Title: Twistor String Theory and Perturbative Yang-Mills 1

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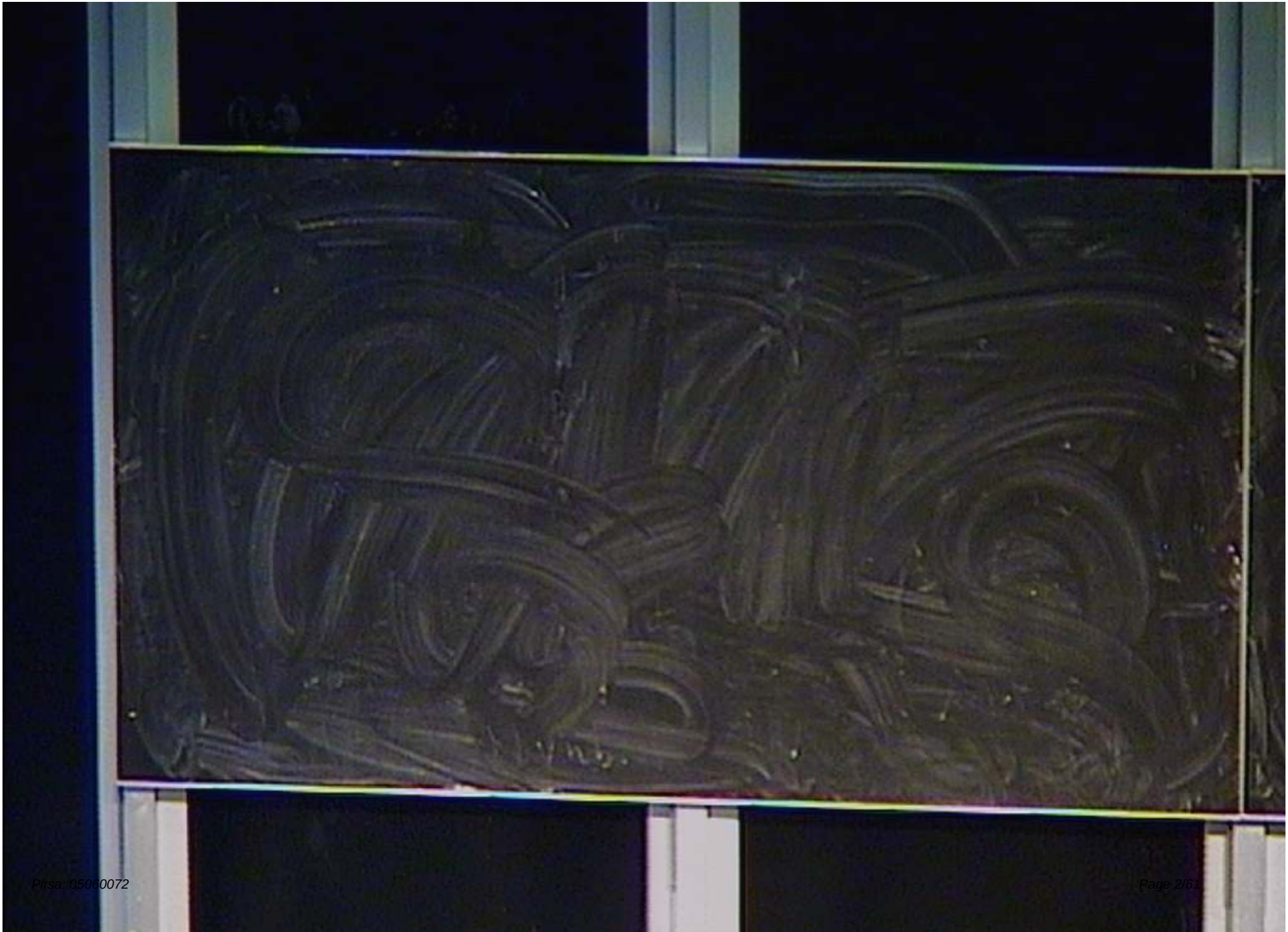
Abstract:



Twistor String Theory & Perturbative YM

① Perturb. QCD.

LHC

$\alpha_s(M_t^2) \sim \frac{1}{10}$

+Hope
New Physics.
Background.
QCD.

Twistor String Theory & Perturbative YM

① Perturb. QCD. LHC

Feynman diag.

New Techniques

$\alpha_s(M_t^2) \sim \frac{1}{10}$

$n = 6 \quad 10^5$

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New Physics.
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② 't Hooft 1974

Twistor String Theory & Perturbative YM

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LHC

$\alpha_s(M_Z^2) \sim \frac{1}{10}$

Hope
New Physics.
Background.
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Feynman diag.
New Techniques

$n = 6$ $\quad 10^5$

(2) 't Hooft 1974

Exp. $\frac{1}{N}$

Fey Diag.

$\lambda_t = g_{YM}^2 N$

Large N

Fixed λ_t

Aura $\quad U(N) \quad N$

$\sum_{g=0}^{\infty} N^{2-2g} f_g(\lambda)$ g: genus $\longrightarrow$ String Theory

Took 23 yrs $\longrightarrow$ 1997

Maldecena $\longrightarrow$ AdS/CFT

$\downarrow$

$\lambda \gg 1$

Pertub. Theory is out of reach.

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(5) Twistor Space Penrose (1967)

Cplx analys. $\mathbb{C}$ 2D. $\partial\bar{\partial} f = 0$

$$f(z,\bar{z}) = g(z) + h(\bar{z})$$

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4D? (x_1, x_2, x_3, x_4)

$\mathbb{C}^2$

$z = x_1 + i x_2$

$z' = x_3 + i x_4$

$|x|^2 = z\bar{z} + z'\bar{z}'$ $\quad O(4)/U(2) \cong S^2 = \mathbb{CP}^1$

Each point S^4, a choice $\mathbb{CP}^1$

$\mathbb{CP}^1$ bundle over $S^4 \cong \mathbb{CP}^3$ = Twistor Space.

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$Z^I = (\underbrace{\lambda_1, \lambda_2}_{\mathbb{CP}^1}, \mu^1, \mu^2)$ $\quad X_{\alpha\dot{\alpha}} = x^\mu (\sigma_\mu)_{\alpha\dot{\alpha}}$ $\quad \sigma^\mu = (\mathbb{1}, \vec{\sigma})$

$$\boxed{(\mu_\alpha)_{\dot{a}} = (X_{\alpha\dot{a}} \lambda^\alpha)}$$

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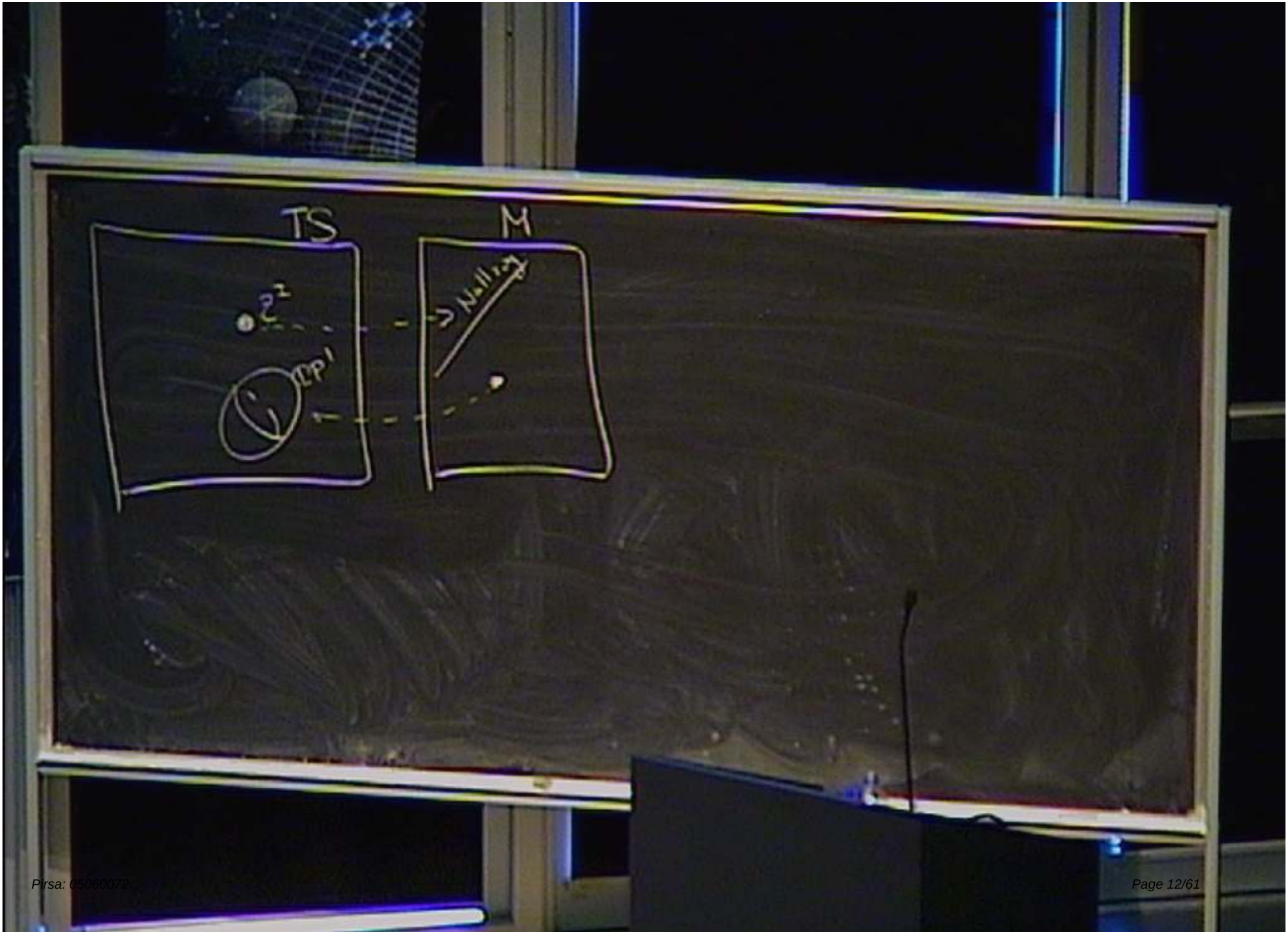
(a) Fix $Z^I \rightarrow$ Null ray in Minkowski Space

(b) Fix $x_{\alpha\dot{a}}$ $\Rightarrow$ $\mathbb{CP}^1 \subset \mathbb{CP}^3$


TS
M
z^2
LP1
Null

④ Topological String theory. (Ooguri)

- 10 Dim, 26 Dim. Infinite # of fields.
- Baby versions. A-, B-model.
  B-model. ① Target space. Super $\mathbb{CP}^3$
  Finite # of fields. → Mink.
  Spectr $\mathcal{N}=4$ SYM

Goal:

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④ Topological String theory. (Ooguri)

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Goal:

① Twistor String Theory.
B-model $\mathbb{CP}^{3|4}$ → pMSYM
+ N D5 branes +
D1 Instantons

(2) MHV Diagrams
Connected in
BCFW Recursion R } Tree-level

Quadruple cuts } One-loop
ABDK conjecture } 2- & higher-loop.

Twistor Space
Cplx analys. $\mathbb{C}$

Penrose (1967)
2D. $\partial\bar{\partial} f = 0$

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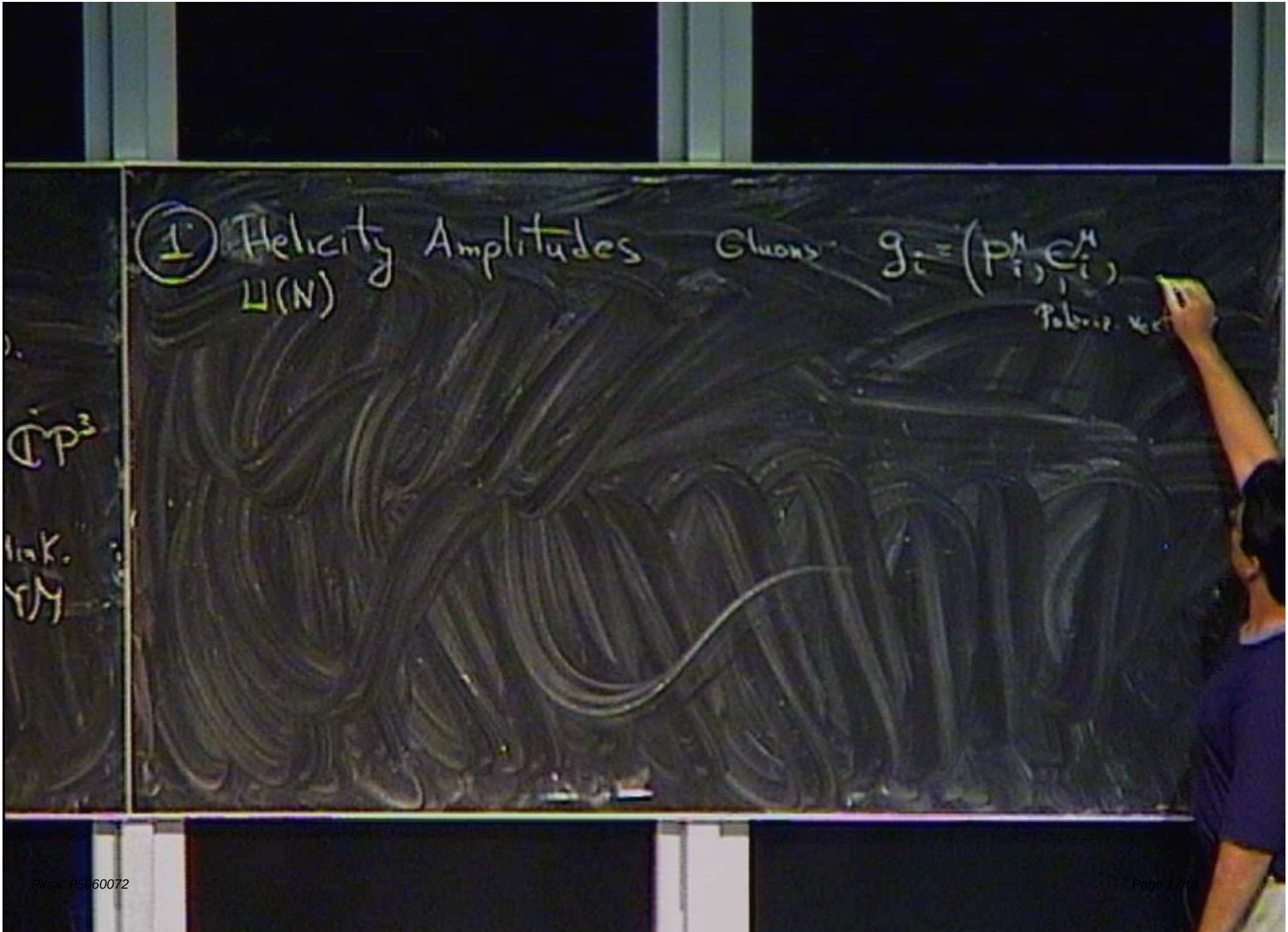
1 Helicity Amplitudes Gluons
U(N)

(1) Helicity Amplitudes Gluons $g_i = (p_i^\mu, \epsilon_i^\mu, a_i)$ Adjoint.
$U(N)$
Polarization vector

$\mathbb{CP}^3$

link.

(1) Helicity Amplitudes

$U(N)$

Gluons: $g_i = (p_i^\mu, \epsilon_i^\mu, a_i)$ Adjoint. Polarization vector

Spinor Helicity Formalism:

$SO(3,1) \xrightarrow{\mathbb{C}} SL_L(2,\mathbb{C}) \times SL_R(2,\mathbb{C})$

(p, q)

Finite dim. Rep.

λ_a $(\frac{1}{2}, 0)$ — Chiral Spinors

$\tilde{\lambda}_{\dot{a}}$ $(0, \frac{1}{2})$ + chirality

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$SL(2,\mathbb{C})$ invariant tensor ϵ_{ab}, $\epsilon_{\dot{a}\dot{b}}$

Lorentz invariant $\langle \lambda, \lambda' \rangle = \epsilon_{ab} \lambda^a \lambda'^b$

$[\tilde{\lambda}, \tilde{\lambda}'] = \epsilon_{\dot{a}\dot{b}} \tilde{\lambda}^{\dot{a}} \tilde{\lambda}'^{\dot{b}}$

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$\langle 1,2 \rangle$

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$[\tilde{\lambda}', \lambda]$

① Helicity Amplitudes Gluons $g_i = (p_i^\mu, \epsilon_i^\mu, a_i)$ Adjoint

$U(N)$ Polariz. vector

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$$SO(3,1) \xrightarrow[\mathbb{C}]{} SL_L(2,\mathbb{C}) \times SL_R(2,\mathbb{C})$$

Finite dim. Rep. (p, q) λ^a $(\frac{1}{2}, 0)$ — chirality Spinors

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$\langle 1, 2 \rangle \to \langle 1\, 2 \rangle$

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Adjoint

i)

ctor

Chirality

Spinors

chirality

$= 1, 2$

$$\langle \lambda, \lambda' \rangle = -\langle \lambda', \lambda \rangle$$

Vectors $\left(\frac{1}{2}, \frac{1}{2}\right)$ p^μ Bispinor

$$p_{a\dot{a}} = p^\mu (\sigma_\mu)_{a\dot{a}} = \lambda_a \tilde{\lambda}_{\dot{a}} + \lambda'_a \tilde{\lambda}'_{\dot{a}}$$

- Adjoint

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chirality

Spinors

chirality

$p^2 = 0 \qquad P_{a\dot{a}} = \lambda_a \tilde{\lambda}_{\dot{a}}$

Ex: $p^\mu p_\mu = \det(P_{a\dot{a}})$

Obs:

$= 1, 2$

Minkowski $(+---)$ P^μ is real

$\lambda, \tilde{\lambda}$ are both complex $\quad \tilde{\lambda} = \pm \bar{\lambda}$

↳ Future pos. Past Dir. Null vector.

$(++--)$, $SO(2,2) \simeq SL(2,\mathbb{R}) \times SL(2,\mathbb{R})$

$\lambda, \tilde{\lambda}$ are real & independent.

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Obs: Two vectors $p^2 = 0 = q^2$

$p_{a\dot{a}} = \lambda_a \tilde{\lambda}_{\dot{a}} \qquad q_{a\dot{a}} = \lambda'_a \tilde{\lambda}'_{\dot{a}}$

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Ex: $2p \cdot q = \langle \lambda, \lambda' \rangle [\tilde{\lambda}, \tilde{\lambda}']$

Spinor

Gluon : (p_i', ϵ_i')

Gluon: (p_i, ϵ_i^r) $(\lambda_i, \tilde{\lambda}_i, h_i)$ Helicity of the gluon h_i:

$(p_{a\dot{a}})_i = \lambda_{ai} \tilde{\lambda}_{\dot{a}i}$

$$\epsilon^{(+)}_{i\,a\dot{a}} = \frac{\eta_a \tilde{\lambda}_{\dot{a}i}}{\langle \eta_i \lambda_i \rangle}$$

where $\eta, \tilde{\eta}$ are arbitrary reference spinors

$$\epsilon^{(-)}_{i\,a\dot{a}} = \frac{\lambda_{ai} \tilde{\eta}_{\dot{a}}}{[\tilde{\lambda}_i, \tilde{\eta}]}$$

Gluon: $(p_i^\mu, \epsilon_i^\mu)$ Helicity of the gluon h_i.

$(\lambda_i, \tilde{\lambda}_i, h_i)$

$(p_{a\dot{a}})_i = \lambda_{a_i} \tilde{\lambda}_{\dot{a}_i}$

$\epsilon_i^{(+)} \cdot p_i = 0$

$$\frac{\langle \eta, \lambda \rangle [\tilde{\lambda}, \tilde{\lambda}]}{\langle \eta, \lambda \rangle} = 0$$

$$\epsilon^{(+)}_{i\,a\dot{a}} = \frac{\eta_a \tilde{\lambda}_{\dot{a}_i}}{\langle \eta_a, \lambda_i \rangle}$$

$$\epsilon^{(-)}_{i\,a\dot{a}} = \frac{\lambda_{a_i} \tilde{\eta}_{\dot{a}}}{[\tilde{\lambda}_i, \tilde{\eta}]}$$

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Gluon: (p_i, ϵ_i) $(\lambda_i, \tilde{\lambda}_i, h_i)$

$(p_{a\dot{a}})_i = \lambda_{ai} \tilde{\lambda}_{\dot{a}i}$

$\epsilon_i^{(+)} \cdot p_i = 0$

$\frac{\langle \eta | \not{\epsilon} | \lambda \rangle [\tilde{\lambda}, \tilde{\lambda}]}{\langle \eta, \lambda \rangle} = 0$

$\epsilon_i \to \epsilon_i + \omega p_i$

Gauge Transf.

Helicity of the gluon h_i:

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Ex: Check that diff. choices of η or $\tilde{\eta}$ $\Rightarrow$ Gauge Transf.

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Hint: $\Delta\eta = \alpha\eta + \beta\lambda$

Scattering Amplitudes

Scattering Amplitudes

Scalars $A(p_1, \ldots, p_n) = F(p_i \cdot p_j)$

ω. Spin $A(\{p_i, \lambda_i\}) =$

Feynman

Scattering Amplitudes

Scalers $A(p_1, \ldots, p_n) = F(p_i \cdot p_j)$

ω. Spin $A(\{p_i, \chi_i\}) =$ Multilinear in χ_i.

Fermions

$A(\{\lambda_i, \tilde{\lambda}_i, h_i\})$

Scattering Amplitudes

Scalers $A(p_1, \ldots, p_n) = F(p_i \cdot p_j)$

ω. Spin $A(\{p_i, \eta_i\}) =$ Multilinear in η_i.

Feynman diags

$A(\{\lambda_i, \tilde{\lambda}_i, h_i\})$

$(\lambda_i, \tilde{\lambda}_i) \sim (t\lambda_i, t^{-1}\tilde{\lambda}_i)$

t^{-2} $h_i = +$

t^{2} $h_i = -$

$$\left(\lambda_i^2 \frac{\partial}{\partial \lambda_i} - \tilde{\lambda}_i^2 \frac{\partial}{\partial \tilde{\lambda}_i}\right) A(\{\lambda_j, \tilde{\lambda}_j, h_j\}) = -2h_i A(\{\lambda_j, \tilde{\lambda}_j, h_j\})$$

$$\left(\lambda_i^a \frac{\partial}{\partial \lambda_i^a} - \tilde{\lambda}_i^a \frac{\partial}{\partial \tilde{\lambda}_i^a}\right) A(\{\lambda_j, \tilde{\lambda}_j, h_j\}) = -2 h_i A(\{\lambda_j, \tilde{\lambda}_j, h_j\})$$

$U(N)$ $\quad [T^a, T^b] = f^{abc} T^c$

$$f^{abc} = \mathrm{tr}(T^a T^b T^c) - \mathrm{tr}(T^a T^c T^b)$$

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Color decomp.

Ex:

Color decomp.

Ex:

Tree-level

Color Ordered Partial Amplitude

$$A_n(\{\lambda_i,\tilde{\lambda}_i,h_i,a_i\}) = g_{YM}^{n-2}\,(2\pi)^4\,\delta^{(4)}\Big(\sum_{i=1}^{n}\lambda_i\tilde{\lambda}_i\Big)\,\mathrm{Tr}(T^{a_1}\cdots T^{a_n})\,\overbrace{\mathcal{A}(\{\lambda_i,\tilde{\lambda}_i,h_i\})}$$

$+$ Permutations

Inequivalent

S_n/Z_n

Parke-Taylor

Parke-Taylor or MHV amplitudes

Maximally Helicity Violating

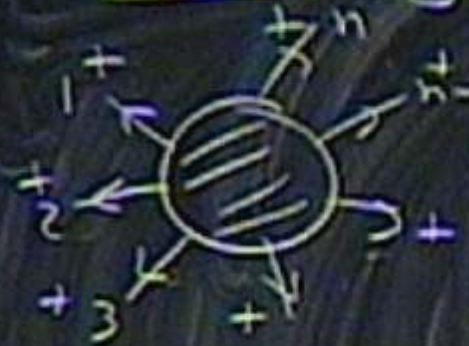
Parke-Taylor or MHV amplitudes

Maximally Helicity Violating

$$\mathcal{A}(1^+, 2^+, \ldots, n^+) = \{\lambda_i, \tilde{\lambda}_i, h_i = +\}$$

Parke-Taylor or MHV amplitudes

Maximally Helicity Violating



$$\mathcal{A}(1^+, 2^+, \ldots, n^+) = 0$$

$\{\lambda_i, \tilde{\lambda}_i, h_i = +\}$

Parke-Taylor or MHV amplitudes

Maximally Helicity Violating

$$\mathcal{A}(1^+, 2^+, \ldots, n^+) = 0$$

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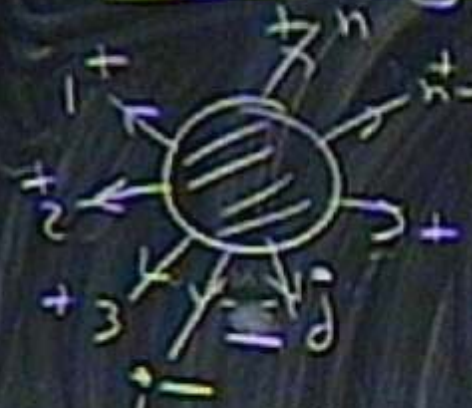
$$\mathcal{A}(1^+, 2^+ \ldots j^- \ldots, n^+) = 0$$

Parke-Taylor or MHV amplitudes

Maximally Helicity Violating



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$$\mathcal{A}(1^+, 2^+ \ldots, j^-, \ldots, n^+) = 0$$

$$\mathcal{A}(1^+, 2^+ \ldots, i^-, \ldots, j^-, \ldots, n^+) =$$

Parke-Taylor or MHV amplitudes

Maximally Helicity Violating

$$\mathcal{A}(1^+, 2^+, \ldots, n^+) = 0$$

$\{\lambda_i, \tilde{\lambda}_i, h_i = +\}$ $\quad \langle \lambda_i, \lambda_j \rangle = \langle i\, j \rangle$

$$\mathcal{A}(1^+, 2^+ \ldots, j^-, \ldots, n^+) = 0$$

$$\mathcal{A}(1^+, 2^+ \ldots, i^-, \ldots, j^-, \ldots, n^+) = \frac{\langle i\, j \rangle^4}{\langle 1\, 2 \rangle \langle 2\, 3 \rangle \cdots \langle n-1\, n \rangle \langle n\, 1 \rangle}$$

At tree-level $A^{\mathcal{N}=4}_{(\text{gluons})} = A^{QCD}_{(\text{gluons})}$

At tree-level $A^{(\text{gluons})}_{N=4} = A^{(\text{gluons})}_{\text{QCD}}$

$\neq$

At tree-level $A_{\mathcal{N}=4}(\text{gluons}) = A_{QCD}(\text{gluons})$

$\neq 0$

$A(-,-,-,+,+,\ldots,+) \equiv \text{Mess}(\langle ij\rangle, [ij])$

Kosower

Conformal Invariant $\longrightarrow$ Twistor Space

Lorentz generators.

$$J_{ab} = \left(\lambda_a \frac{\partial}{\partial \lambda^b} + \lambda_b \frac{\partial}{\partial \lambda^a}\right)$$

$$J_{\dot{a}\dot{b}} = \left(\tilde{\lambda}_{\dot{a}} \frac{\partial}{\partial \tilde{\lambda}^{\dot{b}}} + \tilde{\lambda}_{\dot{b}} \frac{\partial}{\partial \tilde{\lambda}^{\dot{a}}}\right)$$

$$P_{a\dot{b}} = \lambda_a \tilde{\lambda}_{\dot{b}}$$

$[D, P] = iP$ $\quad$ $[D, K] = -iK$ $\quad$ $\dim K = -1$

$\dim P = 1$

$\dim \lambda = \dim \tilde{\lambda} = \frac{1}{2}$

$$K_{a\dot{a}} = \frac{\partial^2}{\partial \lambda^a \partial \tilde{\lambda}^{\dot{a}}}$$

$$[K_{ab}, P^{cd}] = \left(\delta_a^{\,c} J_b^{\,d} + \delta_b^{\,c} J_a^{\,d} + \delta_a^{\,c}\delta_b^{\,d} D\right)$$

Ex: $D = \lambda^a \frac{\partial}{\partial \lambda^a} + \lambda^{ab} \frac{\partial}{\partial \lambda^{ab}} + 2$

$$[K_a, P^b] = \left(\delta_a^b J_i^{\ b} + \delta_i^{\ b} J_a^{\ b} + \delta_a^{\ b}\delta_i^{\ b} D\right)$$

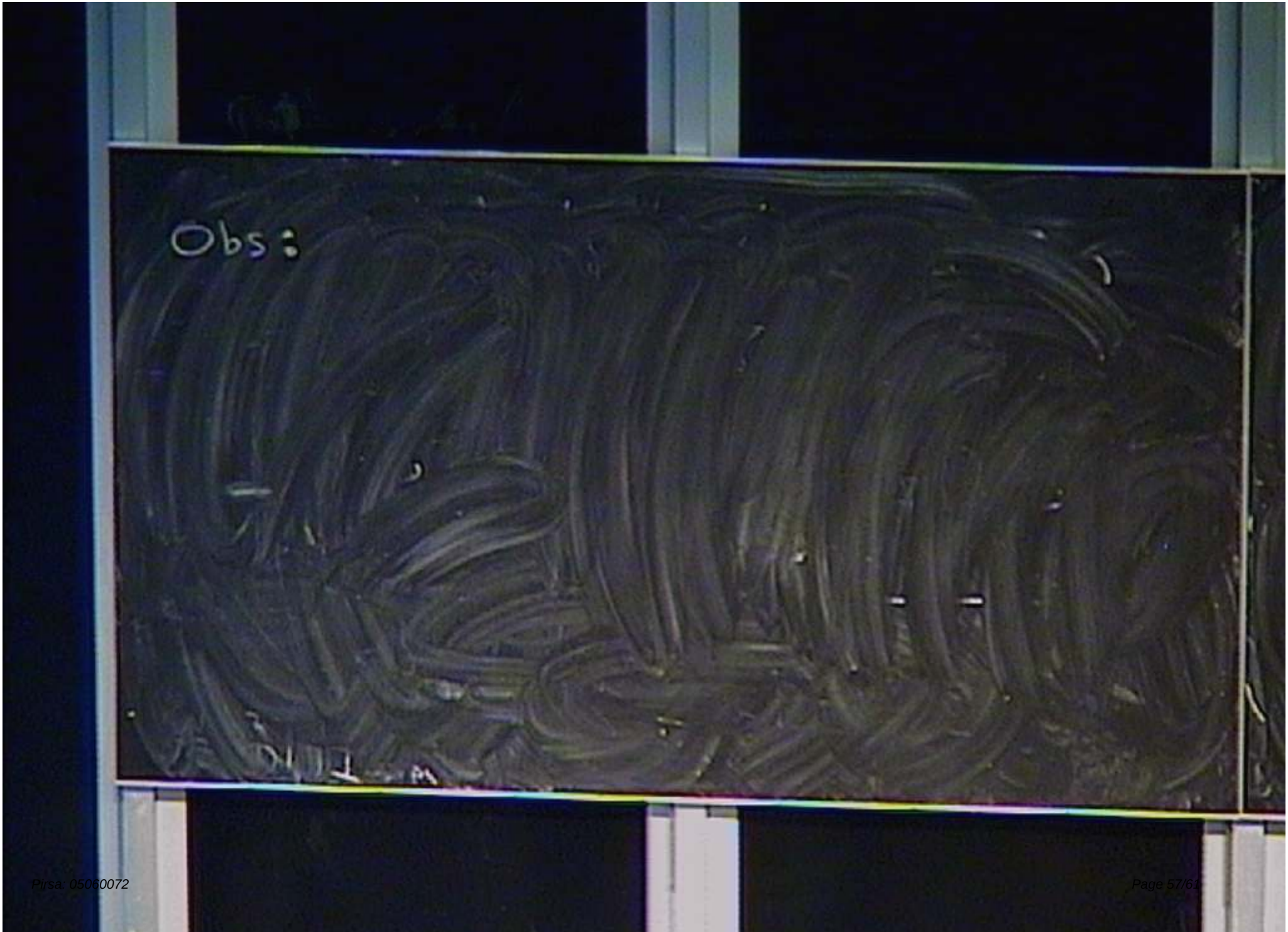
Ex: $D = \lambda^a \frac{\partial}{\partial \lambda^a} + \lambda^{\dot a} \frac{\partial}{\partial \lambda^{\dot a}} + 2$

Ex: Check that

$$A_{ij}^{M+V} = \delta^4\left(\sum_{k=1}^{n} \lambda_k \tilde\lambda_k\right) \frac{\langle i\ j\rangle^4}{\prod_{m=1}^{n} \langle m\ m+1\rangle}$$

Motly + MHV

is conform. Invariant


Obs:

Obs: $SO(4,2) \xrightarrow{\mathbb{C}} SO(\mathbb{C}) \simeq SU(4) \simeq SL(4,\mathbb{C})$

$\hookrightarrow \mathbb{C}^4$

Twistor transform: $\tilde{\lambda}_{\dot{a}} \longrightarrow i\frac{\partial}{\partial \mu^{\dot{a}}}$

$\mu^{\dot{a}} \longrightarrow i\frac{\partial}{\partial \tilde{\lambda}_{\dot{a}}}$

Ex: $D = \lambda^a \frac{\partial}{\partial \lambda^a} - \mu^{\dot{a}} \frac{\partial}{\partial \mu^{\dot{a}}}$

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$P_{a\dot{a}} = \lambda_a \frac{\partial}{\partial \mu^{\dot{a}}}$ $\quad K_{a\dot{a}} = \mu_{\dot{a}} \frac{\partial}{\partial \lambda^a}$

$J_{ab} = \lambda_a \frac{\partial}{\partial \lambda^b} + \lambda_b \frac{\partial}{\partial \lambda^a}$

$J_{\dot{a}\dot{b}} = \mu^{\dot{a}} \frac{\partial}{\partial \mu^{\dot{b}}} + \mu^{\dot{b}} \frac{\partial}{\partial \mu^{\dot{a}}}$

Obs: $SO(4,2) \xrightarrow{\mathbb{C}} SO(6) \simeq SU(4) \simeq SL(4,\mathbb{C})$

$\hookrightarrow \mathbb{C}^4$

Twistor transform: $\lambda_\alpha \longrightarrow i\frac{\partial}{\partial \mu^\alpha}$ $\quad Z^I = (\lambda_1, \lambda_2, \mu_1, \mu_2)$

$\mu_\dot{\alpha} \longrightarrow i\frac{\partial}{\partial \lambda^\dot{\alpha}}$

Ex: $D = \lambda^a \frac{\partial}{\partial \lambda^a} - \mu^\dot{a} \frac{\partial}{\partial \mu^\dot{a}}$ $\quad J_{ab} = \lambda_a \frac{\partial}{\partial \lambda^b} + \lambda_b \frac{\partial}{\partial \lambda^a}$

$P_{a\dot{a}} = \lambda_a \frac{\partial}{\partial \mu^\dot{a}}$ $\quad K_{a\dot{a}} = \mu_\dot{a} \frac{\partial}{\partial \lambda^a}$ $\quad J_{\dot{a}\dot{b}} = \mu_\dot{a} \frac{\partial}{\partial \mu^\dot{b}} + \mu_\dot{b} \frac{\partial}{\partial \mu^\dot{a}}$

$\mathcal{N} \perp \mathcal{M}$

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$\mathbb{CP}^3$

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