

Title: Cosmology

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Abstract:

Problems of the Standard Cosmology

$$H^2 = \frac{8\pi G}{3} \rho - \frac{k}{a^2} \Leftrightarrow \Omega - 1 = \frac{k}{a^2 H^2} \quad \frac{d}{da}$$

Use 2nd E.E.

$$\Rightarrow \frac{d\Omega}{da} = (1+3w) \frac{\Omega(\Omega-1)}{a}$$

$$p = w\rho$$

Problems of the Standard Cosmology

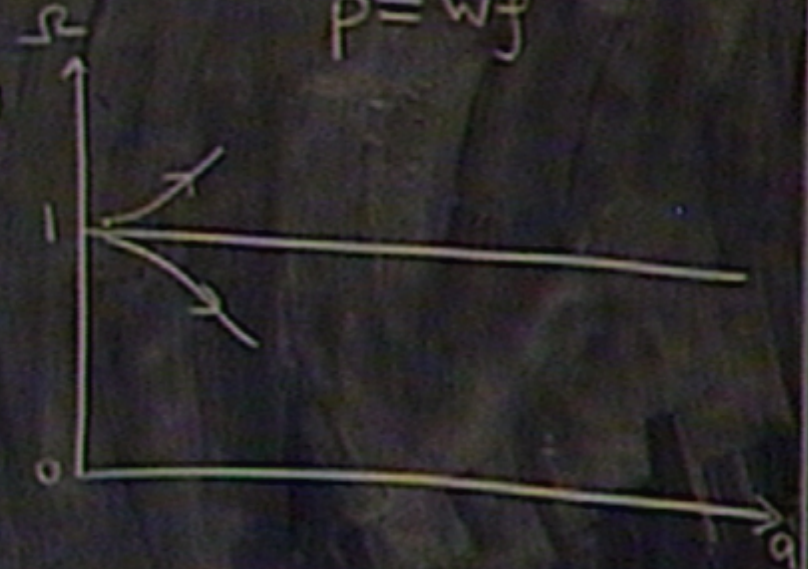
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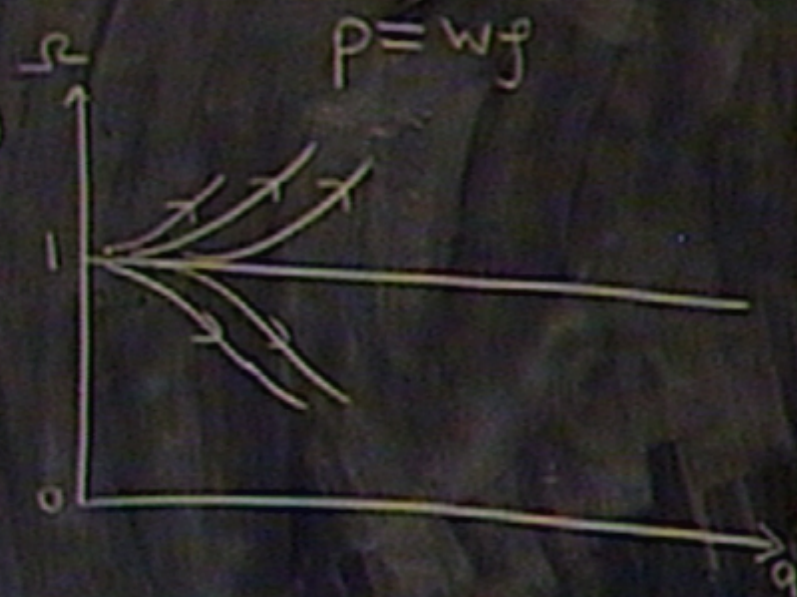
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1st & 2nd E.E.

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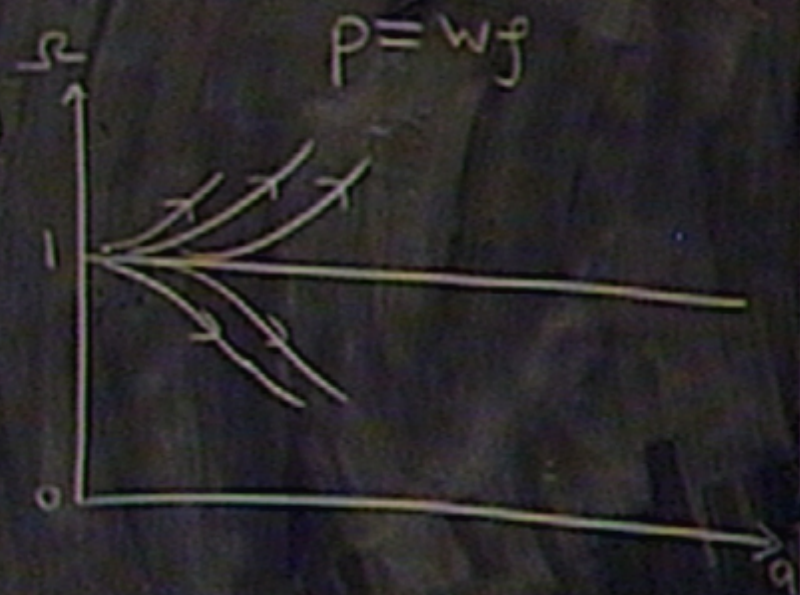
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 $0 \leq 1 - \Omega \leq 10^{-60}$

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FLATNESS PROBLEM.

Consider a photon along a radial trajectory $[k=0]$ $a_0 \equiv 1$

Radial null path :

$$0 = ds^2 = -dt^2 + a^2(t)dr^2$$

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No explanation of this.
FLATNESS PROBLEM.

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$\therefore$ Comoving distance traveled between time

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$$\Delta r = \int_{t_1}^{t_2} \frac{dt}{a(t)}$$

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$$\Rightarrow H(t) = \frac{2}{3} t^{-1}$$

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$$\Rightarrow H(t) = \frac{2}{3} t^{-1} = a^{-3/2} H_0$$

$$\therefore \Delta r = 2 H_0^{-1} (\sqrt{a_2} - \sqrt{a_1})$$

$$a_i \equiv a(t_i)$$

Comoving horizon size at some time t_* .

$$r_{\text{hor}}(a_*) = 2H_0^{-1}\sqrt{a_*}$$

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In any almost flat
cosmology w/ rad^e, matter

$$dn \sim H^{-1}$$

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$$a_i \equiv a(t_i)$$

$$H^{-1} \equiv \text{HUBBLE DISTANCE}$$

Problems of the Standard Cosmology

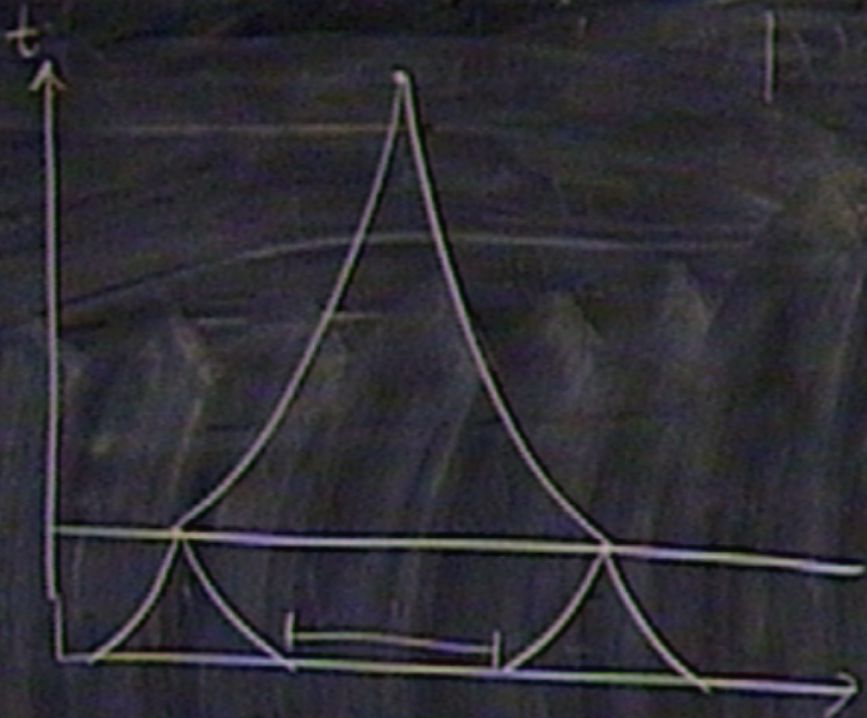
CMB released at $a_{\text{CMB}} \approx \frac{1}{1200}$

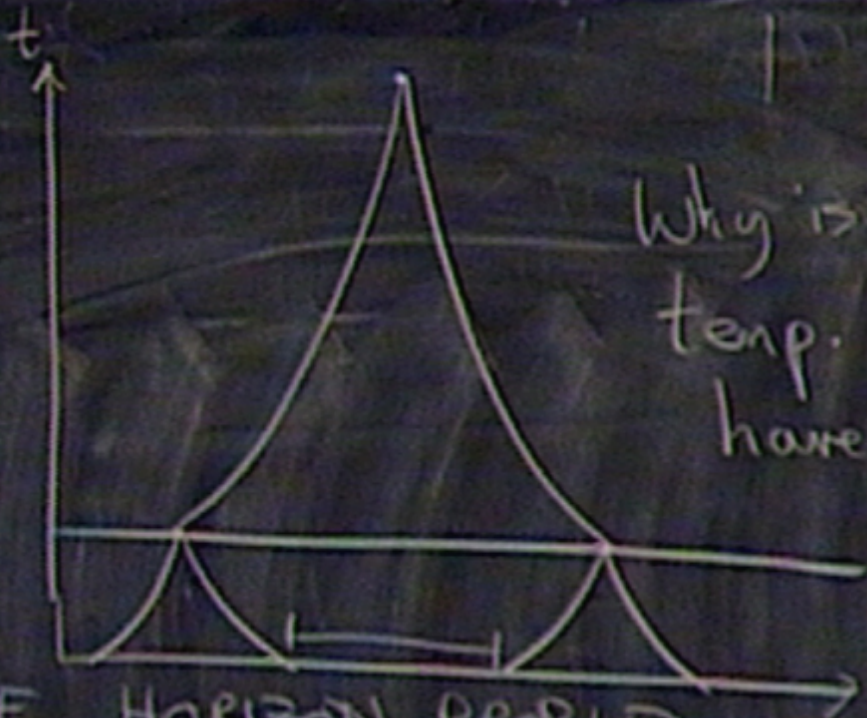
Comoving dist between CMB and us.

$$\Delta r = 2H_0^{-1} (1 - \sqrt{a_{\text{CMB}}})$$

horizon distance at t_{CMB}

$$r_{\text{hor}}(a_{\text{CMB}}) = 2H_0^{-1} \sqrt{a_{\text{CMB}}} \approx 6 \times 10^{-2} H_0^{-1}$$





Why is the CMB the same temp. in directions that have never been in causal contact?

THE HORIZON PROBLEM

Comoving horizon size at some time t_*

$$r_{\text{hor}}(a_*) = 2H_*^{-1}\sqrt{a_*}$$

$$d_{\text{hor}}(a_*) = a_* r_{\text{hor}}(a_*) = 2H_*^{-1}$$

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Slowly-Rolling Scalar Fields

Field ϕ , $V(\phi)$

$$T_{\mu\nu} = (\nabla_\mu \phi)(\nabla_\nu \phi) - g_{\mu\nu} \left[\frac{1}{2} g^{\alpha\beta} (\nabla_\alpha \phi)(\nabla_\beta \phi) + V(\phi) \right]$$

For simplicity set $k=0$, assume homogeneity.

Homogeneous scalar field behaves as a perfect fluid

$$w) \quad \rho_\phi = \frac{1}{2} \dot{\phi}^2 + V(\phi) \quad ; \quad P_\phi = \frac{1}{2} \dot{\phi}^2 - V(\phi)$$

Problems of the Standard Cosmology

$$\ddot{\phi} + 3\frac{\dot{a}}{a}\dot{\phi} + \frac{dV}{d\phi} = 0$$

Friedmann eq: $H^2 = \frac{8\pi G}{3} \left[\frac{1}{2}\dot{\phi}^2 + V(\phi) \right]$

Acc eq: $\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p)$

$$\text{Acc}^{\wedge} \Rightarrow \rho + 3p < 0$$
$$p < -\frac{1}{3}\rho$$

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Accⁿ $\Rightarrow \rho + 3p < 0$
 $p < -\frac{1}{3}\rho$

Easy way for a scalar field to satisfy this is if $\dot{\phi}^2 \ll V(\phi)$

$$p \approx -\rho$$

~~cosm.~~ cosm. constant

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$$< 0 \text{ if } p < -\frac{1}{3} \text{ or } \left(\frac{\ddot{a}}{a} > 0\right)$$

$$0 < 1 - \Omega < 10^{-4}$$



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Slow-roll approx. $0 < 1 - \Omega < 10^{-6}$

Neglect $\ddot{\phi}$

$$\Rightarrow \dot{\phi} \approx -\frac{V'}{3H}$$

$$H^2 \approx \frac{8\pi G}{3} V(\phi)$$

Sufficient conditions for this are.

$$|\epsilon| \ll 1, \quad |\eta| \ll 1$$

where the SLOW-ROLL PARAMETERS ARE

$$\epsilon \equiv \frac{M_{\text{Pl}}^2}{2} \left(\frac{V'}{V} \right)^2; \quad \eta \equiv M_{\text{Pl}}^2 \frac{V''}{V}$$

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$$\epsilon \equiv \frac{M_p^2}{2} \left(\frac{V'}{V} \right)^2; \quad \eta \equiv M_p^2 \frac{V''}{V}$$

Useful to have a general expression for how much inflation takes place.

Simple Model (Chaotic Inflation)

$$0 < p < -\frac{1}{3} \quad \left(\frac{\ddot{a}}{a} > 0 \right)$$

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Simple Model (Chaotic Inflation)

$$V(\phi) = \frac{1}{2} m^2 \phi^2$$

$$t=0 \quad g \sim M_P^4$$

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$$\frac{2}{3} \epsilon \ll 1$$

$$H = \alpha \phi$$

$$\alpha = \sqrt{\frac{4\pi}{3}} \epsilon$$

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$$\frac{2}{3}\epsilon \equiv \epsilon \ll 1$$

$$H = \alpha \phi$$

$$\alpha \equiv \sqrt{\frac{\Delta\pi}{3}} \frac{f}{M_P}$$

$$\ddot{\phi} + 3\alpha\phi\dot{\phi} + m^2\phi = 0$$

$$\phi = \phi_0 - \beta t$$

$$\beta \equiv \frac{m^2}{3\alpha}$$

$$\text{Slow-roll} \Rightarrow \phi \gg \frac{\beta}{m} = \frac{M_P}{\sqrt{12\pi}}$$

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$$\beta = \frac{m^2}{3\alpha}$$

$$\text{Slow-roll} \Leftrightarrow \phi \gg \frac{\beta}{m} = \frac{M_P}{\sqrt{12\pi}}$$

$$\rightarrow \text{valid for } \Delta t \sim \frac{\phi_0}{\beta} \sim \frac{t_P}{\epsilon^2}$$

→ Attractor solution.

$$a(t) = a(0) \exp \left[\frac{2\pi}{M_f} (\phi_0^2 - \phi^2) \right]$$

$$a(\Delta t) \sim a(0) \exp \left(\frac{2\pi}{\varepsilon^2} \right)$$

Choose m s.t. $\varepsilon < 10^{-4}$
 2.2×10^3

$$a(\Delta t) \sim a(0) \times 10$$

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Proper distance L_p at $t=0 \rightarrow 10^{10^8} \text{ cm}$
 at $\Delta t \sim 10^{-36} \text{ s}$

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Proper distance L_p at $t=0 \rightarrow 10^{10^8} \text{ cm}$
 at $\Delta t \sim 10^{-36} \text{ s}$
 c.f. horizon today 10^{28} cm