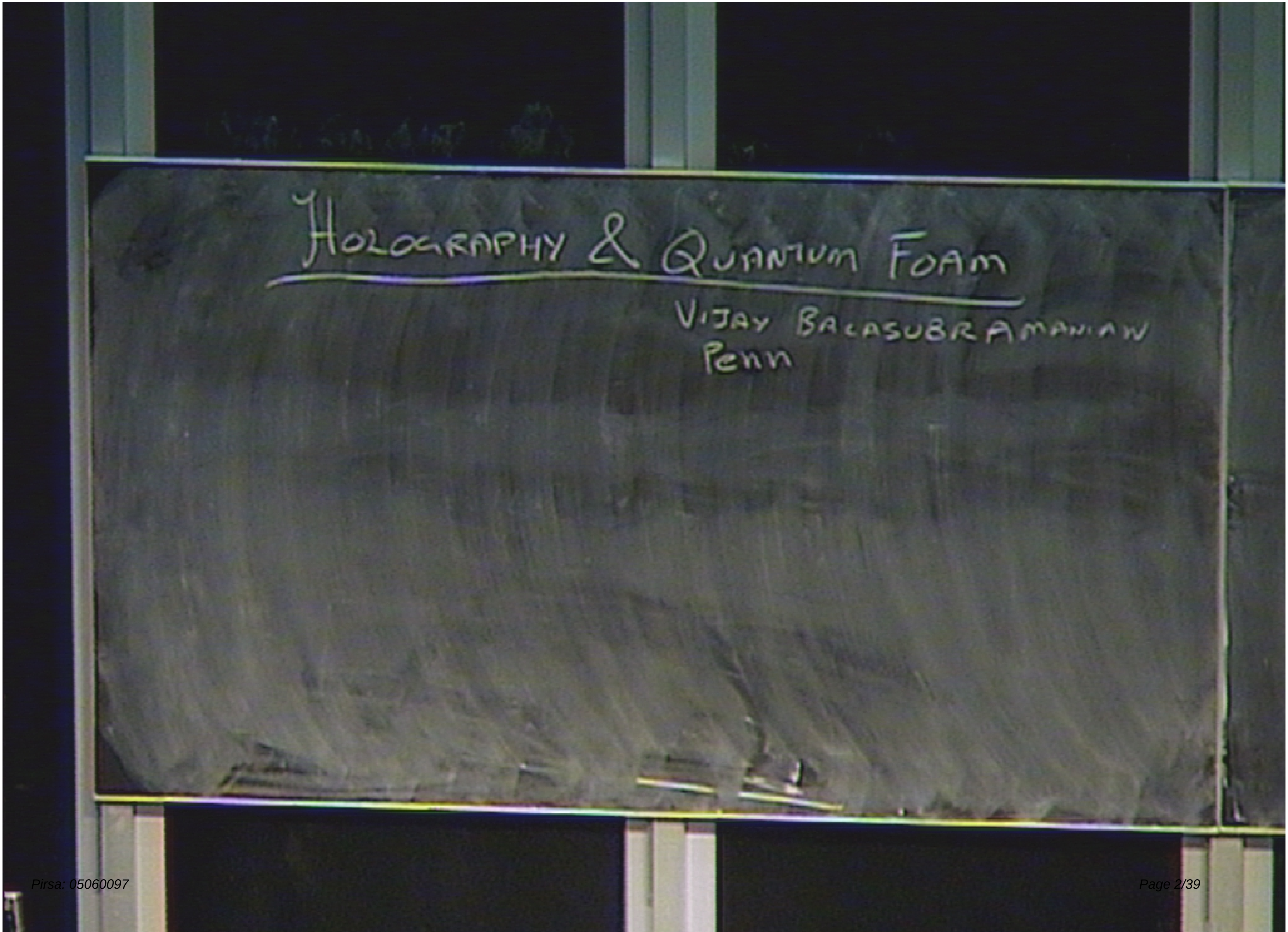
Title: AdS/CFT Correspondence

Date: Jun 27, 2005 02:30 PM

URL: http://pirsa.org/05060097

Abstract: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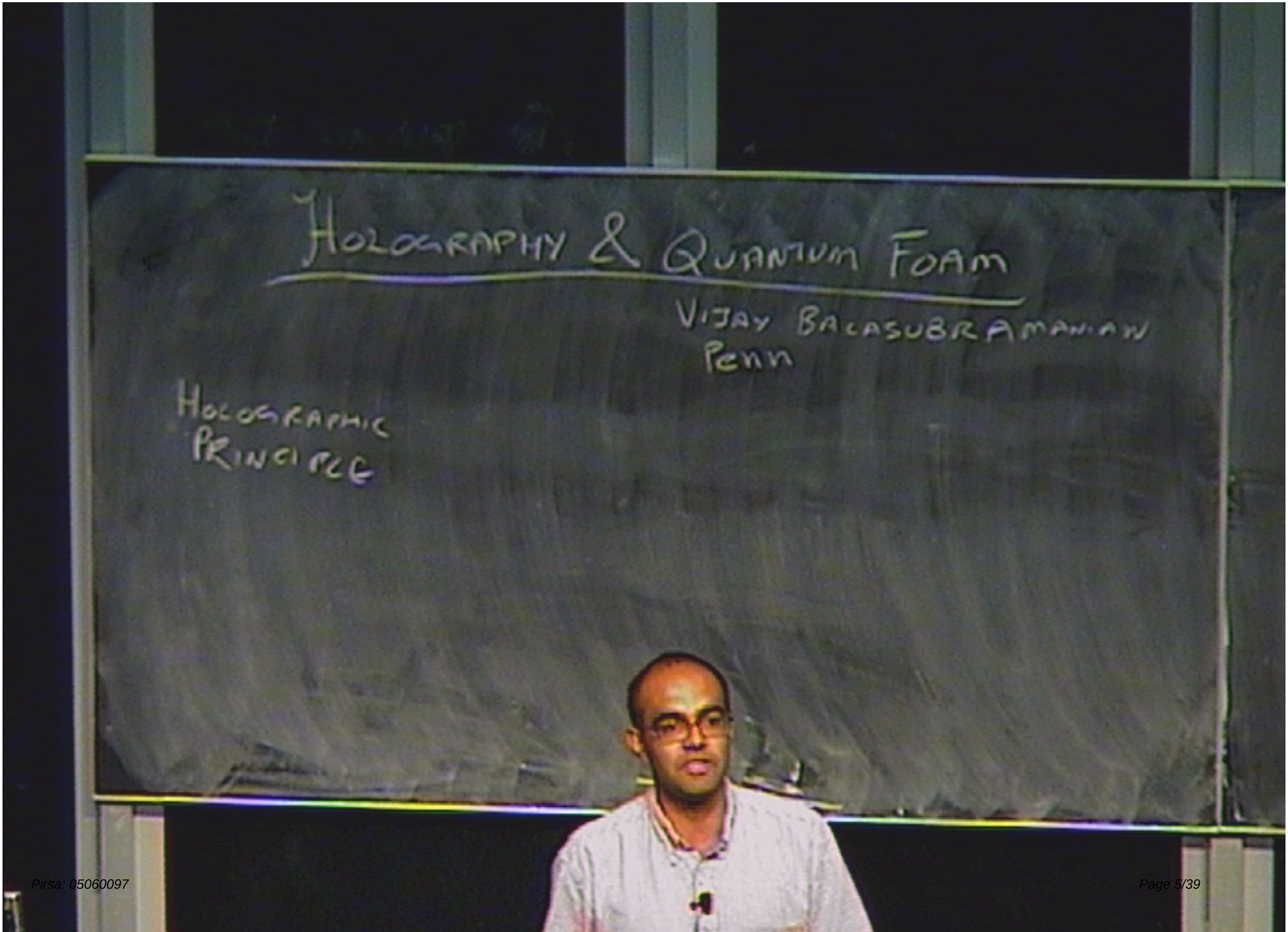
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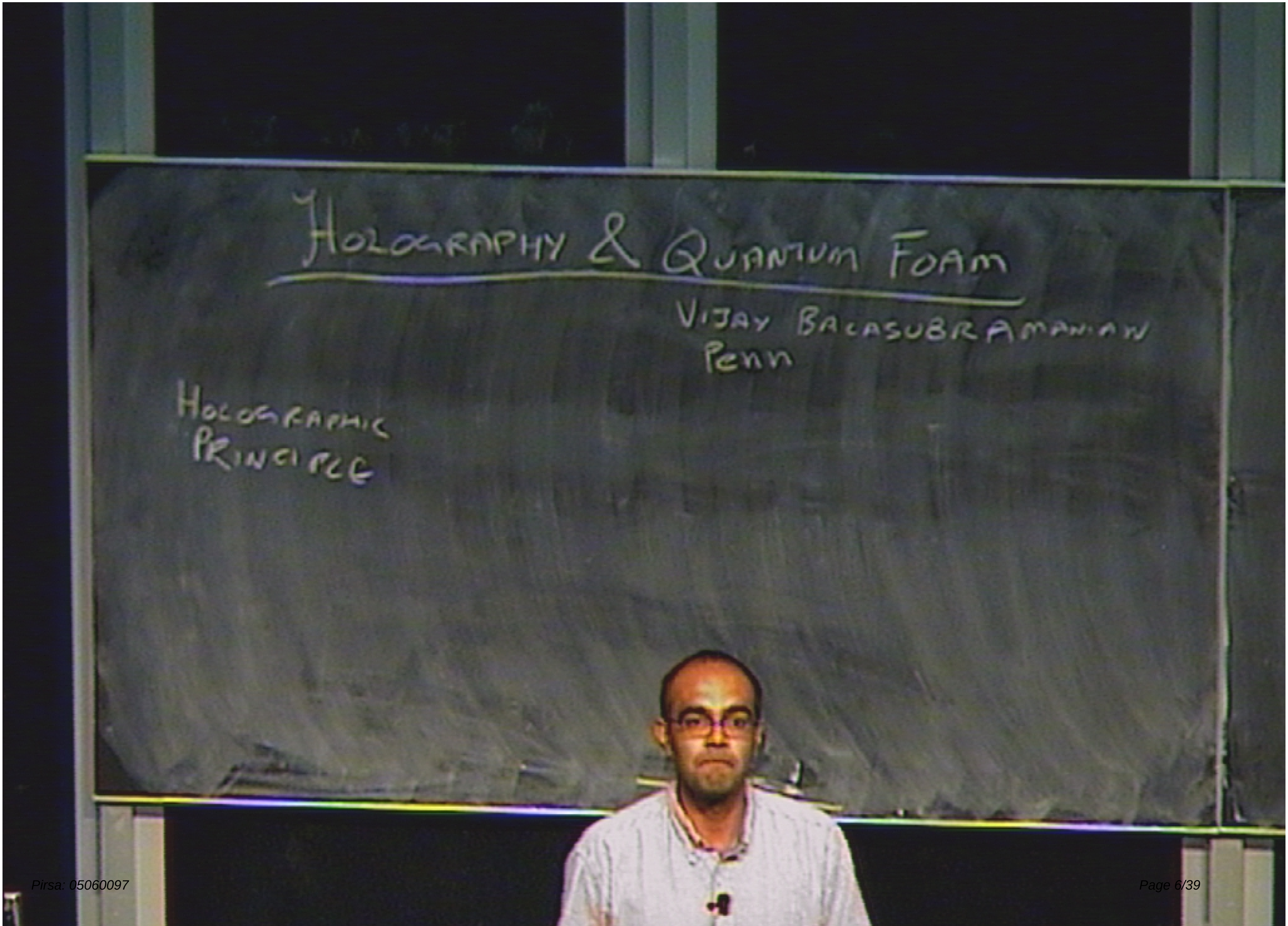
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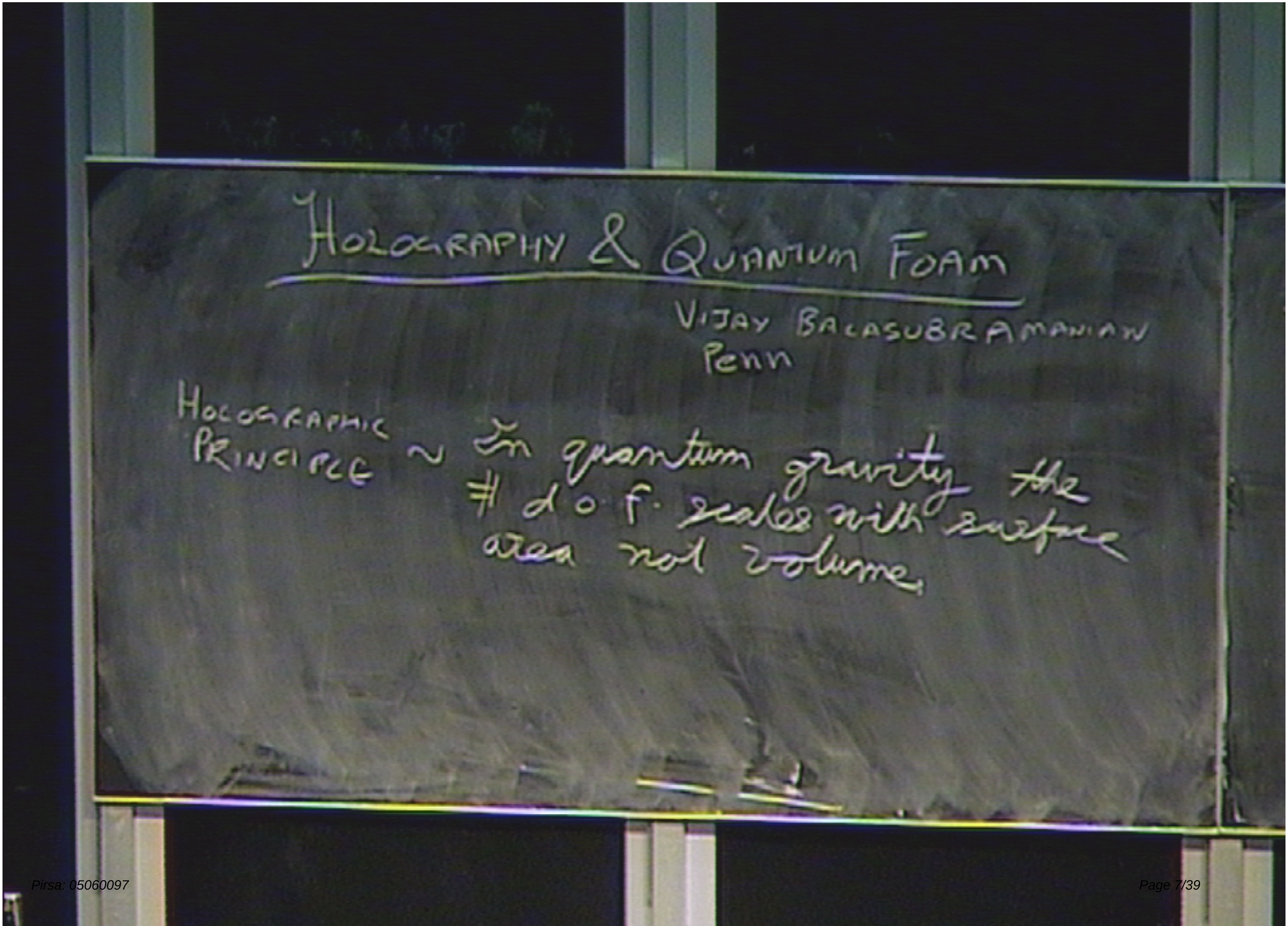
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HOLOGRAPHIC PRINCIPLE ~ In quantum gravity the # d.o.f. scales with surface area not volume.

BLACK HOLES	Thermodynamics
$dM = \kappa\, dA + \cdot$	$dE = T dS +$
$\Delta A \geq 0$	$\Delta S \geq 0$
$\kappa =$ const on a horizon	$T =$ constant in equilibrium

$$ds^2 = -\left(1-\frac{2m}{r}\right)dt^2 + \left(\quad\right)^{-1} dr^2 + r^2\, d\Omega^2$$

BLACK HOLES	Thermodynamics
$dM = \kappa\, dA + \cdots$	$dE = T dS + \cdots$
$\Delta A \geq 0$	$\Delta S \geq 0$
κ = CONST on a horizon	T = Constant in equilibrium

$$ds^2 = -\left(1-\frac{2m}{r}\right)dt^2 + \left(\quad\right)^{-1} dr^2 + r^2\, d\Omega^2$$

$$S_{BH} = \frac{A}{4G_N}$$

$$T \sim \kappa$$

Idea: $S \propto A$ — fundamental property of gravity

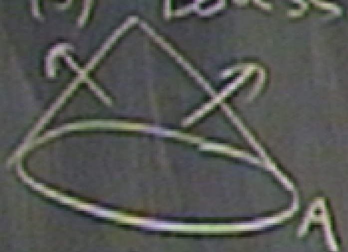
Generalization

(1) Rindler

(2) de Sitter

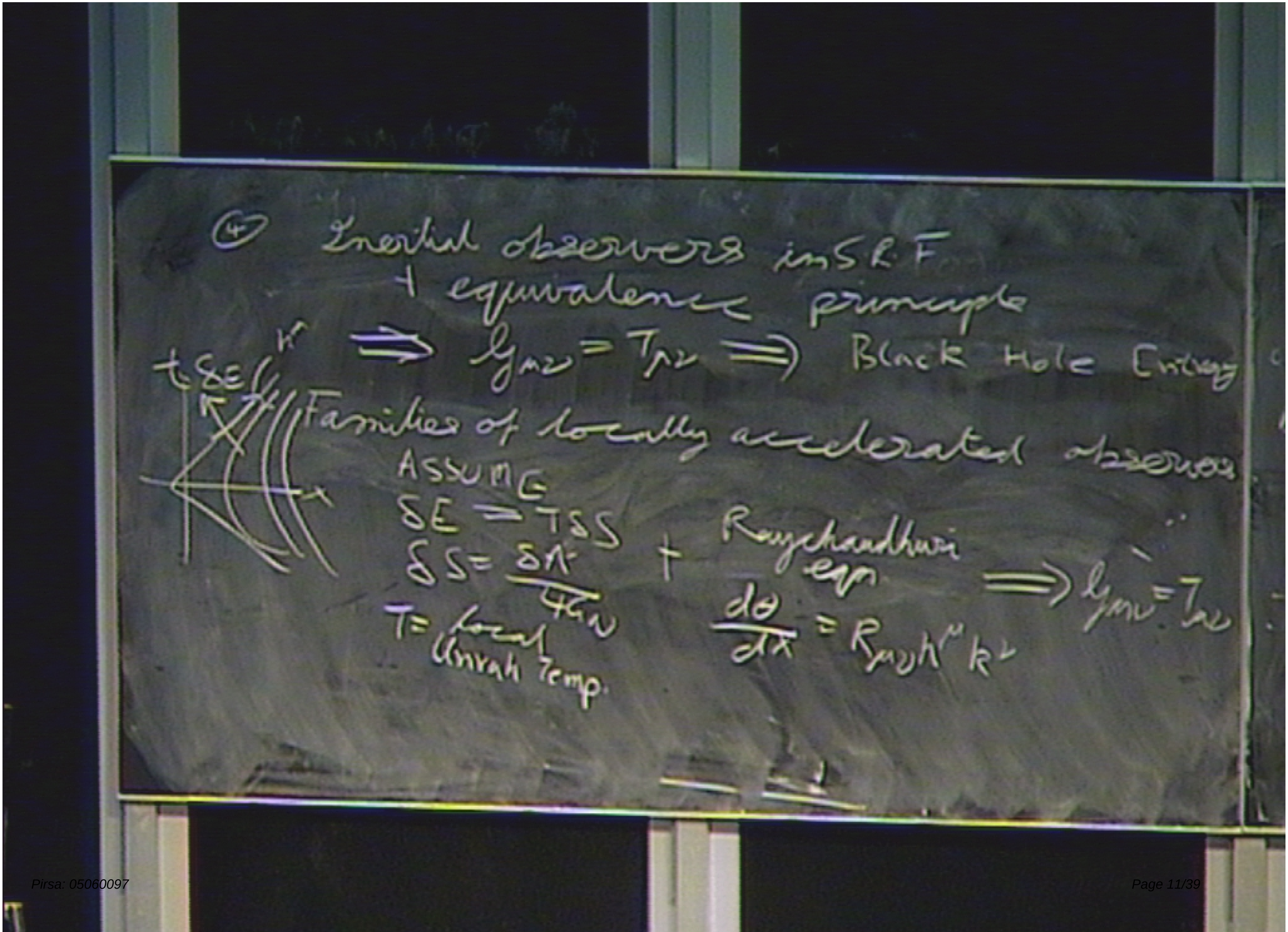
$$ds^2 = -\left(1-\frac{r^2}{l^2}\right)dt^2 + (\;)^{-1} dr^2 + r^2 d\Omega^2$$

(3) Bousso: Covariant Bound



$\frac{A}{4G} \geq$ total entropy passing through lightsheet

(4) Jacobson.


4 Inertial observers in S.R.F
+ equivalence principle
⇒ G_{μν} = T_{μν} ⇒ Black Hole Entropy
Families of locally accelerated observers
ASSUME
δE = TδS
δS = δA / 4G_N
T = local Unruh Temp.
+ Raychaudhuri eqn
dθ/dλ = R_{μν} k^μ k^ν
⇒ G_{μν} = T_{μν}

Laws of Horizon Thermo $\Longleftrightarrow$ Equations of Motion

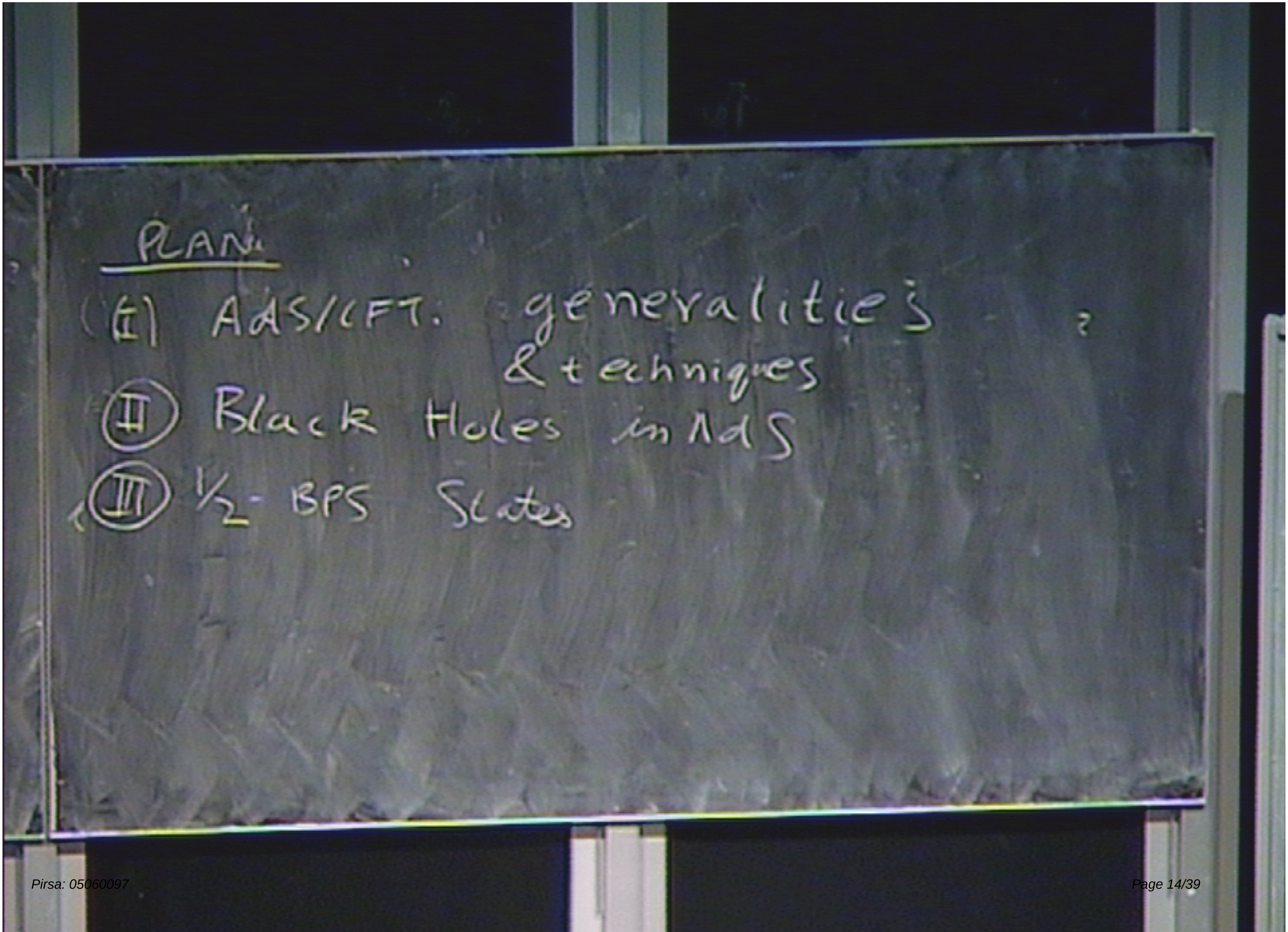
Meaning? gravity & spacetime are effective emergent concepts

Laws of Horizon Thermo $\Longleftrightarrow$ Equations of Motion

<u>Meaning?</u> Gravity & spacetime are effective emergent concepts

(1) What are atoms of spacetime

(2) " " microstates of smooth & singular spacetime geometries


PLAN.
(I) AdS/CFT: generalities
& techniques
(II) Black Holes in AdS
(III) 1/2 - BPS States

PLAN.

(I) AdS/CFT: generalities & techniques

(II) Black Holes in AdS

(III) ½-BPS States

(IV) Quantum Foam

(II) AdS/CFT

Global AdS spacetimes $(\Lambda<0)$

$S^d \times R$

$$ds^2 = -\left(1+\frac{r^2}{\ell^2}\right)dt^2 + (\quad)^{-1} dr^2 + r^2\, d\Omega^2$$

cylinder

$$= -\sec^2\rho\, dt^2 + \ell^2 d\rho^2 + \ell^2 \tan^2\rho\, d\Omega^2 \qquad (r = \ell\tan\rho)$$

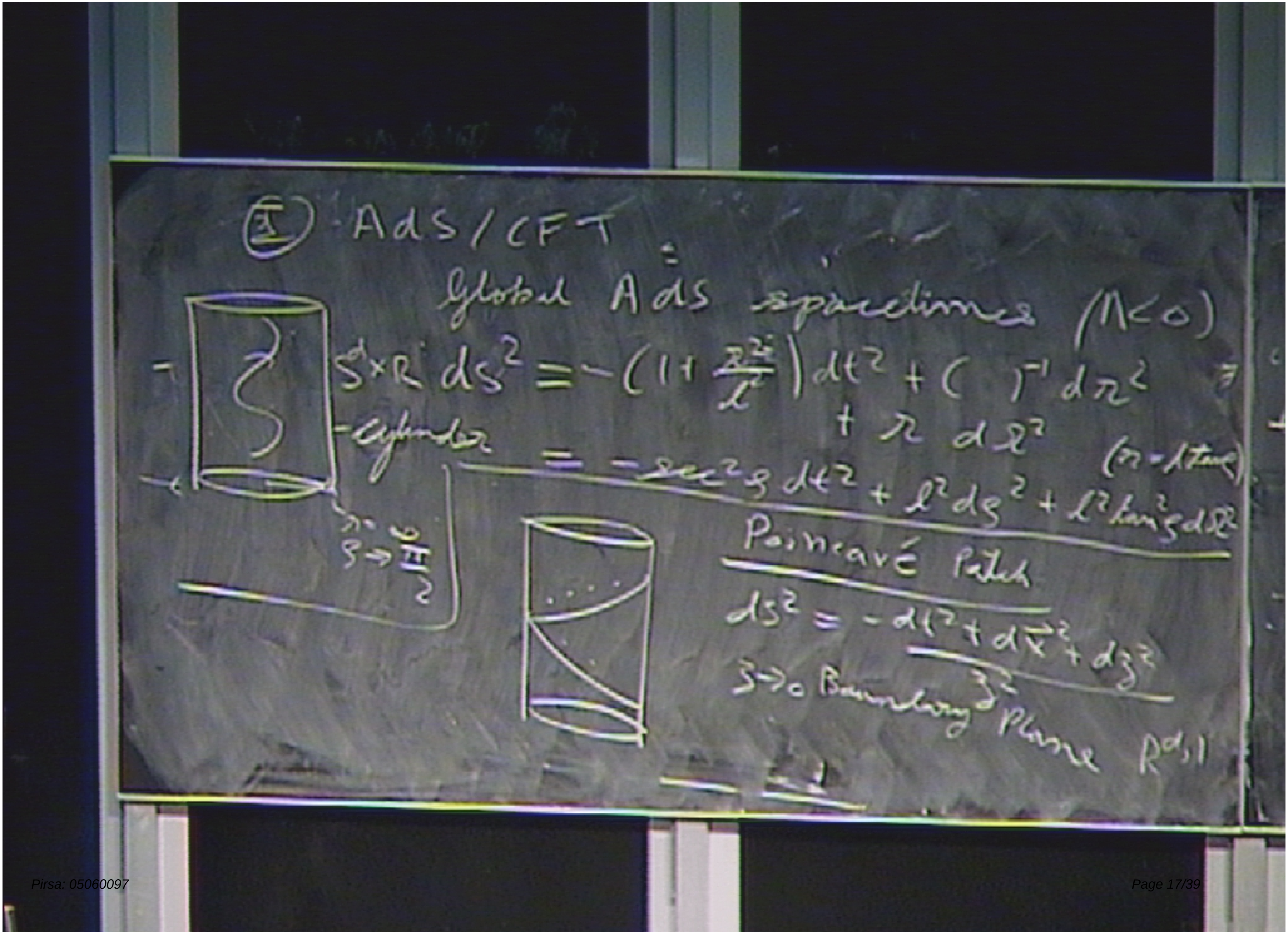
$r \to \infty$

$\rho \to \frac{\pi}{2}$

Poincaré Patch

$$ds^2 = \frac{-dt^2 + d\vec{x}^2 + dz^2}{z^2}$$

$z \to 0$ Boundary Plane $R^{d,1}$


(II) AdS/CFT
Global AdS spacetime (Λ<0)
S^d×R ds^2 = -(1+r^2/ℓ^2) dt^2 + ( )^{-1} dr^2 + r^2 dΩ^2
cylinder
= -sec^2ρ dt^2 + ℓ^2 dρ^2 + ℓ^2 tan^2ρ dΩ^2
(r = ℓ tan ρ)
r → ∞
ρ → π/2
Poincaré Patch
ds^2 = -dt^2 + dx^2 + dz^2 / z^2
z→0 Boundary Plane R^{d,1}

In string theory

$\mathrm{AdS} \times \mathcal{M} = \begin{matrix} 10d \\ 11d \end{matrix}$

eg. IIB $\left\{ \begin{matrix} \mathrm{AdS}_5 \times S^5 \\ \mathrm{AdS}_3 \times S^3 \times T^4 \end{matrix} \right.$ $\left. \begin{matrix} \mathrm{AdS}_4 \times S^7 \\ \mathrm{AdS}_7 \times S^4 \end{matrix} \right\}$ 11d

R^2

In string theory

$AdS \times \mathcal{M} = \begin{matrix} 10d \\ 11d \end{matrix}$

eg. IIB $\begin{cases} AdS_5 \times S^5 \\ AdS_3 \times S^3 \times T^4 \end{cases}$ $\quad \left. \begin{matrix} AdS_4 \times S^7 \\ AdS_7 \times S^4 \end{matrix} \right\}$ 11d

can orbifold, replace S^n with other things

AdS/CFT

Field Theory | String Theory | CFT

$$\int_{\phi \to \phi_0} \mathcal{D}\phi \; e^{iS(\phi)} = \left\langle e^{i\int_{\partial m} \phi_0 \mathcal{O}} \right\rangle$$

field: $\phi \Longleftrightarrow \mathcal{O}$ operator

BC $\phi_0 \Longleftrightarrow \phi_0$ source

Isometries $\Longleftrightarrow$ Global Symmetries

AdS/CFT

Field Theory

String Theory = CFT

$$\int_{\phi\to\phi_0} \mathcal{D}\phi \; e^{iS(\phi)} = \left\langle e^{i\int_{\partial M}\phi_0 \mathcal{O}} \right\rangle$$

field: $\phi \Longleftrightarrow \mathcal{O}$ operator

BC $\phi_0 \Longleftrightarrow \phi_0$ source

Isometries $\Longleftrightarrow$ Global Symmetries

(2) $AdS_5 \times S^5 \Longleftrightarrow \mathcal{N}=4$ superconformal $SU(N)$ gauge theory on ∂M

AdS/CFT

String Theory   CFT

Field Theory

$$\int_{\phi\to\phi_0} \mathcal{D}\phi\, e^{iS(\phi)} = \left\langle e^{i\int_{\partial M} \phi_0 \mathcal{O}} \right\rangle$$

field: $\phi \Longleftrightarrow \mathcal{O}$ operator

B.C. $\phi_0 \Longleftrightarrow \phi_0$ source

Isometries $\Longleftrightarrow$ Global Symmetries

eg. $AdS_5 \times S^5 \Longleftrightarrow$ $\mathcal{N}=4$ superconformal $SU(N)$ gauge theory on ∂M

- $SO(4,2) \times SO(6)$
- $\ell = (g^2_{YM} N)^{1/4} \ell_s$

- $g^2_{YM} = g_s$
- $g_s N =$ 't Hooft coupling

Semiclassical limit:

$$\frac{\ell}{\ell_p} \to \infty \quad \left[\begin{array}{c} \hbar \to 0 \\ N \to \infty \end{array}\right] \quad \left[\begin{array}{l} e^{iS_{cl}(\phi_0)} = e^{iW[\phi_0]} \\ \frac{1}{n!}\frac{\delta^n S_{cl}(\phi_0)}{\delta \phi_0^n}\Big|_{\phi_0=0} = \langle \mathcal{O}\,\mathcal{O} \cdots \mathcal{O} \rangle \end{array}\right]$$

Lorentzian: States, Sources & S-Matrices

Semiclassical limit:

$$\frac{\ell}{\ell_P} \to \infty \quad \left[\begin{array}{l} \hbar \to 0 \\ N \to \infty \end{array}\right]$$

$$\left[\begin{array}{l} e^{iS_{cl}(\phi_0)} = e^{iW[\phi_0]} \\ \dfrac{1}{n!}\dfrac{\delta^n S_{cl}(\phi_0)}{\delta \phi_0^n}\Big|_{\phi_0 = 0} = \langle \mathcal{O}\,\mathcal{O} \cdots \mathcal{O} \rangle \end{array}\right]$$

Lorentzian: States, Sources & S-Matrices

- Many solns $\phi \longrightarrow \phi_0 \longleftrightarrow$ There are many states
- ϕ = scalars of mass m $\Longrightarrow$ $\Box\phi = m^2\phi$

Poincaré

$$ds^2 = \frac{L^2}{z^2}\left(-dt^2 + d\vec{x}^2 + dz^2\right)$$

$$\phi = e^{-i\omega t + i\vec{k}\cdot\vec{x}}\,\chi$$

$$z^2 \partial_z^2 \chi + z\,\partial_z \chi - \left[\left(m^2 + \frac{d^2}{4}\right) + (\vec{k}^2 - \omega^2) z^2\right]\chi = 0$$

$$\phi = e^{i\vec{k}\vec{x} - i\omega t}\, z^{d/2}\left[A\, J_\nu(|q| z) + \phi_0\, Y_\nu(|q| z)\right]$$

Poincaré

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$$q^2 = \vec{k}^2 - \omega^2$$

$$\phi = e^{i\vec{k}\vec{x} - i\omega t}\, z^{d/2}\left[A\, J_\nu(|q| z) + \phi_0\, Y_\nu(|q| z)\right]$$

$$\xrightarrow{z \to 0} A\, z^{2h_+} e^{ikx - i\omega t} + \phi_0\, z^{2h_-} e^{ikx - i\omega t}$$

$$2h_\pm = \frac{d}{2} \pm \frac{\sqrt{d^2 + 4m^2}}{2} = \frac{d}{2} \pm \nu$$

Poincaré

$$ds^2 = \frac{L^2}{z^2}\left(-dt^2 + d\vec{x}^2 + dz^2\right)$$

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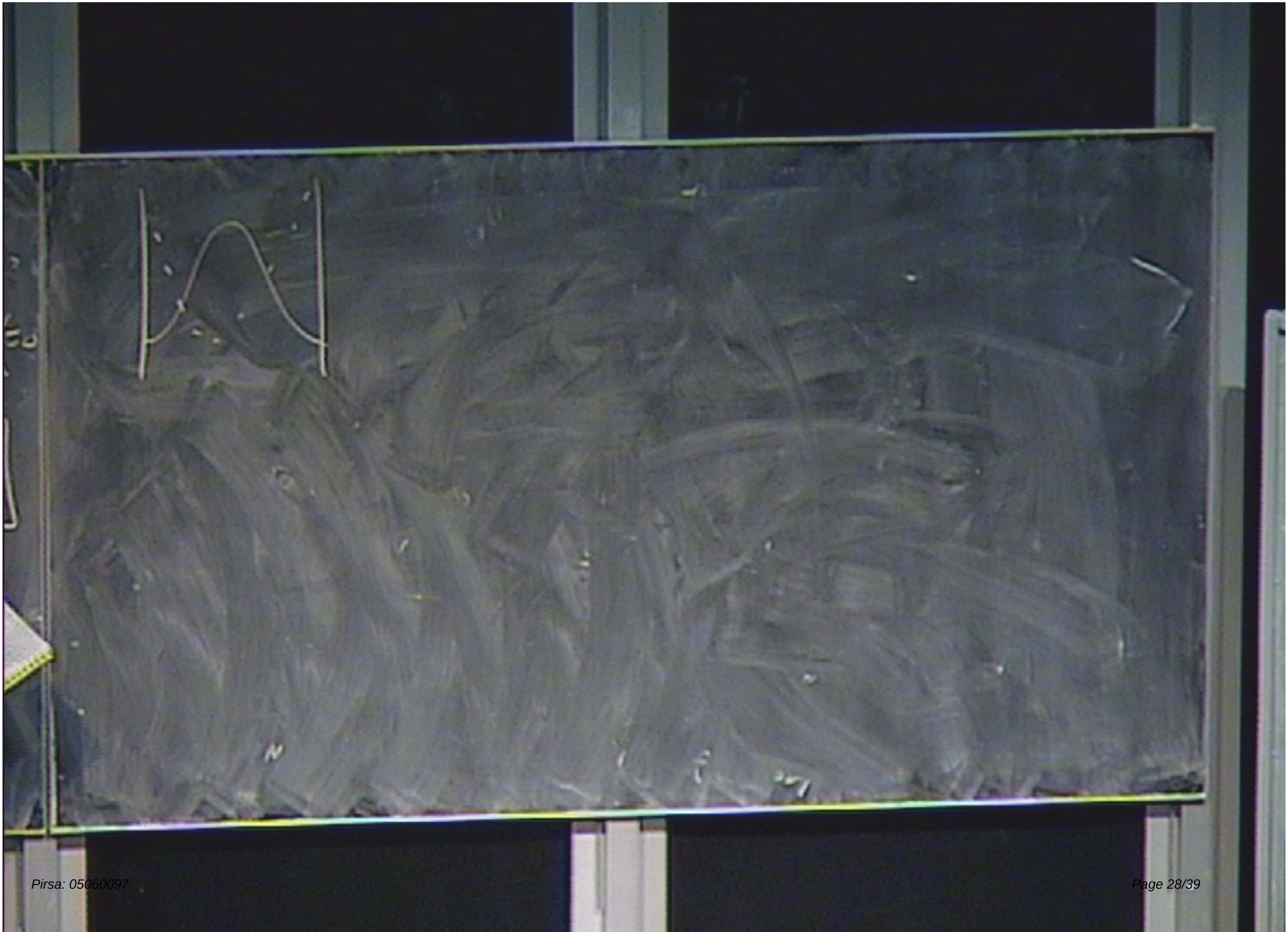
$$\xrightarrow[z \to 0]{} A\, z^{2h_+}\, e^{ikx - i\omega t} + \phi_0\, z^{2h_-}\, e^{ikx - i\omega t}$$

$$2h_\pm = \frac{d}{2} \pm \frac{\sqrt{d^2 + 4m^2}}{2} = \phi_N + \phi_{NN}$$

$$= \frac{d}{2} \pm \nu$$



$$\Rightarrow \phi_{NN} \Rightarrow S_{CFT} \longrightarrow S_{CFT} + \int \phi_0 \, \mathcal{O}$$

$$\phi_N \Longleftrightarrow \mathcal{O}|0\rangle \quad \text{state}$$

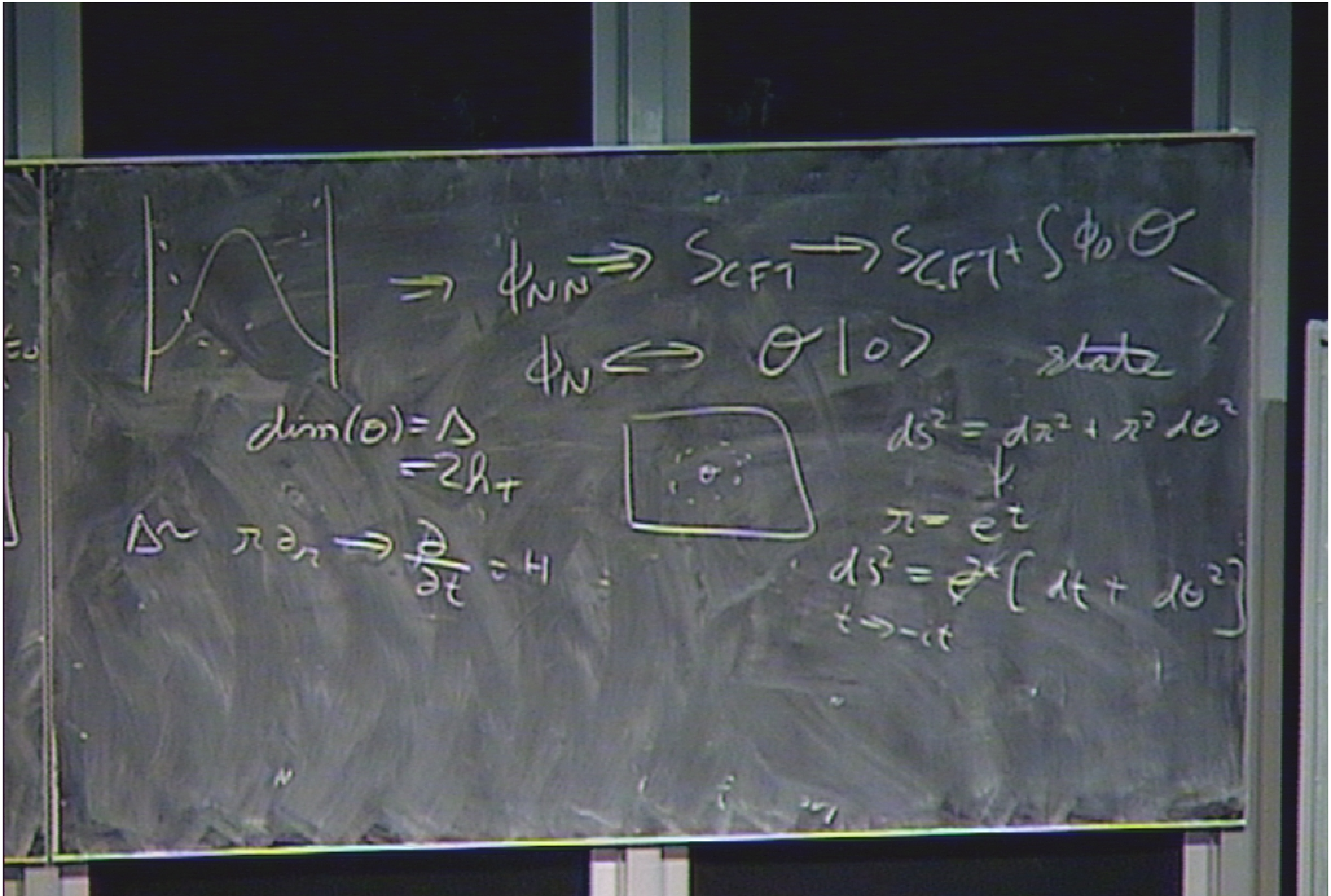
$$\dim(\mathcal{O}) = \Delta$$
$$= 2h_+$$

$$ds^2 = dr^2 + r^2 d\theta^2$$

$$r = e^t$$

$$ds^2 = e^{2t}\left[dt + d\theta^2 \right]$$

$$t \to -it$$


$\Rightarrow \phi_{NN} \Rightarrow S_{CFT} \rightarrow S_{CFT} + \int \phi_0 \mathcal{O}$
$\phi_N \Longleftrightarrow \mathcal{O}|0\rangle$ state
$\dim(\mathcal{O}) = \Delta$
$= 2h_+$
$\Delta \sim r\partial_r \Rightarrow \frac{\partial}{\partial t} = H$
$ds^2 = dr^2 + r^2 d\theta^2$
$r = e^t$
$ds^2 = e^{2t}[dt^2 + d\theta^2]$
$t \rightarrow -it$

<u>Also:</u> • $|0\rangle$ is unique in AdS

• $G_F(b,z\,;\,b',z')$ = Feynman

$$\lim_{z\to 0} z^{-2h_+}\, G_F(b,z,\,b',z')$$

$$= G_{B\partial} = \text{Bulk-Boundary Prop.}$$

Earlier $\Rightarrow e^{iS_{cl}(\phi_0)} = e^{iW[\phi_0]}$

$$\langle \theta(b_1)\cdots\theta(b_n)\rangle$$

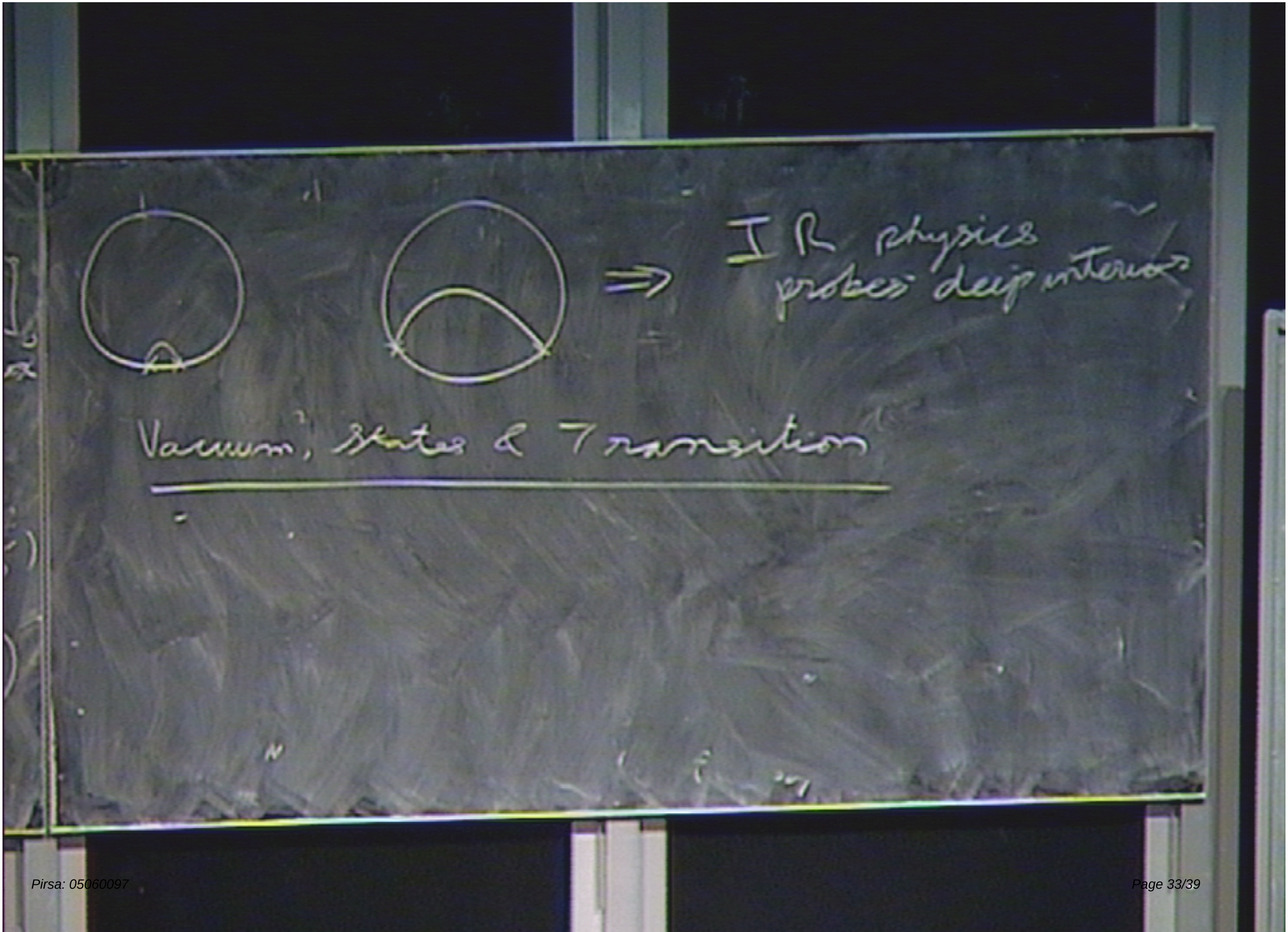
$$= \int \left[G_{BD}[b_1, x_1]\, G_{BD}[b_2, x_2] \times G_F(x, x_1) \right]_{\text{vertex}}$$

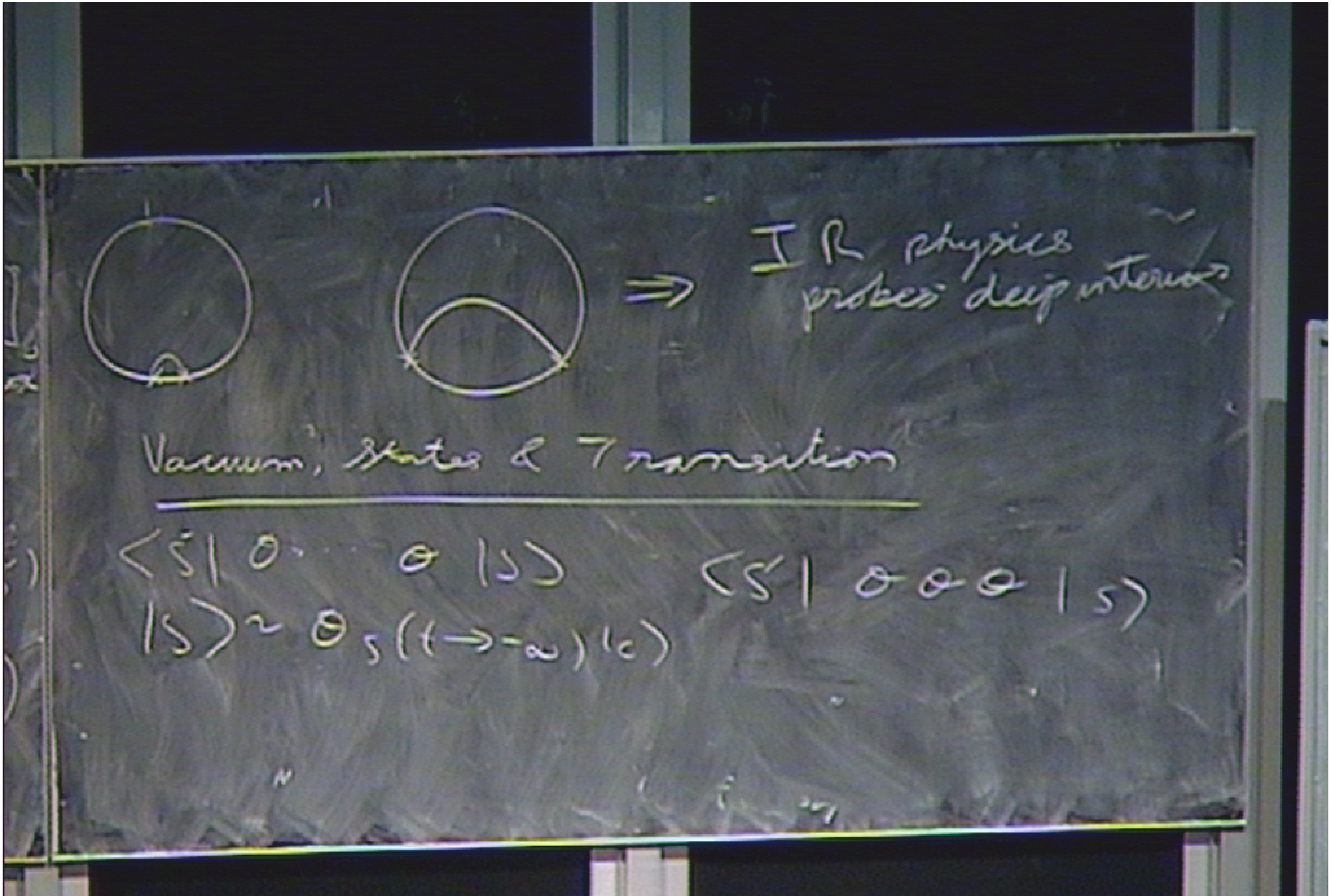
LOCAL INFO about spacetime

$$\langle \theta(b_1)\,\theta(b_2)\rangle \sim \lim_{\xi, \xi' \to 0} (\;)\, G_F(b, \xi, b', \xi')$$

$$\sim \int \mathcal{D}p \; e^{i \frac{m}{\hbar} l(p)}$$

$$\Rightarrow \langle \theta\theta \rangle = \sum_\gamma e^{i \frac{m}{\hbar} l_\gamma}$$


IR physics
probes deep interior
Vacuum, States & Transition


IR physics
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<S| O O O |S>
|S> = O_S(t→-∞)|c>

Spectroscopy of AdS

$$\left.\begin{array}{l}\text{AdS} \\ SO(d,2)\end{array}\right\} \quad \begin{array}{l}\Box \phi = m^2 \phi \\ \text{irrep of } SO(d,2)\end{array}$$

AdS_3

$$\left.\begin{array}{l} -u^2 - v^2 + x^2 + y^2 = -\ell^2 \\ -du^2 - dv^2 + dx^2 + dy^2 = ds^2 \end{array}\right\} \quad \begin{array}{l} SO(2,2) \\ = SL(2,R) \times SL(2,R) \end{array}$$

$$ds^2 = \ell^2 \left[-\cosh^2\rho \, dt^2 + d\rho^2 + \sinh^2\rho \, d\phi^2 \right]$$

Spectroscopy of AdS

$$\left.\begin{array}{l}\text{AdS}\\ SO(d,2)\end{array}\right\}\quad\begin{array}{l}\Box\phi = m^2\phi\\ \text{irrep of } SO(d,2)\end{array}$$

AdS_3

$$\left.\begin{array}{l}-u^2 - v^2 + x^2 + y^2 = -\ell^2\\ -du^2 - dv^2 + dx^2 + dy^2 = ds^2\end{array}\right\}\quad\begin{array}{l}SO(2,2)\\ = SL(2,R)\times SL(2,R)\\ \text{Conf. group in 2d}\end{array}$$

$$ds^2 = \ell^2\left[-\cosh^2\rho\, dt^2 + d\rho^2 + \sinh^2\rho\, d\sigma^2\right]$$

$SL(2,R)$: $[L_0, L_\pm] = \mp L_\pm$

$[L_+, L_-] = 2L_0$

Casimir: $L^2 = \frac{1}{2}(L_+L_- + L_-L_+) - L_0^2$

Construct group: $\{L_0, L_+, L_-\}_R$ as gens of isometry

$SL(2,R)$: $[L_0, L_\pm] = \mp L_\pm$

$[L_+, L_-] = 2L_0$

Casimir: $L^2 = \frac{1}{2}(L_+L_- + L_-L_+) - L_0^2$

Construct group $\{L_0, L_+, L_-\}_R$ as gens of isometry

(Then, $\Box\phi = m^2\phi$ $\quad -2(L_L^2 + L_R^2) = m^2\ell^2$

$L_0\phi = h\phi$ = conformal weight

Highest weight state:

$$L_{+L}\,\phi = L_{+R}\,\phi = 0 \implies L_{0L}\,\phi = L_{0R}\,\phi = h$$

and $$h(h-1) = \frac{m^2\ell^2}{4} \implies h_\pm = \frac{d}{2} \pm \frac{\sqrt{d^2+4m^2}}{2}$$

SOLVE: $$\phi_h^\pm = e^{i(2h_\pm)t}\,\frac{1}{(\cosh\rho)^{2h_\pm}}$$

FULL BASIS. $$(L_{-L})^p\,(L_{-R})^q\,\phi_h = \phi_h^{p,q}$$