Title: String Cosmology

Date: Jun 24, 2005 10:45 AM

URL: http://pirsa.org/05060087

Abstract:

# PROPERTIES OF COSMIC STRINGS
# (COSMIC SUPERSTRINGS)

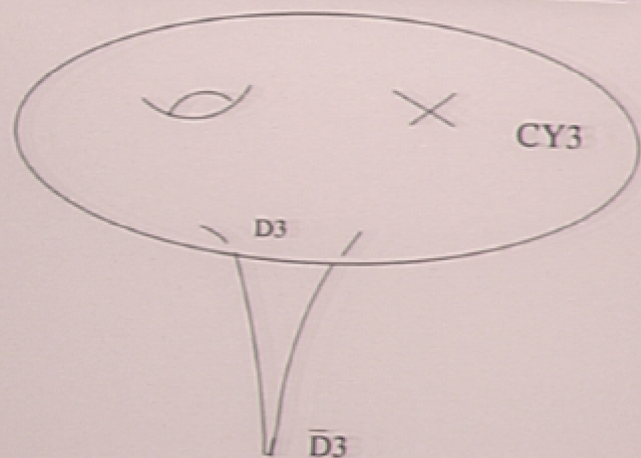
- STABILITY

- EVOLUTION

- SPECTRUM/TENSION

- PRODUCTION

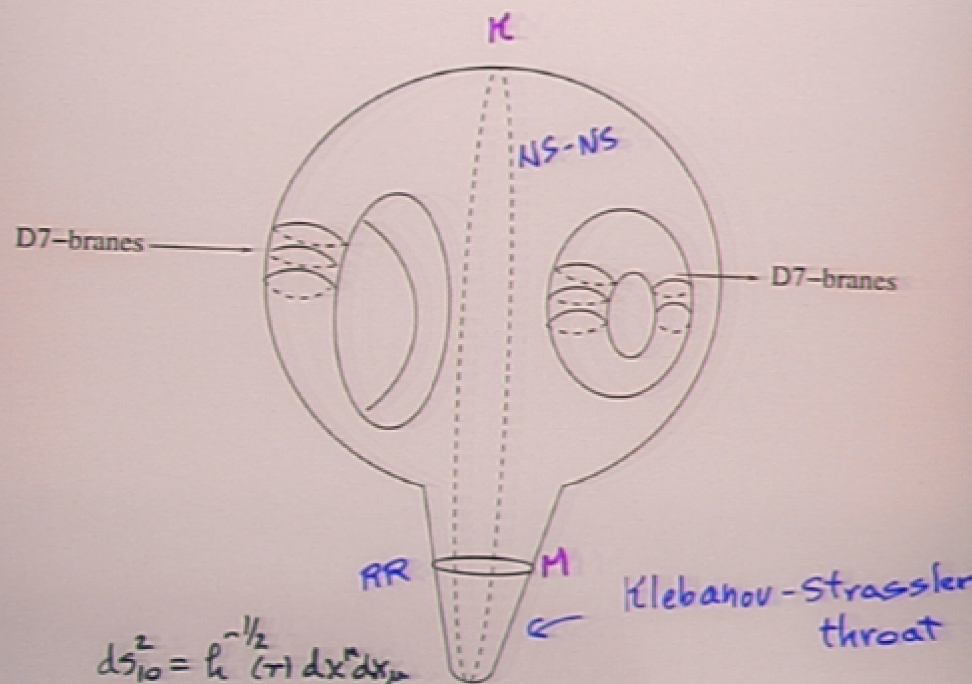
## Inflation in Realistic String Vacua




Giddings, Kachru and Polchinski hep-th/0105097
Kachru, Kallosh, Linde and Trivedi hep-th/0301240
Kachru, Kallosh, Linde, Maldacena, McAllister and Trivedi hep-th/0308055
Firouzjahi and H.T., hep-th/0312020

Other scenarios are emerging.

Kachru, Giddings, Polchinski
Kachru, Kallosh, Linde, Trivedi




$$ds_{10}^2 = h^{-1/2}(r)\, dx^\mu dx_\mu + h^{1/2}(r)\, ds_6^2$$

$$h = h_A^{-4} \simeq \frac{L^4}{r^4}$$

$$\ddot{\phi} + 3H\dot{\phi} + V'(\phi) = 0$$

$$H^2 = \frac{8\pi G}{3} V(\phi) \sim \text{constant}$$

$\ddot{\phi}$ may be neglected in slow roll

$$\dot{\phi} = -\frac{V'(\phi)}{3H}$$

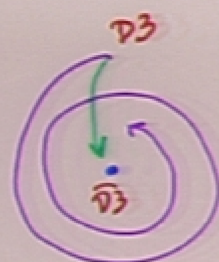
$$a(t) \propto e^{Ht}$$

$$N_e = H\int_{t_i}^{t_f} dt = -H \int_{\phi(t_i)}^{\phi(t_f)} 3HV' d\phi$$

$$= \frac{1}{m_p^2}\int_{\phi_f}^{\phi_i} \frac{V'}{V} d\phi$$

$$\frac{d\phi}{dt} = -\frac{V'}{3H}$$

$$dt = -3HV' d\phi$$




$$\frac{dJ}{dt} = 0$$

$$\downarrow$$

$$\frac{d(Ja^3)}{dt} = 0$$

$$J \simeq \frac{1}{a^3}$$

KKLMMT

$$K = -3 \ln \left[ -i(\rho - \bar{\rho}) + \phi\bar{\phi} \right]$$

$$V(\phi) = \frac{1}{2}\beta H^2 \phi^2 + 2 h_0^4 T_3 + V_{D\bar{D}}$$

$$= \frac{1}{2}\beta H^2 \phi^2 + 3 h_0^4 T_3 \left( 1 - \frac{1}{N}\frac{\phi_0^4}{\phi^4} \right) + \cdots$$

$$ds^2 = h_0^2(r)\left(-dt^2 + a(t)^2 d\vec{x}^2\right) + h_0(r)^{-2} dr^2$$

$$\phi = \sqrt{\frac{T_3}{2}}\, r$$

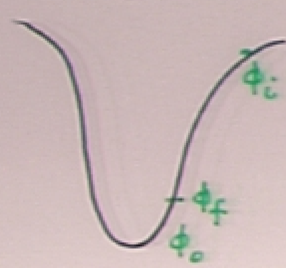
$$h_0 = e^{-2\pi K / 3 M g_s} \qquad N = KM$$

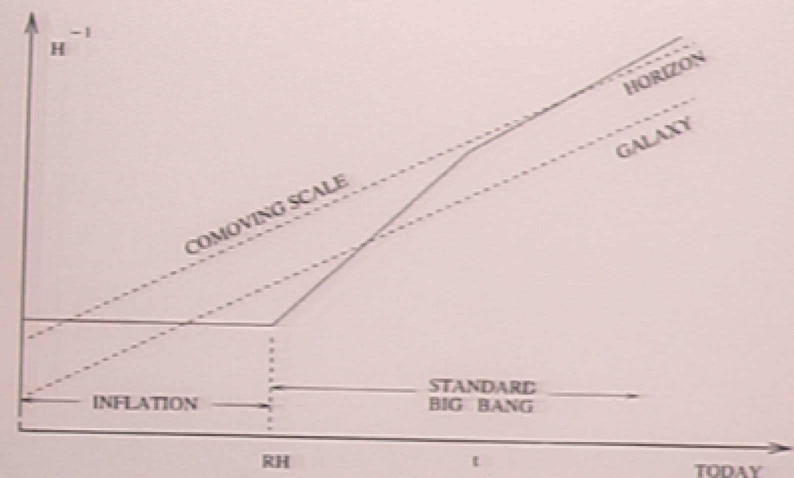
$$h(r) = h_0^{-4}(r) = 1 + \frac{L}{r^4}$$

$$G\mu = G T_1 h_0^2$$

$$\eta = M_P^2 \frac{V''}{V} = \frac{\beta}{3} - \frac{20 M_P^2 \phi_0^4}{N \phi^6}$$

$\beta < 1/30$






**Inflation**

solves

1. Flatness Problem
2. Horizon Problem
3. Defects Overabundance

and generates scale iavariant density perturbations

Number of e-folds $N_e$

$$N_e = \frac{1}{M_p^2}\int \frac{V\,d\phi}{V'} = 55$$

$$\delta_H = 1.9 \times 10^{-5}$$

COBE
WMAP

$$= \frac{1}{\sqrt{75}\,\pi}\,\frac{1}{M_p^3}\,\frac{V^{3/2}}{V'}$$

$$\sim f(T_3 R_0^4, \beta)$$

$$\parallel$$

$$(T_1 R_0^2)^2$$

$$T_3 \sim \frac{1}{\alpha'^2}$$

$$T_1 \sim \frac{1}{\alpha'}$$

$$5 \times 10^{-7} > G\mu > 4 \times 10^{-10}$$

$\beta = 1/7$ $\quad$ $\beta < \frac{1}{7}$ $\quad$ $\beta = 0$

w H. Firouzjahi
hep-th/0501099

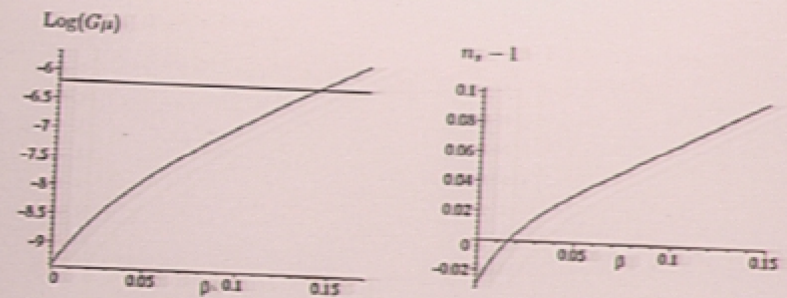



**FIG 1.** The cosmic string tension $G\mu$ and the density perturbation power spectrum index $n_s$ are shown as functions of $\beta$ in the KKLMMT-like brane inflationary scenario. Here $g_s = 1$. The horizontal line at -6.2 in the Log($G\mu$) graph indicates the present observational bound on the cosmic string tension, which corresponds to $\beta \lesssim 1/7$. The observational constraint on $n_s \le 1.086$ also gives $\beta \lesssim 1/7$.

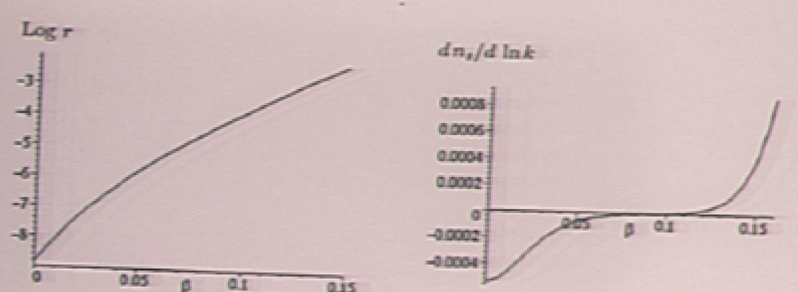



**FIG 2.** The ratio of the tensor to scalar perturbations, $r$, and the running of the density perturbation power spectrum index, $dn_s/d\ln k$, are shown as functions of $\beta$.

$$h^{-4}(\phi) = 1 + \frac{\lambda}{\phi^4}$$

$$S = -T_3 \int d^4x \, a(t)^3 \left[ h^4 \sqrt{1 - \frac{\dot\phi^2}{T_3 h^4}} + \frac{1}{T_3} V(\phi) - h^4 \right]$$

Silverstein, Tong
Alishahiha

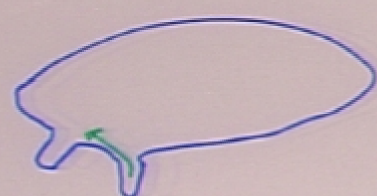
Sarah Shandera

$\dot\phi$ small (slow roll)

$$\longrightarrow \left[ -\frac{1}{2} \frac{\dot\phi^2}{T_3} + \frac{1}{T_3} V(\phi) \right]$$

$\beta \simeq 1$ fast roll

$\beta < 0$ rolling out of a throat



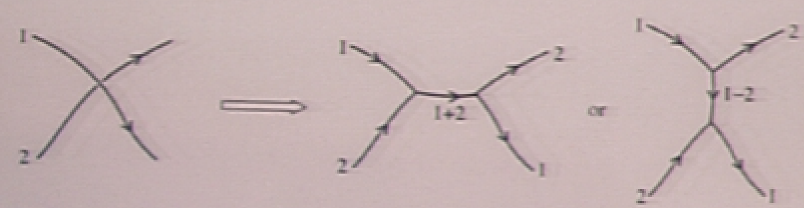
X. Chen

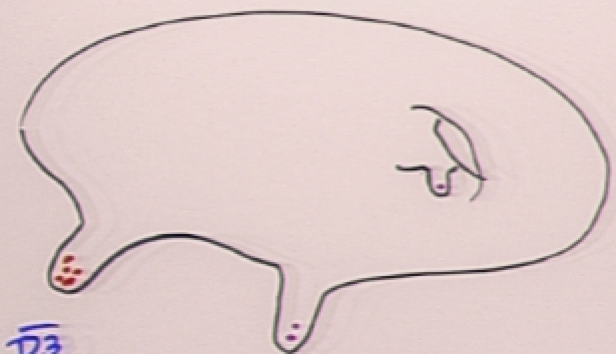
## Cosmic strings in Superstring theory can be rich:

In the simpliest scenario of $D3-\bar{D}3$ brane inflation in the KKLMMT scenario, their annihilation yields $(p,q)$-strings, with tension $G\mu_{pq} \simeq \sqrt{p^2+aq^2}$, where

$$p,q = 0, \pm1, \pm2, \pm3, ...$$

E. Copeland, R.C. Myers and J. Polchinski, hep-th/0312067
M. Jackson, N. Jones and J. Polchinski, hep-th/0405229





$\overline{D3}$

↓

warped to
TeV scale

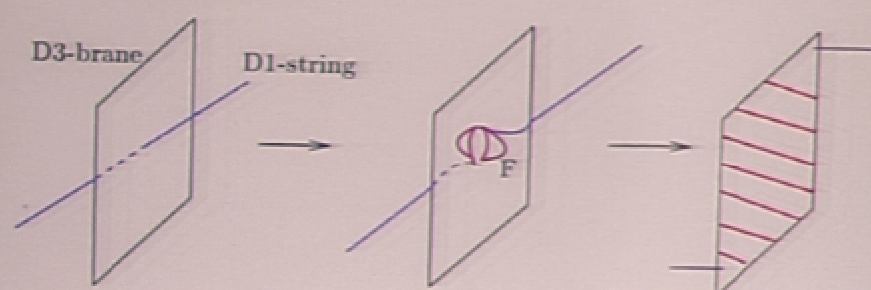
F-D strings

↓

warped to lower
than GUT scale
$\simeq 10^{14}$ GeV

## Breakage

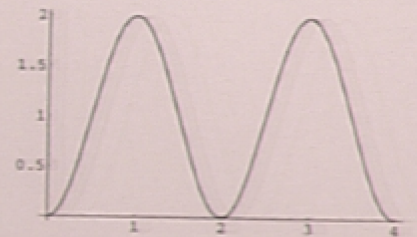
- D1-string could dissolve in a D3-brane. It is still non-BPS, unless the angle is 90°.



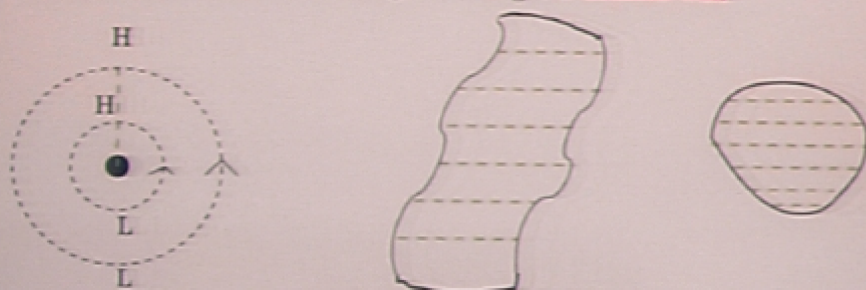

- CMP Puzzle (Copeland, Myers, Polchinski)
  There is two types of instability. Either the string is the boundary of a domain wall which tension makes the string shrink or collide, or it develops endpoints and breaks on a D3-brane. But the boundary of a boundary is zero. Hence both instabilities cannot occur.

$$\epsilon^{\alpha\mu\nu\lambda}\partial_\mu C_{\nu\lambda} \simeq \partial^\alpha \phi$$

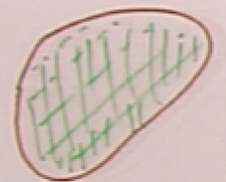
- Generically, instanton effects create a potential for $\phi$ of the form $e^{i\phi}$.



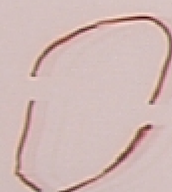

- As $\phi$ goes around the string, we will get a domain wall.




- Cosmic strings are the boundaries of domain walls (Witten). The tension of the domain walls makes the string wants to collide or shrinks.



boundary
of a
domain wall



breaks in
D3-brane

But
boundary of a boundary is zero

CMP puzzle

## Abelian Higgs $\qquad\qquad$ $C_2$

axion $\quad \phi \qquad\qquad A_\mu \quad C_2$

$$^*dC_2 = d\phi$$

(1) $\quad A_\mu$ eats $\phi$ to become massive $A_\mu$

$\quad 2 + 1 = 3$

(2) $\quad C_2$ eats $A_\mu$ to become massive $C_2$

$\quad 1 + 2 = 3$


$$\int d\vec{l} \cdot \vec{A} = \int d\vec{s} \cdot \vec{B} = \phi_{flux}$$

$\phi \to 0$ to $2\pi$

Louis Leblond

Gia Dvali 0505172

Blanco-Pillado, Redi

$U(1) \times U(1) \longrightarrow U_+(1) \times U_-(1)$

$T, T^+$

$T$ behaves as the Higgs field of $U_-(1)$

$\rightarrow$ vortices $\rightarrow$ D1-branes

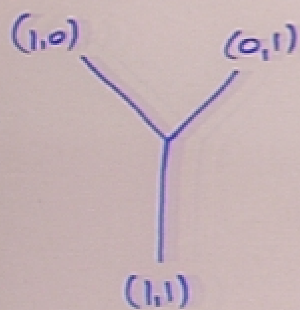
D1-strings

D-strings $(q)$

---

$U_+(1) \longrightarrow$ F-strings $(p)$

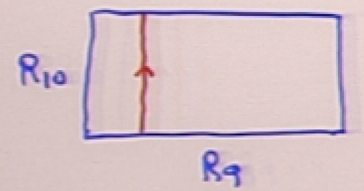
$(p,q)$ strings : $T = \mu \sqrt{p^2 g_s^2 + q^2}$




$U_+(1)$ confining ?

Dvali–Vilenkin
Copeland–Myers–Polchinski

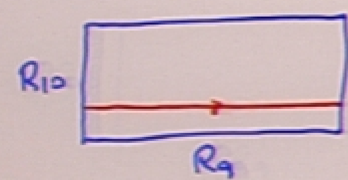
$pR_{10}$



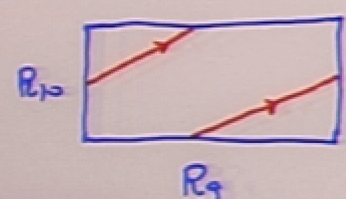

D2 wraps $x_{10}$

$p=1$ F1

$qR_9$




$q=1$ D2 wraps $x_9$




$p=1$ D2 wraps

$q=1$ both once

$$\sqrt{p^2R_{10}^2+q^2R_9^2}$$

Another way to see the origin of $(p,q)$

M theory $\qquad$ IIA $\quad R_9 \to \frac{l_s^2}{R_9} = R_9'$ $\qquad$ IIB

M2 $\qquad R_{10} \qquad$ F1 $\qquad\qquad$ F1

M2 $\qquad\qquad$ D2 $\quad$ wrap $R_9 \to$ D1

winding mode becomes momentum mode

$$\tau_{F1} = \frac{1}{2\pi l_s^2} \qquad g_A = \frac{R_{10}}{l_s}$$

$$\tau_{2A} = \frac{1}{(2\pi)^2 l_s^3 g_A} \qquad \text{wrap } x_9 \text{ with } R_9$$

$$\tau = \tau_{2A}\, 2\pi R_9 \longrightarrow \frac{1}{2\pi l_s^2 g_A}\frac{l_s}{R_9'} = \frac{1}{2\pi l_s^2 g_B}$$

$$\parallel$$

$$\frac{1}{2\pi l_s^2 g_A}\frac{R_9}{l_s}$$

$$g_B = g_A \frac{l_s}{R_9} = \frac{R_{10}}{R_9} = \frac{R_{10}R_9'}{l_s^2}$$

$$\tau_{pq} = \frac{1}{2\pi l_s^2}\sqrt{p^2 + q^2/g_B^2}$$

$$= \frac{1}{2\pi R_{10}}\sqrt{\frac{p^2 R_{10}^2}{l_s^4} + \frac{q^2}{R_9'^2}}$$

$R_9' \to \infty$

$R_{10} \to 0$

$g_B$ finite

$$\tau_{p,q} = \frac{1}{2\pi R_{10}} \sqrt{\frac{p^2 R_{10}^2}{l_s^4} + \frac{q^2}{R_q'^2}}$$

$p=0$ $\quad \tau_q = \frac{1}{2\pi R_0} \frac{q}{R_q'}$ K.K. mode in $X_q$

Suppose we compactify $X_4 \cdots X_9$

$y_i = X_{3+i}$ with radius $R_i'$

$$\tau_{p,q_1,q_2,q_3,q_4,q_5,q_6} =$$

$$\frac{1}{2\pi R_{10}} \sqrt{\frac{p^2 R_{10}^2}{l_s^4} + \sum_1^6 \frac{q_i^2}{R_i'^2}}$$

$$q \to q_6$$

$(p,\vec{q})$ $\quad \vec{q} = (q_1, q_2, q_3, q_4, q_5, q_6)$

$$(\vec{p}^2 + \mu^2)\Psi = (-\nabla^2 + \mu^2)\Psi = E^2 \Psi$$

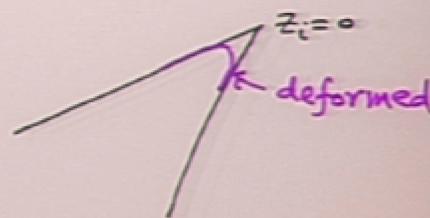
$\tau_{(p,\vec{q})} = \frac{E}{2\pi R_{10}}$ $\quad \mu = \frac{p R_{10}}{l_s^2}$

$$\vec{p} = \frac{\vec{q}}{R_i'}$$

$$ds_{10}^2 = h^{-1/2}(r)\,\eta_{\mu\nu}\,dx^\mu dx^\nu + h^{1/2}(r)\,ds_6^2$$

$$ds_6^2 = dr^2 + r^2 ds_5^2$$

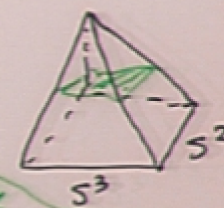
$$\sum_1^4 z_i^2 = 0$$




$$\sum_1^4 z_i^2 = \epsilon^2$$

Base :

$$\sum_1^4 |z_i|^2 = r^2$$




$\vec{z}_i = \vec{x}_i + i\vec{y}_i$ $\quad x\cdot x = r^2/2 \quad y\cdot y = r^2/2 \quad x\cdot y = 0$

non-compact $CY_3$ :

Kähler and Ricci-flat

$$T^{1,1} = \frac{SU(2)\times SU(2)}{U(1)} = \frac{S^3\times S^3}{U(1)}$$

$= S^1$ fibered over $S^2\times S^2$

$$ds_{T^{11}}^2 = \frac{1}{9}\left(d\psi + \sum_1^2 \cos\theta_i\, d\phi_i\right)^2 + \frac{1}{6}\sum_1^2\left(d\theta_i^2 + \sin^2\theta_i\, d\phi_i^2\right)$$

Candelas & de la Ossa
Klebanov Strassler
Herzog, Klebanov, Ouyang
Minasian Tsimpis, ...

find $\tau$ such that $\tau = 0$ is at the tip "$\epsilon$"

$$ds_6^2 = \frac{1}{2}\epsilon^{4/3} K(\tau)\Big[\frac{1}{3K(\tau)^3}(d\tau^2 + (g^5)^2)$$

$$+ \cosh^2\left(\frac{\tau}{2}\right)\left[(g^3)^2 + (g^4)^2\right]$$

$$+ \sinh^2\left(\frac{\tau}{2}\right)\left[(g^1)^2 + (g^2)^2\right] \Big]$$

$$K(\tau) = \frac{(\sinh(2\tau) - 2\tau)^{1/3}}{2^{1/3}\sinh\tau} \xrightarrow{\tau\to 0} (2/3)^{1/3}, \quad \xrightarrow{\tau\to\infty} e^{-\tau/3}$$

$$h(\tau) = 2^{2/3}(g_s M\alpha')^2 \epsilon^{-8/3} I(\tau)$$

$$I(\tau) = \int_\tau^\infty dx \, \frac{x\coth x - 1}{\sinh^2 x}(\sinh(2x) - 2x)^{1/3} \xrightarrow{\tau\to 0} 0.718, \quad \xrightarrow{\tau\to\infty} (\tau - \tfrac{1}{4})e^{-\frac{4\tau}{3}}$$

$$h(0) = e^{+2\pi K/3Mg_s}$$

$$h(r) \doteq 1 + \frac{L^4}{r^4}$$

$$-\partial_r^2\psi + V_{eff}(r, E, p, j, \ell)\psi = 0$$

$$\tau(p, n, j, \ell', m', \ell, m)$$

$$ds_{10}^2 = h^{-1/2}(r)\,\eta_{\mu\nu}\,dx^\mu dx^\nu + h^{1/2}(r)\,ds_6^2$$

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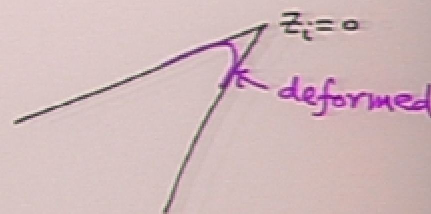


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Base:

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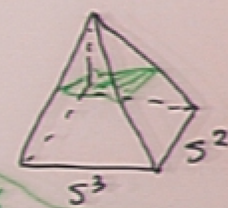


$\vec{z}_i = \vec{x}_i + i\,\vec{y}_i$ $\quad x\cdot x = r^2/2 \quad y\cdot y = r^2/2 \quad x\cdot y = 0$

non-compact $CY_3$ :

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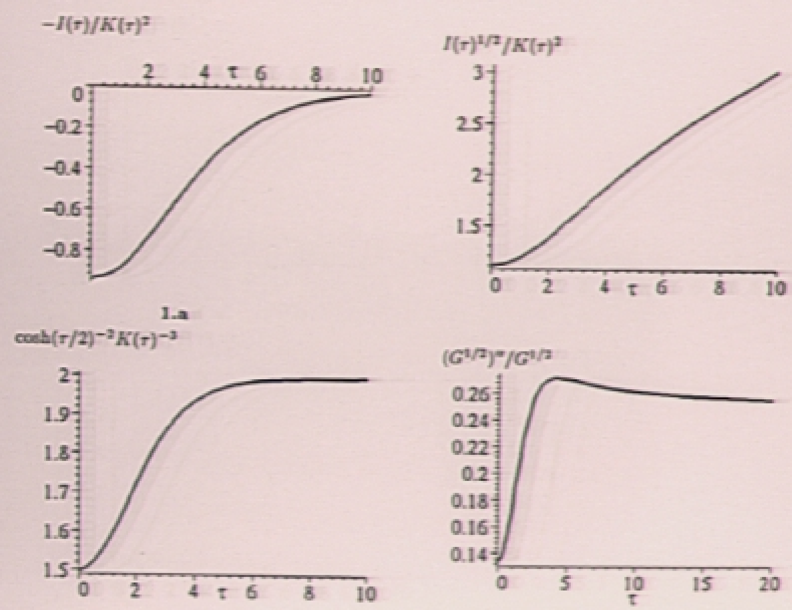
Candelas & de la Ossa
Klebanov Strassler

$$V_{eff} = \frac{I(\tau)}{K(\tau)^2}\left(-\bar{E}^2 + \frac{\bar{\mu}^2}{I(\tau)^{1/2}}\right) + \frac{j^2-1}{3\,K(\tau)^3\cosh^2(\frac{\tau}{2})} + \frac{l(l+1)}{3\,K(\tau)^c\sinh^2(\frac{\tau}{2})} + \frac{(G^{1/2})''}{G^{1/2}}$$

$$G(\tau) = I(\tau)^{1/4}(\sinh(2\tau) - 2\tau)^{7/6}\cosh(\frac{\tau}{2})\sinh(\frac{\tau}{2})^{-3/2}.$$



WKB :

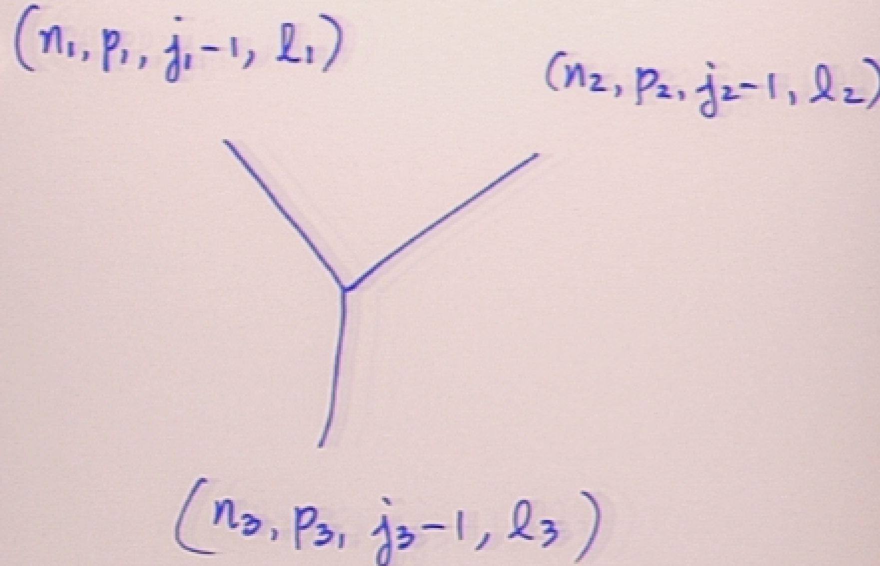
$$\tau \simeq \tau_0 \left[ n + \frac{1}{2} + \frac{\delta_{0\ell}}{4} + Q \right]$$

$$Q^2 = s p^2 + d(j^2 - 1) + f\, \ell(\ell+1) + b$$

| $n$ | $\mu$ | $j-1$ | $l$ | $E$ | IWKB |
|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 1.73 | 1.62 |
| 0 | 1 | 0 | 0 | 3.26 | 3.36 |
| 0 | 5 | 0 | 0 | 11.64 | 11.82 |
| 1 | 5 | 0 | 0 | 12.99 | 13.49 |
| 0 | 10 | 0 | 0 | 22.21 | 22.06 |
| 0 | 0 | 1 | 0 | 3.50 | 3.45 |
| 1 | 0 | 1 | 0 | 4.60 | 4.69 |
| 0 | 0 | 4 | 0 | 7.93 | 7.52 |
| 1 | 5 | 1 | 0 | 11.92 | 12.04 |
| 0 | 0 | 0 | 1 | 3.87 | 3.67 |
| 2 | 0 | 0 | 1 | 5.98 | 6.21 |
| 0 | 5 | 1 | 1 | 13.13 | 11.88 |
| 2 | 5 | 2 | 2 | 17.30 | 16.24 |

p

$$\mathcal{I}\,(n, p, j, \ell', m', \ell, m)$$


$(n_1, p_1, j_1-1, l_1)$
$(n_2, p_2, j_2-1, l_2)$
$(n_3, p_3, j_3-1, l_3)$

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(COSMIC SUPERSTRINGS)

- STABILITY

- EVOLUTION

- SPECTRUM/TENSION

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# PROPERTIES OF COSMIC STRINGS (COSMIC SUPERSTRINGS)

- STABILITY

  STABLE IN SOME CASES

  BELIEVE SOME ARE ALWAYS STABLE

- EVOLUTION

  COSMOLOGICALLY, THE COSMIC STRING NETWORK RAPIDLY APPROACHES A SCALING LIMIT

- SPECTRUM / TENSION

  ABELIAN HIGGS-LIKE VORTICES

  $\downarrow$

  (p,q) strings

  $\downarrow$

  $(\overset{F \nearrow}{p}, n, \underbrace{j, l', m'}_{S^3}, \underbrace{l, m}_{S^2})$ strings

- PRODUCTION