Title: Advanced AdS/CFT Topics

Date: Jun 25, 2005 03:45 PM

URL: http://pirsa.org/05060093

Abstract:

Selected Topics in AdS/CFT
Lecture 2

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Perimeter Institute Summer School

June 2005

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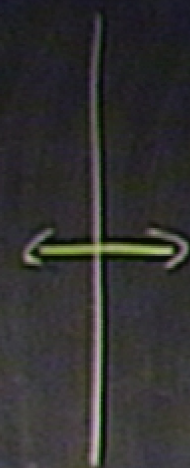
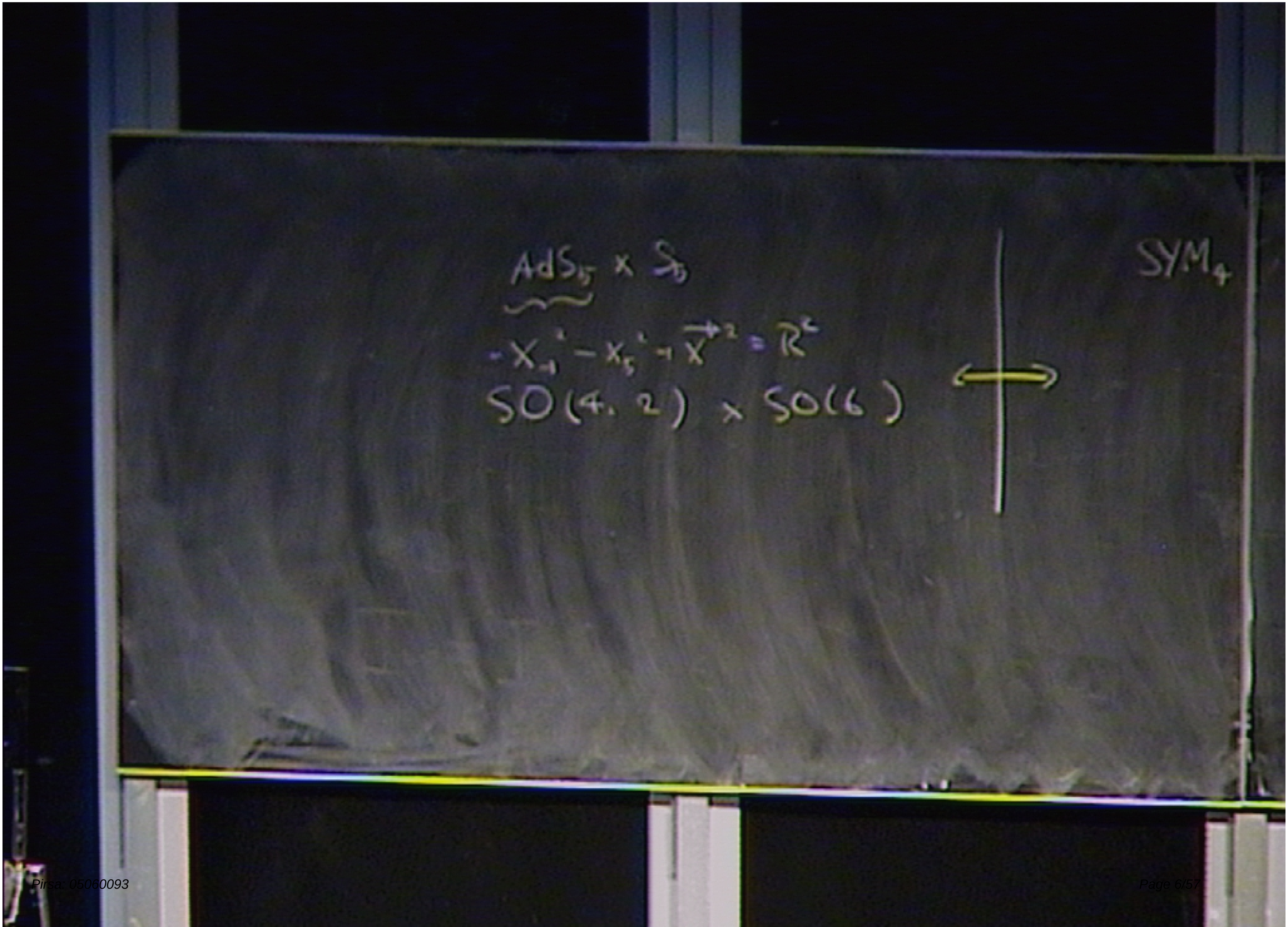
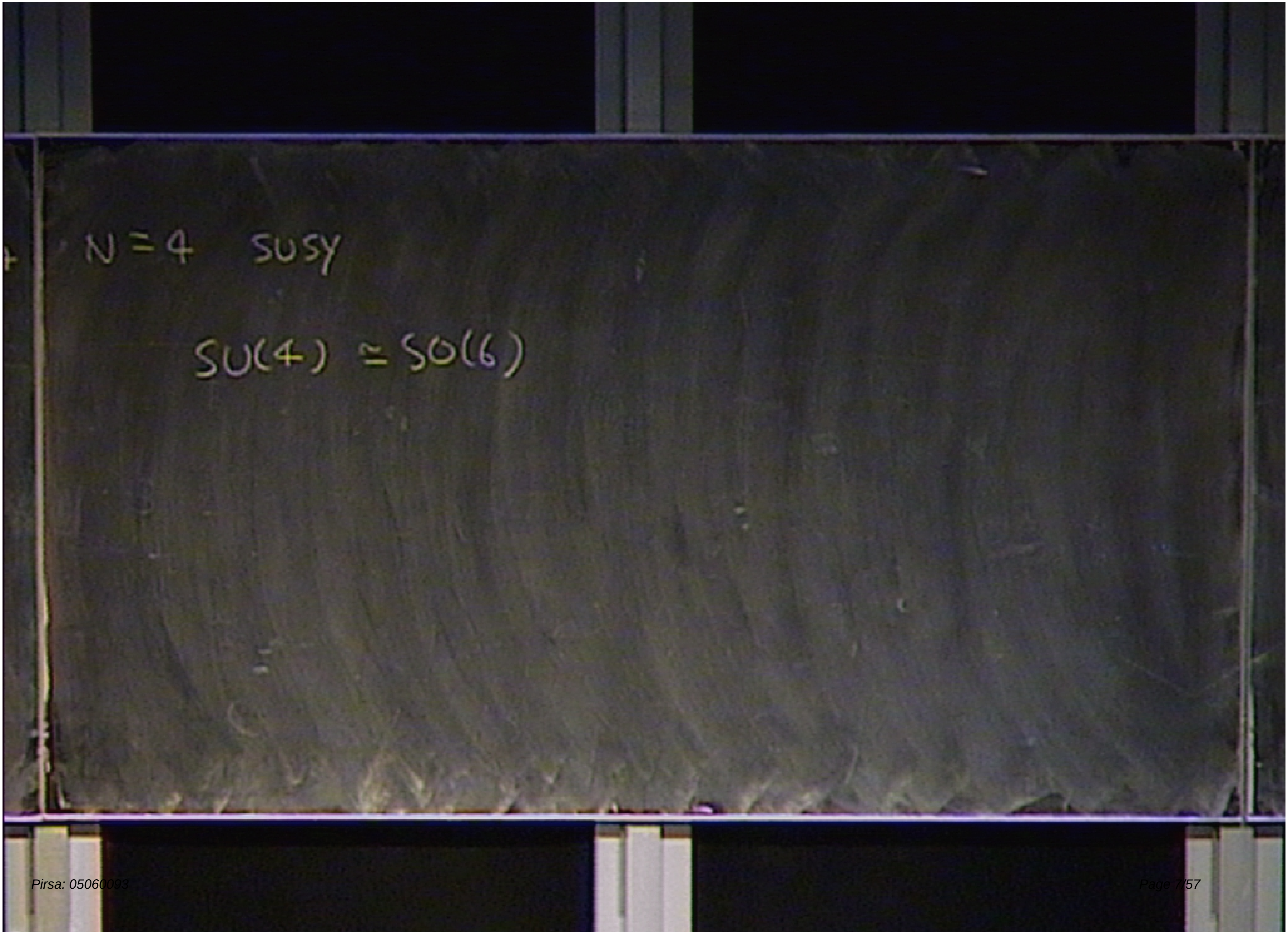
$$-X_{-1}^2 - X_5^2 + \vec{X}^2 = R^2$$

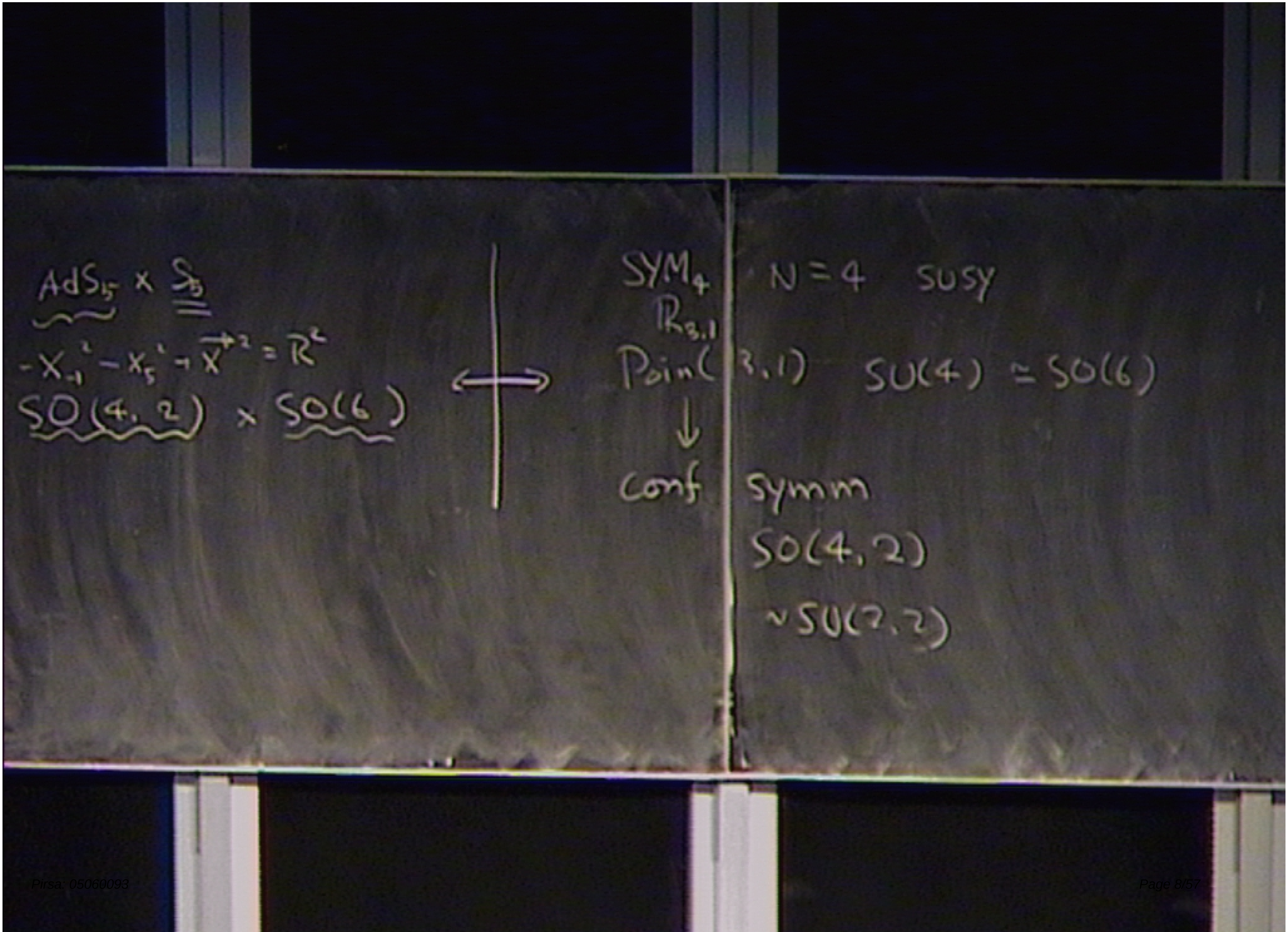
$$SO(4,2) \times SO(6)$$





$AdS_5 \times S_5$
$-X_{-1}^2 - X_5^2 + \vec{X}^2 = R^2$
$SO(4,2) \times SO(6)$
SYM_4


N = 4 susy
SU(4) ≃ SO(6)


AdS5 × S5
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SO(4, 2) × SO(6)
SYM4
R3,1
Poin(3,1)
conf symm
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T=0 vs. T>0 SYM Theory

T=0 (zero-temperature 4d N=4 SYM):

$$ds^2 = r^2 (-dt^2+dx^2) + dr^2/r^2 + (dS_5)^2$$

r = 5th dim // 4d energy scale: $0 < r < \infty$

T>0 (finite-temperature 4d N=4 SYM):

$$ds^2 = r^2 (- F\, dt^2+dx^2) + F^{-1}\, dr^2/r^2+(dS_5)^2$$

$F = (1 - (kT)^4/r^4)$: $kT < r < \infty$

5d AdS Schwarzschild BH = 4d heat bath

Static Quark Potential at Finite Temperature

$$V(r) \sim -(1.254..)\sqrt{g_{YM}^2 N}\left(\frac{1}{r}-\frac{1}{r_*}\right)\theta(r-r_*)$$

Notice:

- Nonanalyticity in λ persists
- exact 1/r persists
- potential vanishes beyond r_*

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- geometry produced by non-BPS D3-branes:

$$ds^2 = \frac{1}{\sqrt{Z}}(-H dt^2 + d\mathbf{x}^2) + \sqrt{Z}\frac{1}{H}dr^2$$

$$H = 1 - \frac{M^4}{r^4} \qquad M^4 = \frac{2^7\pi^4}{3}(g^2N)^2\frac{\mathcal{F}}{N^2}$$

N=4 SYM free energy $\quad \mathcal{F} = \frac{4\pi^2}{45}N^2T^4$

- first integral of motion:

$$\left(\frac{Z}{H}\right)^2 U'^2 + \frac{Z}{H} = \left(\frac{Z}{H}\right)_* = \frac{g^2N}{M^4}\frac{1}{(U_*/M)^4 - 1}$$

- $T \neq 0$ geometric UV-IR relation:

$$L = \frac{\sqrt{g^2N}}{U_*} f\left(\frac{U_*}{M}\right) \quad \text{with} \quad U_* \geq M$$

UV-IR relation for T>0

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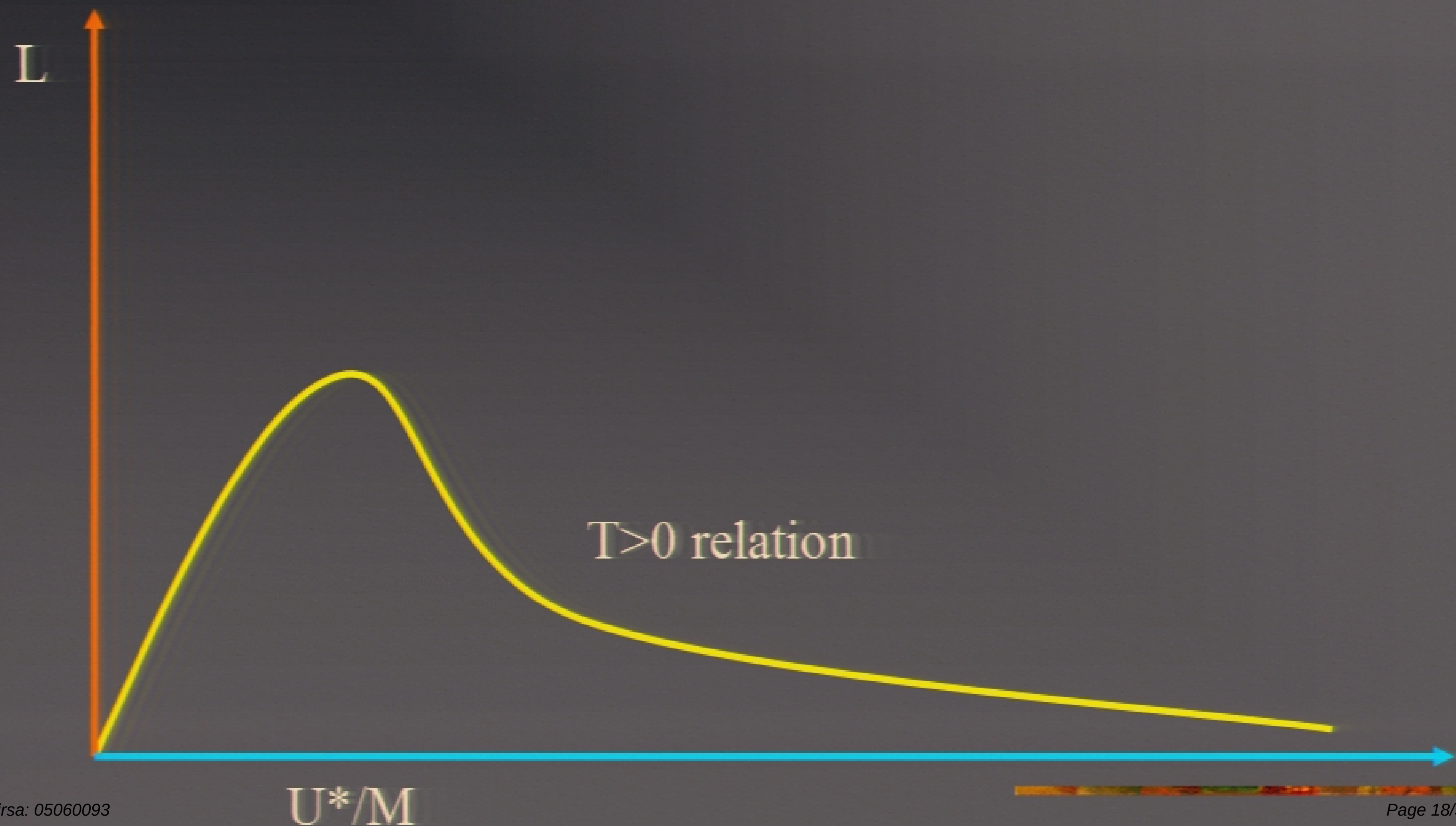

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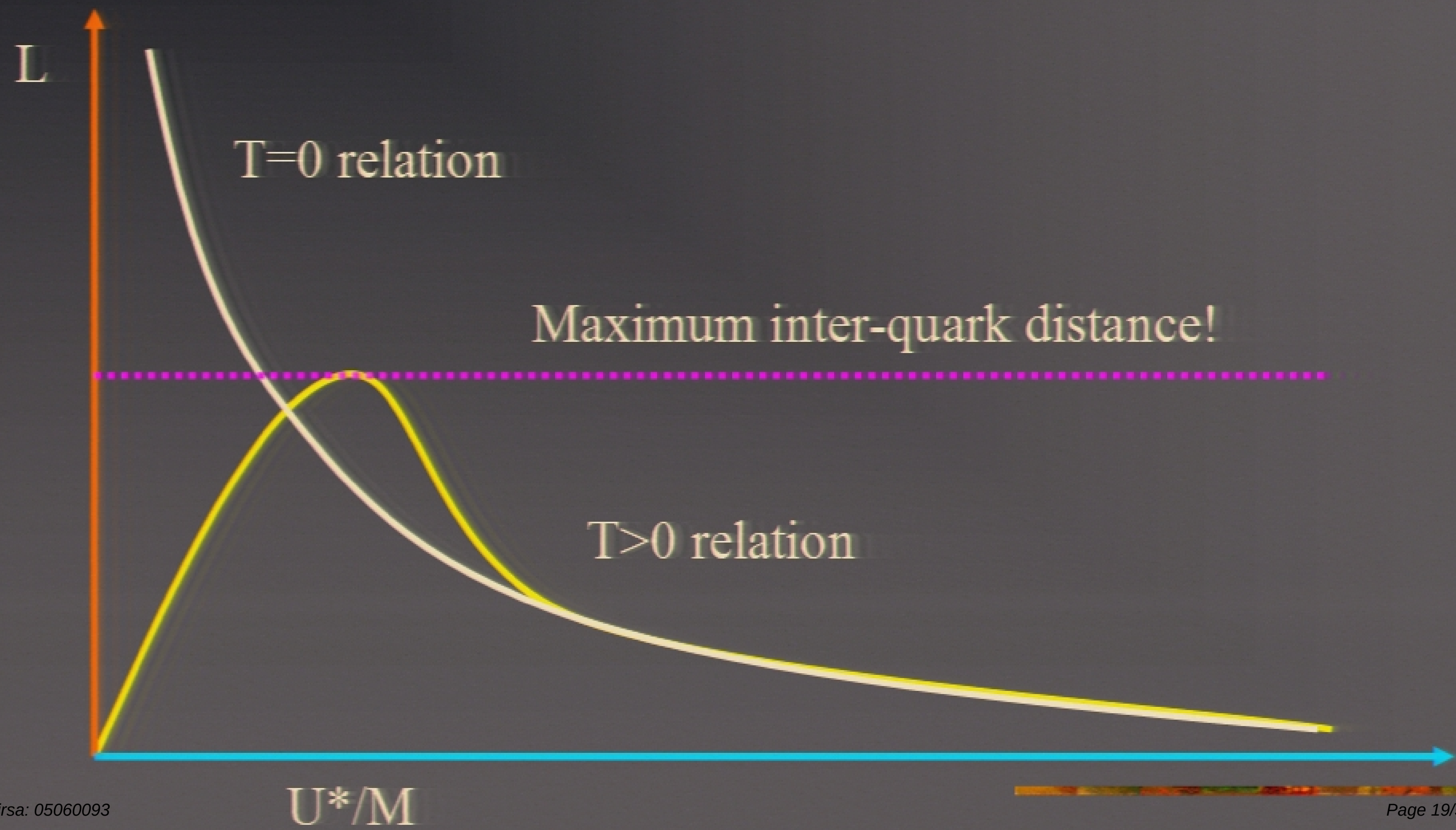
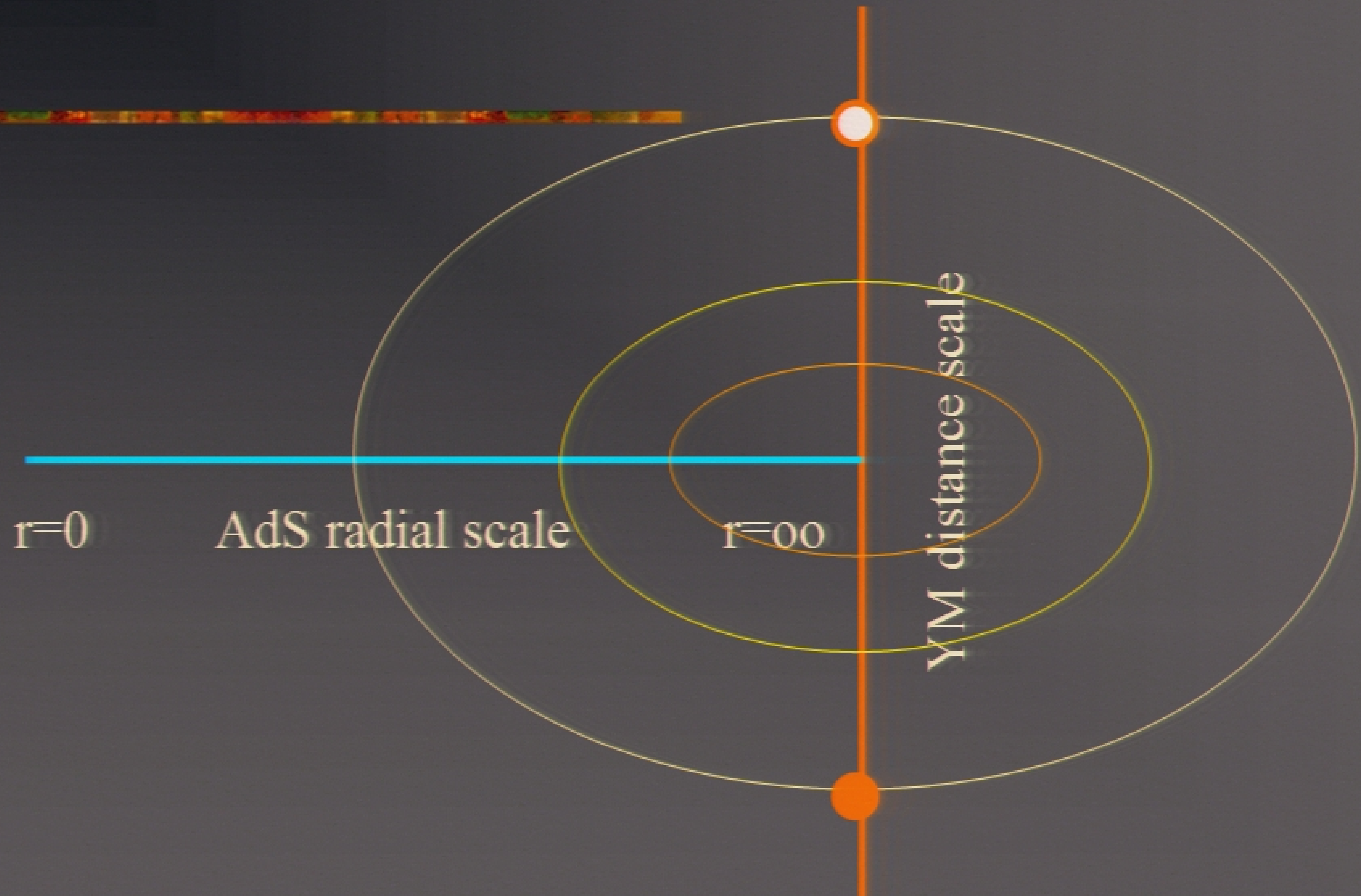


Heavy Meson Configuration (T > 0)



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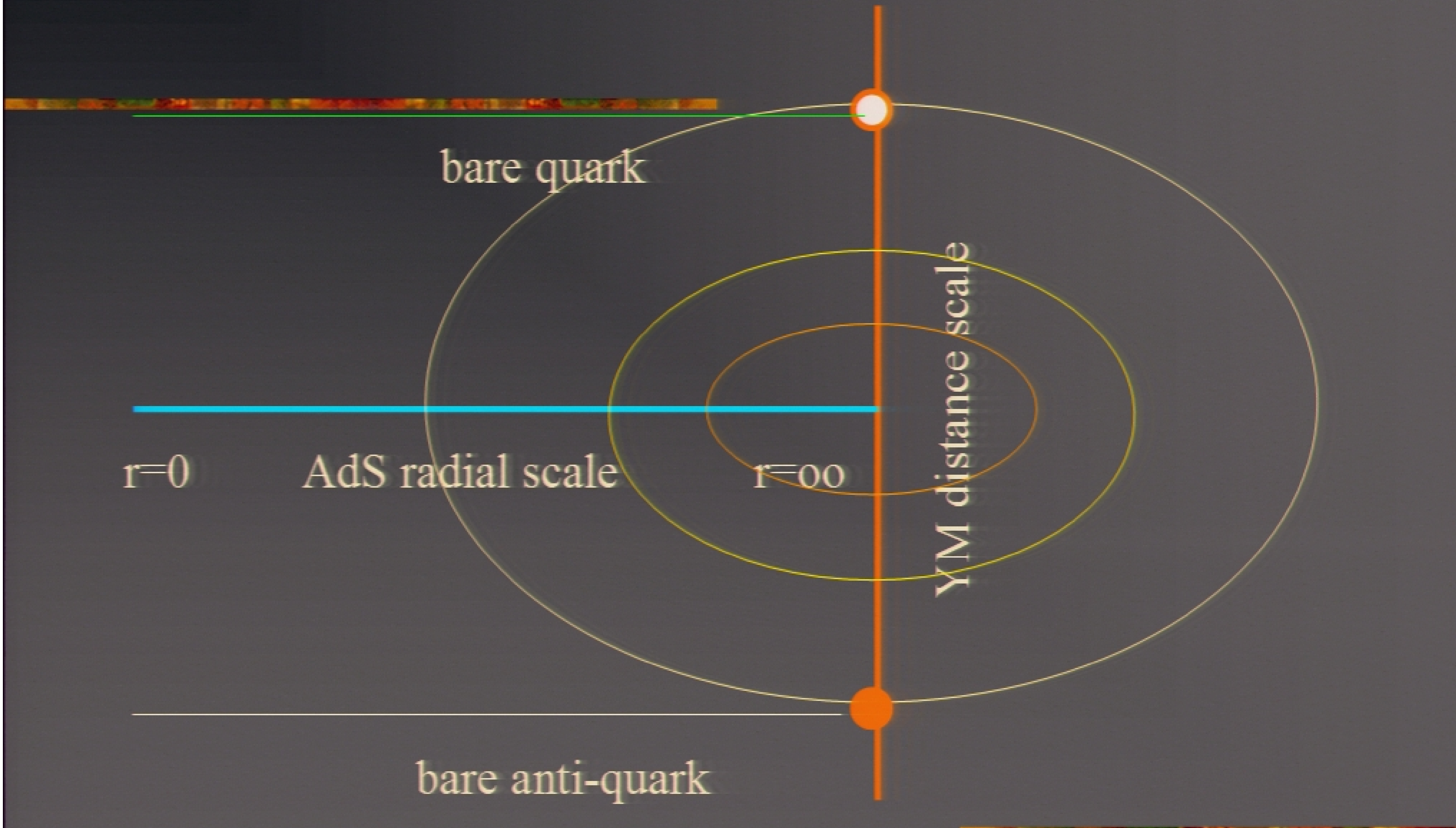

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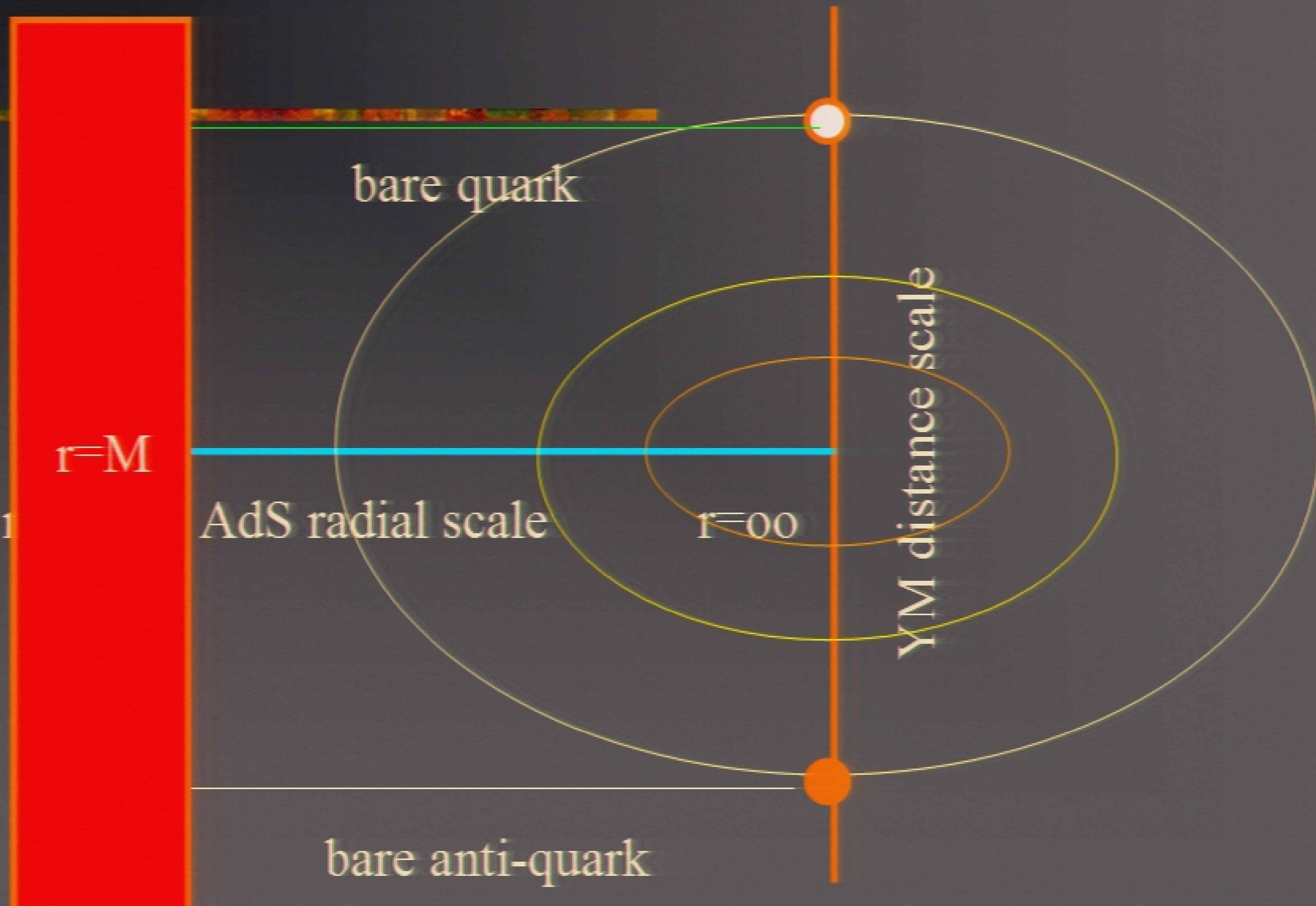

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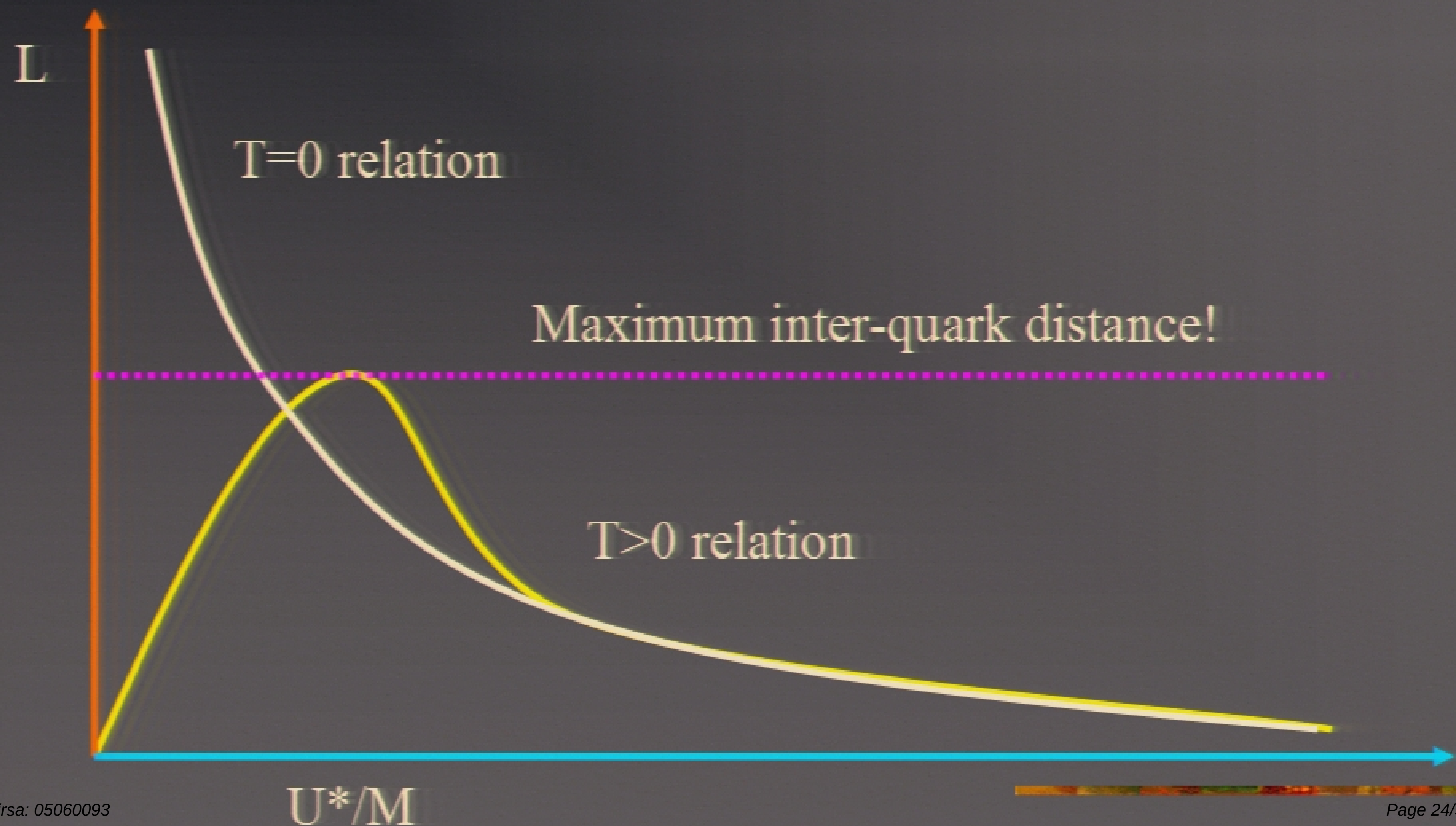

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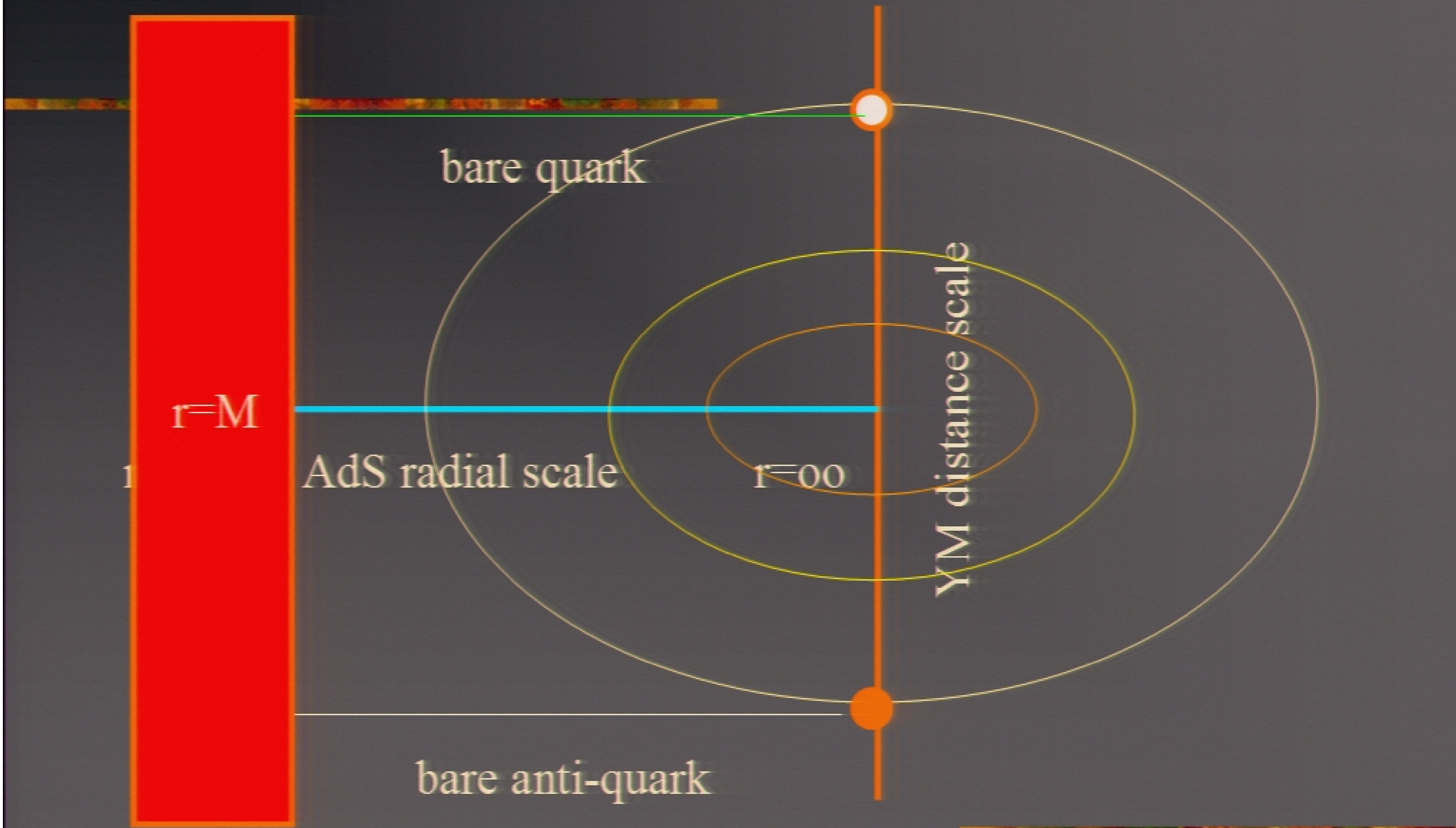

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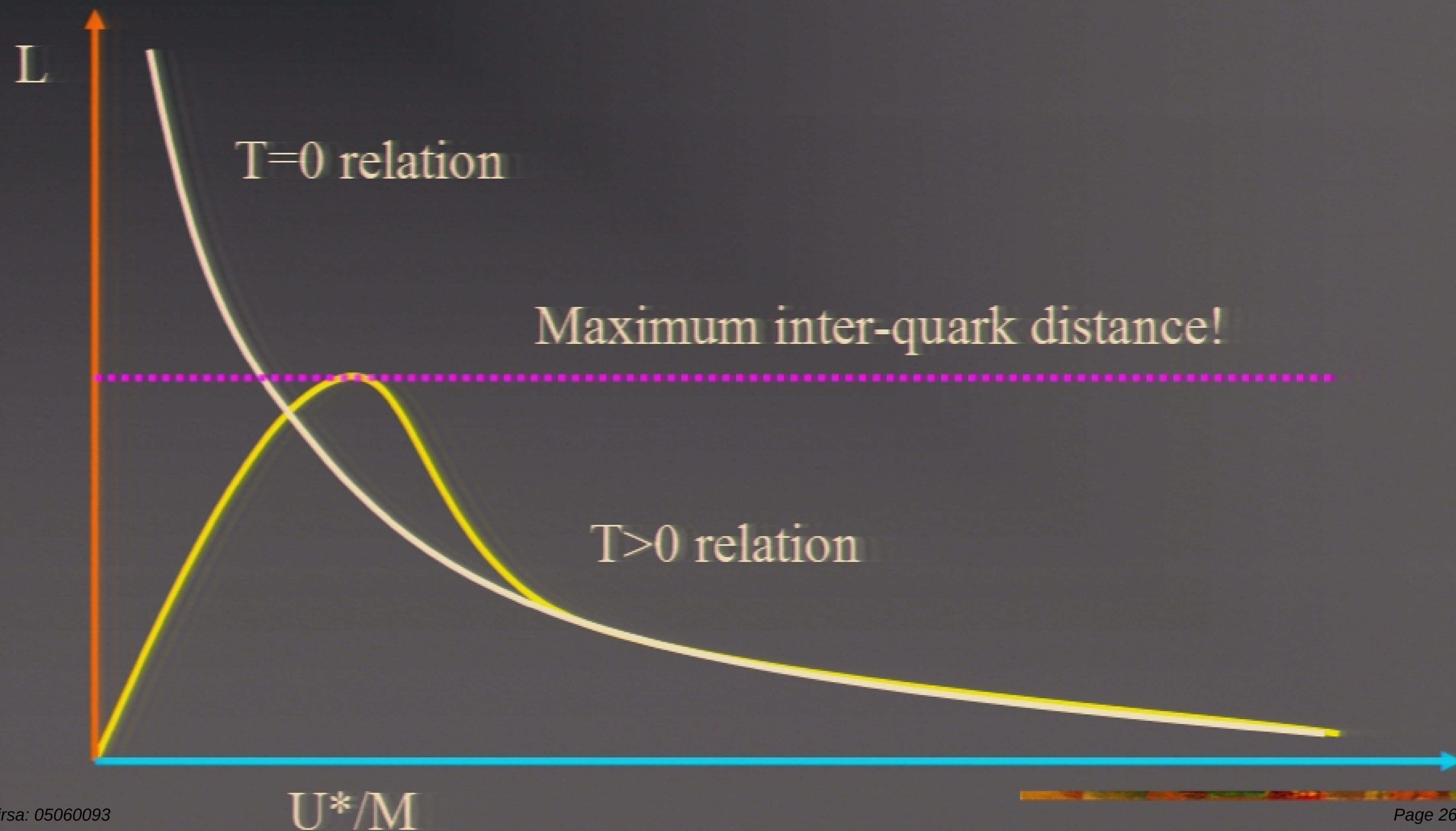

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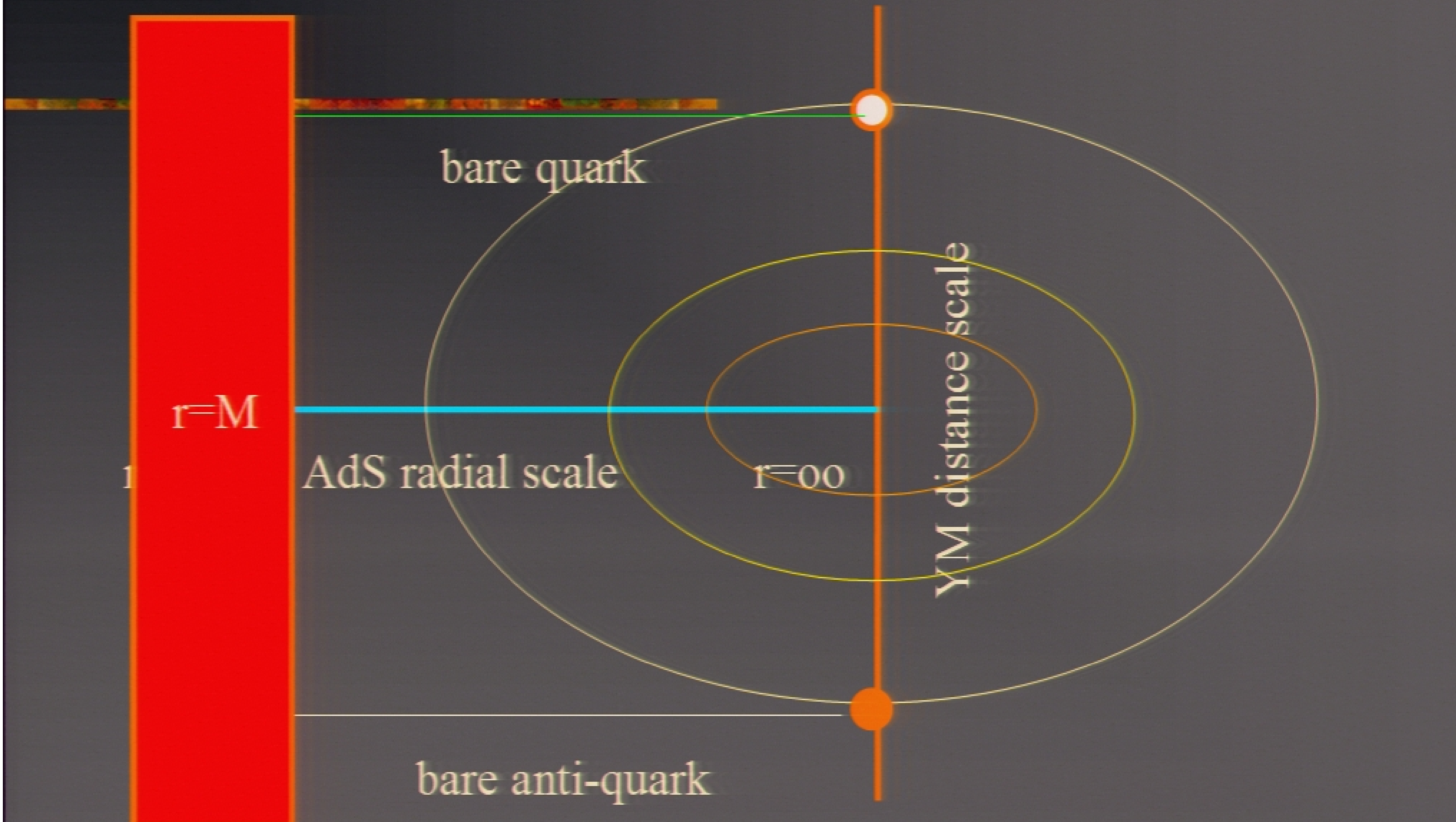

- string energy:

$$E_{(Q\overline{Q})}(a = U_*/M) = \frac{1}{2\pi\alpha'}\int d\sigma\sqrt{U'^2 + HZ^{-1}}$$

$$= \lim_{u\to\infty} 2M\int_a^u dx\sqrt{\frac{x^4-1}{x^4-a^4}}$$

- heavy-quark potential:

$$V_{(Q\overline{Q})}(U_*) \equiv E_{(Q\overline{Q})}(a) - (M_Q + M_{\overline{Q}})_{T>0}$$

- using "geometric UV-IR duality",

$$V_{Q\overline{Q}}(L) = -C\frac{\sqrt{g^2N}}{L}h(LM) \qquad C = 1.254...$$

$$h(LM) = 1 - \frac{1}{2C}\frac{LM}{\sqrt{g^2N}} + \mathcal{O}\left(\frac{LM}{\sqrt{g^2N}}\right)^4$$

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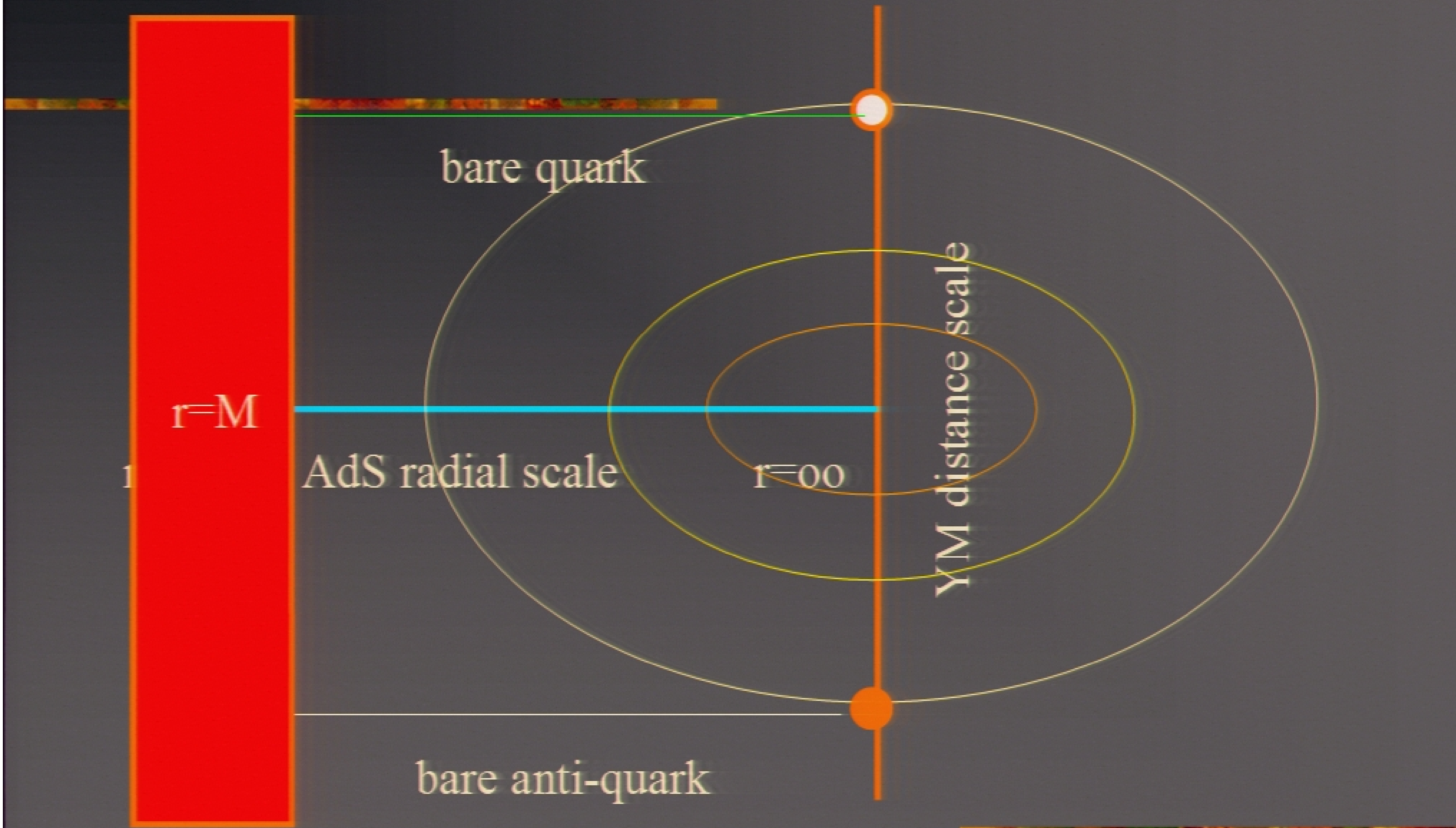

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Compactify on "thermal" S1

* Euclidean AdS5 Schwarzschild black hole:

$$ds^2 = r^2 (F\, dt^2 + dx^2) + F^{-1}\, dr^2/r^2 + (dS_5)^2$$

$$F = (1 - (kT)^4/r^4): \qquad kT < r < \infty$$

* antiperiodic boundary condition for fermions on τ
* 3d x-space is now Lorentzian
* Similar construction works for nonextreme D4-brane wrapped on "thermal" S1 → 4d Lorentzian x-space

Application: D-branes on "thermal" S_1

- "thermal S_1" breaks N=4 susy completely
- At low-energy, 3d Yang-Mills + (junks)
- 5d AdS replaced by
  5d Euclidean black hole (time ↔ space)
- glueball spectrum is obtainable by studying
  bound-state spectrum of gravity modes

Note:

4d space-time, topology: gravity // gauge

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YM_{2+1} glueball spectrum

- 0^{++}: solve dilaton eqn=2nd order linear ode
- result:

	N=3 lattice	N=oo lattice	AdS/CFT
.	4.329(41)	4.065(55)	4.07(input)
*	6.52 (9)	6.18 (13)	7.02
**	8.23 (17)	7.99 (22)	9.92
***	-	-	12.80

[M. Teper]

YM_{3+1} glueball spectrum

- Use T>0 D4-brane instead
- 0^{++}: solve dilaton eqn=2nd-order linear ODE
- result:

	N=3 lattice	AdS/CFT
.	1.61(15)	1.61(input)
*	2.8	2.38
**	-	3.11
***	-	3.82

[M. Teper]

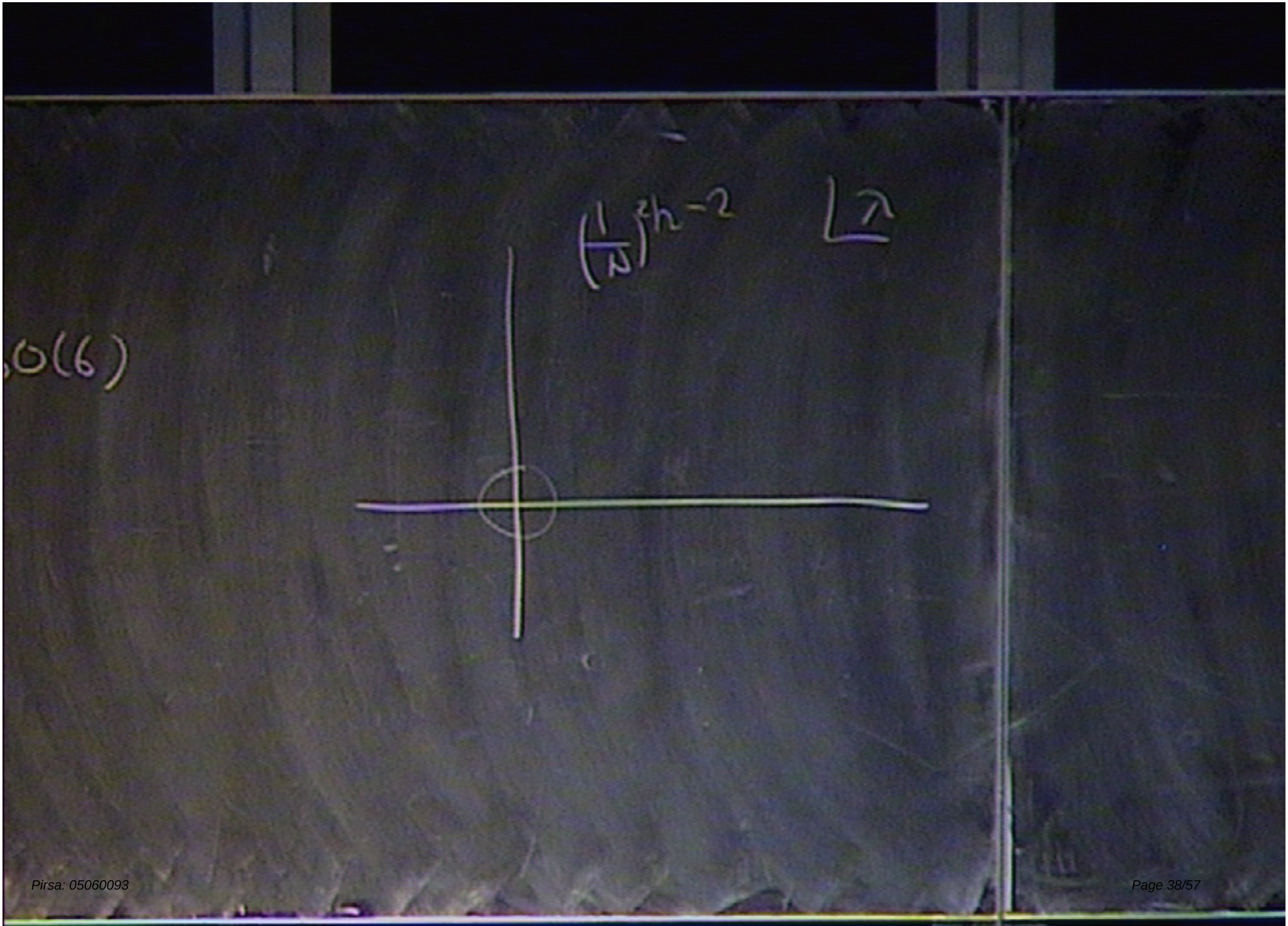
other glueballs fit reasonably well (why??)

The Story of Square-Root

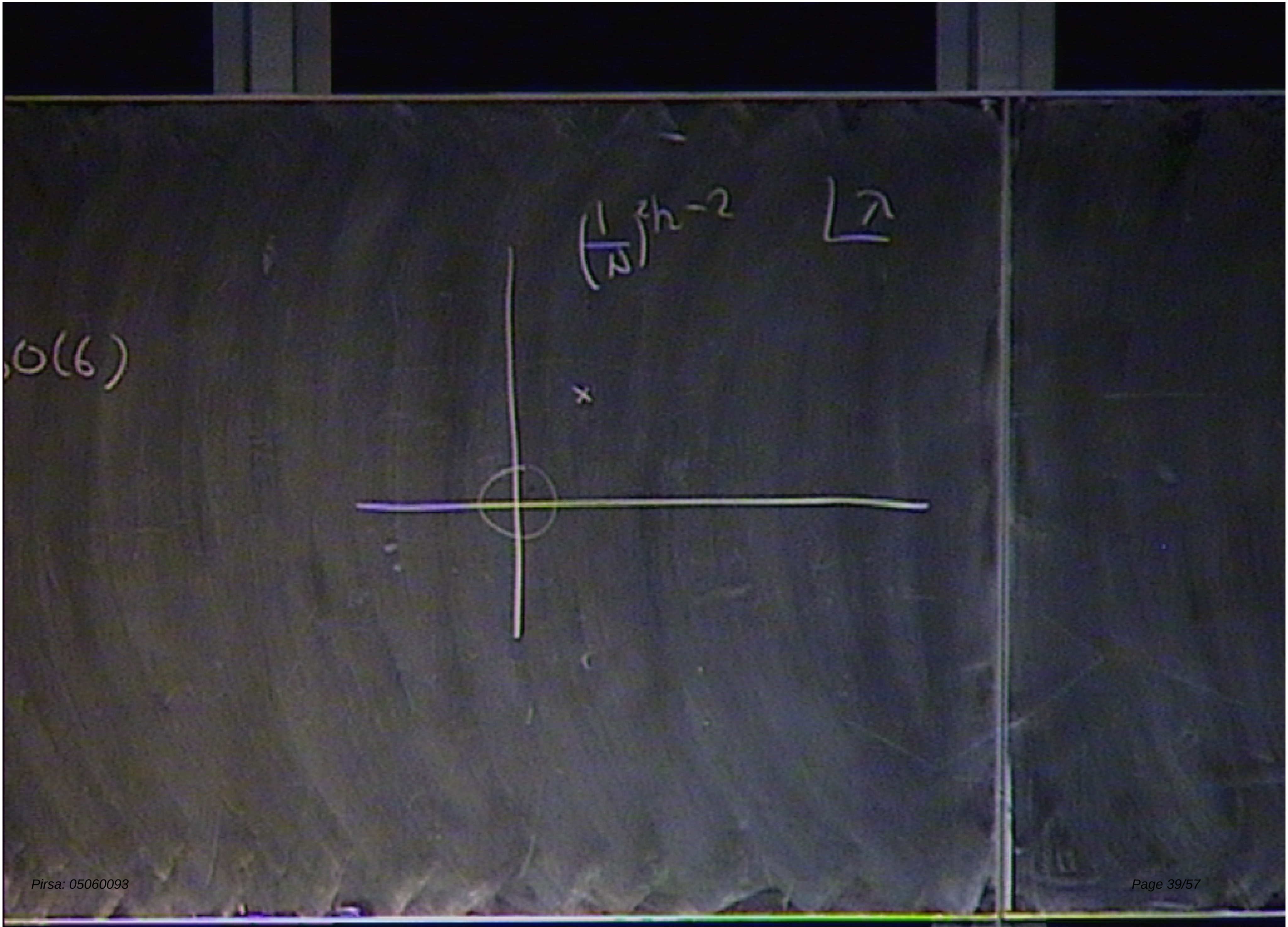
- branch cut from strong coupling?
- artifact of $N \to \infty$ limit
- heuristically, saddle-point of matrices

$$\langle \mathrm{Tr}\, e^M \rangle = \int [dM]\, (\mathrm{Tr}\, e^M) \exp(-\lambda^{-2}\, \mathrm{Tr}\, M^2)$$

Recall modified Bessel function



$\left(\frac{1}{N}\right)^{2h-2}$
O(6)



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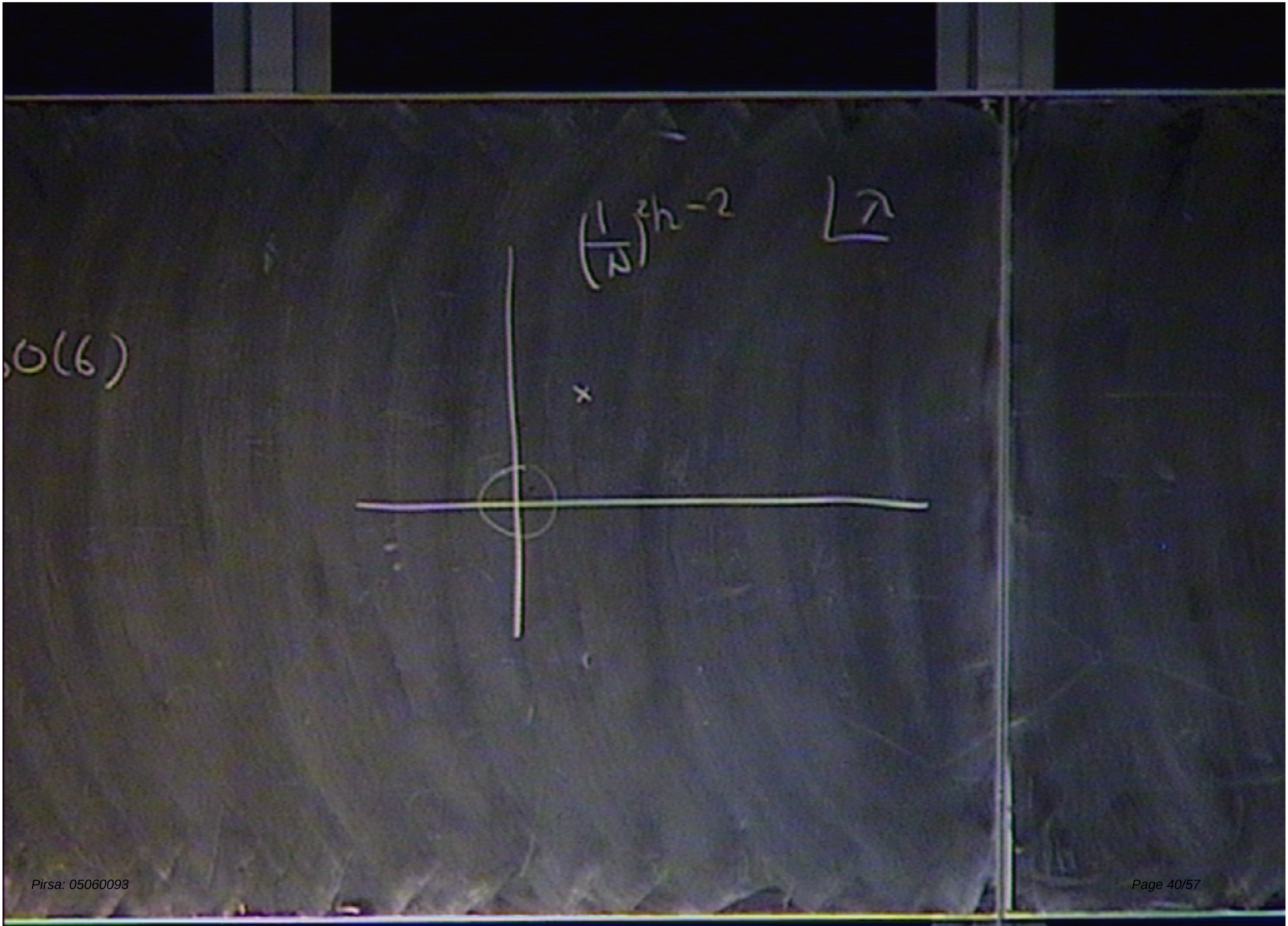
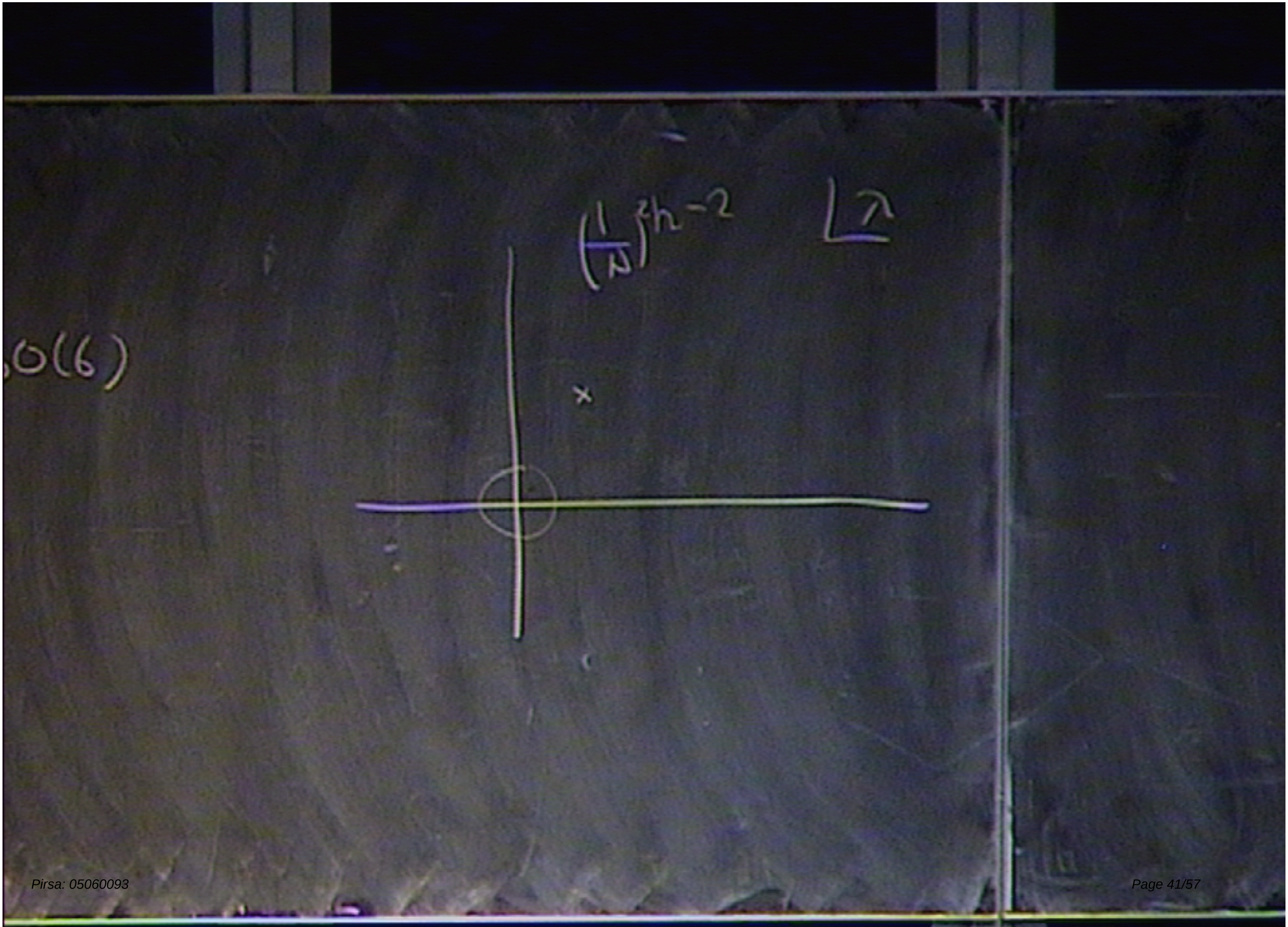

(1/w)^{2h-2}
L λ
O(6)



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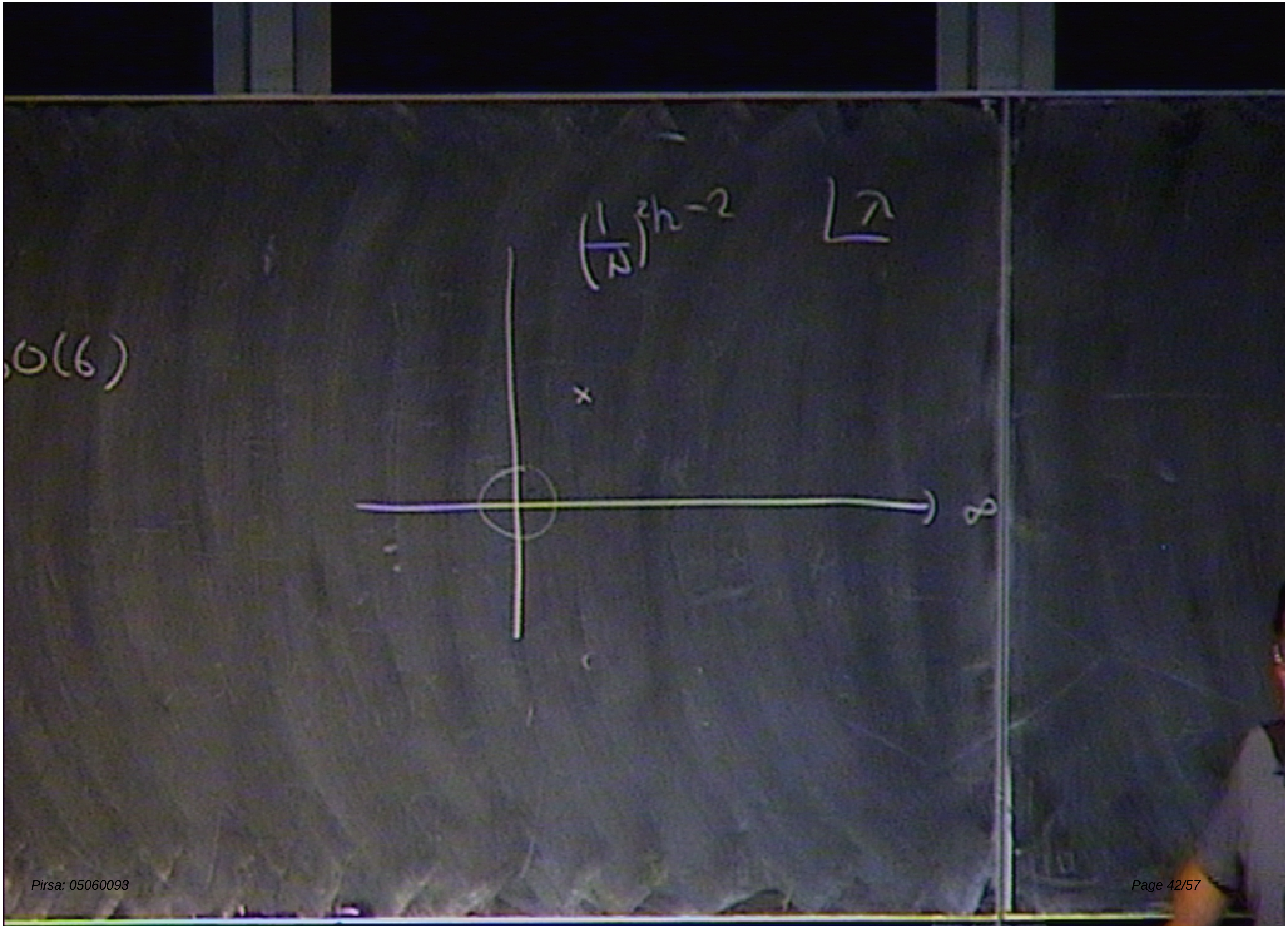
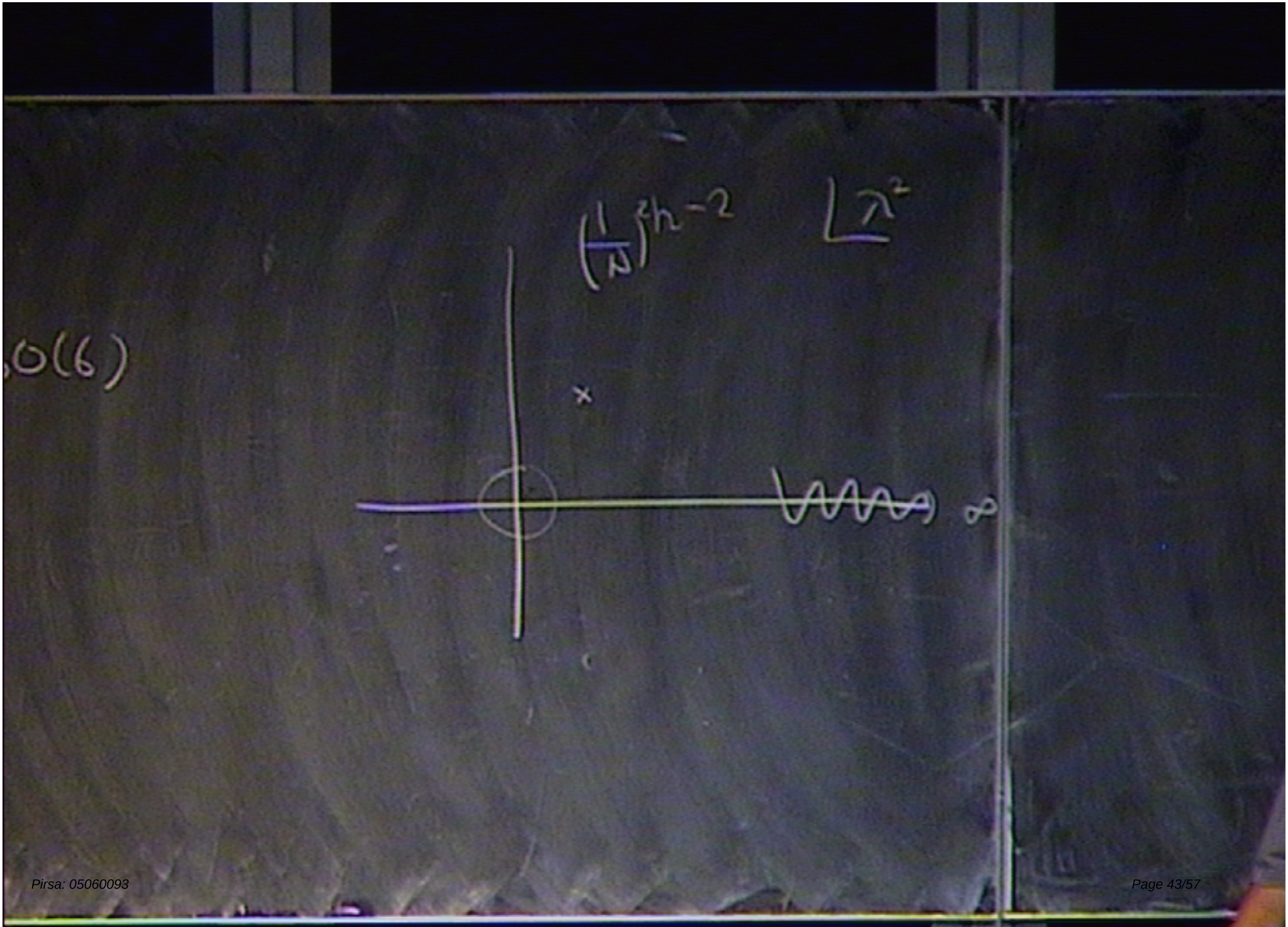
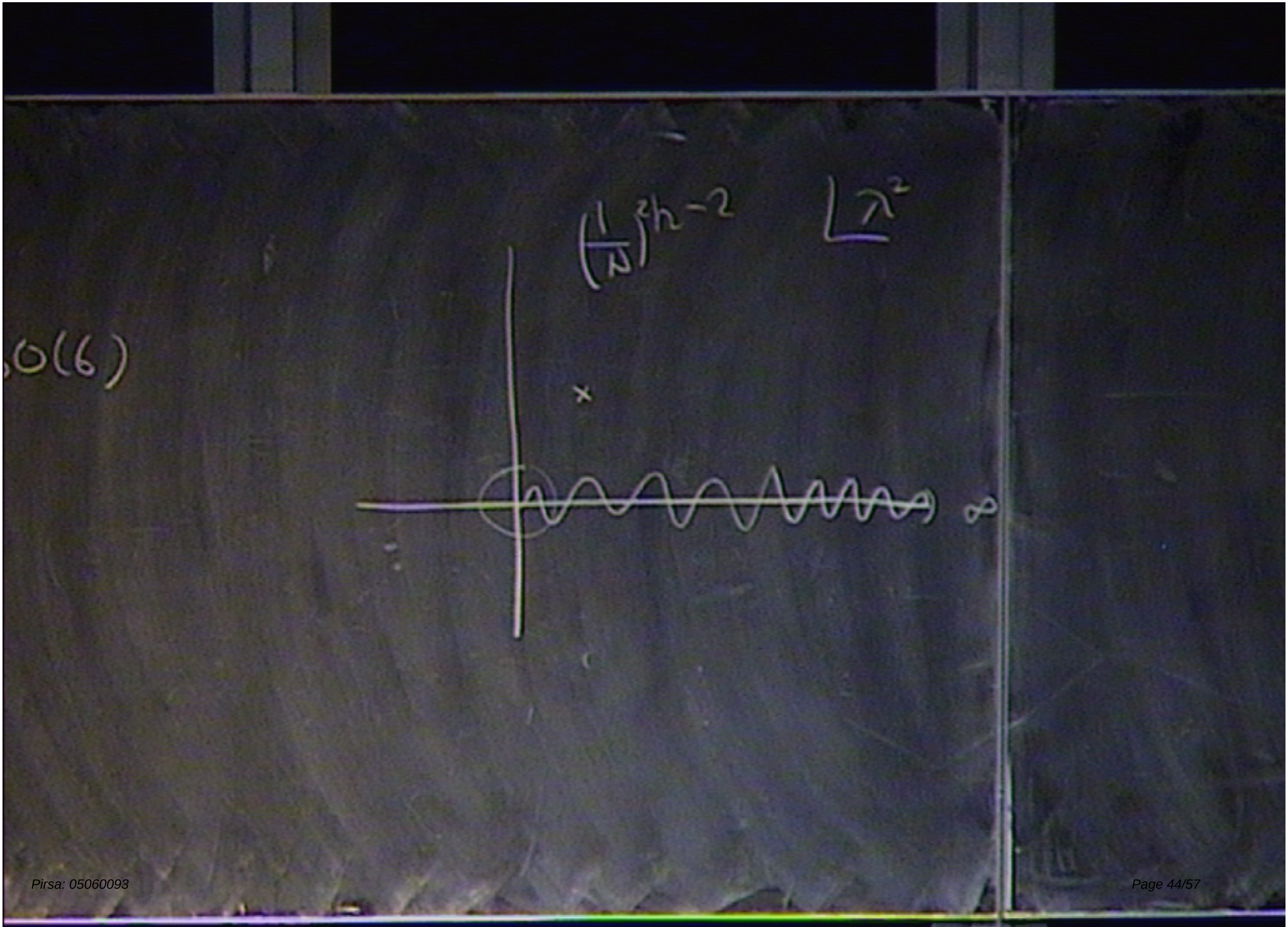

O(6)
$\left(\frac{1}{N}\right)^{2h-2}$
∞


O(6)
$\left(\frac{1}{N}\right)^{2h-2}$
λ^2
∞


O(6)
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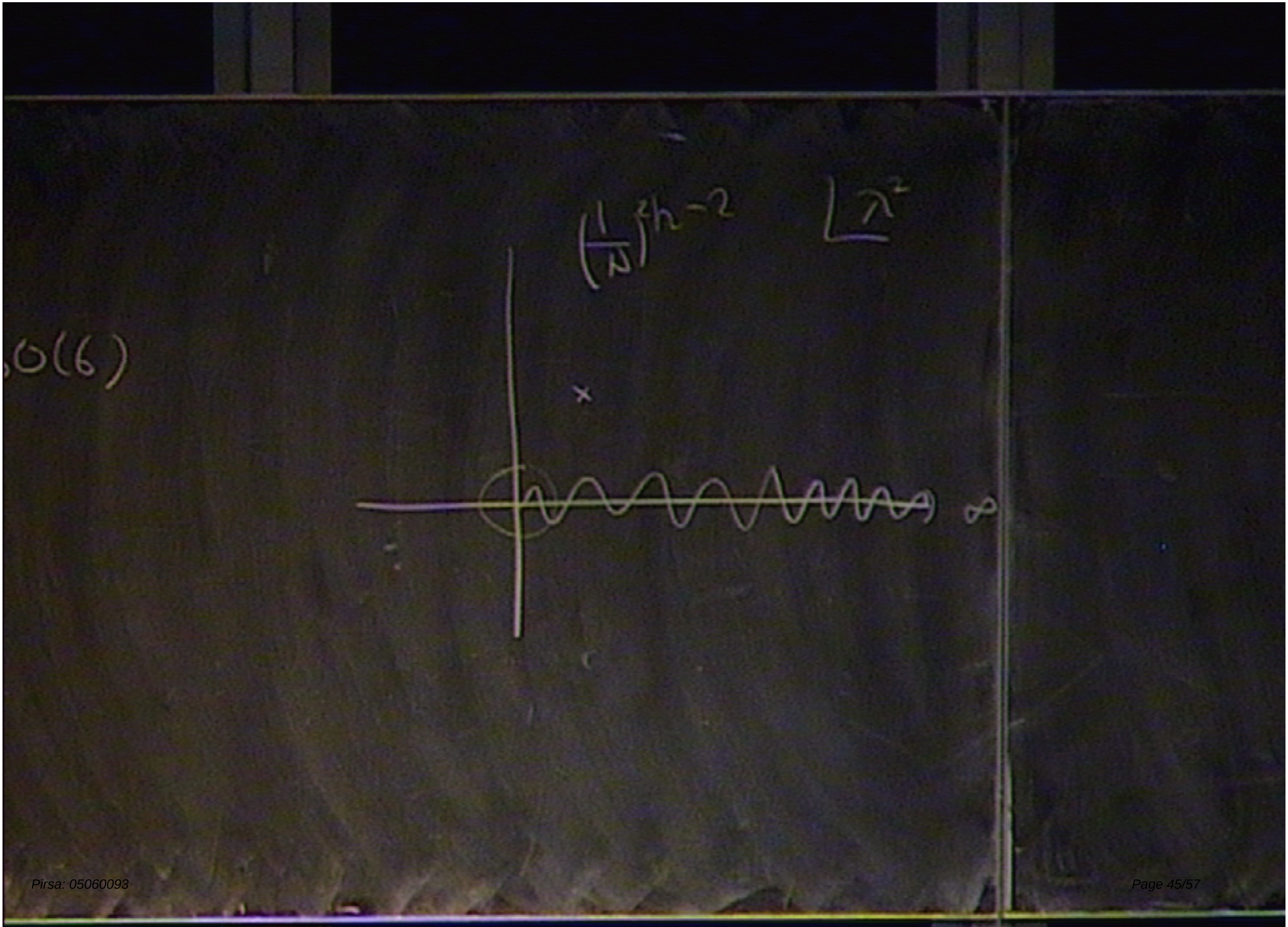
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Baryons

- In SU(3) QCD, baryon is a color singlet state of 3 quarks $\varepsilon_{abc} Q^a Q^b Q^c$
- In SU(N) SYM, baryon is a color singlet state of N "static" quarks $\varepsilon_{ab\ldots c} Q^a Q^b \Lambda Q^c$
- N-ality symmetry: $Q^a \to \omega Q^a \quad (\omega^N = 1)$
- U(N) vs. SU(N):

center of U(N) is U(1); center of SU(N) is Z_N

only SU(N) has baryon of N-ality

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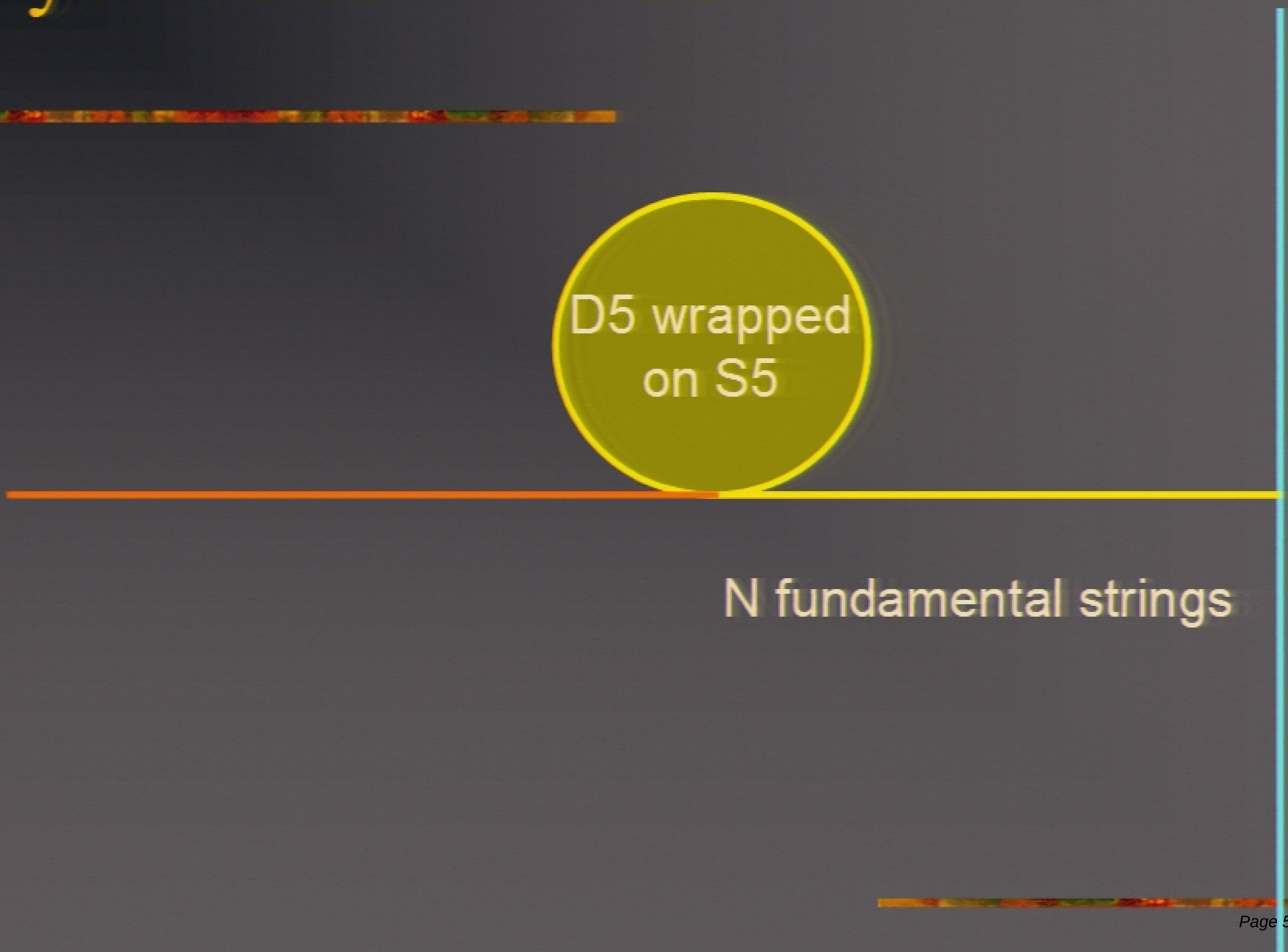
Baryon vertex in AdS5



On D5-brane worldvolume……

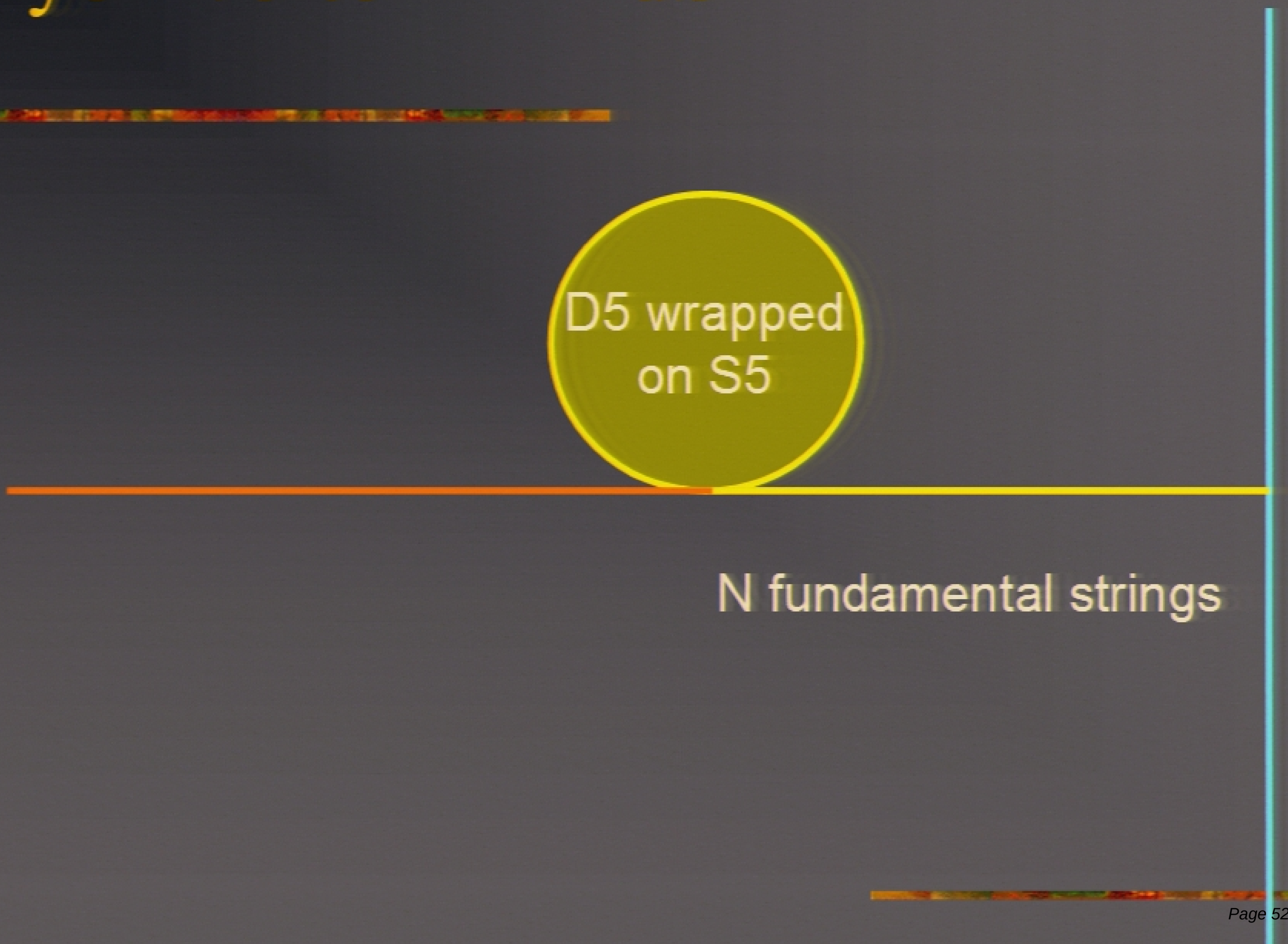
- D5-brane couples to external B_2 and G_5 as

  $L_{D5} \sim - (dA + B_2)^2 + A\wedge G_5$
- D5 is wrapped on S5, so Gauss' law for A_0 is

  $\int_{D5} \nabla \cdot E_{D5} = N$ where $E_{D5} = d A_0$
- bulk+D5-brane EOM for B_2 field

  $\nabla \cdot H_3 = \delta(D5) E_{D5}$ source for B_2 field in the bulk
- Combining the two equations, we conclude that <u>N bundles of fundamental string ended on D5-brane</u>
- Where are these strings from?

Baryon vertex in AdS5



Closer look at AdS5 SUGRA

- 10d IIB supergravity action contains Chern-Simons term

  $S_{CS} = \int (B_2 \wedge G_3 - C_2 \wedge H_3) \wedge G_5$ on AdS5*S5 yields

  $S_{5d} = \int_{AdS5} [\, e^{-2\Phi} |H_3|^2 + |G_3|^2 + N\,(B_2 \wedge G_3 - C_2 \wedge H_3)]$
- Inside AdS5, D5 in the bulk and D3-brane near infinity:

  $L_{D3} = -\,(dA' + B_2)^2 + A' \wedge G_3$
- Solve combined AdS SUGRA+D5+D3 EOM for C_2

  $\nabla \cdot (\wedge G_3 + N\, B_2 - E_{D3}) = 0$ (but B_2 sourced by D5)
- G_3 is trivial, so induces N unit of electric field on D3!!

8 is the magic number….

- Bring D5-brane from flat 10d infinity into N D3-branes

before crossing after crossing

Like quarks inside baryons, N F-strings obey Fermi statistics !

Intuitive picture

- Consider SU(2) → U(1) → ∅ gauge theory
- Add θ–term

$$S_{U(1)} = \int [E^2 + B^2 + \theta\, E \cdot B]$$

- At nonzero θ, Gauss' law reads

$$\nabla \cdot (E + \theta\, B) = 0$$

- So, monopole acquires nonzero electric charge proportional to θ (Witten effect)

$$Q_E = -\,\theta\, Q_M / 2\pi$$

- By electric charge conservation, - Q_E must be deposited on θ-jump
- Between $+Q_E$ and – Q_E, a flux tube is formed

Baryon in AdS5

- D5-brane wrapped on S5 minimizes energy by falling into AdS5 geometry
- Since S5 size is constant inside AdS5, energy is constant (BPS state)
- Large worldvolume E field deforms the shape of D5-brane
- Test of finite N through N-ality property