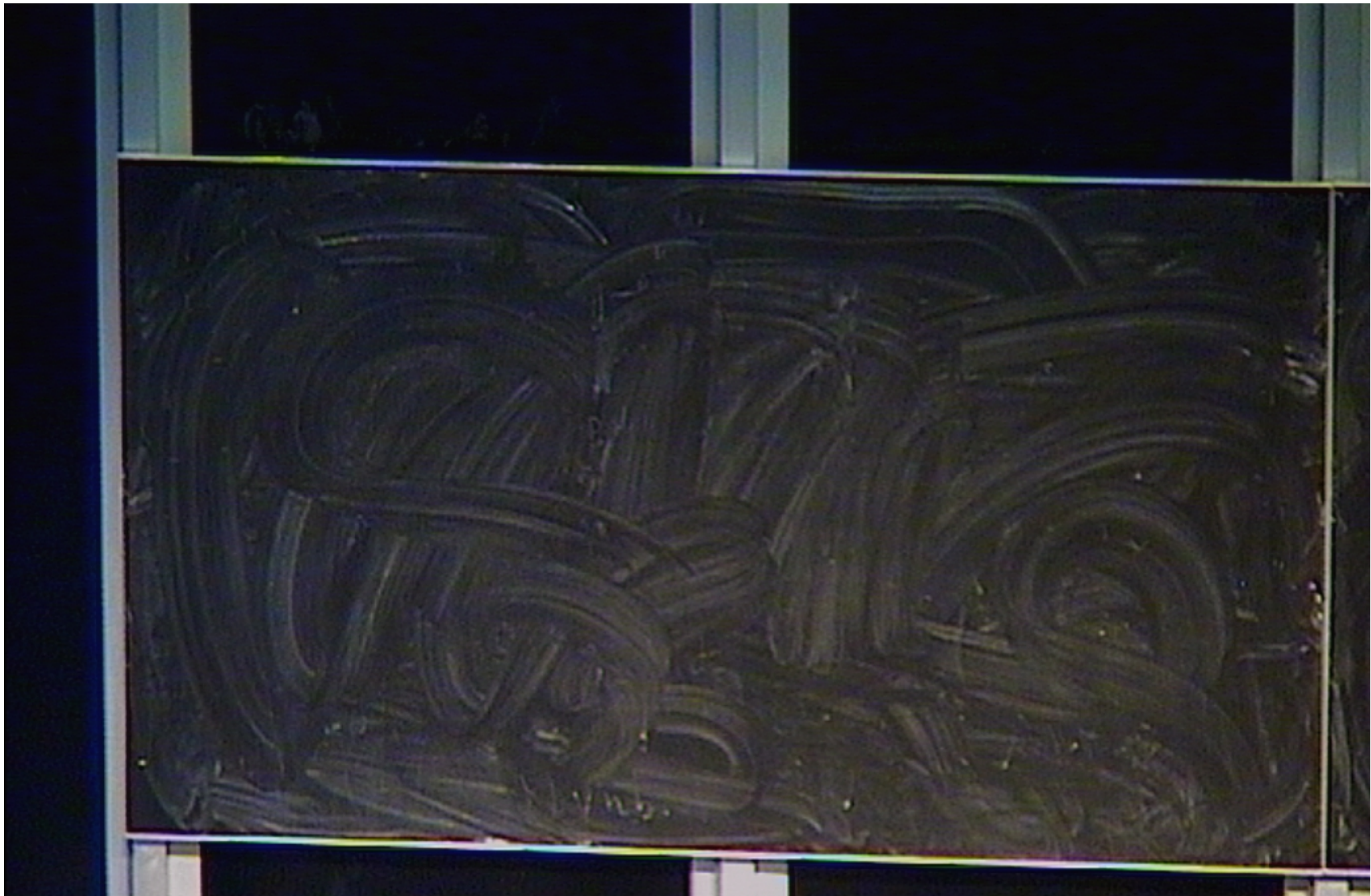


Title: Twistor String Theory and Perturbative Yang-Mills 1

Date: Jun 20, 2005 04:30 PM

URL: <http://pirsa.org/05060072>

Abstract:



Twistor String Theory & Perturbative YM

① Perturb. QCD.



LHC

$$\alpha_s(M_t^2) \sim \frac{1}{10}$$

Hope
New Physics.
Background.
QCD.

Twistor String Theory & Perturbative YM

① Perturb. QCD.



Feynman diag.
New Techniques

② 't Hooft 1974

LHC

$$\alpha_s(M_F^2) \sim \frac{1}{10}$$

$$n=6$$

$$\nabla$$

$$e$$

$$10^5$$

Hope
New Physics.
Background.
QCD.

Twistor String Theory & Perturbative YM

① Perturb. QCD.



Feynman diag.
New Techniques

LHC

$$\alpha_s(M_Z^2) \sim \frac{1}{10}$$

Hope
New Physics.
Background.
QCD.

$$n=6$$

$$10^5$$

②

't Hooft 1974

$$\text{Exp. } \frac{1}{N}$$

Λ_{QCD}

$U(N)$

N

Feynman Diag.



$$\lambda_t = g_{YM}^2 N$$

Large N

Fixed λ_t

$$\sum_{g=0}^{\infty} N^{2-2g} f_g(\lambda)$$

$g = g_{\text{new}} \rightarrow \text{String Theory}$

Took 23 yrs.
Maldacena

$\rightarrow 1997$

AdS / CFT

$\downarrow$
 $\lambda \gg 1$

Perturb. Theory is out of reach.

$$\sum_{g=0}^{\infty} N^{2-2g} f_g(\lambda)$$

$g = g_{\text{new}} \rightarrow \text{String Theory}$

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$\downarrow$
 $\lambda \gg 1$

Perturb. Theory is out of reach.

⑤ Twistor Space
Cplx analys.

$\mathbb{C}$

Penrose (1967)
 $2D$

$$\partial \bar{\partial} f = 0$$

$$f(z, \bar{z}) = g(z) + h(\bar{z})$$

$$\sum_{g=0}^{\infty} N^{2-2g} f_g(\lambda)$$

$g = g_{\text{new}} \rightarrow \text{String Theory}$

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$\mathbb{C}$

Penrose (1967)

2D.

$$\partial \bar{\partial} f = 0$$

$$f(z, \bar{z}) = g(z) + h(\bar{z})$$

4D? (x_0, x_1, x_2, x_3)

$\mathbb{C}^2$

$$z = x_0 + ix_1$$

$$z' = x_2 + ix_3$$

$$|x|^2 = z\bar{z} + z'\bar{z}' \quad O(4)/U(2) \cong S^2 = \mathbb{CP}^1$$

Each point S^4 a choice $\mathbb{CP}^1$

$\mathbb{CP}^1$ bundle over $S^4 \cong \mathbb{CP}^3$ = Twistor Space.

$$|\bar{x}|^2 = z\bar{z} + z'\bar{z}' \quad O(4)/U(2) \cong S^2 = \mathbb{CP}^1$$

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$\mathbb{CP}^1$ bundle over $S^4 \cong \mathbb{CP}^3$ = Twistor Space.

$$Z^I = (\underbrace{\lambda_1, \lambda_2}_{\mathbb{CP}^1}, \mu_1, \mu_2) \quad X_{aa} = X^I (\nabla_I)_{aa} \quad \nabla^I = (1, \vec{\nabla})$$

$$(\mu_a)_{\dot{a}} = (X_{aa} \lambda^a)$$

$$|\lambda|^2 = z\bar{z} + z'\bar{z}' \quad O(4)/U(2) \cong S^2 = \mathbb{CP}^1$$

Each point S^4 a choice $\mathbb{CP}^1$

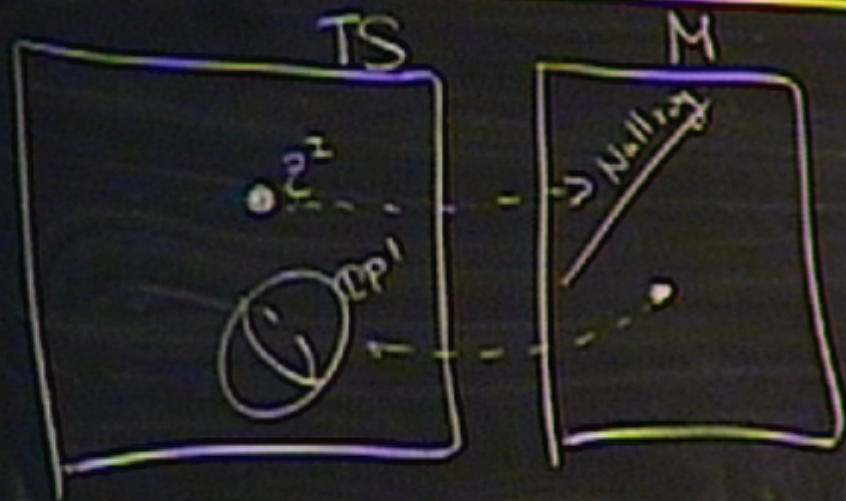
$\mathbb{CP}^1$ bundle over $S^4 \subseteq \mathbb{CP}^3$ = Twistor Space.

$$Z^I = (\underbrace{\lambda_1, \lambda_2}_{\mathbb{CP}^1}, \mu_1, \mu_2) \quad X_{aa} = X^I (\nabla_I)_{aa} \quad \nabla^I = (\mathbb{I}, \vec{\sigma})$$

$$(\mu_a)_{\dot{a}} = (X_{aa} \lambda^a)$$

(a) Fix $Z^I \rightarrow$ Null ray
in Minkowski Space

(b) Fix X_{ab} , $\mathbb{CP}^1 \subset \mathbb{CP}^3$
is a



- ④ Topological String theory. (Aogari)
- 10 Dim, 26 Dim. Infinite $\times$ of fields.
 - Baby versions. A-, B-model.

B-model. ① Target space. Super. $\mathbb{CP}^3$
 Finite $\times$ of fields. $\downarrow$ Mink.
 Spectra $N=4$ SYM

Goal:

① Twistor String Theory

④ Topological String theory. (Aogari)

- 10 Dim, 26 Dim. Infinite $\ast$ of fields.

- Baby versions. A-, B-model.
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Goal:

① Twistor String Theory.

$\downarrow$ Mink.
Spectrum $N=4$ SYM

④ Topological String Theory. (Aguilar)

- 10 Dm, 26 Dm. Infinite * of fields.
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① Target space. Super. $\mathbb{CP}^3$

Finite * of fields. $\downarrow$ Mink.

Spectra $N=4$ SYM

Goal:

① Twistor String Theory.

B-model $\mathbb{CP}^{3|4}$

+ N D5 branes +
D1 Instantons

$\rightarrow$ pMSYM

(2) MHV Diagrams } Tree-level
 Connected or BCFW Recursion R }
 Quadruple cuts } One-loop
 ABDK conjecture } 2- & higher-loop

Twistor Space

Cplx analys. $\mathbb{C}$

4D? (x_0, x_1, x_2, x_3)

$\mathbb{C}^2$

Penrose (1967)

2D. $\partial\bar{\partial}f = 0$

$f(z, \bar{z}) = g(z) + h(\bar{z})$

$z = x_1 + ix_2$

$\bar{z}' = x_3 + ix_4$

① Helicity Amplitudes
 $U(N)$

Gluons

$$g_i = (p_i^\mu, \epsilon_i^\mu)$$

Polarization vector

$\mathbb{CP}^3$

link.
YM

① Helicity Amplitudes $U(N)$

Gluons $g_i = (p_i^M, \epsilon_i^M, a_i)$
 Polar. vector ^{Adjoint}

$\mathbb{P}^3$

link.
YM

① Helicity Amplitudes

$U(N)$

Gluons

$$g_i = (p_i^\mu, \epsilon_i^\mu, a_i)$$

Adjust.
Polarization vector

Spinor Helicity Formalism:

$$SO(3,1) \xrightarrow{\mathbb{C}} SL(2, \mathbb{C}) \times SL(2, \mathbb{C})$$

(p, q)

Finite
dim. Rep.

$$\begin{aligned} \lambda_a &= (\frac{1}{2}, 0) \text{ — chiral spinors} \\ \tilde{\lambda}_{\dot{a}} &= (0, \frac{1}{2}) \text{ + chirality} \end{aligned}$$

$\mathbb{CP}^3$

link.
YM

① Helicity Amplitudes

$U(N)$

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Finite
dim. Rep.

$$\begin{matrix} \lambda_a & (\frac{1}{2}, 0) & - \text{chirality} \\ \tilde{\lambda}_{\dot{a}} & (0, \frac{1}{2}) & + \text{chirality} \end{matrix}$$

Spinors

$SL(2, \mathbb{C})$ invariant tensor

Lorentz invariant $\langle \lambda, \tilde{\lambda}' \rangle = \epsilon_{ab} \lambda^a \tilde{\lambda}'^b, \epsilon^{\dot{a}\dot{b}}$

$[\tilde{\lambda}, \tilde{\lambda}'] = \epsilon_{\dot{a}\dot{b}} \tilde{\lambda}^{\dot{a}} \tilde{\lambda}'^{\dot{b}}$

$\mathbb{CP}^3$

link.
YM

① Helicity Amplitudes

$U(N)$

Gluons: $g_i = (p_i^\mu, \epsilon_i^\mu, a_i)$

Adjust.
Polarization vector

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$\lambda_a \quad (\frac{1}{2}, 0)$ — chirality
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$SL(2, \mathbb{C})$ invariant tensor

Lorentz invariant $\langle \lambda, \tilde{\lambda}' \rangle = \epsilon_{ab} \lambda^a \tilde{\lambda}'^b$

$\langle 1, 2 \rangle$
 $\langle \tilde{\lambda}, \tilde{\lambda}' \rangle$

$$[\tilde{\lambda}, \tilde{\lambda}'] = \epsilon_{\dot{a}\dot{b}} \tilde{\lambda}^{\dot{a}} \tilde{\lambda}'^{\dot{b}}$$

$\mathbb{CP}^3$

link.

YM

① Helicity Amplitudes

$U(N)$

Gluons: $g_i = (p_i^\mu, \epsilon_i^\mu, a_i)$
 Polar. vector, Adj.

Spinor Helicity Formalism:

$$SO(3,1) \xrightarrow{\mathbb{C}} SL(2, \mathbb{C}) \times SL(2, \mathbb{C})$$

Finite dim. Rep.

(p, q)

$\lambda_a^\pm (\frac{1}{2}, 0)$ - chirality
 $\tilde{\lambda}_{\dot{a}}^\pm (0, \frac{1}{2})$ + chirality
 Spinors

$SL(2, \mathbb{C})$ invariant tensor

Lorentz invariant

$$\langle \lambda, \lambda' \rangle = \epsilon_{ab} \lambda^a \lambda'^b, \quad \epsilon_{\dot{a}\dot{b}}$$

$$\langle 1, 2 \rangle \rightarrow \langle 2, 2 \rangle$$

$$[\tilde{\lambda}, \tilde{\lambda}'] = \epsilon_{\dot{a}\dot{b}} \tilde{\lambda}^{\dot{a}} \tilde{\lambda}'^{\dot{b}}$$

$\mathbb{CP}^3$

link. YM

Adjust.

i)

star

chirality
Spinors
chirality

= 1, 2

$$\langle \lambda, \lambda' \rangle = -\langle \lambda', \lambda \rangle$$

Vectors

$$\left(\frac{1}{2}, \frac{1}{2}\right)$$

P^μ

Bispinor

$$P_{a\dot{a}} = P^\mu (\gamma_\mu)_{a\dot{a}} = \lambda_a \tilde{\lambda}_{\dot{a}} + \lambda'_{\dot{a}} \tilde{\lambda}_a$$

Adjust.

i)

star

chirality
Spinors

chirality

$= (1, 2)$

$$\langle \lambda, \lambda' \rangle = -\langle \lambda', \lambda \rangle$$

Vectors $(\frac{1}{2}, \frac{1}{2})$ p^μ Bispinor

$$p_{a\dot{a}} = p^\mu (\sigma_\mu)_{a\dot{a}} = \lambda_a \tilde{\lambda}_{\dot{a}} + \lambda'_{\dot{a}} \tilde{\lambda}_a$$

$$p^2 = 0 \quad p_{a\dot{a}} = \lambda_a \tilde{\lambda}_{\dot{a}}$$

$$\text{Ex: } p^\mu p_\mu = \det(p_{a\dot{a}})$$

Obs:

Minkowski, $(+---)$ p is real

$\lambda, \tilde{\lambda}$ are both complex

$$\tilde{\lambda} = \pm \bar{\lambda}$$

$\hookrightarrow$ Future or

Past Dir.

Null vector.

$(++--)$, $SO(2,2) \cong SL(2,\mathbb{R}) \times SL(2,\mathbb{R})$

$\lambda, \tilde{\lambda}$ are real & independent.

Minkowski, $(+---)$ p is real

$\lambda, \tilde{\lambda}$ are both complex

$$\tilde{\lambda} = \pm \bar{\lambda}$$

↳ Future or

$(++--)$, $SO(2,2) \cong SL(2, \mathbb{R}) \times SL(2, \mathbb{R})$

$\lambda, \tilde{\lambda}$ are real & independent.

Best Dir.
Null vector.

Obs :

Two vectors

$$p^2 = 0 = q^2$$

$$p_{\alpha\dot{\alpha}} = \lambda_{\alpha} \tilde{\lambda}_{\dot{\alpha}}$$

$$q_{\alpha\dot{\alpha}} = \lambda'_{\alpha} \tilde{\lambda}'_{\dot{\alpha}}$$

Minkowski, (+---) p is real

$\lambda, \tilde{\lambda}$ are both complex

$$\tilde{\tilde{\lambda}} = \pm \bar{\lambda}$$

↳ Future or

(++--), $SO(2,2) \cong SL(2, \mathbb{R}) \times SL(2, \mathbb{R})$

Past Dir.

Null vector

$\lambda, \tilde{\lambda}$ are real & independent.

Obs: Two vectors $p^2 = 0 = q^2$

$$p_{\alpha\dot{\alpha}} = \lambda_{\alpha} \tilde{\lambda}_{\dot{\alpha}} \quad q_{\alpha\dot{\alpha}} = \lambda'_{\alpha} \tilde{\lambda}'_{\dot{\alpha}}$$

Ex: $2p \cdot q = \langle \lambda, \lambda' \rangle [\tilde{\lambda}, \tilde{\lambda}']$

Gluon : $(p_i^\mu, \epsilon_i^\mu)$

Gluon: $(p_i^\mu, \underline{e}_i^\mu)$

$(\lambda_i, \tilde{\lambda}_i, h_i)$

$$(p_{ai})_i = \lambda_a; \tilde{\lambda}_a;$$

Helicity of the gluon h_i :

$$\epsilon_{i a \tilde{a}}^{(+)} = \frac{\eta_a \tilde{\lambda}_{\tilde{a}i}}{\langle \eta, \lambda \rangle}$$

$$\epsilon_{i a \tilde{a}}^{(-)} = \frac{\lambda_{ai} \tilde{\eta}_{\tilde{a}}}{[\tilde{\lambda}_i, \tilde{\eta}]}$$

where $\eta, \tilde{\eta}$ are
arbitrary
reference spinors

Given: $(p_i, \underline{\epsilon}_i)$

$(\lambda_i, \tilde{\lambda}_i, h_i)$

$$(p_i)_i = \lambda_a \tilde{\lambda}_i$$

$$\epsilon_i^{(+)} \cdot p_i = 0$$

$$\frac{\langle \eta, \lambda \rangle [\tilde{\lambda}, \tilde{\lambda}]}{\langle \eta, \lambda \rangle} = 0$$

Helicity of the gluon h_i .

$$\epsilon_{i a \tilde{a}}^{(+)} = \frac{\eta_a \tilde{\lambda}_{\tilde{a} i}}{\langle \eta, \lambda \rangle}$$

$$\epsilon_{i a \tilde{a}}^{(-)} = \frac{\lambda_a \tilde{\eta}_{\tilde{a} i}}{[\tilde{\lambda}, \tilde{\eta}]}$$

where $\eta, \tilde{\eta}$ are
arbitrary
reference spinors

Gluon: $(p_i, \underline{\epsilon}_i)$

$(\lambda_i, \tilde{\lambda}_i, h_i)$

$$(p_{ai})_i = \lambda_a \tilde{\lambda}_i$$

$$\epsilon_i^{(+)} \cdot p_i = 0$$

$$\frac{\langle \eta \lambda \rangle [\tilde{\lambda}, \tilde{\lambda}]}{\langle \eta \lambda \rangle} = 0$$

$$\epsilon_i \rightarrow \epsilon_i + \omega p_i$$

Gauge Transp.

Helicity of the gluon h_i :

$$\epsilon_{i\alpha\dot{\alpha}}^{(+)} = \frac{\eta_\alpha \tilde{\lambda}_{\dot{\alpha}i}}{\langle \eta \lambda \rangle}$$

$$\epsilon_{i\alpha\dot{\alpha}}^{(-)} = \frac{\lambda_{\alpha i} \tilde{\eta}_{\dot{\alpha}}}{[\tilde{\lambda}_i, \tilde{\eta}]}$$

where $\eta, \tilde{\eta}$ are
arbitrary
reference spinors

Ex: Check that
diff. choices of η or $\tilde{\eta}$
 $\Rightarrow$ Gauge Transp.

Gluson: $(p_i, \underline{\epsilon}_i)$

$(\lambda_i, \tilde{\lambda}_i, h_i)$

$$(p_{ai})_i = \lambda_a \tilde{\lambda}_i$$

$$\epsilon_i^{(+)} \cdot p_i = 0$$

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Gauge Transp.

Helicity of the gluon h_i :

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where $\eta, \tilde{\eta}$ are
arbitrary
reference spinors

Ex: Check that
diff. choices of η or $\tilde{\eta}$
 $\Rightarrow$ Gauge Transp.

$$\text{Hint: } \Delta\eta = \alpha\eta + \beta\lambda$$

Scattering Amplitudes

Scattering Amplitudes



Scalars

$$A(p_1, \dots, p_n) = \overline{F}(p_1, p_n)$$

ω. Spin

$$A(\{p_i, \gamma_i\}) =$$

Feynman

Scattering Amplitudes



Scalars

$$A(p_1, \dots, p_n) = \overline{F}(p_1, \dots, p_n)$$

ω . Spin

$$A(\{p_i, \lambda_i\}) = \text{Multilinear in } \lambda_i$$

$$A(\{\lambda_i, \tilde{\lambda}_i, h_i\})$$

Feynman diagrams

Scattering Amplitudes



Scalars

$$A(p_1, \dots, p_n) = \overline{F}(p_1, \dots, p_n)$$

Spin

$$A(\{p_i, \gamma_i\}) = \text{Multilinear in } \gamma_i$$

Feynman diag

$$A(\{\lambda_i, \tilde{\lambda}_i, h_i\})$$

$$(\lambda_i, \tilde{\lambda}_i) \sim (t \lambda_i, t^{-1} \tilde{\lambda}_i)$$

$$\begin{matrix} t^{-2} & h_i = + \\ t^2 & h_i = - \end{matrix}$$

$$\left(\lambda_i^a \frac{\partial}{\partial \lambda_i^a} - \tilde{\lambda}_i^a \frac{\partial}{\partial \tilde{\lambda}_i^a} \right) A(\{\lambda_j, \tilde{\lambda}_j, h_j\}) = -2h_i A(\{\lambda_j, \tilde{\lambda}_j, h_j\})$$

$$\left(\lambda_i^a \frac{\partial}{\partial \lambda_i^a} - \tilde{\lambda}_i^a \frac{\partial}{\partial \tilde{\lambda}_i^a} \right) A(\{\lambda_j, \tilde{\lambda}_j, h_j\}) = -2h_i A(\{\lambda_j, \tilde{\lambda}_j, h_j\})$$

$$U(N) \quad [T^a, T^b] = f^{abc} T^c$$

$$f^{abc} = \text{tr}(T^a T^b T^c) - \text{tr}(T^a T^c T^b)$$

$$\left(\lambda_i \frac{\partial}{\partial \lambda_i} - \tilde{\lambda}_i \frac{\partial}{\partial \tilde{\lambda}_i} \right) A(\{\lambda_j, \tilde{\lambda}_j, h_j\}) = -2h_i A(\{\lambda_j, \tilde{\lambda}_j, h_j\})$$

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$$f^{abc} = \text{tr}(T^a T^b T^c) - \text{tr}(T^a T^c T^b)$$

$$\sum_{a=1}^{d=6} (T^a)_i^{\bar{j}} (T^a)_k^{\bar{n}} = \delta_i^{\bar{n}} \delta_k^{\bar{j}}$$

$$\left(\lambda_i \frac{\partial}{\partial \lambda_i} - \tilde{\lambda}_i \frac{\partial}{\partial \tilde{\lambda}_i} \right) A(\{\lambda_j, \tilde{\lambda}_j, h_j\}) = -2h_i A(\{\lambda_j, \tilde{\lambda}_j, h_j\})$$

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$$\sum_{a=1}^{d=6} (T^a)_i^{\bar{j}} (T^a)_k^{\bar{n}} = \delta_i^{\bar{n}} \delta_k^{\bar{j}}$$

$$\text{Feynman diagram} = \text{Feynman diagram} - \text{Feynman diagram}$$

$$\left(\lambda_i^a \frac{\partial}{\partial \lambda_i^a} - \tilde{\lambda}_i^a \frac{\partial}{\partial \tilde{\lambda}_i^a} \right) A(\{\lambda_j, \tilde{\lambda}_j, h_j\}) = -2h_i A(\{\lambda_j, \tilde{\lambda}_j, h_j\})$$

$$U(N) \quad [T^a, T^b] = f^{abc} T^c$$

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$$\sum_{a=1}^{d-6} (T^a)_i^{\bar{j}} (T^a)_k^{\bar{n}} = \delta_i^{\bar{n}} \delta_k^{\bar{j}}$$



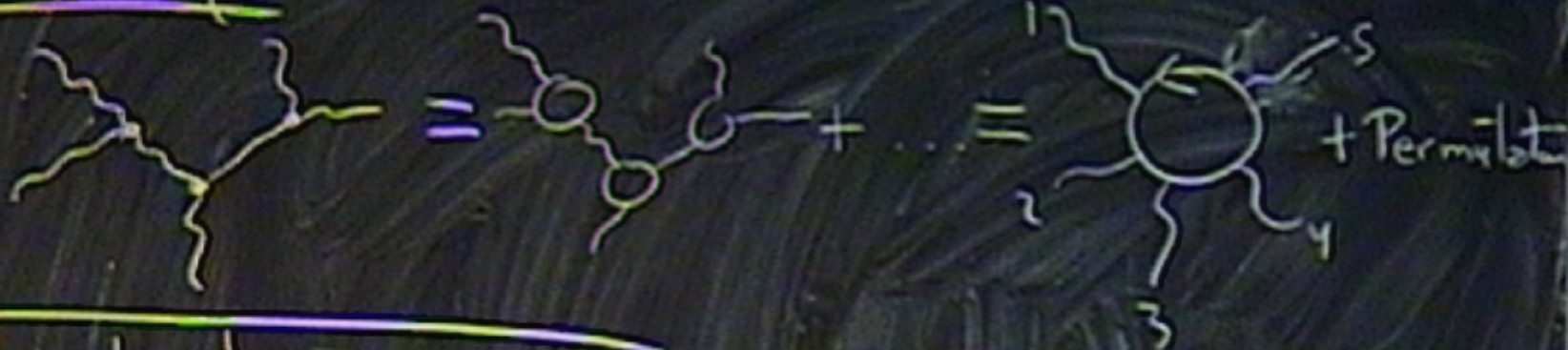
Color decomp.

Ex:



Color decomp.

Ex:



Tree-level

Color Ordered Partial Amplitude

$$A_n(\{\lambda_i, \tilde{\lambda}_i, h_i, a_i\}) = g_{YM}^{n-2} (2\pi)^4 \delta(\sum_{i=1}^n \lambda_i \tilde{\lambda}_i) \text{tr}(T^{a_1} \dots T^{a_n}) \underbrace{A(\{\lambda_i, \tilde{\lambda}_i, h_i\})}_{\substack{+ \text{Permutation} \\ \text{Inequivalent}}} \underbrace{\quad}_{S_n/Z_n}$$

+
Parke-Taylor.

mitata

Amplitude

h_3

Parke-Taylor or MHV amplitudes
Maximally Helicity Violating



mitation

Amplitude

h_3

Parke-Taylor or MHV amplitudes

Maximally Helicity Violating



$$A(1^+, 2^+, \dots, n^+) =$$

$$\{\lambda_1, \lambda_2, h_1, h_2, \dots\}$$

Parke-Taylor or MHV amplitudes

Maximally Helicity Violating

$$A(1^+, 2^+, \dots, n^+) = 0$$

$$\{\lambda_1, \lambda_2, h_1, h_2, \dots\}$$



Parke-Taylor or MHV amplitudes

Maximally Helicity Violating



$$A(1^+, 2^+, \dots, n^+) = 0$$

$$\{\lambda_i, \tilde{\lambda}_i, h_i\}$$

$$A(1^+, 2^+, \dots, j^-, \dots, n^+) = 0$$

Parke-Taylor or MHV amplitudes

Maximally Helicity Violating



$$A(1^+, 2^+, \dots, n^+) = 0$$

$$\{\lambda_1, \lambda_2, \dots, \lambda_n\}$$

$$A(1^+, 2^+, \dots, j^-, \dots, n^+) = 0$$

$$A(1^+, 2^+, \dots, i^-, \dots, j^-, \dots, n^+) =$$

Parke-Taylor or MHV amplitudes

Maximally Helicity Violating

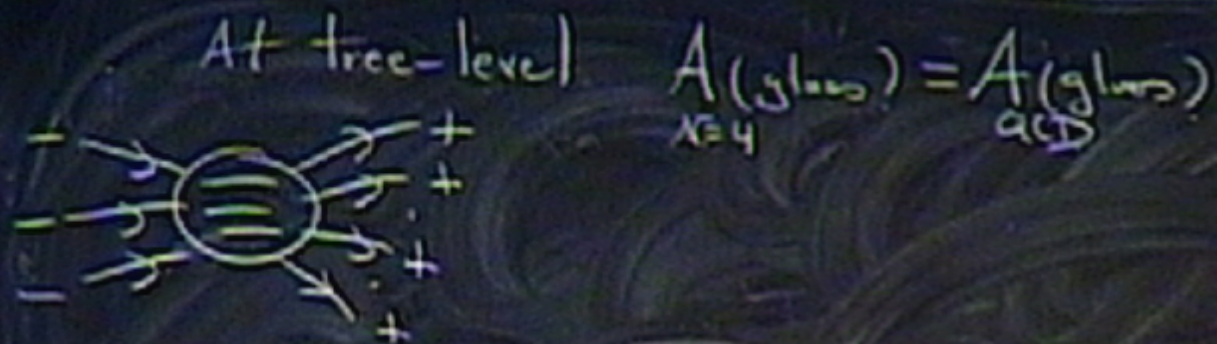


$$A(1^+, 2^+, \dots, n^+) = 0$$

$$\{\lambda_i, \lambda_j, h_{ij}\} \quad \langle \lambda_i, \lambda_j \rangle = \langle i, j \rangle$$

$$A(1^+, 2^+, \dots, j^-, \dots, n^+) = 0$$

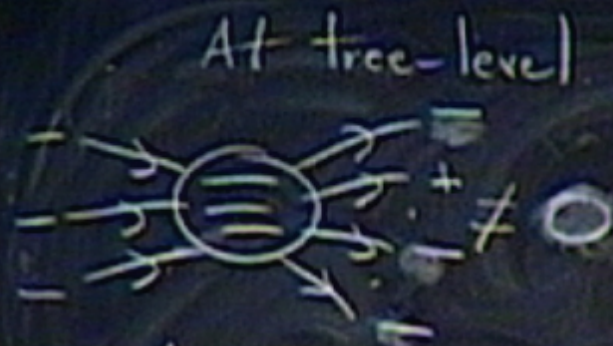
$$A(1^+, 2^+, \dots, i^-, \dots, j^-, \dots, n^+) = \frac{\langle i, j \rangle^4}{\langle 1, 2 \rangle \langle 2, 3 \rangle \dots \langle n-1, n \rangle \langle n, 1 \rangle}$$



At tree-level

$$A_{N=4}(\text{gluon}) = A_{\text{QCD}}(\text{gluon})$$





$$A_{N=4}(\text{gluons}) = A_{\text{QCD}}(\text{gluons})$$



$$A(-, -, -, +, +, \dots, +) = \text{Mess}(\langle ij \rangle, [ij])$$

Kosower

Conformal Invariant $\longrightarrow$ Twistor Space

lorentz generators.

$$J_{ab} = \left(\lambda_a \frac{\partial}{\partial \lambda_b} + \lambda_b \frac{\partial}{\partial \lambda_a} \right)$$

$$J_{\tilde{a}\tilde{b}} = \left(\tilde{\lambda}_{\tilde{a}} \frac{\partial}{\partial \tilde{\lambda}_{\tilde{b}}} + \tilde{\lambda}_{\tilde{b}} \frac{\partial}{\partial \tilde{\lambda}_{\tilde{a}}} \right)$$

$$P_{ab} = \lambda_a \tilde{\lambda}_b$$

$$[D, P] = i P$$

$$d\ln P = 1$$

$$d\ln \lambda = d\ln \tilde{\lambda} = \frac{1}{2}$$

$$[D, K] = -i K$$

$$d\ln K = -1$$

$$K_{a\tilde{a}} = \frac{\partial^2}{\partial \lambda^a \partial \tilde{\lambda}^{\tilde{a}}}$$

$$[K_{ab}, P^{ab}] = (S_a^b J_b^c + S_c^b J_a^c + S_a^b S_c^b D)$$

$$\text{Ex: } D = \lambda^a \frac{\partial}{\partial \lambda^a} + \lambda^{ab} \frac{\partial}{\partial \lambda^{ab}} + 2$$

$$[K_{ab}, P^b] = (S_a^b J_b^c + S_c^b J_a^c + S_a^b S_c^b D)$$

$$\text{Ex: } D = \lambda^a \frac{\partial}{\partial \lambda^a} + \tilde{\lambda}^{\dot{a}} \frac{\partial}{\partial \tilde{\lambda}^{\dot{a}}} + 2$$

Ex: Check that

$$A_{ij}^{MHV} = \frac{\delta(\sum_{k=1}^n \lambda_k \tilde{\lambda}_k)}{\prod_{m=1}^n \langle m, m+1 \rangle} \frac{\langle i, j \rangle^4}{\prod_{m=1}^n \langle m, m+1 \rangle}$$

Motly + MHV

is conform. Invariant.

Obs:

$$\text{Obs: } SO(4,2) \xrightarrow{\mathbb{C}} SO(4) \simeq SU(4) \simeq SL(4, \mathbb{C})$$

$$\text{Twistor transform: } \tilde{\lambda}_a \rightarrow i \frac{\partial}{\partial \mu^a}$$

$$\mu_a \rightarrow -i \frac{\partial}{\partial \tilde{\lambda}^a}$$

$$\text{Ex: } \mathbb{D} = \lambda^a \frac{\partial}{\partial \lambda^a} - \mu^a \frac{\partial}{\partial \mu^a}$$

$$\text{Obs: } SO(4,2) \xrightarrow{\mathbb{C}} SO(4) \simeq SU(4) \simeq SL(4, \mathbb{C})$$

$$\hookrightarrow \mathbb{C}^4$$

$$\text{Twistor transform: } \tilde{\lambda}_a \rightarrow i \frac{\partial}{\partial \mu^a}$$

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$$\text{Ex: } \mathbb{D} = \lambda^a \frac{\partial}{\partial \tilde{\lambda}^a} - \mu^a \frac{\partial}{\partial \mu^a}$$

$$P_{aa'} = \lambda_a \frac{\partial}{\partial \mu^{a'}}$$

$$K_{aa'} = \mu_a \frac{\partial}{\partial \tilde{\lambda}^{a'}}$$

$$J_{ab} = \lambda_a \frac{\partial}{\partial \tilde{\lambda}^b} + \lambda_b \frac{\partial}{\partial \tilde{\lambda}^a}$$

$$J_{a'b'} = \mu_a \frac{\partial}{\partial \mu^{b'}} + \mu_{a'} \frac{\partial}{\partial \mu^{b'}}$$

$$\text{Obs: } SO(4,2) \xrightarrow{\mathbb{C}} SO(4) \simeq SU(4) \simeq SL(4, \mathbb{C})$$

$$\text{Twistor transform: } \begin{array}{l} \tilde{\lambda}_i \rightarrow i \frac{\partial}{\partial \mu^i} \\ \mu_i \rightarrow -i \frac{\partial}{\partial \lambda^i} \end{array} \quad \begin{array}{l} \searrow \mathbb{C}^4 \\ \downarrow \\ \mathcal{Z}^I = (\lambda_1, \lambda_2, \mu_1, \mu_2) \end{array}$$

$$\text{Ex: } \mathbb{D} = \lambda^a \frac{\partial}{\partial \lambda^a} - \mu^a \frac{\partial}{\partial \mu^a}$$

$$P_{a\dot{a}} = \lambda_a \frac{\partial}{\partial \mu^{\dot{a}}}$$

$$K_{a\dot{a}} = \mu_{\dot{a}} \frac{\partial}{\partial \lambda^a}$$

$$J_{ab} = \lambda_a \frac{\partial}{\partial \lambda^b} + \lambda_b \frac{\partial}{\partial \lambda^a}$$

$$J_{\dot{a}\dot{b}} = \mu_{\dot{a}} \frac{\partial}{\partial \mu^{\dot{b}}} + \mu_{\dot{b}} \frac{\partial}{\partial \mu^{\dot{a}}}$$

$$\text{Obs: } SO(4,2) \xrightarrow{\sim} SO(4) \simeq SU(4) \simeq SL(4, \mathbb{C})$$

$$\begin{array}{lcl} \text{Twistor transform: } & \tilde{\lambda}_i \rightarrow i \frac{\partial}{\partial \mu^i} & \downarrow \mathbb{C}^4 \\ & \mu_i \rightarrow -i \frac{\partial}{\partial \lambda^i} & \downarrow \\ & & \mathbb{CP}^3 \end{array}$$

$$Ex: \quad \mathbb{D} = \lambda^a \frac{\partial}{\partial \lambda^a} - \mu^a \frac{\partial}{\partial \mu^a}$$

$$P_{a\dot{a}} = \lambda_a \frac{\partial}{\partial \mu^{\dot{a}}}$$

$$K_{a\dot{a}} = \mu_{\dot{a}} \frac{\partial}{\partial \lambda^a}$$

$$J_{ab} = \lambda_a \frac{\partial}{\partial \lambda^b} + \lambda_b \frac{\partial}{\partial \lambda^a}$$

$$J_{\dot{a}\dot{b}} = \mu_{\dot{a}} \frac{\partial}{\partial \mu^{\dot{b}}} + \mu_{\dot{b}} \frac{\partial}{\partial \mu^{\dot{a}}}$$