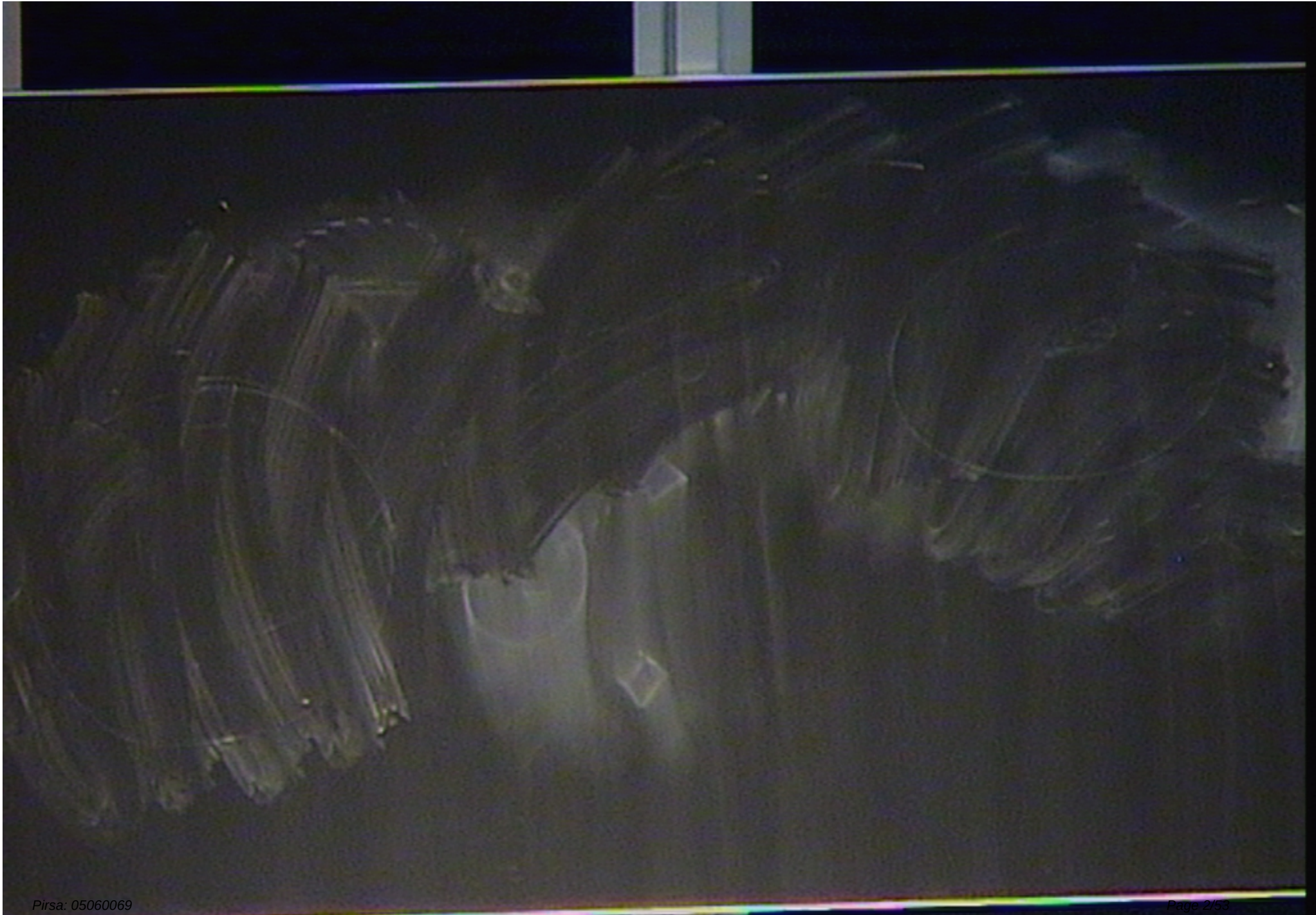


Title: Supersymmetry Basics

Date: Jun 20, 2005 09:30 AM

URL: <http://pirsa.org/05060069>

Abstract:



PHYSICS.UTORONTO.CA/

oppitz/SUSY-PI.pdf

&

SUSY-PI031.pdf

SUSY - relates bosons & fermions

WHY?

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WHY?

PARTICLE THEORY
typically weakly coupled -

Y - beyond-the-SM-physics,

MSSM,
GUTs,

SUSY - relates bosons & fermions

WHY?

(a) PARTICLE THEORY

typically weakly coupled, but

(b) TOY MODELS FOR STRONG CO

Y - beyond-the-SM-physics, not always COUPLING

MSSM
GUTs,

SUSY - relates bosons & fermions

WHY?

(a) PARTICLE THEORY

typically weakly coupled, but

(b) TOY MODELS FOR STRING CO

Y - beyond-the-SM-physics,

not always

COUPLING — dynamical SUSY breaking; confi

"MSSM"

GUTs,

confinement, dualities

"MSSM"

GUTs,

element, dualities --

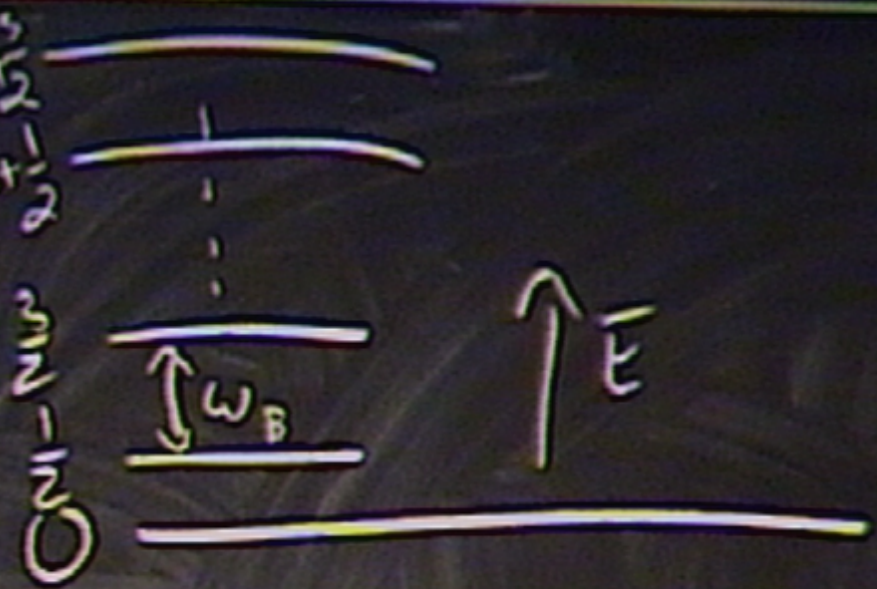
both in 4d as well as
5d & 6d

& lower $d=2,3$

(c) string theory

(d) mathematical results - index theorems

(e) getting jobs

$$E_n = \omega_B \left(n + \frac{1}{2} \right)$$


1) fermionic oscillator

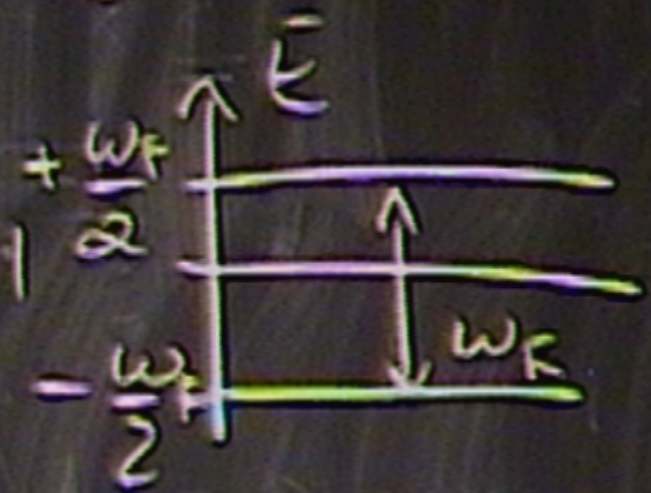
$f, f^\dagger$

$$f^2 = f^{\dagger 2} = 0$$

$$\{f, f^\dagger\} =$$

$$\omega_F \left(f^+ f - \frac{1}{2} \right) = \frac{\omega}{2} [f^+, f]$$

$$= \omega_F \left(k - \frac{1}{2} \right), \quad k = 0, 1, 2, \dots$$



oscillator = $a + b$, with $\omega_B = \omega_F$

$$H_{\text{SUSY}} = H_B + H_F \Big|_{\omega_B = \omega_F} = \omega (b^\dagger b + f^\dagger f)$$

Energy levels (n, k) , $n=0, \dots, \infty$
 $k=0, 1$

<u>$(n, 0)$</u>	<u>$(n, 1)$</u>
$(3, 0)$	$(2, 1)$
$(2, 0)$	$(1, 1)$
$(1, 0)$	$(0, 1)$
$(0, 0)$	

* all $E \geq 0$ levels are
doubly degenerate

* $E_0 = 0$

$$H_{\text{SUSY}} = H_B + H_F \Big|_{\omega_B = \omega_F} = \omega (b^\dagger b + f^\dagger f)$$

Supersymmetry is responsible for these (*) properties (take b, f to commute)

$$Q = b^\dagger f$$

$$Q^\dagger = b f^\dagger$$

$$H_{\text{SUSY}} = H_B + H_F \Big|_{\omega_B = \omega_F} = w(b^\dagger b + f^\dagger f)$$

Supersymmetry is responsible for these (*) properties (take b, f to commute)

$$Q = b^\dagger f \quad \text{nilpotent} \quad Q^2 = (Q^\dagger)^2 = 0$$

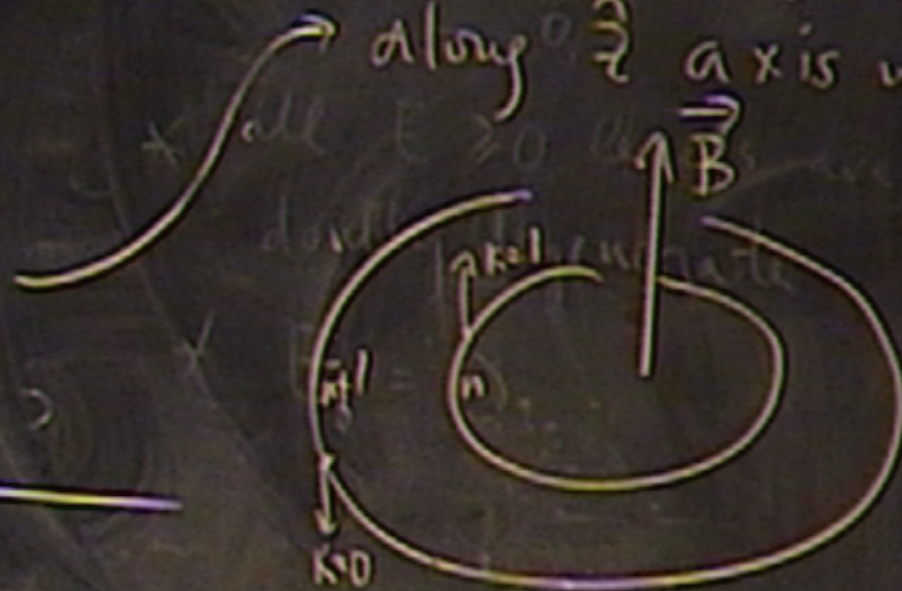
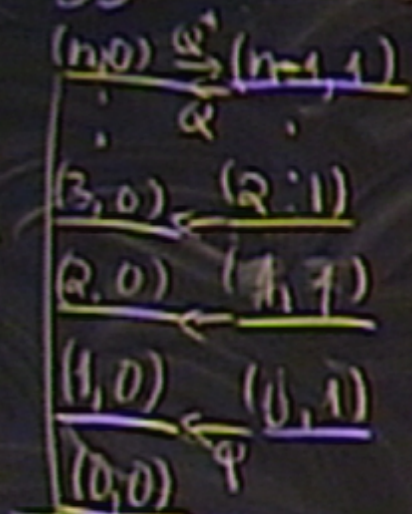
$$Q^\dagger = b f^\dagger \quad \left. \begin{array}{l} Q = b^\dagger f \\ Q^\dagger = b f^\dagger \end{array} \right\} H = w \{Q, Q^\dagger\} \leftarrow \boxed{\text{SHOW!}}$$

$$[Q, H] = [Q^+, H] =$$

$$Q^2 = Q^{+2} = 0$$

$$\omega \{Q, Q^+\} = H$$

Energy levels (n, k) 2d electrons in B
 along $\hat{z}$ axis w/ $g=2$



$$1) \quad [Q^+, H] = 0$$

$$Q^{+2} = 0$$

$$Q^- = H$$

for a system
def'd like this

(1) all energy levels
are ≥ 0

(2) all energy levels
w/ $E > 0$ are doubly
degenerate

$$[H, L^{\pm}] = 0$$



for a system
def'd like this

(1) all energy levels
are > 0

(2) all energy levels
w/ $E > 0$ are doubly
degenerate

to show (1), note that

$$H = (Q + Q^\dagger)^2 (= \{Q, Q^\dagger\})$$

$$H|E\rangle = |E\rangle E$$

$$\langle E|H|E\rangle = E\langle E|E\rangle = \langle E|(Q + Q^\dagger)(Q + Q^\dagger)|E\rangle$$

to show (1), note that

$$H = (Q + Q^\dagger)^2 (= \{a, a^\dagger\})$$

$$H|E\rangle = |E\rangle E$$

$$\langle H|E\rangle = E \langle Q|E\rangle = \langle E|(Q + Q^\dagger)|Q + Q^\dagger|E\rangle = \frac{1}{2} \langle E|(Q - Q^\dagger)^2|E\rangle \geq 0 \rightarrow E \geq 0 \quad \text{iff } (Q + Q^\dagger)|E\rangle = 0.$$

to show (1), note that

$$H = (Q + Q^\dagger)^2 (= \{a, a^\dagger\})$$

$$H|E\rangle = |E\rangle E$$

$$\langle E|H|E\rangle = E \langle E|E\rangle = \langle E|(Q+Q^\dagger)(Q+Q^\dagger)|E\rangle$$

$$H = \left(\frac{1}{2}(Q-Q^\dagger)\right)^2 \rightarrow \text{same chain implies } E=0 \text{ iff } (Q-Q^\dagger)|E\rangle=0$$

$$(2) \quad E=0 \text{ iff } Q|E\rangle=0 \text{ and } Q^\dagger|E\rangle=0$$

$$\| (Q+Q^\dagger)|E\rangle \| \geq 0 \rightarrow E=0 \text{ iff } (Q+Q^\dagger)|E\rangle=0$$

(1')

$$E=0 \text{ iff } \begin{cases} Q|E\rangle=0 \\ Q^\dagger|E\rangle=0 \end{cases}$$

Supersymmetric ground states

in a system w/ unbroken SUSY, $E_0 = 0$.

$$\|(Q+Q^\dagger)|E\rangle\| \geq 0 \Rightarrow E=0 \text{ iff } (Q+Q^\dagger)|E\rangle=0$$

$$(Q-Q^\dagger)|E\rangle=0$$

our system

$N=2$ SUSY QM

$$Q_1 = Q + Q^\dagger$$

$$Q_2 = i(Q - Q^\dagger)$$

$$H = Q_1^2 = Q_2^2$$

$$\{Q_1, Q_2\} = 0$$

$SO(2)$
automorphism
(acts on (i, j))

$$\{Q_i, Q_j\} = \delta_{ij} H, \quad i, j = 1, 2$$

More complicated SUSY QM w/ "inte

$$[\phi, x] = -i;$$

$$\psi(x) = \begin{pmatrix} \psi_1(x) \\ \psi_2(x) \end{pmatrix}$$

$$H = \frac{1}{2} p^2 + \frac{1}{2} (W'(x))^2 + \frac{1}{2} \sigma_3 W''(x)$$

$W(x)$ — superpotential

More complicated SUSY QM w/ "interactions"

$$[\phi, x] = -i;$$

$$\psi(x) = \begin{pmatrix} \psi_1(x) \\ \psi_2(x) \end{pmatrix}$$

states w/ $\psi_2 = 0$

"Fermionic"

states w/ $\psi_1 = 0$

"Bosonic"

$$H = \frac{1}{2} P^2 + \frac{1}{2} (W'(x))^2 + \frac{1}{2} \sigma_3 W''(x)$$

$W(x)$ - superpotential

$$Q_1 = \frac{1}{2} \sigma_1 P + \frac{1}{2} \sigma_2 W'(x)$$

$$Q_2 = \frac{1}{2} \sigma_2 P - \frac{1}{2} \sigma_1 W'(x)$$

↓ check!

$$\{Q_1, Q_2\} = \delta_{12} H$$

↓ "fermion #"

$$F = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} = \frac{1}{2} (\mathbb{I}_2 + \sigma_3)$$

$$\text{also } [F, H] = 0$$

$$[F, Q] = Q$$

$$[F, Q^+] = -Q^+$$

↑ CHECK 4 signs !!
(h.s. signs !!)

define operator $(-1)^F$ = +1 on bosonic states

-1 on fermionic states

$$(-1)^F H = H (-1)^F \rightarrow \text{all states are either B or F.}$$

$$(-1)^F Q = -Q (-1)^F \quad \text{— when } Q \text{ acts on a B(F) state it makes it into F(B) state}$$

$$(-1)^F |B\rangle = |B\rangle$$

$$(-1)^F |F\rangle = -|F\rangle$$

So ready to show that all $E > 0$ ^{eigen}states
 are doubly degenerate ($\forall |B\rangle$ -eigenstate
 is an $|F\rangle$ eigenstate w/ same energy).

let $|E\rangle$ be an eigenstate $H|E\rangle = |E\rangle E$

then $\langle E|E\rangle = 1$

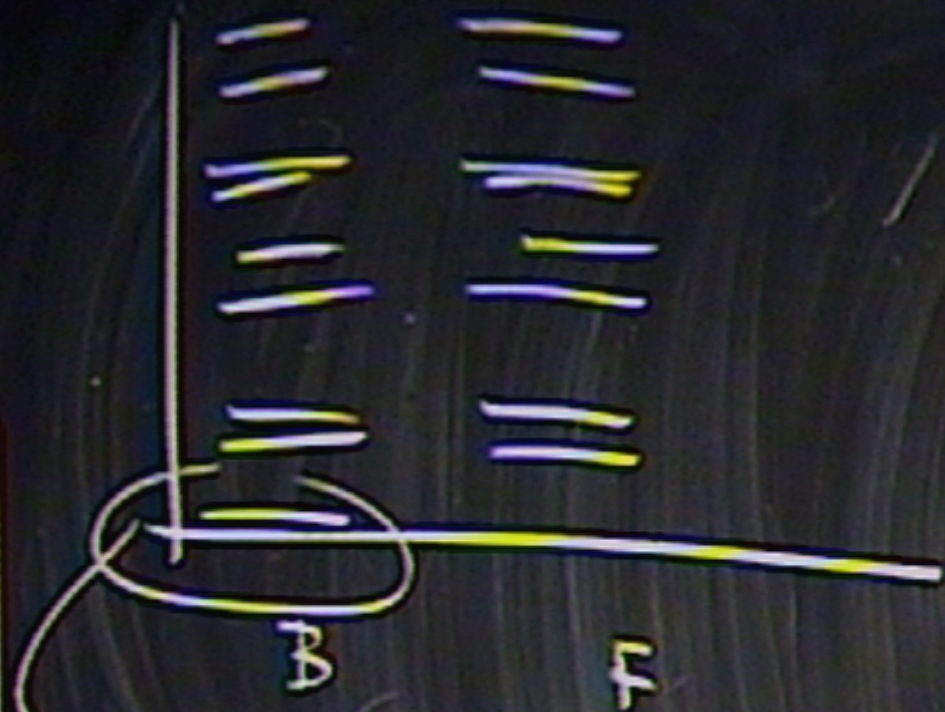
$|E,1\rangle = \frac{Q_1|E\rangle}{\sqrt{E}}$ is also an eigenstate of same E

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$|E,1\rangle = \frac{Q_1|E\rangle}{\sqrt{E}}$ is also an eigensstate of same E & opposite $(-)^K$



$E \leq 0$ states need not be paired (B/F)
 $E > 0$ states are always paired!



$\bar{E} \leq 0$ states need not be paired (B/F)
 $\bar{E} > 0$ states are always paired!



$\bar{E} \leq 0$ states need not be paired (B/F)
 $\bar{E} > 0$ states are always paired!

For a system with given H ($W(x)$, $\hbar$)
about the properties of its ground state?

(this case) can we say smth
(i.e. does it have $E_0 = 0$
- is the ground state?)

(B/F)

For a system with given H ($W(x)$), sh
about the properties of its ground state

(turns out we can ~~in SUSY theories~~ a general
criterion can be given based on Witten index)
 $H|\psi\rangle = 0 \rightarrow$ since $Q_1^2 = Q_2^2 = H \Rightarrow H|\psi\rangle = 0$ iff
(if $|\psi\rangle$ is a SUSY ground state, then must hold d_1)

this case) can we say smthg (w/out solving
the system)
? (i.e. does it have $E_0 = 0$ or not —
— is the ground state SUSY c ?)
criterion can be given based on Witten index

$$Q_1|\psi\rangle = 0$$

For a system with given H ($W(x)$), the
about the properties of its ground state

(turns out we can ~~use~~ SUSY theories a general
criterion can be given based on W within which
 $H|\psi\rangle=0 \rightarrow$ since $Q_1^2=Q_2^2=H \Rightarrow H|\psi\rangle=0$ iff
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this case) can we say something (w/out solving
the system)
? (i.e. does it have $E_0=0$ or not —
— is the ground state SUSY c. ?)
criterion can be given based on W within which
 $Q_1|\psi\rangle=0$ — 2nd order $\rightarrow$ 1st order!

$$Q_1 |\psi\rangle = 0 \Rightarrow \left(i \sigma_1 \frac{d}{dx} + \sigma_2 W'(x) \right) \begin{pmatrix} \psi_1(x) \\ \psi_2(x) \end{pmatrix} = 0$$

Show:

$$\psi_0(x) = \begin{pmatrix} \psi_1(x) \\ \psi_2(x) \end{pmatrix} = e^{\sigma_3 W(x)} \begin{pmatrix} c_1 \\ c_2 \end{pmatrix} = \begin{pmatrix} e^{W(x)} c_1 \\ e^{-W(x)} c_2 \end{pmatrix}$$

$$Q_1 |\psi\rangle = 0 \Rightarrow \left(i \sigma_1 \frac{d}{dx} + \sigma_2 W'(x) \right) \begin{pmatrix} \psi_1(x) \\ \psi_2(x) \end{pmatrix} = 0$$

Show:

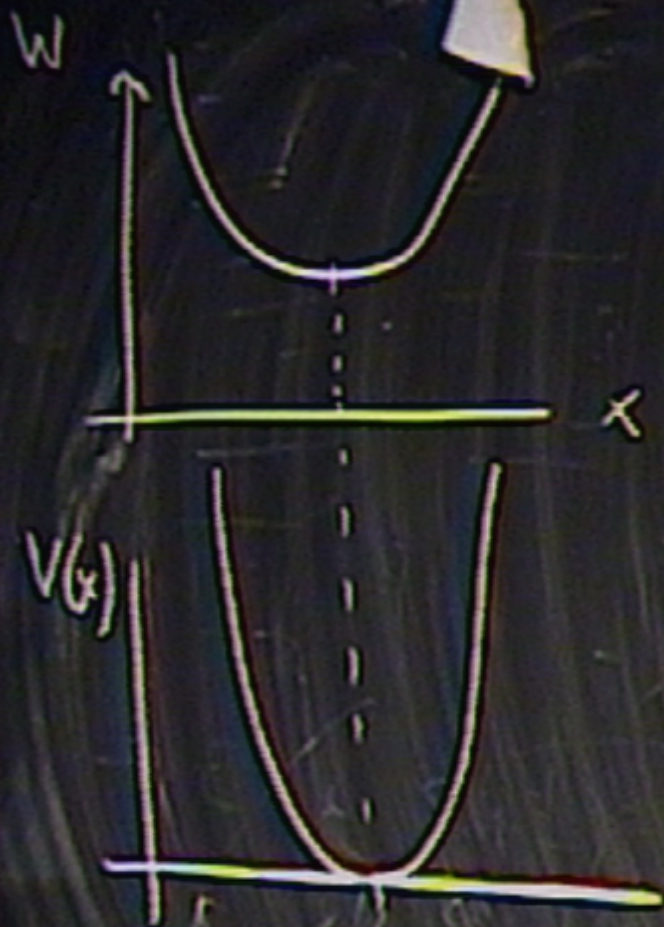
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Is it normalizable?

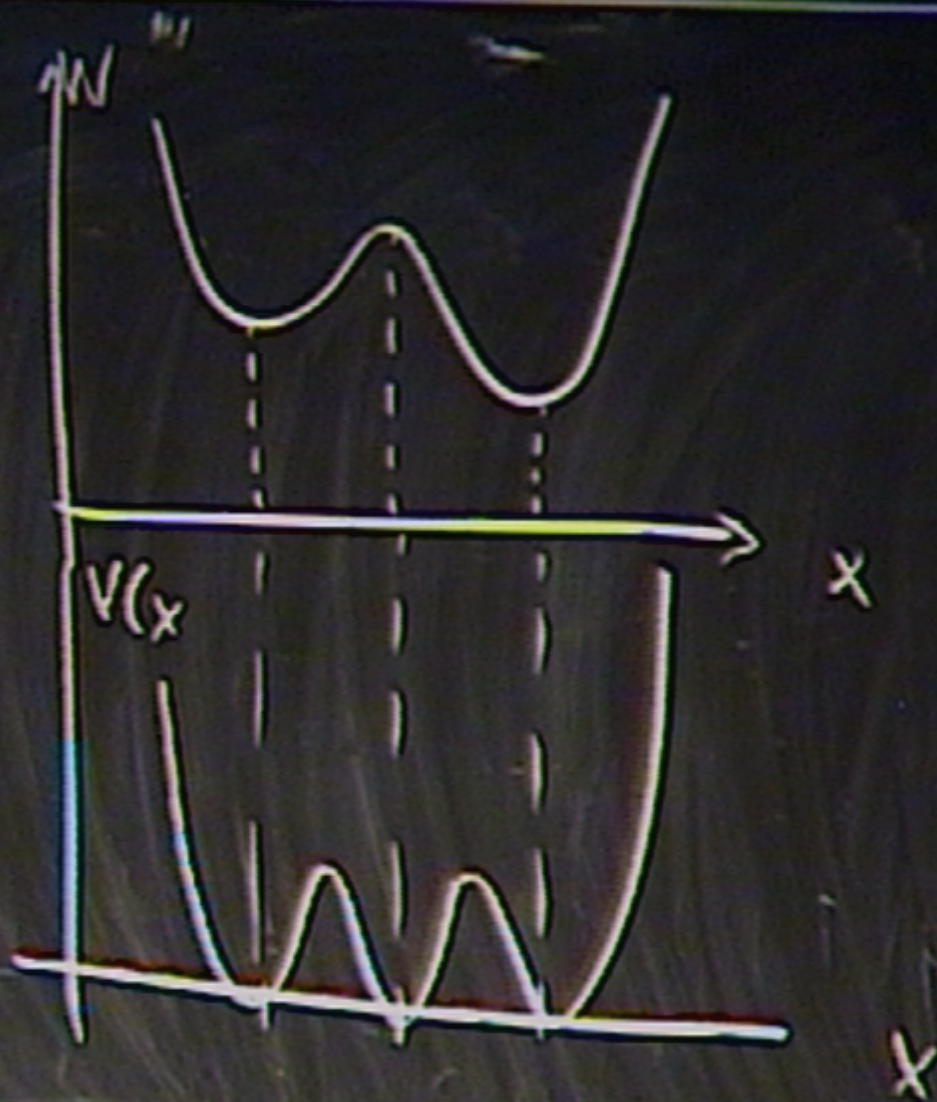
(1) If $c_1 = 0$, then $W(x) \rightarrow +\infty$ as $|x| \rightarrow \infty$

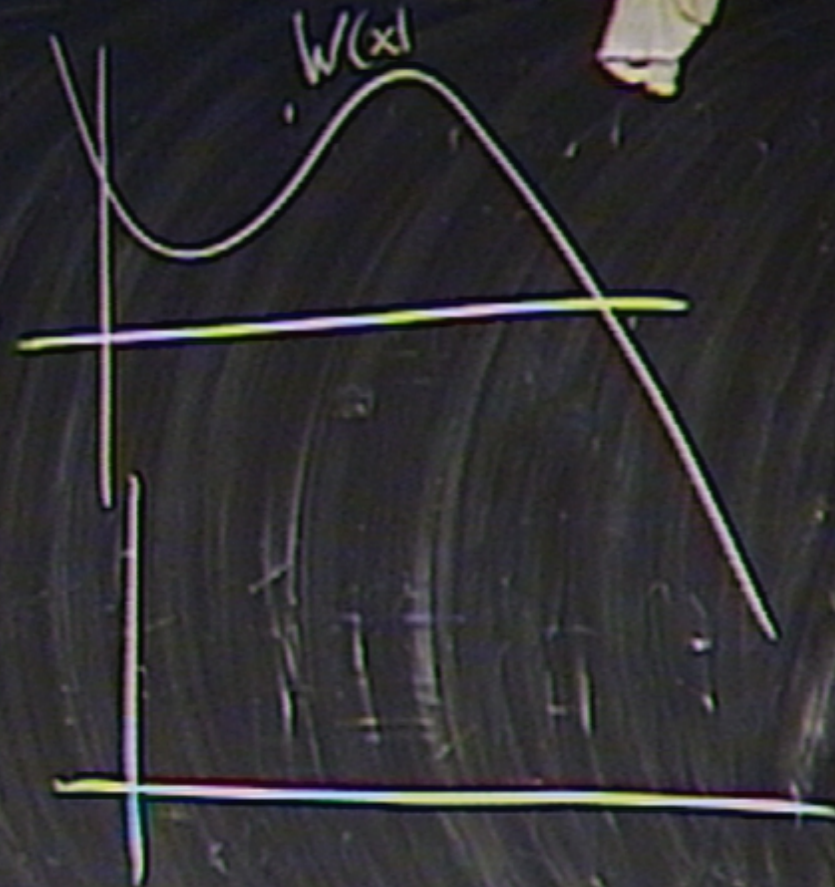
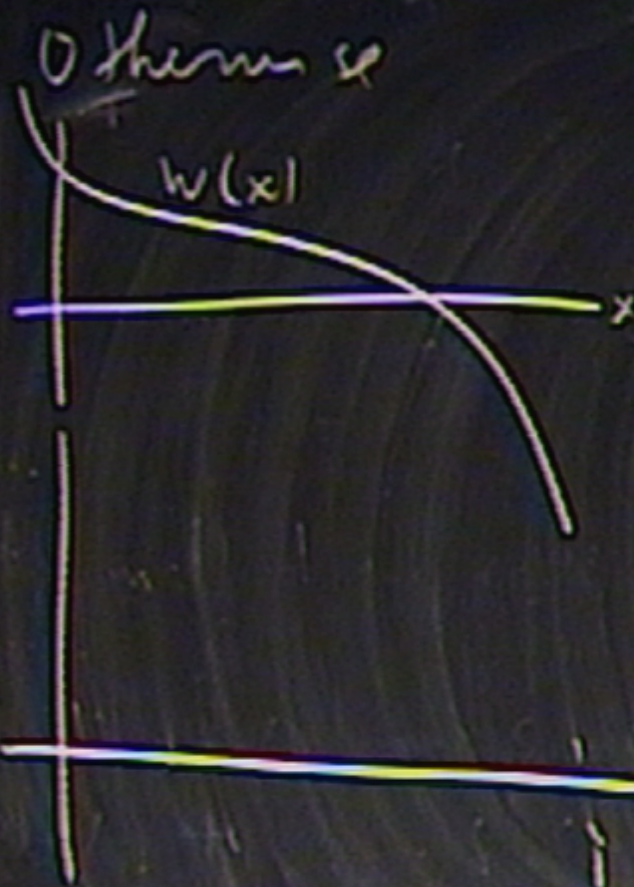
(2) If $c_2 = 0$, then $W(x) \rightarrow -\infty$ as $|x| \rightarrow \infty$

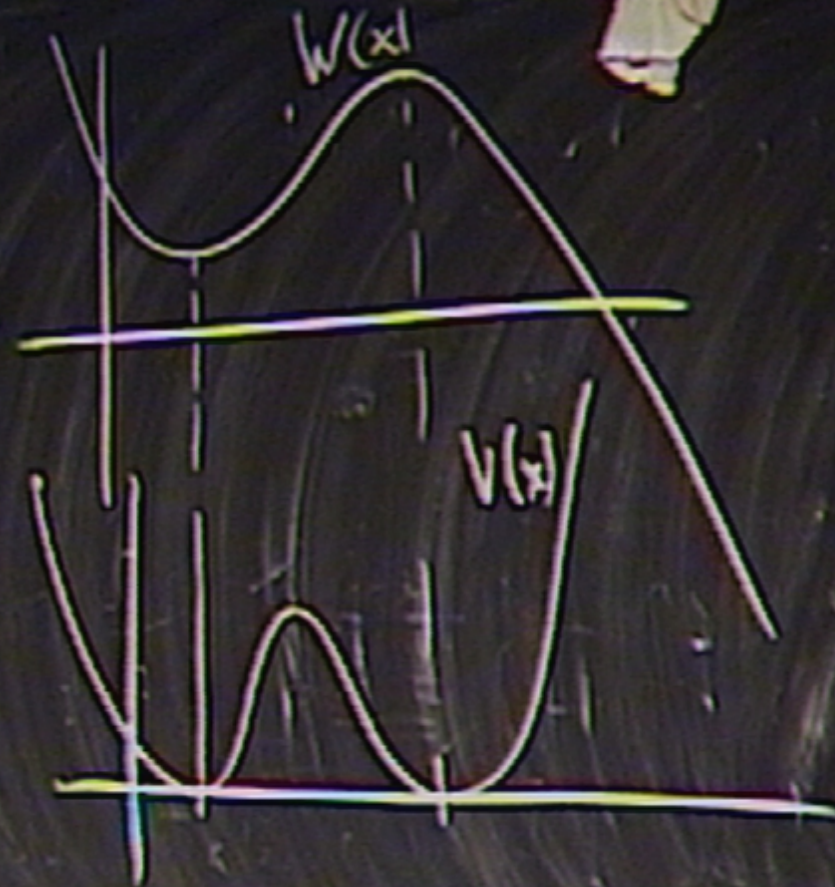
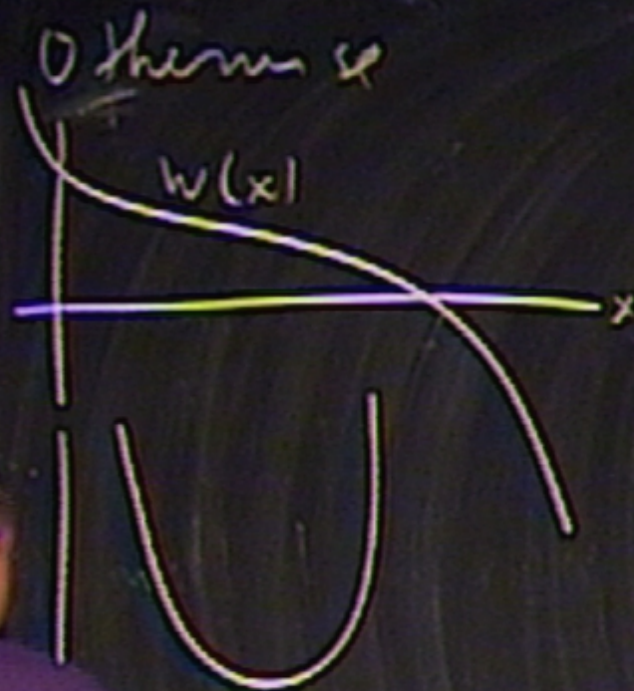
$\Rightarrow$ Summary: $W(x)$ is "even at ∞ "

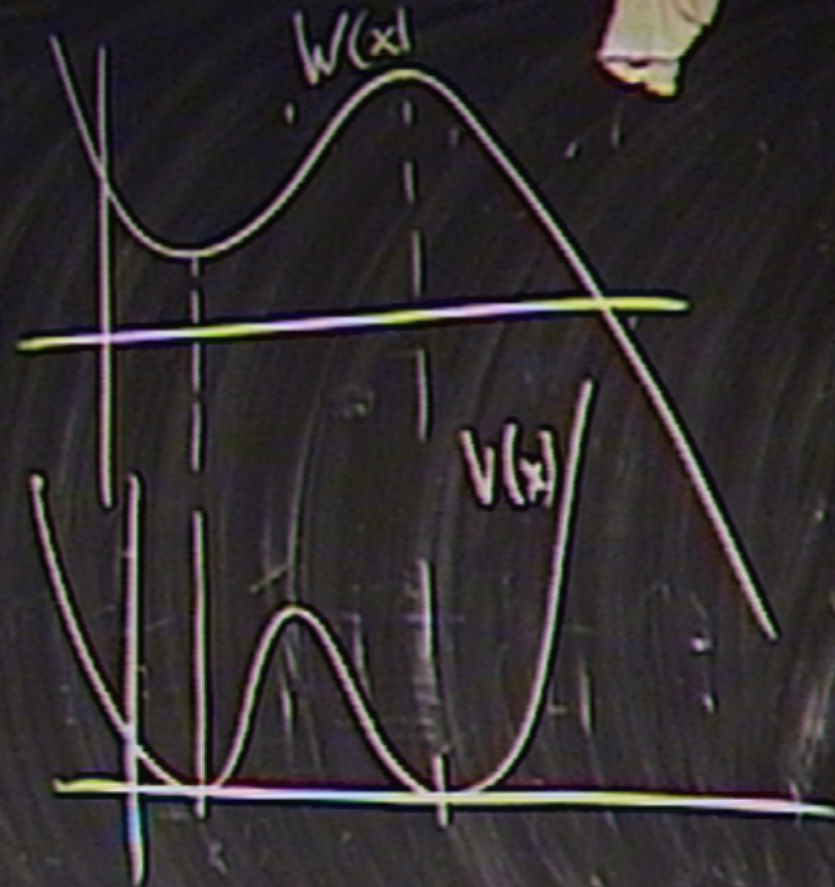
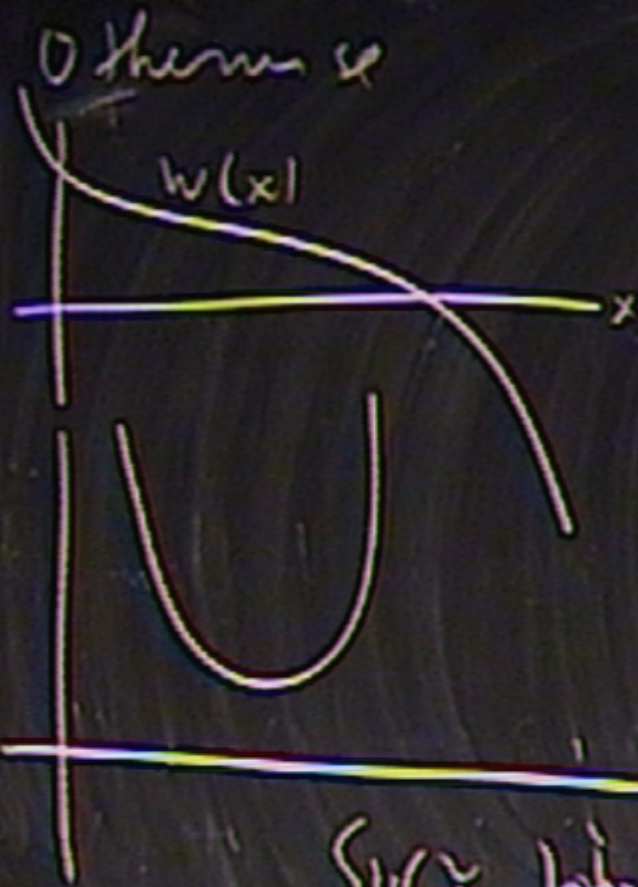


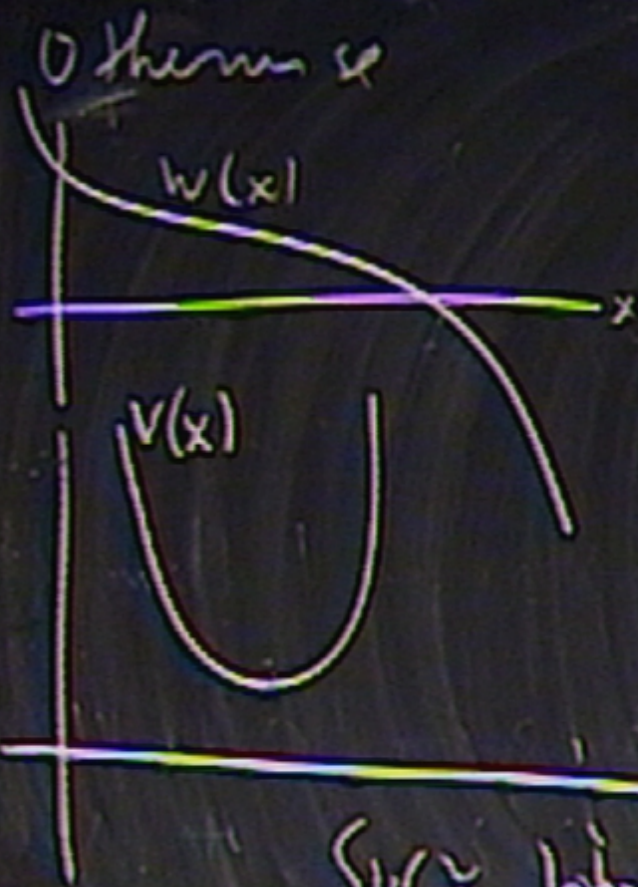
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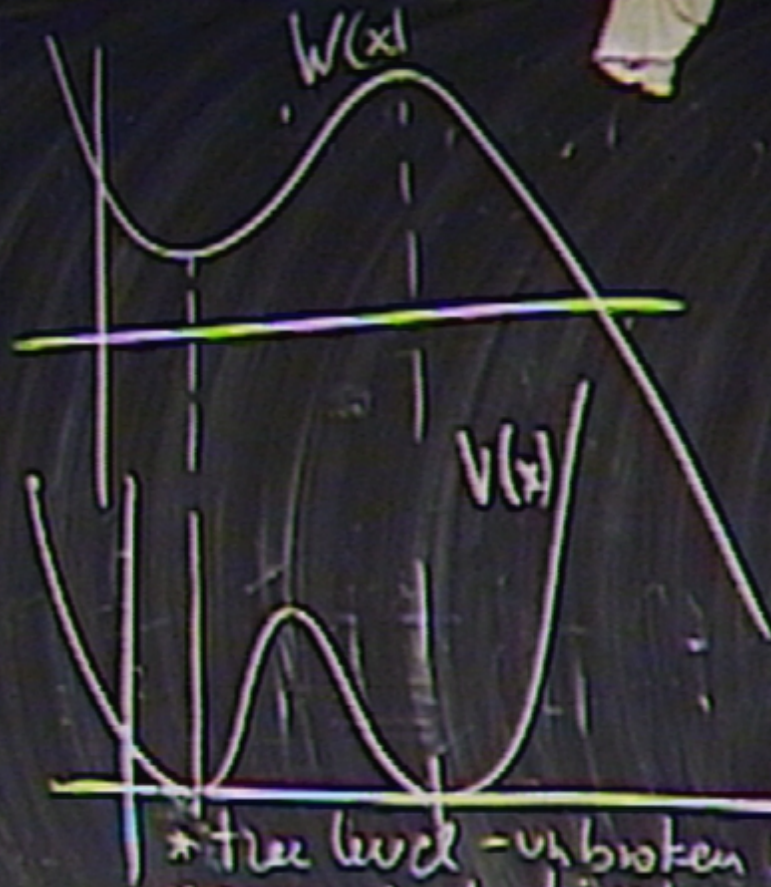








Susy broken



* tree level - unbroken Susy
 * nonperturbatively - broken

Witten index

$$\text{tr}(-1)^F =$$

Witten index

$$\begin{aligned}\text{tr}(-1)^F &= \sum_E n_B(E) - n_F(E) = \\ &= n_B(0) - n_F(0)\end{aligned}$$

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$$\Rightarrow \text{if } \text{tr}(-1)^F = 0$$

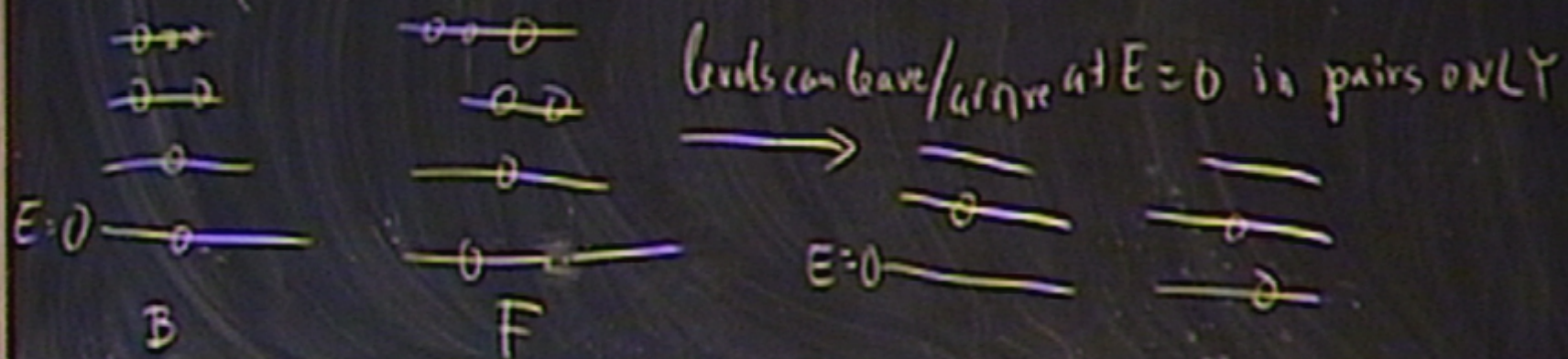
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$$\Rightarrow \text{if } \text{tr}(-1)^F = 0 \text{ SUSY can break}$$

$\text{tr}(-)^F$ is invariant under continuous deformations of the theory -



Witten index

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In 4d $\mathcal{N}=1$
pure SYM

$$\Rightarrow \text{if } \text{tr}(-1)^F = 0 \text{ SUSY can break}$$

SUSY
broken

Witten index

$$\text{tr}(-1)^F = \sum_E n_B(E) - n_F(E) =$$

$$= n_B(0) - n_F(0) \Rightarrow \underline{\text{if } \text{tr}(-1)^F \neq 0, \text{ then SUSY is unbroken}}$$

In 4d QFT

pure SYM

$$\text{tr}(-1)^F = r + 1$$

↑
rank of the group

$$\Rightarrow \text{if } \text{tr}(-1)^F = 0 \text{ SUSY can break}$$