + r'}{2}\right)$$

$$(t' - r') = \tan\left(\frac{t' - r'}{2}\right)$$

$$ds^2 = g'(t'^2 - dr'^2)$$

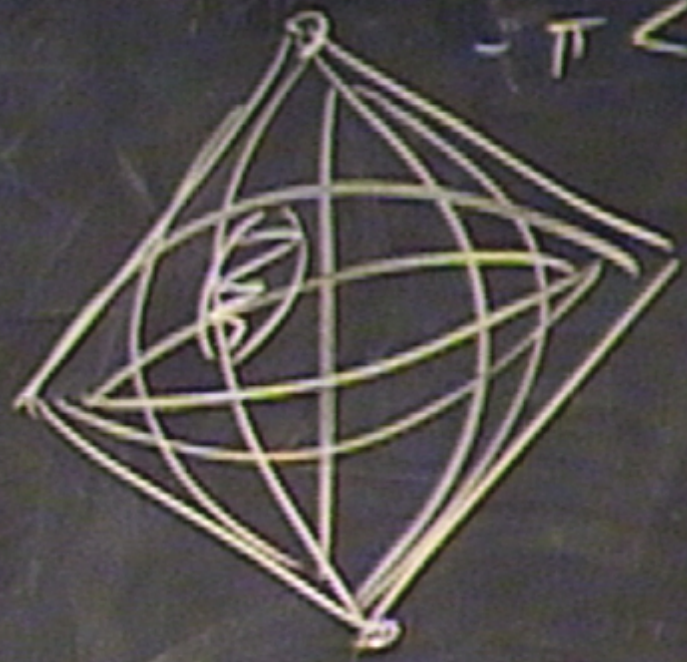
$$\begin{aligned}
 & - \pi < \ell' + \nu' < \pi \\
 & - \pi < \ell' - \nu' < \pi
 \end{aligned}$$



rams

$$-\pi < t' + r' < \pi$$

$$-\pi < t' - r' < \pi$$



z_2^2

$$r = f(t' + r') = \tan\left(\frac{t' + r'}{2}\right)$$

$$r = g(t' - r') = \tan(t' - r')$$

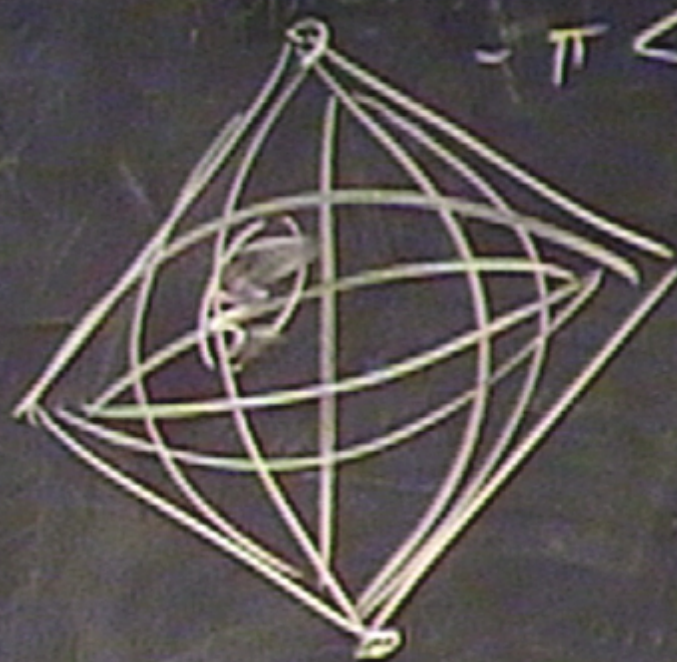
$\delta \varphi$

rams

z_2^2

$$-\pi < t' + v' < \pi$$

$$-\pi < t' - v' < \pi$$



$$\tan\left(\frac{t' + v'}{2}\right) = \tan\left(\frac{t' + v'}{2}\right)$$

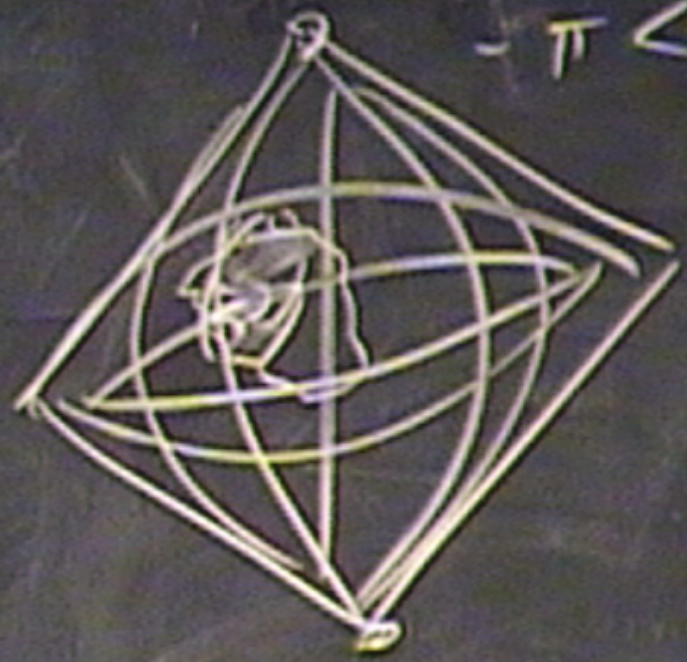
$$\tan(t' - v') = \tan(t' - v')$$

$\delta \varphi$

rams

$$-\pi < t' + r' < \pi$$

$$-\pi < t' - r' < \pi$$



z_2^2

$$r = f(t' + r') = \tan\left(\frac{t' + r'}{2}\right)$$

$$r = g(t' - r') = \tan(t' - r')$$

$\delta \varphi$

2 diagrams

$$\frac{dr^2}{t^2} + r^2 d\Omega_2^2$$

$t = \pm r$



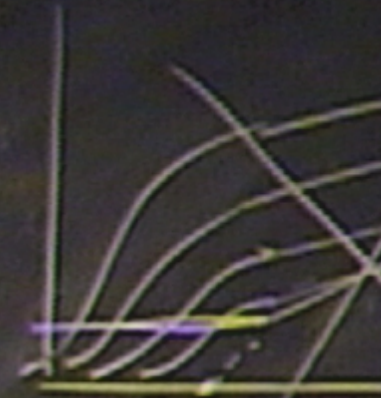
$$-\pi < t' + r' < \pi$$

$$-\pi < t' - r' < \pi$$

$$\begin{cases} t + r = f(t' + r') = \tan\left(\frac{t' + r'}{2}\right) \\ t - r = g(t' - r') = \tan\left(\frac{t' - r'}{2}\right) \end{cases}$$

$$ds^2 = dt^2 - dr^2 = f'g'(dt'^2 - dr'^2)$$

$$\sqrt{\frac{f'}{g'}}$$



$$\delta \varphi(t, x) = \int d$$

$$\lambda = \frac{2\pi}{k}$$

$$\Delta h \cdot \Delta h_r = 0$$

2 diagrams

$$\sqrt{dr^2 + r^2 d\Omega^2}$$

$$t = \pm r$$



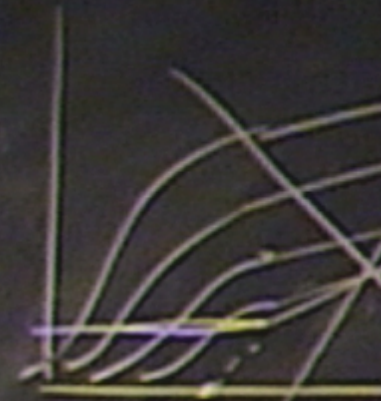
$$-\pi < t' + r' < \pi$$

$$-\pi < t' - r' < \pi$$

$$\begin{cases} t + r = f(t' + r') = \tan\left(\frac{t' + r'}{2}\right) \\ t - r = g(t' - r') = \tan\left(\frac{t' - r'}{2}\right) \end{cases}$$

$$ds^2 = dt^2 - dr^2 = f'g'(dt'^2 - dr'^2)$$

$$\sqrt{\frac{f'}{g'}}$$

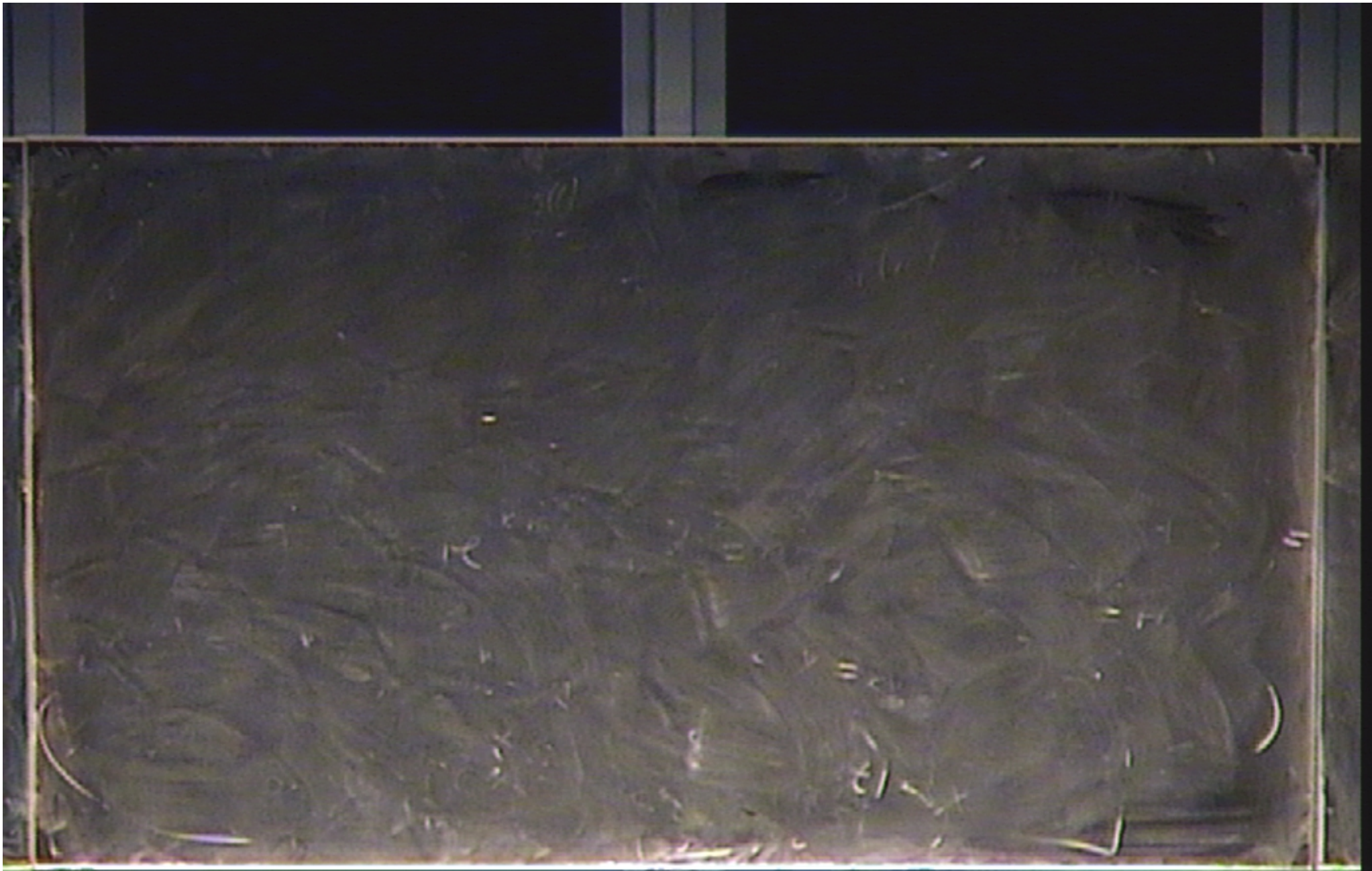


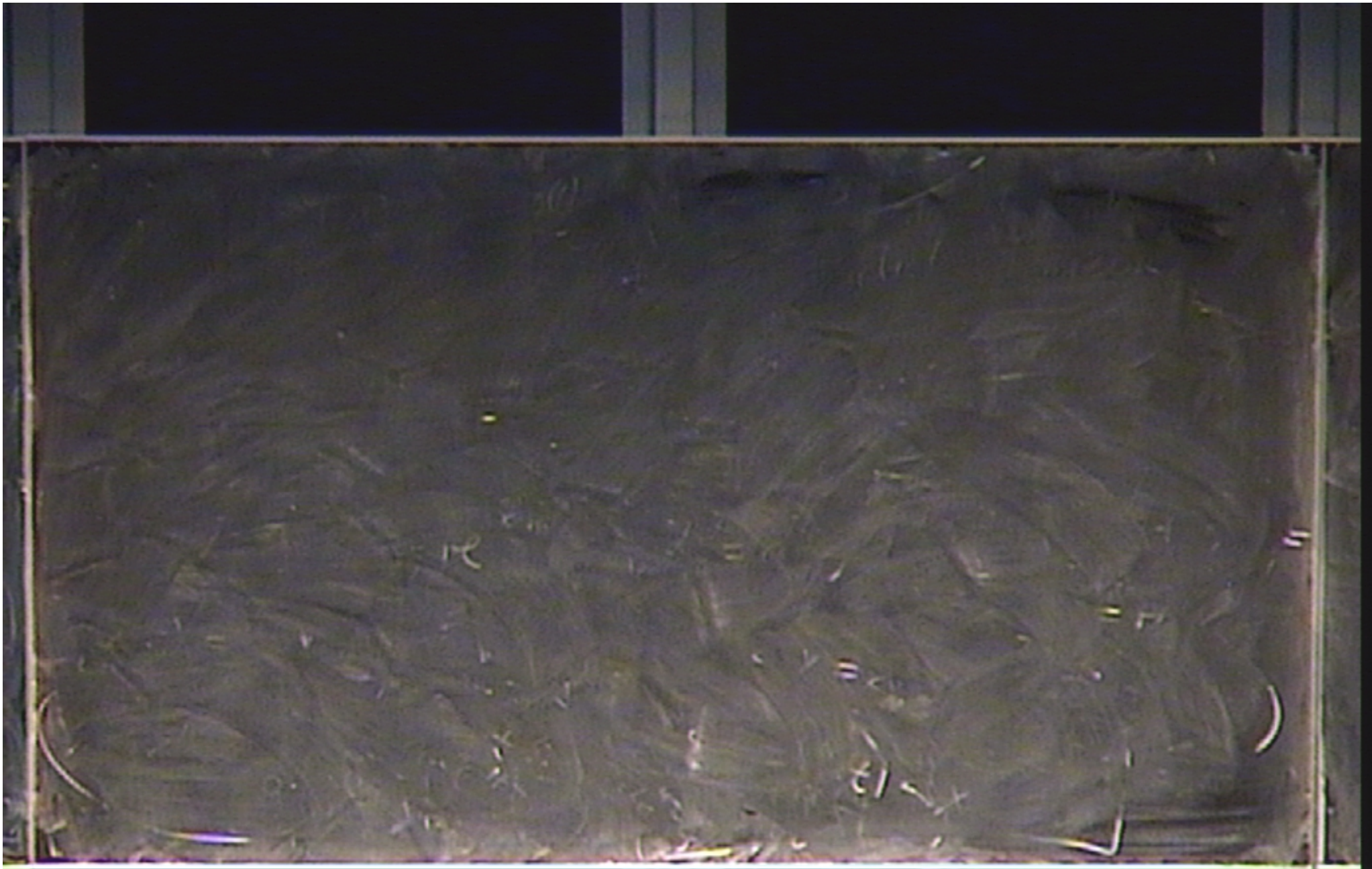
$$\delta \varphi(t, x) = \int d$$

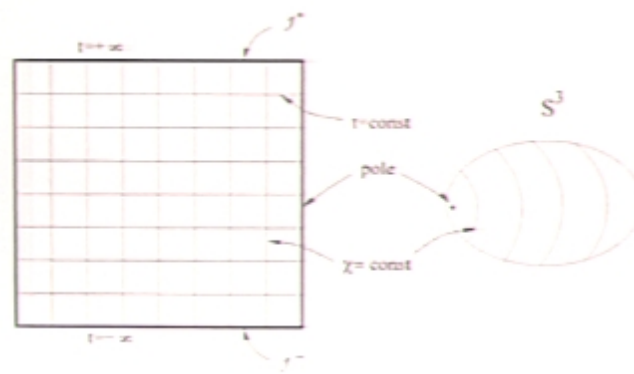
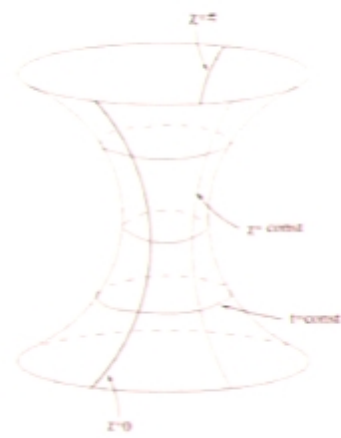
$$\lambda = \frac{2\pi}{k}$$

$$\Delta h \cdot \Delta h_r = 0$$









R^5

$$ds^2 = -dv^2 + dw^2 + dx^2 + dy^2 + dz^2$$

$$+v^2 + w^2 + x^2 + y^2 + z^2 = \frac{1}{4\kappa}$$

$$v \rightarrow iv$$

$$S^5 \quad S(1,4)$$

R^5

$$ds^2 = -dv^2 + dw^2 + dx^2 + dy^2 + dz^2$$

$$+v^2 + w^2 + x^2 + y^2 + z^2 = \frac{1}{4c}$$

$$v \rightarrow iv$$

$$S^5 \quad S(1,4)$$

d

$$R^5 \quad ds^2 = -dv^2 + dw^2 + dx^2 + dy^2 + dz^2$$

$$+v^2 + w^2 + x^2 + y^2 + z^2 = \frac{1}{4c}$$

$$v \rightarrow iv$$

$$S^5 \quad S(1,4)$$

$$ds^2$$

R^5

$$ds^2 = -dv^2 + dw^2 + dx^2 + dy^2 + dz^2$$

$$+v^2 + w^2 + x^2 + y^2 + z^2 = \frac{1}{4c}$$

$$v \rightarrow iv$$

$$S^5 \quad S(1,4)$$

$$ds^2 =$$

$$R^5 \quad ds^2 = -dv^2 + dw^2 + dx^2 + dy^2 + dz^2$$

$$+v^2 + w^2 + x^2 + y^2 + z^2 = \frac{1}{4c}$$

$v \rightarrow i$

$$S^5 \quad S(1,4)$$

$$R^5 \quad ds_5^2 = -dv^2 + dw^2 + dx^2 + dy^2 + dz^2$$

$$+v^2 + w^2 + x^2 + y^2 + z^2 = \frac{1}{4\epsilon}$$

$$v \rightarrow iv$$

$$S^5 \quad S(1,4)$$

$$ds_4^2$$

R^5

$$ds^2 = -dv^2 + dw^2 + dx^2 + dy^2 + dz^2$$

$$+v^2 + w^2 + x^2 + y^2 + z^2 = \frac{1}{H^2}$$

$$v \rightarrow iv$$

S^5

4)

$$-\frac{1}{H^2} \cos^2 \theta +$$

R^5

$$ds^2 = -dv^2 + dw^2 + dx^2 + dy^2 + dz^2$$

$$+v^2 + w^2 + x^2 + y^2 + z^2 = \frac{1}{H^2}$$

$$v \rightarrow iv$$

S^5

4)

$$= \frac{1}{H^2} \cos^2 Ht$$

$$R^5 \quad ds_5^2 = -dv^2 + dw^2 + dx^2 + dy^2 + dz^2$$

$$+v^2 + w^2 + x^2 + y^2 + z^2 = \frac{1}{H^2}$$

$$v \rightarrow i\sqrt{v}$$

$$S^5 \quad S(1,4)$$

$$ds_4^2 = dt^2 - \frac{1}{H^2} \omega_A^2 H t + \left[\frac{1}{H^2} \omega_A^2 H t + (i \odot dp^2) \right]$$

$$R^5 \quad ds_5^2 = -dv^2 + dw^2 + dx^2 + dy^2 + dz^2$$

$$+v^2 + w^2 + x^2 + y^2 + z^2 = \frac{1}{H^2}$$

$$v \rightarrow iv$$

$$S^5 \quad S(1,4)$$

$$ds_4^2 = dt^2 - \frac{1}{H^2} [$$

$$x(d\theta^2 + \sin^2\theta dp^2)]$$

$$R^5 \quad ds_5^2 = -dv^2 + dw^2 + dx^2 + dy^2 + dz^2$$

$$+v^2 + w^2 + x^2 + y^2 + z^2 = \frac{1}{H^2}$$

$$v \rightarrow i\sqrt{v}$$

$$S^5 \quad S(1,4)$$

$$ds_4^2 = dt^2 - \frac{1}{H^2} \cos^2 \theta \left[dx^2 + \sin^2 \theta (d\theta^2 + \sin^2 \theta dp^2) \right]$$

$$R^5 \quad ds_5^2 = -dv^2 + dw^2 + dx^2 + dy^2 + dz^2$$

$$+v^2 + w^2 + x^2 + y^2 + z^2 = \frac{1}{H^2}$$

$$v \rightarrow i\sqrt{v}$$

$$S^4 \quad S(1,4)$$

$$ds_4^2 = dt^2 - \frac{1}{H^2} \cos^2 Ht \left[dx^2 + \sin^2 x (d\theta^2 + \sin^2 \theta dp^2) \right]$$

$$R^5 \quad ds_5^2 = -dv^2 + dw^2 + dx^2 + dy^2 + dz^2$$

$$+v^2 + w^2 + x^2 + y^2 + z^2 = \frac{1}{H^2}$$

$$v \rightarrow iv$$

$$S^4 (1, 4)$$

$$ds_4^2 = \frac{1}{H^2} \cos^2 \theta \left[dx^2 + \sin^2 \theta (d\theta^2 + \sin^2 \theta d\varphi^2) \right]$$

$$R^5 \quad ds_5^2 = -dv^2 + dw^2 + dx^2 + dy^2 + dz^2$$

$$+v^2 + w^2 + x^2 + y^2 + z^2 = \frac{1}{H^2}$$

$$v \rightarrow i\sqrt{}$$

$$S^4 \quad S(1,4)$$

$$ds_4^2 = dt^2 - \frac{1}{H^2} \cos^2 \theta \left[dx^2 + \sin^2 \theta (d\theta^2 + \sin^2 \theta dp^2) \right]$$

$$g(t, r, \theta, \phi)$$

$$t_1$$

$$R^5 \quad ds_5^2 = -dv^2 + dw^2 + dx^2 + dy^2 + dz^2$$

$$+v^2 + w^2 + x^2 + y^2 + z^2 = \frac{1}{H^2}$$

$$v \rightarrow i\sqrt{v}$$

$$S^4 \quad S(1,4)$$

$$ds_4^2 = dt^2 - \frac{1}{H^2} \cos^2 \theta \left[dx^2 + \sin^2 \theta (d\theta^2 + \sin^2 \theta d\varphi^2) \right]$$

$$g_{ij}(x)$$

$$R^5 \quad ds_5^2 = -dv^2 + dw^2 + dx^2 + dy^2 + dz^2$$

$$+v^2 + w^2 + x^2 + y^2 + z^2 = \frac{1}{H^2}$$

$$v \rightarrow iv$$

$$S^4 \quad S(1,4)$$

$$ds_4^2 = dt^2 - \frac{1}{H^2} \cos^2 Ht \left[dx^2 + \sin^2 x (d\theta^2 + \sin^2 \theta dp^2) \right]$$

$$g_{ij}(x)$$

$$R^5 \quad ds_5^2 = -dv^2 + dw^2 + dx^2 + dy^2 + dz^2$$

$$+v^2 + w^2 + x^2 + y^2 + z^2 = \frac{1}{H^2}$$

$$v \rightarrow iv$$

$$S^4 \quad S(1,4)$$

$$ds_4^2 = dt^2 - \frac{1}{H^2} \cos^2 Ht \left[dx^2 + \sin^2 x (d\theta^2 + \sin^2 \theta dp^2) \right]$$

$$g(t, x)$$

$$R^5 \quad ds_5^2 = -dv^2 + dw^2 + dx^2 + dy^2 + dz^2$$

$$+v^2 + w^2 + x^2 + y^2 + z^2 = \frac{1}{H^2}$$

$$v \rightarrow iv$$

$$S^4 \quad S(1,4)$$

$$ds_4^2 = dt^2 - \frac{1}{H^2} \cos^2 Ht \left[dx^2 + \sin^2 x (d\theta^2 + \sin^2 \theta dp^2) \right]$$

$$g_{ij}(x)$$

$$R^5 \quad ds_5^2 = -dv^2 + dw^2 + dx^2 + dy^2 + dz^2$$

$$+v^2 + w^2 + x^2 + y^2 + z^2 = \frac{1}{H^2}$$

$$v \rightarrow i\sqrt{}$$

$$S^4 \quad S(1,4)$$

$$ds_4^2 = dt^2 - \frac{1}{H^2} \cos^2 Ht \left[dx^2 + \sin^2 x (d\theta^2 + \sin^2 \theta dp^2) \right]$$

$$g(t, x)$$

π
 π

R^5

$$ds_5^2 = -dv^2 + dw^2 + dx^2 + dy^2 + dz^2$$

$$+v^2 + w^2 + x^2 + y^2 + z^2 = \frac{1}{H^2}$$

$$v \rightarrow iv$$

$$S^4 \quad S(1,4)$$

$$ds_4^2 = dt^2 - \frac{1}{H^2} \cos^2 Ht \left[dx^2 + \sin^2 Ht \right]$$

$$a^2(t) \propto t^2$$

Penrose diagrams

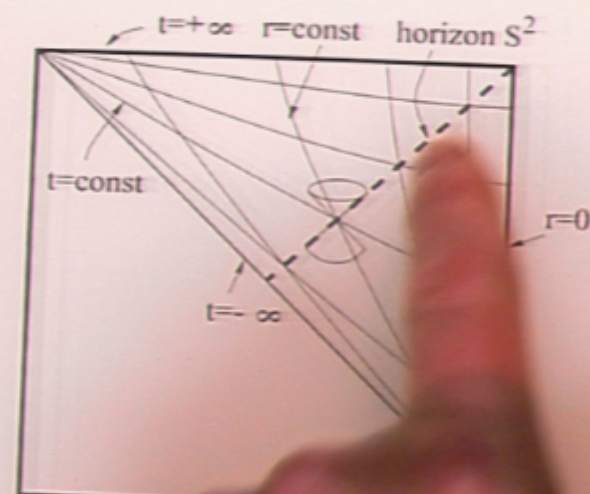
$$ds^2 = dt^2 - e^{Ht} dx^2$$

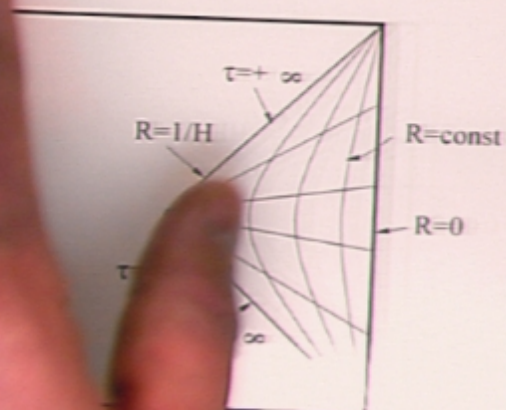
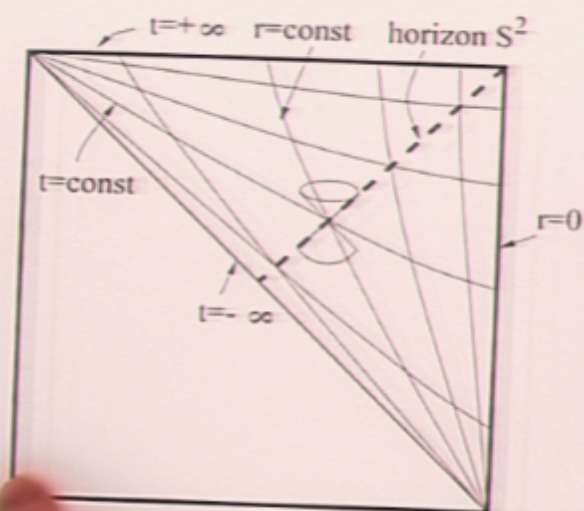


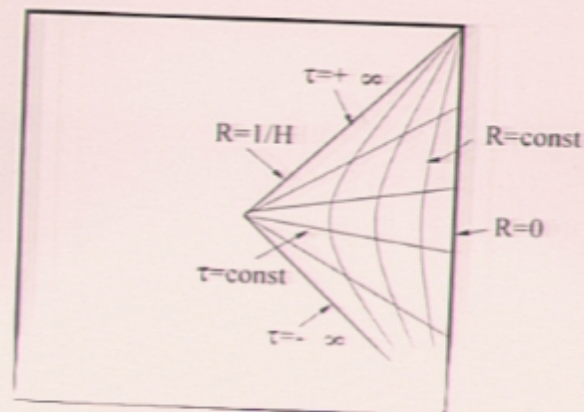
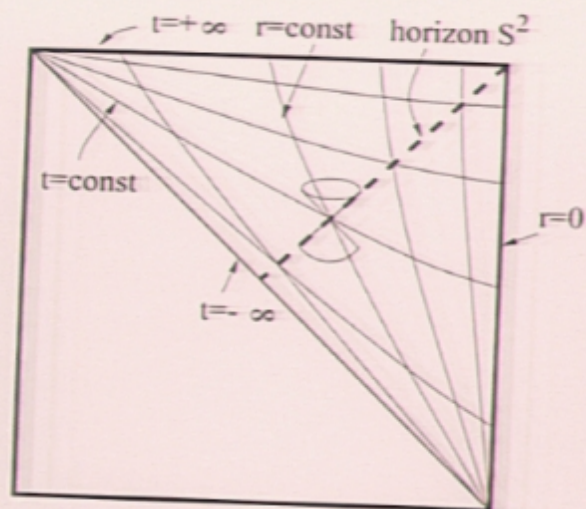
Penrose diagrams

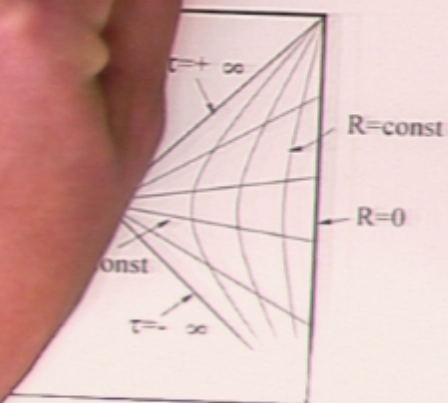
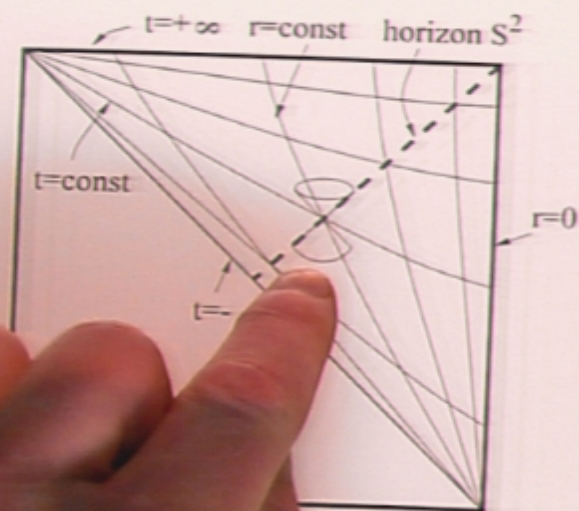
$$ds^2 = dt^2 - e^{2Ht} d\vec{x}^2$$

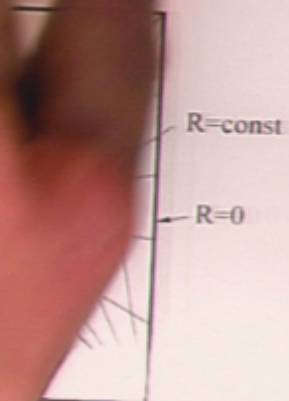


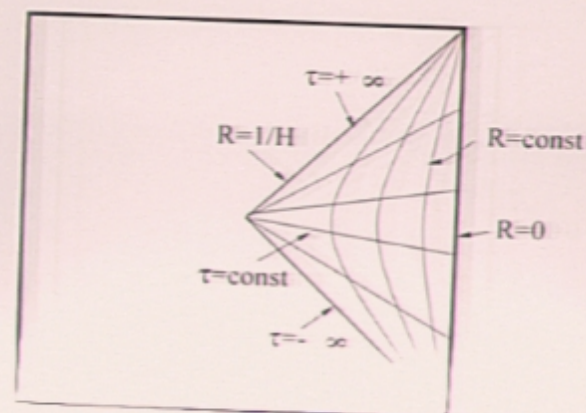
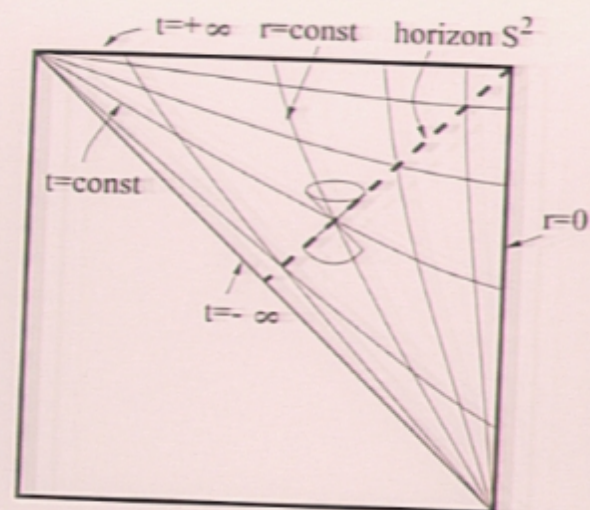


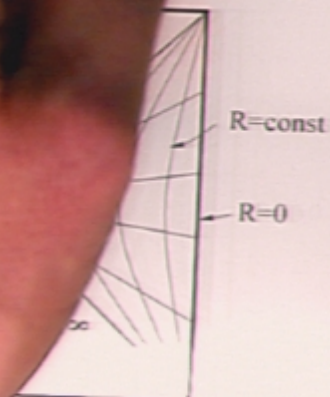
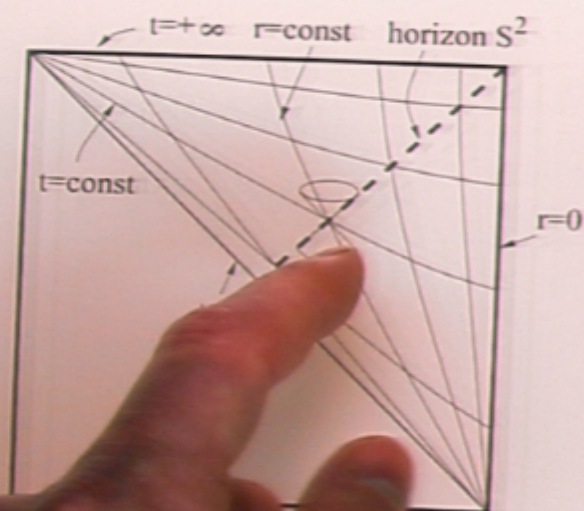


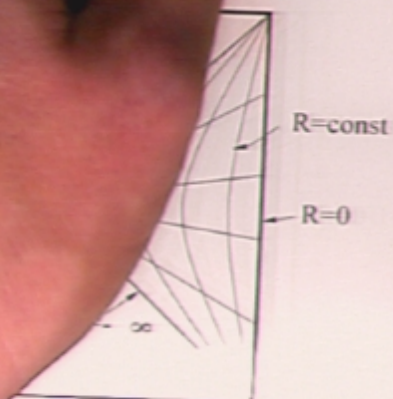
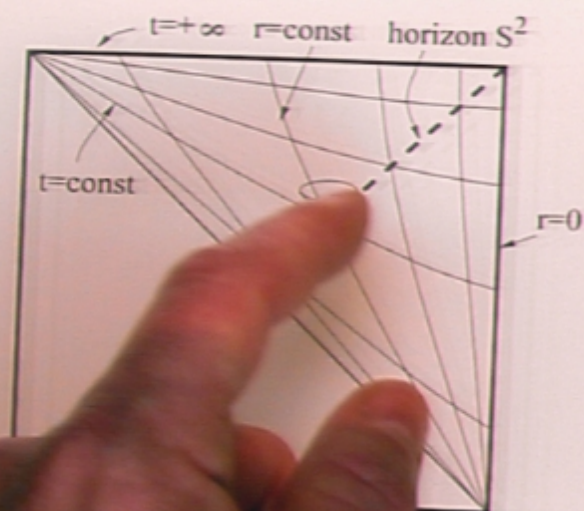


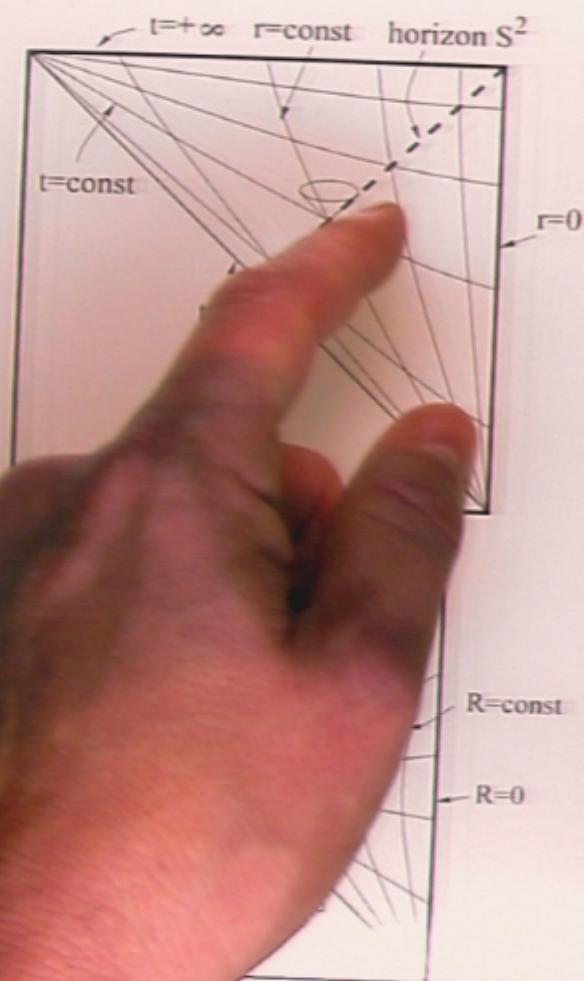


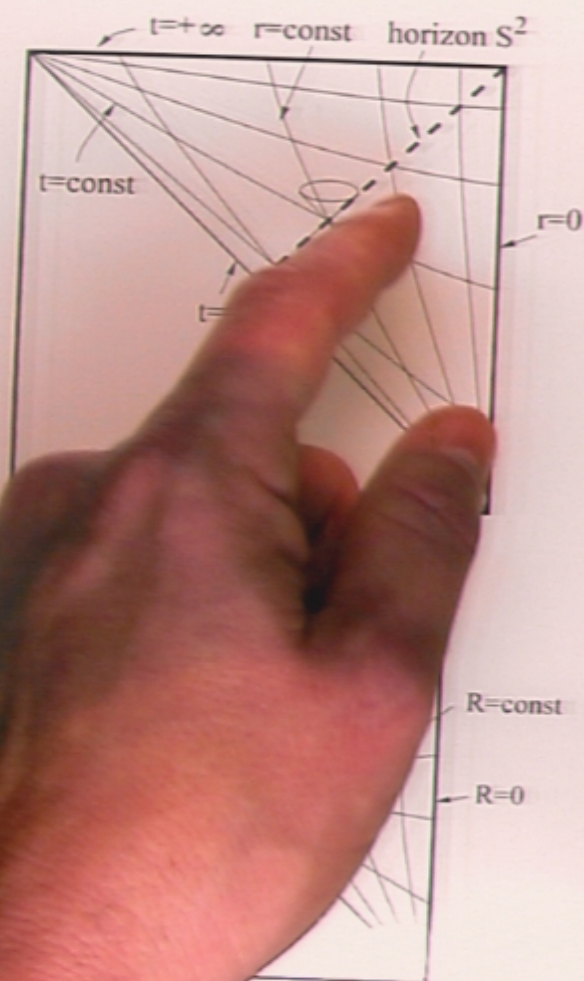


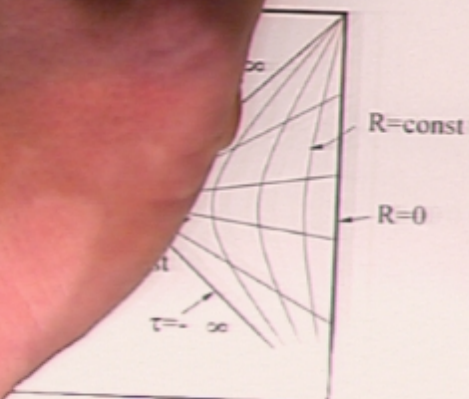
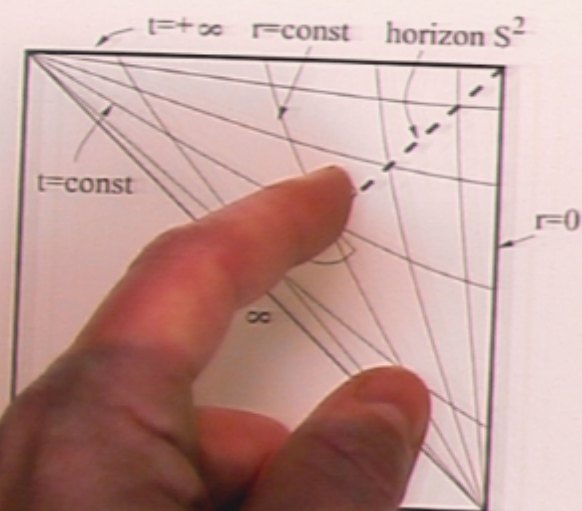


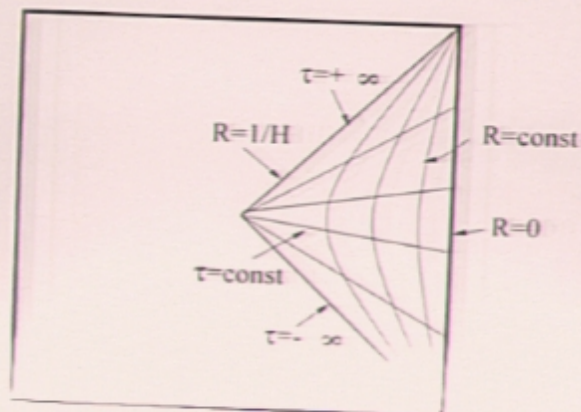
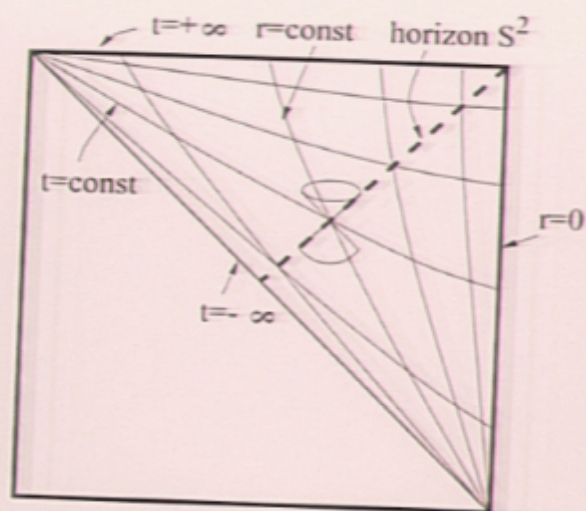


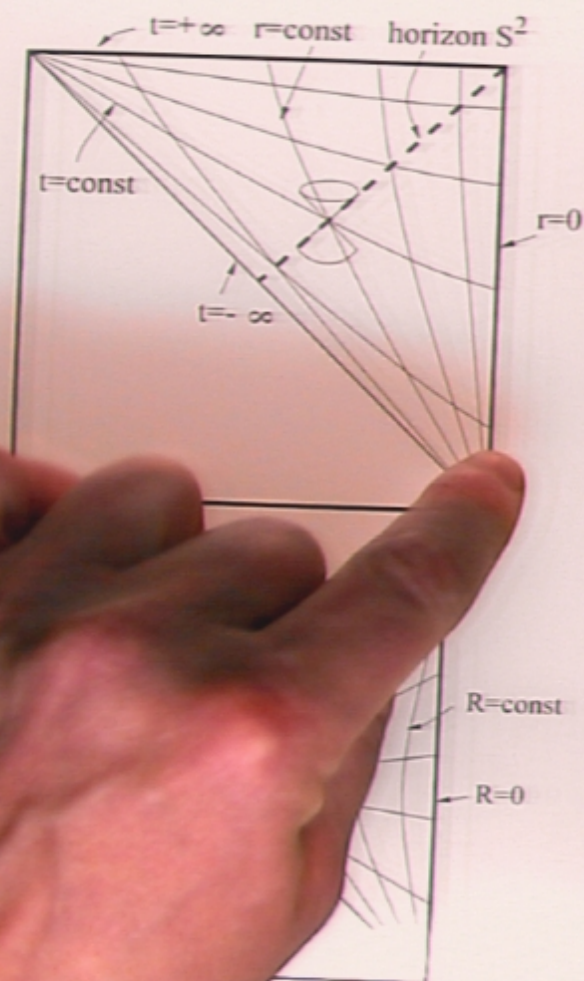


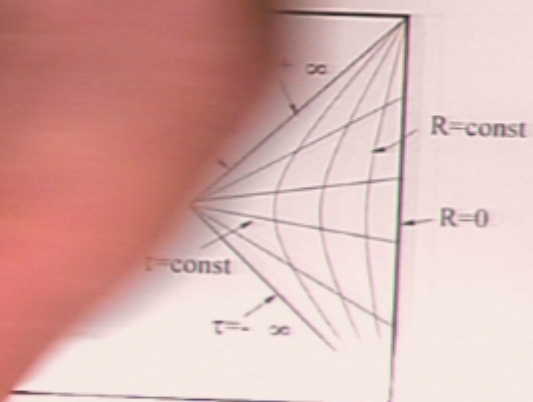
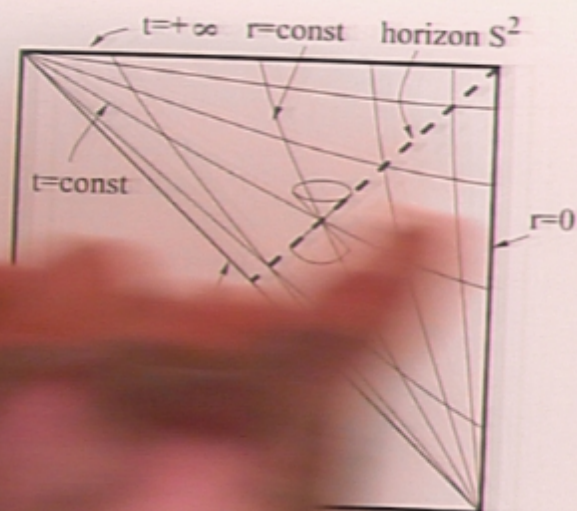


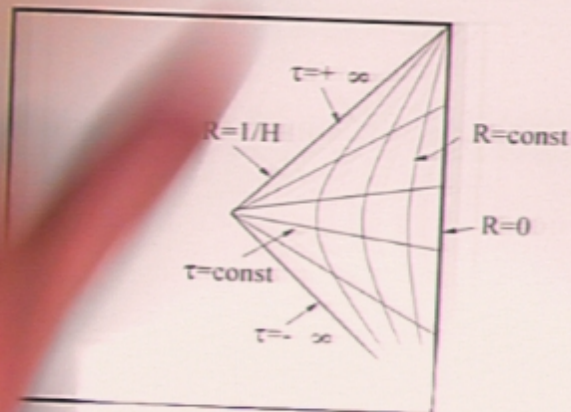
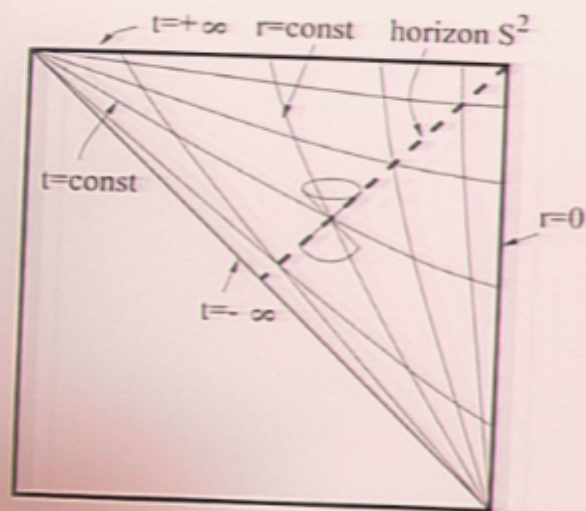


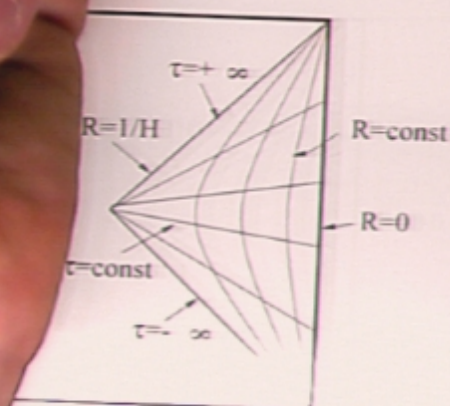
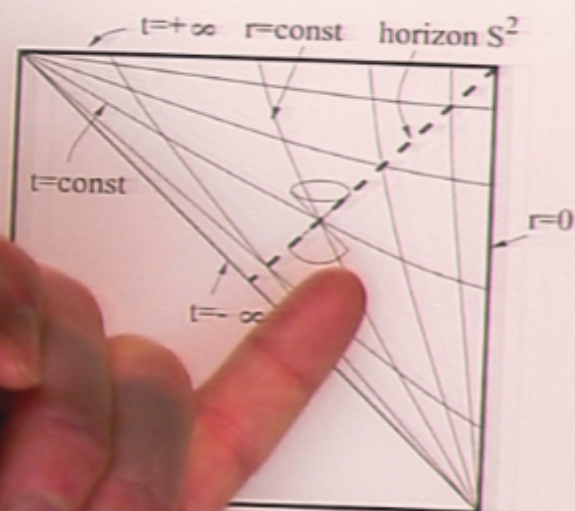


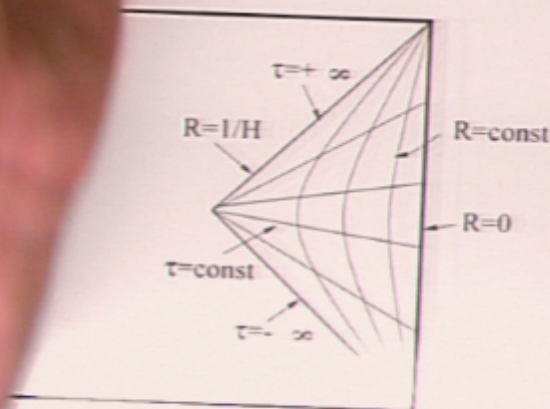
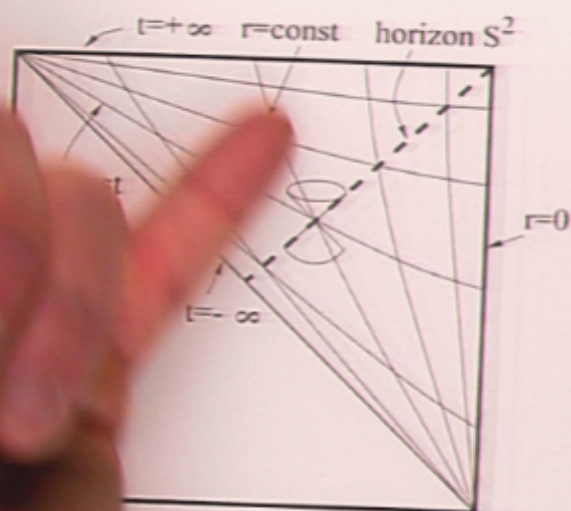


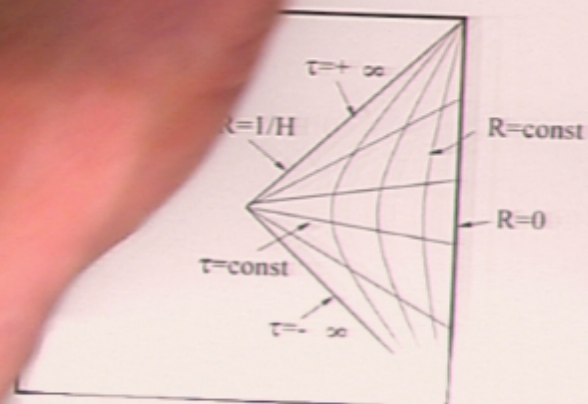


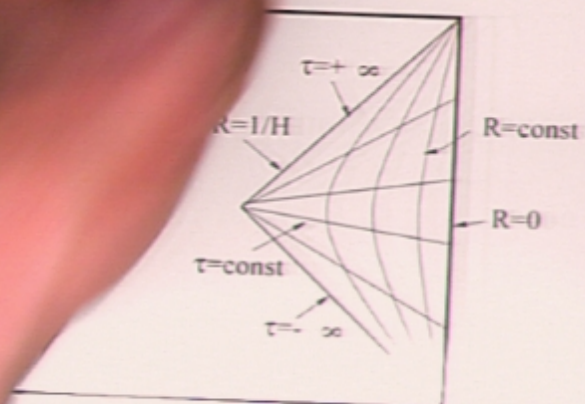
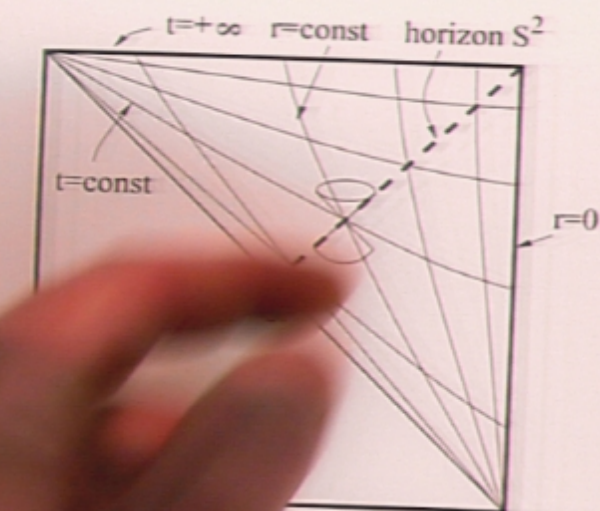


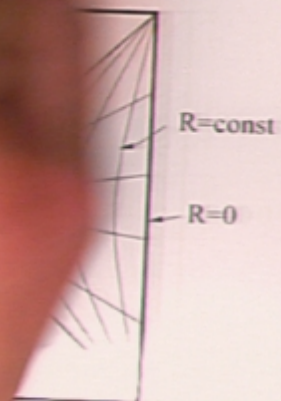
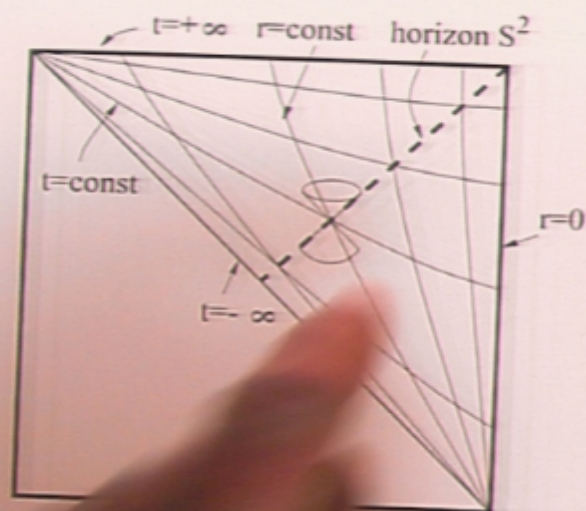




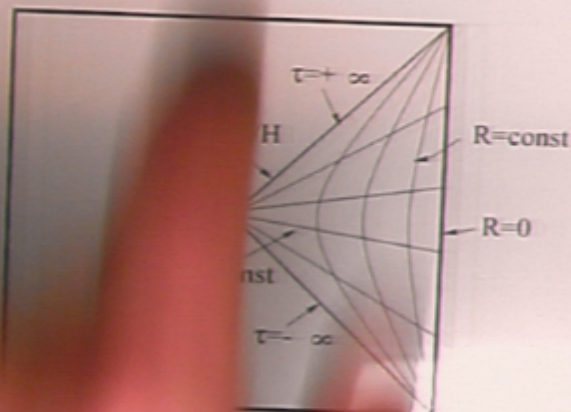
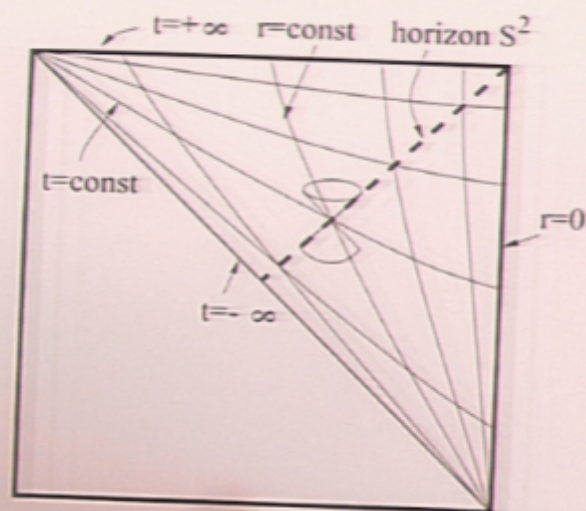


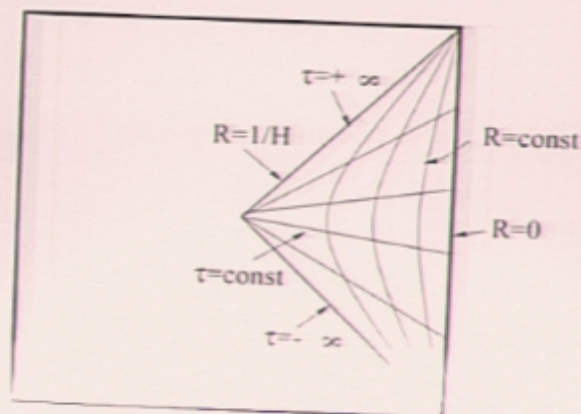
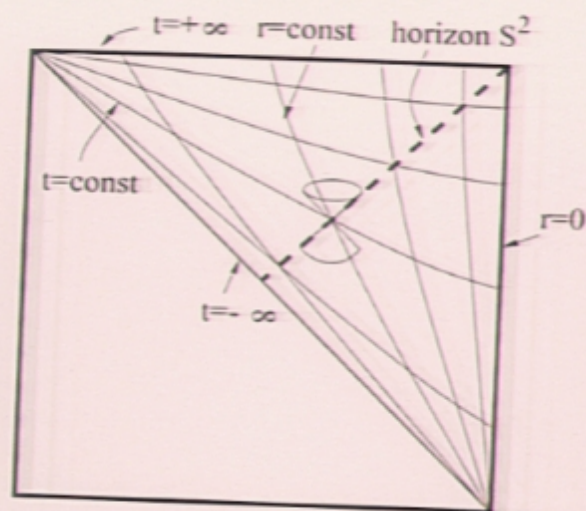






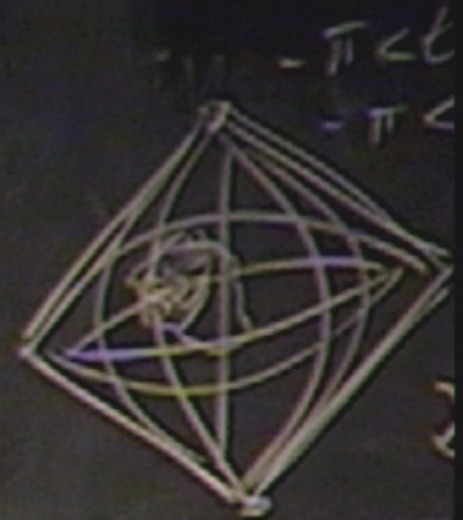






Penrose diagrams

$$ds^2 = dt^2 - e^{2Ht} d\vec{x}^2$$



Penrose diagrams

$$1 - \frac{2H^2 t^2}{c^2} = e^{-2\chi} d\vec{x}^2$$

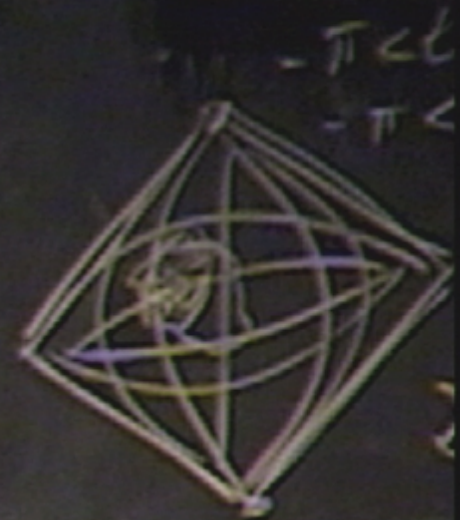
$$= 1 - H^2 R^2$$



Penrose diagrams

$$ds^2 = dt^2 - d\vec{x}^2$$

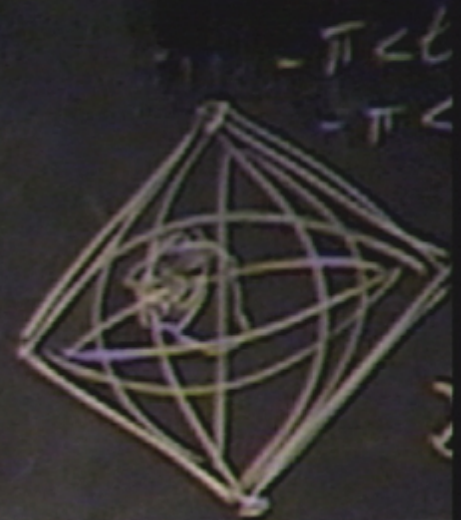
$$ds^2 = (H^2 R^2) d\tau^2$$



Penrose diagrams

$$ds^2 = dt^2 - e^{2Ht} (dx^2 + dy^2 + dz^2)$$

$$ds^2 = (1 - \frac{r^2}{R^2}) dt^2 - \frac{dr^2}{1 - \frac{r^2}{R^2}} - r^2 d\Omega^2$$



Penrose diagrams

$$ds^2 = dt^2 - e^{2Ht} d\vec{x}^2$$



$$L^2(1 - H^2 R^2) dR^2 - \frac{dR^2}{1 - H^2 R^2} - R^2 d\Omega_2^2$$

Penrose diagrams

$$ds^2 = dt^2 - e^{2Ht} d\vec{x}^2$$



$$ds^2 = R^2(dR^2 - \frac{dR^2}{1-H^2R^2} - R^2 d\Omega_2^2)$$

Penrose diagrams

$$ds^2 = dt^2 - e^{2Ht} d\vec{x}^2$$

$$ds^2 = (1 - H^2 R^2) dR^2 - \frac{dR^2}{1 - H^2 R^2} - R^2 d\Omega_2^2$$



Penrose diagrams

$$ds^2 = dt^2 - e^{2Ht} d\vec{x}^2$$

$$ds^2 = (1 - H^2 R^2) dR^2 - \frac{dR^2}{1 - H^2 R^2} - R^2 d\Omega_2^2$$

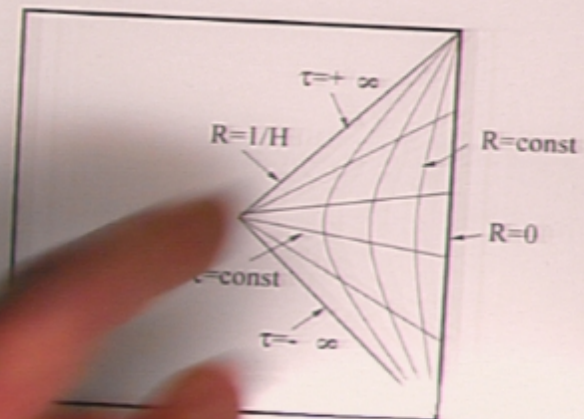
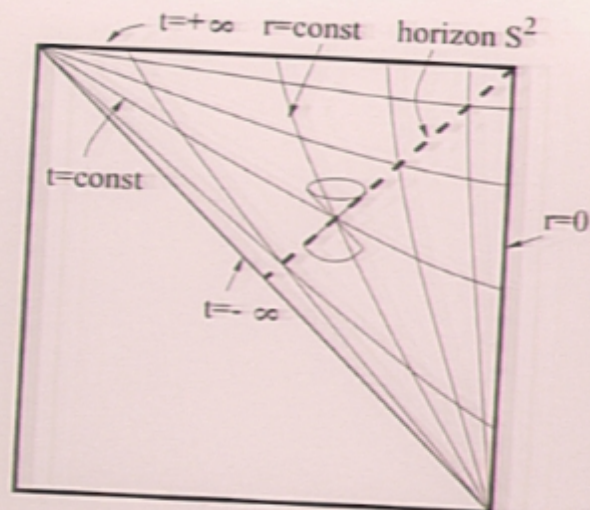


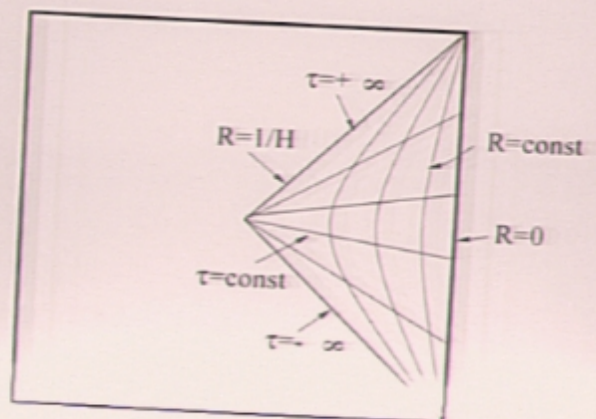
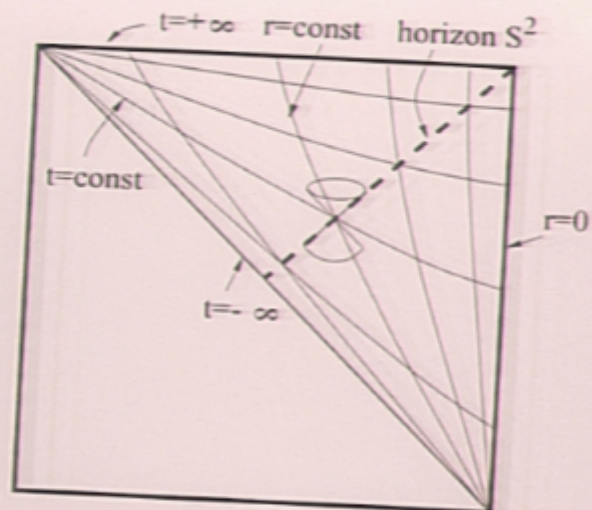
Penrose diagrams

$$ds^2 = dt^2 - e^{2Ht} d\vec{x}^2$$

$$ds^2 = (1 - H^2 R^2) dR^2 - \frac{dR^2}{1 - H^2 R^2} - R^2 d\Omega_2^2$$







Penrose diagrams

$$ds^2 = dt^2 - e^{2Ht} d\vec{x}^2$$

$$ds^2 = (1 - H^2 R^2) dR^2 - \frac{dR^2}{1 - H^2 R^2} - R^2 d\Omega_2^2$$

$$R = \frac{1}{H}$$



$$R^5 \quad ds_5^2 = -dv^2 + dw^2 + dx^2 + dy^2 + dz^2$$

$$+v^2 + w^2 + x^2 + y^2 + z^2 = \frac{1}{H^2}$$

$$v \rightarrow i v$$

$$S^4 \quad S(1,4)$$

$$ds_4^2 = dt^2 - \frac{1}{H^2} \omega_H^2 + \left[dx^2 + \sin^2 x (d\theta^2 + \sin^2 \theta dp^2) \right]$$

$$g(r, \theta)$$



R^5

$$ds_5^2 = -dv^2 + dw^2 + dx^2 + dy^2 + dz^2$$

$$+v^2 + w^2 + x^2 + y^2 + z^2 = \frac{1}{H^2}$$



$$v \rightarrow i v$$

$$S^7 \quad S(1,4)$$

$$ds_4^2 = dt^2 - \frac{1}{H^2} \cos^2 Ht \left[dx^2 + \sin^2 x (d\theta^2 + \sin^2 \theta dp^2) \right]$$

$g_{ij}(x)$

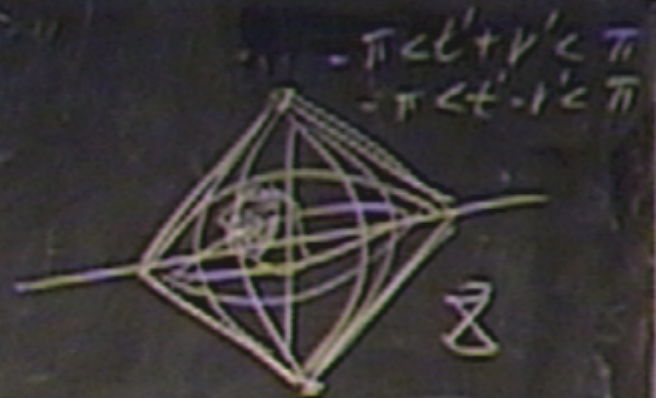


Penrose diagrams

$$ds^2 = dt^2 - e^{2Ht} d\vec{x}^2$$

$$ds^2 = (1 - H^2 R^2) dR^2 - \frac{dR^2}{1 - H^2 R^2} - R^2 d\Omega^2$$

$$R = \frac{1}{H}$$



$$R^5 \quad ds^2 = -dv^2 + dw^2 + dx^2 + dy^2 + dz^2$$

$$+v^2 + w^2 + x^2 + y^2 + z^2 = \frac{1}{H^2}$$



$$v \rightarrow i v$$

$$S^4 \quad S(1,4)$$

$$ds^2 = dt^2 - \frac{1}{H^2} \left[dx^2 + \sin^2 x (d\theta^2 + \sin^2 \theta dp^2) \right]$$

$$S^3 \quad S^2$$



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R^5

$$ds_5^2 = -dv^2 + dw^2 + dx^2 + dy^2 + dz^2$$

$$v^2 + w^2 + x^2 + y^2 + z^2 = \frac{1}{H^2}$$

$$v \rightarrow iv$$

$$S^4 \quad S(1,4)$$

$$ds_4^2 = dt^2 - \frac{1}{H^2} \cos^2 Ht \left[dx^2 + dy^2 + dz^2 + d\phi^2 \right]$$

$$a(r, t)$$



R^5

$$ds_5^2 = -dv^2 + dw^2 + dx^2 + dy^2 + dz^2$$

$$+v^2 + w^2 + x^2 + y^2 + z^2 = \frac{1}{H^2}$$

$$v \rightarrow iv$$

$$S^4 \quad S(1,4)$$

$$ds_4^2 = dt^2 - \frac{1}{H^2} \cos^2 Ht \left[dx^2 + \dots \right]$$

$$a(r, t)$$



R^5

$$ds_5^2 = -dv^2 + dw^2 + dx^2 + dy^2 + dz^2$$

$$v^2 + w^2 + x^2 + y^2 + z^2 = \frac{1}{H^2}$$

$$v \rightarrow iv$$

$$S^4 \quad S(1,4)$$

$$ds_4^2 = dt^2 - \frac{1}{H^2} \cos^2 Ht \left[dx^2 + \right.$$

$$a^2(x, y)$$



$$R^5 \quad ds_5^2 = -dv^2 + dw^2 + dx^2 + dy^2 + dz^2$$

$$+v^2 + w^2 + x^2 + y^2 + z^2 = \frac{1}{H^2}$$

$$v \rightarrow i v$$

$$S^7 \quad S(1,4)$$

$$ds_4^2 = dt^2 - \frac{1}{H^2} \omega$$

$$dx^2 + \sin^2 x (d\theta^2 + \sin^2 \theta dy^2)$$

$$R^5 \quad ds_5^2 = -dv^2 + dw^2 + dx^2 + dy^2 + dz^2$$

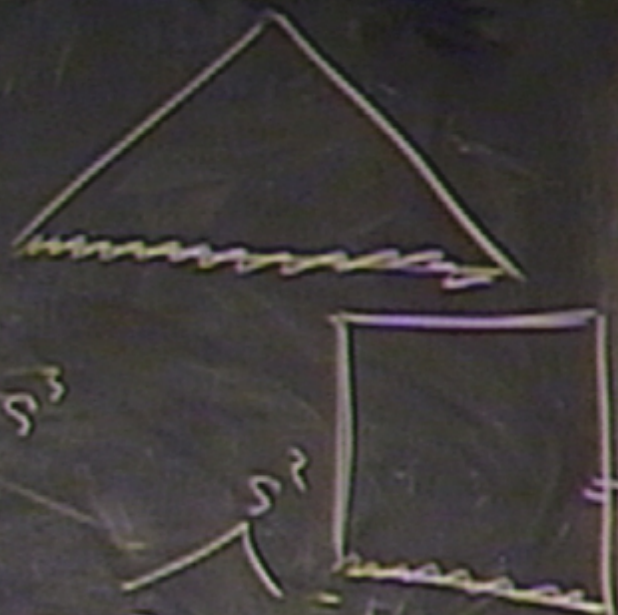
$$+v^2 + w^2 + x^2 + y^2 + z^2 = \frac{1}{H^2}$$

$$v \rightarrow iv$$

$$S^4 \quad S(1,4)$$

$$ds_4^2 = dt^2 - \frac{1}{H^2} \omega_H^2 dt + \left[dx^2 + \sin^2 x (d\theta^2 + \sin^2 \theta dy^2) \right]$$

$$g_{ij}(x)$$

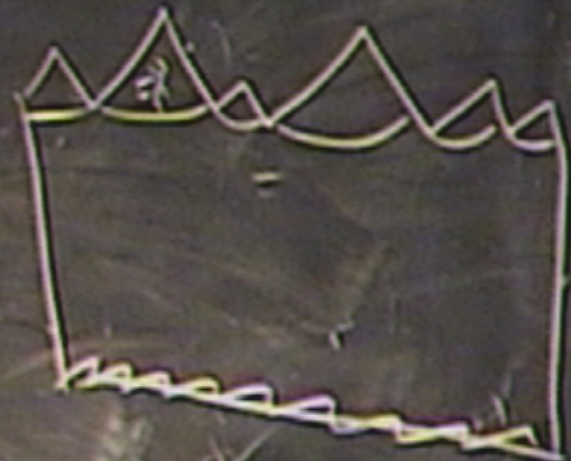
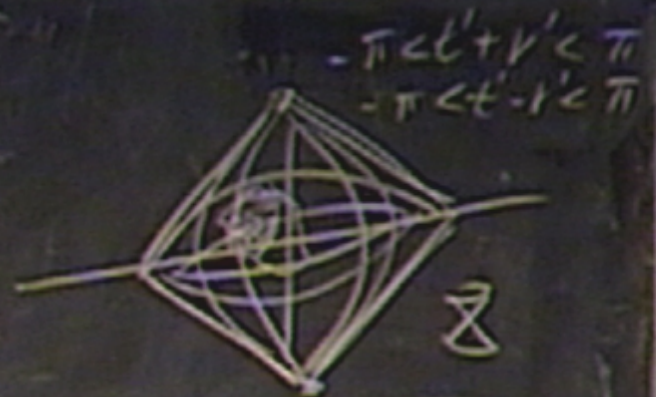


Penrose diagrams

$$ds^2 = dt^2 - e^{2Ht} d\vec{x}^2$$

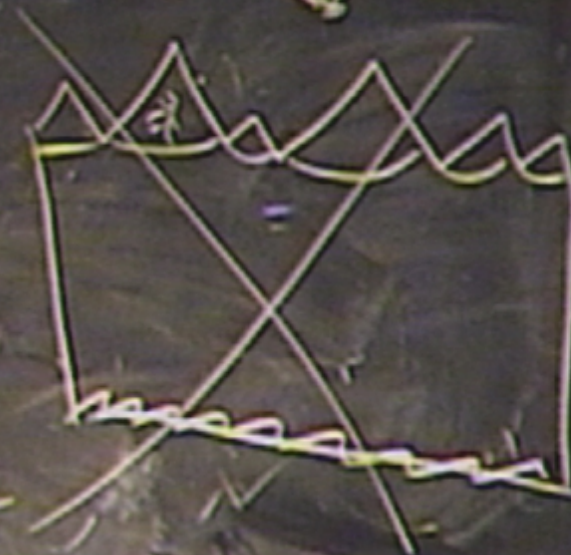
$$ds^2 = (1 - H^2 R^2) dR^2$$

$$R = \frac{1}{H}$$



$$-\pi < t' + v' < \pi$$

$$-\pi < t' - v' < \pi$$



Penrose diagrams

$$\begin{aligned} -\pi < t' + v' < \pi \\ -\pi < t' - v' < \pi \end{aligned}$$

Penrose diagrams

$$-\pi < t' + v' < \pi$$

$$-\pi < t' - v' < \pi$$

Penrose diagrams

$$\begin{aligned} & -\pi \leq t' \leq \pi \\ & -\pi \leq t' \leq \pi \end{aligned}$$

R^M

Penrose diagrams

$$R^{\mu}_{\nu} - \frac{1}{2} \delta^{\mu}_{\nu} R = 8\pi G T^{\mu}_{\nu}$$

$$\begin{aligned} & -\pi \epsilon' + \\ & -\pi \epsilon' \end{aligned}$$

Penrose diagrams

$$R^{\mu}_{\nu} - \frac{1}{2} \delta^{\mu}_{\nu} R = 8\pi G T^{\mu}_{\nu}$$

$$- \pi \epsilon' +$$

$$- \pi \epsilon'$$

Penrose diagrams

$$R^{\mu}_{\nu} - \frac{1}{2} \delta^{\mu}_{\nu} R = 8\pi G \cdot T^{\mu}_{\nu}$$

Penrose diagrams

$$R^{\mu}_{\nu} - \frac{1}{2} \delta^{\mu}_{\nu} R = 8\pi G T^{\mu}_{\nu}$$

$$\begin{aligned} & -\pi \epsilon' + \\ & -\pi \epsilon' \end{aligned}$$

Penrose diagrams

$\rightarrow$
 $-\pi < t' + 1$
 $-\pi < t'$

$$R^{\mu}_{\nu} - \frac{1}{2} \delta^{\mu}_{\nu} R = 8\pi G T^{\mu}_{\nu}$$

Penrose diagrams

$$R^{\mu}_{\nu} - \frac{1}{2} \delta^{\mu}_{\nu} R = 8\pi G T^{\mu}_{\nu}$$

$$ds^2 = dt^2$$

$$\begin{aligned} & -\pi < t' + 1 \\ & -\pi < t' \end{aligned}$$

Penrose diagrams

$$R^{\mu}_{\nu} - \frac{1}{2} \delta^{\mu}_{\nu} R = 8\pi G \cdot T^{\mu}_{\nu}$$

$$ds^2 = dt^2 - t^2 d\alpha^2$$

Penrose diagrams

$$R^{\mu}_{\nu} - \frac{1}{2} \delta^{\mu}_{\nu} R = 8\pi G \cdot T^{\mu}_{\nu}$$

$$ds^2 = dt^2 - t^2$$

$$a \sim \sqrt{t}$$

Penrose diagrams

$$R^{\mu}_{\nu} - \frac{1}{2} \delta^{\mu}_{\nu} R = 8\pi G T^{\mu}_{\nu}$$

$$= dt^2 - t d\vec{x}$$

$$a \sim \sqrt{t}$$
$$a^2 \sim t$$

Penrose diagrams

$$R^M_{\nu} - \frac{1}{2} \delta^M_{\nu} R = 8\pi G \cdot T^M_{\nu}$$

$$ds^2 = dt^2 - t^2 d\vec{x}^2$$

$$a \sim \sqrt{t}$$

$$a^2 \sim t$$

Penrose diagrams

$$\begin{aligned} & -\pi \leq t' + x' \leq \pi \\ & -\pi \leq t' - x' \leq \pi \end{aligned}$$

$$R^M_{\nu} - \frac{1}{2} \delta^M_{\nu} R = 8\pi G T^M_{\nu}$$

$$ds^2 = dt^2 - t^2 d\vec{x}^2$$

$$\begin{aligned} a &\sim \sqrt{t} \\ a^2 &\sim t \end{aligned}$$

Penrose diagrams

$$R^{\mu}_{\nu} - \frac{1}{2} \delta^{\mu}_{\nu} R = 8\pi G \cdot T^{\mu}_{\nu}$$

$$ds^2 = dt^2 - t^2 d\vec{x}^2$$

$$ds^2 = dt^2$$

$$a \sim \sqrt{t}$$

$$a^2 \sim t$$

Penrose diagrams

$$R^{\mu}_{\nu} - \frac{1}{2} \delta^{\mu}_{\nu} R = 8\pi G \cdot T^{\mu}_{\nu}$$

$$P = \frac{1}{3}$$

$$ds^2 = dt^2 - t^2 d\vec{x}^2$$

$$a \sim \sqrt{t}$$

$$a^2 \sim t$$

$$ds^2 = dt^2$$

$$t=0$$

Penrose diagrams

$$-\pi < t' + 1$$

$$-\pi < t'$$

$$R^{\mu}_{\nu} - \frac{1}{2} \delta^{\mu}_{\nu} R = 8\pi G \cdot T^{\mu}_{\nu}$$

$$P = \frac{1}{3} \epsilon$$

$$P = 0$$

$$t=0$$

$$ds^2 = dt^2 - t^2 d\vec{x}^2$$

$$a \sim \sqrt{t}$$

$$a^2 \sim t$$

$$ds^2 = dt^2$$

Penrose diagrams

$$-\pi < t' + 1$$

$$-\pi < t'$$

$$R^{\mu}_{\nu} - \frac{1}{2} \delta^{\mu}_{\nu} R = 8\pi G \cdot T^{\mu}_{\nu}$$

$$P = \frac{1}{3} \epsilon$$

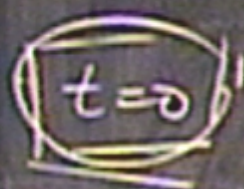
$$P = 0$$

$$ds^2 = dt^2 - t^2 d\vec{x}^2$$

$$a \sim \sqrt{t}$$

$$a^2 \sim t$$

$$ds^2 = dt^2$$



t

Penrose diagrams

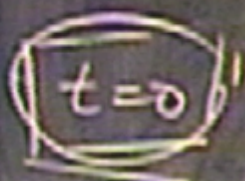
$$-\pi < t' + \dots$$

$$-\pi < t'$$

$$R^{\mu}_{\nu} - \frac{1}{2} \delta^{\mu}_{\nu} R = 8\pi G \cdot T^{\mu}_{\nu}$$

$$P = \frac{1}{3} \epsilon$$

$$P = 0$$



t

$$ds^2 = dt^2 - t^2 d\vec{x}^2$$

$$ds^2 = dt^2$$

$$a \sim \sqrt{t}$$

$$a^2 \sim t$$



Penrose diagrams

$$\begin{aligned} & -\pi < t' + 1 \\ & -\pi < t' \end{aligned}$$

$$R^{\mu}_{\nu} - \frac{1}{2} \delta^{\mu}_{\nu} R = 8\pi G \cdot T^{\mu}_{\nu}$$

$$\begin{aligned} P &= \frac{1}{3} \epsilon \\ P &= 0 \end{aligned}$$

$$ds^2 = dt^2 - t^2 d\vec{x}^2$$

$$\begin{aligned} a &\sim \sqrt{t} \\ a^2 &\sim t \end{aligned}$$

$$ds^2 = dt^2 - \left(t a_{\alpha\beta} + t^2 \beta_{\alpha\beta} \right) dx^{\alpha} dx^{\beta}$$

$$t=0$$

t

Penrose diagrams

$-\pi < t' + 1$
 $-\pi < t'$

$$R^{\mu}_{\nu} - \frac{1}{2} \delta^{\mu}_{\nu} R = 8\pi G \cdot T^{\mu}_{\nu}$$

$$P = \frac{1}{3} \epsilon$$

$$P = 0$$

$t=0$

$$ds^2 = dt^2 - t^2 d\vec{x}^2$$

$$a \sim \sqrt{t}$$

$$a^2 \sim t$$

$$ds^2 = dt^2 - (t a_{\alpha\beta} + t^2 \beta_{\alpha\beta}) dx^{\alpha} dx^{\beta}$$

t

$$-g_{0\alpha} = 1$$

$$a_{\alpha\beta}(t)$$

Penrose diagrams

$$-\pi < t' + 1$$

$$-\pi < t'$$

$$R^{\mu}_{\nu} - \frac{1}{2} \delta^{\mu}_{\nu} R = 8\pi G \cdot T^{\mu}_{\nu}$$

$$P = \frac{1}{3} \epsilon$$

$$P = 0$$

$$t=0$$

$$ds^2 = dt^2 - t^2 d\vec{x}^2$$

$$a \sim \sqrt{t}$$

$$a^2 \sim t$$

$$ds^2 = dt^2 - \left(t a_{\alpha\beta} + t^2 \beta_{\alpha\beta} \right) dx^{\alpha} dx^{\beta}$$

$$a_{\alpha\beta}(t)$$

$$-g_{0\alpha} = 1$$

Penrose diagrams

$$\begin{aligned} & -\pi < t' + \dots \\ & -\pi < t' \dots \end{aligned}$$

$$R^{\mu}_{\nu} - \frac{1}{2} \delta^{\mu}_{\nu} R = 8\pi G \cdot T^{\mu}_{\nu}$$

$$\begin{aligned} P &= \frac{1}{3} \epsilon \\ P &= 0 \end{aligned}$$

$$ds^2 = dt^2 - t^2 d\vec{x}^2$$

$$\begin{aligned} a &\sim \sqrt{t} \\ a^2 &\sim t \end{aligned}$$

$$ds^2 = dt^2 - \left(t a_{\alpha\beta} + t^2 \beta_{\alpha\beta} \right) dx^{\alpha} dx^{\beta}$$

$a_{\alpha\beta}(t)$

$$1 - g_{00} = 1$$

Penrose diagrams

$$R^{\mu}_{\nu} - \frac{1}{2} \delta^{\mu}_{\nu} R = 8\pi G T^{\mu}_{\nu}$$

$$P = \frac{1}{3} \epsilon$$

$$P = 0$$

$$ds^2 = dt^2 - t^2 d\vec{x}^2$$

$$a \sim \sqrt{t}$$

$$a^2 \sim t$$

$$ds^2 = dt^2 - \left(t a_{\alpha\beta} + t^2 \beta_{\alpha\beta} \right) dx^{\alpha} dx^{\beta}$$

$$g_{0\alpha} = 0$$

$$a_{\alpha\beta}(t)$$

Penrose diagrams

$$-\pi < t' + 1$$

$$-\pi < t'$$

$$R^{\mu}_{\nu} - \frac{1}{2} \delta^{\mu}_{\nu} R = 8\pi G T^{\mu}_{\nu}$$

$$P = \frac{1}{3} \epsilon$$

$$P = 0$$

$$t=0$$

$$s^2 = dt^2 - t^2 d\vec{x}^2$$

$$a \sim \sqrt{t}$$

$$a^2 \sim t$$

$$ds^2 = dt^2 - \left(t a_{\alpha\beta} + t^2 \beta_{\alpha\beta} \right) dx^{\alpha} dx^{\beta}$$

$$a_{\alpha\beta}(t)$$

Penrose diagrams

$$\begin{aligned} & -\pi < t' + \\ & -\pi < t' \end{aligned}$$

$$R^{\mu}_{\nu} - \frac{1}{2} \delta^{\mu}_{\nu} R = 8\pi G \cdot T^{\mu}_{\nu}$$

$$\begin{aligned} P &= \frac{1}{3} \epsilon \\ P &= 0 \end{aligned}$$

$$\boxed{t=0}$$

$$ds^2 = dt^2 - t^2 d\vec{x}^2$$

$$a \sim \sqrt{t}$$

$$a^2 \sim t$$

$$ds^2 = dt^2 - \left(t a_{\alpha\beta} + t^2 \beta_{\alpha\beta} \right) dx^{\alpha} dx^{\beta}$$

$$-g_{0\alpha} = 1$$

$$a_{\alpha\beta}(t)$$

Penrose diagrams

$$-\pi < t' + 1$$

$$-\pi < t'$$

$$R^{\mu}_{\nu} - \frac{1}{2} \delta^{\mu}_{\nu} R = 8\pi G \cdot T^{\mu}_{\nu}$$

$$P = \frac{1}{3} \epsilon$$

$$P = 0$$

$$(t=0)$$

$$ds^2 = dt^2 - t^2 d\vec{x}^2$$

$$a \sim \sqrt{t}$$

$$a^2 \sim t$$

$$ds^2 = dt^2 - \left(t a_{\alpha\beta} + t^2 \beta_{\alpha\beta} \right) dx^{\alpha} dx^{\beta}$$

$$a_{\alpha\beta}(t)$$

$$-g_{0\alpha} = 1$$

Penrose diagrams

$$\begin{aligned} & -\pi < t' + \\ & -\pi < t' \end{aligned}$$

$$R^{\mu}_{\nu} - \frac{1}{2} \delta^{\mu}_{\nu} R = 8\pi G \cdot T^{\mu}_{\nu}$$

$$\begin{aligned} P &= \frac{1}{3} \epsilon \\ P &= 0 \end{aligned}$$

$$t=0$$

$$ds^2 = dt^2 - t^2 d\vec{x}^2$$

$$\begin{aligned} a &\sim \sqrt{t} \\ a^2 &\sim t \end{aligned}$$

$$ds^2 = dt^2 - \left(t a_{\alpha\beta} + t^2 \beta_{\alpha\beta} \right) dx^{\alpha} dx^{\beta}$$

t

$$1 - g_{0\alpha} = 1$$

$$a_{\alpha\beta}(t)$$

$$ds^2 = \alpha t^2 - g_{ij}(t) dx^i dx^j$$

Penrose diagrams

$$\begin{aligned} \gamma &= -\pi < t' + 1 \\ \gamma &= -\pi < t' \end{aligned}$$

$$R^{\mu}_{\nu} - \frac{1}{2} \delta^{\mu}_{\nu} R = 8\pi G \cdot T^{\mu}_{\nu}$$

$$\begin{aligned} P &= \frac{1}{3} \epsilon \\ P &= 0 \end{aligned}$$

t=0

$$ds^2 = dt^2 - t^2 d\vec{x}^2$$

$$\begin{aligned} a &\sim \sqrt{t} \\ a^2 &\sim t \end{aligned}$$

$$ds^2 = dt^2 - \left(t a_{\alpha\beta} + t^2 \beta_{\alpha\beta} \right) dx^{\alpha} dx^{\beta}$$

t

$$-g_{0\alpha} = 1$$

$$a_{\alpha\beta}(t)$$

$$ds^2 = \alpha t^2 - \rho(t) dx^2 - \frac{1}{4} dy^2 - c^2/l$$

Penrose diagrams

$$\begin{aligned} & -\pi < t' + 1 \\ & -\pi < t' \end{aligned}$$

$$R^{\mu}_{\nu} - \frac{1}{2} \delta^{\mu}_{\nu} R = 8\pi G \cdot T^{\mu}_{\nu}$$

$$\begin{aligned} P &= \frac{1}{3} \epsilon \\ P &= 0 \end{aligned}$$

$$ds^2 = dt^2$$

$$d\vec{x}$$

$$\begin{aligned} a &\sim \sqrt{t} \\ a^2 &\sim t \end{aligned}$$

$$ds^2 =$$

$$-(t a_{\alpha\beta} + t^2 \beta_{\alpha\beta}) dx^{\alpha} dx^{\beta}$$

$$a \sim R(t)$$

$$ds^2 = \alpha(t) dt^2 - \beta(t) dx^2 - \gamma(t) dy^2 - \delta(t) dz^2$$

Penrose diagrams

$$\begin{aligned} & -\pi < t' + 1 \\ & -\pi < t' \end{aligned}$$

$$R^{\mu}_{\nu} - \frac{1}{2} \delta^{\mu}_{\nu} R = 8\pi G \cdot T^{\mu}_{\nu}$$

$$\begin{aligned} P &= \frac{1}{3} \varepsilon \\ P &= 0 \end{aligned}$$

$$ds^2 = dt^2 - t^2 d\vec{x}^2$$

$$\begin{aligned} a &\sim \sqrt{t} \\ a^2 &\sim t \end{aligned}$$

$$ds^2 = dt^2 - \left(t a_{\alpha\beta} + t^2 \beta_{\alpha\beta} \right) dx^{\alpha} dx^{\beta}$$

$$g_{0\alpha} = 1$$

$$a_{\alpha\beta}(t)$$

$$ds^2 = \alpha(t)^2 - \rho(t) dx^2 - b(t) dy^2 - c^2(t) dz^2$$

Penrose diagrams

$$\begin{aligned} & -\pi < t' + \dots \\ & -\pi < t' \dots \end{aligned}$$

$$R^\mu_{\nu} = \frac{1}{2} \delta^\mu_{\nu} R = 8\pi G \cdot T^\mu_{\nu}$$

$$\begin{aligned} P &= \frac{1}{3} \varepsilon \\ P &= 0 \end{aligned}$$

$$ds^2 = dt^2 - t^2 d\vec{x}^2$$

$$\begin{aligned} a &\sim \sqrt{t} \\ a^2 &\sim t \end{aligned}$$

$$ds^2 = dt^2 - (t a_{\alpha\beta} + t^2 \beta_{\alpha\beta}) dx^\alpha dx^\beta$$

$$g_{0\alpha} = 1$$

$$a_{\alpha\beta}(t)$$

$$ds^2 = \alpha(t)^2 dt^2 - a(t)^2 dx^2 - b(t)^2 dy^2 - c(t)^2 dz^2$$

Penrose diagrams

$$\begin{aligned} -\pi < t' + \dots \\ -\pi < t' \dots \end{aligned}$$

$$R^{\mu}_{\nu} - \frac{1}{2} \delta^{\mu}_{\nu} R = 8\pi G \cdot T^{\mu}_{\nu}$$

$$\begin{aligned} P &= \frac{1}{3} \epsilon \\ P &= 0 \end{aligned}$$

$$(t=0)$$

$$ds^2 = dt^2 - t^2 d\vec{x}^2$$

$$\begin{aligned} a &\sim \sqrt{t} \\ a^2 &\sim t \end{aligned}$$

$$ds^2 = dt^2 - (t a_{\alpha\beta} + t^2 \beta_{\alpha\beta}) dx^{\alpha} dx^{\beta}$$

$$= 1 \quad a_{\alpha\beta}(t)$$

$$ds^2 = \alpha(t)^2 dt^2 - a(t) dx^2 - b(t) dy^2 - c(t) dz^2$$

Penrose diagrams

$$\begin{aligned} & -\pi < t' + 1 \\ & -\pi < t' \end{aligned}$$

$$R^{\mu}_{\nu} - \frac{1}{2} \delta^{\mu}_{\nu} R = 8\pi G \cdot T^{\mu}_{\nu}$$

$$\begin{aligned} P &= \frac{1}{3} \epsilon \\ P &= 0 \end{aligned}$$

$$(t=0)$$

$$ds^2 = dt^2 - t^2 d\vec{x}^2$$

$$\begin{aligned} a &\sim \sqrt{t} \\ a^2 &\sim t \end{aligned}$$

$$ds^2 = dt^2 - (t^2 a_{\alpha\beta} + t^2 \beta_{\alpha\beta}) dx^{\alpha} dx^{\beta}$$

t

$$1 - g_{00} = 1$$

$$a \sim R(t)$$

$$ds^2 = \alpha(t)^2 - a(t) dx^2 - b(t) dy^2 - c(t) dz^2$$

Penrose diagrams

$$\begin{aligned} & -\pi < t' + 1 \\ & -\pi < t' \end{aligned}$$

$$R^{\mu}_{\nu} - \frac{1}{2} \delta^{\mu}_{\nu} R = 8\pi G \cdot T^{\mu}_{\nu}$$

$$\begin{aligned} P &= \frac{1}{3} \epsilon \\ P &= 0 \end{aligned}$$

$$(t=0)$$

$$ds^2 = dt^2 - t^2 d\vec{x}^2$$

$$\begin{aligned} a &\sim \sqrt{t} \\ a^2 &\sim t \end{aligned}$$

$$ds^2 = dt^2 - (t^2 a_{\alpha\beta} + t^2 \beta_{\alpha\beta}) dx^{\alpha} dx^{\beta}$$

$$g_{0\alpha} = 1 \quad a_{\alpha\beta}(t)$$

$$ds^2 = \alpha(t)^2 dt^2 - a(t)^2 dx^2 - b(t)^2 dy^2 - c(t)^2 dz^2$$

Penrose diagrams

$$-\pi < t' + r' < \pi$$

$$-\pi < t' - r' < \pi$$

$$R^{\mu}_{\nu} - \frac{1}{2} \delta^{\mu}_{\nu} R = 8\pi G \cdot T^{\mu}_{\nu} + 8\pi$$

$$(t=0)$$

$$ds^2 = dt^2 - t^2$$

$$a \sim \sqrt{t}$$

$$a^2 \sim t$$

$$ds^2 = dt^2 - (t^2)$$

t

$$g_{00} = 1$$

$$ds^2 = \alpha t^2$$

$$t^2 \beta_{AB} = \sqrt{t} dx^A dx^B$$

$$(t) ds^2 = c^2(t) d\tau^2$$

Penrose diagrams

$$\begin{aligned} -\pi < t' + v' < \pi \\ -\pi < t' - v' < \pi \end{aligned}$$

$$R^{\mu}_{\nu} - \frac{1}{2} \delta^{\mu}_{\nu} R = 8\pi G T^{\mu}_{\nu} + 8\pi G \delta^{\mu}_{\nu} \epsilon$$

$$ds^2 = dt^2 - t^2 d\vec{x}^2 \quad \begin{aligned} a &\sim \sqrt{t} \\ a^2 &\sim t \end{aligned}$$

$$ds^2 = dt^2 - (t a_{\alpha\beta} + t^2 \beta_{\alpha\beta}) dx^{\alpha} dx^{\beta}$$

$$g_{0\alpha} = 1$$

$$ds^2 = dt^2 - a(t)^2 dx^2 - b(t)^2 dy^2 - c(t)^2 dz^2$$

Penrose diagrams

$$\begin{aligned} -\pi < t' + v' < \pi \\ -\pi < t' - v' < \pi \end{aligned}$$

$$R^{\mu}_{\nu} - \frac{1}{2} \delta^{\mu}_{\nu} R = 8\pi G T^{\mu}_{\nu} + 8\pi G \delta^{\mu}_{\nu} \epsilon$$

$$ds^2 = dt^2 - t^2 d\vec{x}^2 \quad \begin{aligned} a &\sim \sqrt{t} \\ a^2 &\sim t \end{aligned}$$

$$ds^2 = dt^2 - (t a_{\alpha\beta} + t^2 \beta_{\alpha\beta}) dx^{\alpha} dx^{\beta}$$

$$g_{0\alpha} = 1$$

$$a_{\alpha\beta}(t)$$

$$ds^2 = dt^2 - a(t)^2 dx^2 - b(t)^2 dy^2 - c(t)^2 dz^2$$

Penrose diagrams

$$\begin{aligned} -\pi < t' + v' < \pi \\ -\pi < t' - v' < \pi \end{aligned}$$

$$R^{\mu}_{\nu} - \frac{1}{2} \delta^{\mu}_{\nu} R = 8\pi G T^{\mu}_{\nu} + 8\pi G \delta^{\mu}_{\nu} \epsilon$$

$t=0$

$$ds^2 =$$

dx^i

$$a \sim \sqrt{t}$$

$$a^2 \sim t$$

$$ds^2 =$$

$$(a_{\alpha\beta} + t^2 \beta_{\alpha\beta})$$

$$dx^{\alpha} dx^{\beta}$$

$$g_{0\alpha} = 1$$

$$ds^2 =$$

$(\vec{x})$

$$-b(t)dy^2 - c^2(t)dz^2$$

Penrose diagrams

$$\begin{aligned} -\pi < t' + v' < \pi \\ -\pi < t' - v' < \pi \end{aligned}$$

$$R^{\mu}_{\nu} - \frac{1}{2} \delta^{\mu}_{\nu} R = 8\pi G T^{\mu}_{\nu} + 8\pi G \delta^{\mu}_{\nu} \epsilon$$

$$\boxed{t=0}$$

$$ds^2 = dt^2 -$$

$$a \sim \sqrt{t}$$

$$a^2 \sim t$$

$$ds^2 = dt^2 -$$

t

$$g_{00} = 1$$

a

$$\beta + t^2 \beta_{,n} = \sqrt{x} dx \beta$$

$$ds^2 = \alpha$$

$$ds^2 = c^2(t) dz^2$$

Penrose diagrams

$$\begin{aligned} -\pi < t' + v' < \pi \\ -\pi < t' - v' < \pi \end{aligned}$$

$$R^{\mu}_{\nu} - \frac{1}{2} \delta^{\mu}_{\nu} R = 8\pi G T^{\mu}_{\nu} + 8\pi G \delta^{\mu}_{\nu} \epsilon$$

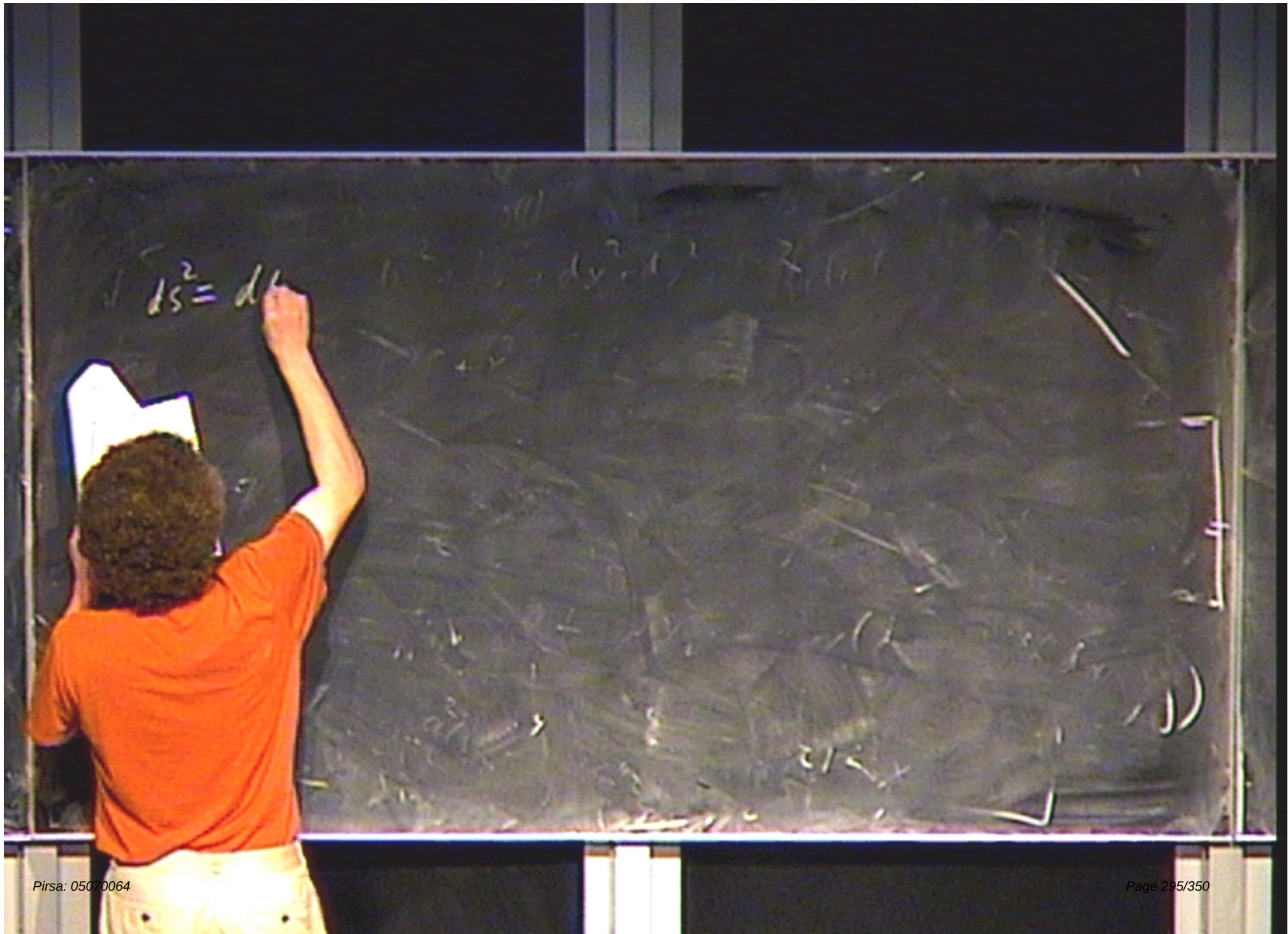
$t=0$

$$ds^2 = dt^2 - t^2 d\vec{x}^2 \quad \begin{aligned} a &\sim \sqrt{t} \\ a^2 &\sim t \end{aligned}$$

$$ds^2 = dt^2 - (t a_{\alpha\beta} + t^2 \beta_{\alpha\beta}) dx^{\alpha} dx^{\beta}$$

$$g_{0\alpha} = 1 \quad a_{\alpha\beta}(t)$$

$$ds^2 = dt^2 - a(t) dx^2 - b(t) dy^2 - c(t) dz^2$$



$$ds^2 = dt^2 - (a_{\alpha\beta} e^{2Ht} + b_{\alpha\beta} + c_{\alpha\beta} e^{-4t}) dx^\alpha dx^\beta$$

$$ds^2 = dt^2 - (a_{\alpha\beta} e^{2Ht} + b_{\alpha\beta} + c_{\alpha\beta} e^{-4t}) dx^\alpha dx^\beta$$

$$ds^2 = dt^2 - (a_{\alpha\beta} e^{2Ht} + b_{\alpha\beta} + c_{\alpha\beta} e^{-4t}) dx^\alpha dx^\beta$$

$$ds^2 = dt^2 - (a_{\alpha\beta} e^{2Ht} + b_{\alpha\beta} + c_{\alpha\beta} e^{-4t}) dx^\alpha dx^\beta$$

$$ds^2 = dt^2 - (a_{\alpha\beta} e^{2Ht} + b_{\alpha\beta} + c_{\alpha\beta} e^{-4t}) dx^\alpha dx^\beta$$

$$c_{\mu\nu\rho} c^{\mu\nu\rho} = e^{-6Ht}$$

$$ds^2 = dt^2 - (a_{\alpha\beta} e^{2Ht} + b_{\alpha\beta} + c_{\alpha\beta} e^{-4t}) dx^\alpha dx^\beta$$

$$c_{\mu\nu\sigma\rho} = e^{-6Ht}$$

$R_{\mu\nu\sigma\rho}$

$R_{\mu\nu}$

$$ds^2 = dt^2 - (a_{\alpha\beta} e^{2Ht} + b_{\alpha\beta} + c_{\alpha\beta} e^{-4t}) dx^\alpha dx^\beta$$

$$C_{\mu\nu\sigma\rho} C^{\mu\nu\sigma\rho} = e^{-6Ht}$$

$$R_{\mu\nu\sigma\rho}$$

$$R_{\mu\nu}$$

$$C_{\mu\nu\sigma\rho}$$

$$ds^2 = dt^2 - (a_{\alpha\beta} e^{2Ht} + b_{\alpha\beta} + c_{\alpha\beta} e^{4t}) dx^\alpha dx^\beta$$

$$c_{\mu\nu\sigma\rho} c^{\mu\nu\sigma\rho} = e^{-6Ht}$$

$$R_{\mu\nu\sigma\rho} = R_{\mu\nu} c_{\sigma\rho}$$

$$ds^2 = dt^2 - (a_{\alpha\beta} e^{2Ht} + b_{\alpha\beta} + c_{\alpha\beta} e^{-4t}) dx^\alpha dx^\beta$$

$$c_{\mu\nu\rho} c^{\mu\nu\rho} = e^{-6Ht}$$

$$R_{\mu\nu\rho} = R \quad R_{\mu\nu} \quad c_{\mu\nu\rho}$$

$$R_{\mu\nu} = 0$$

$$ds^2 = dt^2 - (a_{\alpha\beta} e^{2Ht} + b_{\alpha\beta} + c_{\alpha\beta} e^{-4t}) dx^\alpha dx^\beta$$

$$c_{\mu\nu\sigma\rho} c^{\mu\nu\sigma\rho} = e^{-6Ht}$$

$$R_{\mu\nu\sigma\rho} = R \quad R_{\mu\nu} \quad c_{\mu\nu\sigma\rho}$$

$$R_{\mu\nu} = 0$$

$$ds^2 = dt^2 - (a_{\alpha\beta} e^{2Ht} + b_{\alpha\beta} + c_{\alpha\beta} e^{-4t}) dx^\alpha dx^\beta$$

$$c_{\mu\nu\sigma\rho} c^{\mu\nu\sigma\rho} = e^{-6Ht}$$

$$R_{\mu\nu\sigma\rho} = R_{\mu\nu} c_{\sigma\rho}$$

$$R_{\mu\nu} = 0$$

$$ds^2 = dt^2$$



$$ds^2 = (dt^2 - \cosh^2 Ht)$$

$$ds^2 = (dt^2 - \cosh^2 Ht) + b_0 (d\theta^2 + \sinh^2 \theta d\phi^2)$$

$$ds^2 = (dt^2 - \cosh^2 Ht) + b_0 (d\theta^2 + \sin^2 \theta d\phi^2)$$

$S^2 \times S$

$$ds^2 = (dt^2 - \cosh^2 Ht) + b_0 (d\theta^2 + \sin^2 \theta d\phi^2)$$

$$ds^2 \propto s^2$$

$$R^\mu{}_\nu - \frac{1}{2} g^\mu{}_\nu R = 8\pi G \epsilon_\mu{}^\nu$$

$$ds^2 = (dt^2 - \cosh^2 H (dx)^2) - b_0 (d\theta^2 + \sin^2 \theta d\phi^2)$$

$$ds^2 \propto s^2$$

$$R^\mu{}_\nu - \frac{1}{2} g^\mu{}_\nu R = 8\pi G \epsilon_\nu$$

$$ds^2 = dt^2 - a(t)dx^2$$

$$ds^2 = (dt^2 - \cosh^2 H t dx^2) - b_0 (d\theta^2 + \sin^2 \theta d\phi^2)$$

$$dS^2 \propto S^2$$

$$R^\mu{}_\nu - \frac{1}{2} \delta^\mu{}_\nu R = 8\pi G \epsilon_\mu{}^\nu$$

$$ds^2 = dt^2 - a^2(t) dx^2 - b(t) d\Omega_2^2$$

$$ds^2 = (dt^2 - \cosh^2 H (dx)^2) + b_0 (d\theta^2 + \sin^2 \theta d\phi^2)$$

$$d\Omega_2^2 \propto d\Omega^2$$

$$R^\mu{}_\nu - \frac{1}{2} g^\mu{}_\nu R = 8\pi G \epsilon_\nu^\mu$$

$$ds^2 = dt^2 - a^2(t) dx^2 - b(t) d\Omega_2^2$$

$$\frac{2\ddot{b}}{b} + \left(\frac{\dot{b}}{b}\right)^2 + \frac{1}{b^2} = \Lambda$$

$$ds^2 = (dt^2 - \cosh^2 H (dx)^2) + b_0 (d\theta^2 + \sin^2 \theta d\phi^2)$$

$$d\Omega_2^2 \times S^2$$

$$R^\mu{}_\nu - \frac{1}{2} g^\mu{}_\nu R = 8\pi G \epsilon_\mu{}^\nu$$

$$ds^2 = dt^2 - a^2(t) dx^2 - b(t) d\Omega_2^2$$

$$\frac{2\ddot{b}}{b} + \left(\frac{\dot{b}}{b}\right)^2 + \frac{1}{b^2} = \Lambda$$

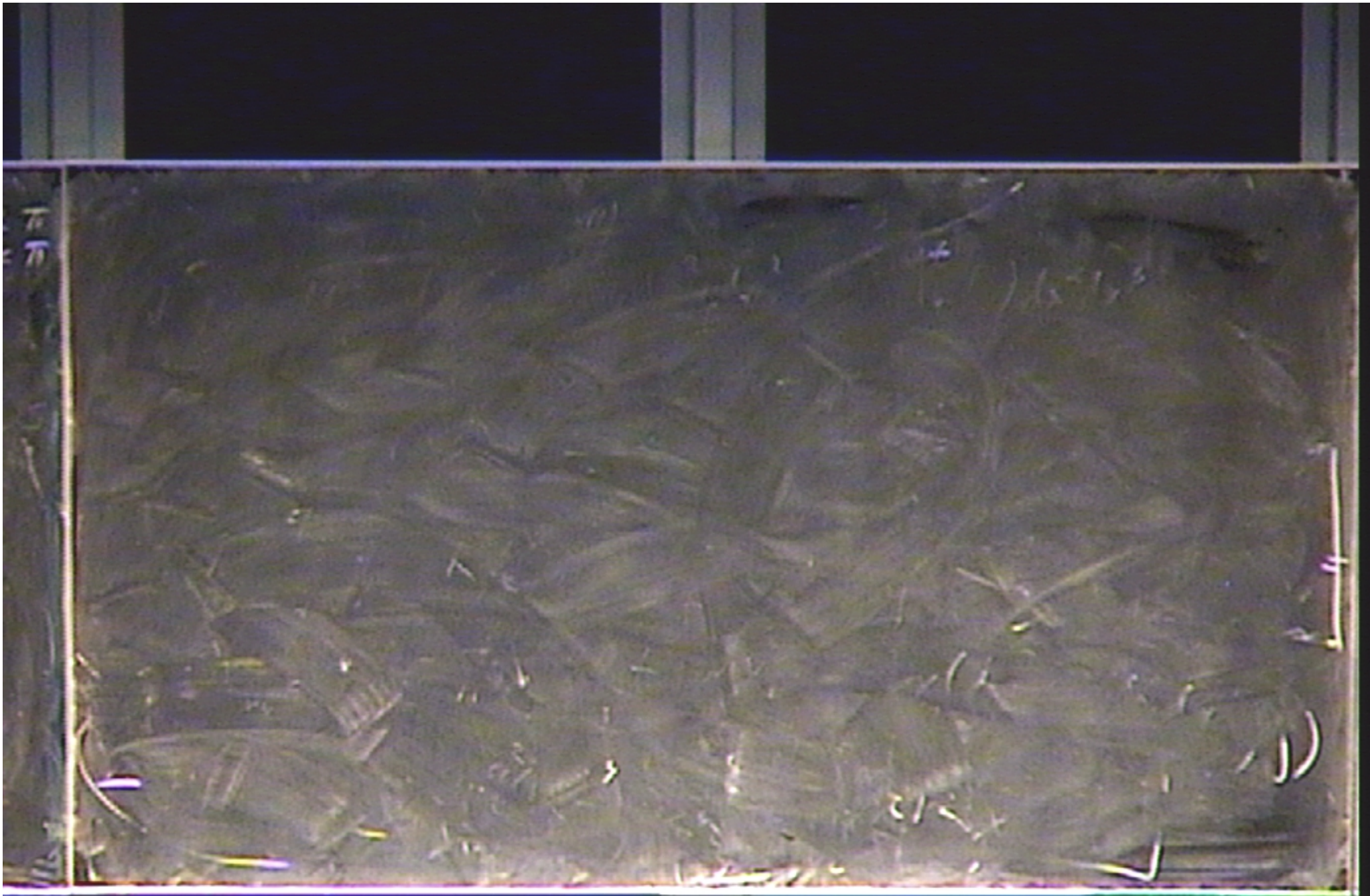
$$ds^2 = (dt^2 - \cosh^2 H (dx^2)^2) - b_0 (d\theta^2 + \sin^2 \theta d\phi^2)$$

$$ds^2 \propto s^2$$

$$R^\mu{}_\nu - \frac{1}{2} g^\mu{}_\nu R = 8\pi G \epsilon_\mu{}^\nu$$

$$ds^2 = dt^2 - a^2(t) dx^2 - b(t) d\Omega_2^2$$

$$\left| \frac{2\ddot{b}}{b} + \left(\frac{\dot{b}}{b}\right)^2 + \frac{1}{b^2} = 1 \right| \quad \frac{\ddot{a}}{a} = \frac{\dot{b}}{b}$$



$$\frac{dy}{dt} = Q(y, x)$$

$$\frac{dx}{dt} = P(y, x)$$

$$\frac{dy}{dt} = Q(y, x)$$

$$\frac{dx}{dt} = P(y, x)$$

$$P(x, y) = 0$$

$$Q(x, y) = 0$$

$$(x_*, y_*)$$

$$\frac{dy}{dt} = Q(y, x)$$

$$\frac{dx}{dt} = P(y, x)$$

$$P(x, y) = 0$$

$$Q(x, y) = 0$$

$$(x_*, y_*)$$

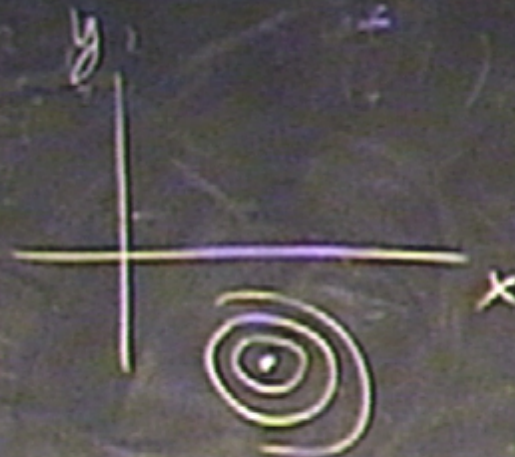


$$\frac{dy}{dt} = Q(y, x)$$

$$\frac{dx}{dt} = P(y, x)$$

$$P(x, y) = 0$$

$$Q(x, y) = 0$$



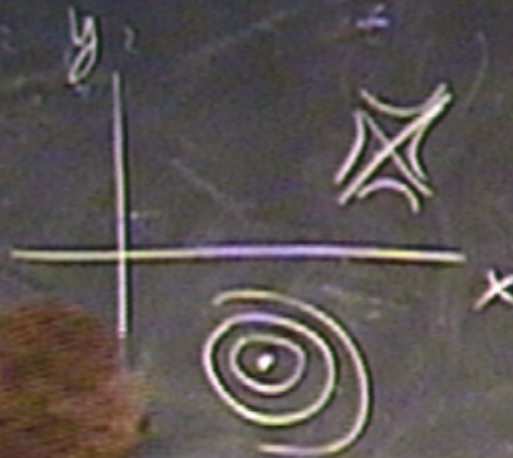
$$\frac{dy}{dt} = Q(y, x)$$

$$\frac{dx}{dt} = P(y, x)$$

$$P(x, y) = 0$$

$$Q(x, y) = 0$$

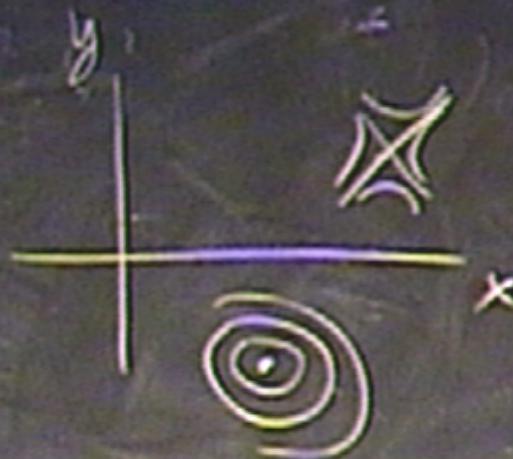
$$(x_*, y_*)$$



$$\frac{dy}{dt} = Q(y, x)$$

$$\frac{dx}{dt} = P(y, x)$$

$P(x, y)$
 $Q(x, y)$
 (x_*, y_*)



$$\frac{dy}{dt} = Q(y, x)$$

$$\frac{dx}{dt} = P(y, x)$$

$$P(x, y) = 0$$

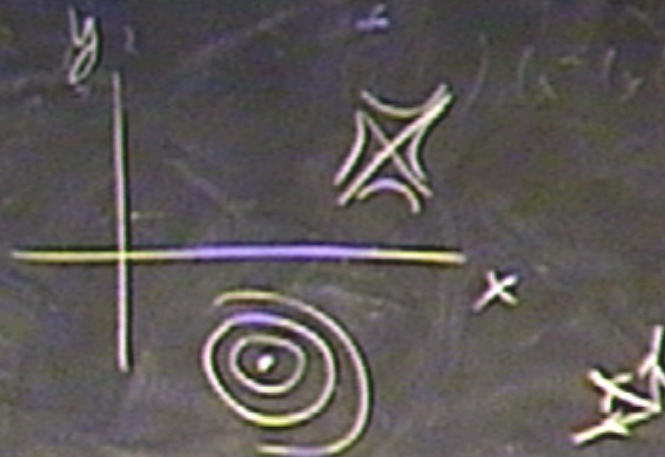
$$Q(x, y) = 0$$

$$(x_*, y_*)$$



$$\frac{dy}{dt} = Q(y, x)$$

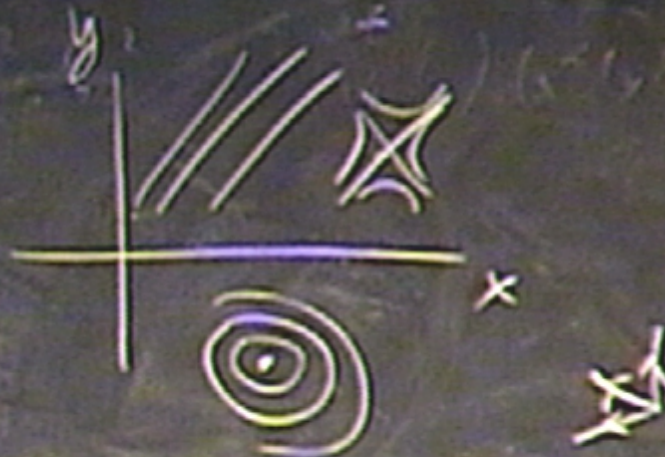
$$\frac{dx}{dt} = P(y, x)$$



$$\frac{dy}{dt} = Q(y, x)$$

$$\frac{dx}{dt} = P(y, x)$$

$P(x, y)$
 $Q(x, y)$
 (x, y)



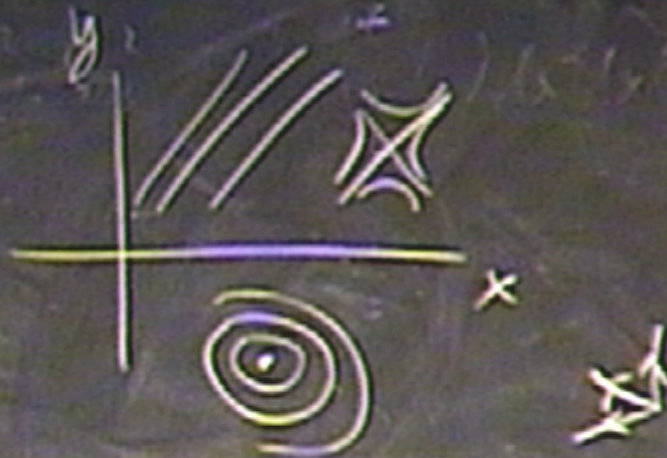
$$\frac{dy}{dt} = Q(y, x)$$

$$\frac{dx}{dt} = P(y, x)$$

$$P(x, y) = 0$$

$$Q(x, y) = 0$$

$$(x_*, y_*)$$



$$\frac{dy}{dt} = Q(y, x)$$

$$\frac{dx}{dt} = P(y, x)$$

$$P(x, y) = 0$$

$$Q(x, y) = 0$$

$$(x_*, y_*)$$



$$\frac{dy}{dt} = Q(y, x)$$

$$\frac{dx}{dt} = P(y, x)$$

$$P(x, y) = 0$$

$$Q(x, y) = 0$$

$$(x_*, y_*)$$



$$\frac{dy}{dt} = Q(y, x)$$

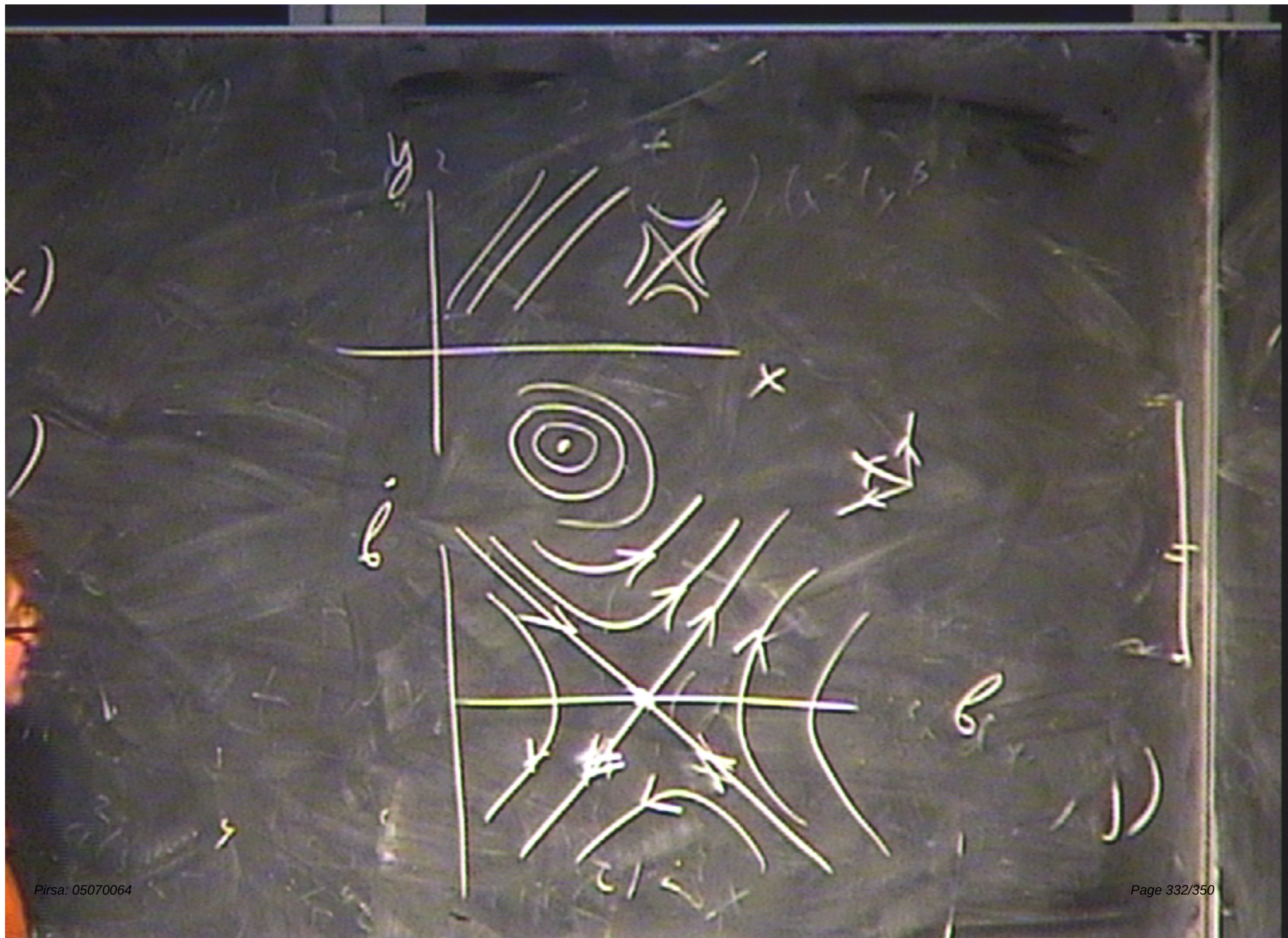
$$\frac{dx}{dt} = P(y, x)$$

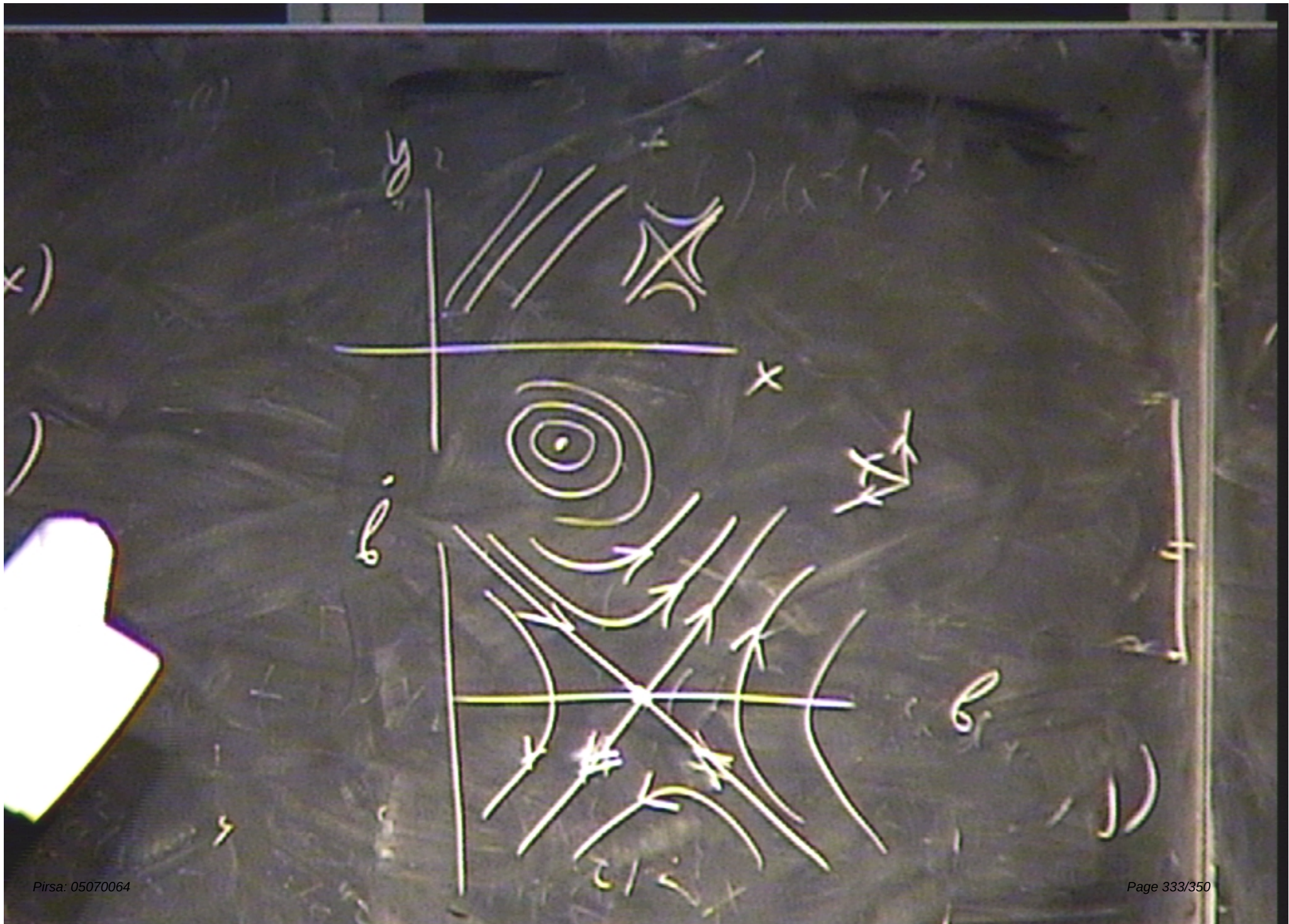
$$P(x, y) = 0$$

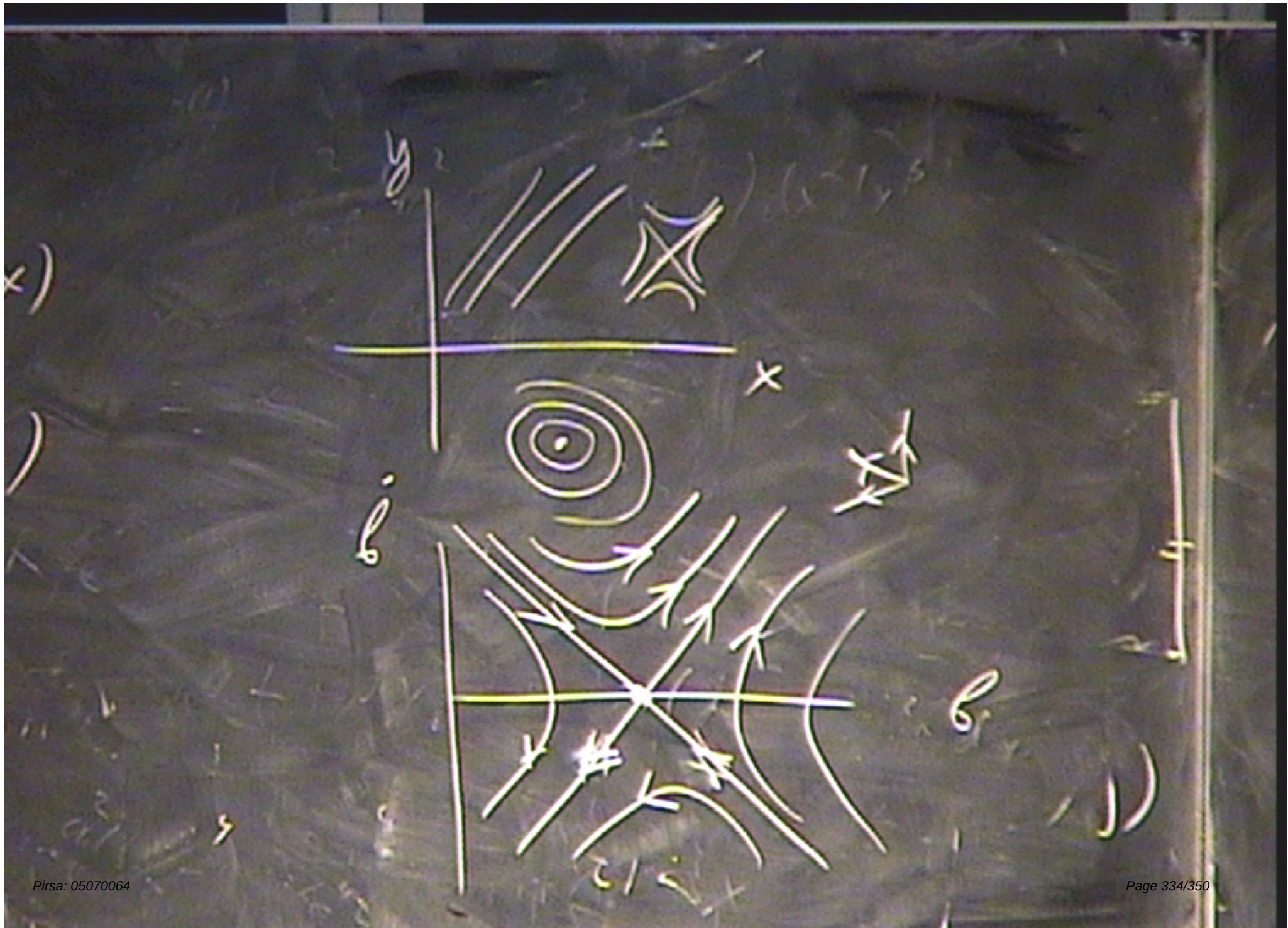
$$Q(x, y) = 0$$

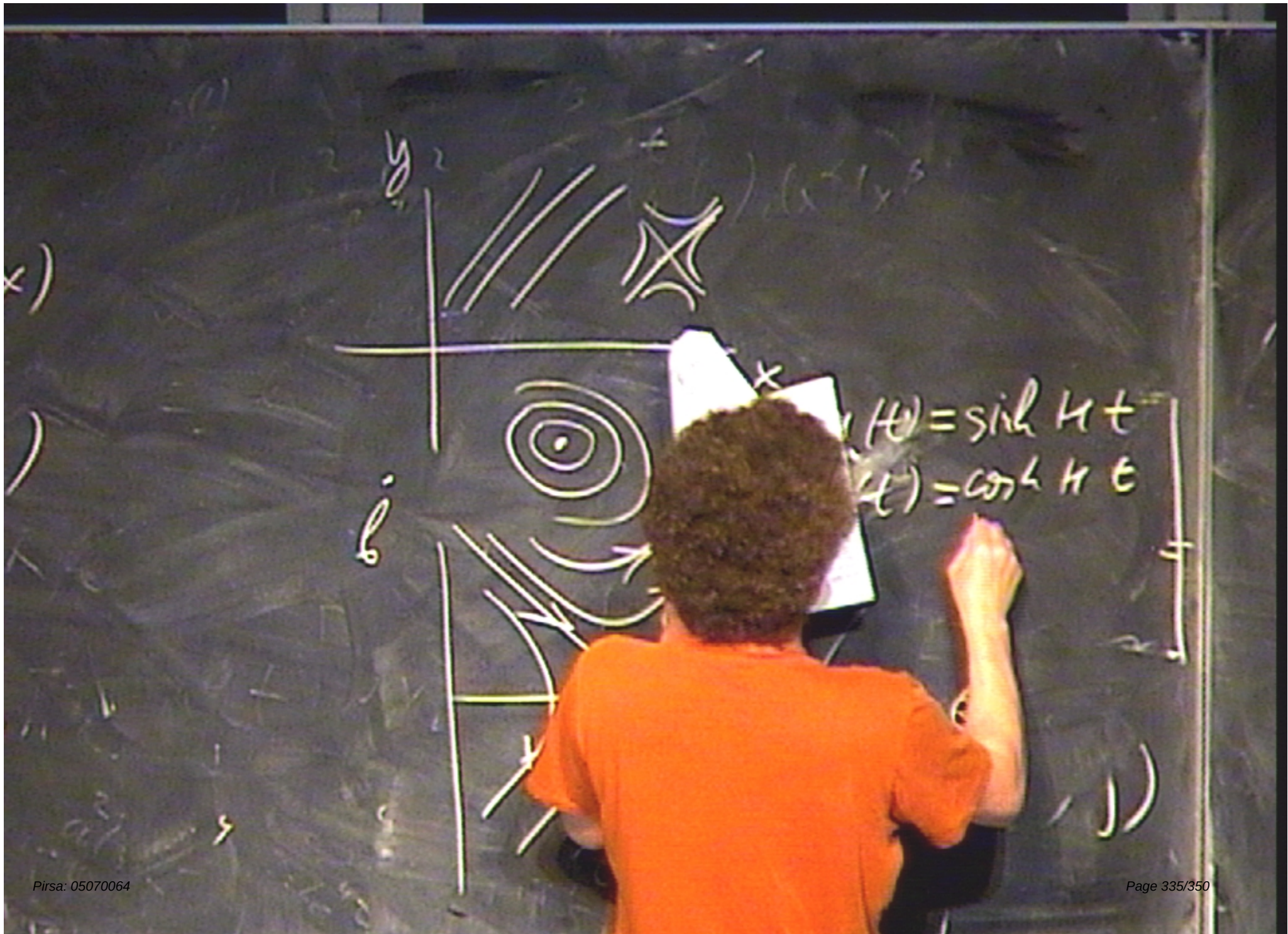
$$(x_*, y_*)$$

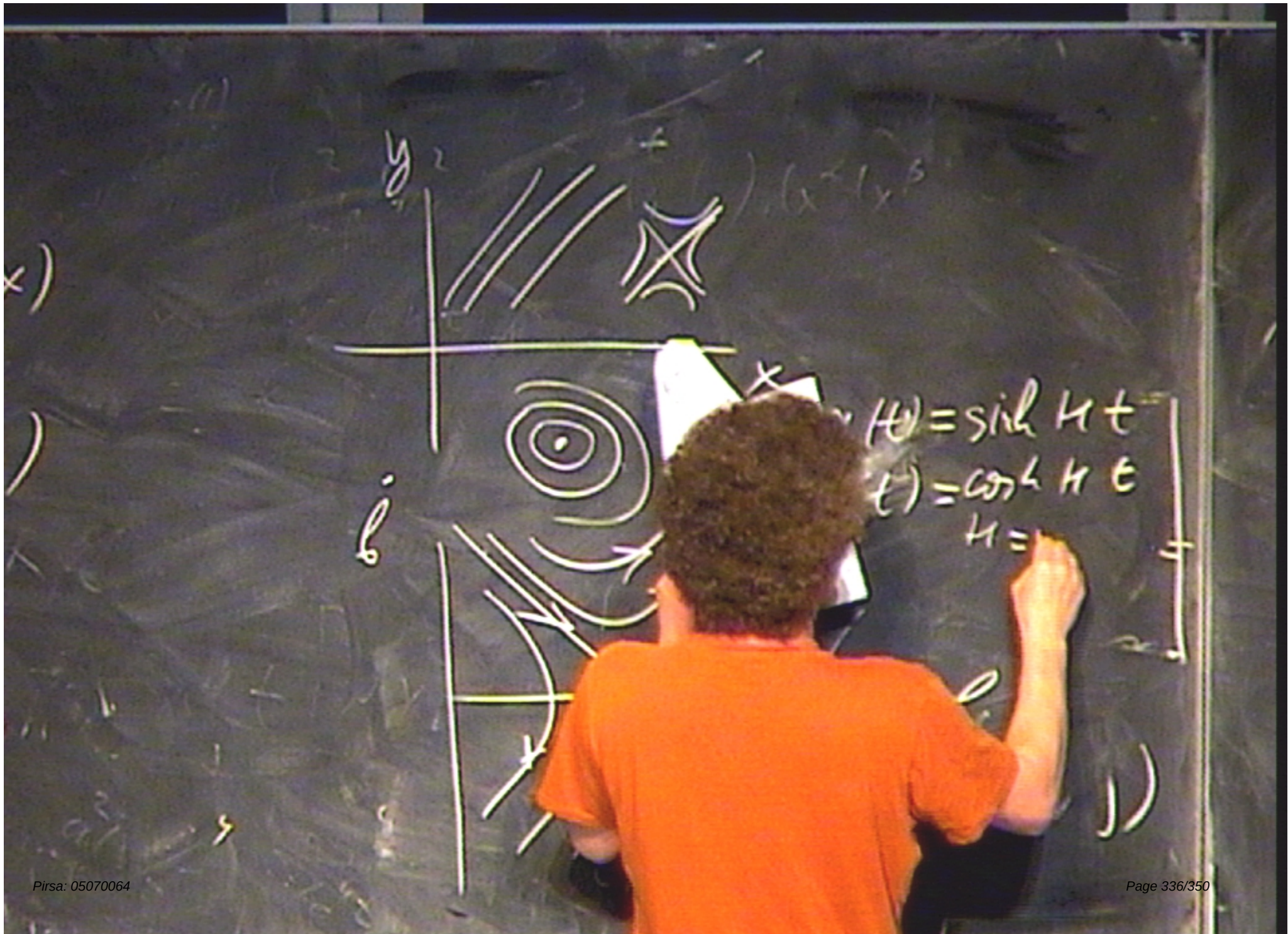






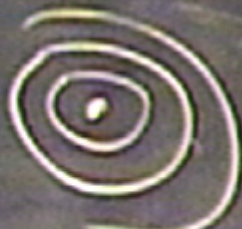




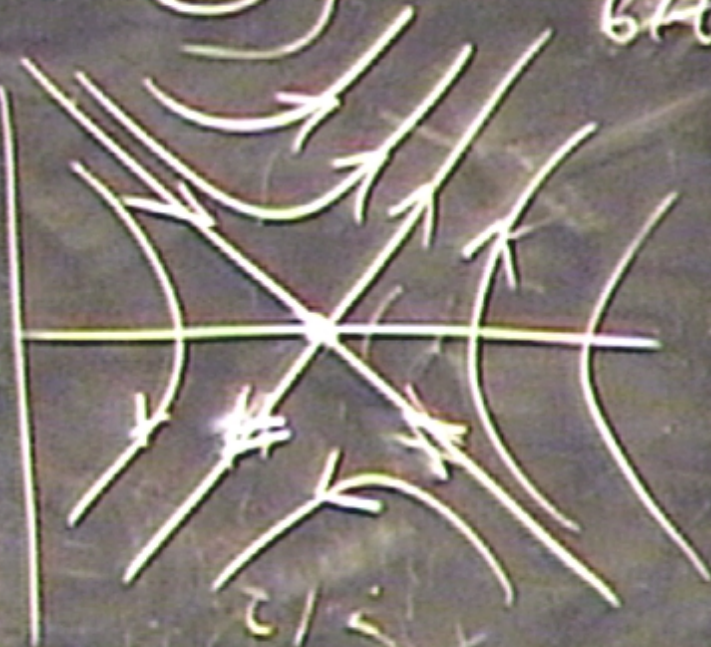








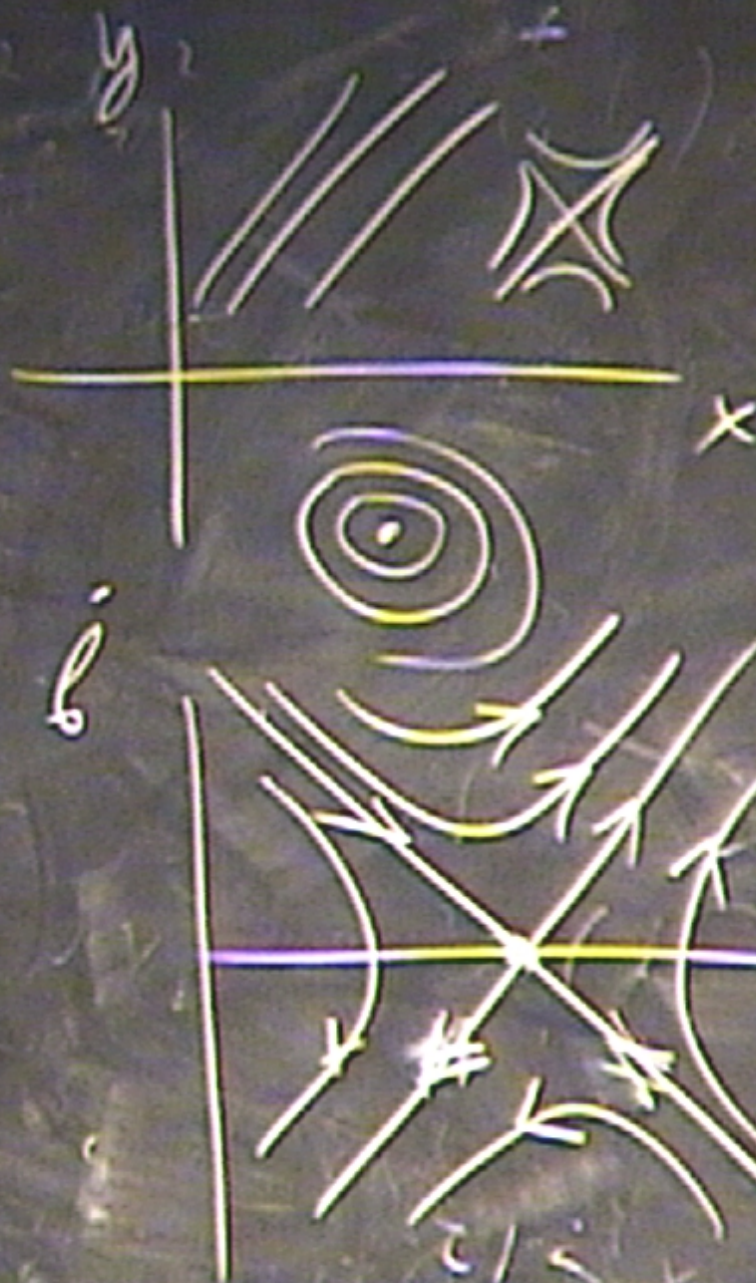
$$\begin{aligned} a(t) &= \sinh Ht \\ \dot{b}(t) &= \cosh Ht \frac{H}{c} \\ H &= \sqrt{\frac{\Lambda}{3}} \end{aligned}$$



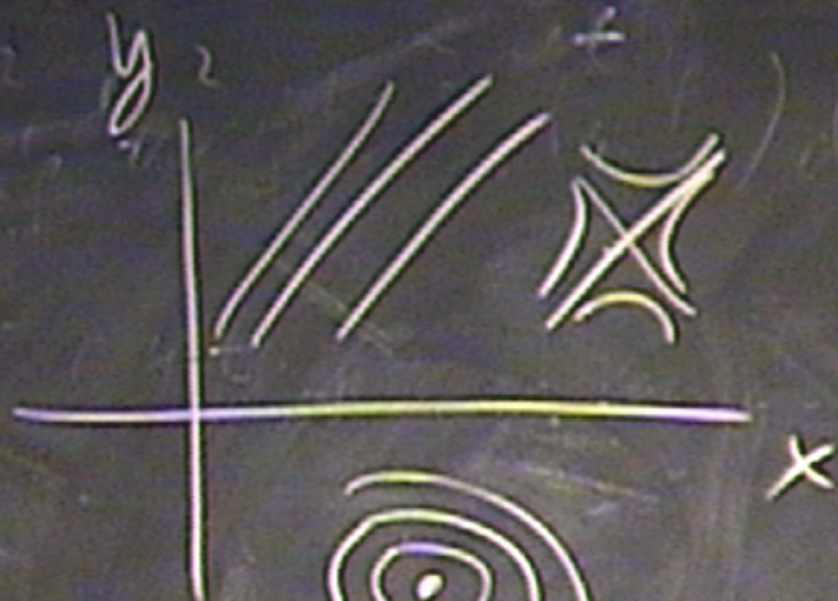


$$\begin{aligned} a(t) &= \sinh Ht \\ b(t) &= \cosh \frac{H}{3} t \\ H &= \sqrt{\frac{\Lambda}{3}} \end{aligned}$$

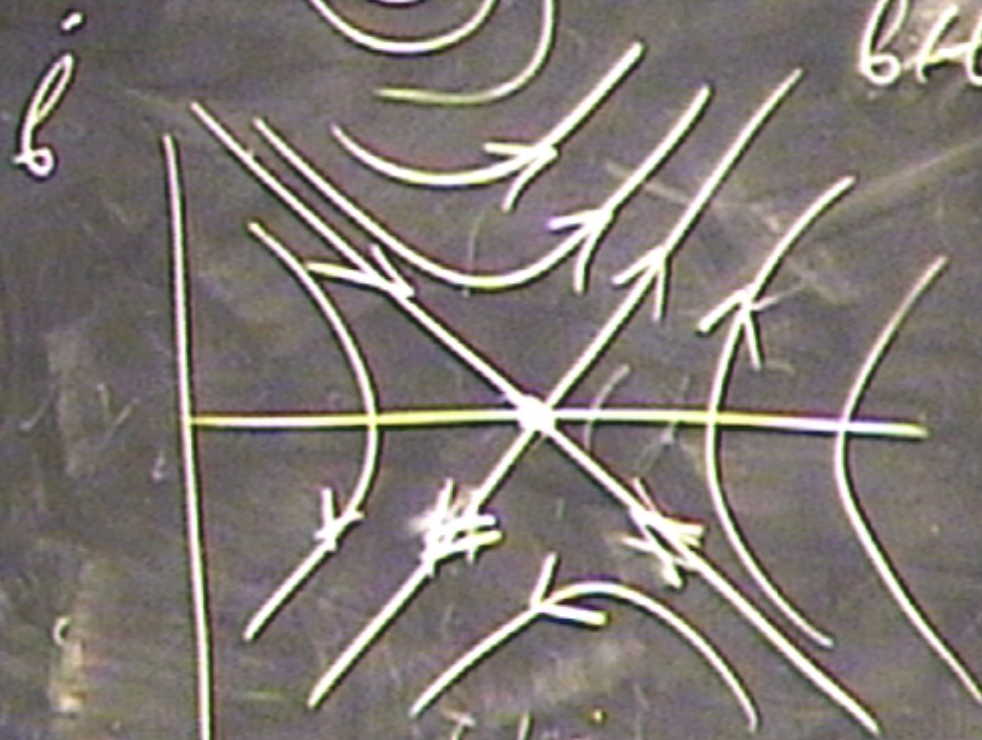


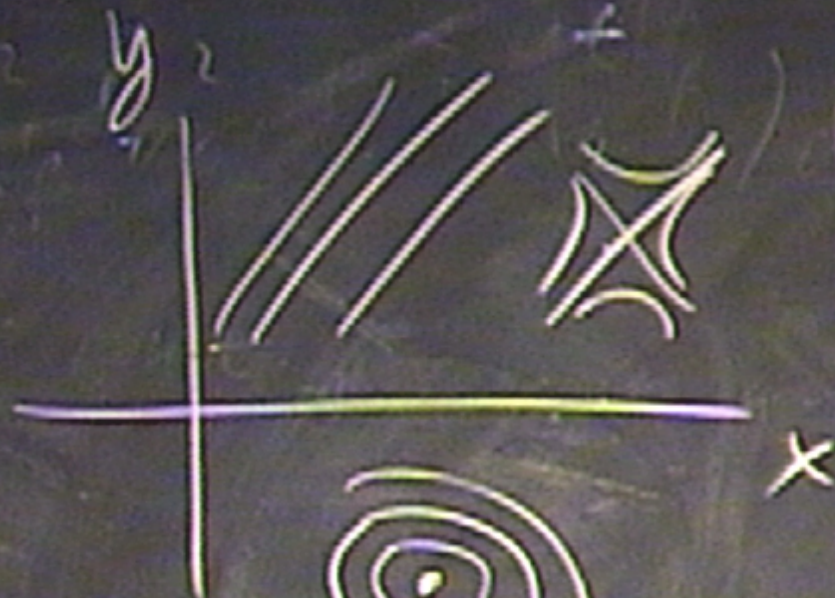


$$\begin{aligned} a(H) &= \sinh H + \\ b(H) &= \cosh \frac{H}{3} \\ H &= \sqrt{\frac{A}{3}} \end{aligned}$$

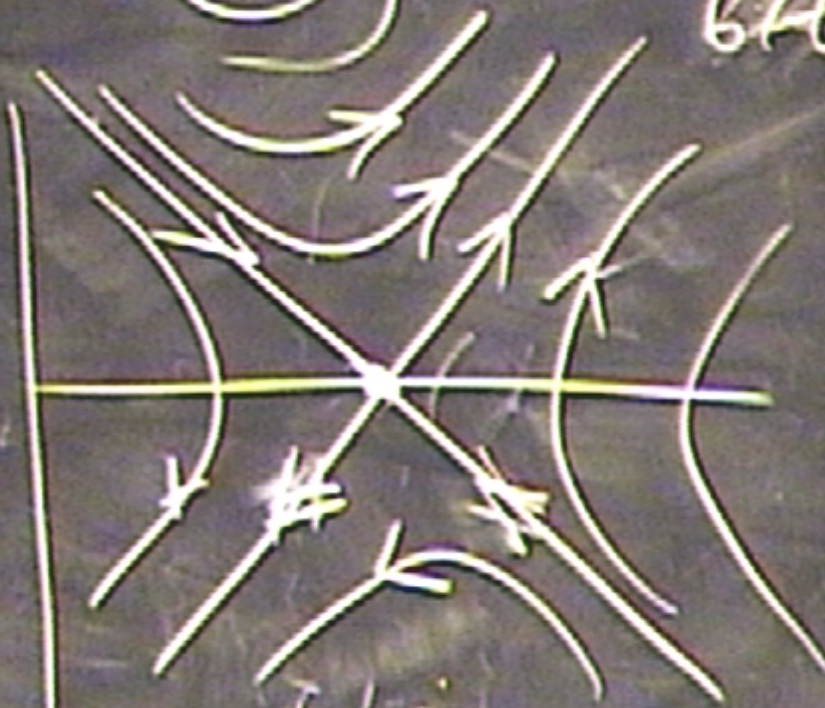


$$\begin{aligned} a(H) &= \sinh Ht \\ b(H) &= \cosh Ht \\ H &= \sqrt{\frac{\Lambda}{3}} \end{aligned}$$





$\dot{b}$



$$\begin{aligned} a(t) &= \sinh Ht \\ b(t) &= \cosh Ht \\ H &= \sqrt{\frac{\Lambda}{3}} \end{aligned}$$

$\dot{b}$

Penrose diagrams

$$-\pi < t' + v' < \pi$$

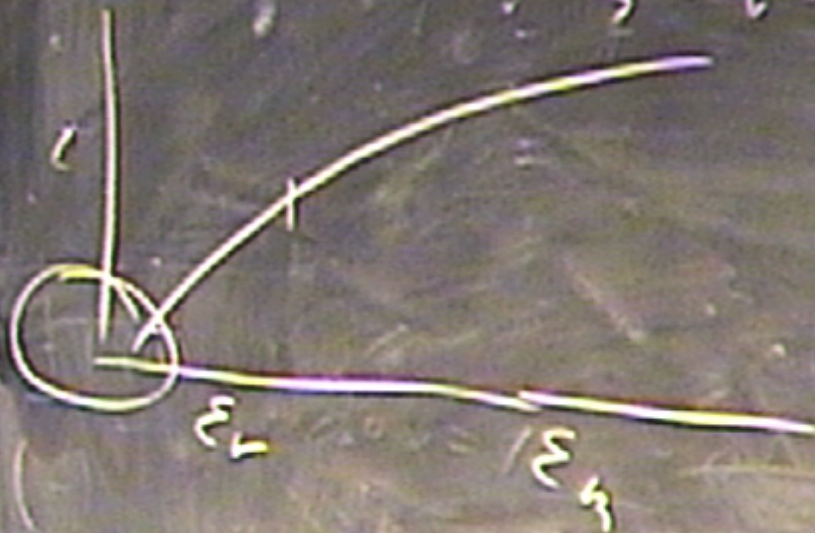
$$-\pi < t' - v' < \pi$$

$$\frac{1}{2} \delta^\mu_\nu R = 8\pi G T^\mu_\nu + 8\pi G \delta^\mu_\nu \epsilon$$

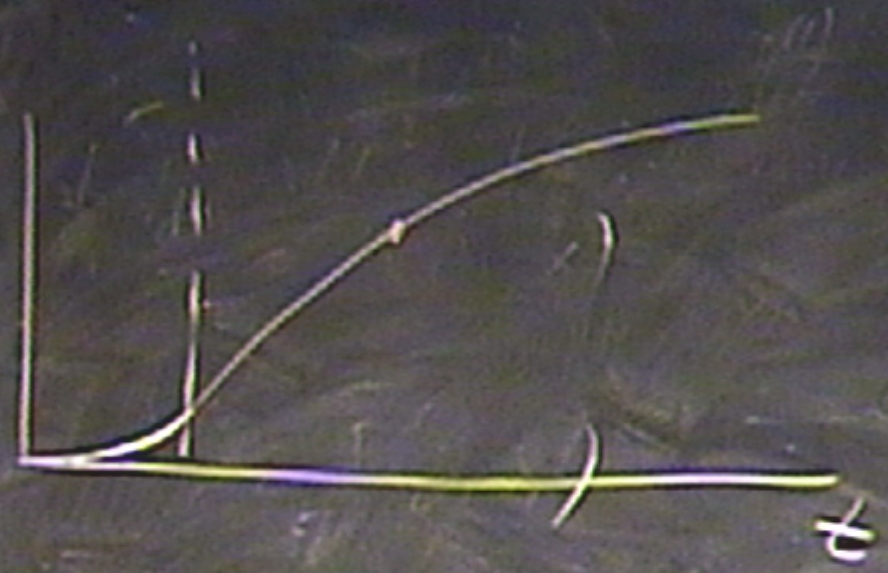


Penrose diagrams

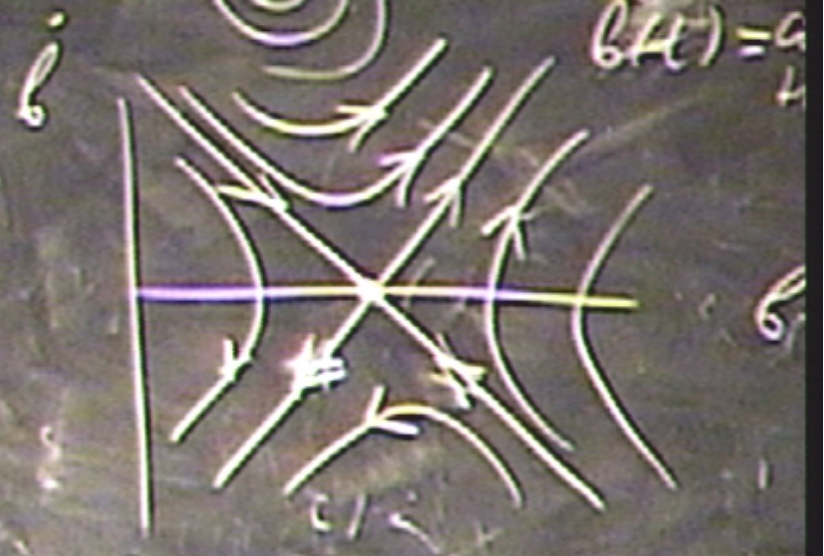
$$\frac{1}{2} \delta^{\mu}_{\nu} R = 8\pi G \cdot T^{\mu}_{\nu} + 8\pi$$



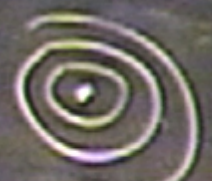
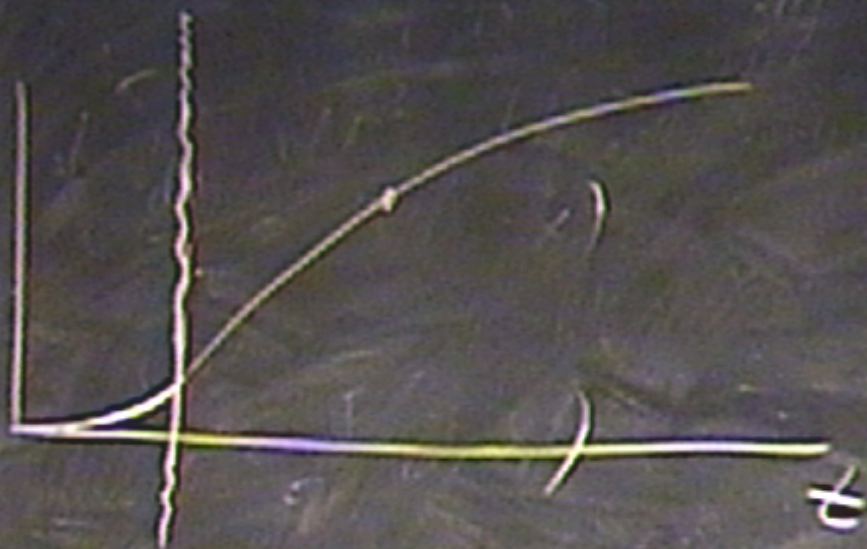
$\nu' < \pi$
 $\nu' < \pi$



$\alpha(H) = S$
 $\beta(H) = a$



$$\begin{aligned} \psi' &< \pi \\ \psi' &< \pi \end{aligned}$$

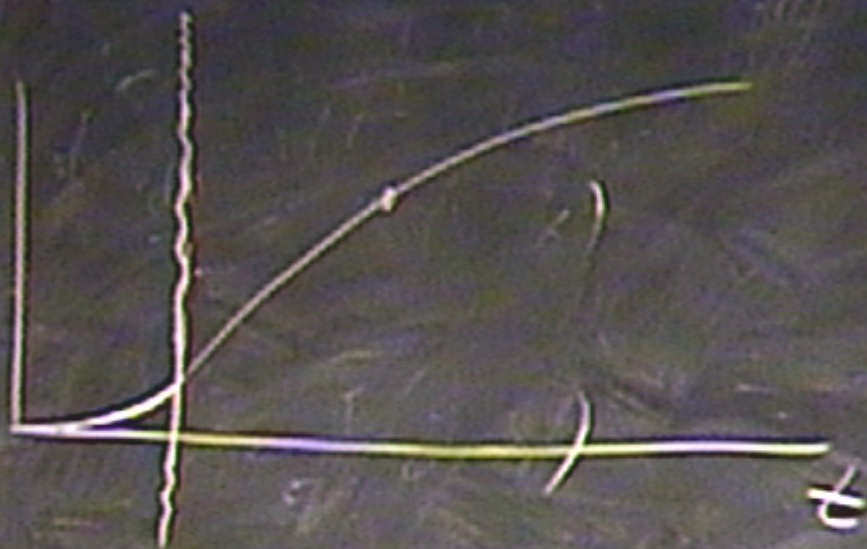


$$\begin{aligned} a(t) &= s \\ b(t) &= a \end{aligned}$$

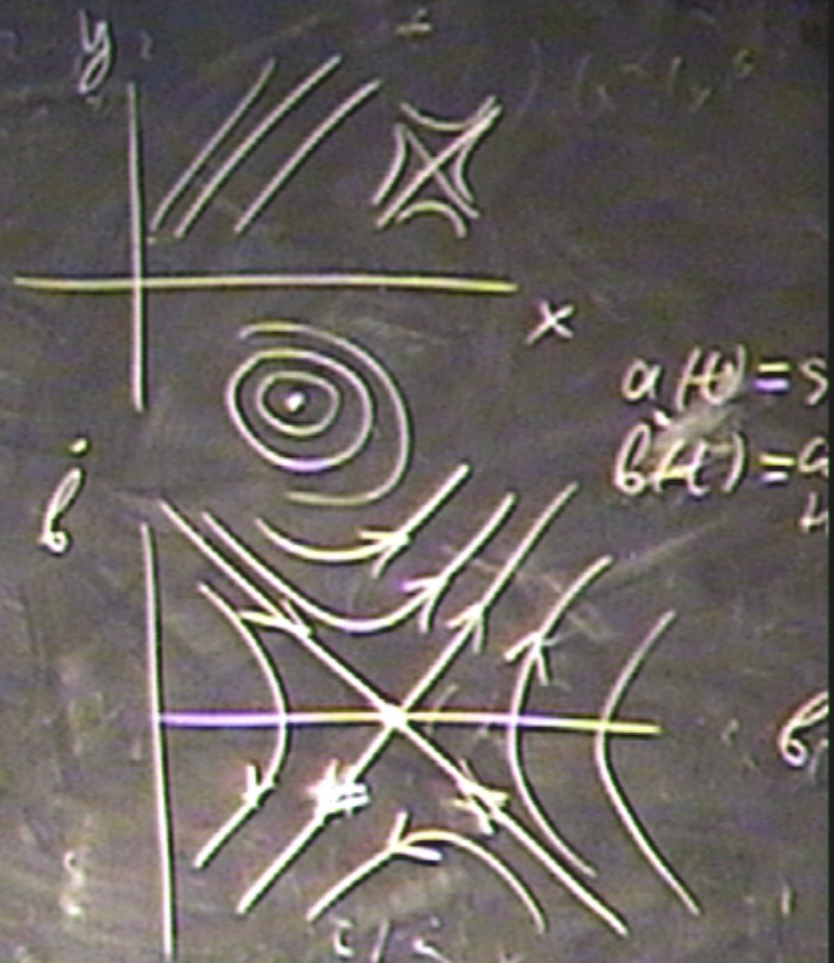


$$\nu' < \pi$$

$$-1' < \pi$$

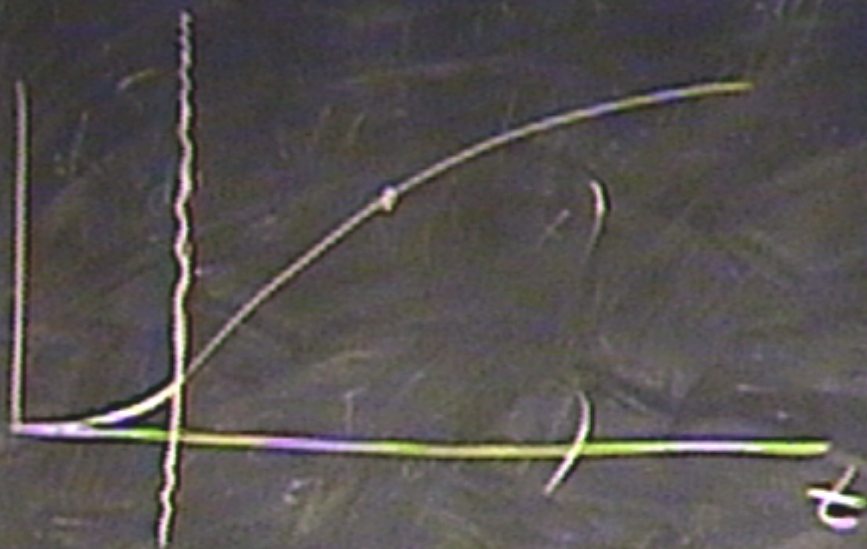


$$P \approx -\varepsilon$$

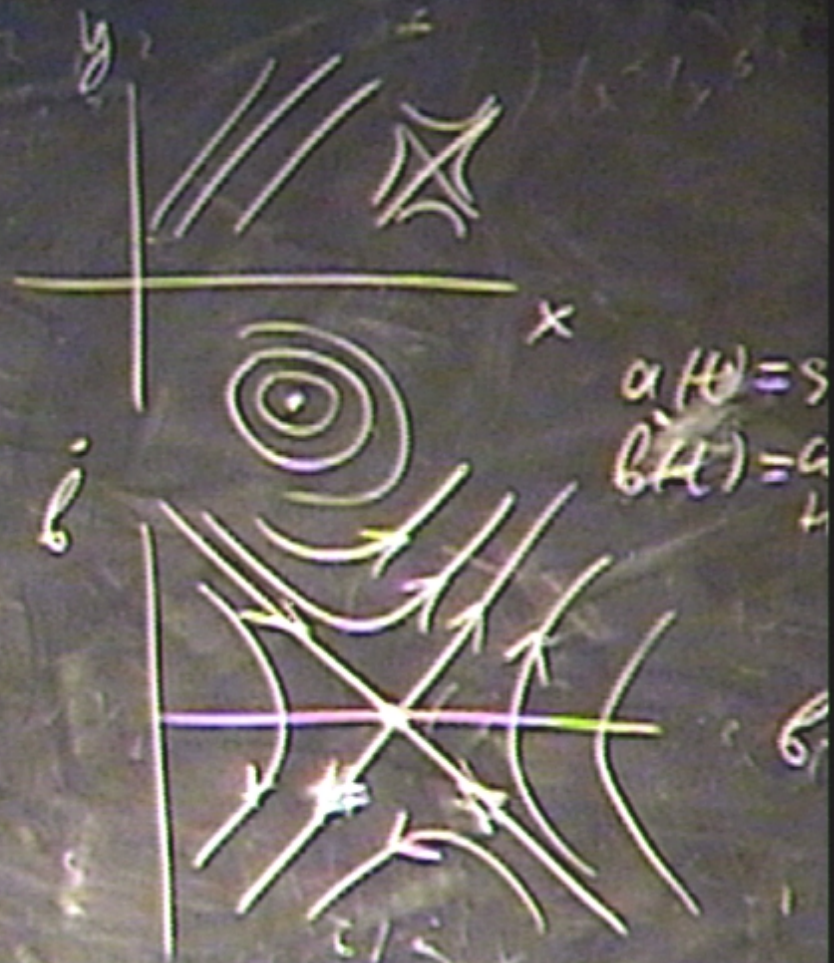


$$\nu' < \pi$$

$$-1' < \pi$$

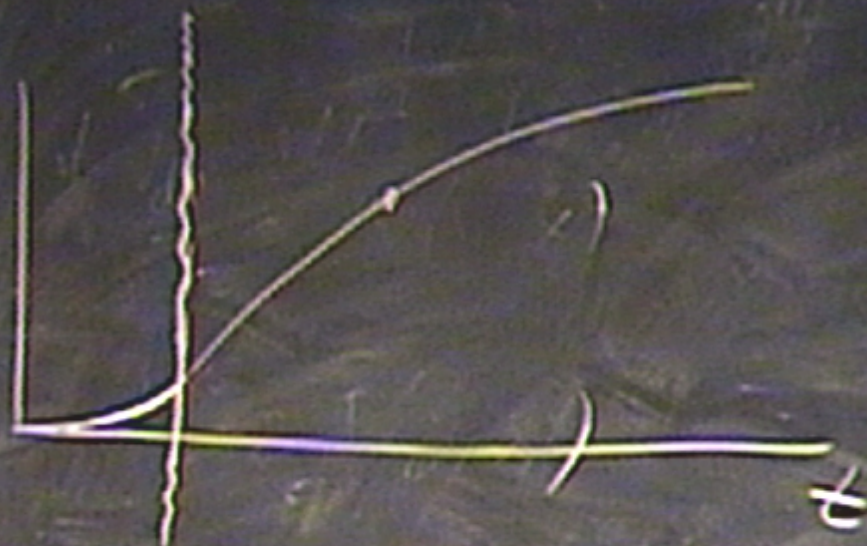


$$p \approx -\varepsilon$$

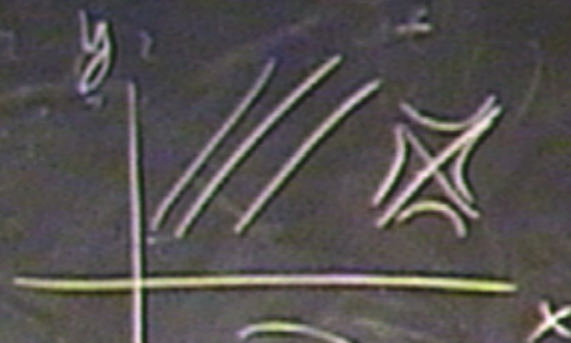


$$\nu' < \pi$$

$$-1' < \pi$$



$$p \approx -\varepsilon$$



$$a(H) = s$$

$$b(H) = a$$

