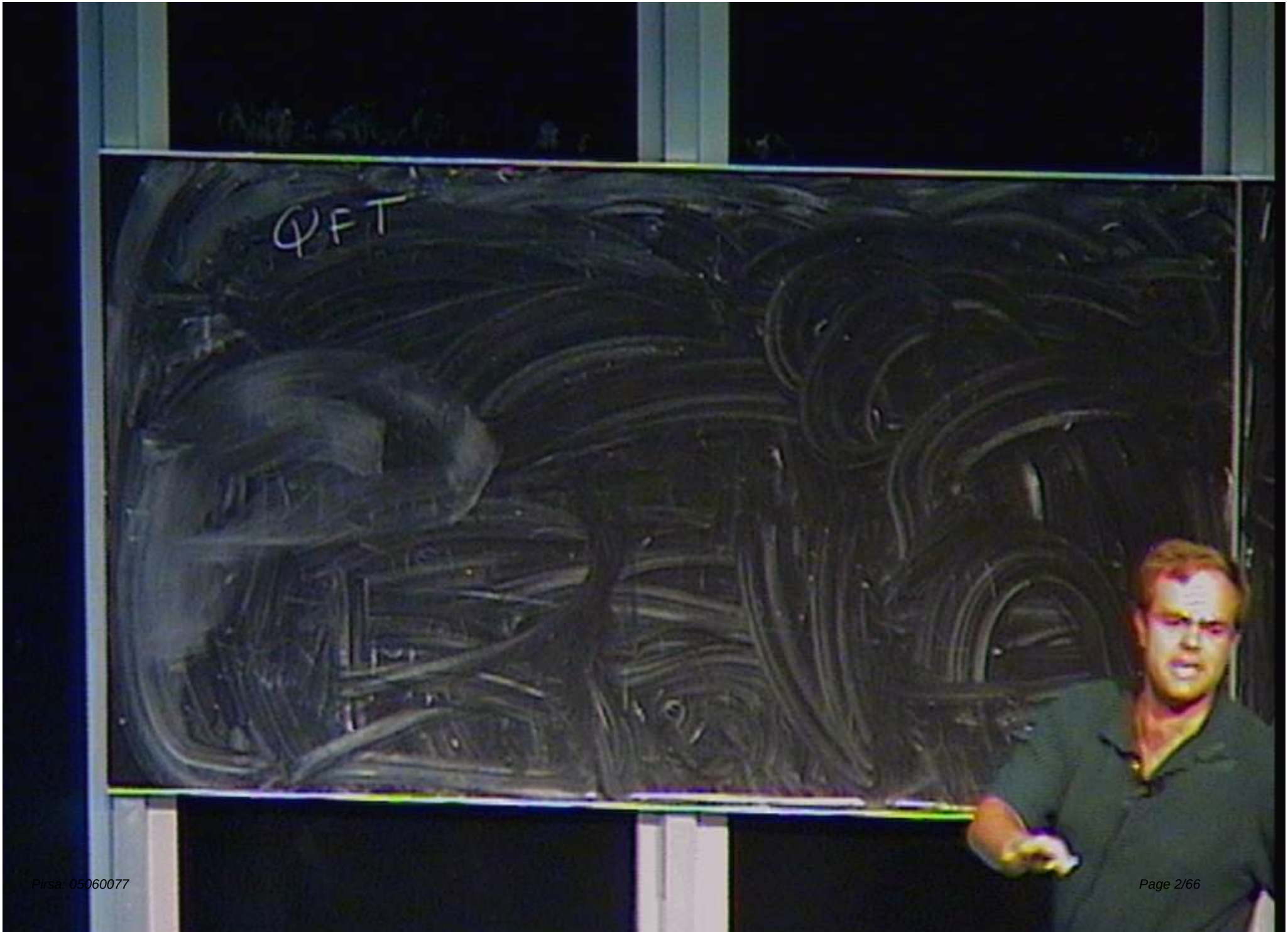
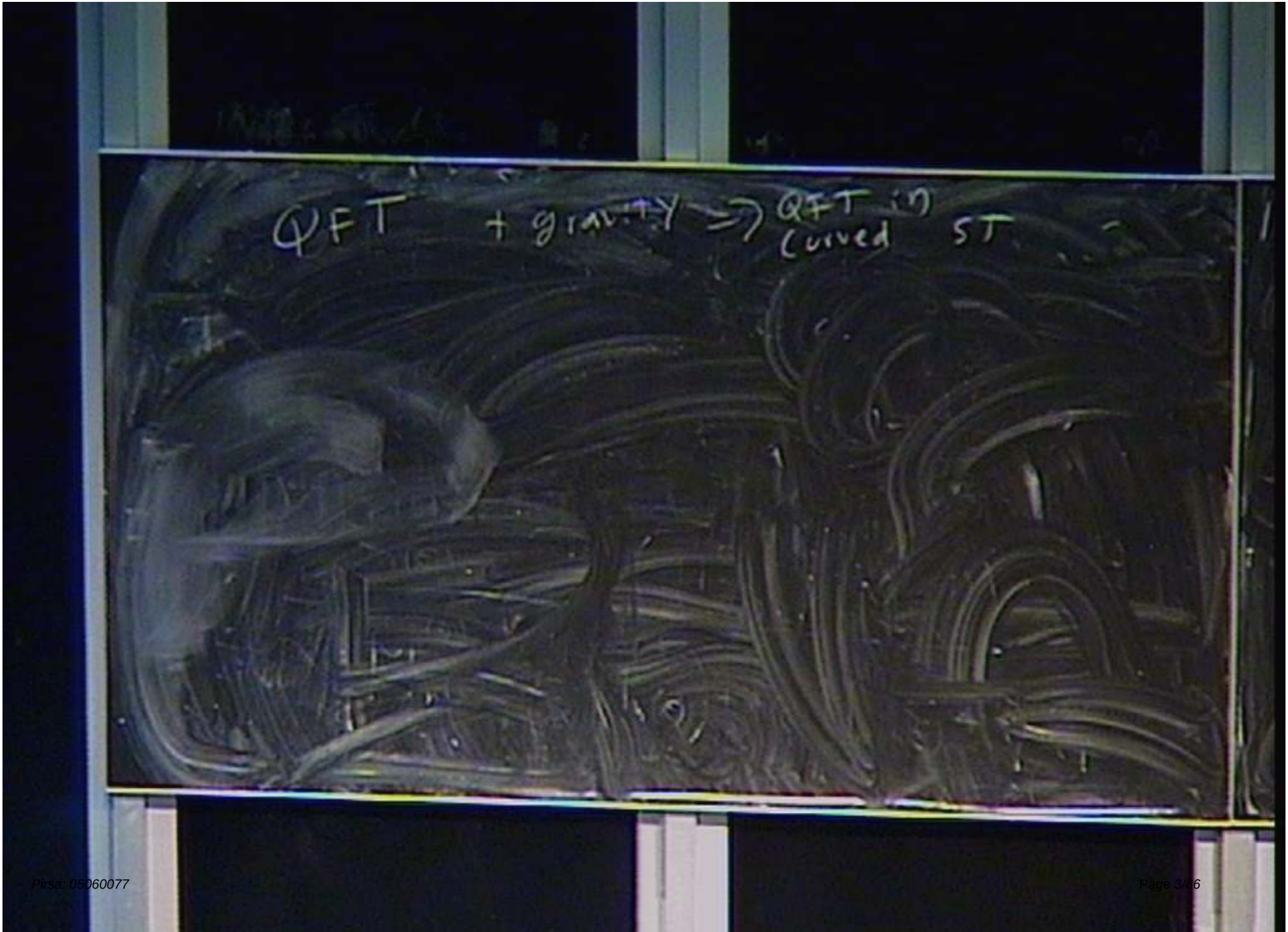
Title: QFT in Curved Space

Date: Jun 22, 2005 09:30 AM

URL: http://pirsa.org/05060077

Abstract:


QFT


QFT + gravity ⇒ QFT in Curved ST

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Jacobson hep-th/0308048

Ross hep-th/0502195

QFT + gravity ⇒ QFT in Curved ST

Jacobson hep-th/0308048

Ross hep-th/0502195

I. Covariant QFT w/o Poincaré

II. Unruh

III. Hawking

QFT + gravity ⇒ QFT in Curved ST

Jacobson hep-th/0308048

Ross hep-th/0502195

I. Covariant QFT w/o Poincaré

→ II. Unruh

III. Hawking

QFT

F

QFT

Free scalar Field.

$$S = -\frac{1}{2}\int d^d x \sqrt{-g}\left(g^{ab}\nabla_a\phi\nabla_b\phi + m^2\phi^2\right)$$

are $\Rightarrow (\Box - m^2)\phi = 0$

QFT

Free scalar Field.

$$S = -\frac{1}{2}\int d^d x \sqrt{-g}\left(g^{ab}\nabla_a\phi\nabla_b\phi + m^2\phi^2\right)$$

are $\Rightarrow (\Box - m^2)\phi = 0$

Solve using Green's Functions:

$$(\Box - m^2)G^{\pm}(x,y) = -\delta(x,y)$$

$G^{+}(x,y) = 0$ for

QFT

Free scalar Field.

$$S = -\frac{1}{2}\int d^d x \sqrt{-g}\left(g^{ab}\nabla_a\phi\nabla_b\phi + m^2\phi^2\right)$$

are $\Rightarrow (\Box - m^2)\phi = 0$

Solve using Green's Functions:

$$(\Box - m^2)G^{\pm}(x,y) = -\delta(x,y)$$

$G^{+}(x,y) = 0$ for $x > y$

$G^{-}(x,y) = 0$ for $y > x$

QFT

Free scalar Field.

$$S = -\frac{1}{2}\int d^d x \sqrt{-g}\left(g^{ab}\nabla_a\phi\nabla_b\phi + m^2\phi^2\right)$$

are $\Rightarrow (\Box - m^2)\phi = 0$

Solve using Green's Functions:

$$(\Box - m^2)G^{\pm}(x,y) = -\frac{\delta(x,y)}{\sqrt{-g}}$$

bi-scalar

$$\int \sqrt{-g}\, f(x)\delta(x,y)\, dx = f(y)$$

$G^{+}(x,y) = 0$ for $x > y$

$G^{-}(x,y) = 0$ for $y > x$

QFT

Free scalar Field.

$$S = -\frac{1}{2}\int d^d x \sqrt{-g}\left(g^{ab}\nabla_a\phi\nabla_b\phi + m^2\phi^2\right)$$

$$\text{Scalar} \Rightarrow (\Box - m^2)\phi = 0$$

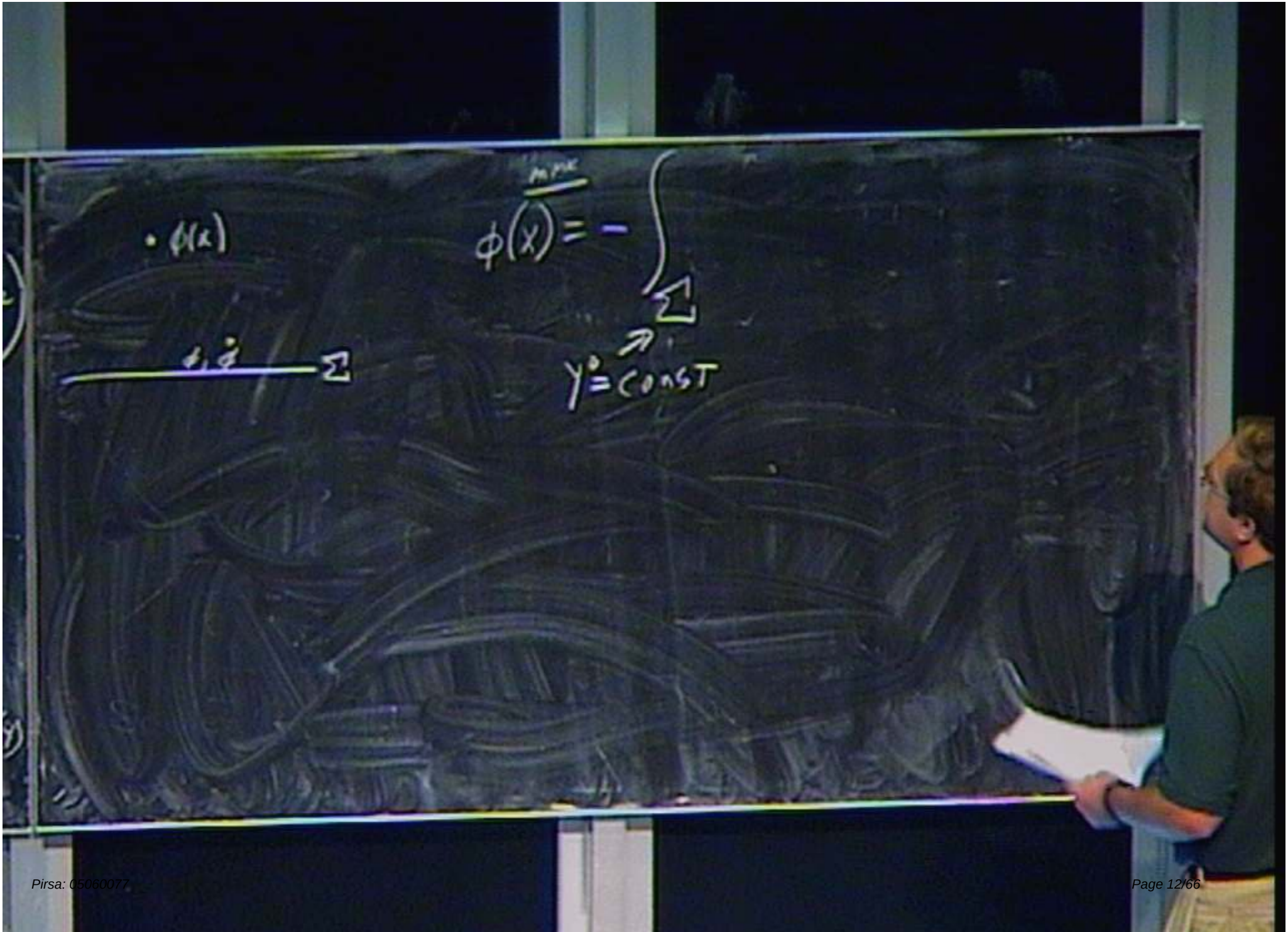
Solve using Green's Functions:

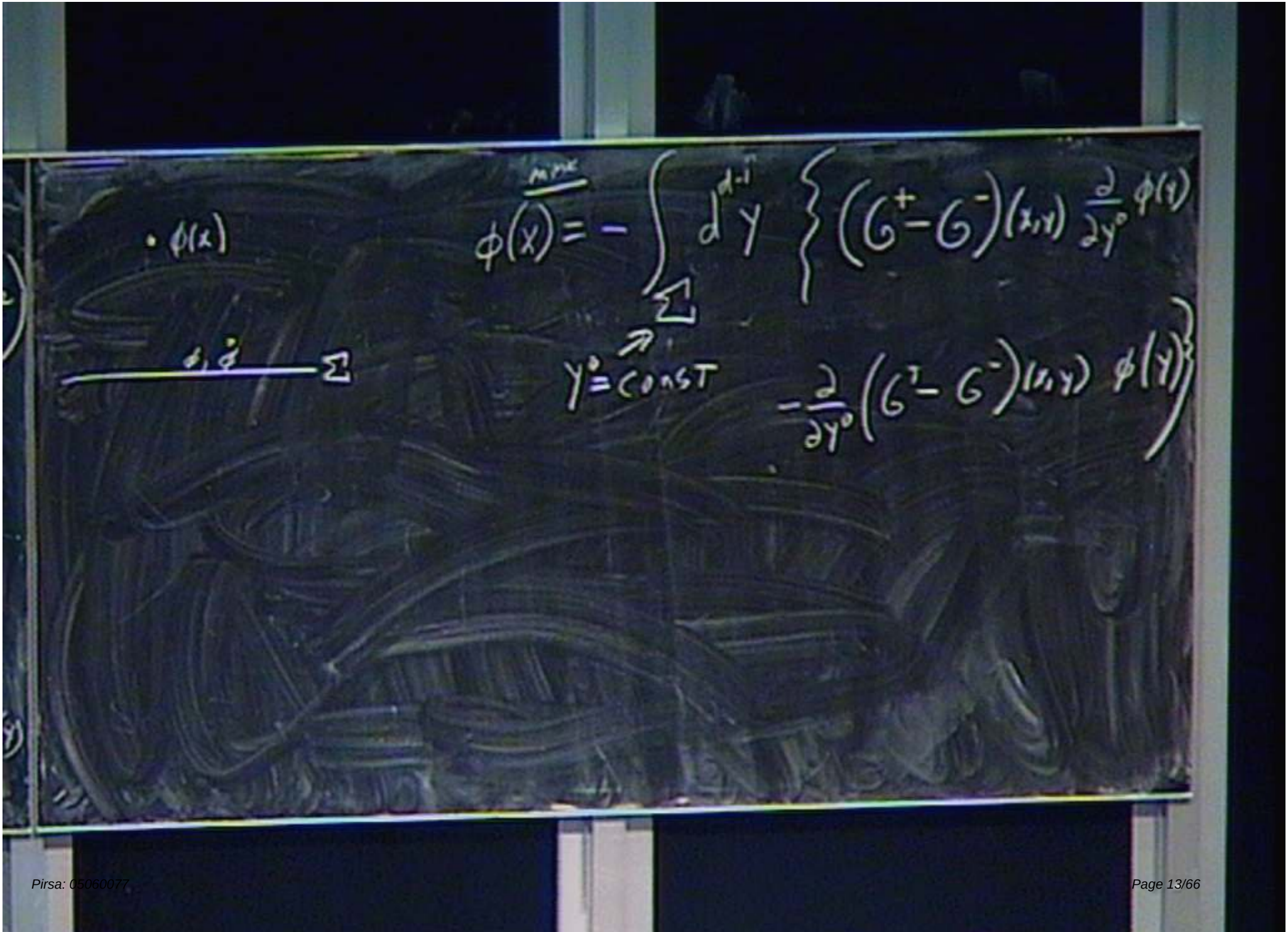
$$(\Box - m^2)G^{\pm}(x,y) = -\delta(x,y)$$

$$\int \sqrt{-g}\, f(x)\,\delta(x,y)\,dx = f(y)$$

$G^{+}(x,y) = 0$ for $x > y$

$G^{-}(x,y) = 0$ for $y > x$


$\phi(x)$
$\phi(x) = -\int_{\Sigma}$
Σ
$Y^0 = \text{CONST}$


$\phi(x) \overset{\text{on shell}}{=} -\int_\Sigma d^{d-1}y \left\{ (G^+ - G^-)(x,y) \frac{\partial}{\partial y^0} \phi(y) - \frac{\partial}{\partial y^0} (G^+ - G^-)(x,y)\, \phi(y) \right\}$
$\phi(x)$
$\phi, \dot{\phi}$
Σ
$y^0 = \text{const}$

• $\phi(x)$

$$\phi(x) \overset{\text{mink}}{=} -\int_{\Sigma} d^{d-1}y \left\{ (G^+ - G^-)(x,y) \frac{\partial}{\partial y^0} \phi(y) - \frac{\partial}{\partial y^0} (G^+ - G^-)(x,y)\, \phi(y) \right\}$$

$\phi, \dot{\phi}$ — Σ ($y^0 = \text{const}$)

$$(\Box - m^2)(G^+ - G^-) = -\delta + \delta = 0$$

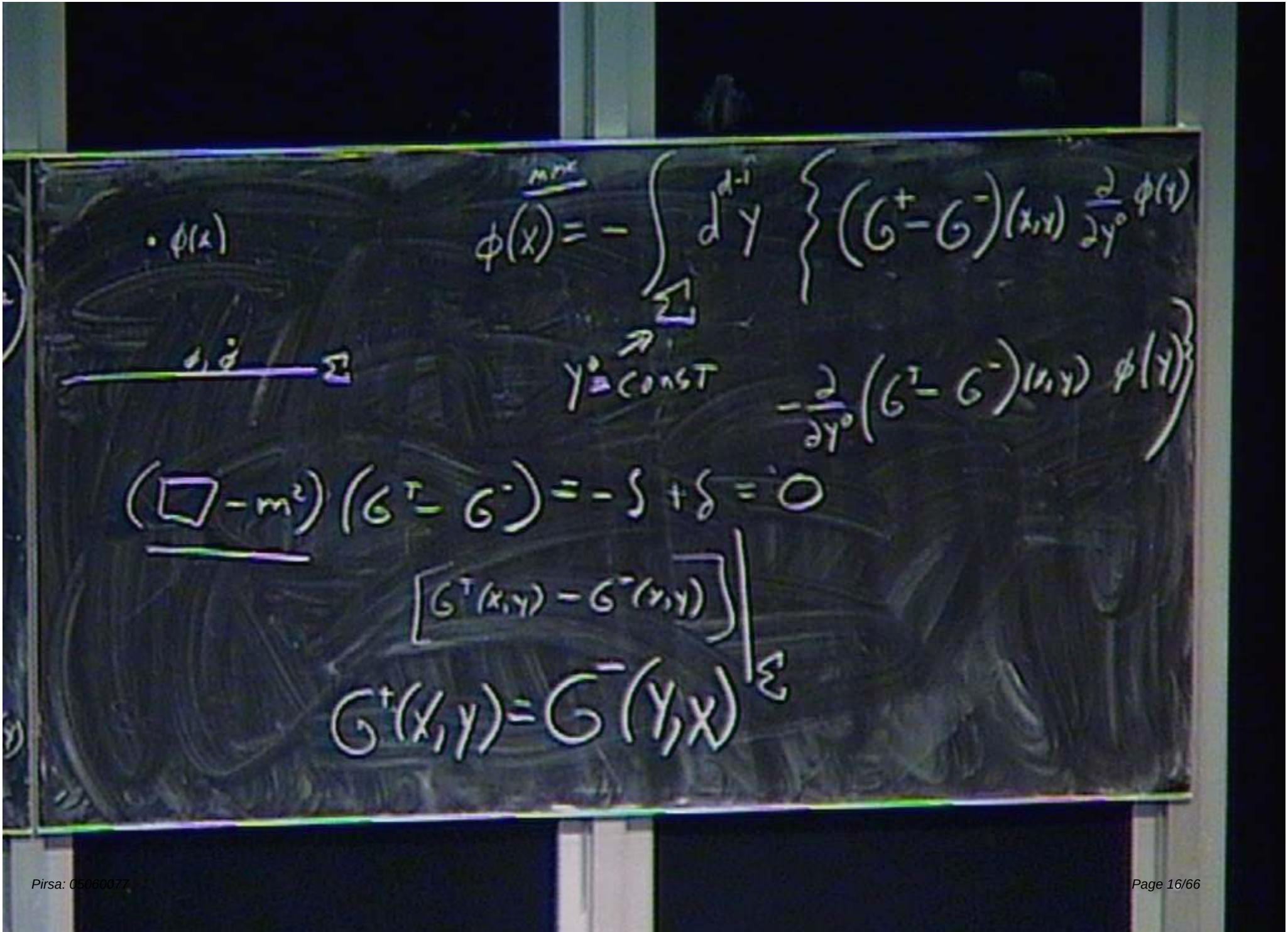
• $\phi(x)$

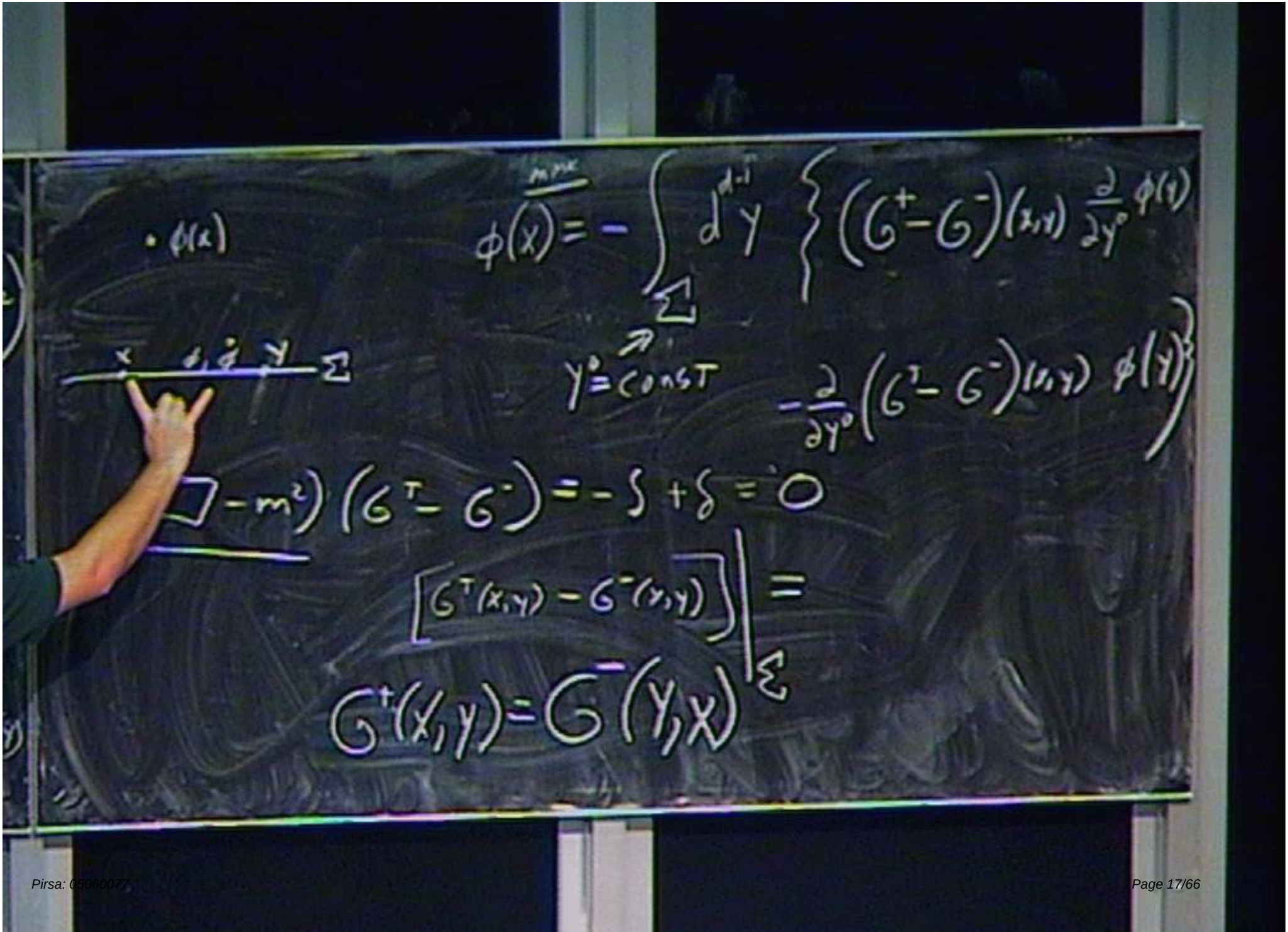
$$\phi(x) \overset{\text{mmc}}{=} -\int_{\Sigma} d^{d-1}y \left\{ (G^+ - G^-)(x,y) \frac{\partial}{\partial y^0} \phi(y) - \frac{\partial}{\partial y^0}(G^+ - G^-)(x,y)\, \phi(y) \right\}$$

$\phi, \dot{\phi}$ — Σ ; $y^0 = \text{const}$

$$(\Box - m^2)(G^+ - G^-) = -\delta + \delta = 0$$

$$\left[G^+(x,y) - G^-(x,y) \right]\Big|_{\Sigma}$$


$\phi(x) \overset{\text{mnc}}{=} -\int_\Sigma d^{d-1}y \left\{ (G^+ - G^-)(x,y) \frac{\partial}{\partial y^0} \phi(y) - \frac{\partial}{\partial y^0}(G^+ - G^-)(x,y)\, \phi(y) \right\}$
$\phi(x)$
Σ
$y^0 = \text{const}$
$(\Box - m^2)(G^+ - G^-) = -\delta + \delta = 0$
$\left[G^+(x,y) - G^-(x,y) \right]\Big|_\Sigma$
$G^+(x,y) = G^-(y,x)$


$\phi(x) = -\int_\Sigma d^{d-1}y \left\{ (G^+ - G^-)(x,y) \frac{\partial}{\partial y^0} \phi(y) - \frac{\partial}{\partial y^0} (G^+ - G^-)(x,y) \phi(y) \right\}$
$y^0 = \text{const}$
$(\Box - m^2)(G^+ - G^-) = -\delta + \delta = 0$
$\left[G^+(x,y) - G^-(x,y) \right]\Big|_\Sigma =$
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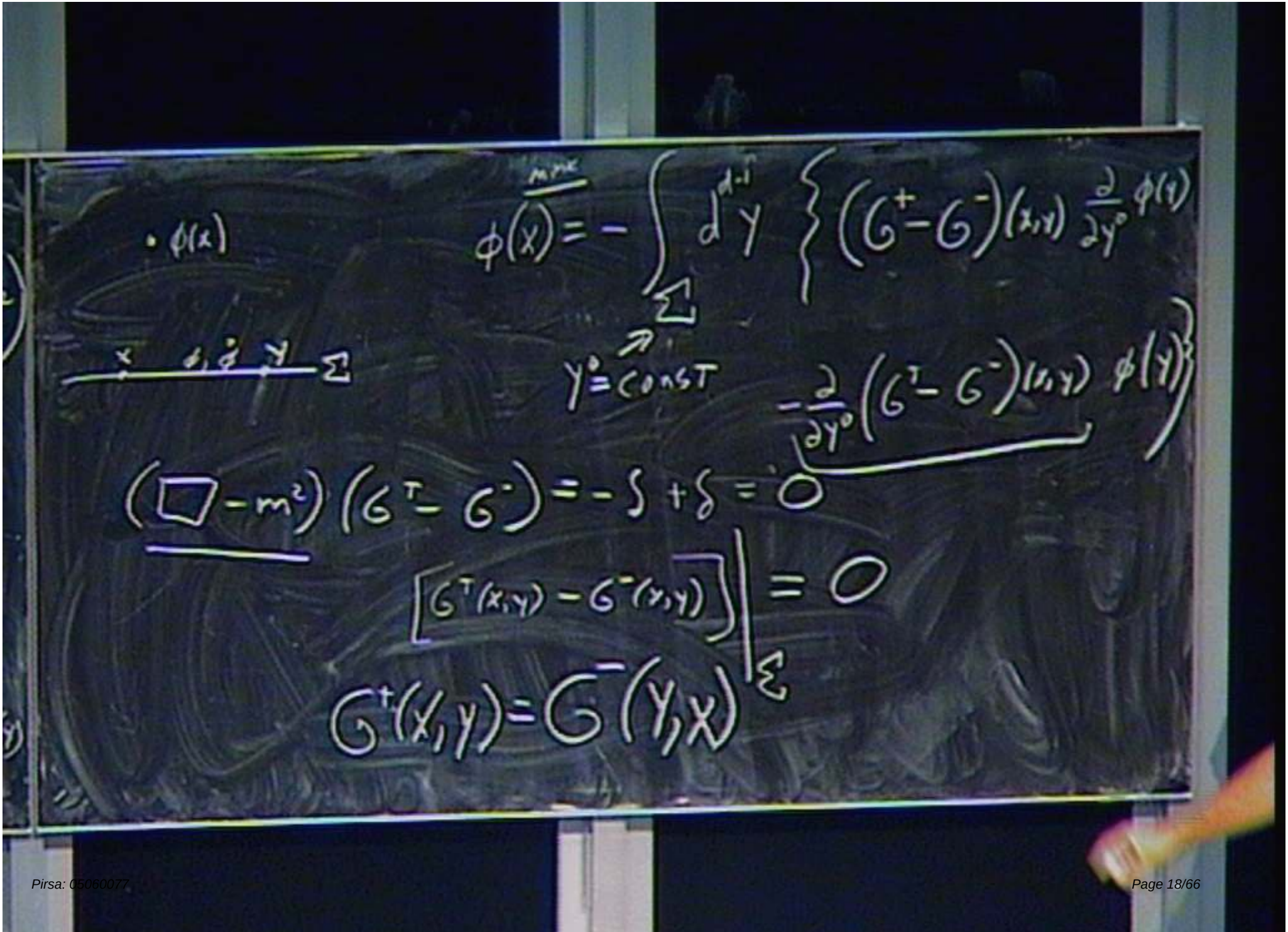
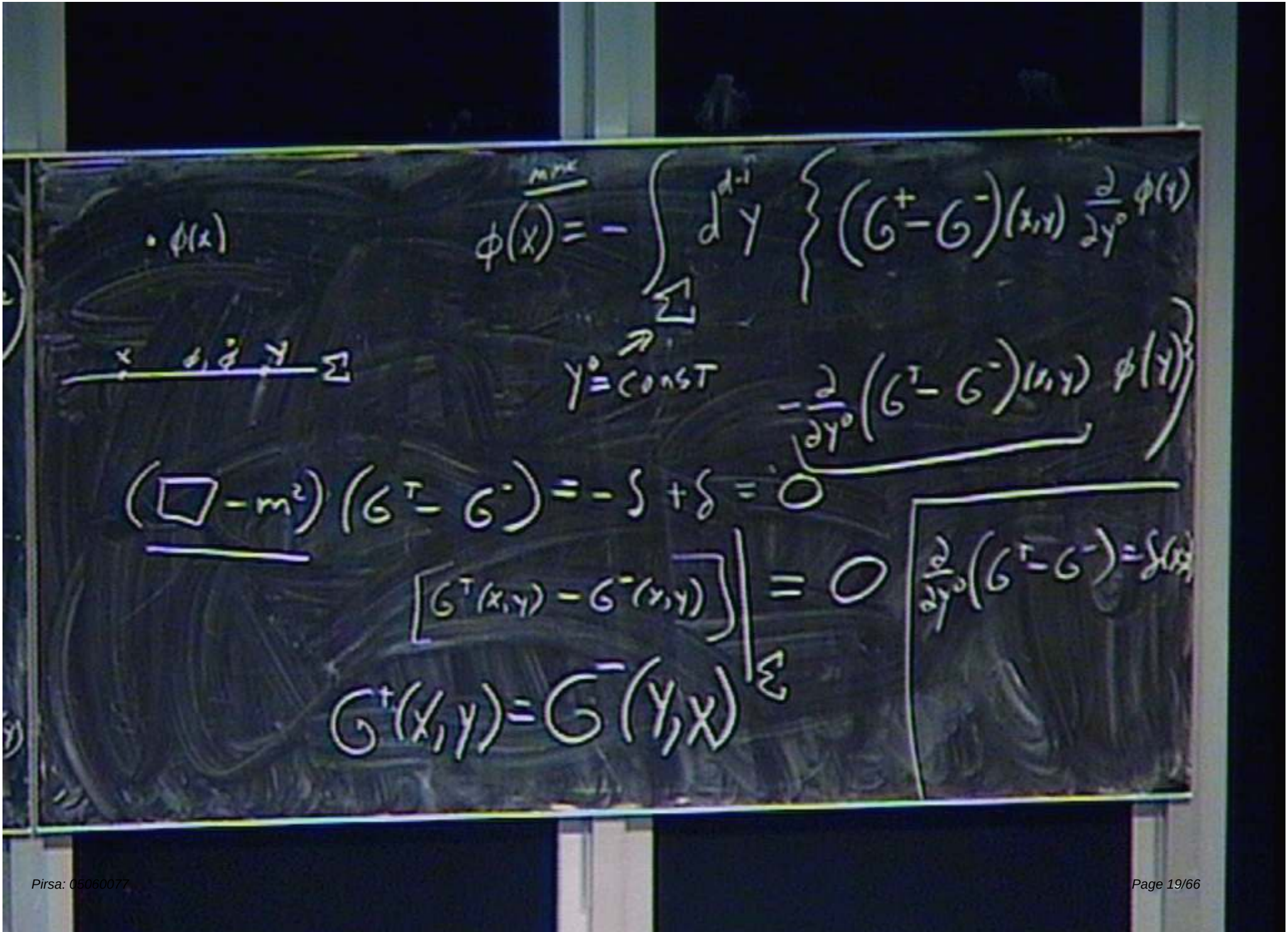


• φ(x)
x φ, φ̇ y Σ
mink
φ(x) = − ∫_Σ d^{d-1}y { (G⁺ − G⁻)(x,y) ∂/∂y⁰ φ(y)
y⁰ = const
− ∂/∂y⁰ (G⁺ − G⁻)(x,y) φ(y) }
(□ − m²)(G⁺ − G⁻) = −δ + δ = 0
[G⁺(x,y) − G⁻(x,y)]|_Σ = 0
G⁺(x,y) = G⁻(y,x)



$\cdot\ \phi(x)$

$x \quad \phi, \dot{\phi} \quad y \quad \Sigma$

$$\phi(x) \overset{\text{mix}}{=} -\int_{\Sigma} d^{d-1}y \left\{ (G^+ - G^-)(x,y)\, \frac{\partial}{\partial y^0}\phi(y) - \frac{\partial}{\partial y^0}(G^+ - G^-)(x,y)\, \phi(y) \right\}$$

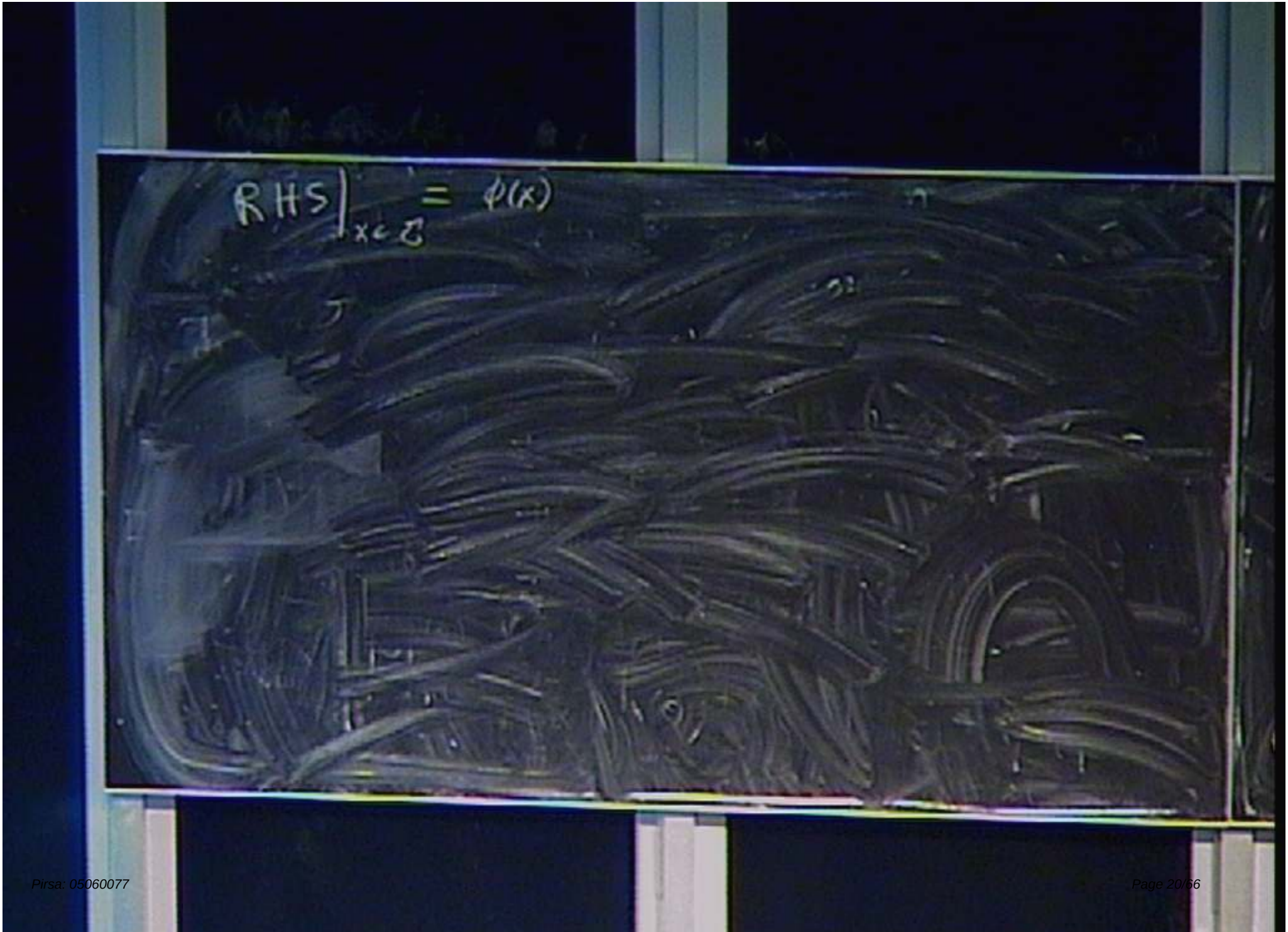
$y^0 = \text{const}$

$$(\Box - m^2)(G^+ - G^-) = -\delta + \delta = 0$$

$$\left[G^+(x,y) - G^-(x,y)\right]\Big|_{\Sigma} = 0$$

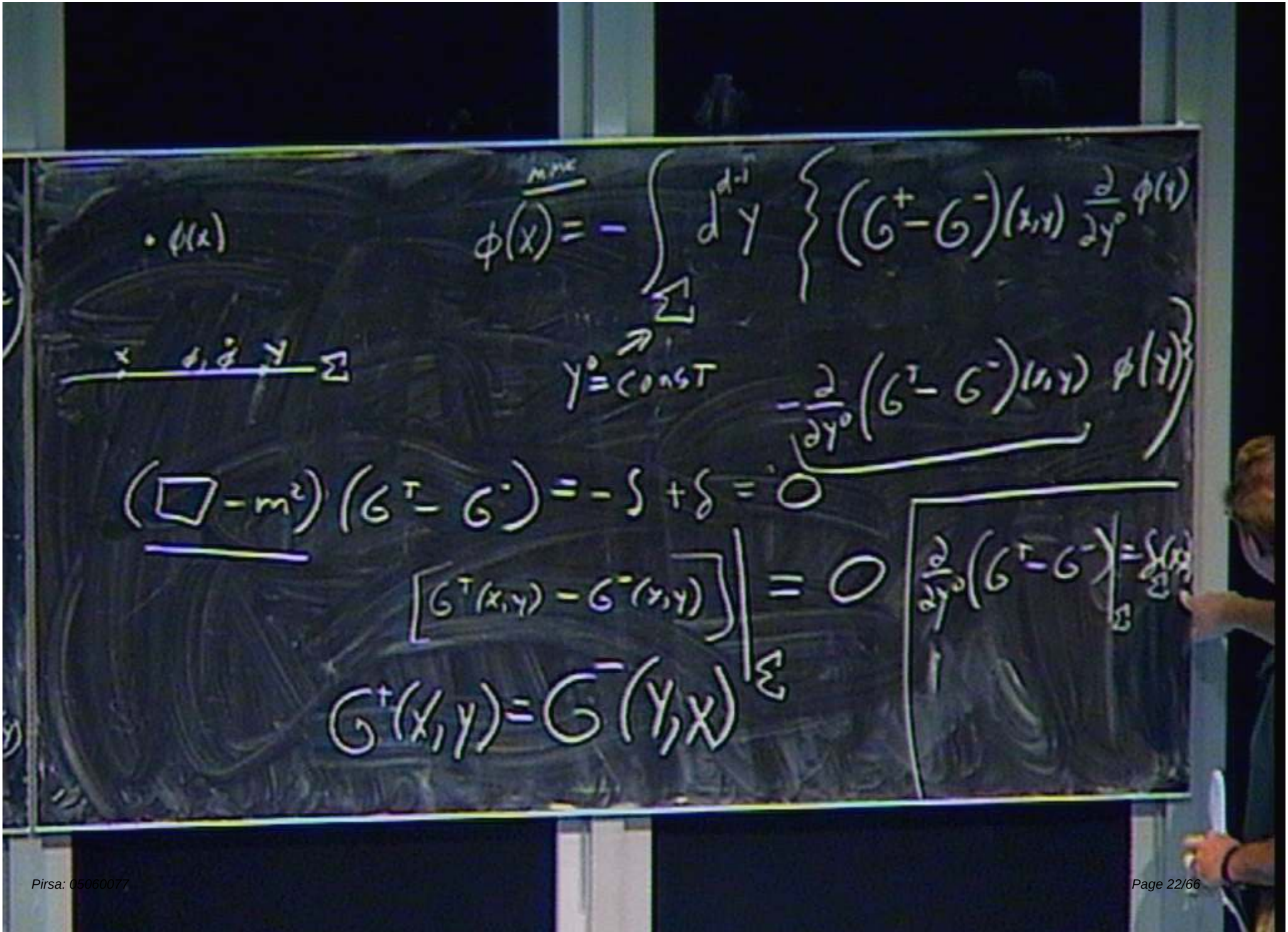
$$\frac{\partial}{\partial y^0}(G^+ - G^-) = \delta(x,y)$$

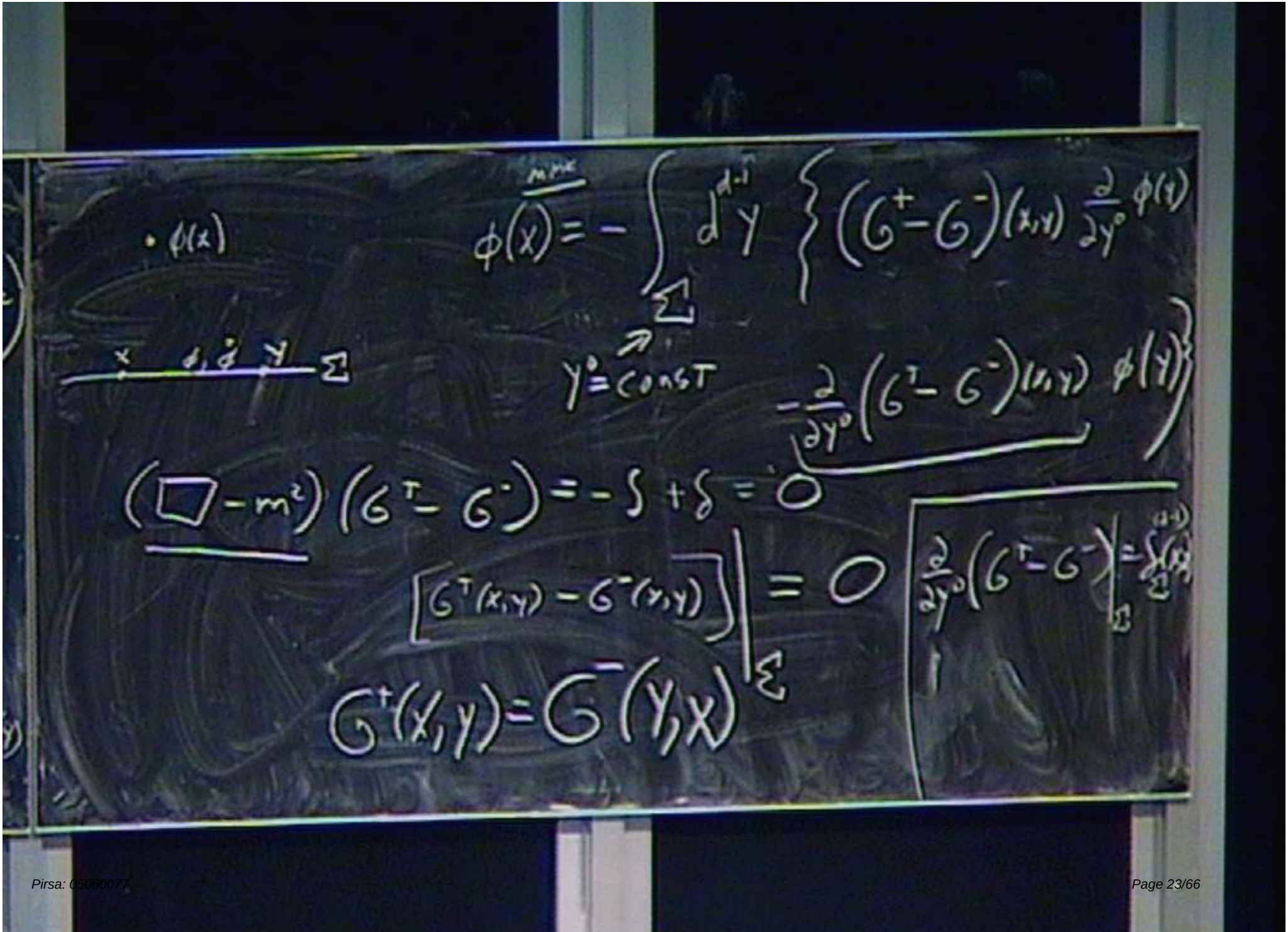
$$G^+(x,y) = G^-(y,x)$$

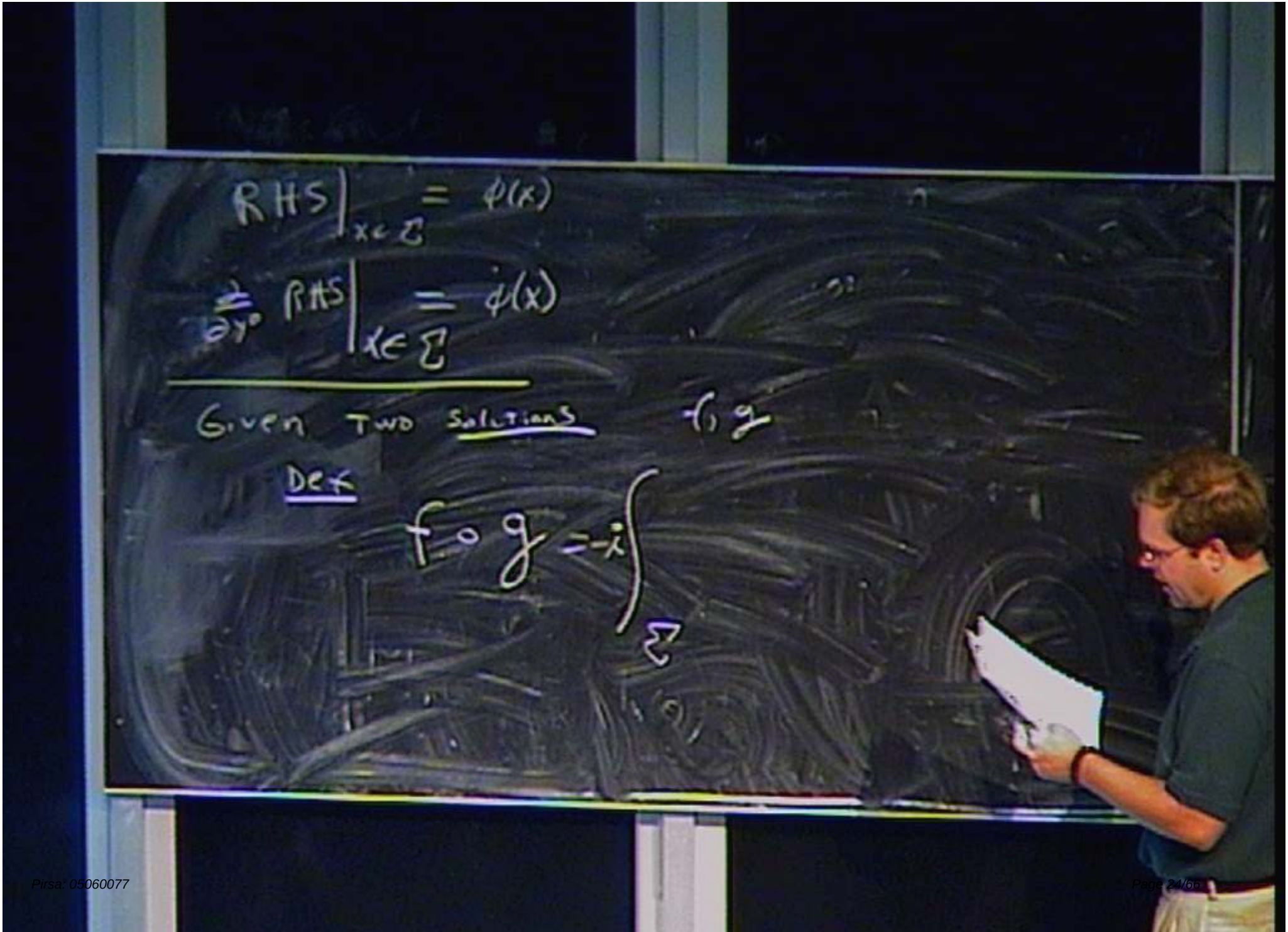

RHS $|_{x \in \Sigma} = \phi(x)$

$$\text{RHS}\Big|_{x\in\Sigma} = \phi(x)$$

$$\frac{\partial}{\partial y^0}\,\text{RHS}\Big|_{x\in\Sigma} = \dot{\phi}(x)$$


$\phi(x)$
$\phi(x) = -\int_\Sigma d^{d-1}y \left\{ (G^+ - G^-)(x,y) \frac{\partial}{\partial y^0} \phi(y) - \frac{\partial}{\partial y^0}(G^+ - G^-)(x,y)\, \phi(y) \right\}$
$x \quad \phi, \partial \quad y \in \Sigma$
$y^0 = \text{const}$
$(\Box - m^2)(G^+ - G^-) = -\delta + \delta = 0$
$\left[G^+(x,y) - G^-(x,y) \right]_\Sigma = 0$
$G^+(x,y) = G^-(y,x)$
$\frac{\partial}{\partial y^0}(G^+ - G^-)\Big|_\Sigma = \delta(x)$


$\phi(x)$
$\phi(x) = -\int_\Sigma d^{d-1}y \left\{ (G^+ - G^-)(x,y) \frac{\partial}{\partial y^0} \phi(y) - \frac{\partial}{\partial y^0}(G^+ - G^-)(x,y)\, \phi(y) \right\}$
$x \neq y \in \Sigma$
$y^0 = \text{const}$
$(\Box - m^2)(G^+ - G^-) = -\delta + \delta = 0$
$\left[G^+(x,y) - G^-(x,y) \right]\Big|_\Sigma = 0$
$\frac{\partial}{\partial y^0}(G^+ - G^-)\Big|_\Sigma = \delta^{(d-1)}(x-y)$
$G^+(x,y) = G^-(y,x)$


RHS $|_{x \in \Sigma} = \phi(x)$
$\frac{\partial}{\partial y^0}$ RHS $|_{x \in \Sigma} = \dot{\phi}(x)$
Given Two Solutions f, g
Def
$f \circ g = -i \int_\Sigma$

$$\text{RHS}\Big|_{x \in \Sigma} = \phi(x)$$

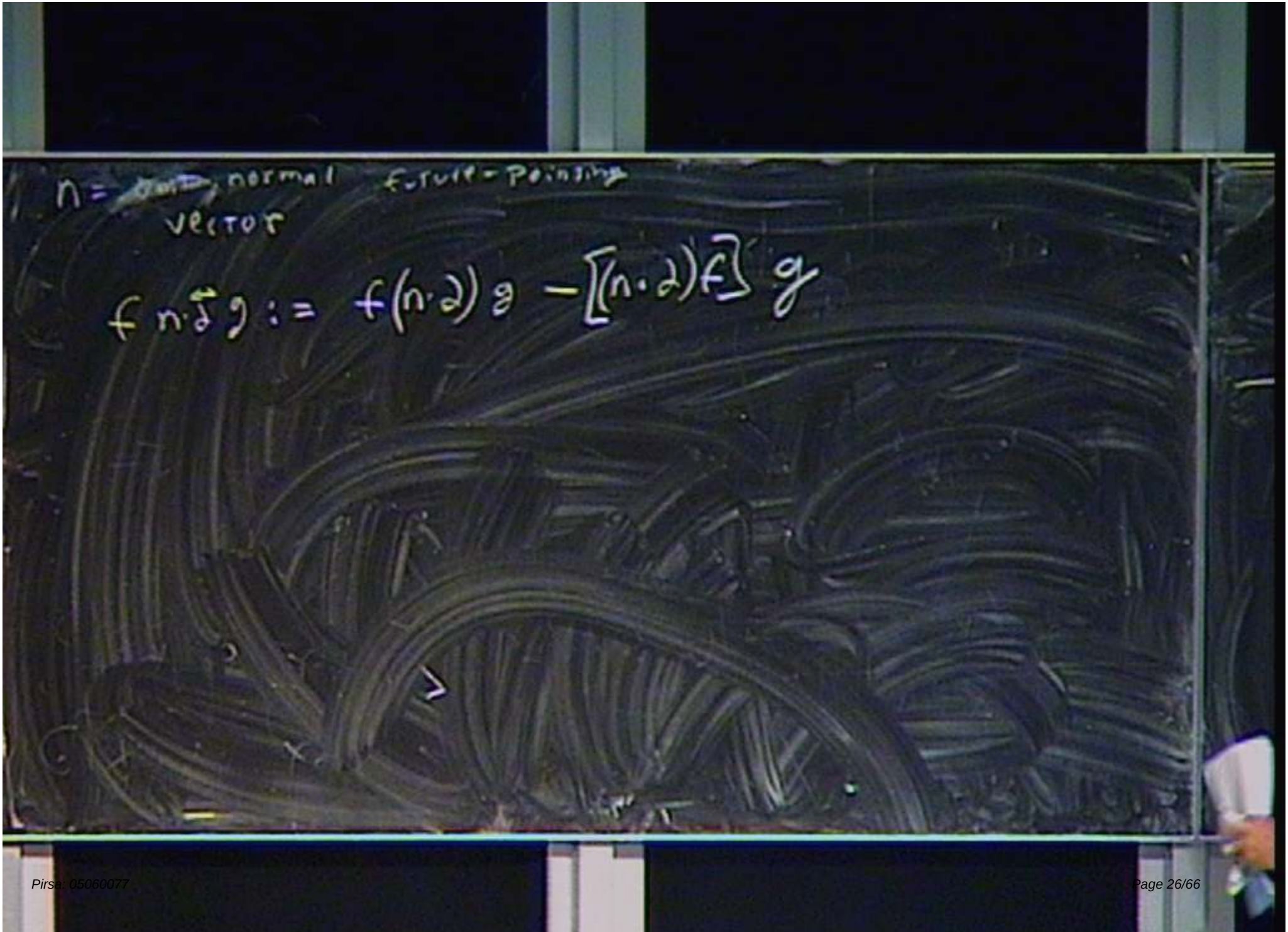
$$\frac{\partial}{\partial y^0}\,\text{RHS}\Big|_{x \in \Sigma} = \dot{\phi}(x)$$

Given Two Solutions f, g

Def

$$f \circ g = -i \int_\Sigma d\Sigma \sqrt{h_\Sigma}\; f\, n \cdot \overleftrightarrow{\partial}\, g$$

h_Σ = induced metric on Σ



$n =$ normal vector future-pointing

$$f\, n\cdot\overleftrightarrow{\partial}\, g := f(n\cdot\partial)\, g - [(n\cdot\partial) f]\, g$$

$$\text{RHS}\Big|_{x\in\Sigma} = \phi(x)$$

$$\frac{\partial}{\partial y^0}\,\text{RHS}\Big|_{x\in\Sigma} = \dot{\phi}(x)$$

Given Two Solutions f, g

Def

Klein-Gordon Inner Product

$$f \circ g = -i\int_\Sigma d\Sigma\,\sqrt{h_\Sigma}\; f\, n\cdot\overleftrightarrow{\partial}\, g$$

h_Σ = induced metric on Σ

n = normal vector future-Pointing

$$f \overleftrightarrow{n\cdot\partial} g := f(n\cdot\partial)g - [(n\cdot\partial)f]\,g$$

$$\overleftrightarrow{\partial_a} = f\partial_a g - g\partial_a f$$

n = normal future-pointing vector

$$f\, n\cdot\vec{\partial}\, g := f(n\cdot\partial)g - [(n\cdot\partial)f]\, g$$

$$\vec{\partial}_a = f\partial_a g - g\partial_a f$$

$$\boxed{\nabla_a \vec{\partial}^a = 0}$$

$n =$ ~~[illegible]~~ normal vector future-pointing

$$f\, n\cdot\overleftrightarrow{\partial}\, g := f(n\cdot\partial)g - [(n\cdot\partial)f]\, g$$

$$j_a = f\partial_a g - g\partial_a f$$

$$\boxed{\nabla_a j^a = 0}$$

Mink $\partial_a j^a = \cancel{\partial_a f\, \partial^a g} + f\partial_a\partial^a g - \cancel{\partial_a g\, \partial^a f} - g\partial_a\partial^a f$

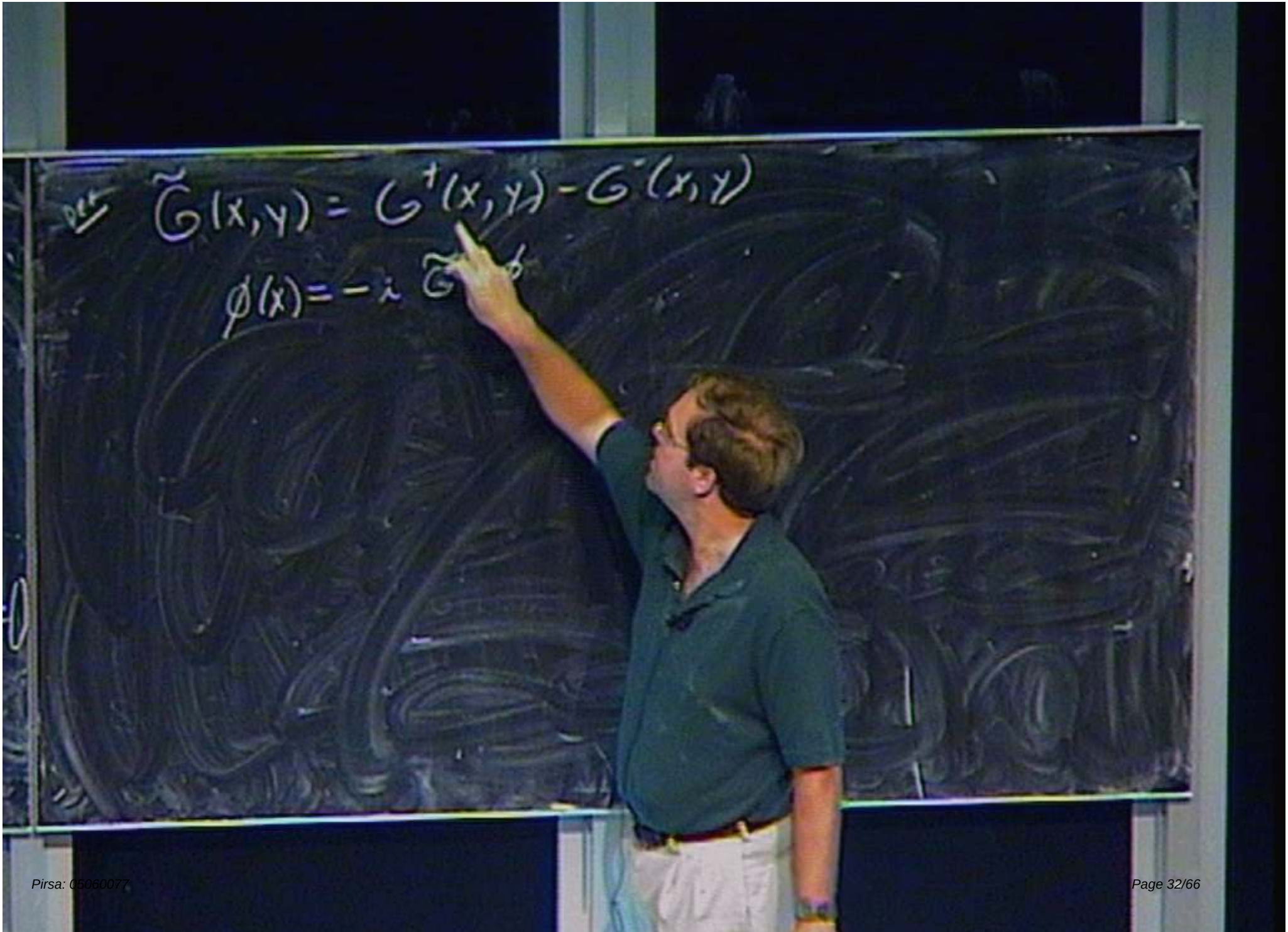
$n =$ ~~[illegible]~~ normal future-pointing vector

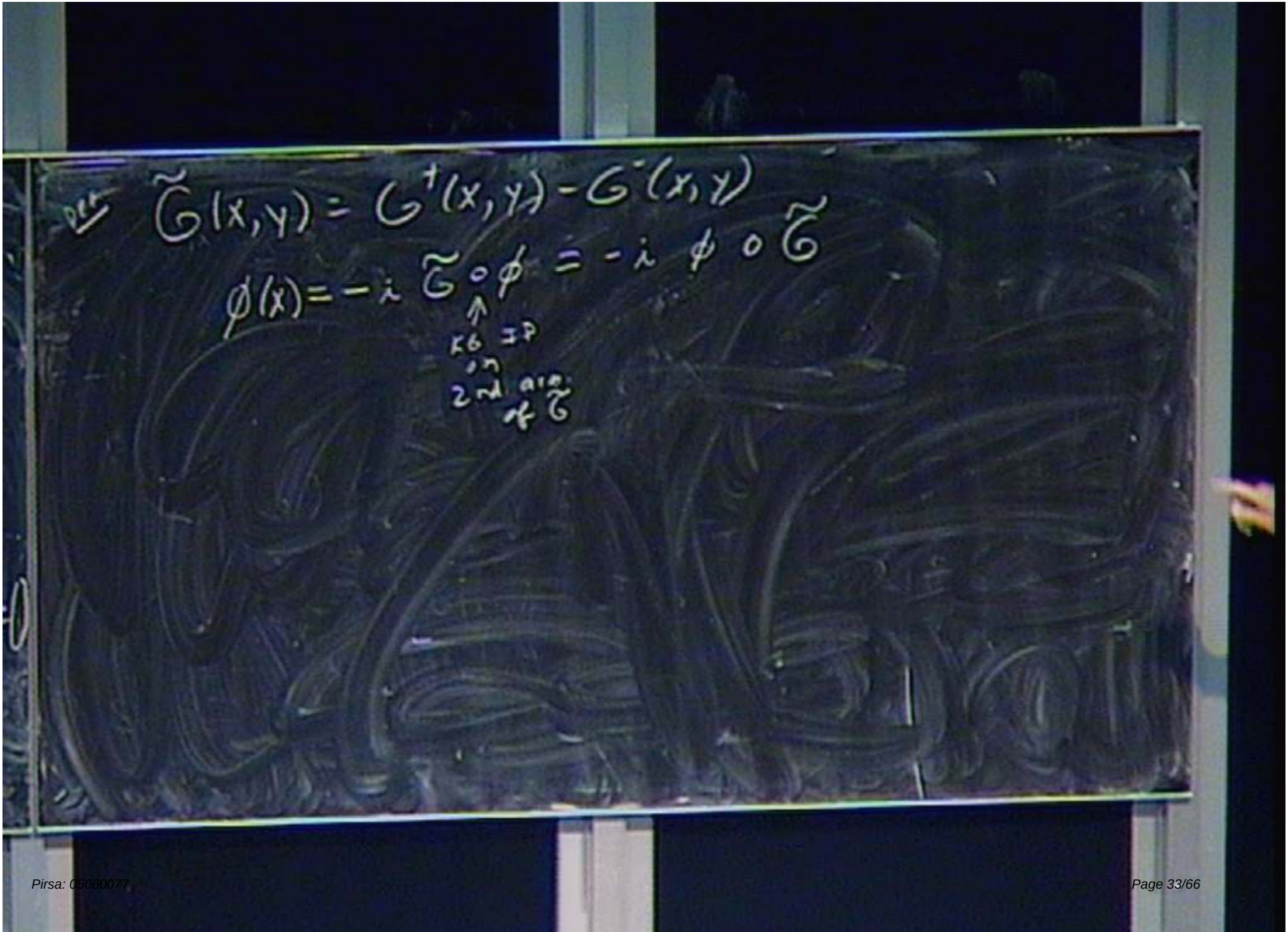
$$f\, n\cdot\overleftrightarrow{\partial}\, g := f(n\cdot\partial)g - [(n\cdot\partial)f]\, g$$

$$j_a = f\partial_a g - g\partial_a f$$

$$\boxed{\nabla_a j^a = 0}$$

Mink $\partial_a j^a = \cancel{\partial_a f\, \partial^a g} + f\underbrace{\partial_a\partial^a}_{m^2} g - \cancel{\partial_a g\, \partial^a f} - g\underbrace{\partial_a\partial^a}_{m^2} f = 0$

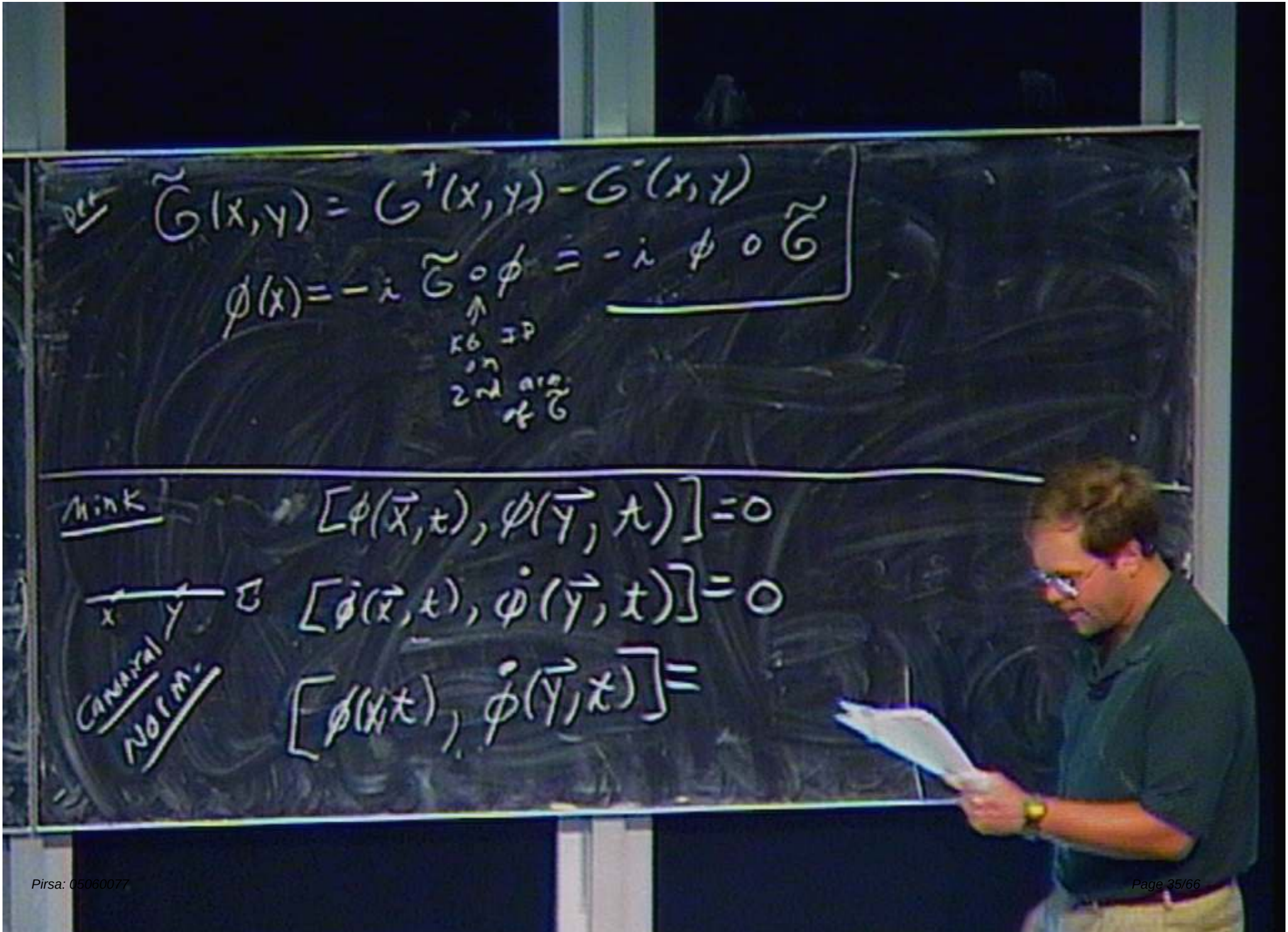

$\tilde{G}(x,y) = G^{+}(x,y) - G^{-}(x,y)$
$\phi(x) = -i$


$\tilde{G}(x,y) = G^{+}(x,y) - G^{-}(x,y)$
$\phi(x) = -i \tilde{G} \circ \phi = -i \phi \circ \tilde{G}$
KG in 2nd quant. $\tilde{G}$

$$\tilde{G}(x,y) = G^{+}(x,y) - G^{-}(x,y)$$

$$\phi(x) = -i\,\tilde{G} \circ \phi = -i\,\phi \circ \tilde{G}$$

KG IP on 2nd arg of $\tilde{G}$


Def $\tilde{G}(x,y) = G^+(x,y) - G^-(x,y)$
$\phi(x) = -i \tilde{G} \circ \phi = -i \phi \circ \tilde{G}$
KG eq in 2nd arg of $\tilde{G}$
Mink
$[\phi(\vec{x},t), \phi(\vec{y},t)] = 0$
x y t
$[\dot{\phi}(\vec{x},t), \dot{\phi}(\vec{y},t)] = 0$
Canonical Norm.
$[\phi(\vec{x},t), \dot{\phi}(\vec{y},t)] =$

Def $\tilde{G}(x,y) = G^+(x,y) - G^-(x,y)$

$\phi(x) = -i \tilde{G} \circ \phi = -i \phi \circ \tilde{G}$

KG EP

2nd order $\tilde{G}$

Mink

$[\phi(\vec{x},t), \phi(\vec{y},t)] = 0$

$[\dot{\phi}(\vec{x},t), \dot{\phi}(\vec{y},t)] = 0$

$[\phi(\vec{x},t), \dot{\phi}(\vec{y},t)] = i\delta(x,y)$

bi-scalar

x y z

Canonical Norm.

Calculate

$$[\phi(x), \phi(y)] = (-\lambda)^2 \left[(\tilde{G} \circ \phi)(x), (\phi \circ \tilde{G})(y) \right]$$

Hint

$$= \left[\int_\Sigma dz_1 \left(\tilde{G}(x, z_1)\, \dot{\phi}(z_1) - \dot{\tilde{G}}(x, z_1)\, \phi(z_1) \right), \right.$$

$$\left. \int_\Sigma dz_2 \left(\phi(z_2)\, \dot{\tilde{G}}(z_2, y) - \dot{\phi}(z_2)\, \tilde{G}(z_2, y) \right) \right]$$

Main

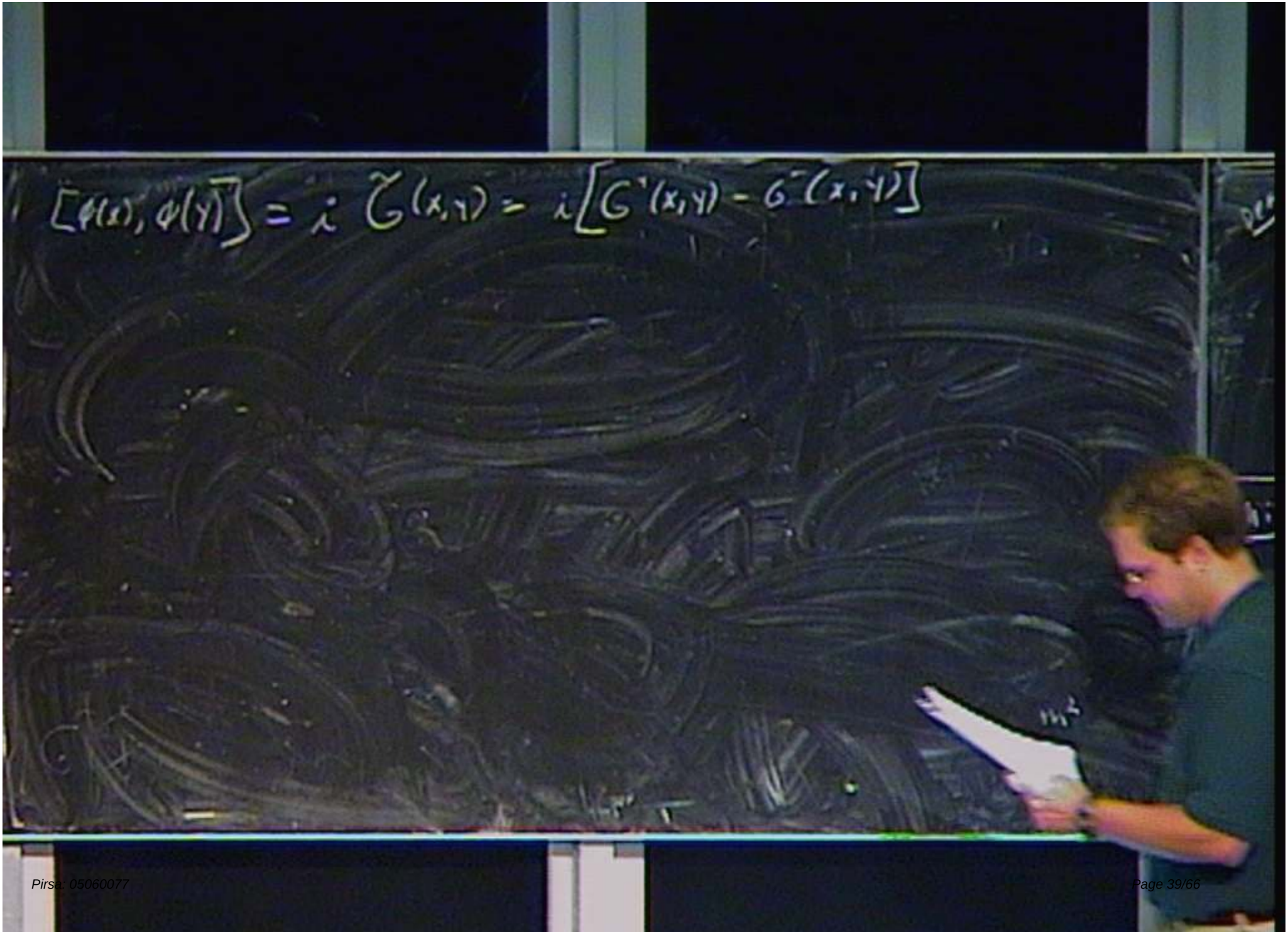
Calculate

$$[\phi(x), \phi(y)] = (-\lambda)^2 \left[(\tilde{G} \circ \phi)(x), (\phi \circ \tilde{G})(y) \right]$$

$$= \left[\int_\Sigma dz_1 \left(\tilde{G}(x,z_1)\, \dot{\phi}(z_1) - \dot{\tilde{G}}(x,z_1)\, \phi(z_1) \right), \right.$$

$$\left. \int_\Sigma dz_2 \left(\phi(z_2)\, \dot{\tilde{G}}(z_2,y) - \dot{\phi}(z_2)\, \tilde{G}(z_2,y) \right) \right]$$

$$= -i \int_\Sigma dz \left[\tilde{G}(x,z)\, \dot{\tilde{G}}(z,y) - \dot{\tilde{G}}(x,z)\, \tilde{G}(z,y) \right] = \tilde{G} \circ \tilde{G}$$


$[\phi(x), \phi(y)] = i\,\mathcal{C}(x,y) = i\left[G^{+}(x,y) - G^{-}(x,y)\right]$

$$[\phi(x), \phi(y)] = i\,G(x,y) = i\left[G^{+}(x,y) - G^{-}(x,y)\right]$$

Recall in Mink

$$\phi(x) = \sum_n \left(a_n^{+} e^{-i\omega x} e^{ikx} + a_n \ldots - \right)$$

$$e^{ik\cdot x} e^{-i\omega x_0}\, \phi(x) =$$

$$[\phi(x), \phi(y)] = i\,\tilde{G}(x,y) = i\left[G^{+}(x,y) - G^{-}(x,y)\right]$$

Consider any Solution $U(x)$

Def

$$a_U := U \circ \phi$$

$$[a_U, a_{U'}] \equiv -[U\circ\phi,\ \phi\circ U'] = -U\circ\overbrace{[\phi(x),\phi(y)]}^{i\tilde{G}(x,y)}\circ U'$$

$$= U\cdot U'$$

$$[\phi(x), \phi(y)] = i\,\Delta(x,y) = i\left[G^{+}(x,y) - G^{-}(x,y)\right]$$

Consider any Solution $U(x)$

Def

$$a_U := U \circ \phi$$

$$[a_U, a_{U'}] = -[U\circ\phi,\ \phi\circ U'] = -U\circ\overbrace{[\phi(x),\phi(y)]}^{i\Delta(x,y)}\circ U'$$

$$= U\cdot U'$$

$$[\phi(x), \phi(y)] = i\,\tilde{G}(x,y) = i\left[G^{+}(x,y) - G^{-}(x,y)\right]$$

Consider any Solution $U(x)$

Def

$$a_U := U \circ \phi$$

$$[a_U, a_{U'}] \equiv -[U\circ\phi,\ \phi\circ U'] = -U\circ\overbrace{[\phi(x), \phi(y)]}^{i\tilde{G}(x,y)}\circ U'$$

$$\equiv U\cdot U'$$

Suppose
$U' = U^{*}$
$U\cdot U^{*} \neq 0$
$U\cdot U^{*} \not\equiv 0$,

$$\boxed{U\cdot U^{*} = -1}$$

$$[a_U, a_U] = 0$$

$$a_U^{\dagger} = (U\circ\phi)^{\dagger} = \phi$$

$$[\phi(x), \phi(y)] = i\,\tilde{G}(x,y) = i\left[G^{+}(x,y) - G^{-}(x,y)\right]$$

Consider any Solution $U(x)$

Def

$$a_U := U \circ \phi$$

$$[a_U, a_{U'}] = -[U\circ\phi, \phi\circ U'] = -U\circ\underbrace{[\phi(x),\phi(y)]}_{i\tilde{G}(x,y)}\circ U'$$

$$= U\cdot U'$$

Suppose $U' = U^{*}$

$U \cdot U^{*} \neq 0$

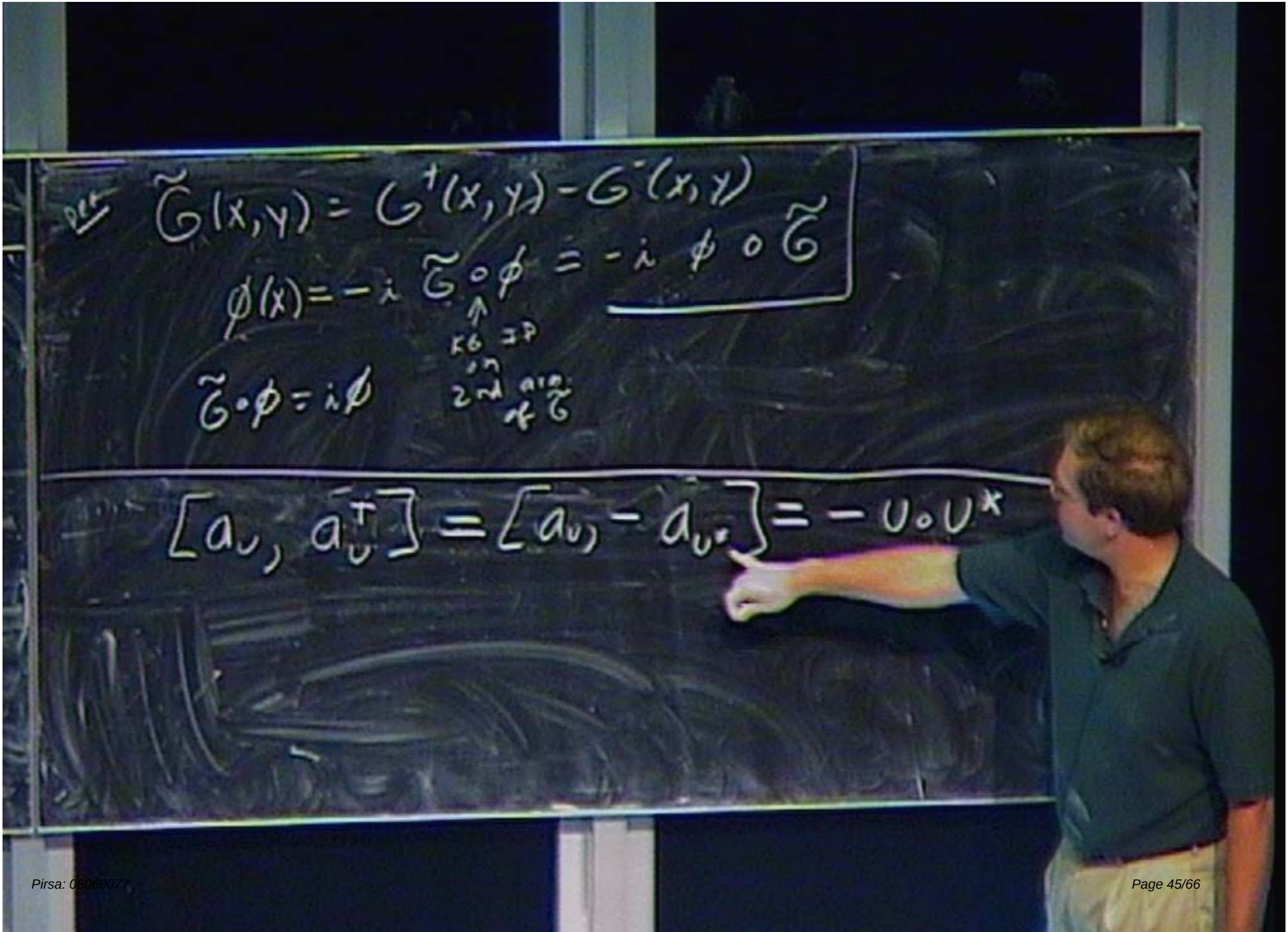
$U \cdot U^{*} \not\equiv 0$,

$\boxed{U\cdot U^{*} = -1}$

$[a_U, a_U] = 0$

$a_U^{\dagger} = (U\circ\phi)^{\dagger} = -U^{*}$

$= a_{U^{*}}$


$\tilde{G}(x,y) = G^{+}(x,y) - G^{-}(x,y)$
$\phi(x) = -i\,\tilde{G}\circ\phi = -i\,\phi\circ\tilde{G}$
KG eqn
$\tilde{G}\cdot\phi = i\phi$
2nd arg of $\tilde{G}$
$[a_U, a_U^\dagger] = [a_U, -a_{U^*}] = -U\circ U^*$

$$\tilde{G}(x,y) = G^{+}(x,y) - G^{-}(x,y)$$

$$\phi(x) = -i\,\tilde{G}\circ\phi = -i\,\phi\circ\tilde{G}$$

KG eq. in 2nd arg. of $\tilde{G}$

$$\tilde{G}\cdot\phi = i\phi$$

$$[a_U, a_U^{\dagger}] = [a_U, -a_{U^*}] = -U\circ U^* = 1$$

$$\tilde{G}(x,y) = G^{+}(x,y) - G^{-}(x,y)$$

$$\phi(x) = -i\,\tilde{G}\circ\phi = -i\,\phi\circ\tilde{G}$$

KG eq in 2nd arg of $\tilde{G}$

$$\tilde{G}\cdot\phi = i\phi$$

$$[a_\nu, a_\nu^\dagger] = [a_\nu, -a_{\nu^*}] = -U\circ U^* = 1$$

$$[a_\nu^\dagger, a_\nu^\dagger] = 0$$

$$[\phi(x), \phi(y)] = i\,\tilde{G}(x,y) = i\left[G^{+}(x,y) - G^{-}(x,y)\right]$$

Consider any Solution $U(x)$

Def

$$a_U := U \circ \phi$$

$$[a_U, a_{U'}] = -[U\circ\phi, \phi\circ U'] = -U\circ\underbrace{[\phi(x),\phi(y)]}_{i\tilde{G}(x,y)}\circ U'$$

$$= U\cdot U'$$

Suppose $U' = U^{*}$

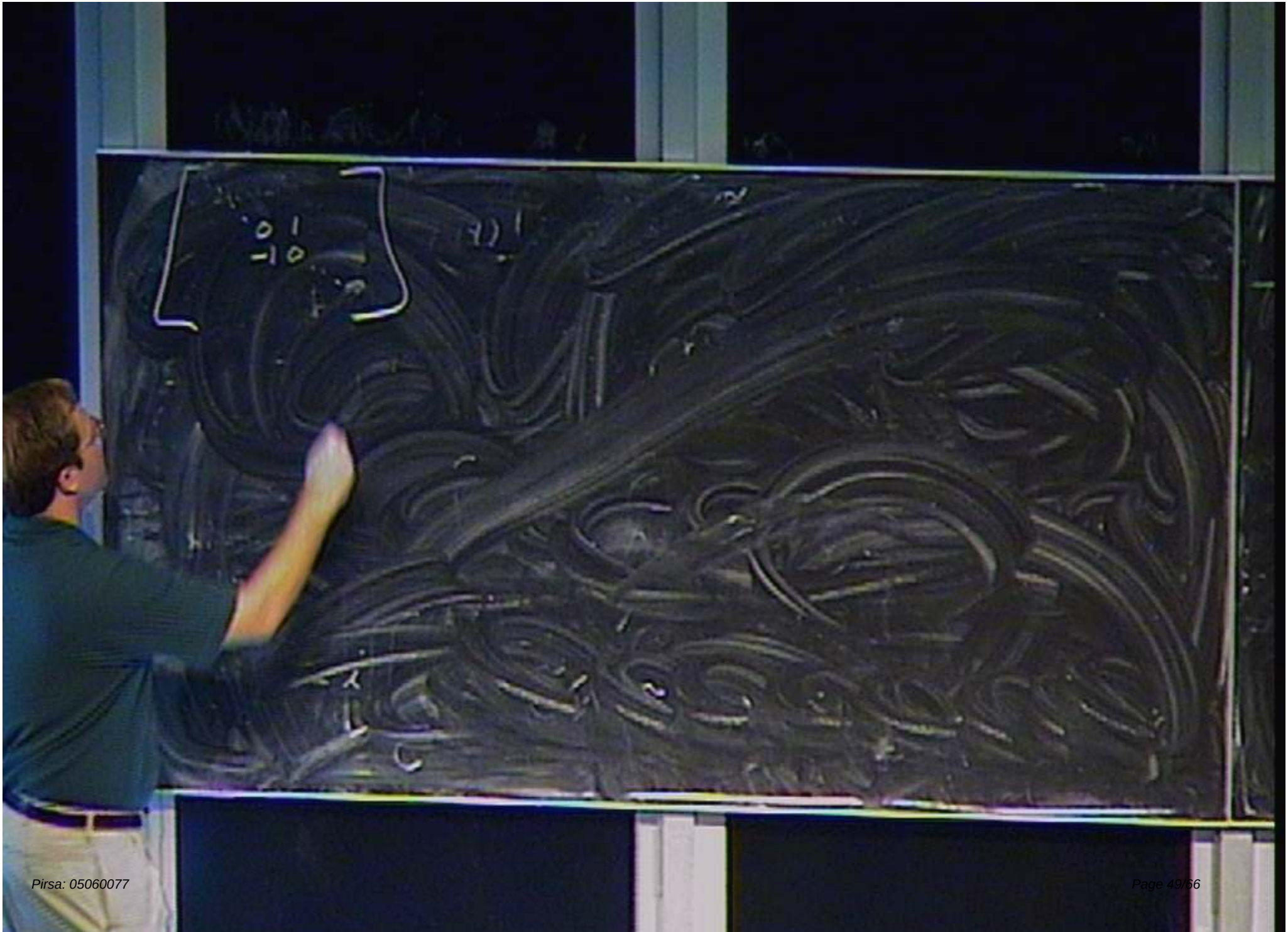
$U \cdot U^{*} \neq 0$,

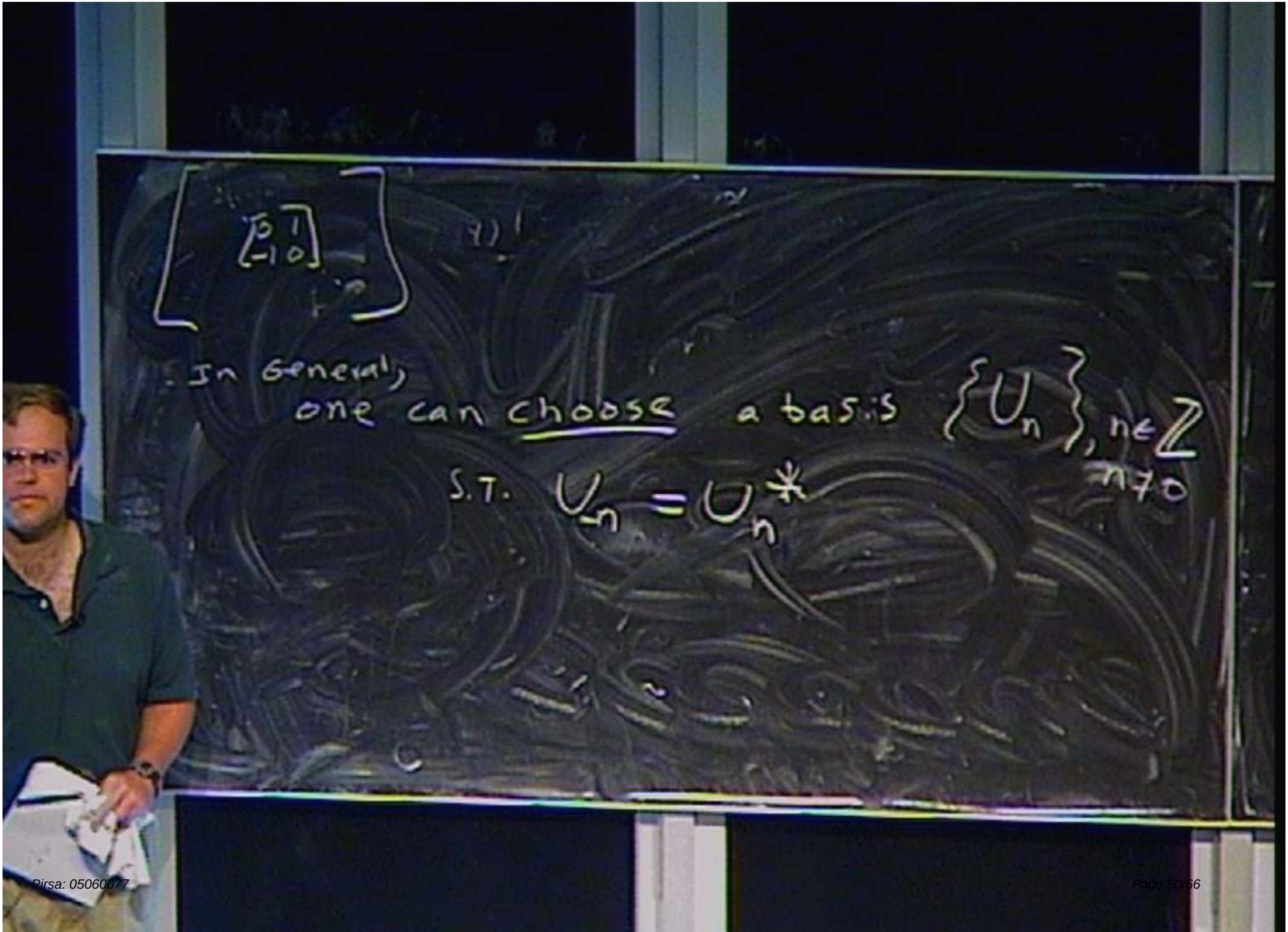
$U \cdot U^{*} = -U$

$[a_U, a_U] = 0$

$a_U^{\dagger} = (U\circ\phi)^{\dagger} = -U^{*}$

$= a_{U^{*}}$




In General,
one can choose a basis
s.t.

$\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}^n$

In General,
one can choose a basis $\{U_n\}_{n \in \mathbb{Z}, n \neq 0}$

s.t. $U_{-n} = U_n^*$

& s.t. $U_n \circ U_m = -\delta_{n,-m} \operatorname{sign}(n)$

Given U_j can define a "zero-particle state"

$$a_n |0\rangle_U = 0 \quad \text{for } n > 0$$

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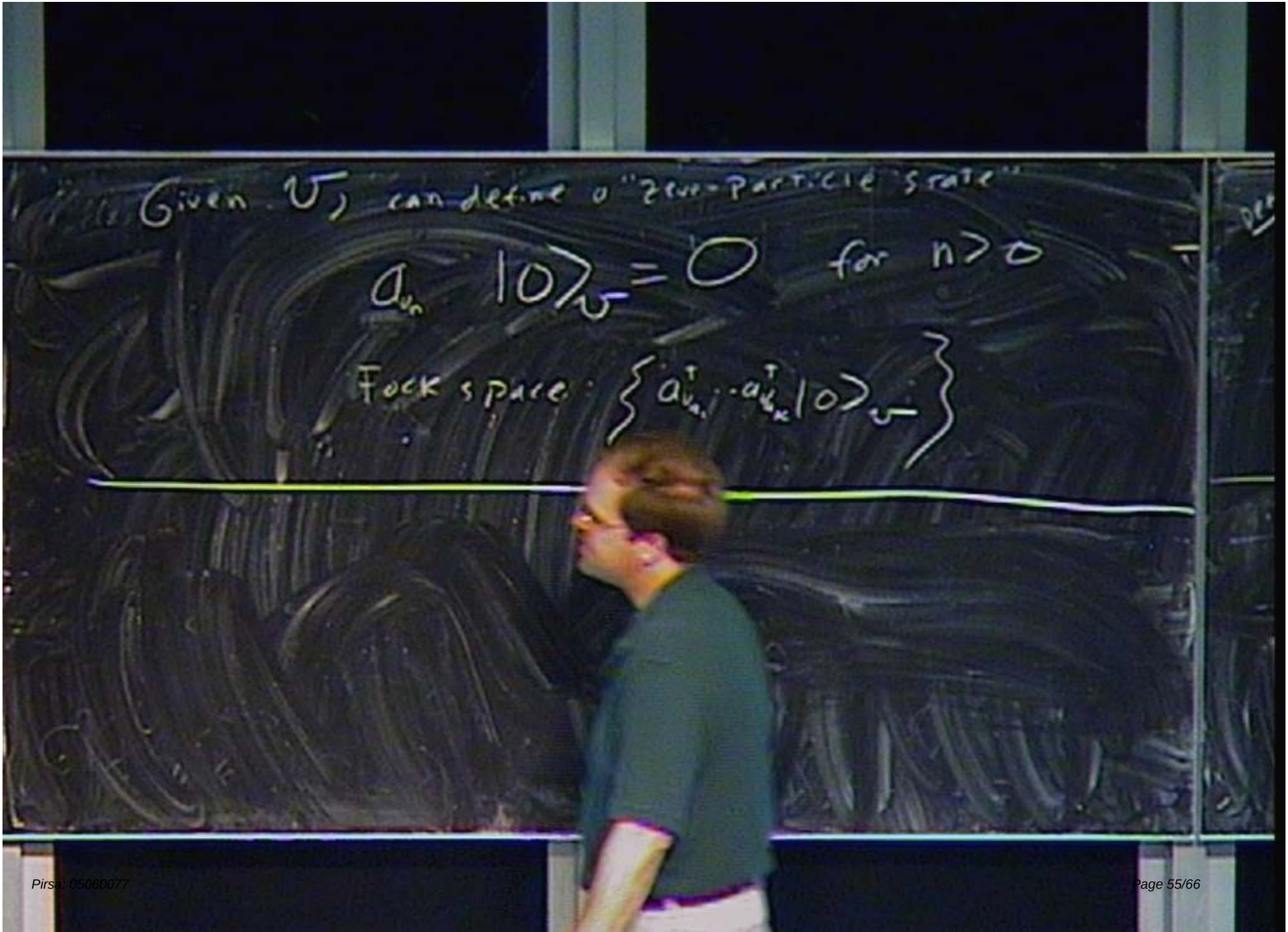
Fock space: $\left\{ a^\dagger_{U_{n_1}} \cdots a^\dagger_{U_{n_k}} |0\rangle_U \right\}$

Given v, can define a "zero-particle state"

$$a_{v_n} |0\rangle_v = 0 \quad \text{for } n > 0$$

Fock space: $\left\{ a^\dagger_{v_{n_1}} \cdots a^\dagger_{v_{n_k}} |0\rangle_v \right\}$

$$\langle \psi | :T(\phi): | \psi \rangle$$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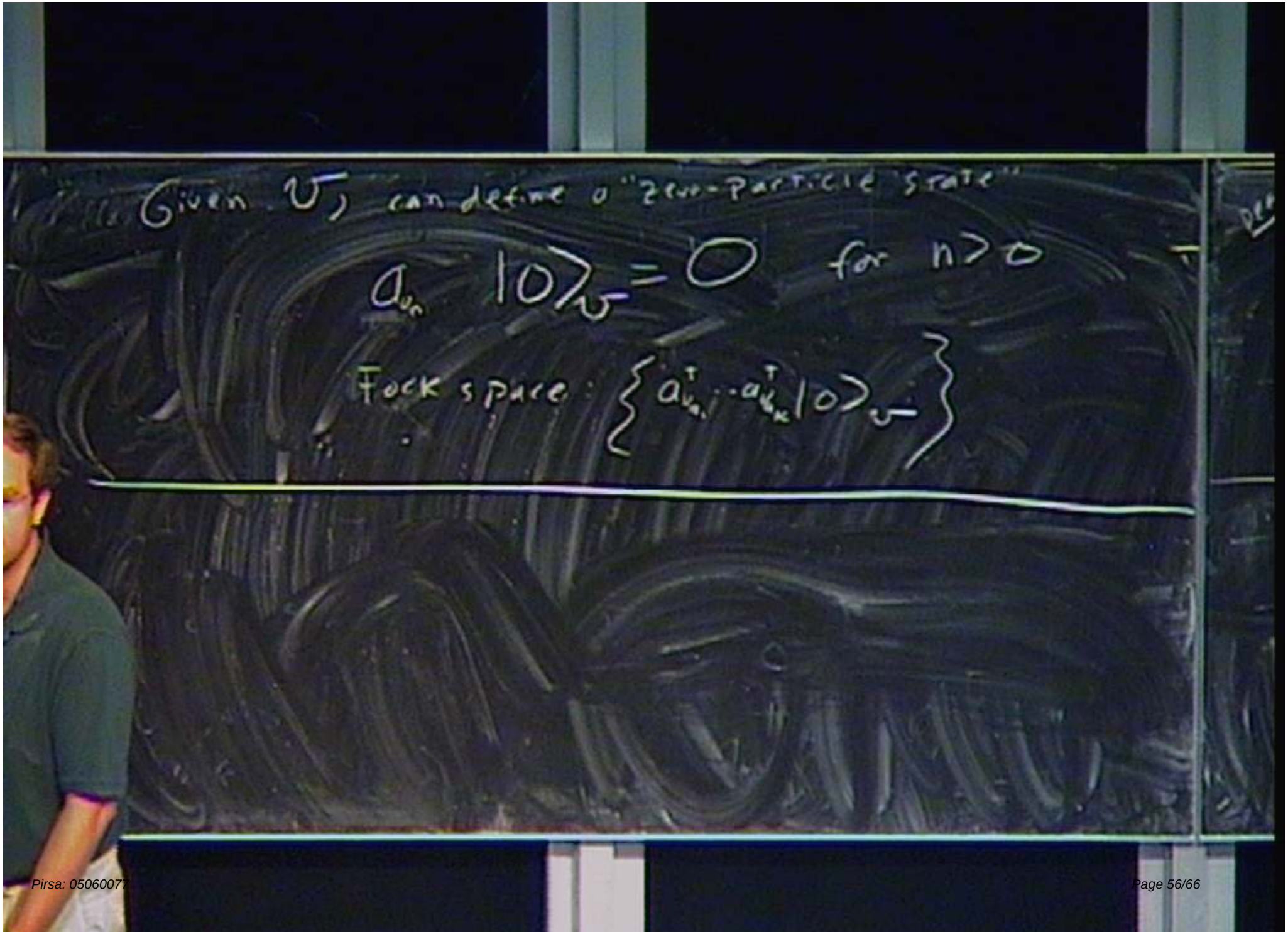
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Compare U vs. V

Given U, can define a "zero-particle state"

$$a_{un}\,|0\rangle_U = 0 \quad \text{for } n > 0$$

Fock space: $\left\{ a^\dagger_{u_{n_1}} \cdots a^\dagger_{u_{n_k}} |0\rangle_U \right\}$

Compare U vs. $V = \{v_n\}$

$$V_n = \sum_{m>0} \alpha_{nm} U_m - \beta_{nm} U_m^*$$

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$$a_{V_n} = V_n \circ \phi = \sum_{m>0} \left(\alpha_{nm} a_{U_m} + \beta_{nm} a^\dagger_{U_m} \right)$$

$a_{\nu_n}|0\rangle_T = 0 \quad \text{for } n>0$

$\sum_{n>0} (\alpha_{nm} a_{\nu_m} + \beta_{nm} a^T_{\nu_m})|0\rangle_T = 0$

$a_{\nu_n}|0\rangle_T = 0 \quad \text{for } n>0$

$\sum_{n>0}\left(\alpha_{nm}\, a_{\nu_m} + \beta_{nm}\, a^T_{\nu_m}\right)|0\rangle_T = 0$

$\Rightarrow |0\rangle_\nu \neq |0\rangle_T$

$a_{V_n}|0\rangle_V = 0 \quad \text{for } n>0$

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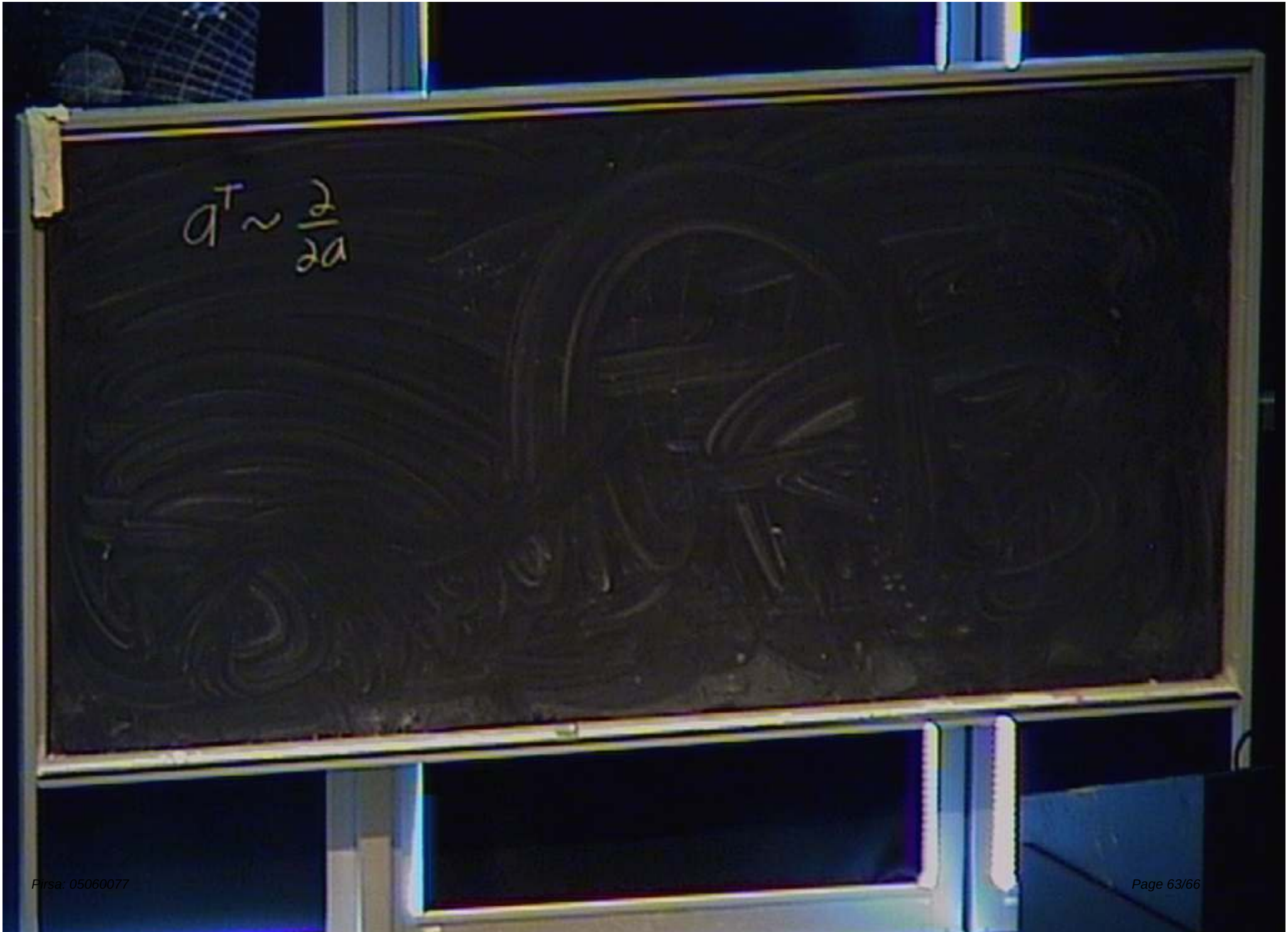
$a^T \sim \frac{2}{2a}$

$a_{V_n}|0\rangle_V = 0 \quad \text{for } n>0$

$$\sum_{n>0}\left(\alpha_{nm}\, a_{V_m} + \beta_{nm}\, a^{\dagger}_{V_m}\right)|0\rangle_V = 0$$

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iff

$\beta_{nm} \neq 0$

$$a^T \sim \frac{\partial}{\partial a}$$

$$|0\rangle_T = N \exp\left(-\frac{1}{2} \sum_{n,m} (\alpha^{-1}\beta)_{nm} a^T_{\omega_n} a^T_{\omega_m}\right) |0\rangle_T$$

$$N = |\det \Gamma^{-1/2}| = \left(\sum_{n,m} \beta^*_{nm} \beta_{nm}\right)^{-1/4}$$

$$a^T \sim \frac{\partial}{\partial a}$$

$$|0\rangle_V = N \exp\left(-\frac{1}{2}\sum_{n,m}(\alpha^{-1}\beta)_{nm}\, a^T_{v_n} a^T_{v_m}\right)|0\rangle_V$$

$$N = |\alpha|^{-1/4} = \left(\sum_{n,m} \alpha^{*}_{nm}\beta_{nm}\right)^{-1/4}$$