Title: Advanced AdS/CFT Topics

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Abstract:

# Selected Topics in AdS/CFT
# lecture 1

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# Topics:

- Lightning Review of AdS5/SYM4
- Wilson loop (hep-th/9803001, 9803002)
- Polyakov loop (hep-th/9803135)
- Baryons
- Instantons
- Time-varying phenomena
- homeworks: to be distributed separately

# Starting Point

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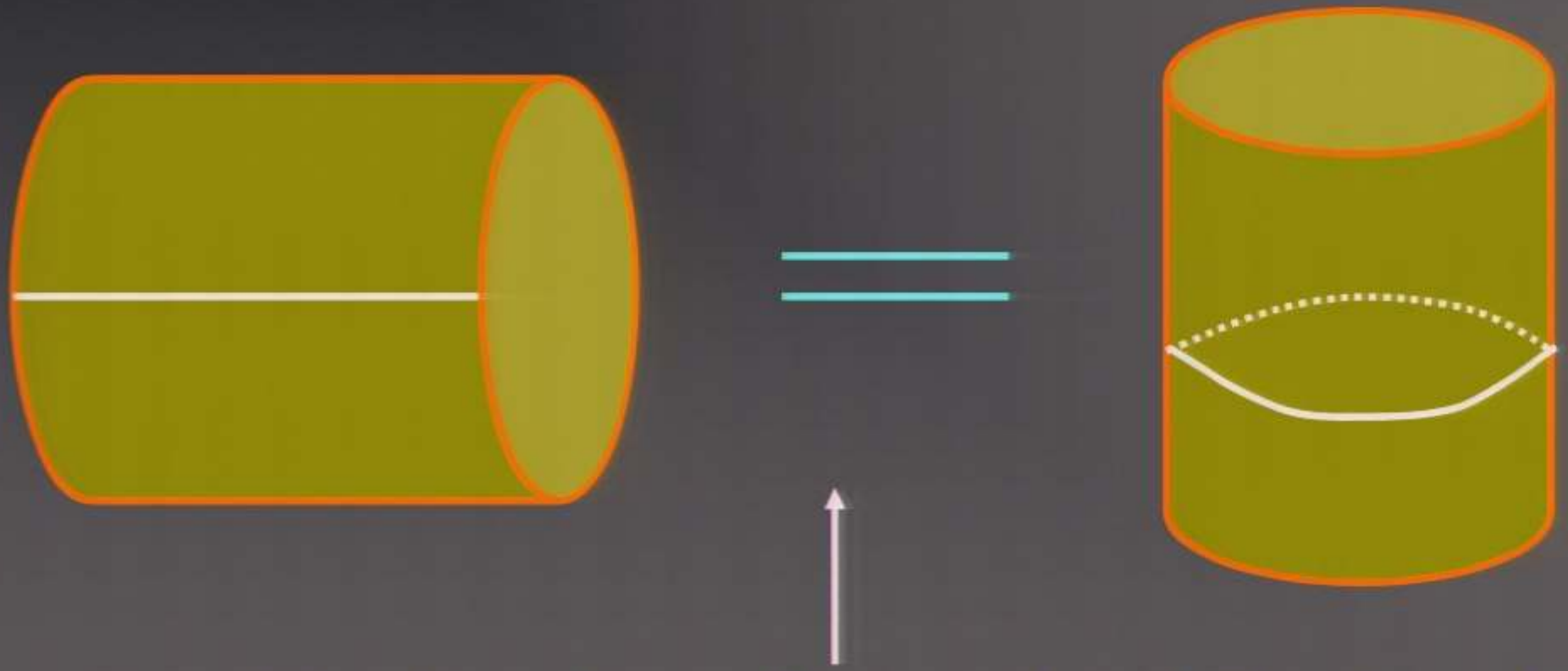
- string theory: open string + closed string
- low-energy limit of string theory:

  open string --> gauge theory (spin=1)

  closed string --> gravity theory (spin=2)
- channel duality: open $ closed

# Channel Duality



# Gauge – Gravity Duality

- keypoint:

"gauge theory dynamics"

(low energy limit of open string dynamics)

is describable by

"gravity dynamics"

(low-energy limit of closed string dynamics)

and vice versa

# Tool Kits for AdS/CFT

- elementary excitations:
  open string + closed string
- solitons:
  D-branes (quantum) + NS-branes (classical)
- Mach's principle:
  spacetime Ã elementary + soliton sources
- "holography":
  gravity fluctuations = source fluctuations

# p-Brane (gravity description)

- string effective action
- $S = s\, d^{10} x\, [e^{-2\Phi} (R_{(10)} + (r\,\Phi)^2 + |H_3|^2 + \ldots) + 1\,¢\, (\sum_p |G_{p+2}|^2 + \ldots)]$

  $= (1/g^2_{st})(\text{NS-NS sector}) + (\text{R-R sector})$
- $H_3 = d\, B_2$, $G_{p+2} = d\, C_{p+1}$ etc.
- R-R sector is "quantum" of NS-NS sector

# elementary and solitons

- fixed “magnetic” charge, energy-minimizing static configurations

(necessary condition for BPS states)

- NS 5-brane:

$E = s\, g_{st}^{-2} [(r\Phi)^2 + H_3^2] \;>\; g_{st}^{-2}\, s_{S3}\, e^{-2\Phi}\, H_3$

- F-string:

$E = s\, [g_{st}^{-2}\, (r\Phi)^2 + g_{st}^{2}\, K_7^2] \;>\; g_{st}^{0}\, s_{S7}\, K_7$

- Mass: M(string) ~ 1 ; M(NS5) ~ $(1/g_{st}^2)$

“elementary” ; “soliton”

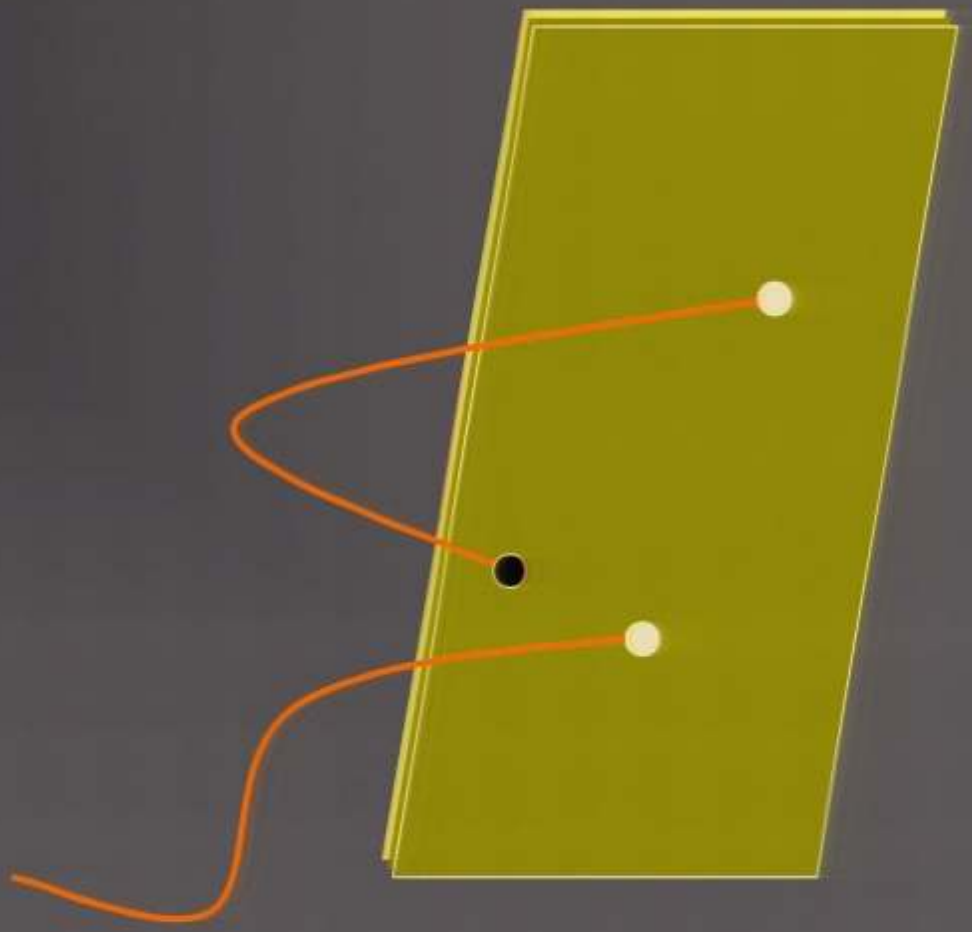
# p-brane: in between

- p-brane:

  $E = s\, g_{st}^{-2}\, (r\, \Phi)^2 + (G_{p+2})^2 > g_{st}^{-1}\, s\, e^{-\Phi}\, G_{p+2}$
- $M(\text{p-brane}) \gg (1/g_{st})$
- p-brane = “quantum soliton”:

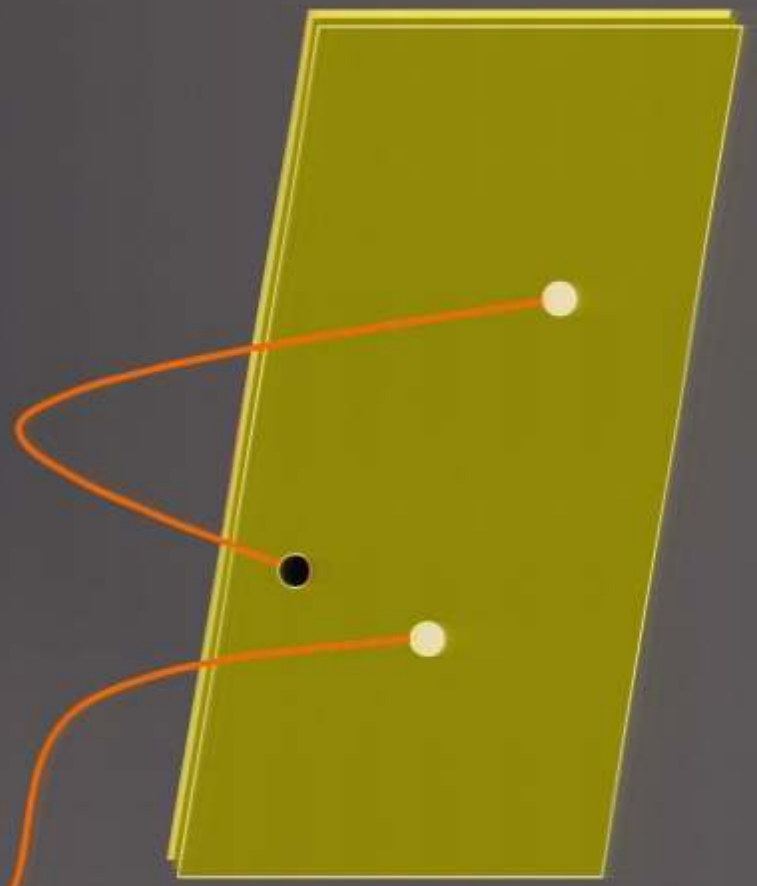
  more solitonic than F-string but less solitonic than NS5-brane
- F-string $ p-brane $ NS5-brane
- quantum treatment of p-brane is imperative

# Dp-brane (CFT description)



# Dp-brane (CFT description)

- strings can end on it
  - open string
- string endpoints labelled by Chan-Paton factors
  - ( ) = (i,j)
- mass set by disk diagram
  - $M \sim 1/g_{st}$
  - ---- identifiable with p-brane
- $Q_L = Q_R$ (half BPS object)



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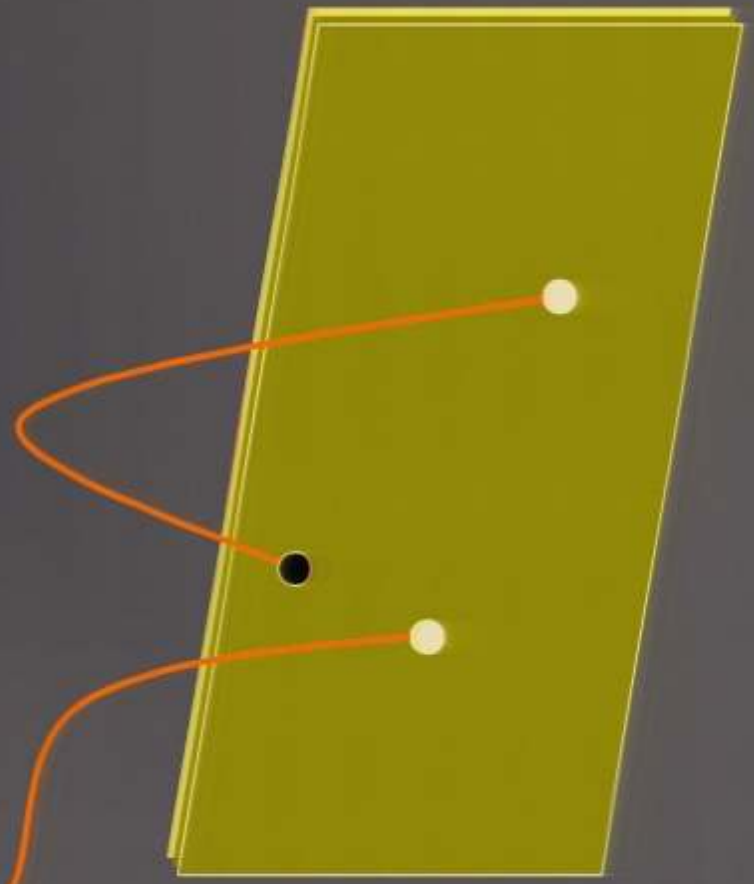

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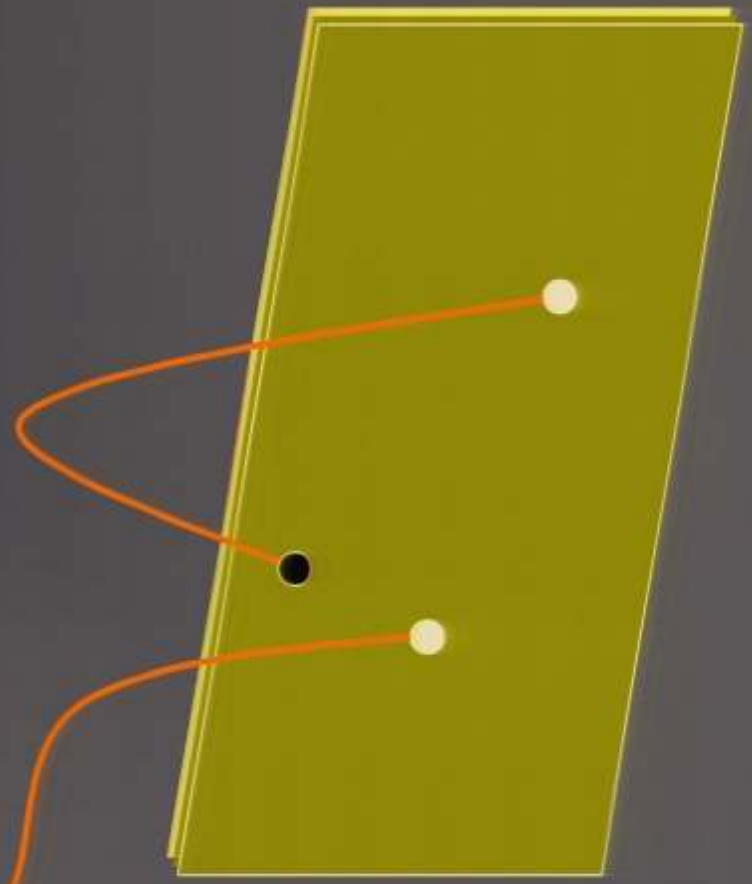

# $SYM_{p+1}$ at Low-Energy

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- infinite tension limit

  $(\alpha' \rightarrow 0)$

- open strings → rigid rods

  $(Mw \sim \Delta r/\alpha')$

- (N,N) string dynamics

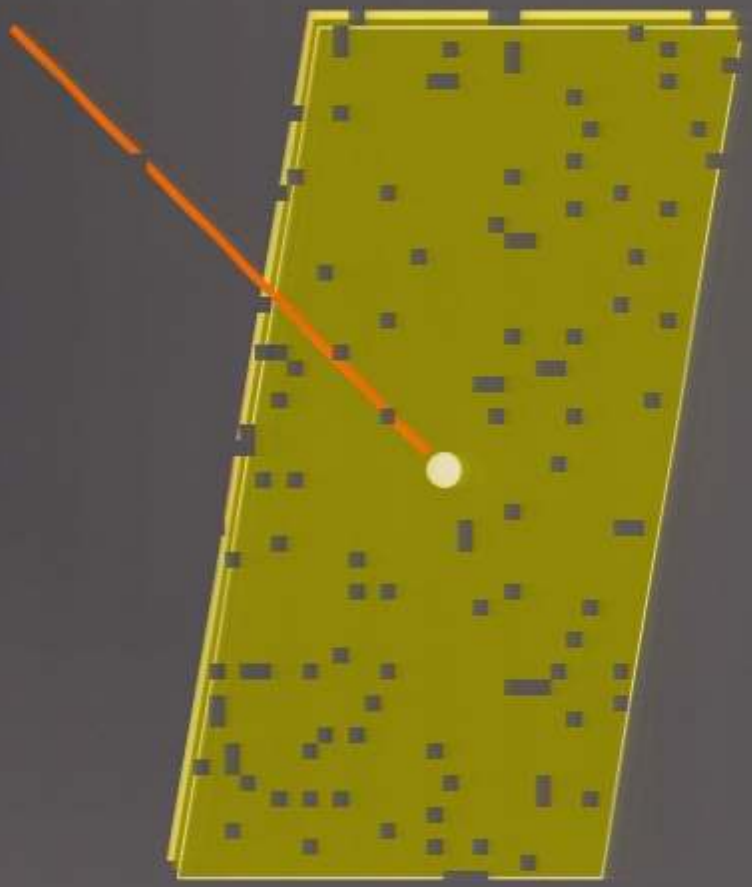
  U(N) SYM(P+1)

$$L = g^{-2}_{YM}\, Tr\,( F_{mn}^2 + (D_m \Phi^a)^2 + [\Phi^a, \Phi^b]^2 + \ldots )$$

"||"

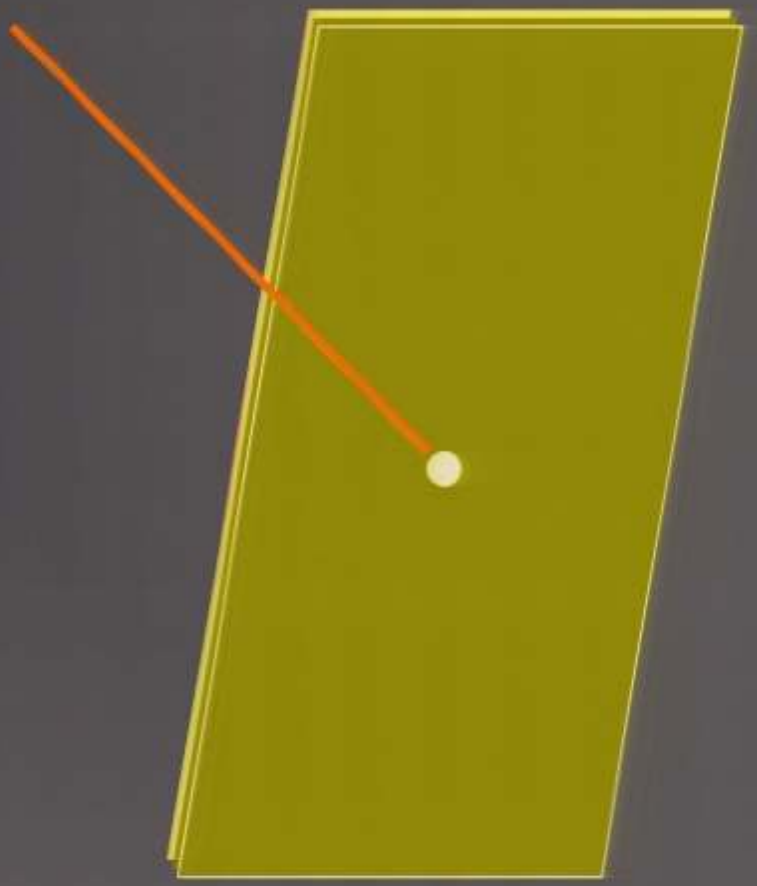
"?"

# Static source: heavy quarks



# Static source: heavy quarks

- semi-infinite string
  (M ~ 1)
- at rest or constant velocity
  (static source)
- labelled by a single
  Chan-Paton factor
  ( ) = (i)
  heavy (anti)quark
  in (anti)fundamental rep.



# Perturbation theory (1)

- large N conformal gauge theory :
  double expansion in 1/N and $\lambda^2 = g_{YM}^2 N$
- $S_{YM} = (N/\lambda^2)\, s\, d^4 x\, Tr\, (F_{mn}^2 + (D_m \Phi)^2 + \ldots)$
- planar expansion: $(N/\lambda^2)^{V-E} N^F = (1/N)^{2h-2} (\lambda^2)^{E-V}$
- $\lambda^2$ : nonlinear interactions
- 1/N: quantum fluctuations (effective ~)
- observable: $\sum_{h=0}^{1} \sum_{l=0}^{1} (1/N)^{2h-2} (\lambda^2)^l C_{l,h}$

# Perturbation theory (2)

- *closed string theory*:

  double expansion in $g_{st}$ and $\alpha'$
- $\alpha'$ : string worldsheet fluctuation

  $S_{string} = (1/2\pi\alpha') \int d^2\sigma\, \partial X \cdot \partial X$
- $g_{st}$ : string quantum loop fluctuation
- $S_{SFT} = \Phi * Q_B \Phi + g_{st}\, \Phi * \Phi * \Phi$
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# Identifying the Two Sides….

- large-N YM and closed string theory have the same perturbation expansion structure
- $g_{st}$ \$ (1/N)
- $(\alpha'/R^2)$ \$ $1/\lambda$ where R = characteristic scale
- Maldacena's AdS/CFT correspondence:

  "near-horizon"(R) geometry of D3-brane

  = large-N $SYM_{3+1}$ at large but fixed $\lambda$
- not only perturbative level but also nonperturbatively (evidence?)

# D3-Brane Geometry

- 10d SUGRA(closed string)+4d SYM (D3-brane):

$$S_{total} = S_{10d} + s\, d^4 x\, (-e^{-\Phi} TrF_{mn}{}^2 + C_4 \ldots.)$$

- Solution
- $ds^2 = Z^{-1/2}\, dx^2{}_{3+1} + Z^{1/2}\, dy^2{}_6$
- $G_5 = (1+Æ)\, dVol_4\, Æ\, d\, Z^{-1}$ where

$Z = (1+R^4/r^4)$; $r = |y|$ and $R^4 = 4\pi\, g_{st}\, N\, \alpha'^2$

- r ! 1: 10d flat spacetime
- r ! 0 : characteristic curvature scale = R

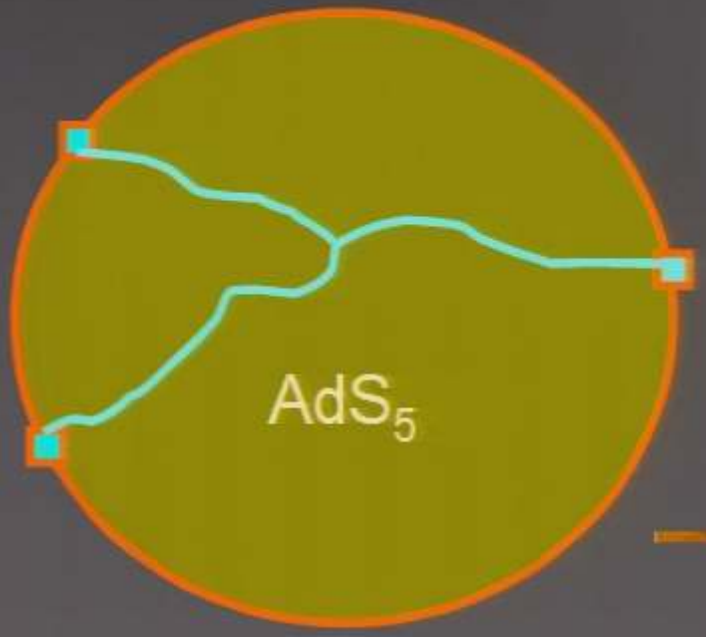
# Identifications

- D-brane stress tensor grows with $(N/g_{st})$
- At large N, curvedness grows with $g^2_{st}(N/g_{st})=g_{st}N$
- Near D3-brane, spacetime $= AdS_5 \times S^5$
- 4d D3-brane fluct. \$ 10d spacetime fluct.
  (from coupling of D3-brane to 10d fields)
  $Tr\,(F_{mp}F_{np})$ \$ metric $g_{mn}$
  $Tr\,(F_{mn}F_{mn})$ \$ dilaton $\Phi$
  $Tr(F_{mp}F^*_{np})$ \$ Ramond-Ramond C, $C_{mn}$

# AdS/CFT correspondence

- Dirichlet problem in $AdS_5$ or $EAdS_5 = H_5$
- $Z_{gravity} \approx \exp(-S_{AdS5}(\phi_{bulk,a}, \phi^1{}_a))$

$= Z_{SYM} \approx \int [dA]_? \exp(-S_{SYM} - \int \sum_a \phi^1{}_a O_a)$



# $AdS_5$

- In flat 4+2 dimensional space

$$ds^2 = - dX_0^2 - d X_5^2 + \sum_{a=1}^{4} d X_a^2$$

- embed hyperboloid

$$X_0^2 + X_5^2 - \sum_{a=1}^{4} X_a^2 = R^2$$

- SO(4,2) inv, homogeneous, isotropic
- $AdS_5$ = induced geometry on hyperboloid
- <homework>

- Global coordinates:

  $ds^2 = R^2(-\cosh^2\rho\, d\tau^2 + d\rho^2 + \sinh^2\rho\, d\Omega_3^2)$

  boundary = $R_t$ £ $S_3$

- Poincare coordinates:

  $ds^2 = R^2[r^2(-dt^2 + dx^2 + dy^2 + dz^2) + r^{-2}dr^2]$

  boundary = $R_t$ £ $R_3$

# Why AdS Throat = D3-Brane?

- D-brane absorption cross-section:

SUGRA computation = SYM computation

AdS_5 region in gravity description
= N D3-branes in gauge description

# Another argument

- D-instantons probing (Euclidean) AdS5
- For U(N) gauge group, "homogeneous" instanton number < N (otherwise inhomogeneous)
- Q D-instanton cluster in approx. flat region
  $S_{Dinstanton} = -(1/g_{st}\,\alpha'^2)\,Tr_Q[\Phi^1, \Phi^2]^2 + \ldots.$

- $< Tr(\Phi^1)^2 > \sim Q L^2, \quad <Tr(\Phi^2)^2 > \sim Q^2\, g_{st}\, \alpha'^2 / L^2$
- rotational symmetry ! $L^4 = Q\, g_{st}\, \alpha'^2 = N\, g_{st}\, \alpha'^2$

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# How can it be that 5d = 4d?

- extensive quantities in 4d SYM theory scales as $[\text{length}]^4$
- Extensive quantities in 5d AdS gravity scales as $[\text{length}]^5$
- So, how can it be that quantities in 4d theory is describable by 5d theory??
- [Question] Show both area and volume of a ball of radius X in $AdS_d$ scales as $X^{d-1}$!

- Answer:

Consider $(d+1)$-dimensional AdS space. Take a finite comoving $d$-volume $(\mathrm{vol})_d = \Delta t \Delta^{d-1}\mathbf{x}$.

"Volume" $V_{d+1}(r_*)$ of ball of radius $r = r_*$:

$$V_{d+1}(r_*) = \int_0^{r_*} Z^{1/4}(r)\mathrm{d}r \int_{\mathrm{vol}_d} Z^{-d/4}(r)\mathrm{d}t\mathrm{d}^{d-1}\mathbf{x}$$
$$= \frac{1}{4} r_*^4 R^{-3}\mathrm{vol}_4 \qquad \text{for} \quad d = 4.$$

"Area" $A_d(r)$ of ball of radius $r = r_*$:

$$A_d(r_*) = \int_{r=r_*} Z^{-d/4}(r)\mathrm{d}t\mathrm{d}^{d-1}\mathbf{x}$$
$$= r_*^4 R^{-4}\mathrm{vol}_4 \qquad \text{for} \quad d = 4.$$

Comparing the two, we find

$$\frac{1}{4}\frac{A_5(r_*)}{V_4(r_*)} = \frac{1}{R} = \text{AdS curvature radius.}$$

Notice that the ratio is independent of $r_*$, and holds for both $r_* \ll 1$ and $r_* \gg 1$!

# Shall we test AdS/CFT?

- Recall that heavy quarks are represented by fundamental strings attached to D3-brane
- now strings are stretched and fluctuates inside $AdS_5$
- Let's compute interaction potential between quark and antiquark
- Do we obtain physically reasonable answers?

# Static Quark Potential at Zero Temperature

$$V(r) = -(1.254...)\frac{\sqrt{g_{YM}^2 N}}{r}$$

Notice:

- Square-Root --- non-analyticity for $\lambda$
- exact 1/r --- conformal invariance

- geometry produced by D3-branes:

$$ds^2 = G_{mn}(x)dx^m dx^n = \frac{1}{\sqrt{Z}}(-dt^2 + d\mathbf{x}^2) + \sqrt{Z}dr^2$$

$$Z = 1 + \frac{R^4}{r^4} \quad \rightarrow \quad \frac{R^4}{r^4} \quad \text{near-horizon}$$

Notice: $R^4 = g_{st} N \alpha'^2$

- string dynamics in the D3-brane background:

$\alpha'^2$ cancels out!

$$L_{\text{string}} = -\frac{1}{2\pi\alpha'}\int d\sigma\sqrt{-h} \quad \text{(Nambu-Goto)}$$

$$h_{ab}(\sigma,\tau) = G_{mn}(X)\partial_a X^m \partial_b X^n.$$

- look for static string configuration:

static gauge : $\quad X^0 = \tau, \quad \mathbf{X} \equiv \mathbf{X}_{||} + U\hat{r}$

- compute $h_{ab}$ → plug back to Nambu-Goto

- string Lagrangian:

$$L_{\text{string}} = -\frac{1}{2\pi\alpha'}\int d\sigma\sqrt{U'^2 + Z^{-1}\mathbf{X}_{\|}'^2}$$

- static-gauge constraint = energy-conservation

- eqns of motion:

$$\left(\frac{U'}{\sqrt{U'^2 + Z^{-1}\mathbf{X}_{\|}'^2}}\right)' = \mathbf{X}_{\|}'^2\left(\partial_U Z^{-1}\right)$$

$$\left(\frac{Z^{-1}\mathbf{X}_{\|}'}{\sqrt{U'^2 + Z^{-1}\mathbf{X}_{\|}'^2}}\right)' = 0.$$

- heavy quark

$$X^0 = \tau, \quad \mathbf{X}_{\parallel}(\sigma) = 0, \quad \frac{1}{2\pi\alpha'} U(\sigma) = \sigma$$

- string energy = quark mass

$$E_{\text{string}} = \frac{1}{2\pi\alpha'} \int d\sigma \sqrt{U'^2} = \Delta\sigma$$

- interpretation: radial distance = energy scale

$$r(\text{gravity}) \quad \leftrightarrow \quad \frac{1}{\lambda}(\text{gauge})$$

geometric UV-IR duality, holography

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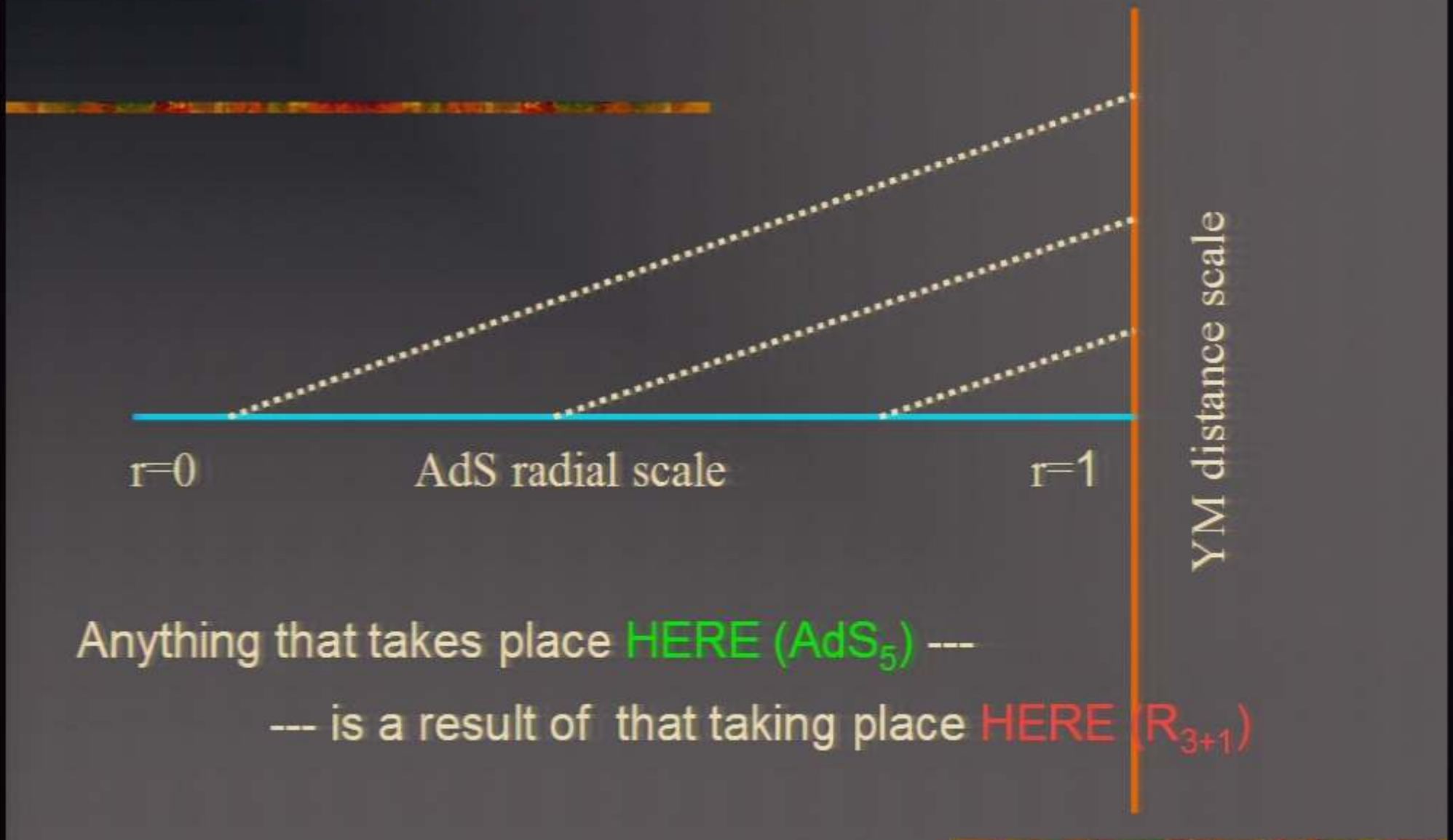
# Holography (boundary = bulk)




Anything that takes place HERE ($AdS_5$) ---

--- is a result of that taking place HERE ($R_{3+1}$)

- heavy meson

$$X^0 = \tau, \quad \mathbf{X}_{||}(\sigma) = \sigma \hat{\mathbf{r}}_3, \quad U = U(\sigma)$$

- $U$-eqn of motion

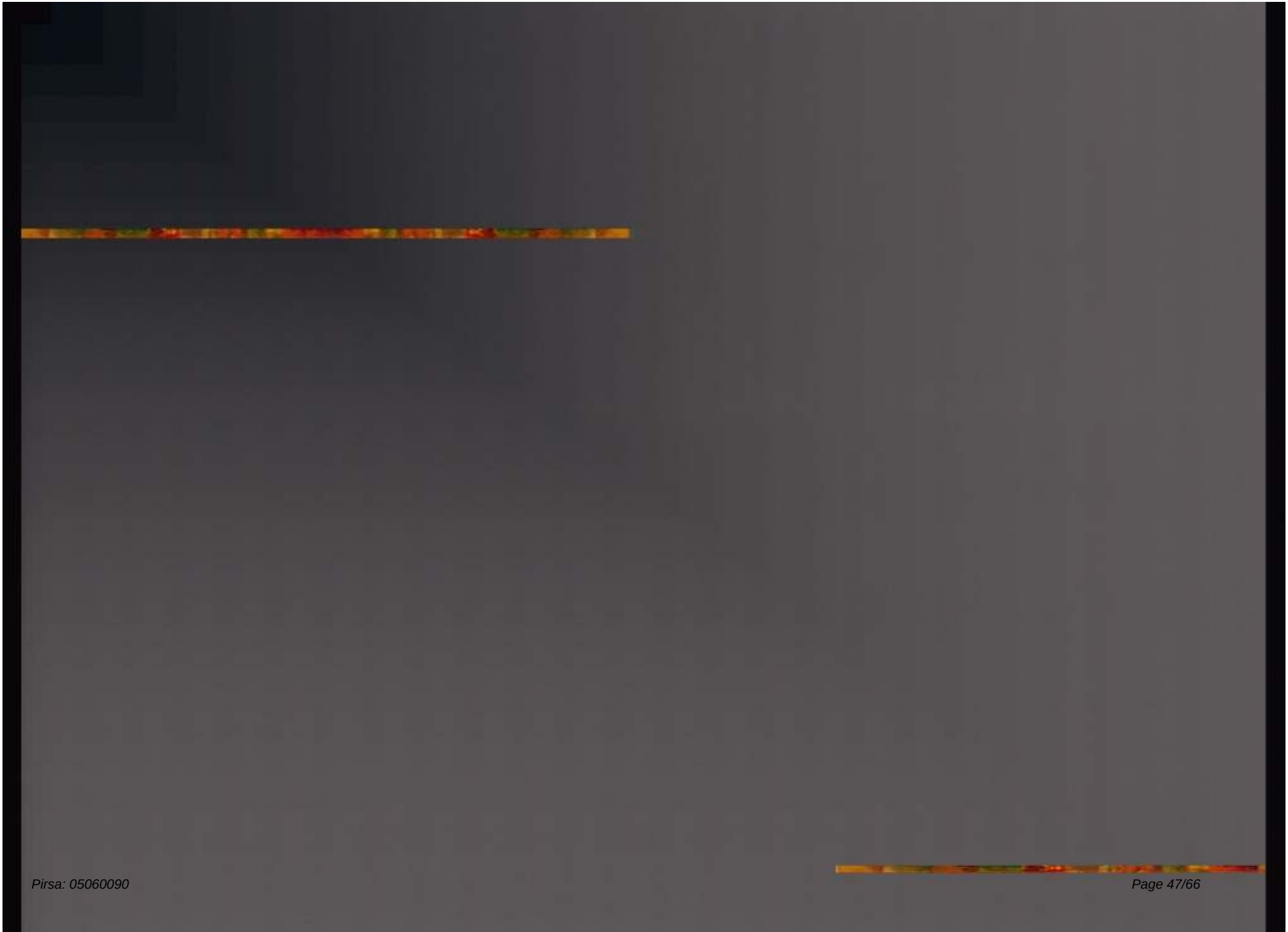
$$-2Z^{-1}U'' + \left(\partial_U Z^{-1}\right)(2U'^2 + Z^{-1}) = 0$$

- first integral of motion:

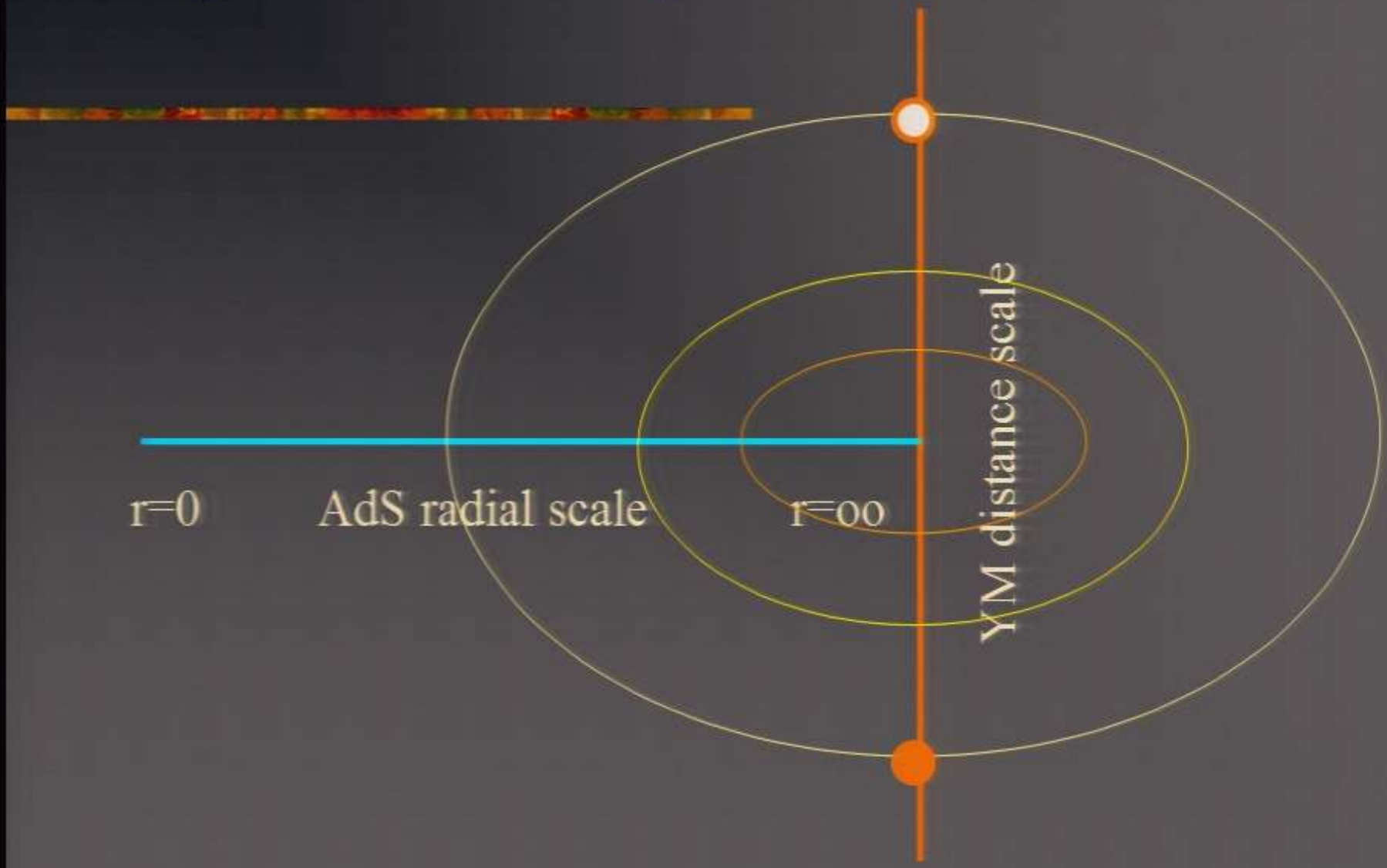
$$Z^2 U'^2 + Z = \frac{g^2 N}{U_*^4} \quad \leftarrow \text{int. const.}$$

- solution $U(\sigma)$ via implict function form:
  easy! mechanical particle analog

$$q(t) := U_*/U(\sigma) \quad \rightarrow \text{bounded orbit}$$



# Heavy Meson Configuration



- exact solution:

$$r - \frac{L}{2} = \pm \frac{\sqrt{g^2 N}}{U_*} \left[ \sqrt{2}\mathbf{E} - \frac{1}{\sqrt{2}}\mathbf{F} \right] \left( \cos^{-1} \frac{U_*}{U}, \frac{1}{\sqrt{2}} \right)$$

- inter-quark distance = semi-minor axis of "closed orbit":

$$L = C \frac{\sqrt{g^2 N}}{U_*} \qquad C = \frac{2\sqrt{\pi}\Gamma(3/4)}{\Gamma(1/4)} = 1.19814...$$

→ geometric UV-IR relation

- Note!

non-analyticity : $\sqrt{g^2 N}$

holography : $L \propto 1/U_*$

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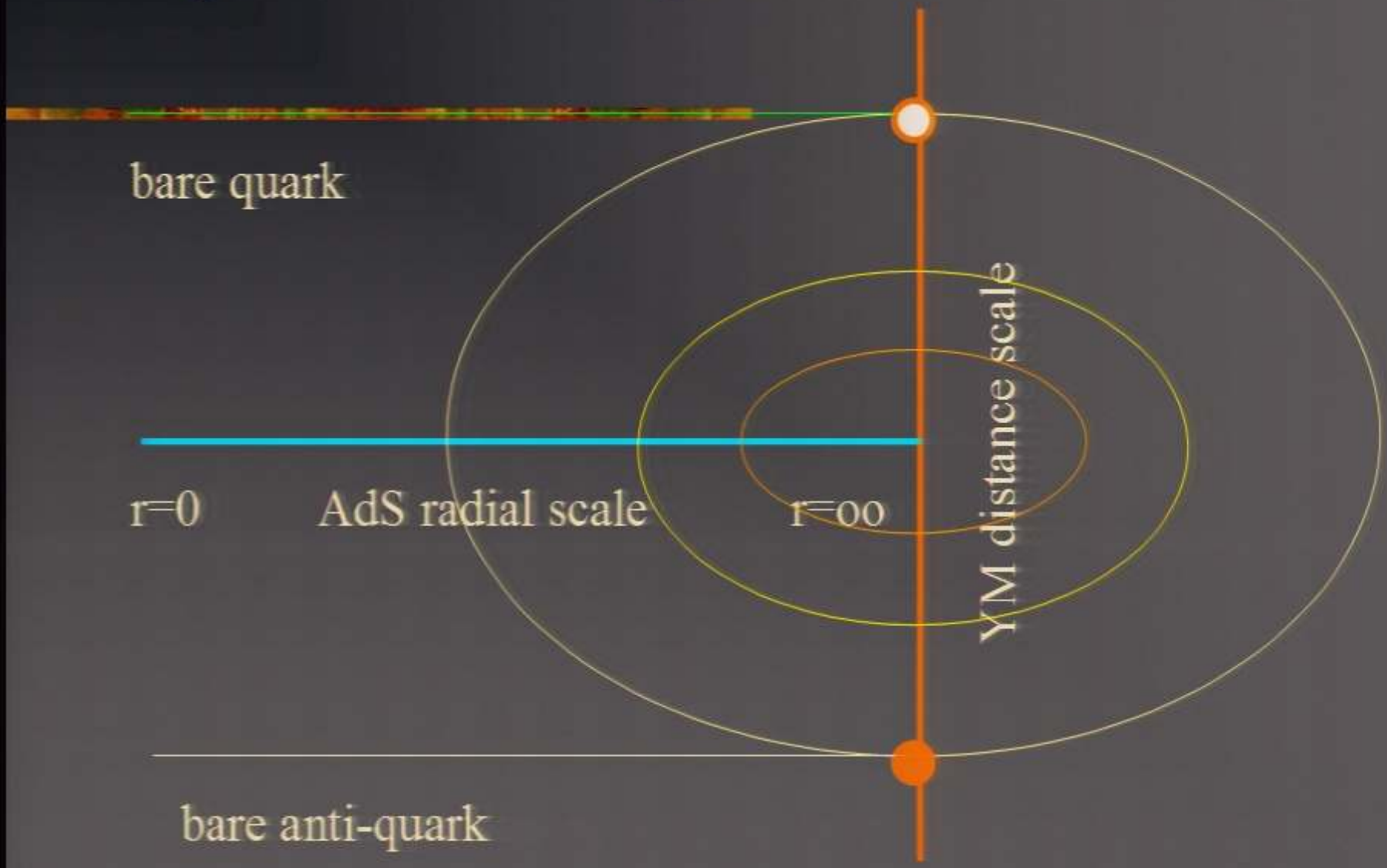

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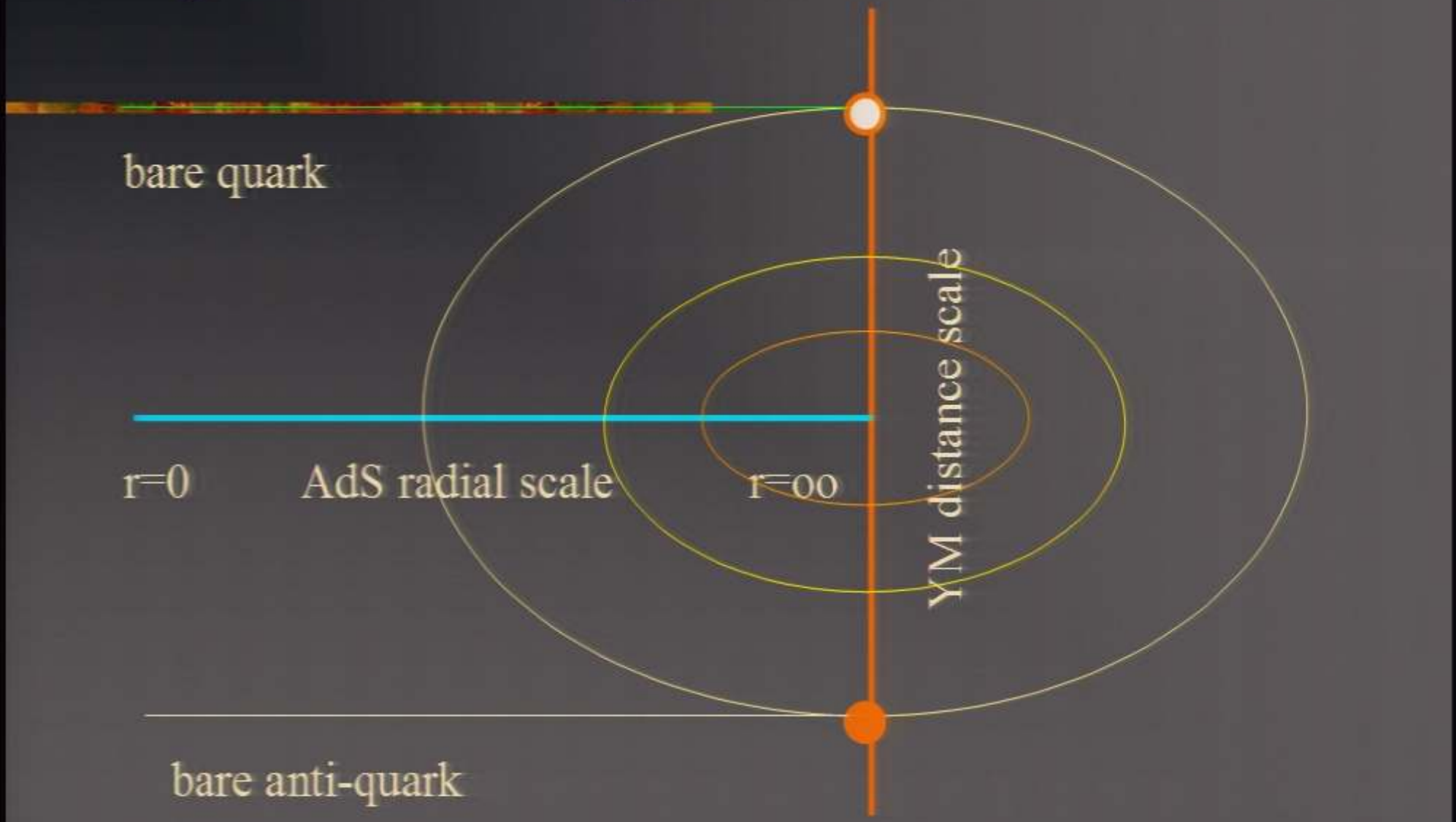

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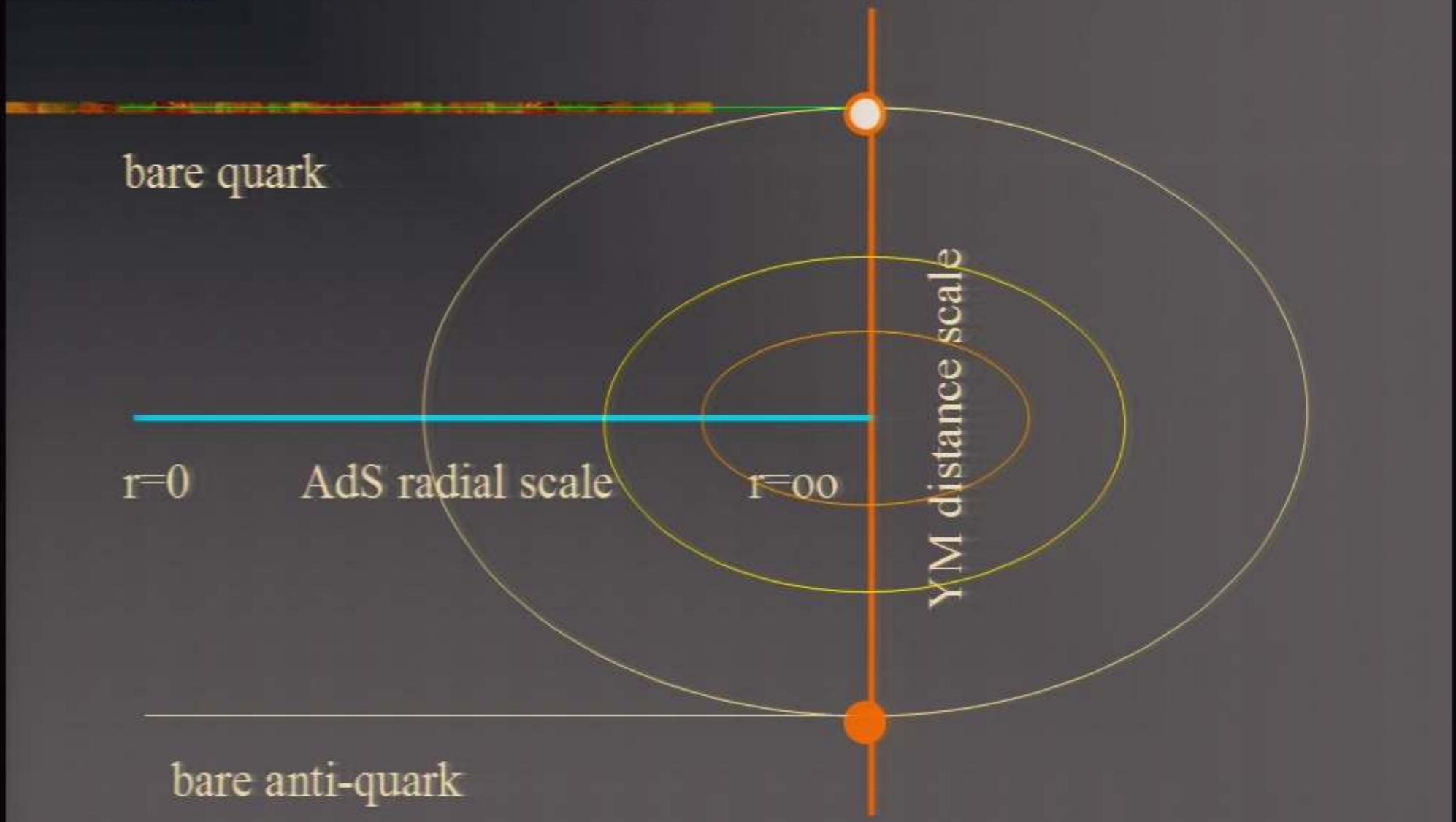
# Heavy Meson Configuration



- exact solution:

$$r - \frac{L}{2} = \pm \frac{\sqrt{g^2 N}}{U_*} \left[ \sqrt{2}\mathbf{E} - \frac{1}{\sqrt{2}}\mathbf{F} \right] \left( \cos^{-1} \frac{U_*}{U}, \frac{1}{\sqrt{2}} \right)$$

- inter-quark distance = semi-minor axis of "closed orbit":

$$L = C \frac{\sqrt{g^2 N}}{U_*} \qquad C = \frac{2\sqrt{\pi}\Gamma(3/4)}{\Gamma(1/4)} = 1.19814...$$

→ geometric UV-IR relation

- Note!

non-analyticity : $\sqrt{g^2 N}$

holography : $L \propto 1/U_*$

- string energy:

$$E_{(Q\overline{Q})}(U_*) = \frac{1}{2\pi\alpha'}\int d\sigma\sqrt{U'^2+Z^{-1}}$$
$$= \lim_{u\to\infty} 2U^*\int_0^u dx\frac{x^2}{\sqrt{x^4-1}}$$

[used the first integral ("conserved energy")]

- heavy-quark potential:

$$V_{(Q\overline{Q})}(U_*) \equiv E_{(Q\overline{Q})}(U_*) - (M_Q + M_{\overline{Q}})$$
$$= -2U_*\left[\frac{\sqrt{2}\pi^{3/2}}{\Gamma^2(1/4)}\right]$$

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$$V_{Q\overline{Q}}(L) = -C\frac{\sqrt{g^2N}}{L} \qquad C = 1.254...$$

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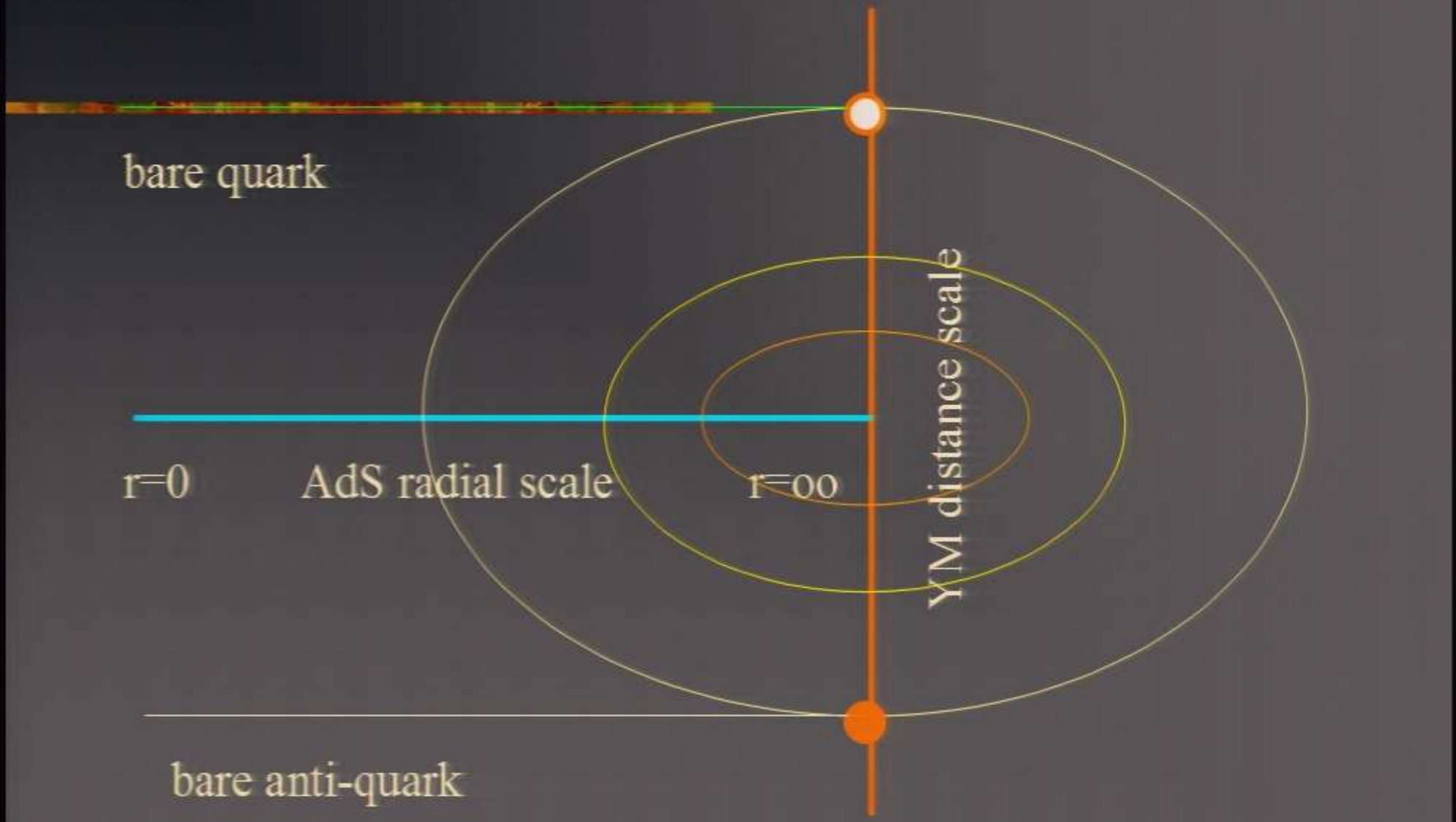

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# What have we evaluated?

- rectangular Wilson loop in N=4 SYM
- $W[C] = \mathrm{Tr}\, P \exp \oint_C (i A_m dx^m + \Phi^a d y^a)$
- gauge field part = Aharonov-Bohm phase
- scalar field part = W-boson mass
- unique N=4 supersymmetric structure

- heavy meson

$$X^0 = \tau, \quad \mathbf{X}_{\|}(\sigma) = \sigma \hat{\mathbf{r}}_3, \quad U = U(\sigma)$$

- $U$-eqn of motion

$$-2Z^{-1}U'' + \left(\partial_U Z^{-1}\right)(2U'^2 + Z^{-1}) = 0$$

- first integral of motion:

$$Z^2 U'^2 + Z = \frac{g^2 N}{U_*^4} \quad \leftarrow \text{int. const.}$$

- solution $U(\sigma)$ via implict function form:
  easy! mechanical particle analog

$$q(t) := U_*/U(\sigma) \quad \rightarrow \text{bounded orbit}$$

- heavy quark

$$X^0 = \tau, \quad \mathbf{X}_{\parallel}(\sigma) = 0, \quad \frac{1}{2\pi\alpha'} U(\sigma) = \sigma$$

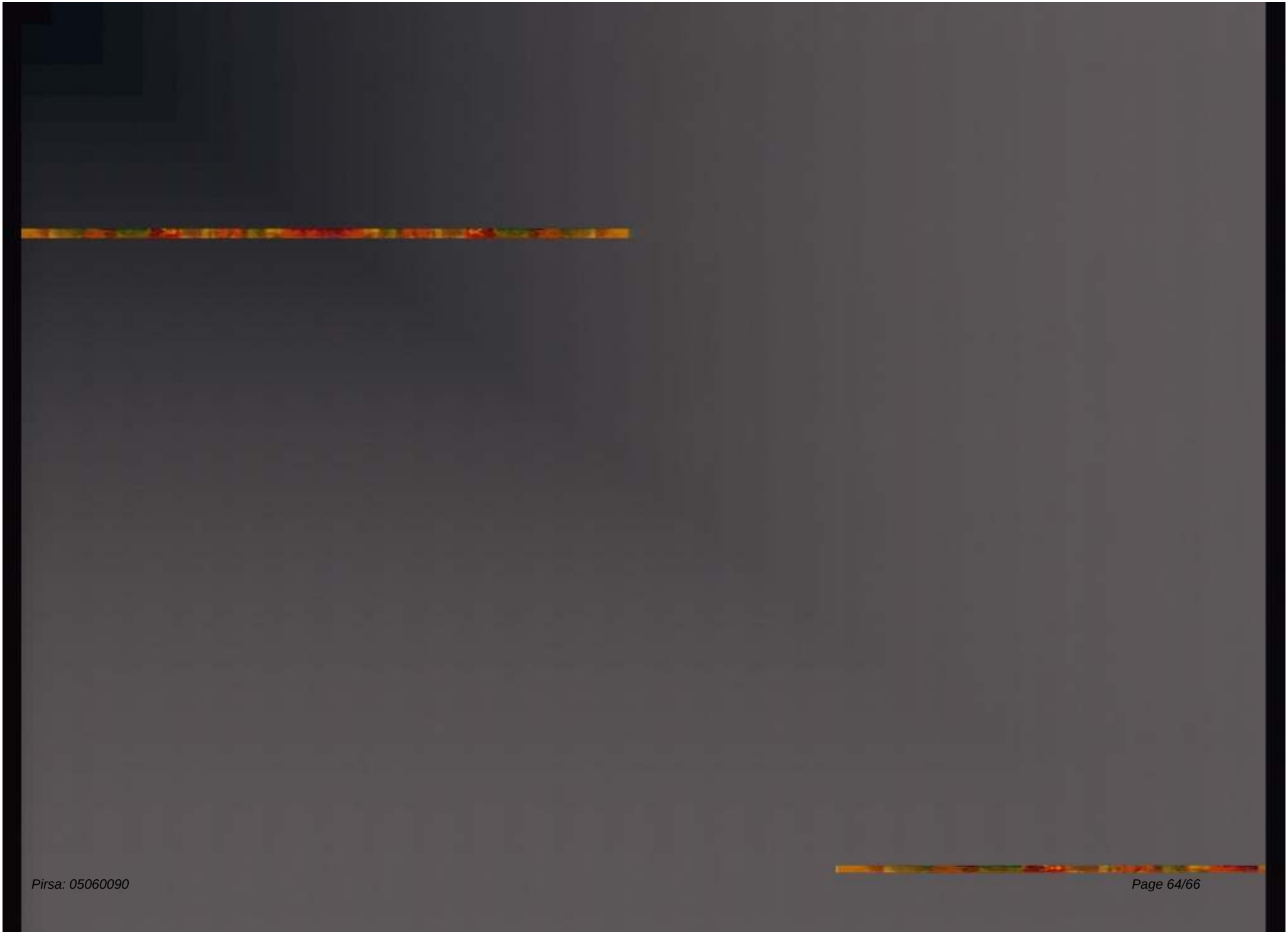
- string energy = quark mass

$$E_{\text{string}} = \frac{1}{2\pi\alpha'} \int d\sigma \sqrt{U'^2} = \Delta\sigma$$

- interpretation: radial distance = energy scale

$$r(\text{gravity}) \quad \leftrightarrow \quad \frac{1}{\lambda}(\text{gauge})$$

geometric UV-IR duality, holography



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