Title: Fault tolerant quantum dynamical decoupling

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Abstract:

Perimeter Institute

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Fault Tolerant Quantum Dynamical Decoupling

quant-ph/0408128

Daniel Lidar
with Kaveh Khodjasteh

Center for Quantum Information and Quantum Control CQIQC

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Outline

Motivation: Preserving an unknown quantum state in the presence of decoherence

- **Context: Open quantum systems & decoherence**
- Solution 1: Periodic Dynamical decoupling (DD) with ideal pulses
- Solution 2: Periodic DD with real (faulty) pulses – it doesn't work so well
- Solution 3: Concatenated DD
  - ★ Analytical theory
  - ★ Numerical simulations on a spin-bath
- Generalizations & Implications

Open Systems Control Problem

Controlled evolution of **system** + **bath** given by the following **ideal** Hamiltonian:

$$H = (H_S^{\text{int}} + H_{\text{Control}}) \otimes I_B + I_S \otimes H_B + \overbrace{\sum S_\alpha \otimes B_\alpha}^{H_{SB}}$$

H_{SB} causes **decoherence** (non-unitary evolution due to entanglement w/ bath) and (unitary) **control errors**

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Instrument control errors

$$H_{\text{Control}} \mapsto H_C + \underbrace{W_C}_{\text{stochastic and/or systematic}}$$

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Pulse: $U(\delta) = T\exp(-i\int_0^\delta H(t)dt)$

How to overcome ***both*** decoherence and faulty controls?

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Methods to overcome decoherence

- Quantum error correcting codes
- Continuous feedback control
- Decoherence-free subspaces
- Dynamical decoupling pulses

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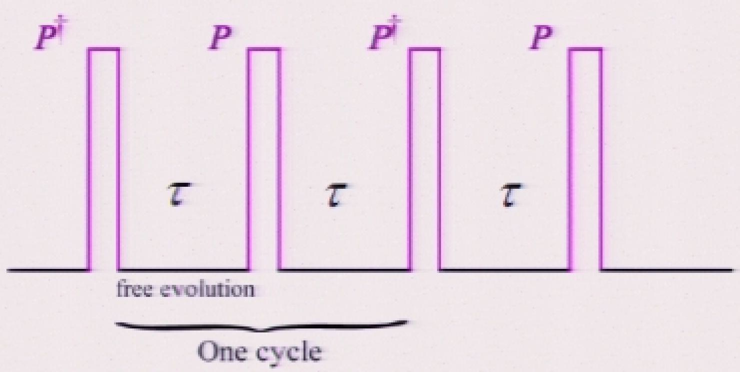
Time Reversal: The Canonical DD cycle

A pulse producing a unitary evolution P, such that

$$PHP^{\dagger} = -H \qquad \text{i.e., } \{P, H\} = 0$$

(Carr-Purcell)



The ***time reversal one***(two)***-liner***:

$$P \exp(-i\tau H) P^{\dagger} \exp(-i\tau H) = \exp(-i\tau PHP^{\dagger}) \exp(-i\tau H)$$
$$= \exp(i\tau H) \exp(-i\tau H) = I$$

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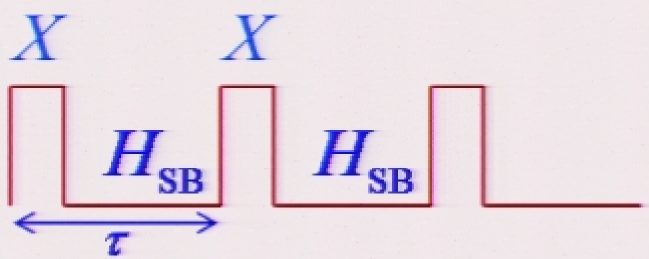

$$H_{SB} = \lambda Z \otimes B$$

$XZX = -Z \quad \Rightarrow$
"time reversal",
H_{SB} averaged to zero
(in 1st order Magnus expan.)

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Universal Dynamical Decoupling

Approximately remove a **general** H_{SB} from the evolution:

"Symmetrizing group" of pulses $\{ g_i \}$ and their inverses are applied in series:

$$(g_N^\dagger \mathtt{f} g_N)\cdots(g_2^\dagger \mathtt{f} g_2)(g_1^\dagger \mathtt{f} g_1) \approx \exp(-i\tau \sum_i g_i^\dagger H_{SB} g_i)$$

$$\mathtt{f} \equiv \exp(-iH_{SB}\tau)$$

first order Magnus expansion

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Dynamical Decoupling Condition

(more generally: projection into group commutant)

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Dynamical Decoupling Condition

(more generally: projection into group commutant)

For a qubit the Pauli group $G=\{X,Y,Z,I\}$ (π pulses around all three axes) removes an *arbitrary* H_{SB} :

$$(\mathtt{XfX})(\mathtt{YfY})(\mathtt{ZfZ})(\mathtt{IfI}) = \underline{\mathtt{XfZfXfZf}}$$

(we will focus on qubits)

Periodic DD: periodic repetition of the universal DD pulse sequence

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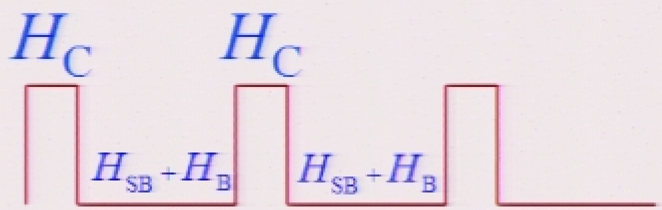
Ideal vs Real Pulses

Ideal:



$H_{SB}=\lambda Z\otimes B,\quad H_C=X$

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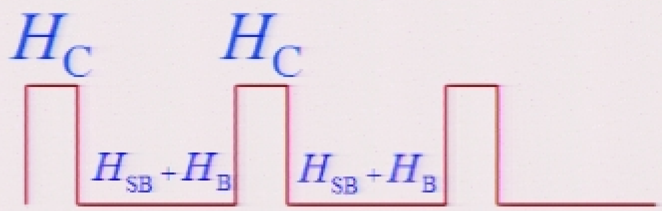

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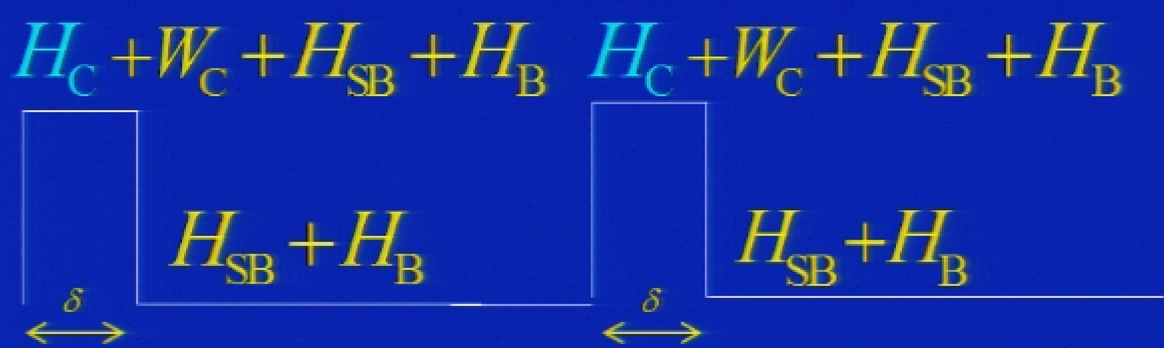
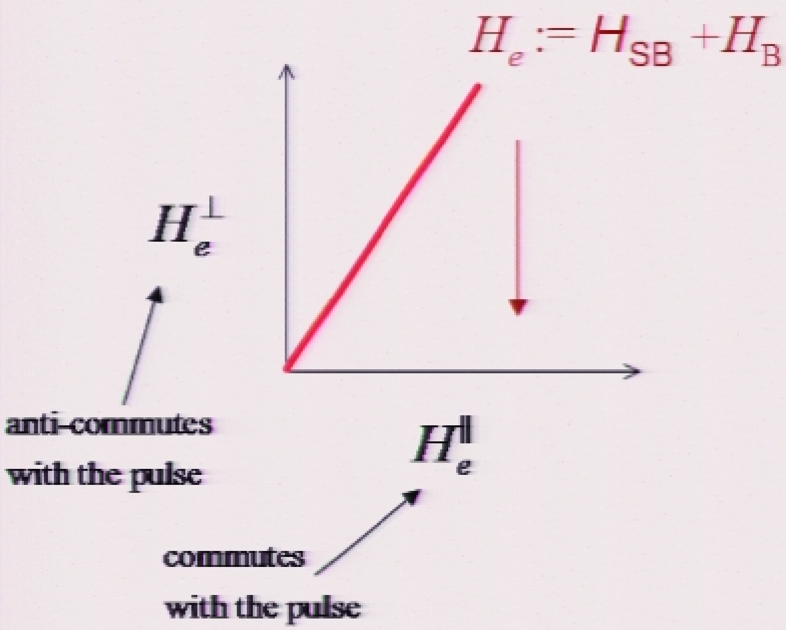
Real:



Note: true also for QEC, considered in fault-tolerance setting.

$$\left.\begin{matrix}[H_{SB}, H_B] \neq 0 \\ [H_{SB}, H_C] \neq 0 \\ [H_{SB}, W_C] \neq 0\end{matrix}\right\}$$ **control errors!** (show up in 2nd order Magnus expansion)

The effect of 2nd order Magnus errors for **zero-width** pulses: Canonical DD as projection + rotation



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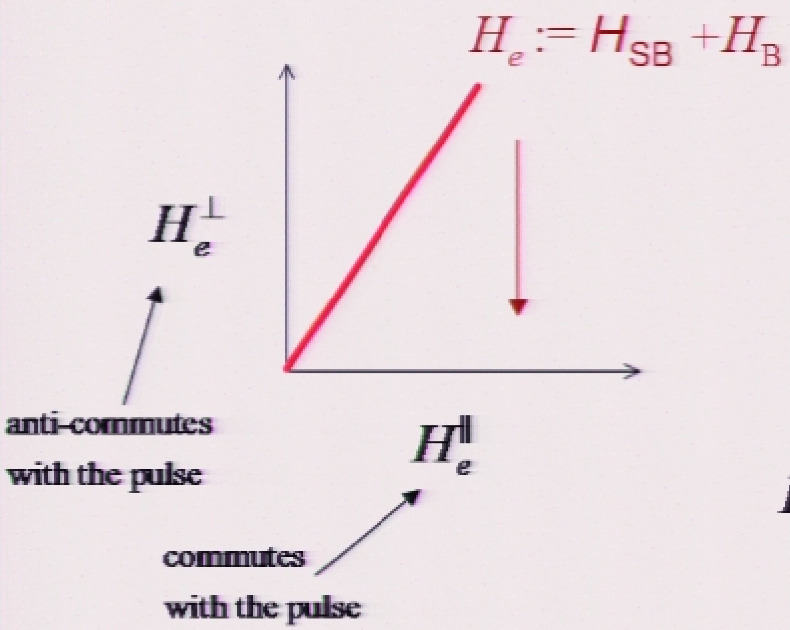
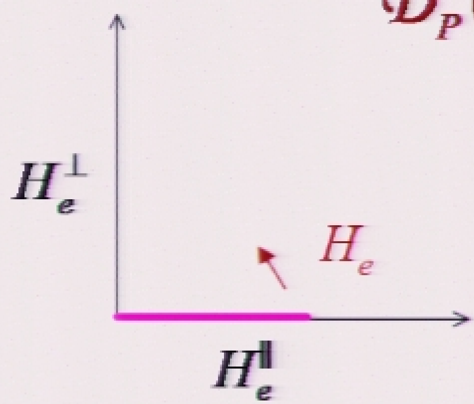


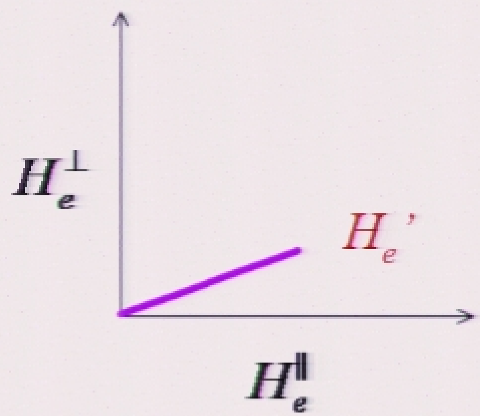
Suzuki-Trotter expansion:

$$H_e \mapsto H_e' = \mathcal{D}_P(H_e) + O(\lambda^3)$$

$$\mathcal{D}_P(H_e) = e^{-i\tau H_e^{\perp}/2} H_e^{\parallel} e^{+i\tau H_e^{\perp}/2}$$

$$\lambda^3 := \tau^3 \left\| H_e^{\perp} \right\|^2 \left\| H_e^{\parallel} \right\|$$





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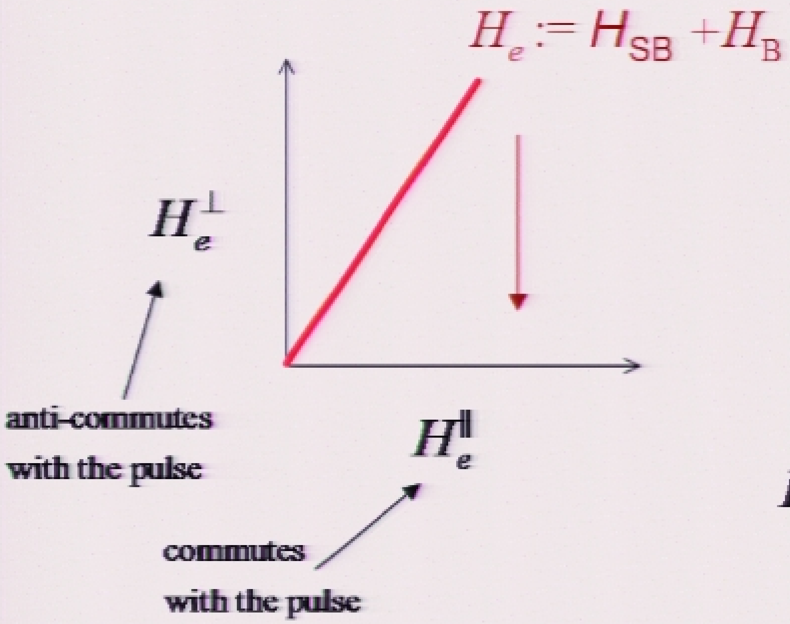

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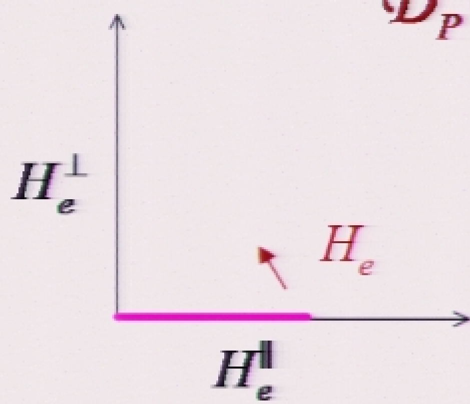
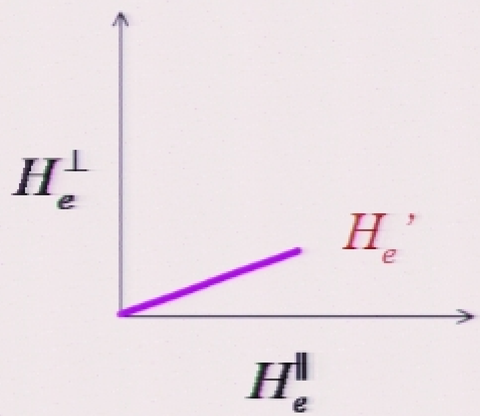


Lemma:

$D_P(H_e)$ is a norm-decreasing map

$\Rightarrow$ H_e is renormalized by the DD procedure



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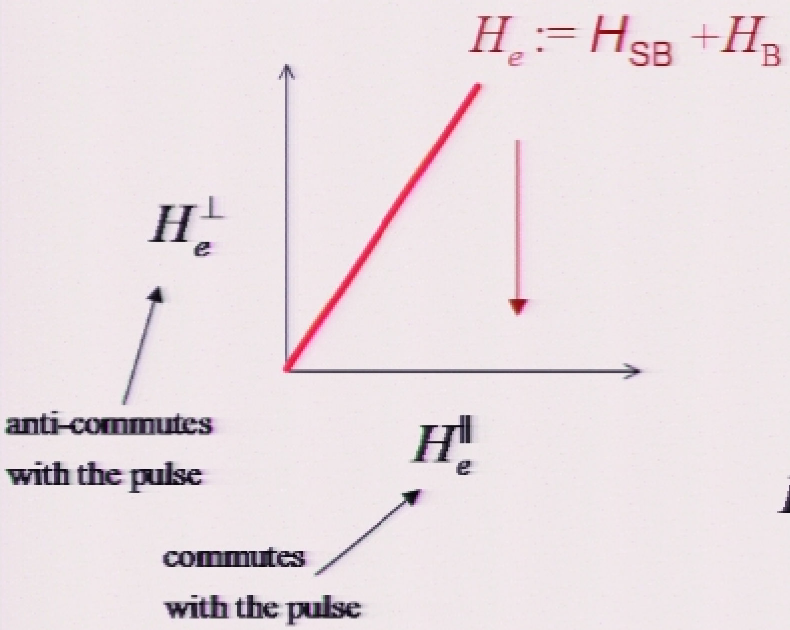

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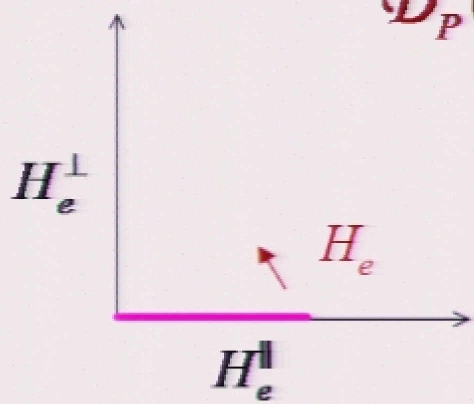

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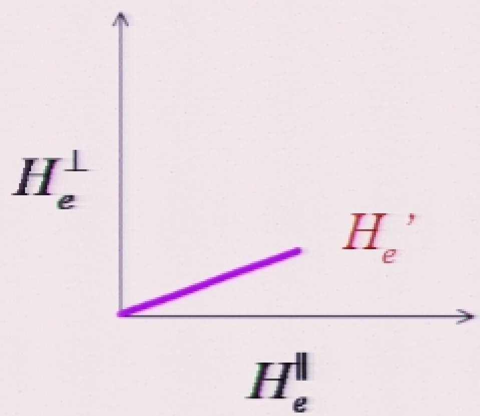

$\Rightarrow$ H_e is renormalized by the DD procedure

Concatenate BB sequences! (as in QEC)

Renormalization $\Rightarrow$ effective H_e shrinks super-exponentially(?)

total pulse sequence time grows exp.

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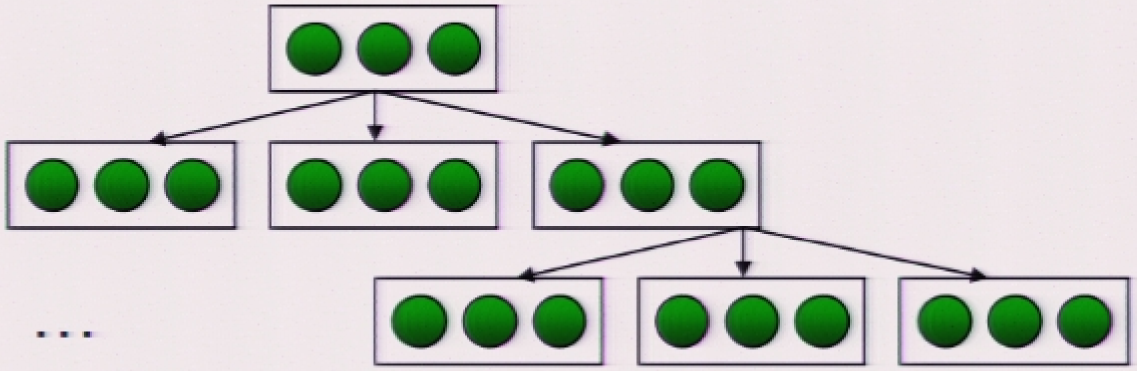
Concatenation and Renormalization

- Why do concatenated Quantum Error Correcting Codes work so well?

$n=0 \quad p_{\text{eff}} = p$

$n=1 \quad p_{\text{eff}} = p^2$

$n=2 \quad p_{\text{eff}} = p^{2^2} \quad \ldots$



$p_{\text{eff}} = p^{2^n}$, code size grows only(!) exponentially

$\therefore$ Effective system-bath interaction is renormalized (?)

Can this be done without encoding, fault-tolerantly?

Yes: Repeat concatenation idea in time rather than space.

Concatenated Universal Dynamical Decoupling

Nest the universal DD pulse sequence into its own free evolution periods `f` :

```
p(1)= X f   Z f   X f   Z f
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p(1)= X f   Z f   X f   Z f
p(2)= X p(1)Z p(1)X p(1)Z p(1)
```

etc.

Level	Concatenated DD Series after multiplying Pauli matrices
1	XfZfXfZf
2	fZfXfZfYfZfXfZffZfXfZfYfZfXfZf
3	XfZfXfZfYfZfXfZffZfXfZfYfZfXfZfZfZfXfZfYfZfXfZffZfXfZfYfZfXfZfXfZf XfZfYfZfXfZffZfXfZfYfZfXfZfZfZfXfZfYfZfXfZffZfXfZfYfZfXfZf

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Geometry of Concatenated DD Pulse Sequences

Successive projections and rotations converge quickly to zero:

Convergence of Concatenated vs Periodic Zero-Width Pulses

$$H_e = I_S \otimes B_0 + \sum_{\alpha = X,Y,Z} \sigma_\alpha \otimes B_\alpha$$

Assumptions:

$$\|B_0\| > \|B_X\|, \|B_Y\|, \|B_Z\|$$

$$\|B_0\| T = c \ll 1$$

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$$H_e = I_S \otimes B_0 + \sum_{\alpha=X,Y,Z} \sigma_\alpha \otimes B_\alpha$$

"Infidelity" after max. # of levels of concatenation ($T = 4^n\tau = N\tau$)

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Super-polynomial advantage for CDD!

$$\frac{1-f_{\mathrm{CDD}}}{1-f_{\mathrm{PDD}}} \leq \frac{\left(c\|B_0\|\tau\right)^{-\log_4 \|B_0\|\tau/c}}{4\left(\|B_0\|\tau\right)^2} \xrightarrow{\|B_0\|\tau \to 0} 0$$

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Why the difference?

-In PDD errors can build up if not completely removed in the basic cycle.

-In CDD the next-layer-up removes errors that were left from the lower layer; this will work if errors are not too large: threshold (just as in concatenated QEC)

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Test System: The *Spin-Bath* (Model for GaAs, P/Si, …)

A spin-1/2 qubit as the system, coupled to N-1 other interacting spin-1/2s:

$$H = \omega_s Z_1 + \omega_b \sum_{i=2}^{N} Z_i + \sum_{i>j}^{i,j<N} j_{ij} H_{ij}$$

where $H_{ij} = X_i X_j + Y_i Y_j + Z_i Z_j$ is the Heisenberg interaction

and j_{ij} is an exponentially decaying exchange constant.

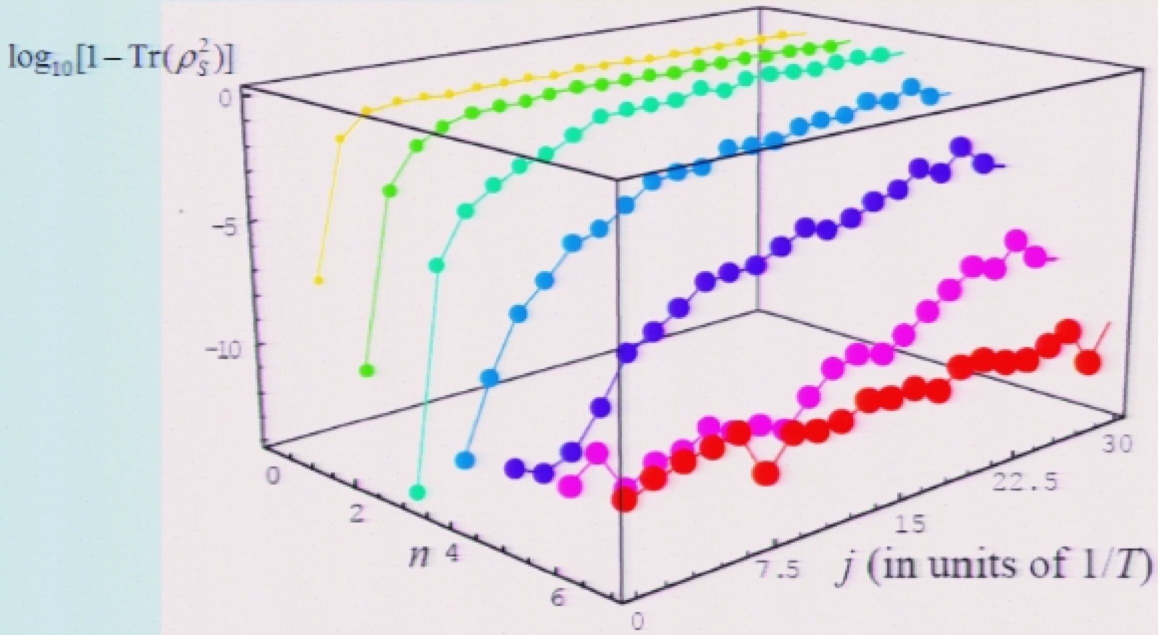
Numerically exact solution,
compute Error Measure: $e = 1\text{-}\mathrm{Tr}[\rho_s^{\,2}]$.
(Non-Markovian bath : revivals, so e oscillates)

Pulsing Apparatus:

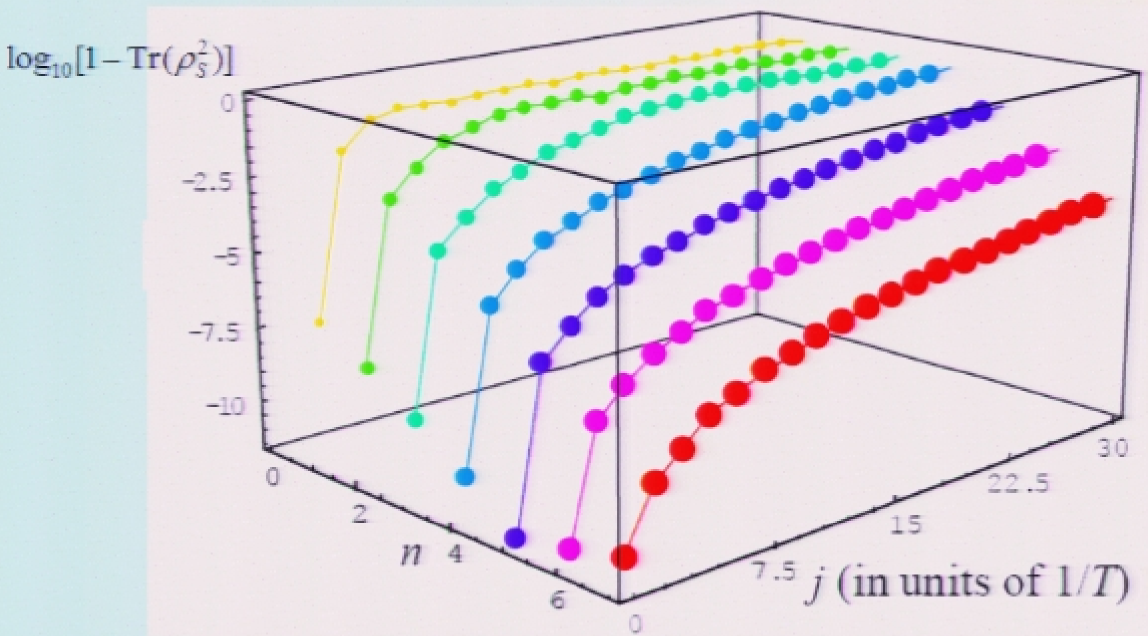
- Limited switching times
- Faulty controls

We compare Periodic/Concatenated DD/Pulse-free
- pulse-free evolution, total time T
- concatenated DD, and
- periodic DD with the same pulse interval τ, pulse width δ

Numerical Results – CDD vs PDD as Function of Coupling j



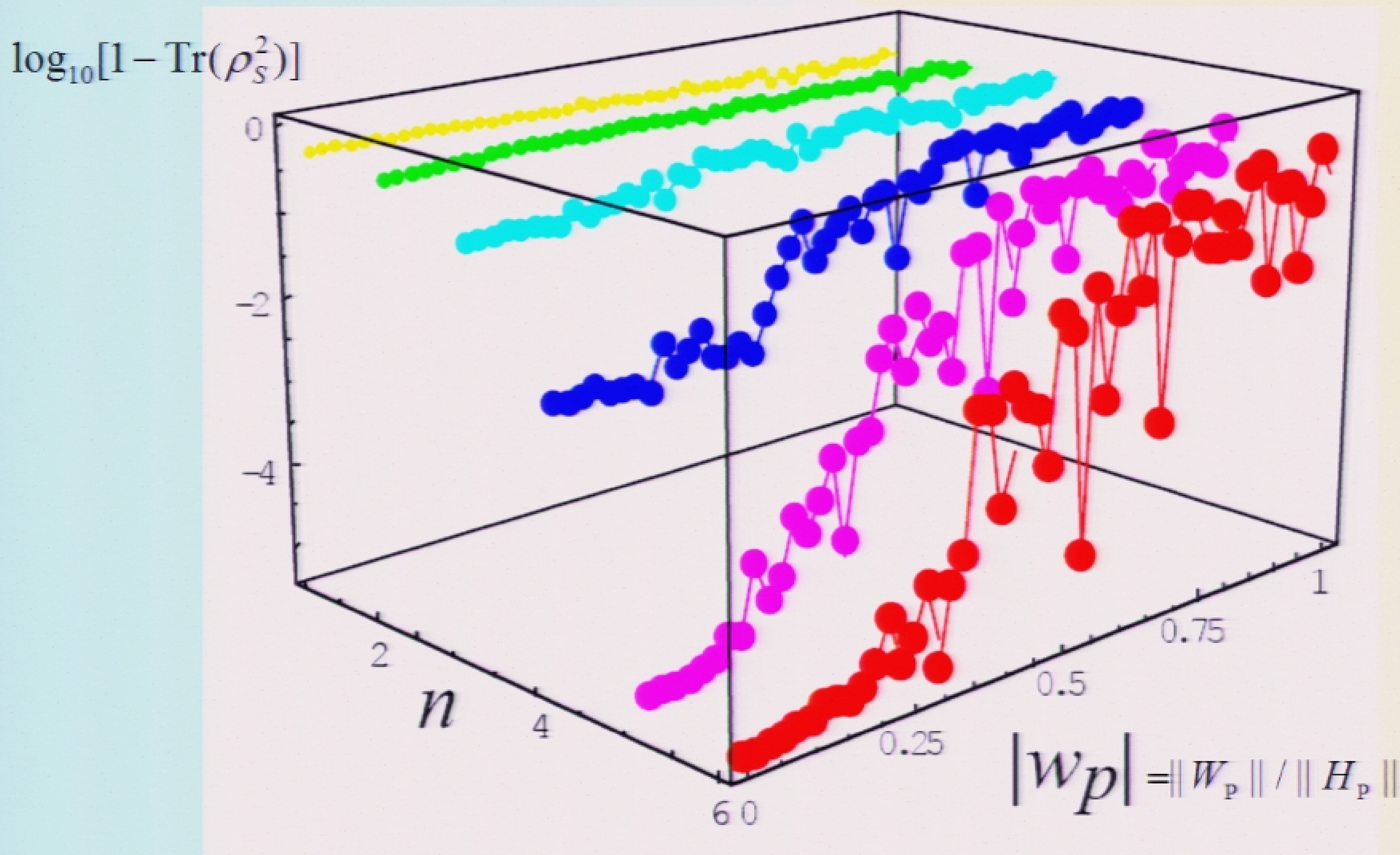
Concatenated DD



Periodic DD

$\delta = 10^{-4} T, \;\; N = 5$

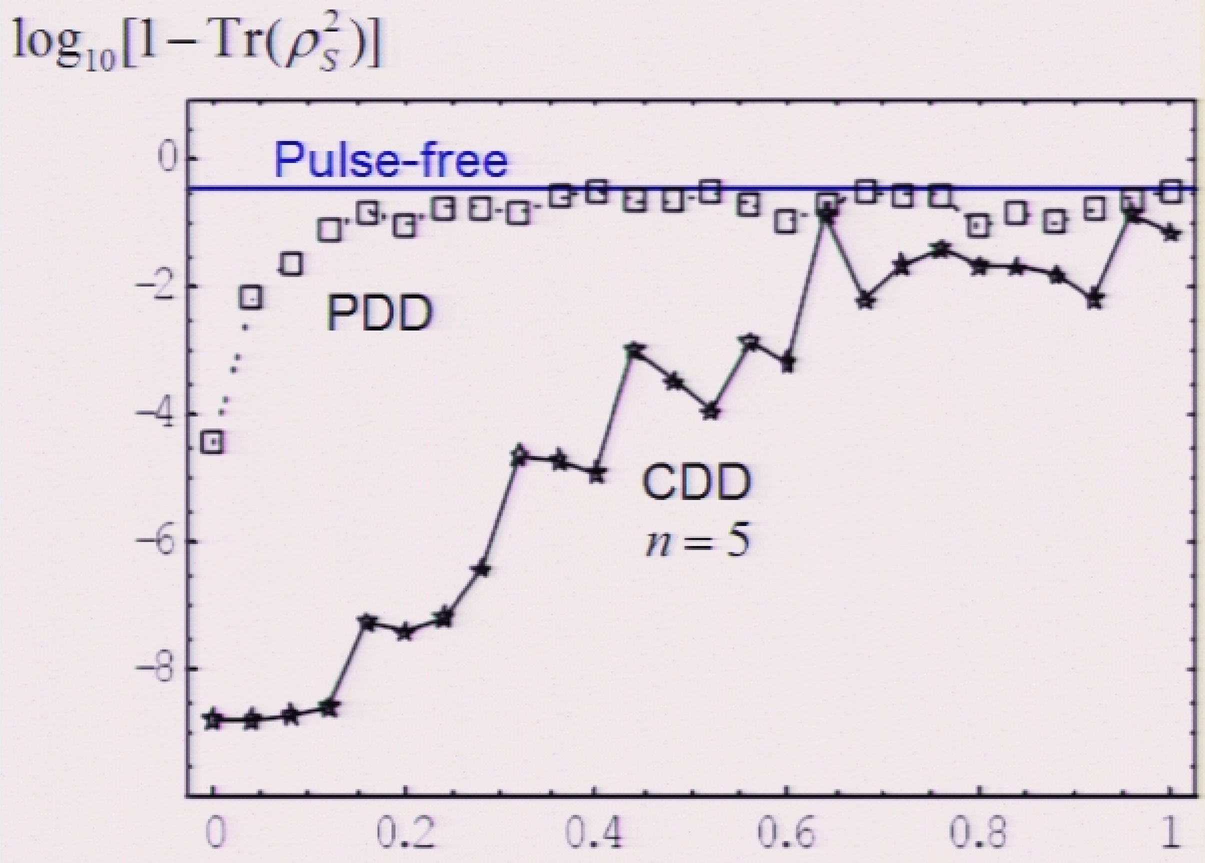
Numerical Results – Concatenated DD for Systematic Noise



$\delta = 10^{-4}T,\ \ jT = 15.0,\ \ N = 5$

averaged over 3 noise realizations

Numerical Results -- CDD vs PDD for Systematic Noise

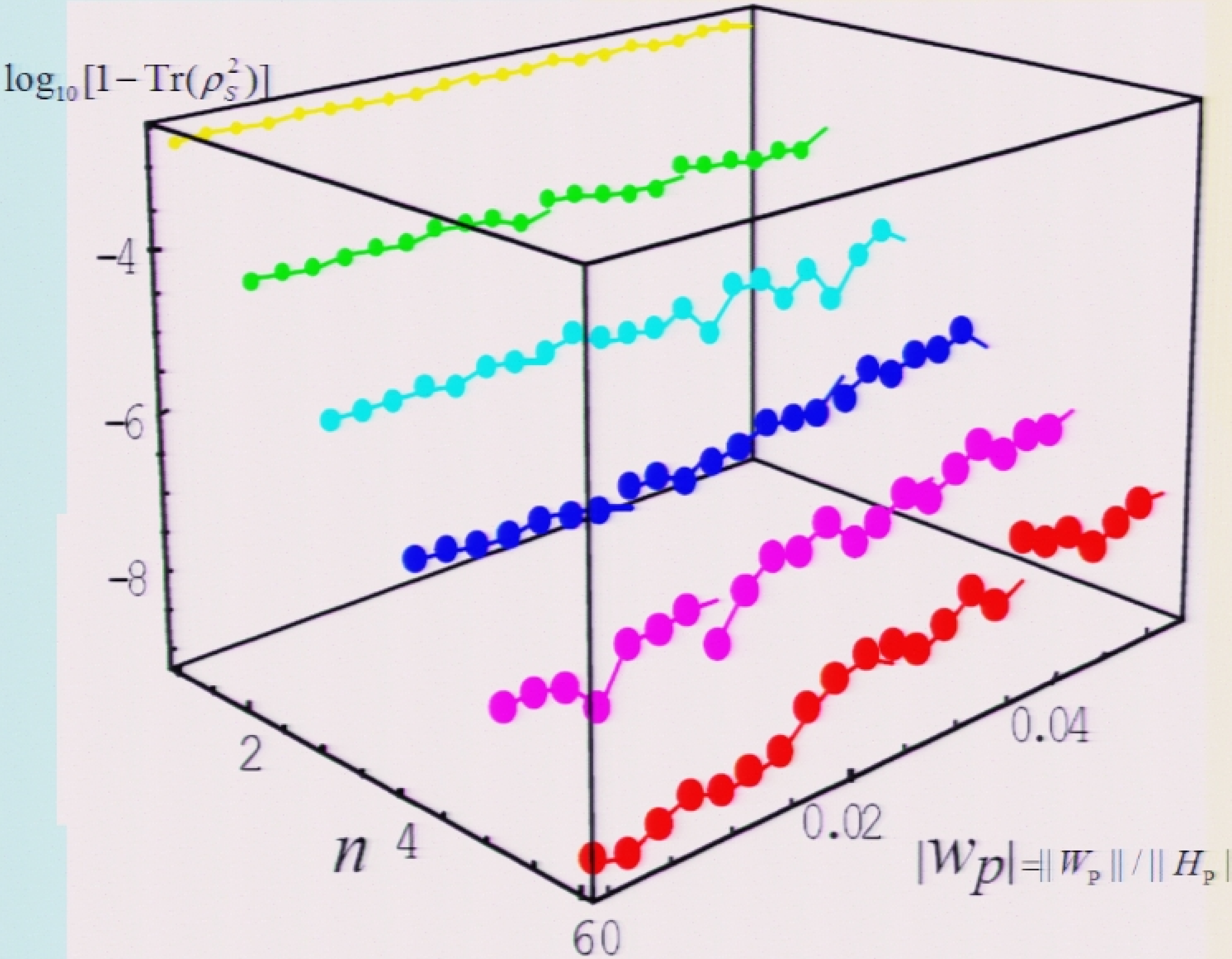


$|w_P| = \| W_P \| / \| H_P \|$

$\delta = 10^{-5}T,\ \ jT = 3.0,\ \ N = 5$

averaged over 10 noise realizations

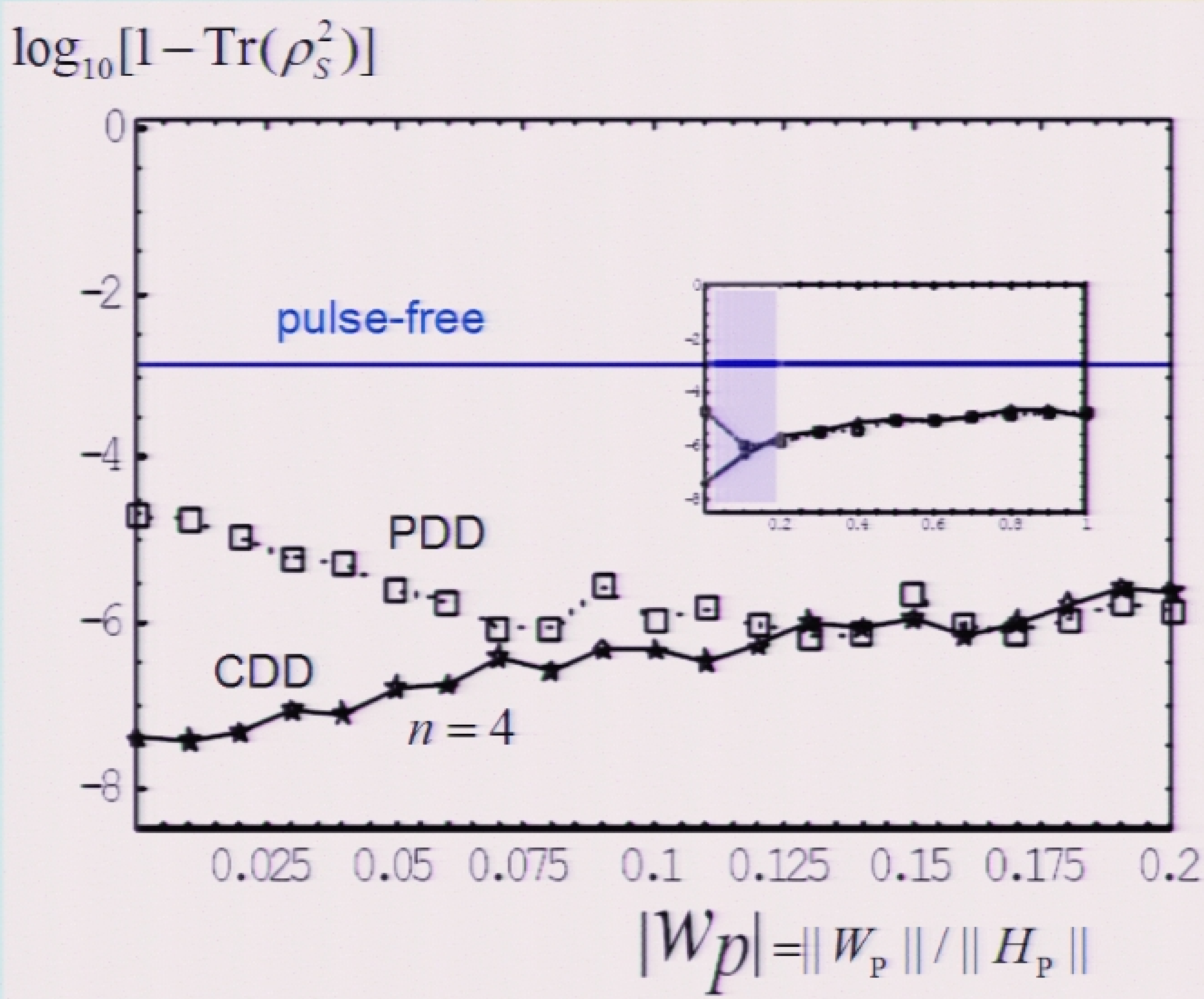
Numerical Results – CDD with Random Noise



$\delta = 10^{-5} T,\ jT = 0.2,\ N = 2$

averaged over 30 noise realizations

Numerical Results – CDD vs PDD with Random Noise



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Directions for the Future – What About **Computation**?

DD pulses can interfere with computational pulses.

How can they be reconciled?

- Use encoded qubits from a stabilizer error-correcting code.

  Then DD pulses can be chosen as stabilizer elements (time-reversal requires they anti-commute with errors), logic gates can be chosen as normalizer elements (they commute with stabilizer).

- Hybrid method: CDD combined with composite pulses.

Concluding Remarks

quant-ph/0408128

- Typical pulse-based decoupling methods use a given pulse-sequence periodically.
- A concatenated (recursively nested) pulse sequence is strictly advantageous at equal cost.
  - More robust against systematic and random control errors
  - Improved performance over wide range of couplings
- Can be used to dynamically generate symmetries, then combined with decoherence-free subspace encoding for (almost) full decoherence protection.
- Quantum error correcting codes needed against Markovian uncorrelated errors, where DD/DFS does not apply.

XfZfXfZfYfZfXfZffZfXfZfYfZfXfZfZfZfXfZfYfZfXfZffZfXfZfYfZfXfZfXfZfXfZf
YfZfXfZffZfXfZfYfZfXfZfZfZfXfZfYfZfXfZffZfXfZfYfZfXfZf