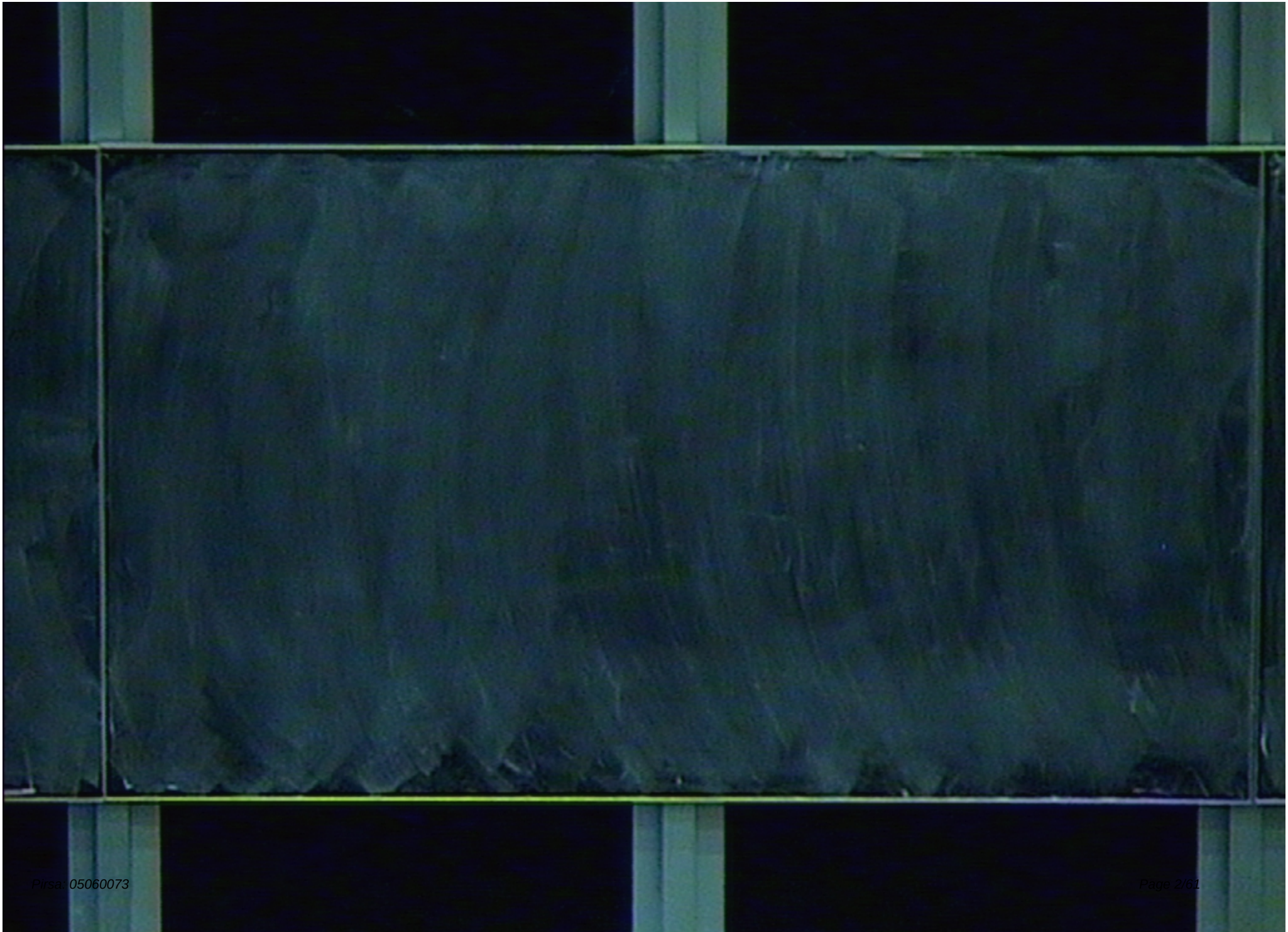
Title: Supersymmetry Basics

Date: Jun 21, 2005 02:30 AM

URL: http://pirsa.org/05060073

Abstract:



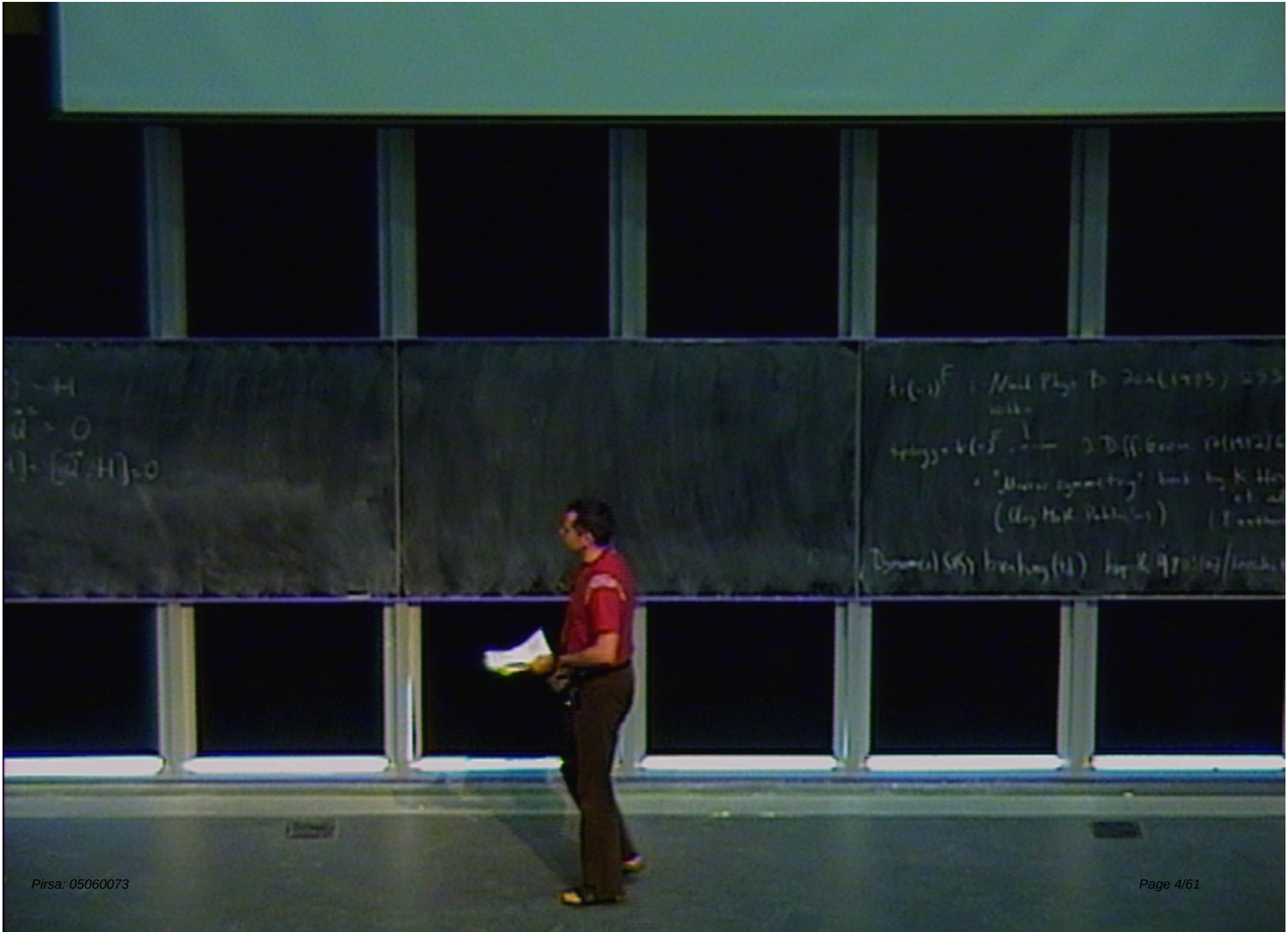
$tr(-1)^F$: Nucl Phys B 202(1983) 253

Witten

topology + $tr(-)^F \cdot \frac{1}{-\Delta}$ J. Diff. Geom. 17(1982) 661

+ "Mirror symmetry" book by K. Hori et al
(Clay Math. Publications) (8 authors)

(Dynamical SUSY breaking (4d) hep-th 9803107/Intrilligator, E.T.)



$$H = \frac{1}{2}p^2 + \frac{1}{2}\left(W'(x)\right)^2 + \frac{1}{2}\sigma_3 W''(x) \text{ acting on } \begin{pmatrix}\psi_1(x)\\ \psi_2(x)\end{pmatrix}$$

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$$\sigma_3 = \begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix} = \underbrace{\begin{pmatrix}0 & 1\\ 0 & 0\end{pmatrix}}_{\bar{\psi}}\underbrace{\begin{pmatrix}0\\ 1\end{pmatrix}}_{\psi} - \underbrace{\begin{pmatrix}0 & 0\\ 1 & 0\end{pmatrix}}_{\psi}\underbrace{\begin{pmatrix}0 & 1\\ 0 & 0\end{pmatrix}}_{\bar{\psi}}$$

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$\psi^2 = \bar{\psi}^2 = 0$, and also

$$\{\psi, \bar{\psi}\} = 1$$

$$H = \frac{1}{2} p^2 + \frac{1}{2}\left(W'(x)\right)^2 + \frac{1}{2} W''(x)\left(\bar{\psi}\psi - \psi\bar{\psi}\right)$$

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$$= \frac{1}{2}P^2 + \frac{1}{2}\left(W'(x)\right)^2 - \frac{1}{2}W''\left(\pi_\psi\psi - \psi\pi_\psi\right)$$

$$\dot{\psi} = \frac{\partial H}{\partial \pi_\psi} = -\tfrac{1}{2}W''(x)\psi \longrightarrow \mathcal{L}_\psi = i\bar{\psi}\dot{\psi} - \frac{1}{2}W''(x)\left(\bar{\psi}\psi - \psi\bar{\psi}\right)$$

$$\mathcal{L} = \frac{1}{2}\dot{x}^2 - \frac{1}{2}\left(W'(x)\right)^2 + \frac{i}{2}\left(\bar{\psi}\dot{\psi} - \dot{\bar{\psi}}\psi\right) - \frac{1}{2}W''(x)\left(\bar{\psi}\psi - \psi\bar{\psi}\right)$$

$x(t), \psi(t) \longrightarrow$ one B & fermionic field

$$\mathcal{L} = \frac{1}{2}\dot{x}^2 - \frac{1}{2}\left(W'(x)\right)^2 + \frac{i}{2}\left(\bar{\psi}\dot{\psi} - \dot{\bar{\psi}}\psi\right) - \frac{1}{2}W''(x)\left(\bar{\psi}\psi - \psi\bar{\psi}\right)$$

$x(t), \psi(t) \longrightarrow$ one B & fermionic field

$\{\psi, \bar{\psi}\} = \psi^2 = \bar{\psi}^2 = 0$

$(\psi\chi)^+ = \chi^+\psi^+$

$\frac{\partial}{\partial\psi}\psi\chi = \chi \qquad \frac{\partial}{\partial\chi}\psi\chi = -\psi$

$\{Q, \vec{Q}^+\} = H$

$Q^2 = \vec{Q}^{+2} = 0$

$[Q, H] = [\vec{Q}^+, H] = 0$

$\downarrow$

reduction of $N=1$ 3d algebra

$H = \frac{1}{2}P^2 + \frac{1}{2}(W'(x))$

$\sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} = \underbrace{\begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}}_{\bar{\psi}} \underbrace{\begin{pmatrix} 0 & \\ 1 & \end{pmatrix}}_{\psi}$

$\psi^2 = \bar{\psi}^2 = 0$, and also

$\{Q, \bar{Q}\} = -i 2 \frac{\partial}{\partial t}$

$Q^2 = \bar{Q}^2 = 0$

$+H] = 0$

3d

$$\{Q, \bar{Q}\} = -i 2 \frac{\partial}{\partial t}$$

$$Q^2 = \bar{Q}^2 = 0$$

Superspace $(t, \theta, \bar{\theta})$

$$\bar{\theta} = (\theta)^+$$

$$\theta^2 = \bar{\theta}^2 = 0$$

$$\{Q, \theta\} = 0$$

$\{Q, \bar{Q}\} = -i 2 \frac{\partial}{\partial t}$

$Q^2 = \bar{Q}^2 = 0$

Superspace $(t, \theta, \bar{\theta})$

$\bar{\theta} = (\theta)^\dagger$

$\theta^2 = \bar{\theta}^2 = 0$

$\{\theta, \theta\} = 0$

$H = -i \frac{\partial}{\partial t}$

$Q = \frac{\partial}{\partial \theta} + i \bar{\theta} \frac{\partial}{\partial t}$

$\bar{Q} = -\frac{\partial}{\partial \bar{\theta}} - i \theta \frac{\partial}{\partial t}$

$$H = -i\frac{\partial}{\partial t}$$

$$Q = \frac{\partial}{\partial \theta} + i\bar{\theta}\frac{\partial}{\partial t}$$

$$\bar{Q} = -\frac{\partial}{\partial \bar{\theta}} - i\theta\frac{\partial}{\partial t}$$

check

$$Q^2 = 0, \quad \bar{Q}^2 = 0$$

$$Q\bar{Q} = \left(\frac{\partial}{\partial\theta} + i\bar{\theta}\frac{\partial}{\partial t}\right)\left(-\frac{\partial}{\partial\bar{\theta}} - i\theta\frac{\partial}{\partial t}\right) =$$

$$= -\frac{\partial}{\partial\theta}\frac{\partial}{\partial\bar{\theta}} + i\theta\frac{\partial}{\partial\theta}\frac{\partial}{\partial t} - i\frac{\partial}{\partial t} + \bar{\theta}\theta\frac{\partial^2}{\partial t^2} + i\bar{\theta}\frac{\partial}{\partial\bar{\theta}}\frac{\partial}{\partial t}$$

$$Q\bar{Q} = -\frac{\partial}{\partial\theta}\frac{\partial}{\partial\bar{\theta}} - i\frac{\partial}{\partial t} + i\bar{\theta}\frac{\partial}{\partial\bar{\theta}}\frac{\partial}{\partial t} + \theta\bar{\theta}\frac{\partial^2}{\partial t^2} - i\theta\frac{\partial}{\partial\theta}\frac{\partial}{\partial t}$$

$H =$

Q

$\bar{Q}$

$$Q\bar{Q} = \left(\frac{\partial}{\partial\theta} + i\bar{\theta}\frac{\partial}{\partial t}\right)\left(-\frac{\partial}{\partial\bar{\theta}} - i\theta\frac{\partial}{\partial t}\right) =$$

$$= -\frac{\partial}{\partial\theta}\frac{\partial}{\partial\bar{\theta}} + i\theta\frac{\partial}{\partial\theta}\frac{\partial}{\partial t} - i\frac{\partial}{\partial t} + \bar{\theta}\cancel{\frac{\partial^2}{\partial t^2}} + i\bar{\theta}\frac{\partial}{\partial\theta}\frac{\partial}{\partial t}$$

$$Q\bar{Q} = -\frac{\partial}{\partial\bar{\theta}}\frac{\partial}{\partial\theta} - i\frac{\partial}{\partial t} + i\theta\cancel{\frac{\partial}{\partial\bar{\theta}}\frac{\partial}{\partial t}} + \theta\bar{\theta}\cancel{\frac{\partial^2}{\partial t^2}} - i\bar{\theta}\frac{\partial}{\partial\theta}\frac{\partial}{\partial t}$$

$H =$

Q

$\bar{Q}$

$$f(t,\theta,\bar{\theta}) = f_0(t) + \theta f_1(t) + \bar{\theta} f_2(t) + \theta\bar{\theta} f_3(t)$$

$$f(t,\theta,\bar{\theta}) = f_0(t) + \theta f_1(t) + \bar{\theta} f_2(t) + \theta\bar{\theta} f_3(t)$$

$f(t,\theta,\bar{\theta})$ — **superfield**

$f_0(t)$ — **lowest**

$f_3(t)$ — **highest**

$f(t,\theta,\bar\theta) = f_0(t) + \theta f_1(t) + \bar\theta f_2(t) + \theta\bar\theta f_3(t)$

$H = -i\frac{\partial}{\partial t}$

Superfield — lowest — highest

Real bosonic SF

$\Phi(t,\theta,\bar\theta) = \underset{\text{real}}{x(t)} + \theta\bar\psi(t) - \bar\theta\psi(t) + \bar\theta\theta \underset{\text{real}}{f(t)}$

$\Phi^+ = \Phi$

c.c.

SUSY transform of $\phi(t,\theta,\bar{\theta})$

$$\delta\phi(t,\theta,\bar{\theta}) = (\epsilon Q - \bar{\epsilon}\bar{Q}) \equiv \hat{\delta}_{\epsilon} + \hat{\delta}_{\bar{\epsilon}}$$

Grassman parameters of SUSY

ϵ $\bar{\epsilon} = (\epsilon)^{+}$

$$\bar{\theta}\psi(t) + \bar{\theta}\theta f(t)$$

C.C.

real

i

SUSY transform of $\phi(t,\theta,\bar\theta)$

$$\delta\phi(t,\theta,\bar\theta) = (\epsilon Q - \bar\epsilon \bar Q) \equiv \hat\delta_\epsilon + \hat\delta_{\bar\epsilon}$$

Grassman parameters of SUSY

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real

SUSY transform of $\phi(t,\theta,\bar\theta)$

$$\delta\phi(t,\theta,\bar\theta) = (\epsilon Q - \bar\epsilon \bar Q)\phi \equiv (\hat\delta_\epsilon + \hat\delta_{\bar\epsilon})\phi$$

$\hat\delta_\epsilon$ $\hat\delta_{\bar\epsilon}$

Grassman parameters of SUSY

$\bar\epsilon = (\epsilon)^+$

$\bar\theta\psi(t) + \bar\theta\theta f(t)$

real

C.C.

$$\delta\phi = (\epsilon Q - \bar{\epsilon}\bar{Q}).\phi =$$

$$= \delta x(t) + \theta\delta\bar{\psi}(t) - \bar{\theta}\delta\psi(t) + \theta\bar{\theta}\delta f(t)$$

USEFUL EXCERSIZE:

$$\delta_\epsilon x = \epsilon\bar{\psi}, \quad \delta_\epsilon\bar{\psi} = -i\epsilon\dot{x} + \epsilon f$$

$$\delta_\epsilon\psi = \not{\partial}, \quad \delta_\epsilon f = -i\epsilon\dot{\bar{\psi}}$$

+ c.c to get $\bar{\epsilon}$ transforms

if Φ is a SF, so is Φ^2, Φ^3, etc., in fact any $f(\Phi)$ is a superfield

$f(t)$

$+\epsilon$

if Φ is a SF, so is Φ^2, Φ^3, etc., in fact

any $f(\Phi)$ is a superfield

Φ_i – many fields

$$S = \int dt\, d\theta\, d\bar{\theta}\, \mathcal{F}\left(\Phi_i(t, \theta, \bar{\theta})\right)$$

$\mathcal{F}$ – arbitrary f-n

$$\delta S = \int dt\, d\theta\, d\bar{\theta} \left(\mathcal{F}(\Phi_i + \delta\Phi_i) - \mathcal{F}(\Phi_i)\right) = \int dt\, d\theta\, d\bar{\theta}\, (\epsilon Q - \bar{\epsilon}\bar{Q})$$

if Φ is a SF, so is Φ^2, Φ^3, ... in fact

any $f(\Phi)$ is a superfield

Φ_i – many fields

$$S = \int dt\, d\theta\, d\bar{\theta}\, \mathcal{F}\left(\Phi_i(t,\theta,\bar{\theta})\right)$$

$\mathcal{F}$ – arbitrary f-n

$$\delta S = \int dt\, d\theta\, d\bar{\theta}\left(\mathcal{F}(\Phi_i + \varepsilon\Phi_i) - \mathcal{F}(\Phi_i)\right) = \int dt\, d\theta\, d\bar{\theta}\left(\varepsilon Q - \bar{\varepsilon}\bar{Q}\right)\Phi_i \frac{\partial \mathcal{F}}{\partial \Phi_i} = \int dt\, d\theta\, d\bar{\theta}\left(\varepsilon a - \bar{\varepsilon}\bar{a}\right)\mathcal{F}(\Phi_i)$$

$+ \frac{1}{2}(\bar{\Psi}\Psi -$

B + fermion

$\Psi, \bar{\Psi}$

$$\mathcal{L} = \dots + \frac{i}{2}\left(\bar\psi\dot\psi - \dot{\bar\psi}\psi\right) - \frac{1}{2}W''(x)\bar\psi\psi - 4\bar\psi$$

2 B & fermionic field

$$\{\psi, \bar\psi\} = \psi^2 = \bar\psi^2 = 0$$

$$\epsilon\frac{\partial}{\partial\theta} + i\epsilon\bar\theta\frac{\partial}{\partial t} \qquad \int d\theta\,\theta = 1 \qquad \int d\theta = 0$$

$$\dots(\phi_i)\cdot\frac{\partial\mathcal{F}}{\partial\phi_i} = \int dt\,d\theta\,d\bar\theta\,(\epsilon Q - \bar\epsilon\bar Q)\,\mathcal{F}(\phi_i)$$

$$\int d\theta d\bar{\theta}\ \bar{\theta}\theta = 1$$

$$+ \frac{i}{2}\left(\bar{\psi}\dot{\psi} - \dot{\bar{\psi}}\psi\right) - \frac{1}{2}W''(x)(\bar{\psi}\psi$$

$$-\psi\bar{\psi})$$

z B & fermionic field

$$\{\psi, \bar{\psi}\} = \psi^2 = \bar{\psi}^2 = 0$$

$$\epsilon\frac{\partial}{\partial\theta} + i\epsilon\bar{\theta}\frac{\partial}{\partial t}$$

$$\int d\theta\,\theta = 1$$

$$\int d\theta = 0$$

$$\left(\phi_i\right)\cdot\frac{\partial F}{\partial\phi_i} = \int dt\, d\theta\, d\bar{\theta}\left(\epsilon Q - \bar{\epsilon}\bar{Q}\right)F\left(\phi_i\right)$$

$$S_W = \int dt\, d\theta\, d\bar{\theta}\; W(\phi(t,\theta,\bar{\theta}))$$

$$W(\phi) = W(x) + (\theta\bar{\psi} - \bar{\theta}\psi)W'(x) + \theta\bar{\theta}\left(f\,W'(x) + W''(x)\bar{\psi}\psi\right)$$

$$\int d\theta d\bar{\theta}\;\bar{\theta}\theta$$

$$\phi_i)\cdot\frac{\partial F}{\partial \phi_i} = \int d$$

$$S_W = \int dt\, d\theta\, d\bar{\theta}\; W(\phi(t,\theta,\bar{\theta}))$$

$$W(\phi) = W(x) + (\theta\bar{\psi} - \bar{\theta}\psi)W'(x) + \theta\bar{\theta}\left(fW'(x) + W''(x)\bar{\psi}\psi\right)$$

$$S_W = \int dt\left(-fW'(x) - W''(x)\bar{\psi}\psi\right)$$

$$\int d\theta d\bar{\theta}\; \bar{\theta}\theta$$

$$\phi_i)\cdot\frac{\partial F}{\partial \phi_i} = \int d$$

$[t] = -1$ $\left[\int dt\ \dot{x}^2\right] = 0$ $[x] = -\frac{1}{2}$

$[\psi] = 0$

$[\theta] = -\frac{1}{2}$

$+\frac{1}{2} - [d\theta] = \left[\frac{\partial}{\partial\theta}\right]$

$[t] = -1$ $\left[\int dt\ \dot{x}^2\right] = 0$ $[x] = -\frac{1}{2}$

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$\left[dt\, d\theta\, d\bar{\theta}\right] = 0$

$[\theta] = -\frac{1}{2}$

$[\phi] = [x] = -\frac{1}{2}$ $+\frac{1}{2} - [d\theta] = \left[\frac{\partial}{\partial\theta}\right]$

Superspace action must have dim 0.

$[t] = -1$ $\quad \left[\int dt\ \dot{x}^2\right] = 0 \quad [x] = -\frac{1}{2}$

$[dt\, d\theta\, d\bar{\theta}] = 0$ $\quad [\psi] = 0$

$[\theta] = -\frac{1}{2}$

$[\phi] = [x] = -\frac{1}{2}$ $\quad +\frac{1}{2} - [d\theta] = \left[\frac{\partial}{\partial\theta}\right]$

Superspace action must have dim 0.

$\frac{\partial}{\partial\theta}\phi \quad \frac{\partial}{\partial\bar{\theta}}\phi\,\bar{\psi}$

$[t] = -1$ $\left[\int dt\ \dot{x}^2\right] = 0$ $[x] = -\frac{1}{2}$

$[\psi] = 0$

$\left[dt\, d\theta\, d\bar{\theta}\right] = 0$

$[\theta] = -\frac{1}{2}$

$[\phi] = [x] = -\frac{1}{2}$: $+\frac{1}{2} - [d\theta] = \left[\frac{\partial}{\partial\theta}\right]$

Superspace action must have dim ϕ.

$\frac{\partial}{\partial\theta}\phi$ $\frac{\partial}{\partial\bar{\theta}}\phi$ — real ϕ has right dimensions to be

$S_W = \int dt\, d$

$W(\phi) \equiv W($

$S_W = \int dt\, (- f W$

a kinetic term.

$\bar{\psi}\psi)$

Problem: $\frac{\partial}{\partial\theta}\phi$ is NOT a

$\delta\left(\frac{\partial}{\partial\theta}\phi\right) \equiv \frac{\partial}{\partial\theta}(\delta\phi) = \frac{\partial}{\partial\theta}(\epsilon Q - \bar{\epsilon}\bar{Q}$

because $\left\{\frac{\partial}{\partial\theta}, Q\right\} \neq 0$ $\neq (\epsilon Q - \bar{\epsilon}\bar{Q})\frac{\partial}{\partial\theta}\phi$

Goal: $\frac{\partial}{\partial\theta} \to D$

$\frac{\partial}{\partial\bar{\theta}} \to \bar{D}$

such that $\{D, Q\} = \{\bar{D}, \bar{Q}\} = 0$

Goal: $\frac{\partial}{\partial\theta} \to D$ $\quad \{D, \bar{D}\} = 2i\frac{\partial}{\partial t}$; $D^2 = \bar{D}^2 = 0$

$\frac{\partial}{\partial\bar{\theta}} \to \bar{D}$ $\quad$ such that $\{D, Q\} = \{D, \bar{Q}\} = 0$

+ c.c.

by demanding $\{\frac{\partial}{\partial\theta} + \ldots, \bar{Q}\} = 0$ we arrive at

$$D = \frac{\partial}{\partial\theta} - i\bar{\theta}\frac{\partial}{\partial t} \qquad \bar{Q} = -\frac{\partial}{\partial\bar{\theta}} - i\theta\frac{\partial}{\partial t}$$

$$\bar{D} = -\frac{\partial}{\partial\bar{\theta}} + i\theta\frac{\partial}{\partial t} \qquad Q = -\frac{\partial}{\partial\theta} + i\bar{\theta}\frac{\partial}{\partial t}$$

$$S_{kin} = (\#) \int dt\, d\theta\, d\bar{\theta}\, \bar{D}\phi\, D\phi$$

- supersymmetric
- real
- no dimensionful parameters

$$D\phi = \bar{\psi} + \bar{\theta}(f - i\dot{x}) - i\bar{\theta}\theta\,\dot{\bar{\psi}}$$

$$\bar{D}\phi = \psi + \theta(f + i\dot{x}) + i\bar{\theta}\theta\,\dot{\psi}$$

$$S_{kin} = (\#) \int dt\, d\theta\, d\bar{\theta}\; \bar{D}\phi\, D\phi \;\Big|_{\text{bosonic part}} = (\#)\left(f^2 + \dot{x}^2\right)$$

- supersymmetric
- real
- no dimensionful parameters

$$D\phi = \bar{\psi} + \bar{\theta}(f - i\dot{x}) - i\bar{\theta}\theta\,\dot{\bar{\psi}}$$

$$\bar{D}\phi = \psi + \theta(f + i\dot{x}) + i\bar{\theta}\theta\,\dot{\psi}$$

$$\mathcal{L} = \frac{1}{2}\dot{x}^2 = \frac{1}{2}(W'(x))^2 + \frac{i}{2}\left(\bar{\psi}\dot{\psi} - \dot{\bar{\psi}}\psi\right) - \frac{1}{2}W''(x)(\bar{\psi}\psi - 4\bar{\psi})$$

$$S = \int dt\, d\theta\, d\bar{\theta}\left(\frac{1}{2}D\phi\bar{D}\phi + W(\phi)\right)$$

$$S_{\text{kin+W}}$$

$$\mathcal{L} = \frac{1}{2}\dot{x}^2 = \frac{1}{2}(W'(x))^2 + \frac{i}{2}\left(\bar{\psi}\dot{\psi} - \dot{\bar{\psi}}\psi\right) - \frac{1}{2}W''(x)(\bar{\psi}\psi - \psi\bar{\psi})$$

$$S = \int dt\, d\theta\, d\bar{\theta}\left(\frac{1}{2}D\phi\bar{D}\phi + W(\phi)\right)$$
$$S_{\text{kin+W}}$$

$$= \int dt\left(\frac{1}{2}\dot{x}^2 + \frac{1}{2}f^2 - f\,W'(x) - W''\bar{\psi}\psi + i\bar{\psi}\dot{\psi}\right)$$

f has no dynamics in the action $\rightarrow$ auxiliary

$$\mathcal{L} = \frac{1}{2}\dot{x}^2 = \frac{1}{2}(W'(x))^2 + \frac{i}{2}\left(\bar{\psi}\dot{\psi} - \dot{\bar{\psi}}\psi\right) - \frac{1}{2}W''(x)(\bar{\psi}\psi - 4\bar{\psi})$$

$$S = \int dt\, d\theta\, d\bar{\theta}\left(\frac{1}{2}D\phi\bar{D}\phi + W(\phi)\right)$$

$$= \int dt\left(\frac{1}{2}\dot{x}^2 + \frac{1}{2}f^2 - f\,W'(x) - W''\bar{\psi}\psi + i\bar{\psi}\dot{\psi}\right)$$

$\mathcal{D}x\,\mathcal{D}f\,\mathcal{D}\psi\,\mathcal{D}\bar{\psi}$

f has no dynamics in the action $\rightarrow$ auxiliary

$f = W'$, so $\frac{1}{2}f^2 = \int W'' = -\frac{1}{2}(W')^2$

so get action you want

SF formalism allows it to write most general susy action with a given SF content

$f = W'$, so $\frac{1}{2} f^2 = -f W' = -\frac{1}{2}(W')^2$

so get action you want

SF formalism allows us to write most general susy action with a given SF content

$$S = \int dt\, d\theta\, d\bar{\theta} \left(\frac{1}{2} g_{ij}(\phi) D\phi^i \bar{D}\phi^j + W(\phi) \right)$$

$$S = \int dt\, d\theta\, d\bar{\theta} \left(\frac{1}{2} g_{ij}(\phi) D\phi^i \bar{D}\phi^j + W(\phi) \right)$$

going to $t \to -it$, slight modifications

$$\int dt\, dx\, d\theta\, d\bar{\theta} \left(D\phi \bar{D}\phi\, g(\phi) + W(\phi) \right)$$

- $(1,1)$ supersymmetry

$\downarrow$

3d, two (N=1 in 3d).

$V(\phi)$

$V(\phi)$

$\mathcal{N}=1$ SUSY in 4d

* simplest (smallest amount of SUSY)

* in 4d, $\mathcal{N}=1$ theories are the only SUSY ones that allow chiral fermions

$\mathcal{N}=1$ SUSY in 4d

* simplest (smallest amount of SUSY)
* in 4d, $\mathcal{N}=1$ theories are the only SUSY ones that allow chiral fermions
* can dynamically break supersymmetry & can generate small scales (hierarchy problem)
* lots of interesting dynamics — confinement, χSB, ...

Seiberg duality, ...!

$N=1$ algebra generated by Q_α, $\bar{Q}_{\dot\alpha}$:

$$(\bar{Q}_{\dot\alpha})^\dagger = (Q_\alpha)$$

$N=1$ algebra generated by Q_α, $\bar{Q}_{\dot\alpha}$:

$$\{Q_\alpha, \bar{Q}_{\dot\alpha}\} = 2i\,\sigma^m_{\alpha\dot\alpha} P_m \qquad (\bar{Q}_{\dot\alpha} = (Q_\alpha)^+)$$

$m = 0,1,2,3$ — space time vector index; $\alpha, \dot\alpha$ — $SL(2,\mathbb{C})_L$ or R indices

Seiberg duality [illegible]

$W(\phi)$

Follow similar path:

$$(x^m, \theta_\alpha, \bar{\theta}_{\dot\alpha}) \;;\quad \bar{\theta}_{\dot\alpha} = (\theta_\alpha)^\dagger$$

$$Q_\alpha = \frac{\partial}{\partial \theta^\alpha} - i\sigma^m_{\alpha\dot\alpha}\bar{\theta}^{\dot\alpha}\partial_m$$

$W(\phi)$

$$\bar{Q}_{\dot\alpha} = -\frac{\partial}{\partial \bar{\theta}^{\dot\alpha}} + i\theta^\alpha \sigma^m_{\alpha\dot\alpha}\partial_m$$

Follow similar path:

$$(x^m, \theta_\alpha, \bar{\theta}_{\dot\alpha}) \;;\quad \bar{\theta}_{\dot\alpha} = (\theta_\alpha)^\dagger$$

$$Q_\alpha = \frac{\partial}{\partial\theta^\alpha} - i\sigma^m_{\alpha\dot\alpha}\bar{\theta}^{\dot\alpha}\partial_m$$

$$\bar{Q}_{\dot\alpha} = -\frac{\partial}{\partial\bar{\theta}^{\dot\alpha}} + i\theta^\alpha\sigma^m_{\alpha\dot\alpha}\partial_m$$

$N=1$ in 4d

↓ dim reduce to 3d, $\partial_2 = 0$

$N=2$ in 3d

↓ reduce to 2d, $\partial_2 = \partial_1 = 0$

(2,2) susy in 2d

Simplest rep. of $N=1$, 4d SUSY:

$\theta^\alpha\theta^\beta \sim \epsilon^{\alpha\beta}\,\theta^1\theta_2$ — 1 $\mathbb{C}$ scalar + Weyl fermion

or

— Weyl fermion + vector (massless)

$$F = f(x) + \theta^\alpha \varphi_\alpha(x) + \bar\theta_\dot\alpha \bar\chi^\dot\alpha + \theta^\alpha\theta_\alpha\, m(x) + \bar\theta_\dot\alpha\bar\theta^\dot\alpha\, n(x) + \theta^\alpha\sigma^m_{\alpha\dot\alpha}\bar\theta^\dot\alpha v_m$$
$$+ \theta^\alpha\theta_\alpha\bar\theta_\dot\alpha\bar\lambda^\dot\alpha + \bar\theta_\alpha\bar\theta^\dot\alpha\,\theta^\beta\psi_{\beta\dot\alpha} + \theta^\alpha\theta_\alpha\,\bar\theta_\dot\beta\bar\theta^\dot\beta\, d(x)$$

→ too many components!

Simplest rep. of $N=1$, 4d SUSY:

$\theta^\alpha\theta^\beta \sim \epsilon^{\alpha\beta}\theta^\gamma\theta_\gamma$ — 1 ℂ scalar + Weyl fermion

or

— Weyl fermion + vector (massless)

$$F = f(x) + \theta^\alpha \varphi_\alpha(x) + \bar\theta_{\dot\alpha}\bar\lambda^{\dot\alpha} + \theta^\alpha\theta_\alpha m(\lambda) + \bar\theta_{\dot\alpha}\bar\theta^{\dot\alpha} n(x) + \theta^\alpha\sigma^m_{\alpha\dot\alpha}\bar\theta^{\dot\alpha} v_m$$
$$+ \theta^\alpha\theta_\alpha\bar\theta_{\dot\alpha}\bar\lambda^{\dot\alpha} + \bar\theta_{\dot\alpha}\bar\theta^{\dot\alpha}\theta^\beta\psi_\beta + \theta^\alpha\theta_\alpha\bar\theta_{\dot\beta}\bar\theta^{\dot\beta} d(x)$$

→ too many components! need to impose constraints ⇒ (a) SUSY ↺ (b) not involved

Follow similar path:

$(x^m, \theta_\alpha, \bar{\theta}_{\dot\alpha}) \;;\; \bar{\theta}_{\dot\alpha} = (\theta_\alpha)^\dagger$

$\{D_\alpha, \bar{\theta}_{\dot\beta}\} = \{D_\alpha, Q_\beta\} = 0$ + ...

$Q_\alpha = \frac{\partial}{\partial\theta^\alpha} - i\sigma^m_{\alpha\dot\alpha}\bar{\theta}^{\dot\alpha}\partial_m \qquad D_\alpha = \frac{\partial}{\partial\theta^\alpha} + i\sigma^m_{\alpha\dot\alpha}\bar{\theta}^{\dot\alpha}\partial_m$

$\bar{Q}_{\dot\alpha} = -\frac{\partial}{\partial\bar{\theta}^{\dot\alpha}} + i\theta^\alpha\sigma^m_{\alpha\dot\alpha}\partial_m \qquad \bar{D}_{\dot\alpha} = -\frac{\partial}{\partial\bar{\theta}^{\dot\alpha}} - i\theta^\alpha\sigma^m_{\alpha\dot\alpha}\partial_m$

Diff'l eqn's in x^m

Follow similar path:

$(x^m, \theta_\alpha, \bar{\theta}_{\dot\alpha}) \;\to\; \bar{\theta}_{\dot\alpha} \to (\theta_\alpha)^\dagger$

$\{D_\alpha, \bar{\theta}_{\dot\beta}\} = \{D_\alpha, Q_\beta\} = 0$ trivial

$\{D_\alpha, D_\beta\} = 0$ trivial, $\{D_\alpha, \bar{D}_{\dot\alpha}\} = -2i\sigma^m_{\alpha\dot\alpha}\partial_m$

$$Q_\alpha = \frac{\partial}{\partial\theta^\alpha} - i\sigma^m_{\alpha\dot\alpha}\bar{\theta}^{\dot\alpha}\partial_m \qquad D_\alpha = \frac{\partial}{\partial\theta^\alpha} + i\sigma^m_{\alpha\dot\alpha}\bar{\theta}^{\dot\alpha}\partial_m$$

$$\bar{Q}_{\dot\alpha} = -\frac{\partial}{\partial\bar{\theta}^{\dot\alpha}} + i\theta^\alpha\sigma^m_{\alpha\dot\alpha}\partial_m \qquad \bar{D}_{\dot\alpha} = -\frac{\partial}{\partial\bar{\theta}^{\dot\alpha}} - i\theta^\alpha\sigma^m_{\alpha\dot\alpha}\partial_m$$

SUSY (cov't derivatives).

diff'l eqns in x^m

chiral superfields $\Phi(x,\theta,\bar{\theta})$

- bosonic
- complex

$$\bar{D}_{\dot{\alpha}} \Phi(x,\theta,\bar{\theta}) = 0$$

chiral superfields $\Phi(x,\theta,\bar{\theta})$

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$$\equiv \bar{D}_{\dot{\alpha}} \Phi(x,\theta,\bar{\theta}) = 0$$

$\{D_\alpha, D_\dot{\beta}\} = -2i\sigma^m_{\alpha\dot{\beta}}\partial_m$

chiral superfields

- bosonic
- complex

SUSY cov't derivatives

c.f. – antichiral $= \bar{D}_{\dot{\alpha}}\phi(x,\theta,\bar{\theta}) = 0$

$\phi^\dagger$ obeys $D_\alpha \phi^\dagger = 0$