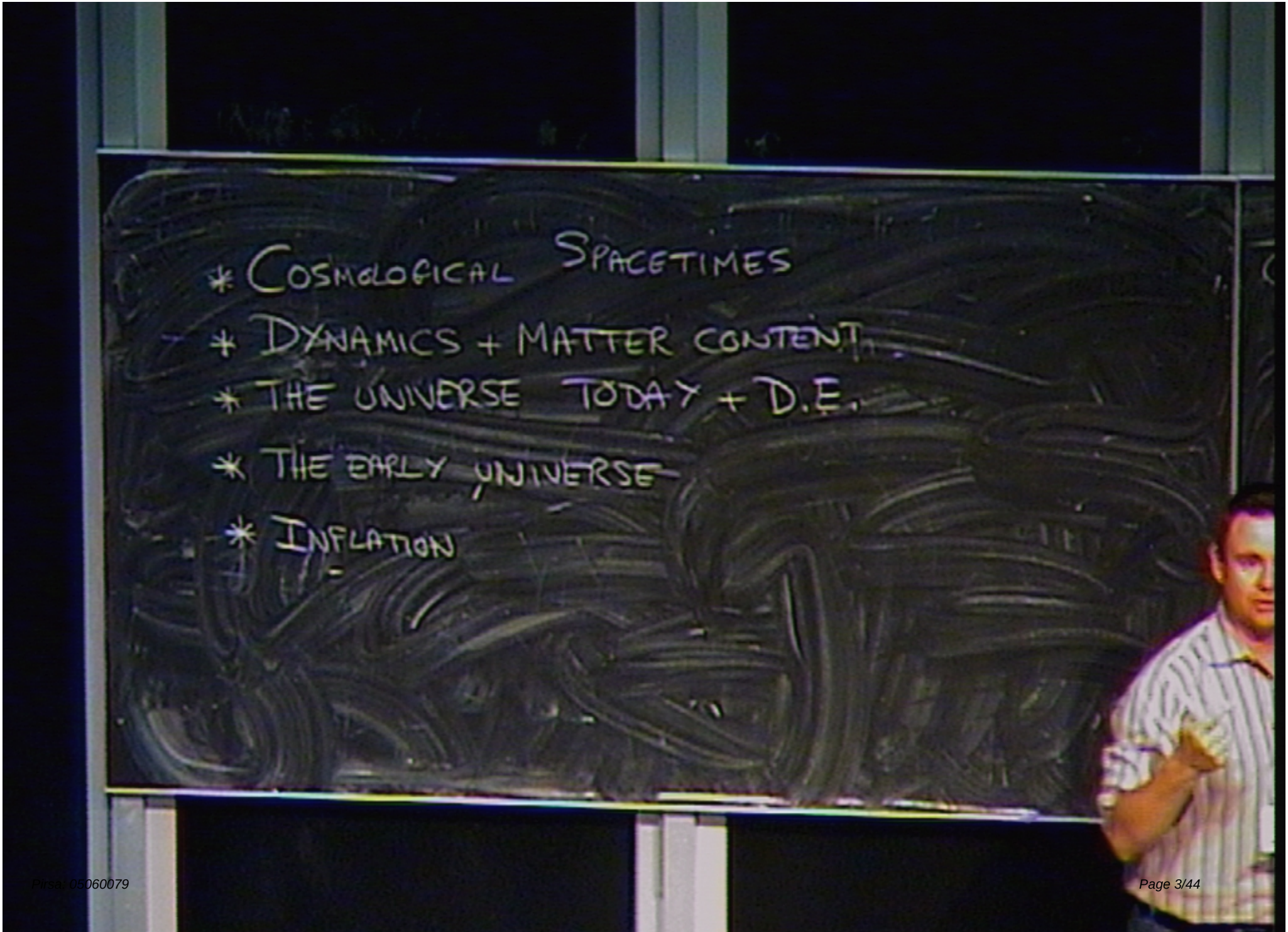
Title: Cosmology

Date: Jun 22, 2005 02:30 PM

URL: http://pirsa.org/05060079

Abstract:




* COSMOLOGICAL SPACETIMES
* DYNAMICS + MATTER CONTENT
* THE UNIVERSE TODAY + D.E.
* THE EARLY UNIVERSE
* INFLATION

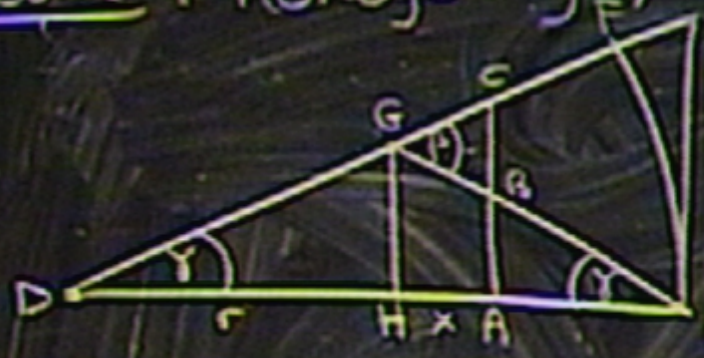
Cosmological Spacetimes

Assume: homogeneity + isotropy.



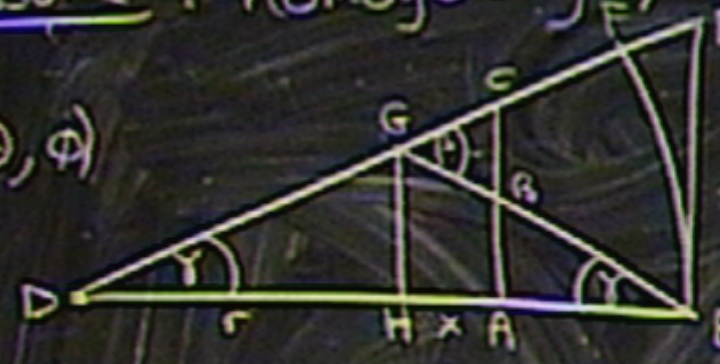
Cosmological Spacetimes

Assume : homogeneity + isotropy.



Cosmological Spacetimes

Assume : homogeneity + isotropy



$EF \simeq EF' = f(2r)\,\gamma$

$= f(r)\,\beta$

$AC = \gamma f(r+x)$

$= AB + BC = \gamma f(r-x) + \beta f(x)$

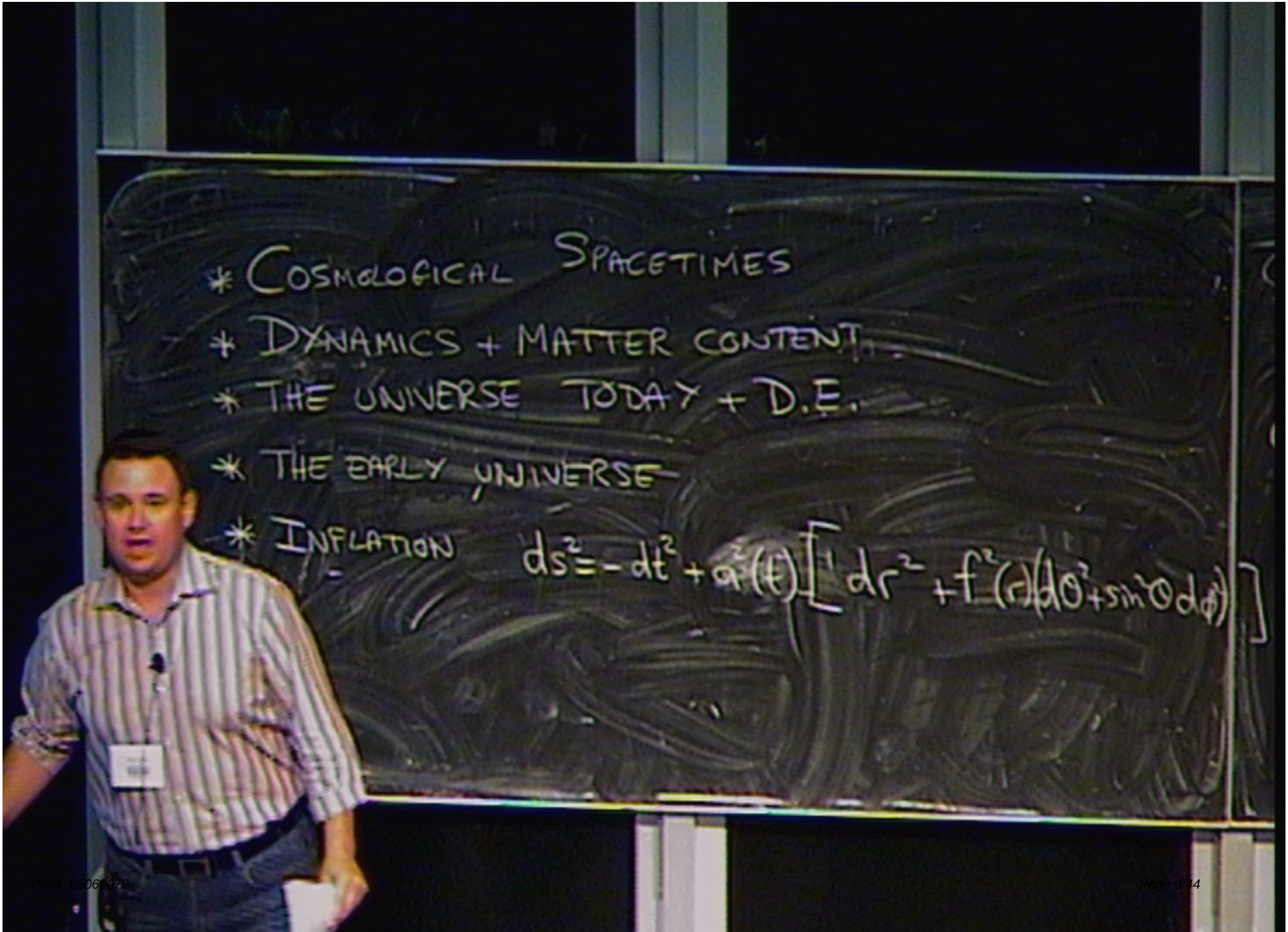
$d\sigma^2 = dr^2 + f^2(r)(d\theta^2 + \sin^2\theta\, d\phi^2)$

$f(r) \sim r \quad r \to 0$

$$\frac{df}{dr} = \frac{f(2r)}{2f(r)}$$

$$\frac{df}{dr} = \frac{f(2r)}{2f(r)} \quad f \to r \quad r \to 0$$

$f(r) = r, \sin(r), \sinh(r)$


* COSMOLOGICAL SPACETIMES
* DYNAMICS + MATTER CONTENT
* THE UNIVERSE TODAY + D.E.
* THE EARLY UNIVERSE
* INFLATION
$ds^2 = -dt^2 + a^2(t)\left[\frac{dr^2}{1} + f^2(r)(d\theta^2 + \sin^2\theta\, d\phi^2)\right]$

$$ds^2 = -dt^2 + a^2(t)\left[\frac{dr^2}{1-kr^2} + r^2(d\theta^2 + \sin^2\theta\, d\phi^2)\right]$$

FRW

$$ds^2 = -dt^2 + a^2(t)\left[\frac{dr^2}{1-kr^2} + r^2(d\theta^2 + \sin^2\theta\, d\phi^2)\right]$$

$a_0 = 1$

FRW

Define CONFORMAL TIME $\tau(t) \equiv \int_{-\infty}^{t} \frac{dt'}{a(t')}$

$$\Rightarrow ds^2 = a^2(\tau)\left[-d\tau^2 + \frac{dr^2}{1-kr^2} + r^2(d\theta^2 + \sin^2\theta\, d\phi^2)\right]$$

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HUBBLE PARAMETER $H(t) \equiv \frac{\dot{a}}{a}$ $\quad v \simeq Hd$

DYNAMICS

Use Einstein equation $G_{\mu\nu} = 8\pi G T_{\mu\nu}$

Need hom. + iso. ansatz for matter.

Use PERFECT FLUID

$$T_{\mu\nu} = (\rho + p) U_\mu U_\nu + p g_{\mu\nu}$$

ρ ← energy density; p ← pressure (in the rest frame)

$U^\mu = (1,0,0,0)$

$$T_{\mu\nu} = \begin{pmatrix} \rho & 0 \\ \hline 0 & p g_{ij} \end{pmatrix}$$

Einstein eqns give:

$$H^2 \equiv \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\sum_i \rho_i - \frac{k}{a^2}$$

FRIEDMANN EQN

$|k| = 1$ or 0

Einstein eq^{n} gives:

$$H^2 \equiv \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\sum_i \rho_i - \frac{k}{a^2}$$

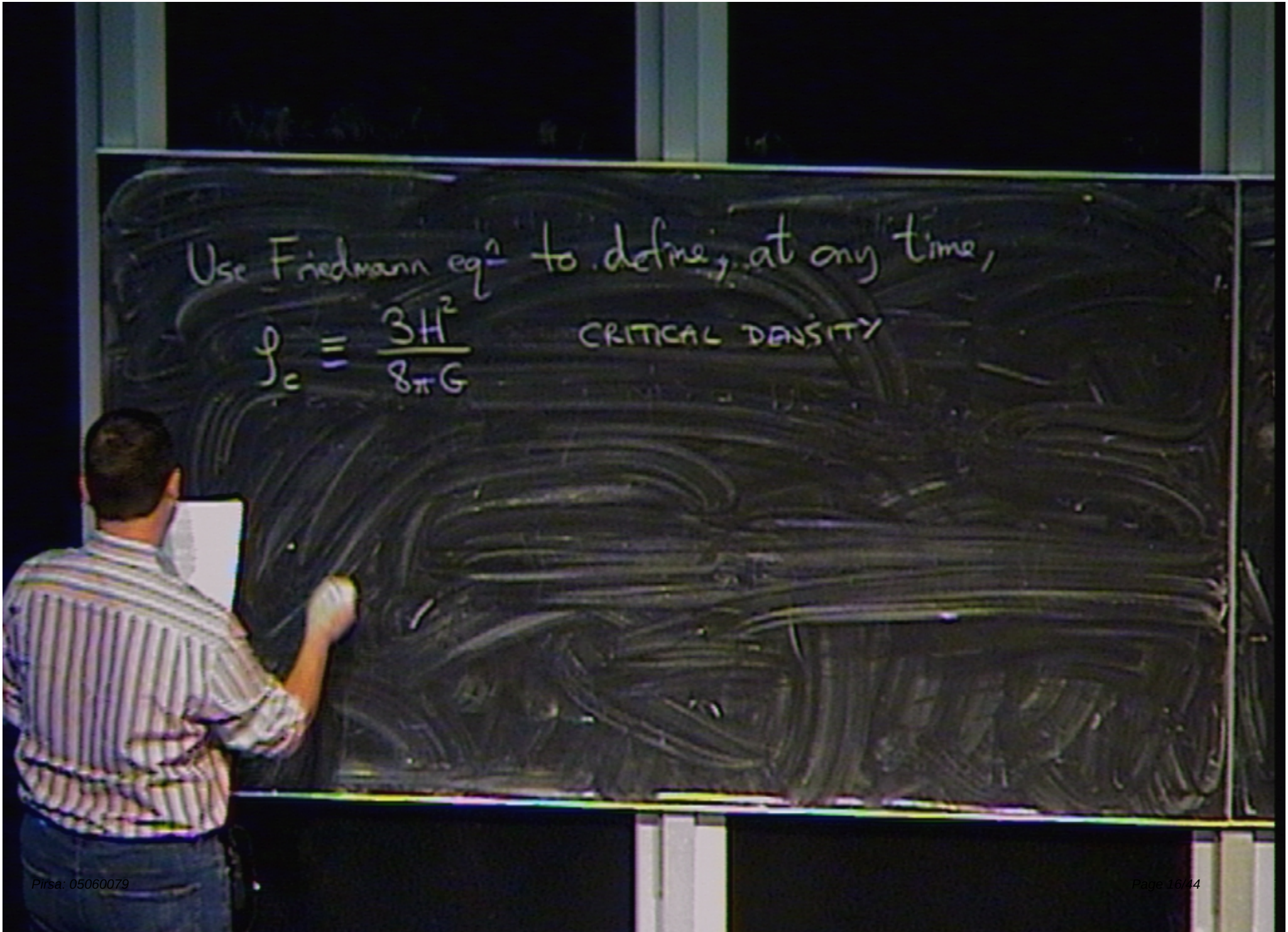
FRIEDMANN EQ^{N}

$|k| = 1$ or 0

$$\frac{\ddot{a}}{a} + \frac{1}{2}\left(\frac{\dot{a}}{a}\right)^2 = -4\pi G \sum_i p_i - \frac{k}{2a^2}$$

Combine to give ACCELERATION EQ^{N}

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\sum_i (\rho_i + 3p_i)$$


Use Friedmann eq^n to define, at any time,
$\rho_c = \frac{3H^2}{8\pi G}$
CRITICAL DENSITY

Use Friedmann eq^n to define, at any time,

$$\rho_c \equiv \frac{3H^2}{8\pi G}$$ CRITICAL DENSITY

If $\Sigma\rho = \rho_c$ then $k=0$

Define density parameter

$$\Omega_{tot} \equiv \frac{\rho}{\rho_c}$$ FRACTIONS OF ρ_c

$\Omega_{tot} > 1 \Leftrightarrow k=+1$

$\Omega_{tot} = 1 \Leftrightarrow k=0$

$\Omega_{tot} < 1 \Leftrightarrow k=-1$

$$\Omega_i \equiv \frac{\rho_i}{\rho_c}$$ ...

Energy-Momentum conservation:

$\nabla_\mu T^{\mu\nu} = 0 \Rightarrow \dot{\rho} + 3H(\rho + p) = 0$

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Need to supply an EQUATION OF STATE

$p(\rho) \longrightarrow p = w\rho$ (INSTANTANEOUS DEF^N OF w)

↳ eq^n of state parameter.

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Need to supply an EQUATION OF STATE

$p(\rho) \xrightarrow[\text{DEF}^N \text{ OF } w]{\text{INSTANTANEOUS}} p = w\rho$ ↳ eq^n of state parameter.

DUST $\rho_m(a) \propto a^{-3}$

RAD^N $\rho_r(a) \propto a^{-4}$

$$\frac{d\rho}{da} = -3(1+w)\frac{\rho}{a} \Rightarrow \rho(a) = \rho_0 \left(\frac{a_0}{a}\right)^{3(1+w)}$$

DUST : pressureless matter $\Rightarrow P_m \equiv 0 \Rightarrow w_m = 0$

RAD^N : Gas of photons $P_r = \frac{1}{3}\rho_r \Rightarrow w_r = \frac{1}{3}$

Einstein eqns give:

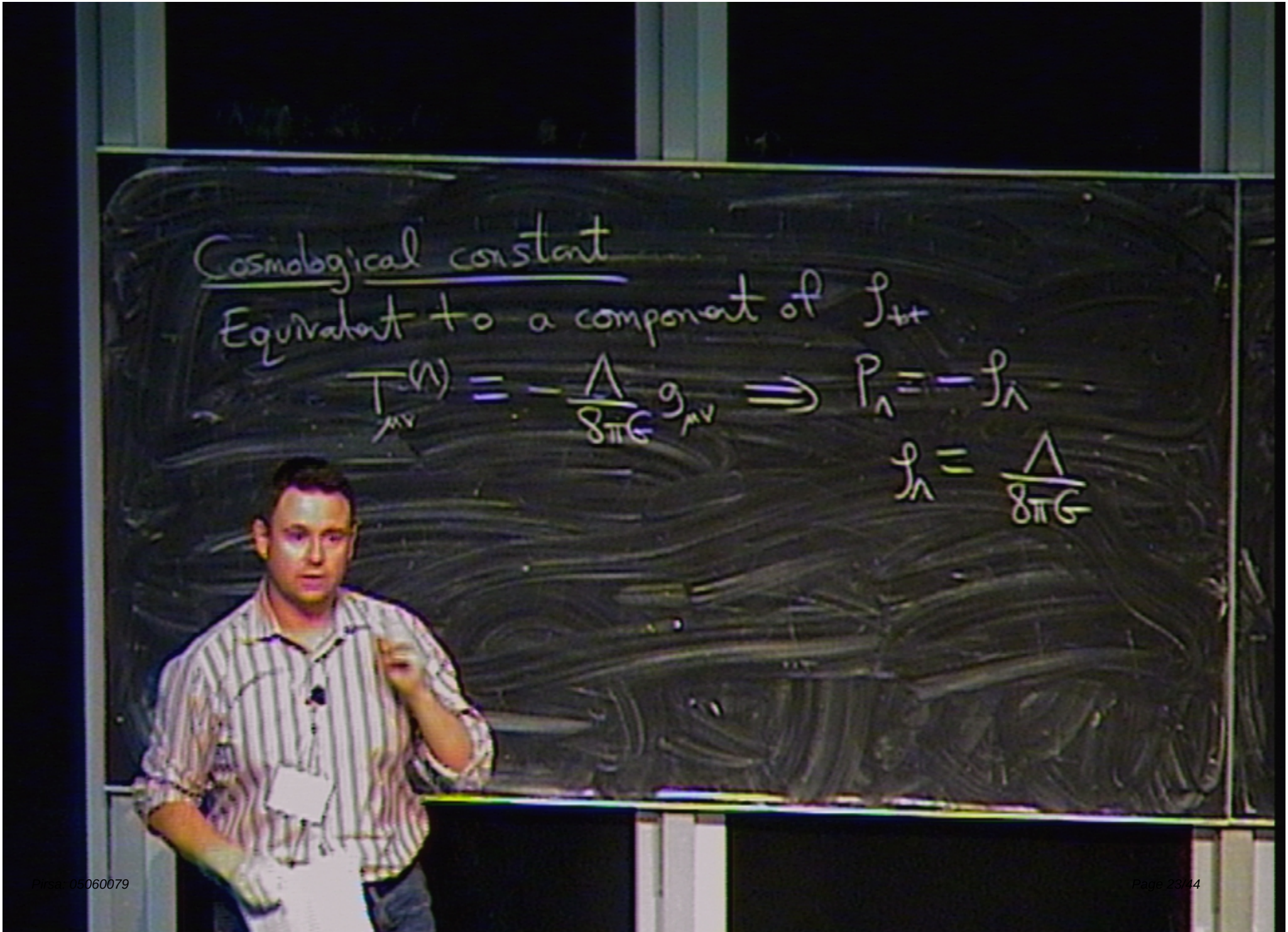
$$H^2 \equiv \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\sum_i \rho_i - \frac{k}{a^2}$$ FRIEDMANN EQN

$$|k| = 1 \text{ or } 0$$

$$\frac{\ddot{a}}{a} + \frac{1}{2}\left(\frac{\dot{a}}{a}\right)^2 = -4\pi G \sum_i p_i - \frac{k}{2a^2}$$

Combine to give ACCELERATION EQN

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\sum_i (\rho_i + 3p_i)$$


Cosmological constant
Equivalent to a component of $T_{\mu\nu}$
$T^{(\Lambda)}_{\mu\nu} = -\frac{\Lambda}{8\pi G} g_{\mu\nu} \Rightarrow p_\Lambda = -\rho_\Lambda$
$\rho_\Lambda = \frac{\Lambda}{8\pi G}$

Cosmological constant

Equivalent to a component of ρ_{tot}

$$T^{(\Lambda)}_{\mu\nu} = -\frac{\Lambda}{8\pi G} g_{\mu\nu} \Rightarrow \left.\begin{array}{l} P_\Lambda = -\rho_\Lambda \\ \rho_\Lambda = \dfrac{\Lambda}{8\pi G} \end{array}\right\} w_\Lambda = -1$$

Sometimes useful to think of curvature as an energy component

$$\rho_{curv} = -\frac{3k}{8\pi G a^2} \; ; \; P_{curv} = \frac{k}{8\pi G a^2} \quad w_{curv} = -\frac{1}{3}$$

$$\frac{d\rho}{da} = -3(1+w)\frac{\rho}{a} \Rightarrow \rho(a) = \rho_0 \left(\frac{a_0}{a}\right)^{3(1+w)}$$

DUST : pressureless matter $\rightarrow P_m \equiv 0 \Rightarrow w_m = 0$

RAD^{N} : Gas of photons $P_r = \frac{1}{3}\rho_r \Rightarrow w_r = \frac{1}{3}$

$$\frac{\lambda_0}{\lambda_e} = \frac{a_0}{a_e} \equiv 1+z \qquad 1+z = \left(\frac{a_0}{a}\right)$$

Flat Universes ($k=0$)

If universe contains only one component of ρ_{tot} (or is dominated by one) then

$$a(t) = a_0 \left(\frac{t}{t_0}\right)^{\frac{2}{3(1+w)}}$$

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[Unless $w = -1$, $a(t) \propto e^{Ht}$]

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Note that $a \to 0$ as $t \to 0$, BIG BANG

Age of Universe

$$t_0 = \int_0^{t_0} dt = \int_0^1 \frac{da}{a H(a)}$$

$$\boxed{t_0 = \frac{2}{3(1+w)H_0}}$$

This is $\sim \frac{1}{H_0}$

Age of Universe

$$t_0 = \int_0^{t_0} dt = \int_0^1 \frac{da}{\dot{a} H(a)}$$

$$\boxed{t_0 = \frac{2}{3(1+w)H_0}}$$

This is $\sim \frac{1}{H_0}$

↑ HUBBLE TIME

Including Curvature

MATTER ($w=0$)

Work in conformal time τ

$$3(k+h^2) = 8\pi G \rho a^2 \qquad h(\tau) = \frac{a'}{a} \qquad ()' \equiv \frac{d()}{d\tau}$$

$$k + h^2 + 2h' = 0$$

$$\Rightarrow h(\tau) = \begin{cases} \cot(\tau/2) \\ 2/\tau \\ \coth(\tau/2) \end{cases} \qquad a(\tau) = \begin{cases} 1-\cos(\tau) & k=+1 \\ \tau^2/2 & 0 \\ \sinh(\tau)-1 & -1 \end{cases}$$

$$t(\tau) \propto \begin{cases} \tau - \sin(\tau) & +1 \\ \tau^3/6 & 0 \\ \sinh(\tau) - \tau & -1 \end{cases}$$

Including Curvature

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$$t(\tau) \propto \begin{cases} \tau - \sin(\tau) \\ \tau^3/6 \\ \sinh(\tau) - \tau \end{cases} \qquad \begin{matrix} +1 \\ 0 \\ -1 \end{matrix}$$

Horizons

Suppose an emitter, e, sends a light signal to an observer, o, at $r=0$ (θ, ϕ const)

Have Radial null rays $\Rightarrow \tau_o - \tau = r$

$\Rightarrow \tau_o - \tau_e = r_e$

Suppose τ_e bdd below by $\overline{\tau}_e$ (e.g. BIG BANG)

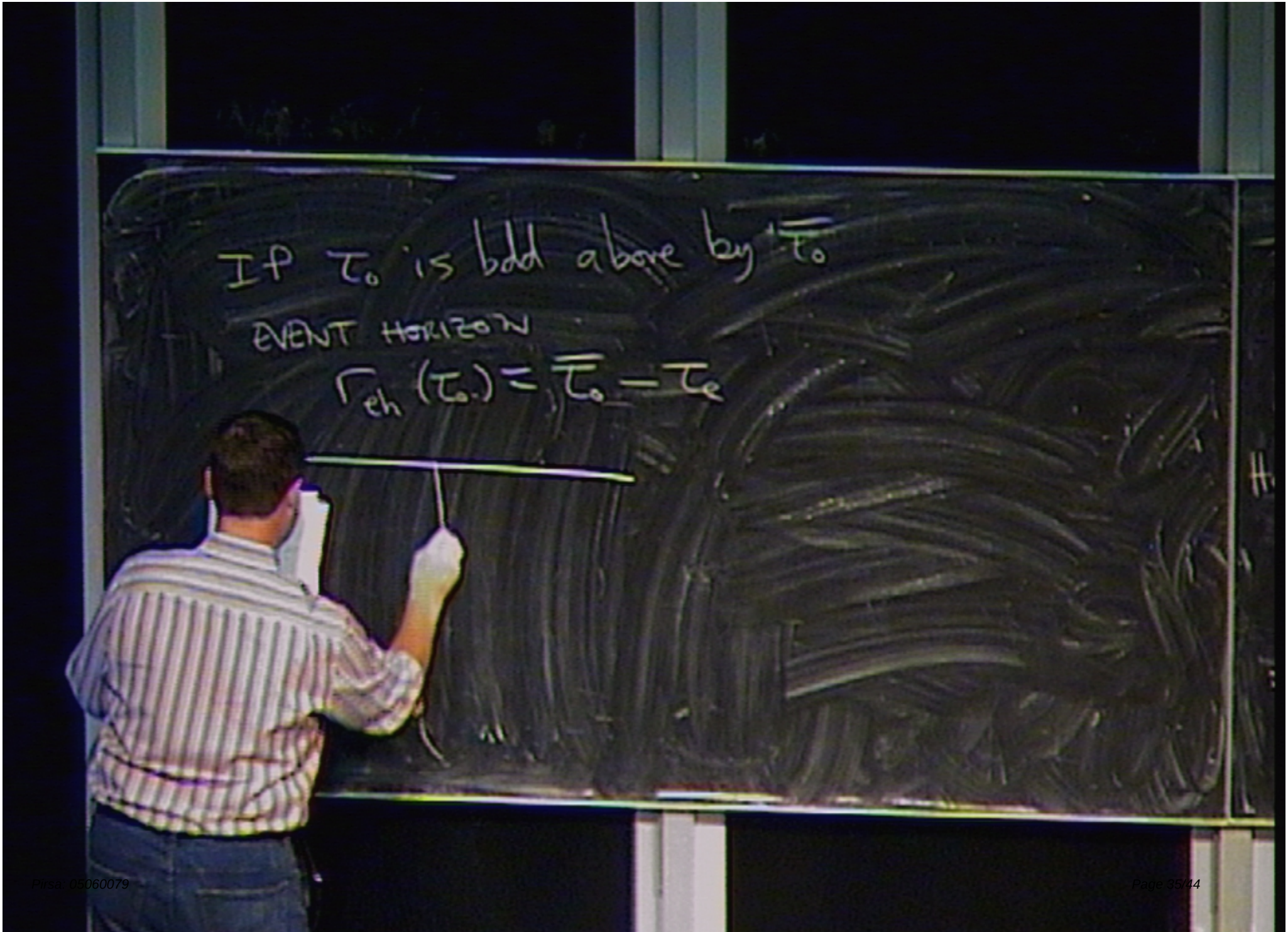
$\Rightarrow \exists$ a max. distance to which observer can see

.....

PARTICLE HORIZON DISTANCE

$$r_{ph}(\tau_0) = \tau_0 - \bar{\tau}_e$$




If τ_0 is bdd above by $\bar{\tau}_0$
EVENT HORIZON
$\Gamma_{eh}(\tau_0) = \bar{\tau}_0 - \tau_0$

If τ_0 is bdd above by $\bar{\tau}_0$

EVENT HORIZON

$$r_{eh}(\tau_0) = \bar{\tau}_0 - \tau_e$$



CONVERT TO PROPER DISTANCES

$$d_H = a(\tau) r_{ph}$$

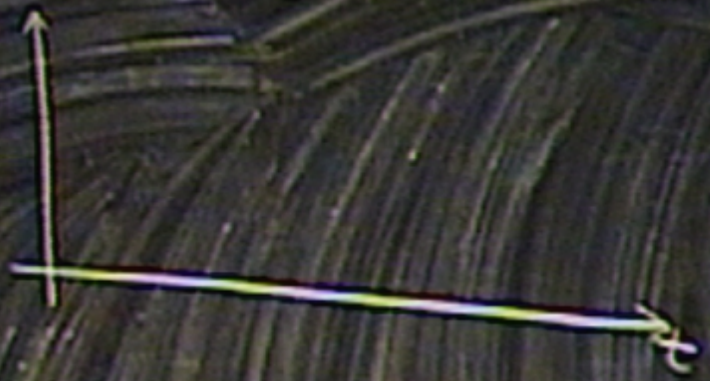
$$= a(\tau)(\tau - \bar{\tau}_e)$$

$$= a(t) \int_{t_e}^{t} \frac{dt'}{a(t')}$$

Geometry + Destiny

(Dark energy)

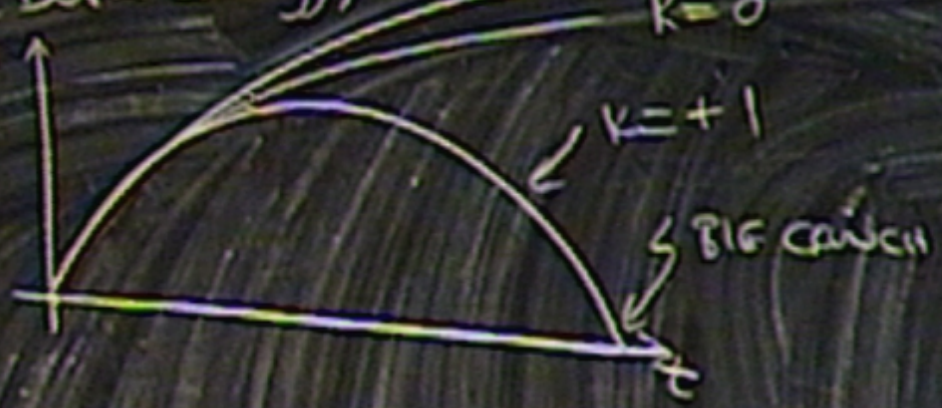
$$a(t) = a_0 \left(\frac{t}{t_0}\right)^{\frac{2}{3(1+w)}}$$



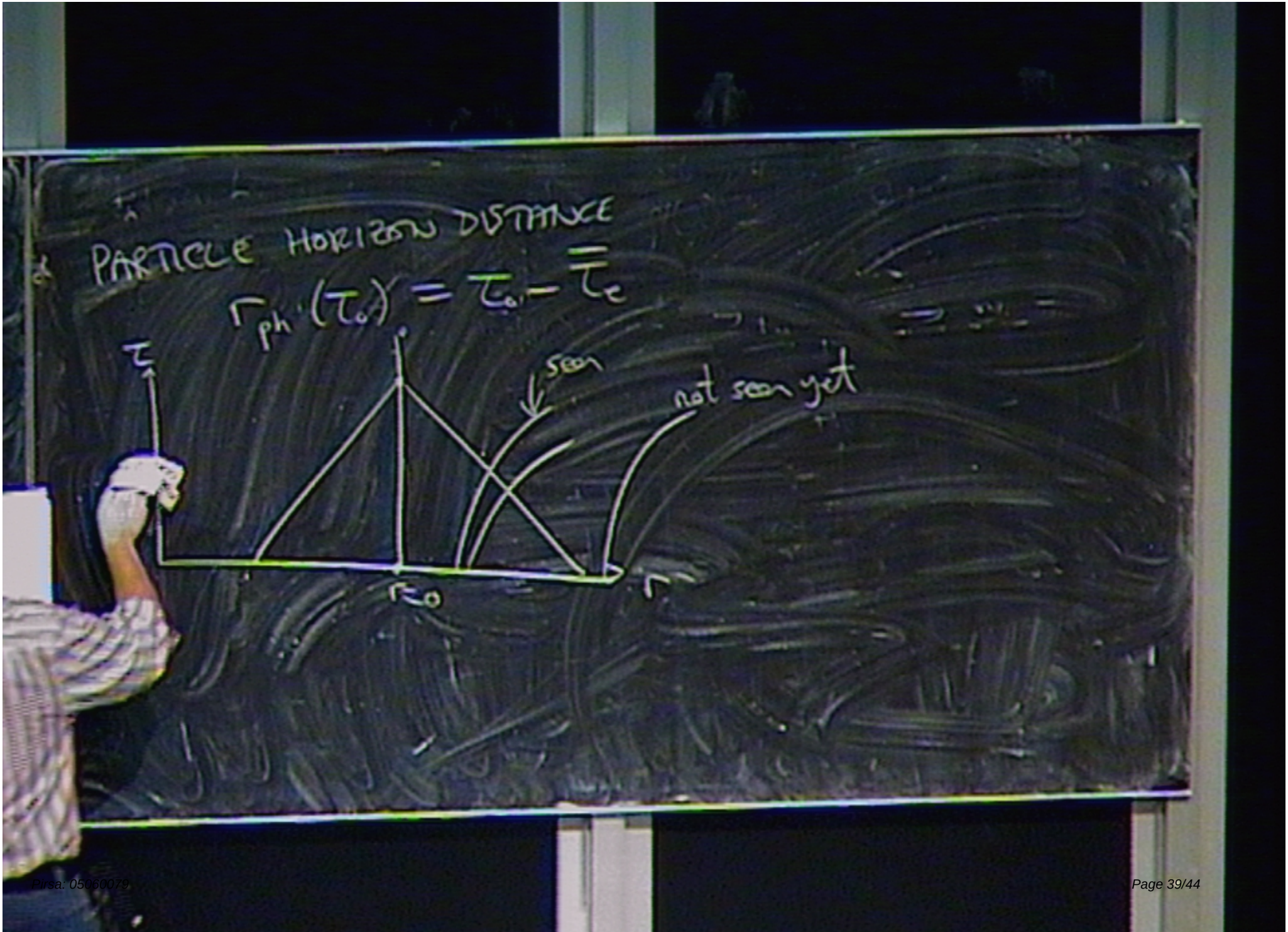
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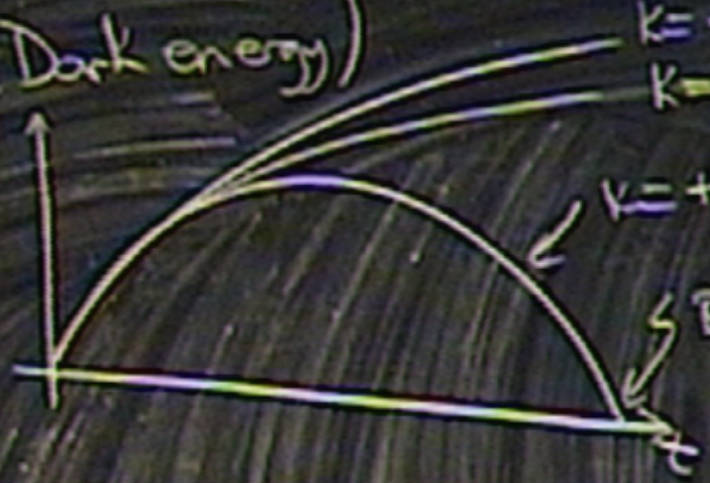


RAD^N + MATTER


PARTICLE HORIZON DISTANCE
$r_{ph}(\tau_0) = \tau_0 - \bar{\tau}_e$
τ
seen
not seen yet
$r=0$
r

Geometry + Destiny

$$a(t) = a_0 \left(\frac{t}{t_0}\right)^{\frac{2}{3(1+w)}}$$



The Universe Today

$H_0 = 100h$ km/s/Mpc

$h = 0.71 \pm 0.06$

THE UNIVERSE TODAY

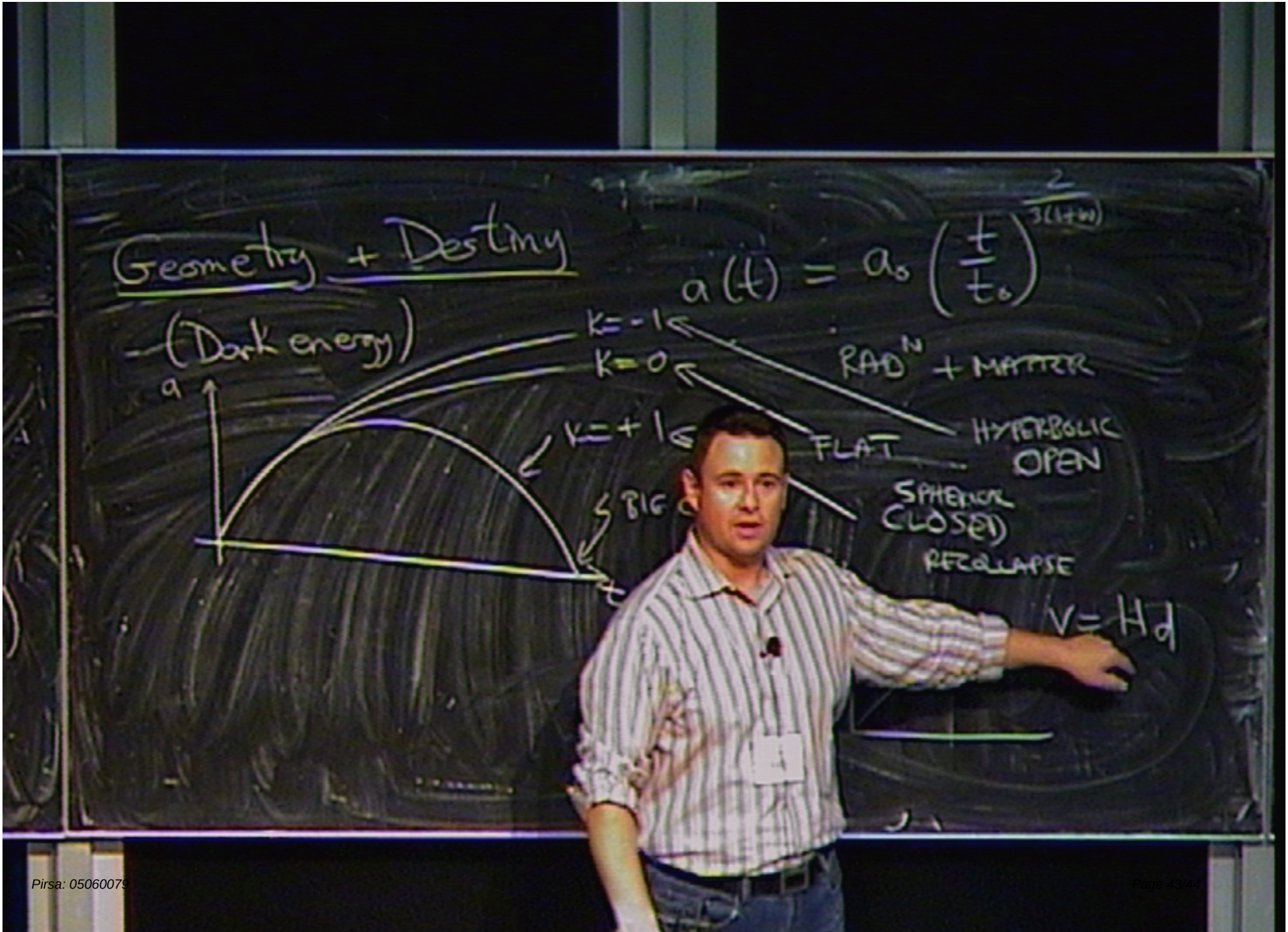
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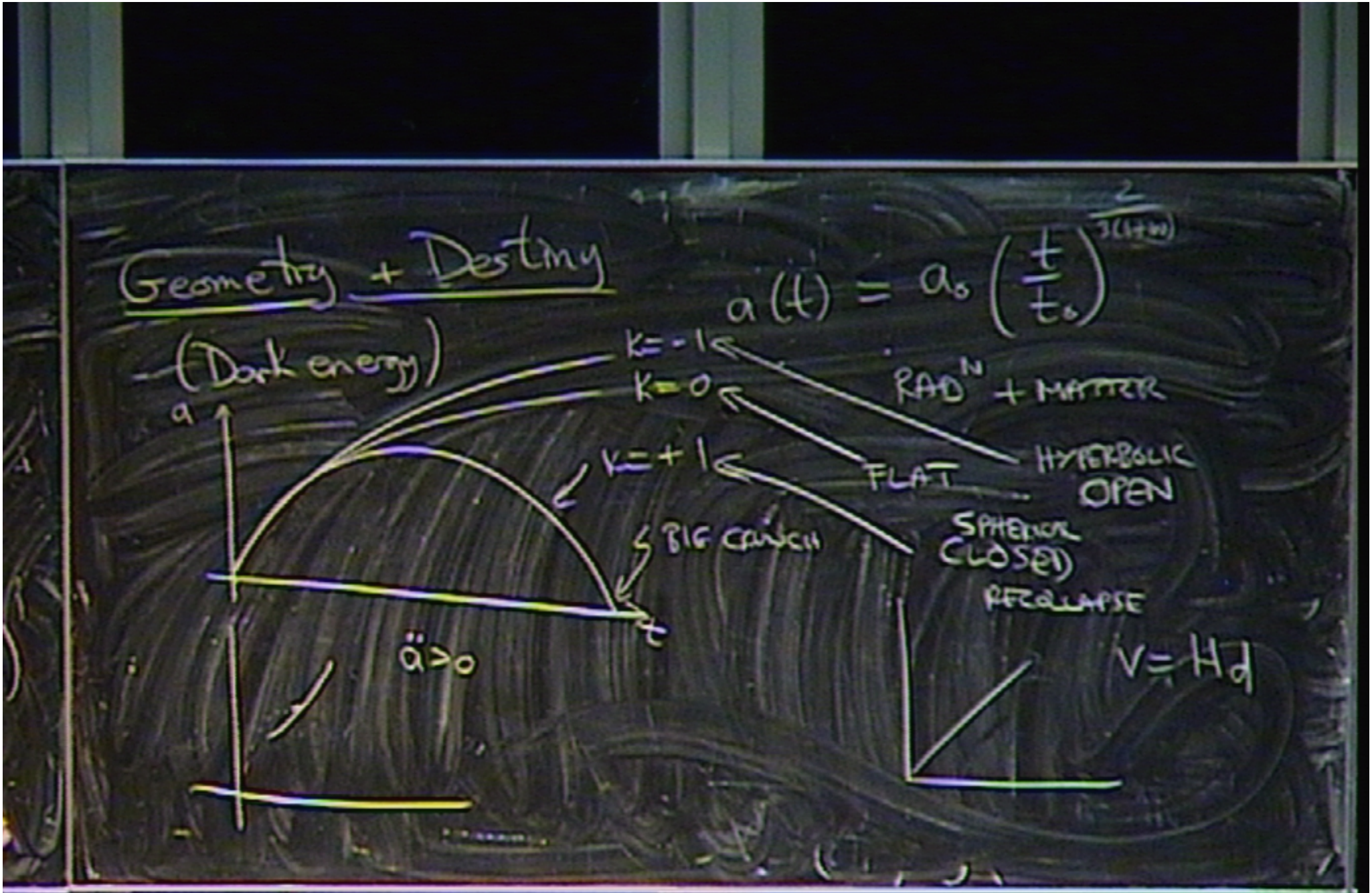
$h = 0.71 \pm 0.06$

$\Omega_M \approx 0.25$

BARYONS
DARK MATTER

$0.05 = \Omega_b \subset \Omega_M$


Geometry + Destiny
$a(t) = a_0 \left(\frac{t}{t_0}\right)^{\frac{2}{3(1+w)}}$
(Dark energy)
a
K=-1
K=0
K=+1
RAD^N + MATTER
HYPERBOLIC OPEN
FLAT
SPHERICAL CLOSED
RECOLLAPSE
V=Hd


Geometry + Destiny
$a(t) = a_0 \left(\frac{t}{t_0}\right)^{\frac{2}{3(1+w)}}$
(Dark energy)
a
K=-1
K=0
K=+1
RAD^N + MATTER
HYPERBOLIC OPEN
FLAT
SPHERICAL CLOSED
BIG CRUNCH
RECOLLAPSE
$\ddot{a}>0$
$V = Hd$