Title: Strong magnetic limit of string theory

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Abstract:

hep-th/0504187

$$t^2 = k_i^2 + g F_{12} (2\ell + 1 + S_i)$$

$\ell = 0, 1, 2, \ldots$

$g t_{ii} > 0$

hep-th/0504127

$$t^2 = k_1^2 + g\,\mathrm{Tr}(2\ell+1+S_z)$$

$\ell = 0, 1, 2, \ldots$

$g\,t_{11} > 0$

Type I D-strings in magnetic field.

$\alpha' = \frac{1}{2}$ $\quad S = \frac{1}{2\pi}\int d\tau\, d\sigma\,(\dot{X}^\mu X'_\mu - X'_\mu \dot{X}^\mu) = \frac{1}{\pi}\int d\tau\,\left[g_1 A_\mu \dot{X}^\mu(\sigma=0) + g_2 A_\mu \dot{X}^\mu(\sigma=\pi)\right]$

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$$E^2 = k_z^2 + q F_{12} (2l + 1 + S_z)$$

$l = 0, 1, 2,$

$q F_{12} > 0$

Type I dring in magnetic field.

$\alpha' = \frac{1}{2}$ $$S = \frac{1}{2\pi} \int d\tau\, d\sigma \left(X^\mu \dot{X}_\mu - X'_\mu X'^\mu \right) - \frac{1}{\pi} \int d\tau \left(q_1 A_\mu \dot{X}^\mu(\sigma=0) + q_2 A_\mu \dot{X}^\mu(\sigma=\pi) \right)$$

$$X'_\mu = q_1 F_{\mu\nu} \dot{X}^\nu \quad \sigma = 0$$

$$X'_\mu = -q_2 F_{\mu\nu} \dot{X}^\nu \quad \sigma = \pi$$

$$Z_1 = \frac{1}{\sqrt{2}} (X_1 + i X_2)$$

$$Z(0) = -i\kappa \dot{Z}(0) \qquad \kappa = q_1 F_{12}$$

$$Z(\pi) = i\rho \dot{Z}(\pi) \qquad \rho = q_2 F_{12}$$

$$X'_\mu = -q_\nu F_{\nu\mu} X^\nu \qquad \sigma = \pi$$

$$\dot{z}(0) = -i\alpha\, \dot{z}(0) \quad , \quad \alpha = q_1 F_{12}$$

$$\dot{z}(\pi) = i\beta\, \dot{z}(\pi) \qquad \beta = q_2 F_{12}$$

$$z = i \sum_{n>0} a_n \psi_n(z,\sigma) - \sum_{n>0} b_n^\dagger \psi_n(z,\sigma) \quad , \quad [a_n, a_m^\dagger] = \delta_{nm}$$

$$[b_n, b_m^\dagger] = \delta_{nm}$$

$$\psi_n = \frac{1}{|n-\varphi|^{1/2}} \cos[(n-\varphi)\sigma + \pi]\, e^{-i(n-\varphi)\tau}$$

$$\varphi = \frac{1}{\pi}(\pi + \sigma') = \frac{1}{\pi}\left(\arctan q_1 F_{12} + \arctan q_2 F_{12}\right)$$

$X'_\mu = -q_2 F_{\mu\nu} X \qquad \sigma = \pi \qquad \dot{z}(\pi) = i\beta\, \dot{z}(\pi) \qquad \beta = q_2 F$

$$z = i \sum_{n=1}^{\infty} a_n \psi_n(z,\sigma) - \sum_{m=0}^{\infty} b'_m \psi_m(z,\sigma) \quad , \quad [a_n, a'_m] = \delta_{nm} \quad [b_n, b'_m] = \delta_{nm}$$

$$\psi_n = \frac{1}{|n-\varphi|^{1/2}} \cos[(n-\varphi)\sigma + \pi]\, e^{-i(n-\varphi)\tau}$$

$$\varphi = \frac{1}{\pi}(\pi + \sigma') = \frac{1}{\pi}\left(\arctan q_1 F_{12} + \arctan q_2 F_{12}\right)$$

<u>Spectrum</u>

$$\boxed{\alpha' M^2 = \hat{N} - \varphi J_{12}} \qquad 0 < \varphi < 1$$

$$\hat{N} = N - a \quad , \; a = \begin{cases} \frac{1}{2} & \text{NS} \\ 0 & \text{R} \end{cases}$$

$$J_{12} = S_{12} - l - \frac{1}{2}$$

$$S_{12} = \sum_{n=1}^{\infty} \left(a^{\dagger}_{n_1} a_{n_1} - a^{\dagger}_{n_2} a_{n_2} \right) + \text{fermion oscillators}$$

$$\hat{N} = \sum_{n=1}^{\infty} n \left(a^{\dagger}_{n_1} a_{n_1} + a^{\dagger}_{n_2} a_{n_2} \right) + \cdots$$

$$\hat{N} = N - a \quad , \quad a = \begin{cases} \frac{1}{2} & NS \\ 0 & R \end{cases}$$

$$J_{12} = S_{12} - \ell - \frac{1}{2}$$

$$S_{12} = \sum_{n=1}^{\infty} \left(a^{\dagger}_{n+} a_{n+} - a^{\dagger}_{n-} a_{n-} \right) + \text{fermion oscillators}$$

$$\hat{N} = \sum_{n=1}^{\infty} n \left(a^{\dagger}_{n+} a_{n+} + a^{\dagger}_{n-} a_{n-} \right) + \cdots$$

oscillator $2\pi\alpha' q\, F_{12} \sim 2\pi\alpha' T_{12}$

$$\alpha' M^2 = \hat{N} - 2\alpha' T_{12}\, q \left(-\ell - \frac{1}{2} + S_{12} \right)$$

$\underline{\ell = 0}$

$S_z = 2$

$$E^2 = k_z^2 - 2 T_{12}$$

$$\hat{N} = N - a \quad , \quad a = \begin{cases} \frac{1}{2} & NS \\ 0 & R \end{cases}$$

$$J_{12} = S_{12} - \ell - \frac{1}{2}$$

$$S_{12} = \sum_{n=1}^{\infty} \left(a^{\dagger}_{n+} a_{n+} - a^{\dagger}_{n-} a_{n-} \right) + \text{(zero oscillator)}$$

$$\hat{N} = \sum_{n=1}^{\infty} n \left(a^{\dagger}_{n+} a_{n+} + a^{\dagger}_{n-} a_{n-} \right) + \cdots$$

oscillator $2\pi\alpha' q F_{12} \sim 2\pi\alpha' F_{12}$

$$\alpha' M^2 = \hat{N} - 2\alpha' F_{12}\, q \left(-\ell - \frac{1}{2} + S_{12} \right)$$

$\underline{\ell = 0}$

$S_z = 2$

$$E^2 = k_z^2 - 2 F_{12}$$

Highest spin

$$S = \hat{N} + 1$$

$$\hat{N} = N - a \quad , \quad a = \begin{cases} \frac{1}{2} & NS \\ 0 & R \end{cases}$$

$$J_{12} = S_{12} - \ell - \frac{1}{2}$$

$$S_{12} = \sum_{n=1}^{\infty} \left(a^{\dagger}_{n,-} a_{n,-} - a^{\dagger}_{n,+} a_{n,+} \right) + \text{fermion oscillators}$$

$$\hat{N} = \sum_{n=1}^{\infty} n \left(a^{\dagger}_{n,+} a_{n,+} + a^{\dagger}_{n,-} a_{n,-} \right) + \cdots$$

oscillator $2\pi\alpha' q F_{12} \sim 2\pi\alpha' F_{12}$

$$\alpha' M^2 = \hat{N} - 2\alpha' F_{12}\, q \left(-\ell - \frac{1}{2} + S_{12} \right)$$

$\underline{\ell = 0}$

$S_z = 2$

$$E^2 = k_z^2 - 2 F_{12}$$

Highest spin

$$S = \hat{N} + 1$$

$$|\phi_{k_{\mu}k}\rangle = \left(a^{\dagger}_{1+} \right)^{N} b^{\dagger}_{-1/2} |0\rangle$$

$$|\phi_{R,n,k}\rangle = (a^\dagger_{1+})^n \, b^\dagger_{-1/2} \, |0\rangle$$

$$\alpha' M^2 = n - \varphi\left(n + \tfrac{1}{2}\right)$$

$$M^2 < 0 \quad \text{if} \quad \varphi > \varphi_{cr} = \frac{n}{n + \frac{1}{2}}$$

$$\hat{F}_{ab} = \frac{1}{4} \gamma^{(1)}_{ab} F_{ij}$$

$$S^a \to S^a + \epsilon^a$$

$$F_{23} \pm F_{13} \pm F_{12} \pm F_{03} = 0$$

$$F_{03} = F_{12} \qquad \frac{1}{2} \text{ of maxwell.}$$

Spectrum

$$\alpha' M^2 = \hat{N} - \underset{\substack{| \\ E_{45}}}{\varphi_1} \hat{J}_1 - \underset{\substack{| \\ E_{67}}}{\varphi_2} \hat{J}_2$$

$J_1 = J_{45}$

$J_2 = J_{67}$

Spectrum

$$\alpha' M^2 = \hat{N} - \underset{\substack{| \\ E_{45}}}{\varphi_1} \hat{J}_1 - \underset{\substack{| \\ E_{67}}}{\varphi_2} \hat{J}_2 \qquad \begin{array}{l} J_1 = J_{45} \\ J_2 = J_{67} \end{array}$$

$$\alpha' M^2 = m(1-\varphi_1) - \frac{(\varphi_1 - \varphi_2)}{2} \Rightarrow M^2 < 0 \qquad m < \frac{\varphi_1 - \varphi_2}{2(1-\varphi_1)}$$

$$\boxed{\varphi_1 = \varphi_2 \Rightarrow M^2 \geq 0}$$

$$S_1 + S_2 = N + 1$$

Spectrum

$$\alpha' M^2 = \hat{N} - \underset{\substack{| \\ E_{45}}}{\varphi_1} \hat{J}_1 - \underset{\substack{| \\ E_{67}}}{\varphi_2} \hat{J}_2 \qquad \begin{array}{l} J_1 = J_{45} \\ J_2 = J_{67} \end{array}$$

$$\alpha' M^2 = n(1-\varphi_1) - \frac{(\varphi_1 - \varphi_2)}{2} \Rightarrow M^2 < 0 \qquad n < \frac{\varphi_1 - \varphi_2}{2(1-\varphi_1)}$$

$$\varphi_1 = 0 = \varphi_2$$

$$\boxed{\varphi_1 = \varphi_2 \Rightarrow M^2 \geq 0}$$

$$\therefore s_1 + s_2 = N+1 \Rightarrow \alpha' M^2 = n(1-\varphi_1)$$

Spectrum

$$\alpha' M^2 = \hat{N} - \varphi_1 \hat{J}_1 - \varphi_2 \hat{J}_2$$

$\varphi_1 \to E_{45}$, $\varphi_2 \to E_{67}$

$J_1 = J_{45}$

$J_2 = J_{67}$

$$\alpha' M^2 = m(1-\varphi_1) - \frac{(\varphi_1-\varphi_2)}{2} \Rightarrow M^2 < 0 \qquad m < \frac{\varphi_1-\varphi_2}{2(1-\varphi_1)}$$

$\ell_1 = 0 = \ell_2$

$$\boxed{\varphi_1 = \varphi_2 \Rightarrow M^2 \geq 0}$$

$$S_1 + S_2 = N+1 \Rightarrow \alpha' M^2 = m(1-\varphi_1)$$

$\varphi = 1-\epsilon$, $\epsilon \to 0$

$\alpha' = \epsilon \alpha'_{\parallel}$,

$$\Rightarrow \boxed{\alpha'_{\parallel} M^2 = N}$$

Spectrum

$$\alpha' M^2 = \hat{N} - \varphi_1 \hat{J}_1 - \varphi_2 \hat{J}_2$$

$\varphi_1 \to E_{67}$, $\varphi_2 \to E_{47}$

$J_1 = J_{45}$
$J_2 = J_{47}$

$$\alpha' M^2 = m(1-\varphi_1) - \frac{(\varphi_1 - \varphi_2)}{2} \Rightarrow M^2 < 0 \qquad m < \frac{\varphi_1 - \varphi_2}{2(1-\varphi_1)}$$

$\varphi_1 = 0 = \varphi_2$

$$\boxed{\varphi_1 = \varphi_2 \Rightarrow M^2 \geq 0}$$

$$\alpha' M^2 = m(1-\varphi_1)$$

$S_1 + S_2 = N+1 \Rightarrow$

$\varphi = 1 - \epsilon \qquad \epsilon \to 0$

$\alpha' = \epsilon \alpha'_{\parallel}$,

$$\Rightarrow \boxed{\alpha'_{\parallel} M^2 = N}$$

Any state with $J_1 + J_2 < N$

Spectrum

$$\alpha' M^2 = \hat{N} - \varphi_1 \hat{J}_1 - \varphi_2 \hat{J}_2$$

$\varphi_1 \to F_{45}$, $\varphi_2 \to F_{67}$

$J_1 = J_{45}$

$J_2 = J_{67}$

$$\alpha' M^2 = m(1-\varphi_1) - \frac{(\varphi_1 - \varphi_2)}{2} \Rightarrow M^2 < 0 \qquad m < \frac{\varphi_1 - \varphi_2}{2(1-\varphi_1)}$$

$\varphi_1 = 0 = \varphi_2$

$$\boxed{\varphi_1 = \varphi_2 \Rightarrow M^2 \geq 0}$$

$$\alpha' M^2 = m(1-\varphi_1)$$

$S_1 + S_2 = N+1 \Rightarrow$

$\varphi = 1 - \epsilon \qquad \epsilon \to 0$

$\alpha' = \epsilon \alpha'_{\parallel}$,

$$\Rightarrow \boxed{\alpha'_{\parallel} M^2 = N}$$

Any state with $J_1 + J_2 < N$

$$\alpha'_{\parallel} M^2 = \frac{1}{\epsilon}(\hat{N} - J_1 - J_2)$$

$$\Rightarrow M^2 < 0 \qquad n < \frac{\varphi_1 - \varphi_2}{2(1-\varphi_1)}$$

$$\ell_1 = 0 = \ell_2$$

$$n(1-\varphi_1)$$

$$\epsilon \to 0$$

$$\boxed{M^2 = N}$$

Any other state has

$$J_1 + J_2 < N$$

$$\alpha'_{2\parallel} M^2 = \frac{1}{\epsilon}(\hat{N} - J_1 - J_2)$$

$$\to \infty$$

$q + q' = 0$

$\alpha' M^2 = N$

$\Rightarrow \alpha'_{(1)} M^2 = \frac{N}{2}$

$q + q' = 0$

$\alpha' M^2 = N$

$\Rightarrow \alpha'_{eff} M^2 = \frac{N}{2}$

$m_1 + m_2 = N \rightarrow$ N+1 states

$$|\Phi^J\rangle = (a^\dagger_{11})^{m_1} (a^\dagger_{12})^{m_2} \, b^\dagger_{1/2} |0\rangle_{NS}$$

NS

$$|\Phi^J\rangle = (a^\dagger_1)^{m_1} (a^\dagger)^{m_2} |\mu\rangle_R$$

R

$q + q' = 0$

$\alpha' M^2 = N$

$\Rightarrow \alpha'_{eff} M^2 = \frac{N}{2}$

$m_1 + m_2 = N \rightarrow N+1$ states

NS $\quad |\phi^J\rangle = (a^\dagger_{11})^{m_1} (a^\dagger_{11})^{m_2} \, b^\dagger_{1/2} |0\rangle_{NS}$

$2(N+1)$ states

R $\quad |\phi^J\rangle = (a^\dagger_1)^{m_1} (a^\dagger)^{m_2} |\mu\rangle_R$

8N bosons + 8N fermions.

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Type II in magnetic field

Melvin magnetic flux tube. 75 02 068

$$ds^2 = -dt^2 + dx_1^2 + d\rho^2 + \rho^2 (d\varphi + b\, dy) + dy^2 \quad , \quad y \sim y + 2\pi R$$

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Type II in magnetic field

Melvin magnetic flux tube. 75 DR 068

$$ds^2_{10} = -dt^2 + dx_i^2 + d\rho^2 + \rho^2 (d\varphi + b\, dy)^2 + dy^2 \quad , \quad y \sim y + 2\pi R$$

$$\rightarrow ds^2_7 = F^{1/2} \left(-dt^2 + dx_i^2 + dz^2 \right) + F^{-1/2} \rho^2 d\varphi^2$$

$$e^{\frac{\phi}{2}} = F \qquad A_\varphi = \frac{b\rho^2}{2F} \quad , \quad F = 1 + b^2 \rho^2 \; .$$

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Type II in magnetic field

Melvin magnetic flux tube. 75 02 068

$$ds^2_{10} = -dt^2 + dx_i^2 + d\rho^2 + \rho^2 (d\varphi + b\, dy)^2 + dy^2 \quad , \quad y \sim y + 2\pi R$$

$$\rightarrow ds^2_7 = F^{1/2} \left(-dt^2 + dx_i^2 + dz^2 \right) + F^{-1/2} \rho^2 d\varphi^2$$

$$e^{-\frac{\phi}{2}} = F \qquad A_\varphi = \frac{b\rho^2}{2F} \quad , \quad F = 1 + b^2 \rho^2 .$$

$z = \rho e^{i\varphi}$

$$L = \partial_i x_i \partial_i x_i + (\partial_x + ib\partial_y) z (\partial_x - ib\partial_y) z^* + \partial_y \partial_y$$

$$= \qquad z \to e^{-iby} z$$

$z = \rho e^{iy}$

$$\mathcal{L} = \partial x_i \partial x_i + (\partial_1 + ib\partial y) z (\partial - ib\partial y) z^* + \partial y \partial y$$

<u>Spectr</u> $\quad z = e^{-iby} z$

$$\alpha' M^2 = 2(N_R - \hat{\gamma} J_R) + 2(N_L + \hat{\gamma} J_L) + \frac{\alpha'}{R^2}(m - bRJ)^2 + \frac{\omega^2 R^2}{\alpha'}$$

$$N_R - N_L = \omega m$$

$z = \rho e^{iy}$

$$\mathcal{L} = \partial_i x_i \partial x_i + (\partial_t + ib\partial_y) z (\partial - ib\partial_y) z^* + \partial_y \partial_y$$

<u>Spectrum</u>

$$z = e^{-iby} z$$

$$\alpha' M^2 = 2(N_R - \hat{\gamma} J_R) + 2(N_L + \hat{\gamma} J_L) + \frac{\alpha'}{R^2}(m - bRJ)^2 + \frac{\omega^2 R^2}{\alpha'}$$

$N_R - N_L = \omega m$

$\hat{\gamma} = \gamma - L(\sigma)$

$\gamma = 2\sigma bR$

$J_R = S_R - \ell_z - \frac{1}{2}$

$J_L = S_L + \ell + \frac{1}{2}$

Susy configuration

$m = 0$

$N_R = N_L$

$S_R = N_R + 1$

$S_L = -N_L - 1$

$F_{45} = F_{67}$

$b_1 = b_2$

Susy configuration

$m = 0$

$N_R = N_L$

$S_R = N_R + 1$

$S_L = -N_L - 1$

$F_{\mu\nu} = \tilde{F}_{\theta\tau}$

$b_1 = b_2$

$$r^2 M^2 = 2\left(N_R - \hat{\theta}(\lambda_R + \lambda_{R2})\right) + 2\left(\hat{N}_L + \hat{\gamma}(\lambda_{L1} + \lambda_{L2})\right) + \frac{r^2}{R^2}\left(\omega - bR(\lambda_1 + \lambda_2)\right)^2 + \frac{\omega^2 R^2}{r^2}$$

$r = c\,\alpha_{\mu\nu}$, $bR = 1 - c$ $\Rightarrow$ $\hat{\gamma} = 1 - \omega c$

$N_R = N_L$

$S_R = N_R + 1$

$S_L = -N_L - 1$

$F_{45} = F_{67}$

$b_1 = b_2$

$$\alpha' M^2 = 2\left(N_R - \hat{\delta}(J_{R1} + J_{R2})\right) + 2\left(\hat{N}_L + \hat{\delta}(J_{L1} + J_{L2})\right) + \frac{\alpha'}{R^2}\left(m - bR(J_1 + J_2)\right)^2 + \frac{\omega^2 R^2}{\alpha'}$$

$\alpha' = \epsilon\, \alpha'_{(1)}$, $bR = 1 - \epsilon \Rightarrow \hat{\delta} = 1 - \omega\epsilon$, $R = \epsilon \bar{R}$

$$\boxed{\alpha'_{(1)} M^2 = 2(\hat{N}_R + N_L) + \frac{\alpha'_{(1)}}{\bar{R}^2}(\hat{N}_R - \hat{N}_L)^2 + \frac{\bar{R}^2}{\alpha'_{(1)}}}$$

$m = J_1 + J_2 = N_R - N_L$

$\Leftrightarrow \omega = 1$

$\alpha' M^2 = 2(N_R - \omega(\ldots))$

$\alpha' = \epsilon\,\alpha'_{\parallel}$, $bR = 1-\epsilon$ $\Rightarrow$ $\hat{\gamma} = 1 - \omega\epsilon$, $R = \epsilon \bar{R}$

$$\boxed{\alpha'_{\parallel} M^2 = 2(\hat{N}_R + N_L) + \frac{\alpha'_{\parallel}}{\bar{R}^2}(\hat{N}_R - \hat{N}_L)^2 + \frac{\bar{R}^2}{\alpha'_{\parallel}}}$$

$m = J_1 + J_2 = N_R - N_L$

$\Leftrightarrow \omega = 1$

$\omega = 0$

$\alpha' M^2 = 2(N_R + N_L)$

$\alpha'_{\parallel} M^2 = \frac{2}{\epsilon}(N_R + N_L) \to \infty$

except for $N_R = N_L = 0$

Finite temperature Partition function

$$Z = a_1 \beta \int_0^\infty \frac{dt}{t^4}\, \Theta_2\left(0 \middle| \frac{i\beta^2}{2\pi\alpha' t}\right) \prod_{I=1}^{4} f_I \frac{\Theta_2(it\hat{\varphi}_I|it)}{\Theta_1(it|\varphi_I(it))}$$

$$\varphi_1 = \varphi_2 = \varphi \qquad f = f_1 = f_2 = 2\pi\alpha' F_{\mu\nu}(q+\tilde{q})$$

$$\hat{\varphi}_1 = \hat{\varphi}_2 = 0$$

$$\hat{\varphi}_3 = \hat{\varphi}_4 = \gamma$$

$$\Rightarrow Z = \text{const}\, \beta \int_0^\infty \frac{dt}{t^4}\, \Theta_2\left(0 \middle| \frac{i\beta^2}{2\pi\alpha' t}\right) \left(\frac{1+e^{-\pi t}}{1-e^{-\pi t}}\right)^2$$

Finite temperature Partition function

$$Z = \exp\beta \int_0^\infty \frac{dt}{t}\, \Theta_2\left(0 \middle| \frac{i\beta^2}{2\pi\alpha' t}\right) \prod_{I=1}^{4} \frac{f_I\, \Theta_2(it|\hat{\varphi}_I|it)}{\Theta_1(it|\varphi_I|it)}$$

diverges if $\beta < 2\pi\sqrt{2\alpha'}$

$$\varphi_1 = \varphi_2 = \varphi$$

$$\hat{\varphi}_1 = \hat{\varphi}_2 = 0$$

$$\hat{\varphi}_3 = \hat{\varphi}_4 = 4$$

$$f = f_1 = f_2 = 2\pi\alpha' F_{ss}(q|q')$$

$M^2 = N$

Convergent for all β

$$\rightarrow Z = \text{const}\, \beta \int_0^\infty \frac{dt}{t^{4}}\, \Theta_2\left(0 \middle| \frac{i\beta^2}{2\pi\alpha' t}\right) \left(\frac{1+e^{-2\pi t}}{1-e^{-2\pi t}}\right)^2$$

$$\Theta_2 \rightarrow 2\exp\left(\frac{-\beta^2}{8\pi\alpha' t}\right)$$

$$\Theta_2 \sim \sqrt{\pi}$$