Title: Perturbative String Theory

Date: Jun 21, 2005 11:30 AM

URL: http://pirsa.org/05060076

Abstract:

Spacetime Physics

$G_{\mu\nu}(x)$
$B_{\mu\nu}(x)$
$\Phi(x)$

Those field equations can be derived from this spacetime action:

$$
\begin{aligned}
\mathrm{S} \;=\; \frac{1}{2\kappa_0^2}\int d^D X(-G)^{1/2}e^{-2\Phi}\Big[& R + 4\nabla_\mu\Phi\nabla^\mu\Phi - \frac{1}{12}H_{\mu\nu\lambda}H^{\mu\nu\lambda} \\
& - \frac{2(D-26)}{3\alpha'} + O(\alpha')\Big]
\end{aligned}
$$

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Notice the implicit power of the string coupling that appears. Everything was computed at the sphere level. Tree level in string pert. theory.

Language of Perturbation Theory

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Sphere?

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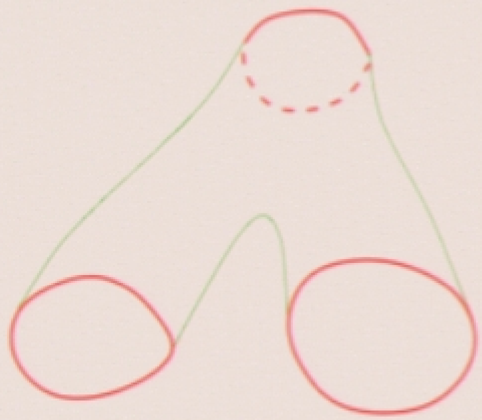
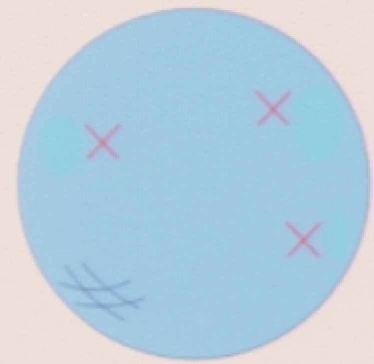
Sphere?

Can always choose conformal factor to map all external states to "vertex operators" at points....





Language of Perturbation Theory

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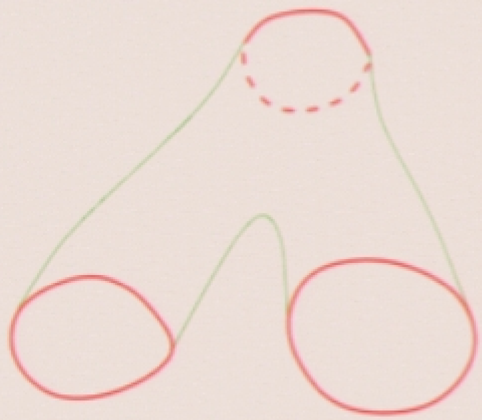
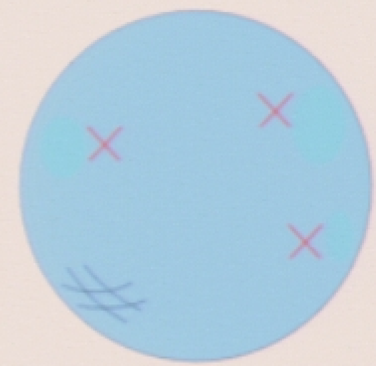
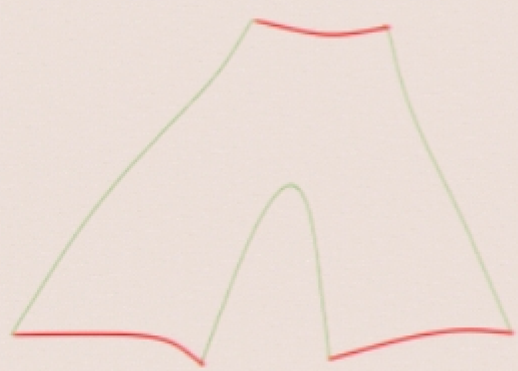
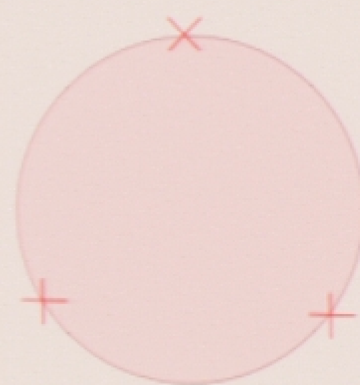




$$g_s^{-2}$$

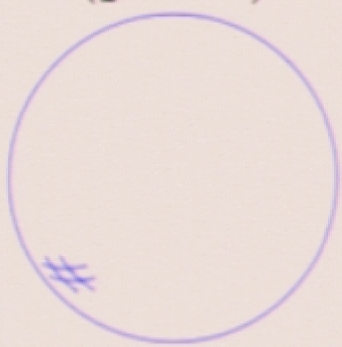
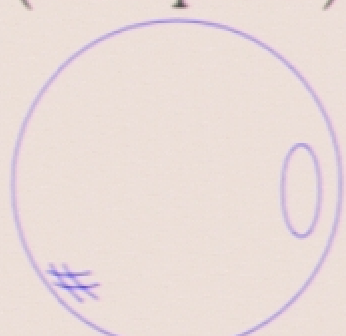
Disc (see later)





Language of Perturbation Theory

Some more diagrams...

	g_s^{-2}	g_s^{-1}	g_s^{0}
closed oriented	sphere S^2 (plane) 	.	torus T^2 
open oriented	.	disc D_2 (half–plane) 	cylinder C_2 (annulus) 

Spacetime Physics

Can also rescale metric to present action differently:

set this to be asymptotic value of dilaton

$$\tilde{G}_{\mu\nu}(X) = e^{2\Omega(X)} G_{\mu\nu} = e^{4(\Phi_0 - \Phi)/(D-2)} G_{\mu\nu}$$

$G_{\mu\nu}(x)$

$B_{\mu\nu}(x)$

$\Phi(x)$

Spacetime Physics

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$B_{\mu\nu}(x)$
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$$\begin{aligned} \mathrm{S} \;=\; & \frac{1}{2\kappa^2} \int d^D X (-\tilde{G})^{1/2} \Big[R - \frac{4}{D-2} \nabla_\mu \tilde{\Phi} \nabla^\mu \tilde{\Phi} \\ & - \frac{1}{12} e^{-8\tilde{\Phi}/(D-2)} H_{\mu\nu\lambda} H^{\mu\nu\lambda} - \frac{2(D-26)}{3\alpha'} e^{4\tilde{\Phi}/(D-2)} + O(\alpha') \Big] \end{aligned}$$

This is the low energy effective action in "Einstein frame". (Previous was "String frame").

Closed string coupling sets Newton's constant $\quad \kappa \equiv \kappa_0 e^{\Phi_0} = (8\pi G_N)^{1/2}$

Spacetime Physics

Similar story for the open string (all Neumann):

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$$A_\mu(x)$$

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$$\int_{\partial\mathcal{M}} d\tau\, A_\mu \partial_t X^\mu$$

Coupling to boundary of the worldsheet.

$$\mathrm{S} = -\frac{\mathrm{C}}{4}\int d^D X\, e^{-\Phi}\mathrm{Tr} F_{\mu\nu}F^{\mu\nu} + O(\alpha')$$

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Effective spacetime action is Maxwell, with coupling set by dilaton.

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Effective spacetime action is Maxwell, with coupling set by dilaton.

g_s^{-1} "disc order"

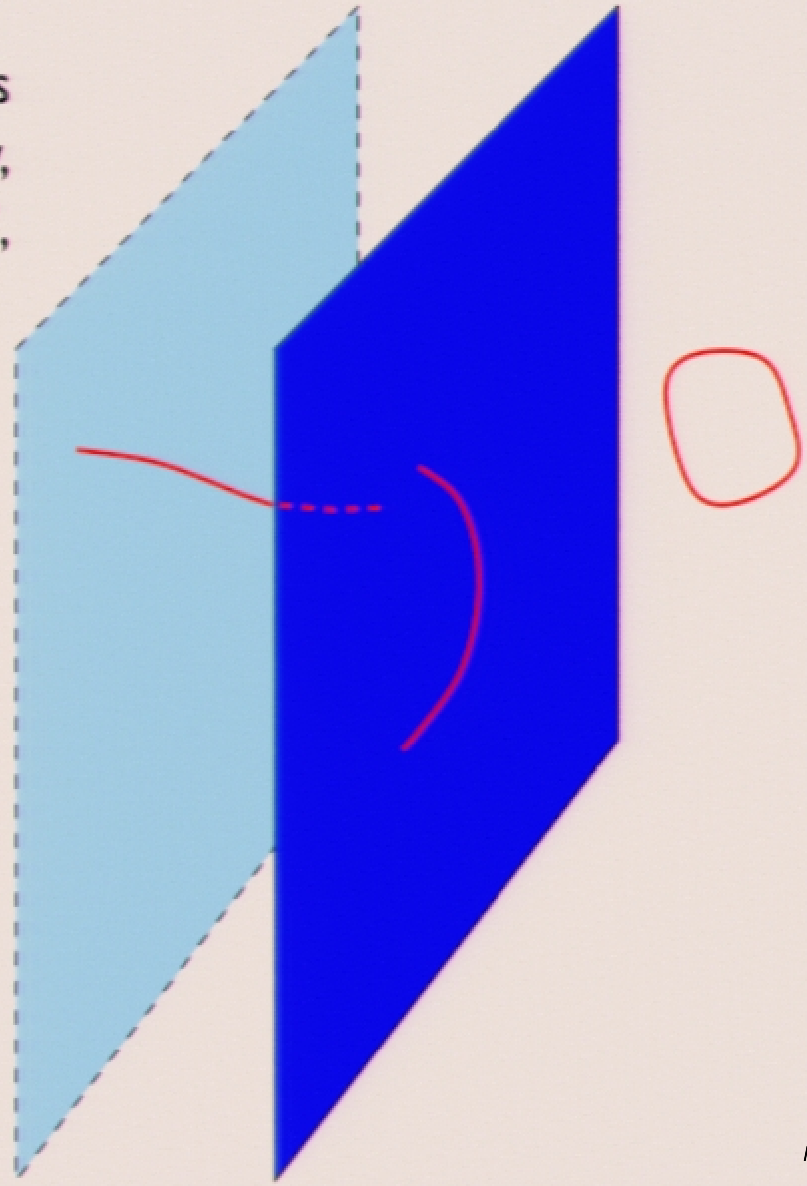
Spacetime Physics

$A_\mu(x)$

When there are Dirichlet-Dirichlet and Dirichlet-Neumann directions also, there is a more geometrical language:

D-Branes

Think of open string sectors as existing within a closed string theory, as hypersurfaces called “D-branes”, where the endpoints lie.



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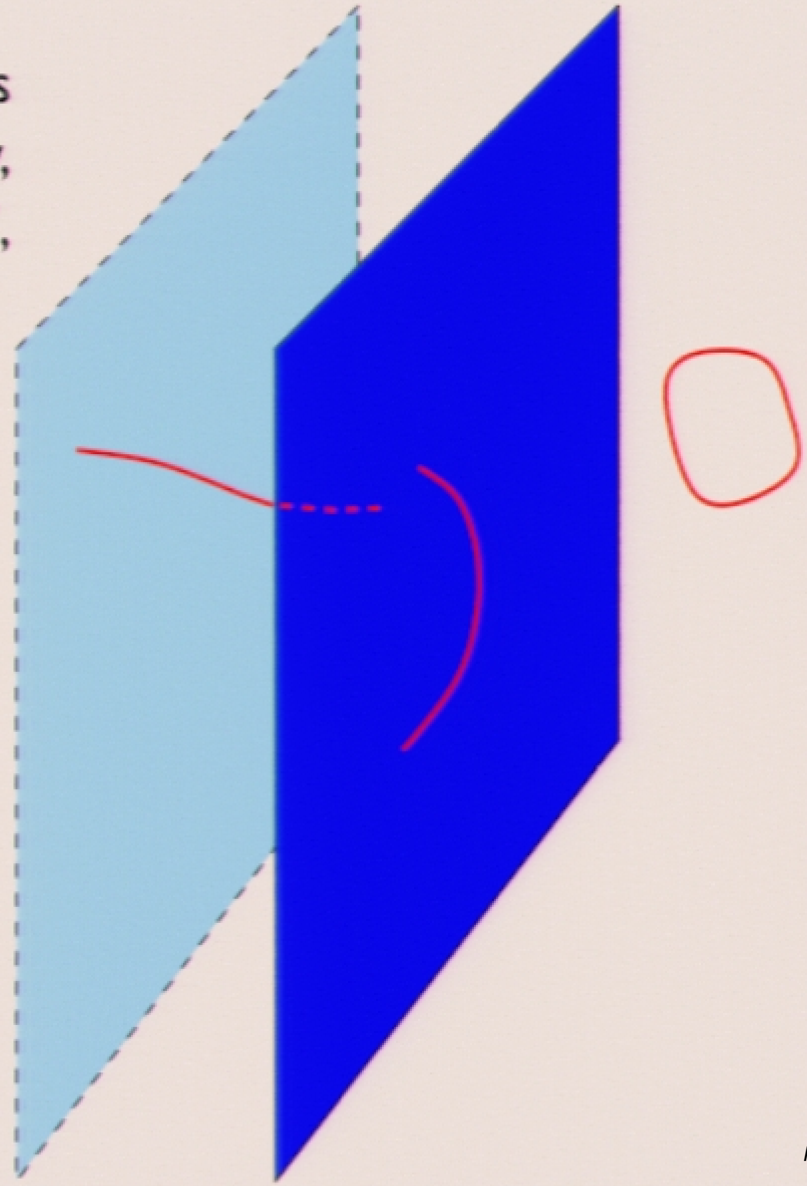
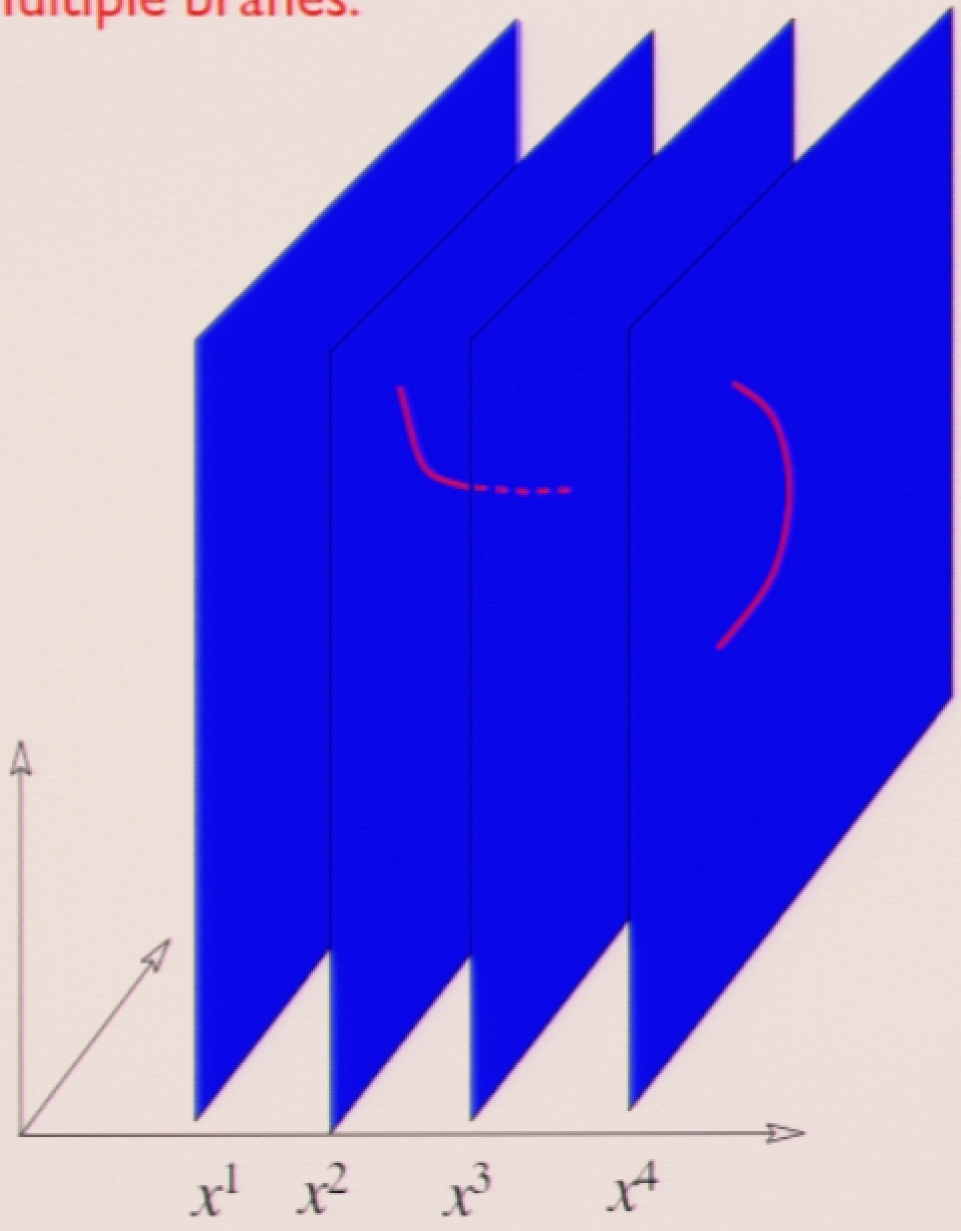
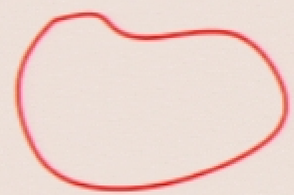


p extended directions: **Dp-brane**

Multiple branes:



$U(N)$ gauge theory
on world-volume instead



Multiple branes:

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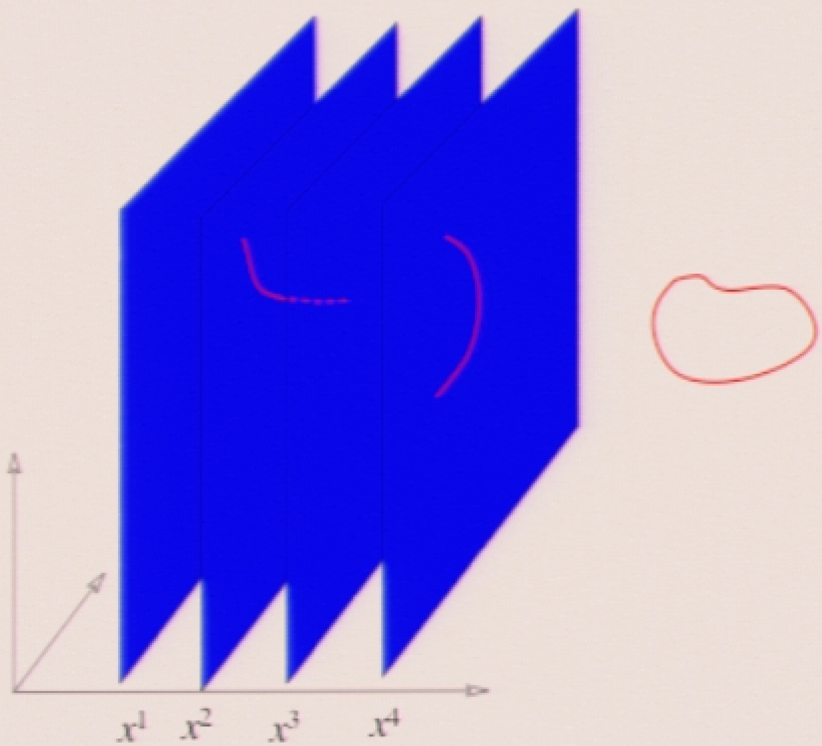
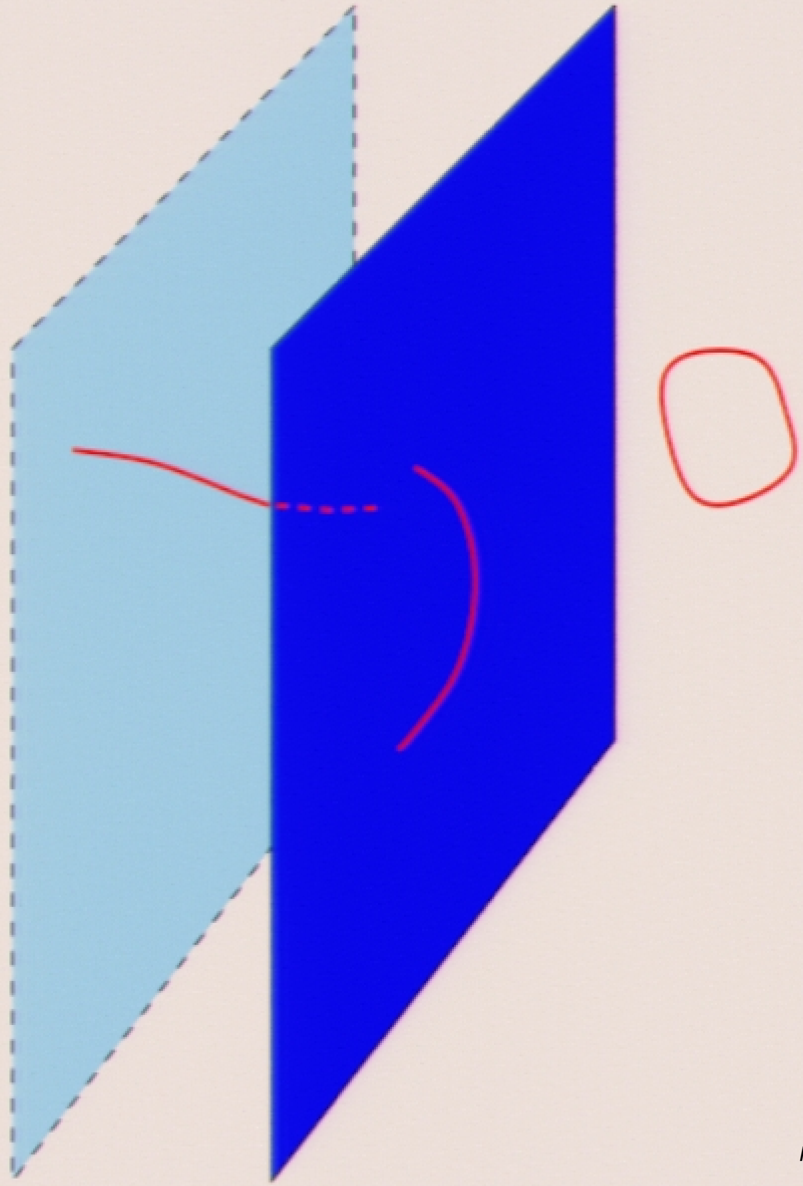


D-Branes

Open string degrees of freedom give a $U(1)$ gauge theory in the $(p+1)-$dimensions of its "worldvolume"

Vibrations of DN-strings and DD-strings appear as (charged) scalars in this world-volume theory.



Supersymmetric Strings

$d^2\sigma = d\tau\, d\sigma \qquad 0 \le \sigma \le \pi$ open $\qquad \sigma^1, \sigma^2 = \tau, \sigma$

Consider: $\qquad -\infty \le \tau \le \infty$

$(0 \le \sigma \le 2\pi$ closed$)$

$$S = \frac{1}{4\pi\alpha'} \int d^2\sigma \left\{ \partial X^\mu \bar{\partial} X_\mu + \alpha' \left(\psi^\mu \bar{\partial} \psi_\mu + \tilde{\psi}^\mu \partial \tilde{\psi}_\mu \right) \right\}$$

$$\partial = \frac{\partial}{\partial z} \qquad \bar{\partial} = \frac{\partial}{\partial \bar{z}}$$

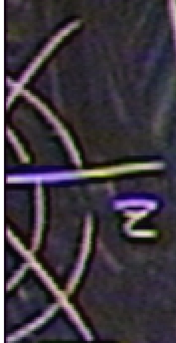
$z, \bar{z} \qquad z = e^{\tau - i\sigma}$ exponential map

We had:

$$X^\mu(z,\bar z) = X_L^\mu(z) + X_R^\mu(\bar z)$$

$$X_L^\mu(z) = \frac{1}{2}x^\mu - i\left(\frac{\alpha'}{2}\right)^{1/2}\alpha_0^\mu \ln z + i\left(\frac{\alpha'}{2}\right)^{1/2}\sum_{n\neq 0}\frac{1}{n}\alpha_n z^{-n}$$

$$X_R^\mu(\bar z) = \frac{1}{2}x^\mu - i(\quad)^{1/2}\tilde\alpha_0^\mu \ln \bar z + i\left(\frac{\alpha'}{2}\right)^{1/2}\sum \frac{1}{n}\tilde\alpha_n \bar z^{-n}$$



We had:

$$X^\mu(z,\bar z) = X_L^\mu(z) + X_R^\mu(\bar z)$$

x^μ is c.m position

p^μ is c.m. momentum

$$p^\mu = \frac{1}{\sqrt{2\alpha'}}\left(\alpha_0^\mu + \tilde\alpha_0^\mu\right)$$

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Stress tensor

$$T_B(z) = -\frac{1}{\alpha'}\partial X^\mu \partial X_\mu - \frac{1}{2}\psi^\mu \partial \psi_\mu$$

$$\bar T_B(\bar z) =$$

$$T_F(z) = i\frac{\sqrt{2}}{\alpha'}\psi^\mu \partial X_\mu$$

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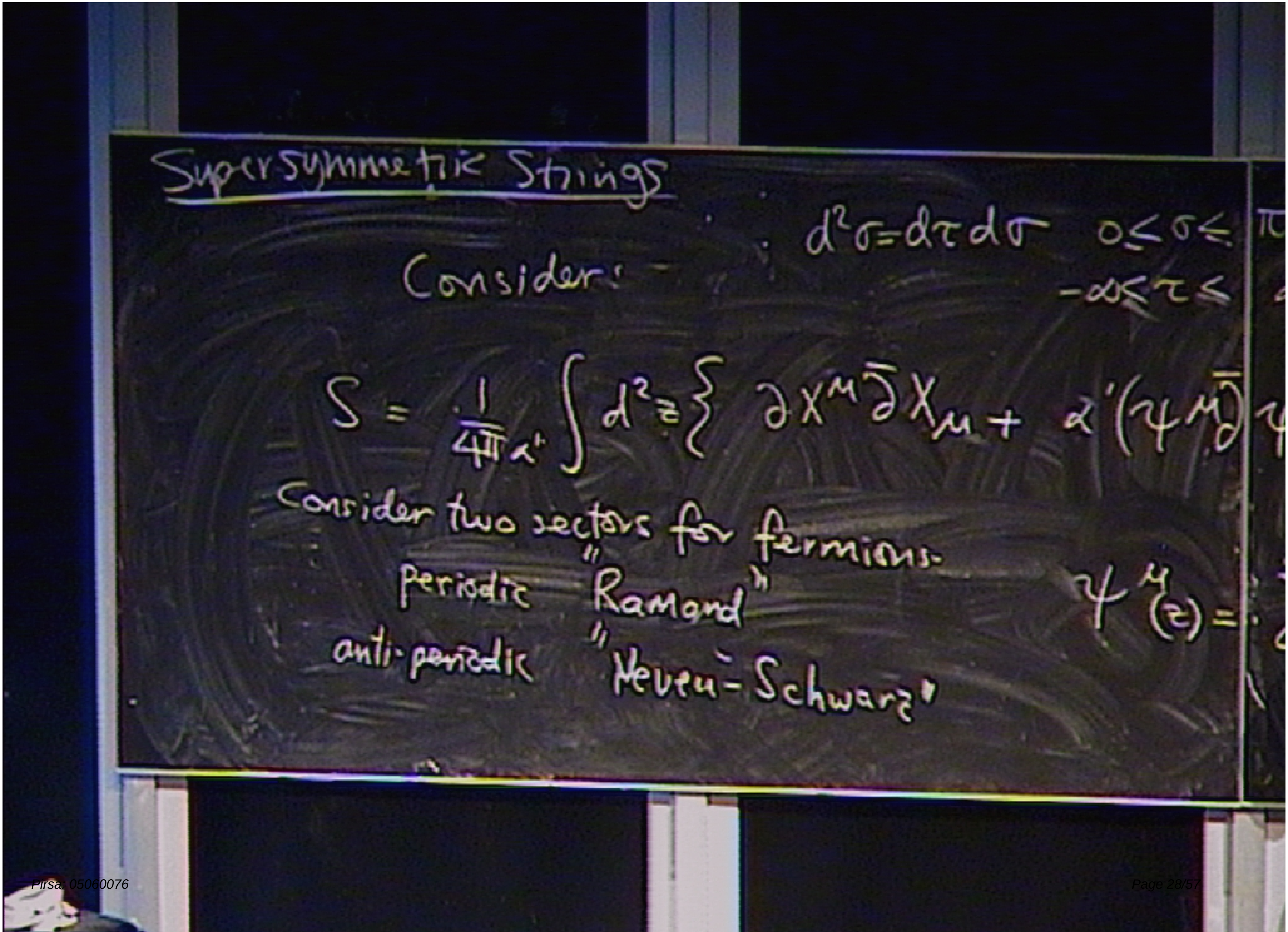
Supersymmetric Strings
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Consider:
$-\infty \le \tau \le$
$S = \frac{1}{4\pi\alpha'} \int d^2z \{ \partial X^\mu \bar\partial X_\mu + \alpha' (\psi^\mu \bar\partial$
Consider two sectors for fermions.
periodic "Ramond"
$\psi^\mu(z) =$
anti-periodic "Neveu-Schwarz"

$d\tau\, d\sigma \quad 0 \leq \sigma \leq \pi$ open

$\sigma^1, \sigma^2 = \tau, \sigma$

$-\infty \leq \tau \leq \infty$

$(0 \leq \sigma \leq 2\pi$ closed$)$

$X_\mu + \alpha'(\psi^\mu \bar{\partial} \psi_\mu + \tilde{\psi}^\mu \partial \tilde{\psi}_\mu)\}$

ns.

$$\psi^\mu(z) = \sum \frac{\psi_r^\mu}{z^{r+1/2}} \qquad \begin{array}{ll} r \in \mathbb{Z} & (R) \\ r \in \mathbb{Z}+\frac{1}{2} & (NS) \end{array}$$

z'

We' h

$X^\mu(z, \bar{z}$

$X_L^\mu($

X_R^μ

Stress

T

Doubling trick

have $0 < \sigma \leq \pi$

for open

Use a single

$\psi^\mu(\sigma, \tau)$ on $0 < \sigma \leq 2\pi$

and recover

$$\tilde{\psi}(\sigma, \tau) = \psi(2\pi - \sigma, \tau).$$

sed)

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Supersymmetric Strings

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Consider:

$$S = \frac{1}{4\pi\alpha'}\int d^2\sigma \left\{ \partial X^\mu \bar{\partial} X_\mu + \alpha'\left(\psi^\mu \bar{\partial}\psi_\mu + \tilde{\psi}^\mu \partial \tilde{\psi}_\mu\right)\right\}$$

Doubling trick
have $0 < \sigma < \pi$
for open
Use a single
$\psi^\mu(\sigma,\tau)$
and recover
$\tilde{\psi}(\sigma,\tau) = \psi(2\pi$

Consider two sectors for fermions:

periodic "Ramond"

anti-periodic "Neveu-Schwarz"

$$\psi^\mu(z) = \sum \frac{\psi_r^\mu}{z^{r+1/2}} \qquad r \in \mathbb{Z} \ \ (R) \qquad r \in \mathbb{Z}+\tfrac{1}{2} \ \ (NS)$$

$$\{\psi_r^\mu, \psi_s^\nu\} = \{\tilde\psi_r^\mu, \tilde\psi_s^\nu\} = \eta^{\mu\nu}\delta_{r+s}$$

$$+\tilde\alpha_0^\mu)$$

$$[\alpha_m^\mu, \alpha_n^\nu] = [\sim \sim] = m\,\delta_{m+n}\,\eta^{\mu\nu}$$

$$[x^\mu, p^\mu] = i\eta^{\mu\nu}$$

$$\partial X\, \partial X_\mu - \frac{1}{2}\psi^\mu \partial \psi_\mu.$$

$$T_F(z) = i\frac{2}{\alpha'}\,\psi^\mu \partial X_\mu$$

$$\bar T_F(\bar z) = i\frac{2}{\alpha'}\,\tilde\psi^\mu \bar\partial X_\mu$$

$$\{\psi_r^\mu, \psi_s^\nu\} = \{\tilde\psi_r^\mu, \tilde\psi_s^\nu\} = \eta^{\mu\nu}\delta_{r+s}$$

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$$[\alpha_m^\mu, \alpha_n^\nu] = [\tilde{\ }\ \tilde{\ }] = m\,\delta_{m+n}\,\eta^{\mu\nu}$$

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So

$$T_B(z) = \sum_{m=-\infty}^{+\infty} \frac{L_m}{z^{m+2}} \qquad T_F(z) = \sum_r \frac{G_r}{z^{r+3/2}}$$

$$\{\psi_r^\mu, \psi_s^\nu\} = \{\tilde\psi_r^\mu, \tilde\psi_s^\nu\} = \eta^{\mu\nu}\delta_{r+s}$$

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So

$$T_B(z) = \sum_{m=-\infty}^{+\infty} \frac{L_m}{z^{m+2}} \qquad T_F(z) = \sum_r \frac{G_r}{z^{r+3/2}}$$

$$[L_n, L_m] = (m-n)L_{m+n} + \frac{c}{12}(m^3-m)\,\delta_{m+n} \qquad \{L_m, G_r\} = \frac{1}{2}(m-2r)\,G_{m+r}$$

$$\{G_r, G_s\} = 2L_{r+s} + \frac{c}{12}(4r^2-1)\,\delta_{r+s}$$

$$R \to P$$

$\leq \sigma \leq$

$\leq \tau \leq$

$$L_m = \frac{1}{2}\sum :\alpha_{m-n}\cdot\alpha_n: + \frac{1}{4}\sum_r (2r-m) :\psi_{m-r}\cdot\psi_r: + a\,\delta_{m,0}$$

$$G_r = \sum :\alpha_n\cdot\psi_{r-n}:$$

$$L_m = \frac{1}{2}\sum :\alpha_{m-n}\alpha_n: + \frac{1}{4}\sum_r (2r-m) :\psi_{m-r}\psi_r: + a\delta_{m,0}$$

$$G_r = \sum :\alpha_n \cdot \psi_{r-n}:$$

$$\boxed{c = D + \frac{D}{2}}$$

a?

$$\boxed{D = 10}$$

Bosons

NS fermions

NS z.p.e.

a

$$8\left(-\frac{1}{24}\right) + 8\left(-\frac{1}{48}\right) = -\frac{1}{2}$$

R fermion

R z.p.e

$$8\left(-\frac{1}{24}\right) + 8\left(\frac{1}{24}\right) = 0$$

Formula	Boson	fermion
z.p.e.	$\frac{1}{2}\omega$	$-\frac{1}{2}\omega$

$$\omega = \frac{1}{24} - \frac{1}{8}(2\theta-1)^2$$

$\theta = 0$ integer modes

$\frac{1}{2}$ halfinteger

Supersymmetric Strings

physical states

$$|\phi\rangle$$

$$L_n |\phi\rangle = 0$$

$$G_r |\phi\rangle = 0$$

$$L_0 = 1$$

$$L_0 = \frac{1}{2}\sum :\alpha_{-n}\alpha_n: + \frac{1}{4}\sum_r (2r-0) :\psi_{-r}\psi_r: + a$$

$$G_r = \sum :\alpha_n \psi_{r-n}:$$

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$$\boxed{D = 10}$$

Bosons — NS fermions

NS z.p.e.

$$8\left(-\frac{1}{24}\right) + 8\left(-\frac{1}{48}\right) = -\frac{1}{2}$$

R z.p.e. — R fermion

$$8\left(-\frac{1}{24}\right) + 8\left(\frac{1}{24}\right) = 0$$

Supersymmetric Strings

physical states

$$|\phi\rangle \qquad L_0 \to$$

$$M^2 = \frac{1}{\alpha'}\left[\sum(\alpha_{-n}\cdot\alpha_n + r\,\psi_{-r}\,\psi_r) - \begin{matrix}\tfrac{1}{2}\\ 0\end{matrix}\right]$$

count the oscillators

$$L_n\,|\phi\rangle = 0$$

$$G_r\,|\phi\rangle = 0$$

$$L_0 = $$

Supersymmetric Strings

physical states

$|\phi\rangle$

$L_0 \to$

$L_n |\phi\rangle = 0$

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$L_0 = 1$

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count the oscillators

Massless states

$\psi_{-\frac{1}{2}}|\ \rangle \qquad M^2 = 0$

Basic thing is just have momentum $p^\mu = k^\mu$

$|0,k\rangle \qquad M^2 = -\frac{1}{2\alpha'}$ tachyon $T(X)$

$\psi^\mu_{-\frac{1}{2}}|0,k\rangle \qquad M^2 = 0 \qquad A^\mu(X)$ photon.

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choose that 0, 1 directions don't contribute

"GSO projection"

Define operation $(-1)^F$ ← of fermion #

Assign $(-1)^F = -1$ to the vacuum.

$\Rightarrow$ Project onto even fermion number.

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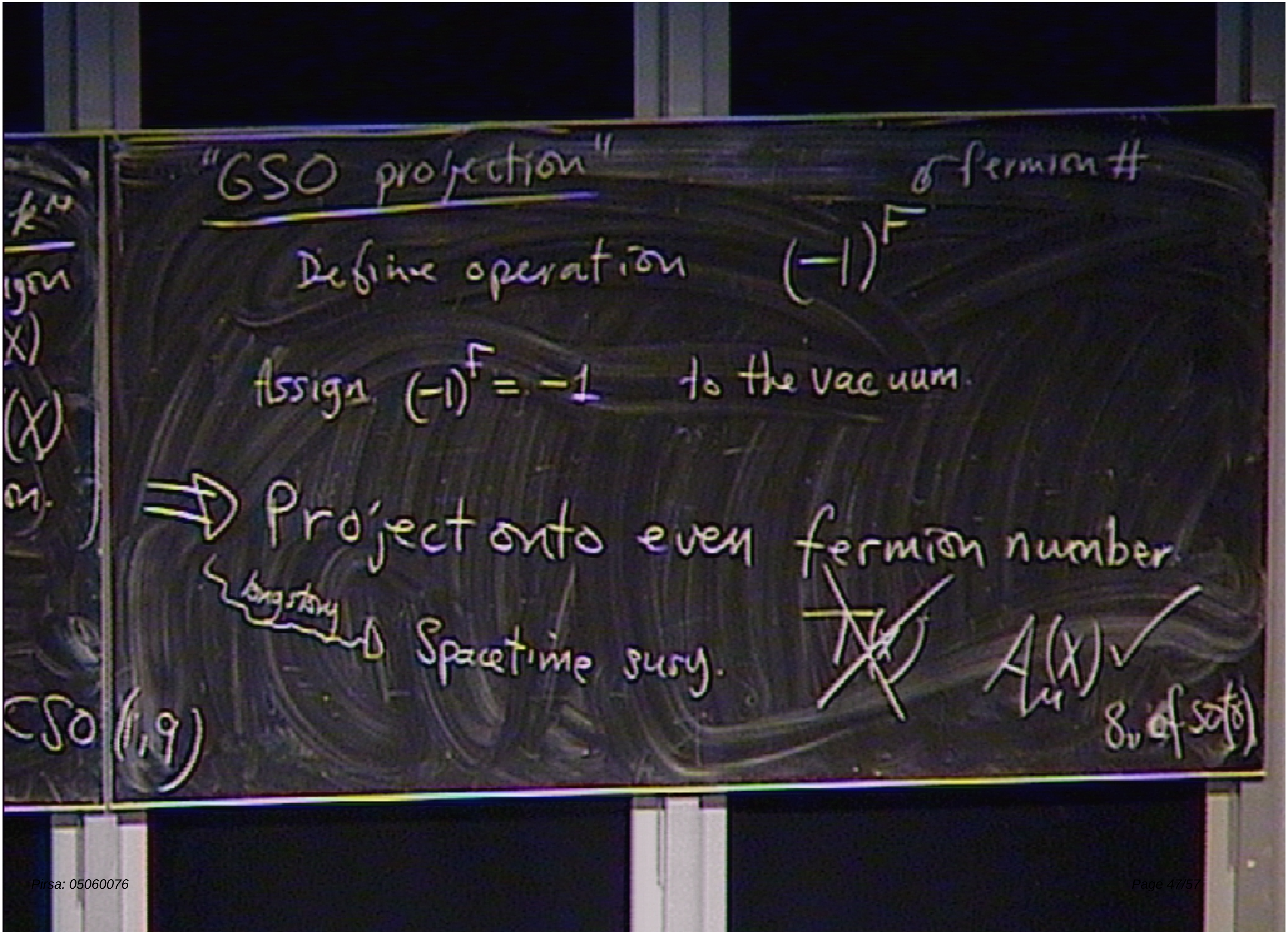
$\psi^\mu_{-\frac{1}{2}}|0,k\rangle \qquad M^2 = 0$ $A^\mu(X)$ photon

choose that 0, 1 directions don't contribute

Classify states in spacetime according to $SO(8) \subset SO$


"GSO projection"
σ fermion #
Define operation $(-1)^F$
Assign $(-1)^F = -1$ to the vacuum
⇒ Project onto even fermion number
long story ⇒ Spacetime susy.
$\cancel{T(x)}$
$A_\mu(x)$ ✓
8_v of $SO(8)$
$SO(1,9)$

Supersymmetric Strings

Notice $\{\psi_0^\mu, \psi_0^\nu\} = \eta^{\mu\nu}$

$$M^2 = \frac{1}{\alpha'}\left[\sum(\alpha_{-n}\cdot\alpha_n + r\psi_{-r}\psi_r) - \underset{0}{\tfrac{1}{2}}\right]$$

count the oscillators

if def $\psi_0^\mu = \frac{\Gamma^\mu}{\sqrt{2}}$

Then $\{\Gamma^\mu, \Gamma^\nu\} = 2\eta^{\mu\nu}$

So there are $2^{10/2} = 32$ states

Massless states

$\psi_0^\mu |\,\rangle \quad M^2 = 0$

$\alpha_{-n}\cdot\alpha_n + r\psi_{-r}\psi_r) - \frac{1}{2}$

$0]$

count the oscillators

ss states

$M^2 = 0$

Lorentz can be built from Γ^{μ}

$$[J_{\mu\nu}, J_{\rho\sigma}] = -i(\eta_{\mu\rho} J_{\nu\sigma} + \eta J - \eta J$$

$$J_{\mu\nu} = -J_{\nu\mu}$$

$$J^{\mu\nu} = -\frac{i}{4}[\Gamma^\mu, \Gamma^\nu]$$

Classify states in spacetime according to

Lorentz can be built from Γ^{μ}

$$[J_{\mu\nu}, J_{\rho\sigma}] = -i(\eta_{\mu\rho} J_{\nu\sigma} + \eta J - \eta J - \eta J)$$

$$J_{\mu\nu} = -J_{\nu\mu}$$

$$J^{\mu\nu} = -\frac{i}{4} [\Gamma^{\mu}, \Gamma^{\nu}]$$

So construct $S^{\mu\nu} = -\frac{i}{2} \sum_r [\psi^{\mu}_{-r}, \psi^{\nu}_{r}]$

Classify states in spacetime according to $SO(8) \subset SO($

Lorentz can be built from Γ^{μ}

$$[J_{\mu\nu}, J_{\rho\sigma}] = -i\left(\eta_{\mu\rho} J_{\nu\sigma} + \eta J - \eta J - \eta J\right)$$

$$J_{\mu\nu} = -J_{\nu\mu}$$

$$J^{\mu\nu} = -\frac{i}{4}[\Gamma^{\mu}, \Gamma^{\nu}]$$

So construct $S^{\mu\nu} = -\frac{i}{2}\sum_{r}[\psi^{\mu}_{-r}, \psi^{\nu}_{r}]$

Choose this nice basis:

$$d_i^{\pm} = \frac{1}{\sqrt{2}}\left(\psi_0^{2i} \pm i\psi_0^{2i+1}\right) \quad i = 1..4 \qquad d_0^{\pm} = \frac{1}{\sqrt{2}}\left(\psi_0^{1} \mp \psi_0^{0}\right)$$

d_i^+ are creation ops

d_i^- are annihilation

Clifford algebra:

$$\{d_i^+, d_j^-\} = \delta_{ij}$$

$$|s_0, s_1, s_2, s_3, s_4\rangle$$

$s_i = \pm\frac{1}{2}$

iS^{01} has eigenvalue s_0

S^{23} s_1

d_i^+ are creation ops
d_i^- are annihilation

Clifford algebra:

$$\{d_i^+, d_j^-\} = \delta_{ij}$$

$$|s_0, s_1, s_2, s_3, s_4\rangle$$

iS^{01} has eigenvalue s_0

S^{23} s_1

since $s_i = \pm\frac{1}{2}$

32 states

$$\boxed{d_i^- \left|-\tfrac{1}{2} -\tfrac{1}{2} -\tfrac{1}{2} -\tfrac{1}{2} -\tfrac{1}{2}\right\rangle = 0}$$

Supersymmetric Strings

Notice $\{\psi_0^\mu, \psi_0^\nu\} = \eta^{\mu\nu}$

$$M^2 = \frac{1}{\alpha'}\left[\sum(\alpha_{-n}\cdot\alpha_n + r\,\psi_{-r}\psi_r) - \begin{matrix}\frac{1}{2}\\ 0\end{matrix}\right]$$

if def $\psi_0^\mu = \frac{\Gamma^\mu}{\sqrt{2}}$

Then $\{\Gamma^\mu, \Gamma^\nu\} = 2\eta^{\mu\nu}$

So there are $2^{10/2} = 32$ states

$32 \to 16 + 16'$

even # $b + \frac{1}{2}$ — odd

physical state condition

$G_0|\,\rangle = 0$

$\alpha_0 \cdot \psi_0 \to p_\mu \psi_0^\mu$

Supersymmetric Strings

Notice $\{\psi_0^\mu, \psi_0^\nu\} = \eta^{\mu\nu}$

$$M^2 = \frac{1}{\alpha'}\left[\sum_0 (\alpha_{-n}\cdot\alpha_n + r\psi_{-r}\psi_r) - \frac{1}{2}\right]$$

expand G_0 some more.

$$G_0 = \alpha'^{1/2}(p_0\Gamma_0 + p_1\Gamma_1)$$

$$2\alpha'^{1/2} p_1\Gamma_0\left(\frac{1}{2} - S_0\right) = 0 \quad S_0 = \frac{1}{2}.$$

picking a basis $p^0 = p^1$

$32 \to 16 + 16'$

16: even # $S + \frac{1}{2}$

16': odd

$G_0|\ \rangle = 0$

8_c 8_s

GSO

$$\sum_{i=1}^{4} S_i = 0 \mod 2$$

$\boxed{8_s}$

Supersymmetric Strings

Notice $\{\psi_0^\mu, \psi_0^\nu\} = \eta^{\mu\nu}$

$M^2 = \frac{1}{\alpha'}\left[\sum(\alpha_{-n}\cdot\alpha_n + r\psi_{-r}\psi_r) - \frac{1}{2}, 0\right]$

expand G_0 some more.

$G_0 = \alpha'^{1/2}(p_0\Gamma_0 + p_1\Gamma_1)$

$\downarrow$

$2\alpha'^{1/2} p_1 \Gamma_0 \left(\frac{1}{2} - S_0\right)$

$= 0 \quad S_0 = \frac{1}{2}$

picking a basis $p^0 = p^1$

$32 \to 16 + 16'$

even # $8 + \frac{1}{2}$ ... odd

$\langle \det \rangle = 0$

GSO: 8_c ~~X~~, 8_s

$\sum_{i=1}^{4} S_i = 0 \mod 2$

R: $\boxed{8_s}$

Lore...

$[J$...

S_0

choos...

NS $\boxed{8_v}$

$\boxed{N=1 \quad D=10 \quad \text{Super YM}}$ $\Gamma^{\mu\nu}$

$x J - x J - x J$

$\boxed{8s \oplus 8v}$

$-\frac{i}{4} \quad [\Gamma^\mu, \Gamma^\nu]$

$= -\frac{i}{2} \sum_r [\psi^\mu_{-r}, \psi^\nu_r]$

NS

$\pm i \psi_0^{2i+1}) \quad i = 1\text{-}4 \quad d_0^\pm = \frac{1}{\sqrt{2}}(\psi_0^1 \mp \psi_0^0)$

$\boxed{8v}$