Title: Perturbative String Theory

Date: Jun 24, 2005 02:30 PM

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Abstract:

Compactifying on a Circle

Closed Strings

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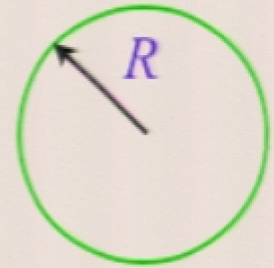
Closed Strings

$$X^\mu(z,\bar{z}) = \frac{x^\mu}{2} + \frac{\tilde{x}^\mu}{2} - i\sqrt{\frac{\alpha'}{2}}(\alpha_0^\mu + \tilde{\alpha}_0^\mu)\tau + \sqrt{\frac{\alpha'}{2}}(\alpha_0^\mu - \tilde{\alpha}_0^\mu)\sigma + \text{oscillators.}$$

Compactifying on a Circle

Now consider placing one of the directions on a circle:



$$X^9 \rightarrow X^9 + 2\pi R$$

So now when we go around the string: $\sigma \longrightarrow \sigma + 2\pi$

String can wind, i.e.:

$$X^\mu(z,\bar{z}) \rightarrow X^\mu(z,\bar{z}) + 2\pi\sqrt{\frac{\alpha'}{2}}(\alpha_0^\mu - \tilde{\alpha}_0^\mu)$$

$$2\pi\sqrt{\frac{\alpha'}{2}}(\alpha_0^9 - \widetilde{\alpha}_0^9) = 2\pi w R$$

$$p^\mu = \frac{1}{\sqrt{2\alpha'}}(\alpha_0^\mu + \tilde{\alpha}_0^\mu)$$

Compactifying on a Circle

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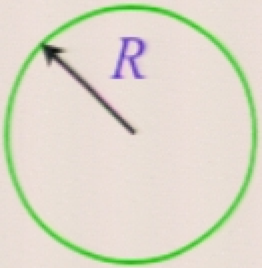
$$\frac{1}{\sqrt{2\alpha'}}(\alpha_0^9 + \tilde{\alpha}_0^9) = \frac{n}{R}$$



...and we get:

$$\alpha_0^9 = \sqrt{\frac{\alpha'}{2}}\left(\frac{n}{R} + \frac{wR}{\alpha'}\right)$$

$$\tilde{\alpha}_0^9 = \sqrt{\frac{\alpha'}{2}}\left(\frac{n}{R} - \frac{wR}{\alpha'}\right)$$

Compactifying on a Circle

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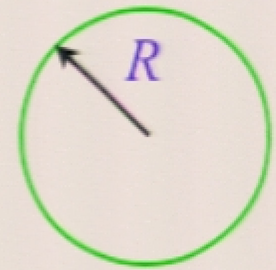
$$M^2 = -p_\mu p^\mu = \frac{2}{\alpha'}(\alpha_0^9)^2 + \frac{4}{\alpha'}(N + a_L)$$

$$M^2 = -p_\mu p^\mu = \frac{2}{\alpha'}(\tilde{\alpha}_0^9)^2 + \frac{4}{\alpha'}(\tilde{N} + a_R)$$

$$a_{L,R} = -\frac{1}{2}, 0 \ldots$$



$$M^2 = \frac{n^2}{R^2} + \frac{w^2 R^2}{(\alpha')^2} + \frac{2}{\alpha'}(N + \tilde{N} + a_L + a_R)$$

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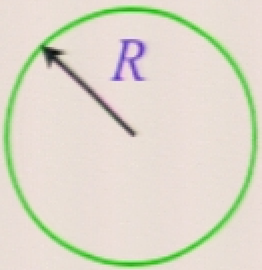

$R \longrightarrow \infty$

"decompactify"

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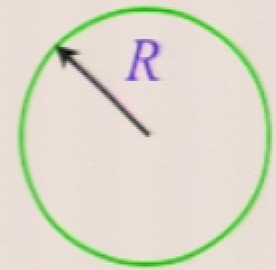

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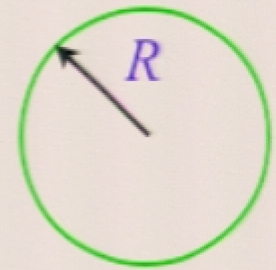

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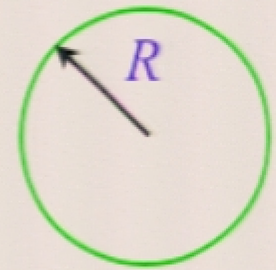

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"decompactify"

spacing of ladder of momentum states becomes smaller recover a continuum of momenta

winding states become massive. They leave the theory.

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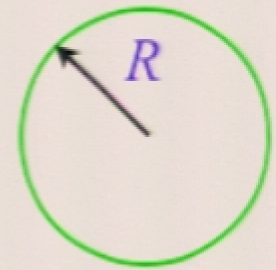

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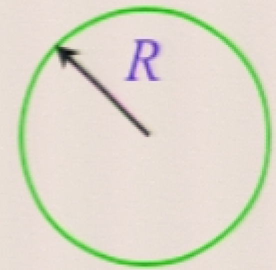

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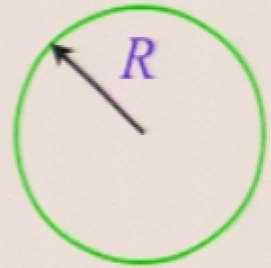

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So we recover an uncompactified theory again!

T-Duality

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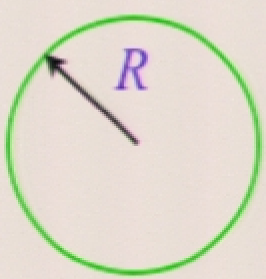

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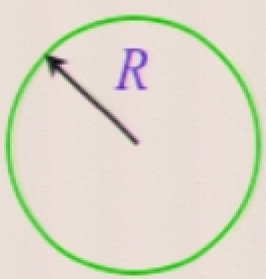

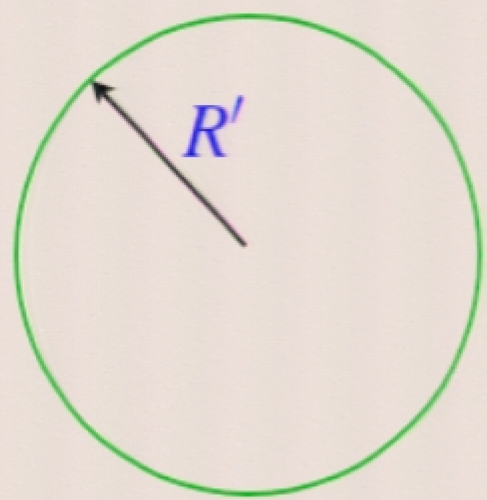

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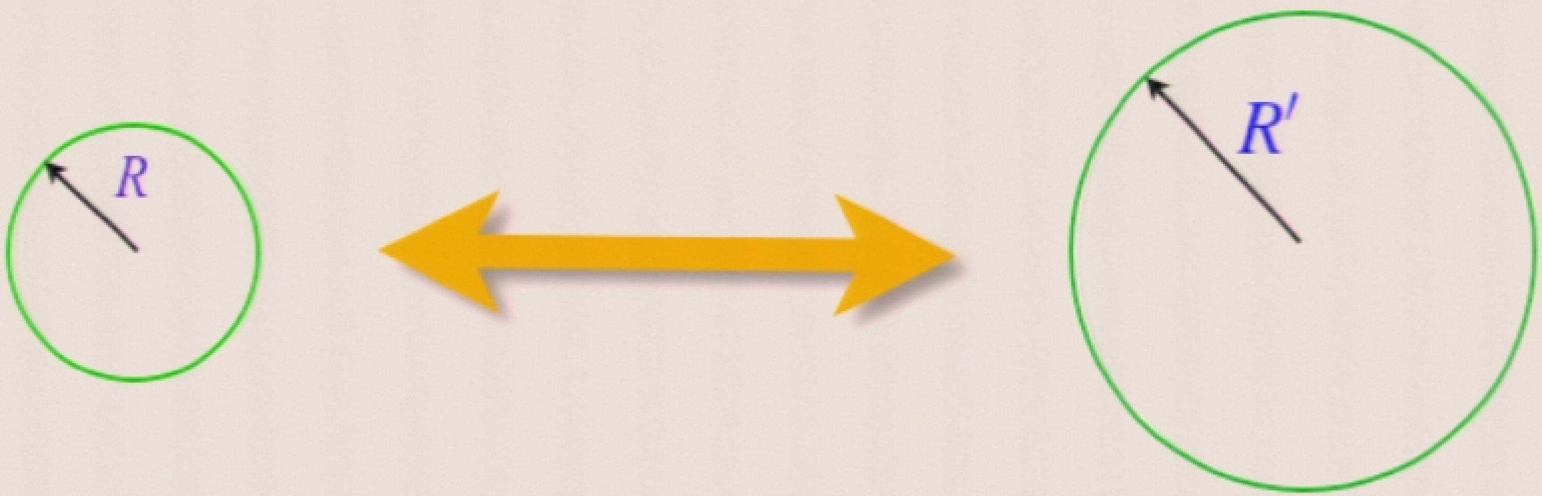

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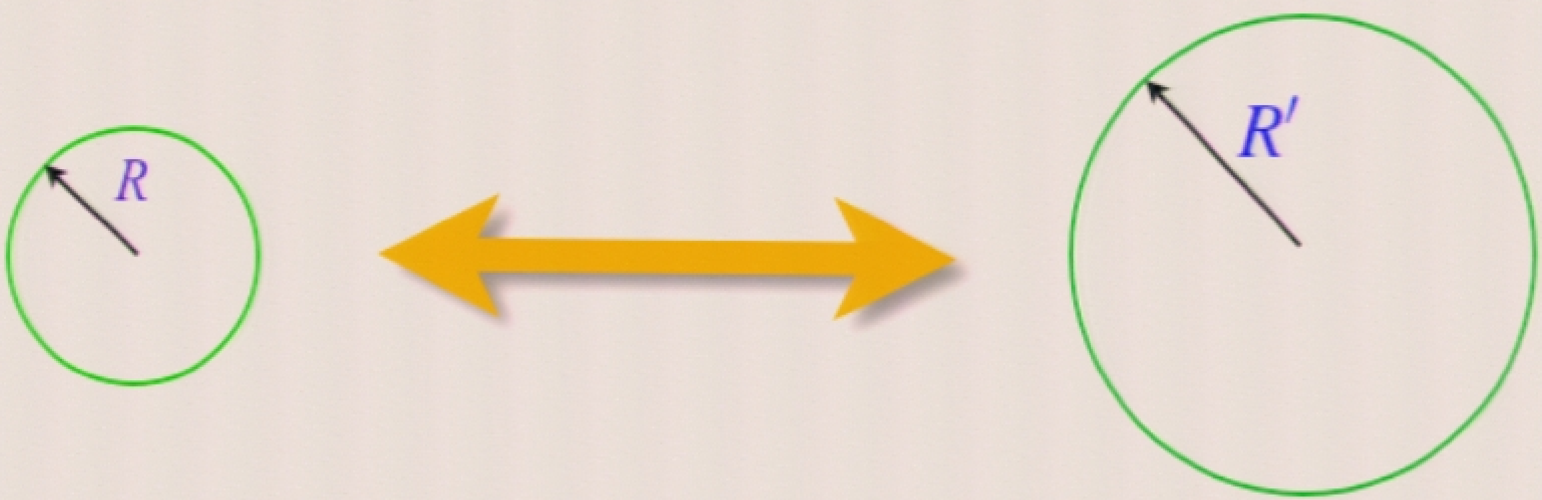

Large and small circles are dual theories!

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$$\alpha_0^9 \rightarrow \alpha_0^9$$

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Write things in terms of "dual" coordinate:

$$X'^9(z,\bar{z}) = X^9(z) - X^9(\bar{z})$$

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Entire theory cares little about this sign, so a trivial world-sheet symmetry can make a big difference to spacetime interpretation.

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trap the ends somehow?

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How to square this circle?

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Write the mode expansion as:

T-Duality for Open Strings

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$$X^\mu(z,\bar{z}) = X^\mu(z) + X^\mu(\bar{z})$$

$$X^\mu(z) = \frac{x^\mu}{2} + \frac{x'^\mu}{2} - i\alpha' p_{\rm o}^\mu \ln z + i\left(\frac{\alpha'}{2}\right)^{1/2} \sum_{n\neq 0} \frac{1}{n}\alpha_n^\mu z^{-n}$$

$$X^\mu(\bar{z}) = \frac{x^\mu}{2} - \frac{x'^\mu}{2} - i\alpha' p_{\rm o}^\mu \ln \bar{z} + i\left(\frac{\alpha'}{2}\right)^{1/2} \sum_{n\neq 0} \frac{1}{n}\alpha_n^\mu \bar{z}^{-n}$$

T-Duality for Open Strings

Now place theory on circle, and explore the dual coordinate:

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$$X'^9(z,\bar{z}) = X^9(z) - X^9(\bar{z})$$

$$= x'^9 - i\alpha' p^9 \ln\left(\frac{z}{\bar{z}}\right) + \text{oscillators}$$

$$= x'^9 + 2\alpha' p^9 \sigma + \text{oscillators}$$

T-Duality for Open Strings

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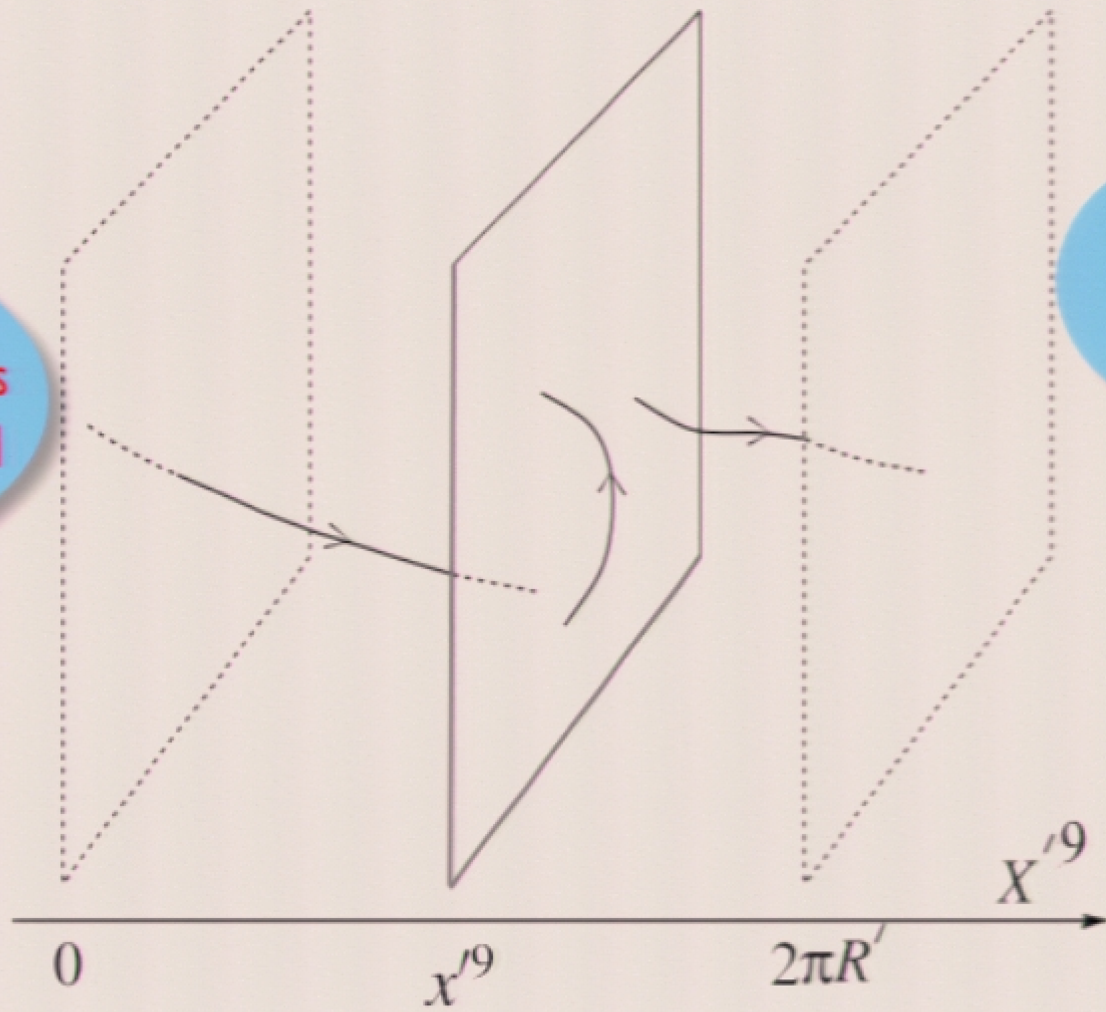
Where are the endpoints of the string?

$$X'^9(\sigma = \pi) - X'^9(\sigma = 0) = 2\pi \frac{\alpha'}{R} n = 2\pi n R'$$

T-Duality for Open Strings



Yes, only endpoint physics is 9-dimensional

So the ends did get trapped somehow.

$n = 1$

$n = 0$

D-brane

Dirichlt-brane... D8-brane

Starting theory had D9-brane, Neumann in all directions.

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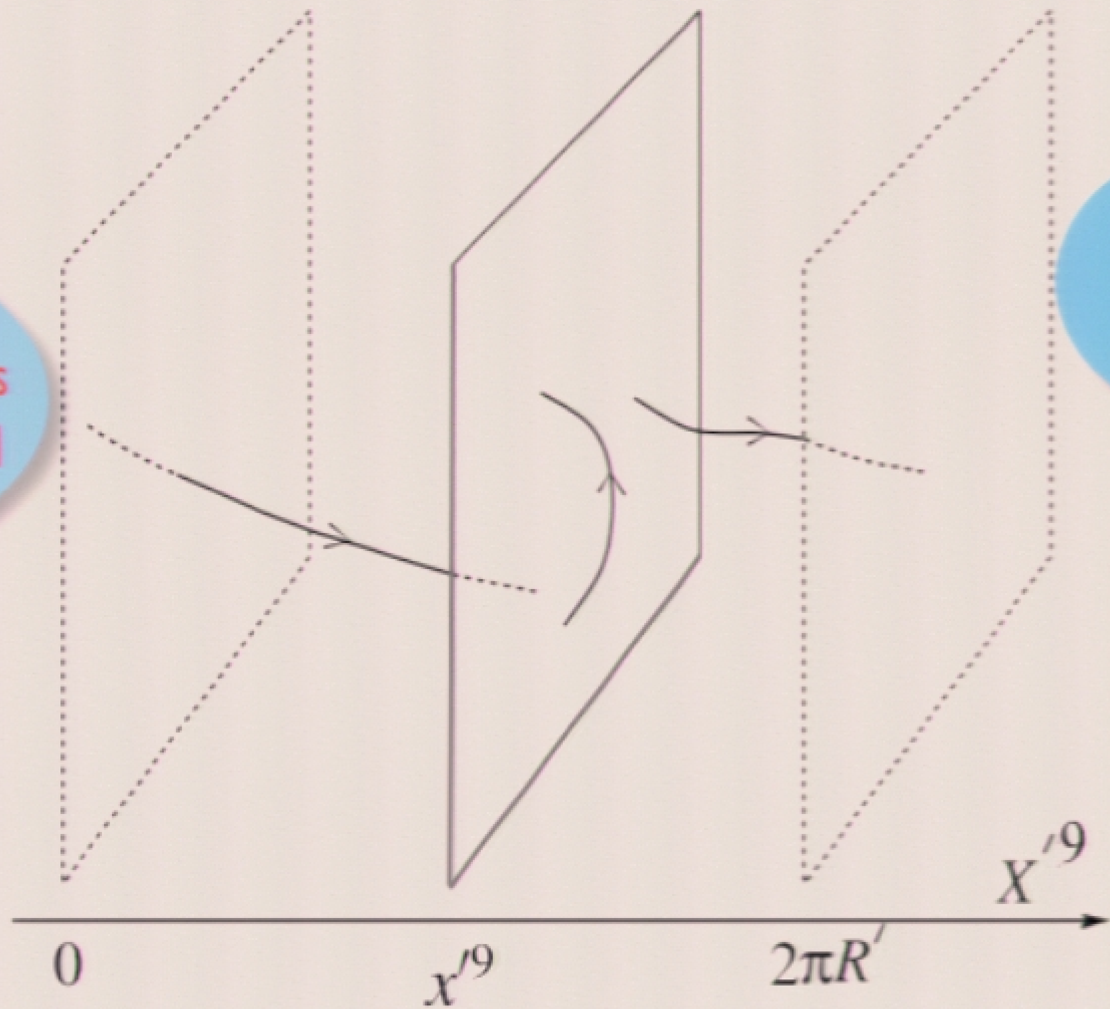

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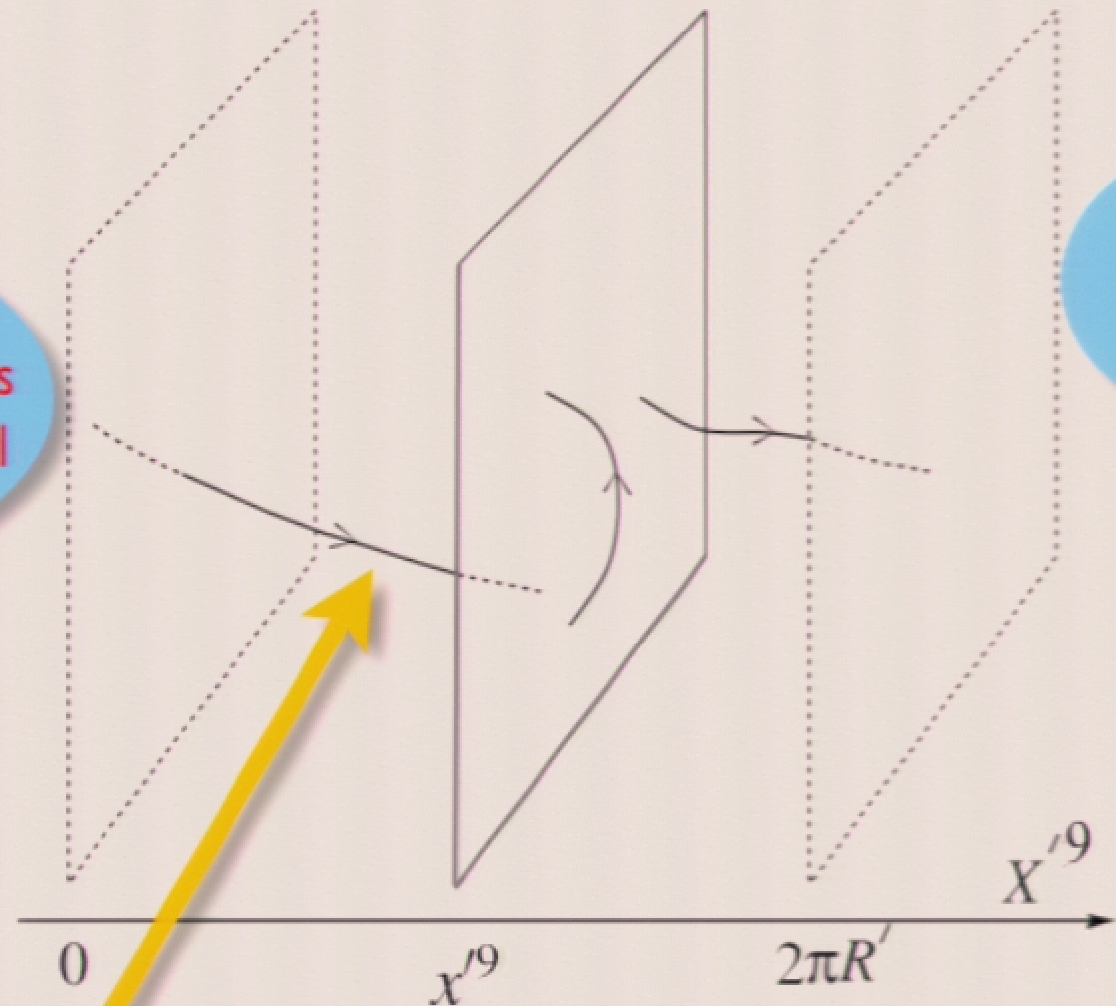

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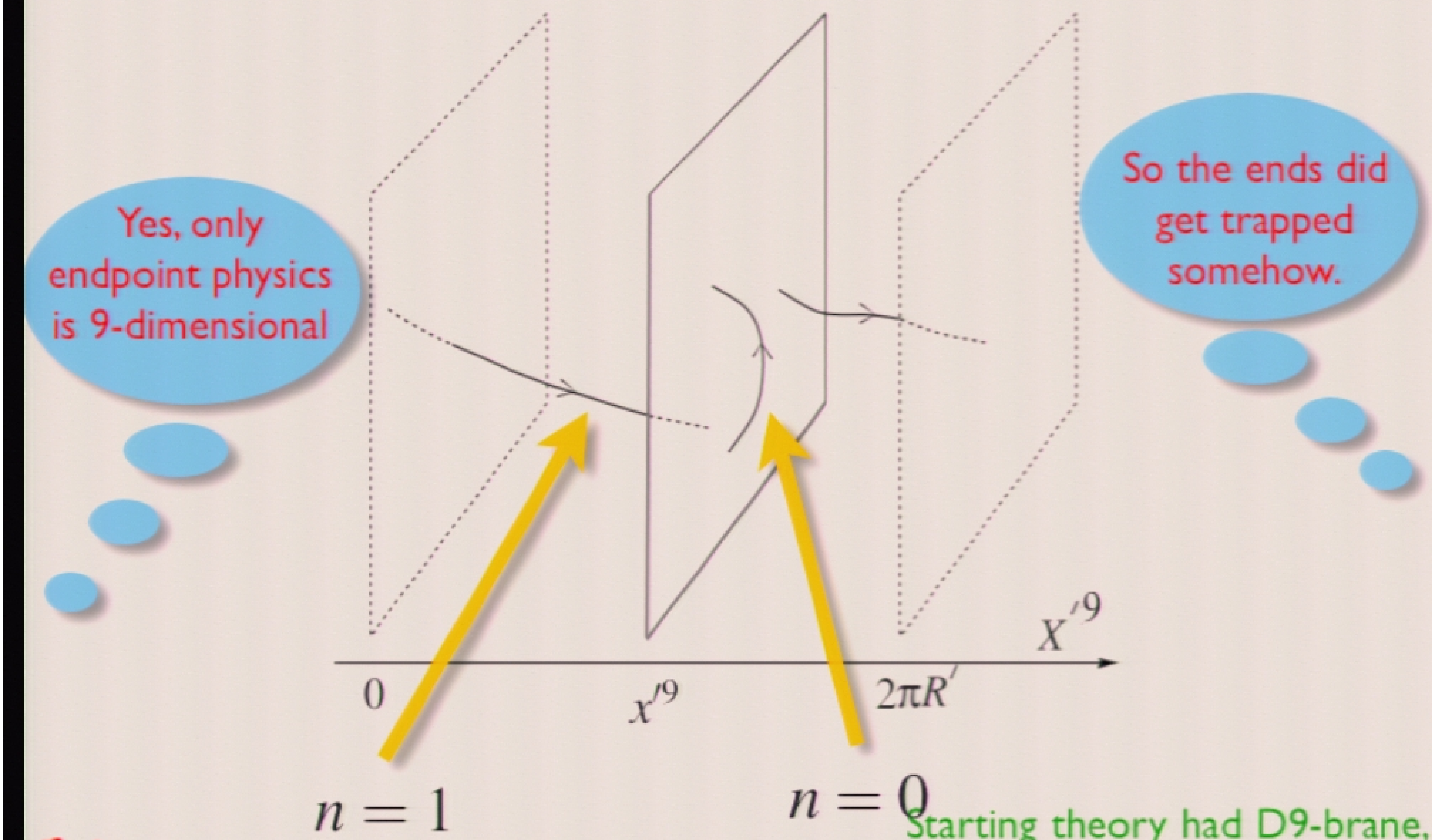

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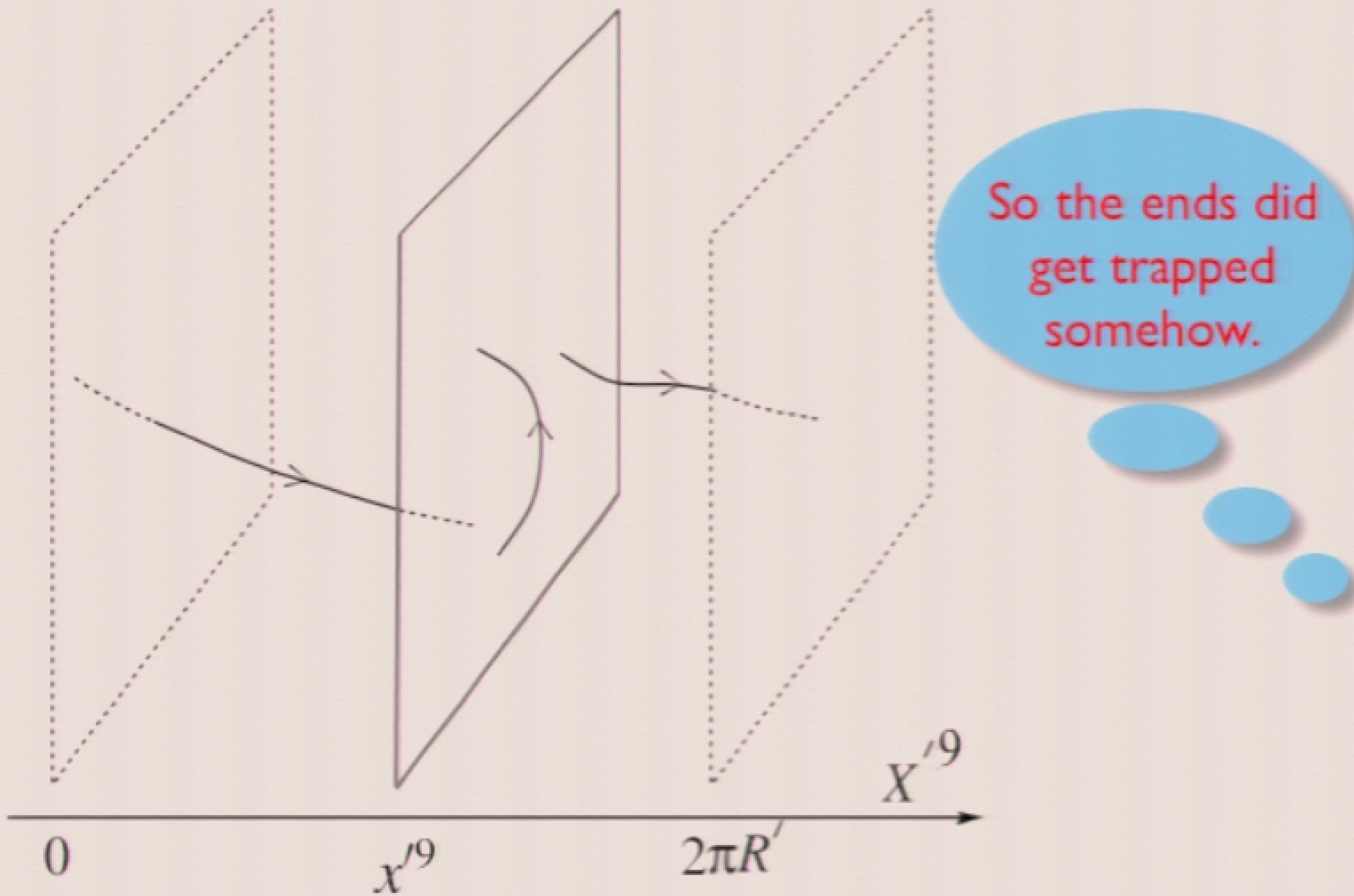

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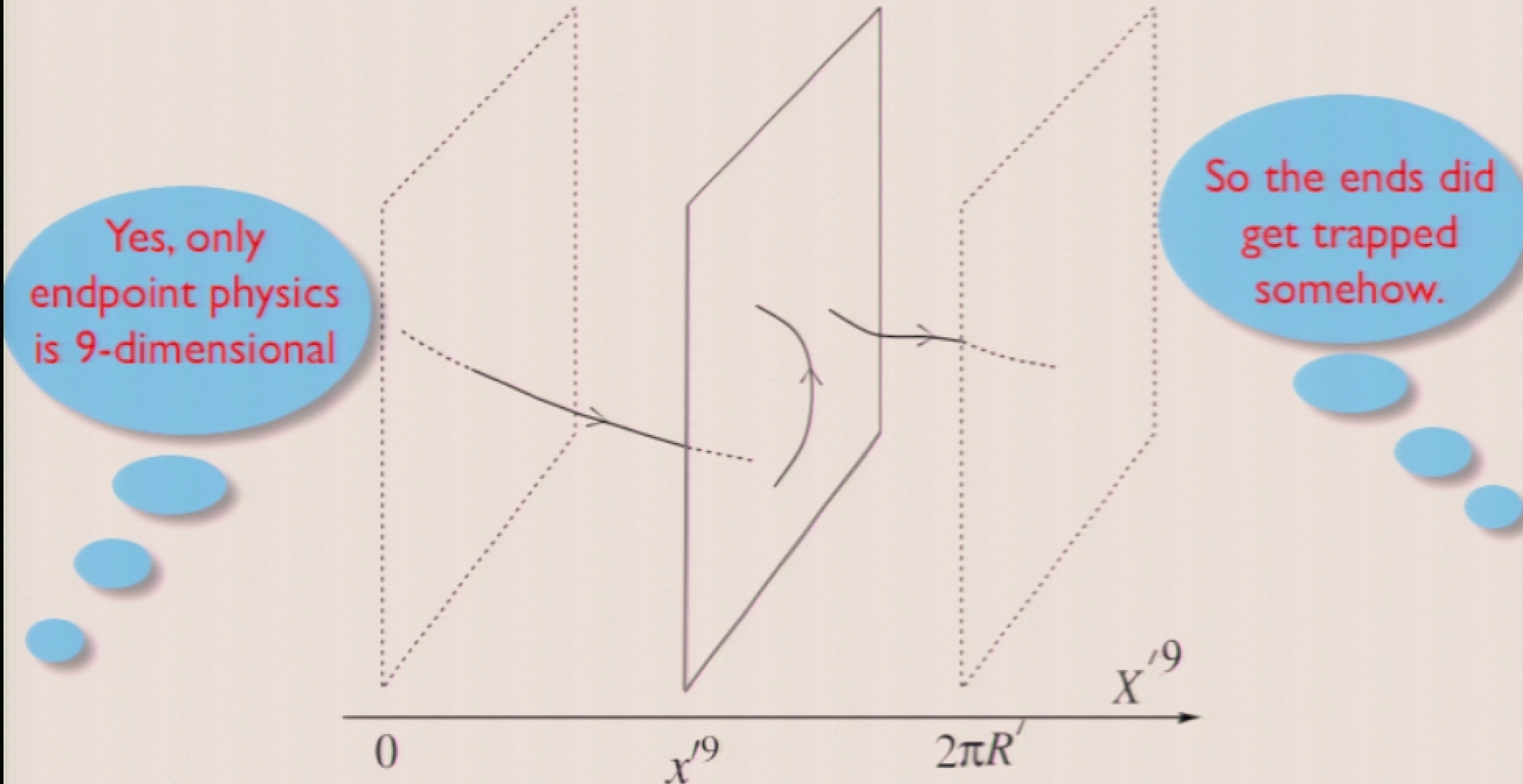

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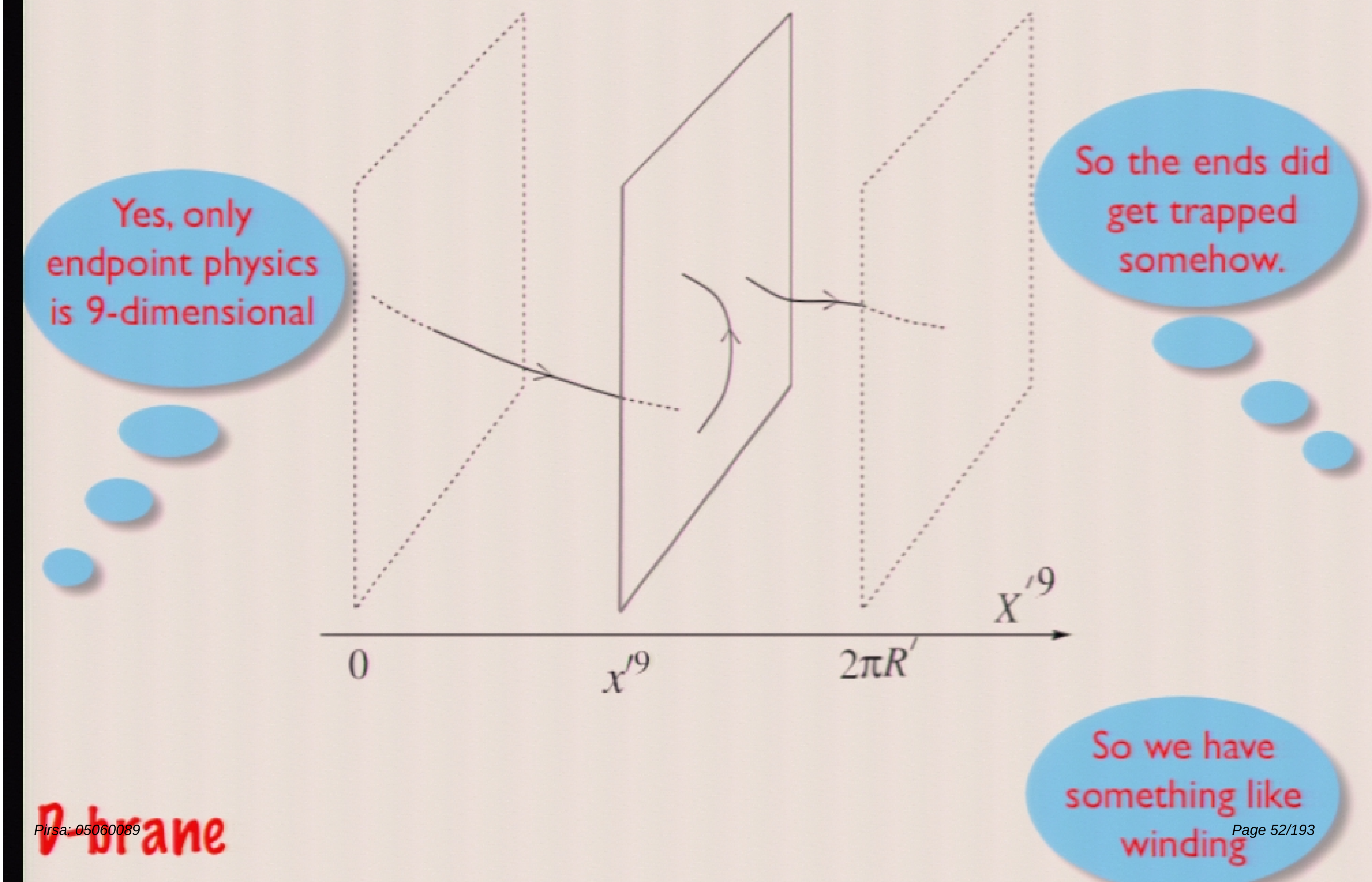

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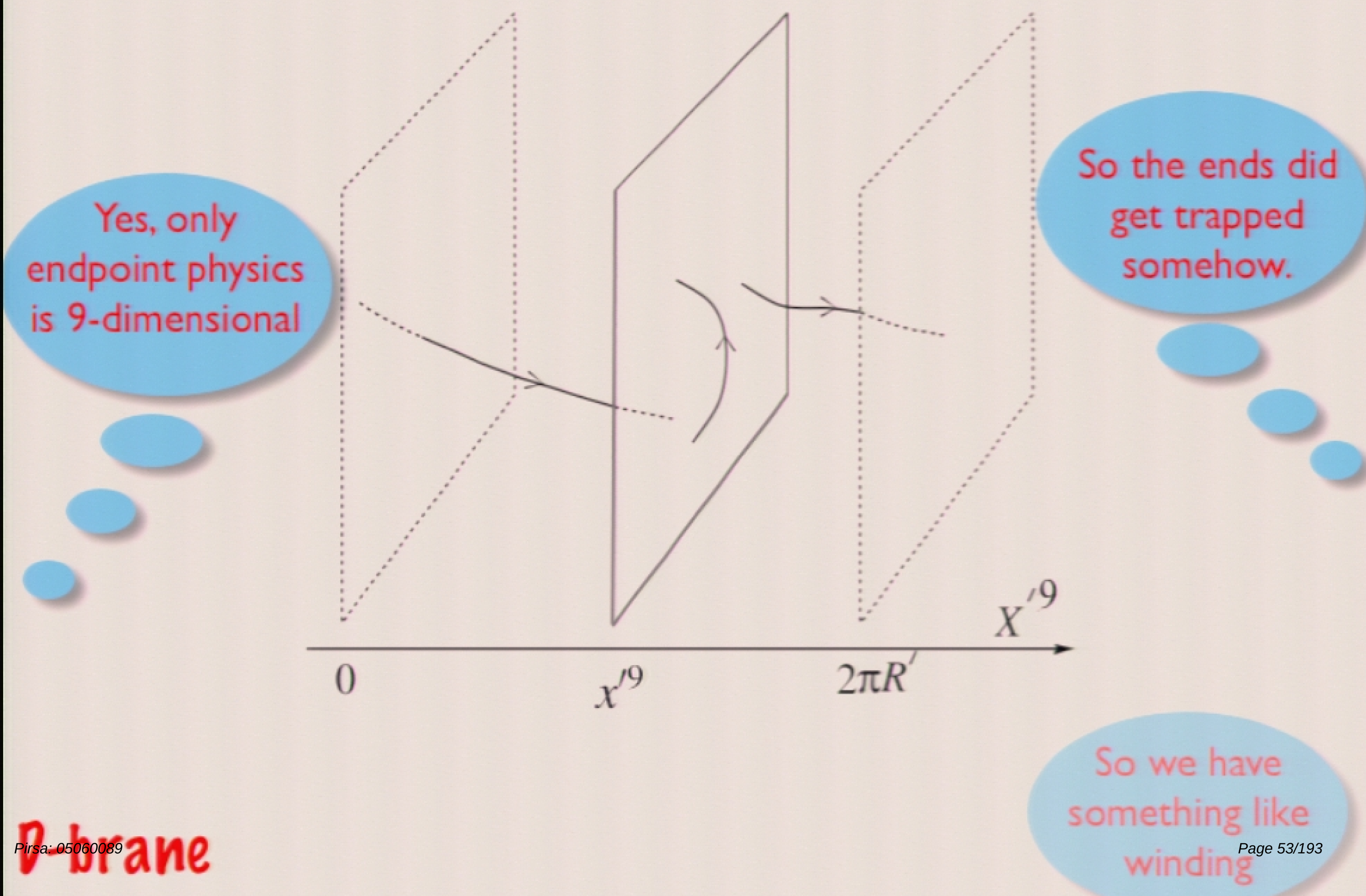

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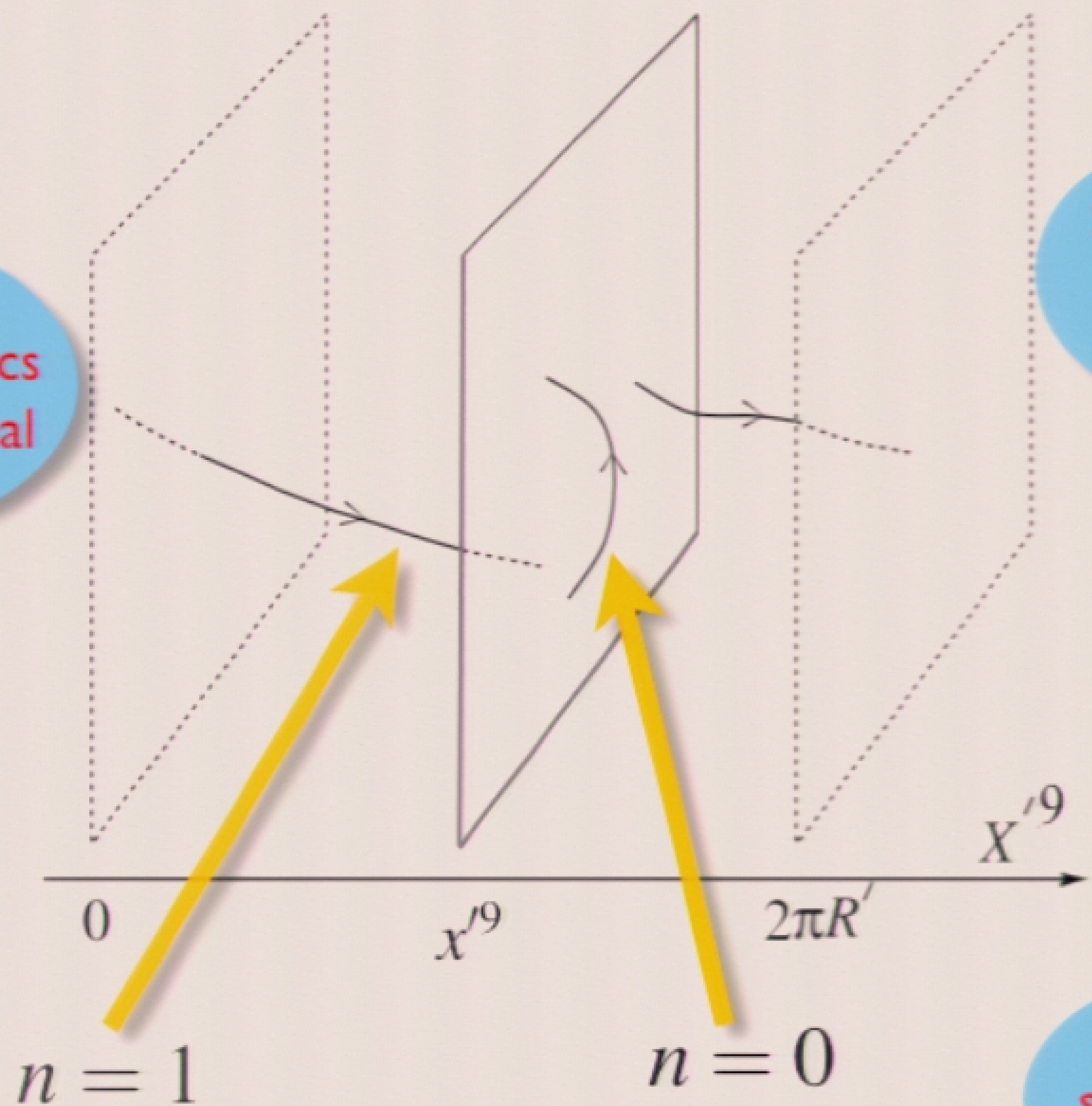

So the ends did get trapped somehow.

So we have something like winding

D-brane

What of Chan-Paton Factors?

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Oriented case first $U(N)$

Put theory on circle and pick Wilson line: $U(N) \to U(1)^N$

$$A_9 = \mathrm{diag}\{\theta_1, \theta_2, \ldots, \theta_N\}/2\pi R$$

This is pure gauge locally: $A_9 = i\Lambda^{-1}\dfrac{\partial\Lambda}{\partial X^9}$

But as we go around circle, there'll be a phase:

$$\mathrm{diag}\left\{e^{-i\theta_1}, e^{-i\theta_2}, \ldots, e^{-i\theta_N}\right\}$$

since a charge couples to give action: $W = \exp\left(iq\int dX^9 A_9\right)$

What of Chan-Paton Factors?

So canonical momentum gets shifted: $p^9 = \frac{n}{R} + \frac{q\theta}{2\pi R}$

String endpoint charges normalised as ± 1

$$X'^9 = \theta_i R = 2\pi\alpha'(A_9)_{ii}$$

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So $|ij\rangle$ has $p^9 = (2\pi n + \theta_j - \theta_i)/2\pi R$

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Multiple
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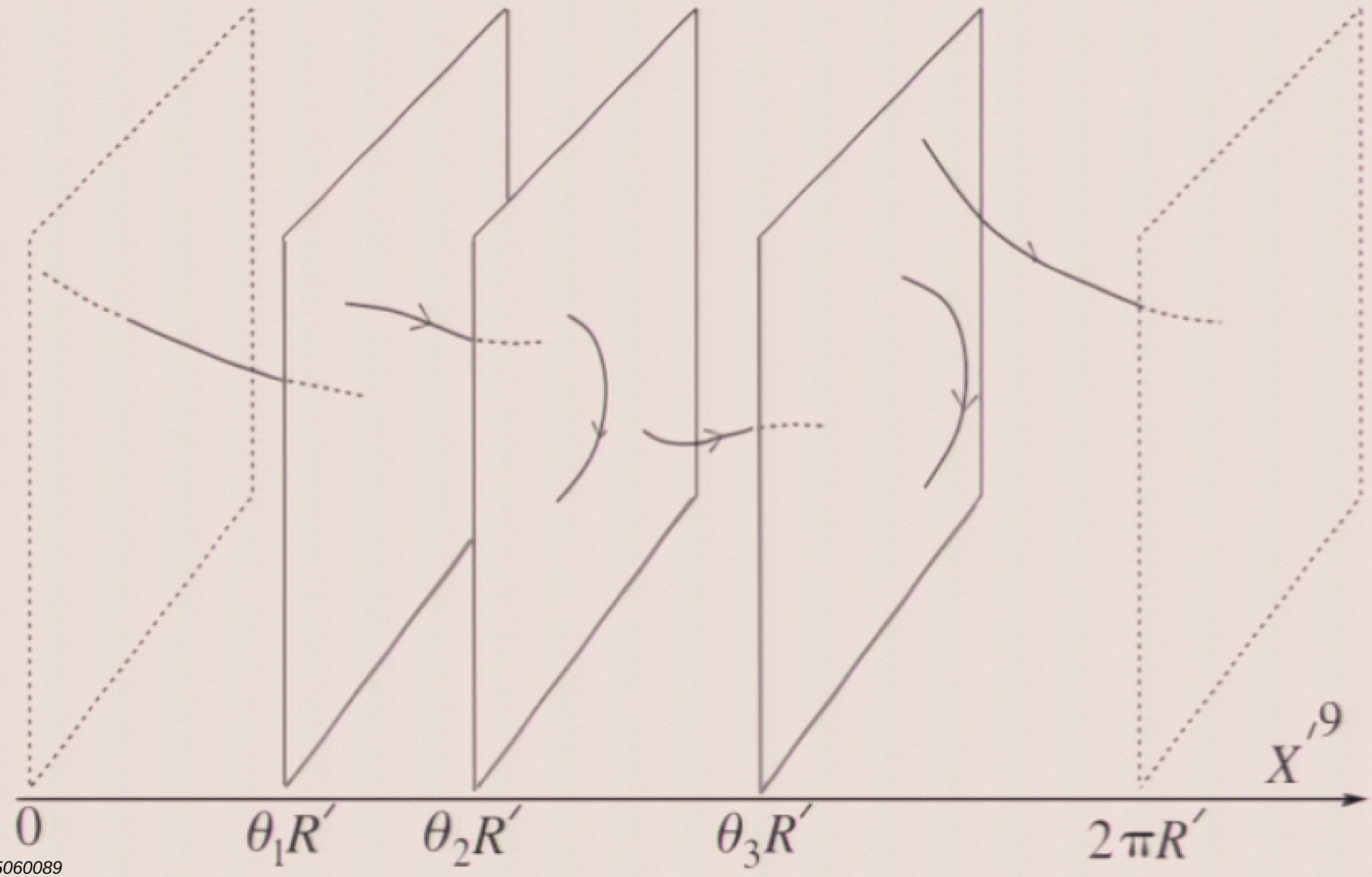

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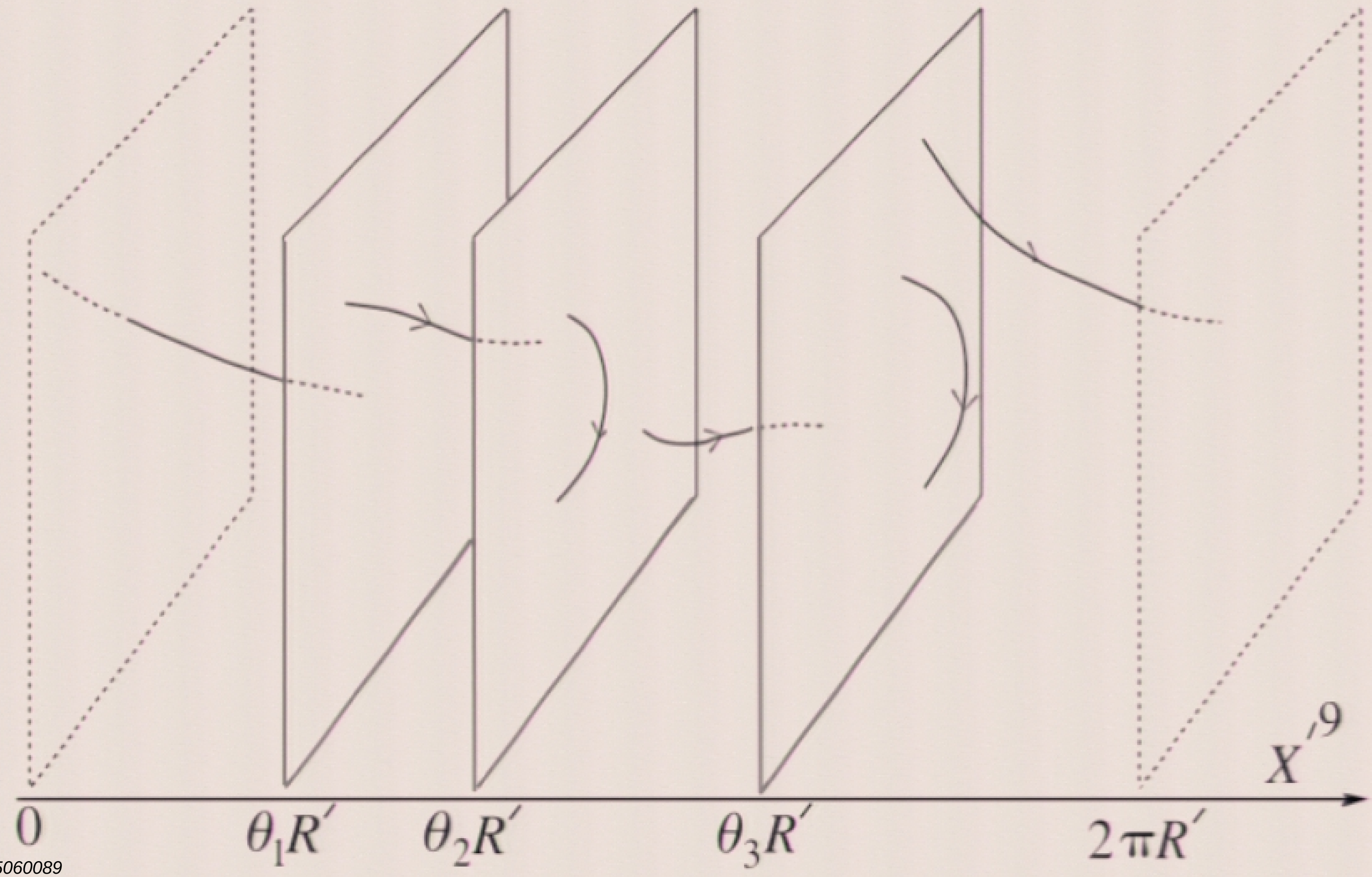


D8-brane's position is T-dual to gauge fields in compact direction!

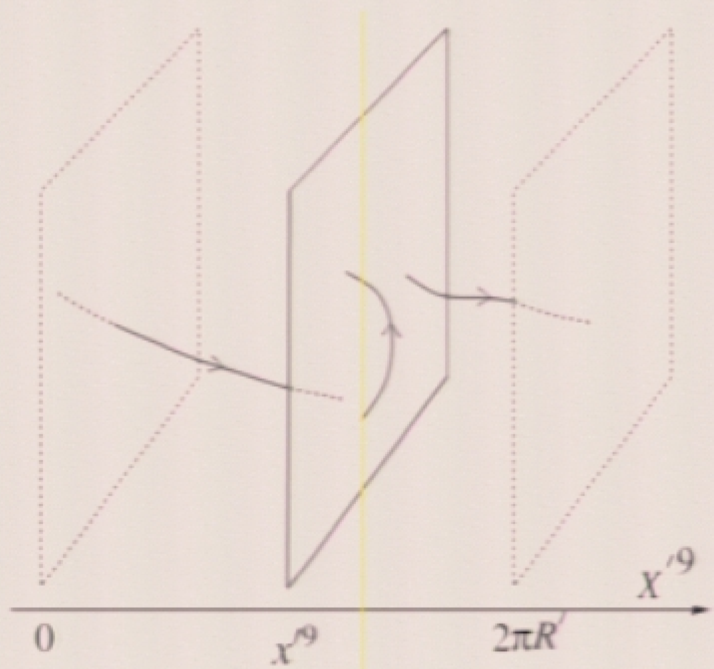
Collective Dynamics of a D-brane

Lorentz on "world-volume":

$$SO(1,8) \subset SO(1,9)$$

$$M^2 = (p^{25})^2 + \frac{1}{\alpha'}(N-1)$$

$$= \left(\frac{[2\pi n + (\theta_i - \theta_j)]R'}{2\pi\alpha'}\right)^2 + \frac{1}{\alpha'}(N-1)$$



$\psi^{\mu}_{-\frac{1}{2}}|ii,k>$ $\qquad A^{\mu}(x^{\mu})$

$\psi^{9}_{-\frac{1}{2}}|ii,k>$ $\qquad A^{9}(x^{\mu}) \equiv \phi(x^{\mu})$

$\mu = 0, 1, \ldots, 8$

Gauge theory with scalar on brane's "world-volume"

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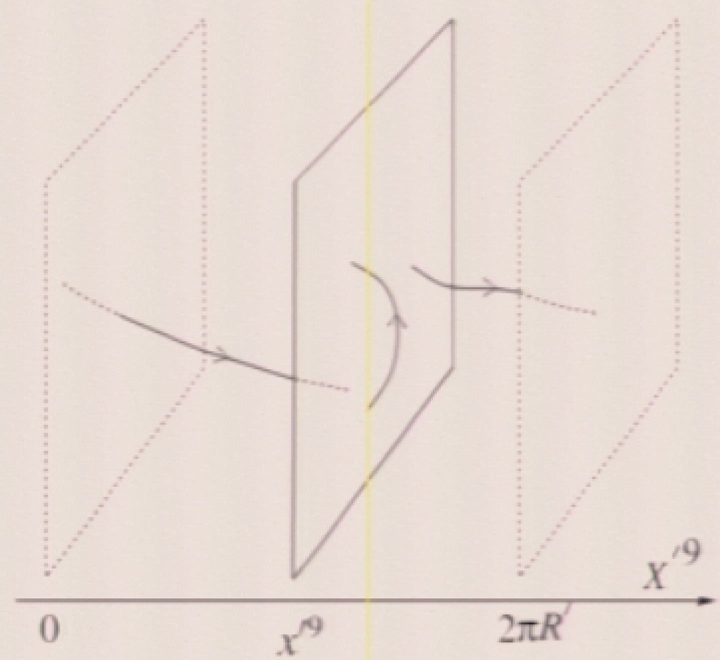

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$\mu = 0,1,\ldots,8$ Gauge theory with scalar on brane's "world-volume"

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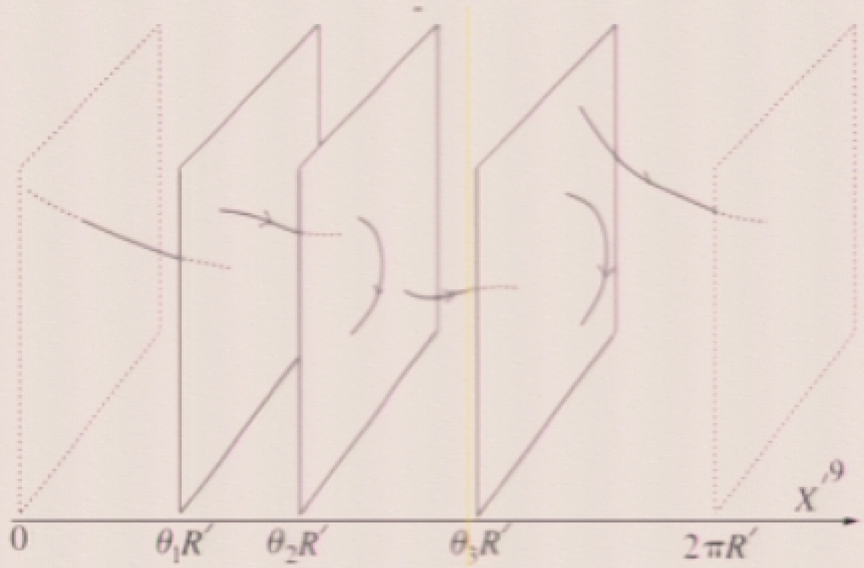

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$U(n)$ gauge theory with adjoint scalar on n coincident brane's "world-volume"

$\mu = 0, 1, \ldots, 8$

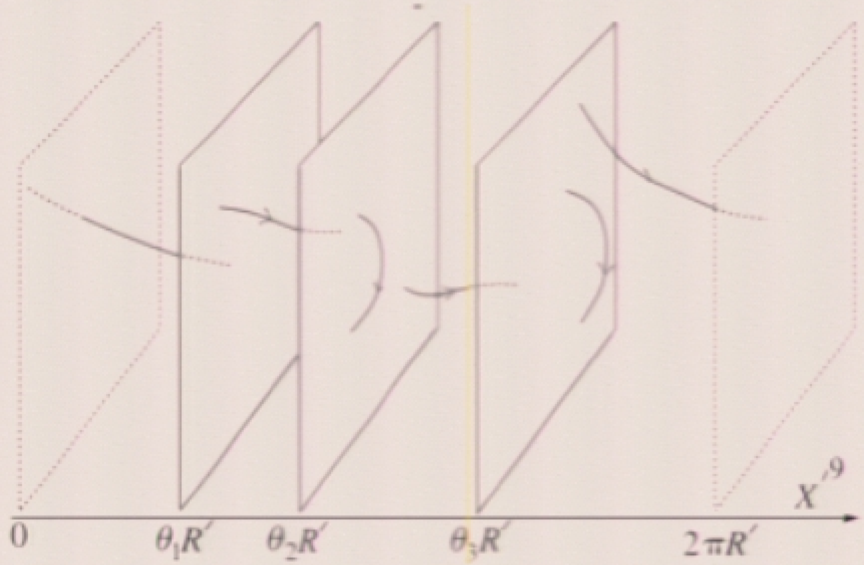
Collective Dynamics of Several D-branes

Keep looking at mass formula:

$$M^2 = (p^{25})^2 + \frac{1}{\alpha'}(N-1)$$

$$= \left(\frac{[2\pi n + (\theta_i - \theta_j)]R'}{2\pi\alpha'}\right)^2 + \frac{1}{\alpha'}(N-1)$$

Get massless states also from strings stretching between branes when the branes coincide:



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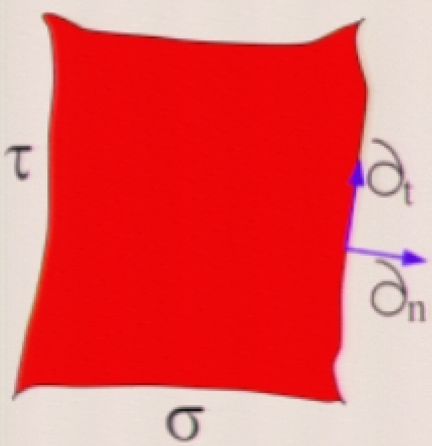

Dp-brane in D-dimensions

$$X^M(\sigma,\tau)\ ; \quad M = 0,\ldots,D-1$$

T-Duality and Unoriented Strings

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For closed strings, recall:

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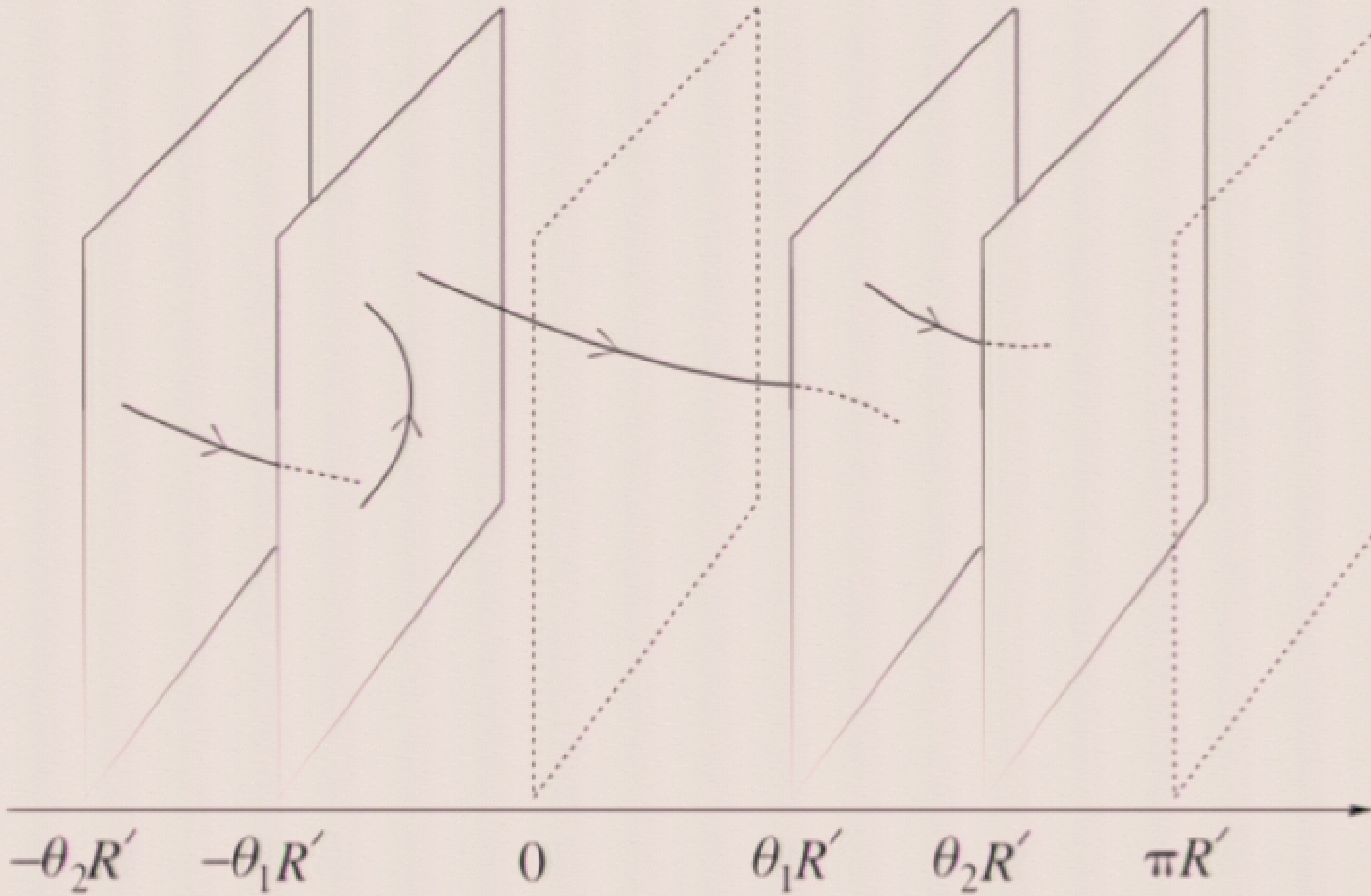
Action of Ω: exchange $X^\mu(z)$ and $X^\mu(\bar{z})$

$$X'^m(z,\bar{z}) \leftrightarrow -X'^m(\bar{z},z)$$

T-Duality and Unoriented Strings

Orientifold Plane:



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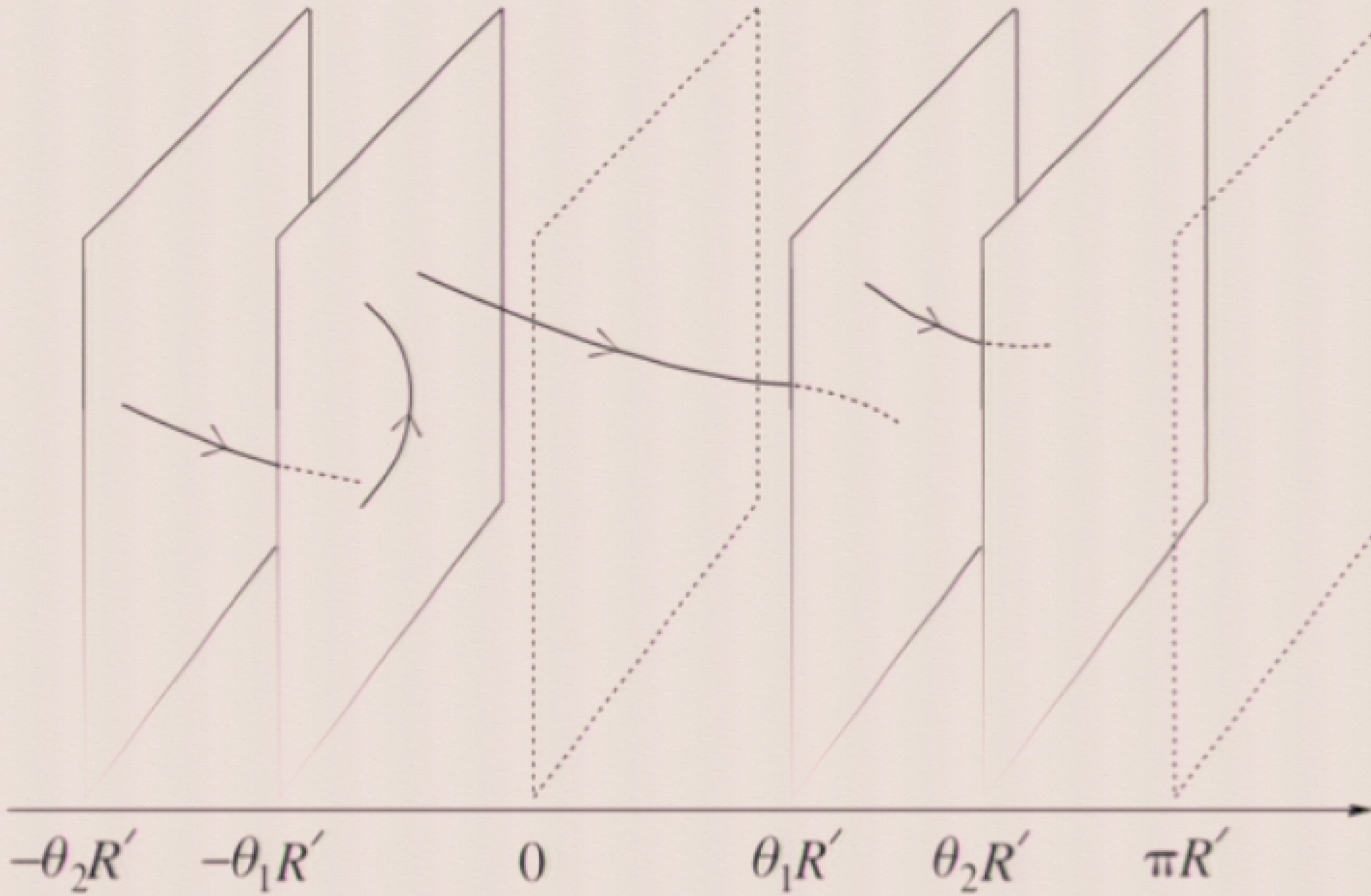

S^1/Z_2 orbifold

T-Duality and Unoriented Strings

This gives new view on what Type I string theory is:

16 D9-branes + Orientifold 9-Plane

D9-branes have a certain "10-form" charge. This is cancelled (satisfying Gauss' Law) using O9-plane which has 16 units of opposite charge.

Return to Supersymmetric Strings

Well, here they are again:

Name	O/C	Comments	Content (low energy massless fields)
			$G_{\mu\nu}, B_{\mu\nu}, \Phi$
Type IIA	(closed)	Very (super) symmetric (*N*=2)	$C_{\mu}\ , C_{\mu\nu\kappa}\ , C_{\mu\nu\kappa\sigma\rho}$
Type IIB	(closed)	Very (super) symmetric (*N*=2)	$C\ , C_{\mu\nu}\ , C_{\mu\nu\kappa\sigma}$
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Heterotic (E8xE8)	(closed)	Less (super) symmetric (*N*=1)	
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			A_{μ}
Type I (SO(32))	(open)	Less (super) symmetric (*N*=1)	$G_{\mu\nu}, C_{\mu\nu}, \Phi \quad A_{\mu}$

Supersymmetric Strings

The fields in black are from the "NS-NS" sector

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The fields in red are from the "R-R" sector

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The language describes combining different types of left and right

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Notice that they all have a metric, two-index antisymmetric tensor, and a dilaton.

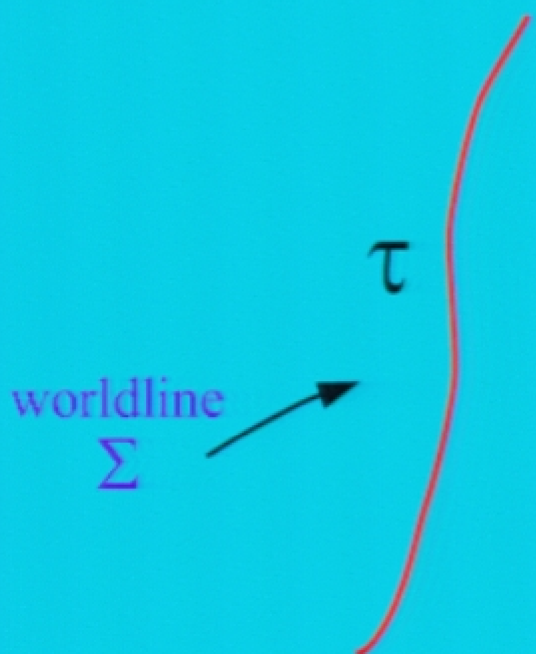
Why is there always that Multiplet?

Whenever there's a closed string, there'll always be that multiplet.

Why is there always that Multiplet?

B is there to give dynamical weight to the string's ability to wind.

Particle



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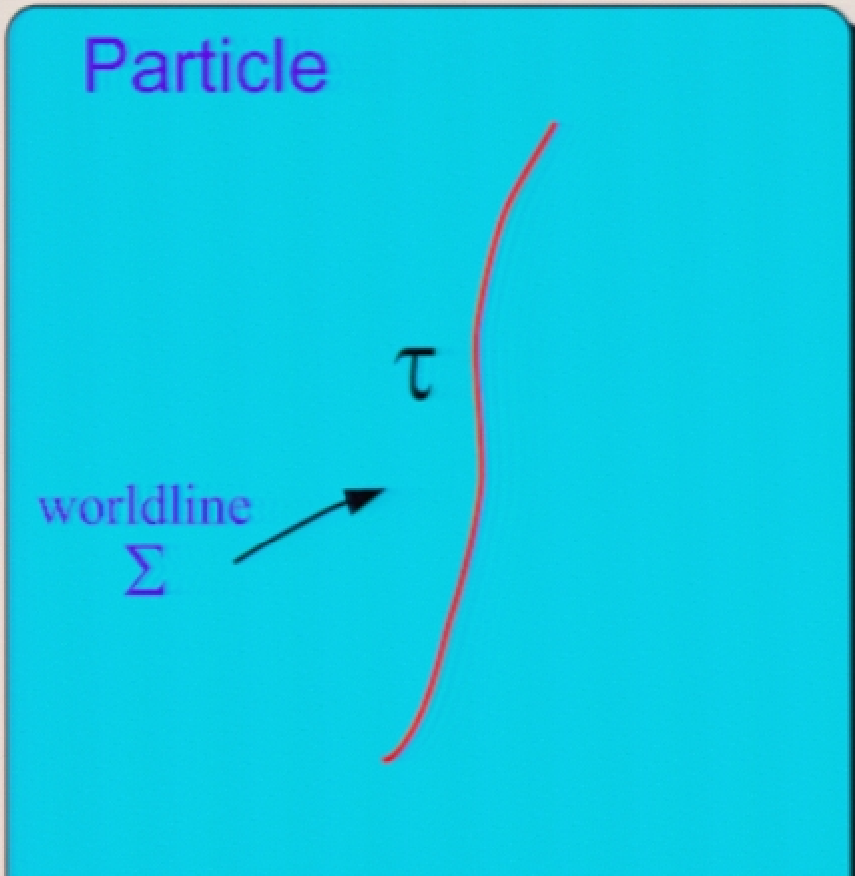

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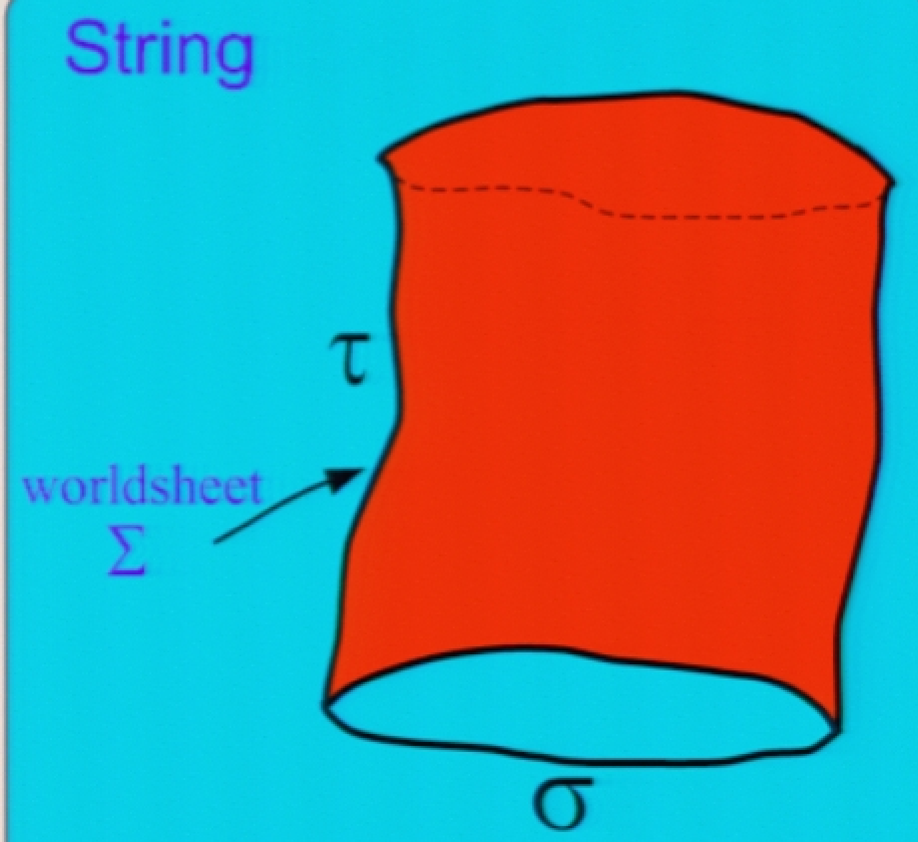

$$\int_\Sigma B = \int d^2\sigma \, \varepsilon^{ab} \frac{\partial X^\mu}{\partial \sigma^a} \frac{\partial X^\nu}{\partial \sigma^b} B_{\mu\nu}$$

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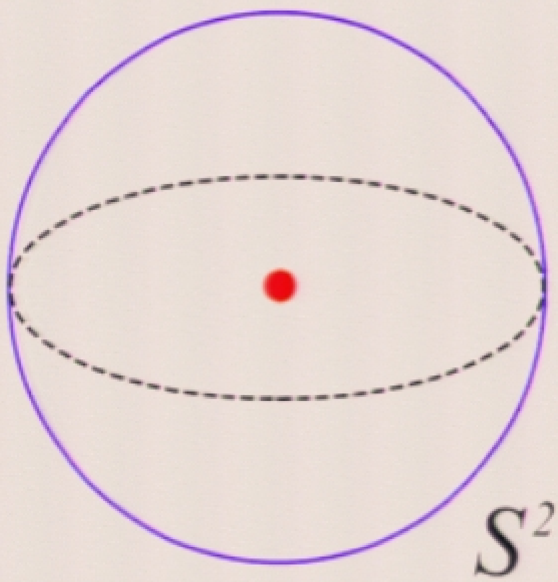
Measure the "A charge" carried by a particle in D=4 by surrounding it with a two-sphere....



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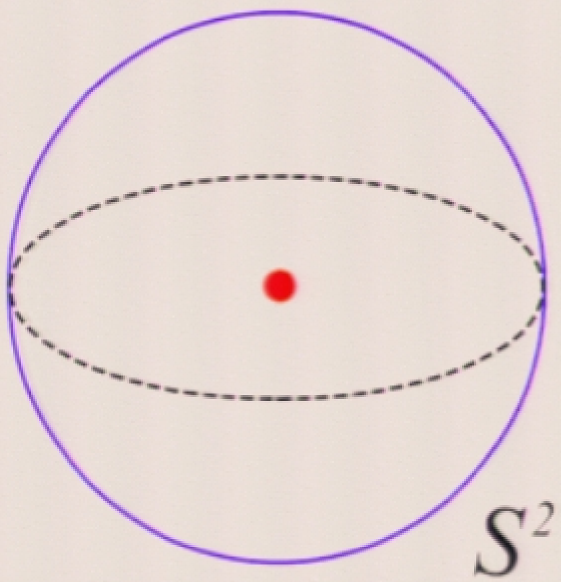

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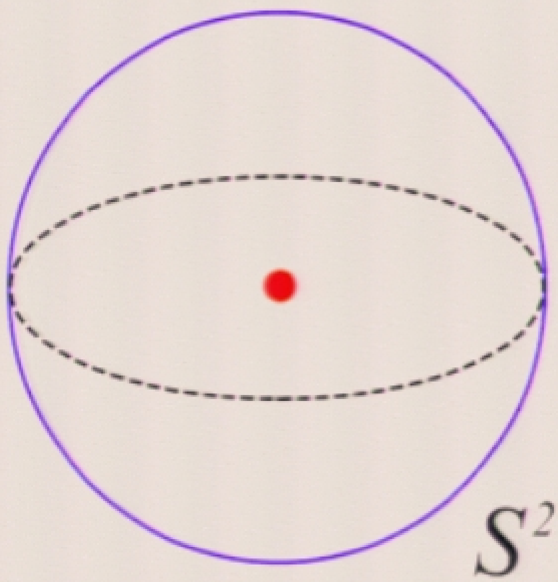

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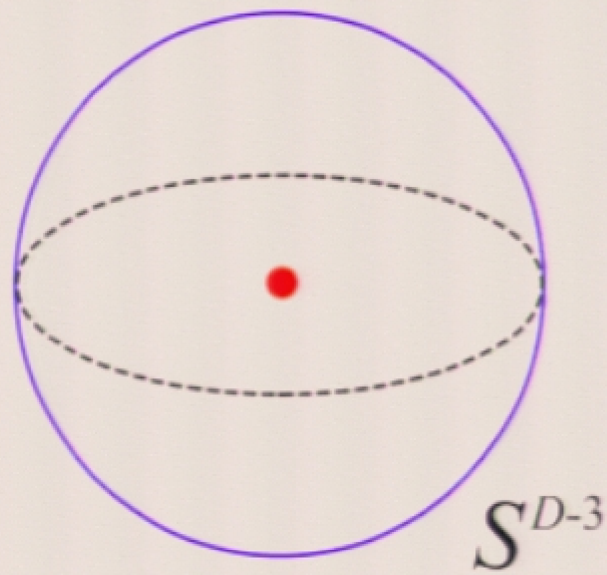
Measure the "B charge" carried by a string in D by surrounding it with a (D-3)-sphere....



$$H^{(3)} = dB^{(2)}$$

$$\widetilde{H}^{(D-3)} = {}^*H^{(3)}$$

$$\text{total charge} = \int_{S^{D-3}} \widetilde{H}^{(D-3)}$$

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Type I, Type IIA and Type IIB all have R-R sector, where higher rank forms arise....

Might there be fundamental charge carriers for them too?

Supersymmetric Strings

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Branes as Sources

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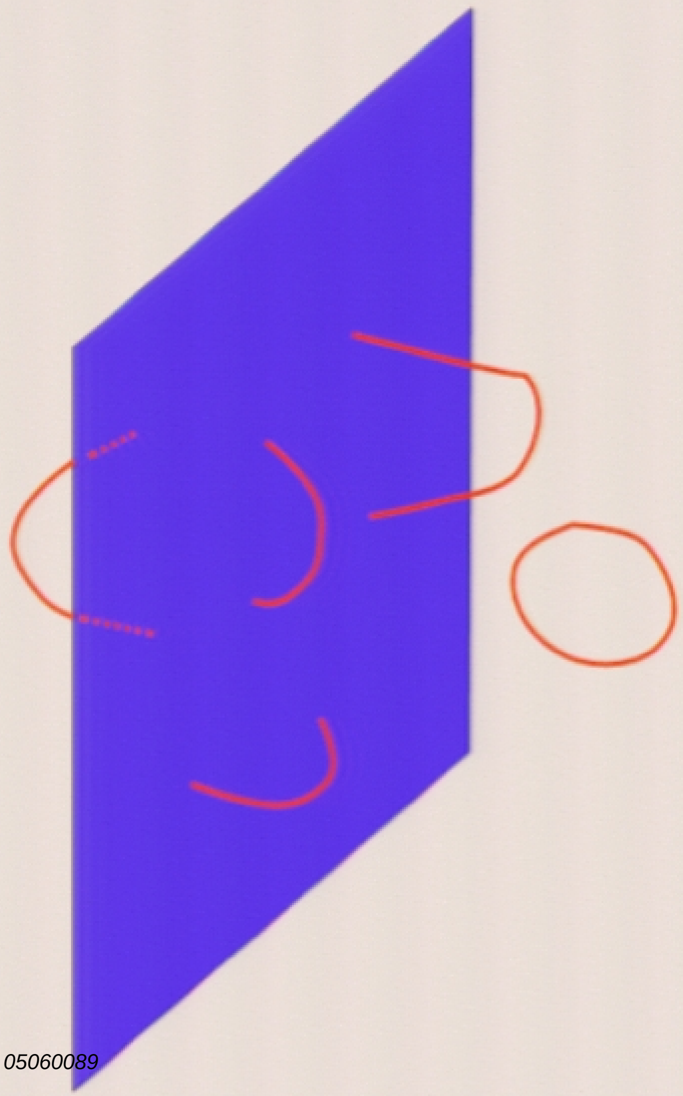
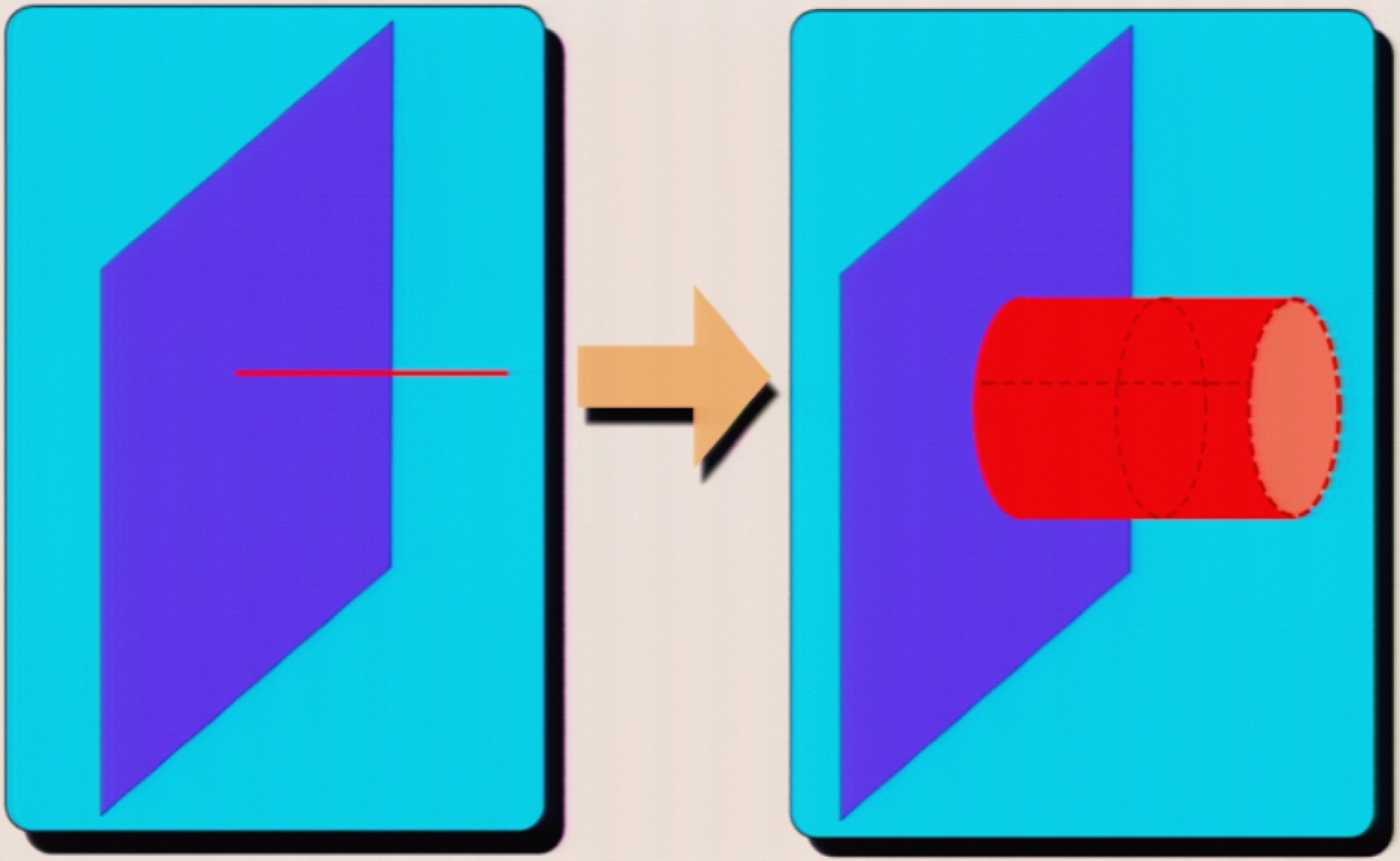
Single:



Within the open/closed description, one can describe the D-branes' natural sourcing of closed string fields including R-R sector!



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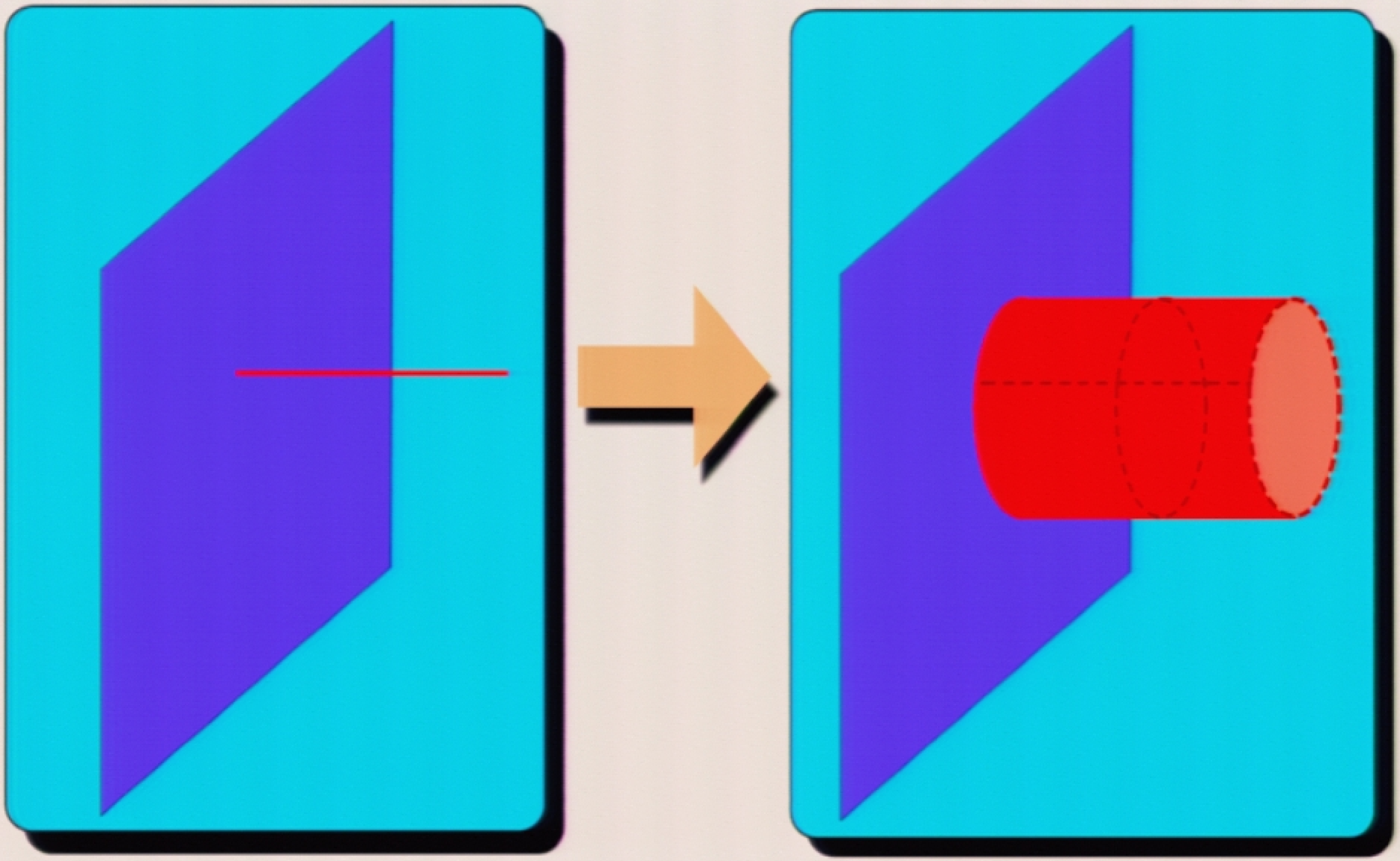

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NS-NS: → $G_{\mu\nu}, \Phi$

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So we have odd D-branes in Types IIB and I and even in Type IIA....

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	$G_{\mu\nu}, B_{\mu\nu}, \Phi$	IIA or IIB string itself couples to B
Type IIA :	C_μ , $C_{\mu\nu\kappa}$, $C_{\mu\nu\kappa\sigma\rho}$, $C_{\mu\nu\kappa\sigma\rho\lambda\gamma}$	D0, D2, D4,D6
Type IIB :	C , $C_{\mu\nu}$, $C_{\mu\nu\kappa\sigma}$, $C_{\mu\nu\kappa\sigma\rho\lambda\gamma}$, $C_{\mu\nu\kappa\sigma\rho\lambda\gamma\tau}$	D(-1), D1, D3, D5, D7
Type I :	$C_{\mu\nu}$, $C_{\mu\nu\kappa\sigma\rho\lambda\gamma}$	D1, D5
	$G_{\mu\nu}$ Φ A_μ	(ends of string couple to the SO(32) gauge field)

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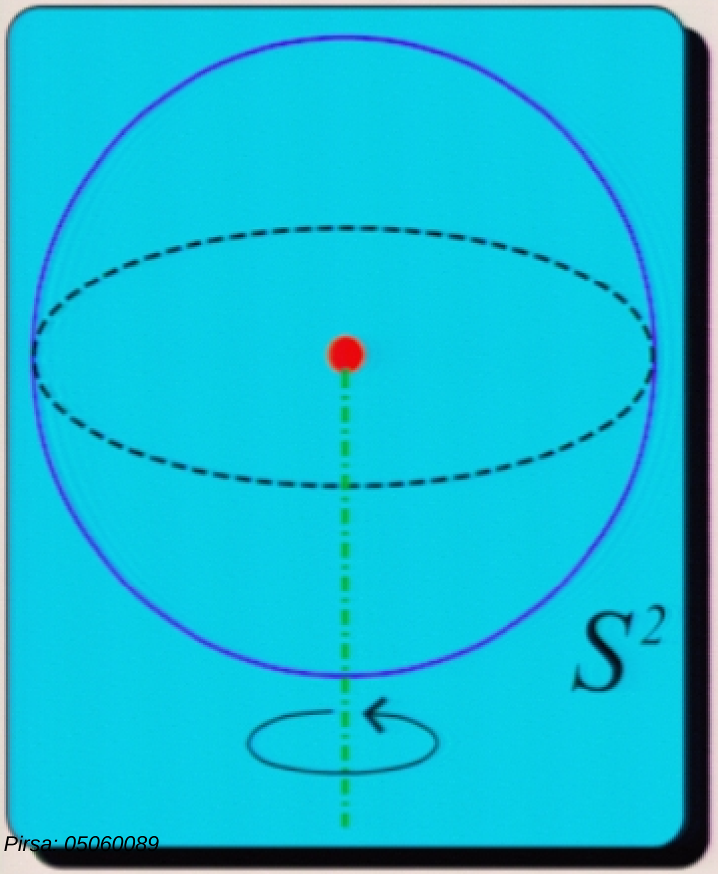

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$$F_{\mu\nu} \longleftrightarrow \tilde{F}_{\mu\nu}$$

electron $\longleftrightarrow$ monopole

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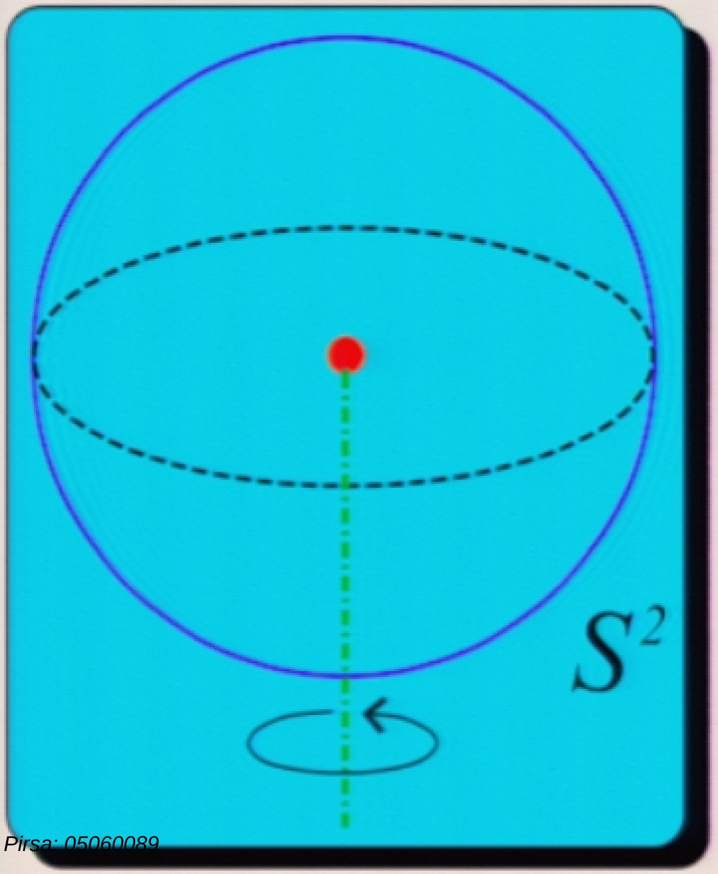

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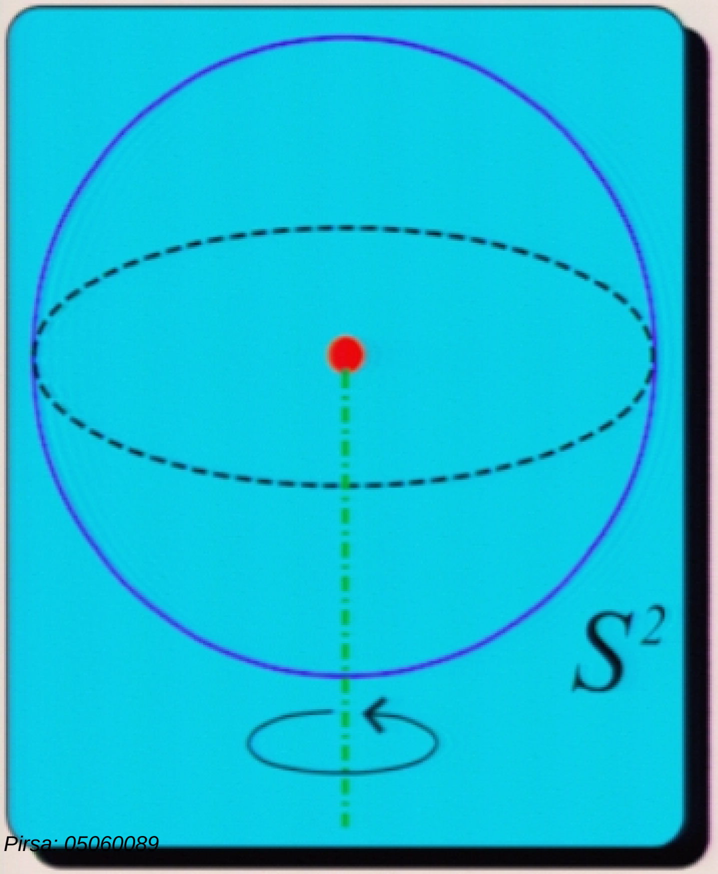

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For this "Dirac String" to be absent in the theory, we must therefore have....

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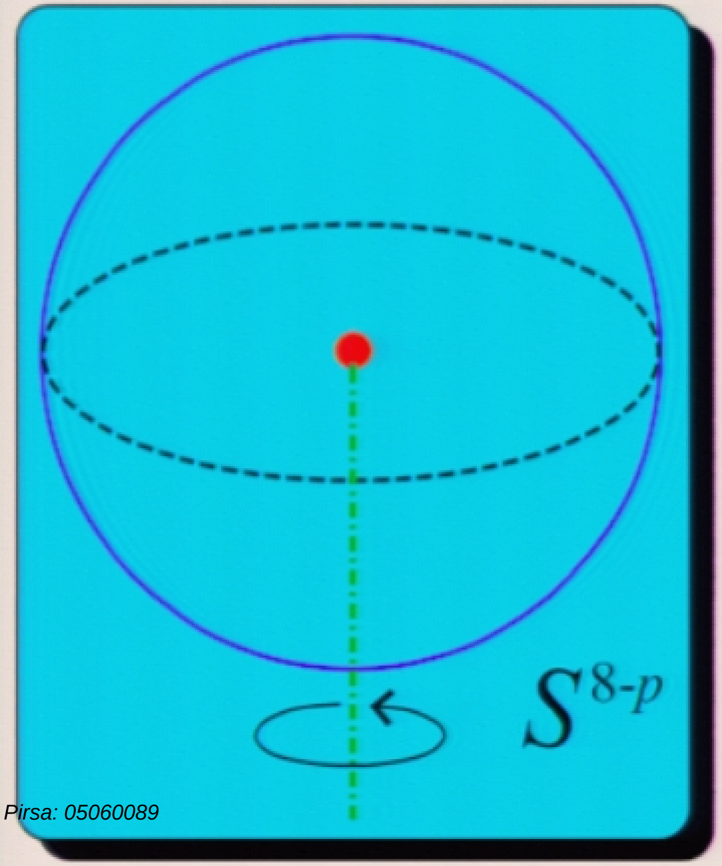

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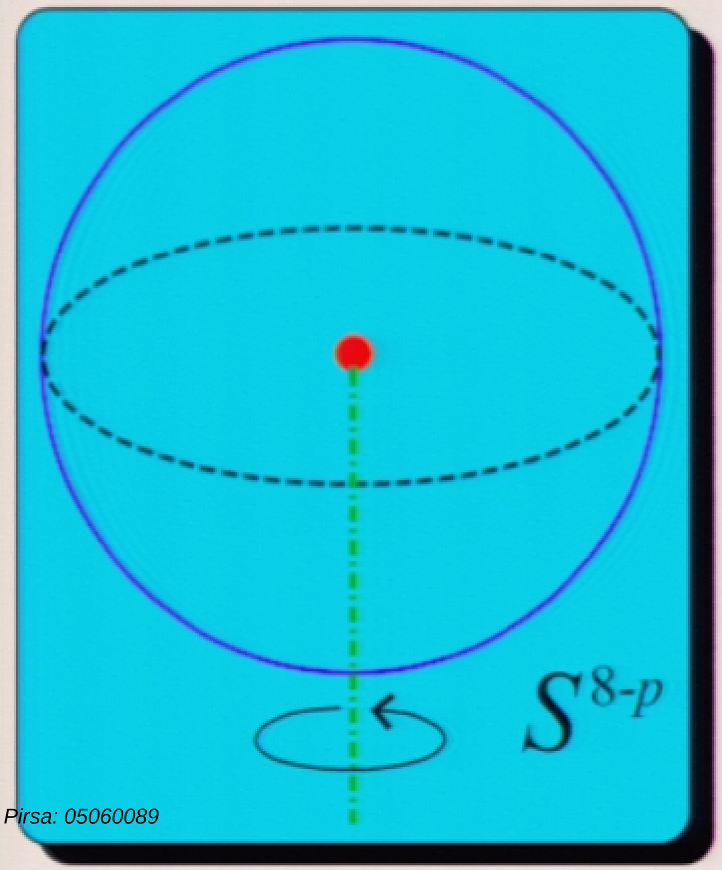
$$G^{(p+2)} = dC^{(p+1)} \longleftrightarrow G^{(8-p)} = dC^{(7-p)}$$

$$\mathrm{D}p\mathrm{-brane} \longleftrightarrow \mathrm{D}(6-p)\mathrm{-brane}$$



Constructing the monopole produces an infinite "Dirac Sheet"... *The Dp-brane potentially sees this string as a phase change of the wavefunction upon circulation...*

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Quantum Consistency

Dirac Quantisation

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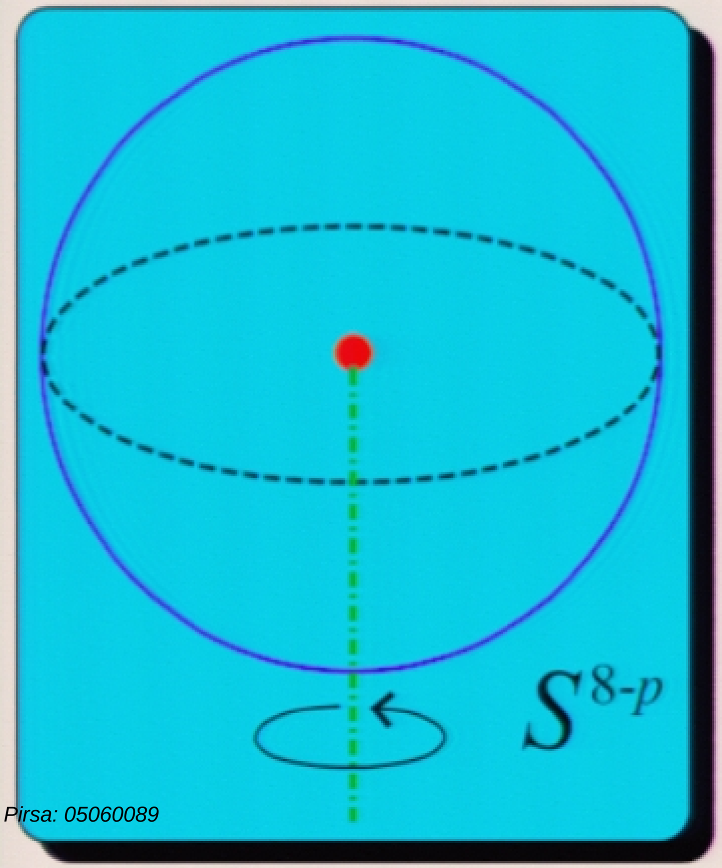

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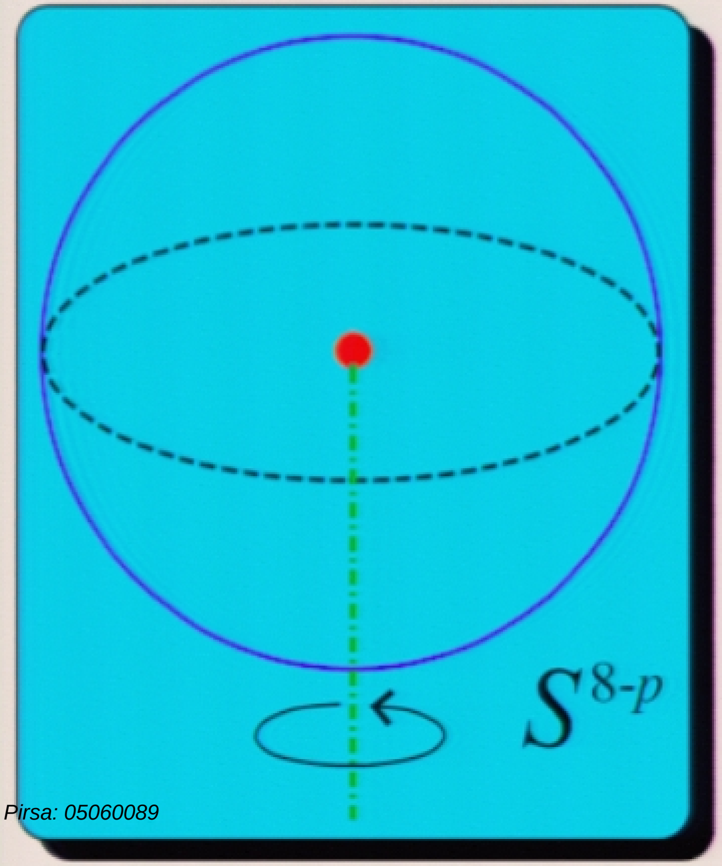

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For this "Dirac Sheet" to be absent in the theory, we must therefore have....

$$\mu_p \mu_{6-p} 2\kappa_0^2 = 2\pi n \longrightarrow \mu_p = \frac{2\pi n}{\mu_{6-p}} \frac{1}{2\kappa_0^2}$$

Quantum Consistency

Dirac Quantisation

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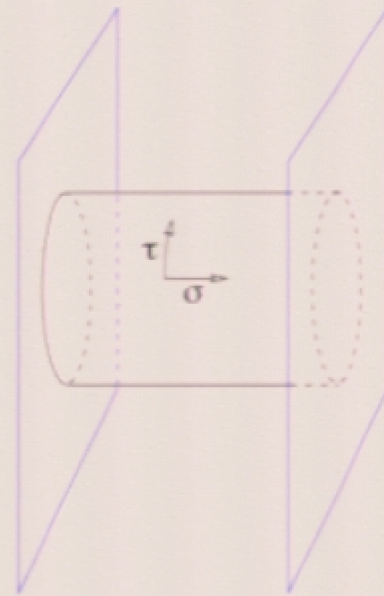

Quantum Consistency

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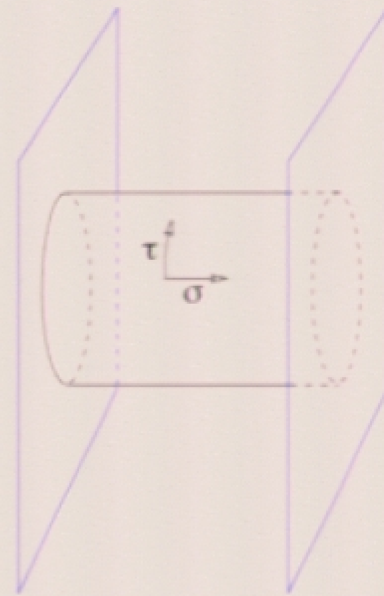

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Quantum Consistency

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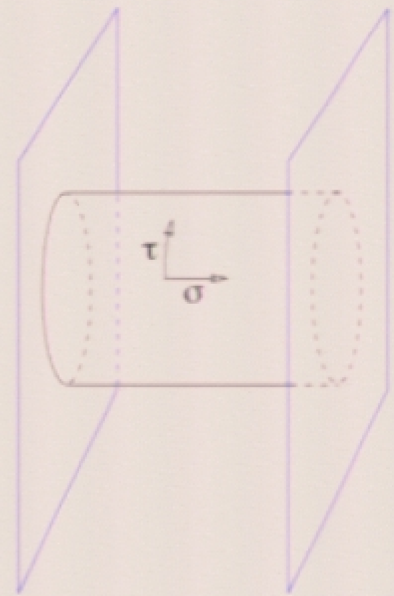

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R-R charge

$$\tau_p = g_s^{-1} \mu_p$$

Quantum Consistency

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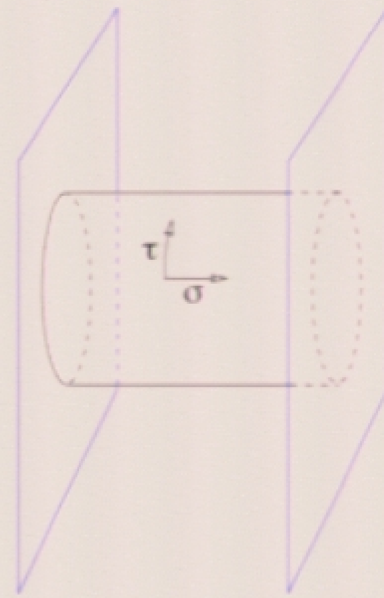
Compute the charges μ_p for the D-branes,



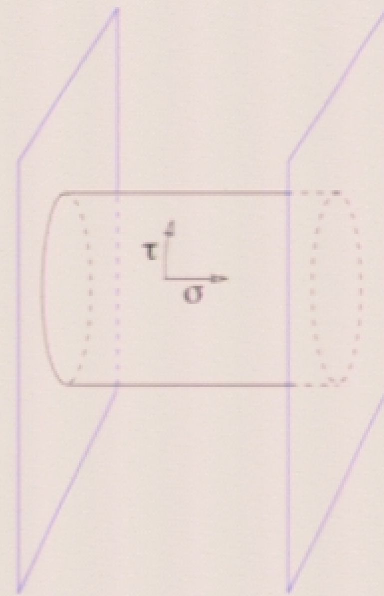
$$\mu_p = (2\pi)^{-p} \alpha'^{-\frac{(p+1)}{2}}$$ R-R charge

$$\tau_p = g_s^{-1} \mu_p$$ tension

and find that they satisfy the contraint with $n = 1$!!

Quantum Consistency

Dirac Quantisation



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$$2\kappa^2 \equiv 2\kappa_0^2 g_s^2 = 16\pi G_N = (2\pi)^7 \alpha'^4 g_s^2$$

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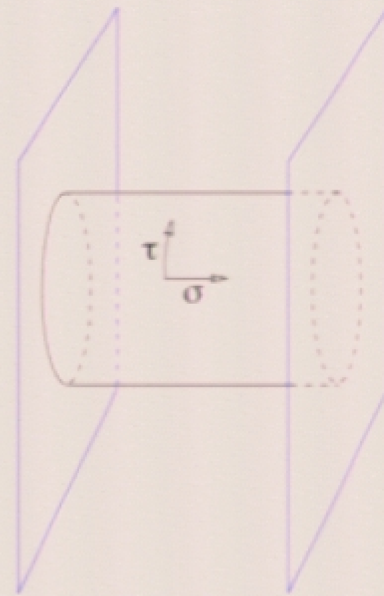
In other words, D-branes carry the smallest possible charges of the R-R sector.

D-Branes as BPS States



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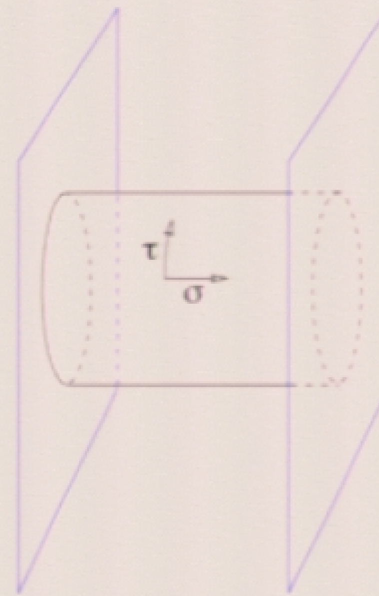
Polchinski '95

D-Branes as BPS States

Mass Equals Charge



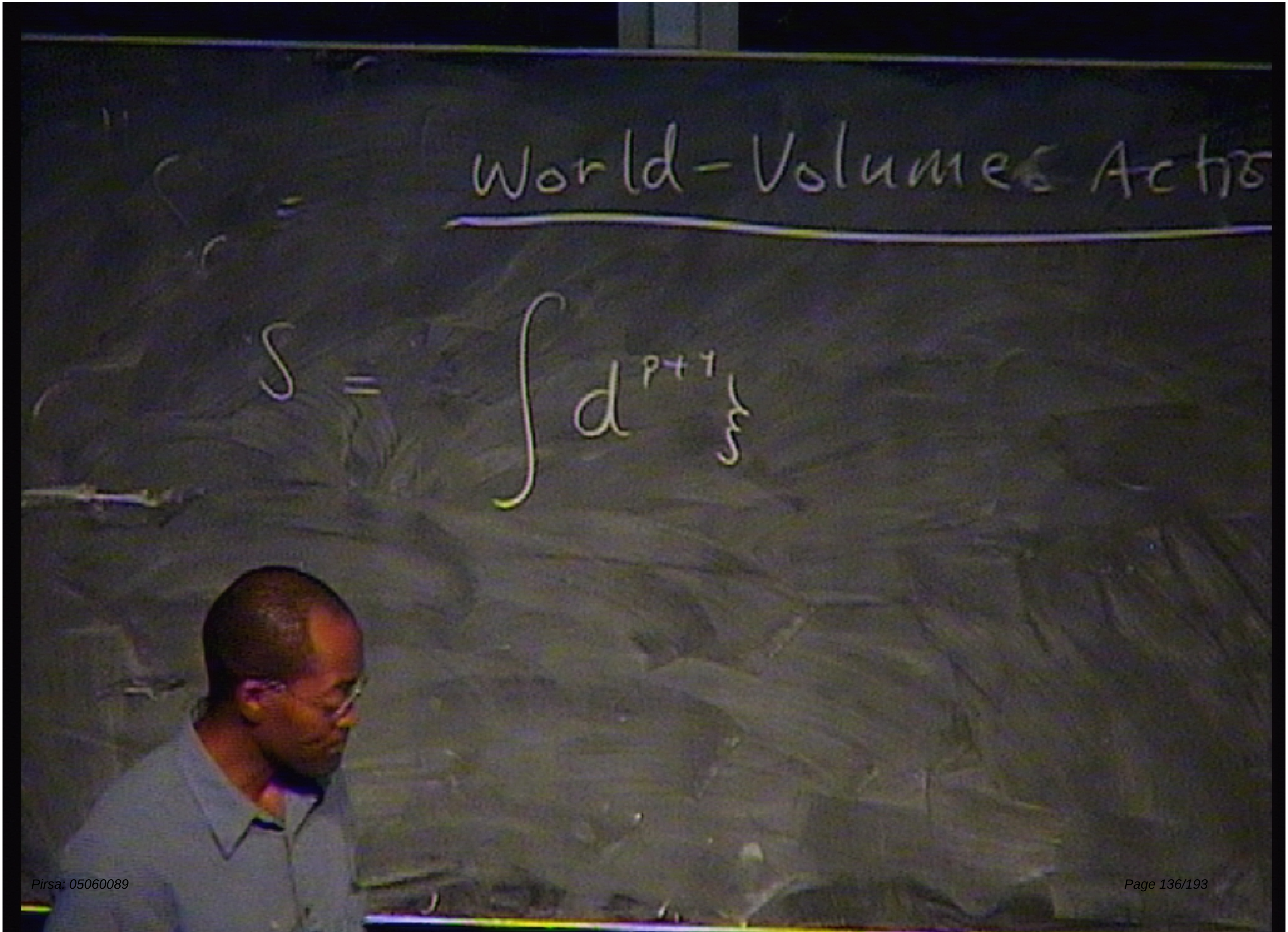
$$\mu_p = (2\pi)^{-p}\alpha'^{-\frac{(p+1)}{2}}$$ R-R charge

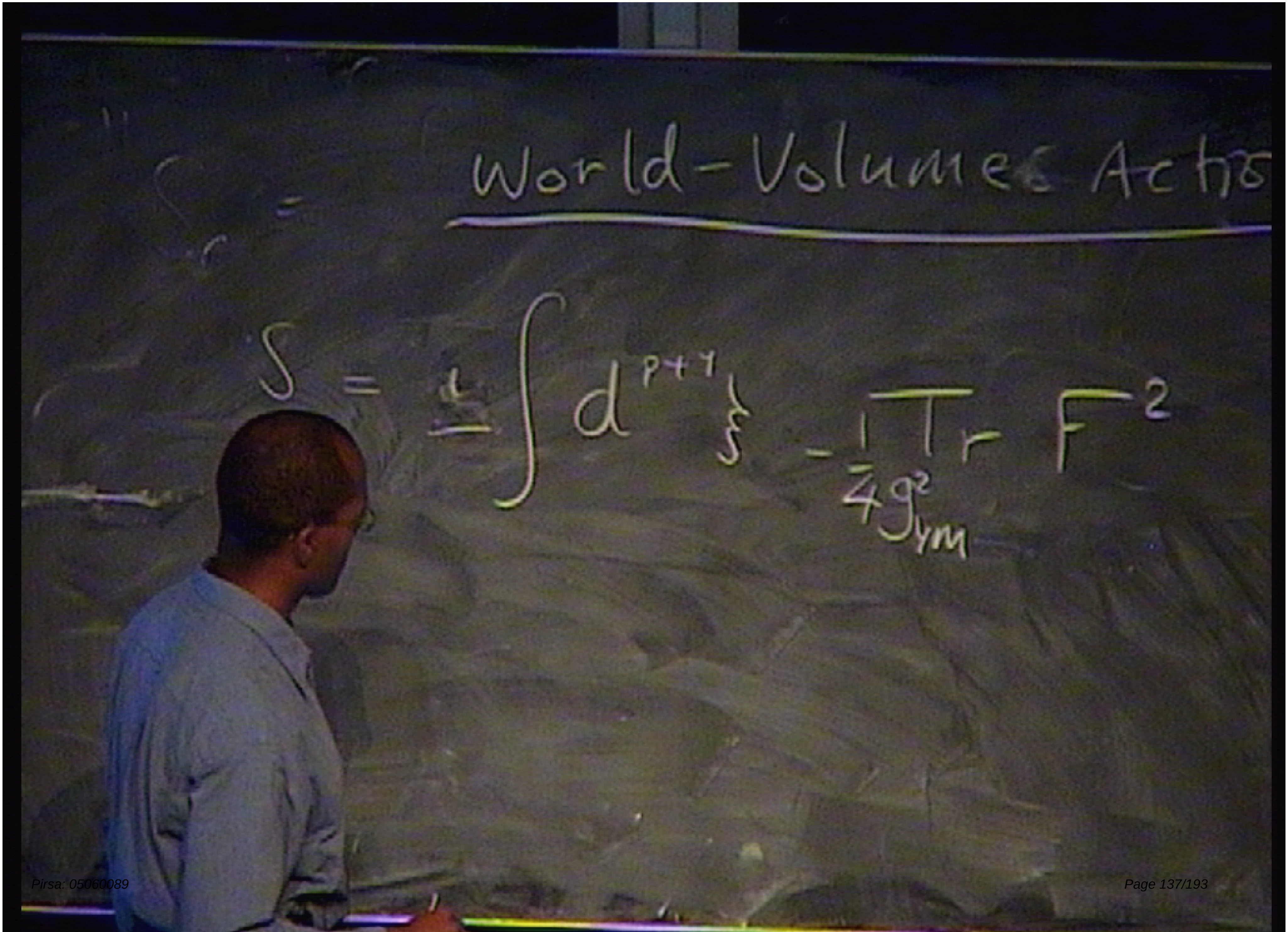
$$\tau_p = g_s^{-1}\mu_p$$ tension

D-branes saturate the Bogolmo'nyi-Prasad-Sommerfeld bound: $\tau_p \geq g_s^{-1}\mu_p$

BPS states are the lightest states for that given charge.
They cannot decay.
They do not interact with each other (attraction balances repulsion)
The perturbative computation of their mass-charge is exact.

Polchinski '95


World-Volumes Actio
$S = \int d^{p+1}\xi$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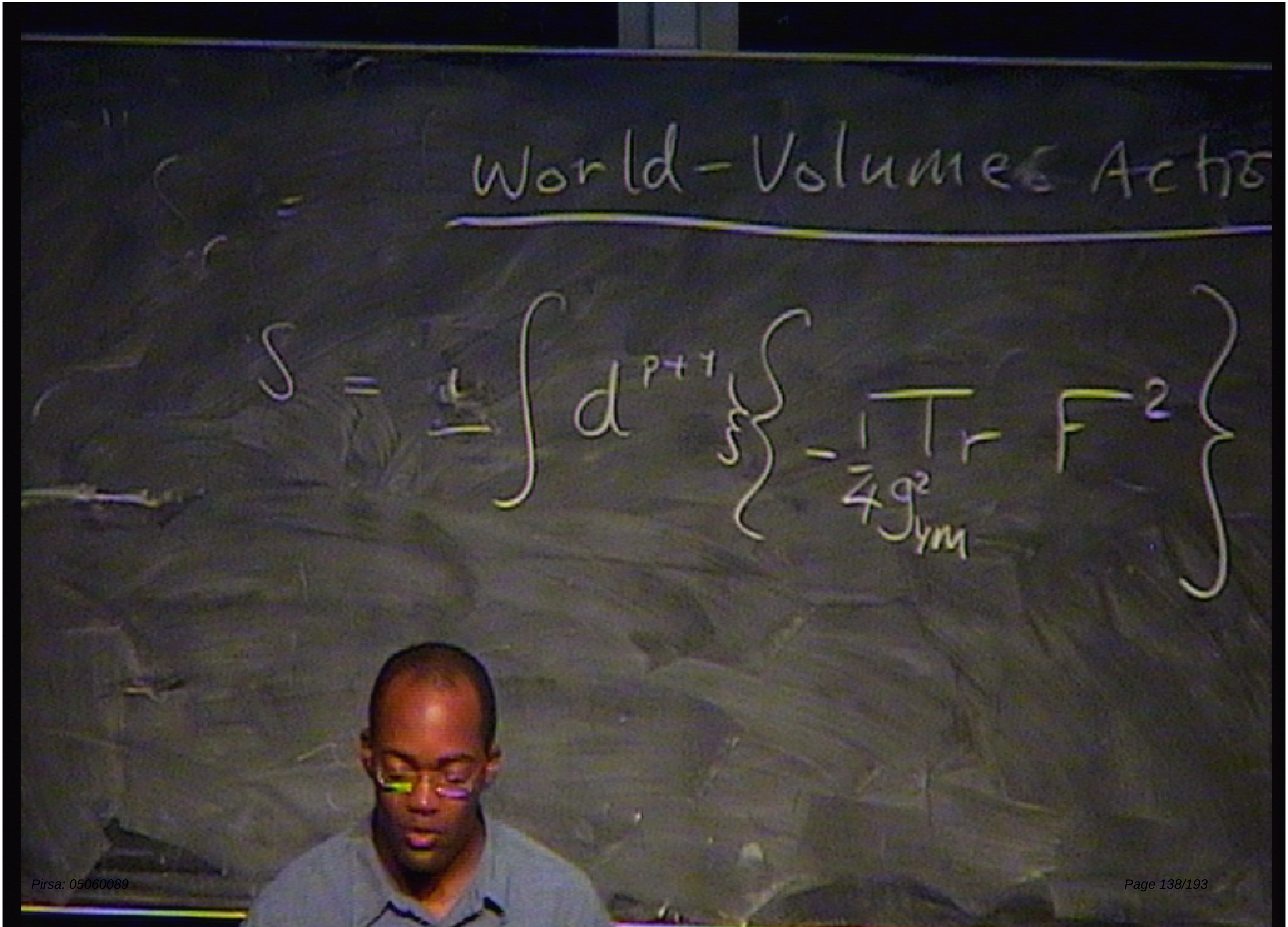
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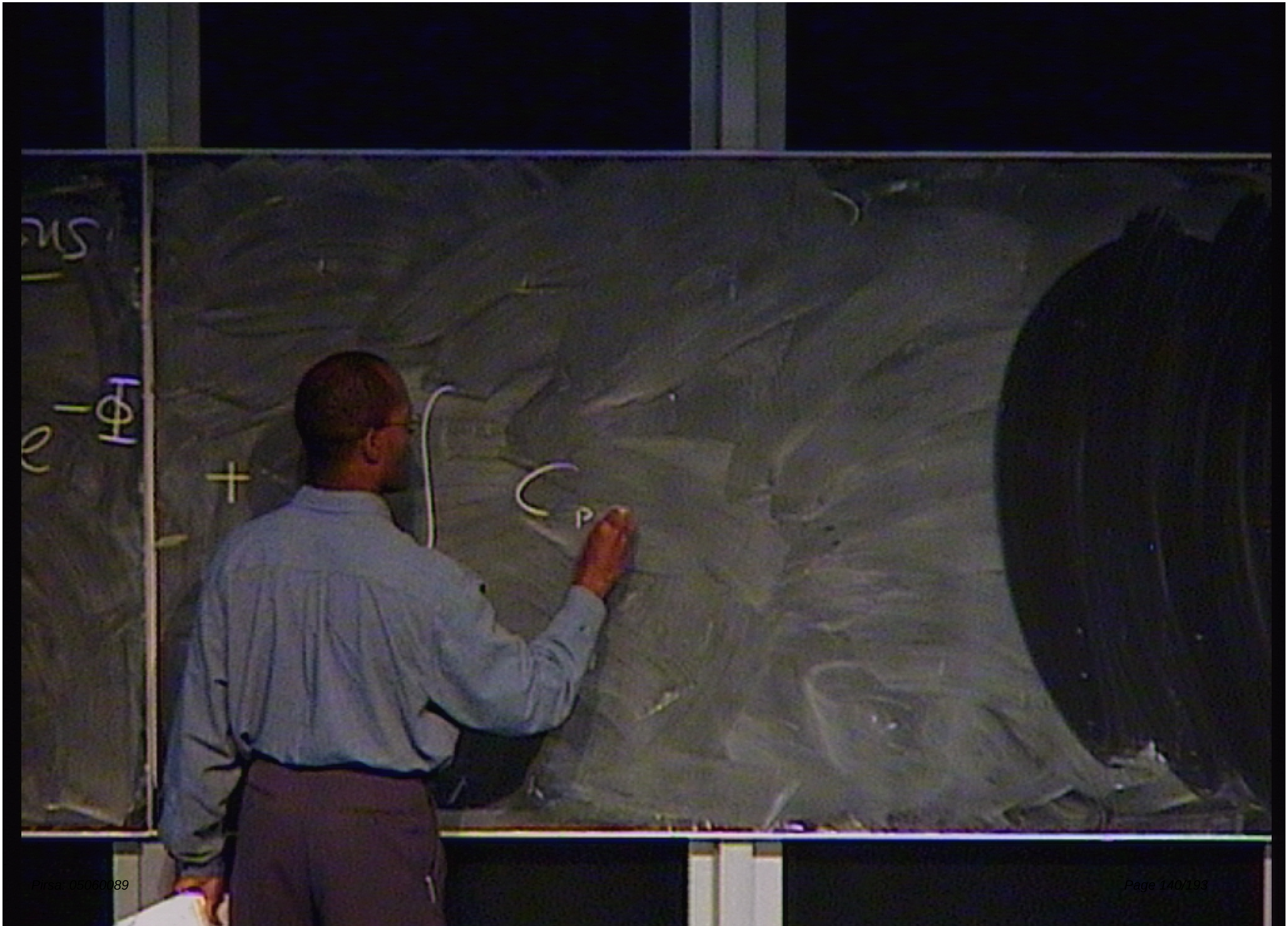
World-Volume Action

$$S = [illegible] \int d^{p+1}\xi \left\{ -\frac{1}{4g_{YM}^2} \mathrm{Tr}\, F^2 \right\} e$$



ons

$e^{-\Phi}$

$$+ \mu_p \int C_{(p+1)}$$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+ μp ∫ C(p+1)

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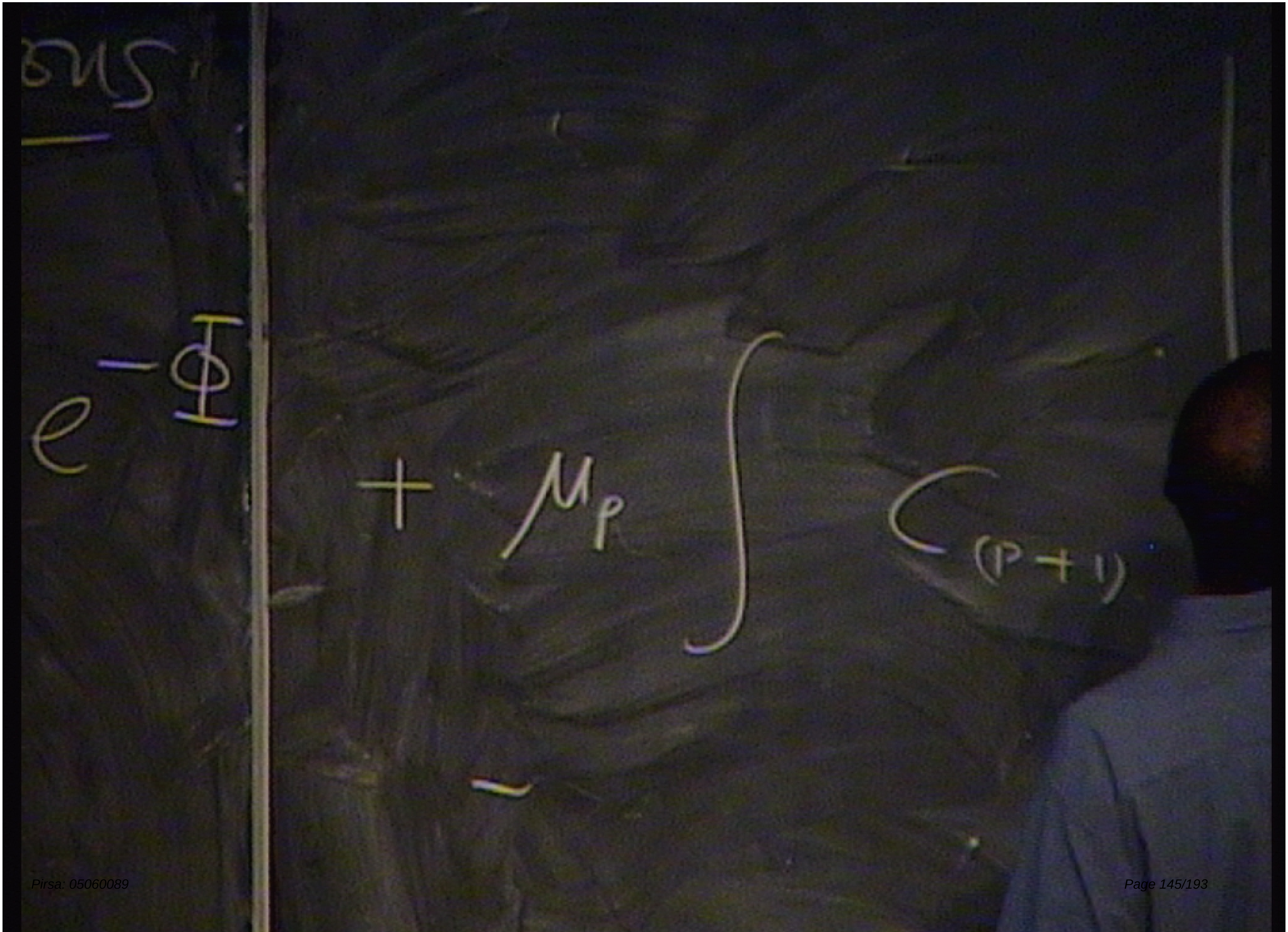
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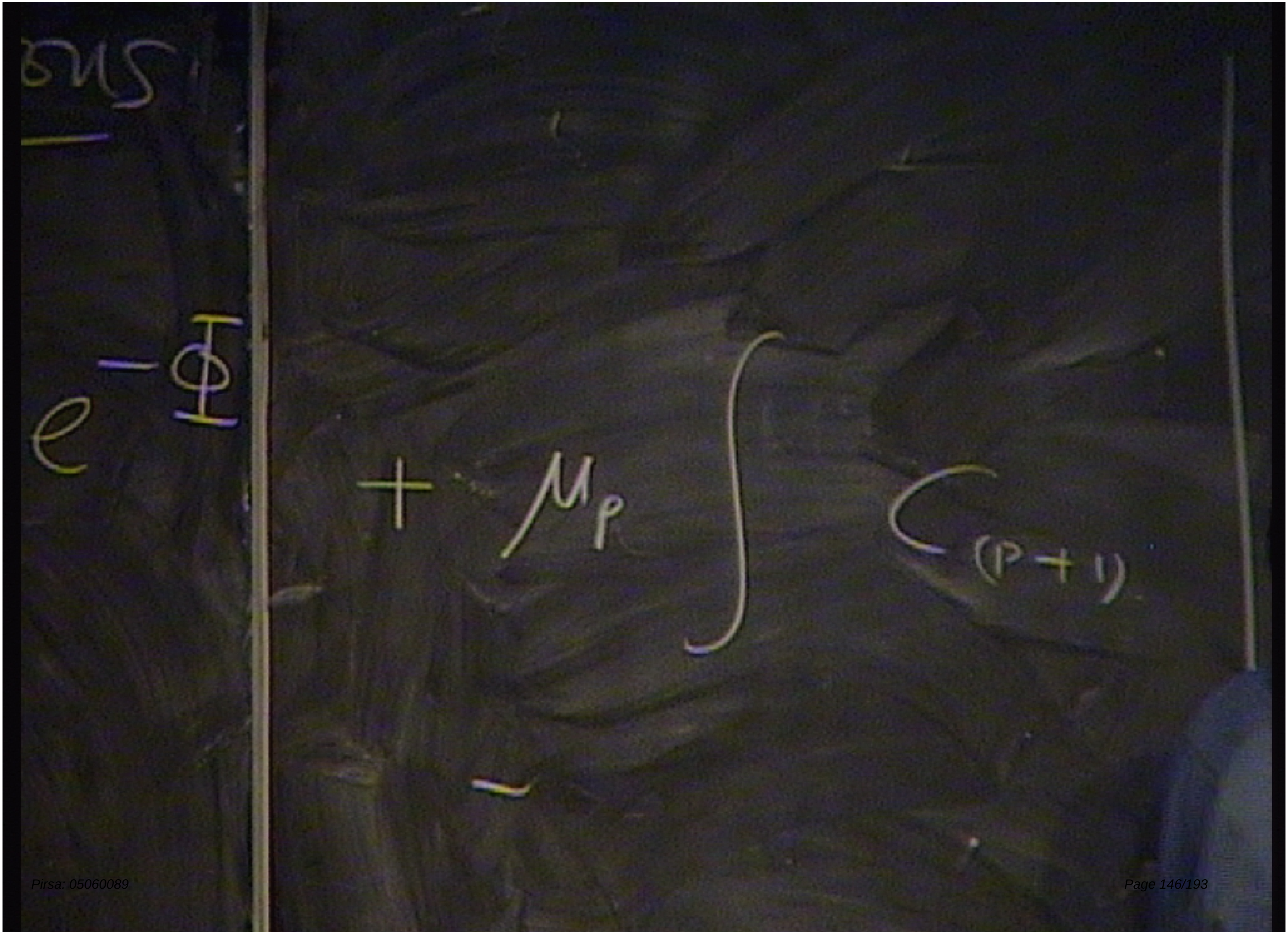
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$$S = \tau_p \int d^{p+1}\xi \, (\det G_{ab})^{1/2}$$

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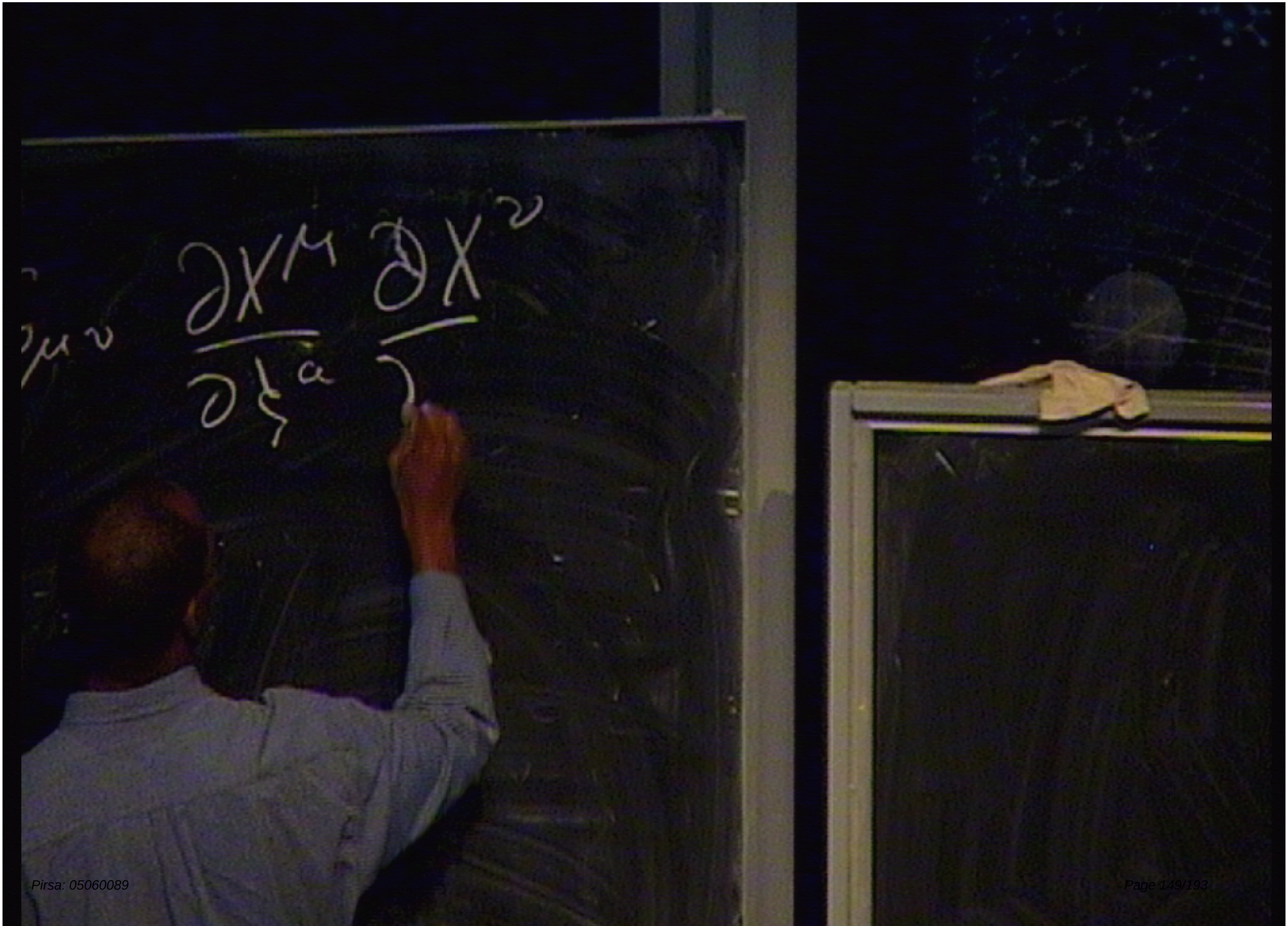
$g_{\mu\nu} \frac{\partial X^\mu}{\partial \xi^a} \frac{\partial X^\nu}{\partial}$

$$G_{ab} = G_{\mu\nu} \frac{\partial X^{\mu}}{\partial \xi^{a}} \frac{\partial X^{\nu}}{\partial \xi^{b}}$$

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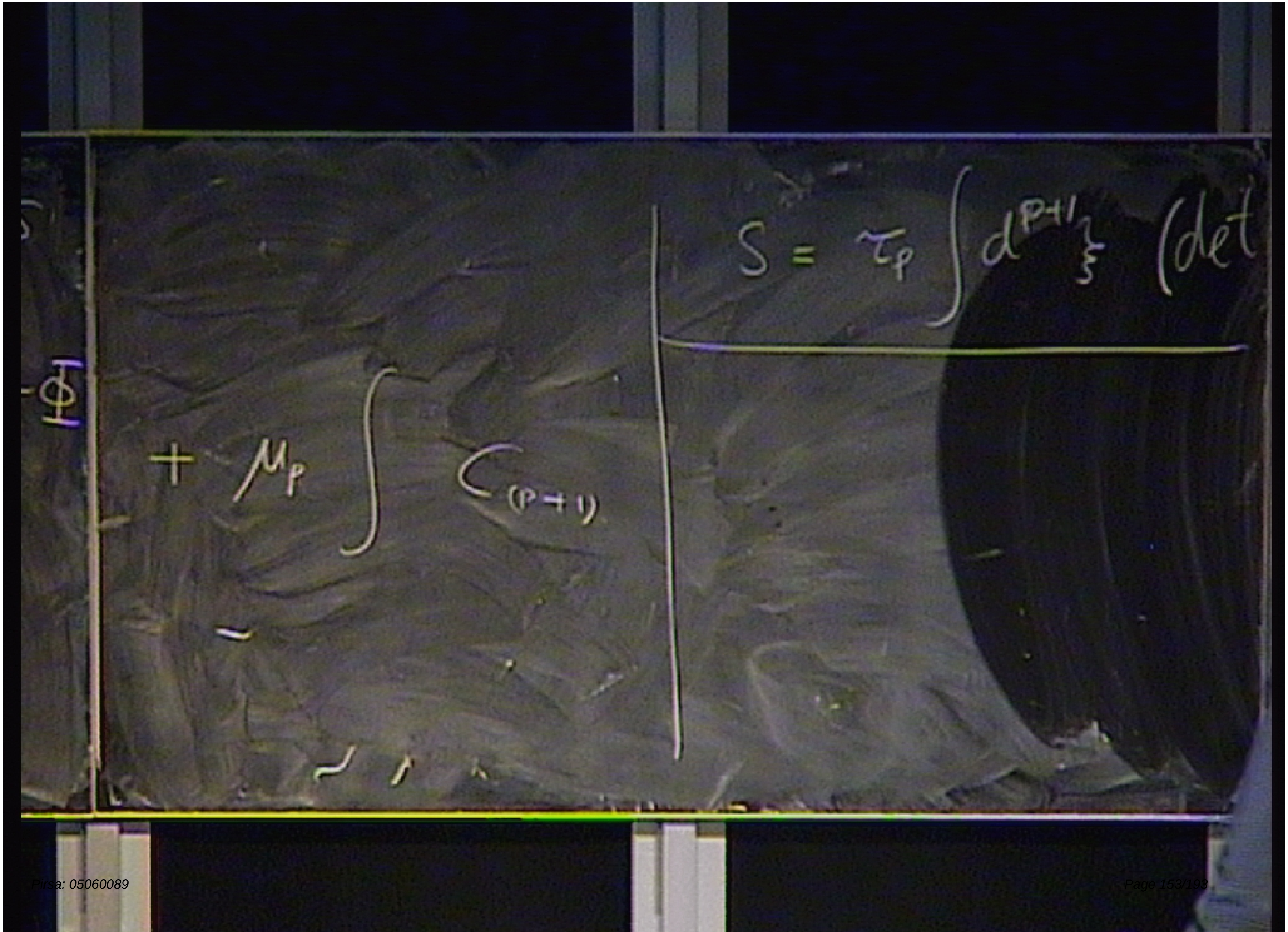

Φ
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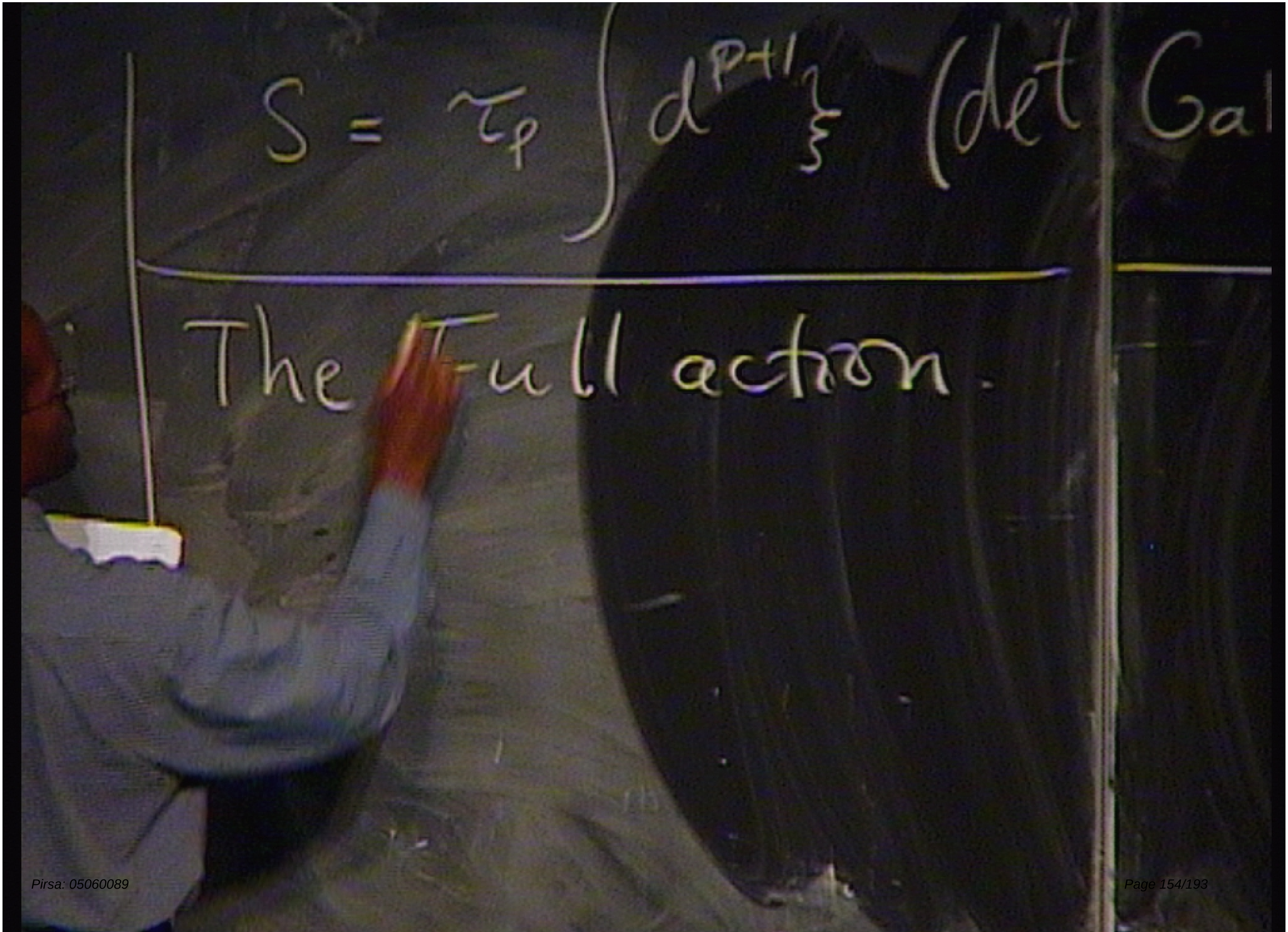
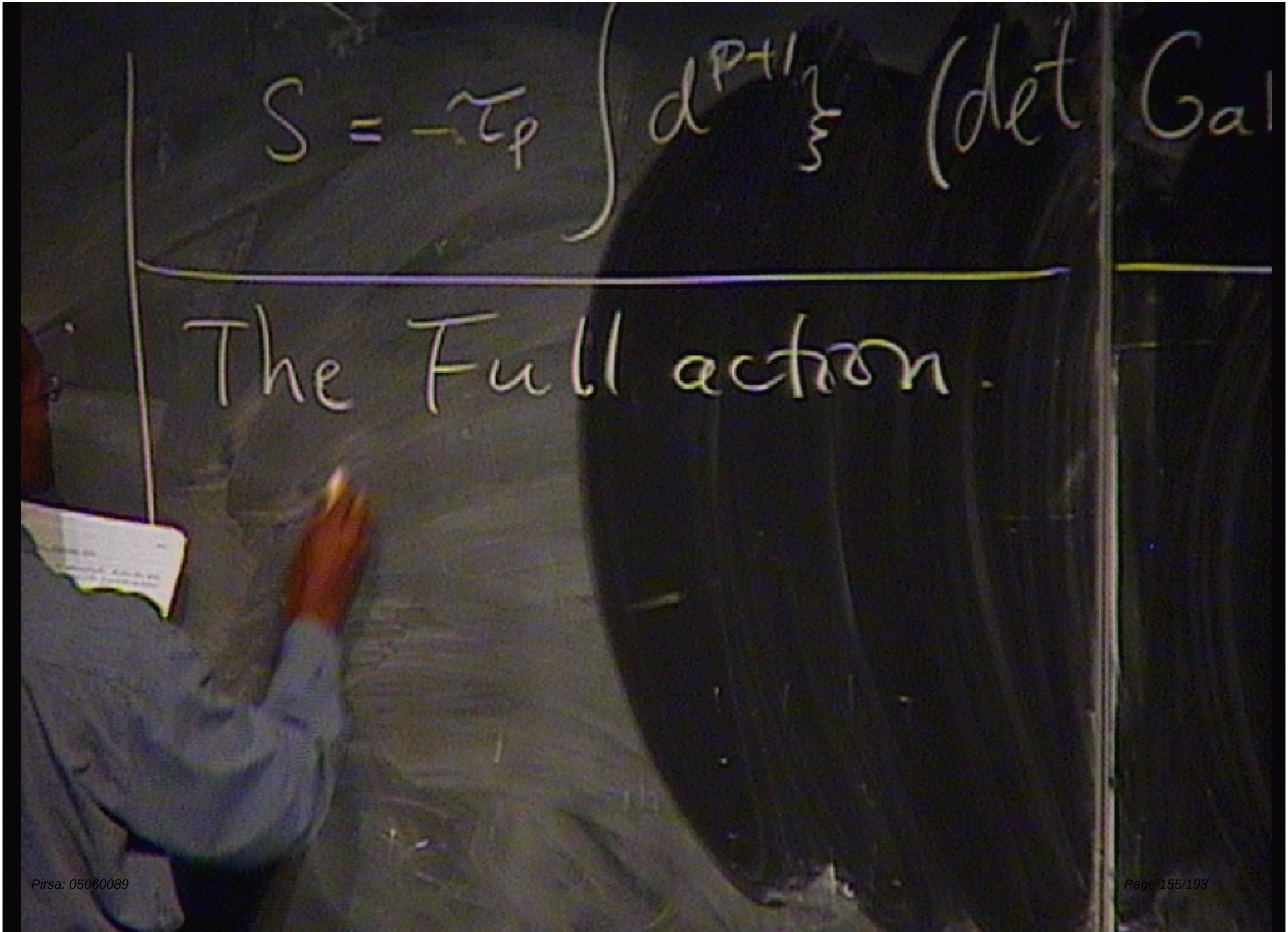

$S = \tau_p \int d^{p+1}\xi \; (\det Ga$
The Full action.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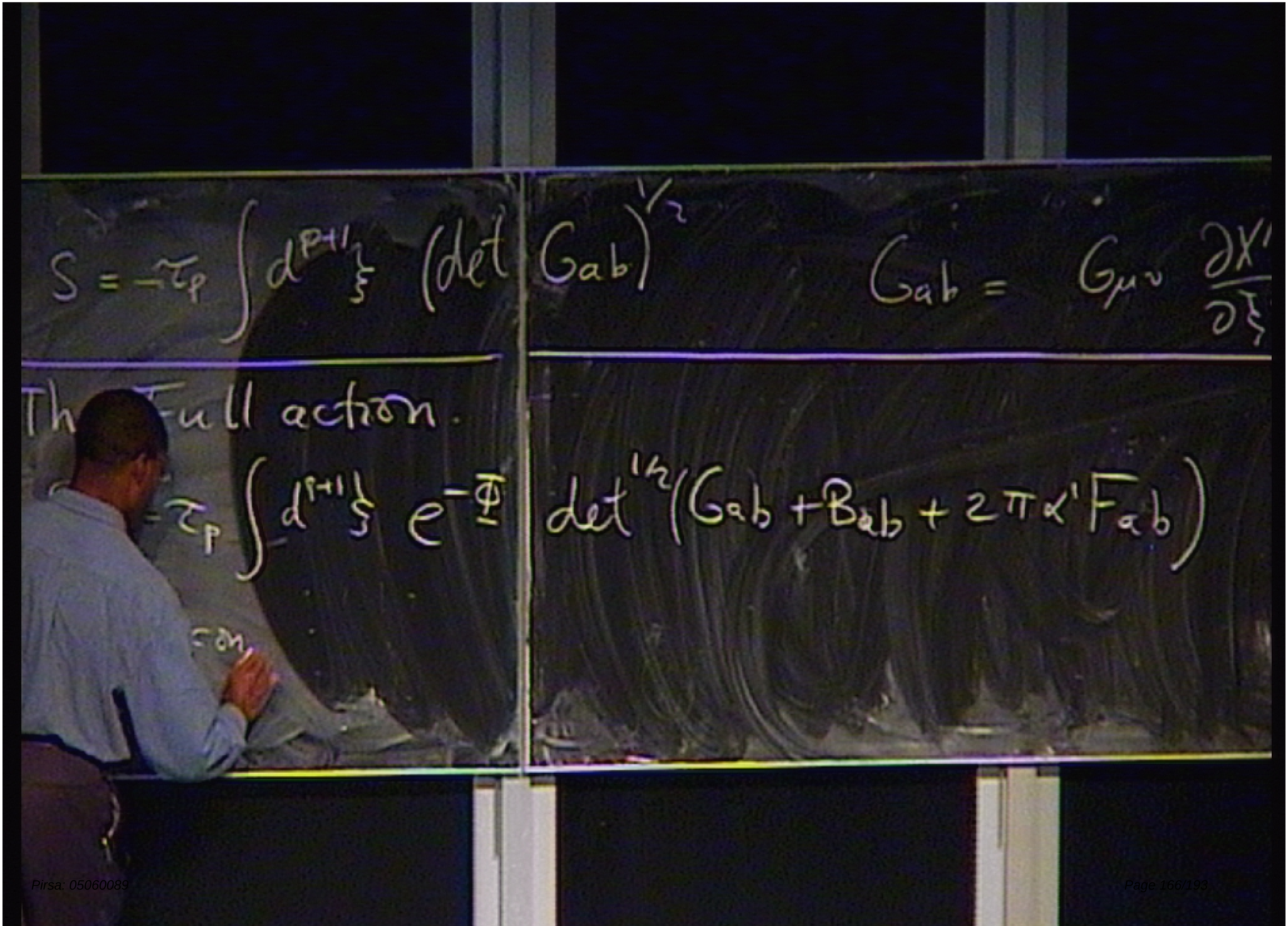
S = -τp ∫ d^{p+1} ξ (det Gab)^{1/2}
Gab = Gμν ∂X
∂ξ
Full action.
τp ∫ d^{p+1} ξ e^{-Φ} det^{1/2}(Gab + Bab + 2πα' Fab)

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The Full action.

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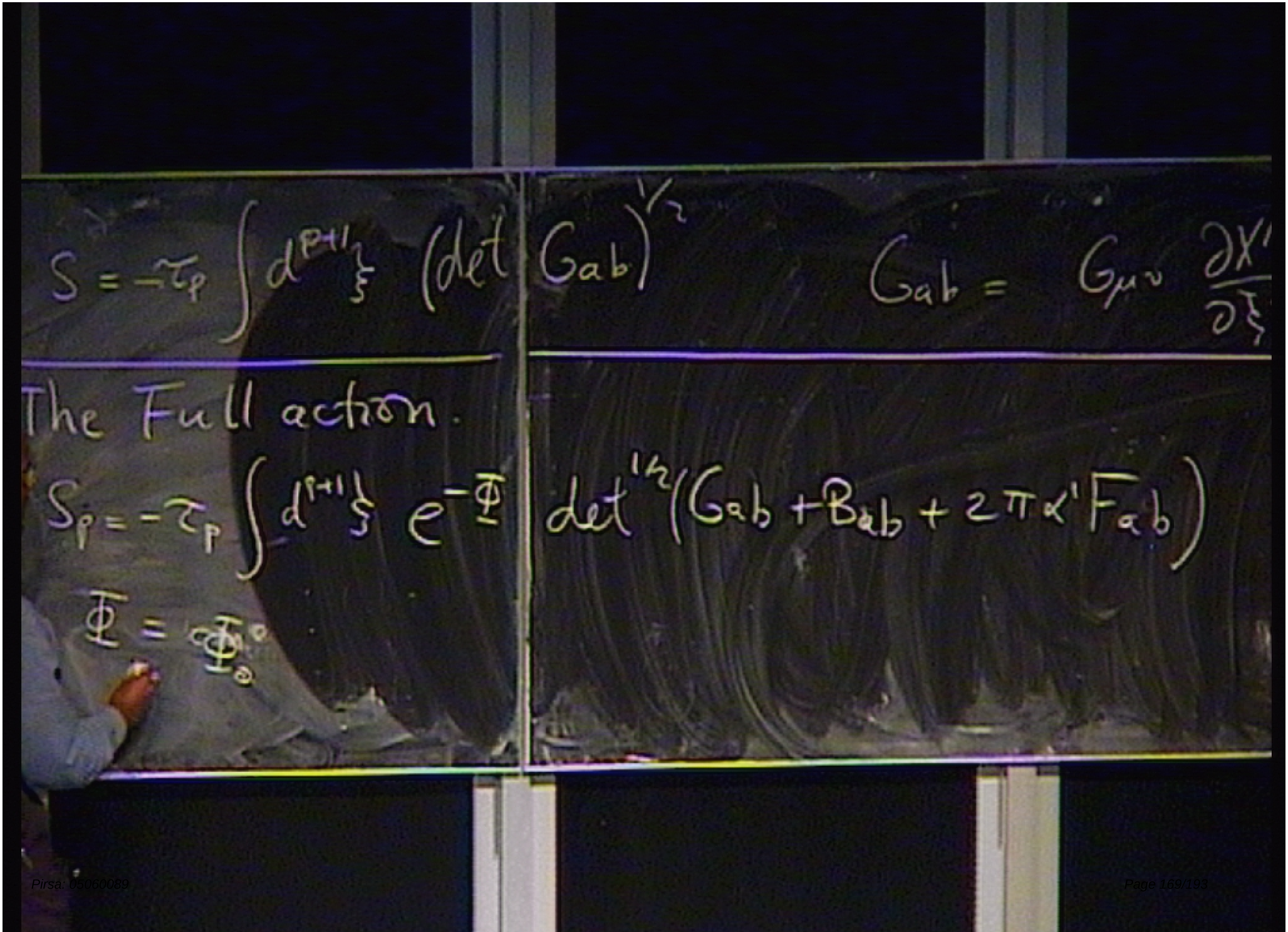
S = -τp ∫ d^{p+1}ξ (det Gab)^{1/2}
Gab = Gμν ∂X
The Full action.
Sp = -τp ∫ d^{p+1}ξ e^{-Φ} det^{1/2}(Gab + Bab + 2πα' Fab)
Φ = Φ0

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$\Phi = \Phi_0$ expand.

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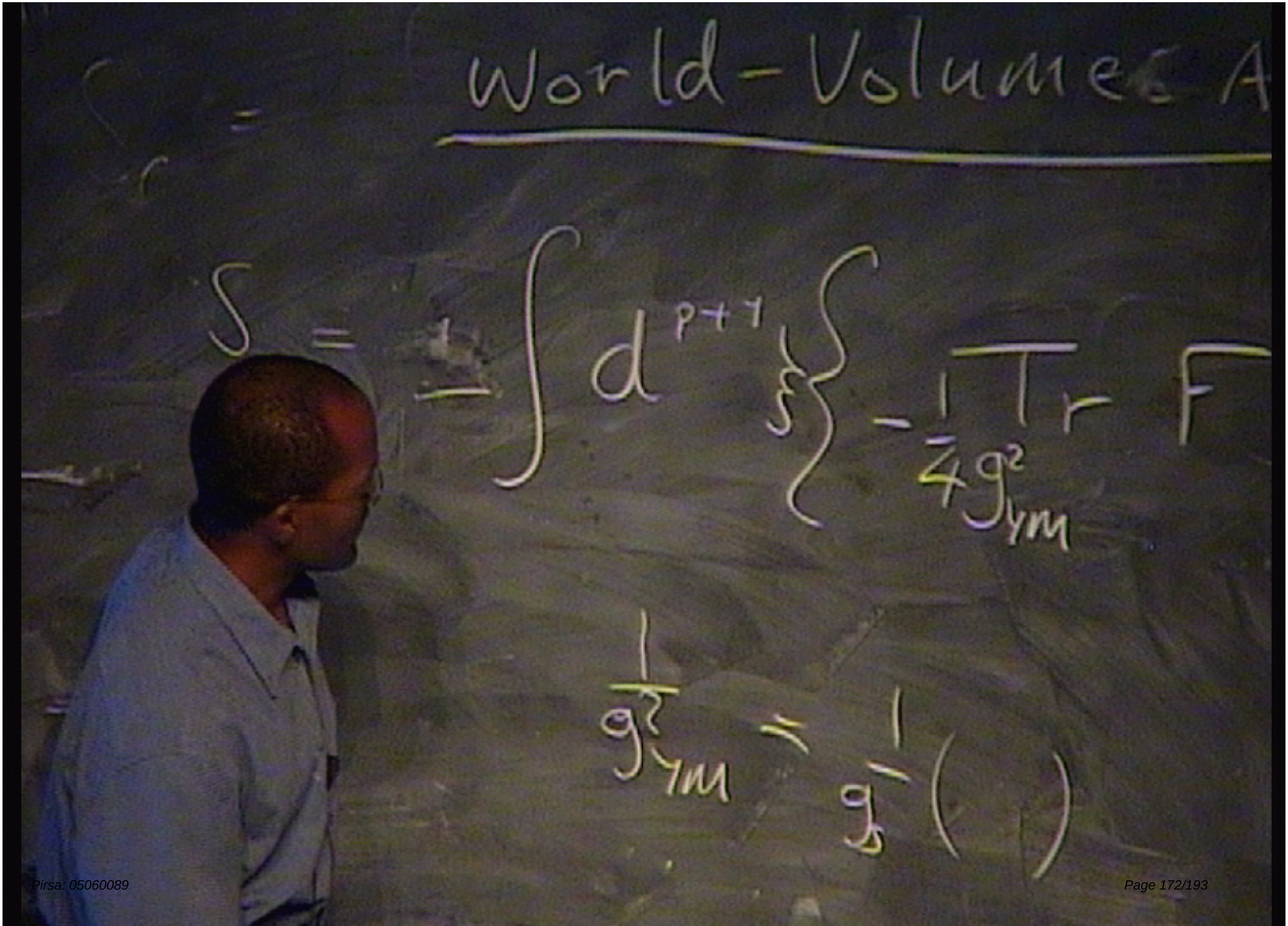
World-Volume A
$S = -\int d^{p+1}\xi \left\{ -\frac{1}{4g_{YM}^2} \mathrm{Tr}\, F \right.$
$\frac{1}{g_{YM}^2} = \frac{1}{g_s}(\ \ )$

World-Volumes A

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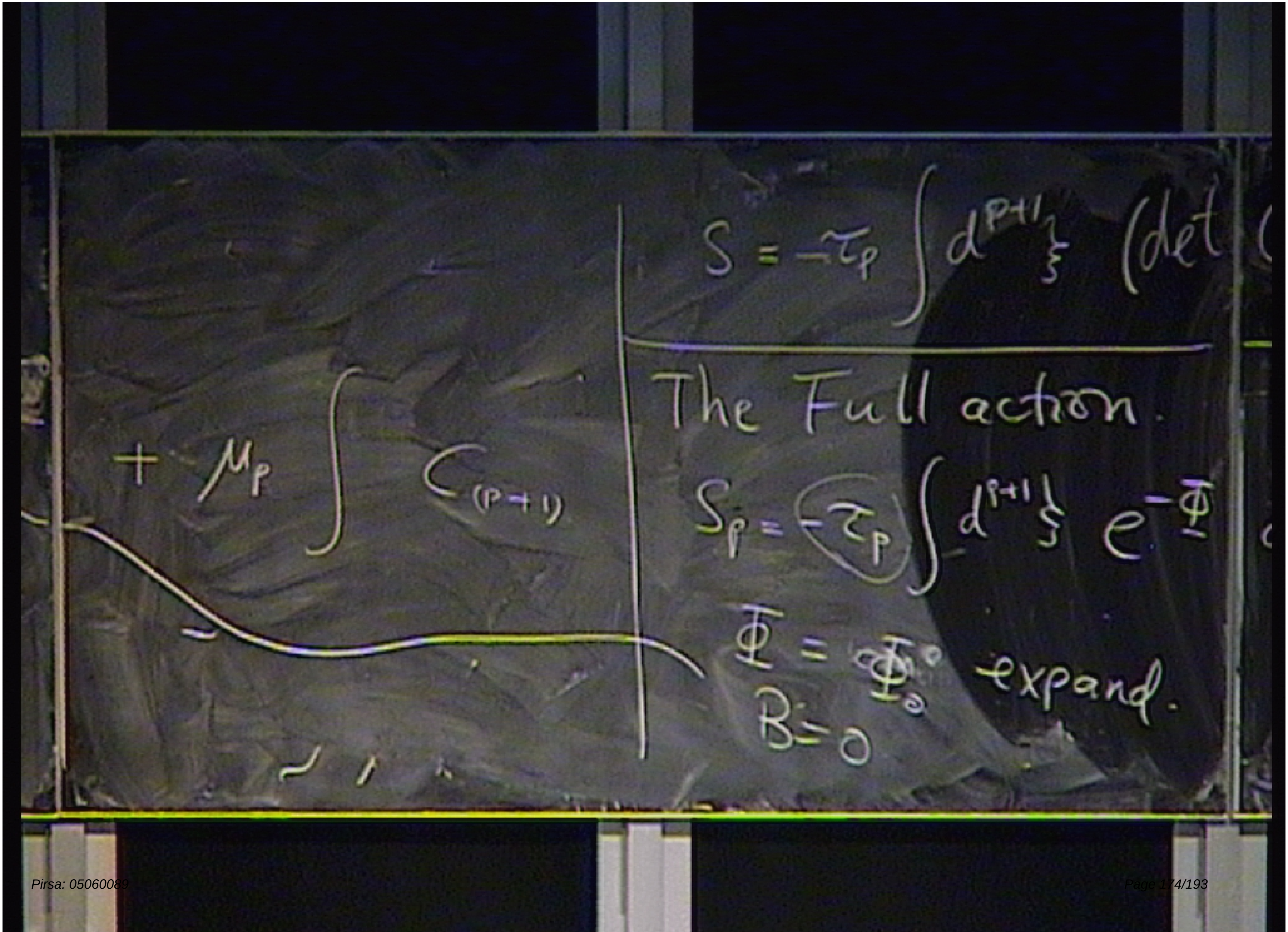
$$\frac{1}{g_{YM}^2} = \frac{1}{g_s} (\quad)$$


$+ \mu_p \int C_{(p+1)}$
$S = -\tau_p \int d^{p+1}\xi \, (\det$
The Full action.
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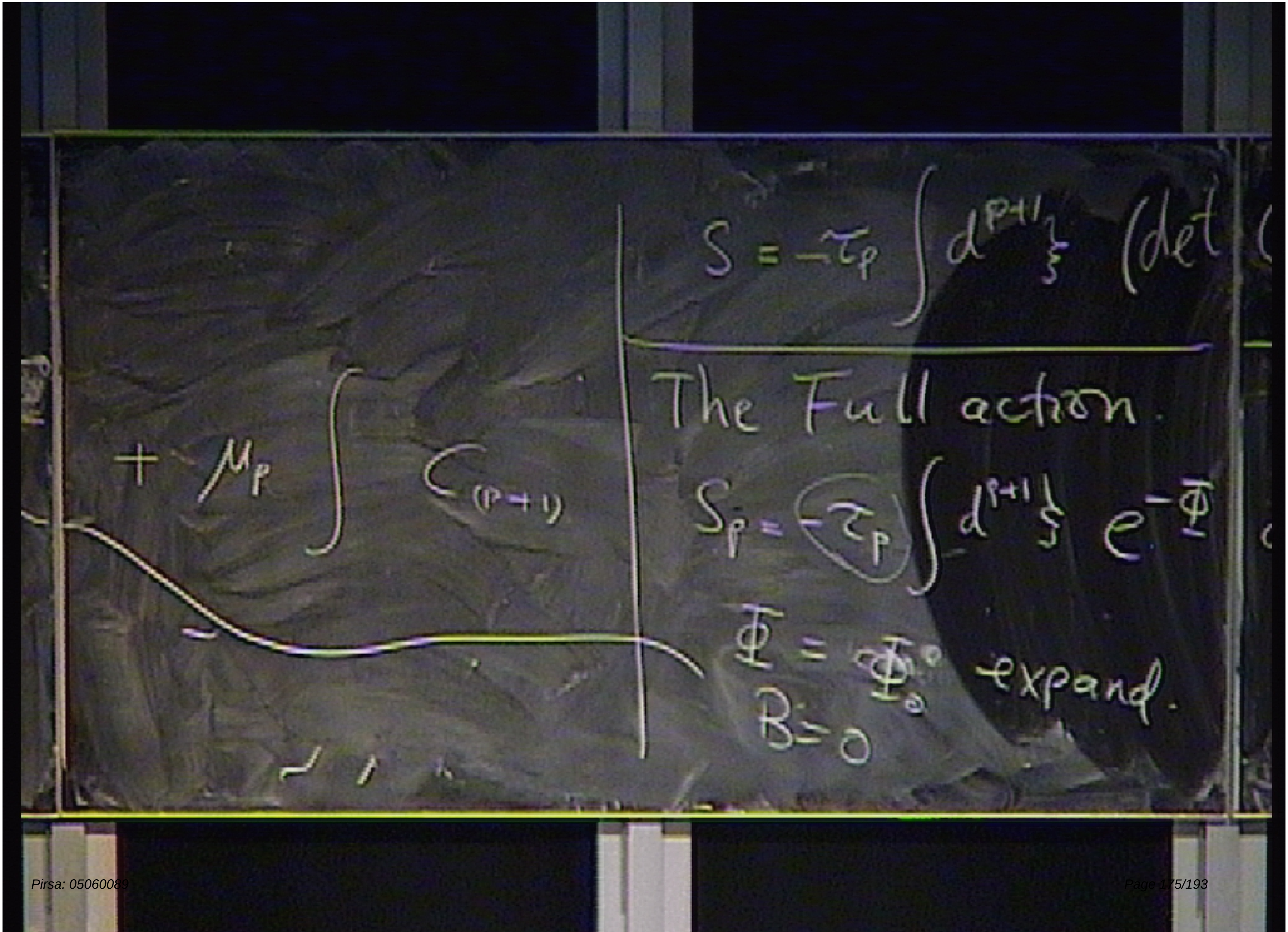
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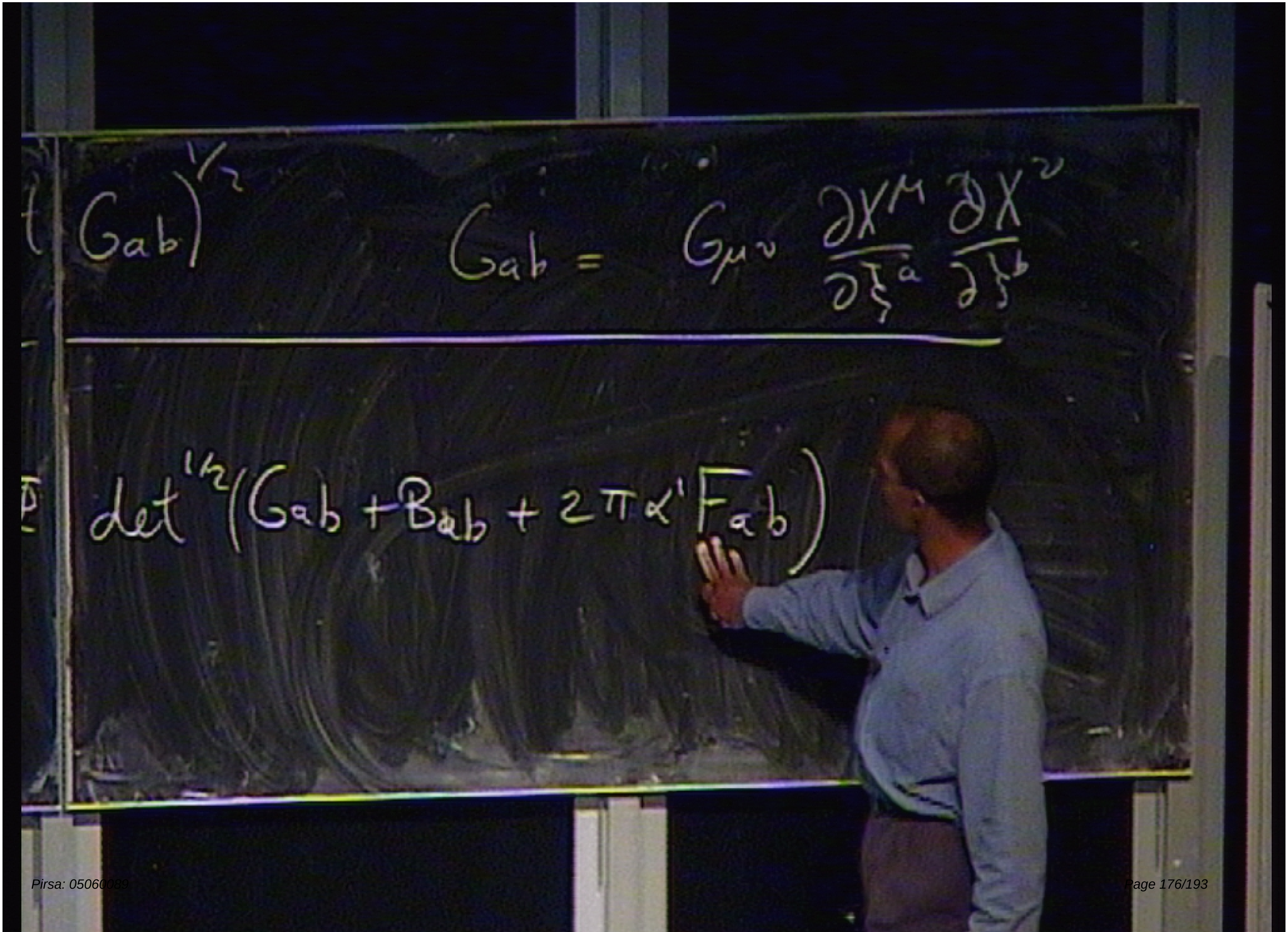
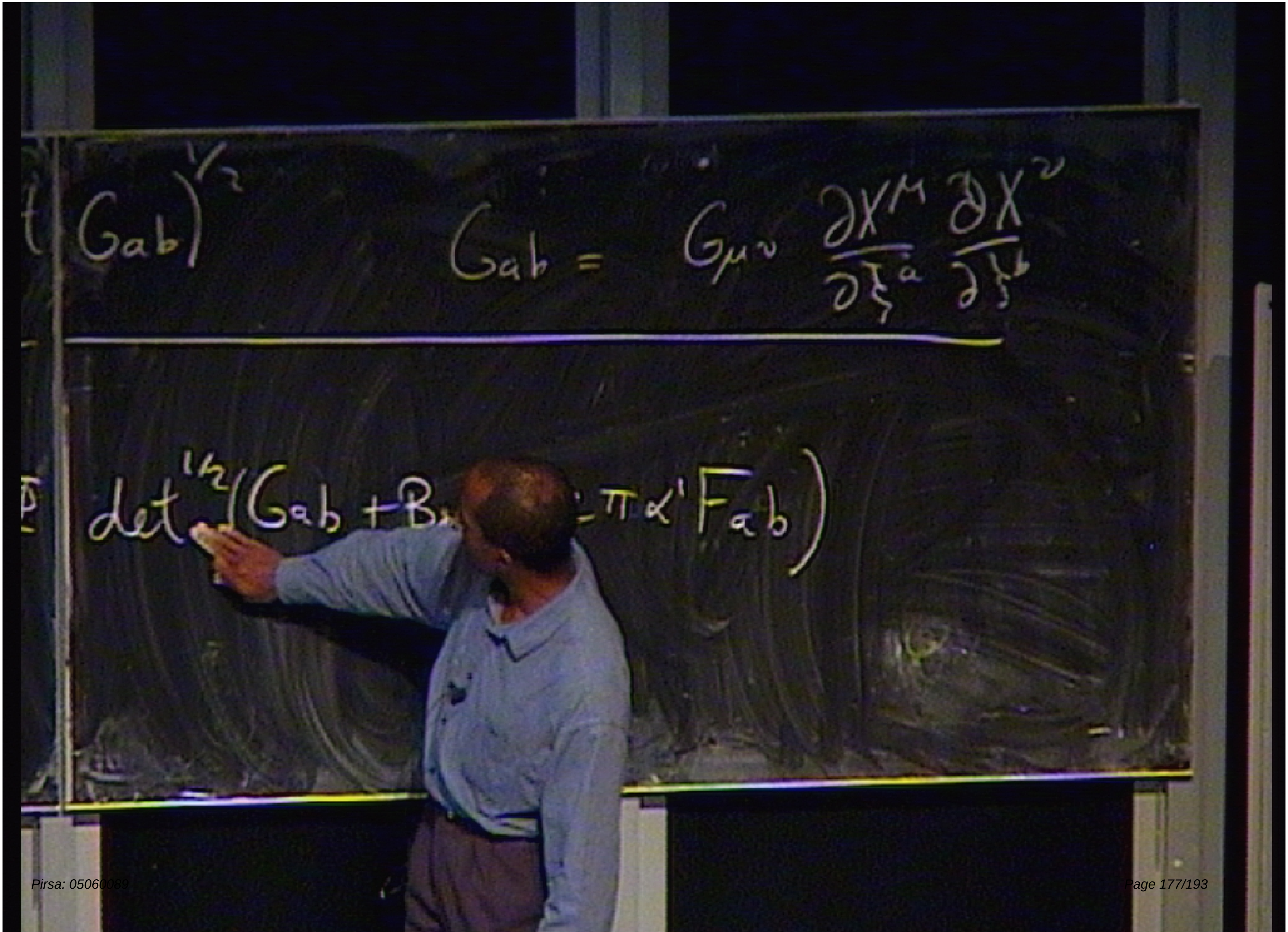

(Gab)^1/2
Gab = Gμν ∂X^μ/∂ξ^a ∂X^ν/∂ξ^b
det^1/2 (Gab + Bab + 2πα' Fab)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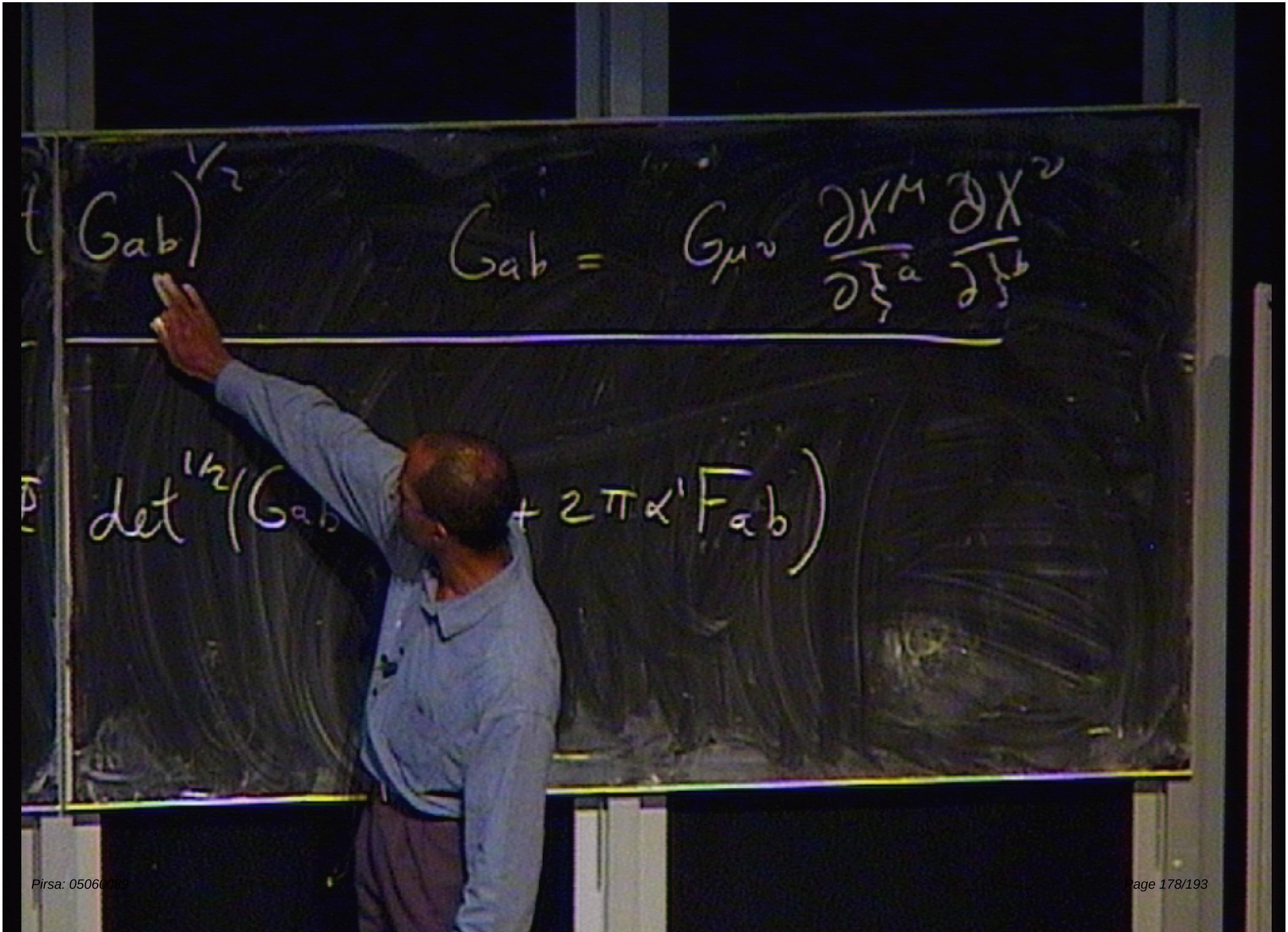
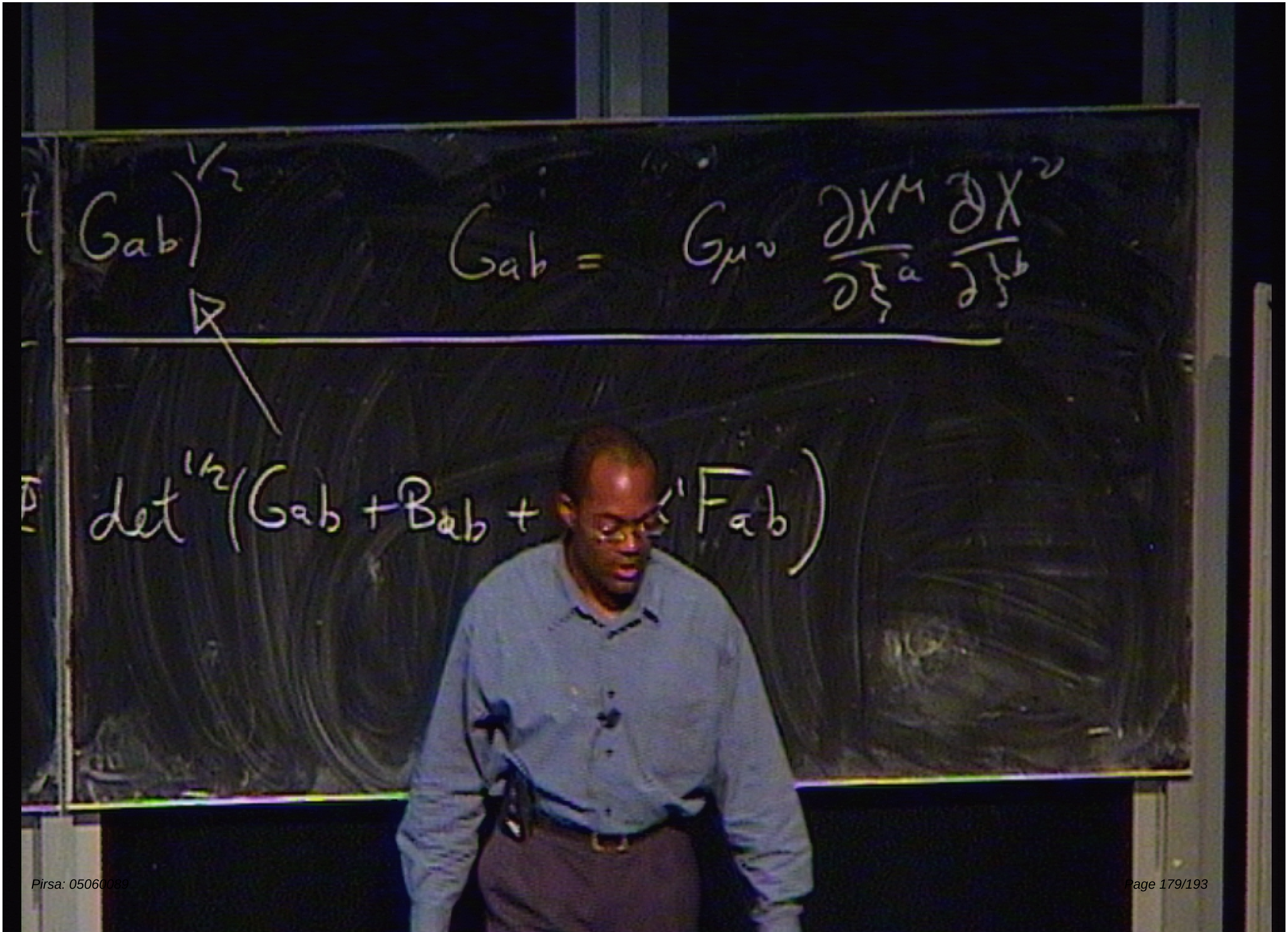

t (Gab)^1/2
Gab = Gμν ∂X^μ/∂ξ^a ∂X^ν/∂ξ^b
det^1/2(Gab + 2πα'Fab)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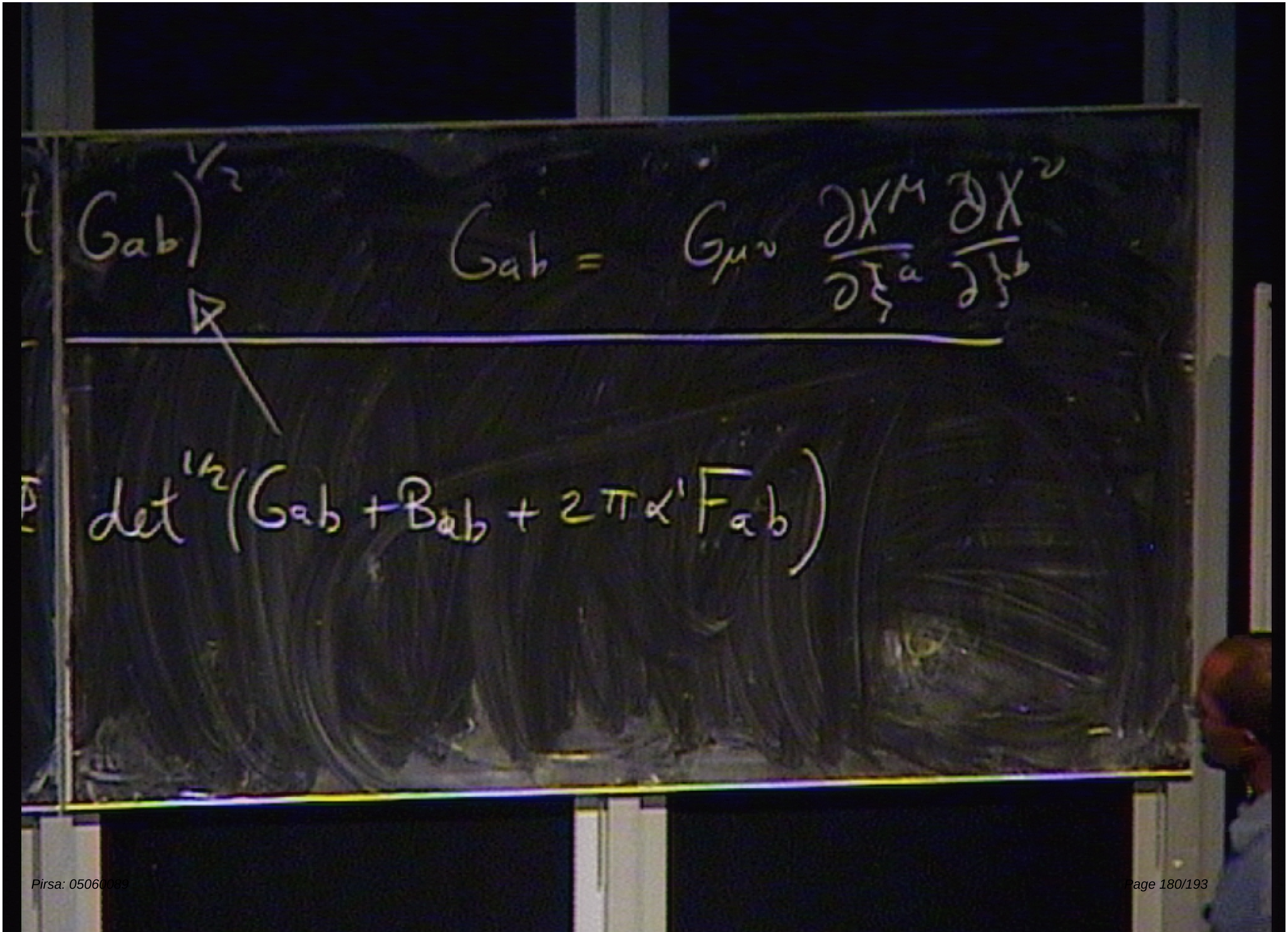
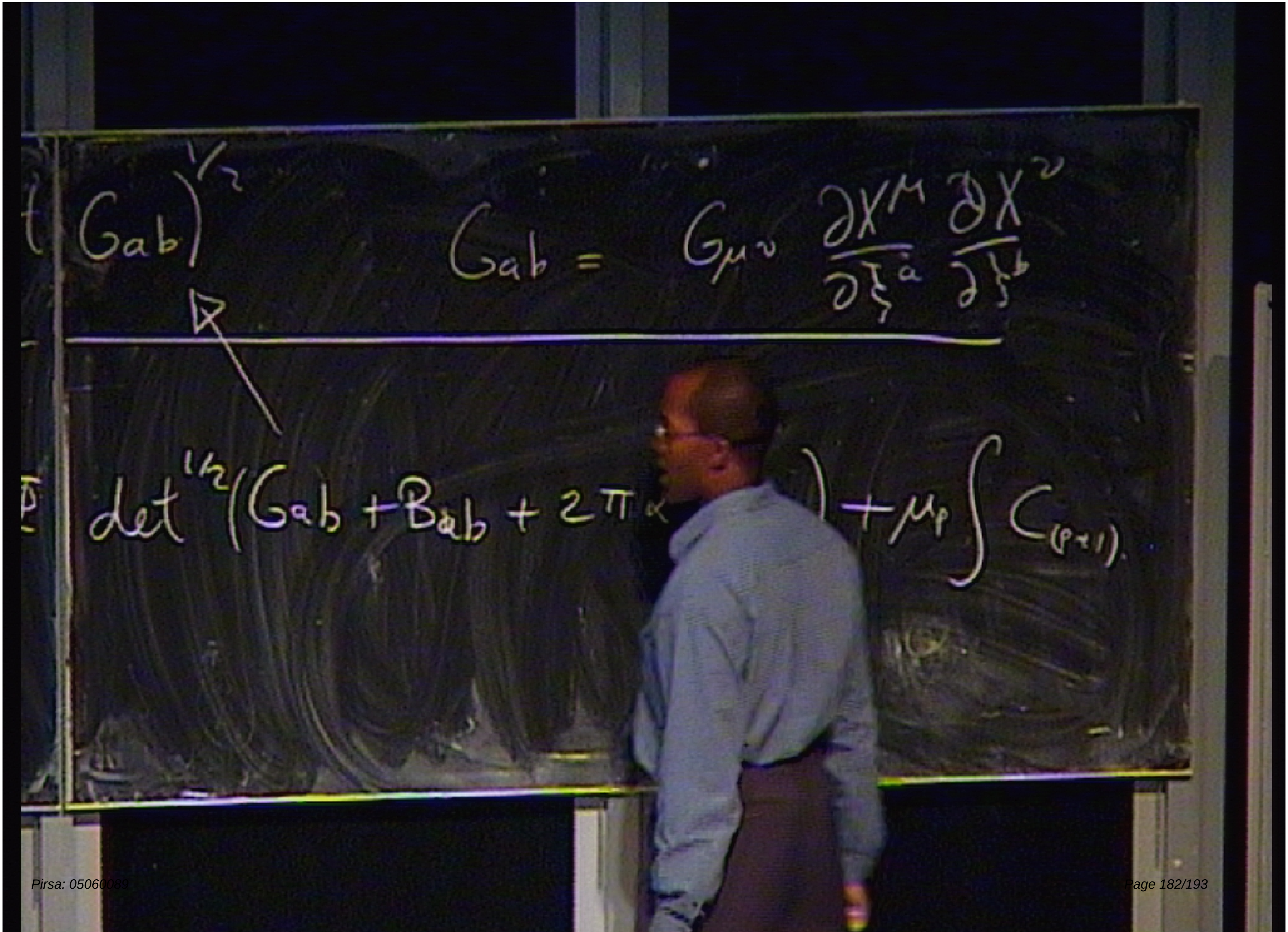
'Fab)


$(G_{ab})^{1/2}$
$G_{ab} = G_{\mu\nu} \frac{\partial X^\mu}{\partial \xi^a} \frac{\partial X^\nu}{\partial \xi^b}$
$\det^{1/2}(G_{ab} + B_{ab} + 2\pi\alpha' F_{ab})$

$$t\,(G_{ab})^{1/2}$$

$$G_{ab} = G_{\mu\nu}\,\frac{\partial X^{\mu}}{\partial \xi^{a}}\,\frac{\partial X^{\nu}}{\partial \xi^{b}}$$

$$\det{}^{1/2}\left(G_{ab} + B_{ab} + 2\pi\alpha' F_{ab}\right)$$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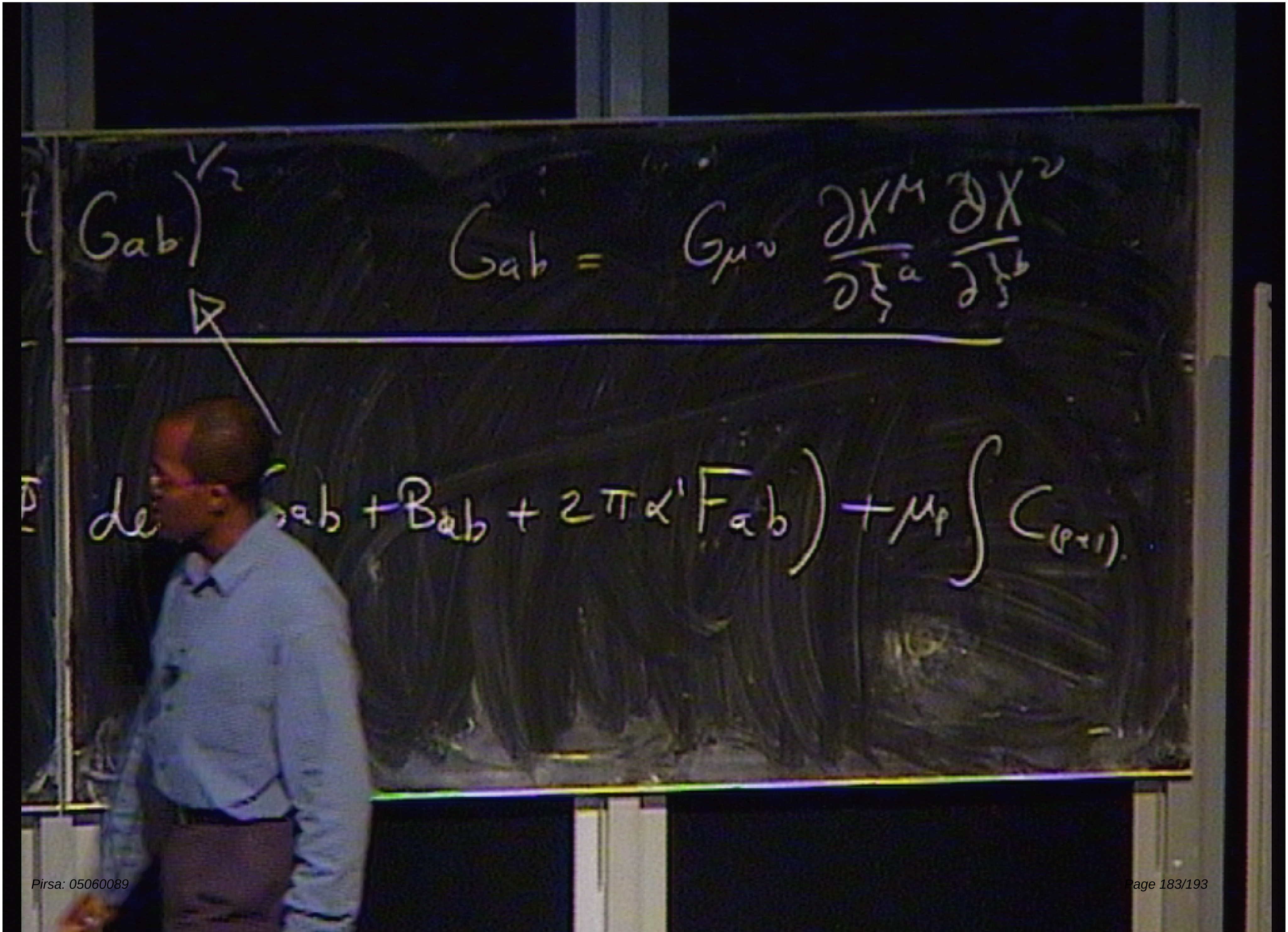
Gab = Gμν ∂X^μ/∂ξ^a ∂X^ν/∂ξ^b
det^1/2 (Gab + Bab + 2πα
) + μp ∫ C(p+1)


Gab)^1/2
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ab + Bab + 2πα'Fab) + μp ∫ C(p+1)

$$t(G_{ab})^{1/2}$$

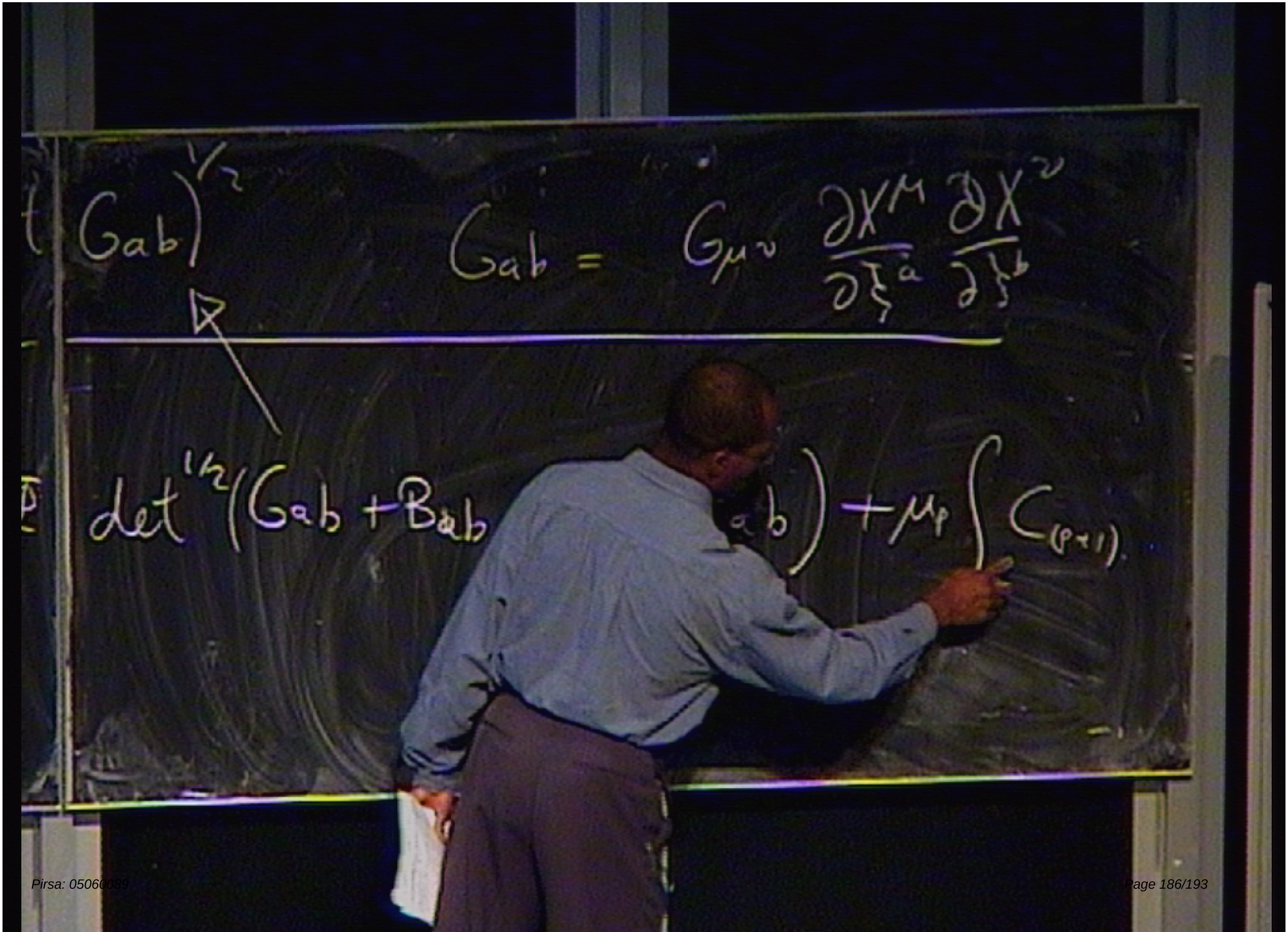
$$G_{ab} = G_{\mu\nu} \frac{\partial X^{\mu}}{\partial \xi^{a}} \frac{\partial X^{\nu}}{\partial \xi^{b}}$$

$$\Phi \ \det{}^{1/2}\left(G_{ab} + B_{ab} + 2\pi\alpha' F_{ab}\right) + \mu_p \int C_{(p+1)}$$

$$t(G_{ab})^{1/2}$$

$$G_{ab} = G_{\mu\nu} \frac{\partial X^{\mu}}{\partial \xi^{a}} \frac{\partial X^{\nu}}{\partial \xi^{b}}$$

$$\det{}^{1/2}\left(G_{ab} + B_{ab} + 2\pi\alpha' F_{ab}\right) + \mu_p \int C_{(p+1)}$$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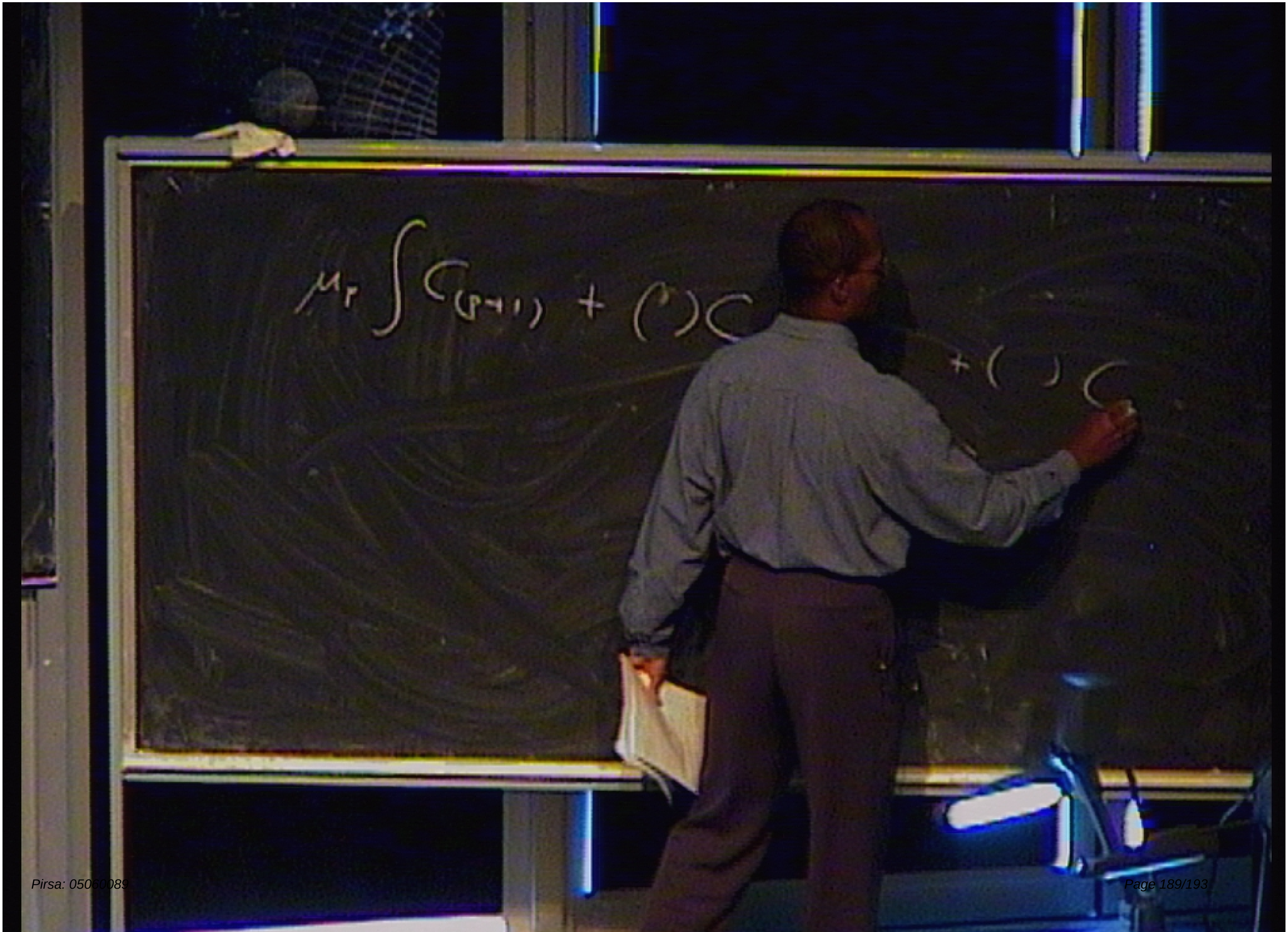
$$\ldots t\,(G_{ab})^{1/2} \qquad G_{ab} = G_{\mu\nu}\frac{\partial X^{\mu}}{\partial \xi^{a}}\frac{\partial X^{\nu}}{\partial \xi^{b}}$$

$$\det{}^{1/2}\left(G_{ab} + B_{ab} + 2\pi\alpha' F_{ab}\right) + \mu_p \int C_{(p+1)}$$

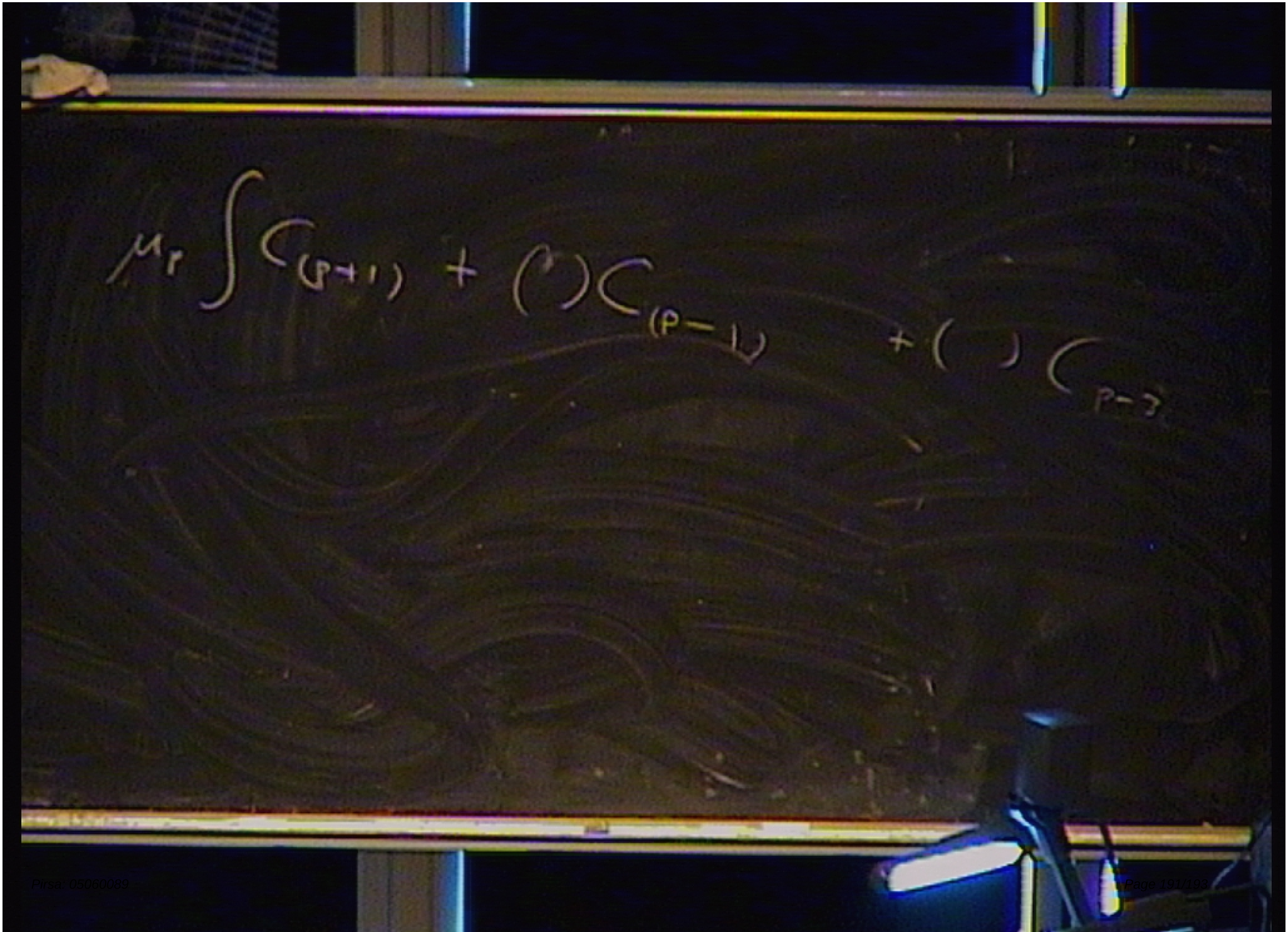
$$\ldots t\,(G_{ab})^{1/2}$$

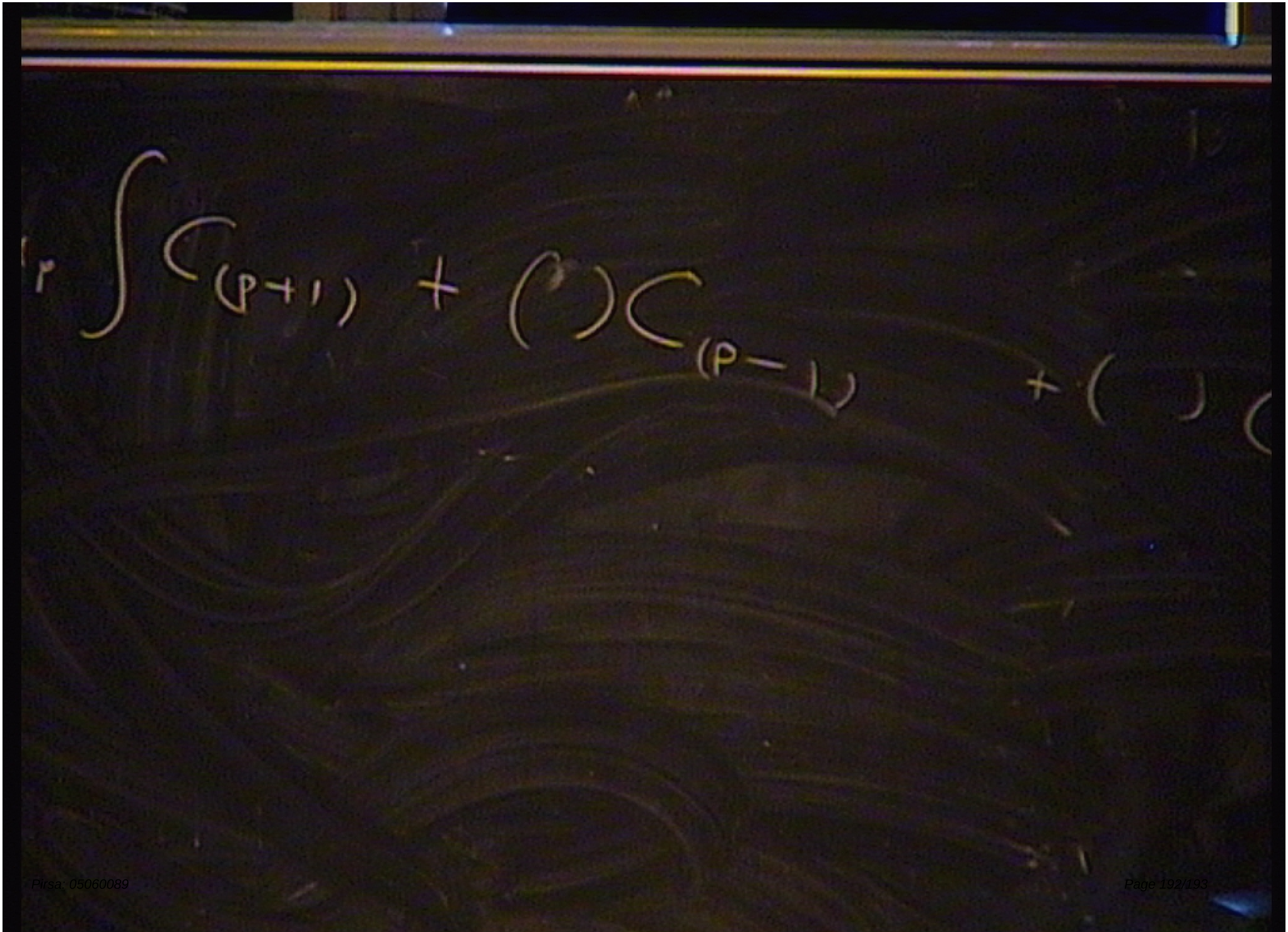
$$G_{ab} = G_{\mu\nu} \frac{\partial X^\mu}{\partial \xi^a} \frac{\partial X^\nu}{\partial \xi^b}$$

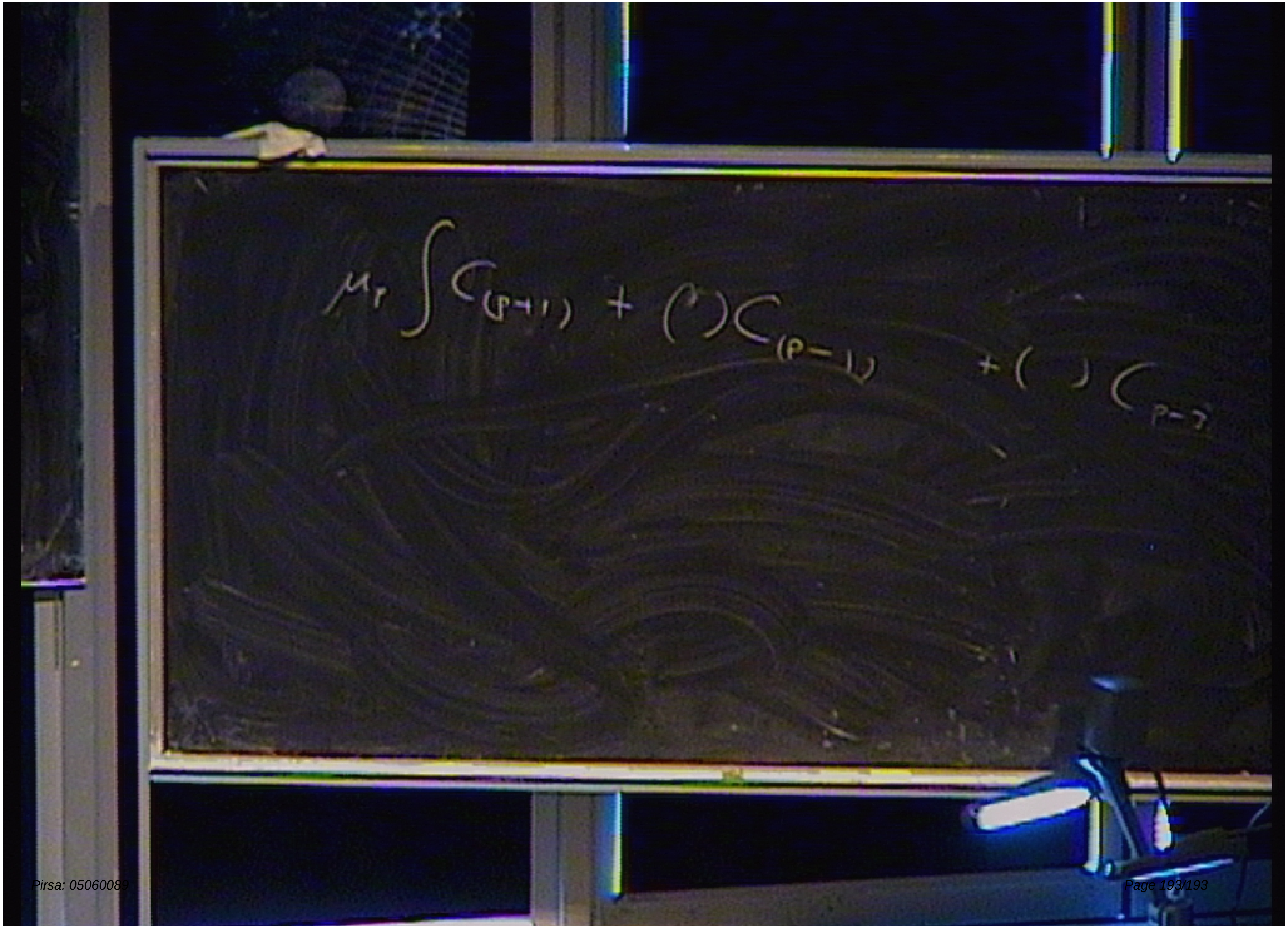
$$\ldots \Phi \; \det{}^{1/2}\left(G_{ab} + B_{ab} + 2\pi\alpha' F_{ab}\right) + \mu_p \int C_{(p+1)}$$



$$\mu_p \int C_{(p+1)} + (\;)\,C_{(p-1)} + (\;)\,C_{p-3}$$


$\mu_r \int C_{(p+1)} + ( ) C_{(p-1)} + ( ) C_{p-3}$


$\int C_{(p+1)} + (\ ) C_{(p-1)} + (\ ) ($


μr ∫ C(p+1) + ( ) C(p−1) + ( ) C(p−3)