

Title: SUSY Gauge Theory

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Abstract:

III. $N=2$ SQCD

A. $N=2$ algebra

$$i=1,2 \quad \{Q_{\alpha i}, \bar{Q}_{\dot{\alpha} j}\} = \delta_{ij} \sigma_{\alpha\dot{\alpha}}^{\mu} P_{\mu}$$

III. $N=2$ SQCD

A. $N=2$ algebra

$i=1,2$

$$\{ \bar{Q}_{i,j} \} = \delta_{ij} \sum_{\mu} P_{\mu}$$
$$\{ Q_{\mu,j} \} = 1$$

III. $N=2$ SQCD

A. $N=2$ algebra

$i=1,2$

$$\{Q_{\alpha i}, \bar{Q}_{\dot{\alpha} j}\} = \delta_{ij} \sigma_{\alpha\dot{\alpha}}^{\mu} P_{\mu}$$

$$\{Q_{\alpha i}, Q_{\beta j}\} = \epsilon_{ij} \epsilon_{\alpha\beta} \mathbb{Z}$$

III. $N=2$ SQCD

A. $N=2$ algebra

$i=1,2$

$$\{Q_{\alpha i}, \bar{Q}_{\dot{\alpha} j}\} = \delta_{ij} \sigma_{\alpha\dot{\alpha}}^{\mu} P_{\mu}$$

$$\{Q_{\alpha i}, Q_{\beta j}\} = \epsilon_{ij} \epsilon_{\alpha\beta} \mathbb{Z}$$

$$[\mathbb{Z}, Q_{\alpha i}] = [\mathbb{Z}, P_{\mu}] = 0$$

"central charge"

$$m \geq |Z|$$

Go to form $P^u = (m, \vec{\sigma})$

$$\{c_a, c_b^+\} \quad S_{ab}(M + |Z|)$$

$$m \geq |z|$$

Go to frame $P^u = (m, \vec{0})$

$$\{c_a, c_b^+\} = \delta_{ab} (m + |z|)$$

$$\{\tilde{c}_a, \tilde{c}_b^+\} = \delta_{ab} (m - |z|)$$

$$m \geq |z|$$

Go to form $P^u = (m, \vec{\sigma})$

$$\{c_a, c_b^+\} = \delta_{ab} (m + |z|)$$

$$\{\tilde{c}_a, \tilde{c}_b\} = \delta_{ab} (m - |z|)$$

$$m \geq |z|$$

$$G_0 \text{ to form } P^\mu = (m, \vec{0})$$

$$\{c_a, c_b^+\} = \delta_{ab} (m + |z|)$$

$$\{\tilde{c}_a, \tilde{c}_b^+\} = \delta_{ab} (m - |z|)$$

III. $N=2$ SQCD

A. $N=2$ algebra

$= 1, 2$

$$\{Q_{\alpha i}, \bar{Q}_{\dot{\alpha} j}\} = \delta_{ij} \sigma^{\mu}_{\alpha \dot{\alpha}} P_{\mu}$$

$$\{Q_{\alpha i}, Q_{\beta j}\} = \epsilon_{ij} \epsilon_{\alpha\beta} Z$$

$$[Z, Q_{\alpha}] = [Z, P_{\mu}] = 0$$

"central charge"

$$m \geq |Z|$$

Go to form $P^u = (m, \vec{0})$

$$\{c_a, c_b^+\} = \delta_{ab} (m + |Z|)$$

$$\{\tilde{c}_a, \tilde{c}_b^+\} = \delta_{ab} (m - |Z|)$$

$$m \geq |z|$$

Go to frame $P^u = (m, \vec{0})$

$$\{c_a, c_b^+\} = \delta_{ab} (m + |z|)$$

$$\{\tilde{c}_a, \tilde{c}_b^+\} = \delta_{ab} (m - |z|)$$

$$m \geq |z|$$

Go to form $P^u = (m, \vec{0})$

$$a, b \in \{1, 3\} \quad \{C_a, C_b^+\} = \delta_{ab} (m + |z|)$$

$$\{\tilde{C}_a, \tilde{C}_b\} = \delta_{ab} (m - |z|)$$

$$\Rightarrow m \geq |z|$$

from $P^u = (m, \vec{0})$

$$\{c_a, c_b^+\} = \delta_{ab} (m + |\vec{z}|)$$

$$\{\tilde{c}_a, \tilde{c}_b^+\} = \delta_{ab} (m - |\vec{z}|)$$

$$\{c, c^+\} \geq 0 \Rightarrow m \geq |\vec{z}|$$

$$m \geq |z|$$

to form $P^u = (m, \vec{0})$

$$\{c_a, c_b^+\} = \delta_{ab} (m + |z|)$$

$$\{\tilde{c}_a, \tilde{c}_b^+\} = \delta_{ab} (m - |z|)$$

$$\{c^-, c^+\} \geq 0 \Rightarrow m \geq |z|$$

Go to frame $P^u = (m, \vec{0})$

$a, b \in \mathbb{Z}$ $\{c_a, c_b^+\} = \delta_{ab} (M + |\mathbb{Z}|)$

$\{\tilde{c}_a, \tilde{c}_b^+\} = \delta_{ab} (M - |\mathbb{Z}|)$

$\{c, c^+\} \geq 0 \Rightarrow m \geq |\mathbb{Z}|$

Go to frame $P^u = (m, \vec{0})$

$$\{c_a, c_b^+\} = \delta_{ab} (M + |\vec{z}|)$$

$$\{\tilde{c}_a, \tilde{c}_b^+\} = \delta_{ab} (M - |\vec{z}|)$$

$$\{c_a, \tilde{c}_a^+\} \geq 0 \Rightarrow m \geq |\vec{z}|$$

$\neq$ $m = |\vec{z}|$ BPS state

Go to frame $P^u = (m, \vec{0})$

$$a, b \in \{1, 2\} \quad \{c_a, c_b^+\} = \delta_{ab} (M + |\vec{z}|)$$

$$\{\tilde{c}_a, \tilde{c}_b^+\} = \delta_{ab} (M - |\vec{z}|)$$

$$\{c, c^+\} \geq 0 \Rightarrow m \geq |\vec{z}|$$

If $m = |\vec{z}|$ BPS state

$$\{c, c^\dagger\} \geq 0 \Rightarrow m \geq |\tilde{\epsilon}|$$

$\neq m = |\tilde{\epsilon}|$ BPS state

$$\{|4\rangle, c_q^\dagger |4\rangle, c_i^\dagger c_c^\dagger |4\rangle\}$$

PS bound.

$(m, \vec{\sigma})$

$(m + |\vec{z}|)$

$(m - |\vec{z}|)$

$m \geq |\vec{z}|$

PS state

$$1 \rightarrow 2 + 3$$

$$\vec{z}_1 = \vec{z}_2 - \vec{z}_3$$

PS bound.

$(m, \vec{\sigma})$

$(m + |z|)$

$(m - |z|)$

$m \geq |z|$

PS state

$$1 \rightarrow 2 + 3$$

$$z_1 = z_2 + z_3$$

PS bound.

$(m, \vec{\sigma})$

$(m + |z|)$

$(m - |z|)$

$$1 \rightarrow 2 + 3$$

$$z_1 = z_2 + z_3$$

$$|z_1| \leq |z_2| + |z_3|$$

$$m_1 \leq m_2 + m_3$$

PS bound.

$(m, \vec{\sigma})$

$(m + |z|)$

$(m - |z|)$

$$1 \rightarrow 2 + 3$$

$$z_1 = z_2 + z_3$$

$$|z_1| \leq |z_2| + |z_3|$$

$$m_1 \leq m_2 + m_3$$

PS bound.

$(m, \vec{\sigma})$

$(m + |z|)$

$(m - |z|)$

$m \geq$

PS start

$$1 \rightarrow 2 + 3$$

$$z_1 = z_2 + z_3$$

$$|z_1| \leq |z_2| + |z_3|$$

$$m_1 \leq m_2 + m_3$$

PS bound.

$(m, \vec{o})$

$(m + |z|)$

$(m - |z|)$

$$1 \rightarrow 2 + 3$$

$$z_1 = z_2 + z_3$$

$$|z_1| \leq |z_2| + |z_3|$$

$$m_1 \leq m_2 + m_3$$

PS bound.

$(m, \vec{o})$

$(m + |z|)$

$(m - |z|)$

$$1 \rightarrow 2 + 3$$

$$z_1 = z_2 + z_3$$

$$|z_1| \leq |z_2| + |z_3|$$

$$m_1 \leq m_2 + m_3$$

PS bound.

$(m, \vec{\sigma})$

$|Z_1|$

$|Z_1|$

$$1 \not\rightarrow 2+3$$

$$Z_1 = Z_2 + Z_3$$

$$|Z_1| \leq |Z_2| + |Z_3|$$

BPS

$$m_1 \leq m_2 + m_3$$

PS bound.

$(m, \vec{\sigma})$

$(m + |z|)$

$(m - |z|)$

$$m \geq |z|$$

PS state

$$1 \not\rightarrow 2 + 3$$

$$z_1 = z_2 + z_3$$

$$|z_1| \leq |z_2| + |z_3|$$

BPS

m_1

m_3

PS bound.

$(m, \vec{\sigma})$

$(m + |z|)$

$(m - |z|)$

$$m \geq |z|$$

PS state

$$1 \not\rightarrow 2 + 3$$

$$z_1 = z_2 + z_3$$

$$|z_1| \leq |z_2| + |z_3|$$

BPS

$$m_1 \leq m_2 +$$



$$m \geq |Z| \quad \text{BPS bound.}$$

$$G_0 \text{ to form } P^\mu = (m, \vec{0})$$

$$a, b \in \{1, 2\} \quad \{c_a, c_b^\dagger\} = \delta_{ab} (m + |Z|)$$

$$\{\tilde{c}_a, \tilde{c}_b^\dagger\} = \delta_{ab} (m - |Z|)$$

$$\{c, c^\dagger\} \geq 0 \Rightarrow m \geq |Z|$$

↳ If $m = |Z|$ BPS state

$$\{|4\rangle, c_1^\dagger |4\rangle, c_1^\dagger c_1^\dagger |4\rangle\}$$

$$1 \not\rightarrow 2+3$$

$$Z_1 = Z_2 + Z_3$$

$$|Z_1| \leq |Z_2| + |Z_3|$$

$$\text{BPS} \quad m_1 \leq m_2 + m_3$$

$$m \geq |Z| \quad \text{BPS bound.}$$

$$\text{Go to frame } P^u = (m, \vec{0})$$

$$a, b \in \{1, 2\} \quad \{c_a, c_b^\dagger\} = \delta_{ab} (m + |Z|)$$

$$\{\tilde{c}_a, \tilde{c}_b^\dagger\} = \delta_{ab} (m - |Z|)$$

$$\{c, c^\dagger\} \geq 0 \Rightarrow m \geq |Z|$$

If $m = |Z|$ BPS state

$$\{|4\rangle, c_1^\dagger |4\rangle, c_1^\dagger c_1^\dagger |4\rangle\}$$

$$1 \not\rightarrow 2+3$$

$$Z_1 = Z_2 + Z_3$$

$$|Z_1| \leq |Z_2| + |Z_3|$$

$$\text{BPS} \quad m_1 \leq m_2 + m_3$$

$$\text{IR eff actions} \Rightarrow Z \rightarrow 0$$

Hypers

$$H = Q + \tilde{Q}$$

G_0

$a, b \in \{1, 2\}$

$$\sigma_{\mu}^{\nu} p_{\mu} p_{\nu}$$

$$\epsilon_{\alpha\beta} \mathbb{Z}$$

$$\underline{N = 11}$$

Hypers

$$H = Q + \tilde{Q}$$

Vector

$$V = \Phi$$

$$\underline{N = 1}$$

Hypers

$$H = Q + \tilde{Q}$$

Vector

$$V = \Phi + W_\alpha$$

gauge

R

a_2

BPS

$= (m$

$s = \int_{ab} (M$

gauge

$$R + \bar{R}$$

BPS

ad

$$= (m$$

$$s = \int_{ab} (M$$

gauge

$$R + \bar{R}$$

BPS

$a d_j$

$$= (w$$

$$s = \sum_{ab} (w$$

III. $N=2$ SQCD

B. $N=2$ SQCD - classical

Hypers

$$H = Q + \tilde{Q}$$

Vector

$$V = \Phi + W_\alpha$$

$N=1$

III. $N=2$ SQCD

B. $N=2$ SQCD - classical

$$\mathcal{L}_{uv} = \int d^4\theta \left(Q e^v \bar{\tilde{\Psi}} + \tilde{\Phi} e^v \right)$$

III. $N=2$ SQCD

B. $N=2$ SQCD - classical

$N=1$

Hypers

$$H = Q + \tilde{Q}$$

Vector

$$V = \Phi + W_\alpha$$

$$\mathcal{L}_{uv} = \int d^4\theta \left(Q e^v \bar{Q} + \tilde{Q} e^v \tilde{Q} + \Phi e^v \right) \frac{1}{16\pi} \int d^2\theta$$

III. $N=2$ SQCD

B. $N=2$ SQCD - classical

$N=1$

Hypers

$$H = Q + \tilde{Q}$$

Vector

$$V = \Phi + W_\alpha$$

$$\mathcal{L}_{uv} = \int d^4\theta \left(Q e^v \bar{Q} + \tilde{Q} e^v \tilde{Q} + \Phi e^v \tilde{\Phi} \right) + \int d^4\theta \left(\text{tr} W^2 + \sqrt{2} \right)$$

III. $N=2$ SQCD

B. $N=2$ SQCD - classical

$N=1$

Hypers

$$H = Q + \tilde{Q}$$

Vector

$$V = \Phi + W_\alpha$$

$$\mathcal{L}_{uv} = \int d^4\theta (Q e^V \bar{Q} + \tilde{Q} e^V \tilde{\bar{Q}} + \Phi e^V \tilde{\Phi})$$

$$- \int d^4\theta (\text{tr} W^2 + \sqrt{2})$$

III. $N=2$ SQCD

B. $N=2$ SQCD - classical

$$\mathcal{L}_{uv} = \int d^4\theta \left(Q e^v \bar{Q} + \tilde{Q} e^v \tilde{Q} + \Phi e^v \right)$$

Hypers

$$H = Q + \tilde{Q}$$

Vector

$$V = \Phi + W_\alpha$$

$$\frac{1}{16\pi} \int d^4\theta \left(\text{tr} W^2 + \sqrt{2} \right)$$

III. $N=2$ SQCD

B. $N=2$ SQCD - classical

$$\mathcal{L}_{uv} = \int d^4\theta \left(Q e^v \bar{Q} + \tilde{Q} e^v \tilde{Q} + \Phi e^v \right)$$

Hypers

$$H = Q + \tilde{Q}$$

Vector

$$V = \Phi + W_\alpha$$

$$\frac{1}{4\pi} \int d^4\theta \left(\text{tr} W^2 + \sqrt{2} Q \Phi \tilde{Q} \right)$$

III. $N=2$ SQCD

B. $N=2$ SQCD - classical

$N=2$

Hypers

$$H = Q + \tilde{Q}$$

Vector

$$V = \Phi + W_\alpha$$

$$\mathcal{L}_{uv} = \int d^4\theta (Q e^\nu \bar{Q} + \tilde{Q} e^\nu \tilde{Q} + \Phi e^\nu \tilde{\chi}) = \int d^4\theta (\text{tr } W^2 + \sqrt{2} Q \Phi \tilde{Q})$$

III. $N=2$ SQCD

B. $N=2$ SQCD - classical

$$\mathcal{L}_{uv} = \int d^4\theta (Q e^v \bar{Q} + \tilde{Q} e^v \tilde{\bar{Q}} + \Phi e^v \tilde{\Xi})$$

Hypers

$$H = Q + \tilde{Q}$$

Vector

$$V = \Phi + W_\alpha$$

$N=1$

$$\int d^4\theta (\text{tr } W^2 + \sqrt{2} Q \Phi \tilde{Q})$$

III. $N=2$ SQCD

B. $N=2$ SQCD - classical

Hypers

$$H = Q + \tilde{Q}$$

Vector

$$V = \Phi + W_\alpha$$

$$\mathcal{L}_{uv} = \int d^4\theta (Q e^v \bar{Q} + \tilde{Q} e^v \tilde{\bar{Q}} + \Phi e^v \tilde{\Xi}) \int d^4\theta (\text{tr } W^2 + \sqrt{2} Q \Phi \tilde{Q})$$

III. $N=2$ SQCD

B. $N=2$ SQCD - classical

$$\mathcal{L}_{uv} = \int d^4\theta (Q e^V \bar{Q} + \tilde{Q} e^V \tilde{\bar{Q}} + \Phi e^V \tilde{\Xi})$$

Hypers

$$H = Q + \tilde{Q}$$

Vector

$$V = \Phi + W_\alpha$$

$$\int d^4\theta (\text{tr } W^2 + \sqrt{2} Q \Phi \tilde{Q})$$

III. $N=2$ SQCD

B. $N=2$ SQCD - classical

Hypers

$$H = Q + \tilde{Q}$$

Vector

$$V = \Phi + W_\alpha$$

$$\mathcal{L}_{\text{cl}} = \int d^4\theta \left(Q e^{\gamma} \tilde{Q} + \Phi e^{\gamma} \tilde{\Phi} \right) + \frac{1}{16\pi} \int d^4\theta \left(\text{tr} W^2 + \sqrt{2} Q \Phi \tilde{Q} \right)$$

III. $N=2$ SQCD

B. $N=2$ SQCD - classical

$N=1$

Hypers

$$H = Q + \tilde{Q}$$

Vector

$$V = \Phi + W_\alpha$$

$$\mathcal{L} = i\bar{\theta} \left(Q e^\nu \bar{\tilde{Q}} + \tilde{Q} e^\nu \bar{Q} + \Phi e^\nu \bar{\tilde{\chi}} \right) + \frac{1}{16\pi} \int d^2\theta \left(\text{tr} W^2 + \sqrt{2} Q \Phi \tilde{Q} \right)$$

III. $N=2$ SQCD

B. $N=2$ SQCD - classical

Hypers

$$H = Q + \tilde{Q}$$

Vector

$$V = \Phi + W_\alpha$$

$$\mathcal{L}_m = \int d^4\theta (Q e^\nu \bar{Q} + \tilde{Q} e^\nu \tilde{\bar{Q}}) + \frac{g^2}{16\pi} \int d^4\theta (\text{tr} W^2 + \sqrt{2} Q \Phi \tilde{Q})$$

III. $N=2$ SQCD

B. $N=2$ SQCD - classical

$N=2$

Hypers

$$H = Q + \tilde{Q}$$

Vector

$$V = \Phi + W_\alpha$$

$$\mathcal{L}_{\text{cl}} = \int d^4x \left(\bar{\Psi} \not{\partial} \Psi + \tilde{Q} \not{\partial} Q + \bar{\Phi} \not{\partial} \Phi \right) + \frac{1}{16\pi} \int d^2\theta \left(\text{tr} W^2 + \sqrt{2} Q \Phi \tilde{Q} \right)$$

III. $N=2$ SQCD

B. $N=2$ SQCD - classical

$N=2$

Hypers

$$H = Q + \tilde{Q}$$

Vector

$$V = \Phi + W_\alpha$$

$$\mathcal{L}_{uv} = \int d^4\theta (Q e^v \bar{Q} + \tilde{Q} e^v \tilde{Q} + \Phi e^v)$$

$$\frac{1}{16\pi} \int d^4\theta (\text{tr} W^2 + \sqrt{2} Q \Phi \tilde{Q})$$

III. $N=2$ SQCD

B. $N=2$ SQCD - classical

Hypers

$$H = Q + \tilde{Q}$$

Vector

$$V = \Phi + W_\alpha$$

$$\mathcal{L}_{\text{cl}} = \int d^4\theta \left(Q e^V \bar{Q} + \Phi e^V \tilde{\Phi} \right) + \frac{1}{16\pi} \int d^4\theta \left(\text{tr} W^2 + \sqrt{2} Q \Phi \tilde{Q} \right)$$

III. $N=2$ SQCD

B. $N=2$ SQCD - classical

$N=2$

Hypers

$$H = Q + \tilde{Q}$$

Vector

$$V = \Phi + W_\alpha$$

$$\mathcal{L}_{uv} = \int d^4\theta \left(Q e^v \bar{Q} + \tilde{Q} e^v \tilde{Q} + \right) + \frac{v^2}{16\pi} \int d^4\theta \left(\text{tr} W^2 + \sqrt{2} Q \Phi \tilde{Q} \right)$$

III. $N=2$ SQCD

B. $N=2$ SQCD - classical

Hypers

$$H = Q + \tilde{Q}$$

Vector

$$V = \Phi + W_\alpha$$

$$\mathcal{L}_{uv} = \int d^4\theta \left(Q e^v \bar{Q} + \tilde{Q} e^v \tilde{Q} + \Phi e^v \tilde{\Phi} \right) + \theta \left(\text{tr} W^2 + \sqrt{2} Q \Phi \tilde{Q} \right)$$

III. $N=2$ SQCD

B. $N=2$ SQCD - classical

$$\mathcal{L}_{uv} = \int d^4\theta \left(Q e^V \bar{Q} + \tilde{Q} e^V \tilde{Q} + \Phi e^V \tilde{Q} \right) \stackrel{z}{=} \int d^4\theta \left(\text{tr} W^2 + \sqrt{2} Q \Phi \tilde{Q} \right)$$

$N=1$

Hypers

$$H = Q + \tilde{Q}$$

Vector

$$V = \Phi + W_\alpha$$

III. $N=2$ SQCD

B. $N=2$ SQCD - classical

Hypers

$$H = Q + \tilde{Q}$$

Vector

$$V = \Phi + W_\alpha$$

$$\mathcal{L}_{\text{cl}} = \int d^4\theta (Q e^V \bar{Q} + \tilde{Q} e^V \tilde{\bar{Q}} + \Phi e^V \tilde{\Phi}) + \frac{e^2}{4} \int d^4\theta \frac{1}{\sqrt{2}} Q \Phi \tilde{Q}$$

III. $N=2$ SQCD

B. $N=2$ SQCD - classical

Hypers

$$H = Q + \tilde{Q}$$

Vector

$$V = \Phi + W_\alpha$$

$$\mathcal{L}_{uv} = \int d^4\theta \left(Q e^v \bar{Q} + \tilde{Q} e^v \tilde{Q} + \Phi e^v \tilde{\Phi} \right) + \frac{v}{\Lambda^2} \left(W^2 + \sqrt{2} Q \Phi \tilde{Q} + m_i^2 Q \right)$$

III. $N=2$ SQCD

B. $N=2$ SQCD - classical

Hypers

$$H = Q + \tilde{Q}$$

Vector

$$V = \Phi + W_\alpha$$

$$\mathcal{L}_{uv} = \int d^4\theta (Q e^v \bar{Q} + \tilde{Q} e^v \tilde{\bar{Q}} + \Phi e^v \tilde{\Phi}) + \frac{v}{\Lambda} \int d^4\theta (W^2 + \sqrt{2} Q \Phi \tilde{Q} + m_j Q)$$

III. $N=2$ SQCD

B. $N=2$ SQCD - classical

Hypers

$$H = Q + \tilde{Q}$$

Vector

$$V = \Phi + W_\alpha$$

$$\mathcal{L}_{\text{cl}} = \int d^4\theta (Q e^\nu \bar{Q} + \tilde{Q} e^\nu \tilde{Q} + \Phi e^\nu \tilde{\Phi}) + \frac{1}{4\pi} \int d^2\theta \left(\sqrt{2} Q \Phi \tilde{Q} + m_j Q_j \tilde{Q}_j \right)$$

III. $N=2$ SQCD

B. $N=2$ SQCD - classical

Hypers

$$H = Q + \tilde{Q}$$

Vector

$$V = \Phi + W_\alpha$$

$$\mathcal{L}_{\text{cl}} = \int d^4\theta (Q e^\nu \bar{Q} + \tilde{Q} e^\nu \tilde{Q} + \Phi e^\nu \tilde{\Phi}) + \frac{1}{4} \int d^4\theta (\dots) + \sqrt{2} Q \Phi \tilde{Q} + m_i Q_i$$

III. $N=2$ SQCD

B. $N=2$ SQCD - classical

$N=1$

Hypers

$$H = Q + \tilde{Q}$$

Vector

$$V = \Phi + W_\alpha$$

$$\mathcal{L}_{uv} = \int d^4\theta (Q e^v \bar{Q} + \tilde{Q} e^v \tilde{\bar{Q}} + \Phi e^v)$$

$$\frac{1}{16\pi} \int d^4\theta (\text{tr} W^2 + \sqrt{2} Q \Phi \tilde{Q} + m_i Q_i \tilde{Q}_i)$$

III. $N=2$ SQCD

B. $N=2$ SQCD - classical

Hypers

$$H = Q + \tilde{Q}$$

Vector

$$V = \Phi + W_\alpha$$

$$\mathcal{L}_{\text{cl}} = \int d^4\theta \left(Q e^V \bar{Q} + \tilde{Q} e^V \right) + \frac{1}{16\pi} \int d^4\theta \left(\text{tr} W^2 + \sqrt{2} Q \Phi \tilde{Q} + m_i Q_i \tilde{Q}_i \right)$$

III. $N=2$ SQCD

B. $N=2$ SQCD - classical

$N=1$

Hypers

$$H = Q + \tilde{Q}$$

Vector

$$V = \Phi + W_\alpha$$

$$\mathcal{L} = \int d^4\theta (Q e^\nu \bar{\Psi} + \tilde{Q} e^\nu \tilde{\Psi} + \Phi e^\nu \tilde{\Xi}) + \frac{1}{16\pi} \int d^2\theta (\text{tr} W^2 + \sqrt{2} Q \Phi \tilde{Q} + m_i Q_i \tilde{Q}_i)$$

III. $N=2$ SQCD

B. $N=2$ SQCD - classical

$N=2$

Hypers

$$H = Q + \tilde{Q}$$

Vector

$$V = \Phi + W_\alpha$$

$$\mathcal{L}_{uv} = \int d^4\theta (Q e^v \bar{Q} + \tilde{Q} e^v \tilde{\bar{Q}} + \Phi e^v \tilde{\bar{Q}})$$

$$+ \int d^4\theta (\text{tr } W^2 + \sqrt{2} Q \Phi \tilde{Q} + m_i Q_i \tilde{Q}_i)$$

III. $N=2$ SQCD

B. $N=2$ SQCD - classical

Hypers

$$H = Q + \tilde{Q}$$

Vector

$$V = \Phi + W_\alpha$$

gauge
 $R + \bar{R}$

adj

$$\mathcal{L}_v = \int d^4\theta (Q e^\vee \bar{Q} + \tilde{Q} e^\vee \tilde{Q} + \Phi e^\vee \tilde{\Phi}) +$$

$$+ \frac{1}{2} W^2 + \sqrt{2} Q \Phi \tilde{Q} + W Q_i \tilde{Q}^i + c.c. +$$

$$[\tilde{Q}, \tilde{Q}^\dagger] = 0$$

III. $N=2$ SQCD

B. $N=2$ SQCD - classical

Hypers

$$H = Q + \tilde{Q}$$

Vector

$$V = \Phi + W_\alpha$$

$$\mathcal{I} = \int d^4\theta \left(Q e^\nu \bar{\Psi} + \tilde{Q} e^\nu \tilde{\Psi} + \Phi e^\nu \tilde{\Phi} \right) + \frac{L\pi}{16\pi} \int d^4\theta \left(\text{tr} W^2 + \sqrt{2} Q \Phi \tilde{Q} + m_i^j Q_i \tilde{Q}^j \right) + \dots$$

$[m, m^+] = 0$

III. $N=2$ SQCD

B. $N=2$ SQCD - classical

Hypers

$$H = Q + \tilde{Q}$$

Vector

$$V = \Phi + W_\alpha$$

$$\mathcal{L}_m = \int d^4\theta (Q e^\vee \bar{Q} + \tilde{Q} e^\vee \tilde{\bar{Q}} + \Phi e^\vee \tilde{\Phi}) + \frac{g^2}{16\pi} \int d^4\theta (\text{tr} W^2 + \sqrt{2} Q \Phi \tilde{Q} + m_i^j Q_i \tilde{Q}^j) + \text{c.c.}$$

$[m, m^*] = 0$

III. $N=2$ SQCD

B. $N=2$ SQCD - classical

Hypers

$$H = Q + \tilde{Q}$$

Vector

$$V = \Phi + W_\alpha$$

$$\mathcal{L} = (Q_e^\dagger \bar{Q} + \tilde{Q}_e^\dagger \bar{\tilde{Q}} + \Phi_e^\dagger \tilde{\Phi}) + \frac{c_F}{16\pi} \int d^4\theta \left(\text{tr} W^2 + \sqrt{2} Q \Phi \tilde{Q} + m_i \cancel{Q_i \tilde{Q}^i} \right) + \text{c.c.}$$

$[m, m^\dagger] = 0$

III. $N=2$ SQCD

B. $N=2$ SQCD - classical

Hypers

$$H = Q + \tilde{Q}$$

Vector

$$V = \Phi + W_\alpha$$

$$\mathcal{L}_{cl} = \int d^4\theta \left(\bar{\psi} + \tilde{Q}^i \psi + \bar{\Phi}^i \psi + \tilde{\Phi}^i \psi \right) + \frac{1}{16\pi} \int d^4\theta \left(\text{tr} W^2 + \sqrt{2} \overbrace{Q \Phi \tilde{Q}}^F + m_i \tilde{Q}^i Q^i \right) + \text{c.c.}$$

$[m, m^*] = 0$

III. $N=2$ SQCD

B. $N=2$ SQCD - classical

Hypers

$$H = Q + \tilde{Q}$$

Vector

$$V = \Phi + W_\alpha$$

$$\mathcal{L}_{cl} = \int d^4\theta (Q e^\vee \bar{Q} + \tilde{Q} e^\vee \bar{\tilde{Q}} + \Phi e^\vee \tilde{\Phi}) + \frac{1}{16\pi} \int d^4\theta (\text{tr} W^2 + \sqrt{2} \overbrace{Q \Phi \tilde{Q}}^F + m_i \cancel{Q_i \tilde{Q}_i}) + \text{c.c.}$$

$[m, m^*] = 0$

III. $N=2$ SQCD

B. $N=2$ SQCD - classical

Hypers

$$H = Q + \tilde{Q}$$

Vector

$$V = \Phi + W_\alpha$$

$$\mathcal{I}_{cl} = \int d^4\theta (Q e^\nu \bar{Q} + \tilde{Q} e^\nu \bar{\tilde{Q}} + \Phi e^\nu \tilde{\Phi}) + \frac{g^2}{16\pi} \int d^4\theta (\text{tr} W^2 + \sqrt{2} \overbrace{Q \Phi \tilde{Q}}^F + m_i \cancel{Q_i \tilde{Q}_i}) + \dots$$

$[m, m^+] = 0$

III. $N=2$ SQCD

B. $N=2$ SQCD - classical

$$\mathcal{L}_m = \int d^4\theta (Q e^V \bar{Q} + \tilde{Q} e^V \bar{\tilde{Q}} + \Phi e^V \tilde{\Phi})$$

Hypers

$$H = Q + \tilde{Q}$$

Vector

$$V = \Phi + W_\alpha$$

$N=1$

$$= \int d^4\theta (tr W^2 + \sqrt{2} Q \Phi \tilde{Q} + m_i \tilde{Q}_i Q_i) + c.c.$$

$[m, m^*] \in 0$

III. $N=2$ SQCD

B. $N=2$ SQCD - classical

$$\mathcal{L}_m = \int d^4\theta (Q e^V \bar{Q} + \tilde{Q} e^V \bar{\tilde{Q}} + \Phi e^V \tilde{\Phi})$$

Hypers

$$H = Q + \tilde{Q}$$

Vector

$$V = \Phi + W_\alpha$$

$\Phi \in \mathbb{C}$

$$\int d^4\theta (\text{tr} W^2 + \dots)$$

$$(\tilde{Q}, Q^i) + \dots$$

$$[m, m^*] = 0$$

III. $N=2$ SQCD

B. $N=2$ SQCD - classical

$$\mathcal{L}_m = \int d^4\theta (Q e^V \bar{Q} + \tilde{Q} e^V \bar{\tilde{Q}} + \Phi e^V \bar{\Phi})$$

Hypers

$$H = Q + \tilde{Q}$$

Vector

$$V = \Phi + W_\alpha$$

$\Phi \bar{\Phi}$

$$\int d^4\theta (\text{tr} W^2 + \bar{\Phi} \Phi \bar{\tilde{Q}} \tilde{Q})$$

$$(\tilde{Q}_i \tilde{Q}^i) + \dots$$

$(m, m^*) \in 0$

III. $N=2$ SQCD

B. $N=2$ SQCD - classical

$$\mathcal{L}_m = \int d^4\theta (Q e^V \bar{Q} + \tilde{Q} e^V \bar{\tilde{Q}} + \Phi e^V \tilde{\Phi})$$

Hypers

$$H = Q + \tilde{Q}$$

Vector

$$V = \Phi + W_\alpha$$

$$\Phi \tilde{\Phi} =$$

$$\int d^4\theta (\text{tr } W^2 + \dots)$$

$$(\tilde{Q}, Q^i) + \dots$$

$$[m, m^*] = 0$$

III. $N=2$ SQCD

B. $N=2$ SQCD - classical

$$\mathcal{L}_m = \int d^4\theta (Q e^V \bar{Q} + \tilde{Q} e^V \bar{\tilde{Q}} + \Phi e^V \tilde{\Phi})$$

Hypers

$$H = Q + \tilde{Q}$$

Vector

$$V = \Phi + W_\alpha$$

$$\Phi \tilde{Q} = 0$$

$\underbrace{F}$

$$\mathcal{L}_m = \int d^4\theta (\text{tr} \sqrt{2} Q \Phi \tilde{Q}$$

$$+ m_i \tilde{Q}_i Q_i)$$

$$[m, m^\dagger] = 0$$

III. $N=2$ SQCD

B. $N=2$ SQCD - classical

$$\mathcal{L}_{cl} = \int d^4\theta (Q e^V \bar{Q} + \tilde{Q} e^V \bar{\tilde{Q}} + \Phi)$$

Hypers

$$H = Q + \tilde{Q}$$

Vector

$$V = \Phi + W_\alpha$$

$$F \Phi \tilde{Q} = 0$$

$$\frac{1}{16\pi} \int d^4\theta (W^2 + \sqrt{2} Q \Phi \tilde{Q})$$

$$+ m (\tilde{Q} Q) + c.c.$$

$$[m, m^\dagger] = 0$$

III. $N=2$ SQCD

B. $N=2$ SQCD - classical

$$\mathcal{L}_m = \int d^4\theta (Q e^V \bar{Q} + \tilde{Q} e^V \bar{\tilde{Q}} + \frac{1}{2} \text{tr} W^2 + \sqrt{2} Q \Phi \tilde{Q} + m \tilde{Q} Q) + \text{c.c.}$$

Hypers

$$H = Q + \tilde{Q}$$

Vector

$$V = \Phi + W_\alpha$$

$$\Phi = \tilde{Q} = 0$$

F

$$\frac{1}{16\pi} \int d^4\theta \text{tr} W^2 + \sqrt{2} Q \Phi \tilde{Q}$$

$$+ m \tilde{Q} Q$$

$$[m, m^*] = 0$$

III. $N=2$ SQCD

B. $N=2$ SQCD - classical

$$\mathcal{L}_m = \int d^4\theta (Q e^V \bar{Q} + \tilde{Q} e^V \bar{\tilde{Q}} + \frac{1}{2} \text{Tr} F^2)$$

Hypers

$$H = Q + \tilde{Q}$$

Vector

$$V = \Phi + W_\alpha$$

$$Q \Phi = \Phi \tilde{Q} = 0$$

$$+ \frac{1}{16\pi} \int d^4\theta (W^\alpha W_\alpha + \sqrt{2} Q \Phi \tilde{Q})$$

$$+ m_i \bar{Q}_i Q_i + \text{c.c.}$$

$$[m, m^\dagger] = 0$$

III. $N=2$ SQCD

B. $N=2$ SQCD - classical

$$\mathcal{L}_{cl} = \int d^4\theta (Q e^\nu \bar{Q} + \tilde{Q} e^\nu \bar{\tilde{Q}} + \Phi)$$

Hypers

$$H = Q + \tilde{Q}$$

Vector

$$V = \Phi + W_\alpha$$

$$Q\Phi = \Phi\tilde{Q} = 0$$

$$+ \frac{g^2}{16\pi} \int d^4\theta (N^2 + \sqrt{2} Q\Phi\tilde{Q})$$

$$+ m_i \bar{Q}_i Q_i + c.c.$$

$$[m, m^\dagger] = 0$$

III. $N=2$ SQCD

B. $N=2$ SQCD - classical

$$\mathcal{L}_m = \int d^4\theta (Q e^\nu \bar{Q} + \tilde{Q} e^\nu \bar{\tilde{Q}} + \Phi e^\nu \bar{\Phi})$$

Hypers

$$H = Q + \tilde{Q}$$

Vector

$$V = \Phi + W_\alpha$$

$$Q \Phi = \Phi \tilde{Q} = 0$$

$$\frac{1}{16\pi} \int d^4\theta (tr W^2 + \sqrt{2} Q \Phi \tilde{Q})$$

$$+ m_i \bar{Q}_i Q_i + c.c.$$

$$[m, m^*] = 0$$

III. $N=2$ SQCD

B. $N=2$ SQCD - classical

$$\mathcal{L}_{\text{cl}} = \int d^4x (Q \not{D} \bar{Q} + \tilde{Q} \not{D} \bar{\tilde{Q}} + \dots)$$

Hypers

$$H = Q + \tilde{Q}$$

Vector

$$V = \Phi + W_\mu$$

$$Q \Phi = \Phi \tilde{Q} = 0$$

$$\frac{1}{16\pi} \int d^4x (F^2 + \sqrt{2} Q \Phi \tilde{Q} + \dots)$$

$$+ m_i (\bar{Q}_i \tilde{Q}_i) + \dots$$

$$[m, m^\dagger] = 0$$

III. $N=2$ SQCD

B. $N=2$ SQCD - classical

$$\mathcal{L}_{cl} = \int d^4\theta (Q e^V \bar{Q} + \tilde{Q} e^V \bar{\tilde{Q}} + \frac{1}{2} \Phi^2)$$

Hypers

$$H = Q + \tilde{Q}$$

Vector

$$V = \Phi + W_\alpha$$

$$Q \Phi + \Phi \tilde{Q} = 0$$

$$\frac{1}{16\pi}$$

$$\int d^4\theta$$

$$+ \sqrt{2} Q \Phi \tilde{Q}$$

$$+ m \cdot \cancel{Q \tilde{Q}} + \dots$$

$$[m, m^\dagger] = 0$$

III. $N=2$ SQCD

B. $N=2$ SQCD - classical

Hypers

$$H = Q + \tilde{Q}$$

Vector

$$V = \Phi + W_\alpha$$

$$Q\Phi = \Phi\tilde{Q} = 0$$

$$\mathcal{L}_{cl} = \int d^4\theta \left(Q^\dagger Q + \tilde{Q}^\dagger \tilde{Q} + \Phi^\dagger \Phi \right) + \frac{1}{16\pi} \int d^4\theta \left(\text{tr} W^\alpha W_\alpha + \sqrt{2} Q \Phi \tilde{Q} + m \Phi \tilde{Q} \right) + \text{c.c.}$$

$[m, m^\dagger] = 0$

III. $N=2$ SQCD

B. $N=2$ SQCD - classical

Hypers

$$H = Q + \tilde{Q}$$

Vector

$$V = \Phi + W_\alpha$$

$$Q\Phi = \Phi\tilde{Q} = 0$$

$$\mathcal{L}_{cl} = \int d^4\theta \left(\tilde{Q}^\dagger Q + \tilde{Q}^\dagger \gamma^\mu \tilde{Q} + \Phi^\dagger \gamma^\mu \Phi \right) + \frac{1}{16\pi} \int d^4\theta \left(\text{tr} W^2 + \sqrt{2} Q \Phi \tilde{Q} + m_i \tilde{Q}_i Q_i \right) + c.c.$$

$[m, m^\dagger] = 0$

III. $N=2$ SQCD

B. $N=2$ SQCD - classical

Hypers

$$H = Q + \tilde{Q}$$

Vector

$$V = \Phi + W_\mu$$

$$Q\Phi = \Phi\tilde{Q} = 0$$

$$\mathcal{L}_v = \int d^4\theta \left(Q \tilde{Q}^\dagger \tilde{Q} + \Phi^\dagger \Phi \right) + \frac{1}{16\pi} \int d^4\theta \left(\text{tr} W^2 + \sqrt{2} Q \Phi \tilde{Q} + m_i \tilde{Q}_i Q_i \right) + \text{c.c.}$$

$(m, m^*) \in \mathbb{C}$

III. $N=2$ SQCD

B. $N=2$ SQCD - classical

Hypers

$$H = Q + \tilde{Q}$$

Vector

$$V = \Phi + W_\alpha$$

$$Q\Phi = \Phi\tilde{Q} = 0$$

$$\mathcal{L}_{cl} = \int d^4\theta \left(Q e^V e^{\tilde{V}} \tilde{Q} + \Phi e^V \tilde{\Phi} \right) + \frac{1}{16\pi} \int d^4\theta \left(\text{tr} W^2 + \sqrt{2} Q \Phi \tilde{Q} + m_i \tilde{Q}_i Q_i \right) + \dots$$

$[m, m^*] = 0$

III. $N=2$ SQCD

B. $N=2$ SQCD - classical

Hypers

$$H = Q + \tilde{Q}$$

Vector

$$V = \Phi + W_\alpha$$

$$Q\Phi = \Phi\tilde{Q} = 0$$

$$\mathcal{L}_{cl} = \int d^4\theta (Qe^\vee\bar{\Psi} + \tilde{Q}e^\vee\bar{\tilde{\Psi}} + \Phi e^\vee\tilde{\Psi}) + \frac{1}{16\pi} \int d^4\theta (\text{tr} W^2 + \sqrt{2} \overbrace{Q\Phi\tilde{Q}}^F$$

$$+ m_i \cancel{Q_i\tilde{Q}_i}) + \dots$$

$[m, m^*] = 0$

III. $N=2$ SQCD

B. $N=2$ SQCD - classical

Hypers

$$H = Q + \tilde{Q}$$

Vector

$$V = \Phi + W_\alpha$$

$$Q\Phi + \Phi\tilde{Q} = 0$$

$$\mathcal{L} = \int d^4\theta (Qe^\vee\bar{\Psi} + \tilde{Q}e^\vee\bar{\tilde{\Psi}} + \Phi e^\vee\tilde{\Psi}) + \frac{1}{16\pi} \int d^4\theta (\text{tr} W^2 + \sqrt{2} Q\Phi\tilde{Q} + m_i \tilde{Q}_i Q_i)$$

$$[m, m^\dagger] = 0$$

$$\mathcal{L}_{uv} = \int d^4\theta \left(Q e^v \bar{\Psi} + \tilde{Q} e^v \bar{\tilde{\Psi}} + \dots \right)$$

(1) Coulomb Branch (CB)

$$L_v = \int d^4\theta \left(Q e^v \bar{Q} + \tilde{Q} e^v \bar{\tilde{Q}} + \Phi \right)$$

(1) Coulomb Branch (CB)

$$\langle \Phi \rangle \neq 0$$

$$\langle Q, \tilde{Q} \rangle = 0$$

$$\mathcal{L} \propto \theta \left(Q e^\nu \bar{\Phi} + \tilde{Q} e^\nu \bar{\tilde{\Phi}} + \dots \right)$$

Coulomb Branch (CB)

$$\langle \Phi \rangle \neq 0 \Rightarrow$$

$$\langle \Phi, \tilde{\Phi} \rangle = 0$$

$$\mathcal{L}_{uv} = \int d^4\theta \left(\dots + \bar{Q} e^{\gamma} \bar{Q} + \dots \right)$$

(1) $\text{anch} (CB)$

$\Rightarrow SU$

$$\mathcal{L}_{uv} =$$

$$\bar{Q} e^{\nu} \bar{\Psi} + \bar{\tilde{Q}} e^{\nu} \bar{\tilde{\Psi}} + \dots$$

mb Branch (CB)

$$\rangle \neq 0 \Rightarrow SU(N_c)$$

$$\tilde{Q} \rangle = 0$$

$$1^4 \theta (Q e^\nu \bar{Q} + \tilde{Q} e^\nu \bar{\tilde{Q}} + \Phi e^\nu \bar{\Phi}) +$$

Coulomb Branch (CB)

$$\langle \Phi \rangle \neq 0 \Rightarrow SU(N_c) \rightarrow U(1)^r$$

$$\langle Q, \tilde{Q} \rangle = 0$$

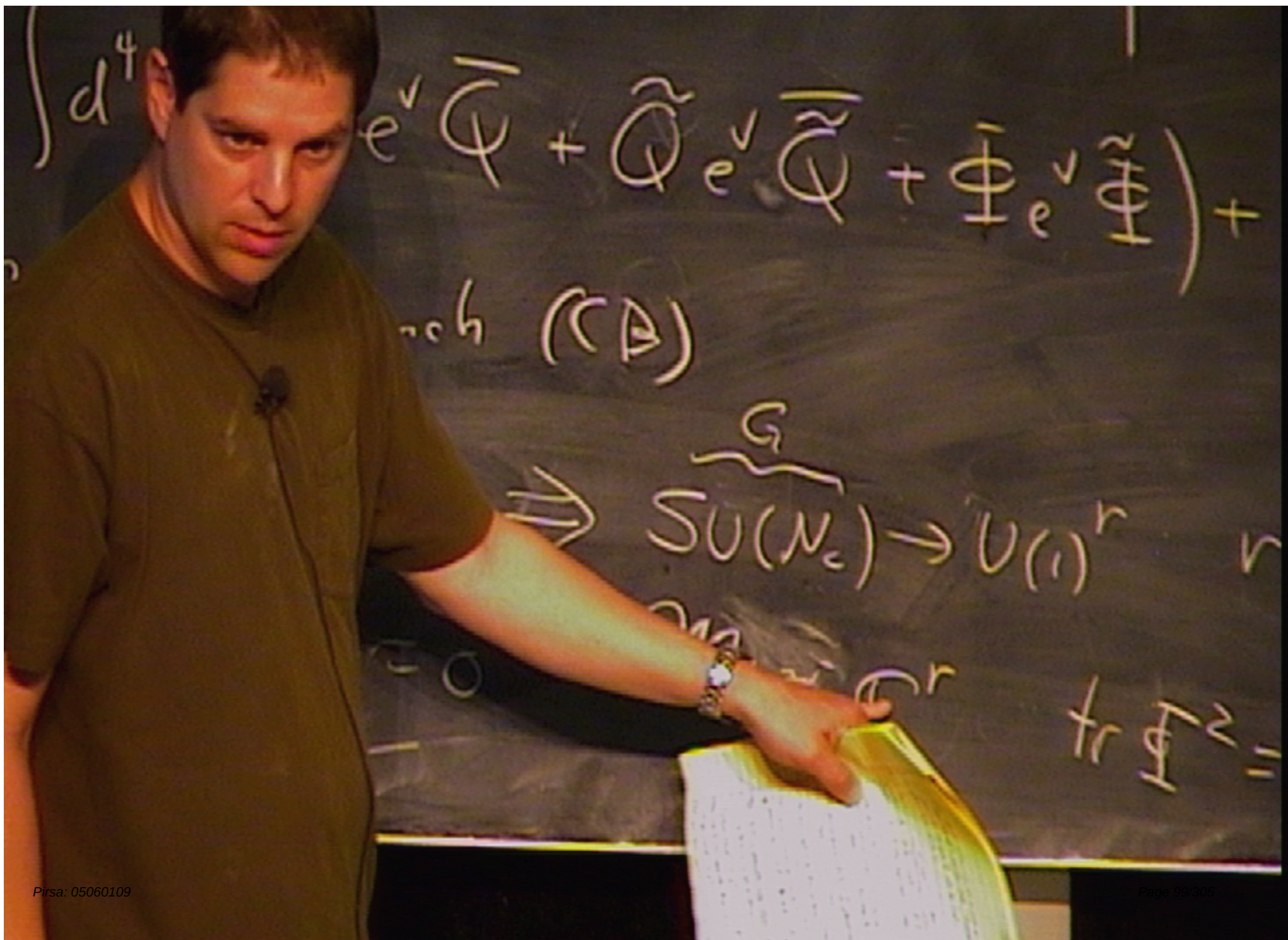
$$\int d^4\theta \left(Q e^\nu \bar{Q} + \tilde{Q} e^\nu \bar{\tilde{Q}} + \Phi e^\nu \tilde{\Phi} \right) +$$

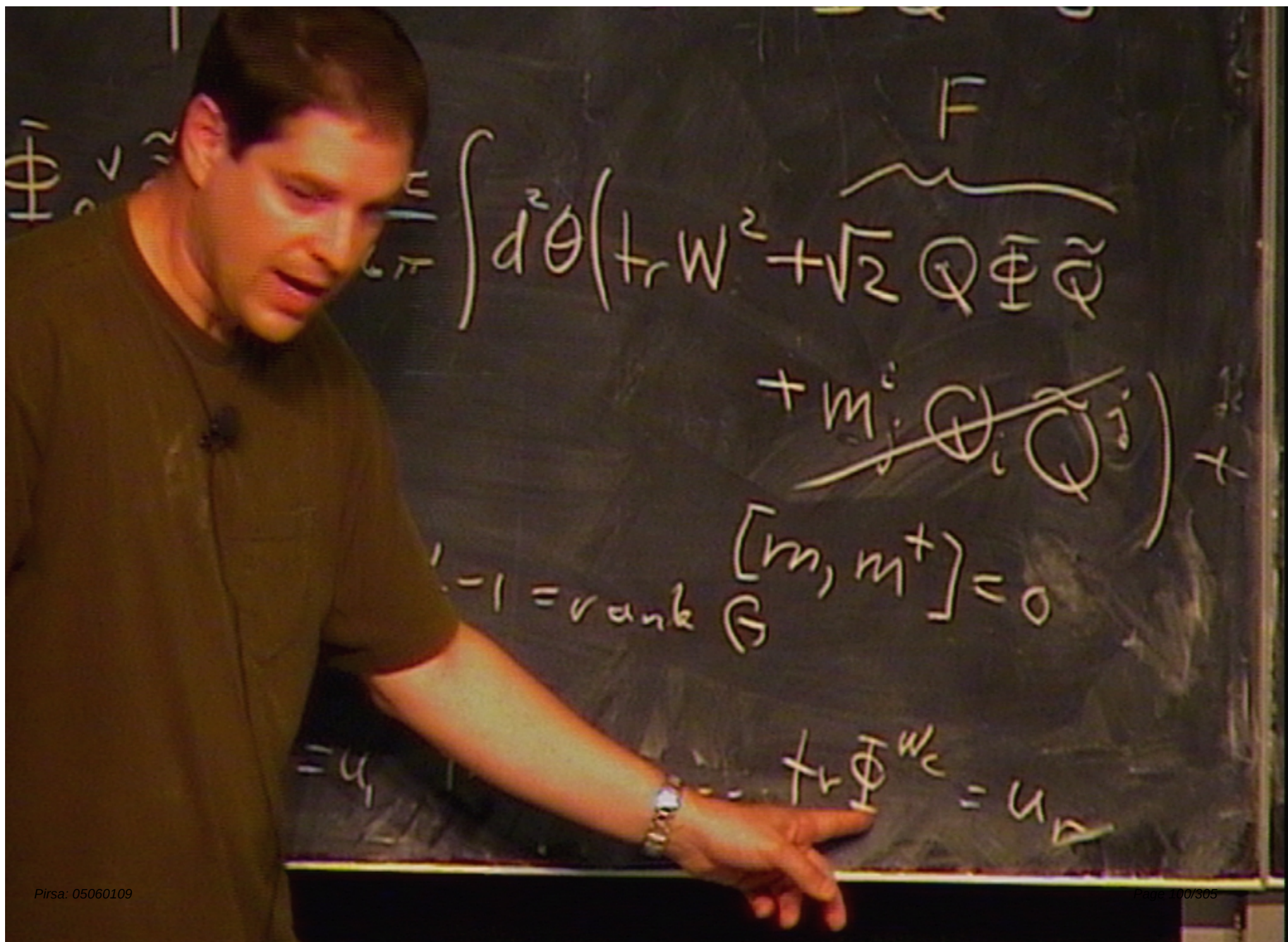
Coulomb Branch (CB)

$$\langle \Phi \rangle \neq 0 \Rightarrow \overbrace{SU(N_c)}^G \rightarrow U(1)^r \quad r$$

$$\langle Q, \tilde{Q} \rangle = 0$$

$$\mathcal{M}_{CB} \cong \mathbb{C}^r$$



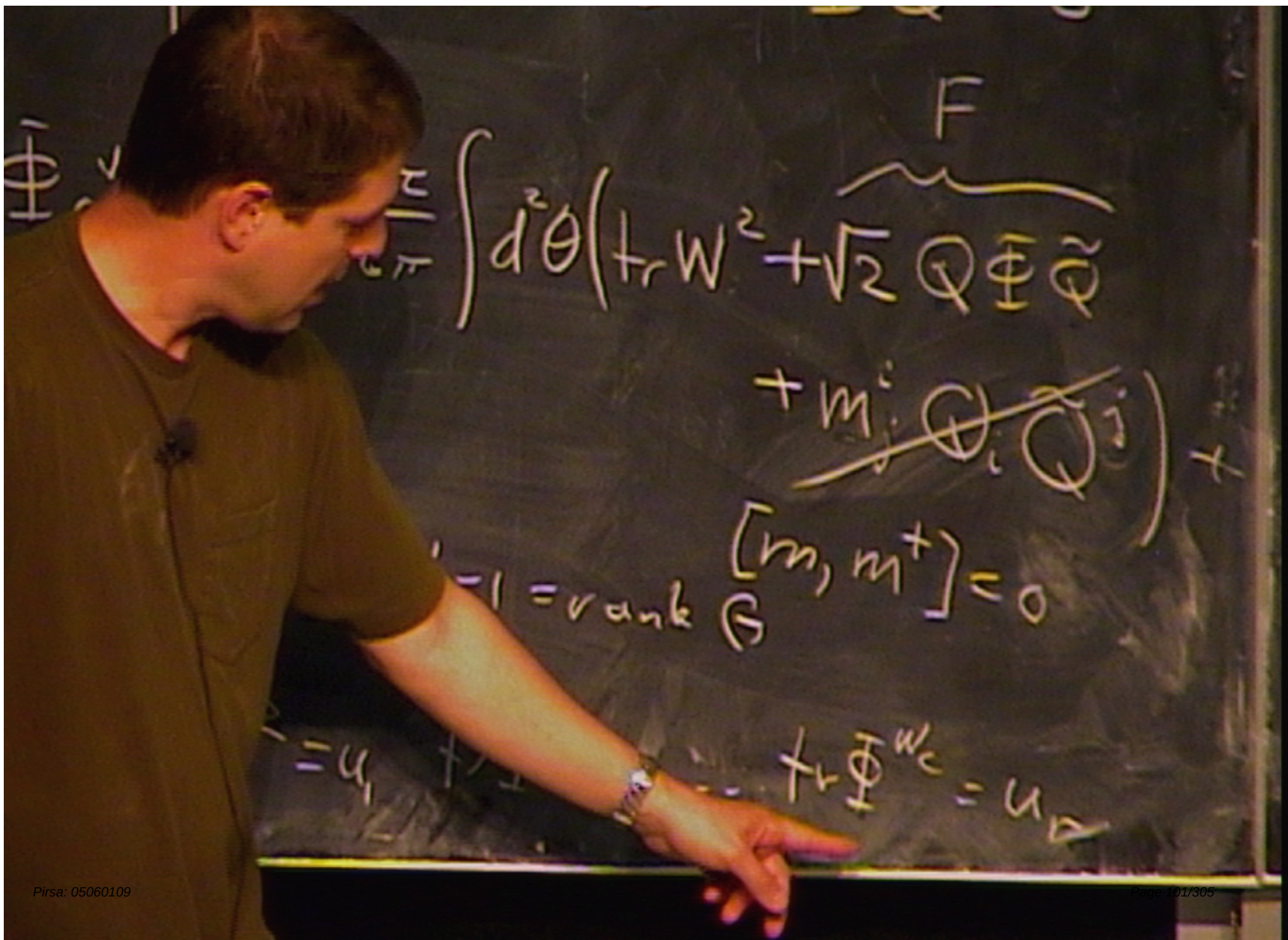


$$\int d^2\theta \left(\text{tr} W^2 + \sqrt{2} Q \bar{\Phi} \tilde{Q} \right)$$

$$+ m_i^c \cancel{(\Phi_i \tilde{Q}^c_i)} +$$

$$-1 = \text{rank } G \quad [m, m^+] = 0$$

$$\text{tr} \Phi W_c = u_D$$

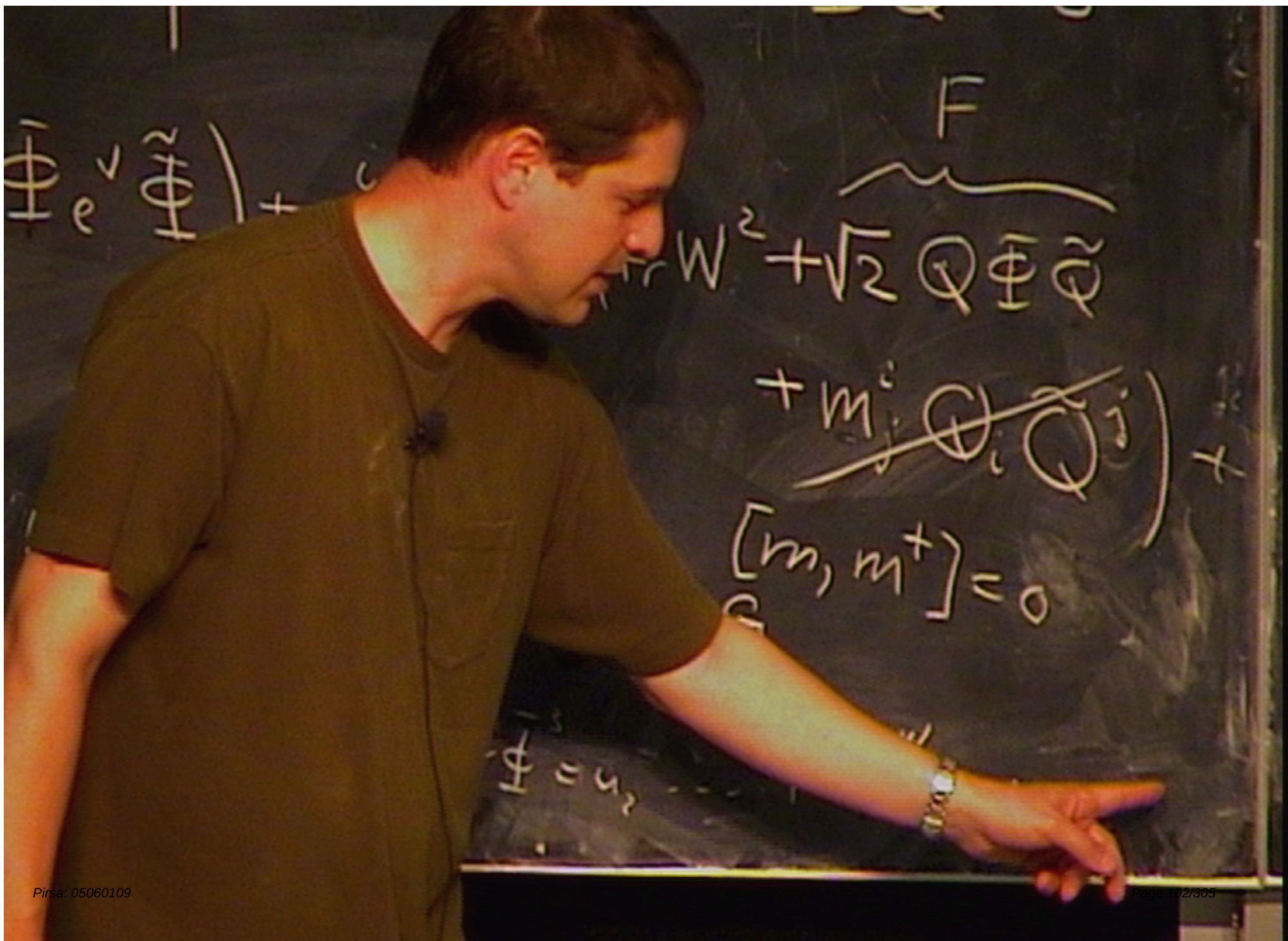


$$\frac{i}{4\pi} \int d^2\theta \left(\text{tr} W^2 + \sqrt{2} \overbrace{Q \Phi \tilde{Q}}^F \right.$$

$$\left. + m_i^j \cancel{\Phi_i \tilde{Q}^j} \right) +$$

$$-1 = \text{rank } G \quad [m, m^+] = 0$$

$$= u_1 \quad \text{tr } \Phi W_c = u_2$$



$$\begin{aligned}
 & \bar{\Phi} e^{\nu} \tilde{\Phi} \Big) + \frac{iz}{16\pi} \int d^2\theta \left(\text{tr} W^2 + \overbrace{\sqrt{2} Q \bar{\Phi} \tilde{Q}}^F \right. \\
 & \quad \left. + \cancel{m_i^j \Phi_i \tilde{Q}^j} \right) +
 \end{aligned}$$

$$U(1)^r \quad r = N_c - 1 = \text{rank } \mathfrak{g} \quad [m, m^+] = 0$$

$$\text{tr } \Phi^2 = u_1, \quad \text{tr } \Phi^3 = u_2, \quad \dots, \quad \text{tr } \Phi^{N_c} = u_{N_c}$$

(2) Higgs branch $\langle Q, \tilde{Q} \rangle \neq 0$
 $\langle \tilde{\Phi} \rangle = 0$

Higgs $G \rightarrow \phi$

$$1 \not\rightarrow 2+3$$

$$Z_1 = Z_2 + Z_3$$

$$|Z_1| \leq |Z_2| + |Z_3|$$

BPS $m_1 \leq m_2 + m_3$

IR eff actions $\Rightarrow Z \rightarrow 0$

(2) Higgs branch $\langle Q, \bar{Q} \rangle \neq 0$
 $\langle \Phi \rangle = 0$

$$\text{higgs } G \rightarrow \phi$$
$$\Rightarrow \cong \mathbb{C}^{2n}$$

$$1 \not\rightarrow 2+3$$

$$Z_1 = Z_2 + Z_3$$

$$|Z_1| \leq |Z_2| + |Z_3|$$

BPS $m_1 \leq m_2 + m_3$

IR eff actions $\Rightarrow Z \rightarrow 0$

(2) Higgs branch $\langle Q, \bar{Q} \rangle \neq 0$
 $\langle \Phi \rangle = 0$

Higgs $G \rightarrow \phi$
 $\Rightarrow \cong \mathbb{C}^{2n}$

$$1 \not\rightarrow 2+3$$

$$Z_1 = Z_2 + Z_3$$

$$|Z_1| \leq |Z_2| + |Z_3|$$

BPS $m_1 \leq m_2 + m_3$

IR eff actions $\Rightarrow Z \rightarrow 0$

(2) Higgs branch $\langle Q, \bar{Q} \rangle \neq 0$
 $\langle \Phi \rangle = 0$

$$\text{higgs } G \rightarrow \phi$$
$$\Rightarrow = \mathbb{C}^{2n}$$

$$1 \not\rightarrow 2+3$$

$$Z_1 = Z_2 + Z_3$$

$$|Z_1| \leq |Z_2| + |Z_3|$$

$$\text{BPS } m_1 \leq m_2 + m_3$$

IR eff actions $\Rightarrow Z \rightarrow 0$

(2) Higgs branch $\langle Q, \bar{Q} \rangle \neq 0$
 $\langle \Phi \rangle = 0$

$$\text{higgs } G \rightarrow \phi$$
$$\Rightarrow = \mathbb{C}^{2n}$$

(3)

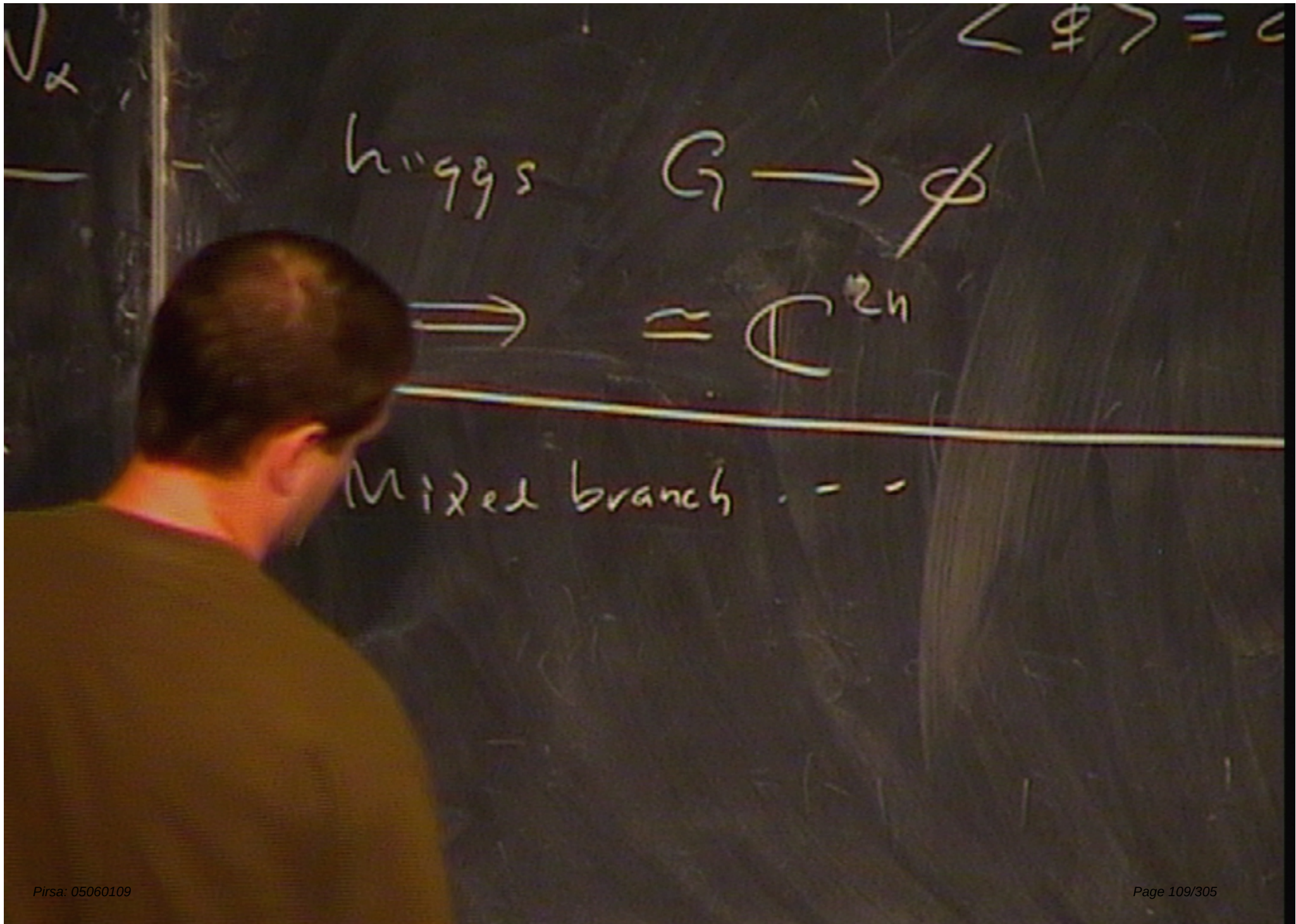
$$1 \not\rightarrow 2+3$$

$$Z_1 = Z_2 + Z_3$$

$$|Z_1| \leq |Z_2| + |Z_3|$$

$$\text{BPS } m_1 \leq m_2 + m_3$$

$$\text{IR eff actions } \Rightarrow Z \rightarrow 0$$



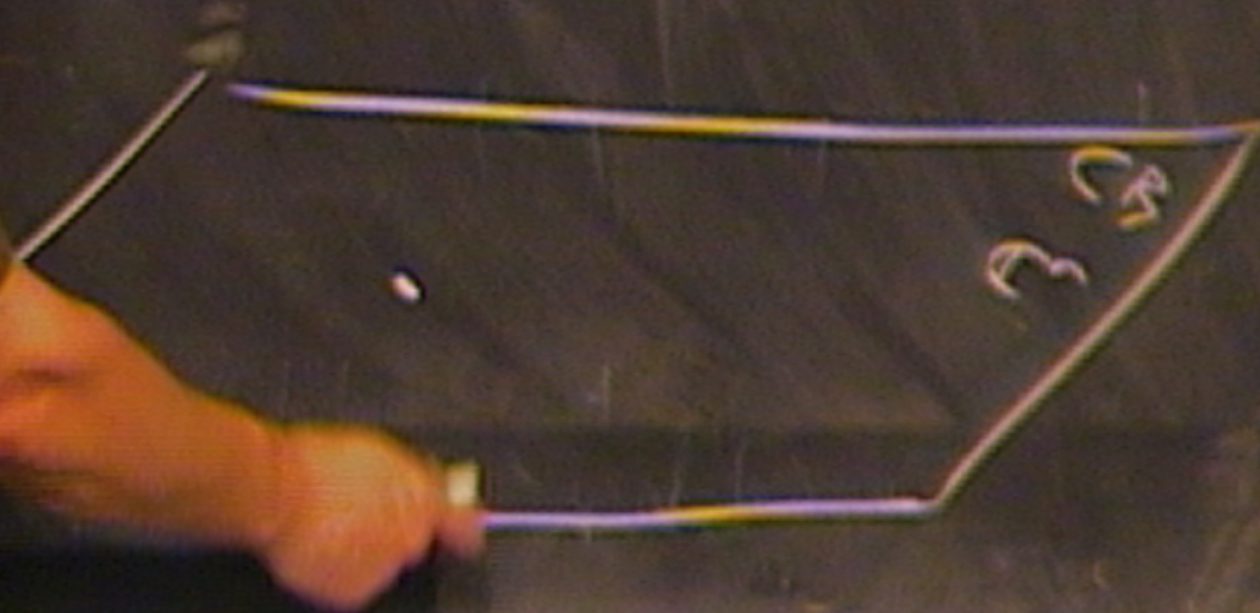
$$\Rightarrow \approx \mathbb{C}^{2n}$$

3) Mixed branch . . .



$$\Rightarrow \cong \mathbb{C}^{2n}$$

Mixed branch . . .



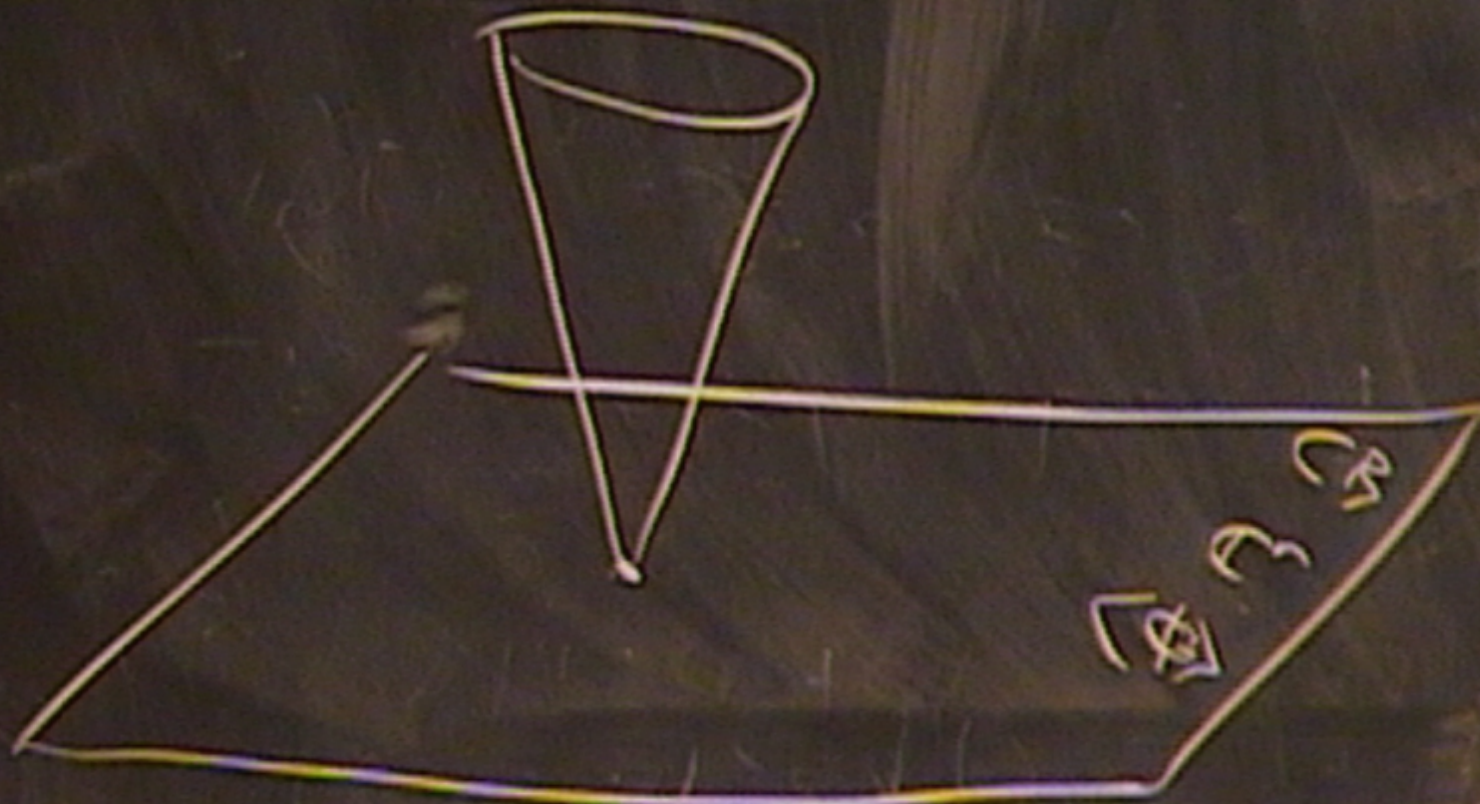
$$\Rightarrow \cong \mathbb{C}^{2n}$$

3) Mixed branch . . .



$$\Rightarrow \cong \mathbb{C}^{2n}$$

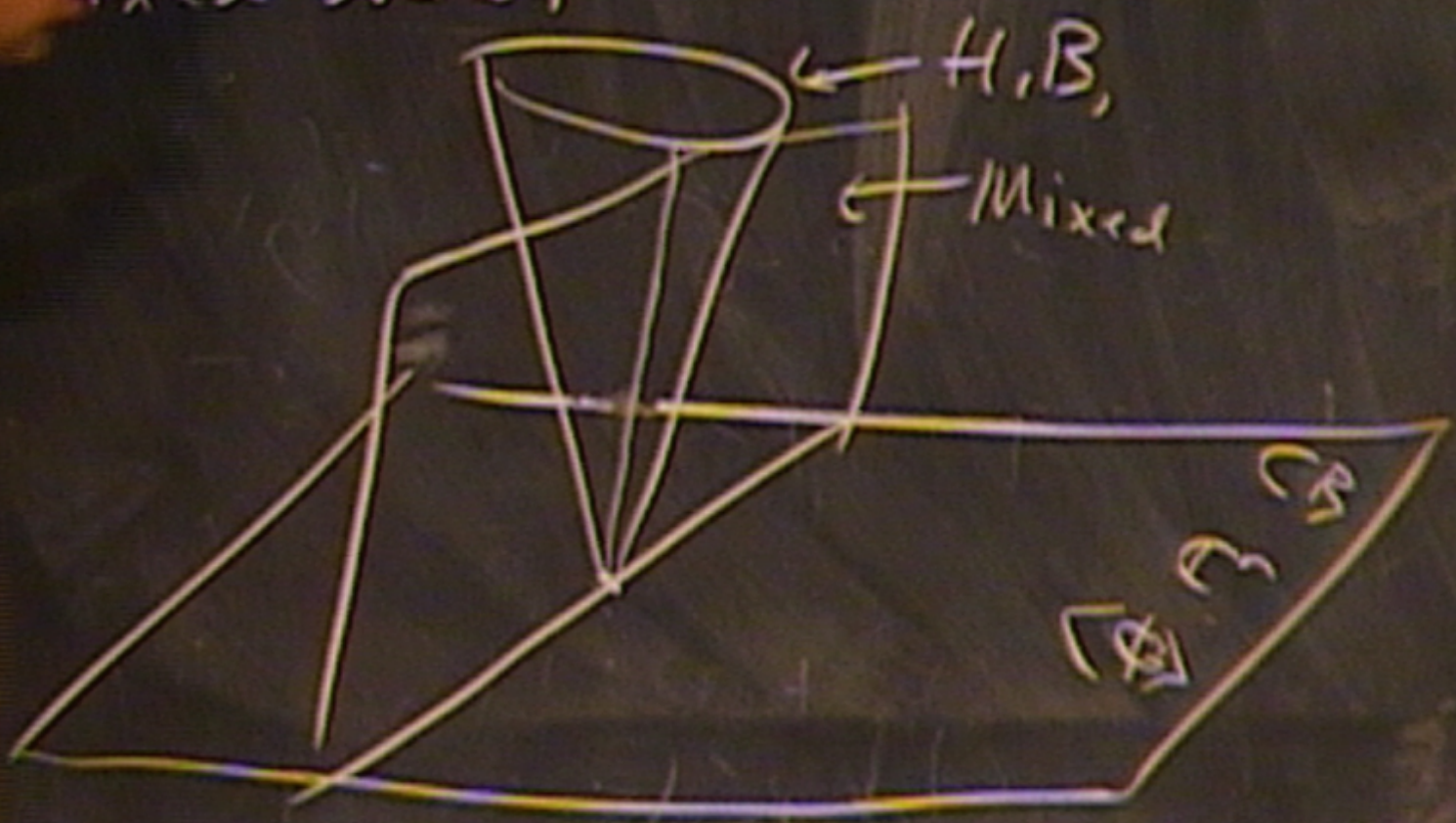
3) Mixed branch . . .



$$19) \quad \varphi \rightarrow \varphi$$

$$\Rightarrow \approx \mathbb{C}^{2n}$$

mixed branch . . .



III. $N=2$ SQCD

B. $N=2$ SQCD - classical

Hypers

$$H = Q + \tilde{Q}$$

Vector

$$V = \Phi + W_\alpha$$

$$Q\Phi = \Phi\tilde{Q} = 0$$

$$\mathcal{L}_{uv} = \int d^4\theta (Q e^\nu \bar{Q} + \tilde{Q} e^\nu \bar{\tilde{Q}} + \Phi e^\nu \bar{\Phi}) + \frac{c_2}{16\pi} \int d^4\theta (\text{tr} W^2 + \underbrace{\sqrt{2} Q \Phi \tilde{Q}}_F + m_i \tilde{Q}_i Q_i)$$

(1) Coulomb Branch (CB)

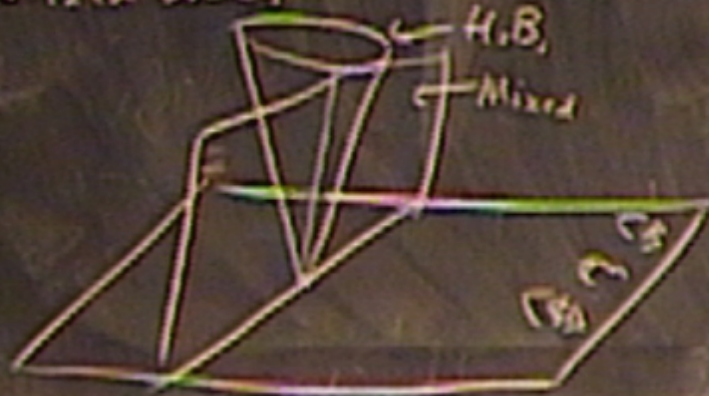
$$\langle \Phi \rangle \neq 0 \Rightarrow \underbrace{SU(N_c)}_G \rightarrow U(1)^r \quad r = N_c - 1 = \text{rank } G$$

$$\langle Q, \tilde{Q} \rangle = 0 \quad \underbrace{\mathcal{M}_{CS} = \mathbb{C}^r}_{\text{Coulomb Branch}} \quad \text{tr} \Phi^2 = u_1, \text{tr} \Phi^3 = u_2, \dots, \text{tr} \Phi^{N_c} = u_{N_c}$$

(2) Higgs branch $\langle Q, \bar{Q} \rangle \neq 0$
 $\langle \Phi \rangle = 0$

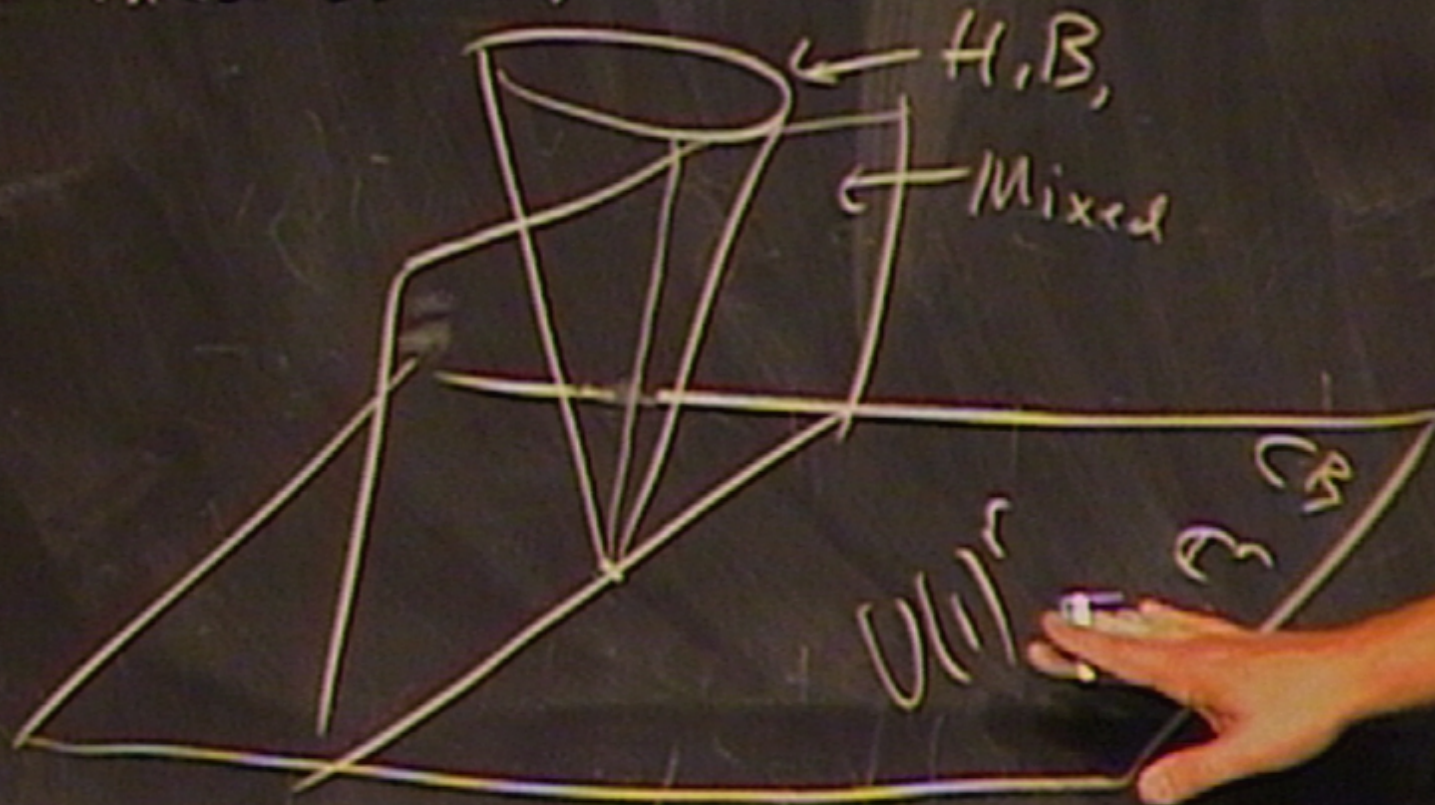
Higgs $G \rightarrow \phi$
 $\Rightarrow \cong \mathbb{C}^{2n}$

(3) Mixed branch - - -

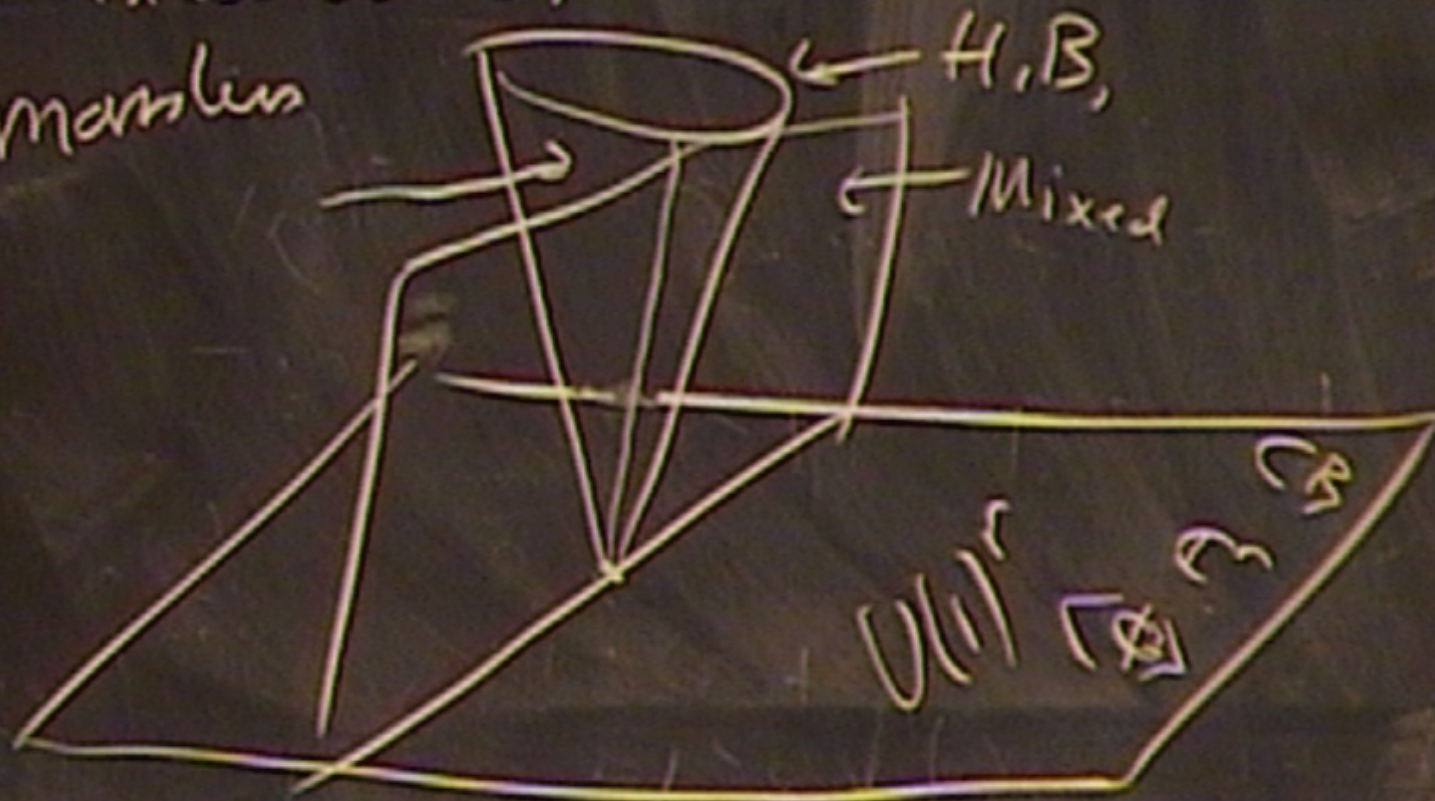


C. Eff.

(3) Mixed branch . . .



(3) Mixed branch . . .
mammas



(3) Mixed branch . . .

mammals
neutral hypens

H.B,

Mixed

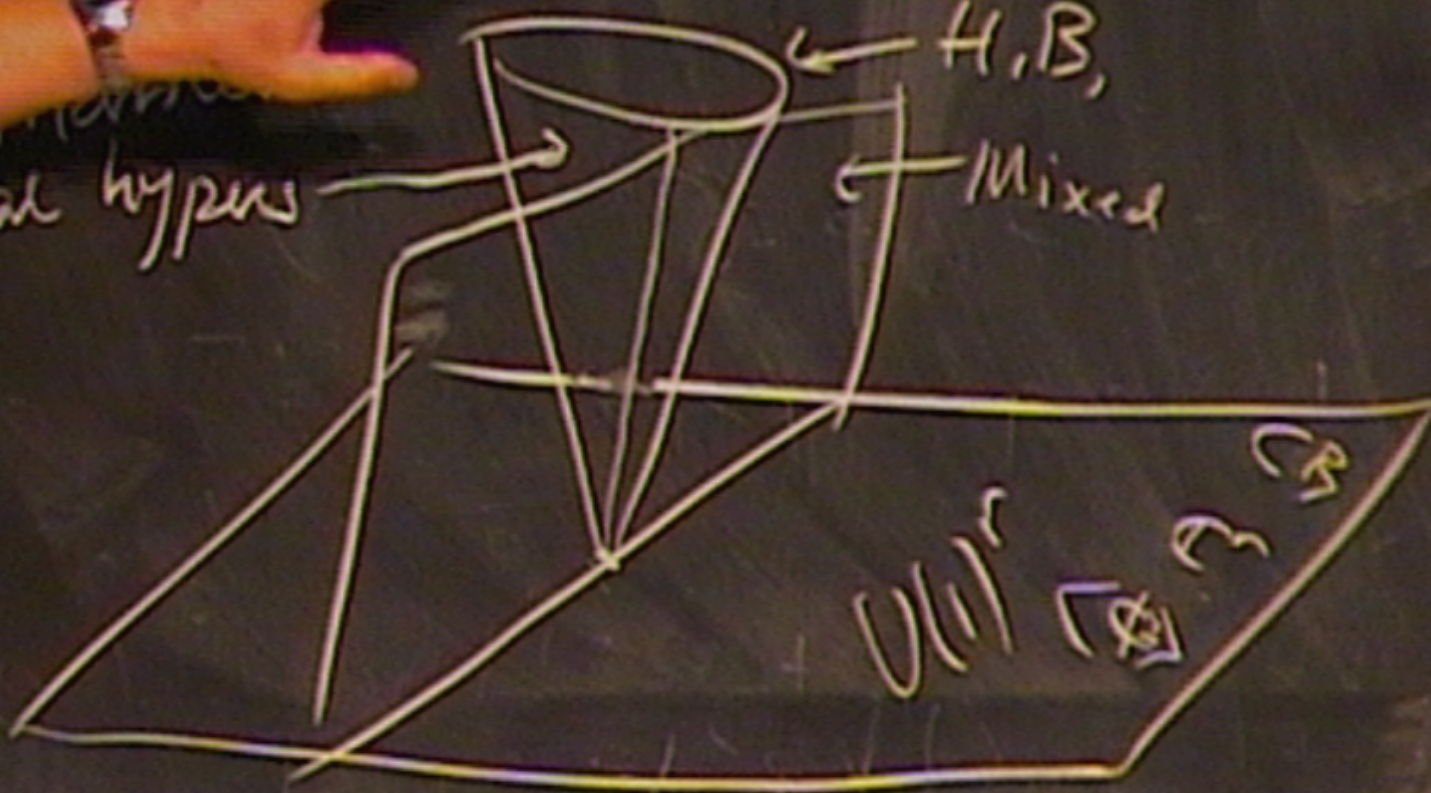
Ullir [X] 3 3

(3) Mixed branch . . .

Neutral hypers

H.B.

Mixed

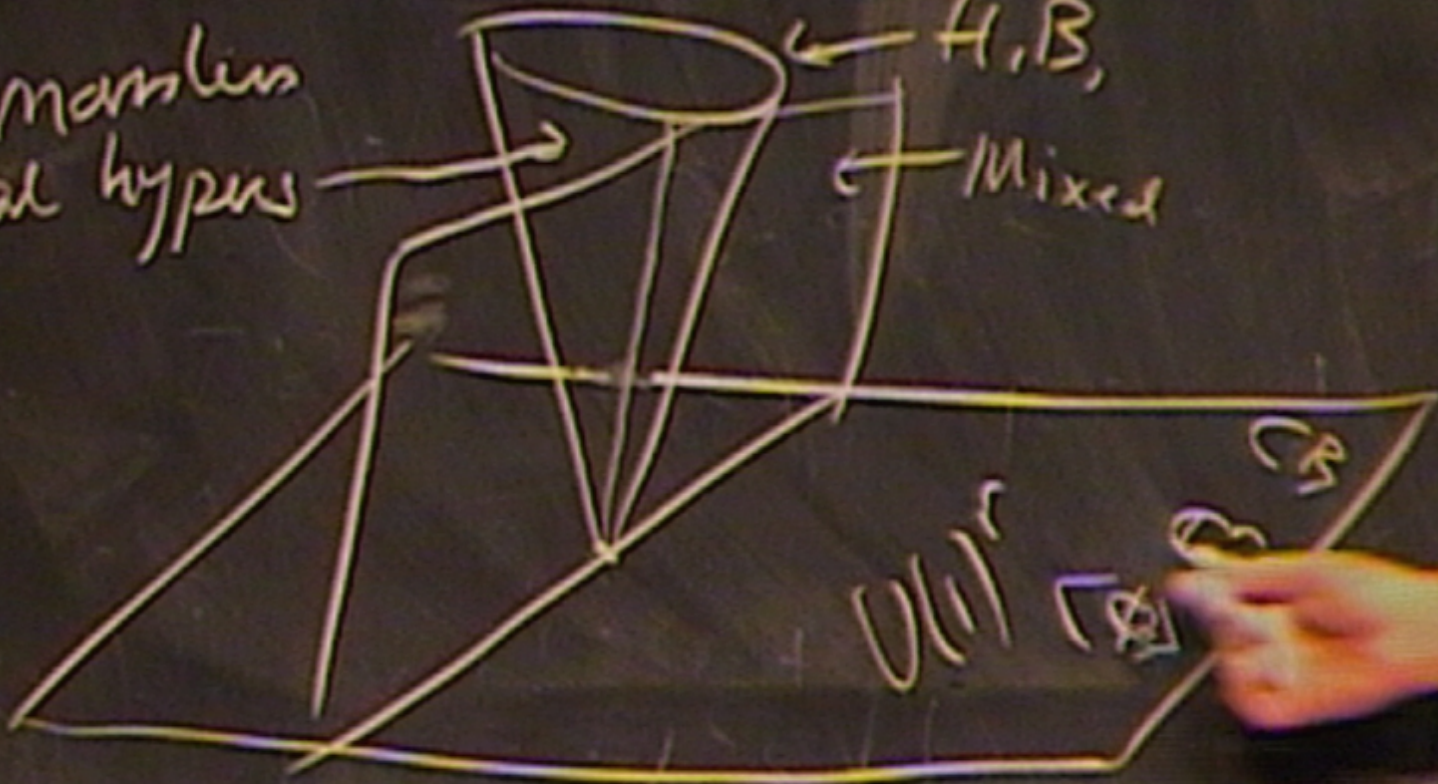


(3) Mixed branch . . .

mammals
neutral hypens

H.B.

Mixed



(2) Higgs branch $\langle Q, \tilde{Q} \rangle \neq 0$

$$\langle \tilde{\Phi} \rangle = 0$$

$$\text{Higgs } G \rightarrow \phi$$

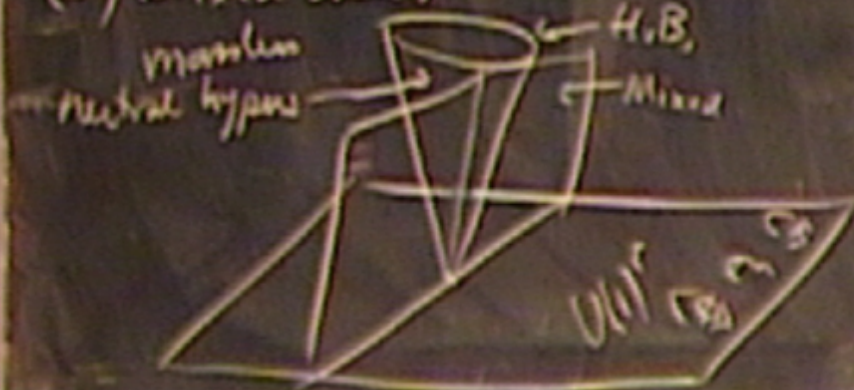
$$\Rightarrow = \mathbb{C}^{2n}$$

C. Eff. Actions

hypers
↓
($q, \tilde{q}$)

vectors
↓
(a)

(3) Mixed branch - -



C. Eff. Actions

hypers



$(q, \tilde{z})$

vectors



$(A, \tilde{w}_\perp)$

C. Eff. Actions

hypers



$(q, \tilde{z})$

vectors



$(A, \bar{w}_\perp)$

C. Eff. Actions

hypers



$(a, \tilde{g})$

vectors



$(A^i, \bar{w}_i)$

$\mathcal{I}_{\text{eff}}$

$$[\mathcal{K}_H(\bar{g}, \tilde{g}, \bar{g}, \tilde{g}) + \mathcal{K}_V(A^i, \bar{w}_i)] +$$

C. Eff. Actions

hypers

$(q, \tilde{q})$

vectors

$(A^i, \bar{w}_i)$

$$\mathcal{I}_H = \int d^4\theta \left[\mathcal{K}_H(q, \tilde{q}, \bar{q}, \tilde{\bar{q}}) + \mathcal{K}_V(A, \bar{A}) \right] + \\ i \int d^2\theta \tau_{ij}(A) W^i W^j + \text{c.c.}$$

C. Eff. Actions

hypers

$(q, \tilde{q})$

vectors

$(A^i, \bar{w}_i)$

$$\mathcal{I}_{eff} = \int d^4\theta \left[\mathcal{K}_H(q, \tilde{q}, \bar{q}, \tilde{\bar{q}}) + \mathcal{K}_V(A, \bar{A}) \right] + \\ i \int d^2\theta \tau_{ij}(A) W^i W^j + c.c.$$

$\epsilon > 0$

$\epsilon = 0$

C. Eff. Actions

hypers



$(q, \tilde{q})$

vectors



$(A^i, \tilde{w}_i)$

$\mathcal{L}$

$$[\mathcal{K}_H(q, \tilde{q}, \bar{q}, \bar{\tilde{q}}) + \mathcal{K}_V(A^i, \tilde{w}_i)] +$$

$$\mathcal{L}_{ij}(A) \tilde{w}^i \tilde{w}^j + \text{c.c.}$$

$\epsilon > 0$

$\epsilon = 0$

C. Eff. Actions

②

hypers

vectors

↓
($q, \tilde{q}$)

↓
($A^i, \bar{w}_i$)

$$\mathcal{I}_{eff} = \int d^4\theta \left[\mathcal{K}_H(q, \tilde{q}, \bar{q}, \tilde{\bar{q}}) + \mathcal{K}_V(A, \bar{A}) \right] + \\ i \int d^2\theta \tau_{ij}(A) W^i W^j + c.c.$$

① no superpot'l $\Rightarrow$ CB, Higgs are not lifted.

$\epsilon > 0$
 $\epsilon = 0$

C. Eff. Actions

②

hypers
 $\downarrow$
 $(q, \tilde{q})$

vectors
 $\downarrow$
 $(A^i, \tilde{W}_i)$

$$\mathcal{I}_{eff} = \int d^4\theta \left[\mathcal{K}_H(q, \tilde{q}, \bar{q}, \bar{\tilde{q}}) + \mathcal{K}_V(A^i, \tilde{A}_i) \right] +$$

$$i \int d^2\theta \tau_{ij}(A) W^i W^j + c.c.$$

① no superpot' $\Rightarrow$ CB, Higgs are not lifted.

C. Eff. Actions

②

hypers

vectors

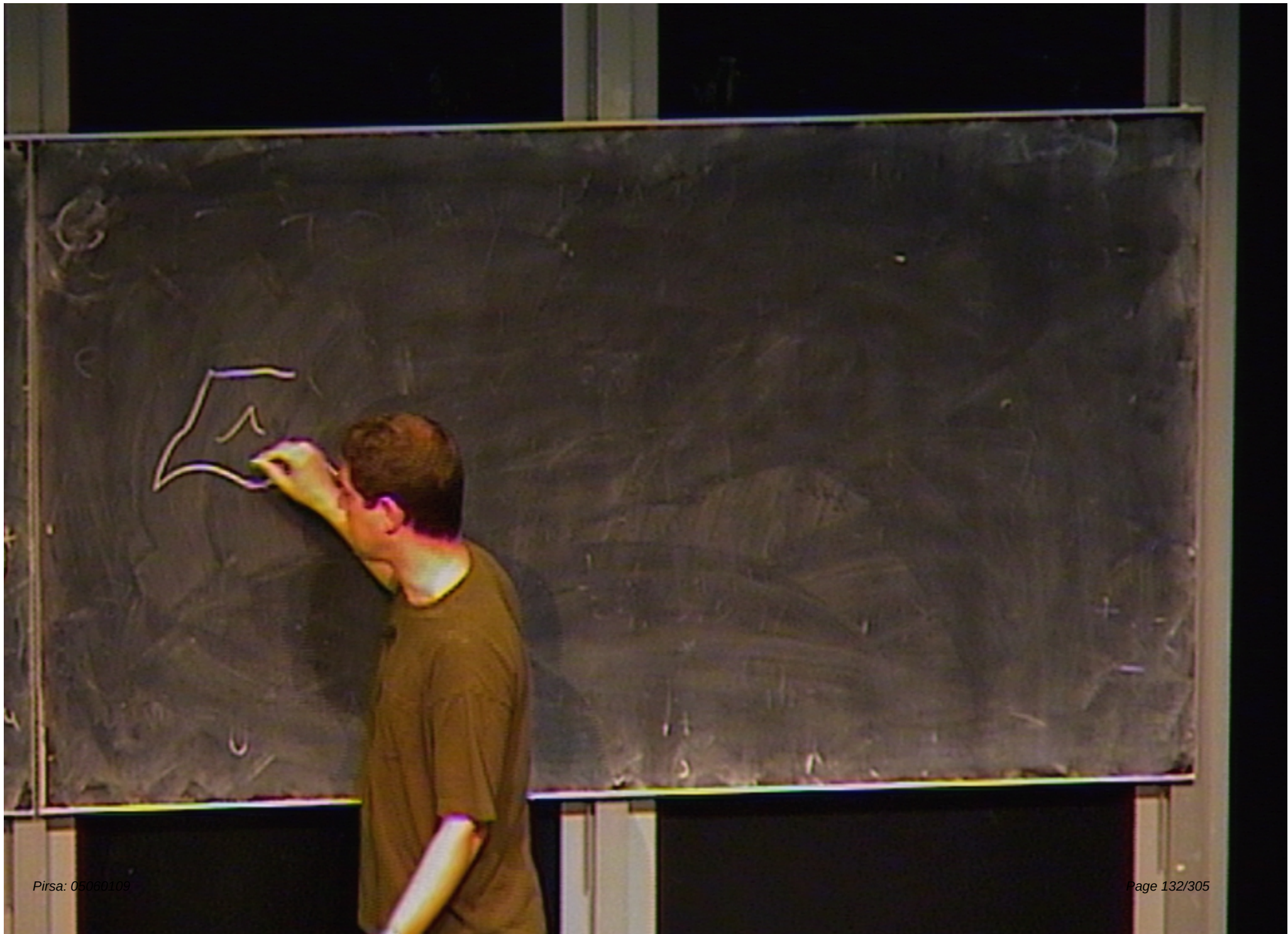
$(q, \tilde{q})$

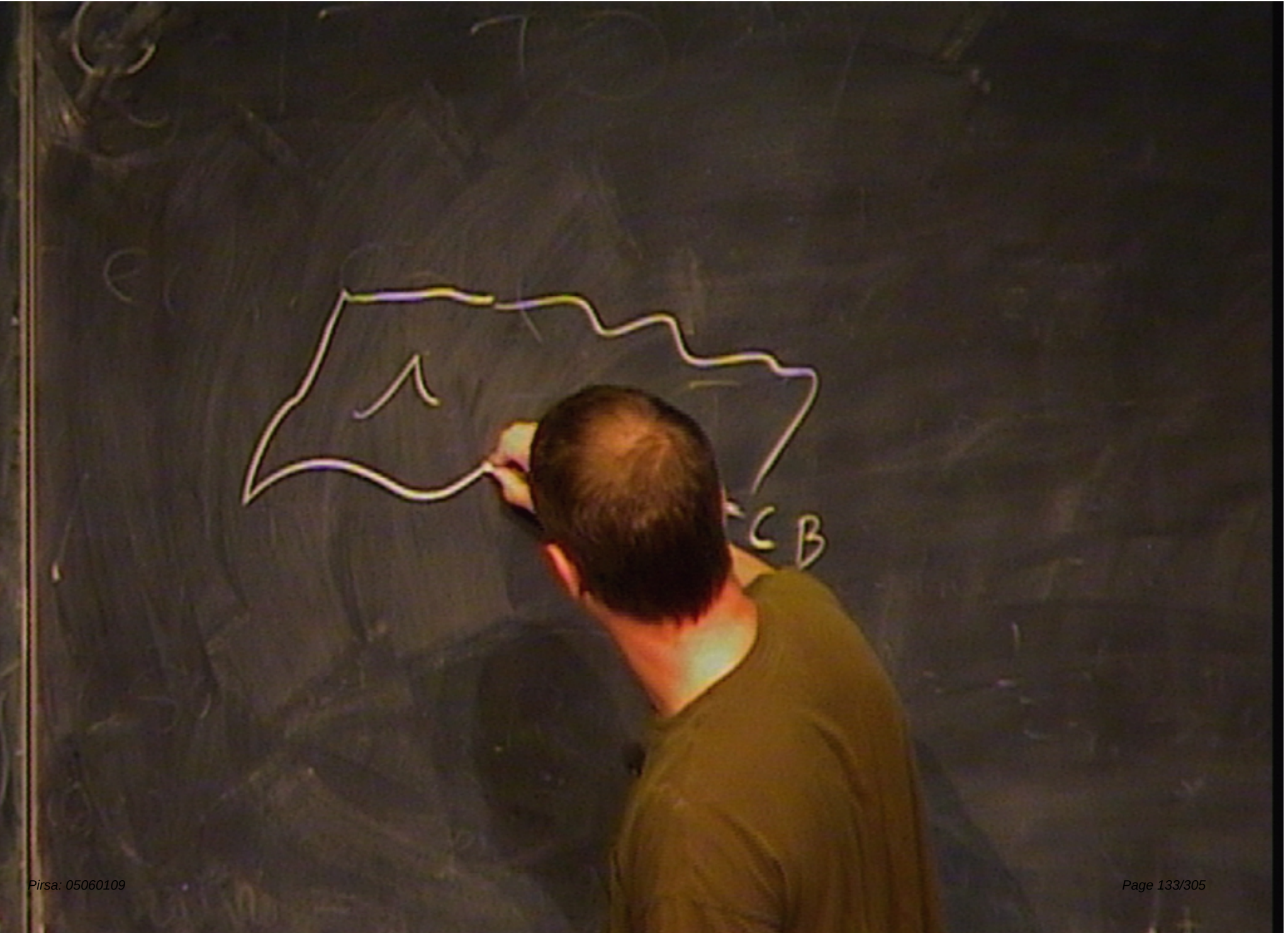
$(A^i, \bar{w}_i)$

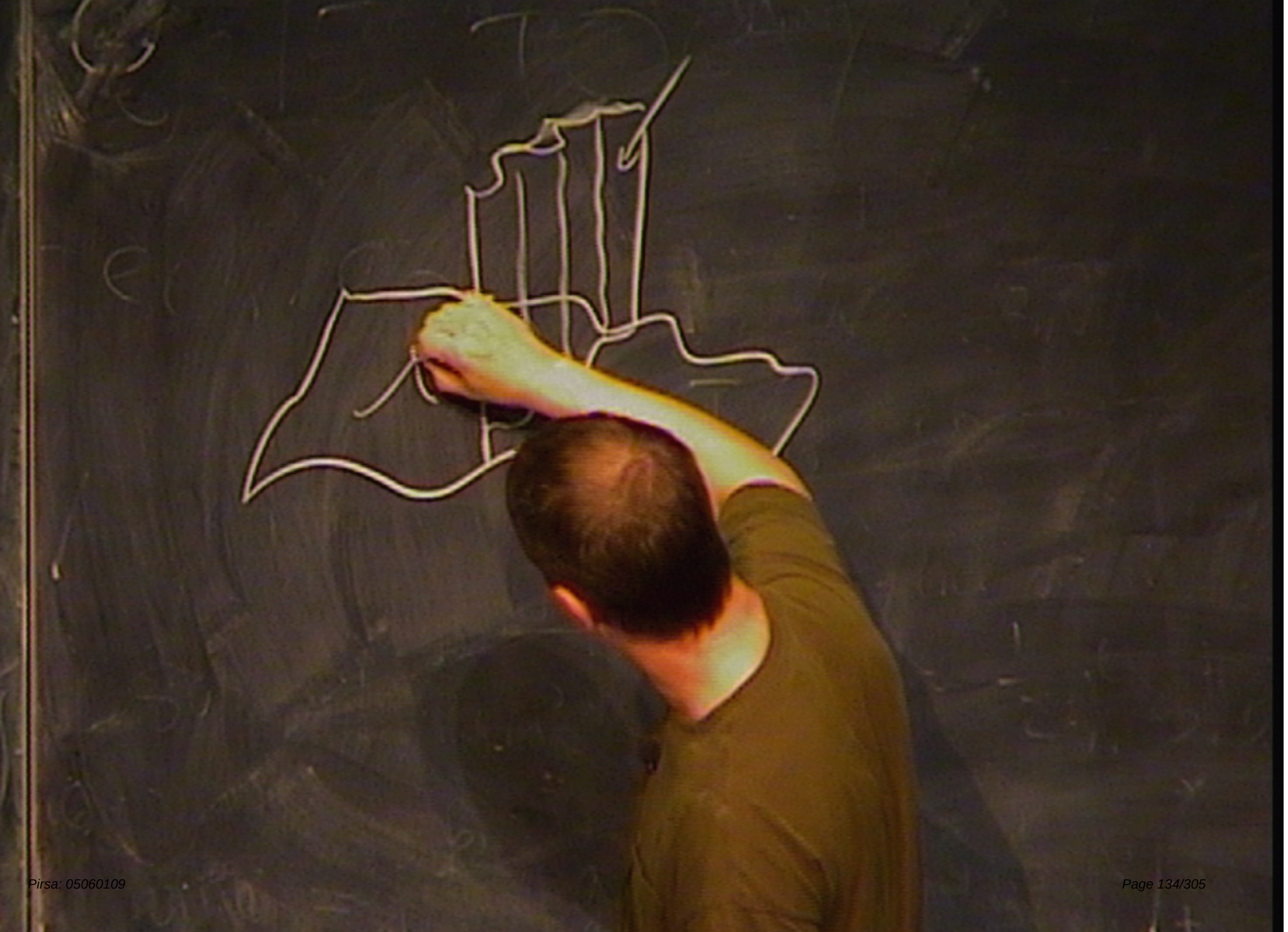
$$\mathcal{I}_{eff} = \int d^4\theta \left[\mathcal{K}_H(q, \tilde{q}, \bar{q}, \tilde{\bar{q}}) + \mathcal{K}_V(A, \bar{A}) \right] +$$

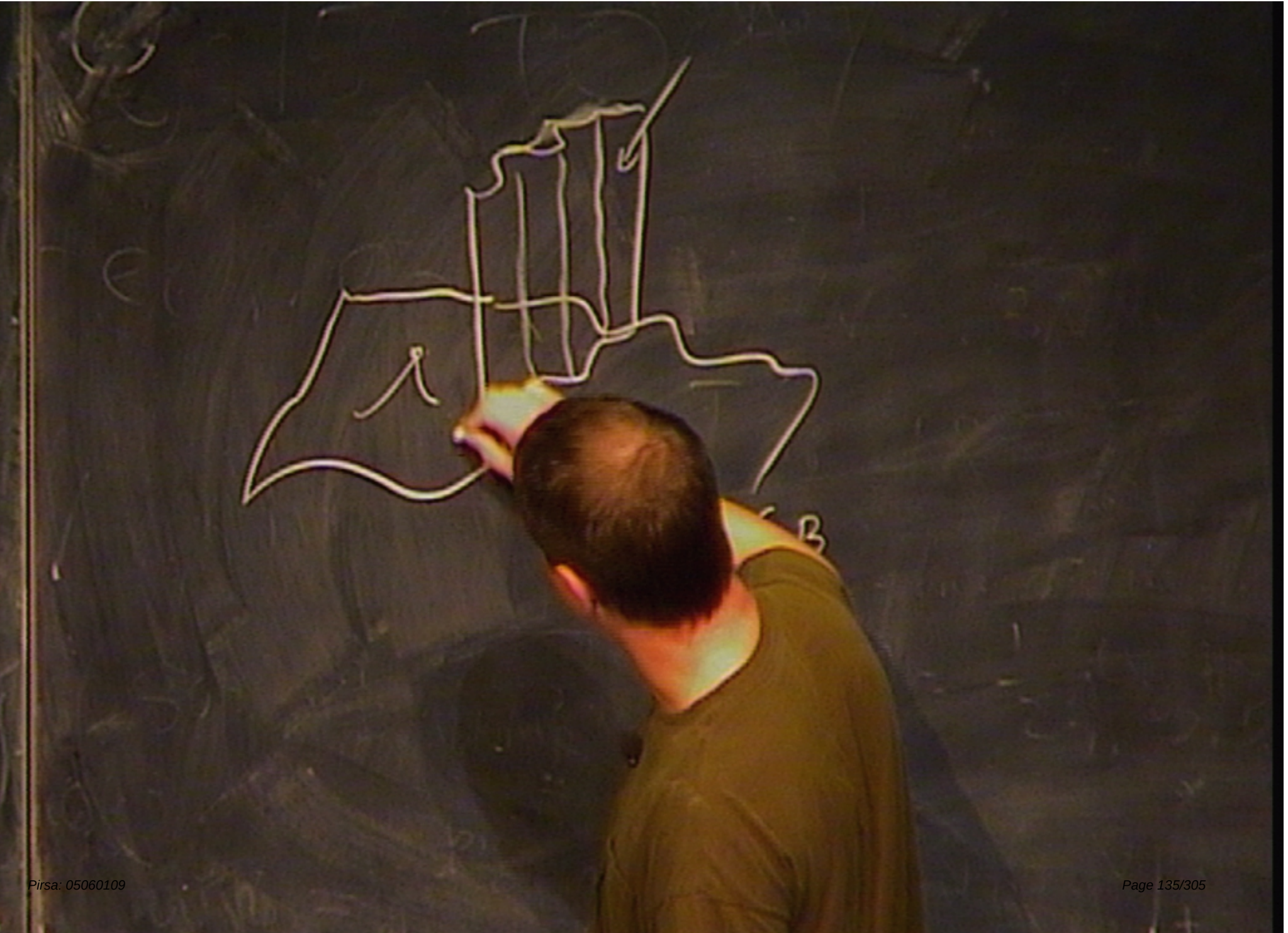
$$i \int d^2\theta \tau_{ij}(A) W^i W^j + c.c.$$

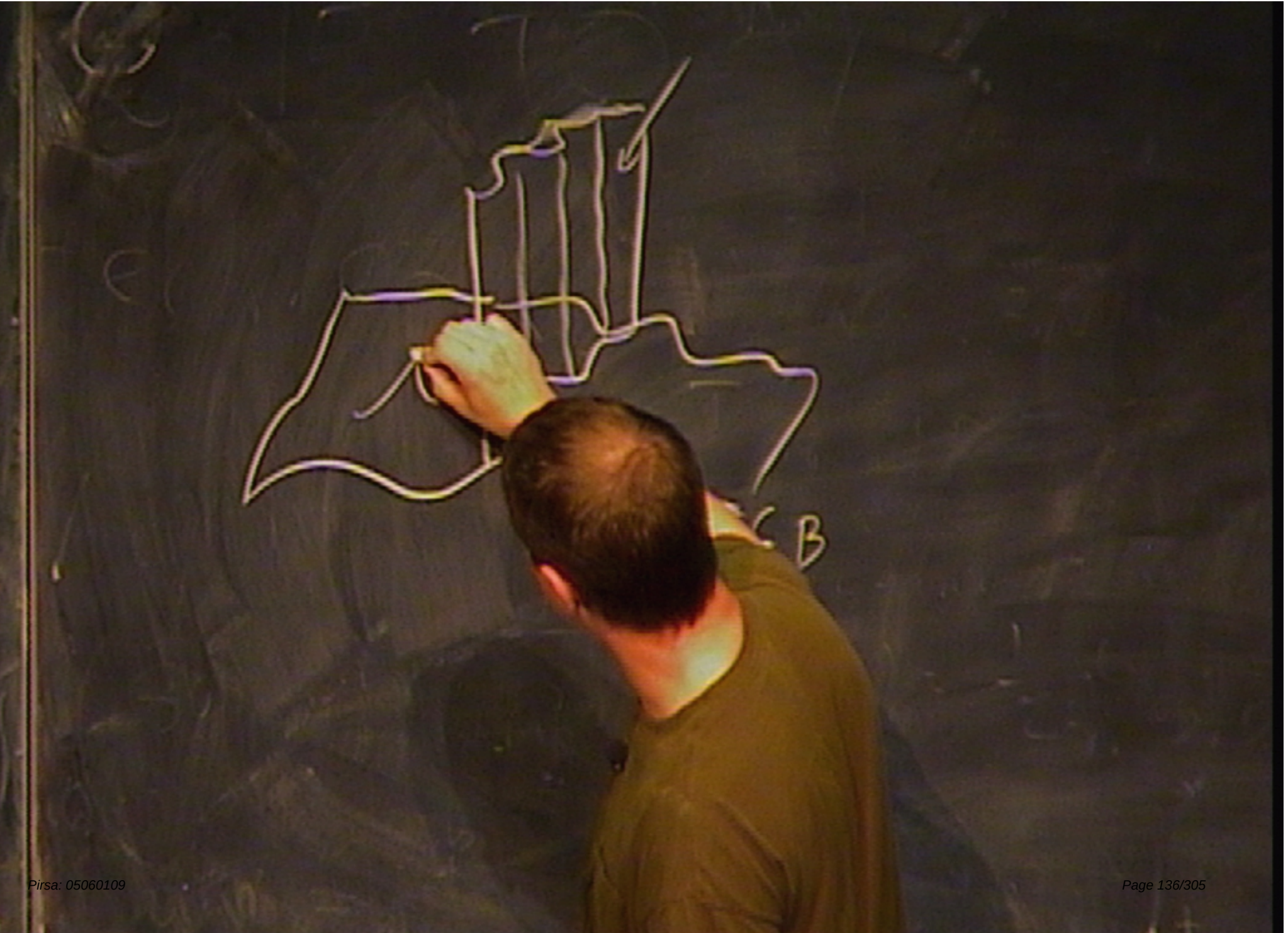
① no superpot' $\Rightarrow$ CB, Higgs are not lifted.

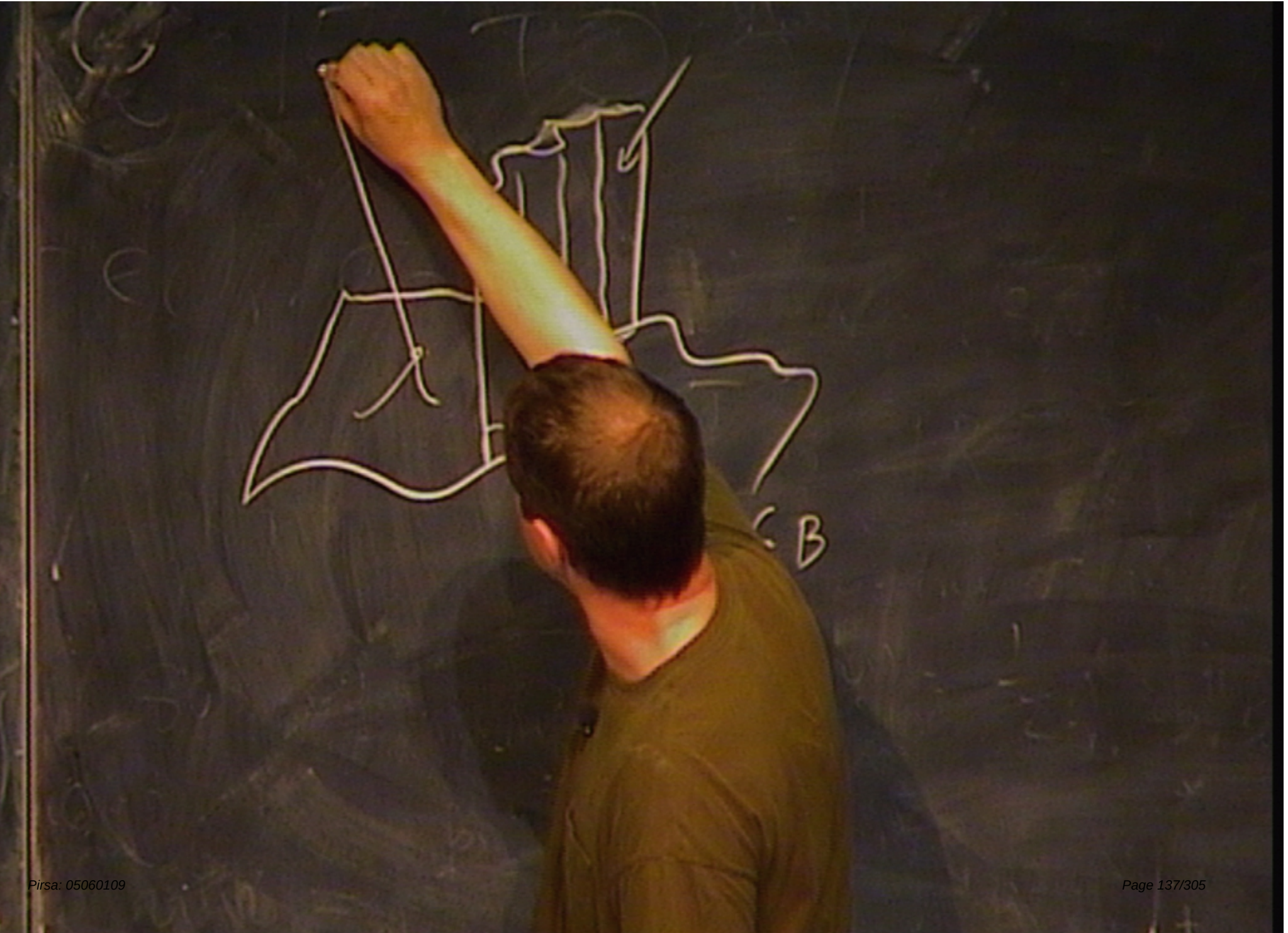


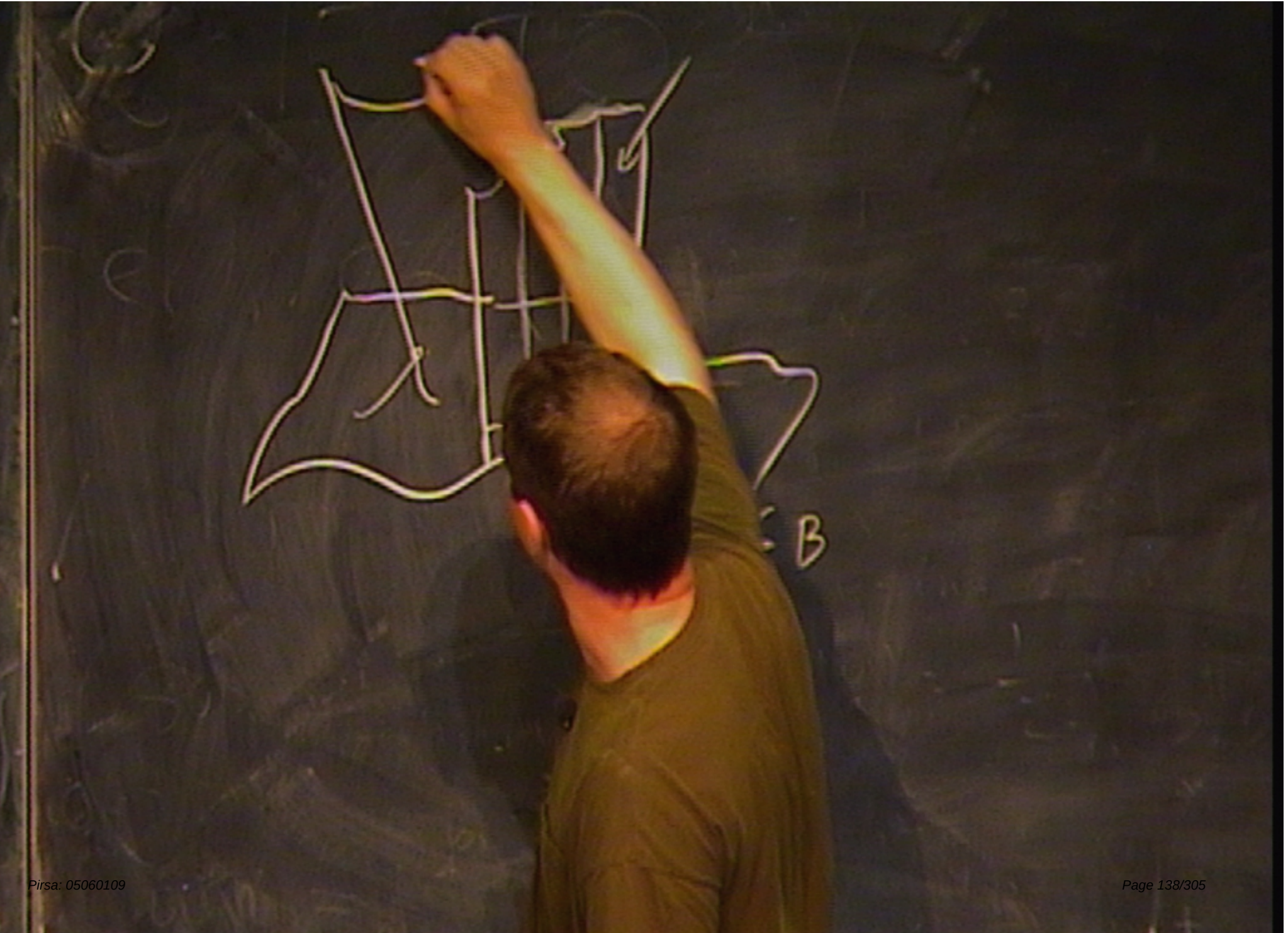


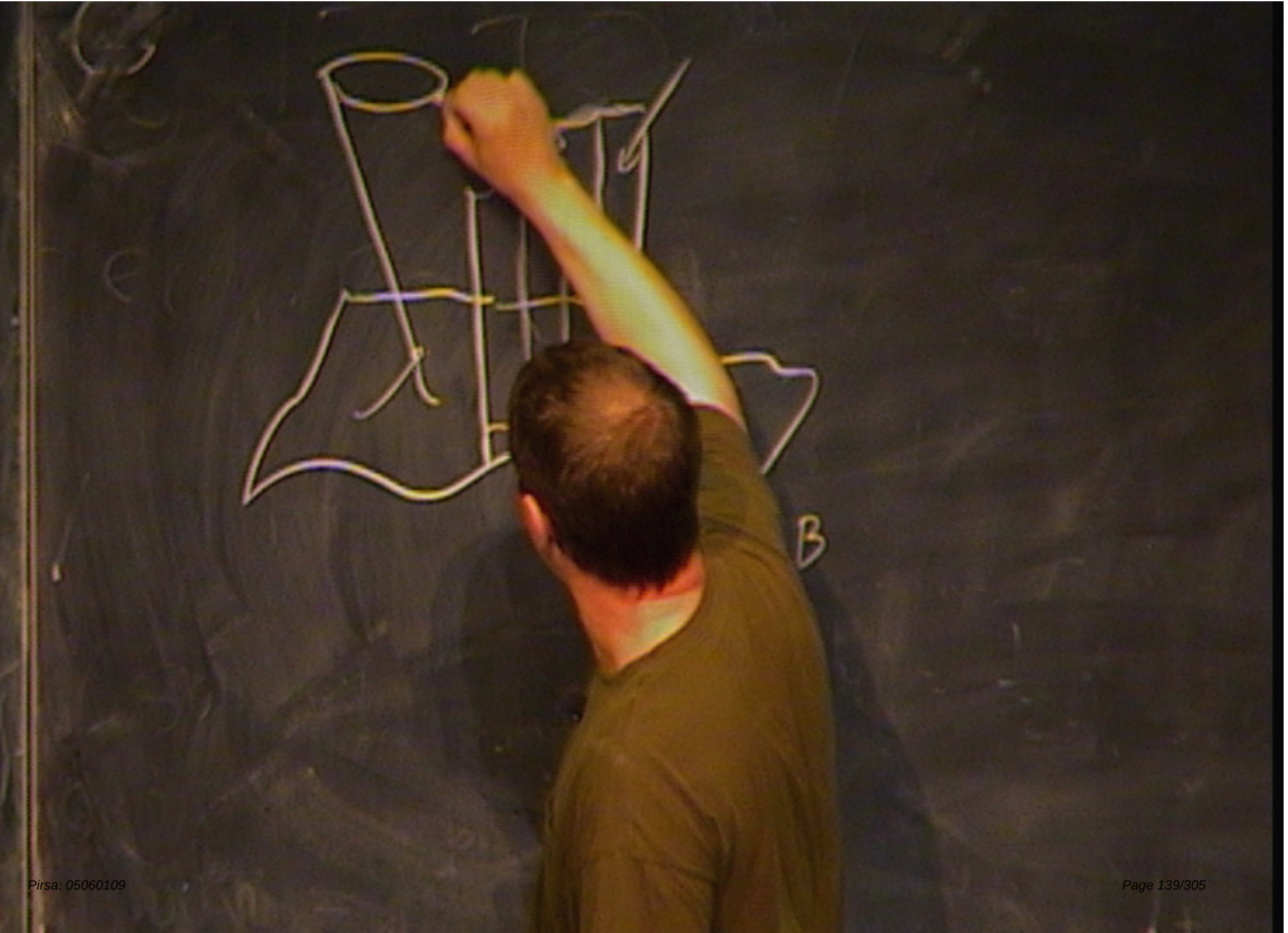


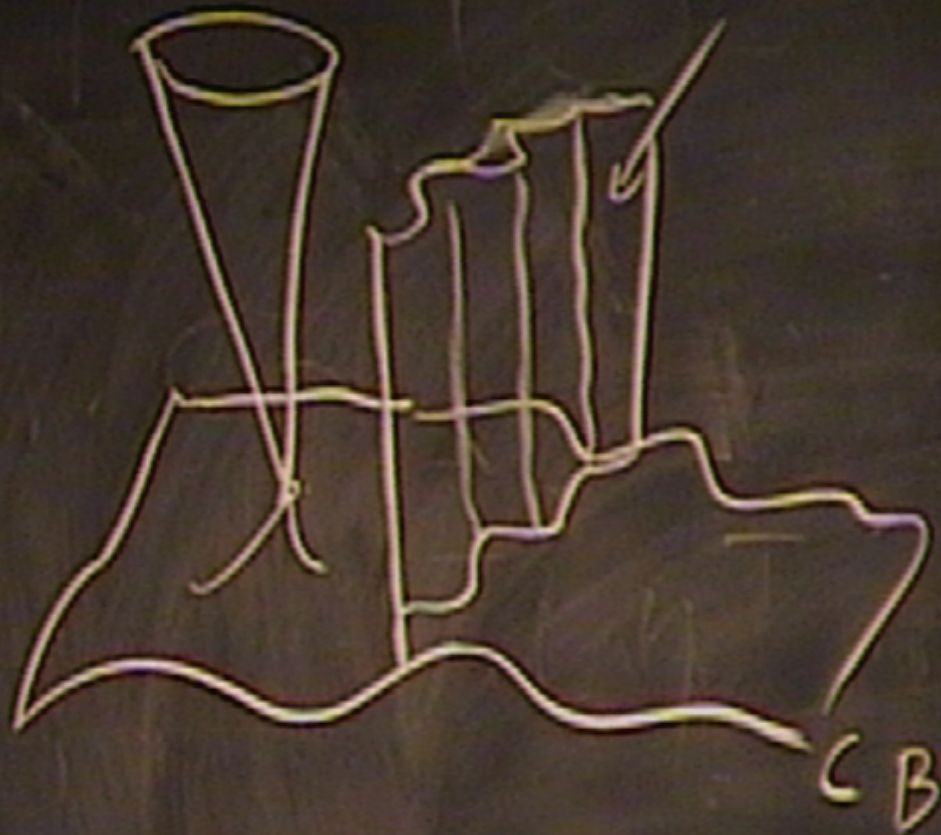


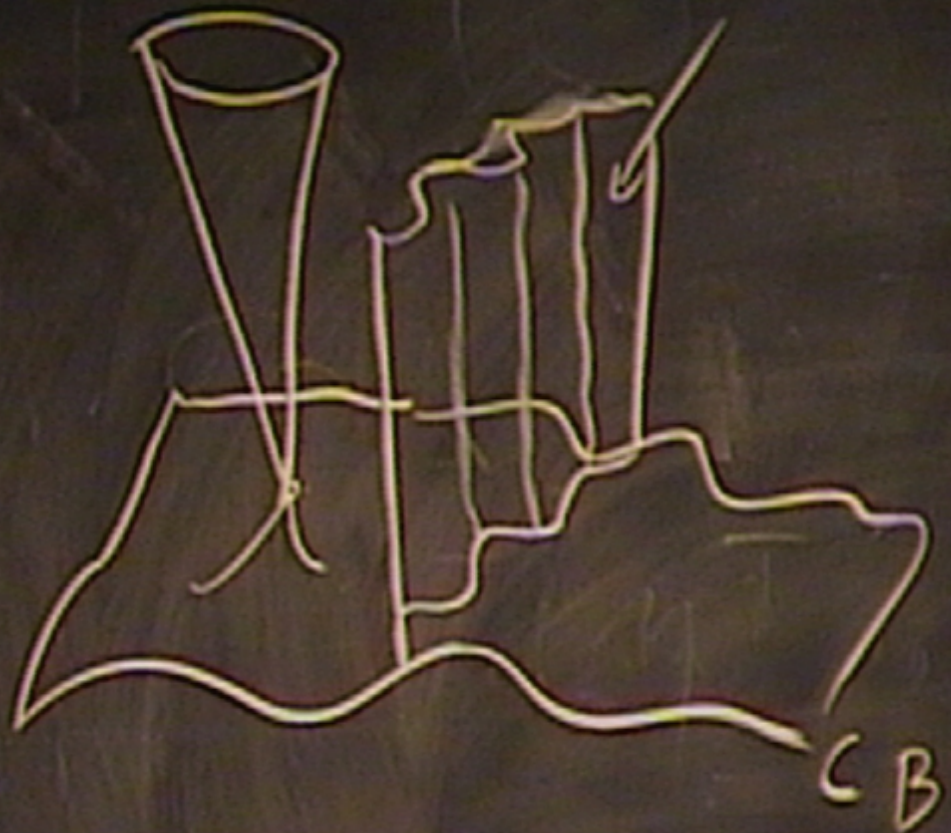






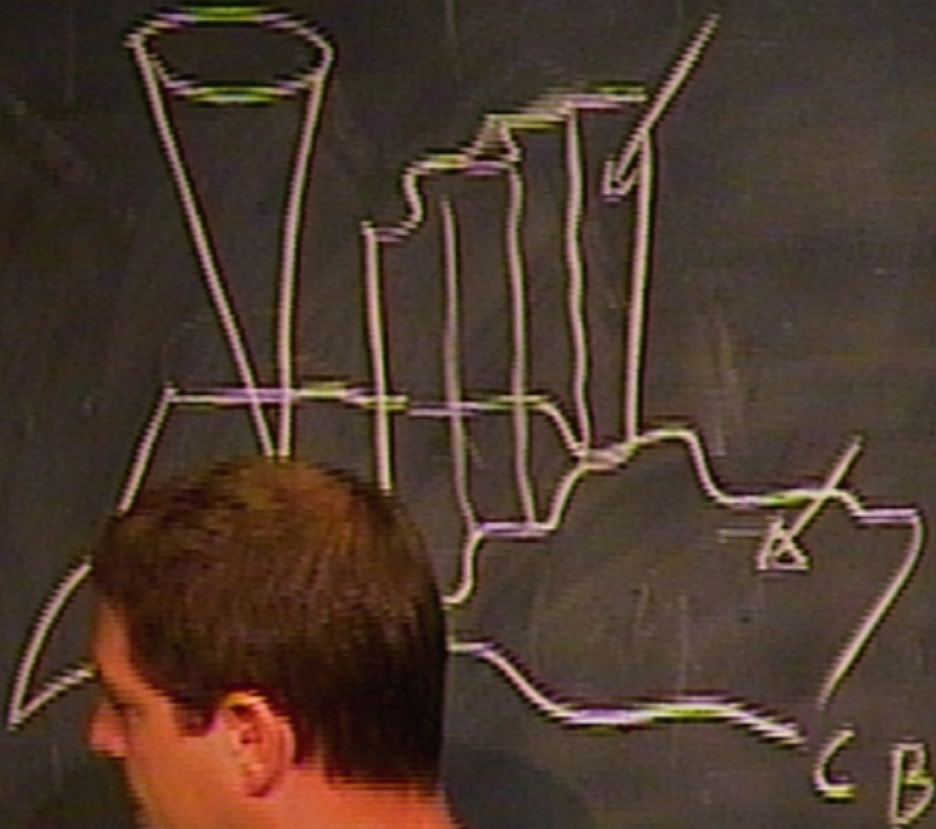








$$K_v \supset I_m$$



$(\varphi, \tilde{\varphi})$ $(A, \bar{w})$

$$\mathcal{I}_H = \int d^4\theta \left[\mathcal{K}_H(\varphi, \tilde{\varphi}, \bar{\varphi}, \tilde{\bar{\varphi}}) + \mathcal{K}_V(A, \bar{A}, W, \bar{W}) \right] \rightarrow \text{hyperkahler}$$

$$+ \int d^2\theta \tau_{ij}(A, \bar{A}) W^i W^{\bar{j}} + \text{c.c.}$$

① no superpot'l $\Rightarrow$ CB, Higgs are not lifted.

$(\psi, \tilde{\psi})$ $(A, \bar{w})$

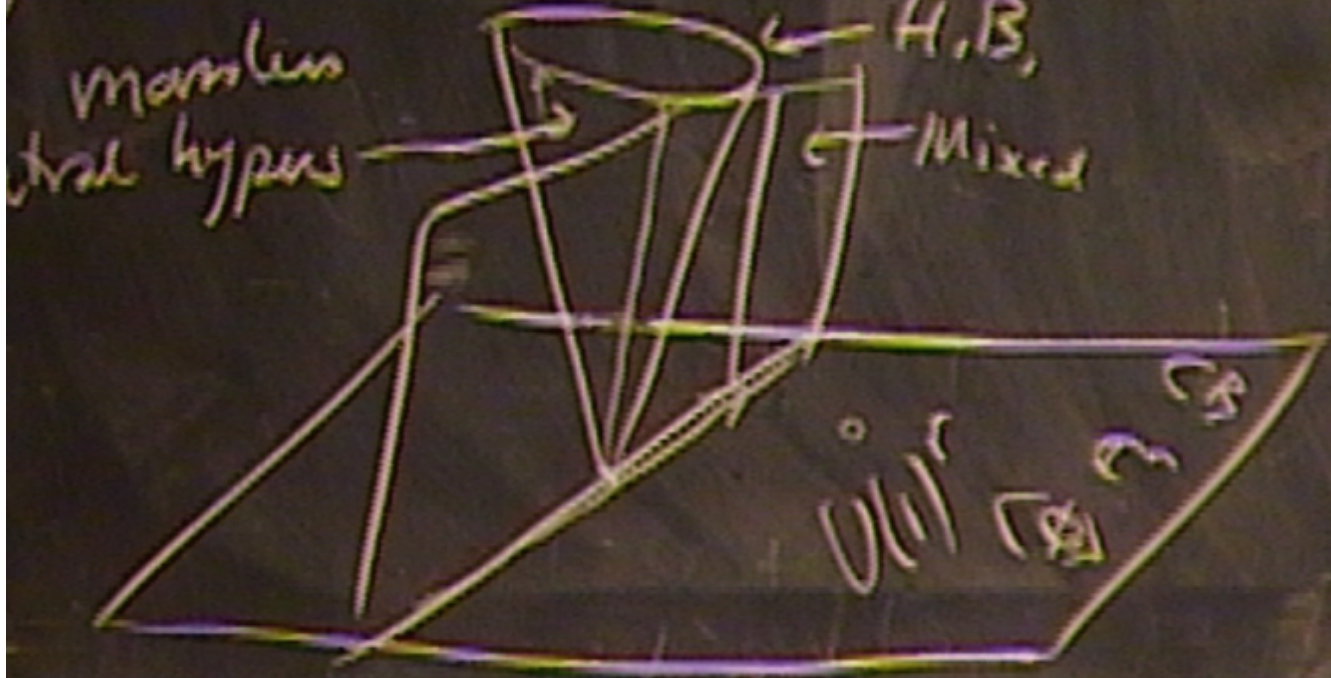
$$\mathcal{L}_H = \int d^4\theta \left[\overset{\text{hyperkahler}}{\mathcal{K}_H(\psi, \tilde{\psi}, \bar{\psi}, \bar{\psi})} + \mathcal{K}_V(A, \bar{A}, W, \bar{W}) \right] + \int d^2\theta \left[\mathcal{L}_V(A, W, W) + \text{c.c.} \right]$$

① no superpot' $\Rightarrow$ CB, Higgs are not lifted.

$\mathbb{C}^n \rightarrow \emptyset$

$\Rightarrow \mathbb{C}^{2n}$

Mixed branch . . .



$I_4 = \int d^4 \theta$

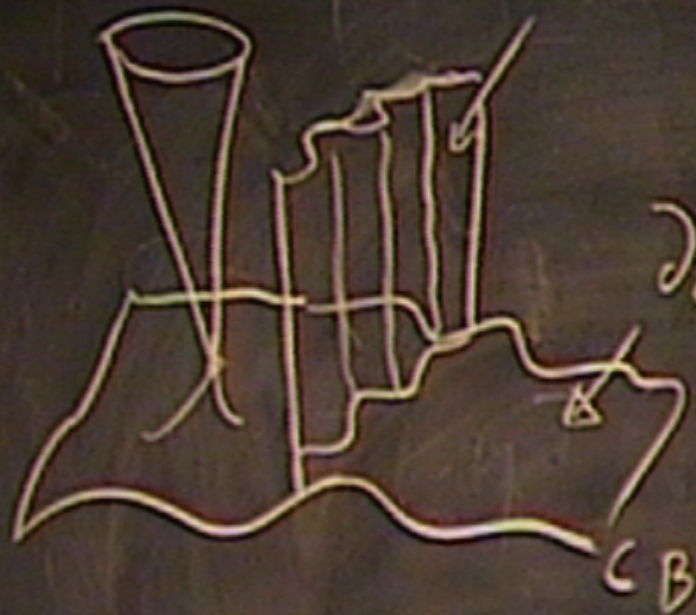
① no 3

$$\mathcal{K}_v \supset \text{Im } \tau_{i,j}(A) \partial_\mu A^i \partial^\mu \bar{A}^j$$

$$\mathcal{K}_v \supset \text{Im } \tau_{i,j}(A) \partial_\mu A^i \partial^\mu \bar{A}^j$$

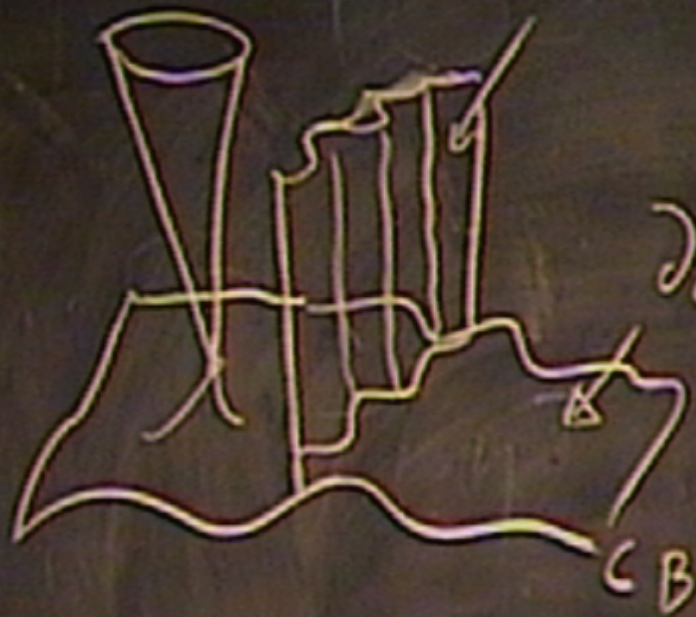
$$0 = \frac{2}{c} \frac{\partial A^i}{\partial x^j} \partial_\mu A^i \partial^\mu \bar{A}^j = 0$$





$$K_v \supset \text{Im } \tau_i(A) \partial_\mu A^i \partial^\mu \bar{A}^j$$

$$\partial_i \equiv \frac{\partial}{\partial A^i} \quad \tau_{ij} = 0$$



$$K_v \supset \text{Im } \tau_{i,j}(A) \partial_\mu A^i \partial^\mu \bar{A}^j$$

$$\partial_i \equiv \frac{\partial}{\partial A^i} \quad \tau_{i,j} = 0$$

$$\mathcal{K}_v \supset \text{Im } \tau_{ij}(A) \partial_\mu A^i \partial^\mu \bar{A}^j$$

$$\partial_i \equiv \frac{\partial}{\partial A^i}$$

$$\tau_{ij} = \tau_{ji} \quad \tau_{ij} = 0$$

$$\text{Im } \tau_{ij} > 0$$

$$\mathcal{K}_v \supset \text{Im } \tau_{ij}(A) \partial_\mu A^i \partial^\mu \bar{A}^j$$

$$\frac{2}{\partial A^i}$$

$$\tau_{ij} \tau_{jk} = 0 \quad \tau_{ij} = \tau_{ji}$$

$$\text{Im } \tau_{ij} > 0$$

Rigid Special Kähler (RSK)

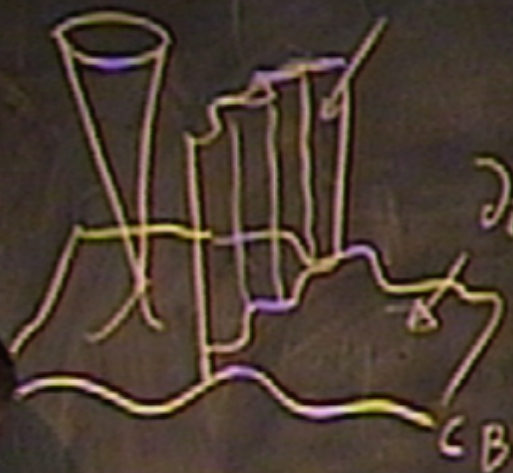
$$g_{\mu\nu} \supset \text{Im } \tau_{ij}(A) \partial_\mu A^i \partial^\nu \bar{A}^j$$

$$\frac{2}{\partial A^i}$$

$$\tau_{ij} = 0 \quad \tau_{ij} = \tau_{ji}$$

$$\text{Im } \tau_{ij} > 0$$

Ricci Special Kahler (RSK)



$$\partial_i \equiv \frac{\partial}{\partial A^i}$$

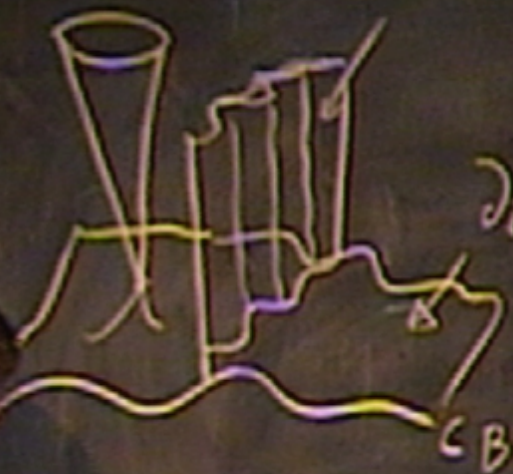
$$\mathcal{K}_v \supset \text{Im } \tau_{ij}(A) \partial_\mu A^i \partial^\mu \bar{A}^j$$

$$\partial_i \tau_{jk} = 0 \quad \tau_{ij} = \tau_{ji}$$

$$\text{Im } \tau_{ij} > 0$$

Rigid Special Kahler (RSK)

F-M duality = redundancy in the description i.e. $U(1)$ actions



$$\partial_i \equiv \frac{\partial}{\partial A^i}$$

$$\mathcal{K}_v \supset \text{Im } \tau_{ij}(A) \partial_\mu A^i \partial^\mu \bar{A}^j$$

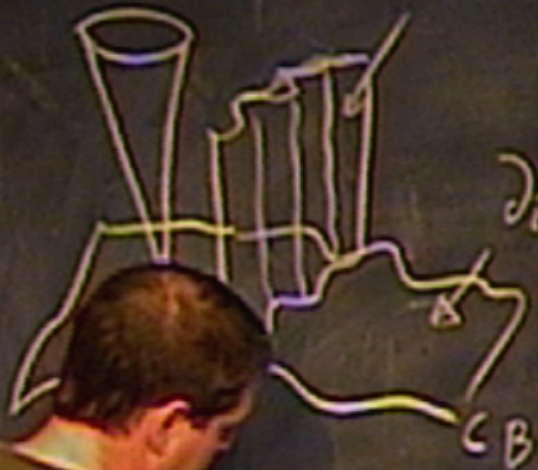
$$\partial_i \tau_{jk} = 0 \quad \tau_{ij} = \tau_{ji}$$

$$\text{Im } \tau_{ij} > 0$$

Rigid Special Kähler (RSK)

M duality = redundancy in the description i.e. $U(1)$ actions

$$\vec{E} \rightarrow \vec{B}$$



$$\partial_i \equiv \frac{\partial}{\partial A^i}$$

$$\mathcal{K}_v \supset \text{Im } \tau_{ij}(A) \partial_\mu A^i \partial^\mu \bar{A}^j$$

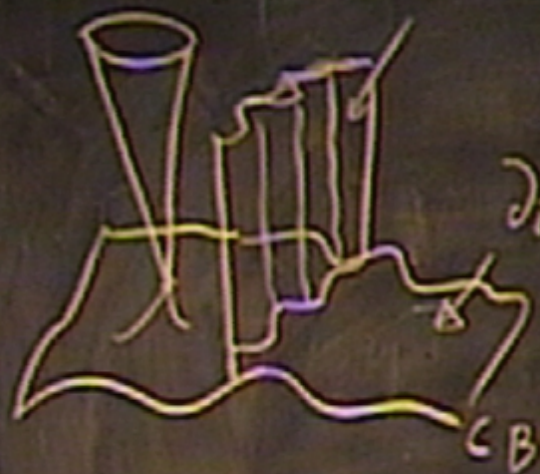
$$\Re \tau_{ij} = 0 \quad \tau_{ij} = \tau_{ji}$$

$$\text{Im } \tau_{ij} > 0$$

Rigid Special Kahler (RSK)

lity = redundancy in the description i.e. $U(1)$ actions

$\mathcal{R} \rightarrow$



$$\mathcal{K}_v \supset \text{Im } \tau_{ij}(A) \partial_\mu A^i \partial^\mu \bar{A}^j$$

$$\partial_i \tau_{jk} = 0 \quad \tau_{ij} = \tau_{ji}$$

$$\text{Im } \tau_{ij} > 0$$

Rigid Special Kahler (RSK)

E-M duality = redundancy in the description i.e. $U(1)$ actions

$$\vec{E} \rightarrow \vec{B}, \vec{B} \rightarrow -\vec{E}$$

$$\tau \rightarrow \frac{a\tau + b}{c\tau + d}$$

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} \in SL(2, \mathbb{Z})$$

$$U(1)^r$$

$$\tau \rightarrow \frac{a\tau + b}{c\tau + d}$$

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} \in SL(2, \mathbb{Z})$$

$$U(1)^r: \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in Sp(2r, \mathbb{Z})$$



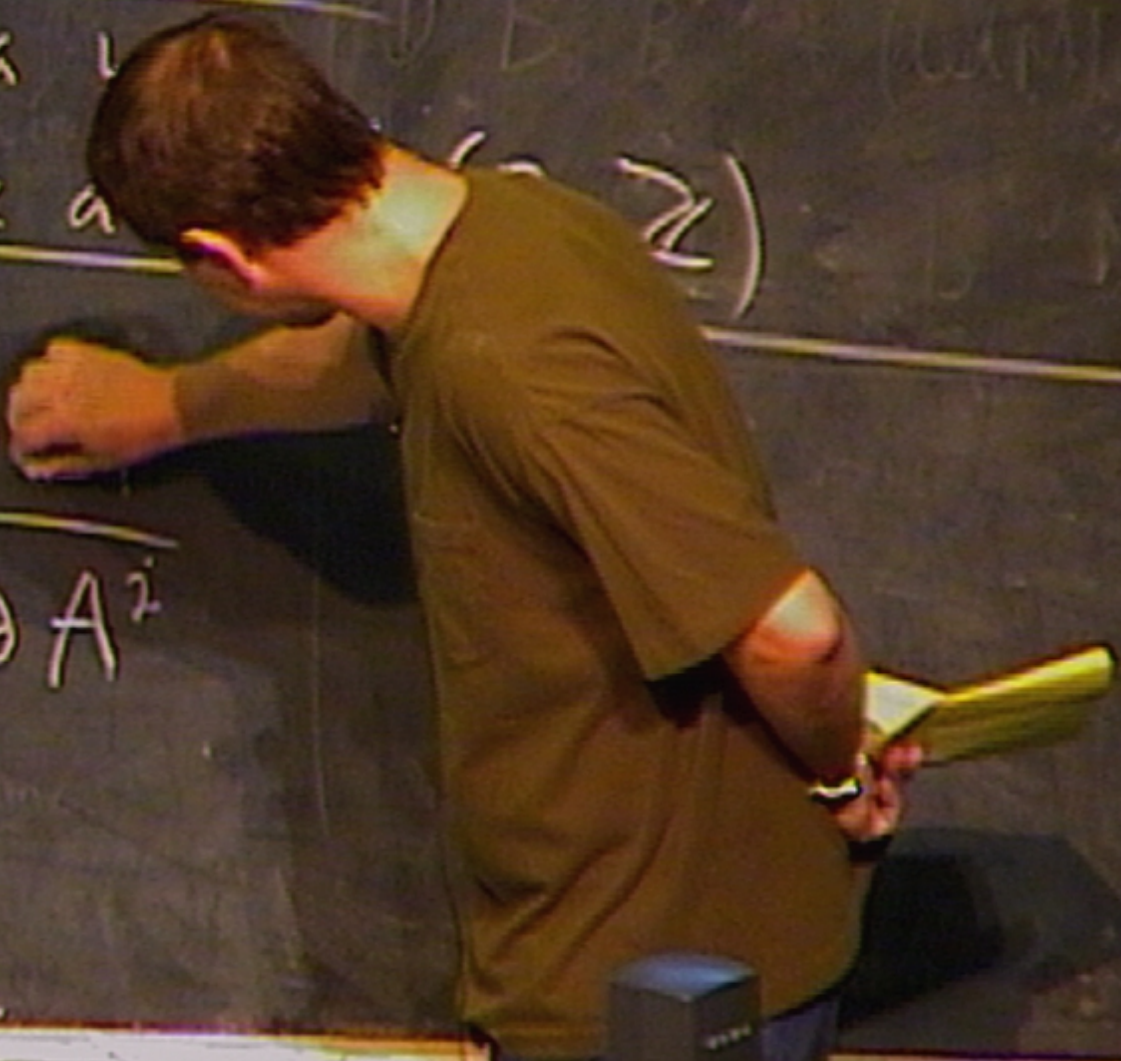
$$\tau \rightarrow \frac{a\tau + b}{c\tau + d}$$

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} \in SL(2, \mathbb{Z})$$

$V(1)^r$:

$$\begin{pmatrix} a & 1 \\ c & a \end{pmatrix} \in SL(2, \mathbb{Z})$$

$$\tau_{ij} = \frac{\partial}{\partial A^i}$$



$$\tau \rightarrow \frac{a\tau + b}{c\tau + d}$$

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} \in SL(2, \mathbb{Z})$$

$$U(1)^r: \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in SL(2, \mathbb{Z})$$

$$\tau_{ij} = \frac{\partial \bar{b}}{\partial A^2}$$

$$\tau \rightarrow \frac{a\tau + b}{c\tau + d}$$

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} \in SL(2, \mathbb{Z})$$

$$V(1)^r: \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in S$$

$$\tau_{ij} = \frac{\partial R}{\partial A^2}$$

$$\tau \rightarrow \frac{a\tau + b}{c\tau + d} \quad \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in SL(2, \mathbb{Z})$$

$$U(1)^r: \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in Sp(2r, \mathbb{Z})$$

$$\tau_{ij} = \frac{\partial \textcircled{B_i}}{\partial A^j}$$

(A^j, B_i) "special co"

$$\frac{a\tau + b}{c\tau + d} \quad \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in SL(2, \mathbb{Z})$$

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} \in Sp(2r, \mathbb{Z})$$

$$j = \frac{\partial \overline{B_i}}{\partial A^i}$$

(A^i, B_i) "special coords"



$$\partial_i \equiv \frac{\partial}{\partial A^i}$$

$$\mathcal{K}_V \supset \text{Im } \tau_{ij}(A) \partial_\mu A^i \partial^\mu \bar{A}^j$$

$$\partial_i \tau_{jk} = 0 \quad \tau_{ij} = \tau_{ji}$$

$$\text{Im } \tau_{ij} > 0$$

Rigid Special Kahler (RSK)

E-M duality = redundancy in the description i.e. $U(1)$ actions

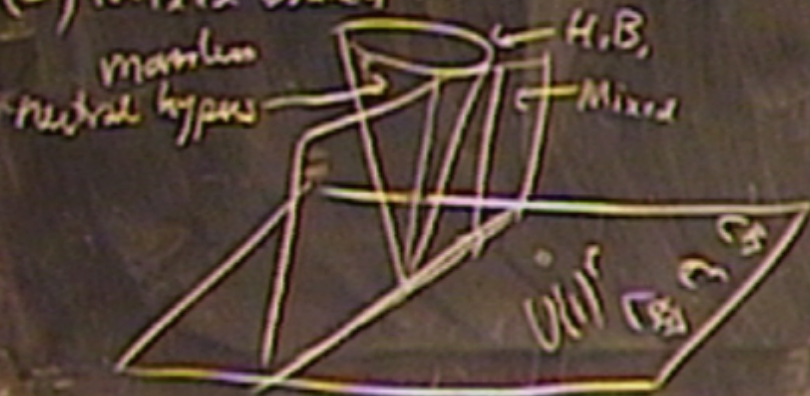
$$\vec{E} \rightarrow \vec{B}, \vec{B} \rightarrow -\vec{E} \quad \tau \rightarrow -\frac{1}{\tau} \sim \text{cplx to imagiz.}$$

$$\tau \rightarrow \tau + 1$$

(2) Higgs branch $\langle Q, \tilde{Q} \rangle \neq 0$

$\hookrightarrow$ No quantum corrections,
 $G \rightarrow \emptyset$
 $\Rightarrow \cong \mathbb{C}^{2n}$

(3) Mixed branch ---



C. Eff. Actions

hypers vectors
 $\downarrow$ $\downarrow$
 $(z, \tilde{z})$ $(A, \tilde{A})$

$$\mathcal{I}_H = \int d^4\theta \left[\overset{\text{hyperkahler}}{\mathcal{K}_H(z, \tilde{z}, \bar{z}, \tilde{\bar{z}})} + \mathcal{K}_V(A, \tilde{A}, \bar{A}, \tilde{\bar{A}}) \right]$$

$$\int d^4\theta \left[\mathcal{L}_V(A, \tilde{A}, \bar{A}, \tilde{\bar{A}}) + \text{c.c.} \right]$$

① no superpot $\Rightarrow$ $\mathbb{B}$, Higgs are not lifted.

TRISK

RSK geometry

Donagi & Witten
9510101

TRSK geometry

Donagi & Witten
9510101

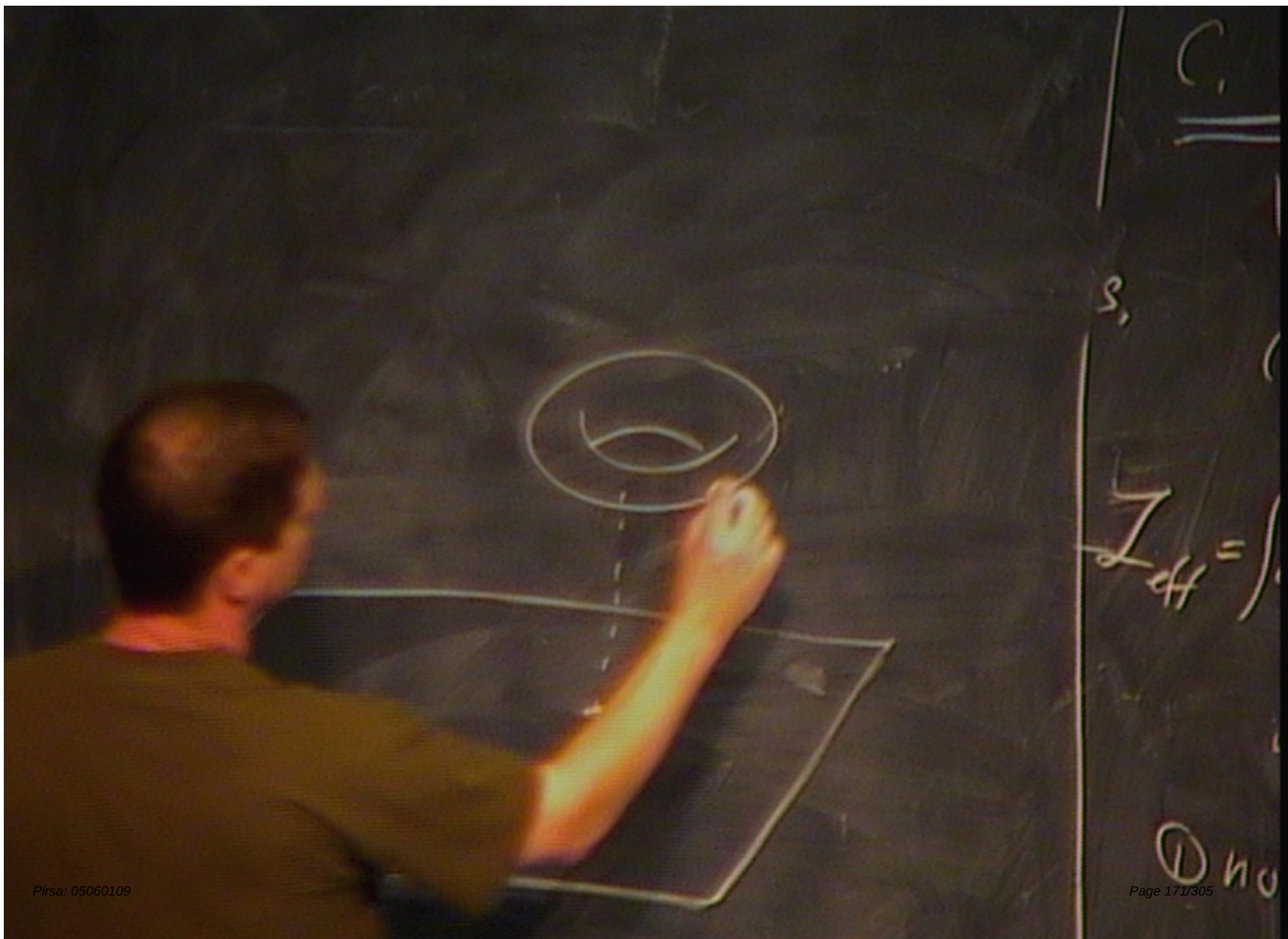
$$\mathcal{Z}_{\text{eff}} = \text{Im} \left[\tau_{ij} (\partial A^i \partial \bar{A}^j + \dots \right]$$

RSK geometry

Donagi & Witten
9510101

$$Z_{\text{eff}} = \text{Im} \left[\tau_{ij} (\partial A^i \cdot \partial \bar{A}^j + \mathcal{F}^i \cdot \mathcal{F}^j) \right]$$

$$\partial_{[i} \tau_{j]k} = 0 \quad \tau_{ij} = \tau_{ji} \quad \text{Im} \tau_{ij} > 0$$

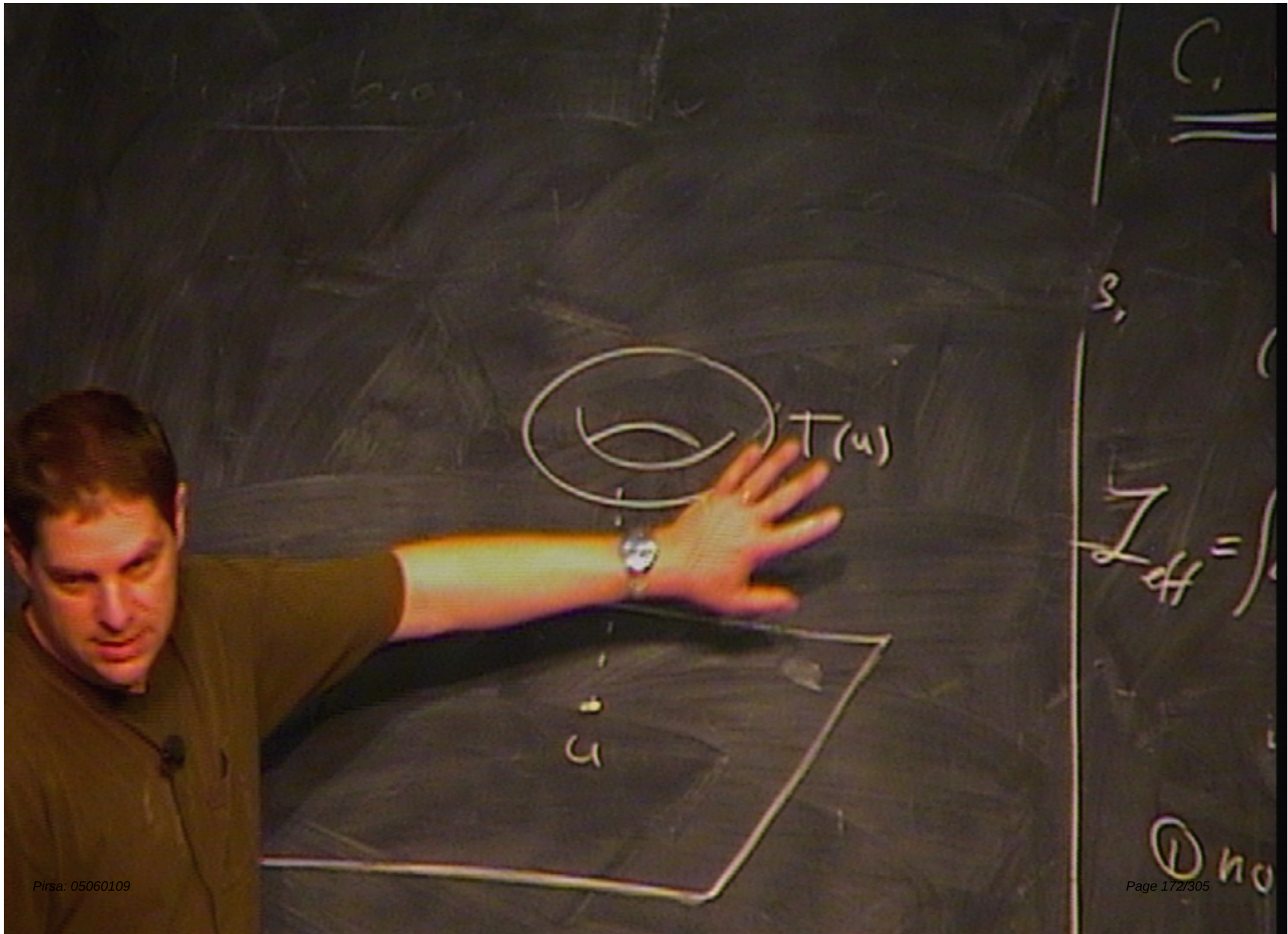


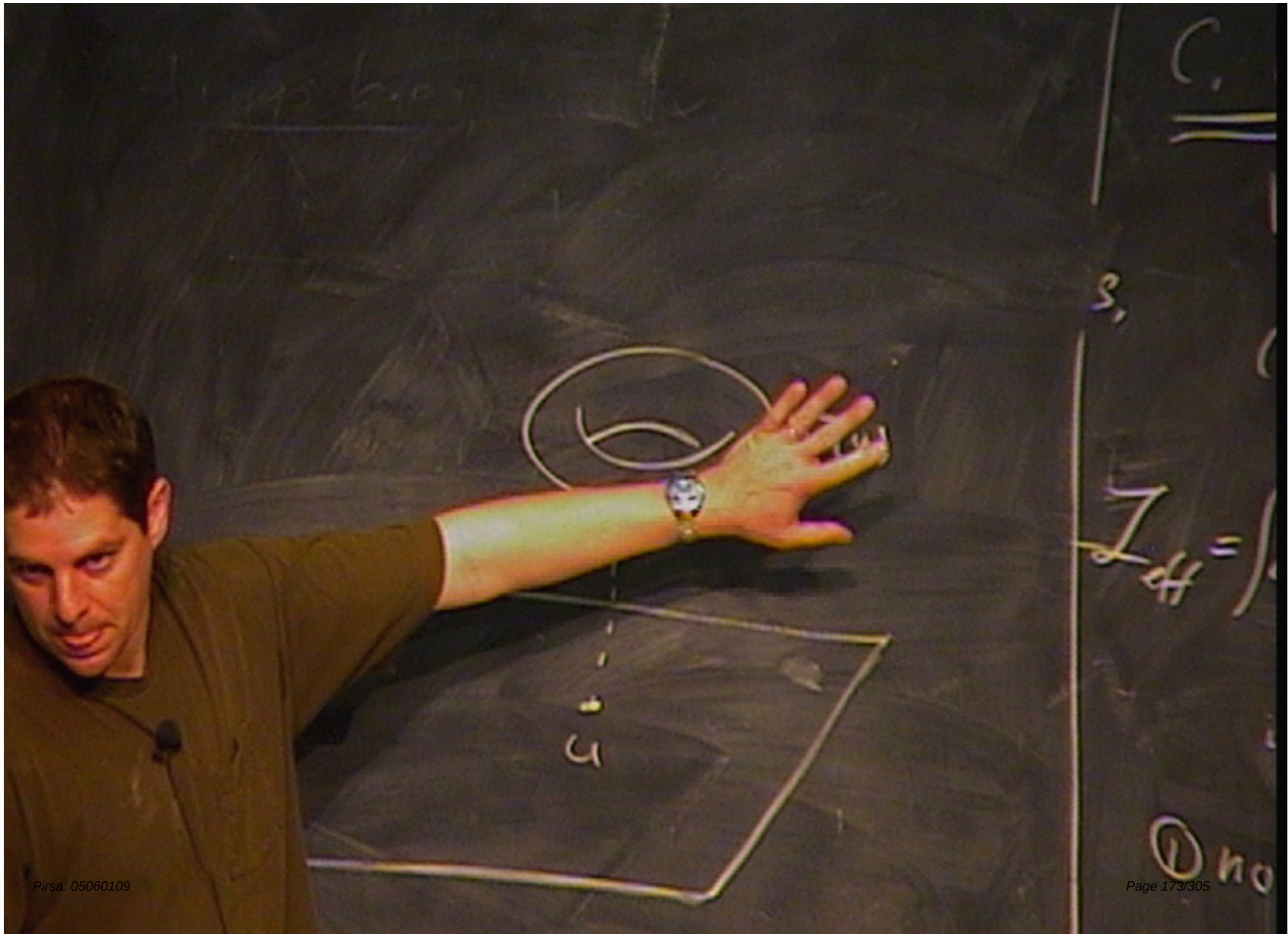
C_1

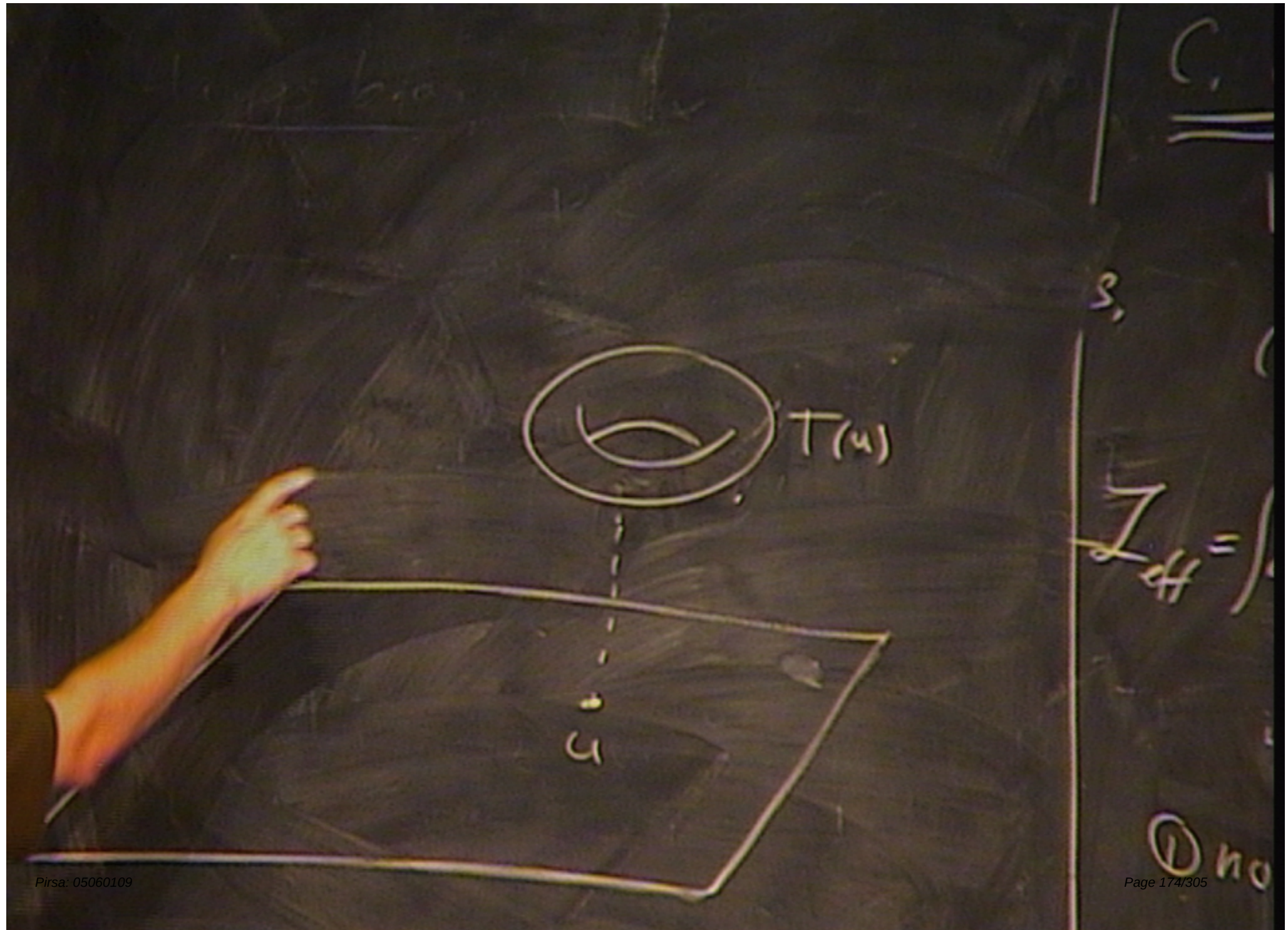
S_1

$$I_{eff} = \int$$

D_{no}







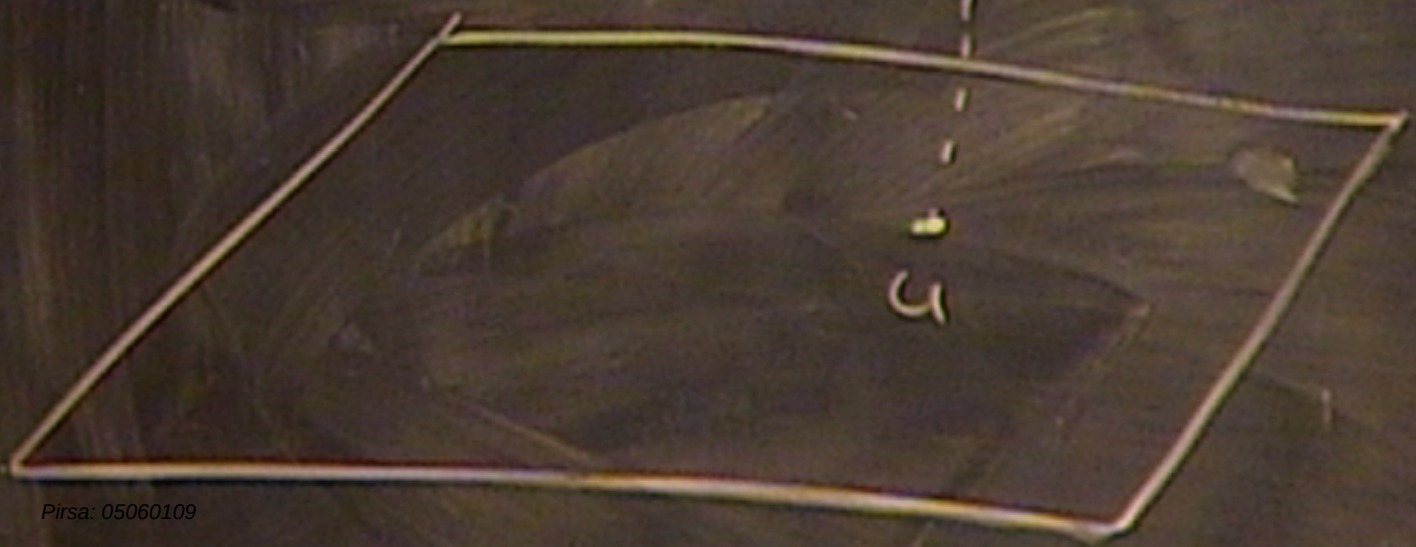
Things to know

C,

S,



$I_H =$



$\mathbb{D}^n$

TRSK geometry

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$$\mathcal{L}_{\text{eff}} = \text{Im} \left[\tau_{ij} (\partial A^i \cdot \partial \bar{A}^j + \mathcal{F}^i \cdot \mathcal{F}^j) \right]$$

$$\partial_{[i} \tau_{j]k} = 0 \quad \tau_{ij} = \tau_{ji} \quad \text{Im} \tau_{ij} > 0$$

(u) r - $\mathbb{C}$ -dim'd torus

TRSK geometry

Donagi & Witten
9510101

$$\mathcal{L}_{\text{eff}} = \text{Im} \left[\tau_{ij} (\partial A^i \cdot \partial \bar{A}^j + \tilde{F}^i \cdot \tilde{F}^j) \right]$$

$$\partial_{[i} \tau_{j]k} = 0 \quad \tau_{ij} = \tau_{ji} \quad \text{Im} \tau_{ij} > 0$$

$T(u)$ r - $\mathbb{C}$ -dim'd torus $\{x_i, y^i\} \quad i=1 \dots r$
 $x_i = x_{i+1} \quad y^i = y^{i+1}$

TRSK geometry

Donagi & Witten
9510101

$$\mathcal{L}_{\text{eff}} = \text{Im} \left[\tau_{ij} (\partial A^i \cdot \partial \bar{A}^j + \mathcal{F}^i \cdot \mathcal{F}^j) \right]$$

$$\partial_{[i} \tau_{j]} = 0$$

$$\tau_{ij} = \tau_{ji} \quad \text{Im} \tau_{ij} > 0$$

$T(u)$ r-C-div

X_i

$\{x, y^i\} \quad i=1 \dots r$

$$= y^{i+1} \Rightarrow Z_i = n_i X_i + \tau_{ij} y^j$$

RSK geometry

Donagi & Witten
9510101

$$\mathcal{Z}_{\text{eff}} = \text{Im} \left[\tau_{ij} (\partial A^i \cdot \partial \bar{A}^j + \bar{\omega}^i \cdot \omega^j) \right]$$

$$\tau_{ij} = 0 \quad \tau_{ij} = \tau_{ji} \quad \text{Im} \tau_{ij} > 0$$

$T(u)$ r.

1 torus

$$\{x_i, y_i\} \quad i=1 \dots r$$

$$x_{i+1}$$

$$y_i = y_{i+1} \Rightarrow$$

$$\mathcal{Z}_i = (n_i) x_i + \tau_{ij} y_j$$

RSK geometry

Donagi & Witten
9510101

$$\mathcal{Z}_{\text{eff}} = \text{Im} \left[\tau_{ij} (\partial A^i \cdot \partial \bar{A}^j - \bar{A}^i \cdot \partial A^j) \right]$$

$$\partial_{[i} \tau_{j]k} = 0$$

$$\tau_{ji} \quad \text{Im} \tau_{ij} > 0$$

$T(u)$ r - $\mathbb{C}$ -dim'd torus

$$X_i \simeq X_i +$$

$$i=1 \dots r$$

$$\Rightarrow \mathcal{Z}_i = (n_i) X_i + \tau_{ij} y_j$$

RSK geometry

Donagi & Witten
9510101

$$\mathcal{L}_{\text{eff}} = \text{Im} \left[\tau_{ij} (\partial A^i \cdot \partial \bar{A}^j + \bar{A}^i \cdot \bar{A}^j) \right]$$

$$\partial_{[i} \tau_{j]k} = 0$$

$$\tau_{ij} \text{ Im } \tau_{ij} > 0$$

$T(u)$ r - $\mathbb{C}$ -dim'd torus

$$X_i = X_i +$$

$i = 1 \dots r$

$\sum$

$$X_i + \tau_{ij} y_j$$

RSK geometry

Donagi & Witten
9510101

$$\mathcal{Z}_{\text{eff}} = \text{Im} \left[\tau_{ij} (\partial A^i \cdot \partial \bar{A}^j + \bar{\tau}^i \cdot \bar{\tau}^j) \right]$$

$$\partial_{[i} \tau_{j]k} = 0$$

τ

$$\text{Im} \tau_{ij} > 0$$

$T(u)$ r - $\mathbb{C}$ -dim'd torus

$$X_i = X_i +$$

$(i=1 \dots r)$

$$Z_i = (n_i) X_i + \tau_{ij} y_j$$

RSK geometry

Donagi & Witten
9510101

$$\mathcal{Z}_{\text{eff}} = \text{Im} [\tau_{ij} (\partial A^i \partial \bar{A}^j + \mathcal{F}^i \mathcal{F}^j)]$$

$$\partial_{[i} \tau_{jk]} = 0 \quad \tau_{ij} = \tau_{ji} \quad \text{Im} \tau_{ij} > 0$$

$T(u)$ r - $\mathbb{C}$ -dim'd for

$$X_i = X_i$$

$$\{y^i\} \quad i=1 \dots r$$

$$y^{i+1} \Rightarrow Z_i = (n)X_i + \tau_{ij} y^j$$

RSK geometry

Donagi & Witten
9510101

$$\mathcal{Z}_{\text{eff}} = \text{Im} [\tau_{ij} (\partial A^i \cdot \gamma^j + \tilde{A}^i \cdot \tilde{\gamma}^j)]$$

$$\partial_{[i} \tau_{jk]} = 0 \quad \tau_{ji} = \tau_{ij} \quad \text{Im} \tau_{ij} > 0$$

$T(u)$ r - $\mathbb{C}$ -dim'd for

$$X_i = X_j$$

$(i=1 \dots r)$

$\mathbb{Z}$

$$X_i + \tau_{ij} y_j$$

RSK geometry

Donagi & Witten
9510101

$$\mathcal{Z}_{\text{eff}} = \text{Im} \left[\tau_{ij} (\partial A^i \cdot \partial \bar{A}^j) \right]$$

$$\partial_{[i} \tau_{j]k} = 0$$

$$\text{Im} \tau_{ij} > 0$$

$T(u)$ r - $\mathbb{C}$ -dim'd torus

$$X_i = X_{i+1}$$

$1 \dots r$

$\sum$

$$X_i + \tau_{ij} y_j$$

RSK geometry

Donagi & Witten
9510101

$$\mathcal{Z}_{\text{eff}} = \text{Im} [\tau_{ij} (\partial A^i \cdot \partial \bar{A}^j + \mathcal{F}^i \cdot \mathcal{F}^j)]$$

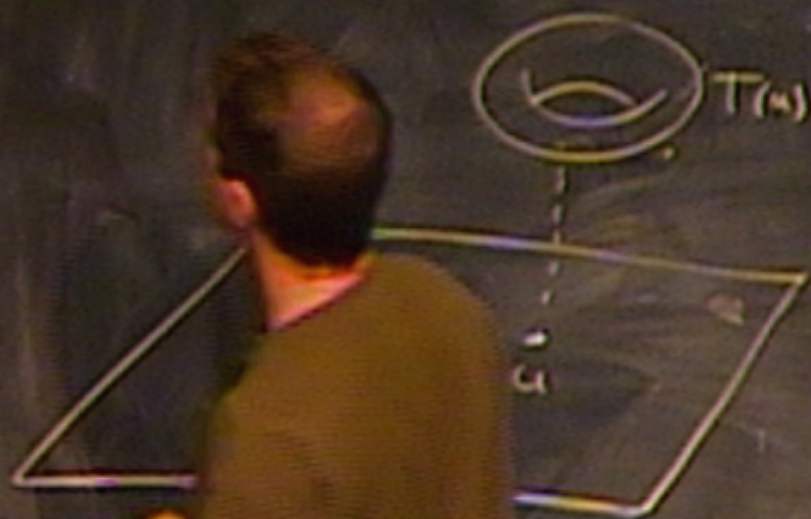
$$\partial \tau_{ij} = 0 \quad \tau_{ij} = \tau_{ji} \quad \text{Im} \tau_{ij} > 0$$

$T(u) \quad r = \mathbb{C} - d$

$$\{x_i, y_i\} \quad i = 1 \dots r$$

$$i = y_i + 1 \Rightarrow Z_i = (n_i) x_i + \tau_{ij} y_j$$

$$t = dx + ndy^2$$



C. Eff. Actions

hyper

vectors

 $(\mathbf{r}, \mathbf{z})$ $(A, \bar{u})$

$$I_4 = \int d^4\theta \left[\underbrace{K_H(\vec{q}, \vec{q}, \vec{q}, \vec{q})}_{\rightarrow \text{hyperkähler}} + \underbrace{K_V(\vec{A}, \vec{A}, \vec{A}, \vec{A})}_{\rightarrow \text{hyperkähler}} \right]$$

$$\int d\theta \left[\mathcal{L}_i(\theta) \dot{w}_i \cdot \dot{w}_i \right] + c.c.$$

① no superpartners $\Rightarrow$ B, Higgs are not lifted

$$\epsilon = dx_1 \wedge dy^1$$

C. Eff. Actions

hyperm

vector

$(\tilde{\chi}, \tilde{\bar{\chi}})$

$(A^i, \bar{W}_i)$

$$\mathcal{L}_{eff} = \int d^4\theta \left[\mathcal{K}_H(\tilde{\chi}, \tilde{\bar{\chi}}, \tilde{\chi}, \tilde{\bar{\chi}}) + \mathcal{K}_V(A^i, \bar{A}^i, W^i, \bar{W}^i) \right] \rightarrow \text{hyperkahler}$$

$$\int d^4\theta \left[\mathcal{L}_{eff}(A^i, \bar{A}^i, W^i, \bar{W}^i) + c.c. \right]$$

superpot's $\Rightarrow$ $\mathcal{L}_B$, Higgs are not lifted.



TRSK geometry

$$\mathcal{F}_{\mu\nu} = \frac{1}{2} \overline{F}_{\mu\nu} +$$

Donald & Witten
9510101

$$\mathcal{L}_{\text{eff}} = \text{Im} \left[\tau_{ij} (\partial A^i - \partial A^j + \dots) \right]$$

$$\partial_{[i} \tau_{j]k} = 0$$

$$\text{Im} \tau_{ij} > 0$$

$T(u)$ r - $\mathbb{C}$ -div'd forms

$$X_i = X_{i+1}$$

$$\sum_i = \sum_{i=1}^r (n_i^2 X_i + \tau_{ij} y_j)$$

RSK geometry

$$\mathcal{Z}_{uv} = \frac{1}{i} F_{uv} + i \Gamma_{uv}$$

Donagi: 2 bitlen
9 0101

$$\mathcal{Z}_{eff} = \text{Im} \left[\tau_{ij} (\partial A^i \cdot \partial \bar{A}^j) \right]$$

$$\partial_{[i} \tau_{j]k} = 0$$

$$\tau_{ij} \text{ in } \tau_{ij} > 0$$

$T(u)$ r-C-dim'd torus

$$X_i = X_{i+1}$$

$$= \binom{1}{n} X_i + \tau_{ij} y_j$$

RSK geometry

$$\mathcal{F}_{\mu\nu} = \frac{1}{i} F_{\mu\nu} + i \bar{F}_{\mu\nu}$$

Donagi & Witten
95101

$$\mathcal{L}_{\text{eff}} = \text{Im} [\tau_{ij} (\partial A^i \cdot \partial \bar{A}^j - \mathcal{F}^i \cdot \mathcal{F}^j)]$$

$$\partial_{[i} \tau_{jk]} = 0$$

$$\tau_{ij} > 0$$

$T(u)$ r- $\mathbb{C}$ -div'd forms

$$X_i = X_{i+1}$$

$$= \hat{h}^i X_i + \tau_{ij} y^j$$

TRSK geometry

$$\mathcal{F}_{\mu\nu} = \frac{1}{i}(\vec{F}_{\mu\nu} + i\vec{F}_{\mu\nu})$$

Donagi & Witten

9510101

$$\mathcal{L}_{\text{eff}} = \text{Im} \left[\tau_{ij} (\partial A^i - \bar{\partial} A^j + \mathcal{F}^i_j \mathcal{F}^j_i) \right]$$

$$\partial_{[i} \tau_{j]k} = 0$$

$$\text{Im} \tau_{ij} > 0$$

$T(u)$ r- $\mathbb{C}$ -div'd torus

$$X_i = X_i +$$

$$\Sigma_i = \hat{\eta}_i X_i + \tau_{ij} y_j$$

TRSK geometry

$$\mathcal{F}_{\mu\nu} = \frac{1}{i}(\vec{F}_{\mu\nu} + i\vec{F}_{\mu\nu})$$

Donagi & Witten

9510101

$$\mathcal{Z}_{\text{eff}} = \text{Im}[\tau_{ij} (\partial A^i \cdot \partial \bar{A}^j + \mathcal{F}^i \cdot \mathcal{F}^j)]$$

$$\partial_{[i} \tau_{j]k} = 0 \quad \tau_{ij} = \tau_{ji} \quad \text{Im} \tau_{ij} > 0$$

$T(u)$

and forms

$$\{x_i, y_i\} \quad i=1 \dots r$$

$$x_i = x_{i+1}$$

$$y_i = y_{i+1} \Rightarrow \sum_i = \hat{n}_i x_i + \tau_{ij} y_j$$

TRSK geometry

$$F_{\mu\nu} = \frac{1}{i}(\tilde{F}_{\mu\nu} + i\tilde{\tilde{F}}_{\mu\nu})$$

Donagi & Witten

9510101

$$\mathcal{L}_{\text{eff}} = \text{Im} \left[\tau_{ij} (\partial A^i \cdot \partial \bar{A}^j + \tilde{F}^i \cdot \tilde{F}^j) \right]$$

$$\partial_{[i} \tau_{j]} = 0$$

$$\tau_{ij} = \tau_{ji}$$

$$\text{Im} \tau_{ij} > 0$$

$$T(u) \text{ r.c.d.}$$

}

$\} \quad i=1 \dots r$

$$y' = y' + 1 \Rightarrow \tilde{Z}_i = \tilde{h}_i^1 X_i + \tau_{ij} y_j$$

TRSK geometry

$$\mathcal{F}_{\mu\nu} = \frac{1}{i}(\vec{F}_{\mu\nu} + i\vec{F}_{\mu\nu})$$

Donagi & Witten

9510101

$$\mathcal{Z}_{\text{eff}} = \text{Im} \left[\tau_{ij} \left(A^i \cdot \partial \bar{A}^j + \omega^i \cdot \mathcal{F}^j \right) \right]$$

$$\partial_{[i} \tau_{j]k}$$

$$\tau_{ij} = \tau_{ji} \quad \text{Im} \tau_{ij} > 0$$

$T(u)$ r.c.-div'd

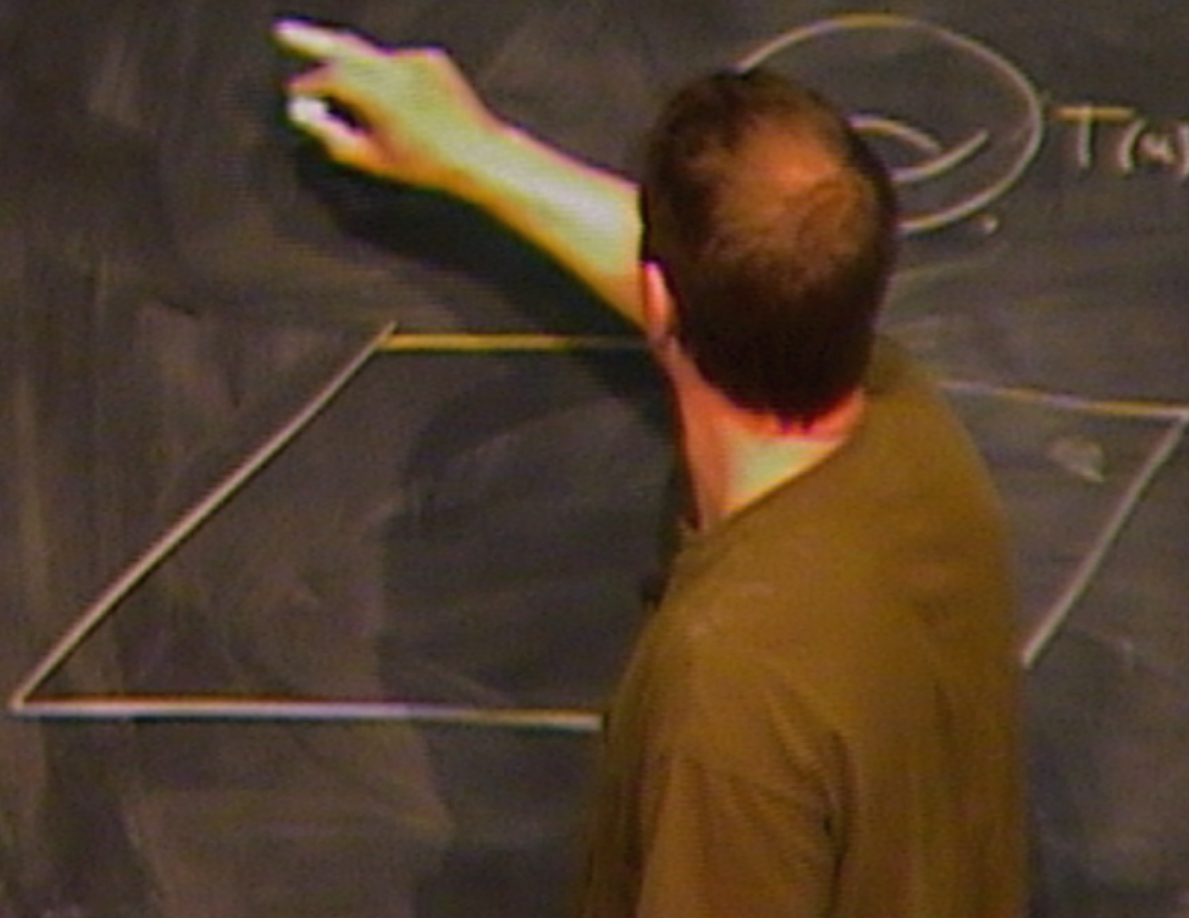
$$X_i =$$

$$i=1 \dots r$$

$$y_i + 1 \Rightarrow \sum_i = \hat{n}_i X_i + \tau_{ij} y_j$$

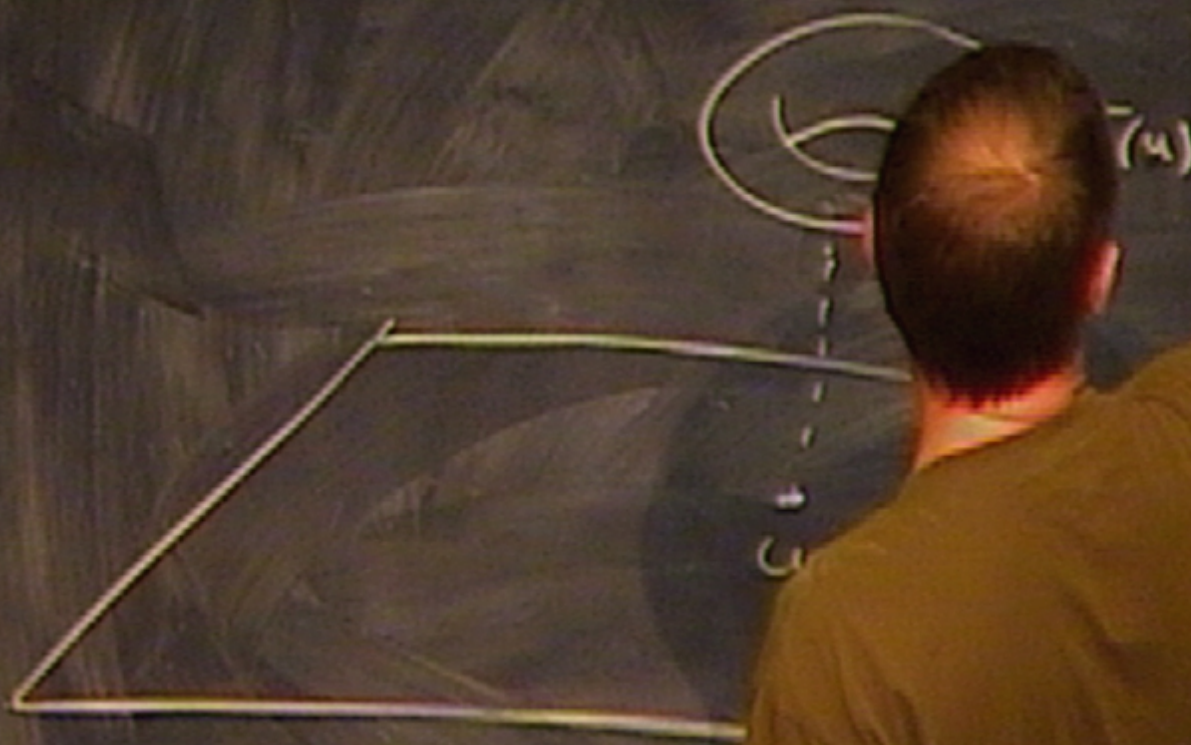
$$\oint = dx; ndy^i$$

closed, integral, positive,



$$\oint = dx; \wedge dy^i$$

closed, integral, positive, (1)



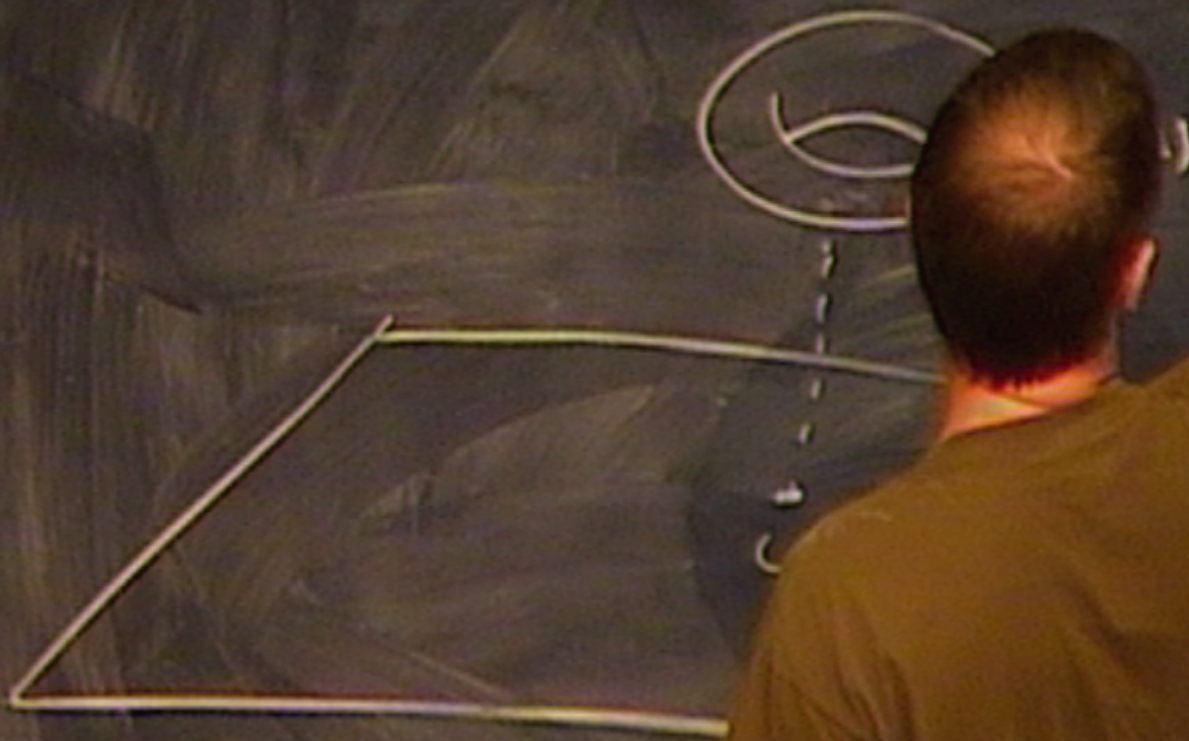
s

$$\int_{\mathcal{H}} =$$

$\textcircled{1} n$

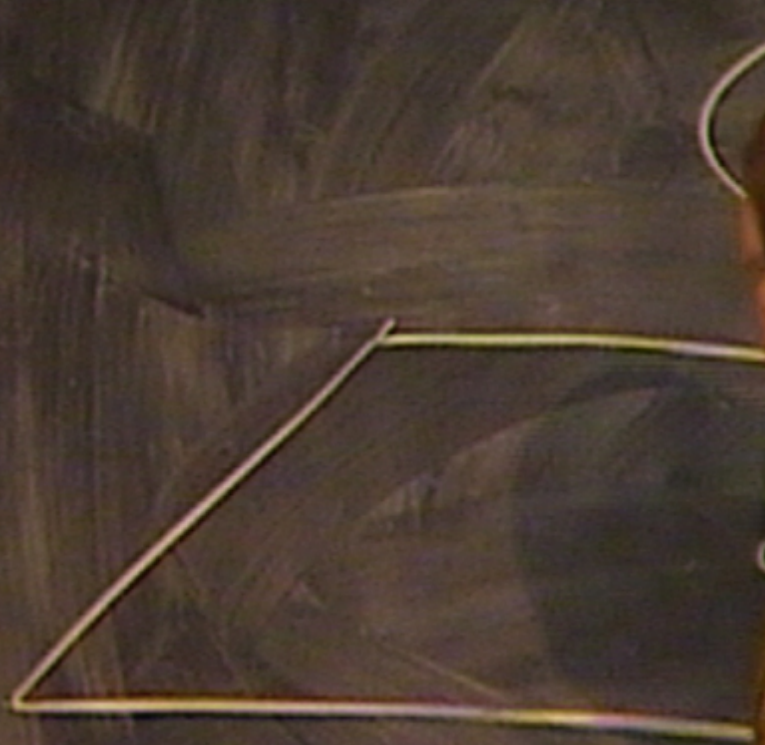
$$\oint = dx; \wedge dy^i$$

closed, integral, positive, (1,1)-



$$\oint = dx; \wedge dy^i$$

closed, integral, positive, (1,1)-form



$S,$

(u)

$\int_{dH} =$

$\textcircled{1} n$

$$\tau = dx_1 \wedge dy_1$$

closed, integral, positive, (1,1)-form



u

S

$$\int_{\mathcal{H}} =$$

$\textcircled{D} n$

$$\oint = dx; \wedge dy^i$$

closed, integral, positive, (1,1)-form



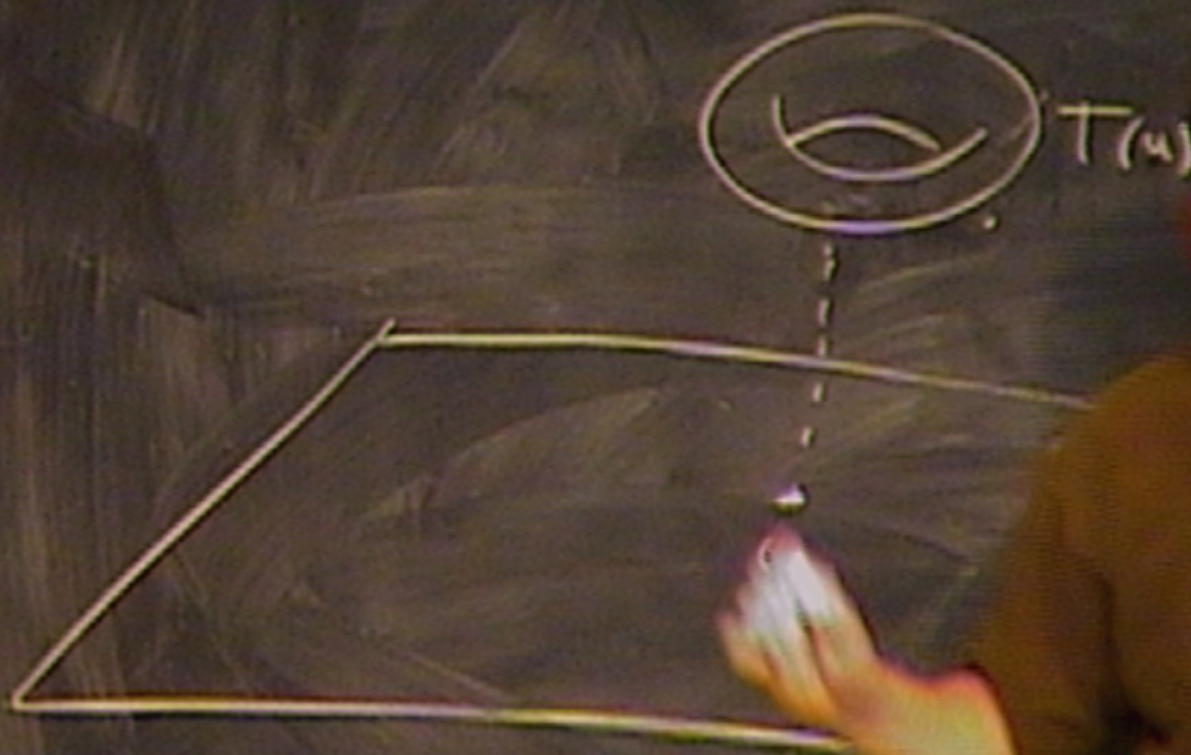
$S,$

$$\int_{dH} =$$

$\textcircled{D} n$

$$t = dx + ndy^i$$

closed, integral, positive, (1,1)-form

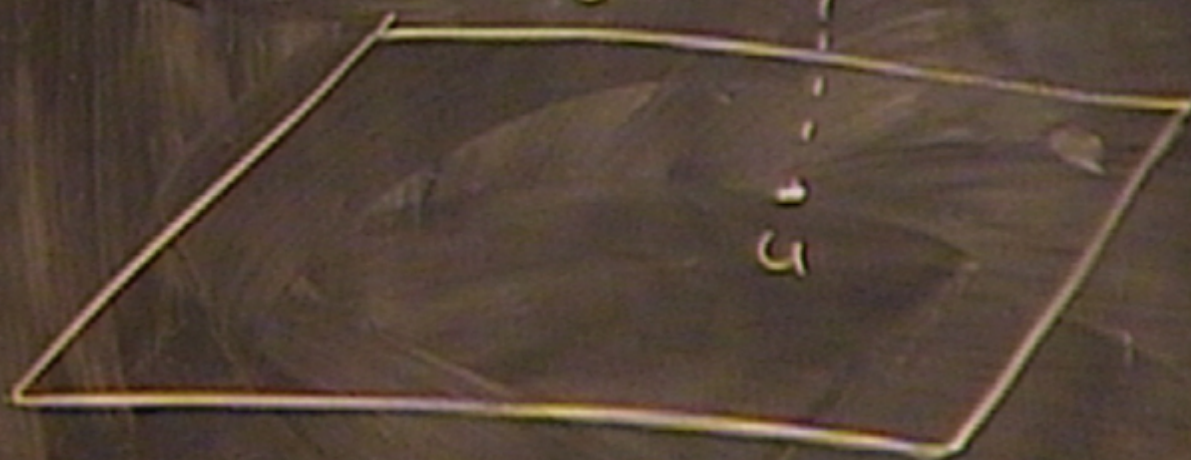


$$t = dx + ndy^i$$

closed, integral, positive, (1,1)-form

"Hodge form", "polarization"

$\Rightarrow T(u) =$ "Abelian variety"



$\mathcal{I}_d =$

$\mathbb{D}_n$

itive, $(1,1)$ -form

$$\tau_{ij} = B_{ik} (A^{-1})^k_j$$

ation"

$T(u)$

sitive, (1,1)-form

ation"

$T(u)$

$$\tau_{ij} = B_{ik} (A^{-1})^k_j$$

$$B_{ik} = \oint_{\beta}$$

itive, $(1,1)$ -form

ation"

τ

$$\tau_{ij} = B_{ik} (A^{-1})^k_j$$

$$B_{ik} = \oint_{\beta_j} \omega_k$$

itive, $(1,1)$ -form

ation"

$T(u)$

$$\tau_{ij} = B_{ik} (A^{-1})^k_j$$

$$B_{ik} = \phi w_k$$

itive, $(1,1)$ -form

ation"

$T(u)$

$$\tau_{ij} = B_{ik} (A^{-1})^k_j$$

$$B_{ik} = \oint_{\beta_j} \omega_k \quad A$$

itive, (1,1)-form

ation"

$T(u)$

$$\tau_{ij} = B_{ik} (A^{-1})^k_j$$

$$B_{ik} = \oint_{\beta_j} \omega_k A$$

itive, $(1,1)$ -form

ation"

$T(u)$

$$\tau_{ij} = B_{ik} (A^{-1})^k_j$$

$$B_{ik} = \oint_{\beta_j} \omega_k$$

$$A_k^j = \oint$$

itive, $(1,1)$ -form

ation"

$T(u)$

$$\tau_{ij} = B_{ik} (A^{-1})^k_j$$

$$B_{ik} = \oint_{\beta_j} \omega_k$$

$$A^j_k = \oint_{\alpha^k} \omega_k$$

$\tau_{ij} = B_{ik} (A^{-1})^k_j$
 $B_{ik} = \oint_{\beta_j} \omega_k \quad A^j_k = \oint_{\alpha^k} \omega_k$
 $\{\omega_k\}$ any basis of hol
 $(1,0)$ - on T

itive, $(1,1)$ -form

ation"

$T(u)$

$$\tau_{ij} = B_{ik} (A^{-1})^k_j$$

$$B_{ik} = \oint_{\beta_j} \omega_k \quad A^j_k = \oint_{\alpha^k} \omega_k$$

$\{\omega_k\}$ any basis of hol
 $(1,0)$ -forms on T
 (z_i)

itive, $(1,1)$ -form

ation"

τ

$$\tau_{ij} = B_{ik} (A^{-1})^k_j$$

$$B_{ik} = \oint_{\beta_j} \omega_k \quad A_k^j = \oint_{\alpha^k} \omega_k$$

$\{\omega_k\}$ any basis of hol
 $(1,0)$ -forms on T

itive, $(1,1)$ -form

ation"

$$\tau_{ij} = B_{ik} (A^{-1})^k_j$$

$$B_{ik} = \oint_{\beta_j} \omega_k \quad A_k^j = \oint_{\alpha^k} \omega_k$$

$\{\omega_k\}$ any basis of hol
 $(1,0)$ -forms on T

$$(\alpha Z_i)$$

itive, $(1,1)$ -form

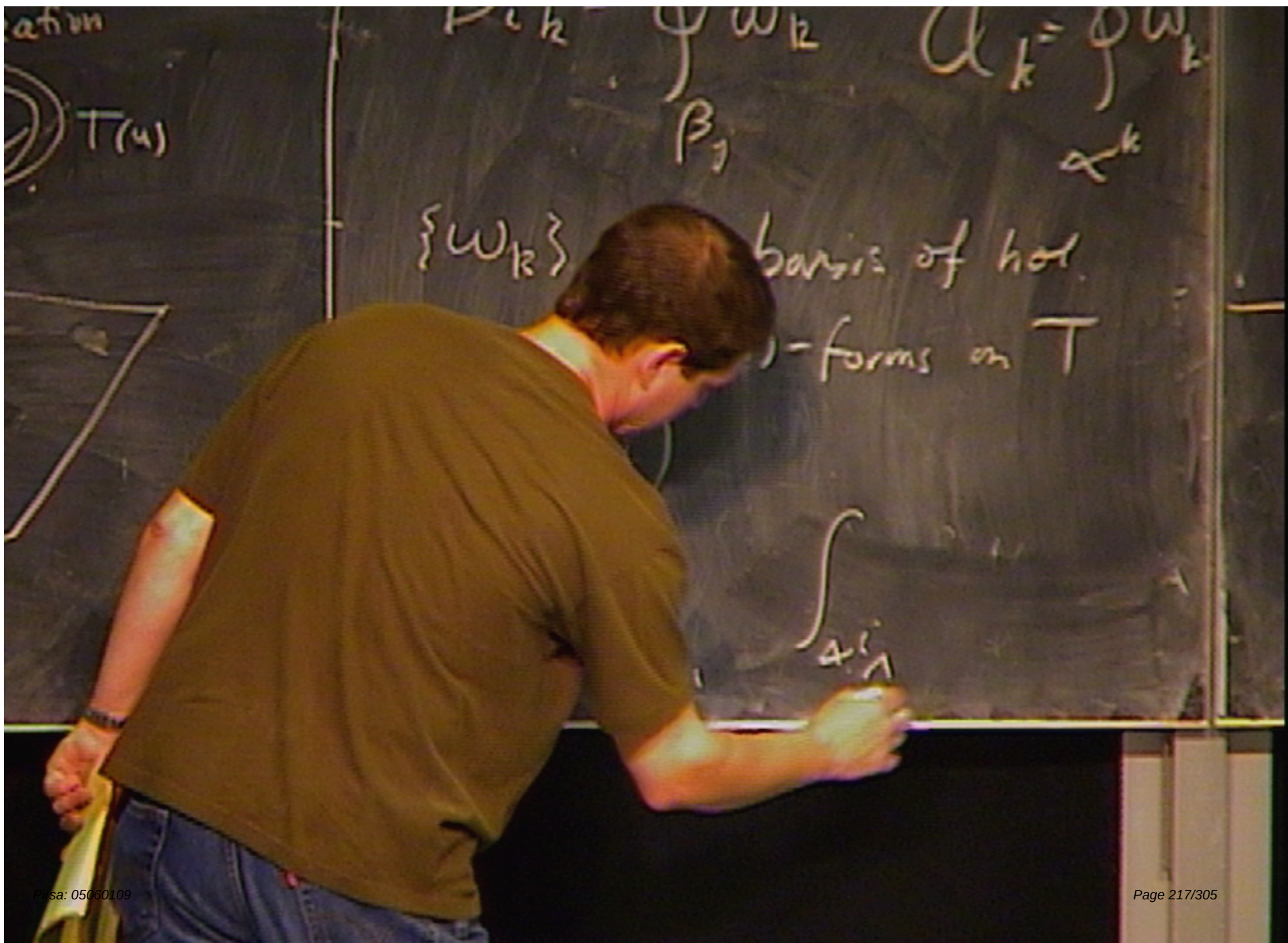
ation"

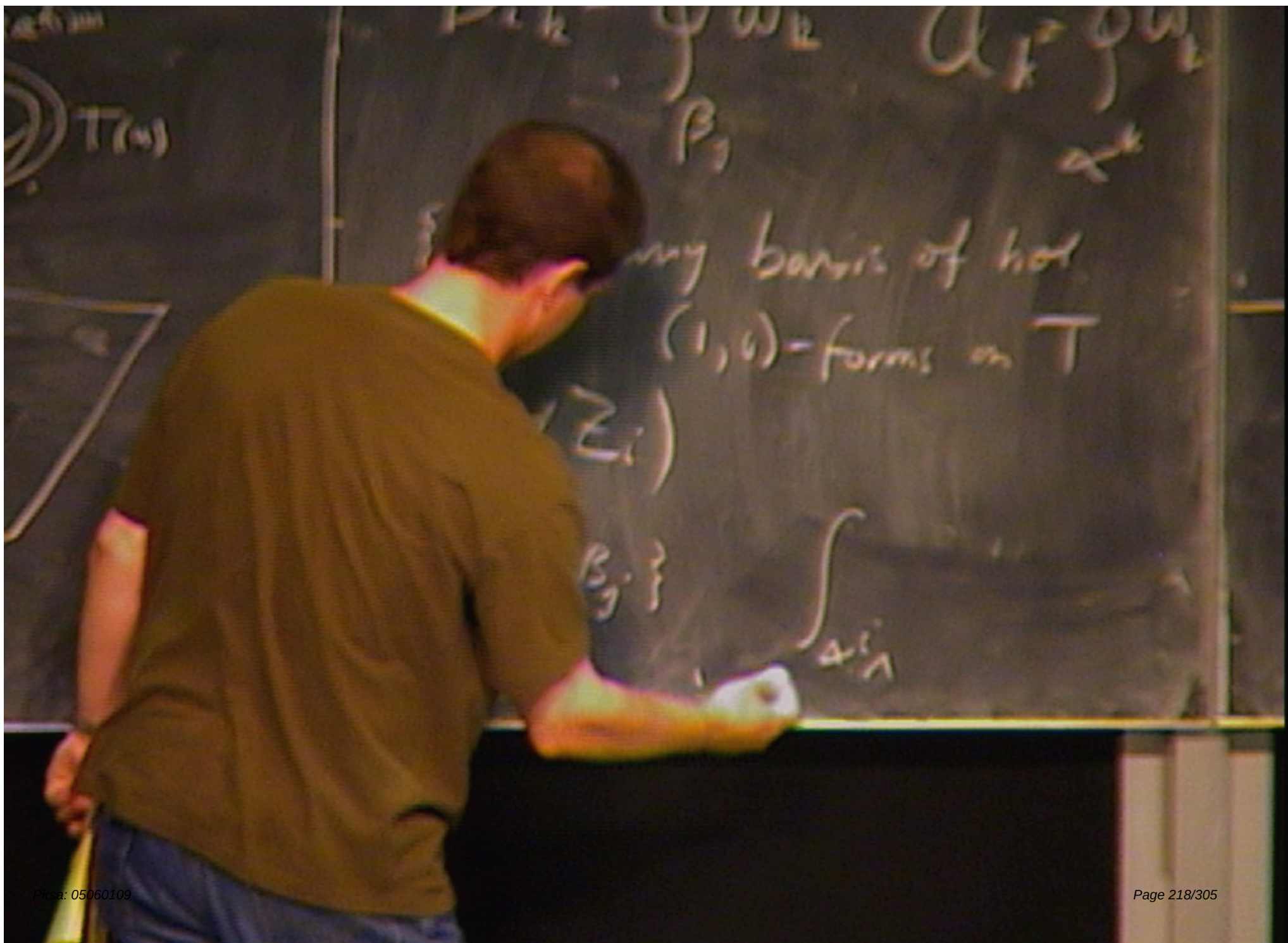
$T(u)$

$$\tau_{ij} = B_{ik} (A^{-1})^k_j$$

$$B_{ik} = \oint_{\beta_j} \omega_k \quad A^j_k = \oint_{\alpha^k} \omega_k$$

$\{\omega_k\}$ basis of hol
 $(1,1)$ -forms on T





ation

ω_k

β_j

$\alpha_k = \phi \omega_k$

$\{\omega_k\}$ any basis of hol
 $(1,0)$ -forms on T

(dz_i)

$\{\alpha^k, \beta_j\}$

$\int_{\alpha^i \wedge \beta_j} t = \delta_{ij}$

TRSK geometry

$$\mathcal{F}_{\mu\nu} = \frac{1}{\hbar} (\vec{F}_{\mu\nu} + i \vec{\tilde{F}}_{\mu\nu})$$

Donagi & Witten

9510101

$$\mathcal{Z}_{\text{eff}} = \text{Im} [\tau_{ij} (\partial A^i \cdot \partial \bar{A}^j + \mathcal{F}^i \cdot \mathcal{F}^j)]$$

$$\partial_{[i} \tau_{j]k} = 0 \quad \tau_{ij} = \tau_{ji} \quad \text{Im} \tau_{ij} > 0$$

$T(u)$ r - $\mathbb{C}$ -dim'd torus $\{x_i, y^i\} \quad i=1 \dots r$

$$x_i = x_{i+1} \quad y^i = y^{i+1} \Rightarrow \sum_i = \sum_i \tau_{ij} x_i + \tau_{ij} y^j$$

$$t = dx_i \wedge dy^i$$

closed, integral, positive, (1,1)-form
 "Hodge form", "polarization"

$\Rightarrow T(u) = \text{"Abelian variety"}$



$$T_{ij} = B_{ik} (A^{-1})^k_j$$

$$B_{ik} = \oint_{\beta_j} \omega_k \quad A_k^j = \oint_{\alpha^k} \omega_k$$

$\{\omega_k\}$ any basis of hol.
 (1,0)-forms on T

$$(dz_i)$$

$$\{\alpha^i, \beta_j\}$$

$$\int_{\alpha^i \wedge \beta_j} t = \delta_{ij}$$

$$\mathcal{K}_v \supset \text{Im } \tau_{ij}(A) \partial_\mu A^i \partial^\mu \bar{A}^j$$

$$\Re \tau_{ij} = 0 \quad \tau_{ij} = \tau_{ji}$$

$$\text{Im } \tau_{ij} > 0$$

Rigid Special Kahler (RSK)

redundancy in the description i.e. $U(1)$ actions

$$\tau \rightarrow \tau + 1 \sim \text{cplx to imag}$$

$$\tau \rightarrow \tau + 1$$

$$\tau \rightarrow \frac{a\tau + b}{c\tau + d} \quad \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in SL(2, \mathbb{Z})$$

$$U(1)^r: \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in Sp(2r, \mathbb{Z})$$

$$T_{ij} = \frac{\partial \langle B_i \rangle}{\partial A^j}$$

(A^j, B_i) "special coords"

$$\tau = dx_1 \wedge dy_1$$

closed, integral, positive, (1,1)-form

"Hodge form", "polarization"

$\Rightarrow T(u) = \text{"Abelian variety"}$



$$T(u) = B(u) \cdot Q \cdot T(u)$$

$$B(u) = \int_{\beta} \omega_u \quad Q = \int_{\alpha} \omega_u$$

$\{\omega_u\}$ any basis of hol
(1,1)-forms on T

$$(\alpha, \beta)$$

$$\int_{\alpha, \beta} \tau = \delta_{ij}$$

$$t = dx_i \wedge dy^i$$

closed, integral, positive, (1,1)-form
 "Hodge form", "polarization"

$(u) = \text{"Abelian variety"}$



$$\tau_{ij} = B_{ik} (A^{-1})^k_j$$

$$B_{ik} = \oint_{\beta_j} \omega_k \quad A_k^j = \oint_{\alpha^k} \omega_k$$

$\{\omega_k\}$ any basis of hol.
 (1,0)-forms on T

$$(\alpha \tilde{z}_i)$$

$$\{\alpha^*, \beta_j\}$$

$$\int_{\alpha_i \wedge \beta_j} t = \delta_{ij}$$

$$t = dx_i \wedge dy^i$$

closed, integral, positive, (1,1)-form
 "Hodge form", "polarization"

$\Rightarrow T(u) = \text{"Abelian variety"}$



$$T_{ij} = B_{ik} (A^{-1})^k_j$$

$$B_{ik} = \oint_{\beta_j} \omega_k \quad A_k^j = \oint_{\alpha^k} \omega_k$$

$\{\omega_k\}$ any basis of hol.
 (1,0)-forms on T

$$(\alpha \tilde{z}_i)$$

$$\{\alpha^i, \beta_j\}$$

$$\int_{\alpha^i \wedge \beta_j} t = \delta_{ij}$$

$$t = dx_i \wedge dy^i$$

closed, integral, positive, (1,1)-form

"Hodge form", "polarization"

$T(u)$ = "Abelian variety"



$$T_{ij} = B_{ik} (A^{-1})^k_j$$

$$B_{ik} = \oint_{\beta_j} \omega_k \quad A_k^j = \oint_{\alpha^k} \omega_k$$

$\{\omega_k\}$ any basis of hol. (1,0)-forms on T

$$(\alpha \tilde{z}_i)$$

$$\{\alpha^i, \beta_j\}$$

$$\int_{\alpha^i \wedge \beta_j} t = \delta_{ij}$$

positive, (1,1)-form
 "normalization"

$T(u)$



$$T_{ij} = B_{ik} (A^{-1})^k_j$$

$$B_{ik} = \int_{\beta_i} \omega_k \quad A_k^j = \int_{\alpha^k} \omega_j$$

(*) any basis of hol.
 (1,1)-forms on T

$$(\alpha \tilde{z}_i)$$

$$\{\alpha^i, \beta_j\}$$

$$\int_{\alpha^i \wedge \beta_j} \tau = \delta_{ij}$$

..., (1,1)-form

ion"

T(u)

$$\tau_{ij} = B_{ik} (A^{-1})^k_j$$

$$B_{ik} = \oint_{\beta_j} \omega_k \quad A^j_k = \oint_{\alpha^k} \omega_k$$

$\{\omega_k\}$ any basis of hol.
(1,0)-forms on T

$(\alpha \mathbb{Z}_i)$

$\{\alpha^k, \beta_j\}$

$$\int t = \int$$

$$\begin{pmatrix} \alpha \\ \beta \end{pmatrix} \rightarrow M \begin{pmatrix} \alpha \\ \beta \end{pmatrix}$$

$$\mathcal{H}_v \supset \text{Im } \tau_{i,j}(A) \partial_\mu A^i \partial^\mu \bar{A}^j$$

$$\partial_i \tau_{j,k} = 0 \quad \tau_{ij} = \tau_{ji}$$

$$\text{Im } \tau_{ij} > 0$$

Rigid Special Kahler (RSK)

$$C \begin{pmatrix} \alpha \\ \beta \end{pmatrix} \rightarrow M \begin{pmatrix} \alpha \\ \beta \end{pmatrix}$$

$$= \phi w_k$$

$$C \begin{pmatrix} \alpha \\ \beta \end{pmatrix} \rightarrow M \begin{pmatrix} \alpha \\ \beta \end{pmatrix}$$

$$C \begin{pmatrix} \alpha \\ \beta \end{pmatrix} \rightarrow M \begin{pmatrix} \alpha \\ \beta \end{pmatrix}$$

RSK geometry

$$F_{\mu\nu} = \frac{1}{2}(\tilde{F}_{\mu\nu} + i\tilde{\tilde{F}}_{\mu\nu})$$

Donagi & Witten

9510101

$$\mathcal{L}_{\text{eff}} = \text{Im} \left[\tau_{ij} (\partial A^i \cdot \partial \bar{A}^j + \tilde{F}^i \cdot \tilde{F}^j) \right]$$

$$\partial_{[i} \tau_{j]k} = 0 \quad \tau_{ij} = \tau_{ji} \quad \text{Im} \tau_{ij} > 0$$

$T(u)$ r - $\mathbb{C}$ -dim'd torus

$\{x_i, y^i\} \quad i=1 \dots r$

$$x_i = x_{i+1}$$

$$y^i = y^{i+1}$$

$$\Rightarrow \sum_i = \sum_i (h_i^1 x_i + \frac{1}{2} y_i y^i)$$

v. complex

$$\begin{pmatrix} \alpha \\ \beta \end{pmatrix} \rightarrow M \begin{pmatrix} \alpha \\ \beta \end{pmatrix}$$

$$\mathcal{K}_v \supset \operatorname{Im} \tau_{i,j}(A) \partial_\mu A^i \partial^\mu \bar{A}^j$$

$$\partial_\mu [\tau_{ij}] = 0$$

$$\Leftrightarrow \text{ex}$$

$$\partial_\mu \tau_{ij} = 0 \quad \tau_{ij} = \tau_{ji}$$

$$\operatorname{Im} \tau_{ij} > 0$$

Rigid Special Kahler (RSK)

$$\begin{pmatrix} \alpha \\ \beta \end{pmatrix} \rightarrow M \begin{pmatrix} \alpha \\ \beta \end{pmatrix}$$

$$K_v \supset \text{Im } \tau_{i,j}(A) \partial_\mu A^i \partial^\mu \bar{A}^j$$

$$\partial [\tau_{i,j}] = 0$$

$\Leftrightarrow$ existence of
closed hol. (2,0) form Ω

$$\begin{aligned} \tau_{i,j} &= 0 & \tau_{i,j} &= \tau_{j,i} \\ \text{Im } \tau_{i,j} &> 0 \end{aligned}$$

Rigid Special Kahler (RSK)

$$G \begin{pmatrix} \alpha \\ \beta \end{pmatrix} \rightarrow M \begin{pmatrix} \alpha \\ \beta \end{pmatrix}$$

$$g_v \supset \text{Im } \tau_{i,j}(A) \partial_\mu A^i \partial^\mu \bar{A}^j$$

$$\partial_{[\mu} \tau_{j]k} = 0$$

$\Leftrightarrow$ existence of closed hol. $(2,0)$ form Ω Rigid Special Kahler (RSK)

$$\Omega = dZ_i \wedge dA^i$$

$$\Re \tau_{ij} = 0 \quad \tau_{ij} = \tau_{ji}$$

$$\text{Im } \tau_{ij} > 0$$

$$t = dx_i \wedge dy^i$$

closed, integral, positive, (1,1)-form

"Hodge form", "polarization"

$\Rightarrow T(u) = \text{"Abelian variety"}$



$$T_{i,j} = B_{ik} (A^{-1})^k_j$$

$$B_{ik} = \oint_{\beta_i} \omega_k \quad A_k^j = \oint_{\alpha^k} \omega_j$$

any basis of hol
(1,0)-forms on T

$\{z_i\}$

$$\int_{\alpha_i \wedge \beta_j} t = \delta_{ij}$$

$$\omega = dx_i \wedge dy^i$$

closed, integral, positive, (1,1)-form

"Hodge form", "polarization"

$\Rightarrow T(u) = \text{"Abelian variety"}$



$$\tau_{ij} = B_{ik} (A^{-1})^k_j$$

$$B_{ik} = \oint_{\beta_j} \omega_k \quad A^j_k = \oint_{\alpha^k} \omega_j$$

$\{\omega_i\}$ basis of hol. forms on T

$$\tau_{ij} = \delta_{ij}$$

$$\begin{pmatrix} \alpha \\ \beta \end{pmatrix} \rightarrow M \begin{pmatrix} \alpha \\ \beta \end{pmatrix}$$

$$q_v \supset \text{Im } \tau_{i,j}(\lambda) \partial_\mu A^i \partial^\mu \bar{A}^j$$

$$\partial_\mu \tau_{ij} = 0$$

$\Leftrightarrow$ existence of

closed hol. (2,0) form Ω

$$\partial_\mu \tau_{ij} = 0$$

$$\tau_{ij} = \tau_{ji}$$

$$\text{Im } \tau_{ij} >$$

Rigid Spec. (RSK)

$$\Omega = dZ_i \wedge dA^i$$

$$\begin{pmatrix} \alpha \\ \beta \end{pmatrix} \mapsto M \begin{pmatrix} \alpha \\ \beta \end{pmatrix}$$

$$K_v \supset \text{Im } \tau_{i,j}(A) \partial_\mu A^i \partial^\mu \bar{A}^j$$

$$\partial_{\bar{z}} \tau_{i,j} = 0$$

$\Leftrightarrow$ existence of
closed holomorphic

$$\partial \tau_{i,j} = 0 \quad \tau_{i,j} = \tau_{j,i}$$

$$\text{Im } \tau_{i,j} > 0$$

Ω Rigid Special Kähler (RSK)

Ω

$\Omega =$

$$\left(\begin{pmatrix} \alpha \\ \beta \end{pmatrix} \right) \rightarrow M \left(\begin{pmatrix} \alpha \\ \beta \end{pmatrix} \right)$$

$$\int \partial_i \tau_j = 0$$

$\Leftrightarrow$ existence of
closed hor. form

$$\Omega = dZ_i \wedge dA^i$$

$$d\Omega = 0 \Rightarrow \Omega = d\lambda \quad \lambda|_{\tau} =$$

$$\left(\begin{pmatrix} \alpha \\ \beta \end{pmatrix} \right) \rightarrow M \left(\begin{pmatrix} \alpha \\ \beta \end{pmatrix} \right)$$

$$\int \partial_i T_{jk} = 0$$

$\Leftrightarrow$ existence of
closed hol. form

$$\Omega = dZ_i \wedge dA^i$$

$$d\Omega = 0 \Rightarrow \Omega = d\lambda \quad \lambda|_{\partial M} = 0$$

$$\begin{pmatrix} \alpha \\ \beta \end{pmatrix} \rightarrow M \begin{pmatrix} \alpha \\ \beta \end{pmatrix}$$

$$\exists \Omega \Leftrightarrow \exists \{w_i, \omega_i\}$$

$$\partial \Gamma_j \omega$$

$$\partial \Gamma_j \omega = 0$$

$\Leftrightarrow$ existence of
closed hol. $(2,0)$ form Ω

$$\Omega = dZ_i \wedge dA^i$$

$$d\Omega = 0 \Rightarrow \Omega = d\lambda \quad \lambda|_K = w$$

$$\exists \Omega \Leftrightarrow \exists \{w_{12}(u)\}$$

$$\partial [j w_k] = \dot{\phi}$$

of
 $(2,0)$ form Ω

idA^i

$$\lambda / \tau = 5 \cdot W / 10$$

$$\exists \Omega \iff \exists \{\omega_k(u)\}$$

$$\partial[j\omega_k] = \partial_T g[jk]$$

$$\exists \Omega \iff \exists \{ \omega_{12}(u) \}$$

$$\partial [j \omega_k] = \partial_T g [jk]$$

g

$$\exists \Omega \iff \exists \{ \omega_{12}(u) \}$$

$$\partial [j \omega_k] = \partial_T g[jk]$$

$$\partial [i \omega_k]$$

$$\omega_m \Omega$$

$$\exists \Omega \iff \exists \{\omega_{12}(u)\}$$

$$\partial[j\omega_k] = \dot{x}_T g[jk]$$

$$\partial[i] = 0$$

from Ω

$$\exists \Omega \iff \exists \{\omega_{12}(u)\}$$

$$\partial_{[j} \omega_{k]} = \dot{x}_\tau g_{[jk]}$$

$$\partial_{[i} \omega_{j]} = 0$$

sum Ω

$$\exists \Omega \iff \exists \{\omega_k(u)\}$$

$$\partial [j^i \omega_k] = d_T g [jk]$$

$$\partial [i] = 0$$

sum Ω

$$\exists \Omega \iff \exists \{\omega_{12}(u)\}$$

$$\partial[j\omega_k] = \partial_\tau g[jk]$$

$$\partial[i] g[jk] = 0$$

$\omega_{12} \Omega$

$$\exists \Omega \iff \exists \{ \omega_{12}(u) \}$$

$$\partial[j \omega_k] = d_T g[jk] \quad \text{merov. func}$$

$$\partial[i g_{jk}] = 0$$

from Ω

$$\exists \Omega \iff \exists \{w_k(u)\}$$

$$\partial[j w_k] = d_T g[jk] \quad \text{merov. func}$$

$$\partial[i g_{jk}] = 0$$

from Ω

$$\begin{pmatrix} \alpha \\ \beta \end{pmatrix} \rightarrow M \begin{pmatrix} \alpha \\ \beta \end{pmatrix}$$

$$\partial[\tau_{jk}] = 0$$

$\Leftrightarrow$ existence of
closed hol. (2,0) form Ω

$$\Omega = dZ^i \wedge dA^i$$

$$d\Omega = 0 \Rightarrow \Omega = d\lambda \quad \lambda|_{\tau} = S \cdot W \text{ 1-form}$$

$$\exists \Omega \Leftrightarrow \exists \{\omega_k(u)\}$$

$$\partial[j\omega_k] = d_{\tau} g[jk] \quad \text{metric}$$

$$\partial[jg_{jk}] = 0 \quad \tau=1$$

$$\begin{pmatrix} \alpha \\ \beta \end{pmatrix} \rightarrow M \begin{pmatrix} \alpha \\ \beta \end{pmatrix}$$

$$\partial[\tau_{jk}] = 0$$

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$$\Omega = dZ^i \wedge dA^i$$

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$$\exists \Omega \Leftrightarrow \exists \{\omega_{jk}(u)\}$$

$$\partial[j\omega_k] = d_T g[jk] \quad \text{metric form}$$

$$\partial[i g_{jk}] = 0 \quad r=1$$

$u: y^2 = x^3 + f(u)x + g(u)$

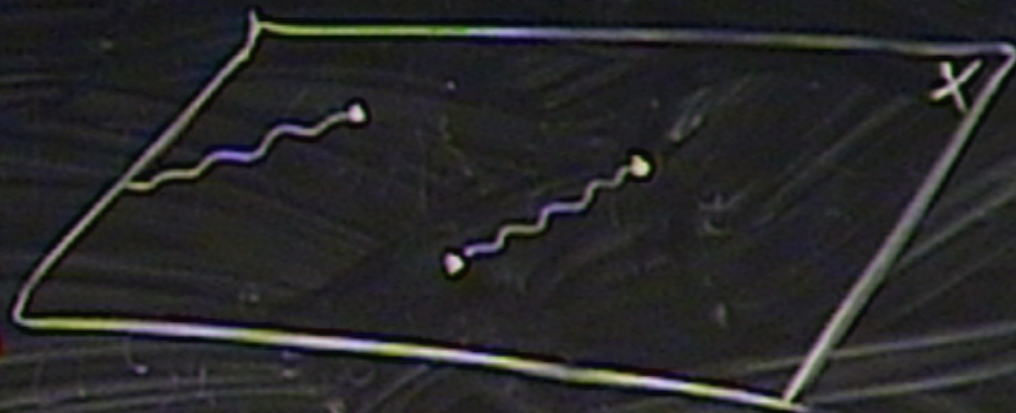
$u: y^2 = x^3 + f(u)x + g(u)$

2-ble cover
of x -plane

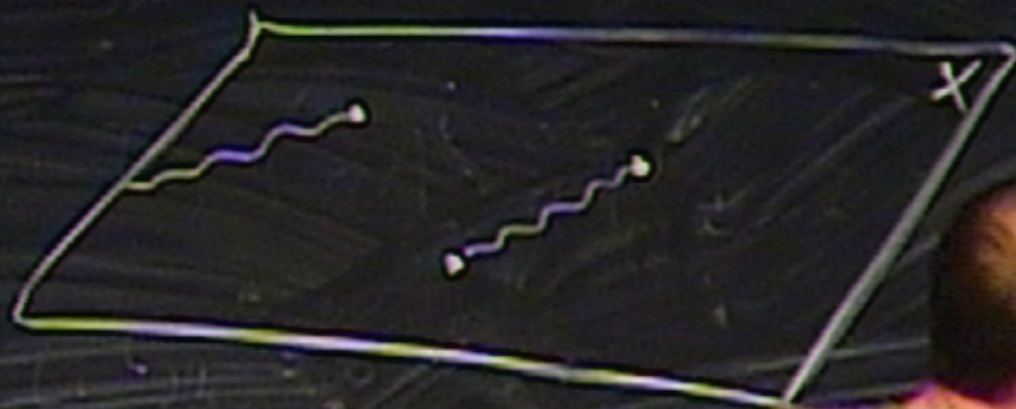


$u: y^2 = x^3 + f(u)x + g(u)$

2-ble cover
of x -plane

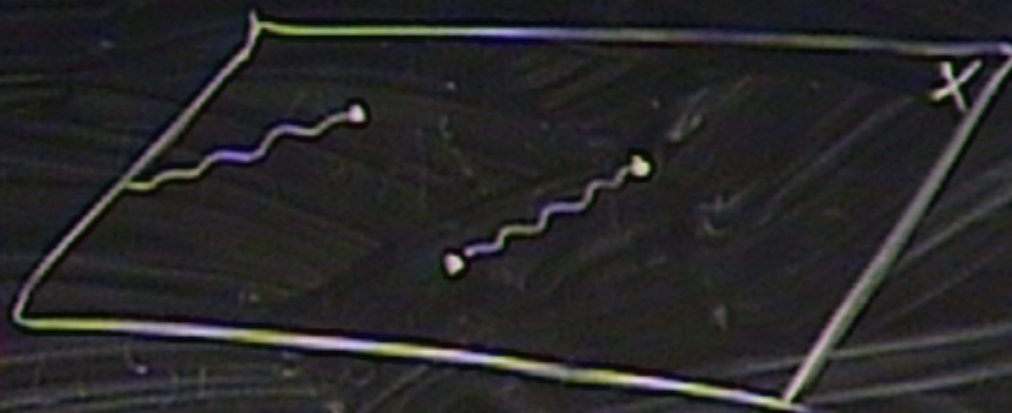


$$u: y^2 = x^3 + f(u)x + g(u)$$



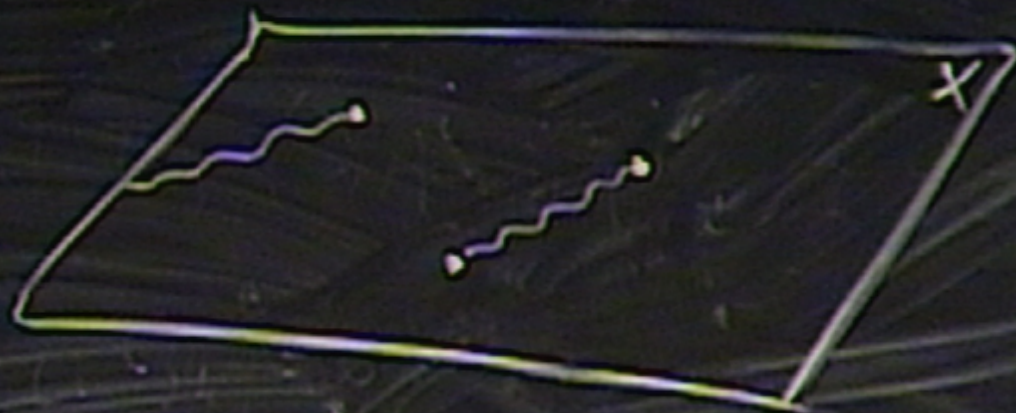
2-ble cover
of x-plane
branched over
3 points -

$$u: y^2 = x^3 + f(u)x + g(u)$$



2-ble cover
of x -plane
branched over
3 points $= \infty$

$$u: y^2 = x^3 + f(u)x + g(u)$$



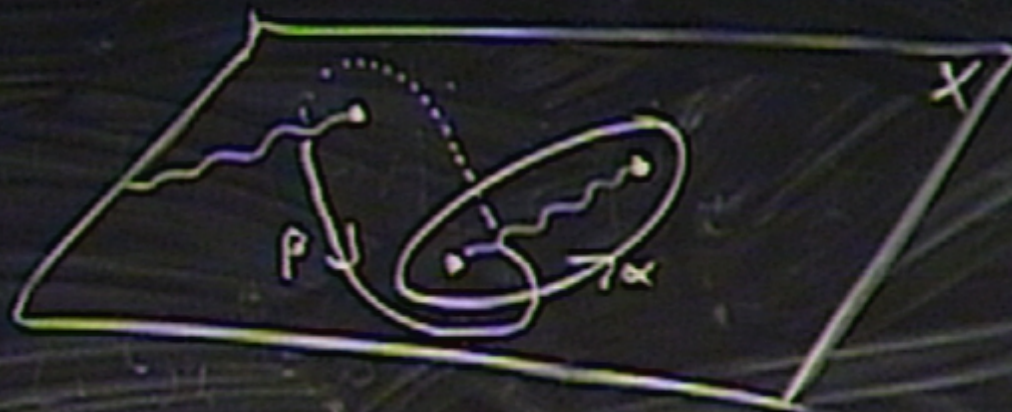
2-ble cover
of x-plane
branched over
3 points + ∞

$$u: y^2 = x^3 + f(u)x + g(u)$$

2-ble cover
of x -plane
branched over
3 points $= \infty$



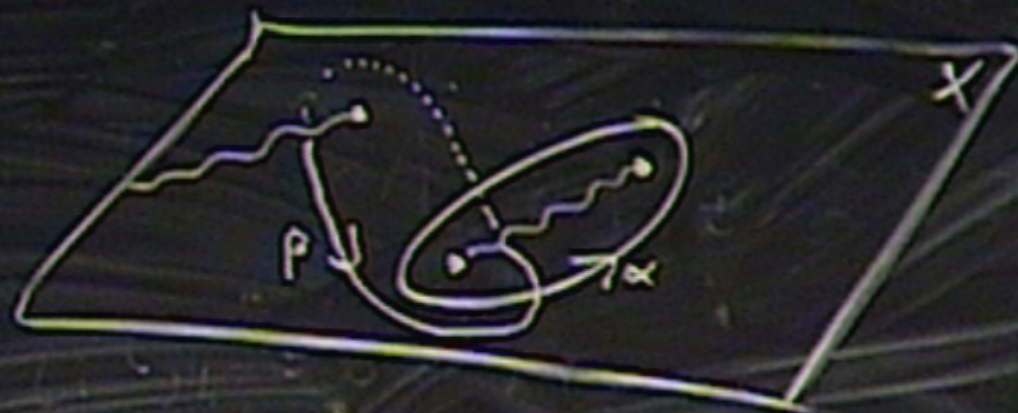
$$u: y^2 = x^3 + f(u)x + g(u)$$



2-ble cover
of x -plane
branched over
3 points $\rightarrow \infty$

$$w = h(u) \frac{dx}{y}$$

$$u: y^2 = x^3 + f(u)x + g(u)$$



2-ble cover
of x -plane
branched over
3 points $= \infty$

$$w = h(u) \frac{dx}{y}$$

Look for scale invariant sol's.

assign scaling dim'n γ

Look for scale invariant sol's.
assign scaling dim'n $[u]$ to u

Look for scale invariant sol's.

assign scaling dim'n $[u]$ to u =

as $u \rightarrow 0$

$$T(u): y^2 = x^3 + \alpha(u)x + \beta(u)$$

2-ble cover
of x -plane
branched over
3 points $= \infty$



$$\omega = \frac{dx}{y}$$

$$u: y^2 = x^3 + \alpha(u) x + \beta(u)$$

2-ble cover
of x -plane
branched over
3 points $= \infty$



$$w = \frac{dx}{\sqrt{f}}$$

$$y^2 = x^3 + \alpha u^n x + \beta u^m$$

2-ble cover
of x -plane
branched over
3 points $= \infty$



$$\omega = \frac{dx}{y}$$

$$u: y^2 = x^3 + \alpha u^n x + \beta u^m$$

2-ble cover
of x -plane
branched over
3 points $= \infty$



$$\frac{dx}{y}$$

$$u: y^2 = x^3 + \alpha u'' x + \beta u'''$$

2-ble cover
of x -plane
branched over
3 points $= \infty$



$$\omega = \frac{dx}{y}$$

$$T_u: y^2 = x^3 + \alpha(u)x + \beta(u)$$

2-ble cover
of x -plane
branched over
3 points $= \infty$



$$\frac{dx}{y}$$

$$u: y^2 = x^3 + \alpha u'' x + \beta u'''$$



2-ble cover
of x -plane
branched over
3 points $= \infty$

$$\omega = \frac{dx}{y}$$

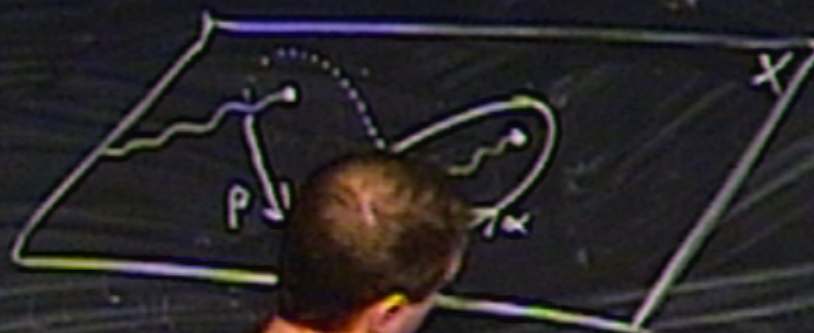
Look for scale invariant sol'ns.

assign scaling dim'n $[u]$ to u =
as $u \rightarrow 0$

$$2[y] = 3[x] \Leftarrow n[u] + [x] = m[u]$$

$$u: y^2 = x^3 + \alpha u^n x + \beta u^m$$

2-ble cover
of x -plane
branched over
3 points $= \infty$



$$\partial \bar{z} = \oint$$

$$u: \quad y^2 = x^3 + \alpha u^n x + \beta u^m$$



2-ble cover
of x-plane
branched over
3 points = ∞

$$w =$$

$$\partial_u \bar{z} = \delta$$

$$u: y^2 = x^3 + \alpha u'' x + \beta u'''$$

2-ble cover
of x -plane
branched over
3 points $= \infty$



$$w = \frac{dx}{y}$$

$$\partial_u \bar{z} = \oint w = \oint \frac{dx}{y}$$

Look for scale invariant sol'ns.

assign scaling dim'n $[u]$ to u =

as $u \rightarrow 0$

$$2[y] = 3[x] \leftarrow n[u] + [x] = m[u]$$

$$[\cancel{x}] - [u] = [x] - [y]$$

$$[u] = \frac{4}{4-n} =$$

$$\exists \Omega \Leftrightarrow \exists \{w_k(u)\}$$

$$\partial [j w_k] = d_\tau g [j k]$$

$$\partial [g_{jk}] = 0 \quad \tau=1$$

$$\left. \begin{array}{l} SO(2) \quad 1.3 \\ SU(2) \quad 4.2 \end{array} \right\}$$

$$[u] = \frac{4}{4-n} = \frac{6}{6-m} > 0$$

$$\exists \Omega \Leftrightarrow \exists \{w_k(u)\}$$

$$\partial[j w_k] = d_\tau g[jk] \quad \text{--- metric tensor}$$

$$\partial[i g_{jk}] = 0 \quad r=1$$

$$\left. \begin{array}{ll} SO(2) & 1.3 \\ SU(2) & 4.2 \end{array} \right\}$$

$$[u] = \frac{4}{4-n} = \frac{6}{6-m} > 0$$

$$[u] \mid y^2 = x^3 + \dots$$

6/5

u

$$\exists \Omega \Leftrightarrow \exists \{w_k(u)\}$$

$$\partial[j w_k] = d_T g[jk]$$

metric form

$$\partial[i g_{jk}] = 0 \quad r=1$$

$$\left. \begin{array}{ll} SO(2) & 1.3 \\ SO(2) & 4.2 \end{array} \right\}$$

1. form

$$[u] = \frac{4}{4-n} = \frac{6}{6-m} > 0$$

$$[u] \quad | \quad y^2 = x^3 + \dots$$

6/5	u
4/3	ux
3/2	u^2
2	$u^2x + u^3$

$$\exists \Omega \Leftrightarrow \exists \{w_k(u)\}$$

$$\partial [j] w_k = d_T g[jk]$$

$$\partial [i] g[jk] = 0 \quad r=1$$

$SO(2)$	1.3
$SU(2)$	4.2

$$[u] = \frac{4}{4-n} = \frac{6}{6-m} > 0$$

$$[u] \quad | \quad y^2 = x^3 + \dots$$

$$6/5$$

$$u$$

$$4/3$$

$$ux$$

$$3/2$$

$$u^2$$

$$2$$

$$u^2x + u^3$$

$$3$$

$$u^4$$

$$4$$

$$u^3x$$

$$6$$

$$u^5$$

$$\exists \Omega \Leftrightarrow \exists \{w_k(u)\}$$

$$\partial [j] w_k = d_T g [j] k$$

$$\partial [i] g [j] k = 0 \quad r=1$$

$$SU(2)$$

$$1.3$$

$$SU(2)$$

$$4.2$$

$$[u] = \frac{4}{4-n} = \frac{6}{6-m} > 0$$

$[u]$	$y^2 = x^3 + \dots$
6/5	u
4/3	ux
3/2	u^2
2	$u^2x + u^3$
3	u^4
4	u^3x
5	u^5

$$\exists \Omega \Leftrightarrow \exists \{w_k(u)\}$$

$$\partial [j] w_k = d_T g [j] k \quad \text{--- metric tensor}$$

$$\partial [i] g [j] k = 0 \quad r=1$$

$$\left. \begin{array}{l} SO(2) \quad 1.3 \\ SU(2) \quad 4.2 \end{array} \right\}$$

$$[u] = \frac{4}{4-n} = \frac{6}{6-m} > 0$$

$[u]$	$y^2 = x^3 + \dots$
6/5	u
4/3	ux
3/2	u^2
$\rightarrow 2$	$u^2x + u^3$
3	u^4
4	u^3x
5	u^5

$$\exists \Omega \Leftrightarrow \exists \{w_k(u)\}$$

$$\partial [j] w_k = d_T g_{[j]k} \quad \text{metric form}$$

$$\partial [i] g_{jk} = 0 \quad r=1$$

$$\left. \begin{array}{l} SO(2) \quad 1.3 \\ SU(2) \quad 4.2 \end{array} \right\}$$

1. form

$$[u] = \frac{4}{4-n} = \frac{6}{6-m} \geq 0$$

$[u]$	$y^2 = x^3 + \dots$
6/5	u
4/3	ux
3/2	u^2
$\rightarrow 2$	$u^2x + u^3$
3	u^4
4	u^3x
6	u^5

$$\exists \Omega \Leftrightarrow \exists \{w_k(u)\}$$

$$\partial[jw_k] = d_T g[jk] \quad \text{metric form}$$

$$\partial[i g_{jk}] = 0 \quad r=1$$

$$\left. \begin{array}{ll} SO(2) & 1.3 \\ SU(2) & 4.2 \end{array} \right\}$$

1-form

$$[u] = \frac{4}{4-n} = \frac{6}{6-m} > 0$$

$[u]$	$y^2 = x^3 + \dots$
6/5	u
4/3	ux
3/2	u^2
$\rightarrow 2$	$u^2x + u^3$
3	u^4
4	u^3x
6	u^5

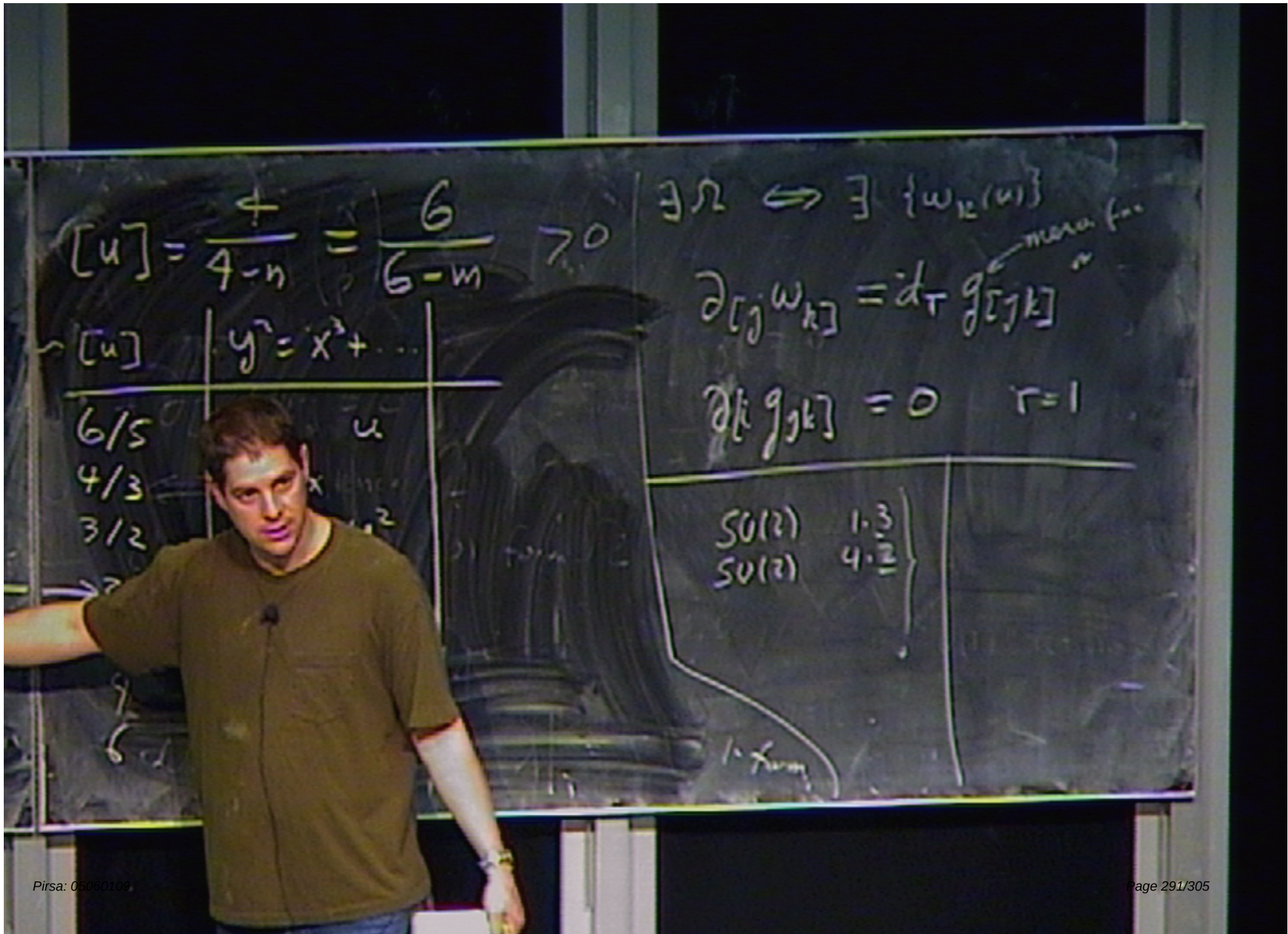
$$\exists \Omega \Leftrightarrow \exists \{w_k(u)\}$$

$$\partial[j w_k] = d_T g[jk] \quad \text{metric form}$$

$$\partial[i g_{jk}] = 0 \quad r=1$$

$$\left. \begin{array}{l} SO(2) \quad 1.3 \\ SO(2) \quad 4.2 \end{array} \right\}$$

1-form



$$[u] = \frac{4}{4-n} = \frac{6}{6-m} > 0$$

$$[u] \mid y^2 = x^3 + \dots$$

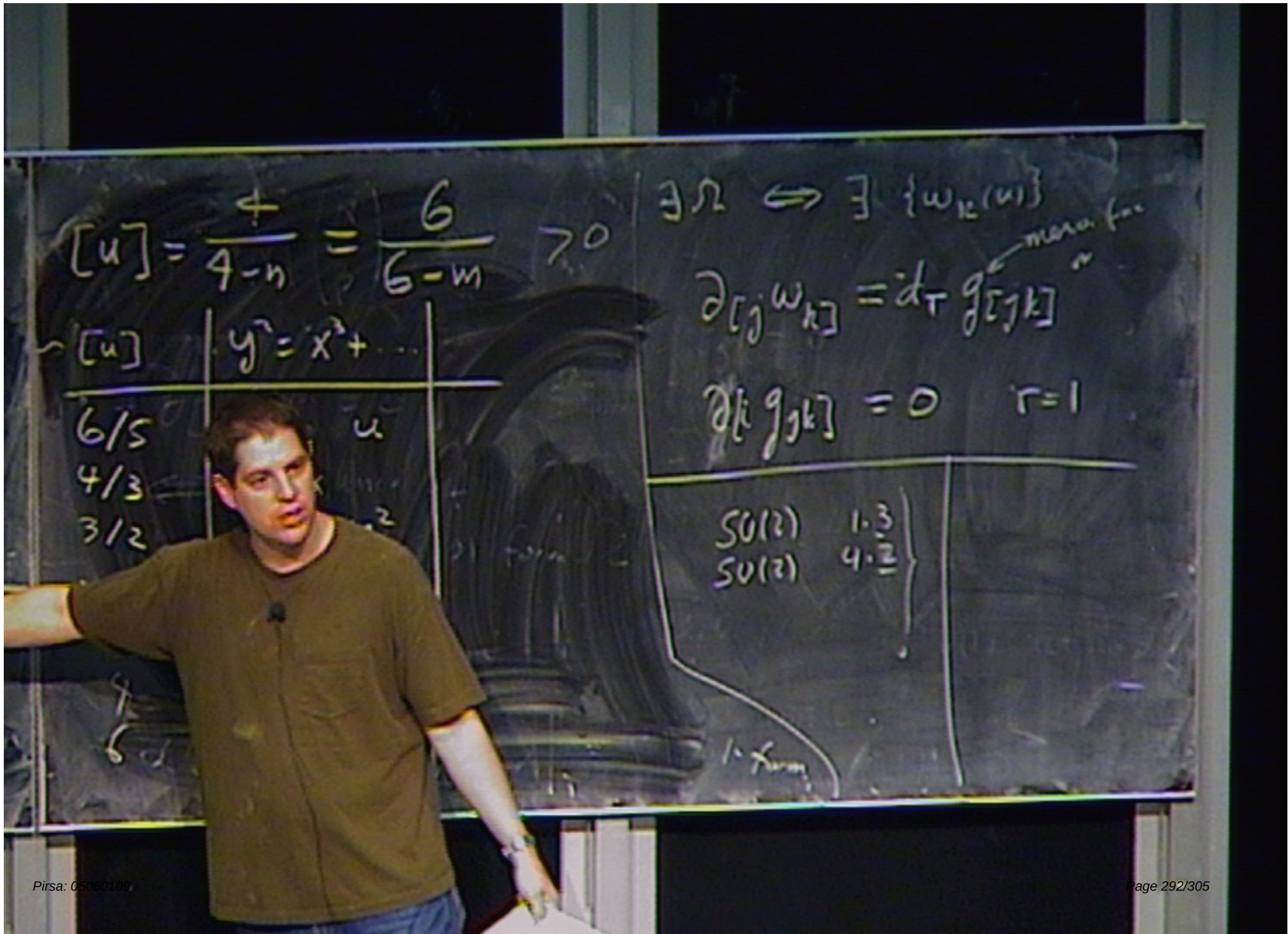
6/5	u
4/3	x
3/2	x^2

$$\exists \Omega \Leftrightarrow \exists \{w_k(u)\}$$

$$\partial [j w_k] = d_T g [j k] \quad \text{max. func.}$$

$$\partial [g_{jk}] = 0 \quad r=1$$

50(2)	1.3
50(2)	4.2



$$[u] = \frac{4}{4-n} = \frac{6}{6-m} > 0$$

$[u]$	$y^2 = x^2 + \dots$
6/5	u
4/3	x
3/2	z

$$\exists \Omega \Leftrightarrow \exists \{w_k(u)\}$$

$$\partial[j w_k] = d_T g[j k] \quad \text{man. for } n$$

$$\partial[k g_{jk}] = 0 \quad r=1$$

$SO(2)$	1.3
$SU(2)$	4.2

$$[u] = \frac{4}{4-n} = \frac{6}{6-m} > 0$$

$$[u] \quad | \quad y = x^2 + \dots$$

$$6/5$$

$$4/3$$

$$3/2$$

$$\rightarrow 2$$

$$3$$

$$4$$

$$5$$

$$6$$

$$u x$$

$$u^2 x + \dots$$

$$\exists \Omega \Leftrightarrow \exists \{w_k(u)\}$$

$$\partial [j] w_k = d_r g [j] k$$

$$\partial [i] g [j] k = 0 \quad r=1$$

$$50(2)$$

$$50(2)$$

$$1.3$$

$$4.2$$

$$[u] = \frac{4}{4-n} = \frac{6}{6-m} > 0$$

$[u]$	$y^2 = x^3 + \dots$
6/5	u
4/3	ux
3/2	u^2
$\rightarrow 2$	$u^2x + u^3$
3	u^4
4	u^3x
6	u^5

$$\exists \Omega \Leftrightarrow \exists \{w_k(u)\}$$

$$\partial[j]w_k = d_T g[jk]$$

$$\partial[k]g[jk] = 0 \quad r=1$$

$$\left. \begin{array}{l} SO(2) \quad 1.3 \\ SO(2) \quad 4.2 \end{array} \right\}$$

$$[u] = \frac{4}{4-n} = \frac{6}{6-m} > 0$$

$$[u] \mid y^2 = x^3 + \dots$$

6/5	u
4/3	ux
3/2	u^2
	u^3
	u^4

SO(8)

$$\exists \Omega \Leftrightarrow \exists \{w_k(u)\}$$

$$\partial_{[j} w_{k]} = d_T g_{[jk]} \quad \text{metric tensor}$$

$$\partial_i g_{jk} = 0 \quad r=1$$

$$\left. \begin{array}{l} SO(2) \quad 1.3 \\ SO(2) \quad 4.2 \end{array} \right\}$$

1/2 form

$$[u] = \frac{4}{4-n} = \frac{6}{6-m} > 0$$

$$\exists \Omega \Leftrightarrow \exists \{w_k(u)\}$$

$$\partial [g] w_k = d_T g [g] k \quad \text{max. func.}$$

$$\partial [g] g_k = 0 \quad r=1$$

$[u]$	$y^2 = x^3 + \dots$	
6/5	u	$U(1)$
4/3	ux	$U(2)$
3/2	u^2	$U(3)$
$\rightarrow 2$	$u^2 x + u^3$	$SO(8)$
3	u^4	E_6
4	$u^3 x$	E_7
6	u^5	E_8

$$\left. \begin{array}{l} SO(2) \\ SU(2) \end{array} \right\} \begin{array}{l} 1.3 \\ 4.2 \end{array}$$

$$[u] = \frac{4}{4-n} = \frac{6}{6-m} > 0$$

$$\exists \Omega \Leftrightarrow \exists \{w_k(u)\}$$

$$\partial [j] w_k = \partial_\tau g[jk] \quad \text{metric tensor}$$

$$\partial [i] g[jk] = 0 \quad r=1$$

$[u]$	$y^2 = x^3 + \dots$	
6/5	u	$U(1)$
4/3	ux	$U(1)$
3/2	u^2	$U(1)$
$\rightarrow 2$	$u^2 x + \dots$	$SO(8)$
3	u^4	E_6
4	$u^3 x$	E_7
6	u^5	E_8

$$\left. \begin{array}{l} SO(2) \\ SU(2) \end{array} \right\} \begin{array}{l} 1.3 \\ 4.2 \end{array}$$

$$[u] = \frac{4}{4-n} = \frac{6}{6-m} > 0$$

$[u]$	$y^2 = x^3 + \dots$	
6/5	u	$U(1)$
4/3	ux	$U(1)$
3/2	u^2	$U(1)$
$\rightarrow 2$	$u^2x + \dots$	$SO(8)$
3	u^4	E_6
4	u^3x	E_7
6	u^5	E_8

$$\exists \Omega \Leftrightarrow \exists \{ \omega_k(u) \}$$

$$\partial [g] \omega_k = d_\tau g [g] \omega_k$$

man. for

$$\partial [g] g_{jk} = 0 \quad \tau = 1$$

$$\left. \begin{matrix} (2) & 1.3 \\ & 4.2 \end{matrix} \right\}$$

$$[u] = \frac{4}{4-n} = \frac{6}{6-m} > 0$$

$[u]$	$y^2 = x^3 + \dots$
6/5	u
4/3	ux
3/2	u^2
$\rightarrow 2$	$u^2x + u^3$
3	u^4
4	u^3x
6	u^5

$$\exists \Omega \Leftrightarrow \exists \{w_k(u)\}$$

$$\partial[jw_k] = d_T g[jk] \quad \text{metric tensor}$$

$$\partial[i g_{jk}] = 0 \quad r=1$$

$U(1)$
 $U(2)$
 $U(3)$
 $SO(8)$
 E_6
 F

1.3
 4.2

$$[u] = \frac{4}{4-n} = \frac{6}{6-m} > 0$$

$[u]$	$y^2 = x^3 + \dots$	
6/5	u	$U(1)$
4/3	ux	$U(1)$
3/2	u^2	$U(1)$
$\rightarrow 2$	$u^2x + \tau u^3$	$SO(8)$
3	u^4	E_6
4	u^3x	E_7
6	u^5	E_8

$$\exists \Omega \Leftrightarrow \exists \{w_k(u)\}$$

$$\partial [g] w_k = d_\tau g [g] k \quad \text{metric tensor}$$

$$\partial [g] k = 0 \quad \tau = 1$$

$$\left. \begin{matrix} 1, 3 \\ 2, 2 \end{matrix} \right\}$$

$$[u] = \frac{4}{4-n} = \frac{6}{6-m} > 0$$

$[u]$	$y^2 = x^3 + \dots$	
6/5	u	$U(1)$
4/3	ux	$U(2)$
3/2	u^2	$U(3)$
$\rightarrow 2$	$u^2x + \dots$	$SO(8)$
3	u^4	E_6
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6	u^5	E_8

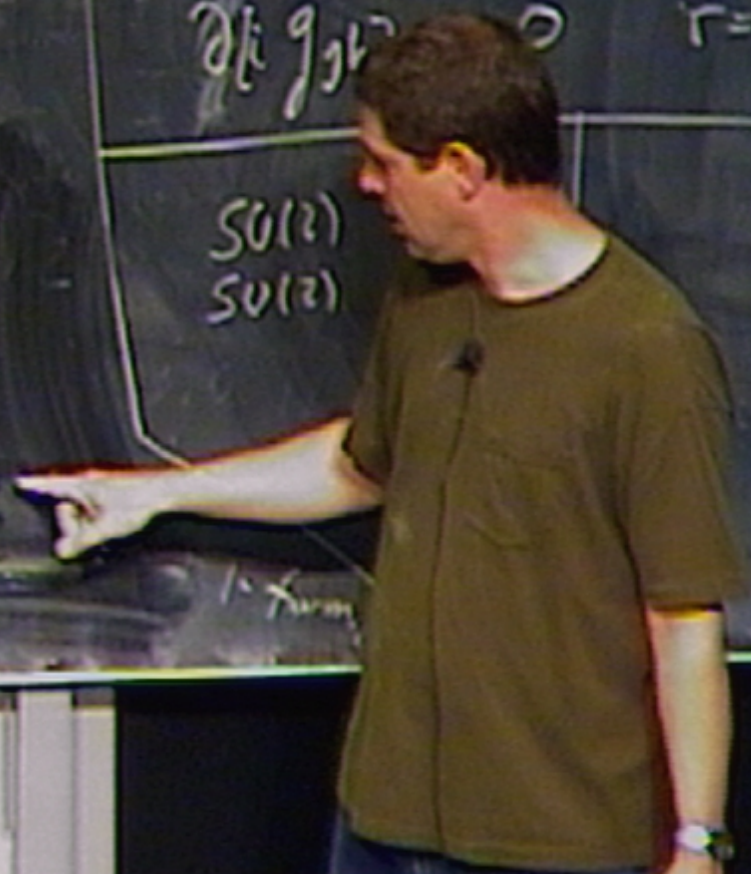
$$\exists \Omega \Leftrightarrow \exists \{w_k(u)\}$$

$$\partial_{[j]} w_k = \partial_\tau g_{[j]k} \quad \text{metric tensor}$$

$$\partial_i g_{jk} = 0 \quad r=1$$

$$SO(2)$$

$$SU(2)$$



$$[u] = \frac{4}{4-n} = \frac{6}{6-m} > 0$$

$[u]$	$y^2 = x^3 + \dots$	
6/5	u	$U(1)$
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3/2	u^2	$U(3)$
$\rightarrow 2$	$u^2x + \tau u^3$	$SO(8)$
3	u^4	E_6
4	u^3x	E_7
6	u^5	E_8

$$\exists \Omega \Leftrightarrow \exists \{ \omega_{jk}(u) \}$$

$$\partial_{[j} \omega_{k]} = d_{\tau} g_{[jk]} \quad \text{metric form}$$

$$\partial_i g_{jk} = 0 \quad r=1$$

$$SO(2)$$

$$SU(2)$$

$$[u] = \frac{4}{4-n} = \frac{6}{6-m} > 0$$

$[u]$	$y^2 = x^3 + \dots$	
6/5	u	$U(1)$
4/3	ux	$U(1)$
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$\rightarrow 2$	$u^2x + \dots$	$SO(8)$
3	u^4	E_6
4	u^3x	E_7
6	u^5	E_8

$$\exists \Omega \Leftrightarrow \exists \{w_k(u)\}$$

$$\partial_{[j]} w_k = d_\tau g_{[j]k}^{\leftarrow \text{metric form}}$$

$$\partial_i g = 0 \quad r=1$$

$SO(2)$
 $SU(1)$

$$[u] = \frac{4}{4-n} = \frac{6}{6-m} > 0$$

$[u]$	$y^2 = x^3 + \dots$	
6/5	u	$U(1)$
4/3	ux	$U(2)$
3/2	u^2	$U(3)$
$\rightarrow 2$	$u^2x + u^3$	$SO(8)$
3	u^4	E_6
4	u^3x	E_7
6	u^5	E_8

$$\exists \Omega \Leftrightarrow \exists \{w_k(u)\}$$

$$\partial[j]w_k = d_T g[jk] \quad \text{metric form}$$

$$\partial[i]g[jk] = 0 \quad r=1$$

$$\left. \begin{array}{l} SO(2) \quad 1.3 \\ SU(2) \quad 4.2 \end{array} \right\}$$

$$[u] = \frac{4}{4-n} = \frac{6}{6-m} > 0$$

$[u]$	$y^2 = x^3 + \dots$	
6/5	u	$U(1)$
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$\rightarrow 2$	$u^2x + \dots$	$SO(8)$
3	u^4	E_6
4	u^3x	E_7
6	u^5	E_8

$$\exists \Omega \Leftrightarrow \exists \{w_k(u)\}$$

$$\partial [g] w_k = d_T g_{[jk]}^{\text{metric form}}$$

$$\partial [g_{jk}] = 0 \quad r=1$$

$$\left. \begin{array}{l} SO(2) \quad 1.3 \\ SU(2) \quad 4.2 \end{array} \right\}$$

1. form