Title: Advanced AdS/CFT Topics

Date: Jun 27, 2005 11:30 AM

URL: http://pirsa.org/05060096

Abstract:

- All lecture notes (powerpoint) and suggested homework problems (ps) will be posted in my webpage:

http://phya.snu.ac.kr/~sjrey/index.html

Instantons

- So far, tested AdS/CFT for perturbative aspects on N=4 SYM and string theory sides.
- How about nonperturbative effects?
- In N=4 SYM theory, instanton effects are exponentially suppressed (cf. in QCD, infrared divergence dominates).

Instanton effects

- Type IIB string theory has D(-1)-brane, a localized D-brane in Euclidean 10d spacetime
- Instanton location is parametrized by (r, X^m) ---

matches with Yang-Mills instanton moduli (size, center)

Instantons in Large-N limit

- On N=4 SYM side, instanton effects are computable precisely in $N \to \infty$ limit
- K-instantons in SU(N) gauge group obey $N \to \infty$ saddle-point configuration for $K \ll N$:
- Pointlike on AdS_5
- Dilutely spread in SU(N) group space
- Fermion biliinears are peaked at S_5

Instanton-induced Amplitudes

- What sort of amplitudes can we test?
- N=4 SYM has 16 supersymmetry and 16 superconformal symmetry
- Charge-K instanton in SU(N) gauge group has 4KN bosonic/fermionic zero modes
- Most of them are lifted up at strong coupling limit except 16 dictated by super(conformal)symmetry

R^4 and superpartners

- Gross-Witten $\alpha'^3 R^4$ terms (from Virasoro-Shapiro 4-point amplitude) receive D-instanton corrections
- Also, superpartners such as sixteen dilatinos $\lambda_1 \Lambda \lambda_{16}$
- not only coupling-parameter dependence but also numerical factors all agree between IIB string and N=4 SYM
- One of the most accurate test of AdS/CFT correspondence

Application of AdS/CFT: D3-branes on "thermal" S_1

Instantons in Large-N limit

- On N=4 SYM side, instanton effects are computable precisely in $N \to \infty$ limit
- K-instantons in SU(N) gauge group obey $N \to \infty$ saddle-point configuration for $K << N$:
- Pointlike on AdS_5
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Application of AdS/CFT: D3-branes on “thermal” S_1

Application of AdS/CFT: D3-branes on "thermal" S_1

- "thermal S_1" breaks N=4 susy completely
- At low-energy, 3d Yang-Mills + (junks)
- 5d AdS replaced by
  5d Euclidean black hole (time ↔ space)
- glueball spectrum is obtainable by studying bound-state spectrum of gravity modes

Note:

4d space-time, topology: gravity // gauge

YM_{2+1} glueball spectrum

- 0^{++}: solve dilaton eqn=2nd order linear ode
- result:

	N=3 lattice	N=oo lattice	AdS/CFT
.	4.329(41)	4.065(55)	4.07(input)
*	6.52 (9)	6.18 (13)	7.02
**	8.23 (17)	7.99 (22)	9.92
***	-	-	12.80

[M. Teper]

τ–direction is compactified
on thermal S^1 of radius $\beta = 1/T$

2+1 dimensions
(after double Wick rotation $\tau \leftrightarrow x^3$ in AdS side)

Here, low-energy excitations $E << 1/\beta = T$
Are described by 2+1 dim pure YM theory
(+junks)

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[M. Teper]

YM_{3+1} glueball spectrum

- Use T>0 D4-brane instead
- 0^{++}: solve dilaton eqn=2nd-order linear ODE
- result:

	N=3 lattice	AdS/CFT
.	1.61(15)	1.61(input)
*	2.8	2.38
**	-	3.11
***	-	3.82

[M. Teper]

other glueballs fit reasonably well (why??)

N=4 SYM in uniform instanton background

- important issues in QCD:
  confinement and (chiral symmetry breaking)
- `t Hooft criterion of confinement:

Wilson loop: $\langle W(C) \rangle \sim \exp(-\tau L \cdot T)$

area-law (linear confining potential)

`t Hooft loop: $\langle H(C) \rangle \sim \exp(-m(L+T))$

perimeter-law (screened potential)

- Any simple AdS/CFT-type model?

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Old (unfinished) idea

- Callan, Dashen, Gross: instanton gas in Yang-Mills theory drives confinement and chiral symmetry breaking
- unsuccessful and unfinshed (beyond analytic treatment -- nonperturbative)
- variant idea: instanton liquid model
- AdS/CFT picture: D3 + D(-1)

SUGRA solution of D3+D(-1)

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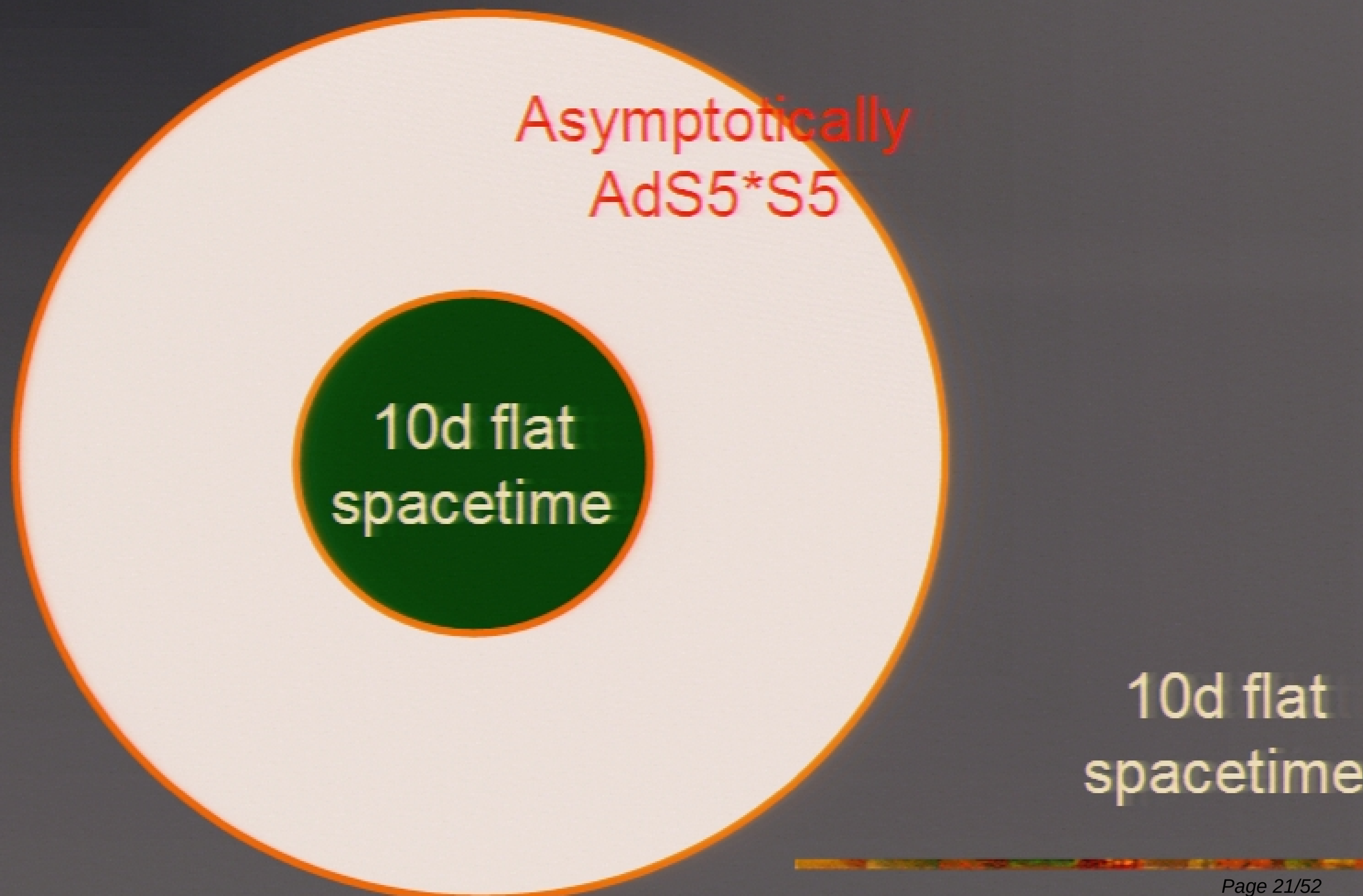
- N D3-branes + uniform D(-1) on R^4
- $ds^2 = F^{1/2}(Z^{-1/2}\, dx_4{}^2 + Z^{1/2}\, dy_6{}^2)$, G_5=same as before

  $e^{\Phi} = F$ and $C_0 |_{\text{Euclidean}} = (F-1)/F$

  $Z = (1 + R^4/r^4)$, $F = (1 + q\, R^4 / r^4)$, $R^4 = g_{st}\, N = \lambda^2$
- $q = (Q/Vol_4)/N$: instanton density per each D3
- string coupling e^{Φ} diverges near r=0
- decoupling limit: $\alpha' \to 0$ while r/α', λ^2, q = fixed
- $Z \to R^4/r^4$ but F remains intact

Geometry looks like…



SUGRA solution of D3+D(-1)

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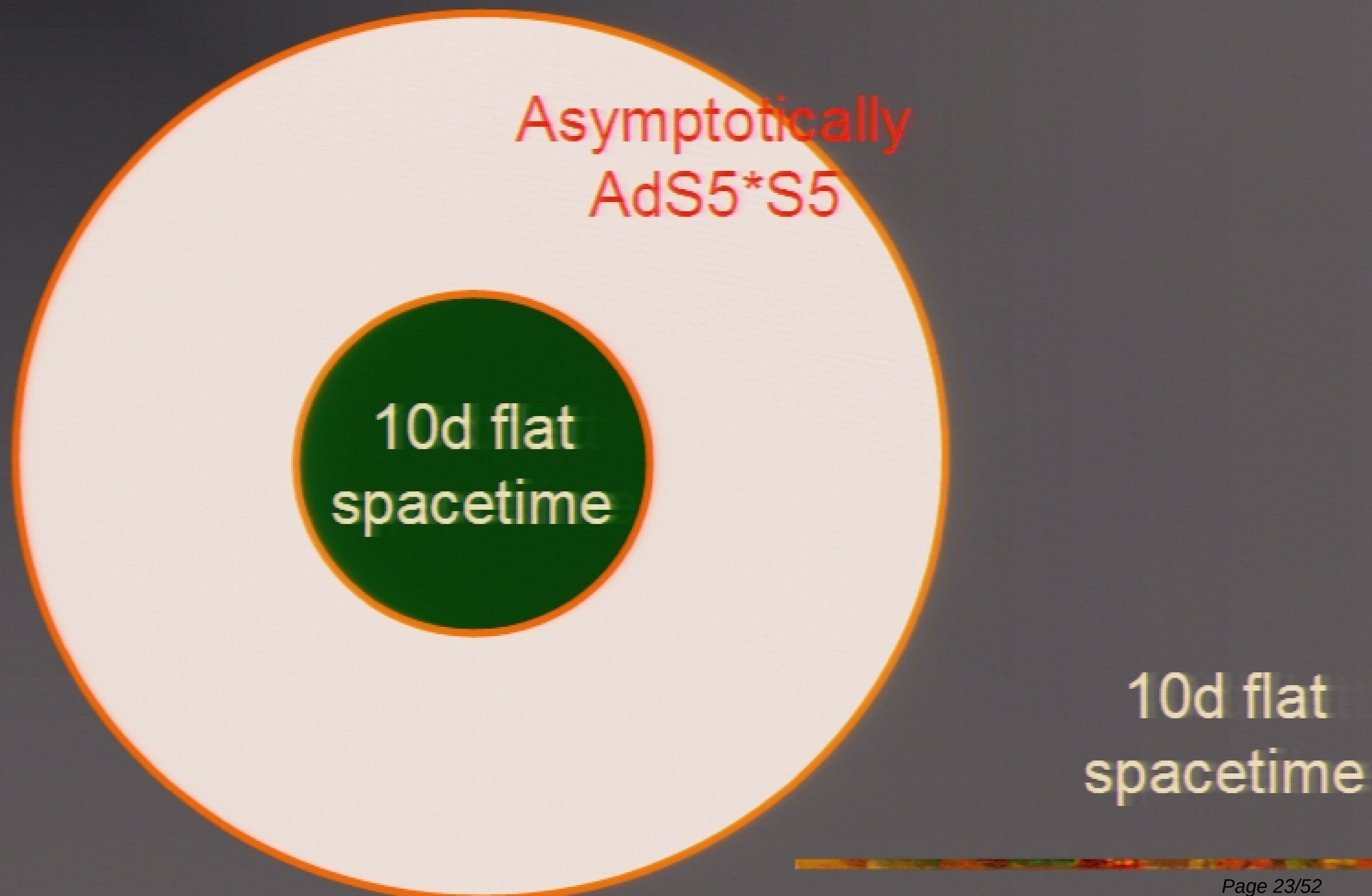

or (poorly drawn)

Near-horizon geometry

- $ds^2 = F^{1/2} (r^2 dx_4{}^2 + dr^2 / r^2 + d\,\Omega_5{}^2)$
- Asymptotically, $r \to \infty$: $AdS_5 \times S_5$
- Near the core, $r \to 0$: flat 10d spacetime
- Interpretation: constant instanton density "q" breaks conformal invariance in the infrared region
- Gravity dual to a certain class of N=2 SYM theory
- Nahm duality (= T-duality in string theory):

  U(N) with Q $\leftrightarrow$ U(Q) with N ($\to$ maximum Q)

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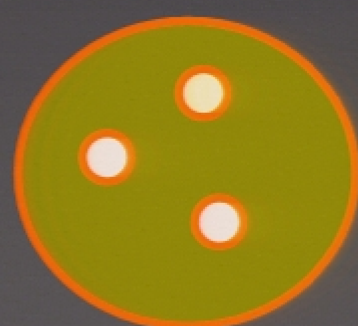
Recall AdS Throat = D3-branes

- D–instantons probing (Euclidean) AdS5
- For U(N) gauge group, “homogeneous” instanton number < N (otherwise inhomogeneous)
- Q D–instanton cluster in approx. flat region $S_{Dinstanton} = -(1/g_{st}\,\alpha'^2)\,\mathrm{Tr}_Q[\Phi^1, \Phi^2]^2 + \ldots.$



- $< \mathrm{Tr}(\Phi^1)^2 > \sim QL^2, \quad <\mathrm{Tr}(\Phi^2)^2 > \sim Q^2\, g_{st}\, \alpha'^2 / L^2$
- rotational symmetry $\rightarrow L^4 = Q\, g_{st}\, \alpha'^2 = N\, g_{st}\, \alpha'^2$

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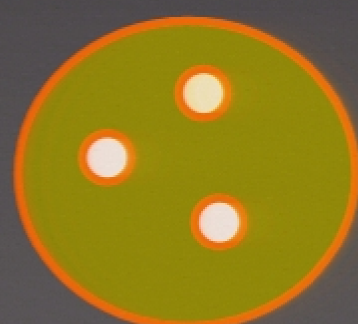

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Coupling parameters

- dilaton $e^{\Phi} \rightarrow \quad \lambda^2 = \lambda_\infty{}^2 (1 + q\, \lambda^2 / r^4)$
- axion C $\rightarrow \theta/2\pi = (1 + q\, \lambda^2/r^4)^{-1} - 1$

$$S_{YM} = \int_V (\lambda^2/N)\, \mathrm{Tr}\, F \cdot F + \theta\, \mathrm{Tr}\, F \wedge F$$

- N=2 supersymmetry implies no α'-corrections
- String loop corrections may be important near the core $\rightarrow$ infrared cutoff set by the 4d volume of the gauge theory

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SL(2, Z) and S-duality

- Type IIB string theory has SL(2, Z) symmetry:
- B_2 (NS-NS) ↔ C_2 (R-R)

  so F-string ↔ D-string (cf. Tye's lecture)

  NS5-brane ↔ D5-brane
- $(e^{-\Phi} + i\, C) \leftrightarrow 1/(e^{-\Phi} + i\, C)$

  so weak coupling ↔ strong coupling
- D3-brane intact but (E, B) ↔ (B, - E) S-duality

  IIB SL(2, Z) symmetry = S-duality on D3-brane

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Instanton-driven confinement

- Instanton condensates drive confinement
- To check this, we should find area-law behavior for <W> and perimeter-law behavior for <H>
- They are evaluated from minimal-area worldsheet for F-string and D-string ending both ends at the boundary

(see lecture notes 1 and 2)

Wilson loop

- Nambu–Goto action for F–string

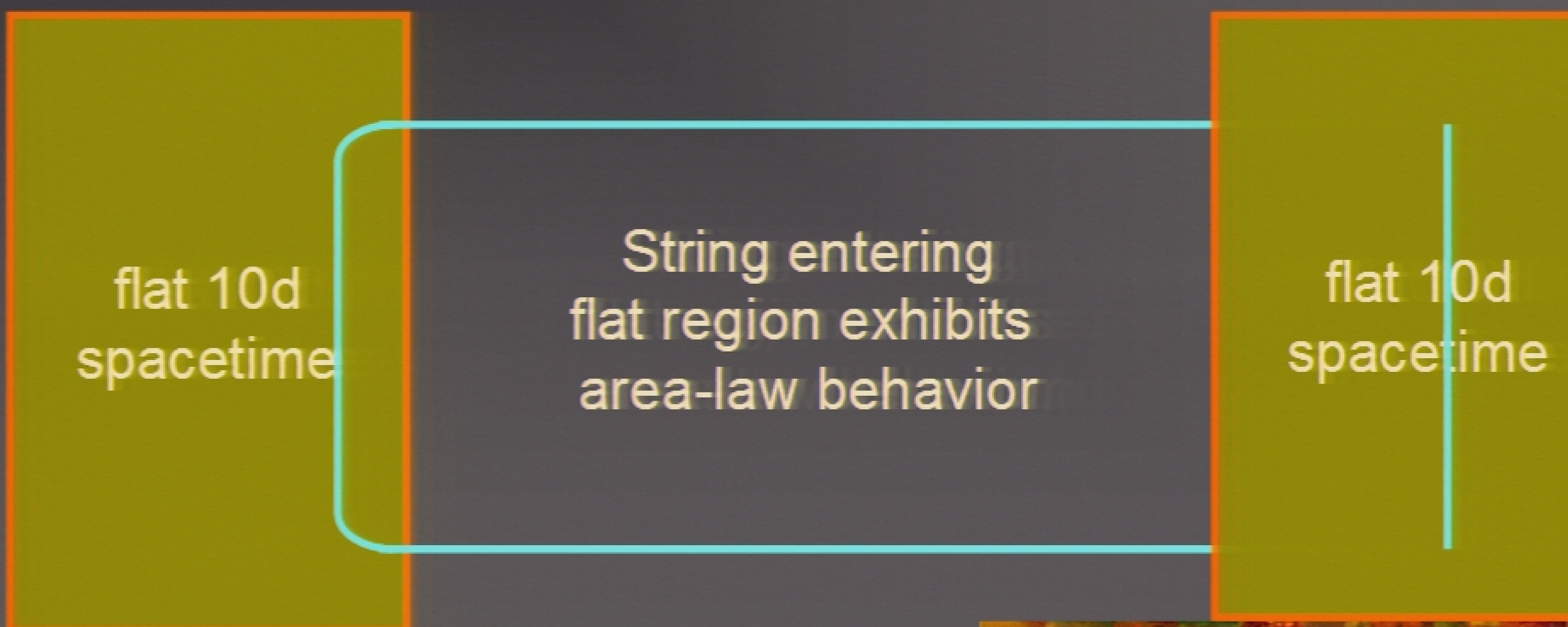
$$S = \int d^2 \sigma \, (g_{mn} \det \partial X^m \partial X^n)^{1/2}$$

- Compared to $AdS_5 \times S_5$ case, extra metric factor $F = (1 + q\, R^4/U^4)$ enters
- $S = \int d^2 \sigma \, F^{1/2} \, (U'^2 + U^4/R^4)^{1/2}$
- [Homework]: solve EOM and construct minimal string worldsheet

Hint: Near $r = 0$, change to $X = \lambda / U$

Confinement from brick wall…

- Without detailed computation, can see "why" confining behavior comes out



Confinement originates from

- In flat spacetime, F–string is confining by definition (highly tensional string)
- Arranging the spacetime deep inside AdS_5 deformed back to flat 10d–like
- Putting together, deformed geometry induces "area–law" behavior of <W>
- Actually, flat 5d is sufficient

How about `t Hooft loop?

- To assert “confinement”, it is also needed to check that `t Hooft loop <H> exhibits “perimeter-law” (screened) behavior
- <H> is evaluatable by computing D-string minimal-area worldsheet
- F-string and D-string are related by S-duality, so <H> can be deduced easily

`t Hooft loop

- $S_{D1} = \int d^2\sigma\, (e^{-2\Phi} - C_E{}^2)^{1/2}\, (g_{mn}\, \partial X^m\, \partial X^n)^{1/2}$
- Extra factor depending on dilaton/axion plays a crucial role:
  $(e^{-2\Phi} - C_E{}^2) = (1 - q\, R^4/r^4)/(1 + q\, R^4/r^4)$
- Inserting,
- $S_{D1} = \int d^2\sigma\, (1 - q\, R^4 / U^4)^{1/2}\, (U'^{\,2} + U^4/R^4)^{1/2}$
- This is the same as F–string action except $q \to -q$ in the funciton F!
- Consistent with SL(2, Z) and S–duality

Screening behavior emerges…

- $q \rightarrow -q$: S–dual spacetime geometry has spacetime singularity at $r = q^{1/4} R$
- The D–string action takes exactly the same form as F–string action in AdS_5 Schwarzschild black hole
- In the latter, string melted as it hits the horizon, where the metric factor $\rightarrow 0$.
- Homework: confirm this by explicitly solving for minimal–surface worldsheet of D–string

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Adding Flavors!

- Add N_F Dp-branes in bulk spacetime (p>3, N_F << N)
- quarks = open string stretching between D3 and Dp
  representation = (N*, N_F), (N, N_F*)
  mass = distance between D3 and Dp

Meson spectrum is again computable in two methods
 * small J: fluctuation of Dp-brane worldvolume shape

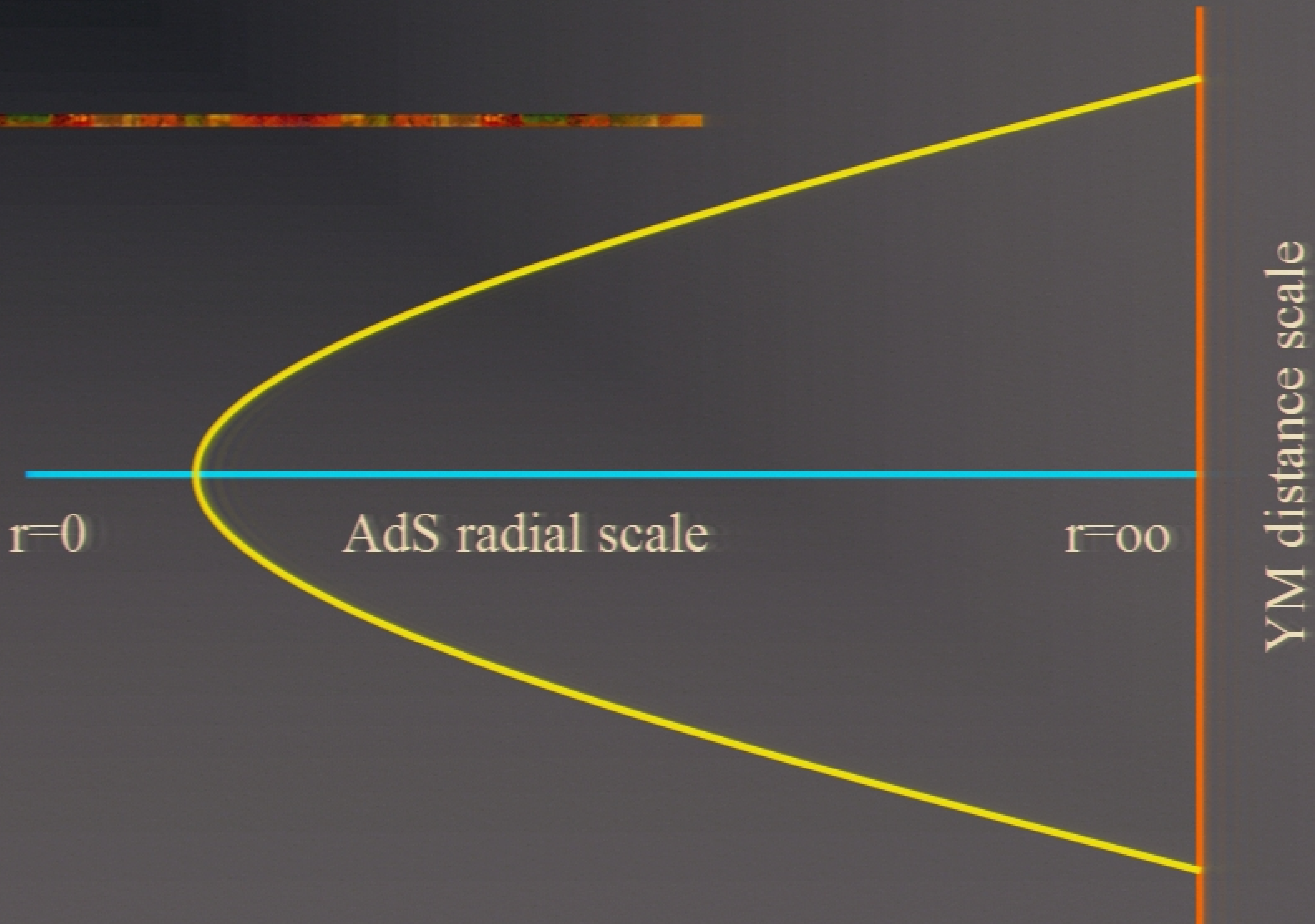
 * large J: open string attached to Dp-brane
 (heavy quark in uniform rotation)

cross-over at $J = (g^2_{YM} N)^{1/2}$: Regge --> Coulomb potential

Dynamical Test

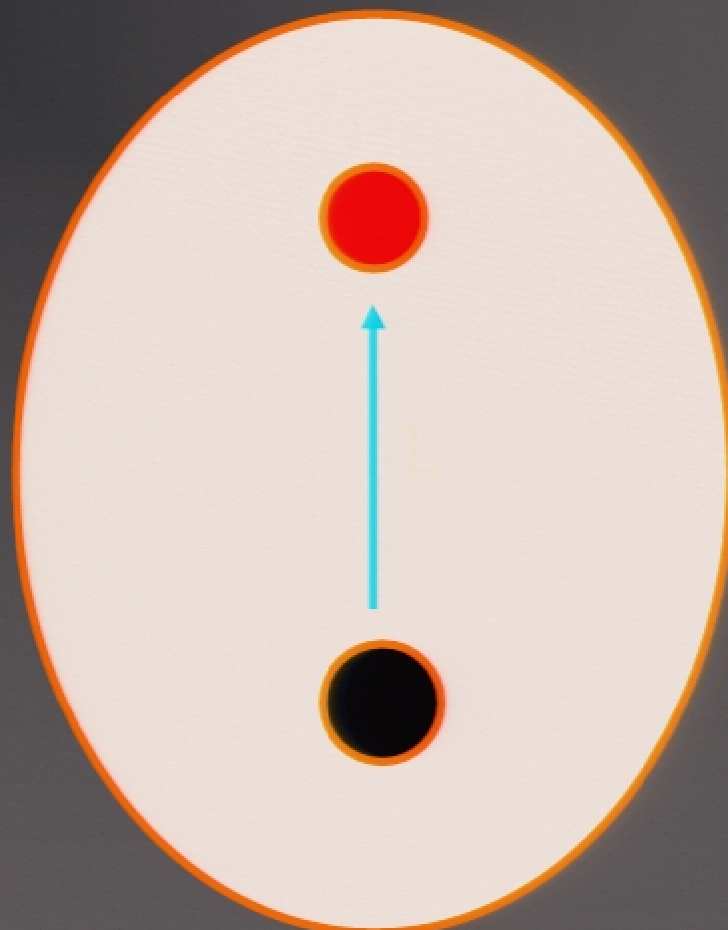
- What about time-varying phenomena?
- OK, take the meson at separation L
- Shake Q and send signal to Q*
  (Thomson scattering)
- How long does it take for signal to reach?
- Does Huygens' principle hold?
- Do Gravity and Gauge theory agree?

Gravity: signal along the string



Gauge: signal between the two quarks

$(\Delta t) = L/c$



- light signal along the string $U = U(\sigma)$:

$$\sqrt{Z}U^2 = \frac{1}{\sqrt{Z}}(-dt^2 + d\sigma^2)$$

$$\rightarrow \quad (\Delta t)_{\text{grav}} = \int_{-L/2}^{L/2} d\sigma \sqrt{ZU'^2 + 1}$$

- string satisfies first integral of motion:

$$Z^2U'^2 + Z \quad = \quad Z_* = \frac{g^2 N}{U_*^4}$$

$$\rightarrow (\Delta t)_{\text{grav}} = B\frac{\sqrt{g^2N}}{U_*} \qquad B = \frac{2\sqrt{\pi}\Gamma(5/4)}{\Gamma(3/4)}$$

- using "geometric UV-IR duality" relation,

$$(\Delta t)_{\text{grav}} = A(\Delta t)_{\text{gauge}}$$

$$A = \frac{\Gamma(5/4)\Gamma(1/4)}{\Gamma^2(3/4)} = 2.188...$$

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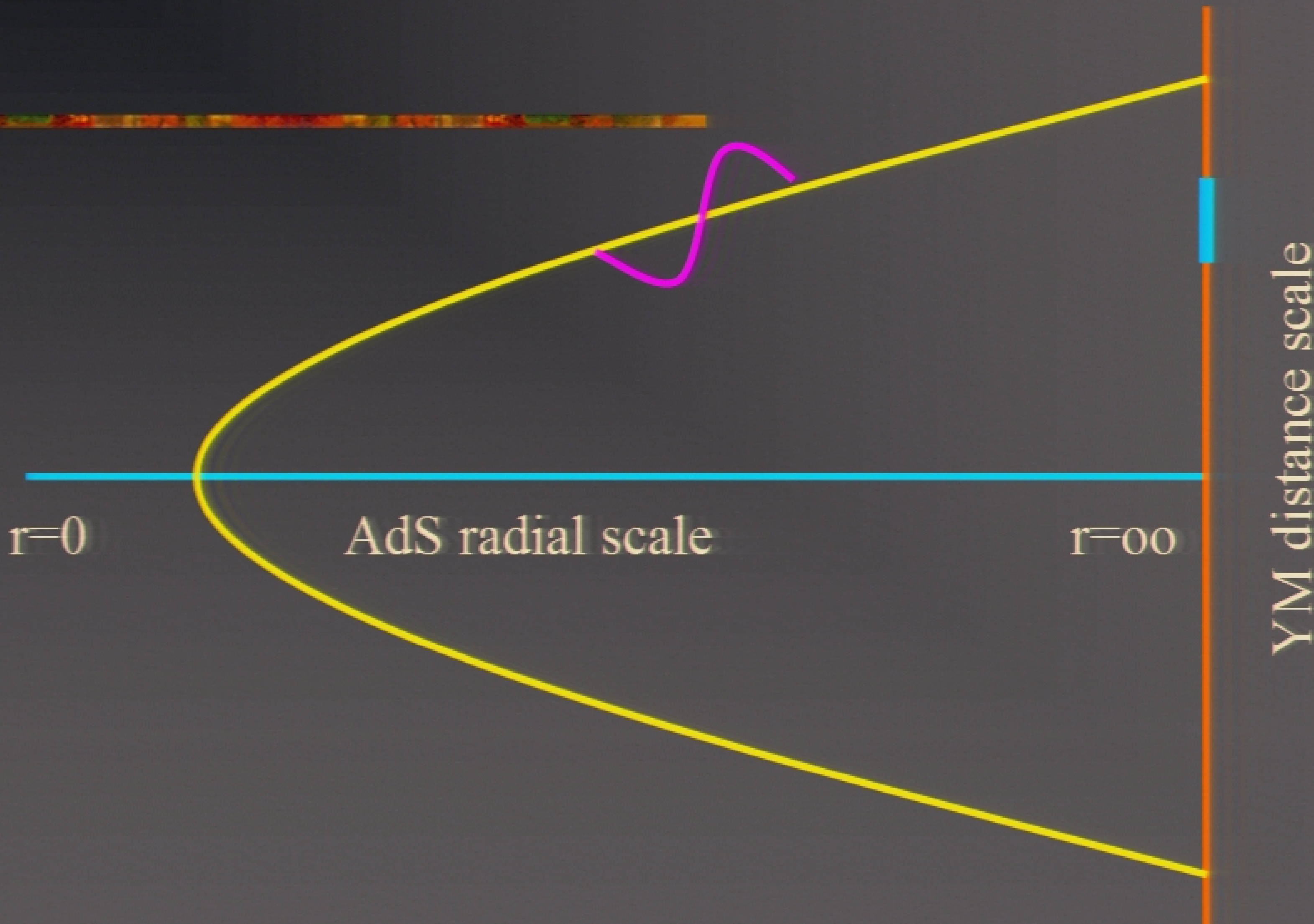

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So……….

- Causal time-delay does not match on both sides (strong field effects, strong coupling dynamics…)
- Huygens' principle?
- More surprise of AdS/CFT may be revealed by understanding time-varying phenomena better

Time to stop...

- AdS/CFT as indispensible tool for string theory and gauge theory altogether
- Act II by Balasubramanian this week
- Act III by Aharony next week
- Enjoy and Have Fun!