Title: Advanced AdS/CFT Topics

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Abstract:

Selected Topics in AdS/CFT lecture 1

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Topics:

- Lightning Review of AdS5/SYM4
- Wilson loop (hep-th/9803001, 9803002)
- Polyakov loop (hep-th/9803135)
- Baryons
- Instantons
- Time-varying phenomena
- homeworks: to be distributed separately

Starting Point

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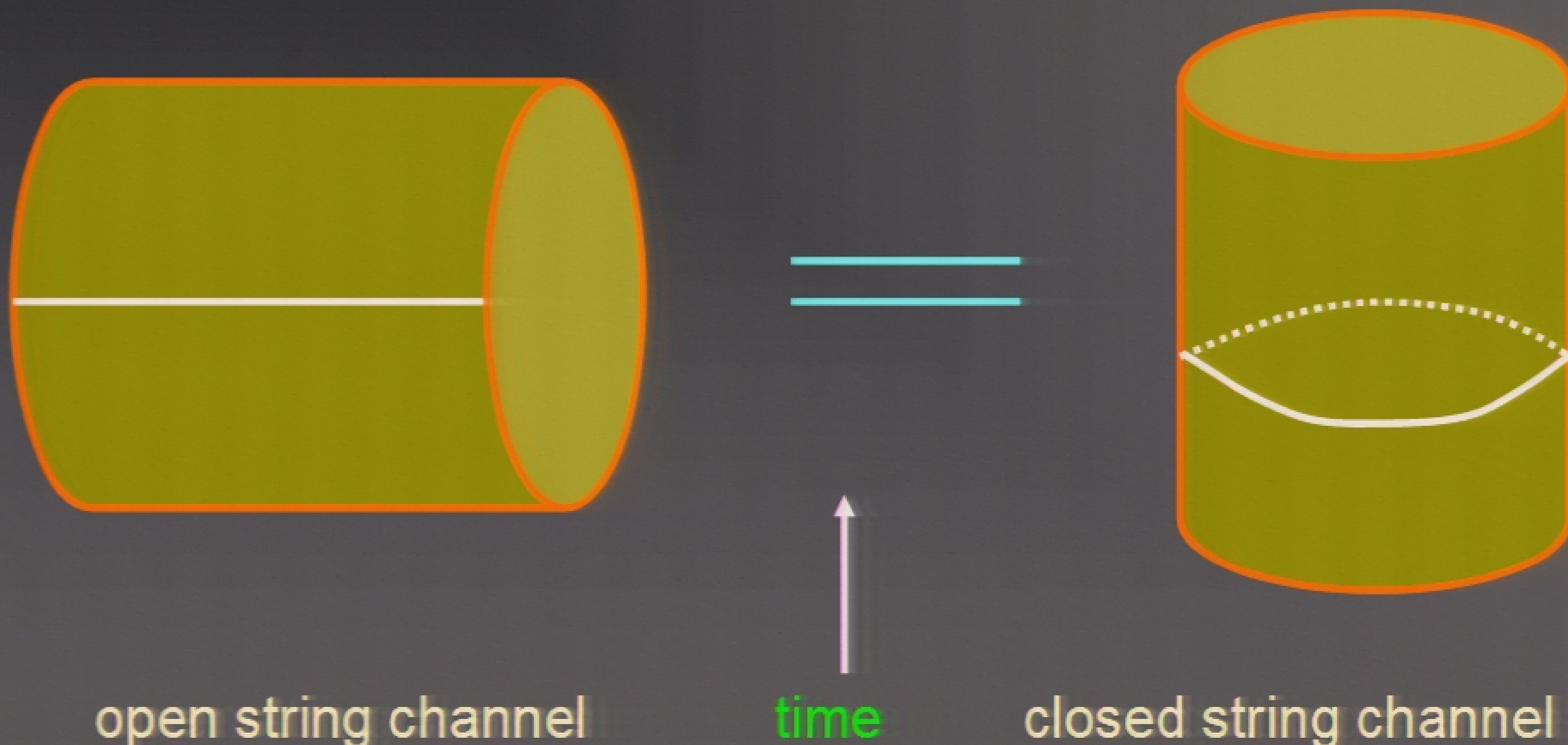
- string theory: open string + closed string
- low-energy limit of string theory:
  - open string --> gauge theory (spin=1)
  - closed string --> gravity theory (spin=2)
- channel duality: open $ closed

Channel Duality



Gauge – Gravity Duality

- keypoint:

"gauge theory dynamics"

(low energy limit of open string dynamics)

is describable by

"gravity dynamics"

(low-energy limit of closed string dynamics)

and vice versa

Tool Kits for AdS/CFT

- elementary excitations:
  open string + closed string
- solitons:
  D-branes (quantum) + NS-branes (classical)
- Mach's principle:
  spacetime Ã elementary + soliton sources
- "holography":
  gravity fluctuations = source fluctuations

p-Brane (gravity description)

- string effective action
- $S = s\, d^{10} x\, [e^{-2\Phi} (R_{(10)} + (r\, \Phi)^2 + |H_3|^2 + \ldots)$

  $+ 1 ¢ (\sum_p |G_{p+2}|^2 + \ldots)]$

  $= (1/g^2_{st})$(NS-NS sector) + (R-R sector)
- $H_3 = d\, B_2, \quad G_{p+2} = d\, C_{p+1}$ etc.
- R-R sector is "quantum" of NS-NS sector

elementary and solitons

- fixed "magnetic" charge, energy-minimizing static configurations (necessary condition for BPS states)
- NS 5-brane:

$$E = s\, g^{-2}_{st}\, [(r\Phi)^2 + H_3^2] \;>\; g^{-2}_{st}\, s_{S3}\, e^{-2\Phi}\, H_3$$

- F-string:

$$E = s\, [g^{-2}_{st}\, (r\Phi)^2 + g^{2}_{st}\, K_7^2] \;>\; g^{0}_{st}\, s_{S7}\, K_7$$

- Mass: M(string) ~ 1 ; M(NS5) ~ $(1/g^2_{st})$

"elementary" ; "soliton"

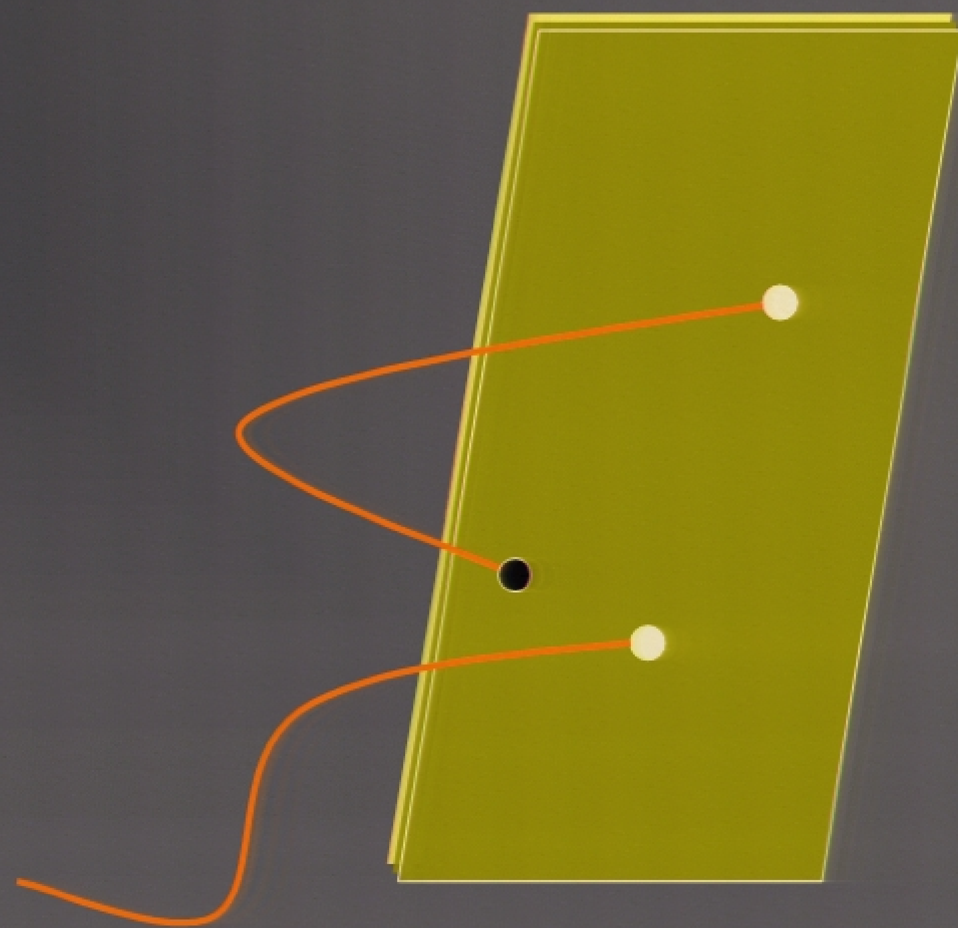
p-brane: in between

- p-brane:

  $E = s\, g^{-2}_{st}\, (r\, \Phi)^2 + (G_{p+2})^2 > g^{-1}_{st}\, s\, e^{-\Phi}\, G_{p+2}$
- M(p-brane) » ($1/g_{st}$)
- p-brane = “quantum soliton”:

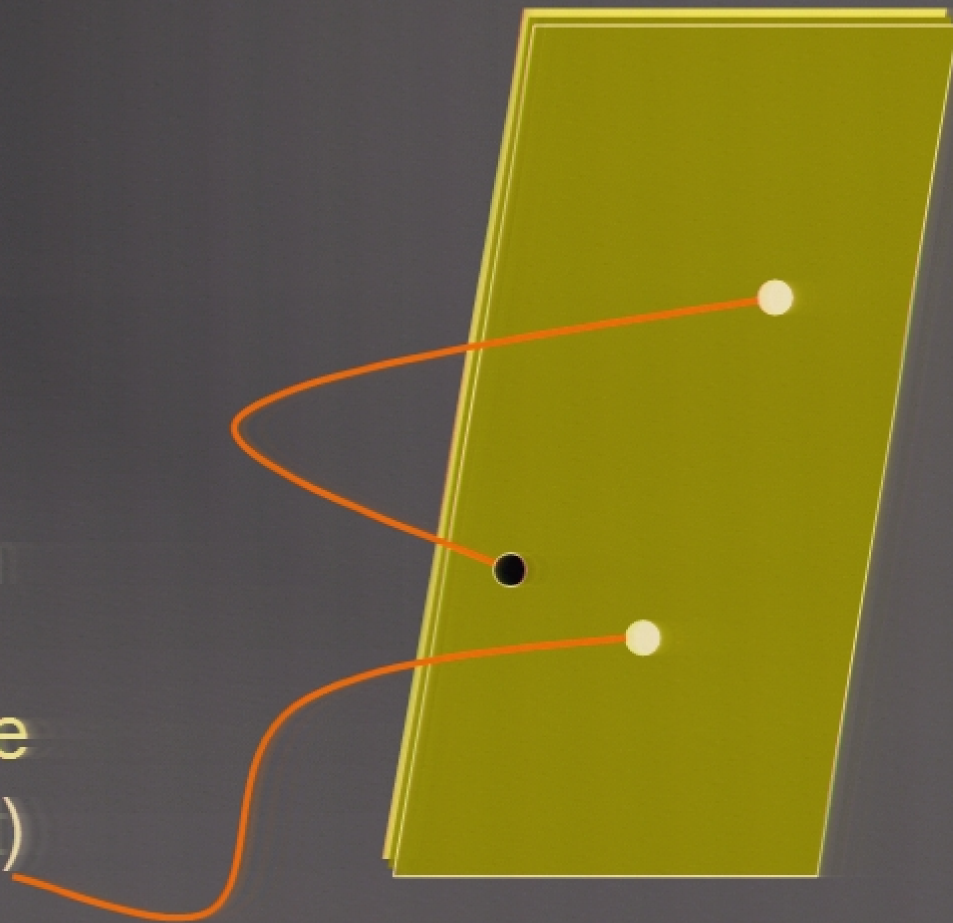
  more solitonic than F-string but less solitonic than NS5-brane
- F-string \$ p-brane \$ NS5-brane
- <u>quantum treatment of p-brane is imperative</u>

Dp-brane (CFT description)



Dp-brane (CFT description)

- strings can end on it
  open string
- string endpoints labelled by Chan-Paton factors
  (   ) = (i,j)
- mass set by disk diagram
  $M \sim 1/g_{st}$
  ---- identifiable with p-brane
- $Q_L = Q_R$ (half BPS object)



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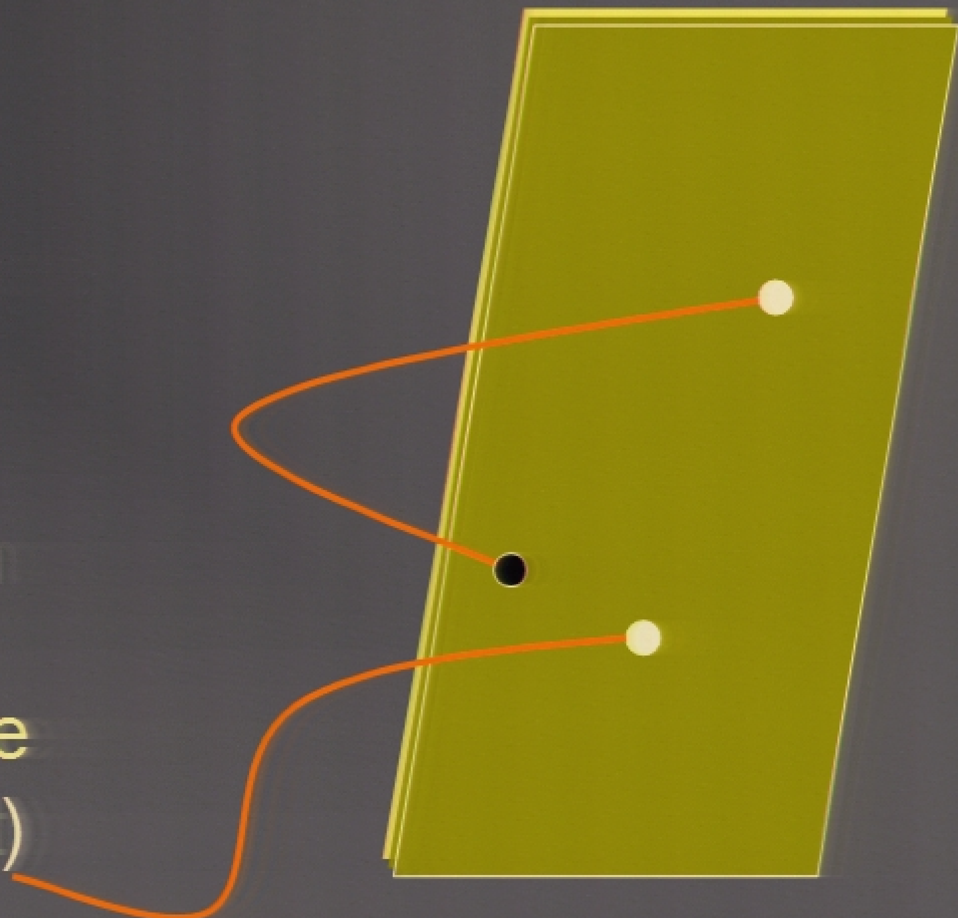

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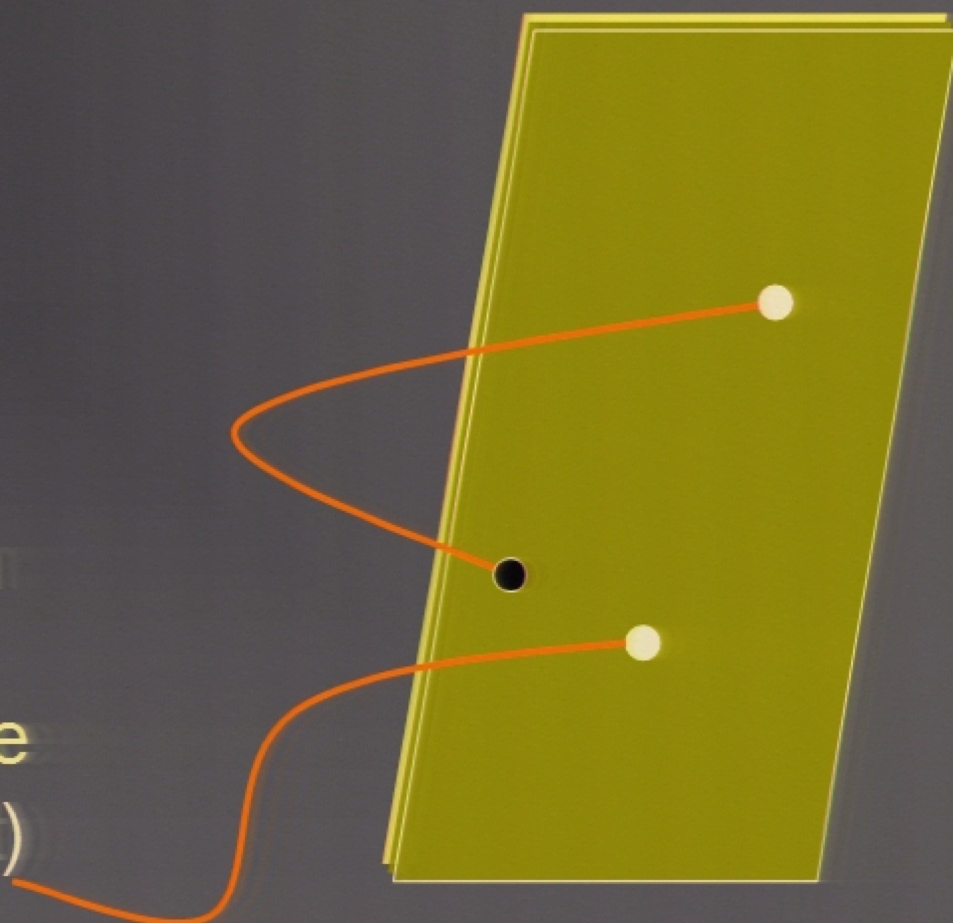
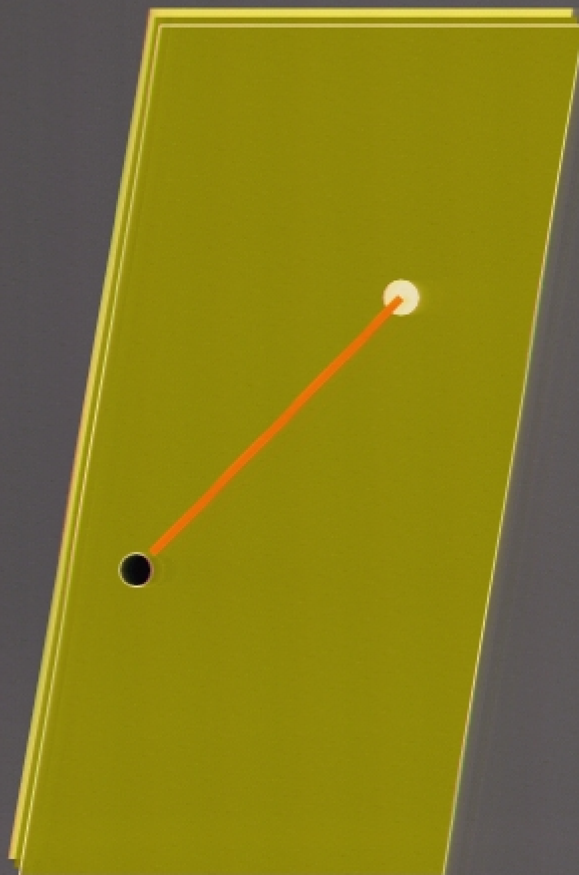


SYM_{p+1} at Low-Energy



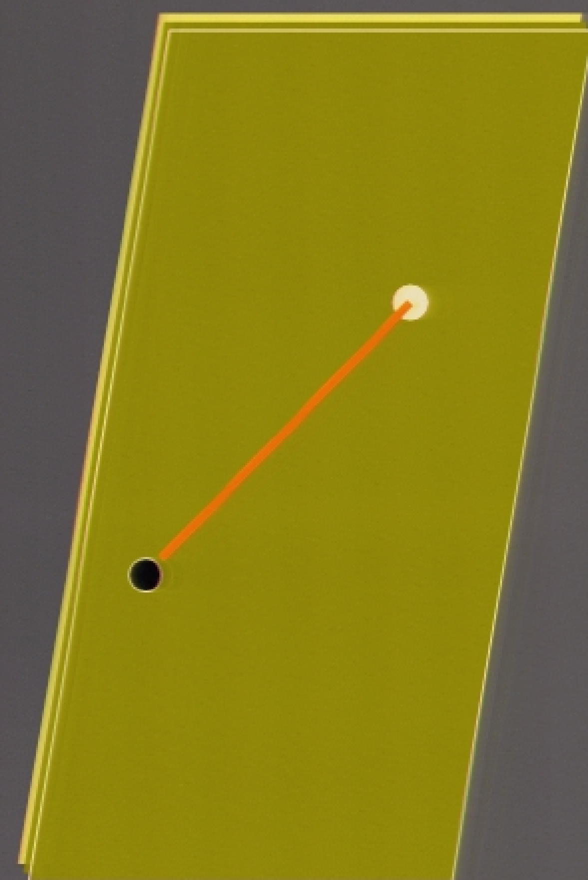
SYM_{p+1} at Low-Energy

- infinite tension limit
  ($\alpha' \rightarrow 0$)
- open strings → rigid rods
  ($M_W \sim \Delta r/\alpha'$)
- (N,N) string dynamics
  U(N) SYM(P+1)

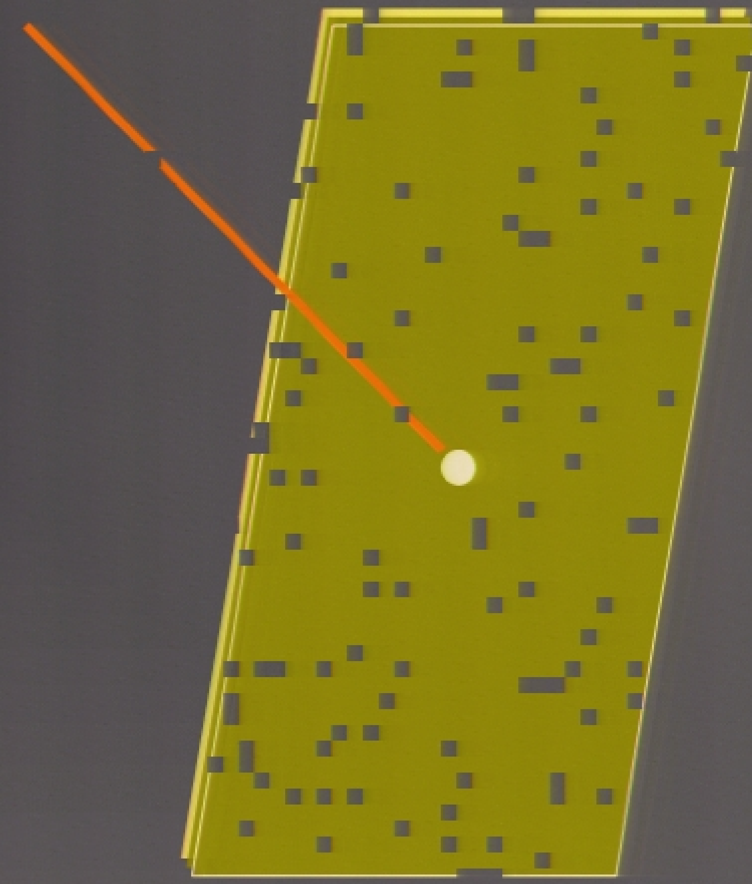
$L = g^{-2}_{YM}\, \mathrm{Tr}\, (F_{mn}^{2}$ "||"

$+ (D_m \Phi^a)^2 + [\Phi^a, \Phi^b]^2$ "?"

$+ \ldots)$



Static source: heavy quarks



Static source: heavy quarks

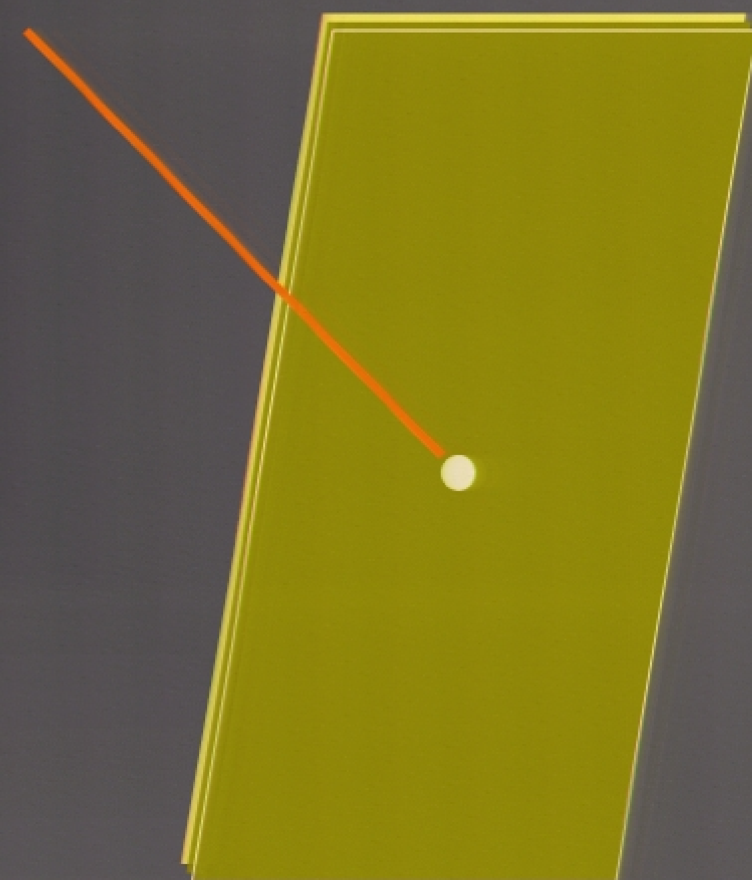
- semi-infinite string

  (M ~ 1)
- at rest or constant velocity

  (static source)
- labelled by a single Chan-Paton factor

  ( ) = (i)

  heavy (anti)quark in (anti)fundamental rep.



Perturbation theory (1)

- large N conformal gauge theory :

  double expansion in 1/N and $\lambda^2 = g_{YM}^2 N$
- $S_{YM} = (N/\lambda^2) \int d^4x \, \mathrm{Tr}\, (F_{mn}^2 + (D_m \Phi)^2 + \ldots)$
- planar expansion: $(N/\lambda^2)^{V-E} N^F = (1/N)^{2h-2} (\lambda^2)^{E-V}$
- λ^2 : nonlinear interactions
- 1/N: quantum fluctuations (effective ~)
- observable: $\sum_{h=0}^{1} \sum_{l=0}^{1} (1/N)^{2h-2} (\lambda^2)^l C_{l,h}$

Perturbation theory (2)

- *closed string theory*:

  double expansion in g_{st} and α'
- α' : string worldsheet fluctuation

  $$S_{string} = (½\pi\alpha')\ s\ d^2\ \sigma\ \partial X ¢ \partial X$$
- g_{st} : string quantum loop fluctuation
- $S_{SFT} = \Phi * Q_B \Phi + g_{st} \Phi * \Phi * \Phi$
- observables = $\Sigma_{h=0}{}^1\ \Sigma_{l=0}{}^1\ g_{st}{}^{2h-2}\ (2\pi\alpha'\ p^2)^l\ D_{l,\ h}$

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Identifying the Two Sides….

- large-N YM and closed string theory have the same perturbation expansion structure
- $g_{st} \ \$ \ (1/N)$
- $(\alpha'/R^2) \ \$ \ 1/\lambda$ where R = characteristic scale
- Maldacena's AdS/CFT correspondence:

"near-horizon"(R) geometry of D3-brane

= large-N SYM_{3+1} at large but fixed λ

- not only perturbative level but also nonperturbatively (evidence?)

D3-Brane Geometry

- 10d SUGRA(closed string)+4d SYM (D3-brane):

$$S_{total} = S_{10d} + s\, d^4 x\, (-e^{-\Phi}\, TrF_{mn}{}^2 + C_4 \ldots.)$$

- Solution
- $ds^2 = Z^{-1/2}\, dx^2{}_{3+1} + Z^{1/2}\, dy^2{}_6$
- $G_5 = (1+Æ)\, dVol_4\, Æ\, d\, Z^{-1}$ where

$Z = (1+R^4/r^4)$; $r = |y|$ and $R^4 = 4\pi\, g_{st}\, N\, \alpha'^2$

- r ! 1: 10d flat spacetime
- r ! 0 : characteristic curvature scale = R

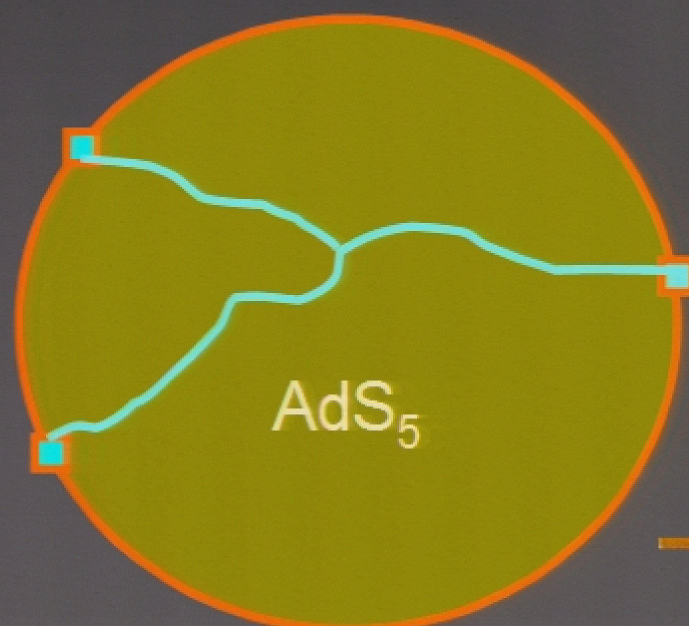
Identifications

- D-brane stress tensor grows with (N/g_{st})
- At large N, curvedness grows with $g^2_{st}(N/g_{st})=g_{st}N$
- Near D3-brane, spacetime= $AdS_5 \times S^5$
- 4d D3-brane fluct. $ 10d spacetime fluct.
  (from coupling of D3-brane to 10d fields)
  $Tr\,(F_{mp}F_{np})$ $ metric g_{mn}
  $Tr\,(F_{mn}F_{mn})$ $ dilaton Φ
  $Tr(F_{mp}F^*_{np})$ $ Ramond-Ramond C, C_{mn}

AdS/CFT correspondence

- Dirichlet problem in AdS_5 or $EAdS_5=H_5$
- $Z_{gravity}$ » $\exp(-S_{AdS5}(\phi_{bulk,a}, \phi^1_a))$

$= Z_{SYM}$ » **s** $[dA]_?$ $\exp(-S_{SYM} -$ **s** $\Sigma_a \phi^1_a O_a)$



AdS_5

- In flat 4+2 dimensional space

  $$ds^2 = -\, dX_0^{\,2} - d\, X_5^{\,2} + \Sigma_{a=1}^{\;\;4}\, d\, X_a^{\,2}$$

- embed hyperboloid

  $$X_0^{\,2} + X_5^{\,2} - \Sigma_{a=1}^{\;\;4}\, X_a^{\,2} = R^2$$

- SO(4,2) inv, homogeneous, isotropic
- AdS_5 = induced geometry on hyperboloid
- <homework>

- Global coordinates:

$$ds^2 = R^2(-\cosh^2\rho\, d\tau^2 + d\rho^2 + \sinh^2\rho\, d\Omega_3{}^2)$$

boundary = R_t £ S_3

- Poincare coordinates:

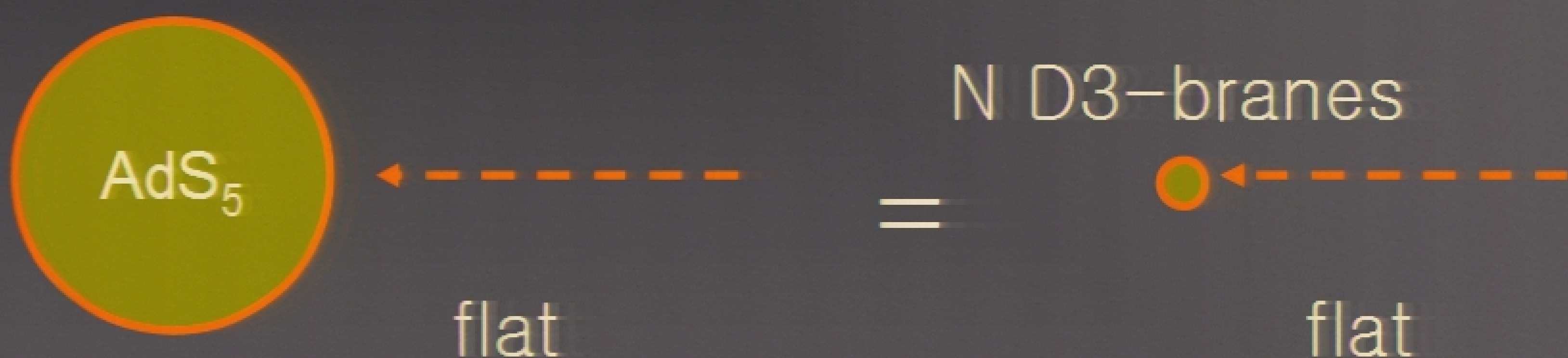
$$ds^2 = R^2[r^2(-dt^2 + dx^2 + dy^2 + dz^2) + r^{-2}dr^2]$$

boundary = R_t £ R_3

Why AdS Throat = D3-Brane?

- D-brane absorption cross-section:

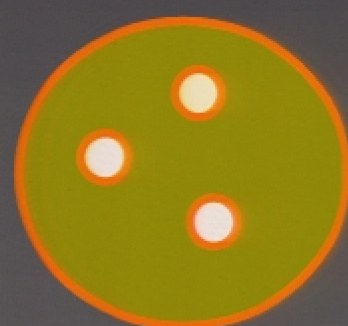
SUGRA computation = SYM computation



AdS_5 region in gravity description
= N D3-branes in gauge description

Another argument

- D–instantons probing (Euclidean) AdS5
- For U(N) gauge group, “homogeneous” instanton number < N (otherwise inhomogeneous)
- Q D–instanton cluster in approx. flat region $S_{Dinstanton} = -(1/g_{st}\,\alpha'^2)\,Tr_Q[\Phi^1, \Phi^2]^2 + \ldots.$



- $< Tr(\Phi^1)^2 >$ » QL^2, $<Tr(\Phi^2)^2 >$ » $Q^2\, g_{st}\, \alpha'^2 / L^2$
- rotational symmetry ! $L^4 = Q\, g_{st}\, \alpha'^2 = N\, g_{st}\, \alpha'^2$

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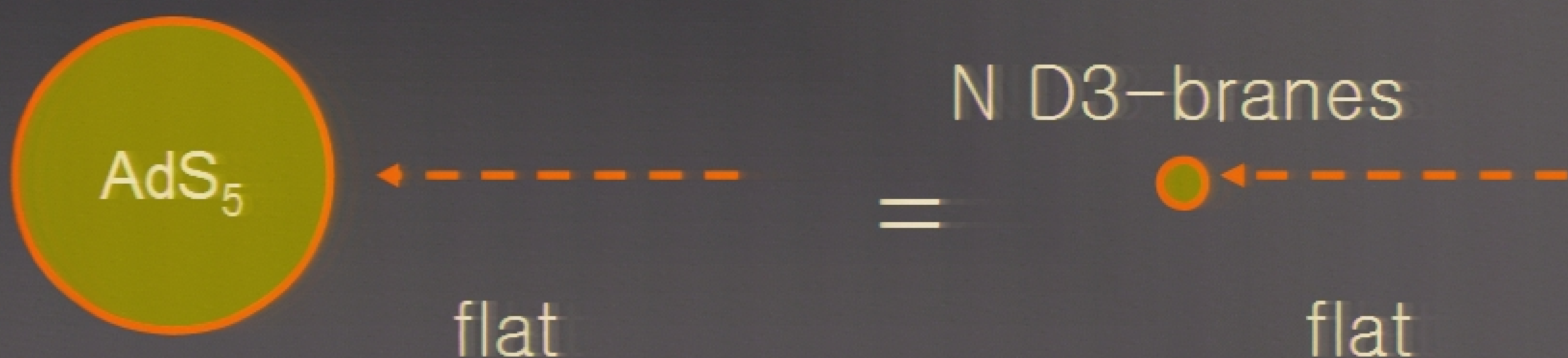
- D-brane absorption cross-section:

SUGRA computation = SYM computation



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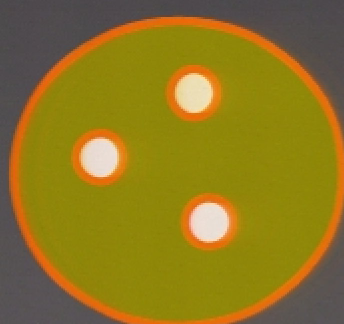

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How can it be that 5d = 4d?

- extensive quantities in 4d SYM theory scales as $[\text{length}]^4$
- Extensive quantities in 5d AdS gravity scales as $[\text{length}]^5$
- So, how can it be that quantities in 4d theory is describable by 5d theory??
- [Question] Show both area and volume of a ball of radius X in AdS_d scales as X^{d-1}!

- Answer:

Consider $(d+1)$-dimensional AdS space. Take a finite comoving d-volume $(\text{vol})_d = \Delta t \Delta^{d-1}\mathbf{x}$.

"Volume" $V_{d+1}(r_*)$ of ball of radius $r = r_*$:

$$V_{d+1}(r_*) = \int_0^{r_*} Z^{1/4}(r)\mathrm{d}r \int_{\text{vol}_d} Z^{-d/4}(r)\mathrm{d}t\mathrm{d}^{d-1}\mathbf{x}$$
$$= \frac{1}{4} r_*^4 R^{-3}\text{vol}_4 \qquad \text{for} \quad d = 4.$$

"Area" $A_d(r)$ of ball of radius $r = r_*$:

$$A_d(r_*) = \int_{r=r_*} Z^{-d/4}(r)\mathrm{d}t\mathrm{d}^{d-1}\mathbf{x}$$
$$= r_*^4 R^{-4}\text{vol}_4 \qquad \text{for} \quad d = 4.$$

Comparing the two, we find

$$\frac{1}{4}\frac{A_5(r_*)}{V_4(r_*)} = \frac{1}{R} = \text{AdS curvature radius.}$$

Notice that the ratio is independent of r_*, and holds for both $r_* \ll 1$ and $r_* \gg 1$!

Shall we test AdS/CFT?

- Recall that heavy quarks are represented by fundamental strings attached to D3-brane
- now strings are stretched and fluctuates inside AdS_5
- Let's compute interaction potential between quark and antiquark
- Do we obtain physically reasonable answers?

Static Quark Potential at Zero Temperature

$$V(r) = -(1.254...)\frac{\sqrt{g_{YM}^2 N}}{r}$$

Notice:

- Square-Root --- non-analyticity for λ
- exact 1/r --- conformal invariance

- geometry produced by D3-branes:

$$ds^2 = G_{mn}(x)dx^m dx^n = \frac{1}{\sqrt{Z}}(-dt^2 + d\mathbf{x}^2) + \sqrt{Z}dr^2$$

$$Z = 1 + \frac{R^4}{r^4} \quad \rightarrow \quad \frac{R^4}{r^4} \quad \text{near-horizon}$$

- string dynamics in the D3-brane background:

$$L_{\text{string}} = -\frac{1}{2\pi\alpha'}\int d\sigma\sqrt{-h} \quad \text{(Nambu-Goto)}$$

$$h_{ab}(\sigma,\tau) = G_{mn}(X)\partial_a X^m \partial_b X^n.$$

- look for static string configuration:

static gauge : $X^0 = \tau, \quad \mathbf{X} \equiv \mathbf{X}_{||} + U\hat{r}$

- compute $h_{ab} \rightarrow$ plug back to Nambu-Goto

Notice:
$R^4 = g_{st} N \alpha'^2$

α'^2 cancels out!

- string Lagrangian:

$$L_{\text{string}} = -\frac{1}{2\pi\alpha'}\int d\sigma\sqrt{U'^2 + Z^{-1}\mathbf{X}_{\parallel}'^2}$$

- static-gauge constraint = energy-conservation

- eqns of motion:

$$\left(\frac{U'}{\sqrt{U'^2 + Z^{-1}\mathbf{X}_{\parallel}'^2}}\right)' = \mathbf{X}_{\parallel}'^2\left(\partial_U Z^{-1}\right)$$

$$\left(\frac{Z^{-1}\mathbf{X}_{\parallel}'}{\sqrt{U'^2 + Z^{-1}\mathbf{X}_{\parallel}'^2}}\right)' = 0.$$

- heavy quark

$$X^0 = \tau, \quad \mathbf{X}_{\|}(\sigma) = 0, \quad \frac{1}{2\pi\alpha'}U(\sigma) = \sigma$$

- string energy = quark mass

$$E_{\text{string}} = \frac{1}{2\pi\alpha'}\int d\sigma\sqrt{U'^2} = \Delta\sigma$$

- interpretation: radial distance = energy scale

$$r(\text{gravity}) \quad \leftrightarrow \quad \frac{1}{\lambda}(\text{gauge})$$

geometric UV-IR duality, holography

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Holography (boundary = bulk)

Anything that takes place HERE (AdS_5) ---

--- is a result of that taking place HERE (R_{3+1})

- heavy meson

$$X^0 = \tau, \quad \mathbf{X}_{||}(\sigma) = \sigma \hat{\mathbf{r}}_3, \quad U = U(\sigma)$$

- U-eqn of motion

$$-2Z^{-1}U'' + \left(\partial_U Z^{-1}\right)(2U'^2 + Z^{-1}) = 0$$

- first integral of motion:

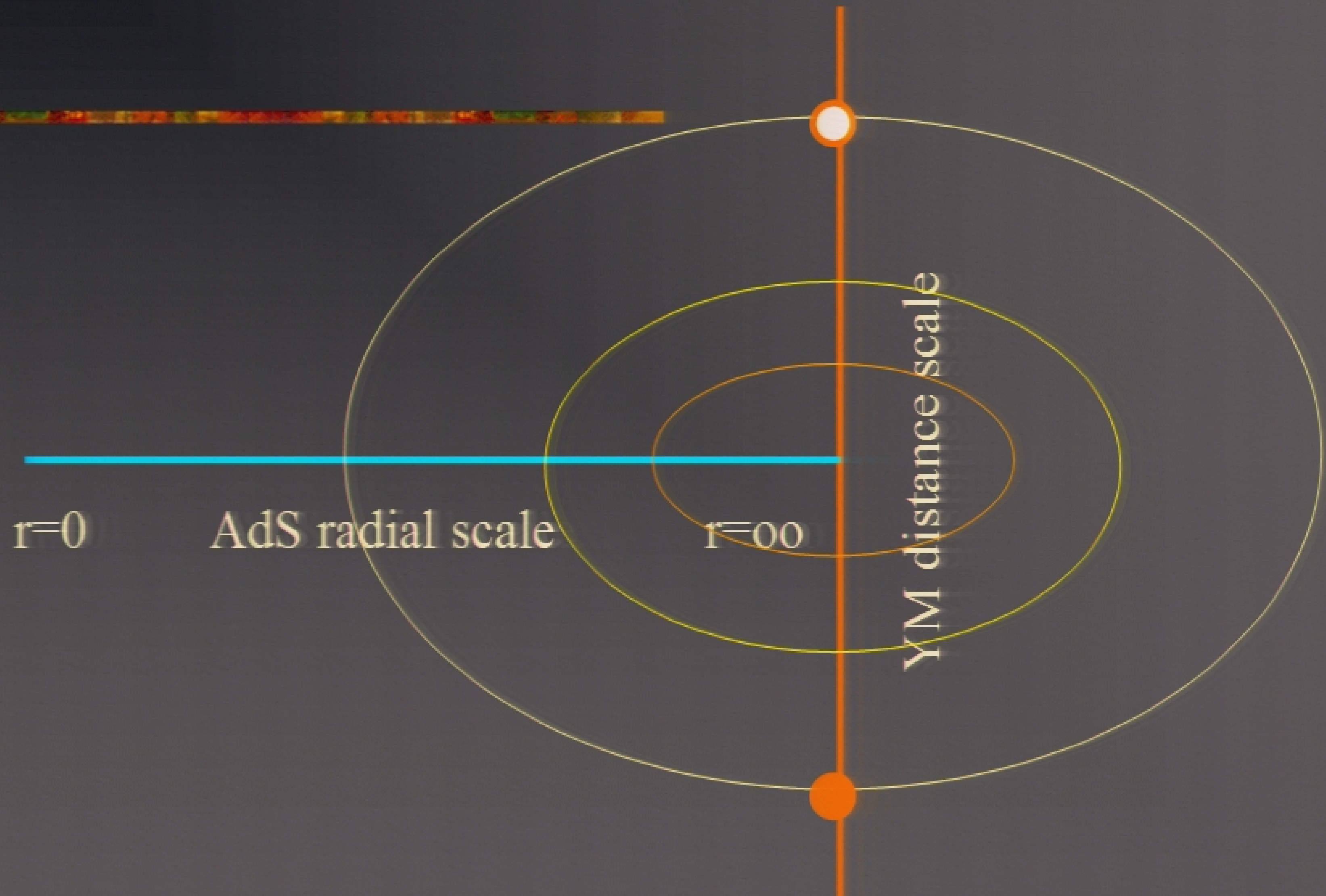
$$Z^2 U'^2 + Z = \frac{g^2 N}{U_*^4} \quad \leftarrow \text{int. const.}$$

- solution $U(\sigma)$ via implict function form:

  easy! mechanical particle analog

$$q(t) := U_*/U(\sigma) \quad \rightarrow \text{bounded orbit}$$

Heavy Meson Configuration



- exact solution:

$$r - \frac{L}{2} = \pm \frac{\sqrt{g^2 N}}{U_*} \left[\sqrt{2}\mathbf{E} - \frac{1}{\sqrt{2}}\mathbf{F} \right] \left(\cos^{-1}\frac{U_*}{U}, \frac{1}{\sqrt{2}} \right)$$

- inter-quark distance = semi-minor axis of "closed orbit":

$$L = C\frac{\sqrt{g^2 N}}{U_*} \qquad C = \frac{2\sqrt{\pi}\Gamma(3/4)}{\Gamma(1/4)} = 1.19814...$$

→ geometric UV-IR relation

- Note!

non-analyticity : $\sqrt{g^2 N}$

holography : $L \;\propto\; 1/U_*$

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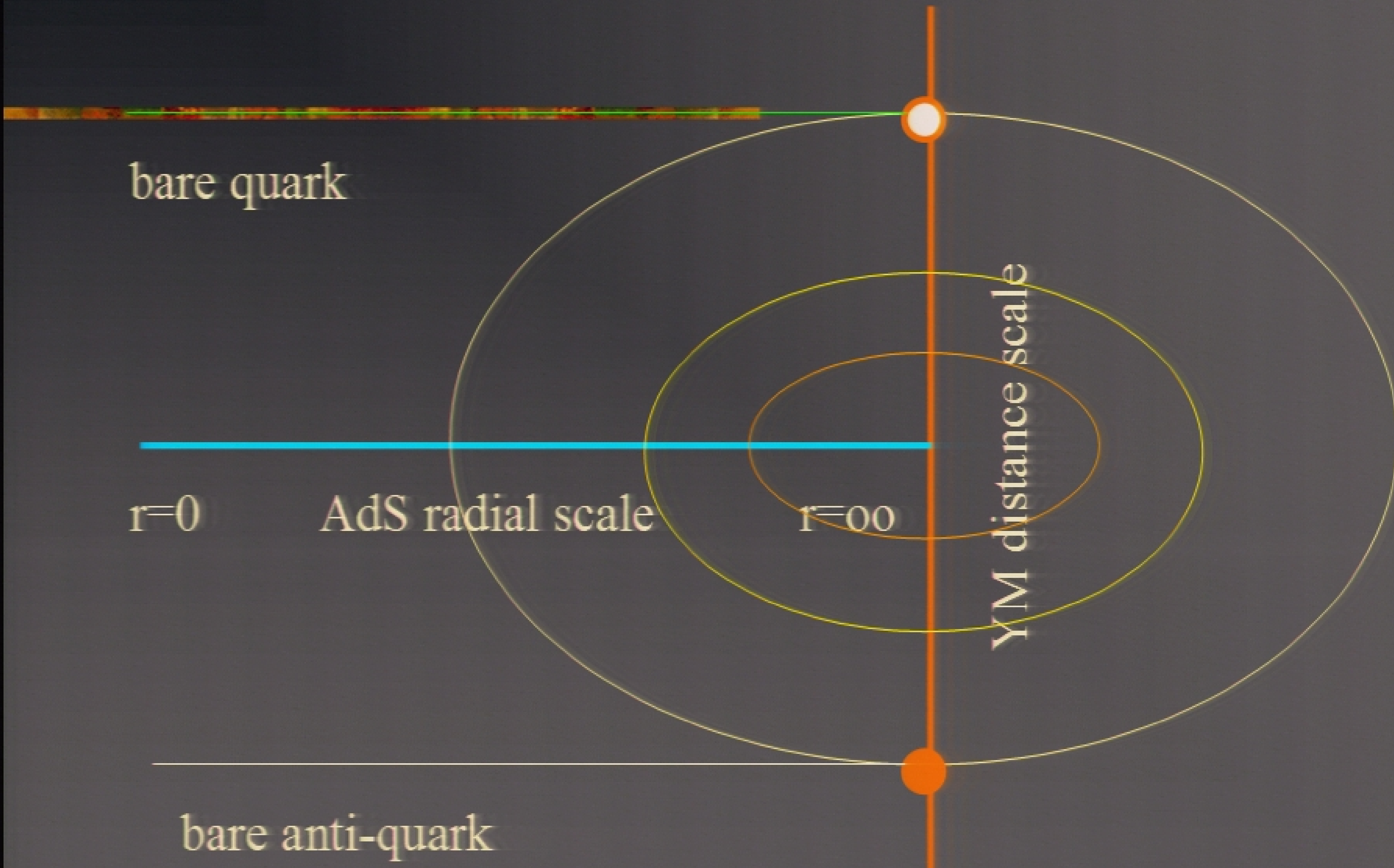

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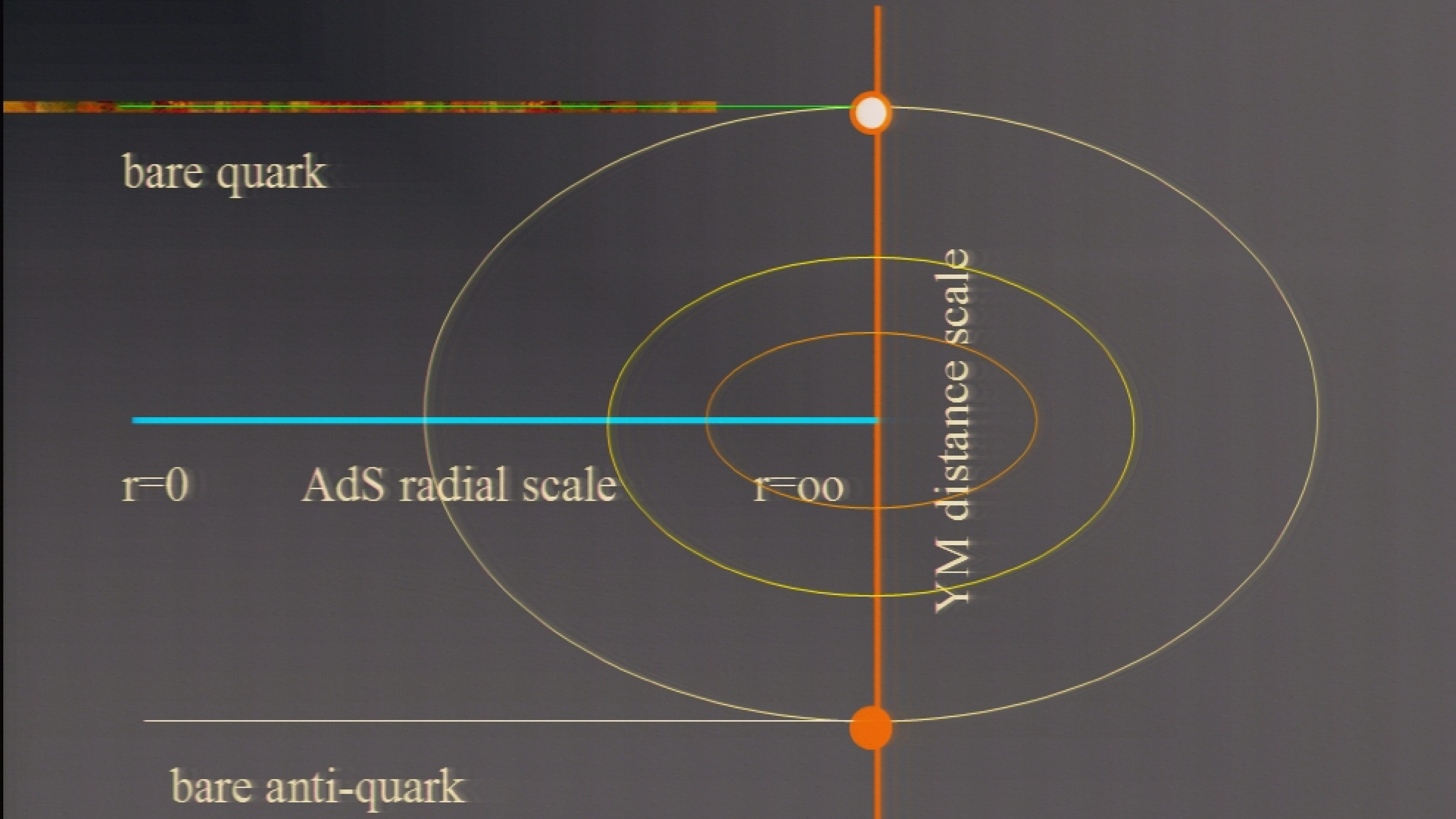

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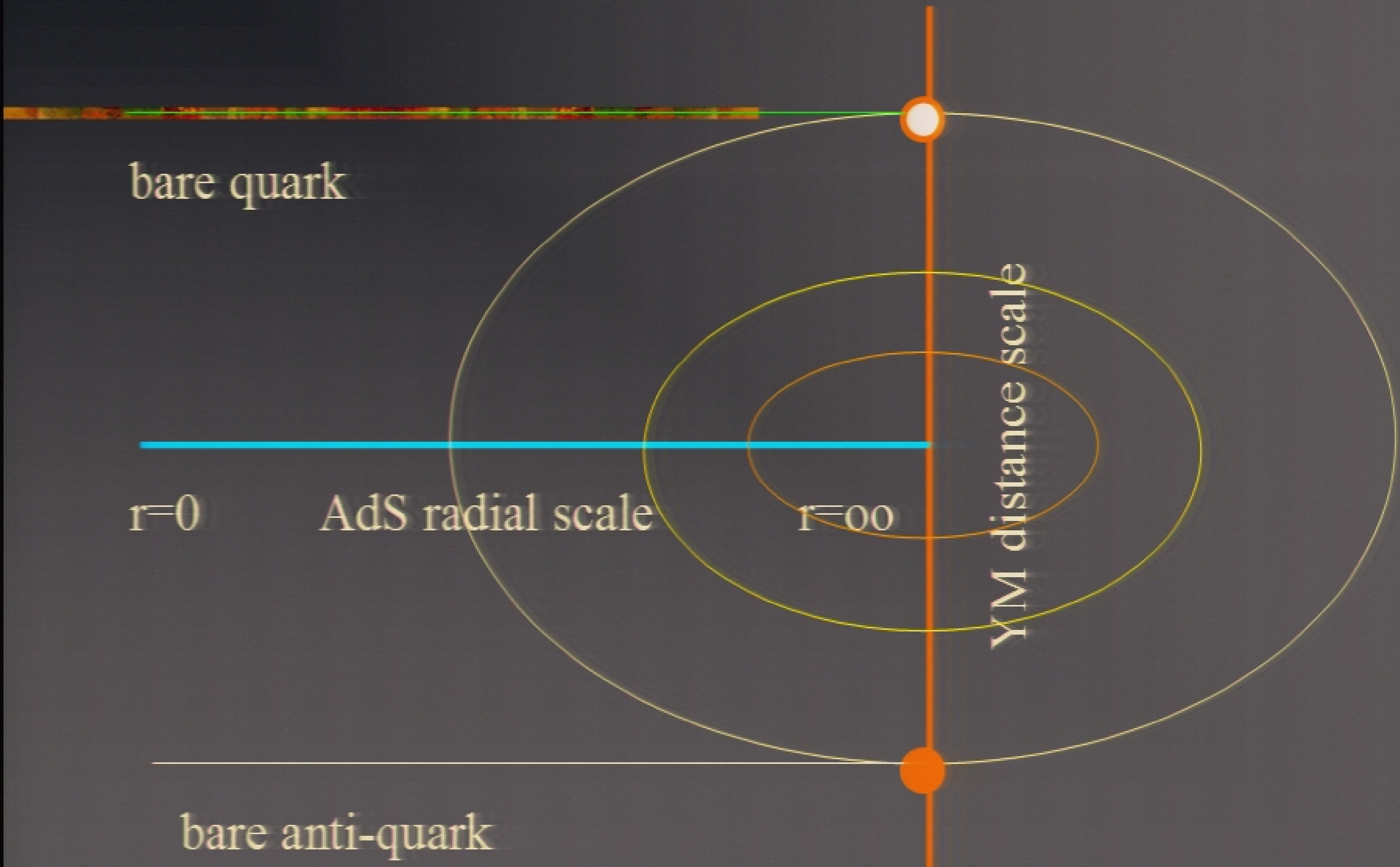

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$$r - \frac{L}{2} = \pm \frac{\sqrt{g^2 N}}{U_*} \left[\sqrt{2}\mathbf{E} - \frac{1}{\sqrt{2}}\mathbf{F} \right] \left(\cos^{-1}\frac{U_*}{U}, \frac{1}{\sqrt{2}} \right)$$

- inter-quark distance = semi-minor axis of "closed orbit":

$$L = C\frac{\sqrt{g^2 N}}{U_*} \qquad C = \frac{2\sqrt{\pi}\Gamma(3/4)}{\Gamma(1/4)} = 1.19814...$$

→ geometric UV-IR relation

- Note!

non-analyticity : $\sqrt{g^2 N}$

holography : $L \propto 1/U_*$

- string energy:

$$E_{(Q\overline{Q})}(U_*) = \frac{1}{2\pi\alpha'}\int d\sigma\sqrt{U'^2 + Z^{-1}}$$
$$= \lim_{u\to\infty} 2U^*\int_0^u dx \frac{x^2}{\sqrt{x^4-1}}$$

[used the first integral ("conserved energy")]

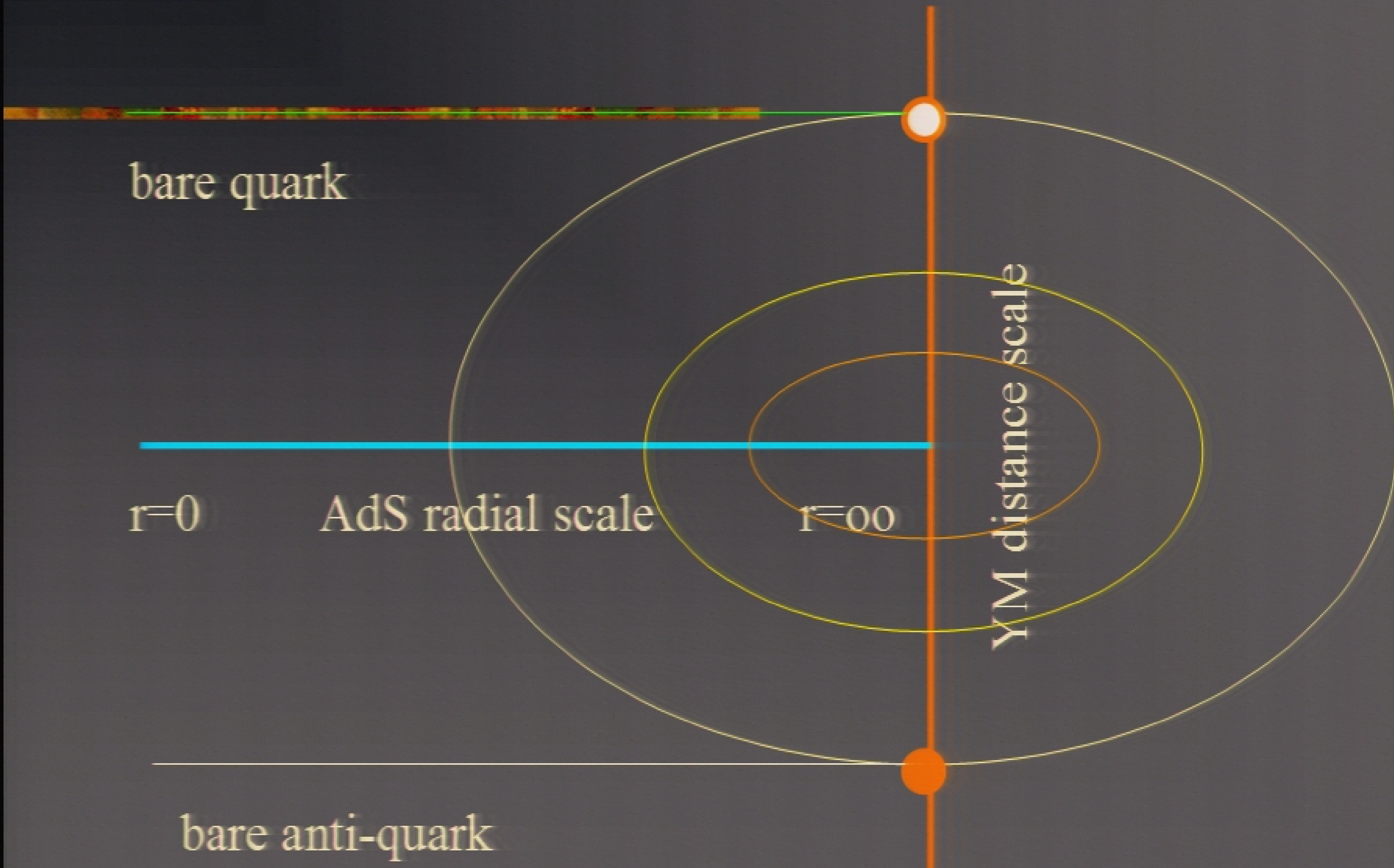
- heavy-quark potential:

$$V_{(Q\overline{Q})}(U_*) \equiv E_{(Q\overline{Q})}(U_*) - (M_Q + M_{\overline{Q}})$$
$$= -2U_*\left[\frac{\sqrt{2}\pi^{3/2}}{\Gamma^2(1/4)}\right]$$

- using "geometric UV-IR duality",

$$V_{Q\overline{Q}}(L) = -C\frac{\sqrt{g^2N}}{L} \qquad C = 1.254...$$

Heavy Meson Configuration



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What have we evaluated?

- rectangular Wilson loop in N=4 SYM

- $W[C] = \mathrm{Tr}\, P \exp \mathbf{s}_C \, (i\, A_m\, dx^m + \Phi^a\, d\, y^a\,)$

- gauge field part = Aharonov-Bohm phase
- scalar field part = W-boson mass
- unique N=4 supersymmetric structure

- heavy meson

$$X^0 = \tau, \quad \mathbf{X}_{||}(\sigma) = \sigma \hat{\mathbf{r}}_3, \quad U = U(\sigma)$$

- U-eqn of motion

$$-2Z^{-1}U'' + \left(\partial_U Z^{-1}\right)(2U'^2 + Z^{-1}) = 0$$

- first integral of motion:

$$Z^2 U'^2 + Z = \frac{g^2 N}{U_*^4} \quad \leftarrow \text{int. const.}$$

- solution $U(\sigma)$ via implict function form:

  easy! mechanical particle analog

$$q(t) := U_*/U(\sigma) \quad \rightarrow \text{bounded orbit}$$

- heavy quark

$$X^0 = \tau, \quad \mathbf{X}_{\|}(\sigma) = 0, \quad \frac{1}{2\pi\alpha'}U(\sigma) = \sigma$$

- string energy = quark mass

$$E_{\text{string}} = \frac{1}{2\pi\alpha'}\int d\sigma\sqrt{U'^2} = \Delta\sigma$$

- interpretation: radial distance = energy scale

$$r(\text{gravity}) \quad \leftrightarrow \quad \frac{1}{\lambda}(\text{gauge})$$

geometric UV-IR duality, holography

- string energy:

$$
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