Title: SUSY Gauge Theory

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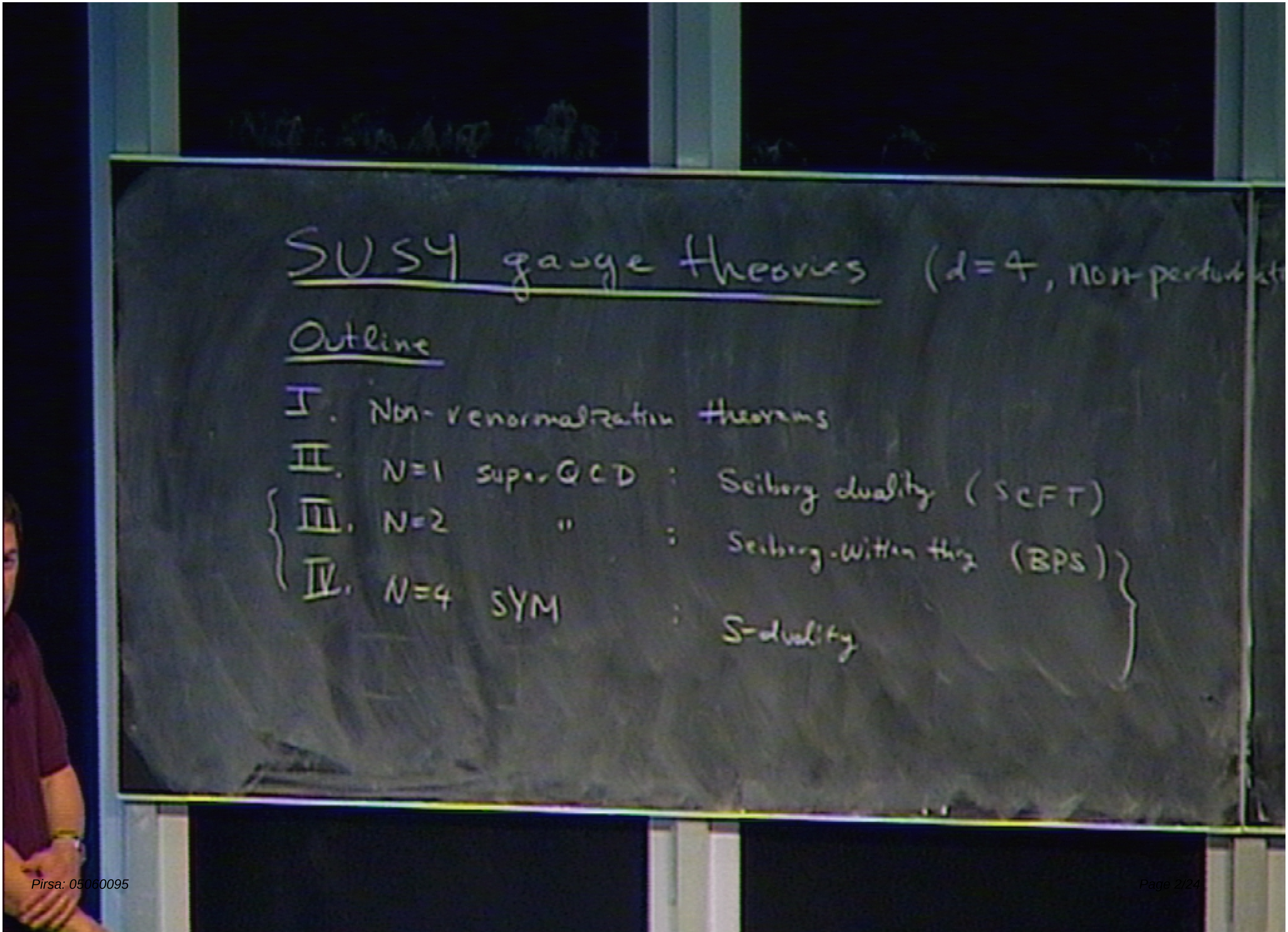
Abstract:


SUSY gauge theories (d=4, non-perturb
Outline
I. Non-renormalization theorems
II. N=1 super QCD : Seiberg duality (SCFT)
III. N=2 " : Seiberg-Witten thy (BPS)
IV. N=4 SYM : S-duality

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Outline

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III. $N=2$ " : Seiberg-Witten thy (BPS)

IV. $N=4$ SYM : S-duality

I. NR thms

A. N=1 review

B. Holomorphy $\int d^2\theta\, W$

C. NR thm χsf

D. YM review

E. NR thm vsf

F. Exactly marginal operators

A. N=1 reminder

chiral superfield

complex

Weyl

$$\Phi(x,\theta) \sim \phi(x) + \theta\cdot\psi + \dots \quad (\bar{D}_{\dot\alpha}\Phi = 0)$$

vector sfield

$$V(x,\theta,\bar\theta) \sim \bar\theta\sigma^\mu\theta A_\mu + \bar\theta\bar\theta\,\theta\cdot\lambda + \dots \quad (V = \bar{V})$$

chiral field strength

$$W_\alpha(x,\theta) \sim \lambda_\alpha + (\sigma^{\mu\nu}\theta)_\alpha \mathcal{F}_{\mu\nu} + \dots$$

$$\mathcal{F}_{\mu\nu} = \tfrac{1}{2}(F_{\mu\nu} + i\tilde{F}_{\mu\nu})$$

$$\tilde{F}_{\mu\nu} = \tfrac{1}{2}\epsilon_{\mu\nu\rho\sigma}F^{\rho\sigma}$$

gauge kinetic term

$$\frac{i\tau}{16\pi}\left(\mathcal{F}\right)^2 + c.c$$

$$\tau = \frac{\theta}{2\pi} + i\,\frac{4\pi}{g^2}$$

gauge invariance

$$e^{-V} \to e^{-i\bar{\Lambda}}\, e^{-V}\, e^{+i\Lambda}$$

$(\alpha = 0)$

$$W_\alpha \to e^{-i\Lambda}\, W_\alpha\, e^{+i\Lambda}$$

$$W_\alpha = -\frac{1}{4}\bar{D}^2 e^{-V} D_\alpha e^{V} \qquad D^\alpha W_\alpha = \bar{D}_{\dot\alpha}\bar{W}^{\dot\alpha} \quad \text{Bianchi}$$

$\underline{N=1 \quad \text{SQCD}} \quad (SU(N_c))$

$$\mathcal{L}_{SQCD} = \int d^4\theta \left(\bar{Q}^{ia} (e^V)_a^b Q_{ib} + \tilde{\bar{Q}}_i e^V \tilde{Q}^i \right) + \left(\int d^2\theta \left(\frac{i\tau}{16\pi} \mathrm{tr}(W^2) \right) + c.c. \right)$$

Q_{ia} "quark" χsf $\qquad \tilde{Q}^{ia}$ "antiquarks" χsf

$i = 1 \cdots N_f$

$a = 1 \cdots N_c \rightarrow N_c\text{-rep} \qquad \bar{N}_c\text{-rep}$

$(W_\alpha)_a^b \qquad ((W_\alpha)_a^a = 0)$

$N=1$ SQCD $(SU(N_c))$

$$\mathcal{L}_{SQCD} = \int d^4\theta \left(\bar{Q}^{\dagger i a} (e^V)_a^{\ b} Q_{ib} + \bar{\tilde{Q}}_i e^V \tilde{Q}^i \right) + \left(\int d^2\theta \left(\frac{i\tau}{16\pi} tr(W^2) \right) + c.c. \right)$$

Q_{ia} "quark" $\chi = f$ $\qquad$ $\tilde{Q}^{ia}$ "antiquarks" $\chi = \bar{f}$

$i = 1 \ldots N_f$

$a = 1 \ldots N_c \rightarrow N_c\text{-rep}$ $\qquad$ $\bar{N}_c\text{-rep}$ $\qquad$ $(W_\alpha)_a^{\ b}$ $\qquad$ $((W_\alpha)_a^{\ a} = 0)$

$$\delta \mathcal{L}_{SQCD} = \int d^2\theta \, m_i^{\ j} Q_j \tilde{Q}^i + c.c.$$

B. $\mathcal{N}=1$ SUSY selection rule: holomorphy of $\mathcal{W}$

$$\mathcal{L}_{eff} = \int d^4\theta \; \mathcal{K}_{eff}\left(\Phi, \bar{\Phi}, m, \tau, \bar{m}, \bar{\tau} \right)$$

$$+ \underset{\substack{K \\ D^2}}{\int d^2\theta} \left[\mathcal{W}_{eff}\left(\Phi, m, \tau, \bar{m}, \bar{\tau} \right) + \tau_{eff}\left(\Phi, \tau, \bar{\tau}, m, \bar{m} \right) \mathrm{tr}\left(\tilde{W}^2 \right) \right] + c$$

$+$ higher-derivative terms...

B. $\mathcal{N}=1$ SUSY selection rule: holomorphy of $\mathcal{W}$

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K

D^2

$+$ higher-derivative terms ...

$N=1$ SQCD $(SU(N_c))$

$$\sim -\frac{1}{4g^2}\operatorname{tr}F^2 + \frac{\theta}{32\pi^2}\operatorname{tr}F\tilde{F}$$

$$\mathcal{L}_{SQCD} = \int d^4\theta \left(\bar{Q}^{\dagger i a}\left(e^{V}\right)_a^b Q_{ib} + \bar{\tilde{Q}}_i e^{V} \tilde{Q}^i \right) + \left(\int d^2\theta \left(\frac{i\tau}{16\pi} \operatorname{tr}(W^2) \right) + c.c. \right)$$

Q_{ia} "quark" χsf $\qquad \tilde{Q}^{ia}$ "antiquarks" χsf

$i = 1 \ldots N_f$

$a = 1 \ldots N_c \rightarrow N_c$-rep $\qquad \bar{N}_c$-rep

$$(W_\alpha)_a^b \qquad ((W_\alpha)_a^a = 0)$$

$$\delta\mathcal{L}_{SQCD} = \int d^2\theta \; m_i^j Q_j \tilde{Q}^i + c.c.$$

B. $\mathcal{N}=1$ SUSY selection rule: holomorphy of $\mathcal{W}$

$$\mathcal{L}_{eff} = \int d^4\theta \; \mathcal{K}_{eff}\left(\Phi, \bar{\Phi}, m, \tau, \bar{m}, \bar{\tau} \right)$$

$$+ \int d^2\theta \left[\mathcal{W}_{eff}\left(\Phi, m, \tau, \cancel{\bar{m}}, \cancel{\bar{\tau}} \right) + \tau_{eff}\left(\Phi, \tau, \cancel{\bar{\tau}}, m, \cancel{\bar{m}} \right) \mathrm{tr}\left(\tilde{W}^2 \right) \right] + c$$

K

D^2

+ higher-derivative terms...

$\vec{E} = (a,0,0)$

$$H = V(\vec{x}\cdot\vec{x}) + \frac{\vec{p}^2}{2m} + a x_1 \Rightarrow \vec{E}\cdot\vec{x}$$

C. χsf NR thm

A

- use R-symmetry $\rightarrow$ Q_α transforms in non-trivial rep of R

$N=1$ SUSY : $U(1)_R$:

$$Q_\alpha \rightarrow e^{-i\varepsilon} Q_\alpha$$

$$\bar{Q}_{\dot\alpha} \rightarrow e^{+i\varepsilon} \bar{Q}_{\dot\alpha}$$

] +c.c

$$[R, Q_\alpha] = -Q_\alpha \qquad [R, \bar{Q}_{\dot\alpha}] = +\bar{Q}_{\dot\alpha}$$

$$Q_\alpha \sim \frac{\partial}{\partial\theta^\alpha} + \text{---}$$

$$R\left(\frac{\partial}{\partial\theta^\alpha}\right) = -1 = R(d\theta^\alpha)$$

C. Xsf NR thm

- use R-symmetry $\rightarrow$ Q_α transforms in non-trivial rep of R

N=1 SUSY : $U(1)_R$: $Q_\alpha \rightarrow e^{-i\alpha} Q_\alpha$

$\bar{Q}_{\dot\alpha} \rightarrow e^{+i\alpha} \bar{Q}_{\dot\alpha}$

$[R, Q_\alpha] = -Q_\alpha \qquad [R, \bar{Q}_{\dot\alpha}] = +\bar{Q}_{\dot\alpha}$

$Q_\alpha \sim \frac{\partial}{\partial\theta^\alpha} + \ldots$

$R\left(\frac{\partial}{\partial\theta^\alpha}\right) = +1 = R(d\theta^\alpha) \qquad R(d\bar{\theta}^{\dot\alpha}) = +1$

$\Rightarrow$ need

$R(K) = 0$

$R(W) = +2$

C. χsf NR thm

• use R-symmetry $\longrightarrow$ Q_α transforms in non-trivial rep of R

N=1 SUSY : $U(1)_R$:

$$Q_\alpha \rightarrow e^{-i\lambda} Q_\alpha$$
$$\bar{Q}_{\dot\alpha} \rightarrow e^{+i\lambda} \bar{Q}_{\dot\alpha}$$

$$[R, Q_\alpha] = -Q_\alpha \qquad [R, \bar{Q}_{\dot\alpha}] = +\bar{Q}_{\dot\alpha}$$

$$Q_\alpha \sim \frac{\partial}{\partial\theta^\alpha} + \ldots$$

$$R\left(\frac{\partial}{\partial\theta^\alpha}\right) = -1 = R(d\theta^\alpha) \qquad R(d\bar\theta^{\dot\alpha}) = +1$$

$\Rightarrow$ need

$$R(K) = 0$$
$$R(W) = +2$$

$$R(\Phi) = r$$
$$\Phi = \phi + \theta\psi + \ldots$$
$$R(\phi) = r$$
$$R(\psi) = r-1$$

C. Xsf NR thm

- use R-symmetry $\rightarrow$ Q_α transforms in nontrivial rep of R

N=1 SUSY $\quad U(1)_R$: $\quad Q_\alpha \to e^{-i\alpha} Q_\alpha$

$\bar{Q}_{\dot\alpha} \to e^{+i\alpha} \bar{Q}_{\dot\alpha}$

+c.c.

$$[R, Q_\alpha] = -Q_\alpha \qquad [R, \bar{Q}_{\dot\alpha}] = +\bar{Q}_{\dot\alpha}$$

$$Q_\alpha \sim \frac{\partial}{\partial\theta^\alpha} + \dots$$

$$R\left(\frac{\partial}{\partial\theta^\alpha}\right) = -1 = R(d\theta^\alpha) \qquad R(d\bar{\theta}^{\dot\alpha}) = +1$$

$\Rightarrow$ need

$R(K) = 0$

$R(W) = +2$

$R(\Phi) = r$

$\Phi = \phi + \theta\psi + \dots$

$R(\phi) = r \Leftrightarrow \phi \to e^{ir\alpha}\phi$

$R(\psi) = r-1 \qquad \psi_\alpha \to e^{i(r-1)\alpha}\psi_\alpha$

of R

$\Rightarrow$ need

$R(K) = 0$

$R(W) = +2$

$R(\Phi) = r$

$\Phi = \phi + \overset{+1}{\theta}\psi + \dots$

$R(\phi) = r \Leftrightarrow \phi \to e^{ir\alpha}\phi$

$R(\psi) = r-1 \qquad \psi_\alpha \to e^{i(r-1)\alpha}\psi_\alpha$

$U(1)_R \qquad U(1)_i$ ← non-R

$\downarrow \qquad \downarrow$

$R \qquad U_i$ ← generators

$R' = R + \sum_i a_i U_i \qquad a_i$ arbitrary

(Weinberg $\underline{\mathrm{III}}$)

'UV' theory $\mathcal{W}_{\mu_0} = \mathcal{W}_{\mu_0}(\Phi_n)$
@ scale μ_0

effective supst's $\mathcal{W}_{\mu} = \mathcal{W}_{\mu}(\bar{\Phi}_n)$
@ scale $\mu < \mu_0$

(Weinberg III)

'UV' theory @ scale μ_0 $\quad \mathcal{W}_{\mu_0} = \mathcal{W}_{\mu_0}(\Phi_n) \xrightarrow{\text{write as}} = Y\, \mathcal{W}_{\mu_0}(\Phi_n)$

	$U(1)_R$
Y	$+2$
Φ_n	0

effective superpot'l @ scale $\mu < \mu_0$ $\quad \mathcal{W}_{\mu} = g(Y, \Phi_n)$ ← assume same set of light d.o.f.

$$= Y\, g(\Phi_n)$$

let $Y \to 0$ becomes free $\Rightarrow g(\Phi_n) = \mathcal{W}_{\mu_0}(\Phi_n)$ (up to canonical scalings)

set $Y = 1 \Rightarrow \mathcal{W}_{\mu} = \mathcal{W}_{\mu_0}(\Phi_n)$

$$\int d^4x \qquad \frac{\partial}{\partial x} \sim \left(\frac{\partial}{\partial\theta}\right)^2$$

$$\mathcal{L}_\mu = \int d^4\theta \, Z_n \, \Phi_n \bar{\Phi}_n + \int d^2\theta \left(\frac{\mu}{\mu_0}\right)^{3-\Delta} \lambda \mathcal{O}$$

$$\mathcal{W}_{\mu_0} = \lambda \, \mathcal{O} \, , \quad \mathcal{O} = \prod_n \Phi_n^{r_n} \qquad \Delta = \sum_n r_n$$

$$\mathcal{W}_\mu = \left(\frac{\mu}{\mu_0}\right)^{3-\Delta} \lambda \mathcal{O}$$

$$\int d^4x \qquad \xi_n \sim \left(\frac{\partial}{\partial\theta}\right)^2$$

$$\mathcal{L}_\mu = \int d^4\theta\, Z_n\, \phi_n \bar{\phi}_n + \int d^2\theta \left(\frac{\mu}{\mu_0}\right)^{3-\Delta} \lambda \mathcal{O}$$

rescale $\quad \phi_n \to \phi_n^{can} = \sqrt{Z_n}\,\phi_n \qquad \mathcal{O} = \prod_n \phi_n^{r_n}$

$$W_\mu = \underbrace{\left(\frac{\mu}{\mu_0}\right)^{3-\Delta_0} \left(\prod_n Z_n^{-r_n/2}\right) \lambda_r}_{\lambda_r^{can}(\mu)} \mathcal{O}_r^{can}$$

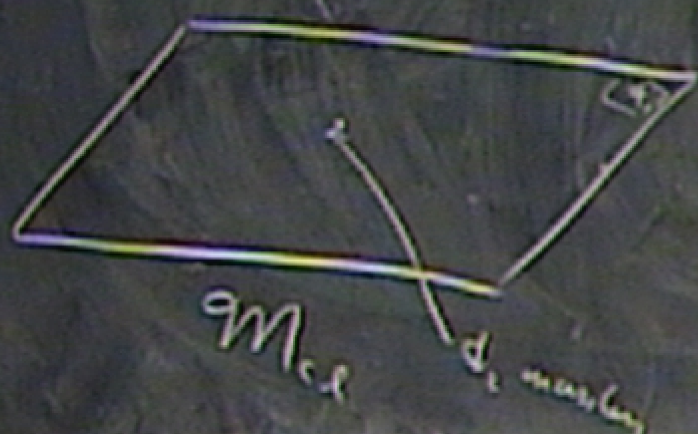
$$\boxed{\mu \frac{d\lambda_r^{can}}{d\mu} = \lambda_r^{can}(\mu)\left(\Delta - 3 - \frac{1}{2}\sum_n r_n \gamma_n\right)} \qquad \gamma_n \equiv \frac{d \ln Z_n}{d \ln \mu}$$

An example theory w/ 2 χsf's

$\phi_2 = 0$ $\quad$ $\phi_1\phi_2 = 0$

- UV: $\quad K_{\mu_0} = \bar{\phi}_1 \phi_1 + \bar{\phi}_2 \phi_2 \qquad W_{\mu_0} = \lambda \phi_1 \phi_2^2$

- classically $\Rightarrow$ moduli space (of vacua) $\mathcal{M}_{cl} = \{ \phi_2 = 0,\ \phi_1 \text{ arbitrary} = \mathbb{C} \}$



- everywhere ϕ_1 massless

  ϕ_2 has mass$^2 = |\lambda \langle \phi_1 \rangle|^2$

• Flow IR $\quad \mu < \mu_0$

NR thm $\quad \mathcal{W}_\mu = \mathcal{W}_{\mu_0} \Rightarrow \mathcal{M}_{q,\mu} \simeq \mathbb{C}$

• 1-loop renormalization of $\mathcal{K}_\mu$

for $\quad \mu < \text{mass}\ \phi_2 < \mu_0$

$$\mathcal{K}_\mu = \bar{\Phi}_1 \Phi_1 - (\#)\, \Phi_1 \bar{\Phi}_1 \, |\lambda|^2 \ln\left|\frac{\phi_1}{\mu_0}\right|^2$$

$$ds^2_\mu = (\partial_{\Phi_1} \partial_{\bar{\Phi}_1} \mathcal{K}_\mu)\, d\Phi_1 \, d\bar{\Phi}_1$$

$\mathcal{M}_{q^v}$

let $Y \to 0$ becomes free $\Rightarrow$ $g(\Phi_n) = \mathcal{W}_\mu(\Phi_n)$

set $Y = 1$ $\Rightarrow$ $W_\mu = \mathcal{W}_{\mu_0}(\Phi_0)$