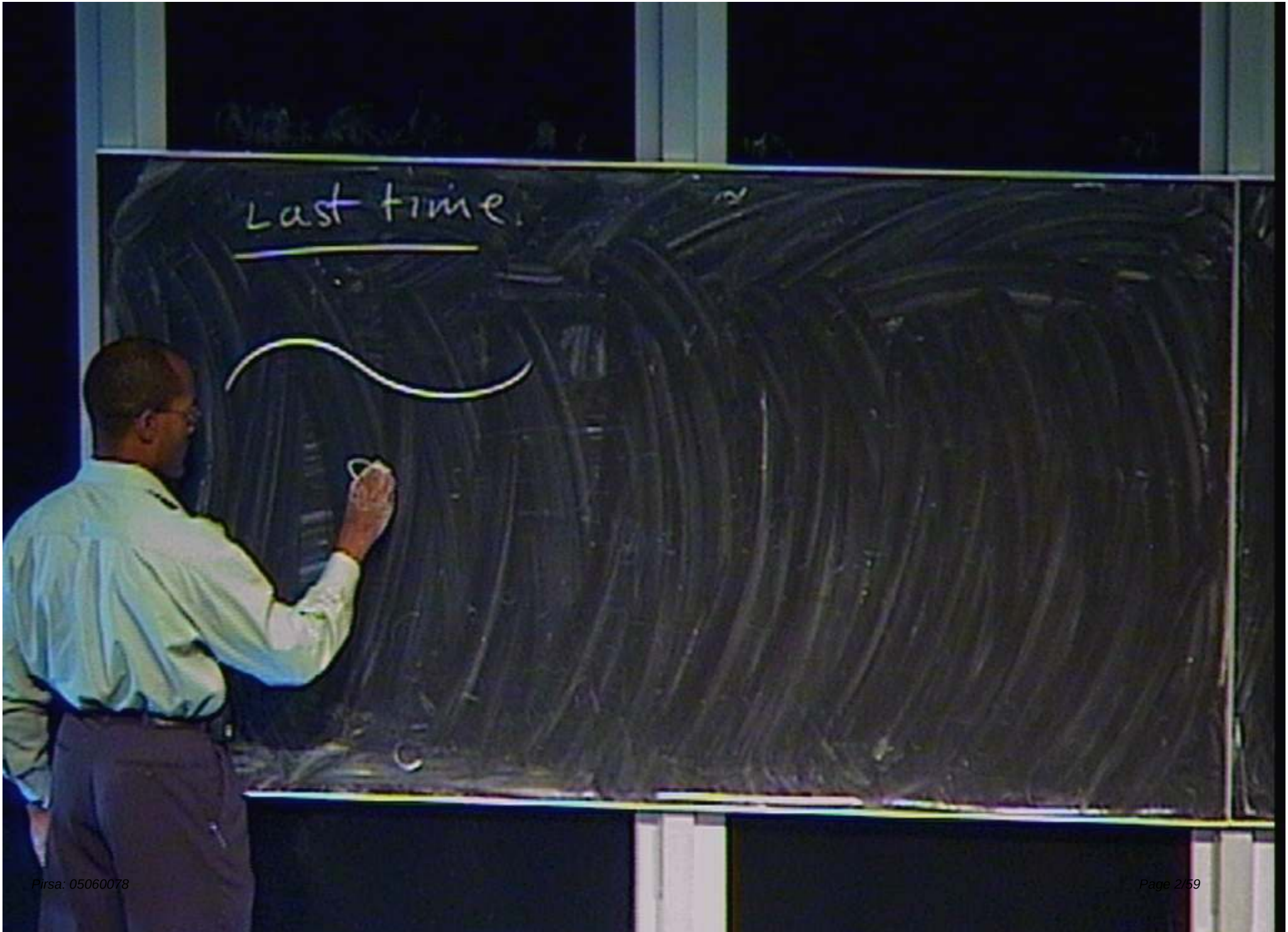
Title: Perturbative String Theory

Date: Jun 22, 2005 11:30 AM

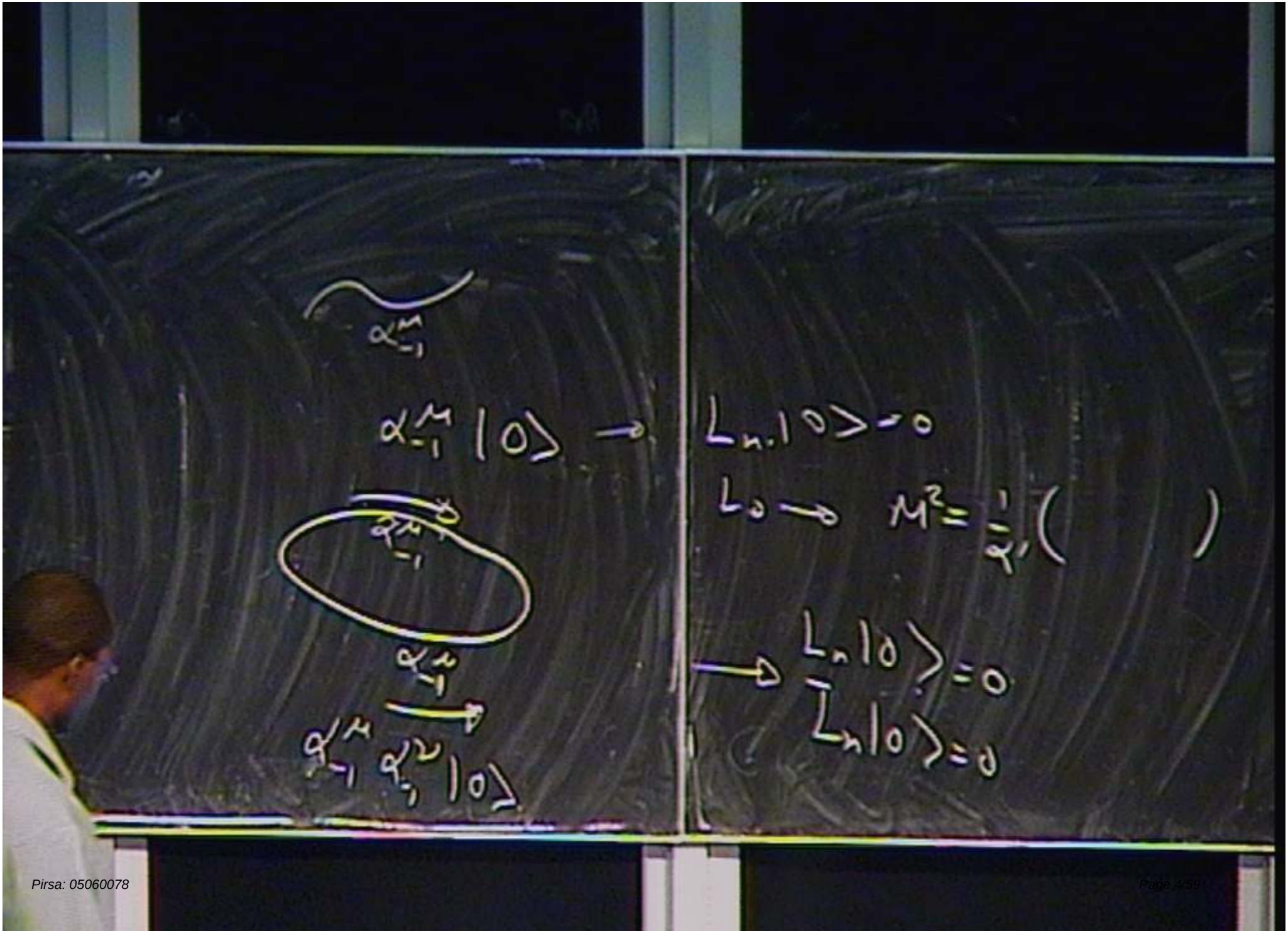
URL: http://pirsa.org/05060078

Abstract:


Last time.

Last time:

$8_2 \oplus 8$


α_{-1}^{μ}
$\alpha_{-1}^{\mu}|0\rangle$
$L_n|0\rangle = 0$
$L_0 \rightarrow M^2 = \frac{1}{\alpha'}($ $)$
α_{-1}^{μ}
α_{-1}^{μ}
$\alpha_{-1}^{\mu}\alpha_{-1}^{\nu}|0\rangle$
$L_n|0\rangle = 0$
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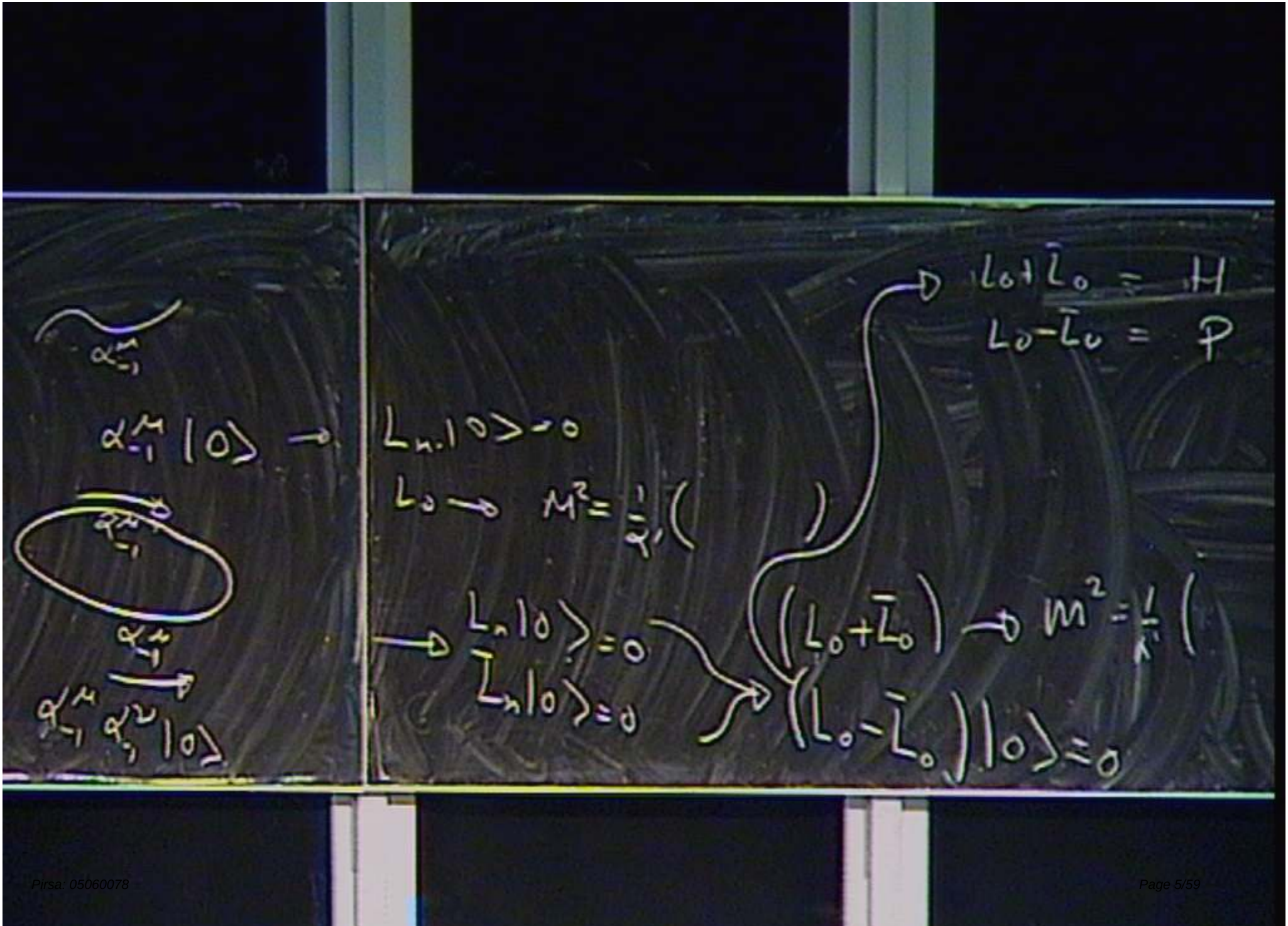

α_{-1}^{μ}
$\alpha_{-1}^{\mu}|0\rangle$
$L_n|0\rangle = 0$
$L_0 \rightarrow M^2 = \frac{1}{\alpha'}($
$L_0 + \bar{L}_0 = H$
$L_0 - \bar{L}_0 = P$
α_{-1}^{μ}
$\alpha_{-1}^{\mu}\alpha_{-1}^{\nu}|0\rangle$
$L_n|0\rangle = 0$
$\bar{L}_n|0\rangle = 0$
$(L_0 + \bar{L}_0) \rightarrow M^2 = \frac{1}{\alpha'}($
$(L_0 - \bar{L}_0)|0\rangle = 0$

$L_0 + \bar{L}_0 = H$

$L_0 - \bar{L}_0 = P_\sigma$ – generates translations on σ

→ "Level matching"

$(L_0 + \bar{L}_0) \rightarrow M^2 = \frac{1}{\alpha'} (\quad)$

$(L_0 - \bar{L}_0)|0\rangle = 0$

Last time.

universal. NS-NS

$8_v \otimes 8_v = 1 \oplus \Phi$

Type IIA $(8_v \oplus 8_s)_{NS} \otimes (8_v \oplus 8_c)_{R}$ — R-R: $8_s \otimes 8_c =$

Type IIB $(8_v \oplus 8_s) \otimes (8_v \oplus 8_s)$

universal NS-NS

$8_v \otimes 8_v = 1 \oplus \Phi$ | $28 \oplus 35$

$B_{\mu\nu}$ $G_{\mu\nu}$

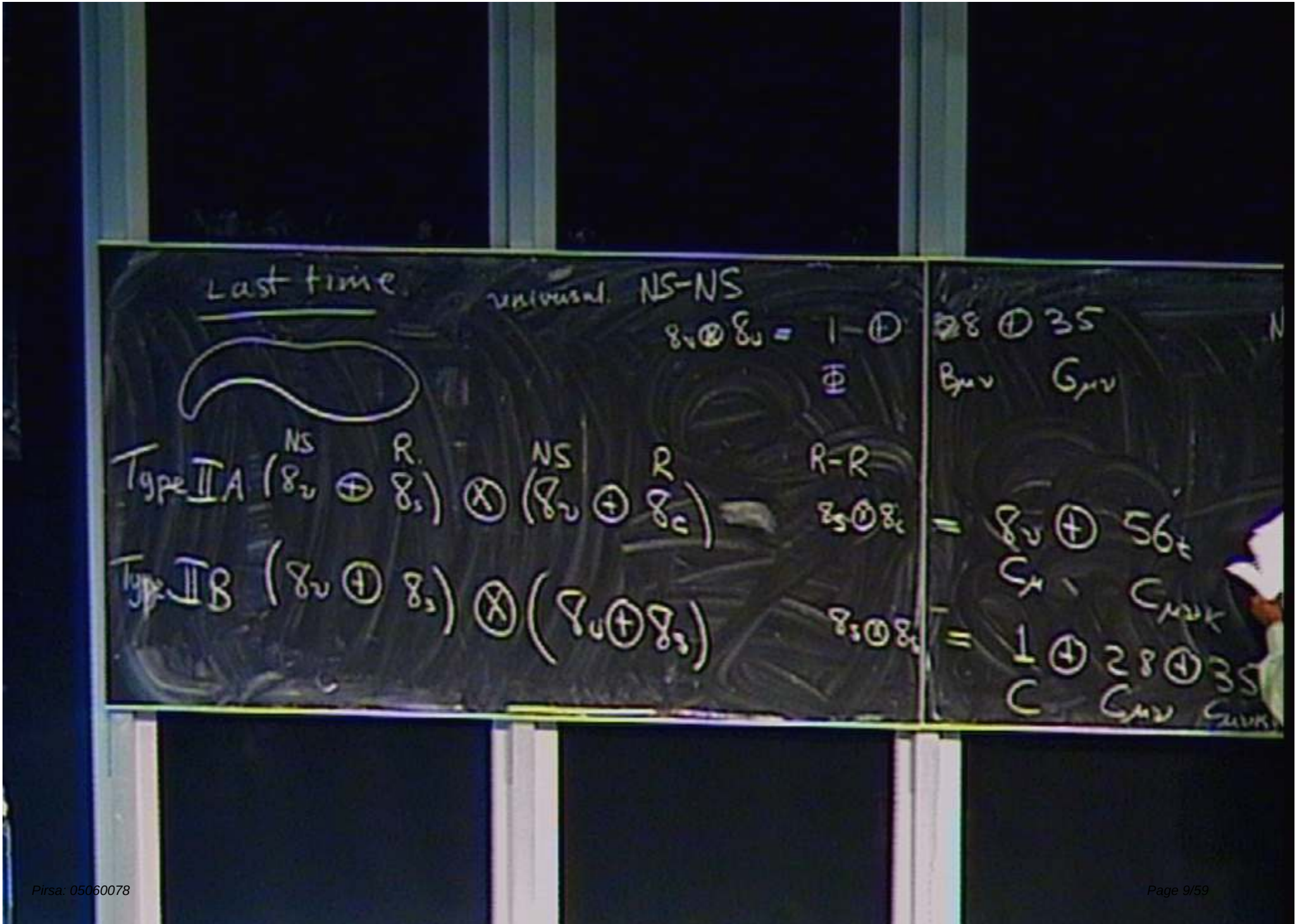
NS R R-R

$(\quad) \otimes (8_v \oplus 8_c)$ $8_s \otimes 8_c = 8_v \oplus 56_t$

C_μ $C_{\mu\nu k}$

$(\quad) \otimes (8_v \oplus 8_s)$ $8_s \otimes 8_s = 1 \oplus 28 \oplus 35_+$

C $C_{\mu\nu}$ $C_{\mu\nu kp}$


Last time.
universal NS-NS
$8_v \otimes 8_v = 1 \oplus \Phi$
$28 \oplus 35$
$B_{\mu\nu}$ $G_{\mu\nu}$
Type IIA $(8_v \oplus 8_s) \otimes (8_v \oplus 8_c)$
NS R NS R
R-R
$8_s \otimes 8_c = 8_v \oplus 56_t$
C_μ $C_{\mu\nu\lambda}$
Type IIB $(8_v \oplus 8_s) \otimes (8_v \oplus 8_s)$
$8_s \otimes 8_s = 1 \oplus 28 \oplus 35$
C $C_{\mu\nu}$ $C_{\mu\nu\lambda\rho}$

NS-R $8_v \otimes 8_c = 8_s \oplus 56_c$

$\lambda_\alpha \quad \psi^\mu_\lambda$

R-NS $8_v \otimes 8_s = 8_c \oplus 56_s$

$\lambda_\alpha \quad \psi^\mu_\alpha$

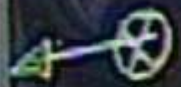
Type IIA has one each. $(1,1)$ super

Type IIB " two " $\otimes$ $(0,2)$ "

35_+

Type II A has one each. (1,1) supersymmetry

Type II B " two of ⊗ (0,2) "



Type IIA has one each. $(1,1)$ supersymmetry

Type IIB " two of ⊗ $(0,2)$ "

These fields (just like in bosonic case) have low-energy effective action...

$$S_{IIA} = \frac{1}{2\kappa_0^2}\int d^{10}x\,(-G)^{1/2}\left\{e^{-2\Phi}\left[R + 4(\nabla\Phi)^2 - \frac{1}{12}(H^{(3)})^2\right] - \frac{1}{4}(G^{(2)})^2 - \frac{1}{48}(G^{(4)})^2\right\}$$

$$- \frac{1}{4\kappa_0^2}\int B^{(2)} \wedge dC^{(3)} \wedge dC^{(3)}$$

Type IIA

Type IIB

These

56_t

$C_{\mu\nu\kappa}$

$28 \oplus 35_+$

$C_{\mu\nu}$ $C_{\mu\nu\kappa\rho}$

$$S_{IIB} = \int (\text{same top line}) - \frac{1}{12}\left(G^{(3)} + C^{(0)} \wedge H^{(3)}\right)^2 - \frac{1}{2}\left(dC^{(0)}\right)^2 - \frac{1}{480}\left(G^{(5)}\right)^2 + \frac{1}{4\kappa_0^2}\int \left(C^{(4)} + \frac{1}{2}B^2 \wedge C^{(2)}\right) \wedge G^{(3)} \wedge H^{(3)}$$

$$S_{IIA} = \frac{1}{2\kappa_0^2}\int d^{10}$$

Go to Einstein frame using $G_{\mu\nu} = e^{\frac{\phi}{2}} G_{\mu\nu}$

Enforce $F^{(5)} = *F^{(5)}$ by hand

$$G^{(5)} = dC^{(4)} + H^{(3)} \wedge C^{(2)}$$

56_+ $C_{\mu\nu k}$

$28 \oplus 1 \oplus 35_+$

$C_{\mu\nu}$ $C_{\mu\nu k\rho}^+$

$$S_{IIB} = \int (\text{same top line})$$

$$-\frac{1}{12}\left(G^{(3)} + C^{(0)} \wedge H^{(3)}\right)^2 - \frac{1}{2}\left(dC^{(0)}\right)^2$$

$$-\frac{1}{480}\left(G^{(5)}\right)^2 + \frac{1}{4k_0^2}\int \left(C^{(4)} + \frac{1}{2} B^2 \wedge C^{(4)}\right) \wedge G^{(4)} \wedge H^{(3)}$$

$G_{\mu\nu}$ Type IIA Type IIB

These

$$S_{IIA} = \frac{1}{2k_0^2}\int d^{10}$$

$G_{\mu\nu}$ $\quad 2\kappa^2 = 2\kappa_0^2 g_s^2 = (2\pi)^7 \alpha'^4 g_s^2 = 16\pi G_N$

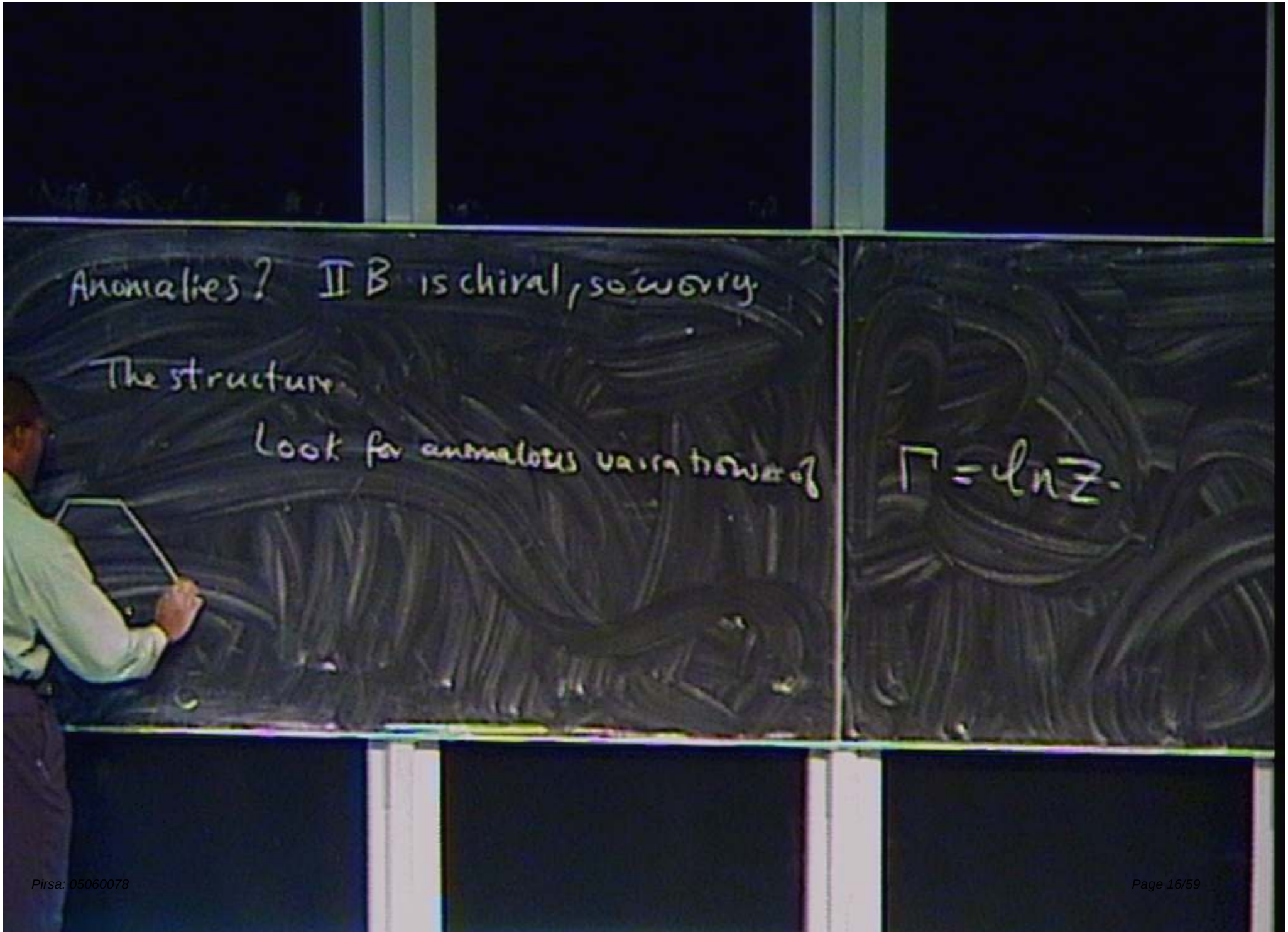
$g_s = e^{\Phi_0}$

These fields (just like in bosonic case) have lowenergy effective action...

$$S_{IIA} = \frac{1}{2\kappa_0^2}\int d^{10}x\,(-G)^{1/2}\left\{e^{-2\Phi}\left[R + 4(\nabla\Phi)^2 - \frac{1}{12}(H^{(3)})^2\right] - \frac{1}{4}(G^{(2)})^2 - \frac{1}{48}(G^{(4)})^2\right\}$$

$(dC^{(0)})^2$

$C^{(4)} \wedge G^{(4)} \wedge H^{(3)} \quad = \frac{1}{4\kappa^2}\int B^{(2)} \wedge dC^{(3)} \wedge dC^{(3)}$


Anomalies? IIB is chiral, so worry.
The structure.
Look for anomalous variations of
$\Gamma = \ln Z$.

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Look for anomalous variations of $\Gamma = \ln Z$.

Anomalies? IIB is chiral, so worry

The structure

Look for anomalous variation of

$$\delta(\quad) = \frac{i}{(2\pi)^{D/2}} \int \hat{I}_D$$

Define $F = dA +$

$\Gamma = \ln Z$.

under gauge transformation

Anomalies? IIB is chiral, so worry

The structure

Look for anomalous variation of

$$\delta \ln Z = \frac{i}{(2\pi)^{D/2}} \int \hat{A} I_D$$

Define $F = dA +$

$$\Gamma = \ln Z.$$

under gauge transformation

Define $F = dA + \Lambda^2$ $\quad R^a{}_{b\mu\nu}\, dx^\mu \wedge dx^\nu$

$R = d\omega + \omega^2$

$\Gamma = \ln Z$.

Descent relation from $\hat{I}_{D+2}$

under gauge transformation

$G_{\mu\nu}$ $\quad 2K^2 = 2K_0^2 g^2$

$g = e^{\phi}$

These fields (i

have low

$S_{II\Lambda} = \frac{1}{2K_0^2}\int d^{10}x\,(-G)^{1/2}\{$

$(dC^{(0)})^2$

$C^{(4)} \wedge G^{(4)} \wedge H^{(3)}$ $\quad -\frac{1}{4K_0^2}\int$

Define $F = dA + A^2$ $\quad R^a{}_{b\mu\nu}\, dx^\mu \wedge dx^\nu$

$R = d\omega + \omega^2$

$\Gamma = \ln Z$.

Descent relation from

$$\hat{I}_{D+2} = d\hat{I}_{D+1}$$

$\hat{I}_{D+1}$ is not gauge invariant

under gauge transformation but $\delta\hat{I}_{D+1} = d\hat{I}_D$

$G_{\mu\nu}$ $\quad 2K^2 = 2K_0^2 g_s^2 =$

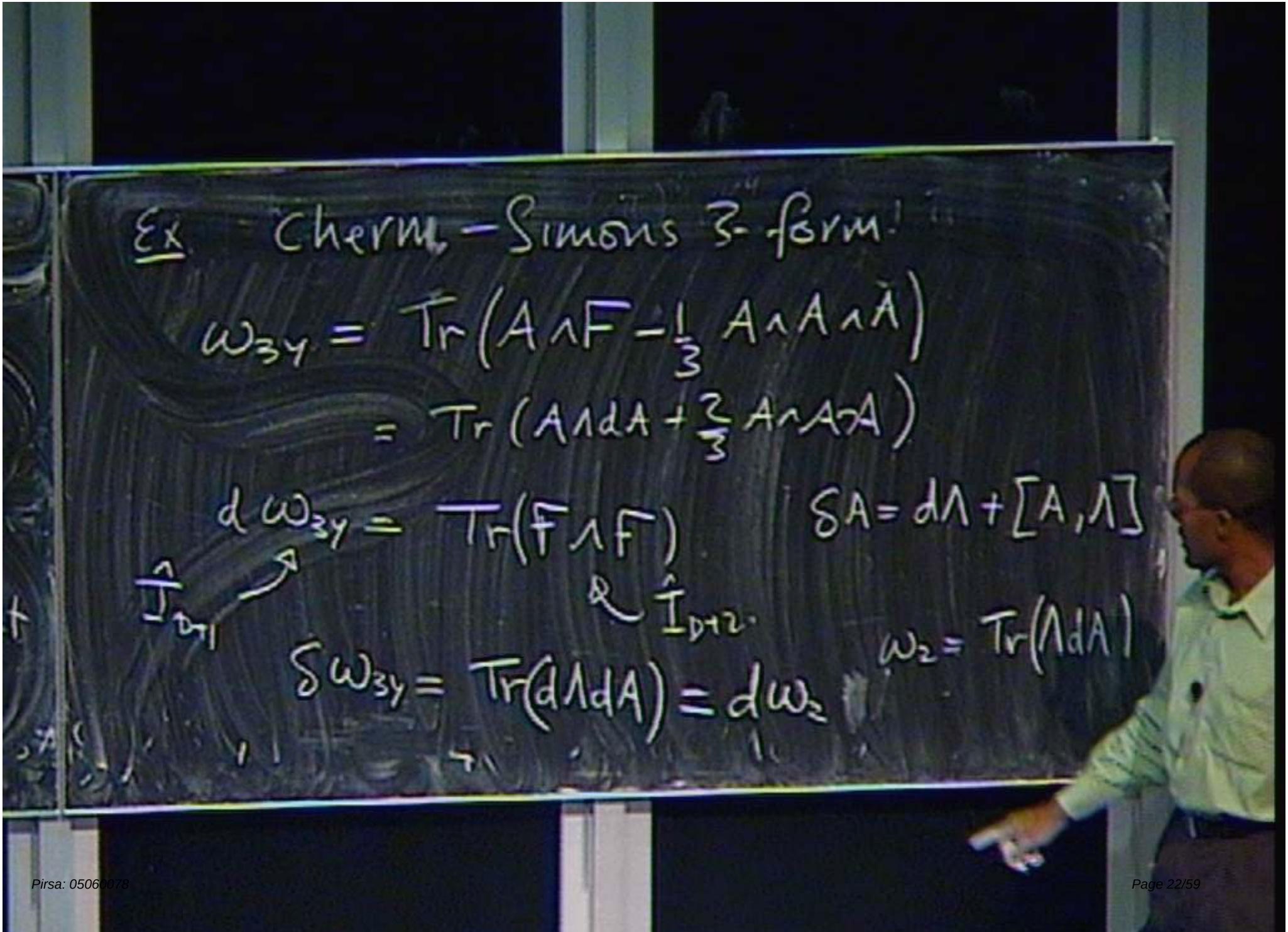
$g_s = e^{\phi}$.

These fields (i
have low

$S_{IA} = \frac{1}{2K^2}\int d^{10}x\,(-G)^{1/2}\{$

$(H^{(p)})^2$

$C^{(4)} \wedge G^{(7)} \wedge H^{(3)}$ $\quad = \frac{1}{4K^2}\int$


Ex Cherm.-Simons 3-form!
$\omega_{3\gamma} = \mathrm{Tr}\left(A\wedge F - \frac{1}{3} A\wedge A\wedge A\right)$
$= \mathrm{Tr}\left(A\wedge dA + \frac{2}{3} A\wedge A\wedge A\right)$
$d\omega_{3\gamma} = \mathrm{Tr}(F\wedge F)$
$\delta A = d\Lambda + [A,\Lambda]$
$\hat{I}_{D+1}$
$\hat{I}_{D+2}$
$\omega_2 = \mathrm{Tr}(\Lambda dA)$
$\delta\omega_{3\gamma} = \mathrm{Tr}(d\Lambda dA) = d\omega_2$

Ex Cherm.-Simons 3-form:

$$\omega_{3\gamma} = \mathrm{Tr}\left(A\wedge F - \frac{1}{3} A\wedge A\wedge A\right)$$

$$= \mathrm{Tr}\left(A\wedge dA + \frac{2}{3} A\wedge A\wedge A\right)$$

$$d\omega_{3\gamma} = \mathrm{Tr}(F\wedge F) \qquad \delta A = d\Lambda + [A,\Lambda]$$

$\hat{I}_{D+1}$ → $d\omega_{3\gamma}$ & $\hat{I}_{D+2}$.

$$\delta\omega_{3\gamma} = \mathrm{Tr}(d\Lambda\, dA) = d\omega_2 \qquad \omega_2 = \mathrm{Tr}(\Lambda\, dA)$$

Anomalies? IIB is chiral, so worry.

The structure.

8_c
56_s } 2
35_+ × 1

Look for anomalous variations of

$$\delta \ln Z = \frac{i}{(2\pi)^{D/2}} \int \hat{I}_D$$

$$I_{12}^{8c} = -\frac{\mathrm{tr}R^6}{725760} - \frac{\mathrm{tr}R^4\,\mathrm{tr}R^2}{552960} - \frac{(\mathrm{tr}R^2)^3}{1327104}$$

$$I_{12}^{35_+} = 992\,(\quad) - 448\,(\quad) + 128\,(\quad)$$

$$I_{12}^{56_s} = -495\,(\quad) + 225\,(\quad) - 63\,(\quad)$$

IIB is chiral, so worry.

8_c, 56_s, 35_+ } 2 $\quad 2I_{12}^{8c} + 2I_{12}^{56s} + I_{12}^{35+}$

$\times 1$

k for anomalous variations of

$$\delta \ln Z = \frac{i}{(2\pi)^{D/2}} \int \hat{I}_D$$

Define $F = dA + A^2$

$$d\omega + \omega^2$$

$$\Gamma = \ln Z.$$

under gauge transformation

Descer

$\hat{I}_{D+}$

$\hat{I}_{D+1}$

but

IIB is chiral, so ~~worry~~.

$$\left.\begin{array}{l} 8_c \\ 56_s \end{array}\right\} 2 \qquad 2I_{12}^{8_c} + 2I_{12}^{56_s} + I_{12}^{35_+} = 0$$

$35_+ \quad \times 1$

k for anomalous variations of

$$\delta \ln Z = \frac{i}{(2\pi)^{D/2}} \int \hat{I}_D.$$

Define $F = dA + \Lambda^2$

$d\omega + \omega^2$

$$\Gamma = \ln Z.$$

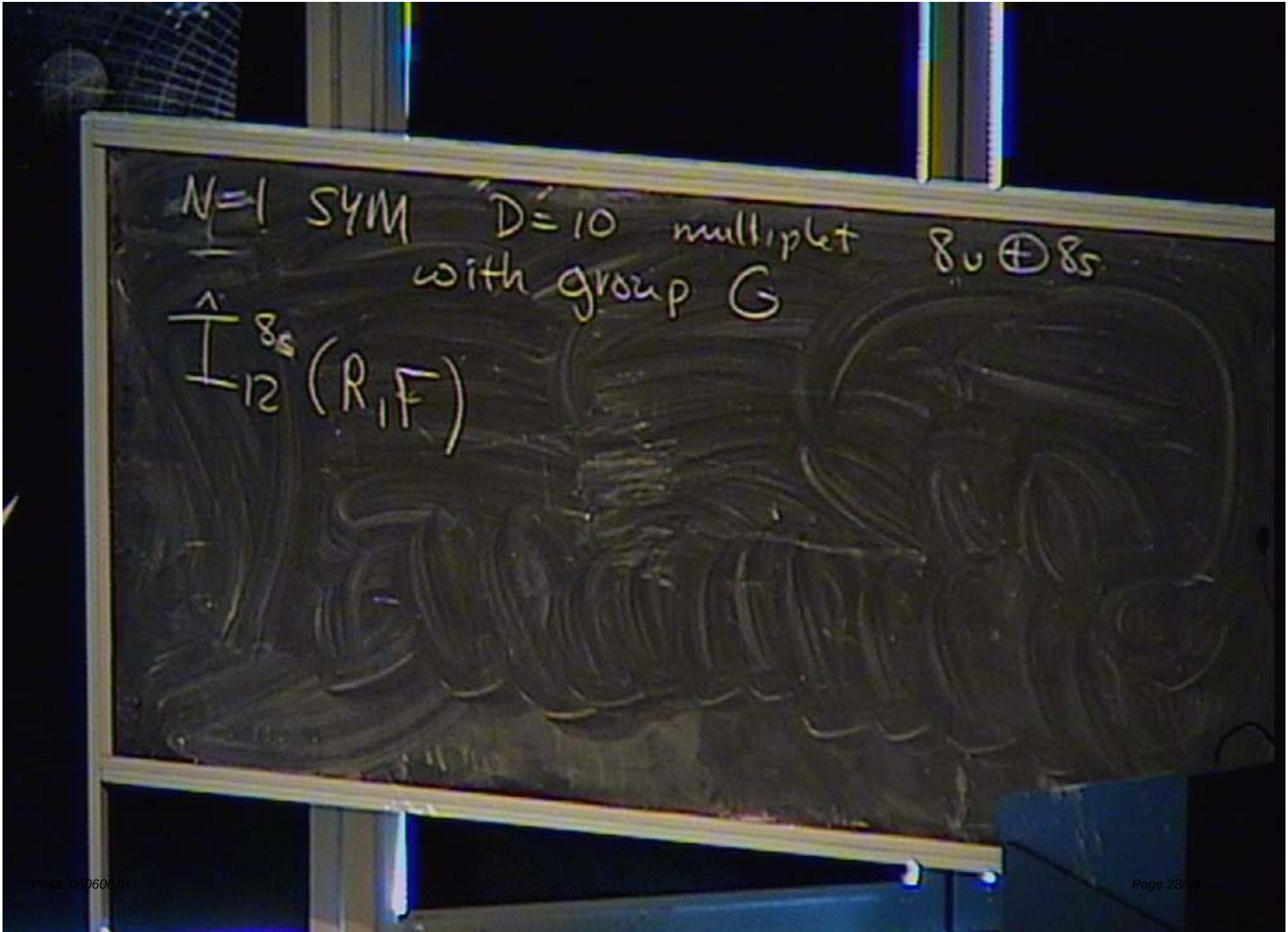
under gauge transformation

Descen

$\hat{I}_{D+}$

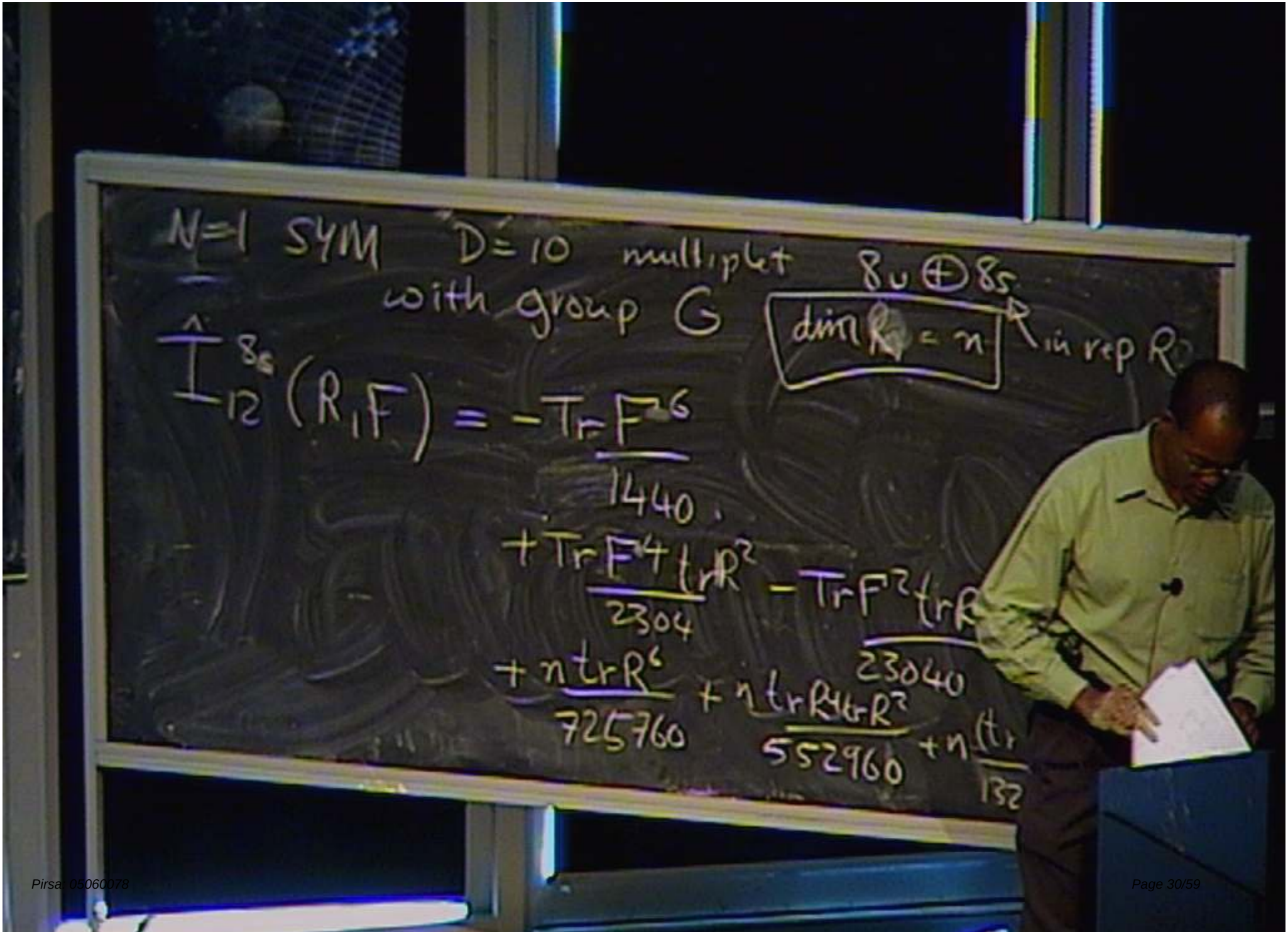
$\hat{I}_{D+1}$

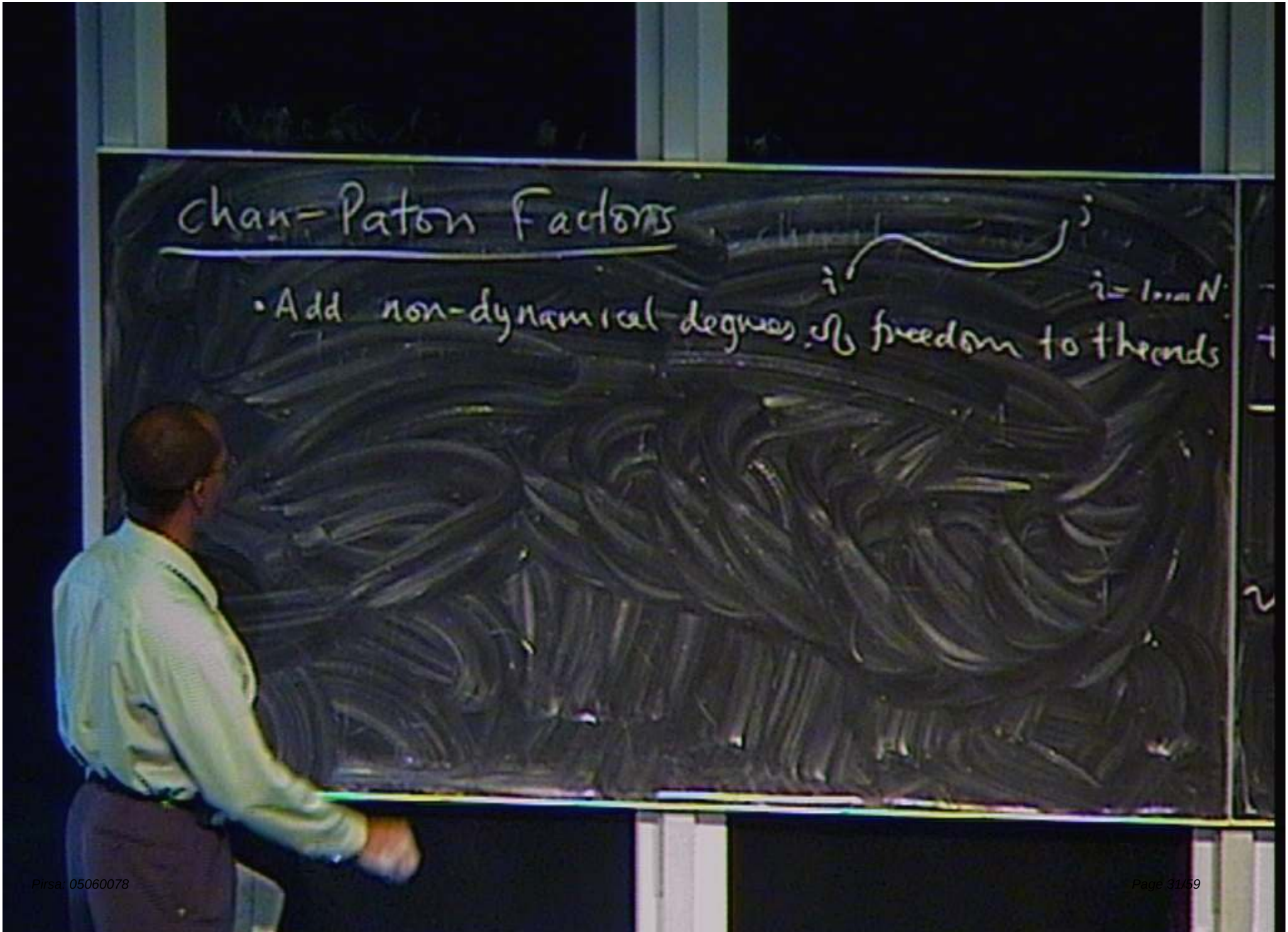
but

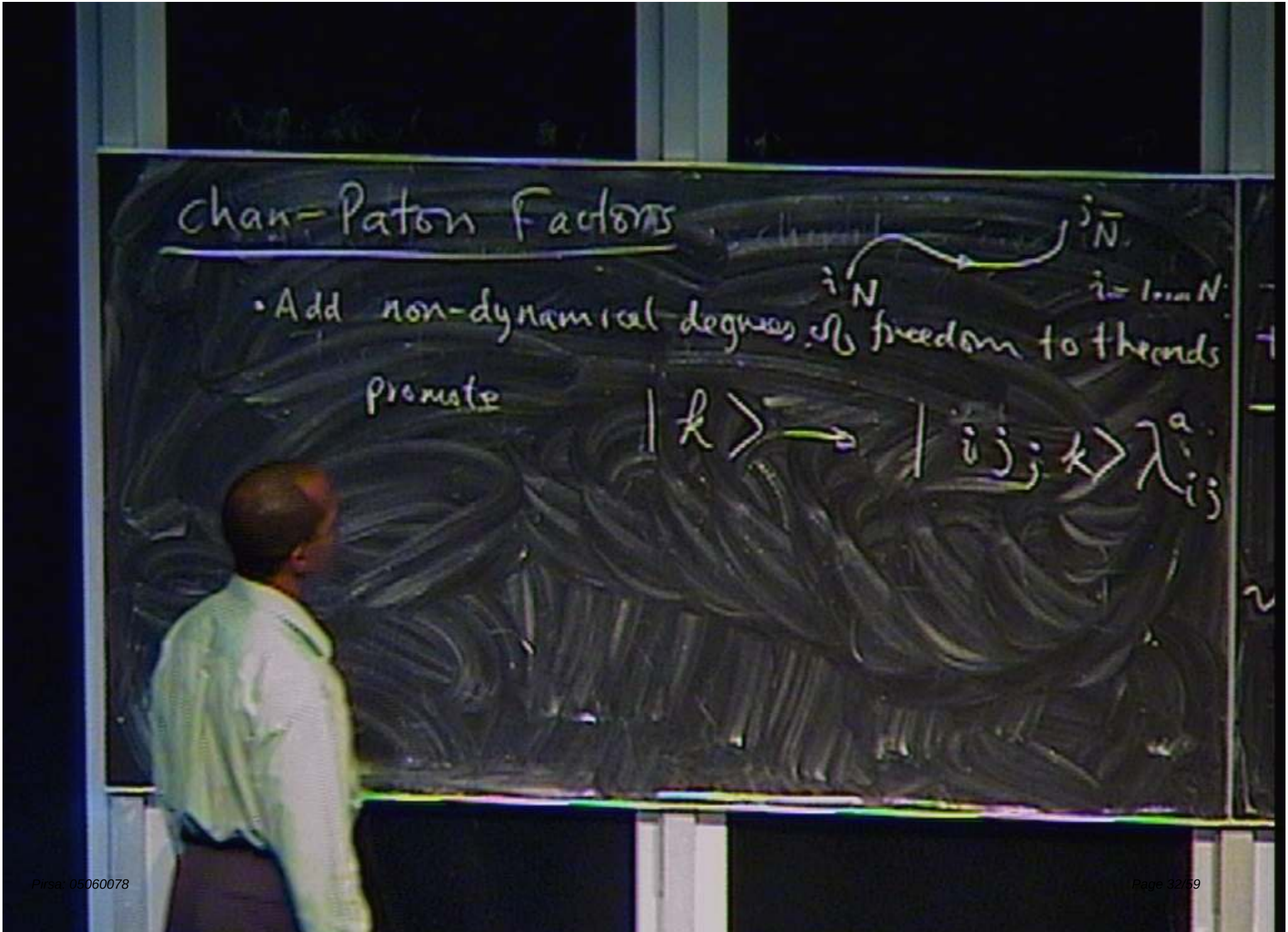

N=1 SYM D=10 multiplet $8_v \oplus 8_s$
with group G
$\hat{I}_{12}^{8_s}(R_1 F)$

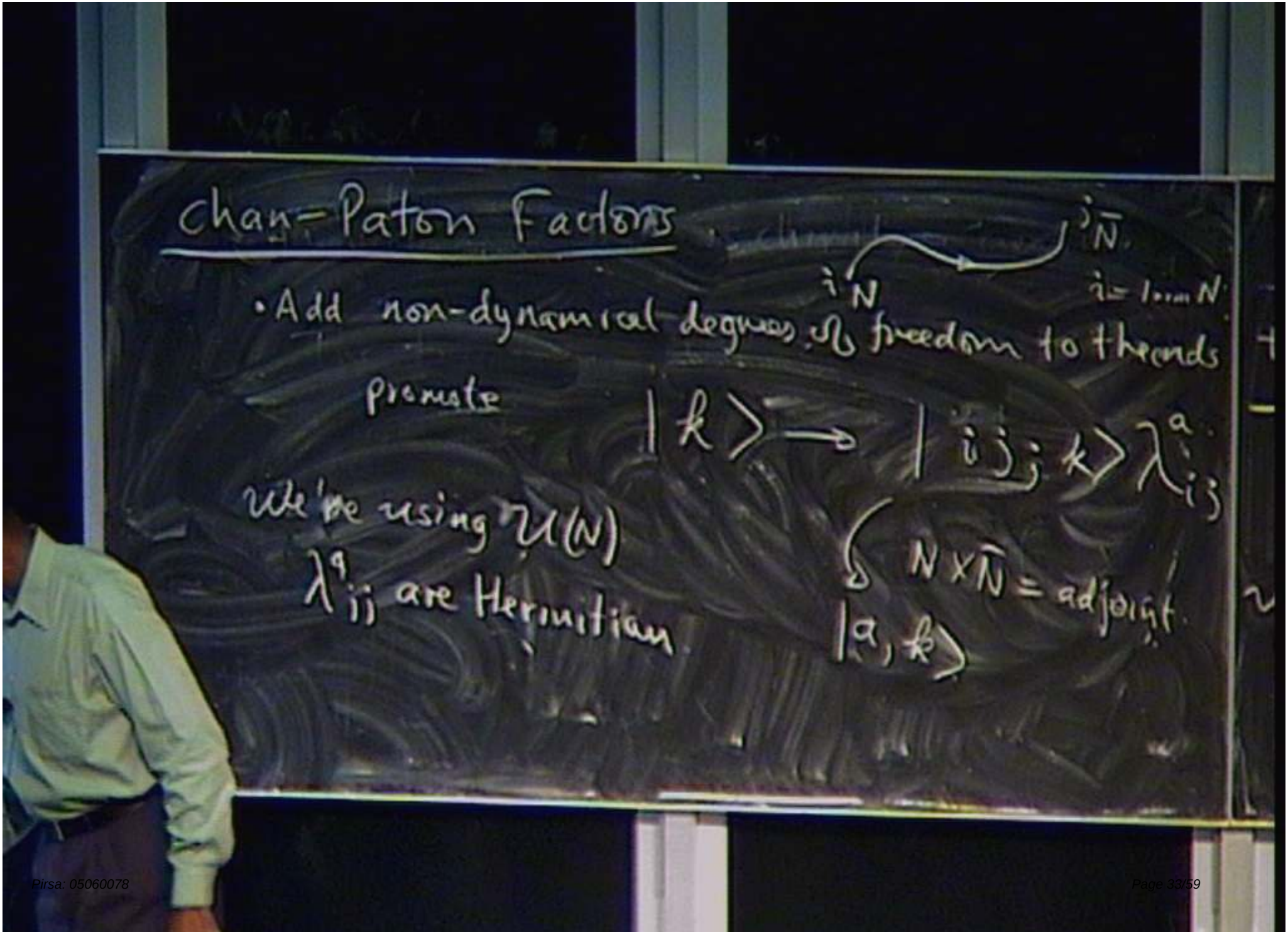
$N=1$ SYM $D=10$ multiplet $8_v \oplus 8_s$ with group G

$$\hat{I}_{12}^{8_s}(R,F) = -\frac{\mathrm{Tr}F^6}{1440} + \frac{\mathrm{Tr}F^4\,\mathrm{tr}R^2}{2304} - \frac{\mathrm{Tr}F^2\,\mathrm{tr}R^4}{23040} - \frac{\mathrm{Tr}F^2(\mathrm{tr}R^2)^2}{18432} + \frac{n\,\mathrm{tr}R^6}{725760} + \frac{n\,\mathrm{tr}R^4\mathrm{tr}R^2}{552960} + \frac{n(\mathrm{tr}R^2)^3}{1327104}$$


N=1 SYM D=10 multiplet 8v ⊕ 8s
with group G
dim R = n
R in rep R
$\hat{I}_{12}^{8s}(R,F) = -\frac{\mathrm{Tr}F^6}{1440}$
$+\frac{\mathrm{Tr}F^4\,\mathrm{tr}R^2}{2304} - \mathrm{Tr}F^2\,\mathrm{tr}R$
$+\frac{n\,\mathrm{tr}R^6}{725760} + \frac{n\,\mathrm{tr}R^4\mathrm{tr}R^2}{552960} + n\frac{(\mathrm{tr}$
132


Chan-Paton Factors
i
j
i = 1...N
• Add non-dynamical degrees of freedom to the ends


Chan-Paton Factors
i_N
$j_{\bar{N}}$
$i = 1 \ldots N$
• Add non-dynamical degrees of freedom to the ends
promote
$|k\rangle \rightarrow |ij;k\rangle \lambda^a_{ij}$


Chan-Paton Factors
$i_{\bar{N}}$
i_N
$i = 1 \ldots N$
• Add non-dynamical degrees of freedom to the ends
promote $|k\rangle \rightarrow |ij;k\rangle \lambda^a_{ij}$
$N \times \bar{N}$ = adjoint
$|a,k\rangle$
We're using $\mathcal{U}(N)$
λ^a_{ij} are Hermitian.

This will give local $U(\vec{v})$ in spacetime

(global on worldsheet)

This will give <u>local</u> $U(V)$ in spacetime

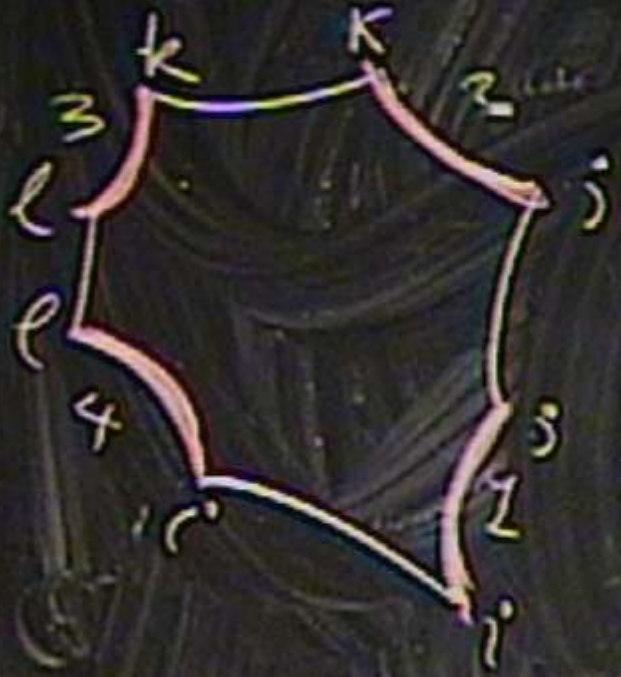
(global on worldsheet)

In any amplitude: ___

This will give local $U(N)$ in spacetime

(global on worldsheet)

In any amplitude:



There'll be a factor $\lambda^1_{ij}\lambda^2_{jk}\lambda^3_{kl}\lambda^4_{li}$

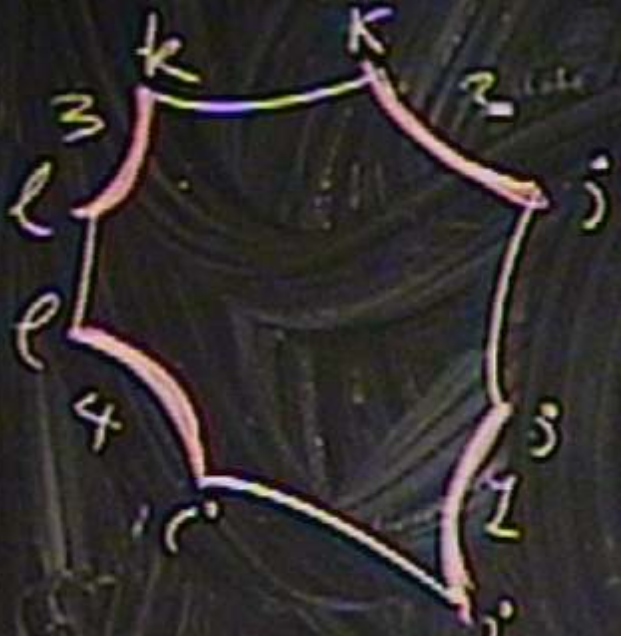
$= \mathrm{Tr}(\lambda^1\lambda^2\lambda^3\lambda^4)$

So $\lambda \to U\lambda U^{-1}$ invariance

This will give local $U(N)$ in spacetime

(global on worldsheet)

In any amplitude:



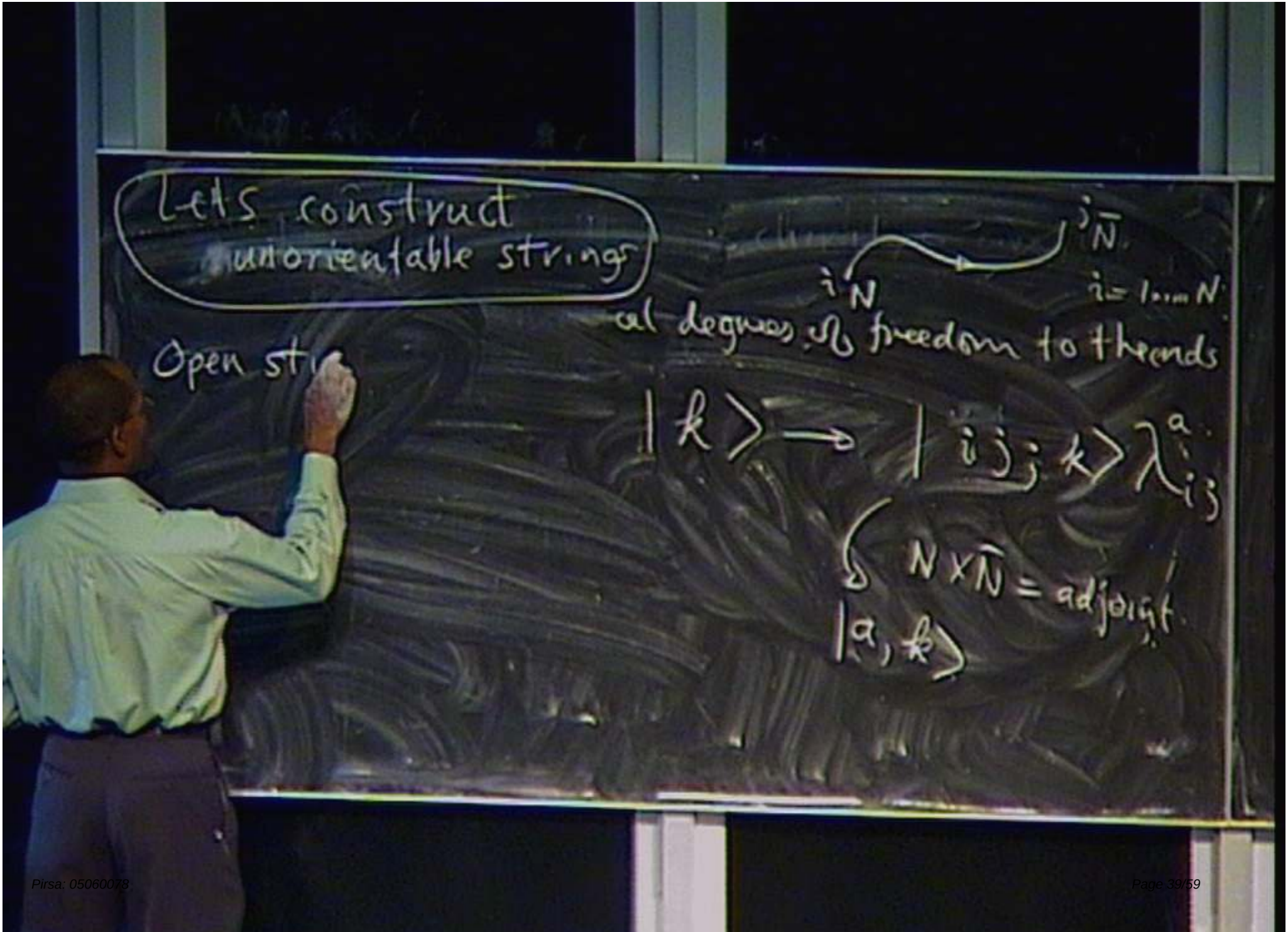
There'll be a factor

$$\lambda^1_{ij}\lambda^2_{jk}\lambda^3_{kl}\lambda^4_{li} = \frac{1}{N}\mathrm{Tr}(\lambda^1\lambda^2\lambda^3\lambda^4)$$

So $\lambda \to U\lambda U^{-1}$ invariance

photon → $\lambda^a_{ij}\ \psi^\mu_{-\frac{1}{2}}\ |ij;k\rangle$

A_μ : A^a_μ


Let's construct
unorientable strings
$i_{\bar{N}}$
i_N
$i = 1 \ldots N$
al degrees of freedom to threads
Open st
$|k\rangle \rightarrow |ij;k\rangle \lambda^a_{ij}$
$N \times \bar{N}$ = adjoint
$|a,k\rangle$

Lets construct unorientable strings

Open strings

$\Omega : \sigma \to \pi - \sigma$

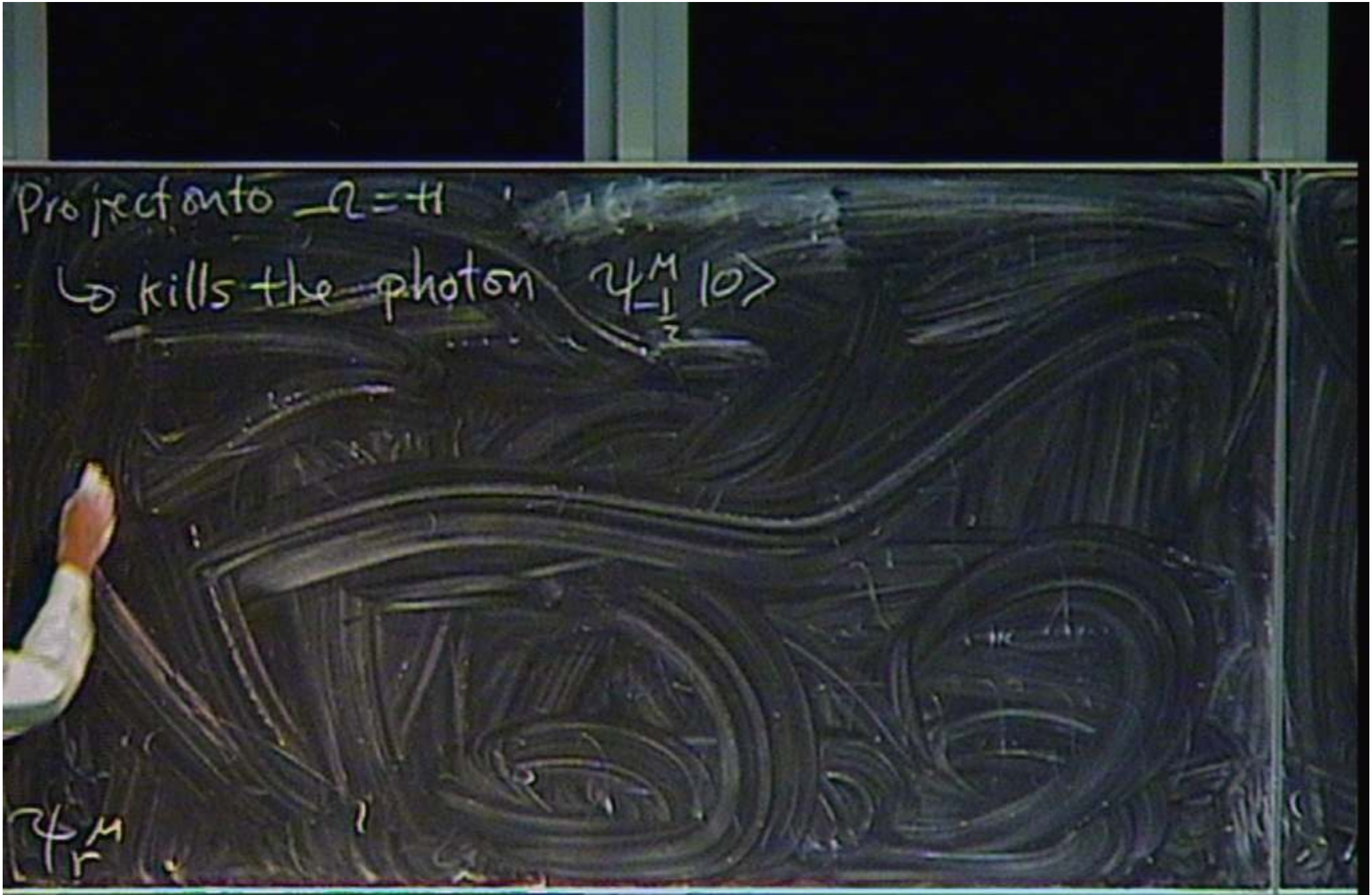
$\tau \to -\tau$

$x^\mu \to x^\mu$

$p^\mu \to p^\mu$

$\alpha_m^\mu \to (-1)^m \alpha_m^\mu$

$\psi_r^\mu \to (-1)^{r+1/2} \psi_r^\mu$


Project onto $\Omega = +1$
↳ kills the photon $\psi^M_{-\frac{1}{2}} |0\rangle$
ψ^M_r

Project onto $\Omega = +1$

↳ kills the photon $\psi^{M}_{-\frac{1}{2}}|0\rangle$

Saved it by transforming Chan-Paton λ

$$\Omega\ \lambda_{ij}\,|k,i,j\rangle \longrightarrow \lambda'_{ij}\,|k,i,j\rangle$$

$$\lambda' = M\lambda^{T}M^{-1}$$

But $\Omega^2 = 1$

$$\therefore \lambda \Rightarrow M(\lambda')^{T}M^{-1} = MM^{-T}\lambda M^{T}M^{-1}$$

$[\psi^{M}_{r}$

We are running over a complete set, so

Schur's lemmas $MM^{-T} \propto \mathbb{I}$

$\Rightarrow$ M is either symmetric or antisymmetric

(a) Symmetric $M = M^T = \mathbb{I}_N$

if $\lambda_{ij} \psi^{\mu}_{-\frac{1}{2}} |k, ij\rangle$ is to survive.

Then $\boxed{\lambda = -\lambda^T}$

We are running over a complete set, so

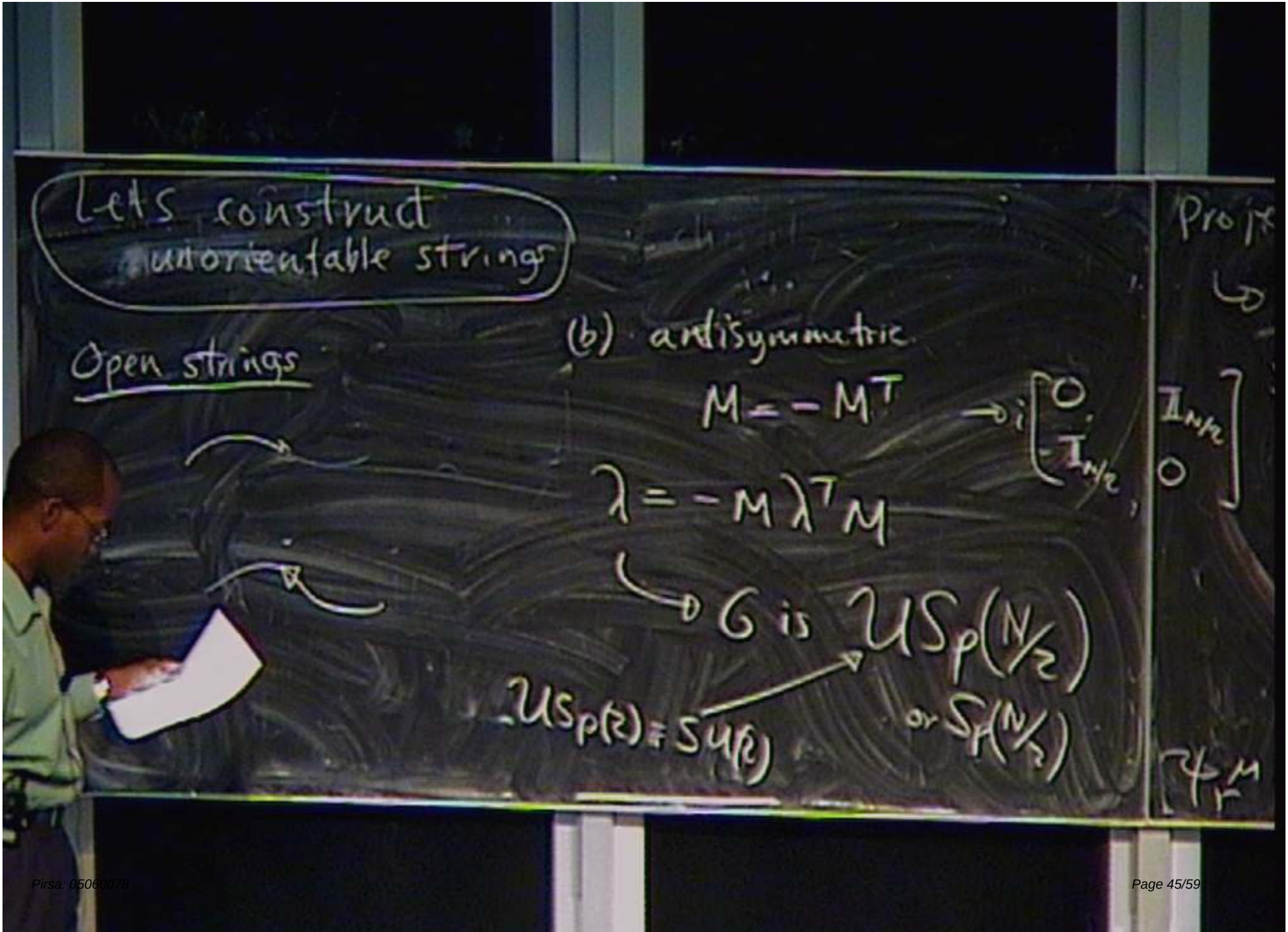
Schur's lemmas $MM^{-T} \propto \mathbb{I}$

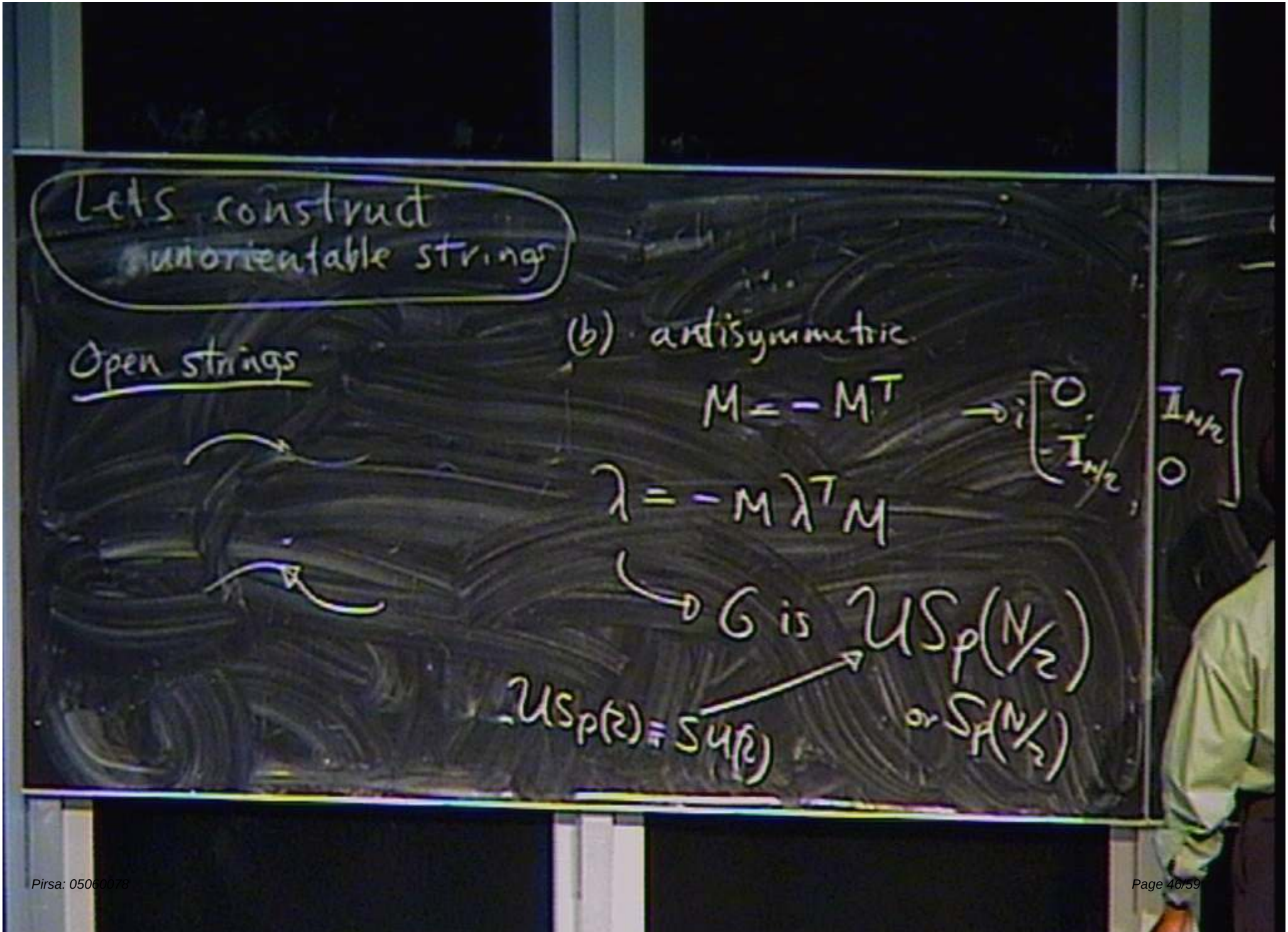
$\Rightarrow M$ is either symmetric or antisymmetric

(a) Symmetric $M = M^T = \mathbb{I}_N$

if $\lambda_{ij} \psi^{\mu}_{-\frac{1}{2}} |k, ij\rangle$ is to survive

$G = SO(N)$ Then $\boxed{\lambda = -\lambda^T}$


Let's construct
unorientable strings
Open strings
(b) antisymmetric
$M = -M^T$
$\lambda = -M\lambda^T M$
G is $USp(N/2)$
$USp(2) = SU(2)$
or $SO(N/2)$
Proj
$\begin{bmatrix} 0 & I_{N/2} \\ -I_{N/2} & 0 \end{bmatrix}$


Let's construct
unorientable strings
Open strings
(b) antisymmetric
$M = -M^T \rightarrow i \begin{bmatrix} 0 & I_{N/2} \\ -I_{N/2} & 0 \end{bmatrix}$
$\lambda = -M \lambda^T M$
G is $USp(N/2)$
$USp(2) = SU(2)$
or $SO(N/2)$

Closed Strings

$\Omega : \sigma \to -\sigma$

$z \to \bar{z}$

$\tilde{\psi}_r^M \longleftrightarrow \psi_r^M$

$\begin{bmatrix} \mathbb{I}_{n/2} \\ 0 \end{bmatrix}$

Ω is a symmetry of IIB

Closed Strings

$\Omega : \sigma \to -\sigma$

$z \to \bar{z}$

$\tilde{\psi}_r^\mu \longleftrightarrow \psi_r^\mu$

$\begin{bmatrix} I_{8\times 8} \\ 0 \end{bmatrix}$

Ω is a symmetry of IIB. So project onto $\Omega = +1$

NS-NS	1	28	35
	Φ	$B_{\mu\nu}$	$G_{\mu\nu}$
Ω	$+1$	-1	$+1$

Closed Strings

$\Omega : \sigma \to -\sigma$ $\qquad \tilde{\psi}_r^\mu \longleftrightarrow \psi_r^\mu$

$z \to \bar{z}$

$\begin{bmatrix} I_{N/2} \\ 0 \end{bmatrix}$

Ω is a symmetry of IIB. So project onto $\Omega = +1$

NS-NS	1	28	35	
	Φ	$B_{\mu\nu}$	$G_{\mu\nu}$	
Ω	+1	-1	+1	
R-R	C	$C_{\mu\nu}$	$C_{\mu\nu\lambda\sigma}$	
Ω	-1	+1	-1	

Closed Strings

$\Omega : \sigma \to -\sigma$

$z \to \bar{z}$

$\tilde{\psi}_r^\mu \longleftrightarrow \psi_r^\mu$

Ω is a symmetry of IIB. So project onto $\Omega = +1$

NS-NS	1	28	35
	Φ	$B_{\mu\nu}$	$G_{\mu\nu}$
Ω	+1	−1	+1

only one dilatino-g survives.

RR			
	C	$C_{\mu\nu}$	$C_{\mu\nu\nu_5}$
Ω	−1	+1	−1

NS-R

R-NS

anor

$8_c \oplus 56_s$

ravitino

anomalous

$8 \oplus 56$ x 1

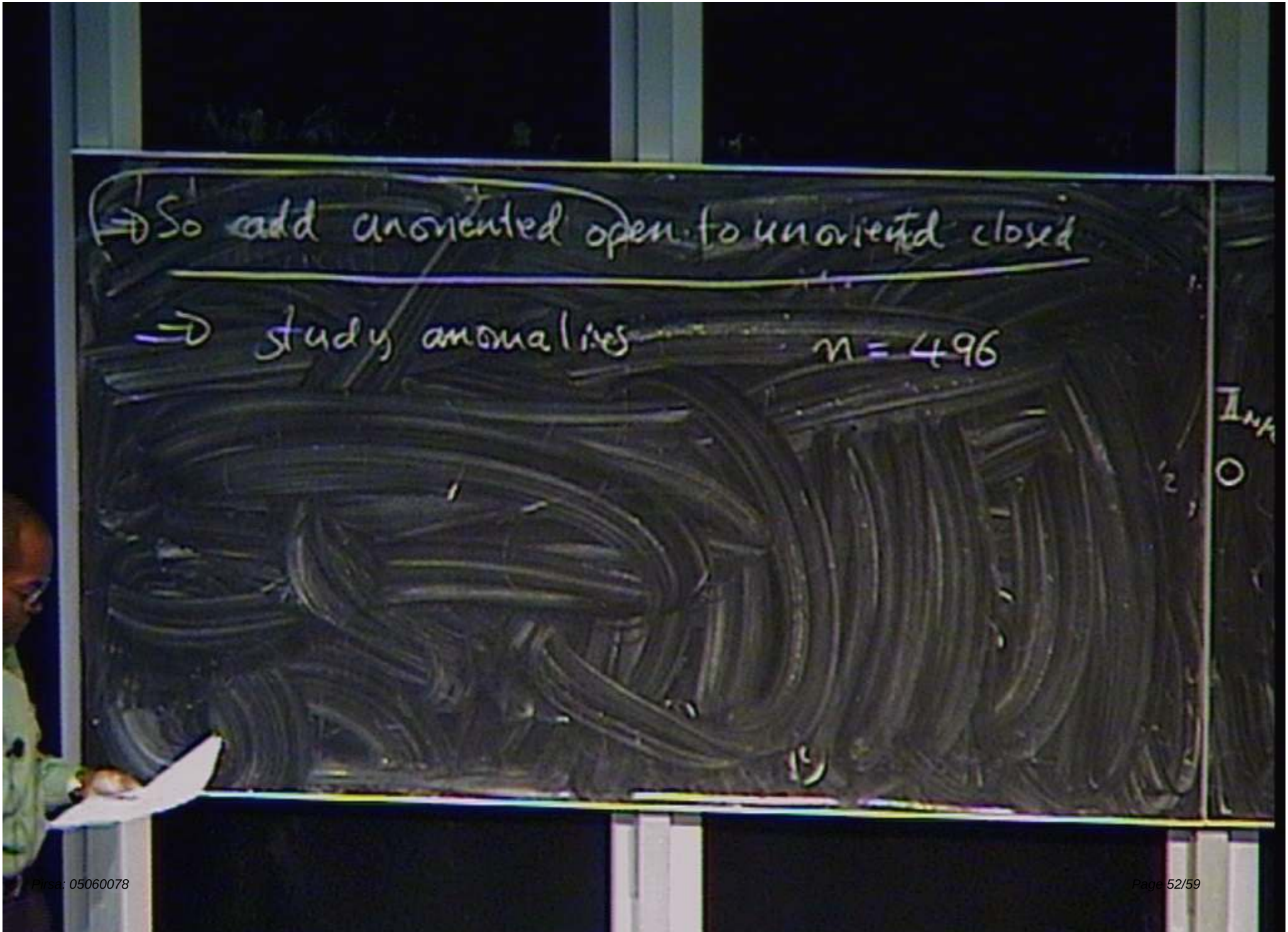
~~3 x 4~~

see an N=1 unorientable closed string.

$8_c \oplus 56_s$

gravitino

R-NS


⇒ So add unoriented open to unoriented closed
⇒ study anomalies
n = 496

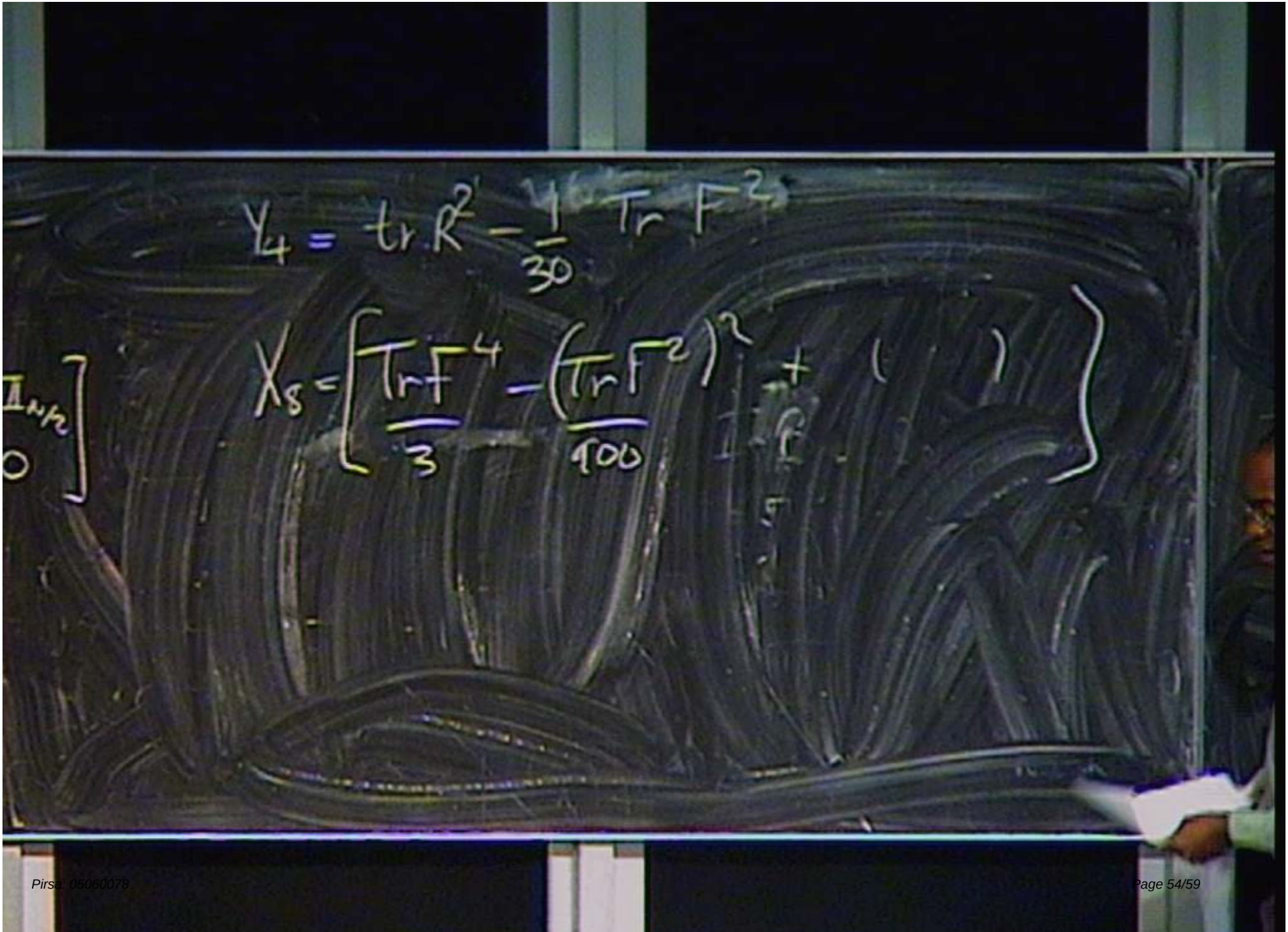
⇒ So add unoriented open to unoriented closed

⇒ study anomalies $n = 496$

pick $n = 496$ vanish if $G = E_8 \times E_8$ or $SO(32)$

$$I_{12} = \frac{1}{3 \times 2^8} Y_4 Y_8 - \frac{1}{1440} \left(\quad f \text{ stuff} \quad \right)$$

$$\left[\begin{matrix} I_{n/2} \\ 0 \end{matrix} \right]$$


$Y_4 = \mathrm{tr}\, R^2 - \frac{1}{30} \mathrm{Tr}\, F^2$
$X_8 = \left[\frac{\mathrm{Tr} F^4}{3} - \frac{(\mathrm{Tr} F^2)^2}{100} + (\quad) \right]$

How to cancel?

use $C_{(2)}$ in a clever way:

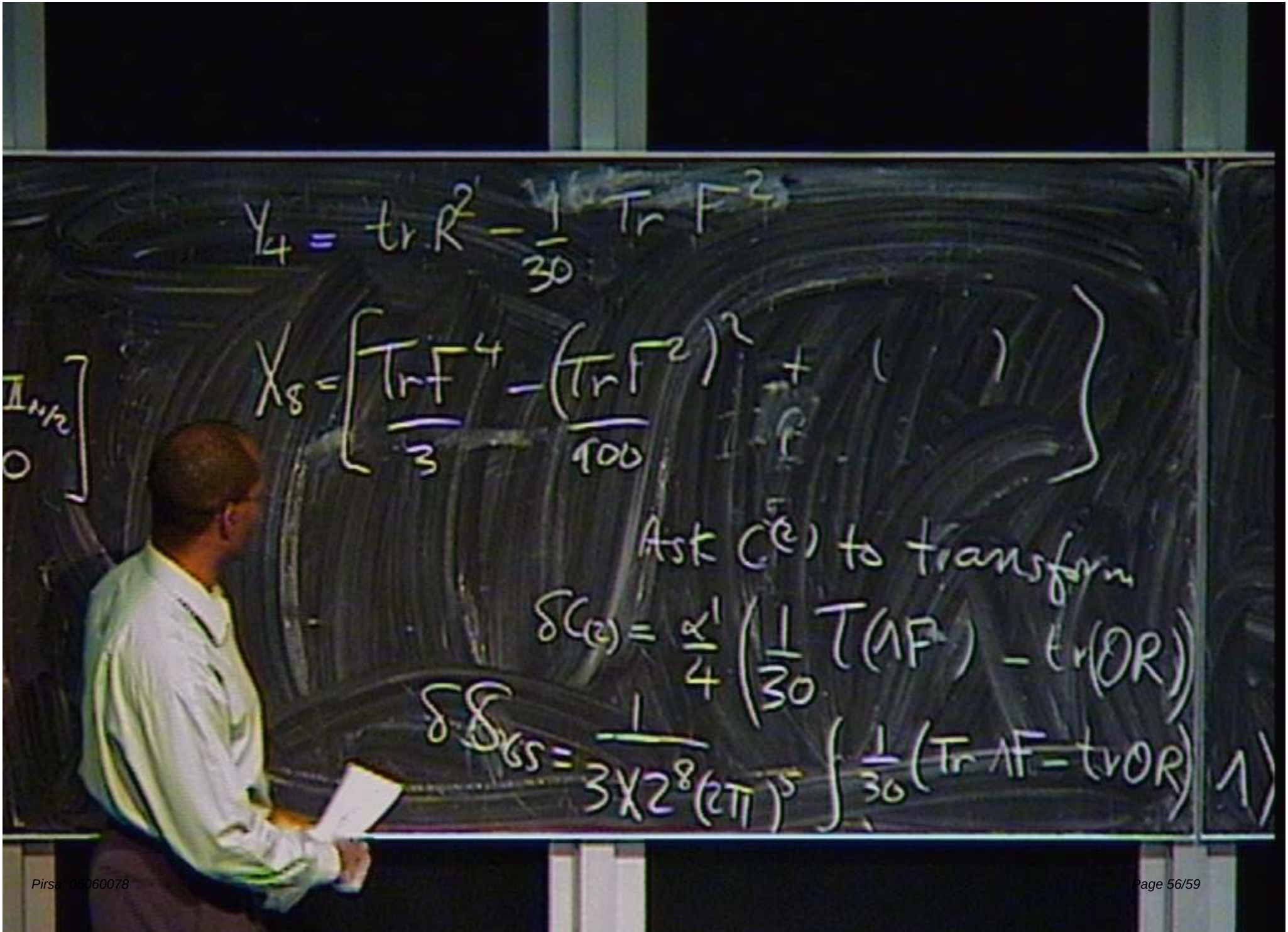
Consider $S_{GS} = \frac{1}{3 \times 2^6 (2\pi)^5 \alpha'} \int C_{(2)} \wedge X_8$

- Invariant under

$$\delta A = d\Lambda + [A, \Lambda]$$

$$\delta \omega = d\theta + [\omega, \theta]$$

$$\delta C_{(2)} = d\lambda_{(1)}$$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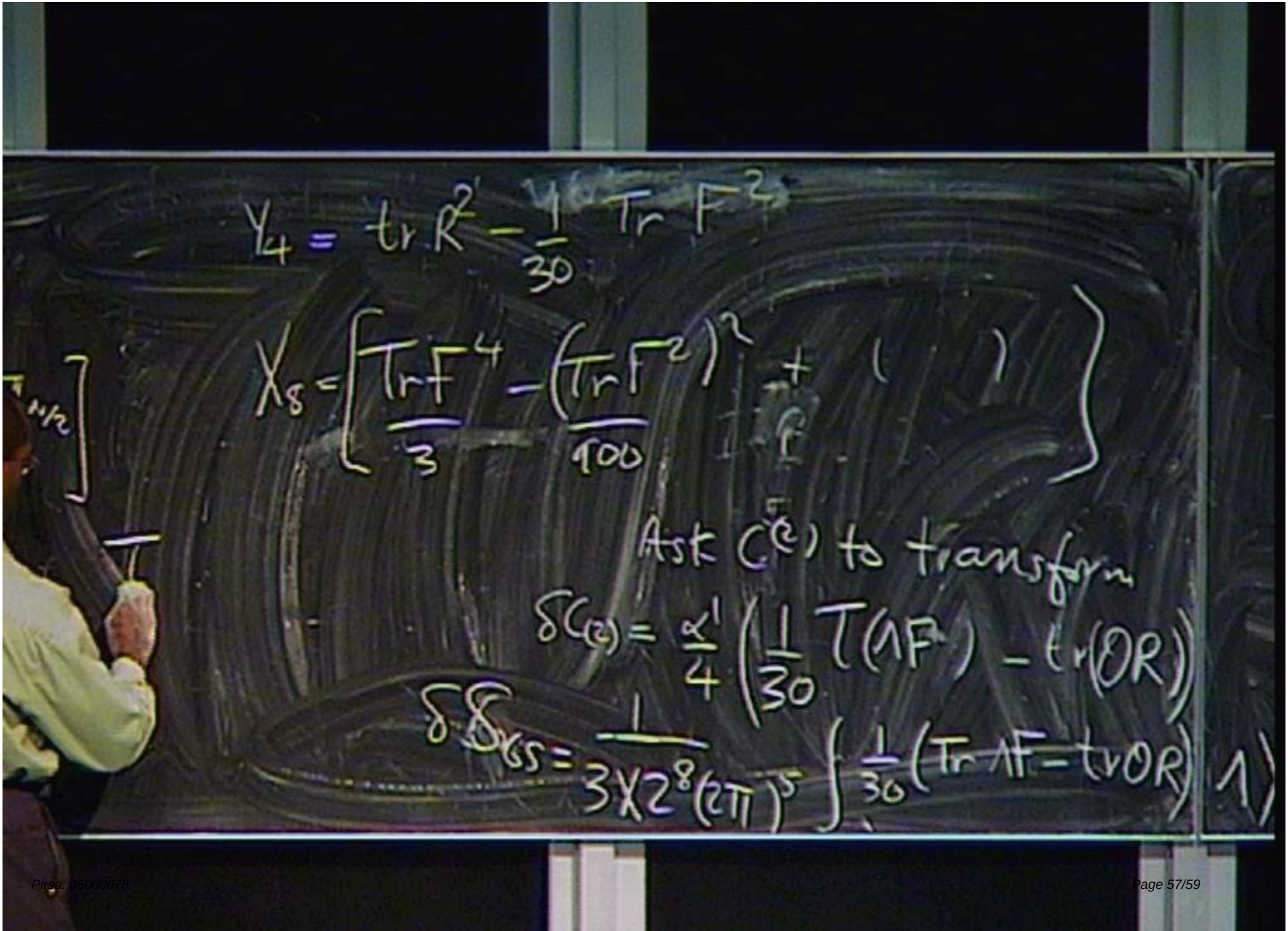
Ask $C^{(2)}$ to transform
$\delta C^{(2)} = \frac{\alpha'}{4} \left(\frac{1}{30} \mathrm{Tr}(\Lambda F) - \mathrm{tr}(\Omega R) \right)$
$\delta S_{GS} = \frac{1}{3 \times 2^8 (2\pi)^5} \int \frac{1}{30} (\mathrm{Tr}\, \Lambda F - \mathrm{tr}\, \Omega R)$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Ask $C^{(2)}$ to transform
$\delta C^{(2)} = \frac{\alpha'}{4}\left(\frac{1}{30}\mathrm{Tr}(\Lambda F) - \mathrm{tr}(\Omega R)\right)$
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$$Y_4 = \mathrm{tr}\, R^2 - \frac{1}{30}\,\mathrm{Tr}\, F^2$$

$$X_8 = \left[\frac{\mathrm{Tr}\, F^4}{3} - \frac{(\mathrm{Tr}\, F^2)^2}{100} + (\quad) \right]$$

$$\hat{I}_{12}^{GS} =$$

Ask $C^{(2)}$ to transform

$$\delta C_{(2)} = \frac{\alpha'}{4}\left(\frac{1}{30}\,\mathrm{T}(\Lambda F) - \mathrm{tr}(\mathcal{O}R)\right)$$

$$\delta S_{GS} = \frac{1}{3 \times 2^8 (2\pi)^5} \int \frac{1}{30}\left(\mathrm{Tr}\,\Lambda F - \mathrm{tr}\,\mathcal{O}R\right) \Lambda$$

$$Y_4 = \mathrm{tr}\, R^2 - \frac{1}{30}\,\mathrm{Tr}\, F^2$$

$$X_8 = \left[\frac{\mathrm{Tr}\, F^4}{3} - \frac{(\mathrm{Tr}\, F^2)^2}{100} + (\quad)\right]$$

$$\hat{I}^{GS}_{12} = -Y_4 X_8$$

Ask $C^{(2)}$ to transform

$$\delta C_{(2)} = \frac{\alpha'}{4}\left(\frac{1}{30}\,\mathrm{Tr}(\Lambda F) - \mathrm{tr}(\mathcal{O}R)\right)$$

$$\delta S_{GS} = \frac{1}{3\times 2^8 (2\pi)^5}\int \frac{1}{30}\left(\mathrm{Tr}\,\Lambda F - \mathrm{tr}\,\mathcal{O}R\right)\Lambda$$

Anomaly free theory!