

Title: QFT in Curved Space

Date: Jun 23, 2005 02:30 PM

URL: <http://pirsa.org/05060084>

Abstract:

Recall: Symplectic Basis for sol^s of $(\square - m^2)\phi = 0$

$$U = \{u_n : n \in \mathbb{Z}, n \neq 0\} \quad U_n = U_{-n}^*$$

$$U_n \circ U_n^* = -1$$

defines

$$|0\rangle_U$$

$$\psi \in T_{\text{symplectic}} \Rightarrow |0\rangle_U = N \exp\left(-\frac{i}{2} \sum_{n \neq 0} (\alpha_n^\dagger \beta_n) a_n^\dagger a_n\right) |0\rangle_V$$

$$\psi_U = \sum_{n \neq 0} (\alpha_n u_n - \beta_n u_n^*) \Rightarrow \alpha^\dagger \alpha - \beta^\dagger \beta = 1$$

$$N = (1 - |\alpha^\dagger \beta|^2)^{1/4} = |\alpha|^{-1/2} = \left(\sum \alpha_n^\dagger \alpha_n\right)^{-1/4}$$

Bogoliubov

Transformation

Recall: Symplectic Basis for sol's of $(\square - m^2)\phi = 0$

$$n \in \mathbb{Z}, n \neq 0 \quad U_n = U_{-n}^\dagger$$

$$U_n \circ U_n^\dagger = -1 \quad \text{for } n > 0$$

$$\text{For } n < 0, U_n \circ U_n^\dagger = +1$$

$$|0\rangle_V$$

$$|0\rangle_V = N \exp\left(-\frac{i}{2} \sum_{n,m} (\alpha^\dagger \beta)_{nm} a_n^\dagger a_m^\dagger\right) |0\rangle_V$$

$$p_{nm} V_{nm}^\dagger \Rightarrow \alpha^\dagger \alpha - \beta^\dagger \beta = 1$$

$$N = |\alpha|^{-1/2} = \left(\sum \alpha_n^\dagger \alpha_n\right)^{-1/4}$$

ubov Transformation

Recall: Symplectic Basis for sol's of $(\square - m^2)\phi = 0$

$$\{u_n, n \in \mathbb{Z}, n \neq 0\} \quad U_n = U_{-n}^*$$

$$U_n \circ U_n^* = -1 \quad \text{for } n > 0$$

$$\text{For } n < 0, U_n \circ U_n^* = +1$$

$$N \exp\left(-\frac{i}{2} \sum_{n,m} (\alpha^\dagger \beta)_{nm} a_n^\dagger a_m^\dagger\right) |0\rangle$$

$$\Rightarrow \alpha^\dagger \alpha - \beta^\dagger \beta = \mathbb{1}$$

$$= \left(\sum_n \alpha_n^\dagger \alpha_n\right)^{-1/4}$$

Bob Transformation

Recall: Symplectic Basis for sol's of $(\square - m^2)\phi = 0$

$$U = \{u_n : n \in \mathbb{Z}, n \neq 0\} \quad U_n = U_{-n}^*$$

$$U_n \circ U_n^* = -1 \quad \text{for } n > 0$$

$$\text{For } n < 0, U_n \circ U_n^* = +1$$

Defines

$$|0\rangle_U$$

$$\tilde{U} \& \tilde{V} \text{ symplectic} \Rightarrow$$

$$|0\rangle_U = N \exp\left(-\frac{i}{2} \sum_{n,m} (\alpha^\dagger \beta)_{nm} a_n^\dagger a_m^\dagger\right) |0\rangle_V$$

$$\psi_U = \sum_{n \geq 0} (\alpha_n a_n - \beta_n a_n^*) \Rightarrow \alpha^\dagger \alpha - \beta^\dagger \beta = 1$$

$$N = (1 - |\alpha^\dagger \beta|^2)^{1/4} = |\alpha|^{-1/2} = \left(\sum \alpha_n^\dagger \alpha_n\right)^{-1/4}$$

Bogoliubov Transformation

$$U^0 U^0 = -1$$

$$U^3 U^3 = +1$$

Split of sol's into
pos. & neg. Freq.'s.

$$U U^\dagger = -1$$

$$U^\dagger U = +1$$

Split of sol's into
pos. & neg. Freq.'s.

Suppose on some Σ ,

$$[n, \partial] u = +i\omega u$$

$$[n, \partial] u^\dagger = -i\omega u^\dagger$$

$$U \circ U^\dagger =$$

" $\bar{u} \sim e^{i\omega t}$ "

$$U \circ U^\dagger = -1$$

$$U^\dagger \circ U = +1$$

Split of sol's into
pos. & neg. Freq. 's.

Suppose on some Σ ,

$$[n, \partial] u = +i\omega u$$

$$[n, \partial] u^\dagger = -i\omega u^\dagger$$

$$U \circ U^\dagger = (-i) \int \mathcal{E} \sqrt{h_0} \quad U(n, \partial) u^\dagger = f.$$

" $U \sim e^{i\omega t}$ "

$$U \circ U^\dagger = -1$$

$$U^\dagger \circ U = +1$$

Split of sol's into
pos. & neg. Freq.'s.

suppose on some Σ ,

$$[n, \partial] u = +i\omega u$$

$$[n, \partial] u^* = -i\omega u^*$$

$$U \circ U^* = (-i) \int \mathcal{E} \sqrt{h_g} \quad U(n^\mu) u^* = (i)$$

" $u \sim e^{i\omega t}$ "

$$\boxed{U \circ U^\dagger = -1}$$

$$U^\dagger \circ U = +1$$

Split of sol's into
pos. & neg. Freq. 's.

Suppose on some Σ ,

$$[n, \partial] u = +i\omega u$$

$$[n, \partial] u^\dagger = -i\omega u^\dagger$$

$$U \circ U^\dagger = (-i) \int \mathcal{E} \sqrt{h_\Sigma} U(n, \partial) u^\dagger = (-i) (-2i\omega) \int \mathcal{E} \sqrt{h_\Sigma} u^\dagger < 0$$

$\Rightarrow q_u$ is an annihilation op, $q_u^\dagger = -q_{u^\dagger} = \text{creation op.}$

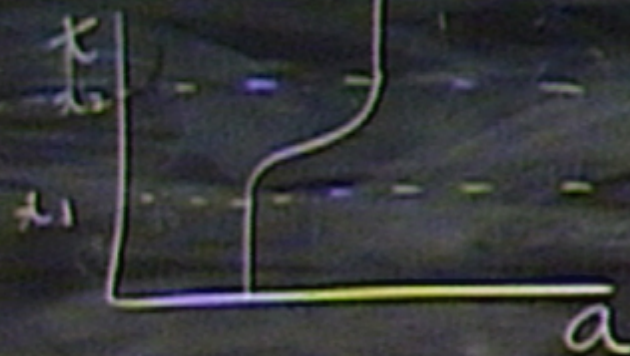
Qualitative Example $d=4$

FRW

$T_{\mu\nu}$

Qualitative Example

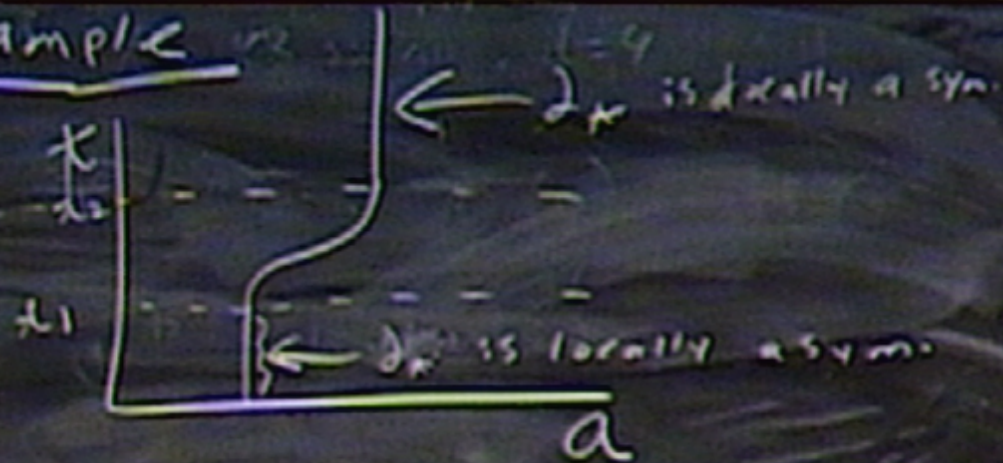
FRW



$\Gamma_{\Sigma} UV$

Qualitative Example

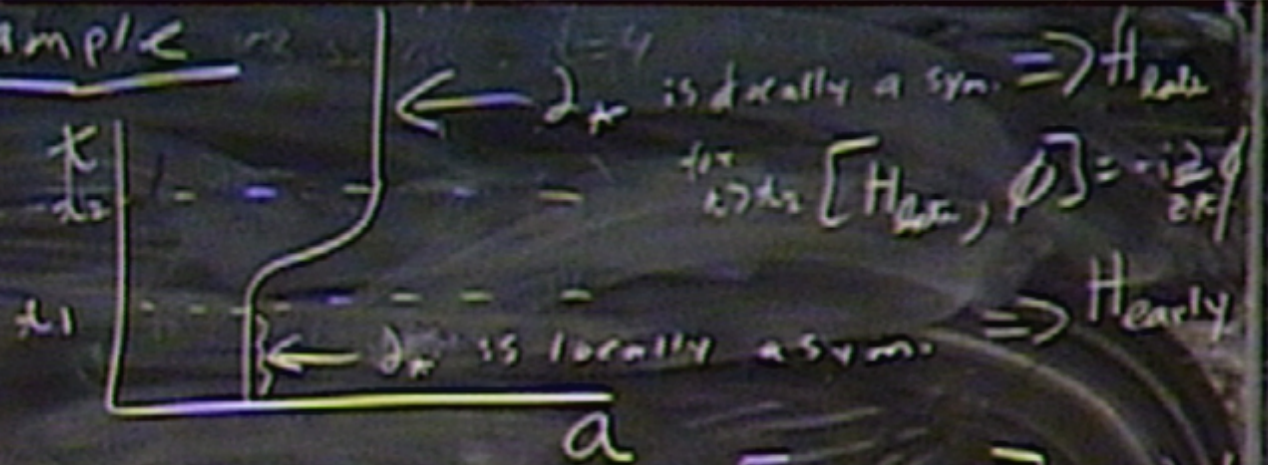
FRW



$\overline{h}_g$ UV^*

Qualitative Example

FRW



$$\text{for } x > x_2 \quad [H_{\text{early}}, \phi] = -i \frac{\partial}{\partial x} \phi$$

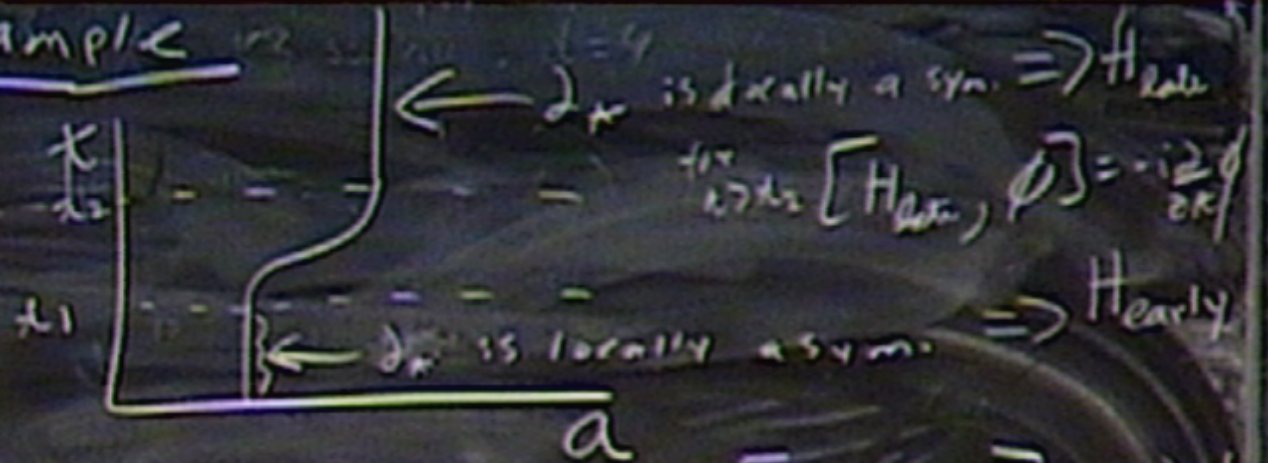
$$[H_{\text{early}}, \phi(x)] = -i \frac{\partial}{\partial x} \phi$$

for $x < x_1$

$\Gamma_{h_2} UV^*$

Qualitative Example

FRW



$$[H_{\text{early}}, \phi(x)] = -i \frac{\partial}{\partial t} \phi$$

for $t < t_1$

$\Gamma_{hg} UV^*$

Consider $\mathcal{U}_{\text{early}}$

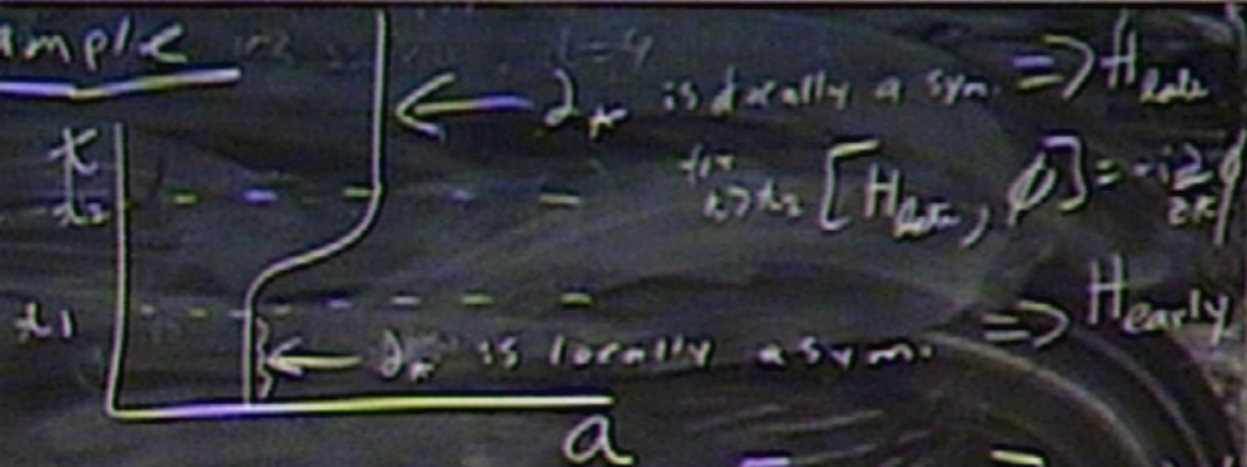
S.T.

$$\partial_t \mathcal{U}_n^{\text{early}} = i \omega_n \mathcal{U}_n^{\text{early}} \text{ for } t < t_1$$

$$\omega_n = -\omega_{-n}$$

Qualitative Example

FRW



$$\text{for } x > x_2 \quad [H_{\text{late}}, \phi] = -i \frac{\partial}{\partial \tau} \phi$$

$$\text{for } x < x_1 \quad [H_{\text{early}}, \phi(x)] = -i \frac{\partial}{\partial \tau} \phi$$

$\Gamma_{hg} \quad UV^*$

Consider $\mathcal{V}_{\text{early}}$ S.T.

$$\partial_{\tau} \mathcal{V}_n^{\text{early}} = i \omega_n \mathcal{V}_n^{\text{early}} \text{ for } t < t_1$$

$$H_{\text{early}} = \sum_{n>0} \omega_n a_n^{\dagger} a_n$$

$$\omega_n = -\omega_{-n}$$

Also ψ^{late} same @ late times
 $\partial_t \psi_n^{\text{late}} = -i\omega_n^{\text{late}} \psi$

$\omega_n > 0$ for $n > 0$

Also ψ^{late} same @ late times
 $\partial_t \psi_n^{\text{late}} = -i\omega_n^{\text{late}} \psi$

$\omega_n > 0$ for $n > 0$

$\psi^{\text{late}} \neq \psi^{\text{early}}$

solve by sep. of var.'s

$$\psi(x,t) = \hat{U}(t) \tilde{\psi}(x)$$

Also ψ^{late} same @ late times
 $\partial_t \psi_n^{\text{late}} = i\omega_n^{\text{late}} \psi$

$\omega_n > 0$ for $n > 0$

$\psi^{\text{late}} \neq \psi^{\text{early}}$

solve by sep. of var.'s

$$\psi(x,t) = \hat{\psi}(t) \tilde{\psi}(x)$$

$$\frac{\partial^2}{\partial x^2} \hat{\psi}(x) = -\omega^2(t) \hat{\psi}(x)$$

Also ψ^{late} same @ late times
 $\partial_t \psi_n^{\text{late}} = i \omega_n^{\text{late}} \psi$

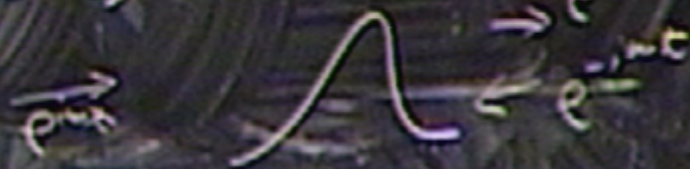
$\omega_n > 0$ for $n > 0$

$\psi^{\text{late}} \neq \psi^{\text{early}}$

solve by sep. of var.'s

$$\psi(x,t) = \hat{\psi}(t) \tilde{\psi}(x)$$

$$\frac{\partial^2}{\partial x^2} \hat{\psi}(x) = -\omega^2(t) \hat{\psi}(x)$$

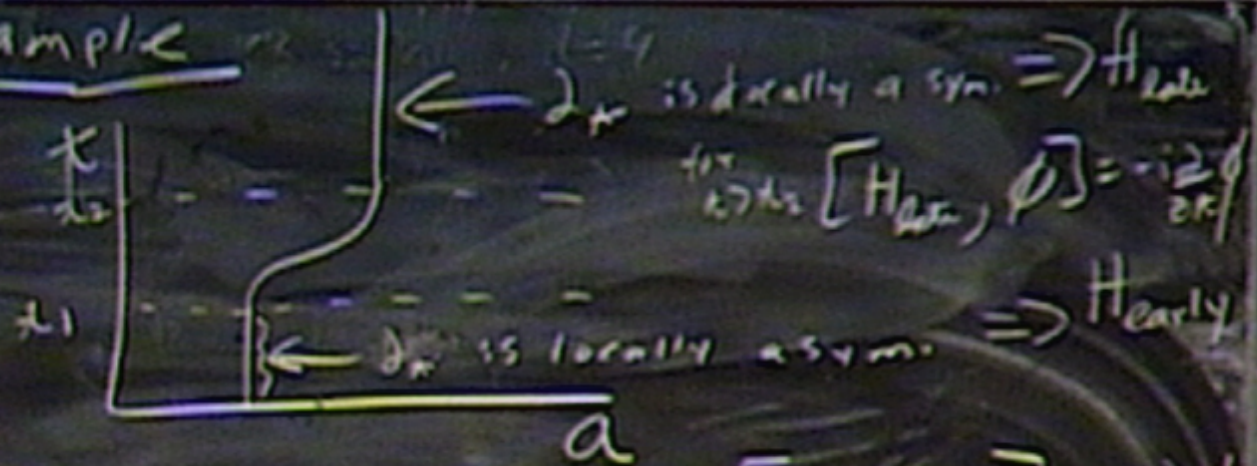


$$a_{U_{early}}^T = \# a_{U_{late}}^+ + \# a_{U_{late}}^-$$

$$1075_{early} \neq 1075_{late}$$

Qualitative Example

FRW



$$[H_{\text{early}}, \phi(x)] = -i \frac{\partial}{\partial x} \phi$$

for $x < x_1$

$\Gamma_{hg} UV^*$

Consider $\mathcal{U}_{\text{early}}$ S.T.

$$\partial_x \mathcal{U}_n^{\text{early}} = i \omega_n \mathcal{U}_n^{\text{early}} \text{ for } t < t_1$$

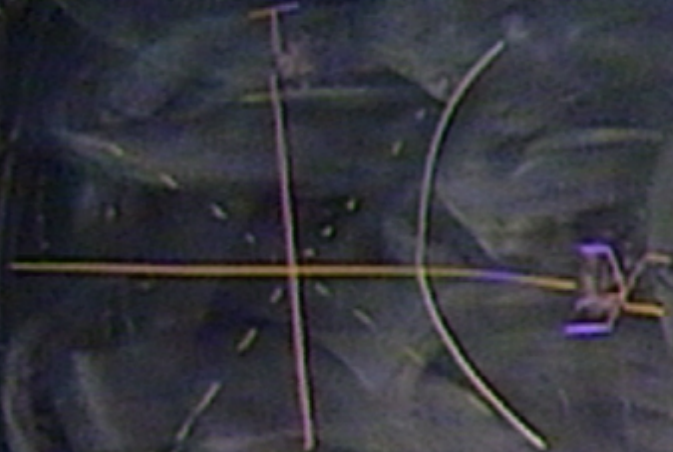
$$H_{\text{early}} = \sum_{n>0} \omega_n^{\text{early}} a_n^{\dagger} a_n$$

$$\omega_n = -\omega_{-n}$$

"Rindler" space

(→ Minkowski space
described by
uniformly accelerating
obs.

$$X^2 - T^2$$



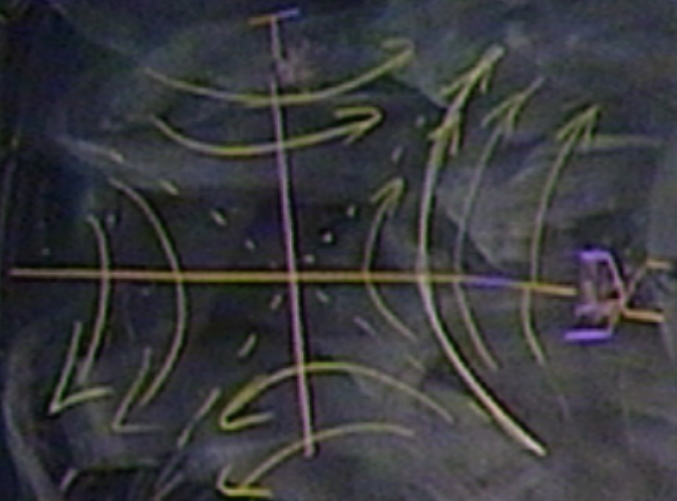
"Rindler space"

(-) Minkowski space

described by

uniformly accelerating
obs.

$$X^2 - T^2 = \frac{1}{a^2} \leftarrow \text{proper acceleration}$$



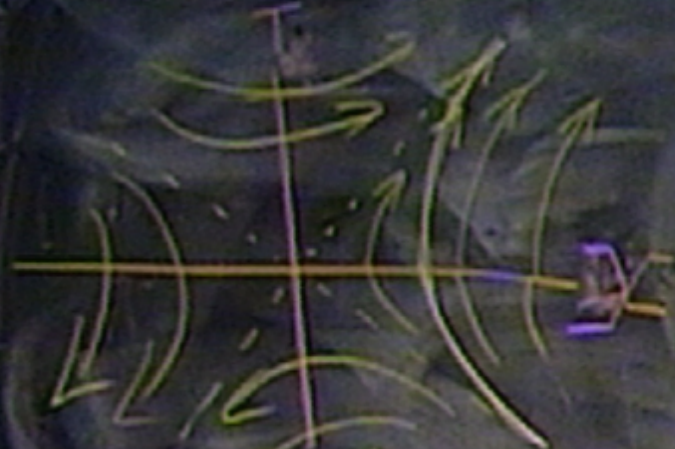
"Rindler space"

↳ Minkowski space

described by

uniformly accelerating
obs.

$$X^2 - T^2 = \frac{1}{a^2} \leftarrow \text{proper acceleration}$$

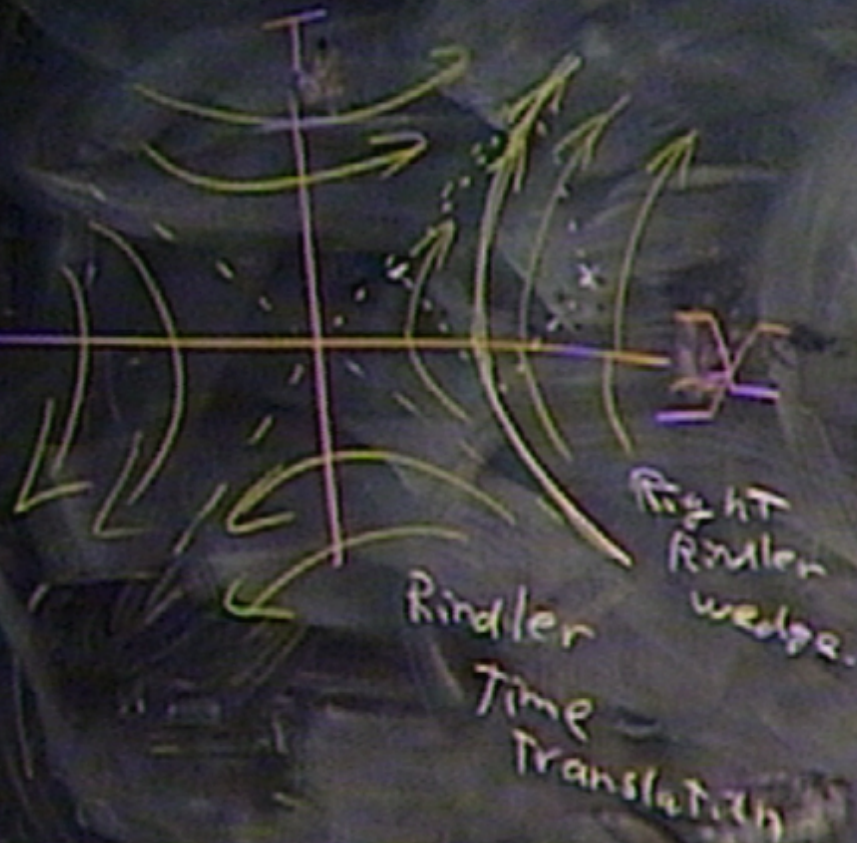


Rindler
Time
Translation

"Rindler space"

($\Rightarrow$) Minkowski space
described by
uniformly accelerating
obs.

$$X^2 - T^2 = \frac{1}{\alpha^2} \leftarrow \text{proper acc.}$$



Inertial	Uniformly acc.
Natural to use Eigenmodes of ∂_T	Natural to use Eigenmodes of Rindler time translation

$$ds^2 = -dt^2 + dx^2 = -X^2 x^2 dt^2 + dx^2$$

$$x = \sqrt{X^2 - T^2}$$

$$\tanh t = \frac{T}{X}$$

$$ds^2 = -dt^2 + dx^2 = -K^2 x^2 dt^2 + dx^2$$

$$x = \sqrt{X^2 - T^2}$$

$$\tanh \chi = \frac{T}{X}$$

uniformly accel. obs.

$$\textcircled{O} x_0 \quad \gamma \alpha = \frac{1}{x}$$

$$K = \frac{1}{x_0} = \alpha$$

$$ds^2 = -dt^2 + dx^2 = -K^2 x^2 dt^2 + dx^2$$

$$x = \sqrt{X^2 - T^2}$$

$$\tanh \chi = \frac{T}{X}$$

uniformly accel. obs.

$$\textcircled{O} x_0 \quad \gamma \alpha = \frac{1}{x}$$

$$K = \frac{1}{x_0} = \alpha$$

$$x = e^{K\rho}$$

$$ds^2 = K^2 e^{2K\rho} (-dt^2 + d\rho^2)$$

$$ds^2 = -dt^2 + dx^2 = -K^2 x^2 dt^2 + dx^2$$

$$x = \sqrt{X^2 - T^2}$$

$$\tanh \left(\frac{Kx}{c} \right) = \frac{T}{X}$$

uniformly accel. obs.

$$@ x_0 \quad \gamma \alpha = \frac{1}{x}$$

$$K = \frac{1}{x_0} = \alpha$$

$$x = e^{Kp}$$

$$ds^2 = K^2 e^{2Kp} (-dt^2 + dp^2)$$

integrate
to future
on
both
 L, R

$$\left\{ \begin{array}{l} v_+ = x + p \\ v_- = x - p \end{array} \right\} \text{ on } R$$

$$v = -(x + p)$$

$$v = -(x - p)$$

on L

$$ds^2 = -dt^2 + dx^2 = -X^2 x^2 d\tau^2 + dx^2$$

$$x = \sqrt{X^2 - T^2}$$

$$\tanh \tau = \frac{T}{X}$$

uniformly accel. obs.

$$\text{at } x_0 \quad \gamma \alpha = \frac{1}{x}$$

$$K = \frac{1}{x_0} = \alpha$$

$$x = e^{K\rho}$$

$$ds^2 = X^2 e^{2K\rho} (-d\tau^2 + d\rho^2)$$

integrate
to future
on
both
 L, R

$$\left\{ \begin{array}{l} v = \tau + \rho \\ u = \tau - \rho \end{array} \right\} \text{ on } R$$

$$v = -(\tau + \rho)$$

$$u = -(\tau - \rho)$$

on L

$$ds^2 = -X^2 e^{2X^2(V-u)} du dv \quad \begin{matrix} \text{near} \\ \text{inL} \end{matrix}$$

Massless K6-cgⁿ

$$P_R = \left\{ e^{i\omega v} \text{ on } R \right.$$

$$ds^2 = -X^2 e^{2\lambda(V-u)} du dv \quad \begin{matrix} \text{on R} \\ \text{on L} \end{matrix}$$

Massless K6-cgⁿ

$$P_R = \begin{cases} e^{-i\omega v} \\ 0 \end{cases}$$

on R

on L

$$ds^2 = -X^2 e^{2\lambda(V-u)} du dv \quad \begin{matrix} \text{from} \\ \text{to} \end{matrix} \quad = -dU dV \quad \begin{matrix} V = T + X \\ U = T - X \end{matrix}$$

Massless KG-cgⁿ

$$p_R = \begin{cases} e^{-i\omega v} \\ 0 \end{cases}$$

on R

on L

$$ds^2 = -X^2 e^{2\kappa(V-U)} du dv$$

from
- on L

$$= -dU dV \quad \begin{matrix} V = T + X \\ U = T - X \end{matrix}$$

Massless KG-cgⁿ

$$p_R = \begin{cases} e^{-i\omega v} \\ 0 \end{cases}$$

on R

on L

$$dV = k e^{\pm k v} dv$$

$$V = \pm e^{\pm k v}$$

+ on R
- on L

$$dV = k e^{\pm k v} dv$$

$$V = \pm e^{\pm k v}$$

+ on R
- on L

$$v = \pm \frac{1}{k} \ln(\pm V)$$

$$dV = \pm k e^{\pm k v} dv$$

$$V = e^{\pm k v}$$

$\begin{matrix} + & \text{on } R \\ - & \text{on } L \end{matrix}$

$$v = \pm \frac{1}{k} \ln(V)$$

$$dV = \pm k e^{\pm k v} dv$$

$$dV = \pm k e^{\pm kv} dv$$

$$V = e^{\pm kv}$$

+ on R
- on L

$$v = \pm \frac{1}{k} \ln(V)$$

$$dV = \pm k e^{\pm kv} dv$$

$$ds^2 = -X^2 e^{2KX(V-U)} dU dV$$

from
-in L

$$= -dU dV \quad \begin{matrix} V = T + X \\ U = T - X \end{matrix}$$

Massless K.G. eqn

$$P_R = \begin{cases} e^{-i\omega V} = \exp\left[-\frac{i\omega}{X} \ln V\right] & \text{on R} \\ 0 & \text{on L} \end{cases}$$

$$ds^2 = -X^2 e^{2\lambda(V-u)} du dv \quad \begin{matrix} \text{on R} \\ \text{on L} \end{matrix} = -dU dV \quad \begin{matrix} V = T + X \\ U = T - X \end{matrix}$$

Massless K.G. eqn

$$p_R = \begin{cases} e^{-i\omega v} = \exp\left[-\frac{i\omega}{X} \ln V\right] & \text{on R} \\ 0 & \text{on L} \end{cases}$$

$$\int dv e^{i\tilde{\omega} v} f$$



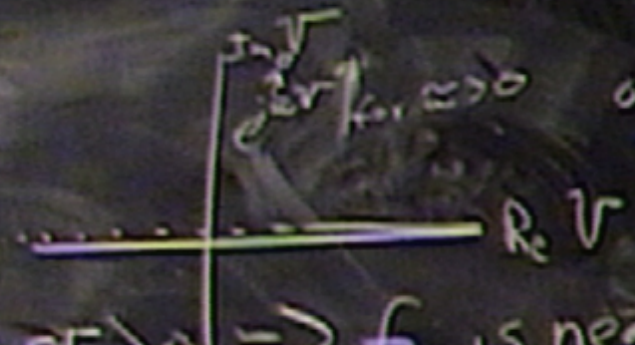
$$ds^2 = -X^2 e^{2\kappa(V-U)} dU dV \quad \begin{matrix} \text{on } R \\ \text{on } L \end{matrix} = -dU dV \quad \begin{matrix} V = T + X \\ U = T - X \end{matrix}$$

Massless KG. eqn

$$p_R = \begin{cases} e^{-i\omega V} = \exp\left[-\frac{i\omega}{\kappa} \ln V\right] & \text{on } R \\ 0 & \end{cases}$$

$$\int dV e^{i\tilde{\omega} V} f$$

f analytic in $\text{Im } V > 0 \Rightarrow f$ is neg. freq.



$$ds^2 = -X^2 e^{2\lambda(V-U)} du dv \quad \begin{matrix} \text{on } R \\ \text{on } L \end{matrix} = -dU dV \quad \begin{matrix} V = T + X \\ U = T - X \end{matrix}$$

Massless K.G. eqn

$$p_R = \begin{cases} e^{-i\omega v} = \exp\left[-\frac{i\omega}{X} \ln V\right] & \text{on } R \\ 0 & \end{cases}$$

$$\int dv e^{i\tilde{\omega} v} f$$

f analytic in $\text{Im } V > 0 \Rightarrow f$ is neg. freq.

$\frac{\ln V}{X} \approx 0$ on L

R, V

$$P = \begin{cases} \exp\left(\frac{i\omega}{\kappa} \ln(-V)\right) & \text{for } V > 0 \\ e^{-\pi\omega/\kappa} \exp\left(\frac{i\omega}{\kappa} \ln V\right) & \text{for } V < 0 \end{cases}$$

$\ln V \rightarrow \ln V + i\pi$

$$P = \begin{cases} \exp\left(\frac{i\omega}{x} \ln(-V)\right) & \text{for } V > 0 \\ e^{-\pi\omega/x} \exp\left(\frac{i\omega}{x} \ln V\right) & \text{for } V < 0 \end{cases}$$

is analytic for
 $\text{Im } V > 0$
 & neg. Freq.

$$P = \begin{cases} \exp\left(\frac{i\omega}{x} \ln(-V)\right) & \text{for } V > 0 \\ e^{-\pi\omega/x} \exp\left(\frac{i\omega}{x} \ln V\right) & \text{for } V < 0 \end{cases}$$

is analytic for
 $\text{Im } V < 0$
 & pos. Freq.

$$P = P_R + e^{-\pi\omega/x} n_L$$

$$n_L = \begin{cases} 0 & \text{on } R \\ \exp\left(\frac{i\omega}{x} \ln V\right) & \text{on } L \end{cases}$$

$$\underline{P} = \begin{cases} \exp\left(\frac{i\omega}{x} \ln(-V)\right) & \text{for } V > 0 \\ e^{-\pi\omega/x} \exp\left(\frac{i\omega}{x} \ln(V)\right) & \text{for } V < 0 \end{cases}$$

is analytic for
In $V < 0$
& pos. Freq.

$$\underline{P} = P_R + e^{-\pi\omega/x} n_L$$

$$n_L = \begin{cases} 0 & \text{on } R \\ \exp\left(\frac{i\omega}{x} \ln(V)\right) & \text{on } L \end{cases} = e^{+i\omega v} \text{ on } R$$

$$\underline{P} = \begin{cases} \exp\left(\frac{i\omega}{x} \ln(-V)\right) & \text{for } V > 0 \\ e^{-\pi\omega/x} \exp\left(\frac{i\omega}{x} \ln(V)\right) & \text{for } V < 0 \end{cases}$$

is analytic for
In $V < 0$
& pos. Freq.

$$\underline{P} = P_R + e^{-\pi\omega/x} n_L$$

$$n_L = \begin{cases} 0 & \text{on } R \\ \exp\left(\frac{i\omega}{x} \ln(V)\right) & \text{on } L \end{cases} = e^{+i\omega v} \text{ on } L$$

$$P = \begin{cases} \exp\left(\frac{i\omega}{x} \ln(-V)\right) & \text{for } V > 0 \\ e^{-\pi\omega/x} \exp\left(\frac{i\omega}{x} \ln(V)\right) & \text{for } V < 0 \end{cases}$$

for $V < 0$

is analytic for
In $V < 0$
& pos. Freq.

$$P = P_R + e^{-\pi\omega/x} n_L$$

$$n_L = \begin{cases} 0 & \text{on } R \\ \exp\left(\frac{i\omega}{x} \ln(V)\right) & \text{on } L \end{cases}$$

$$= e^{+i\omega v} \text{ on } L$$

$$|0\rangle_{\text{Mink}} = \exp\left(-\frac{i}{2} \sum_n (\alpha^{-1}\beta)_n a_n^\dagger a_n\right) |0\rangle_{\text{Rindler}}$$

$$= \exp\left(-\frac{1}{2}\right)$$

$$|0\rangle_{\text{Mink}} = \exp\left(-\frac{i}{2} \sum_n (\alpha^{-1} \beta)_n a_n^\dagger a_n\right) |0\rangle_{\text{Rindler}}$$

$$= \exp\left(-\frac{1}{2} e^{-\pi \omega / \kappa} \sum_n 1\right)$$

$$|0\rangle_{\text{mink}} = \exp\left(-\frac{1}{2} \sum_n (\alpha^{-1}\beta)_{nn} a_n^+ a_n^+\right) |0\rangle_{\text{Rindler}}$$

$$= \exp\left(-\frac{1}{2} e^{-\pi\omega/\kappa} \sum_n a_{n,R}^+ a_{n,L}^+\right)$$

$$|0\rangle_{\text{mink}} = \exp\left(-\frac{1}{2} \sum_n (\alpha^{-1}\beta)_n a_n^\dagger a_n\right) |0\rangle_{\text{Rindler}}$$

$$= \exp\left(-\frac{1}{2} \sum_n e^{-\omega_n \pi / \kappa} a_{n,R}^\dagger a_{n,L}^\dagger\right) |0\rangle_{\text{Rindler}}$$

$$|0\rangle_{\text{Mink}} = \exp\left(-\frac{1}{2} \sum_n (\alpha^{-1}\beta)_n a_n^\dagger a_n^\dagger\right) |0\rangle_{\text{Rindler}}$$

$$= \exp\left(-\frac{1}{2} \sum_n e^{-\omega_n \pi / \kappa} a_{n,R}^\dagger a_{n,L}^\dagger\right) |0\rangle_{\text{Rindler}}$$

$$= |0\rangle_{\text{Rindler}} + |2\text{-particle}\rangle + |4\text{-particle}\rangle + \dots$$

$$|0\rangle_{\text{Mink}} = \exp\left(-\frac{1}{2} \sum_n (\alpha^{-1})_{nn} a_n^\dagger a_n\right) |0\rangle_{\text{Rindler}}$$

$$= \exp\left(-\frac{1}{2} \sum_n e^{-\omega_n \pi / \kappa} a_{n,R}^\dagger a_{n,L}^\dagger\right) |0\rangle_{\text{Rindler}}$$

$$= |0\rangle_{\text{Rindler}} + |2\text{-particle}\rangle + |4\text{-particle}\rangle + \dots$$

Consider any $\mathcal{O}$ built from $\phi(x)$ on R

(Ex 1)

$$|0\rangle_{\text{Mink}} = \exp\left(-\frac{1}{2} \sum_n (\alpha^{-1}\beta)_n a_n^\dagger a_n^\dagger\right) |0\rangle_{\text{Rindler}}$$

$$= \exp\left(-\frac{1}{2} \sum_n e^{-\omega_n \pi / \kappa} a_{n,R}^\dagger a_{n,L}^\dagger\right) |0\rangle_{\text{Rindler}}$$

$$= |0\rangle_{\text{Rindler}} + |2\text{-particle}\rangle + |4\text{-particle}\rangle + \dots$$

Consider any $\mathcal{O}$ built from $\phi(x)$ on R

Ex 1) $\mathcal{O} = N_{\text{Rindler}}$

2) $\mathcal{O} = \int \phi(x) f(x) dx$ w/ $f(x) \neq 0$ only on R

Ex. 3

$$\Theta = \iint_{\substack{x, y \\ \text{on uniformly} \\ \text{accelerating} \\ \text{worldline}}} \phi(x) \phi(y) f(x) f'(y) \lambda(\text{worldline}) d(\omega')$$

$$\langle \Theta \rangle_{\text{minK}} = \iint \langle \phi(x) \phi(y) \rangle f(x) f'(y) d(\cdot) d(\cdot)$$

Ex. 3

$\theta =$

$$\iint_{\substack{x, y \\ \text{on uniformly} \\ \text{accelerating} \\ \text{worldline}}} \phi(x) \phi(y) f(x) f'(y) \lambda(\text{worldline}) d(\omega')$$

$$\langle \phi \rangle_{\text{mink}} = \iint \langle \phi(x) \phi(y) \rangle f(x) f'(y) d(x) d(y)$$

$$\langle \phi | \phi | \phi \rangle_{\text{mink}}$$

Ex. 3

$$\Theta = \iint_{x, y \text{ on uniformly accelerating worldline}} \phi(x) \phi(y) f(x) f'(y) d(\text{worldline}) d(\text{worldline})$$

$$\langle \Theta \rangle_{\text{mink}} = \iint \langle \phi(x) \phi(y) \rangle f(x) f'(y) d(\text{worldline}) d(\text{worldline})$$

$$\langle \Theta \rangle_{\text{mink}} = \langle 0 | \exp(-\frac{1}{2} \alpha^T F \alpha_R \alpha_L) \Theta \exp(-\frac{1}{2} \alpha^T p \alpha_R \alpha_L) | 0 \rangle$$

Ex. 3

$$\Theta = \iint_{\substack{x, y \\ \text{on uniformly} \\ \text{accelerating} \\ \text{worldline}}} \phi(x) \phi(y) f(x) f'(y) \lambda(\text{worldline}) d(\omega')$$

$$\langle \Theta \rangle_{\text{mink}} = \iint \langle \phi(x) \phi(y) \rangle f(x) f'(y) d(\cdot) d(\cdot)$$

$$\langle 0|0|0 \rangle_{\text{mink}} = \langle 0| \exp(-\frac{1}{2} \alpha^T F \alpha_R^T \alpha_L^T) |0 \rangle \exp(-\frac{1}{2} \alpha^T p \alpha_R^T \alpha_L^T)$$

$$[0, a_L] = 0 = [0, a_L^+]$$

Ex. 3

$$\Theta = \iint_{\substack{x, y \\ \text{on uniformly} \\ \text{accelerating} \\ \text{worldline}}} \phi(x) \phi(y) f(x) f'(y) \lambda(\text{worldline}) d(\text{vol})$$

$$\langle \Theta \rangle_{\text{mink}} = \iint \langle \phi(x) \phi(y) \rangle f(x) f'(y) d(x) d(y)$$

$$\langle 0|0|0 \rangle_{\text{mink}} = \langle 0| \exp(-\frac{1}{2} \alpha^T F \alpha_R^T a_L^T) | \Theta | \exp(-\frac{1}{2} \alpha^T P \alpha_R^T a_L^T) |$$

$$[\phi, a_L] = 0 = [\phi, a_L^+]$$

$$\langle 0|0|0 \rangle_{\text{m}} = \text{Tr} \left(\langle 0|0 \rangle_{\text{m}} \langle 0| \right) = \text{Tr}_{\mathcal{H}_R} (\sigma \rho)$$

$\mathcal{H} = \mathcal{H}_R \otimes \mathcal{H}_L$

$$w/g = \text{Tr}_{\mathcal{H}_L} |\mathcal{O}\rangle \langle \mathcal{O}|_{\text{Mink} \text{ Mink}}$$

$|\mathcal{O}\rangle$ Rindler

consider any $\mathcal{O}$ built from $\phi(x)$ on $\mathcal{I}$

Ex 1) $\mathcal{O} = N_{\text{Rindler}}$

2) $\mathcal{O} = \int \phi(x) f(x) dx$ w/ $f(x) \neq 0$ only on R

$$w/g = \text{Tr}_{\mathcal{H}_L} |0\rangle_{\text{Mink}} \langle 0|_{\text{Mink}}$$

$$\underbrace{\langle n_1, n_2, \dots, n_k |}_{\text{occupation \#s for } R\text{-modes}} \underbrace{p |}_{\text{occ. number for } R\text{-modes}} \underbrace{n'_1, n'_2, \dots, n'_k \rangle}_{\text{occ. number for } R\text{-modes}} = \delta_{n_1, n'_1} \delta_{n_2, n'_2} \dots \delta_{n_k, n'_k} \times \exp(-)$$

$|0\rangle_{\text{Rindler}}$

consider any $\mathcal{O}$ built from $\phi(x)$ on $\mathcal{I}$

Ex 1) $\mathcal{O} = N_{\text{Rindler}}$

2) $\mathcal{O} = \int \phi(x) f(x) dx$ w/ $f(x) \neq 0$ only on R

$$w/g = \text{Tr}_{\mathcal{H}_L} |0\rangle_{\text{Mink}} \langle 0|_{\text{Mink}}$$

$$\underbrace{\langle n_1, n_2, \dots, n_k }_{\text{occupations for R-modes}} | \rho | \underbrace{n'_1, n'_2, \dots, n'_k}_{\text{occ. number R-modes}} \rangle = \delta_{n_1, n'_1} \delta_{n_2, n'_2} \dots \delta_{n_k, n'_k} \times \exp\left(-\frac{2\pi}{\kappa} \sum_{j=1}^k n_j \omega_j\right)$$

$|0\rangle_{\text{Rindler}}$

consider any $\mathcal{O}$ built from $\phi(x)$ on $\mathcal{I}$

Ex 1) $\mathcal{O} = N_{\text{Rindler}}$

2) $\mathcal{O} = \int \phi(x) f(x) dx$ w/ $f(x) \neq 0$ only on R

$$w/p = \text{Tr}_{\mathcal{H}_L} |0\rangle_{\text{Mink}} \langle 0|_{\text{Mink}}$$

$$\underbrace{\langle n_1, n_2, \dots, n_k }_{\text{occupation numbers for R-modes}} | p | \underbrace{n'_1, n'_2, \dots, n'_k}_{\text{occ. numbers for R-modes}} \rangle = \delta_{n_1, n'_1} \delta_{n_2, n'_2} \dots \delta_{n_k, n'_k} \times \exp\left(-\frac{2\pi}{\kappa} \sum_{j=1}^k n_j \omega_j\right)$$

$$|0\rangle_{\text{Rindler}}$$

$$\rho = \exp\left(-\frac{2\pi}{\kappa} H_{\text{Rindler}}\right)$$

$$w/p = \text{Tr}_{\mathcal{H}_L} |0\rangle_{\text{Mink}} \langle 0|$$

$$\underbrace{\langle n_1, n_2, \dots, n_k }_{\text{occupation numbers for R-modes}} | p | \underbrace{n'_1, n'_2, \dots, n'_k}_{\text{occupation numbers for R-modes}} \rangle = \delta_{n_1, n'_1} \delta_{n_2, n'_2} \dots \delta_{n_k, n'_k} \times \exp\left(-\frac{2\pi}{\chi} \sum_{j=1}^k n_j \omega_j\right)$$

$|0\rangle_{\text{Rindler}}$

$$\rho = \exp\left(-\frac{2\pi}{\chi} H_{\text{Rindler}}\right)$$

Thermal state

$$w/T = \frac{\chi}{2\pi} = \frac{\alpha}{2\pi}$$