Title: Supergravity

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Abstract:

Exact solutions in supergravity

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Lecture 1: Introduction; Supergravities in ten and eleven dimensions
Lecture 2: Supersymmetric vacua, holonomy and Killing spinors
Lecture 3: BPS branes and backgrounds with fluxes

Supersymmetry

- Supersymmetry relates bosons with fermions

$$\text{Fermion} \quad \longleftrightarrow \quad \text{Boson}$$

or

$$\delta(\text{Fermion}) = \partial(\text{Boson})\epsilon, \qquad \delta(\text{Boson}) = \overline{\epsilon}(\text{Fermion})$$

where $\epsilon =$ spinor parameter of transformation

- For example, in the Wess-Zumino model (on-shell formulation)

$$\mathcal{L} = -\tfrac{1}{2}\partial A^2 - \tfrac{1}{2}\partial B^2 - \tfrac{1}{2}i\overline{\chi}\gamma^\mu\partial_\mu\chi$$

(two real scalars A, B and one Majorana fermion χ)

$$\delta A = \tfrac{1}{2}i\overline{\epsilon}\chi, \qquad \delta B = \tfrac{1}{2}\overline{\epsilon}\gamma_5\chi, \qquad \delta\chi = \tfrac{1}{2}\gamma^\mu\partial_\mu(A - i\gamma_5 B)\epsilon$$

Invariance of the action

- For the Wess-Zumino model, we can check

$$\begin{aligned}\delta\mathcal{L} = \delta\mathcal{L}_B + \delta\mathcal{L}_F &= -\partial^\mu A \partial_\mu \delta A - \partial^\mu B \partial_\mu \delta B - i\overline{\chi}\gamma^\mu\partial_\mu\delta\chi \\ &= -\tfrac{1}{2}i\overline{\epsilon}\partial^\mu(A + i\gamma_5 B)\partial_\mu\chi - \tfrac{1}{2}i\overline{\chi}\gamma^\mu\partial_\mu\gamma^\nu\partial_\nu(A - i\gamma_5 B)\epsilon \\ &= \tfrac{1}{2}i\overline{\epsilon}\,\Box(A + i\gamma_5 B)\chi - \tfrac{1}{2}i\overline{\chi}\,\Box(A - i\gamma_5 B)\epsilon \\ &= 0\end{aligned}$$

(up to integration by parts)

- In addition the commutator of two transformations yields

$$\begin{aligned}[\delta_1, \delta_2]A &= \tfrac{1}{2}\xi^\mu\partial_\mu A \\ [\delta_1, \delta_2]B &= \tfrac{1}{2}\xi^\mu\partial_\mu B \\ [\delta_1, \delta_2]\chi &= \tfrac{1}{2}\xi^\mu\partial_\mu\chi - \tfrac{1}{4}\gamma^\mu\xi_\mu\gamma^\mu\partial_\mu\chi \qquad \text{where} \quad \xi^\mu = i\overline{\epsilon}_2\gamma^\mu\epsilon_1\end{aligned}$$

The supersymmetry algebra

- An extension of the Poincaré algebra

In $D = 4$:

$M_{\mu\nu}$ = Lorentz generators

P_μ = Translations

Q^i_α = "supertranslations" where $i = 1, \ldots, \mathcal{N}$

with

$$
\begin{aligned}
[M_{\mu\nu}, M_{\rho\sigma}] &= i(\eta_{\mu\rho}M_{\nu\sigma} - \eta_{\mu\sigma}M_{\nu\rho} + \eta_{\nu\sigma}M_{\mu\rho} - \eta_{\nu\rho}M_{\mu\sigma}) \\
[M_{\mu\nu}, P_\rho] &= i(\eta_{\mu\rho}P_\nu - \eta_{\nu\rho}P_\mu) \\
[M_{\mu\nu}, Q^i_\alpha] &= \tfrac{1}{2}i(\gamma_{\mu\nu})_{\alpha\beta}Q^{\beta\, i} \\
\{Q^i_\alpha, Q^j_\beta\} &= 2(\gamma^\mu C)_{\alpha\beta}\delta^{ij}P_\mu + C_{\alpha\beta}X^{[ij]} + (\gamma_5 C)_{\alpha\beta}Y^{[ij]}
\end{aligned}
$$

The central charge $Z = X + iY$ commutes with all generators and plays an important role in extended supersymmetry

Extended supersymmetry

- For $D = 4$ there is an upper limit on $\mathcal{N}$

  Consider massless representations of supersymmetry (given by helicities)

  $\mathcal{N} = 1$: $(0, \frac{1}{2})$ Wess-Zumino; $(\frac{1}{2}, 1)$ vector; $(\frac{3}{2}, 2)$ graviton

  $\mathcal{N} = 2$: $(-\frac{1}{2}, 0, \frac{1}{2})$ hypermatter; $(0, \frac{1}{2}, 1)$ vector; $(1, \frac{3}{2}, 2)$ graviton

  $\mathcal{N} = 4$: $(-1, -\frac{1}{2}, 0, \frac{1}{2}, 1)$ vector; $(0, \frac{1}{2}, 1, \frac{3}{2}, 2)$ graviton

  $\mathcal{N} = 8$: $(-2, -\frac{3}{2}, -1, -\frac{1}{2}, 0, \frac{1}{2}, 1, \frac{3}{2}, 2)$ graviton

- Spins less than or equal to 2 give a restriction

$$\mathcal{N} \leq 8 \quad \text{in} \quad D = 4$$

  and $N = 8$ must include gravity

Local supersymmetry is supergravity

- In fact, gravity shows up naturally once we impose local supersymmetry

$$\epsilon \longrightarrow \epsilon(x)$$

Consider the action of two supersymmetries

$$[\delta_1, \delta_2](\text{field}) = [\overline{\epsilon}_1 Q, \overline{\epsilon}_2 Q](\text{field}) = \xi^\mu(x)\partial_\mu(\text{field}) + \cdots$$

where $\xi^\mu(x) = i\overline{\epsilon}_2\gamma^\mu\epsilon_1$

- This is a general coordinate transformation, and the gravitino $\psi_\mu(x)$ acts as a spin-$\frac{3}{2}$ gauge field

Simple supergravity in $D = 4$

- Constructed by Freedman, van Nieuwenhuizen and Ferrara and Deser and Zumino in 1976
- To lowest order in fermions

$$e^{-1}\mathcal{L} = R + \tfrac{1}{2}i\overline{\psi}_\mu \gamma^{\mu\nu\sigma}\nabla_\nu \psi_\sigma$$

with $\quad \delta\psi_\mu = \nabla_\mu \epsilon \qquad \delta e_\mu{}^\alpha = \tfrac{1}{4}i\bar{\epsilon}\gamma^\alpha\psi_\mu$

- We can check

$$\begin{aligned} e^{-1}\delta\mathcal{L} &= (R^{\mu\nu} - \tfrac{1}{2}g^{\mu\nu}R)\delta g_{\mu\nu} + i\overline{\psi}_\mu\gamma^{\mu\nu\sigma}\nabla_\nu\delta\psi_\sigma \\ &= \tfrac{1}{2}i(R^{\mu\nu} - \tfrac{1}{2}g^{\mu\nu}R)\bar{\epsilon}\gamma_\mu\psi_\nu + i\overline{\psi}_\mu\gamma^{\mu\nu\sigma}\nabla_\nu\nabla_\sigma\epsilon \\ &= \tfrac{1}{2}i(R^{\mu\nu} - \tfrac{1}{2}g^{\mu\nu}R)\bar{\epsilon}\gamma_\mu\psi_\nu + \tfrac{1}{8}i\overline{\psi}_\mu\gamma^{\mu\nu\sigma}R_{\nu\sigma\lambda\eta}\gamma^{\lambda\eta}\epsilon \\ &= \tfrac{1}{2}i(R^{\mu\nu} - \tfrac{1}{2}g^{\mu\nu}R)\bar{\epsilon}\gamma_\mu\psi_\nu + \tfrac{1}{2}i(R^{\mu\nu} - \tfrac{1}{2}g^{\mu\nu}R)\overline{\psi}_\mu\gamma_\nu\epsilon = 0 \end{aligned}$$

A general coordinate transformation

- In addition, we verify explicitly that the commutator of two supersymmetries yields a general coordinate transformation

$$[\delta_1, \delta_2] e_\mu{}^\alpha = \frac{1}{4} i (\overline{\epsilon}_2 \gamma^\alpha \nabla_\mu \epsilon_1 - \overline{\epsilon}_1 \gamma^a \nabla_\mu \epsilon_2) = \nabla_\mu \xi^a$$

where $\xi^a = \frac{1}{4} i \overline{\epsilon}_2 \gamma^a \epsilon_1$

- This implies that

$$[\delta_1, \delta_2] g_{\mu\nu} = 2 \nabla_{(\mu} \xi_{\nu)} = \nabla_\mu \xi_\nu + \nabla_\nu \xi_\mu$$

which is the expected result

Closure of the algebra on fermions is more involved, as it involves (fermi)^2 terms in $\delta \psi_\mu$

An upper limit on spacetime dimension

- In four dimensions, we found the restriction $\mathcal{N} \leq 8$

  We can also consider the restriction on spacetime dimension D

- Need to match fermions and bosons in D dimensions

Bosons		Fermions	
scalar	1	Dirac spinor	$\frac{1}{2} \times 2^{\lfloor D/2 \rfloor}$ complex
vector	$D-2$	Dirac gravitino	$\frac{1}{2}(D-3) \times 2^{\lfloor D/2 \rfloor}$ complex
n-form	$\binom{n}{D-2}$		
graviton	$\frac{1}{2}(D-1)(D-2)-1$		

Eventually the fermion degrees of freedom grow too large

For spins less than or equal to 2, the maximum is $D = 11$, $\mathcal{N} = 1$

Eleven dimensional supergravity

- The maximum dimension for (conventional) supergravity is eleven

  A relatively simple theory with field content

g_{MN}	44	metric
A_{MNP}	84	3-form potential
ψ_M	128	gravitino

- This can be related to the $D = 4$, $\mathcal{N} = 8$ theory by dimensional reduction

  Kaluza-Klein reduction on T^7

  In fact, the arguments for maximum $\mathcal{N}$ and maximum D are closely related

$D = 11$ supergravity: the setup

- Eleven dimensional supergravity was constructed in 1976 by Cremmer, Julia and Scherk

For fermions coupled to gravity, introduce a vielbein and spin-connection

$$M, N, P, \ldots = \text{curved indices}, \qquad A, B, C, \ldots = \text{flat indices}$$

Work in signature $(- + \cdots +)$ with

$$\{\Gamma_A, \Gamma_B\} = 2\eta_{AB}, \qquad \overline{\psi}_M = \psi_M^T C^{-1}, \qquad C^{-1}\Gamma_A C = -\Gamma_A^T$$

The spin-connection is given by $\omega_M{}^{AB} = \omega_M{}^{AB}(e) + K_M{}^{AB}$ with contorsion

$$K_M{}^{AB} = -\tfrac{1}{16}(\overline{\psi}_N \Gamma_M{}^{ABNP}\psi_P + 4\overline{\psi}_M \Gamma^{[A}\psi^{B]} + 2\overline{\psi}^A \Gamma_M \psi^B)$$

$D = 11$ supergravity: the Lagrangian

- The supergravity Lagrangian is given by

$$
\begin{aligned}
e^{-1}\mathcal{L}_{11} = {}& R(\omega) - \tfrac{1}{2\cdot 4!}F^2_{MNPQ} + \tfrac{1}{6}\big(\tfrac{1}{4!4!3!}\varepsilon^{MNPQRSTUVWX}F_{MNPQ}F_{RSTU}A_{VWX}\big) \\
& -\tfrac{1}{2}i\overline{\psi}_M\Gamma^{MNP}\nabla_N(\tfrac{1}{2}(\omega+\hat{\omega}))\psi_P \\
& -\tfrac{1}{8\cdot 4!}i(\overline{\psi}_M\Gamma^{MNPQRS}\psi_N + 12\overline{\psi}^P\Gamma^{QR}\psi^S)(\tfrac{1}{2}(F+\hat{F}))_{PQRS}
\end{aligned}
$$

where the supercovariant quantities are

$$
\begin{aligned}
\hat{\omega}_M{}^{AB} &= \omega_M{}^{AB} + \tfrac{1}{16}\overline{\psi}_N\Gamma_M{}^{ABNP}\psi_P \\
\hat{F}_{MNPQ} &= F_{MNPQ} + \tfrac{3}{2}i\overline{\psi}_{[M}\Gamma_{NP}\psi_{Q]}
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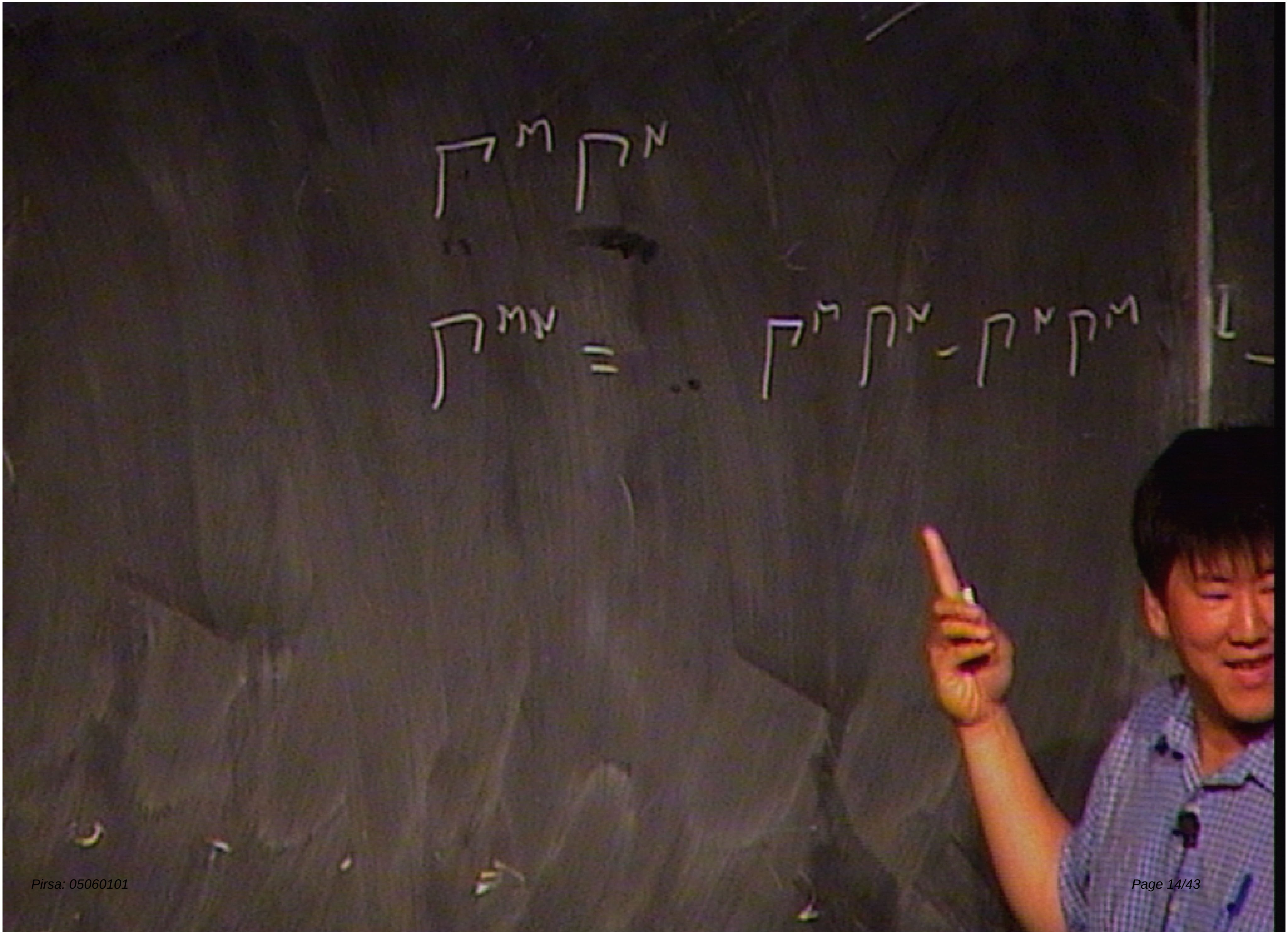
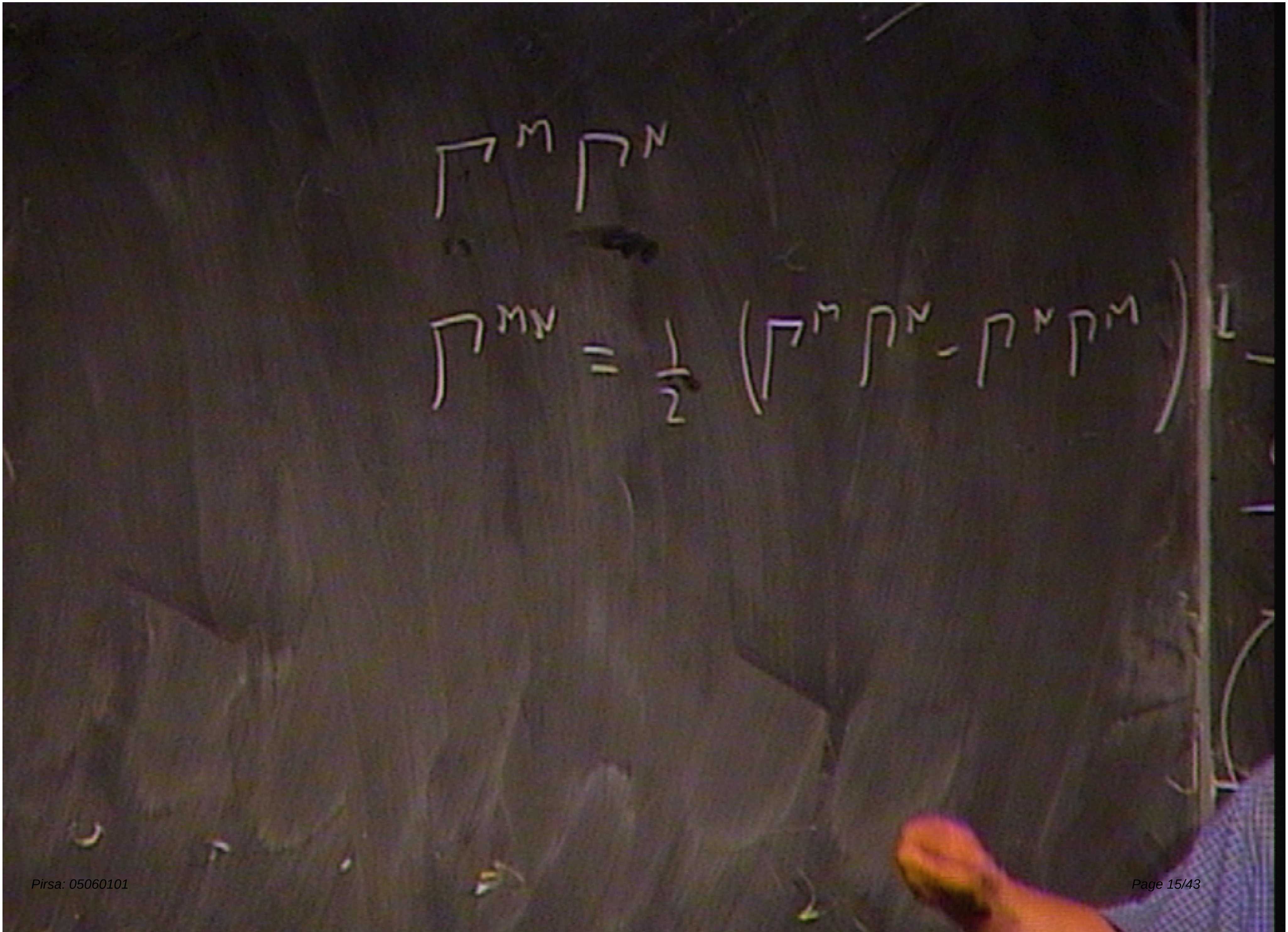
$$

- The bosonic Lagrangian is simple (in a form notation)

$$
\mathcal{L}_{11} = R*1 - \tfrac{1}{2}F_{(4)}\wedge *F_{(4)} + \tfrac{1}{6}F_{(4)}\wedge F_{(4)}\wedge A_{(3)}
$$




$\Gamma^\mu \Gamma^\nu$
$\Gamma^{\mu\nu} = \frac{1}{2}(\Gamma^\mu\Gamma^\nu - \Gamma^\nu\Gamma^\mu)$

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$D = 11$ supergravity: equations of motion

- The above Lagrangian gives rise to the equations of motion

$$R_{MN}(\hat{\omega}) - \frac{1}{2} g_{MN} R(\hat{\omega}) = \frac{1}{12}(\hat{F}_{MPQR}\hat{F}_N{}^{PQR} - \frac{1}{8} g_{MN}\hat{F}^2)$$

$$\Gamma^{MNP}\hat{\nabla}_N(\hat{\omega})\psi_P = 0$$

$$\nabla_M(\hat{\omega})\hat{F}^{MNPQ} + \frac{1}{2}(\frac{1}{4!4!}\varepsilon^{NPQRSTUVWXY}\hat{F}_{RSTU}\hat{F}_{VWXY}) = 0$$

where the supercovariant derivative is

$$\hat{\nabla}_M = \nabla_M + \frac{1}{288}(\Gamma_M{}^{NPQR} - 8\delta_M^N\Gamma^{PQR})F_{NPQR}$$

- Often we are only concerned with the bosonic equations

$$R_{MN} = \frac{1}{12}(F_{MPQR}F_N{}^{PQR} - \frac{1}{12} g_{MN} F^2)$$

$$dF_{(4)} + \frac{1}{2}F_{(4)} \wedge F_{(4)} = 0$$

$D = 11$ supergravity: transformation laws

- The above system has a large set of symmetries
  - general covariance
  - $SO(1,10)$ local Lorentz symmetry
  - abelian 3-form gauge symmetry $\delta A_{(2)} = d\Lambda_{(2)}$
  - $N = 1$ local supersymmetry

$$\begin{aligned} \delta e_M{}^A &= \tfrac{1}{4} i \bar{\epsilon} \Gamma^A \psi_M \\ \delta A_{MNP} &= -\tfrac{3}{4} i \bar{\epsilon} \Gamma_{[MN} \psi_{P]} \\ \delta \psi_M &= \hat{\nabla}_M(\hat{\omega}) \epsilon \end{aligned}$$

- Closure of supersymmetry is expressed as

$$[\delta_1, \delta_2] = \delta_{\mathbf{g.c.}} + \delta_{\mathbf{l.l.}} + \delta_{\mathbf{gauge}} + \delta_{\mathrm{susy}} + \text{e.o.m.}$$

Connection to string theory

- We will return to eleven dimensional supergravity

  But note that supergravity by itself cannot be the ultimate theory

  UV digergences of quantum gravity not really cured by supersymmetry

- May be viewed as the low energy limit of superstring theory (M-theory)

  Useful as an effective theory below the Planck scale

- String theory gives rise to a tower of massive states

$$\tfrac{1}{4}\alpha' M^2 = 0, 1, 2, \ldots \qquad \text{(closed string)}$$

by integrating out the massive fields, we are left with a theory of

- massless fields
- spin-2 graviton
- supersymmetry

This must be supergravity

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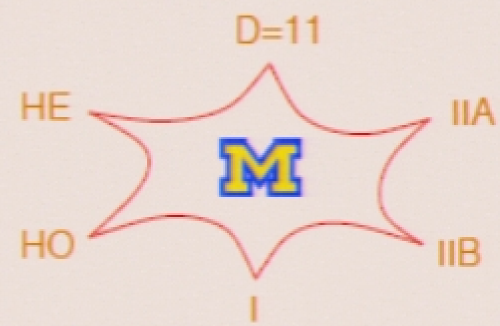
Ten dimensional supersymmetry

- There are five consistent supersymmetric string theories and three supergravities in $D = 10$

SUSY	String	Supergravity
32	Type IIA	$\mathcal{N} = 2A$ (non-chiral)
	Type IIB	$\mathcal{N} = 2B$ (chiral)
16	$E_8 \times E_8$ Heterotic	$\mathcal{N} = 1$
	$SO(32)$ Heterotic	$\mathcal{N} = 1$
	$SO(32)$ Type I	$\mathcal{N} = 1$

- In fact, these $D = 10$ string theories can be related to each other and to $D = 11$ supergravity through dualities



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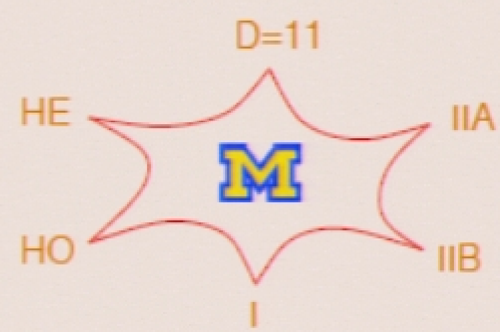

Massless string states

- In the RNS formulation, spacetime supersymmetry is obtained form worldsheet supersymmetry and the GSO projection
- The open string tower of states

$\frac{1}{4}\alpha' M^2$	Bosonic	Fermionic
$\vdots$	$\vdots$	$\vdots$
1	$\psi^\mu_{-1/2}\psi^\nu_{-1/2}\psi^\rho_{-1/2}\lvert NS\rangle_+,\quad \psi^\mu_{-3/2}\lvert NS\rangle_+$ $\psi^\mu_{-1/2}\alpha^\nu_{-1}\lvert NS\rangle_+$	$\alpha^\mu_{-1}\lvert R\rangle_+,\quad \alpha^\mu_{-1}\lvert R\rangle_-$ $\psi^\mu_{-1}\lvert R\rangle_-,\quad \psi^\mu_{-1}\lvert R\rangle_+$
$\frac{1}{2}$	$\psi^\mu_{-1/2}\psi^\nu_{-1/2}\lvert NS\rangle_+,\quad \alpha^\mu_{-1}\lvert NS\rangle_-$	
0	$\psi^\mu_{-1/2}\lvert NS\rangle_+$	$\lvert R\rangle_+,\quad \lvert R\rangle_-$
$-\frac{1}{2}$	$\lvert NS\rangle_-$	

- After GSO, the massless states are $\psi^\mu_{-1/2}\lvert NS\rangle_+$ and $\lvert R\rangle_+$

$8_v + 8_s \quad \rightarrow \quad$ Physical states of $D = 10$, $\mathcal{N} = 1$ super-Yang-Mills

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$\frac{1}{2}$	$\psi^\mu{}_{-1/2}\psi^\nu{}_{-1/2}\|NS\rangle_+, \quad \alpha^\mu{}_{-1}\|NS\rangle_-$	
0	$\psi^\mu{}_{-1/2}\|NS\rangle_+$	$\|R\rangle_+, \quad \|R\rangle_-$
$-\frac{1}{2}$	$\|NS\rangle_-$	

- After GSO, the massless states are $\psi^\mu{}_{-1/2}|NS\rangle_+$ and $|R\rangle_+$

$8_v + 8_s \quad \rightarrow$ Physical states of $D = 10$, $\mathcal{N} = 1$ super-Yang-Mills

Massless string states

- For the closed string, we take two copies (left and right movers)

  Type IIA (non-chiral combination)

$$(8_v + 8_s)_L \otimes (8_v + 8_c)_R = (1 + 28 + 35)_{NSNS} + (8_v + 56_t)_{RR}$$
$$+(8_c + 56_s + 8_s + 56_c)_{RNS+NSR}$$

  Bosonic states: $(g_{\mu\nu}, B_{\mu\nu}, \phi)_{NSNS} + (F_{(2)}, F_{(4)})_{RR}$

  Type IIB (chiral)

$$(8_v + 8_s)_L \otimes (8_v + 8 + s)_R = (1 + 28 + 35)_{NSNS} + (1 + 28 + 35_t)_{RR}$$
$$+(8_c + 56_s + 8_c + 56_s)_{RNS+NSR}$$

  Bosonic states: $(g_{\mu\nu}, B_{\mu\nu}, \phi)_{NSNS} + (F_{(1)}, F_{(3)}, F^{+}_{(5)})_{RR}$

- The $NSNS$ sector is identical (also for the Heterotic string)

Type IIA supergravity

- The IIA theory may be obtained by reduction from eleven dimensions
- The Kaluza-Klein idea

  Split your spacetime

$$\mathcal{M}_D = \mathcal{M}_d \times T^{D-d}, \qquad x^M = (x^\mu, y^m)$$

  Assume fields are independent of the internal coordinates

$$\Phi(x^M) = \varphi(x^\mu)$$

- This may be generalized to non-trivial internal manifolds as well

$$T^{D-d} \to X^{D-d} \quad \text{and} \quad \Phi(x^M) = \varphi_i(x^\mu) f^i(y^m)$$

  where $f^i(y^m)$ are harmonics on X

Type IIA supergravity: reduction from $D = 11$

- Consider the bosonic sector of $D = 11$ supergravity

$$\begin{aligned} g_{MN} &\rightarrow g_{\mu\nu}, g_{\mu\,11}, g_{11\,11} \\ A_{MNP} &\rightarrow A_{\mu\nu\rho}, A_{\mu\nu\,11} \end{aligned}$$

These are the fields of IIA supergravity

- For the actual reduction, write

$$\begin{aligned} ds^2_{11} &= e^{-\frac{2}{3}\phi} ds^2_{10} + e^{\frac{4}{3}\phi}(dy + C_{(1)})^2 \\ A_{(3)} &= C_{(3)} + B_{(2)} \wedge dy \end{aligned}$$

This yields the ten-dimensional fields

$$(g_{\mu\nu}, B_{(2)}, \phi) + (C_{(1)}, C_{(3)})$$

Type IIA supergravity: the bosonic Lagrangian

- We insert the above reduction ansatz into the $D = 11$ Lagrangian

$$\mathcal{L}_{11} = R * 1 - \tfrac{1}{2} F_{(4)} \wedge * F_{(4)} + \tfrac{1}{6} F_{(4)} \wedge F_{(4)} \wedge A_{(3)}$$

to obtain

$$\begin{aligned} \mathcal{L}_{IIA} \;=\; & e^{-2\phi}[R * 1 + 4 d\phi \wedge * d\phi - \tfrac{1}{2} H_{(3)} \wedge * H_{(3)}] \\ & - \tfrac{1}{2} F_{(2)} \wedge * F_{(2)} - \tfrac{1}{2} \widetilde{F}_{(4)} \wedge * \widetilde{F}_{(4)} \\ & - \tfrac{1}{2} F_{(4)} \wedge F_{(4)} \wedge B_{(2)} \end{aligned}$$

where $H_{(3)} = dB_{(2)}$, $F_{(2)} = dC_{(1)}$, $F_{(4)} = dC_{(3)}$ and

$$\widetilde{F}_{(4)} = F_{(4)} - C_{(1)} \wedge H_{(3)}$$

$D = 10$, $\mathcal{N} = 1$ supergravity

- Type IIA supergravity admits a further truncation to $\mathcal{N} = 1$

  We obtain the gravity sector of the Heterotic string by removing the RR sector $(C_{(1)}, C_{(3)}) \to 0$

$$\mathcal{L}_{\mathcal{N}=1} = e^{-2\phi}[R * 1 + 4d\phi \wedge *d\phi - \tfrac{1}{2}H_{(3)} \wedge *H_{(3)}]$$

- This theory is anomalous, but can be cured by adding an $E_8 \times E_8$ or $SO(32)$ gauge sector

  The result is the effective Lagrangian for the Heterotic string

$$\mathcal{L}_{het} = e^{-2\phi}[R * 1 + 4d\phi \wedge *d\phi - \tfrac{1}{2}\widetilde{H}_{(3)} \wedge *\widetilde{H}_{(3)} + \alpha' \mathrm{Tr}_v F_{(2)} \wedge *F_{(2)}]$$

where $\widetilde{H}_{(3)} = dB_{(2)} - \frac{1}{4}\alpha' \omega^{YM}_{(3)}$ with $\omega^{YM}_{(3)} = \mathrm{Tr}_v(A_{(1)}dA_{(1)} + \frac{2}{3}A^3_{(1)})$

Type IIB supergravity

- The final possibility in ten dimensions ($\mathcal{N} = 2B$) cannot be obtained from dimensional reduction
- This theory has a self-dual field strength $F_{(5)} = *F_{(5)}$, and hence does not admit a covariant action formulation

  Self-dual fields also show up in six-dimensional $\mathcal{N} = (1,0)$ and $\mathcal{N} = (2,0)$ supergravity

- However, we can write down an action that gives rise to all equations of motion except the self-dual condition (which must subsequently be imposed by hand)
- Recall that we have the fields

$$(g_{\mu\nu}, B_{(2)}, \phi) + (C_{(0)}, C_{(2)}, C_{(4)})$$

Type IIB supergravity: the bosonic Lagrangian

- The IIB Lagrangian has the form

$$\begin{aligned}\mathcal{L}_{IIB} &= e^{-2\phi}[R*1 + 4d\phi\wedge *d\phi - \tfrac{1}{2}H_{(3)}\wedge *H_{(3)}] \\ &\quad -\tfrac{1}{2}F_{(1)}\wedge *F_{(1)} - \tfrac{1}{2}\widetilde{F}_{(3)}\wedge *\widetilde{F}_{(3)} - \tfrac{1}{4}\widetilde{F}_{(5)}\wedge *\widetilde{F}_{(5)} \\ &\quad -\tfrac{1}{2}C_{(4)}\wedge H_{(3)}\wedge F_{(3)}\end{aligned}$$

where $H_{(3)} = dB_{(2)}$, $F_{(1)} = dC_{(0)}$, $F_{(3)} = dC_{(2)}$, $F_{(5)} = dC_{(4)}$ and

$$\widetilde{F}_{(3)} = F_{(3)} - C_{(0)}\wedge H_{(3)}, \qquad \widetilde{F}_{(5)} = F_{(5)} - \tfrac{1}{2}C_{(2)}\wedge H_{(3)} + \tfrac{1}{2}B_{(2)}\wedge F_{(3)}$$

- In addition, we have to impose $F_{(5)}$ self-duality by hand on the solution

Type IIB supergravity

- The final possibility in ten dimensions ($\mathcal{N} = 2B$) cannot be obtained from dimensional reduction
- This theory has a self-dual field strength $F_{(5)} = *F_{(5)}$, and hence does not admit a covariant action formulation

  Self-dual fields also show up in six-dimensional $\mathcal{N} = (1,0)$ and $\mathcal{N} = (2,0)$ supergravity

- However, we can write down an action that gives rise to all equations of motion except the self-dual condition (which must subsequently be imposed by hand)
- Recall that we have the fields

$$(g_{\mu\nu}, B_{(2)}, \phi) + (C_{(0)}, C_{(2)}, C_{(4)})$$

Type IIB supergravity: the bosonic Lagrangian

- The IIB Lagrangian has the form

$$\begin{aligned}\mathcal{L}_{IIB} &= e^{-2\phi}[R*1 + 4d\phi\wedge *d\phi - \tfrac{1}{2}H_{(3)}\wedge *H_{(3)}] \\ &\quad -\tfrac{1}{2}F_{(1)}\wedge *F_{(1)} - \tfrac{1}{2}\widetilde{F}_{(3)}\wedge *\widetilde{F}_{(3)} - \tfrac{1}{4}\widetilde{F}_{(5)}\wedge *\widetilde{F}_{(5)} \\ &\quad -\tfrac{1}{2}C_{(4)}\wedge H_{(3)}\wedge F_{(3)}\end{aligned}$$

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- In addition, we have to impose $F_{(5)}$ self-duality by hand on the solution

Type IIB supergravity: S-duality

- At the classical level, the IIB Lagrangian is invariant under $SL(2;\mathbb{R})$ transformations

$$\tau \to \frac{a\tau + b}{c\tau + d}, \quad ad - bc = 1$$

where $\tau = C_{(0)} + ie^{-\phi}$ and $(H_{(3)}, F_{(3)})$ transform as a doublet

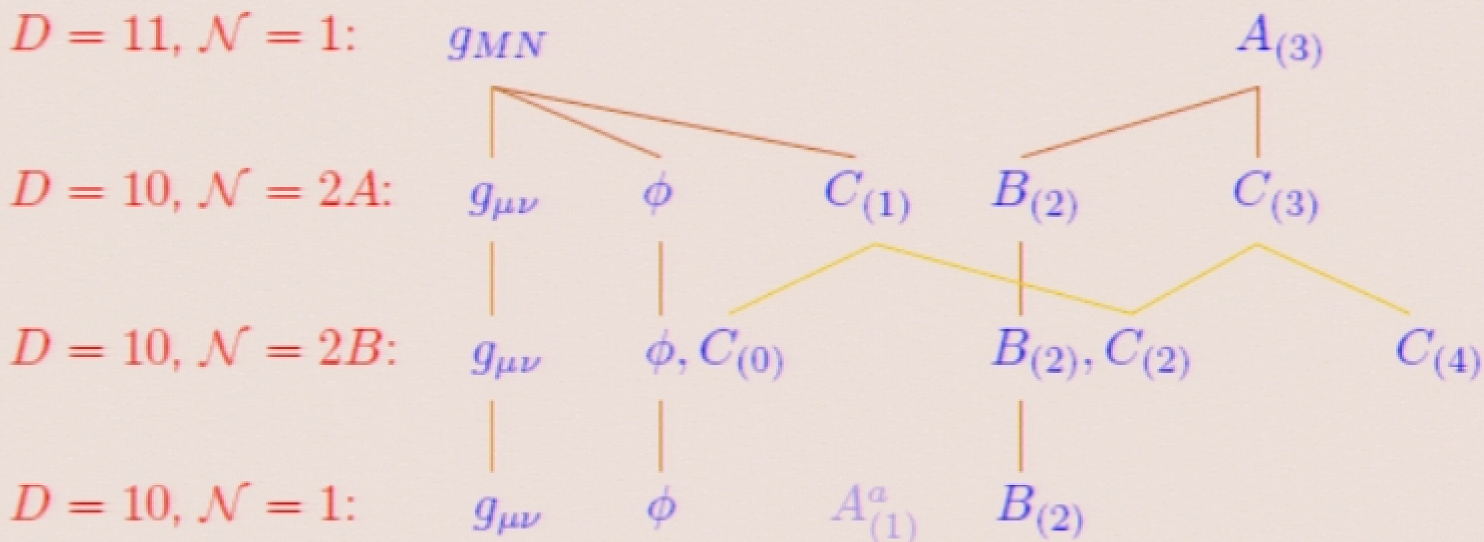
- The subgroup $SL(2;\mathbb{Z})$ is an exact symmetry of the string theory, and is known as the S-duality group of the IIB string

- This is closely related to the symmetries of a two-torus, and suggests a geometrization of S-duality

Type IIB + T^2 $\rightarrow$ twelve dimensional F-theory

Supergravities in ten and eleven dimensions

- A summary of the four supergravities we have considered



Why study supergravity?

- Backgrounds for string compactification

  Calabi-Yau manifolds, flux compactifications

- Low energy dynamics of string theory

  String solutions, black holes, D-brane probes of geometry

- AdS/CFT, gravity duals to supersymmetric gauge theories
- Phenomenology and string cosmology

  Also braneworlds and large extra dimensions

Supergravities in ten and eleven dimensions

- A summary of the four supergravities we have considered

$D = 11, \mathcal{N} = 1$: g_{MN} $A_{(3)}$

$D = 10, \mathcal{N} = 2A$: $g_{\mu\nu}$ ϕ $C_{(1)}$ $B_{(2)}$ $C_{(3)}$

$D = 10, \mathcal{N} = 2B$: $g_{\mu\nu}$ $\phi, C_{(0)}$ $B_{(2)}, C_{(2)}$ $C_{(4)}$

$D = 10, \mathcal{N} = 1$: $g_{\mu\nu}$ ϕ $A^a_{(1)}$ $B_{(2)}$

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Supersymmetric backgrounds

- Many of the exact results in string theory and AdS/CFT depend on understanding backgrounds with some fraction of unbroken supersymmetry, ie BPS configurations

  use of powerful non-renormalization theorems

  $$\{Q, Q\} \sim P + Z \quad \rightarrow \quad M \geq |Z|$$

  Single particle representations with $M = |Z|$ have discretely fewer states than those with $M > |Z|$

  Since we do not expect representations to change discontinuously, these states are protected, and are know as BPS states

- Allows us to investigate connections between strong and weak coupling

No Signal
VGA-1

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Next time

- In the next lecture, we will examine the conditions for unbroken supersymmetry, and will look at some examples of supersymmetric vacua preserving various fractions of supersymmetry
- In the third lecture, we will look at BPS branes and AdS vacua, and also review some techniques for constructing new exact backgrounds