

Title: SUSY Gauge Theory

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URL: <http://pirsa.org/05060099>

Abstract:

D.

D. Semi-classical gauge theory

$$\circ \quad \mathcal{L}_{uv} = \int d^2\theta \left[\frac{i\tau}{16\pi} \text{tr}(W^2) \right] \rightarrow -\frac{1}{4g^2} \text{tr}(F^2) + \frac{\theta}{32\pi^2} \text{tr}(F^2)$$

semi-classical gauge theory

$$\mathcal{I}_{\text{ov}} = \int d^4\theta \frac{i\tau}{16\pi} \text{tr}(W^2) \supset -\frac{1}{4g^2} \text{tr}(F^2) + \frac{\theta}{32\pi^2} \text{tr}(F\tilde{F})$$

D. Semi-classical gauge theory

$$\bullet \quad \mathcal{Z}_{00} = \int d^2\theta \frac{i\tau}{16\pi} \text{tr}(W^2) \supset -\frac{1}{4g^2} \text{tr}(F^2) + \frac{\theta}{32\pi^2} \text{tr}(F\tilde{F})$$

• 1-loop

D. Semi-classical gauge theory

- $\mathcal{Z}_{00} = \int d^2\theta \frac{i\pi}{16\pi} \text{tr}(W^2) \supset -\frac{1}{4g^2} \text{tr}(F^2) + \frac{6}{32\pi^2} \text{tr}(F\tilde{F})$
- 1-loop running

non-abelian gauge theory

$$\mathcal{L}_{uv} = \int d^2\theta \left[\frac{i\tau}{16\pi} \text{tr}(\omega^2) - \frac{1}{4g^2} \right]$$

• 1-loop running

$$\frac{1}{g^2(\mu)} = \frac{-b}{8\pi^2} \ln \left(\frac{|\Lambda|}{\mu} \right)$$

← strong coupling scale

$$b =$$

1-loop - classical gauge theory

$$\circ \quad \mathcal{L}_{00} = \int d^2\theta \quad \frac{i\tau}{16\pi} \text{tr}(\omega^2) \quad \Rightarrow \quad -\frac{1}{4g^2}$$

◦ 1-loop running

$$\frac{1}{g^2(\mu)} = \frac{-b}{8\pi^2} \ln\left(\frac{|\Lambda|}{\mu}\right) \quad \leftarrow \text{strong coupling scale}$$

$$b = \frac{11}{6} T(\text{adj})$$

level - classical gauge theory

• $\mathcal{L}_{\text{ov}} = \int d^2\theta \frac{i\tau}{16\pi} \text{tr}(\omega^2) \supset -\frac{1}{4g^2}$

• 1-loop running

$$\frac{1}{g^2(\mu)} = \frac{-b}{8\pi^2} \ln\left(\frac{|\Lambda|}{\mu}\right)$$

Strong coupling scale

$$b = \frac{11}{6} T(\text{adj}) - \frac{1}{3} \sum_f T(R_f) - \frac{1}{12} \dots$$

classical gauge theory

$$\mathcal{L}_{uv} = \int d^2\theta \frac{i\pi}{16\pi} \text{tr}(W^2) \rightarrow -\frac{1}{4g^2} \text{tr}(F^2) + \dots$$

running

$$= \frac{-b}{8\pi^2} \ln\left(\frac{|\Lambda|}{\mu}\right)$$

← strong coupling scale

$$b = \frac{11}{6} T(\text{adj}) - \frac{1}{3} \sum_f T(R_f) - \frac{1}{12} \sum_s T(R_s)$$

non-classical gauge theory

$$= \int d^2\theta \left[\frac{i\tau}{4g^2} \text{tr}(\omega^2) \right] \supset -\frac{1}{4g^2} \text{tr}(F^2) + \frac{\theta}{32\pi^2} \text{tr}(F\tilde{F})$$

loop running

$$\frac{1}{g^2(\mu)} =$$

strong coupling scale

$$\frac{1}{g^2} = \frac{1}{g_0^2} - \frac{1}{8\pi^2} \sum T(R_i) \ln \frac{\mu}{\Lambda} = \frac{3}{2} T(R) \ln \frac{\mu}{\Lambda}$$

non-classical gauge theory

$$= \int d^2\theta \frac{i\tau}{16\pi} \text{tr}(\omega^2) \supset -\frac{1}{4g^2} \text{tr}(F^2) + \frac{\theta}{32\pi^2} \text{tr}(F\tilde{F})$$

ining

$$= \frac{b}{96\pi^2} \ln\left(\frac{|\Lambda|}{\mu}\right)$$

strong coupling scale

$$T(\text{adj}) - \frac{1}{3} \sum_i T(R_i) - \frac{1}{12} \sum_i T(R_i) = \frac{3}{2} T(\text{adj})$$

4d - classical gauge theory

$$= \int d^2\theta \frac{i\pi}{16\pi} \text{tr}(\omega^2) \supset -\frac{1}{4g^2} \text{tr}(F^2) + \frac{6}{32\pi^2} \text{tr}(F\tilde{F})$$

loop running

$$\frac{1}{g^2(\mu)} = \frac{-b}{8\pi^2} \ln \mu$$

+ empty scale

$$b = \frac{11}{6}$$

$$\sum_i T(R_i) - \frac{1}{12} \sum_i T(R_i) = \frac{3}{2} T(\text{adj})$$

classical gauge theory

$$= \int d^2\theta \left[\frac{1}{16\pi} \text{tr}(W^2) \right] \rightarrow -\frac{1}{4g^2} \text{tr}(F^2) + \frac{6}{32\pi^2} \text{tr}(F\tilde{F})$$

loop running

$$\frac{1}{g^2(\mu)} = \frac{-b}{8\pi^2} \ln\left(\frac{|\Lambda|}{\mu}\right)$$

← strong coupling scale

$$b = \frac{11}{6} T(\text{adj}) - \frac{1}{3} \sum_f T(R_f)$$

$$T(\text{adj}) - \frac{1}{2} \sum_f$$

4d - classical gauge theory

$$= \int d^2\theta \frac{i\tau}{16\pi} \text{tr}(\omega^2) \quad \Rightarrow \quad = \frac{1}{4g^2} \text{tr}(F^2) + \frac{\theta}{32\pi^2} \text{tr}(F\tilde{F})$$

loop running

$$\frac{1}{g^2(\mu)} = \frac{-b}{8\pi^2} \ln\left(\frac{|\Lambda|}{\mu}\right) \quad \leftarrow \text{strong coupling}$$

$$b = \frac{11}{6} T(\text{adj}) -$$

$$\sum_i T(R_i) = \frac{3}{2} T(\text{adj}) - \frac{1}{2} \sum_i T(R_i)$$

the gauge theory

$$\frac{i\pi}{6\pi} \text{tr}(\omega^2) \supset -\frac{1}{4g^2} \text{tr}(F^2) + \frac{\theta}{32\pi^2} \text{tr}(F\tilde{F})$$

→ show

$$\left(\frac{|A|}{N} \right)$$

adj)

$$\frac{1}{2} \text{ST}(R) = \frac{3}{2} T(\mathfrak{g}) - \frac{1}{2} \sum_{\lambda \in \mathfrak{h}} T(R_\lambda)$$

gauge theory

$$\mathcal{L}(w^2) \Rightarrow -\frac{1}{4} \text{tr}(F^2) + \frac{\theta}{32\pi^2} \text{tr}(F \tilde{F})$$

strong coupling

$$SU(N_c): T(N_c) = 1$$

$$T(R_i) = \frac{1}{2} \text{tr}(T_a T_a) = \frac{1}{2} \sum_{a=1}^{N_c^2-1} T_a T_a$$

Gauge theory

$$\mathcal{L}(w^2) \supset -\frac{1}{4g^2} \text{tr}(F^2) + \frac{\theta}{32\pi^2} \text{tr}(F\tilde{F})$$

Strong coupling scale

$$\text{SU}(N_c): \quad T(N_c) = 1$$

$$T(\text{adj}) = 2N_c$$

$$\frac{1}{3} \sum_i T(R_i) - \frac{1}{12} \sum_i T(R_i) = \frac{3}{2} T(\text{adj}) - \frac{1}{2} \sum_i T(R_i)$$

gauge theory

$$\mathcal{L}(w^2) \supset -\frac{1}{4g^2} \text{tr}(F^2) + \frac{\theta}{32\pi^2} \text{tr}(F\tilde{F})$$

strong coupling scale

$$\boxed{\begin{array}{l} \text{SU}(N_c): T(N_c) = 1 \\ T(\mathbf{adj}) = 2N_c \end{array}}$$

$$\frac{1}{3} \sum_i T(R_i) - \frac{1}{12} \sum_i T(R_i) = \frac{3}{2} T(\mathbf{adj}) - \frac{1}{2} \sum_i T(R_i)$$

$$\tau = \frac{Q}{2\pi} - i \frac{4\pi}{g^2}$$

$$= \frac{b}{2\pi i} \ln \left(\frac{1}{\mu} \right)$$

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$$= \frac{b}{2\pi i} \ln\left(\frac{1}{\mu}\right)$$

$$\tau = \frac{Q}{2\pi} - i \frac{4\pi}{g^2}$$

$$= \frac{b}{2\pi i} \ln\left(\frac{\Lambda}{\mu}\right)$$

$$\Lambda = |\Lambda|$$

$$\tau = \frac{\theta}{2\pi} - i \frac{4\pi}{g^2}$$

$$= \frac{b}{2\pi i} \ln\left(\frac{1}{\mu}\right)$$

$$\Lambda = |\Lambda| e^{i\theta/b}$$

$$\tau = \frac{\theta}{2\pi} - i \frac{4\pi}{g^2}$$

$$= \frac{b}{2\pi i} \ln\left(\frac{1}{\mu}\right)$$

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$$= \frac{b}{2\pi i} \ln \left(\frac{1}{\mu} \right)$$

$$\Lambda = |\Lambda| e^{i\theta/b}$$

$U(1)$

$$\tau = \frac{\theta}{2\pi} - i \frac{4\pi}{g^2}$$

$$= \frac{b}{2\pi i} \ln\left(\frac{1}{\mu}\right)$$

$$\lambda = |\lambda| e^{i\theta/b}$$

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$U(1)$

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$U(1)$

$$\tau = \frac{\theta}{2\pi} - i \frac{4\pi}{g^2}$$

$$= \frac{b}{2\pi i} \ln\left(\frac{1}{\mu}\right)$$

$$\Lambda = |\Lambda| e^{i\theta/b}$$

$$U(1)_c:$$

$$\tau = \frac{\theta}{2\pi} - i \frac{4\pi}{g^2}$$

$$= \frac{b}{2\pi i} \ln\left(\frac{1}{\mu}\right)$$

$$\Lambda = |\Lambda| e^{i\theta/b}$$

$$U(1)_i : U_i$$

$$\tau = \frac{\theta}{2\pi} - i \frac{4\pi}{g^2}$$

$$= \frac{b}{2\pi i} \ln\left(\frac{1}{\mu}\right)$$

$$= |\Lambda| e^{i\theta/b}$$

$$U: [4$$

$$\tau = \frac{\theta}{2\pi} - i \frac{4\pi}{g^2}$$

$$= \frac{b}{2\pi i} \ln\left(\frac{\Lambda}{\mu}\right)$$

$$\Lambda = |\Lambda| e^{i\theta/b}$$

$$U(1)_i : U_i[\psi_j] = \delta_{ij}$$

$$\tau = \frac{\theta}{2\pi} - i \frac{4\pi}{g^2}$$

$$= \frac{b}{2\pi i} \ln\left(\frac{1}{\mu}\right)$$

$$\Lambda = |\Lambda| e^{i\theta/b}$$

$$U(1)_i : U_i[\psi_i] = \delta_{ij}$$

$$\tau = \frac{\theta}{2\pi} - i \frac{4\pi}{g^2}$$

$$= \frac{b}{2\pi i} \ln\left(\frac{\Lambda}{\mu}\right)$$

$$\Lambda = |\Lambda| e^{i\theta/b}$$

$$U(1)_i : U_i[4_j] = \delta_{ij}$$

$$\tau = \frac{\theta}{2\pi} - i \frac{4\pi}{g^2}$$

$$= \frac{b}{2\pi i} \ln\left(\frac{1}{\mu}\right)$$

$$\Lambda = |\Lambda| e^{i\theta/b}$$

$$U(1)_i : U_i[\psi_j] = \delta_{ij}$$

$$\tau = \frac{\theta}{2\pi} - i \frac{4\pi}{g^2}$$

$$= \frac{b}{2\pi i} \ln\left(\frac{1}{\mu}\right)$$

$$\Lambda \quad i\theta/b$$

$U(1)_i$

$$[T_j] = \delta_{ij}$$

$$= \frac{1}{2\pi i} \ln \left(\frac{\lambda}{\mu} \right)$$

$$\Lambda = |\Lambda| e^{i\theta/b}$$

$$U(1)_i: \quad U_i[\psi_j] = \delta_{ij} \cdot \psi_i \rightarrow e^{i\varepsilon} \psi_i$$

$$\theta \rightarrow \theta +$$

$$= \frac{1}{2\pi i} \ln\left(\frac{\lambda}{\mu}\right)$$

$$|\Lambda| e^{i\theta/6}$$

$$U_i[\psi_j] = \delta_{ij} \quad (\psi_i \rightarrow e^{i\epsilon} \psi_i)$$

$$\theta \rightarrow \theta +$$

$$= \frac{1}{2\pi i} \ln \left(\frac{\Lambda}{\mu} \right)$$

$$\Lambda = |\Lambda| e^{i\theta/b}$$

$U(1)_c:$

$$U_i[\psi_j] = \delta_{ij} \quad (\psi_i \rightarrow e^{i\varepsilon} \psi_i)$$

$$\theta \rightarrow \theta + \varepsilon T(R_i)$$

$$= \frac{1}{2\pi i} \ln \left(\frac{\Lambda}{\mu} \right)$$

$$\Lambda = |\Lambda| e^{i\theta/b}$$

$$U(1)_i: \quad U_i[\psi_j] = \delta_{ij} \quad (\psi_i \rightarrow e^{i\varepsilon} \psi_i)$$

$$\theta \rightarrow \theta + \varepsilon T(R_i)$$

$$= \frac{1}{2\pi i} \ln \left(\frac{\lambda}{\mu} \right)$$

$$\Lambda = |\Lambda| e^{i\theta/b}$$

$$U(\Lambda)_i : U_i[\psi_j] = \delta_{ij} \quad (\psi_i \rightarrow e^{i\varepsilon} \psi_i)$$

$$\theta \rightarrow \theta + \varepsilon T(R_i)$$

$$U_i(\Lambda) =$$

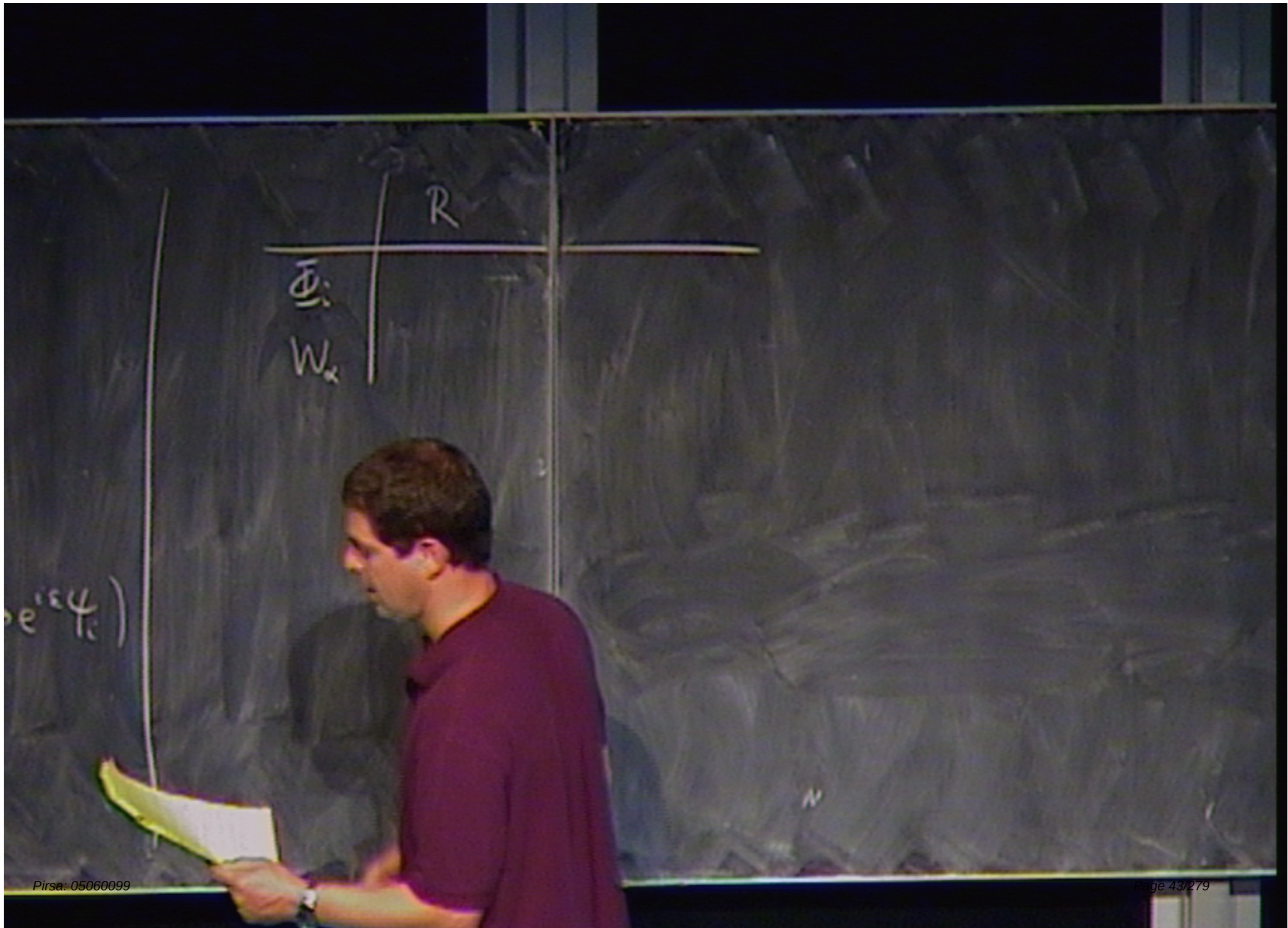
$$= \frac{1}{2\pi i} \ln \left(\frac{\lambda}{\mu} \right)$$

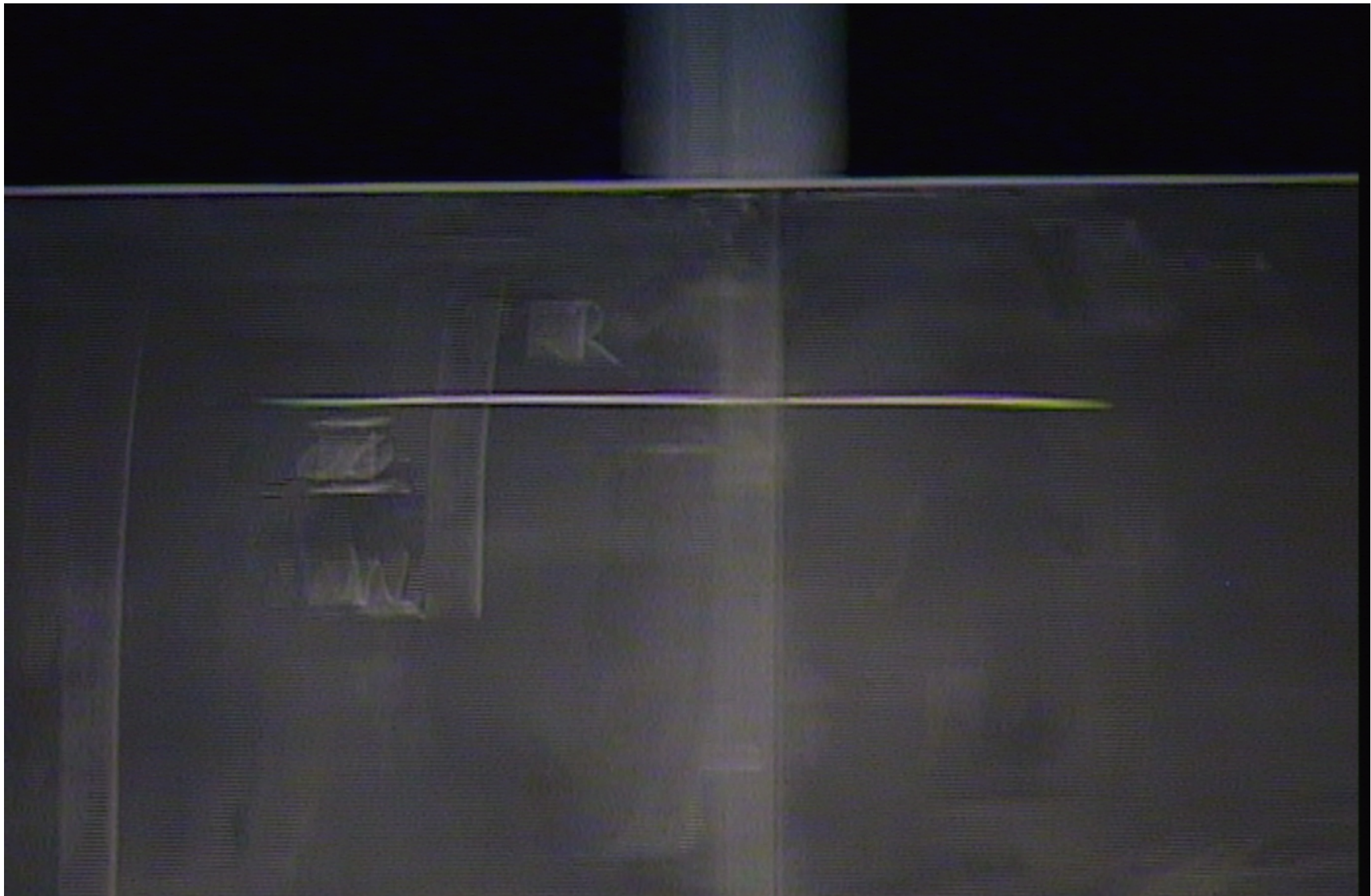
$$\Lambda = |\Lambda| e^{i\theta/b}$$

$$U(1)_i: \quad U_i[\psi_j] = \delta_{ij} \quad (\psi_i \rightarrow e^{i\varepsilon} \psi_i)$$

$$\theta \rightarrow \theta + \varepsilon T(R_i)$$

$$U_i(\Lambda) = T(R_i)$$





	R
$\bar{\Phi}_i$	
W_α	

	R
$\bar{\Phi}_i$	
W_α	1

	R
$\bar{\Phi}_i$	
W_α	1

$$\Rightarrow R[\lambda_\alpha] = 1, \quad R[\lambda_\beta] = 0$$

	$\mathcal{R}$
$\bar{\Phi}_i$	$2/3$
W_α	1

$$\Rightarrow R[\lambda_\alpha] = 1, \quad R[F_{\alpha\beta}] = 0$$

	R
$\bar{\Phi}_i$	$2/3$
W_α	1

$\Rightarrow$

$$\Rightarrow R[\lambda_\alpha] = 1, \quad R[F_\alpha] = 0$$

	$\mathcal{R}$
$\bar{\Phi}_i$	$2/3$
W_α	1

$$\Rightarrow R[\phi_i] = 2/3,$$

$$\Rightarrow R[\lambda_\alpha] = 1, \quad R[F_\alpha] = 0$$

	$\mathcal{R}$
$\bar{\Phi}_i$	$2/3$
W_α	1

$$\Rightarrow R[\phi_i] = 2/3, \quad R[\psi_i] = -1/3$$

$$\Rightarrow R[\lambda_\alpha] = 1, \quad R[F_\alpha] = 0$$

R

Φ_i

$2/3$

W_α

1

$\wedge$

$$\Rightarrow R[\phi_i] = \frac{2}{3}, \quad R[\psi_i] = -\frac{1}{3}$$

$$\Rightarrow R[\lambda_\alpha] = 1, \quad R[F_\alpha] = 0$$

$\mathcal{R}$

$\bar{\Phi}_i$

$2/3$

W_α

1

$\wedge^b$

$$\Rightarrow R[\phi_i] = 2/3, \quad R[\psi_i] = -1/3$$

$$\Rightarrow R[\lambda_\alpha] = 1, \quad R[F_\alpha] = 0$$

R

$\bar{\Phi}_i$

$2/3$

W_α

1

$\wedge^6$

$$\Rightarrow R[\phi_i] = \frac{2}{3}, \quad R[\psi_i] = -\frac{1}{3}$$

$$\Rightarrow R[\lambda_\alpha] = 1, \quad R[F_\alpha] = 0$$

R

$\bar{\Phi}_i$

$2/3$

W_α

1

$\uparrow^b$

$T(\alpha_i)$

$$\Rightarrow R[\phi_i] = 2/3, \quad R[\psi_i] = -1/3$$
$$\Rightarrow \underline{R[\lambda_\alpha]} = 1, \quad R[F_\alpha] = 0$$

R

Φ_i

$2/3$

W_α

1

$\wedge^b$

$$\Rightarrow R[\Phi_i] = 2/3, \quad R[\Psi_i] = -1/3$$

$$\Rightarrow \underline{R[\lambda_\alpha]} = 1, \quad R[F_i] = 0$$

$$-\frac{1}{3} \sum_i T(\Phi_i)$$

R

Φ_i

$2/3$

W_α

1

Λ^b

$T(\alpha_i) =$

$$\frac{1}{3} \sum_i T(R_i)$$

$$\Rightarrow R[\Phi_i] = 2/3, \quad R[\Psi_i] = -1/3$$

$$\Rightarrow R[\lambda_\alpha] = 1, \quad R[F_\alpha] = 0$$

R

Φ_i

$2/3$

W_α

1

Λ^b

$T(ad_1) =$

$$\frac{1}{3} \sum_i T(R_i)$$

$$2 \left(T(ad_1) - \frac{1}{3} \sum_i T(R_i) \right)$$

$$\Rightarrow R[\Phi_i] = 2/3, \quad R[\Psi_i] = -1/3$$

$$\Rightarrow \underline{R[\lambda_\alpha]} = 1, \quad R[F_\alpha] = 0$$

R

Φ_i

$2/3$

W_α

1

$$\Rightarrow R[\Phi_i] = 2/3, \quad R[\Psi_i] = -1/3$$

$$\Rightarrow \underline{R[\lambda_\alpha]} = 1, \quad R[F_\alpha] = 0$$

Λ^b

$T(a_\alpha)$

$$= \frac{1}{3} \sum_i T(R_i)$$

$$b = \frac{2}{3} \left(T(a_\alpha) - \frac{1}{3} \sum_i T(R_i) \right)$$

R

Φ_i

$2/3$

W_α

1

$$\Rightarrow R[\Phi_i] = 2/3, \quad R[\Psi_i] = -1/3$$

$$\Rightarrow \underline{R[\lambda_\alpha]} = 1, \quad R[F_\alpha] = 0$$

Λ^b

$T(ad_1)$

$$= \frac{1}{3} \sum_i T(R_i)$$

$$b = \frac{3}{2} \left(T(ad_1) - \frac{1}{3} \sum_i T(R_i) \right)$$

R

Φ_i

$2/3$

$$\Rightarrow R[\Phi_i] = 2/3, \quad R[\Psi_i] = -1/3$$

W_α

1

$$\Rightarrow R[\lambda_\alpha] = 1, \quad R[F_\alpha] = 0$$

$\uparrow^b$

$$T(ad_1) = \frac{1}{3} \sum_i T(R_i)$$

b

$$= \frac{2}{3} \left(T(ad_1) - \frac{1}{3} \sum_i T(R_i) \right)$$

$\uparrow$

$2/3$

R

Φ_i

$2/3$

$$\Rightarrow R[\Phi_i] = 2/3, \quad R[\Psi_i] = -1/3$$

W_α

1

$$\Rightarrow \underline{R[\lambda_\alpha]} = 1, \quad R[F_\alpha] = 0$$

$\wedge^b$

$$T(ad_1) = \frac{1}{3} \sum_i T(R_i)$$

b

$$= \frac{2}{3} \left(T(ad_1) - \frac{1}{3} \sum_i T(R_i) \right)$$

$\wedge$

$2/3$

$$\overline{[\psi_i]} = \frac{2}{3}, \quad \underline{R[\psi_i]} = -\frac{1}{3}$$

$$\underline{[\lambda_a]} = 1, \quad R[F_{\mu\nu}] = 0$$

$$T(R_c)$$

$$T(R_1)$$

$$R[\Lambda] = \frac{2}{3}$$

$$D[\Lambda] = 1$$

$$\overline{[\psi_i]} = \frac{2}{3}, \quad \underline{R[\psi_i]} = -\frac{1}{3}$$

$$\underline{[\lambda_a]} = 1, \quad R[F_{\mu\nu}] = 0$$

$$T(R_c)$$

$$T(R_1)$$

$$R[\Lambda] = \frac{2}{3}$$

$$D[\Lambda] = 1$$

$$2/3, \underline{R[4]} = -\frac{1}{3}$$

$$1, R[F] = 0$$

$$R[A] = \frac{2}{3}$$

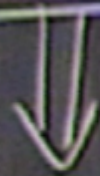
$$D[A] = 1$$

$$2/3, \underline{R[4]} = -1/3$$

$$1, R[F] = 0$$

$$R[A] = 2/3$$

$$D[A] = 1$$



$$R[1] = \frac{2}{3}$$

$$D[1] = 1$$



$$D[4] =$$

$$R[\Lambda] = \frac{2}{3}$$

$$D[\Lambda] = 1$$



$$D[\Phi] =$$

$$R[A] = \frac{2}{3}$$

$$D[A] = 1$$



$$D[\Phi] = \frac{3}{2} R[\Phi]$$

SCFTs $\rightarrow$ XSCFTs

$$R[A] = \frac{2}{3}$$

$$D[A] = 1$$



$$D[\Phi] = \frac{3}{2} R[\Phi]$$

SCFTs $\rightarrow$ X_{SCFTs}

$$R[A] = \frac{2}{3}$$

$$D[A] = 1$$



$$D[\Phi] = \frac{3}{2} R[\Phi]$$

SCFTs $\rightarrow$ X_{SCFTs}

$$\Rightarrow R[\phi_i] = \frac{2}{3}, \quad R[\psi_i] = -\frac{1}{3}$$

$$\Rightarrow \underline{R[\lambda_i]} = 1, \quad R[E_i] = 0$$

$$-\frac{1}{3} \sum_i T(R_i)$$

$$(4) -\frac{1}{3} \sum_i T(R_i)$$

$$R[\Lambda] = \frac{2}{3}$$

$$D[\Lambda] = 1$$



$$\boxed{D[\phi] = \frac{3}{2}}$$

$$\Rightarrow R[\phi_i] = \frac{2}{3}, \quad R[\psi_i] = -\frac{1}{3}$$

$$\Rightarrow \underline{R[\lambda_i]} = 1, \quad R[F_i] = 0$$

$$-\frac{1}{3} \sum_i T(R_i)$$

$$(4) -\frac{1}{3} \sum_i T(R_i)$$

$$R[\Lambda] = \frac{2}{3}$$

$$D[\Lambda] = 1$$



$$\boxed{D[\phi] = \frac{3}{2}}$$

$$R = R'$$

$$R[A] = \frac{2}{3}$$

$$D[A] = 1$$

$$R[\psi] = -\frac{1}{3}$$

$$R[E] = 0$$



$$D[\psi] = \frac{3}{2} R[\psi]$$

softc xch

$$R = R' + \Sigma$$

$$R[\Lambda] = \frac{2}{3}$$

$$D[\Lambda] = 1$$



$$D[\Phi] = \frac{3}{2} R[\Phi]$$

SOFTs $\rightarrow$ XSTs

$$R = R' + \sum_i a_i U$$

$$D[1] = 1$$



$$D[\phi] = \frac{3}{2} R[\phi]$$

SCFTs $\rightarrow$ XFTs

$$R = R' + \sum_i a_i U_i$$

E. NVMR H_{nm} (vst)

E. NVMR Hrm (vstfs)

UV scale $\mu_0 \gg \Lambda$

E. NVMR H_{m} (v s f s)

UV $\mu_0 \gg \Lambda$ (weakly)

E. NVMR μ_{nm} (vstf)

UV scale $\mu_0 \gg \Lambda$ (we

↓

IR scale $\mu < \mu_0$

E. NVMR μ_{nm} (vst)

UV scale $\mu_0 \gg \Lambda$ (weaker)
 $\downarrow$
'IR' scale $\mu < \mu_0$

E. NVMR μ_{nm} (vst)

UV scale $\mu_0 \gg \Lambda$ (weak)

↓
'IR' scale $\mu < \mu_0$

how does $\tau_{\text{eff}}(\lambda)$?

E. NVMR $\hbar m$ (vstf)

UV scale $\mu_0 \gg \Lambda$ (weakly)

↓
'IR' scale $\mu < \mu_0$

how does $\tau_{\text{eff}}(\Lambda, \Lambda)$

E. NVMR $\hbar m$ (v s f s)

UV scale $\mu_0 \gg \Lambda$ (w/ly)

↓
'IR' scale $\mu < \mu_0$

how does $\tau_{\mu}(\Lambda, \Lambda)$

E. NVMR Hm (vst)

UV side $\mu_0 \gg \Lambda$ (w)

↓
'IR' side $\mu < \mu_0$

how does $\tau_\mu(A, \Lambda, \phi_i)$

E. NVMR Hm (vst)

UV side $\mu_0 \gg \Lambda$ (weakly)

↓

'IR' side $\mu < \mu_0$

how does $\tau_\mu(P, \Lambda, \phi_i)$

E. NVMR Hm (vst)

UV side $\mu_0 \gg \Lambda$ (weakly)

↓

'IR'

side

$\mu < \mu_0$

$$\tau_{\mu} = \tau_{\mu}(\Lambda, \Lambda, \phi_i)$$

E. NVMR Hm (vst)

UV scale $\mu_0 \gg \Lambda$ (weakly)

↓

'IR'

scale

$\mu < \mu_0$

$$\tau_{\mu} = \tau_{\mu}(\Lambda, \phi_i)$$

E. NVMR Hm (vstf)

UV scale $\mu_0 \gg \Lambda$ (we

↓

'IR'

scale

$\mu < \mu_0$

$$\tau_{\mu} = \tau_{\mu}(\Lambda, \phi_i)$$

UV scale $\mu_0 \gg \Lambda$ (weak)
 $\downarrow$
 'IR' scale $\mu < \mu_0$

$$\tau_\mu = \tau_\mu(\mu, \Lambda, \phi_i)$$

$$\theta \equiv \theta + 2\pi$$

UV side $\mu_0 \gg \Lambda$ (weak)
 $\downarrow$
 'IR' side $\mu < \mu_0$

$$\tau_\mu = \tau_\mu(\Lambda, \phi_i)$$

$$\Theta \approx \Theta + 2\pi \Rightarrow \tau_{\mu_i} \approx \tau_{\mu_i} + 1$$

UV scale $\mu_0 \gg \Lambda$ (weak)
 $\downarrow$
 'IR' scale $\mu < \mu_0$

$$\tau_\mu = \tau_\mu(\Lambda, \phi_i) =$$

$$\Theta \equiv \Theta + 2\pi \Rightarrow \tau_n \approx \tau_{n+1}$$

$$\Lambda^b \rightarrow e^{2\pi i} \Lambda^b$$

NVNR form (vs fs)

UV scale $\mu_0 \gg \Lambda$ (weakly)

↓
'IR' scale $\mu < \mu_0$

$$\tau_\mu = \tau_\mu(\Lambda, \phi_i) = \frac{1}{2\pi i} \ln \left(\frac{\Lambda^b}{\mu} \right)$$

$$\theta = \theta + 2\pi \Rightarrow \tau_\mu \approx \tau_\mu + 1$$

$$\Lambda^b \rightarrow e^{2\pi i} \Lambda^b$$

MK form (vs fs)

scale $\mu_0 \gg \Lambda$ (weakly)

scale $\mu < \mu_0$

$$\tau_\mu = \tau_\mu(\Lambda, \phi_i) = \frac{b}{2\pi i} \ln\left(\frac{\Lambda}{\mu}\right)$$

$$\theta = \theta + 2\pi \Rightarrow \tau_n \approx \tau_{n+1}$$

$$b \rightarrow e^{2\pi i} b$$

MK Hm (vs fs)

scale $\mu_0 \gg \Lambda$ (weakly)

scale $\mu < \mu_0$

$$\tau_\mu = \tau_\mu(\Lambda, \phi_i) = \frac{b}{2\pi i} \left(\frac{\Lambda}{\mu} \right) +$$

$$\theta = \theta + 2\pi \Rightarrow \tau_\mu \approx \tau_{\mu+1}$$

$$b \rightarrow e^{2\pi i \frac{b}{\mu}}$$

VNR Hm (vs fs)

scale $\mu_0 \gg \Lambda$ (weakly)

scale $\mu < \mu_0$

$$\tau_\mu = \tau_\mu(\Lambda, \phi_i) = \frac{b}{2\pi i} \ln\left(\frac{\Lambda}{\mu}\right) + f(\Lambda^b, \phi_i)$$

$$\theta = \theta + 2\pi \Rightarrow \tau_\mu \approx \tau_{\mu+1}$$

$$\Lambda^b \rightarrow e^{2\pi i} \Lambda^b$$

(λ, f)

$\Rightarrow \Lambda$ (weatuly)

$\leq \mu_0$

$$(\Lambda, \phi_i) = \frac{1}{2\pi i} \ln \left(\frac{\Lambda}{\lambda_i} \right) + f(\Lambda^b, \phi_i)$$

$\Rightarrow \tau_n \approx \tau_{n+1}$

simple

$v s f s)$

$\Rightarrow \Lambda$ (weakly)

$< \mu_0$

$$(\Lambda, \phi_i) = \frac{b}{2\pi i} \ln\left(\frac{\Lambda}{\mu_0}\right) + f(\Lambda^b, \phi_i)$$

$$\Rightarrow \tau_n \approx \tau_{n+1}$$

single-valued in Λ^b

(weakly)

$$\phi_i = \frac{b}{2\pi i} \ln\left(\frac{\Lambda}{\mu}\right) + f(\Lambda)$$

$$\approx \tau_n + 1$$

(weakly)

single-
valued
in Λ^b

$$(\phi_i) = \frac{b}{2\pi i} \ln\left(\frac{\Lambda}{\Lambda^b}\right) + f(\Lambda^b, \phi_i)$$

$$\approx \tau_n + 1$$

$\Lambda \rightarrow 0$ limit

$W(y)$

single
valued
 Λ/b

$\frac{b}{\pi i} \ln \left(\frac{\Lambda}{\mu} \right)$

$sfs)$

$\Lambda \rightarrow 0$ limit

Λ (wreath)

μ_0

$$\Lambda, \phi_i) = \frac{b}{2\pi i} \ln\left(\frac{\Lambda}{\mu_0}\right) + f(\Lambda^b, \phi_i)$$

$$\tau_n \approx \tau_{n+1}$$

single-valued
in Λ

$\rightarrow 0$ limit

single-valued
in Λ^b

$$(\Lambda^b, \phi_i) = \frac{1}{2\pi i} \ln\left(\frac{\Lambda}{\mu}\right) + \sum_{n=1}^{\infty} \Lambda^{nb} c_n(\phi_i)$$

$\rightarrow 0$ limit

single-valued
in Λ^b

$$(\Lambda^b, \phi_i) = \frac{1}{2\pi i} \ln\left(\frac{\Lambda}{\mu}\right) + \sum_{n=1}^{\infty} \Lambda^{nb} c_n(\phi_i)$$

$\rightarrow 0$ limit

single-valued
in Λ^b

$$(\Lambda^b, \phi_i) = \frac{1}{2\pi i} \ln\left(\frac{\Lambda}{\mu}\right) + \sum_{n=1}^{\infty} \Lambda^{nb} c_n(\phi_i)$$

$\rightarrow 0$ limit

single-valued
in Λ^b

$$(\Lambda^b, \phi_i) = \frac{1}{2\pi i} \ln\left(\frac{\Lambda}{\bar{z}}\right) + \sum_{n=1}^{\infty} \Lambda^{nb} c_n(\phi)$$

$\rightarrow 0$ limit

$-n$

e

single-valued
in b

$$(\Lambda^b, \phi_i) = \frac{b}{2\pi i} \ln\left(\frac{\Lambda}{\mu}\right) + \sum_{n=1}^{\infty} \Lambda^{nb} c_n(\phi)$$

0 limit

n -instanton factor

$$e^{-n \frac{8\pi^2}{g^2}}$$

value
in $1/b$

$$\Phi = -\frac{1}{b} \ln(\Lambda) + \sum_{n=1}^{\infty} \Lambda^{nb} \mathcal{F}_n(\Phi)$$

NVMR Hm (vs fs)

UV scale $\mu_0 \gg \Lambda$ (weakly)

$\downarrow$
IR scale $\mu < \mu_0$

$$T_m = \tau_\mu |\Lambda \phi|$$

$$\theta = \theta + 2\pi \Rightarrow T_m \approx \tau_\mu$$

$$\Lambda^b \rightarrow e^{2\pi i} \Lambda^b$$

E. NVMR Hmm (vst)

$\Lambda \rightarrow 0$ limit

UV scale $\mu_0 \gg \Lambda$

$\downarrow$
'IR' scale $\mu < \mu_0$

$$\tau_{\mu} = \tau_{\mu}(\Lambda, \phi_i) = \frac{1}{2\pi i} \ln\left(\frac{\Lambda}{\mu}\right) + \dots$$

$$\theta = \theta + 2\pi \Rightarrow \tau_n = \tau_{n+1}$$

$$\Lambda^b \rightarrow e^{2\pi i} \Lambda^b$$

E. NVMR Hm (vstf)

$\Lambda \rightarrow 0$ limit

UV scale $\mu_0 \gg \Lambda$

$\downarrow$
'IR' scale $\mu < \mu_0$

$$\tau_{\mu} = \tau_{\mu}(\Lambda, \phi_i) = \frac{1}{2\pi i} \ln\left(\frac{\Lambda}{\mu}\right) + \dots$$

$$\theta = \theta + 2\pi \Rightarrow \tau_n = \tau_{n+1}$$

$$\Lambda^b \rightarrow e^{2\pi i} \Lambda^b$$

E. NVMR thm (vst)

$\Lambda \rightarrow 0$ limit

UV scale $\mu_0 \gg \Lambda$ (weakly)

$\downarrow$
'IR' scale $\mu < \mu_0$

$$\tau_{\mu} = \tau_{\mu}(\Lambda, \phi_i) = \frac{b}{2\pi i} \ln\left(\frac{\Lambda}{\mu}\right) +$$

$$\theta = \theta + 2\pi \Rightarrow \tau_n = \tau_{n+1}$$

$$\Lambda^b \rightarrow e^{2\pi i} \Lambda^b$$

E. NVMR Hm (vstf)

$\Lambda \rightarrow 0$ limit

UV scale $\mu_0 \gg \Lambda$ (weakly)

$\downarrow$
'IR' scale $\mu < \mu_0$

$$1 \rightarrow \tau_\mu \equiv \tau_\mu(\Lambda, \phi) =$$

$$\theta \equiv \theta + 2\pi \Rightarrow \tau_n \rightarrow \tau_{n+1}$$

$$\Lambda^b \rightarrow e^{2\pi i} \Lambda^b$$

E. NVMR thm (vstf)

$\Lambda \rightarrow 0$ limit

UV scale $\mu_0 \gg \Lambda$ (weakly)

$\downarrow$
'IR' scale $\mu < \mu_0$

$$\tau_{\mu} = \tau_{\mu}(\Lambda, \phi_i) = \frac{1}{2\pi i} \ln\left(\frac{\Lambda}{\mu}\right) + f(\Lambda^b, \phi_i) = \frac{1}{2\pi i} \ln\left(\frac{\Lambda}{\mu}\right) + f(\Lambda^b, \phi_i)$$

$$\theta = \theta + 2\pi \Rightarrow \tau_n = \tau_{n+1}$$

$$\Lambda^b \rightarrow e^{2\pi i} \Lambda^b$$

single valued
 $\downarrow$
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E. NVMR thm (vs fs)

$\Lambda \rightarrow 0$

UV scale $\mu_0 \gg \Lambda$ (weakly)

$\downarrow$
'IR' scale $\mu < \mu_0$

$$\tau_{\mu} \equiv \tau_{\mu}(\Lambda, \phi_i) = \frac{1}{2\pi i} \ln\left(\frac{\Lambda}{\mu}\right) + f(\Lambda^b, \phi_i)$$

$$\theta = \theta + 2\pi \Rightarrow \tau_n = \tau_{n+1}$$

$$\Lambda^b \rightarrow e^{2\pi i} \Lambda^b$$

E. NVMR thm (vs fs)

UV scale $\mu_0 \gg \Lambda$ (weakly)

↓
'IR' scale $\mu < \mu_0$

$$\tau_{\mu} \equiv \tau_{\mu}(\Lambda, \phi_i) = \frac{1}{2\pi i} \ln\left(\frac{\Lambda}{\mu}\right) + f(\Lambda^b, \phi_i)$$

$$\theta = \theta + 2\pi \Rightarrow \tau_n \approx \tau_{n+1}$$

$$\Lambda^b \rightarrow e^{2\pi i} \Lambda^b$$

$\Lambda \rightarrow 0$

single-valued in

3. NVMR Horn (vs fs)

$\Lambda \rightarrow 0$ limit

UV scale $\mu_0 \gg \Lambda$ (weakly)

$\downarrow$
'IR' scale $\mu < \mu_0$

$$\tau_{\mu} = \tau_{\mu}(\Lambda, \phi_i) = \frac{b}{2\pi i} \ln\left(\frac{\Lambda}{\mu}\right) + f(\Lambda^b)$$

$$\theta = \theta + 2\pi \Rightarrow \tau_n \approx \tau_{n+1}$$

$$\Lambda^b \rightarrow e^{2\pi i} \Lambda^b$$

3. NVMR Hmm (vs fs)

$\Lambda \rightarrow 0$ limit

UV scale $\mu_0 \gg \Lambda$ (weakly)

$\downarrow$
'IR' scale $\mu < \mu_0$

$\ln^2$

$$\tau_{\mu} \equiv \tau_{\mu}(\Lambda, \phi_i) = \frac{b}{2\pi i} \ln\left(\frac{\Lambda}{\mu}\right) + f(\Lambda^b)$$

$$\theta \approx \theta + 2\pi \Rightarrow \tau_n \approx \tau_{n+1}$$

$$\Lambda^b \rightarrow e^{2\pi i} \Lambda^b$$

3. NVMR Hmm (vs fs)

$\Lambda \rightarrow 0$ limit

UV scale $\mu_0 \gg \Lambda$ (weakly)

$\downarrow$
'IR' scale $\mu < \mu_0$

$$\ln^2\left(\frac{\Lambda\bar{\Lambda}}{\mu}\right)$$

$$\tau_{\mu} \equiv \tau_{\mu}(\Lambda, \phi_i) = \frac{b}{2\pi i} \ln\left(\frac{\Lambda}{\mu}\right) + f(\Lambda^b)$$

$$\theta \simeq \theta + 2\pi \Rightarrow \tau_n \simeq \tau_{n+1}$$

$$\Lambda^b \rightarrow e^{2\pi i} \Lambda^b$$

III. NVMR form (vs fs)

$\Lambda \rightarrow 0$ limit

UV scale $\mu_0 \gg \Lambda$ (weakly)

$\downarrow$
'IR' scale $\mu < \mu_0$

$$\tau_\mu \equiv \tau_\mu(\Lambda, \phi_i) = \frac{b}{2\pi i} \ln\left(\frac{\Lambda}{\mu}\right) + f(\dots)$$

$$\theta \simeq \theta + 2\pi \Rightarrow \tau_n \simeq \tau_{n+1}$$

$$\Lambda^b \rightarrow e^{2\pi i} \Lambda^b$$

III. NVMR thm (vs fs)

$\Lambda \rightarrow 0$ limit

UV scale $\mu_0 \gg \Lambda$ (weakly)

$+ \ln^2 \left(\frac{\Lambda \bar{\Lambda}}{\mu^2} \right)$ single
valued

$\downarrow$
'IR' scale $\mu < \mu_0$

$$\tau_\mu \equiv \tau_\mu(\Lambda, \phi_i) = \frac{b}{2\pi i} \ln \left(\frac{\Lambda}{\mu} \right) + f(\Lambda^b, \phi_i)$$

$$\theta \simeq \theta + 2\pi \Rightarrow \tau_n \simeq \tau_{n+1}$$

$$\Lambda^b \rightarrow e^{2\pi i} \Lambda^b$$

thm (v s f s)

$\Lambda \rightarrow 0$ limit

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$\mu_0 \gg \Lambda$ (weakly)

$\mu < \mu_0$

$+ \ln^2 \left(\frac{\Lambda \bar{\Lambda}}{\mu \bar{\mu}} \right)$
 single-valued in Λ

$$\tau_\mu(\Lambda, \phi_i) = \frac{1}{2\pi i} \ln \left(\frac{\Lambda}{\mu} \right) + f(\Lambda^b, \phi_i) = \frac{1}{2\pi i} \ln$$

$$+ 2\pi \Rightarrow \tau_n \approx \tau_{n+1}$$

$$2\pi i \Lambda^b$$

$$\mathcal{W}_\mu = \mathcal{W}_\mu(\Phi),$$

$$\mathcal{W}_\mu = \mathcal{W}_\mu(\Phi, \Lambda)$$

$$\mathcal{W}_\mu = \mathcal{W}_\mu(\Phi, \Lambda)$$

=

$$\mathcal{W}_\mu = \mathcal{W}_\mu(\Phi_i, \Lambda)$$

$$= \mathcal{W}_{\mu_i}(\Phi_i)$$

$$\mathcal{W}_\mu = \mathcal{W}_\mu(\Phi_i, \Lambda)$$

$$= \mathcal{W}_{\mu_0}(\Phi_i) +$$

$$\mathcal{W}_\mu = \mathcal{W}_\mu(\underline{\Phi}, \Lambda)$$

$$= \mathcal{W}_{\mu_0}(\underline{\Phi}) + \sum_{n=1}^N \Lambda^{n_0} \omega_n(\underline{\Phi}_n)$$

$$\mathcal{W}_\mu = \mathcal{W}_\mu(\Phi, \Lambda)$$

$$\mathcal{V}_\mu(\Phi) + \sum_{n=1}^{\infty} \Lambda^n \omega_n(\Phi)$$

$$\mathcal{W}_\mu = \mathcal{W}_\mu(\Phi, \Lambda)$$

$$= \mathcal{W}_\mu(\Phi) + \sum_{n=1}^{\infty} \frac{1}{n!} \mathcal{W}_\mu^{(n)}(\Phi)$$

$$\mathcal{Z}_\mu = \int d^4\theta \, \mathcal{Z}_\mu(\Phi) = \frac{1}{4} \int d^4\theta$$

$$\mathcal{W}_\mu = \mathcal{W}_\mu(\Phi_i, \Lambda)$$

$$= \mathcal{W}_\mu(\Phi_i) + \sum_{n=1}^{\infty} \Lambda^{n6}$$

$$\mathcal{Z}_\mu = \int d^4\theta \, z_i \bar{\Phi}_i e^{\nu \mathcal{I}}$$

$$\mathcal{I}(\theta) \left(\frac{1}{g^2} - \frac{i\theta}{8\pi^2} \right)$$

$$\mathcal{W}_\mu = \mathcal{W}_\mu(\underline{\Phi}, \Lambda)$$

$$= \mathcal{W}_{\mu_0}(\underline{\Phi}) + \sum_{n=1}^{\infty} \Lambda^{n_0} \omega_n(\underline{\Phi})$$

$$\mathcal{Z}_\mu = \int d^4\theta \, z, \bar{\Phi}_i e^\nu \Phi_i - \frac{1}{4} \int d^4\theta \, ($$

1)

$$+ \sum_{n=1}^{\infty} \lambda^{2n} \omega_n(\Phi_n)$$

$$e^{\nu} \Phi_i - \frac{1}{4} \int d\theta \left(\frac{1}{g^2} - \frac{i\theta}{8\pi^2} \right) \text{tr}(u^2) + \dots$$

$$\mathcal{W}_\mu = \mathcal{W}_\mu(\Phi_i, \Lambda)$$

$$= \mathcal{W}_\mu(\Phi_i) + \sum_{n=1}^{\infty} \Lambda^{nL} \omega_n(\Phi_i)$$

$$\int d^4\theta \sum_i \bar{\Phi}_i e^\nu \Phi_i - \frac{1}{4} \int d^4\theta \left(\frac{1}{g^2} - \frac{i\theta}{8\pi^2} \right) \text{tr}(W$$

$$\mathcal{W}_\mu = \mathcal{W}_\mu(\Phi, 1)$$

$$= \mathcal{W}_\mu(\Phi) + \sum_b \gamma^{\mu b} \omega_b(\Phi)$$

$$\mathcal{Z}_\mu = \int d^4\theta \, \mathcal{Z}_\mu(\Phi)$$

$$- \frac{1}{4} \int d^4\theta \left(\frac{1}{g^2} - \frac{i\theta}{8\pi^2} \right) \text{tr}(W^2)$$

$$\mathcal{L}_\mu = \mathcal{W}_\mu(\Phi_i, \Lambda)$$

$$= \mathcal{W}_{\mu_0}(\Phi_i) + \sum_{n=1}^{\infty} \Lambda^{n_b} \omega_n(\Phi_i)$$

$$= \int d^4\theta \, z_i \bar{\Phi}_i e^\nu \Phi_i - \frac{1}{4} \int d^4\theta \left(\frac{1}{g^2} - \frac{i\theta}{8\pi^2} \right) \text{tr}(w^2) + \dots$$

$$\mathcal{W}_\mu = \mathcal{W}_\mu(\Phi_i, \Lambda)$$

$$= \mathcal{W}_\mu(\Phi_i) + \sum_{\alpha=1}^{22} \Lambda^{\alpha 6} \omega_\alpha(\Phi_i)$$

$$\mathcal{Z}_\mu = \int d^4\theta \, z_i \bar{\Phi}_i e^{\nu} \Phi_i - \frac{1}{4} \int d^4\theta \left(\frac{1}{g^2} - \right.$$

$$E_i = -\frac{1}{4} \int d^2\theta \left(\frac{1}{g^2(\mu)} - \frac{i\theta}{8\pi^2} \right) \text{tr}(u^2) + \dots$$

$$\uparrow \frac{dg^{-2}}{d \ln \mu} = \frac{b}{8\pi^2}$$

E. NVMR thm (vst)

$\Lambda \rightarrow 0$ limit

π -instanton factor

UV scale $\mu_0 \gg \Lambda$ (weakly)

↓
'IR' scale $\mu < \mu_0$

$+ \ln\left(\frac{\Lambda}{\mu}\right)$ simple value in

$e^{+n \frac{2\pi i}{g^2}}$

$$\tau_{\mu}(\Lambda) = \tau_{\mu}(\Lambda) = \frac{i}{2\pi} \ln\left(\frac{\Lambda}{\mu}\right) + f\left(\Lambda^b, \phi_i\right) = \frac{i}{2\pi} \ln\left(\frac{\Lambda}{\mu}\right) + \sum_{n=1}^{\infty} \Lambda^{nb} c_n(\phi_i)$$

$$\theta \rightarrow \theta + 2\pi \Rightarrow \tau_n \approx \tau_{n+1}$$

$$\Lambda^b \rightarrow e^{2\pi i} \Lambda^b$$

$$\mathcal{W}_\mu = \mathcal{W}_\mu(\bar{\Phi}, \Lambda)$$

$$= \mathcal{W}_\mu(\bar{\Phi}) + \sum_{a=1}^n \Lambda^a \omega_a(\bar{\Phi})$$

$$\mathcal{Z}_\mu = \int d^4\theta \, \bar{\Phi}_i e^\nu \Phi_i - \frac{1}{4} \int d^4\theta \left(\frac{1}{g^2} - \frac{i\theta}{8\pi^2} \right) \text{tr}(W^2) +$$

$$\uparrow \frac{d\theta}{d\ln\mu} = \frac{b}{8\pi^2}$$

$$\Lambda^{nl} \omega_n(\Phi_r)$$

$$-\frac{1}{4} \int d^2\theta \left(\frac{1}{g_{(n)}} - \frac{i\theta}{8\pi^2} \right) + (n^2)$$

(higher
den)

$$\frac{dg_{(n)}}{d\ln\mu} = \frac{0}{8\pi^2}$$

$$\uparrow^{nb} \omega_n(\underline{\Phi}_n)$$

$$-\frac{1}{4} \int d^2\theta \left(\frac{1}{g_n^2} - \frac{i\theta}{8\pi^2} \right) \text{tr}(u^2) + \dots$$

(higher
dim)
↓

↑ $\frac{dg_n^2}{d\ln\mu} = \frac{b}{8\pi^2}$

$$\uparrow^{nb} \omega_n(\underline{\Phi}_n)$$

$$-\frac{1}{4} \int d^2\theta \left(\frac{1}{g_n^2} - \frac{i\theta}{8\pi^2} \right) \text{tr}(u^2) + \dots$$

(higher den)
↓

↑ $\frac{dg_n^2}{d\ln\mu} = \frac{b}{8\pi^2}$

$$W_\mu = W_\mu(\Phi_i, \Lambda)$$

$$= \tilde{W}_\mu(\bar{\Phi}_i) + \sum_{n=1}^{\infty} \Lambda^{nL} \omega_n(\bar{\Phi}_i)$$

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$$\theta \approx \bar{\Phi}_i \vee \Phi_i - \frac{1}{4} \int d^2\theta \left(\frac{1}{g_{\bar{\Phi}_i}} - \frac{1}{g_{\Phi_i}} \right)$$

$\uparrow \frac{d \log \Lambda}{d \log \Lambda}$

$$W_\mu = W_\mu(\Phi_i, \Lambda)$$

$$= W_{\mu_0}(\Phi_i) + \sum_{n=1}^{\infty} \Lambda^{nL} \omega_n(\Phi_i)$$

$$\mathcal{L} = \frac{1}{4} \partial_\mu \Phi_i \partial^\mu \bar{\Phi}_i e^\nu \Phi_i - \frac{1}{4} \int d^2\theta \left(\frac{1}{g_{ij}^2} - \frac{c}{8} \right)$$

$\uparrow \frac{d \log g}{d \log \Lambda}$

$$W_\mu = W_\mu(\Phi_i, \Lambda)$$

$$= W_\mu(\Phi_i) + \sum_{n=1}^{\infty} \Lambda^{nL} \omega_n(\Phi_i)$$

$$\int d^4\theta \sum_i \bar{\Phi}_i e^\nu \Phi_i - \frac{1}{4} \int d^4\theta \left(\frac{1}{g_{UV}^2} - \frac{c}{8} \right)$$

$$\rightarrow \Phi_i^{\omega}$$

$$\uparrow \frac{d \log}{d \log}$$

$$= \mathcal{W}_{\mu_0}(\Phi_i) + \sum_{n=1}^{\infty} \lambda^{nL} \omega_n(\Phi_i)$$

$$= \int d^4\theta \, z_i \bar{\Phi}_i e^V \Phi_i - \frac{1}{4} \int d^2\theta \left(\frac{1}{g_{YM}^2} - \frac{i\theta}{8\pi^2} \right)$$

$$\Phi_i \rightarrow \Phi_i^{\text{cw}} = \sqrt{z_i} \Phi_i = e^{-i(i \ln \sqrt{z_i})} \Phi_i \quad \uparrow \quad \frac{d g_{YM}^{-2}}{d \ln \mu} = \frac{1}{8}$$

$$= \mathcal{W}_{\mu_0}(\Phi_i) + \sum_{n=1}^{\infty} \lambda^{nb} \omega_n(\Phi_i)$$

$$\int d^4\theta \, z_i \bar{\Phi}_i e^{\nu} \Phi_i - \frac{1}{4} \int d^2\theta \left(\frac{1}{g_{(1)}^2} - \frac{i\theta}{8\pi^2} \right)$$

$$\Phi_i \rightarrow \Phi_i^{\text{cw}} = \sqrt{z_i} \Phi_i = e^{-i(i \ln \sqrt{z_i})} \Phi_i$$

$$\uparrow \frac{d g_{(1)}^{-1}}{d \ln \mu} = \frac{1}{8}$$

$$\rightarrow \frac{1}{g_{(1)}^2}$$

$$\mu \quad W_\mu(\Phi_i, \Lambda)$$

$$= W_{\mu_0}(\Phi_i) + \sum_{n=1}^{\infty} \Lambda^{nb} \omega_n(\Phi_i)$$

$$= \int d^4\theta \, z_i \bar{\Phi}_i e^V \Phi_i - \frac{1}{4} \int d^2\theta \left(\frac{1}{g_{YM}^2} - \frac{i\theta}{8\pi^2} \right)$$

$$\Phi_i \rightarrow \Phi_i^{cw} = \sqrt{z_i} \Phi_i = e^{-i(\ln \sqrt{z_i})} \Phi_i \quad \uparrow \quad \frac{d g_{YM}^{-2}}{d \ln \mu} = \frac{1}{8}$$

$$\rightarrow \frac{1}{g_{YM}^2} \rightarrow$$

$$= \mu_0 (\Phi_i) + \sum_{n=1}^{\infty} \Lambda^n \omega_n (\Phi_i)$$

$$\mathcal{I}_\mu = \int d^4\theta \, z_i \bar{\Phi}_i e^\nu \Phi_i - \frac{1}{4} \int d^4\theta \left(\frac{1}{g_i^2} - \right.$$

$$\Phi_i \rightarrow \Phi_i^w = \sqrt{z_i} \cdot \Phi_i = e^{-i(i \ln \sqrt{z_i})} \Phi_i \quad \uparrow \frac{d}{dz_i}$$

$$\rightarrow \frac{1}{g_i^2} \rightarrow \frac{1}{g_i^w{}^2} = \frac{1}{g_i^2} - \sum_i \frac{\ln \sqrt{z_i} \cdot T(R_i)}{8\pi^2}$$

	$R =$
$\bar{\Phi}_i$	$\left(\frac{2}{3}\right)$
W_k	1
Λ^b	$T(\alpha, \beta) = \frac{1}{3}$
b	$\frac{2}{3} (T(\alpha, \beta) - \frac{1}{3})$
λ	$\frac{2}{3}$

$$\beta^c = \frac{d g_{\mu}^{-2}}{d \ln \mu} =$$

=

$$\beta^c = \frac{d g_{\mu\nu}^{-2}}{d \ln \mu} =$$

=

$$\beta^c = \frac{d g_{\text{ew}}^{-2}}{d \ln \mu} = \frac{b}{8\pi^2}$$

$$\beta^{G_r} = \frac{d g_{G_r}^{-2}}{d \ln \mu} = \frac{1}{8\pi^2} \left(b + \frac{1}{2} \sum_i \gamma_i T(R_i) \right)$$

$$\gamma_i = - \frac{d \ln Z_i}{d \ln \mu}$$

$$\uparrow^{n_b} \omega_n(\underline{\Phi}_n)$$

$$- \frac{1}{4} \int d^2 \theta \left(\frac{1}{g_{\text{eff}}} - \frac{i \theta}{2} \right) + (n^2)$$

(higher
der)
↓

$$\beta^{cc} = \frac{d g_{cc}^{-1}}{d \ln \mu} = \frac{k}{8\pi^2} \left(b + \frac{1}{2} \sum_i \gamma_i T(R_i) \right)$$

$$\beta^G : \frac{d g_{\text{eff}}^{-2}}{d \ln \mu} = \frac{k}{8\pi^2} \left(b + \frac{1}{2} \sum_i \gamma_i T(R_i) \right)$$

$$\beta^{(c)} = \frac{d g_{\text{ren}}^{-2}}{d \ln \mu} = \frac{1}{8\pi^2} \left(b + \frac{1}{2} \sum_i \gamma_i T(R_i) \right)$$

"exact β -function"

$$\sum_i \gamma_i T(R_i) f(g_{en})$$

	R
Φ_i	$\left(\frac{2}{3}\right)$
W_α	1
b	$T(a)$
	$\frac{3}{2}$

$$\sum_i \gamma_i T(R_i) f(g_{en})$$

NSVZ

	R
Φ_i	$\left(\frac{2}{3}\right)$
W_α	1

T(a

3 (

$$\mathcal{W}_\mu(\Phi, \Lambda)$$

$$\gamma_i = - \frac{\delta \ln Z}{\delta \ln \mu}$$

$$= \mathcal{W}_\mu(\Phi) + \sum_{n=0}^{\infty} \Lambda^n \omega_n(\Phi)$$

$$\int d^4\theta \approx \bar{\pi}$$

$$- \frac{1}{4} \int d^4\theta \left(\frac{1}{g_i^2} - \frac{i\theta}{8\pi^2} \right) \text{tr}(W^2) + \dots$$

(higher
dim)

$$\frac{d g_i^{-1}}{d \ln \mu} = \frac{6}{8\pi^2}$$

$$\mu(\Phi_i, \Lambda)$$

$$\gamma_i = - \frac{d \ln Z}{d \ln \mu}$$

$$\mu(\Phi_i) + \sum_{n=1}^{\infty} \Lambda^{nb} \omega_n(\Phi_i)$$

$$\sum_i \bar{\Phi}_i e^\nu \Phi_i - \frac{1}{4} \int d^2 \theta \left(\frac{1}{g_{UV}^2} - \frac{i\theta}{8\pi^2} \right) \text{tr}(W^2) + \dots$$

(higher dim)
↓

$$\rightarrow \Phi_i^w = \sqrt{Z} \cdot \Phi_i = e^{-i(i \ln \sqrt{Z})} \Phi_i$$

↑ $\frac{d \ln \sqrt{Z}}{d \ln \mu} = \frac{b}{8\pi^2}$

$$\rightarrow \frac{1}{g_{UV}^2} = \frac{1}{g^2} - \sum_i \frac{b_i \sqrt{Z_i} \cdot T(R_i)}{8\pi^2}$$

$$\beta^{(2)} = \frac{d g_{\text{ren}}^{-2}}{d \ln \mu} = \frac{1}{8\pi^2} \left(b + \frac{1}{2} \sum_i \gamma_i T(R_i) \right)$$

"exact beta function"

NS

"exact β -function"

NSVZ

$$\beta_g \sim \frac{3}{2} T(\text{adj}) + \frac{1}{2} \sum_i (\gamma_i - 1) T(R_i)$$

$$\mathcal{W}_\mu = \mathcal{W}_\mu(\Phi, \Lambda)$$

$$\chi_i = - \frac{\delta \ln Z}{\delta \ln \mu}$$

$$= \mathcal{W}_\mu(\Phi) + \sum_{n=1}^{\infty} \Lambda^n \omega_n(\Phi)$$

Z

$$Z = \int \bar{\Phi} e^{\nu \Phi} - \frac{1}{4} \int d\theta \left(\frac{1}{g^2} - \frac{i\theta}{8\pi^2} \right) \text{tr}(u^2) + \dots$$

(higher dim)
↓

$$\phi_i^\omega = \sqrt{2} \phi_i = e^{-i(i\hbar \tau)} \phi_i \quad \uparrow \quad \frac{d\phi_i}{d\ln \mu} = \frac{6}{8\pi^2} \frac{1}{\mu}$$

$$\gamma \mathcal{E} = \frac{4\sqrt{c} \cdot T(R)}{8\pi^2}$$

$$\mathcal{W}_\mu = \mathcal{W}_\mu(\Phi, \Lambda)$$

$$\gamma_i = - \frac{d \ln Z}{d \ln \mu}$$

$$= \mathcal{W}_\mu(\Phi) + \sum_{n=1}^{\infty} \Lambda^n \omega_n(\Phi)$$

$$= \int d^4\theta \, \bar{\Phi} e^\nu \Phi - \frac{1}{4} \int d^4\theta \left(\frac{1}{g^2} - \frac{i\theta}{8\pi^2} \right) \text{tr}(u^2) + \dots$$

(higher dim)
↓

$$\Phi_i \rightarrow \Phi_i^\omega = \sqrt{2} \cdot \Phi_i = e^{-i(i\sqrt{2})} \Phi_i \quad \uparrow \quad \frac{d\theta}{d\ln\mu} = \frac{b}{8\pi^2}$$

$$\frac{1}{g^2} \rightarrow \frac{1}{g^2} - \sum \frac{2\sqrt{2} \cdot T(R_i)}{8\pi^2}$$

$$\beta^g = \frac{d g_{\mu}^2}{d \ln \mu} = \frac{L}{8\pi^2} \left(\underbrace{b + \frac{1}{2} \sum_i \gamma_i T(R_i)}_{\text{NSVZ}} \right) f(g_{\mu})$$

"exact" β -function

NSVZ

	$R =$
$\bar{\Phi}_i$	$\left(\frac{2}{3}\right)$
W_α	1
λ^b	$T(\text{adj}) = \frac{N^2-1}{2}$
b	$-\frac{3}{2} \left(T(\text{adj}) - \sum_i T(R_i) \right)$
1	$\frac{N^2-1}{2}$

$N=1$

$$\beta_g \sim \frac{3}{2} T(\text{adj}) + \frac{1}{2} \sum_i (\gamma_i - 1) T(R_i)$$

$$\beta^{cs} = \frac{d g_{cs}}{d \ln \mu} = \frac{1}{8\pi^2} \left(b + \frac{1}{2} \sum_i \gamma_i T(R_i) \right) f(g_{cs})$$

"exact β -function"

NSVZ

= 1

$$\beta_g \sim \frac{3}{2} T(\text{adj}) + \frac{1}{2} \sum_i (\gamma_i - 1) T(R_i)$$

$$\beta_\lambda \sim -3 + \sum_i r_i (1 + \frac{1}{2} \gamma_i)$$

"exact β -function"

NSV

$$T(ad_i) + \frac{1}{2} \sum_i (\gamma_i - 1) T(R_i) \\ + \sum_i r_i \left(1 + \frac{1}{2} \gamma_i\right)$$

$$\frac{1}{n} T(ad_i) + \frac{1}{2} \sum_i (\gamma_i - 1) T(K$$

$$3 + \sum_i r_i (1 + \frac{1}{2} \gamma_i)$$

unction"

NSVZ

$$g_i) + \frac{1}{2} \sum_i (\gamma_i - 1) T(R_i)$$

$$\sum_i r_i (1 + \frac{1}{2} \gamma_i)$$

$$w \sim \lambda \prod \phi_i^{r_i}$$

function"

NSVZ

$$g_i) + \frac{1}{2} \sum_i (\gamma_i - 1) T(R_i)$$

$$\sum_i r_i (1 + \frac{1}{2} \gamma_i) \leftarrow (2W - \lambda \prod \phi_i^{\alpha_i})$$

$$\frac{d g_{\text{ew}}}{d \ln \mu} = \frac{1}{8\pi^2} \left(b + \frac{1}{2} \sum_i \gamma_i T(R_i) \right) f(g_{\text{ew}})$$

"exact β -function"

NSVZ

$\overline{\Phi}_i$

W_α

$\wedge^b$

$$\sim \frac{3}{2} T(\text{adj}) + \frac{1}{2} \sum_i (\gamma_i - 1) T(R_i)$$

b

$$\sim -3 + \sum_i r_i (1 + \frac{1}{2} \gamma_i) \leftarrow (-\mathcal{W} - \lambda \prod_i \phi_i^{\gamma_i})$$

1

$$\beta^{g_{\mu\nu}} = \frac{d g_{\mu\nu}}{d \ln \mu} = \frac{1}{8\pi^2} \left(b + \frac{1}{2} \sum_i \gamma_i T(R_i) \right) f(g_{\mu\nu})$$

"exact β -function"

NSVZ

$$\beta_g \sim \frac{3}{2} T(\text{adj}) + \frac{1}{2} \sum_i (\gamma_i - 1) T(R_i)$$

$$\beta_\lambda \sim -3 + \sum_i r_i (1 + \frac{1}{2} \gamma_i) \leftarrow (-\mathcal{W} - \lambda \prod \phi_i^{r_i})$$

F. Leigh-Strassler

$$\beta^a \frac{d g_{in}^a}{d \ln \mu} = \frac{1}{8\pi^2} \left(b + \frac{1}{2} \sum_i \gamma_i T(R_i) \right) f(g_{in})$$

"exact" function

NSVZ

N=1

$$\beta_g \sim \frac{3}{2} T(ad_1) + \frac{1}{2} \sum_i (\gamma_i - 1) T(R_i)$$

$$\beta_\lambda \sim -3 + \sum_i r_i (1 + \frac{1}{2} \gamma_i) - (2d - \lambda \prod \phi_i^r)$$

F. Le...

(g_{en})

Z

exactly
marginal
operator

$\swarrow$
 uv -free

$F, \underline{Le}$

F. Leigh-Strassler

$$+ \Lambda^{nb} c$$

F. Leigh-Strassler

$$+ \Lambda^{nb} C_n$$

F. Leigh-Strassler

$$\dots + \bigwedge^{n_b} c_n(\phi_i)$$

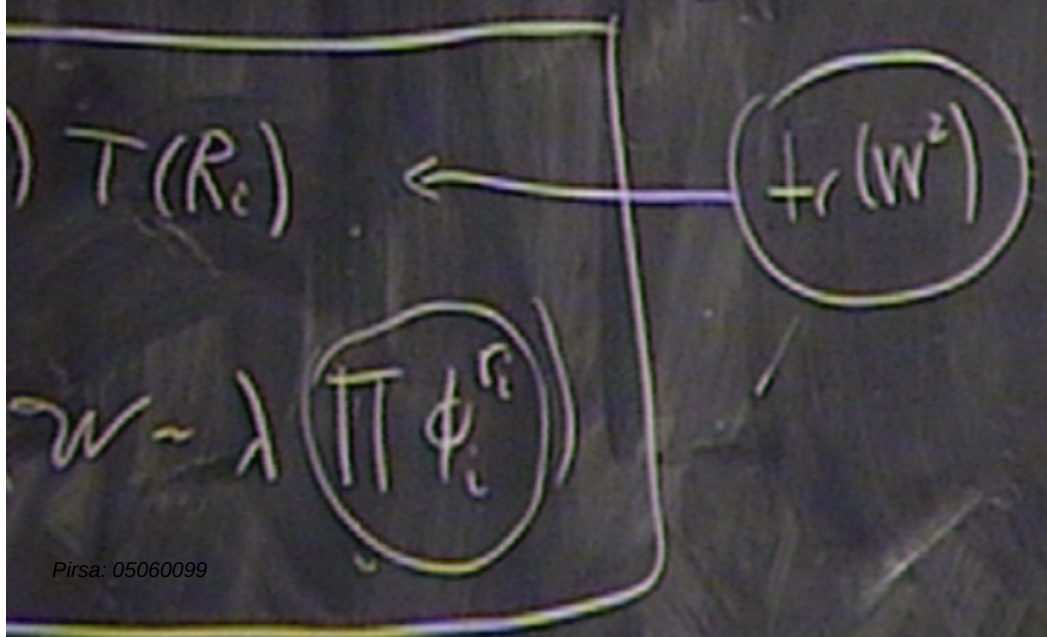
$$T(R_c) f(g_{\text{em}})$$

NSVZ

exactly
marginal
operator
uv-free

F. Leigh

... +



F. Leigh-Strassler

$$\dots + \bigwedge^{nb} c_n(\underline{\phi_i})$$

E. NVMR thm (vst)

$\Lambda \rightarrow 0$ limit

11-instanton factor

UV scale $\mu_0 \gg \Lambda$ (weakly)

↓
'IR' scale $\mu < \mu_0$

$+ \ln^2 \left(\frac{\Lambda \bar{\Lambda}}{\mu^2} \right)$ simple values

$e^{+n \frac{8\pi^2}{g^2}}$

$$\tau_n = \tau_\mu(\Lambda, \phi_i) = \frac{1}{2\pi i} \ln \left(\frac{\Lambda}{\mu} \right) + f(\Lambda^b, \phi_i) = \frac{1}{2\pi i} \ln \left(\frac{\Lambda}{\mu} \right) + \sum_{n=1}^{\infty} \Lambda^{nb} \underline{c_n(\phi_i)}$$

$$\theta = \theta + 2\pi \Rightarrow \tau_n = \tau_{n+1}$$

$$\Lambda^b \rightarrow e^{2\pi i} \Lambda^b$$

F. Leigh-Stratton

$$\dots + \bigwedge^{nb} c_n(\underline{\phi}_i)$$

F. Leigh-Strassler

$$\dots + \Lambda^{nb} c_n(\underline{\phi_i})$$

Examples

$SU(N_c)$

F. Leigh-Strassler

$$\dots + \Lambda^{nb} c_n(\underline{\phi_i})$$

Examples

$SU(N_c)$ w/

F. Leigh-Strassler

$$\dots + \Lambda^{nb} c_n(\underline{\phi_i})$$

Examples

$$SU(N_c) \quad w/ \quad \begin{cases} 2N_c & Q_i & \sim N_c \\ 2N_c & \tilde{Q}_i & \sim \bar{N}_c \\ 1 & \Phi & \end{cases}$$

$$+ \Lambda^{nb} c_n(\underline{\phi_i})$$

Examples

$$SU(N_c) \quad w/ \quad \begin{cases} 2N_c & Q^i & \text{in } \underline{N_c} \\ 2N_c & \tilde{Q}_i & \text{in } \overline{N_c} \\ 1 & \Phi & \text{adjoint} \end{cases}$$

$$+ \Lambda^{nb} c_n(\underline{\phi}_i)$$

Examples

$$SU(N_c) \quad w/ \quad \begin{cases} 2N_c & Q_i & \text{in } \underline{N_c} \\ 2N_c & \tilde{Q}_i & \text{in } \overline{N_c} \\ 1 & \underline{\Phi} & \text{adjoint} \end{cases} \quad \chi \neq f$$

$$+ \Lambda^{nb} c_n(\underline{\phi}_i)$$

plus

$$SU(N_c) \quad w/ \quad \left\{ \begin{array}{lll} 2N_c & Q_i & \text{in } \underline{N_c} \\ 2N_c & \tilde{Q}_i & \text{in } \overline{N_c} \\ 1 & \underline{\Phi} & \text{adjoint} \end{array} \right. \quad \chi^2 f$$

$$w =$$

$$+ \sum^{nb} c_n(\underline{\phi}_i)$$

Examples

$$SU(N_c) \quad w/ \begin{cases} 2N_c & Q^i & \text{in } \underline{N_c} \\ 2N_c & \tilde{Q}_i & \text{in } \overline{N_c} \\ 1 & \Phi & \text{adjoint} \end{cases} \quad \chi^2 f$$

$$w = \lambda \sum_n \text{tr}(\tilde{Q}_n \Phi Q_n)$$

$$+ \sum^{nb} c_n(\underline{\phi}_i)$$

examples

cond.

$$\left\{ \begin{array}{lll} 2N_c & Q_i & \text{in } \underline{N_c} \\ 2N_c & \tilde{Q}_i & \text{in } \overline{N_c} \\ 1 & \underline{\Phi} & \text{adjoint} \end{array} \right. \quad \chi^{\text{sf}}$$

$$\text{Tr}(\tilde{Q}_i \Phi)$$

$$+ \sum^{n_b} c_n(\underline{\phi}_i)$$

$$w/ \begin{cases} 2N_c & Q^i & \text{in } \underline{N_c} \\ 2N_c & \tilde{Q}_i & \text{in } \overline{N_c} \\ 1 & \underline{\Phi} & \text{adjoint} \end{cases} \quad \chi^2 f$$

$$= \lambda \sum_i \text{tr}(\tilde{Q}_i \Phi Q^i)$$

$$+ \sum^{nb} c_n(\underline{\phi}_i)$$

$$w/ \begin{cases} 2N_c & Q^i & \text{in } \underline{N_c} \\ 2N_c & \tilde{Q}_i & \text{in } \overline{N_c} \\ 1 & \Phi & \text{adjoint} \end{cases} \quad \chi^2 f$$

$$\lambda \sum \text{tr}(\tilde{Q}_i \Phi Q^i)$$

$$+ \bigwedge^{n_b} c_n(\underline{\phi}_i)$$

Examples

$$SU(N_c) \quad w/ \begin{cases} 2N_c & Q^i & \text{in } \underline{N_c} \\ 2N_c & \tilde{Q}_i & \text{in } \overline{N_c} \\ 1 & \Phi & \text{adjoint} \end{cases} \quad \chi^2 f$$

$\mathcal{L}$

$$\sum_i \text{tr}(\tilde{Q}_i \Phi Q^i)$$

$$+ \sum^{nb} c_n(\underline{\phi}_i)$$

Examples

$$SU(N_c) \quad w/ \begin{cases} 2N_c & Q^i & \text{in } \underline{N_c} \\ 2N_c & \tilde{Q}_i & \text{in } \overline{N_c} \\ 1 & \underline{\Phi} & \text{adjoint} \end{cases} \quad \chi^{\text{st}}$$

$$w = \lambda \sum_i \text{tr}(\tilde{Q}_i \Phi Q^i)$$

$$+ \Lambda^{nb} c_n(\underline{\phi}_i)$$

examples

$$SU(N_c) \quad w/ \begin{cases} 2N_c & Q^i & \text{in } \underline{N_c} \\ 2N_c & \tilde{Q}_i & \text{in } \overline{N_c} \\ 1 & \underline{\Phi} & \text{adjoint} \end{cases}$$

$$w = \lambda \sum_i \text{tr}(\tilde{Q}_i \Phi Q^i)$$

$$\left\{ \begin{array}{l} 2N_c \\ 1 \end{array} \right. \quad \begin{array}{l} Q_i \\ \Phi \end{array} \quad \text{adjoint}$$

$$\text{tr}(\bar{Q}_i \Phi Q^i) \quad U(2N_c)$$

$$\gamma_{Q_i} = \gamma_{\bar{Q}_i} = \gamma_Q$$

$$\beta_g \sim \frac{3}{2}$$

$$\gamma_{Q_i} = \gamma_{\tilde{Q}_i} = \gamma_Q$$

$$\beta_g \sim \frac{3}{2}$$

$$\gamma_{Q_i} = \gamma_{\tilde{Q}_i} = \gamma_Q$$

$$\beta_g \sim \frac{3}{2} 2N_c +$$

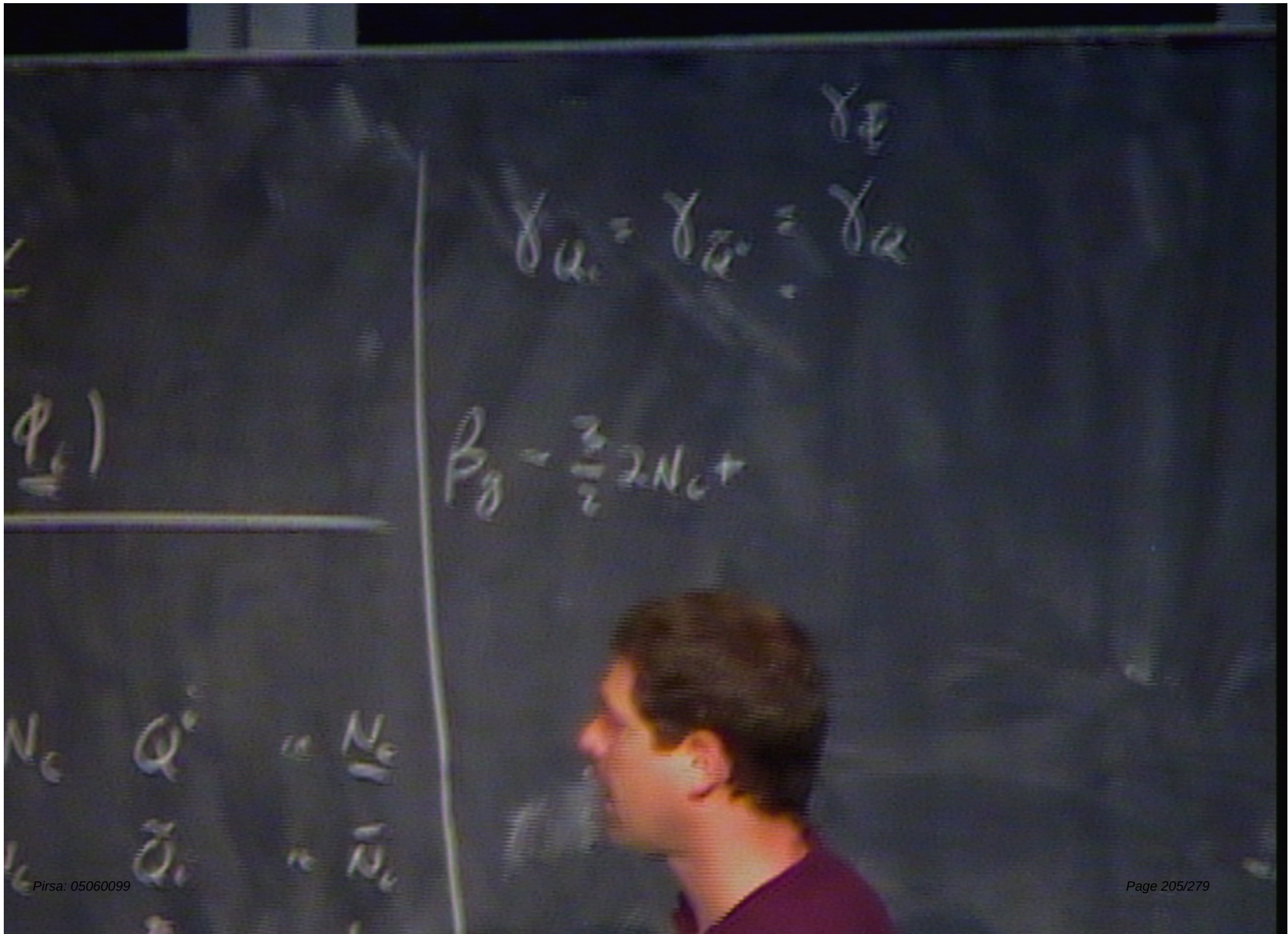
$$\gamma_{Q_i} = \gamma_{\bar{Q}_i} = \gamma$$

$$\beta_g \sim \frac{3}{2} 2N_c +$$

$$1. \frac{N_c}{1}$$

$$2. \frac{N_c}{1}$$

E.



$$\gamma_{a_+} = \gamma_{a_-} = \gamma_a$$

$$\beta_g = \frac{3}{2} 2N_c +$$

ϕ_{+1}

N_c Q_i N_c
 $\tilde{Q}_i$ N_c

$$\gamma_{\Phi}$$

$$\gamma_{Q_i} = \gamma_{\bar{Q}_i} = \gamma_Q$$

$$\underline{\Phi_i}$$

$$\beta_g \sim \frac{3}{2} 2N_c +$$



$$\gamma_{Q_i} = \gamma_{\tilde{Q}_i} = \gamma_Q$$

$$\beta_g \sim \frac{3}{2} 2N_c + \frac{1}{2} 2N_c (\gamma_Q - 1)$$

$$\gamma_{Q_i} = \gamma_{\tilde{Q}_i} = \gamma_Q$$

$$\beta_g \sim \frac{3}{2} 2N_c + \frac{1}{2} 2N_c (\gamma_{\tilde{Q}} - 1) + \frac{1}{2} 4N_c (\gamma_Q - 1)$$

$$\gamma_{\Phi}$$

$$\gamma_{Q_i} = \gamma_{\tilde{Q}_i} = \gamma_Q$$

$$\sim \frac{3}{2} 2N_c + \frac{1}{2} 2N_c (\gamma_{\Phi} - 1) + \frac{1}{2} 4N_c (\gamma_Q - 1)$$

~

$$\gamma_{Q_i} = \gamma_{\tilde{Q}_i} = \gamma_Q$$

$$\beta_g \sim \frac{3}{2} 2N_c + \frac{1}{2} 2N_c (\gamma_{\tilde{Q}} - 1) + \frac{1}{2} 4N_c (\gamma_Q - 1)$$

~

$$\gamma_{Q_i} = \gamma_{\tilde{Q}_i} = \gamma_Q$$

$$\beta_g \sim \frac{3}{2} 2N_c + \frac{1}{2} 2N_c (\gamma_{\tilde{Q}} - 1) + \frac{1}{2} 4N_c (\gamma_Q - 1)$$

$$\gamma_{Q_i} = \gamma_{\tilde{Q}_i} = \gamma_Q$$

$$\beta_g \sim \frac{3}{2} 2N_c + \frac{1}{2} 2N_c (\gamma_{\tilde{Q}} - 1) + \frac{1}{2} 4N_c (\gamma_Q - 1)$$

~

$$\gamma_{Q_i} = \gamma_{\tilde{Q}_i} = \gamma_Q$$

$$\beta_g \sim \frac{3}{2} 2N_c + \frac{1}{2} 2N_c (\gamma_{\tilde{Q}} - 1) + \frac{1}{2} 4N_c (\gamma_Q - 1)$$

$$\sim \gamma_{\tilde{Q}} + 2\gamma_Q$$

$$\gamma_{Q_i} = \gamma_{\tilde{Q}_i} = \gamma_Q$$

$$\beta_g \sim \frac{3}{2} 2N_c + \frac{1}{2} 2N_c (\gamma_{\tilde{Q}} - 1) + \frac{1}{2} 4N_c (\gamma_Q - 1)$$

$$\sim \gamma_{\tilde{Q}} + 2\gamma_Q$$

$$\beta_\lambda \sim \gamma_{\tilde{Q}} + 2\gamma_Q$$

$$\beta_g \sim \frac{3}{2} 2N_c + \frac{1}{2} N_c (\gamma_{\bar{\Phi}} - 1) + \frac{1}{2} 4N_c (\gamma_{\Phi} - 1)$$

$$\sim \gamma_{\bar{\Phi}} + 2\gamma_{\Phi}$$

$$\beta_{\lambda} \sim \gamma_{\bar{\Phi}} + 2\gamma_{\Phi}$$

N_c

N_c

adjoint

f.p.

$$\beta_g = \beta$$

$(2N_c)$

$$\beta_g \sim \frac{3}{2} 2N_c + \frac{1}{2} 2N_c (\gamma_{\bar{\Phi}} - 1) + \frac{1}{2} 4N_c (\gamma_{\Phi} - 1)$$

$$\sim \gamma_{\bar{\Phi}} + 2\gamma_{\Phi}$$

$$\beta_{\lambda} \sim \gamma_{\bar{\Phi}} + 2\gamma_{\Phi}$$

N_c

$\bar{N}_c$

adjoint

f.p.

$$\beta_g = \beta_{\lambda} = 0$$

$(2N_c)$

$$\beta_g \sim \frac{3}{2} 2N_c + \frac{1}{2} 2N_c (\gamma_{\bar{\Phi}} - 1) + \frac{1}{2} 4N_c (\gamma_{\Phi} - 1)$$

$$\sim \gamma_{\bar{\Phi}} + 2\gamma_{\Phi}$$

$$\beta_{\lambda} \sim \gamma_{\bar{\Phi}} + 2\gamma_{\Phi}$$

N_c

$\bar{N}_c$

adjoint

f.p.

$$\beta_g = \beta_{\lambda} = 0$$

λ

$(2N_c)$

$$0\Phi + 0Q$$

$$\text{in } \underline{N_c}$$

$$\beta_\lambda \sim \gamma_{\underline{\Phi}} + 2\gamma_Q$$

$$\text{in } \overline{N_c}$$

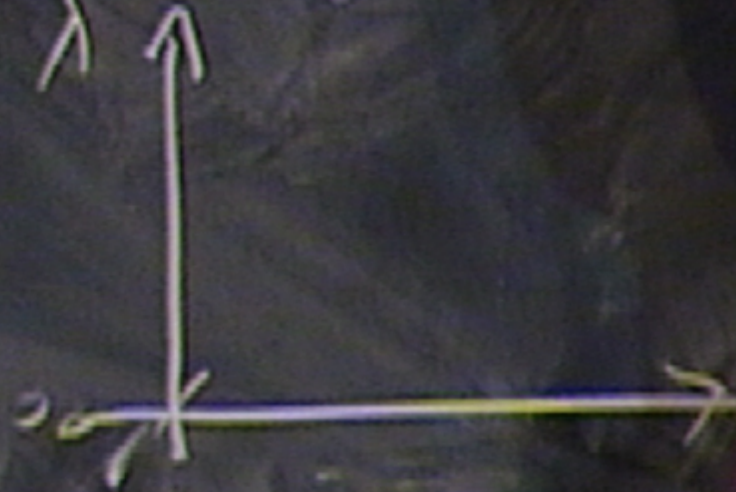
adjoint

$$f \cdot p$$

$$\beta_g = \beta_\lambda = 0$$

$$\lambda$$

$$U(RN_c)$$



$$\beta_\lambda \sim \gamma_{\bar{\Phi}} + 2\gamma_Q$$

in $\bar{N}_c$

adjoint

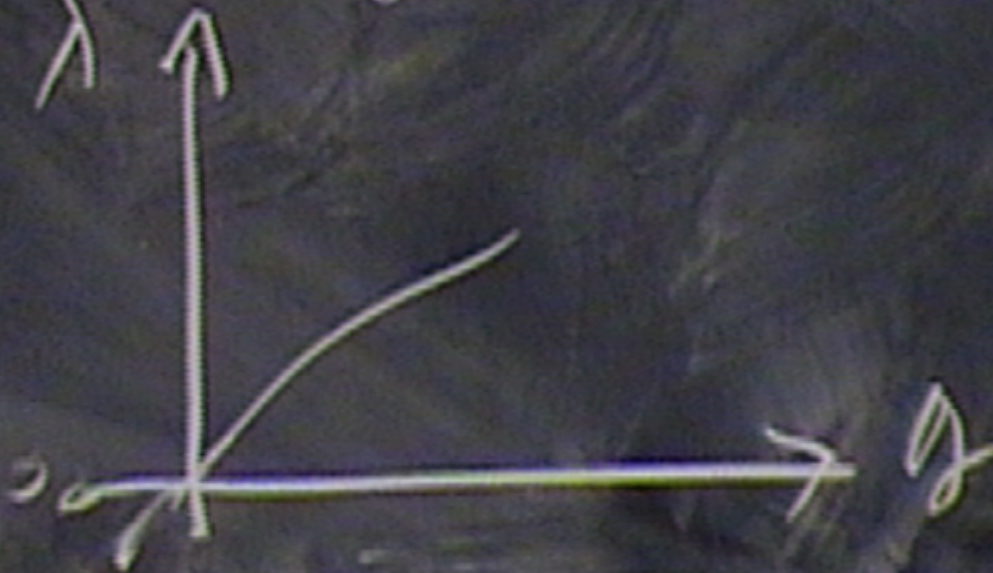
f.p.

$$\beta_g = \beta_\lambda = 0$$

λ

$\bar{N}_c$

f



$$\beta_g \sim \frac{3}{2} 2N_c + \frac{1}{2} 2N_c (\gamma_{\overline{\Phi}} - 1) + \frac{1}{2} 4N_c (\gamma_Q - 1)$$

$$\sim \gamma_{\overline{\Phi}} + 2\gamma_Q$$

$$\beta_{\lambda} \sim \gamma_{\overline{\Phi}} + 2\gamma_Q$$

$$\beta_g = \beta_{\lambda} = 0$$

λ ↑



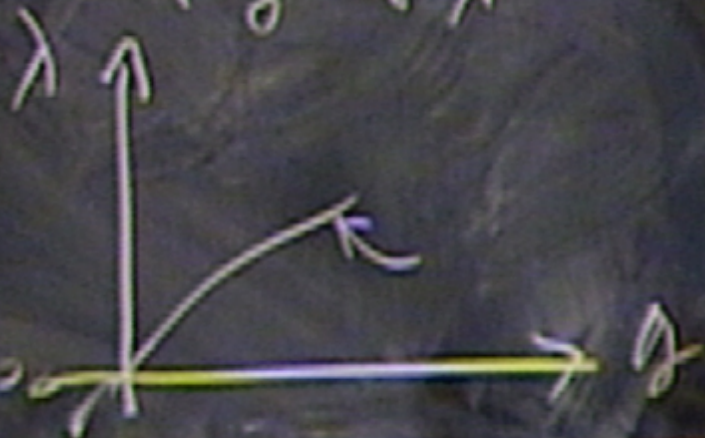
$$\beta_g \sim \frac{3}{2} 2N_c + \frac{1}{2} 2N_c (\gamma_{\Phi} - 1) + \frac{1}{2} 4N_c (\gamma_Q - 1)$$

$$\sim \gamma_{\Phi} + 2\gamma_Q$$

$$\beta_{\lambda} \sim \gamma_{\Phi} + 2\gamma_Q$$

joint f.p.

$$\beta_g = \beta_{\lambda} = 0$$

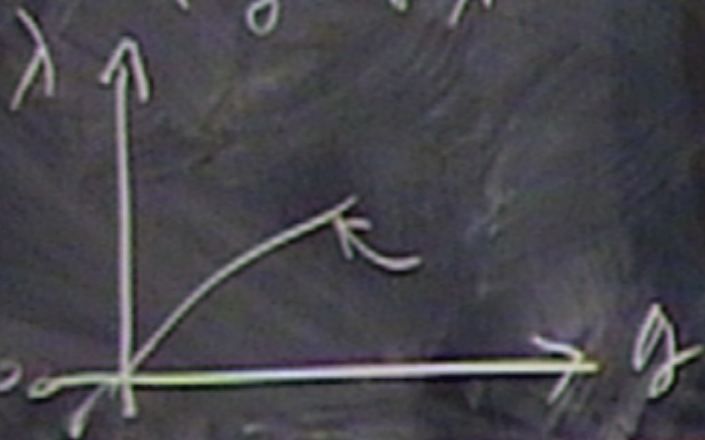


$$\beta_g \sim \frac{3}{2} 2N_c + \frac{1}{2} 2N_c (\gamma_{\Phi} - 1) + \frac{1}{2} 4N_c (\gamma_Q - 1)$$

$$\sim \gamma_{\Phi} + 2\gamma_Q \quad \lambda = \sqrt{2}$$

$$\beta_{\lambda} \sim \gamma_{\Phi} + 2\gamma_Q$$

f.p. $\beta_g = \beta_{\lambda} = 0$



$$\beta_g \sim \frac{3}{2} 2N_c + \frac{1}{2} 2N_c (\gamma_{\Phi} - 1) + \frac{1}{2} 4N_c (\gamma_Q - 1)$$

$$\sim \gamma_{\Phi} + 2\gamma_Q$$

$$\beta_{\lambda} \sim \gamma_{\Phi} + 2\gamma_Q$$

$$\lambda = \sqrt{2} g$$

$$\beta_g = \beta_{\lambda} = 0$$

λ



$$\beta_g \sim \frac{3}{2} 2N_c + \frac{1}{2} 2N_c (\gamma_{\bar{\Phi}} - 1) + \frac{1}{2} 4N_c (\gamma_{\Phi} - 1)$$

$$\sim \gamma_{\bar{\Phi}} + 2\gamma_{\Phi}$$

$$\lambda = \sqrt{2} g$$

$$\beta_{\lambda} \sim \gamma_{\bar{\Phi}} + 2\gamma_{\Phi}$$

$$\beta_g = \beta_{\lambda} = 0$$

λ



transfer

$$\uparrow^{nb} C_n(\underline{\phi}_i)$$

$$w/ \begin{cases} 2N_c & Q^i \\ 2N_c & \tilde{Q}_i \\ 1 & \Phi \end{cases} \quad \begin{matrix} \text{in } N_c \\ \text{in } \bar{N}_c \\ \text{adjoint} \end{matrix}$$

$$\lambda \sum_i \text{tr}(\tilde{Q}_i \Phi Q^i) \quad U(N_c)$$

$$\gamma_{Q_i} = \gamma_{\bar{Q}_i} = \gamma_Q$$

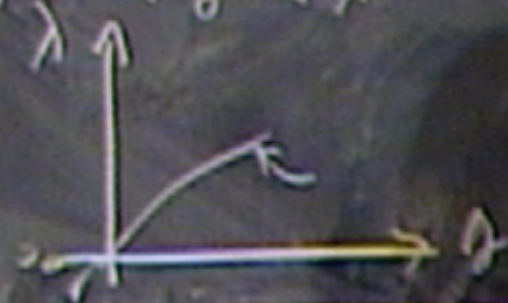
$$\beta_g \sim \frac{3}{2} 2N_c + \frac{1}{2} N_c (\gamma_Q - 1) \Rightarrow \frac{1}{2} 4N_c (\gamma_Q - 1)$$

$$\sim \gamma_{\Phi} + 2\gamma_Q$$

$$\lambda = \sqrt{2} g$$

$$\beta_{\lambda} \sim \gamma_{\Phi} + 2\gamma_Q$$

$$\beta_g = \beta_{\lambda} = 0$$



II. $N=1$ SQCD $\leftrightarrow$ Seiberg-duality

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II. $N=1$ SQCD $\leftrightarrow$ Seiberg-duality

N_f Q^i

N_f

II. $N=1$ SQCD $\leftrightarrow$ Seiberg-duality

$$\begin{array}{cc} N_f & Q_i \\ N_c & \tilde{Q}_i \end{array}$$

II. $N=1$ SQCD $\leftrightarrow$ Seiberg-duality

$$N_f \quad Q_i \quad (N_c)$$

$$N_f \quad \tilde{Q}_i \quad (\bar{N}_c)$$

II. $N=1$ SQCD $\leftrightarrow$ Seiberg-dual

$$\chi_{sf} \left\{ \begin{array}{ll} N_f & Q_i \quad (N_c) \\ N_f & \tilde{Q}_i \quad (\bar{N}_c) \end{array} \right.$$

II . $N=1$ SQCD $\leftrightarrow$ Seiberg-dual

$$\chi_{sf} \left\{ \begin{array}{ll} N_f & Q_i \quad (N_c) \\ N_f & \tilde{Q}_i \quad (\bar{N}_c) \end{array} \right.$$

$$(N=0)$$

II . $N=1$ SQCD $\leftrightarrow$ Seiberg-dual

$$\chi_{sf} \left\{ \begin{array}{ll} N_f & Q_i \quad (N_c) \\ N_f & \tilde{Q}_i \quad (\bar{N}_c) \end{array} \right.$$

$$(N=0)$$

$$\beta_g \propto$$

II. $N=1$ SQCD $\leftrightarrow$ Seiberg-dual

$$\chi_{sf} \left\{ \begin{array}{l} N_f \quad Q^i \quad (N_c) \quad U(N_f) \times U(N_f) \\ \vdots \quad \vdots \quad \vdots \quad \vdots \end{array} \right.$$

$$\tilde{Q}_i$$

$$\beta_g \propto$$

II . $N=1$ SQCD $\leftrightarrow$ Seiberg-dual

$$\begin{array}{lcl}
 \chi_{sf} \left\{ \begin{array}{l} N_f \quad Q^i \quad (N_c) \\ N_f \quad \sim \quad (\bar{N}_c) \end{array} \right. & & U(N_f) \times U(N_f) \\
 (N=0) & &
 \end{array}$$

II. $N=1$ SQCD $\leftrightarrow$ Seiberg-dual

$$\chi_{sf} \left\{ \begin{array}{ll} N_f & Q_i \\ N_f & \tilde{Q}_i \end{array} \right. \begin{array}{ll} (N_c) & \\ & (\bar{N}_c) \end{array} \quad SU(N_f) \times SU(N_f)$$

$$(N=0)$$

$$\beta_g \propto$$

→ Seiberg - duality

$$SU(N_f) \times SU(N_f) \Rightarrow \gamma_a = \gamma_{\bar{a}}$$

→ Seiberg-duality

$$SU(N_f) \times SU(N_f) \Rightarrow \gamma_{\vec{a}} = \gamma_{\vec{a}'}.$$

SQCD $\leftrightarrow$ Seiberg-duality

$$Q: (N_c) \quad SU(N_f) \times SU(N_f) \Rightarrow \gamma_Q = \gamma_{\tilde{Q}}$$

$$\tilde{Q}: (\bar{N}_c)$$

$$\beta_g \propto N_c - N_f + N_f \gamma$$

SQCD $\leftrightarrow$ Seiberg-duality

$$Q: (N_c) \quad SU(N_f) \times SU(N_f) \Rightarrow \gamma_{Q_+} = \gamma_{Q_-}$$

$$\tilde{Q}: (\bar{N}_c)$$

$$N_f < 3N_c \quad AF$$

$$3g \rightarrow 3\gamma + N_f \gamma$$

II. $N=1$ SQCD $\leftrightarrow$ Seiberg-duality

$$\text{(sf)} \left\{ \begin{array}{lll} N_f & Q_i & (N_c) \\ N_f & \tilde{Q}_i & (\bar{N}_c) \end{array} \right. \quad SU(N_f) \times SU(N_f) \Rightarrow \gamma_{Q_i} =$$

$$N_f < 3N_c$$

$$\beta_g \propto 3N_c - N_f + N_f \gamma$$

II. $N=1$ SQCD $\leftrightarrow$ Seiberg-duality

$$\text{(sf)} \left\{ \begin{array}{l} N_f \quad Q_i \quad (N_c) \\ N_f \quad \tilde{Q}_i \quad (\bar{N}_c) \end{array} \right. \quad SU(N_f) \times SU(N_f) \Rightarrow \gamma_{\text{QCD}} =$$

($N=0$)

$$N_f < 3N_c$$

$$\beta_g \propto 3N_c - N_f + N_f \gamma$$

$$D[Q] = 1 + \frac{1}{2} \gamma = \frac{3}{2} R(\phi)$$

II. $N=1$ SQCD $\leftrightarrow$ Seiberg-duality

$$\left\{ \begin{array}{l} N_f \text{ } Q_i \text{ } (N_c) \\ N_f \text{ } \tilde{Q}_i \text{ } (\bar{N}_c) \end{array} \right. \quad SU(N_f) \times SU(N_f) \Rightarrow \gamma_{\text{QCD}} = 0$$

$$N_f < 3N_c$$

$$\beta_g \propto 3N_c - N_f + N_f \gamma$$

$$D[Q_i] = 1 + \frac{1}{2} \gamma = \frac{3}{2} R(Q)$$

$$\sum_i (R(Q_i) + R(\tilde{Q}_i)) = 2(N_f - N_c)$$

II. $N=1$ SQCD $\leftrightarrow$ Seiberg-duality

$$\text{(sf)} \left\{ \begin{array}{ll} N_f & Q_i \\ & \tilde{Q}_i \end{array} \right. \begin{array}{l} (N_c) \\ (\bar{N}_c) \end{array} \quad SU(N_f) \times SU(N_f) \Rightarrow \gamma_Q =$$

($N=0$)

$$N_f < 3N_c$$

$$\beta_g \propto 3N_c - N_f + N_f \gamma$$

$$D[Q] = 1 + \frac{1}{2} \gamma = \frac{3}{2} R(Q)$$

a fixed point

$$\sum_i (R(Q_i) + R(\tilde{Q}_i)) = 2(N_f - N_c)$$

II. $N=1$ SQCD $\leftrightarrow$ Seiberg-duality

$$\text{(sf)} \left\{ \begin{array}{ll} N_f & Q_i \quad (N_c) \\ N_f & \tilde{Q}_i \quad (\bar{N}_c) \end{array} \right. \quad SU(N_f) \times SU(N_f) \Rightarrow \gamma_{Q_i} =$$

$$(N=0)$$

$$\beta_g \propto 3N_c - N_f + N_f \gamma$$

$$D[Q_i] = 1 + \frac{1}{2} \gamma = \frac{3}{2} R(Q_i)$$

at fixed point

$$\sum_i (R(Q_i) + R(\tilde{Q}_i)) = 2$$

II. $N=1$ SQCD $\leftrightarrow$ Seiberg-duality

$$\text{(sf)} \left\{ \begin{array}{ll} N_f & Q_i \\ & \tilde{Q}_i \end{array} \right. \begin{array}{l} (N_c) \\ (\bar{N}_c) \end{array} \quad SU(N_f) \times SU(N_f) \Rightarrow \gamma_Q =$$

($N=0$)

$$N_f < 3N_c$$

$$\beta_g \propto 3N_c - N_f + N_f \gamma$$

$$D[Q] = 1 + \frac{1}{2} \gamma = \frac{3}{2} R(Q)$$

at fixed point

$$\sum_i (R(Q_i) + R(\tilde{Q}_i)) = 2(N_f - N_c)$$

	R
Q_i	$1 - \frac{N_c}{N_f}$

$\tilde{Q}^i$	$1 - \frac{N_c}{N_f}$
---------------	-----------------------

W_α	1
------------	---

Λ	
-----------	--

	R
Q_i	1 - $\frac{N_L}{N_T}$
$\tilde{Q}^i$	1 - $\frac{N_L}{N_T}$
W_2	1
Λ	0

	R
Q_i	$1 - \frac{N_c}{N_f}$
$\tilde{Q}^i$	$1 - \frac{N_c}{N_f}$
W_α	1
Λ	0

$$\gamma_{21} = \gamma$$

4F

	R
$\bar{Q}_1$	$1 - \frac{N_c}{N_f}$
$\bar{Q}_1'$	$1 - \frac{N_c}{N_f}$
W_2	1
$\wedge$	0

	R
Q_i	$1 - \frac{N_c}{N_f}$
$\tilde{Q}^i$	$1 - \frac{N_c}{N_f}$
W_2	1
Λ	0

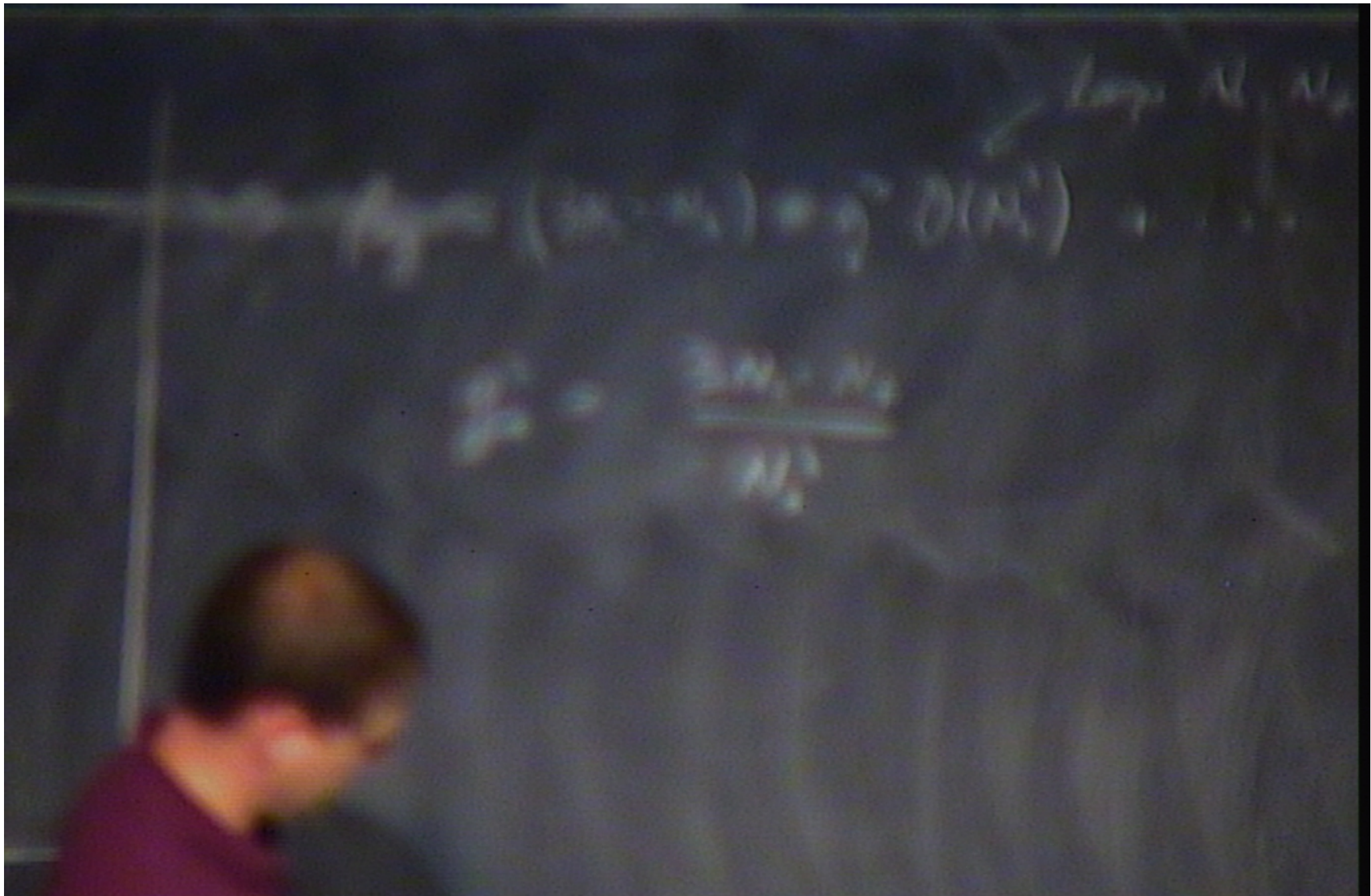
	R
Q_i	$1 - \frac{N_c}{N_f}$
$\tilde{Q}^i$	$1 - \frac{N_c}{N_f}$
W_2	1
Λ	0

$$\beta_g \approx$$

$$\beta_g \approx (3N_c - N_f) - g^2 \mathcal{O}(N_c^3) +$$

$$\beta_g \approx (3N_c - N_f) - g^2 \mathcal{O}(N_c^3) + \dots$$

$$g_*^2 = \frac{3N_c - N_f}{\sim N_c^3}$$



large N_c, N_f

$$\beta_g \approx (3N_c - N_f) - g^2 \mathcal{O}(N_c^3) + \dots$$

$$g_*^2 = \frac{3N_c - N_f}{\sim N_c^3}$$

large N_c, N_f

$$\beta_g \approx (3N_c - N_f) g^2 \mathcal{O}(N_c^3) + \dots$$

$$g_*^2 = \frac{3N_c - N_f}{\sim N_c^3}$$



large N_c, N_f

$$(3N_c - N_f) \sim g^2 \mathcal{O}(N_c^3) + \dots$$

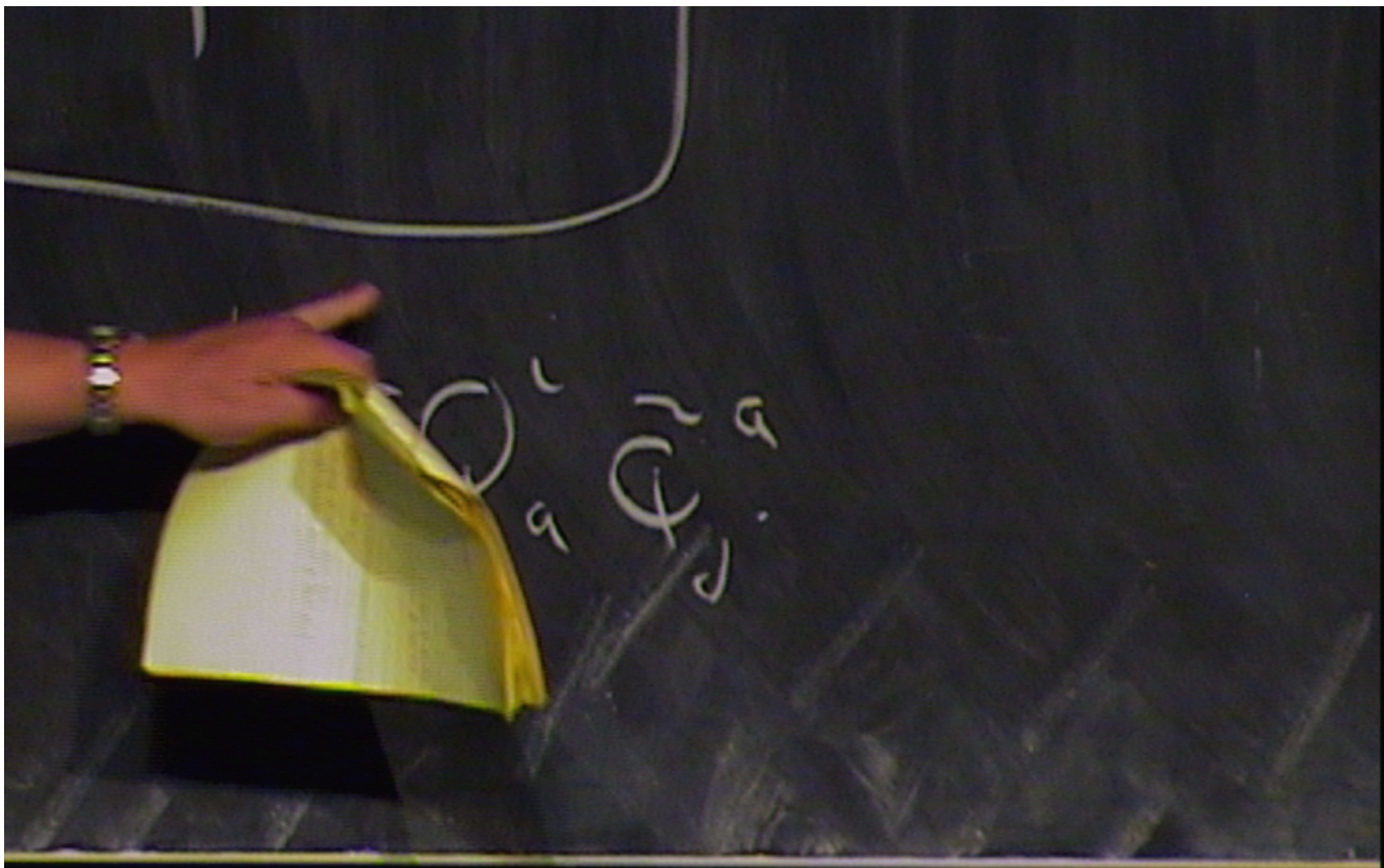
$$= \frac{3N_c - N_f}{\sim N_c^3}$$

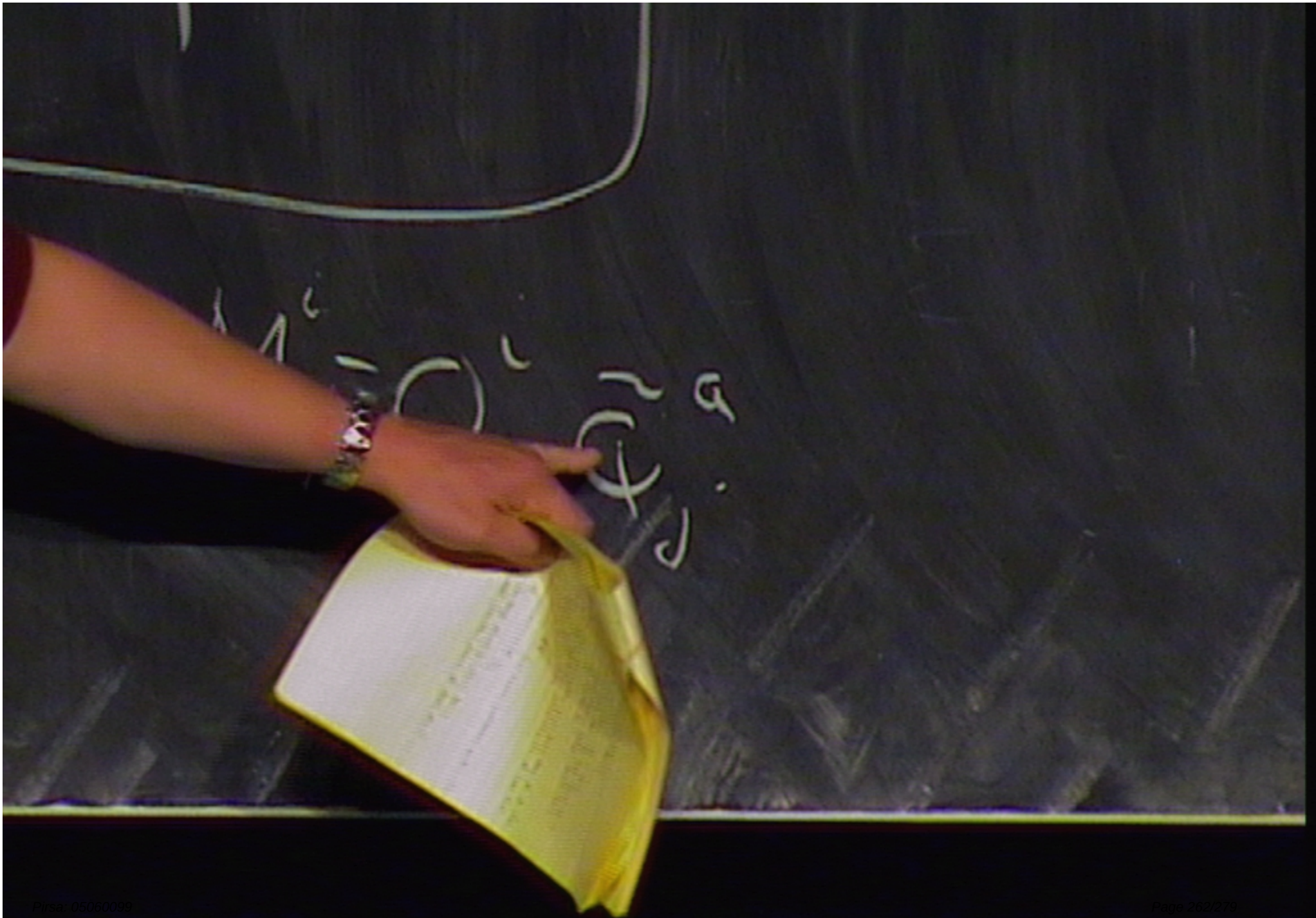


F. Le...

Example

$SU(N_c)$





$$M_i^c = Q_a^c \cdot Q_j^a$$

$$Q: 1 - \frac{N_c}{N_f}$$

$$\tilde{Q}: 1 - \frac{N_c}{N_f}$$

$$W_2$$

$$\wedge$$

$$\beta_g = (3N_c - N_f)$$

$$g_*^2 = \frac{3N_c}{N_f}$$

$$R[M] =$$

$$1 - \frac{N_c}{N_f}$$

$$1 - \frac{N_c}{N_f}$$

$$P_g = (5N_c - N_f) = g \quad 0.11$$

$$g^2 = \frac{3N_c - N_f}{N_c^3}$$

$$R[M] = \frac{2(N_f - N_c)}{N_f}$$

$$1 - \frac{N_c}{N_f}$$

$$1 - \frac{N_c}{N_f}$$

$$\beta_g = (5N_c - N_f) = g \text{ U(1)}$$

$$g_*^2 = \frac{3N_c - N_f}{N_c^3}$$

$$R[M] = \frac{2(N_f - N_c)}{N_f}$$

11
2

$$1 - \frac{N_c}{N_f}$$

$$1 - \frac{N_c}{N_f}$$

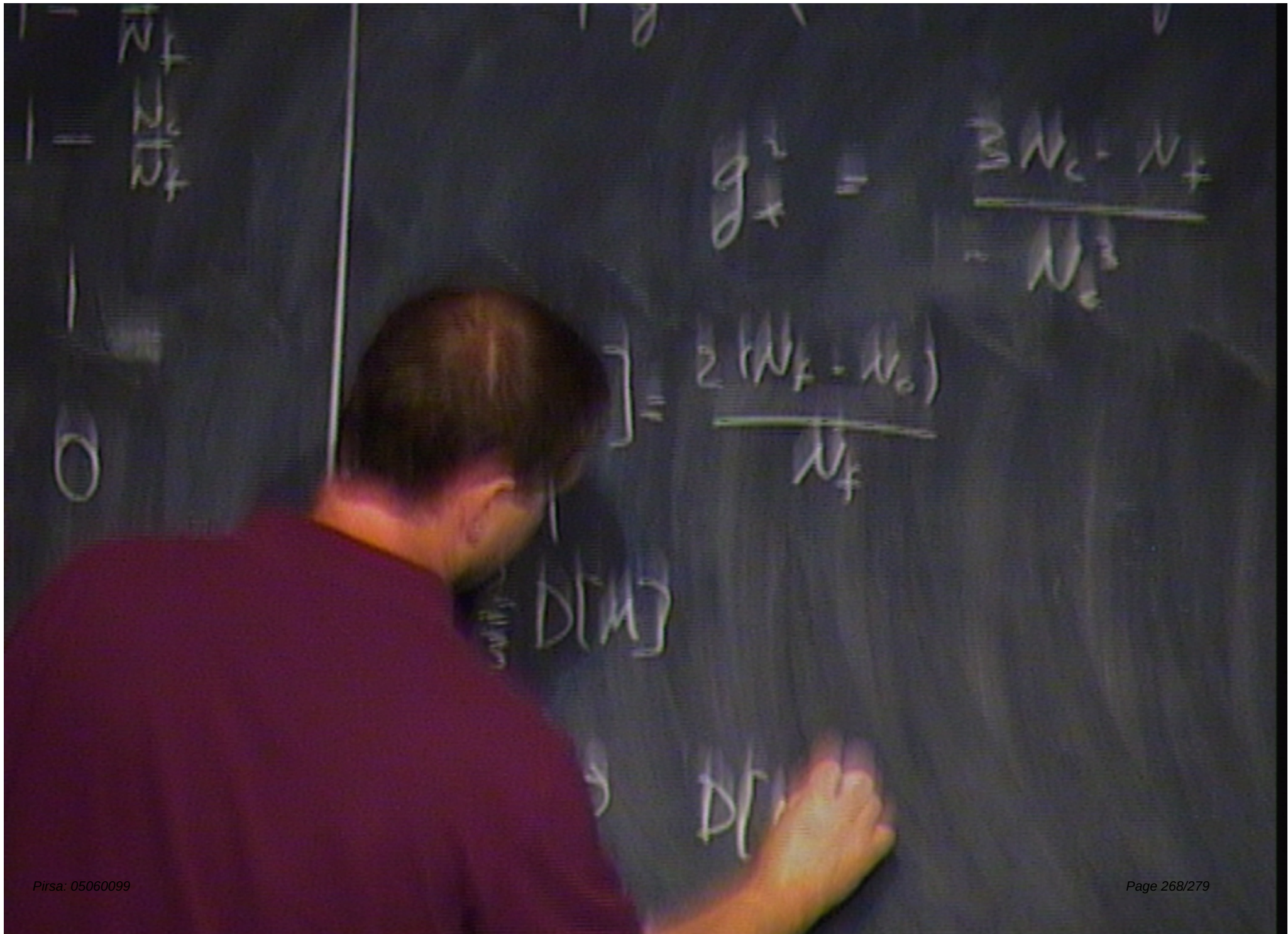
$$15g = (5N_c - N_f) = g \text{ U(1)}$$

$$g_*^2 = \frac{3N_c - N_f}{N_c^3}$$

$$R[M] = \frac{2(N_f - N_c)}{N_f}$$

$$\approx \frac{2}{3} D[M]$$

Q. 9



$$1 - \frac{N_c}{N_f}$$

$$g_+^2 = \frac{3N_c - N_f}{N_c^3}$$

$$] = \frac{2(N_f - N_0)}{N_f}$$

$$D(M)$$

$$D(M)$$

$$R[M] = \frac{2(N_f - N_c)}{N_f}$$

$$\approx \frac{2}{3} D[M]$$

$$\Rightarrow D[M] <$$

$$R[M] = \frac{2(N_f - N_c)}{N_f}$$

$$\approx \frac{2}{3} D[M]$$

$$\Rightarrow D[M] < 1 \text{ when } N_f < \frac{3}{2} N_c$$

$$\sim N_c^3$$

$$R[M] = \frac{2(N_f - N_c)}{N_f}$$

$$\equiv \frac{2}{3} D[M]$$

$$\Rightarrow D[M] < 1 \text{ when } N_f < \frac{3}{2} N_c$$

$$\sim N_c^3$$

$$D[M] = \frac{2(N_f - N_c)}{N_f}$$

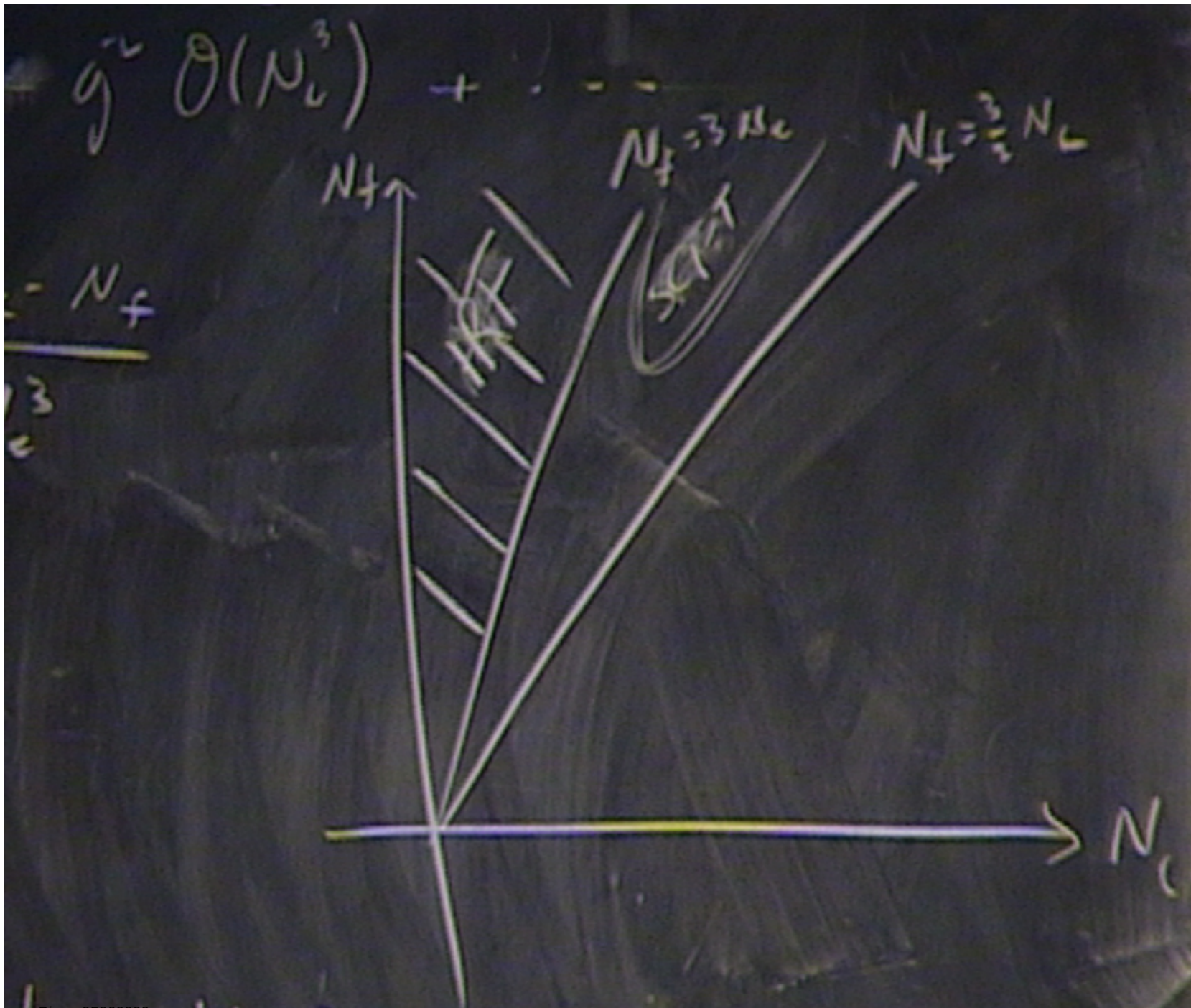
$D[M]$

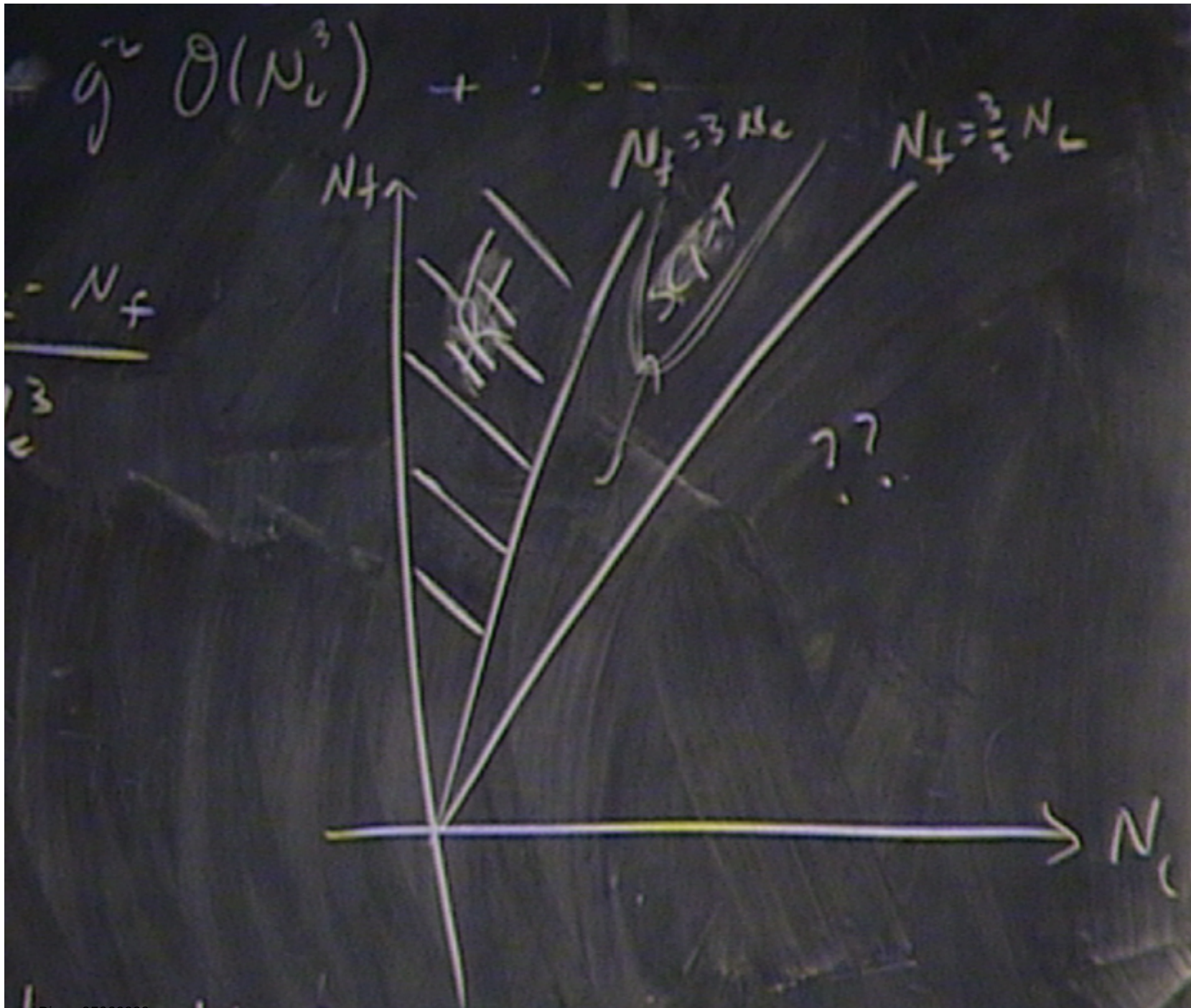
$\Rightarrow D[M] < 1$ when $N_f < \frac{3}{2}N_c$

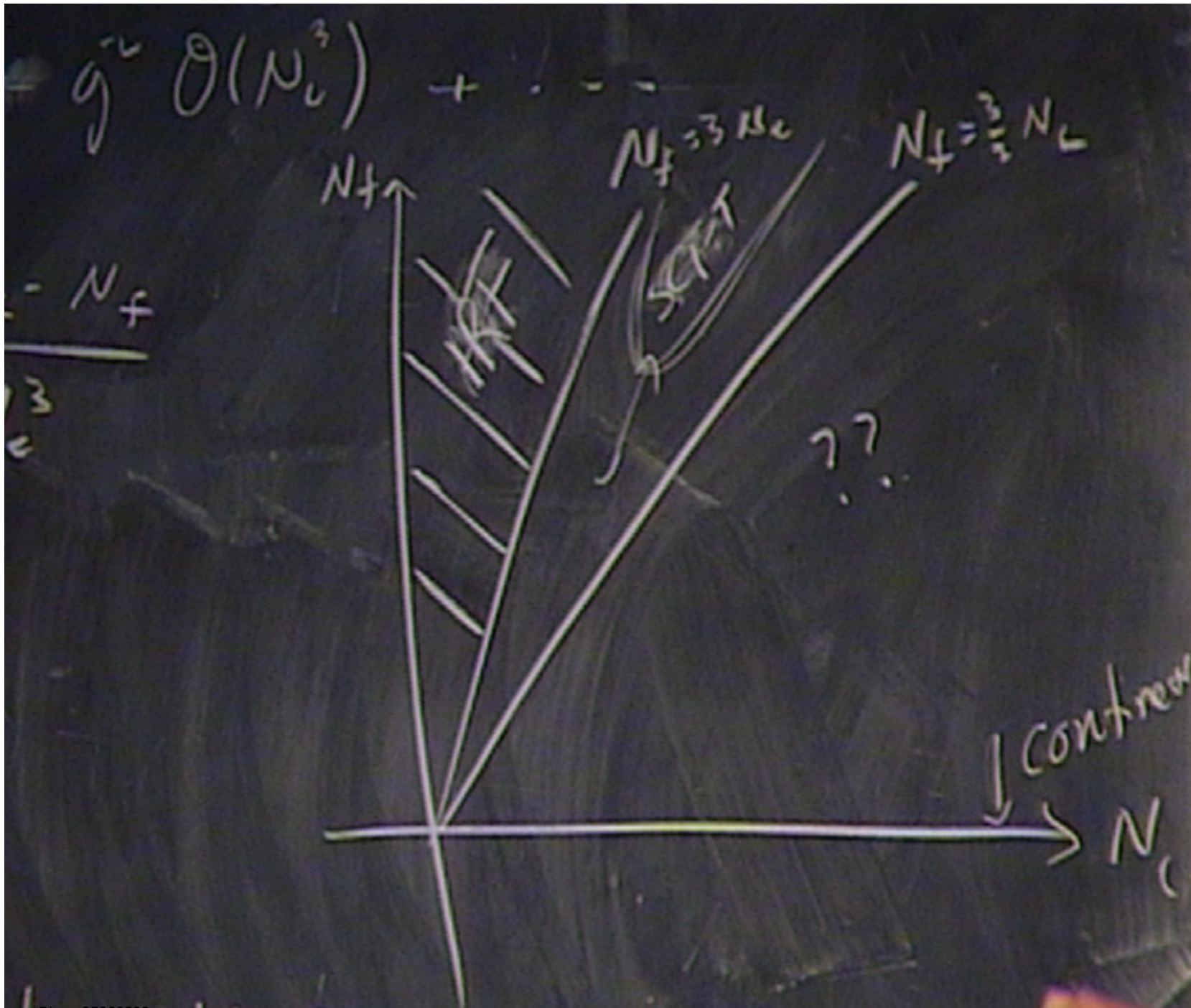
$D[M]$

$\Rightarrow$ $D[M] < 1$ when $N_f < \frac{3}{2} N_c$

↑
violates unitarity



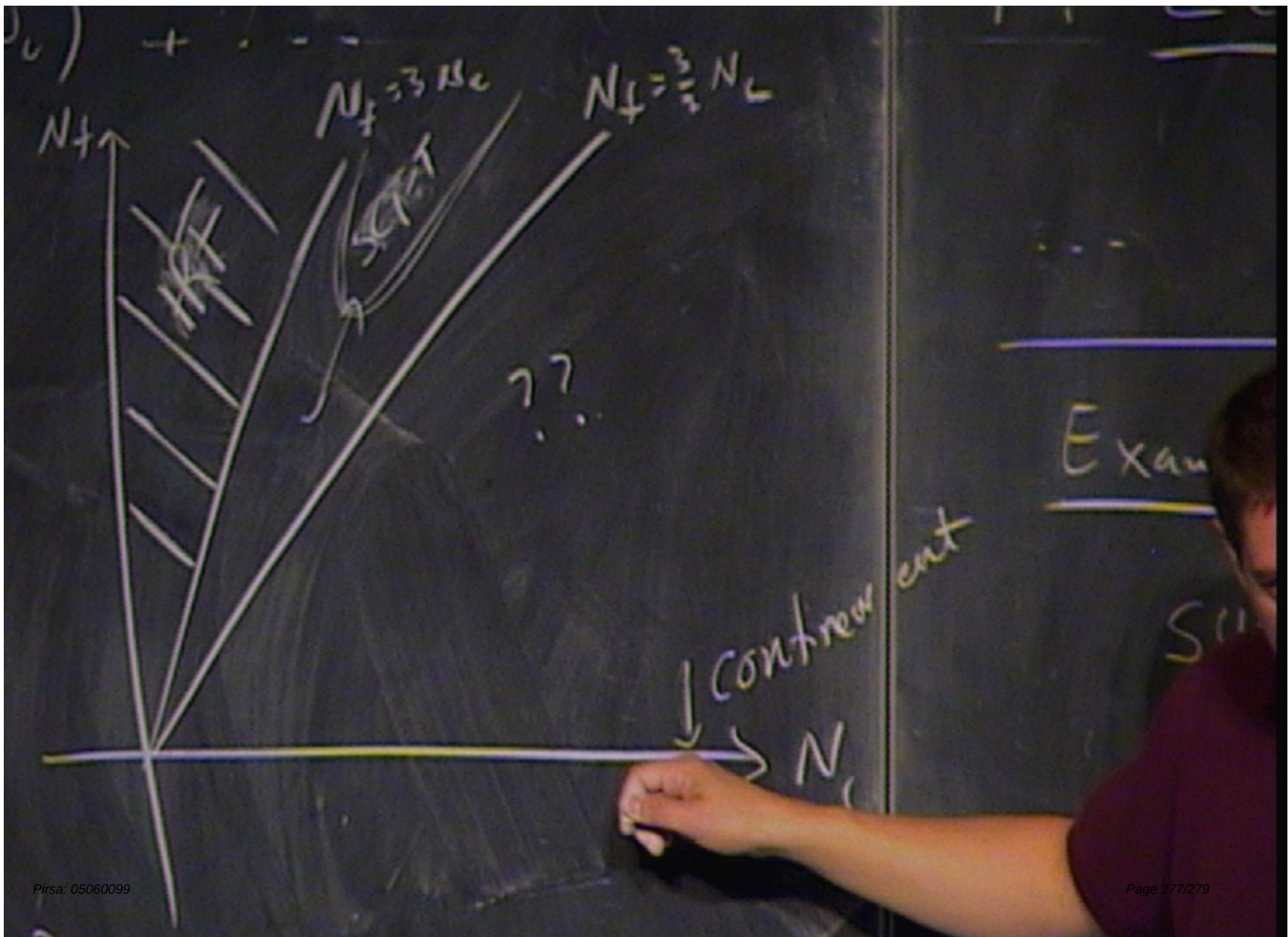




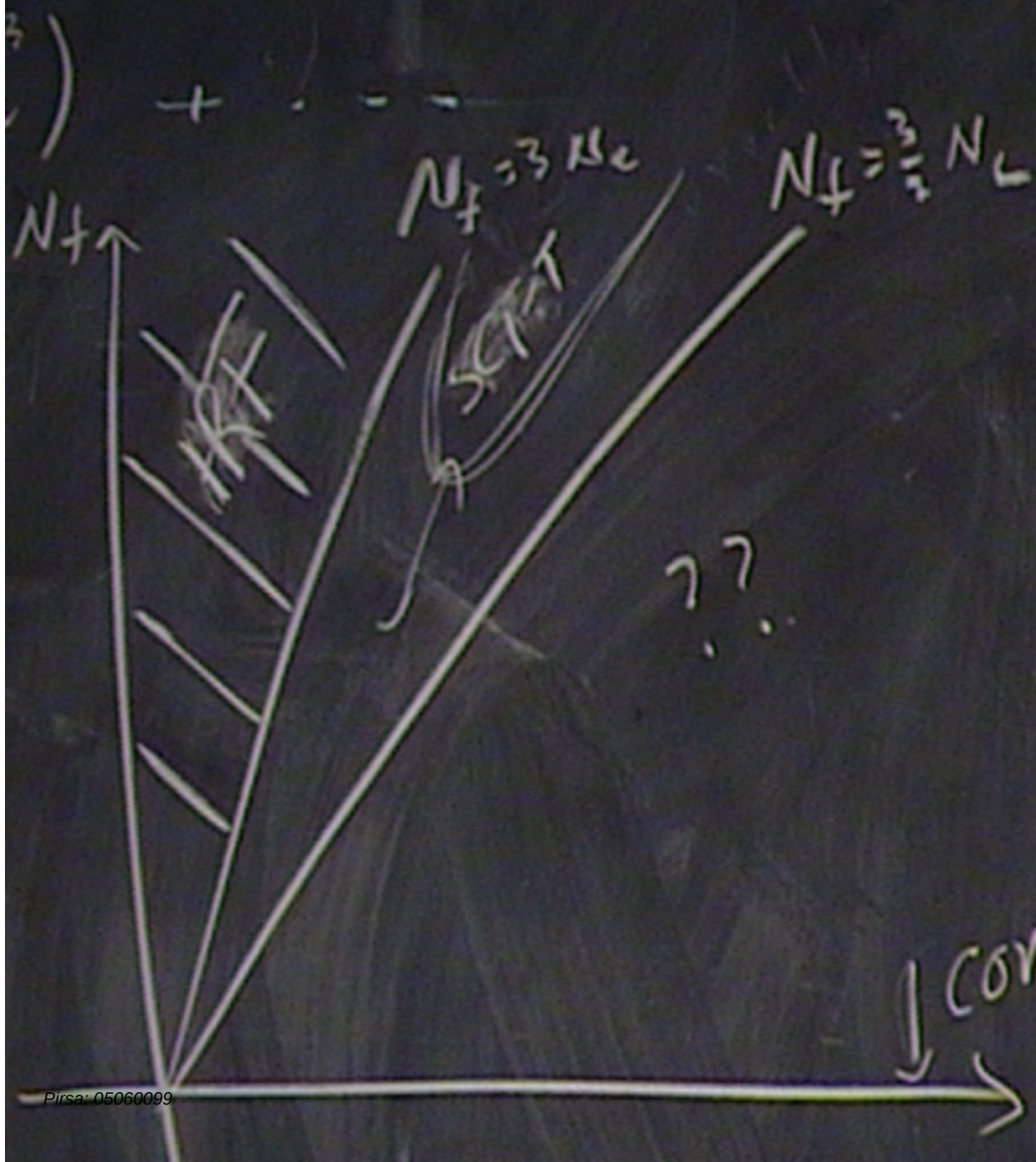
F_1

E_1

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F. L

Example

SU