Title: String Cosmology

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Abstract:

# BRANE INFLATION: FROM SUPERSTRINGS TO COSMIC STRINGS

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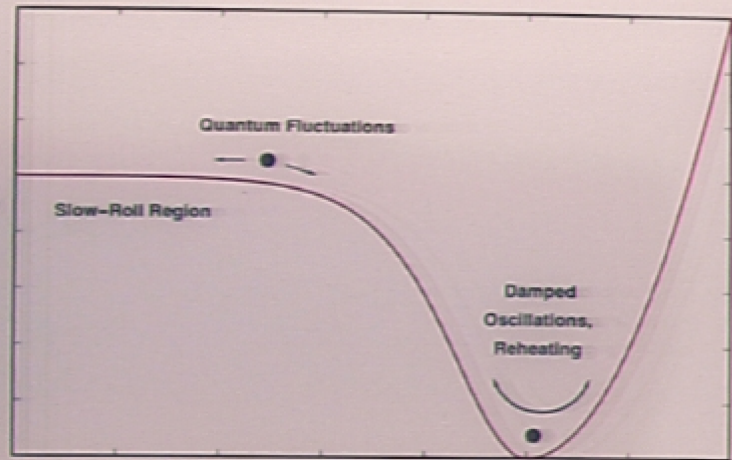
6/23/05

(1)
- INFLATION IN COSMOLOGY
- BRANE INFLATION
- KKLMMT SCENARIO

(2)
- COSMIC STRINGS
- PROPERTIES OF COSMIC STRINGS

(3) DETECTION / TEST




$$H = \frac{\dot{a}}{a}, \quad a(t) = cosmic\ scale\ factor$$

$$H^2 = V(\phi) + \frac{K}{a^2} + \rho, \quad \rho = \begin{cases} 1/a^3 \\ 1/a^4 \end{cases}$$

$$V(\phi) = constant \Longrightarrow H^2 = constant, \quad a(t) = e^{Ht}$$

1. density perturbation $\frac{\delta\rho}{\rho} \simeq 10^{-5}$
2. fall in and damping $\rightarrow$ reheat ← preheat
3. $\delta(k) = k^{n-1}$, $n \sim 1.0 \pm 0.1$

INFLATION:

$n \simeq 1$

$K = 0$

## Benefits of Inflation

Inflation solves a number of problems in Cosmology:

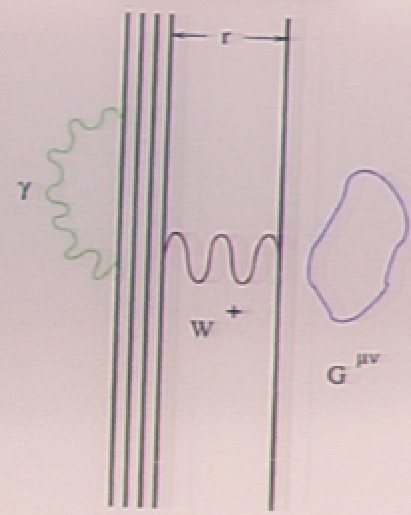
1. Flatness Problem
2. Horizon Problem
3. Defects Overabundance

- Quantum fluctuation of the scalar field during inflation appear as temperature fluctuations of the CMB.
- They also act as seeds for structure formation.
- One signature of inflation is that the spectrum of the density fluctuation is scale-invariant (Harrison-Zeldovich spectrum).
- Tensor mode fluctuations result in background of gravitational waves

### Origin of Inflation

- What is the Inflaton, $\phi$ ?
- Where is the potential $V(\phi)$ coming from ?

## Brane World




**Why brane world?**

- Naturally suggested by string theory;
- Can obtain Standard Model Particles and the graviton: brane separation $\leftrightarrow$ particle mass
- String theory yields many realistic models
- gravity is everywhere (graviton is a closed string), but electrons, photons etc. are open string modes

# Candidates for Inflaton

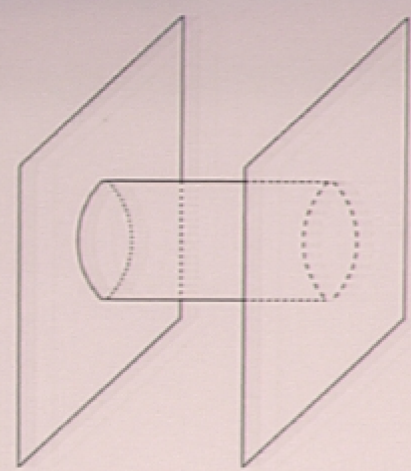
**Light scalar modes in the brane world:**

**Closed string modes**

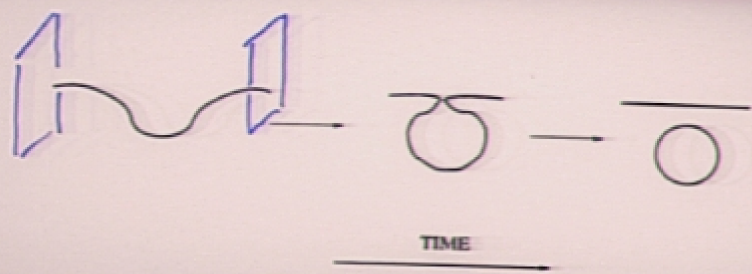
- Radion/dilaton: gravitational strength couplings

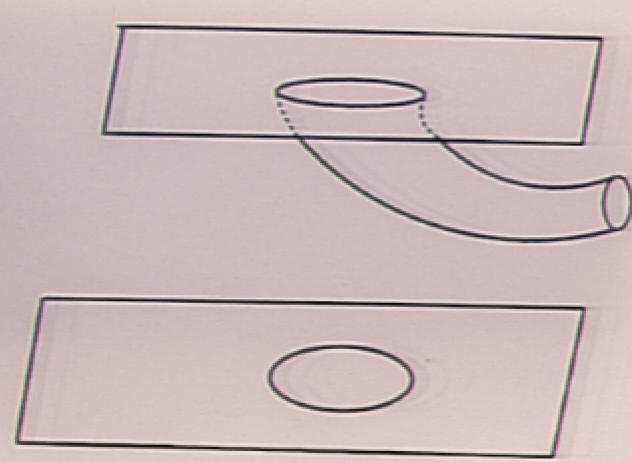
**Open string modes**

- Tachyons: their potential is too steep
- Brane separation as inflaton: brane interaction $\rightarrow$ relatively flat potential



The inflaton: The open string 1-loop dual to closed string exchange. The potential of the inflaton has gravitational strength.



Graviton emission

Quantum processes are dual to tree-level ones. For example the open-string 1-loop is dual to the graviton emission by a standard model particle. Classical gravity is dual to quantum effect in field theory.

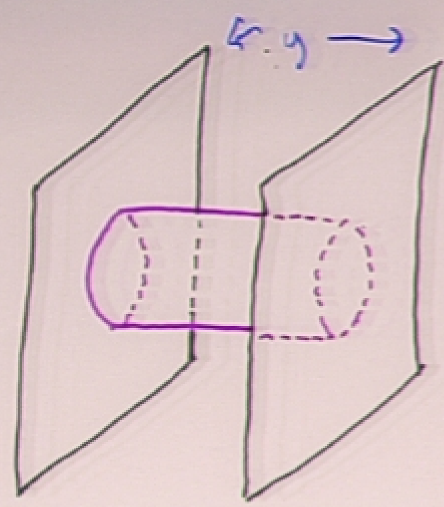
2 parallel branes: open string spectrum is supersymmetric

$$\sum_{\substack{b,f \\ M}} (-1)^f \int dk^l \frac{1}{k^2+M^2}$$

$$\rightarrow \sum_M \int dt \int dk^l \, e^{-(k^2+M^2)t} = 0$$

$V(y) = 0$ for DpDp

$Dp$-$\bar{D}p$ brane pair $V(y) \neq 0$



closed string channel

$$V(y) = -\int_0^\infty \frac{ds}{s}\left(\frac{16\pi^3\alpha'}{s}\right)^{-(p+1)/2} e^{-y^2/s\alpha'} \left(\frac{2\pi}{s}\right)^4$$

$$16\prod_{m=1}^{\infty} \frac{(1+w^m)^8}{(1-w^m)^8} \qquad w = e^{-s}$$

At large $y$, integral dominated by large $s$ (ie, long cylinder), so

$$V(y) = -\frac{8\pi G_{10} T_p^2}{\pi^{(9-p)/2}} \Gamma\left(\frac{7-p}{2}\right) \frac{1}{y^{7-p}}$$

$p=3$ $\qquad V(y) \simeq -\frac{1}{y^{7-p}}$

Bachas

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Write $V(y)$ in terms of open string 1-loop :

## $\boxed{\text{V(y)}}$

Open string one-loop :

$$V(y) \simeq -V_{p+1} \int_0^\infty \frac{dt}{t} (8\pi^2 \alpha' t)^{-(p+1)/2} e^{-t(\frac{y^2}{2\pi\alpha'})} Z(t)$$

as $y \to \infty$,

$$\to -1/y^{7-p}$$

$$Z(t) = \frac{1}{q^{1/2}} \prod_{m=1}^{\infty} \frac{(1-q^{m-1/2})^8}{(1-q^m)^8} \qquad q = e^{-2\pi t}$$

Closed string exchange:

The potential has the large-n behavior ($p = 4$):

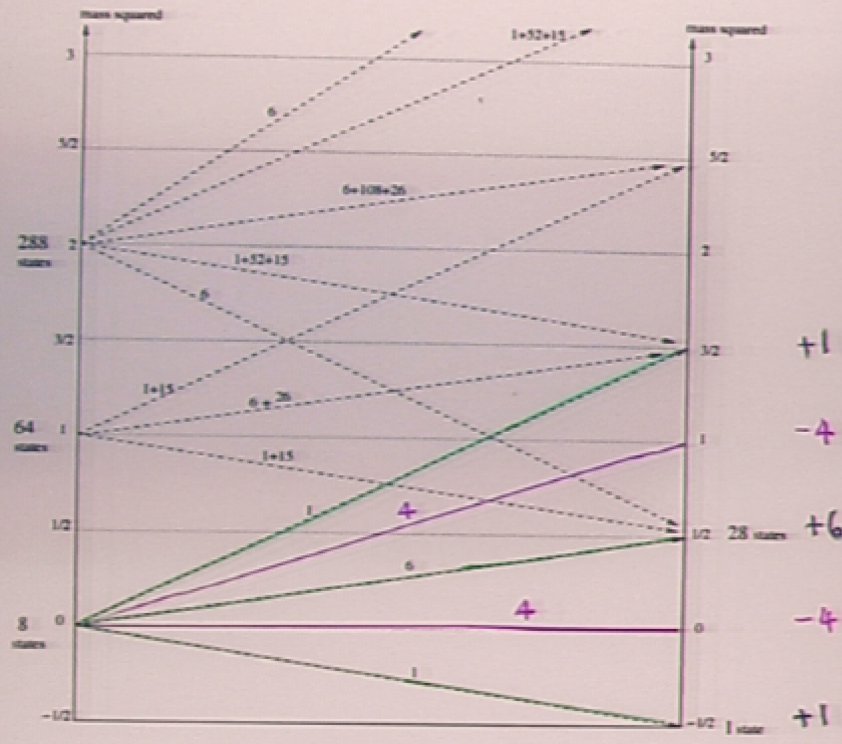
$$V(y) \simeq -\frac{1}{y^2} \int_0^\infty \frac{dn}{n^{9/4}} e^{2\sqrt{n}(\pi\sqrt{2} - y/\sqrt{\alpha'})}$$

It diverges exactly at the tachyon mass threshold.

$$m_T^2 = (\frac{y}{2\pi\alpha'})^2 - \frac{1}{2\alpha'} < 0$$

Open string one-loop :

$$Z(q = e^{-2\pi t}) = \frac{1}{q^{1/2}} - 8 + 36q^{1/2} - 128q + 402q^{3/2} - 1152q^2$$
$$+3064q^{5/2} - 7680q^3 + 18351q^{7/2} - 42112q^4$$
$$+93300q^{9/2} - 200448q^5 + \dots$$

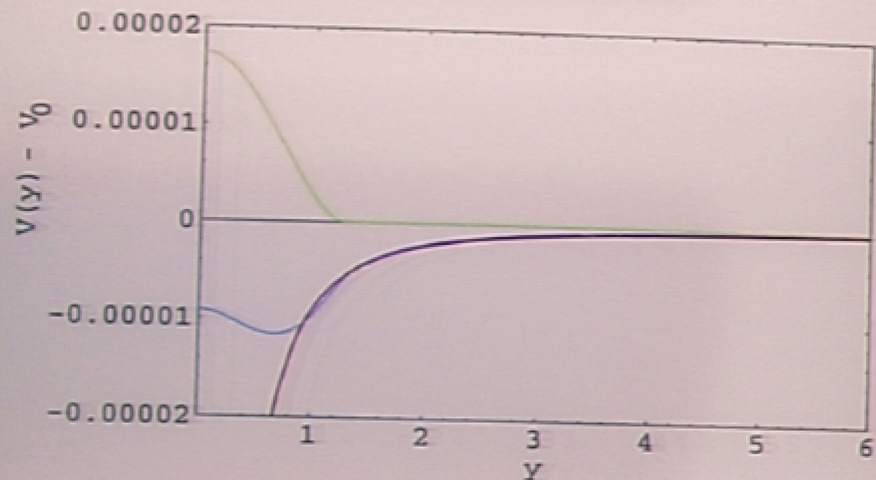



The soft SUSY-breaking →

$$\sum_i (-1)^F m_i^{2n} = 0, n = 1, 2, 3$$

Keeping the supermultiplet, we get a finite potential and an imaginary part from the tachyon one-loop contribution to the effective potential.

## Graceful Exit from Inflation

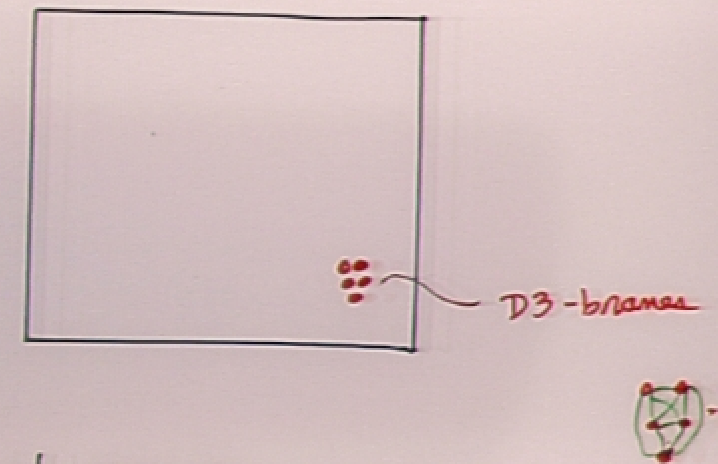



$$V_{1-\text{loop}} = \frac{-1}{(8\pi^2\alpha')^2}\int_0^\infty \frac{dt}{t^3}\sum_i (-1)^{F_i}\, e^{-2\pi\alpha' t\, m_i^2}$$

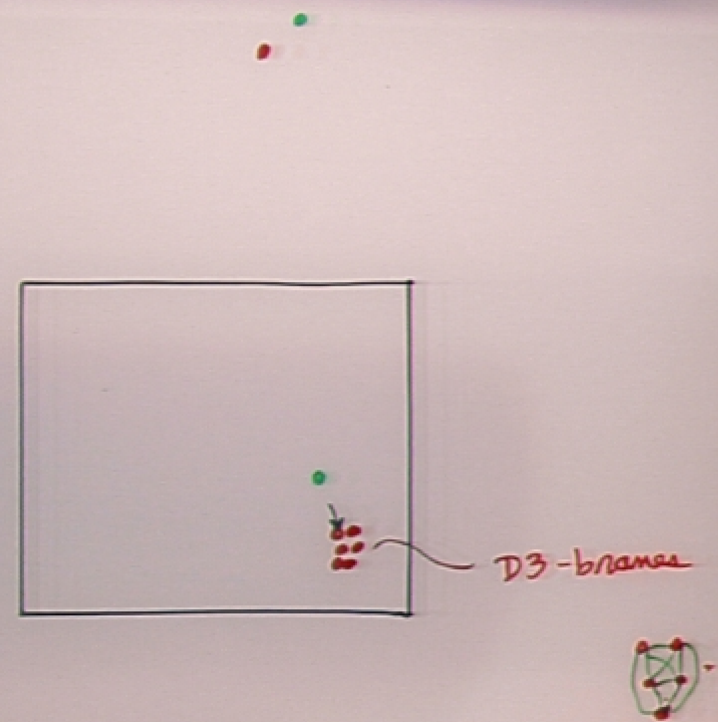
$$= \frac{1}{64\pi^2}\sum_i (-1)^{F_i}\, m_i^4 \log m_i^2 \,,$$

J. Garcia-Bellido, R. Rabadan and F. Zamora, hep-th/0112147

S. Sarangi and H.T. hep-th/0307078

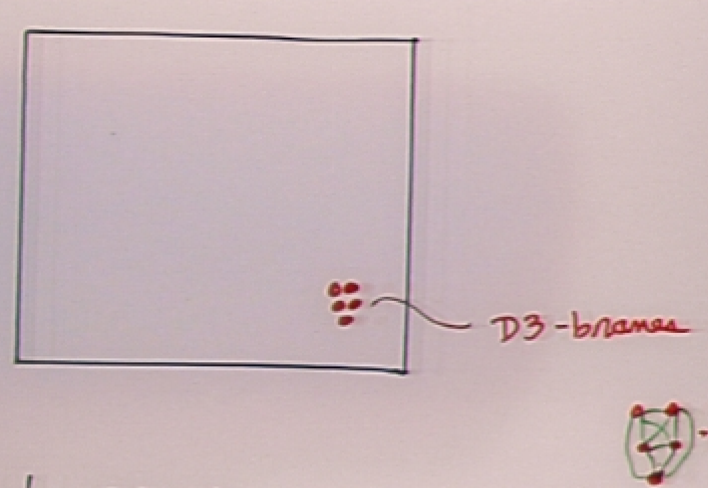



Today's vacuum



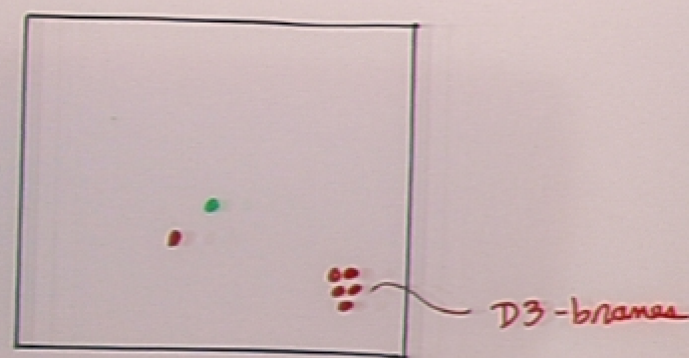

Today's vacuum

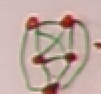
$$V(r) = 2T_3 - \frac{c}{y^4}$$




Today's vacuum

# BRANE INFLATION



Today's vacuum

$$V(r) = 2T_3 - \frac{C}{y^4}$$

$$\Gamma = \int d^4x \sqrt{g} \left\{ \frac{T_3}{2}\left[\partial_\mu y_1 \partial^\mu y_1 + \partial_\mu y_2 \partial^\mu y_2\right] - V(y) + \cdots \right\}$$

$$y = y_1 - y_2$$

$$T_3 = \frac{1}{(2\pi)^3 g_s \alpha'^2} = \frac{M_s^4}{(2\pi)^3 g_s}$$

$$V(y) = 2T_3 - \frac{\kappa^2 T_3^2}{2\pi^2 y^4}$$

$$\kappa^2 = 8\pi G_{10} = \frac{g_s^2 (2\pi)^7}{2 M_s^8}$$

$$\phi = \sqrt{\frac{T_3}{2}}\, y \qquad V(\phi)$$

$$\Gamma \simeq \int d^4x \sqrt{g} \left\{ \frac{R}{16\pi G_4} + \frac{1}{2}\partial_\mu \phi \partial^\mu \phi - V(\phi) \right\}$$

$$G_4 = G_{10} V_6$$

Scales :

2 parameters : $M_s$ , $V_6$

$$M_s^8 V_6 \simeq M_p^2$$

$$\delta_H \sim 10^{-5} \sim \frac{M_s^2}{M_p^2}$$

so $\dfrac{M_s}{M_p} \sim 10^{-3}$

$$H^2 \sim \frac{M_s^4}{M_p^2}$$

$$\frac{H}{M_s} \sim \frac{M_s}{M_p} \sim 10^{-3}$$

so Effective SUGRA is valid

For cosmological purposes,

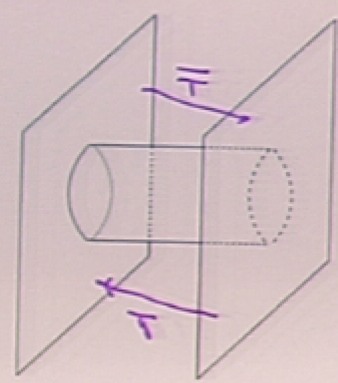
4D Effective theory is valid .

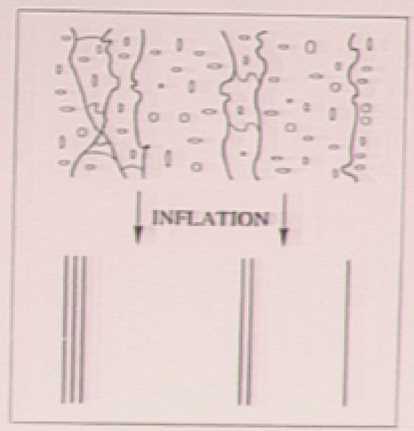
$D3-\overline{D3}$ breaks SUSY, why SUGRA formulation is valid

$$\sum_{B,F} (-1)^{F_i} m_i^{2n} = 0 \qquad n = 1, 2, 3.$$

## Brane Inflation

brane separation as the inflaton






**Inflation "smooths out" the wrinkles.**

$$\nabla_r^2 G(r,r') = \delta(r-r')$$

$r'=0$

$$V(r) = T_1 T_2\, G(r)$$

In a compact manifold $M$ ( $\partial M = 0$ )

$$\nabla_r^2 G(r) = \delta(r) - \frac{1}{V_{ol}}$$

$$\int d\mathrm{vol}\ \nabla_r^2 G(r) = \int d\vec{S}\cdot\vec{\nabla} G = 0$$

$$\int d\mathrm{vol}\left[\delta(r) - \frac{1}{V_{ol}}\right] = 1 - 1 = 0.$$

$\hookrightarrow$ jellium $\quad G(r) \simeq \frac{1}{V_{ol}}\frac{r^2}{d}$

$$V(y) = 2T_3 - \frac{8\pi G_{10} T_3^2 \beta}{y^4} + Cy^2$$

$\phi = \sqrt{T_3}\, y$

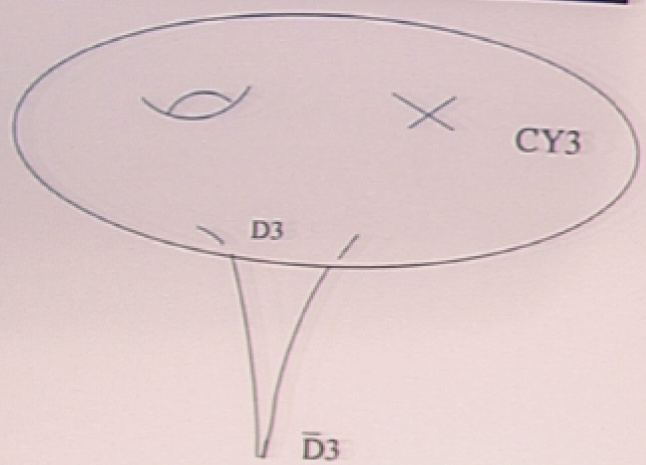
$$\eta = M_P^2 \frac{V''}{V} = -2/d_\perp = -1/3$$

KKLMMT

$$N_e \sim \frac{1}{|\eta|} \quad ?$$

One has to be careful.

## Inflation in Realistic String Vacua




Giddings, Kachru and Polchinski hep-th/0105097
Kachru, Kallosh, Linde and Trivedi hep-th/0301240
Kachru, Kallosh, Linde, Maldacena, McAllister and Trivedi hep-th/0308055
Firouzjahi and H.T., hep-th/0312020

Other scenarios are emerging.

To study open string tachyon dynamics, we need an effective field theory in string theory.

Boundary Superstring Field Theory
BSFT

It extends the $\sigma$-model approach to string theory, in that the disc world-sheet partition function with appropriate boundary insertions gives the classical spacetime action.

Witten
Kutasov, Marino, Moore
Kraus Larsen
Takayanagi, Terashima, Uesugi
Jones HT

$$S_{D\bar{D}} = -\tau_9 \int d^{10}x \; e^{-2\pi\alpha' T\bar{T}} \sum_{i=1,2} \sqrt{|G_i|}\, \mathcal{F}(X_i+\sqrt{Y_i})\,\mathcal{F}(X_i-\sqrt{Y_i})$$

$$(G_{\mu\nu})_{1,2} = g_{\mu\nu} + 2\pi\alpha' F^{1,2}_{\mu\nu}$$

$$X_i = 2\pi\alpha'^2 G_i^{\mu\nu} D_\mu T\, \overline{D_\nu T} \qquad D_\mu T = \partial_\mu T + i(A^1_\mu - A^2_\mu)T$$

$$Y_i = |2\pi\alpha'^2 G_i^{\mu\nu} D_\mu T D_\nu T|^2$$

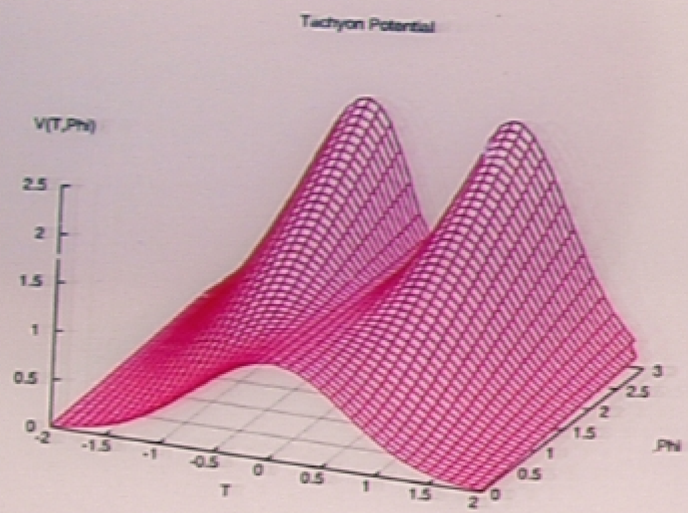
$$\mathcal{F}(x) = \frac{\sqrt{\pi}\,\Gamma(1+x)}{\Gamma(\frac{1}{2}+x)} = \frac{4^x x\, \Gamma(x)^2}{2\Gamma(2x)}$$

$$\mathcal{F}(0) = 1 \qquad \mathcal{F}(-\tfrac{1}{2}) = 0 \qquad \mathcal{F}(x) \longrightarrow \sqrt{\pi x}$$

# Effective Brane-antibrane action from BSFT

$$\mathcal{L} = -2\tau_p V(y) e^{-2\pi\alpha'|T|^2} \mathcal{F}\left(\frac{1}{\pi} y^2 |T|^2\right) \mathcal{F}\left(4\pi\alpha'^2 \partial_\mu T \partial^\mu \overline{T}\right) - \frac{\tau_p}{2} \partial_\mu y \partial^\mu y$$

$$\mathcal{F}(z) = \frac{\sqrt{\pi}\,\Gamma(z+1)}{\Gamma(z+1/2)}$$



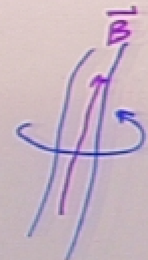

Tunneling results in bubble nucleation → old inflation.

Slow-roll ends before $m_T^2$ becomes zero, and the tachyon rolls fast → bubble nucleation is suppressed.

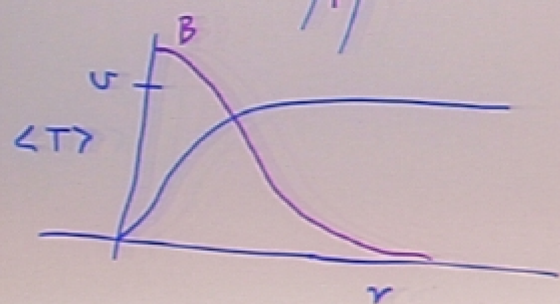
N. Jones and H.T., hep-th/0211180 Hybrid Inflation

# Abelian Higgs Model

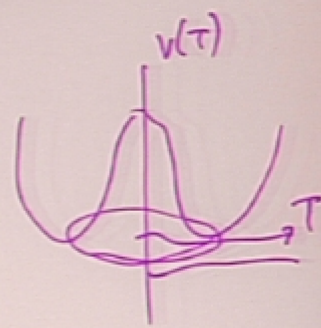
$$\mathcal{L} = \frac{1}{2}\overline{D_\mu T}\, D^\mu T - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{\lambda}{4}(\bar{T}T - v^2)^2$$




$$T = v e^{i\phi}$$




$\mu(\lambda, e, v)$




$$\pi_1(U(1)) = \mathbb{Z}$$

$$T = u \prod_{i=1}^{n} (z - z_i) \prod_{j=1}^{m} (\bar{z} - \bar{z}_j)$$

RR charge :

$$\mu_{p-2} = \tau_{p-2} g_s (n-m)$$

$u \to \infty$

$$\epsilon_{p-2} = (n+m)\, \tau_{p-2}$$

$$\epsilon \to \quad \tau_{p-2} \sum_{i=1}^{n} \frac{1}{\sqrt{1-v_i^2}}$$

(1) Dp-branes have zero classical width.

(2) These solitons have the correct tension and RR charge to be D-branes.

(3) Parallel vortices are BPS with respect to each other.

$D3-\overline{D3}$

$U(1) \times U(1) \longrightarrow U_+(1) \times U_-(1)$

$\downarrow$

$T$

T behaves as the Higgs field in $U_-(1)$ abelian Higgs theory:

vortex solutions

Using BSFT: tension and RR charge of this vortex is that of D1-brane.

D-string $\quad U(1) \times U(1) \longrightarrow U_+(1)$

---

$N\, D3 + (N+M)\, \overline{D3}$

$U(N) \times U(N+M) \longrightarrow U(N) \times U(M)$

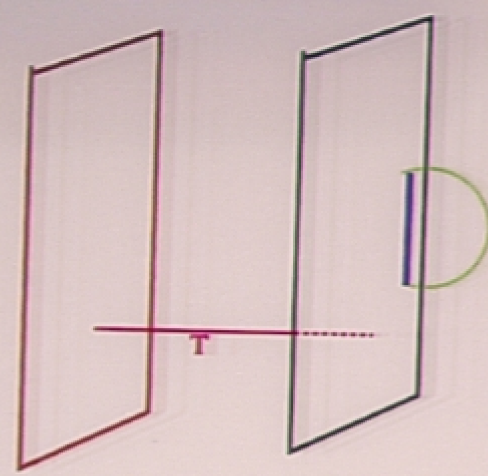
$M=0$ : vacuum manifold:

$$\mathcal{V} = \frac{U(N) \times U(N)}{U(N)}$$

which is topologically equivalent to $U(N)$ and supports co-dim $2e_2$ defects.

Tachyon energy to gravitons (i.e., closed string modes):

- gravitational strength
- $\Gamma \simeq \frac{M_s}{g_s} g_s \to$ finite
- standard model coupling strength




Sen, Yi, Hori, Gibbons, Strominger, .....

PROPERTIES OF COSMIC STRINGS
(COSMIC SUPERSTRINGS)

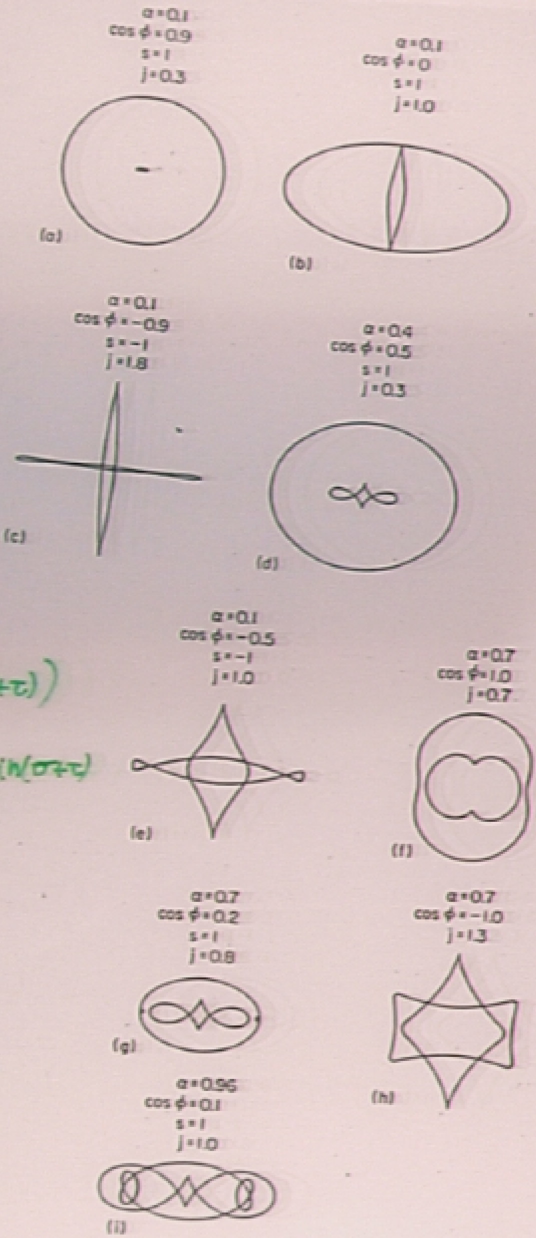
- STABILITY

- EVOLUTION

- SPECTRUM

$$\left(X_L^{\mu}(\sigma-\tau),\ X_R^{\mu}(\sigma+\tau)\right)$$

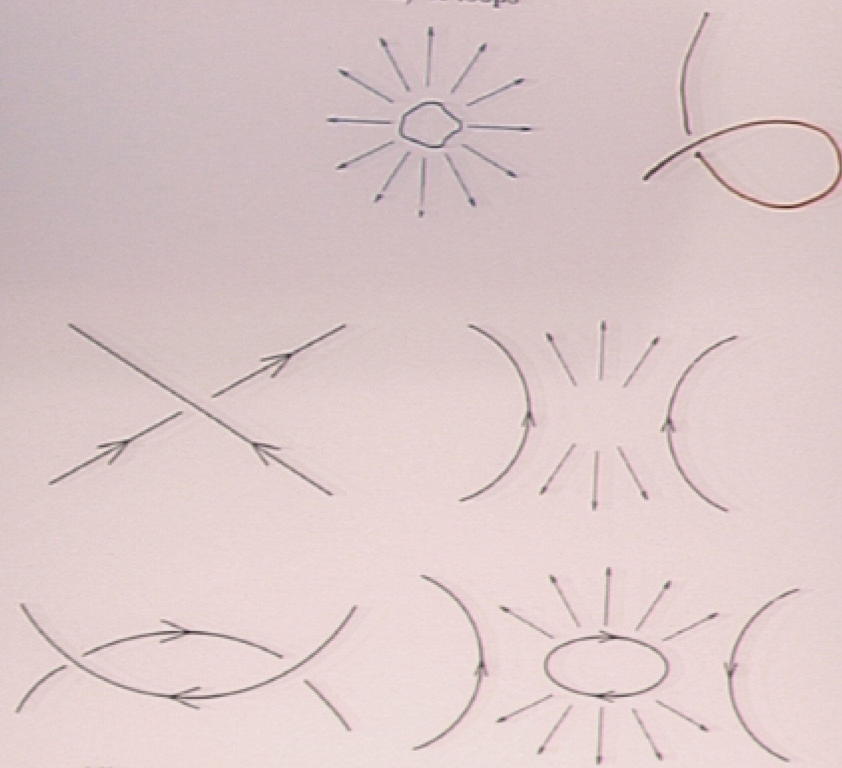
$$e^{\pm in(\sigma-\tau)} \qquad e^{\pm in(\sigma+\tau)}$$




Turok

## Cosmic String Evolution

- Emission of gravitational waves from loops
- Intercommuting
- Formation (and reconnection) of loops



Albrecht, Turok, Bennett, Bouchet, Allen, Shellard, ...

# COSMIC STRING EVOLUTION

$$\rho \simeq \frac{\mu}{L^2} \qquad L(t)$$

$$\dot{\rho} = -2\frac{\dot{a}}{a}\rho - \lambda\frac{\rho}{L}$$

$\lambda = 0$ $\qquad \rho \sim \mu/a^2$

$\lambda \neq 0$ $\qquad$ let $\quad L(t) = \gamma(t)\, t$

$$\dot{\gamma} = -\frac{1}{2t}(\gamma - \lambda)$$

$$\gamma \longrightarrow \lambda$$

$$\rho \simeq \frac{\mu}{\lambda^2 a^4}$$

$$\frac{\rho}{\rho_{radiation}} \simeq G\mu \simeq 10^{-6}$$

↳ independent of $\rho_0$

Kibble
Vilenkin

$\lambda = 1$ $\qquad$ abelian Higgs

$\lambda > 1$ $\qquad$ extra dim

$$\Omega_{c.s} \simeq 10\, G\mu$$

PROPERTIES OF COSMIC STRINGS
(COSMIC SUPERSTRINGS)

- STABILITY

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- SPECTRUM