

Title: Perturbative String Theory

Date: Jun 22, 2005 11:30 AM

URL: <http://pirsa.org/05060078>

Abstract:

Last time



Last time:



$$8_2 \oplus 8$$

$$\alpha_{-1}^{\mu}$$

$$\alpha_{-1}^{\mu} |0\rangle \rightarrow$$

$$\alpha_{-1}^{\mu}$$

$$\alpha_{-1}^{\mu}$$

$$\alpha_{-1}^{\mu} \alpha_{-1}^{\nu} |0\rangle$$

$$L_n |0\rangle = 0$$

$$L_0 \rightarrow M^2 = \frac{1}{\alpha'} ($$

$$\rightarrow \begin{aligned} L_n |0\rangle &= 0 \\ \bar{L}_n |0\rangle &= 0 \end{aligned}$$

$$\alpha_{-1}^\mu$$

$$\alpha_{-1}^\mu |0\rangle \rightarrow$$

$$\alpha_{-1}^\mu$$

$$\alpha_{-1}^\mu \rightarrow \alpha_{-1}^\mu \alpha_{-1}^\nu |0\rangle$$

$$L_n |0\rangle = 0$$

$$L_0 \rightarrow M^2 = \frac{1}{\alpha'} ($$

$$\begin{aligned} L_n |0\rangle &= 0 \\ \bar{L}_n |0\rangle &= 0 \end{aligned}$$

$$\begin{aligned} (L_0 + \bar{L}_0) &\rightarrow M^2 = \frac{1}{\alpha'} (\\ (L_0 - \bar{L}_0) |0\rangle &= 0 \end{aligned}$$

$$\begin{aligned} L_0 + \bar{L}_0 &= H \\ L_0 - \bar{L}_0 &= P \end{aligned}$$

$$L_0 + \bar{L}_0 = H$$

$$L_0 - \bar{L}_0 = P_\sigma \text{ generates translations on } \sigma$$



→ "Level matching"

$$(L_0 + \bar{L}_0) \rightarrow M^2 = \frac{1}{\alpha'} ($$

$$(L_0 - \bar{L}_0) |0\rangle = 0$$

Last time:

universal. NS-NS

$$g_v \otimes g_v = 1 \oplus \oplus$$



$$\text{Type IIA: } \overset{\text{NS}}{g_v} \oplus \overset{\text{R.}}{g_s} \otimes \overset{\text{NS}}{g_v} \oplus \overset{\text{R}}{g_c}$$

$$\text{R-R } g_s \otimes g_c$$

$$\text{Type IIB: } (g_v \oplus g_s) \otimes (g_v \oplus g_s)$$

universal. NS-NS

$$8_v \otimes 8_v = 1 \oplus \oplus$$

$$28 \oplus 35$$

$B_{\mu\nu} \quad G_{\mu\nu}$

$$s) \otimes \left(\begin{matrix} \text{NS} \\ 8_v \oplus 8_c \end{matrix} \right) \left(\begin{matrix} R \\ 8_c \end{matrix} \right)$$

$$R-R \quad 8_s \otimes 8_c$$

$$= 8_v \oplus 56_t$$

$C_\mu \quad C_{\mu\nu\kappa}$

$$s) \otimes (8_v \oplus 8_s)$$

$$8_s \otimes 8_s$$

$$= 1 \oplus 28 \oplus 35_t$$

$C \quad C_{\mu\nu} \quad C_{\mu\nu\kappa\rho}$

Last time



universal NS-NS

$$8_v \oplus 8_v = 1 \oplus \Phi$$

$$28 \oplus 35$$

$$B_{\mu\nu} \quad G_{\mu\nu}$$

$$\text{Type IIA } \overset{\text{NS}}{8_v} \oplus \overset{\text{R}}{8_s} \otimes \overset{\text{NS}}{8_v} \oplus \overset{\text{R}}{8_c}$$

R-R

$$8_s \oplus 8_c = 8_v \oplus 56_t$$

$$\text{Type IIB } (8_v \oplus 8_s) \otimes (8_v \oplus 8_s)$$

$$8_s \oplus 8_s = 1 \oplus 28 \oplus 35$$

$$C \quad C_{\mu\nu} \quad C_{\mu\nu\lambda}$$

$$\text{NS-R} \quad 8_v \otimes 8_c = 8_s \oplus 56_c$$

$$\lambda_1 \quad \psi_2^M$$

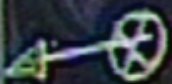
$$\text{R-NS} \quad 8_v \otimes 8_s = 8_c \oplus 56_s$$

$$\lambda_2 \quad \psi_\alpha^M$$

Type IIA has one each $(1,1)$ super
 Type IIB " two $8 \otimes (0,2)$ "

Type IIA has one each $(1,1)$ supersymmetry

Type IIB " two $\otimes$ $(0,2)$ "



Type IIA has one each $(1,1)$ supersymmetry

Type IIB " two $8 \otimes (0,2)$ "

These fields (just like in bosonic case) have low energy effective action.

$$S_{IIA} = \frac{1}{2\kappa_0^2} \int d^{10}x (-G)^{\frac{1}{2}} \left\{ e^{-2\Phi} \left[R + 4(\nabla\Phi)^2 - \frac{1}{12}(H^{(3)})^2 \right] - \frac{1}{4}(G^{(2)})^2 - \frac{1}{48}(G^{(4)})^2 \right\} - \frac{1}{4\kappa_0^2} \int B^{(2)} \wedge dC^{(3)} \wedge dC^{(3)}$$

56

$\zeta_{\mu\nu\kappa}$

28 ④ 35 +
 $\zeta_{\mu\nu} \zeta_{\mu\nu\kappa\rho}$

$$S_{II B} = \int (\text{same top line})$$

$$-\frac{1}{12} \left(G^{(3)} + C^{(0)} \Lambda H^{(3)} \right)^2 - \frac{1}{2} (dC^{(0)})^2$$

$$-\frac{1}{480} (G^{(5)})^2 + \frac{1}{4k_0^2} \left((C^{(4)} + \frac{1}{2} B^2) C^{(3)} \right) \Lambda G^{(3)} \Lambda H^{(3)}$$

Type II A

Type II B

These



$$S_{II A} = \frac{1}{2k_0^2} \int d^{10}$$

Go to Einstein frame using $\tilde{G}_{\mu\nu} = e^{\frac{\phi - \psi}{2}} G_{\mu\nu}$ Type II A

Enforce $F^{(5)} = *F^{(5)}$ by hand.

$$G^{(5)} = dC^{(4)} + H^{(3)} \wedge C^{(2)}$$

Type II B

These

56 $S_{II B} = \int$ (same top line)

$\epsilon_{\mu\nu\kappa}$

$$-\frac{1}{12} (G^{(3)} + C^{(0)} \wedge H^{(3)})^2 - \frac{1}{2} (dC^{(0)})^2$$

$$-\frac{1}{480} (G^{(5)})^2 + \frac{1}{4k_0^2} \left((C^{(4)} + \frac{1}{2} B^2 \wedge C^{(2)}) \wedge G^{(3)} \wedge H^{(3)} \right)$$

28 $\oplus$ 35 $+$
 $\epsilon_{\mu\nu} \epsilon_{\mu\nu\kappa\rho}$

$S_{II A} = \frac{1}{2k_0^2} \int d^{10}$

$$G_{\mu\nu} \quad 2k^2 = 2k_0^2 \alpha_s^2 = (2\pi)^7 \alpha_s^4 g_s^2 = 16\pi G_N$$

$$g_s = e^{\Phi_0}$$

⊗ These fields (just like in bosonic case) have low energy effective action.

$$S_{IIA} = \frac{1}{2k_0^2} \int d^{10}x (-G)^{\frac{1}{2}} \left\{ e^{-2\Phi} \left[R + 4(\nabla\Phi)^2 - \frac{1}{12}(H^{(3)})^2 \right] \right. \\ \left. - \frac{1}{4}(G^{(2)})^2 - \frac{1}{48}(G^{(4)})^2 \right\} \\ - \frac{1}{4k_0^2} \int B^{(2)} \wedge dC^{(3)} \wedge dC^{(3)}$$

Anomalies? ΠB is chiral, so worry.

The structure

Look for anomalous variation of

$$\Gamma = \ln Z.$$

Anomalies? ΠB is chiral, so worry.

The structure



Look for anomalous variations of

$$\Gamma = 4 \ln Z.$$

Anomalies? ΠB is chiral, so worry.

The structure

Look for anomalous variations of



$$\delta \ell_i = \frac{i}{(2\pi)^{D/2}} \int \hat{\mathbb{I}}_D$$

Define $F = dA +$

$$\Gamma = \ln Z.$$

under
gauge transformation

Anomalies? ΠB is chiral, so worry

The structure

Look for anomalous variation of

$$\delta \ln Z = \frac{i}{(2\pi)^{D/2}} \int \hat{I}_D$$



Define $F = dA +$

$$\Gamma = \ln Z$$

under
gauge transformation

Define $F = dA + A^2$

$$R = d\omega + \omega^2$$

$$R^4_{\mu\nu\rho\sigma} dx^\mu \wedge dx^\nu \wedge dx^\rho \wedge dx^\sigma$$

$G_{\mu\nu}$

$$2K^2 = 2K_0^2 g^{\mu\nu} =$$

$$g = e^{\frac{1}{2} \phi}$$

Descent relation from
 $\mathbb{I}_{D+2}$

$$\Gamma = \ln Z$$

under
gauge transformation

These fields (i.e.)
have low

$$S_{IA} = \frac{1}{2K_0^2} \int d^4x (-G)^{\frac{1}{2}} \left\{ \frac{1}{2} (H(\phi))^2 \right.$$

$$- \frac{1}{4} (G^{\mu\nu} \wedge F_{\mu\nu})^2$$

$$= \frac{1}{4K_0^2} \int d^4x$$

$$= \frac{1}{4K_0^2} \int d^4x$$

Define $F = dA + A^2$

$R = d\omega + \omega^2$

$g_{\mu\nu} dx^\mu dx^\nu$

$G_{\mu\nu}$

$2K^2 = 2K_0^2 g_{\mu\nu}^2 =$

$g = e^{\frac{1}{2}}$

of

$\Gamma = \ln Z$

under
gauge transformation

Descent relation from

$\hat{I}_{D+2} = d\hat{I}_{D+1}$

$\hat{I}_{D+1}$ is not gauge invariant
but

$\delta \hat{I}_{D+1} = d\hat{I}_D$

→ ⊗

These fields (i.e.)
have low

$S_{IA} = \frac{1}{2K^2} \int d^{10}x (-G)^{\frac{1}{2}} \{$

$(H^{(0)})^2$

$(G^{(4)}) \wedge G^{(4)} \wedge H^{(2)}$

$-\frac{1}{4K^2} \int$

Ex Chern-Simons 3-form!

$$\omega_{3y} = \text{Tr} \left(A \wedge F - \frac{1}{3} A \wedge A \wedge A \right) \\ = \text{Tr} \left(A \wedge dA + \frac{2}{3} A \wedge A \wedge A \right)$$

$$d\omega_{3y} = \text{Tr}(F \wedge F) \quad \delta A = d\Lambda + [A, \Lambda]$$

$\frac{1}{2} \int_{D_4}$ $\nearrow$ $\mathbb{R} \hat{\mathbb{I}}_{D+2}$

$$\delta \omega_{3y} = \text{Tr}(d\Lambda dA) = d\omega_2 \quad \omega_2 = \text{Tr}(\Lambda dA)$$

Ex Chern-Simons 3-form:

$$\omega_{3y} = \text{Tr} \left(A \wedge F - \frac{1}{3} A \wedge A \wedge A \right)$$

$$= \text{Tr} \left(A \wedge dA + \frac{2}{3} A \wedge A \wedge A \right)$$

$$d\omega_{3y} = \text{Tr}(F \wedge F) \quad \delta A = d\Lambda + [A, \Lambda]$$

$$\hat{\mathbb{I}}_{D+1} \nearrow \quad \mathbb{R} \hat{\mathbb{I}}_{D+2}$$

$$\delta \omega_{3y} = \text{Tr}(d\Lambda dA) = d\omega_2 \quad \omega_2 = \text{Tr}(\Lambda dA)$$

Anomalies? II B is chiral, so worry.

The structure

$$\left. \begin{array}{l} 8c \\ 56s \\ 35+ \end{array} \right\} 2$$

Look for anomalies variation of



$$\delta \ln Z = \frac{i}{(2\pi)^{D/2}} \int \hat{I}_D$$

$$I_{12}^{8c} = \frac{-\text{tr} R^6}{725760} - \frac{\text{tr} R^4 \text{tr} R^2}{552960} - \frac{(\text{tr} R^2)^3}{1327104}$$

$$I_{12}^{35_+} = 992 \left(\begin{array}{c} \swarrow \\ \searrow \end{array} \right) - 448 \left(\begin{array}{c} \swarrow \\ \searrow \end{array} \right) + 128 \left(\begin{array}{c} \swarrow \\ \searrow \end{array} \right)$$

$$I_{12}^{56_5} = -495 \left(\begin{array}{c} \swarrow \\ \searrow \end{array} \right) + 225 \left(\begin{array}{c} \swarrow \\ \searrow \end{array} \right) - 63 \left(\begin{array}{c} \swarrow \\ \searrow \end{array} \right)$$

IB is chiral, so worry.

$$\left. \begin{matrix} 8c \\ 56s \\ 35+ \end{matrix} \right\} \times 2 \quad 2I_{12}^{8c}$$

for anomalies variation of

$$\delta \ln Z = \frac{i}{(2\pi)^{D/2}} \int \hat{I}_D$$

Define $F = dA + A^2$

$$+ 2I_{12}^{56s} + I_{12}^{35+} d\omega + \omega^2$$

$$\Gamma = \ln Z.$$

under
gauge transformation

Descer
 $\hat{I}_{D+1}$

$\hat{I}_{D+1}$
but

IB is chiral, so ~~worry~~.

$$\left. \begin{matrix} 8c \\ 56s \\ 35+ \end{matrix} \right\} \times 1 \quad \left\{ 2 I_{12}^{8c} \right.$$

for anomalies variations of

$$\delta \ln Z = \frac{i}{(2\pi)^{D/2}} \int \hat{I}_D$$

Define $F = dA + A^2$

$$+ 2 I_{12}^{56s} + I_{12}^{35+} \quad dw + w^2 = 0$$

$$\Gamma = \ln Z.$$

under
gauge transformation

Descer
 $\hat{I}_{D+1}$

$\hat{I}_{D+1}$
but

$N=1$ SYM $D=10$ multiplet $8_0 \oplus 8_5$
with group G

$$\hat{I}_{12}^{8_5}(R, F)$$

$N=1$ SYM $D=10$ multiplet $8_0 \oplus 8_1$
with group G

$$\hat{I}_{12}^{8_0}(R, F) = -\frac{\text{Tr} F^6}{1440} + \frac{\text{Tr} F^4 \text{tr} R^2}{2304} - \frac{\text{Tr} F^2 \text{tr} R^4}{23040} - \frac{\text{Tr} F^2 (\text{tr} R^2)^2}{18432} \\ + \frac{n \text{tr} R^6}{725760} + \frac{n \text{tr} R^4 \text{tr} R^2}{552960} + \frac{n (\text{tr} R^2)^3}{1327104}$$

$N=1$ SYM $D=10$ multiplet $8_v \oplus 8_s$
 with group G $\dim R = n$ R in rep R

$$\hat{I}_{12}^{8_s}(R, F) = -\text{Tr} F^6$$

1440

$$+ \text{Tr} F^4 \text{tr} R^2$$

2304

$$+ n \text{tr} R^6$$

725760

$$+ n \text{tr} R^4 \text{tr} R^2$$

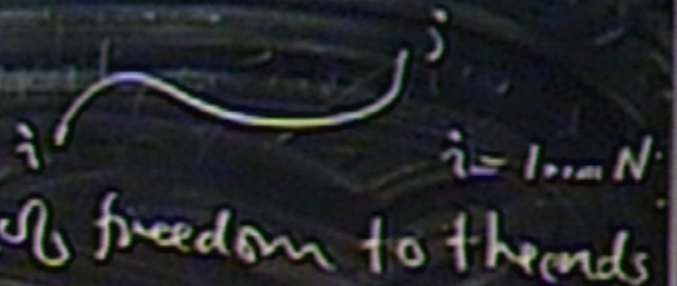
552960

23040

$$+ n \frac{\text{tr}}{132}$$

Chan-Paton Factors

- Add non-dynamical degrees of freedom to the ends



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promote

$$|k\rangle \rightarrow |ij; k\rangle \lambda_{ij}^a$$

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$$|k\rangle \rightarrow |ij; k\rangle \lambda_{ij}^a$$

we're using $U(N)$

λ_{ij}^a are Hermitian

$$\left\{ \begin{array}{l} N \times \bar{N} = \text{adjoint} \\ |a, k\rangle \end{array} \right.$$

This will give local UV in spacetime.
(global on worldsheet)

This will give local $U(1)$ in spacetime.

(global on
worldsheet)

In any amplitude:



This will give local $U(1)$ in spacetime

(global on worldsheet)

In any amplitude:



There'll be a factor

$$\lambda_{ij}^1 \lambda_{jk}^2 \lambda_{ki}^3 \lambda_{ii}^4$$

$$= \text{Tr}(\lambda^1 \lambda^2 \lambda^3 \lambda^4)$$

So $\lambda \rightarrow U \lambda U^{-1}$ invariance

This will give local $U(1)$ in spacetime.

(global on worldsheet)

In any amplitude:



There'll be a factor:

$$\lambda_{ij}^1 \lambda_{jk}^2 \lambda_{kl}^3 \lambda_{li}^4$$

$$= \text{Tr}(\lambda^1 \lambda^2 \lambda^3 \lambda^4)$$

So $\lambda \rightarrow U \lambda U^{-1}$ invariance

photon $\rightarrow$

A_μ

$\sum_a \lambda_{ij}^a \psi_{-\frac{1}{2}}^M |ij, k\rangle$

A_μ^a

Let's construct
nonorientable strings

Open strings

al degrees of freedom to the ends

$i \in N$ $i \in \bar{N}$
 $i = 1, \dots, N$

$$|k\rangle \rightarrow |ij; k\rangle \lambda_{ij}^a$$

$$N \times \bar{N} = \text{adjoint}$$

$$|a, k\rangle$$

Lets construct
unorientable strings

Open strings



$$\Omega : \sigma \rightarrow \pi - \sigma$$

$$\pi \rightarrow \sigma - \pi$$

$$x^\mu \rightarrow x^\mu$$

$$p_\mu \rightarrow p_\mu$$

$$\alpha_m^\mu \rightarrow (-1)^m \alpha_m^\mu$$

$$\psi_r \rightarrow (-1)^{r+1/2} \psi_r$$

Project onto $\sigma_z = +1$
 $\hookrightarrow$ kills the photon $\psi_{\frac{1}{2}}^M |0\rangle$

$\psi_{\frac{1}{2}}^M$

Project onto $\Omega = +1$

$\hookrightarrow$ kills the photon $\psi_{\frac{1}{2}}^M |0\rangle$

Save it by transforming Chen-Paton λ

$$\Omega \lambda_{ij} |k, i, j\rangle \rightarrow \lambda'_{ij} |k, i, j\rangle$$

$$\lambda' = M \lambda^T M^{-1}$$

But $\Omega^2 = 1$

$$\psi_{\frac{1}{2}}^M \therefore \lambda \Rightarrow M(\lambda')^T M^{-1} = M M^T \lambda M^T M^{-1}$$

We are running over a complete set, so

Schur's Lemma $MM^{-T} \propto \mathbb{I}$

$\Rightarrow M$ is either symmetric or antisymmetric

(a) Symmetric $M = M^T = \mathbb{I}_N$

$\Rightarrow \lambda_i; \frac{1}{\sqrt{2}} |k, ii\rangle$ is to survive

Then $\boxed{\lambda = -\lambda^T}$

We are running over a complete set, so

Schur's Lemma $MM^{-T} \propto \mathbb{I}$

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(a) Symmetric $M = M^T = \mathbb{I}_N$

$\Rightarrow \lambda_i; \frac{1}{2} |k, ii\rangle$ is to survive

$G = \text{SO}(N)$

Then $\boxed{\lambda = -\lambda^T}$

Lets construct
unorientable strings

Open strings

(b) antisymmetric

$$M = -M^T$$

$$\rightarrow i \begin{bmatrix} 0 & \mathbb{I}_{N/2} \\ -\mathbb{I}_{N/2} & 0 \end{bmatrix}$$

$$\lambda = -M \lambda^T M$$

$\rightarrow G$ is $USp(N/2)$

$USp(2) = Sp(2)$

or $S_H(N/2)$

Project

$\rightarrow$

$$\begin{bmatrix} \mathbb{I}_{N/2} \\ 0 \end{bmatrix}$$

$$\psi_M$$

Lets construct
unorientable strings

Open strings



(b) antisymmetric

$$M = -M^T$$

$$\rightarrow i \begin{bmatrix} 0 & I_{N/2} \\ -I_{N/2} & 0 \end{bmatrix}$$

$$\lambda = -M \lambda^T M$$

$\rightarrow G$ is $USp(N/2)$
or $S_H(N/2)$

$$USp(2) = Sp(1)$$

Closed Strings

$$\Omega : \begin{aligned} \sigma &\rightarrow -\sigma \\ \tau &\rightarrow -\bar{\tau} \end{aligned}$$

$$\tilde{\psi}_r^M \longleftrightarrow \psi_r^M$$

Ω is a symmetry of IIB

$\left. \begin{array}{c} I_{N/2} \\ \circ \end{array} \right\}$

Closed Strings

$$\Omega : \begin{aligned} \sigma &\rightarrow -\sigma \\ \tau &\rightarrow \tau \end{aligned}$$

$$\tilde{\psi}_r^M \longleftrightarrow \psi_r^M$$

Ω is a symmetry of $II B$. So project onto $\Omega = +1$

NS-NS	1	28	35
	$\oplus$	$\oplus$	$\oplus$
Ω	+1	-1	+1

Closed Strings

$$\Omega : \begin{aligned} \sigma &\rightarrow -\sigma \\ z &\rightarrow \bar{z} \end{aligned}$$

$$\tilde{\psi}_r^M \longleftrightarrow \psi_r^M$$

$\left. \begin{aligned} &\frac{1}{2} \\ &0 \end{aligned} \right\} \text{I}_{N/2}$

Ω is a symmetry of $II B$. So project onto $\Omega = +1$

NS-NS

1

~~28~~

35

$\oplus$

~~B_{μν}~~

G_{μν}

$\Omega = +1$

~~-1~~

R-R

~~$\oplus$~~

C_{μν}

~~+1~~

~~$\psi_{\mu\nu\lambda}$~~

$\Omega = -1$

+1

Closed Strings

$$\Omega : \begin{aligned} \sigma &\rightarrow -\sigma \\ \tau &\rightarrow \tau \end{aligned}$$

$$\tilde{\psi}_r^M \longleftrightarrow \psi_r^M$$

Ω is a symmetry of IIB. So project onto

NS-NS

1

~~28~~

35

$$-\Omega = +1$$

$\oplus$

~~$B_{\mu\nu}$~~

$G_{\mu\nu}$

only one
dilatinio-g
survives.

$8 \oplus 56_5$
gravitino

$$-\Omega = +1$$

$$-\Omega = -1$$

+1

R-R

~~$C_{\mu\nu}$~~

~~$C_{\mu\nu\kappa\sigma}$~~

NS-R

R-NS

$$-\Omega = -1$$

+1

-1

anomalous

$$8 \oplus 56 \times 1$$

~~354~~

$\tau \uparrow$



see an $N=1$
unorientable
closed string.

$8 \oplus 56_s$
gravitino

R-NS

⇒ So add unoriented open to unoriented closed

⇒ study anomalies $n = 496$

⇒ So add unoriented open to unoriented closed

⇒ study anomalies $n = 496$

pick $n = 496$

vanish if $G = E_8 \times E_8$ or $SO(32)$ I_{496}

$$I_{12} = \frac{1}{3 \times 2^8} Y_4 Y_8 - \frac{1}{440} \left(\begin{array}{c} \text{F stuff} \end{array} \right)$$

$$Y_4 = \text{tr} R^2 - \frac{1}{30} \text{Tr} F^2$$

$$X_8 = \left[\frac{\text{Tr} F^4}{3} - \frac{(\text{Tr} F^2)^2}{100} + \dots \right]$$

How to cancel?

use $C_{(2)}$ in a clever way.

Consider
$$S_{GS} = \frac{1}{3 \times 2^6 (2\pi)^5 \alpha'} \int C_{(2)} \wedge X_8$$

• Invariant under

$$\delta A = d\lambda + [A, \lambda]$$

$$\delta \omega = d\theta + [\omega, \theta]$$

$$\delta C_{(2)} = d\lambda_{(1)}$$

$$Y_4 = \text{tr} R^2 - \frac{1}{30} \text{Tr} F^2$$

$$X_8 = \left[\frac{\text{Tr} F^4}{3} - \frac{(\text{Tr} F^2)^2}{100} + \dots \right]$$

Ask $C(\bar{c})$ to transform

$$\delta C(\bar{c}) = \frac{\alpha'}{4} \left(\frac{1}{30} \text{Tr}(F^2) - \text{tr}(OR) \right)$$

$$\delta \delta_{GS} = \frac{1}{3 \times 2^8 (2\pi)^5} \left\{ \frac{1}{30} (\text{Tr} F^2 - \text{tr} OR) \right\}$$

$$Y_4 = \text{tr} R^2 - \frac{1}{30} \text{Tr} F^2$$

$$X_8 = \left[\frac{\text{Tr} F^4}{3} - \frac{(\text{Tr} F^2)^2}{100} + \dots \right]$$

Ask $C(\epsilon)$ to transform

$$\delta C(\epsilon) = \frac{\alpha'}{4} \left(\frac{1}{30} \text{Tr}(F^2) - \text{tr}(OR) \right)$$

$$\delta \delta X_8 = \frac{1}{3 \times 2^8 (2\pi)^5} \left\{ \frac{1}{30} (\text{Tr} F^2 - \text{tr} OR) \right\}$$

$$Y_4 = \text{tr} R^2 - \frac{1}{30} \text{Tr} F^2$$

$$X_8 = \left[\frac{\text{Tr} F^4}{3} - \frac{(\text{Tr} F^2)^2}{100} + \dots \right]$$

$$\frac{\Delta_{GS}}{12} =$$

Ask $C(\epsilon)$ to transform

$$\delta C(\epsilon) = \frac{\alpha'}{4} \left(\frac{1}{30} \text{Tr}(AF) - \text{tr}(OR) \right)$$

$$\delta \delta_{GS} = \frac{1}{3 \times 2^8 (2\pi)^5} \left\{ \frac{1}{30} (\text{Tr} AF - \text{tr} OR) \right\}$$

$$Y_4 = \text{tr} R^2 - \frac{1}{30} \text{Tr} F^2$$

$$\chi_8 = \left[\frac{\text{Tr} F^4}{3} - \frac{(\text{Tr} F^2)^2}{100} + \dots \right]$$

$$\frac{\hat{I}_{12}^{GS}}{12} = - \frac{1}{24} \chi_8$$

Anomaly free theory!

Ask $C(\bar{e})$ to transform

$$\delta C(\bar{e}) = \frac{\alpha'}{4} \left(\frac{1}{30} \text{Tr}(F^2) - \text{tr}(R^2) \right)$$

$$\delta \mathcal{L}_{GS} = \frac{1}{3 \times 2^8 (2\pi)^5} \left\{ \frac{1}{30} (\text{Tr} F^2 - \text{tr} R^2) \right\}$$