Title: Clarifying holographic charges

Date: Jun 07, 2005 02:00 AM

URL: http://pirsa.org/05060059

Abstract:

$$Z_{AdS} = Z_{CFT}\left(\text{Sources}\right)$$

background fields

$$\exp(-S)$$
$$\exp(iS) \cong Z_{AdS}\left(\begin{array}{c}\text{asymptotic}\\ \text{values}\\ \text{of Fields}\end{array}\right) = Z_{CFT}\left(\begin{array}{c}\text{Sources}\\ \underbrace{\text{background Fields}}\end{array}\right)$$

$$\Omega \to 0 \qquad \Omega^2 g_{ab} = h_{ab}$$

Pick

$\Omega^2 g_{ab} = h_{ab}$
Pick
$T^{ab}_{holographic} := \frac{\delta S_{AdS}}{\delta h_{ab}}$

$\lim_{\Omega \to 0} \Omega^2 g_{ab} = h_{ab}$
$T^{ab}_{\text{holographic}} := \frac{\delta S_{AdS}}{\delta h_{ab}}$
well-defined
"on-shell"

For ξ KVF of h_{ab}...

$$Q_\xi = \int \sqrt{h}\, T^{ab} \xi_b \, n_a$$

For ξ KVF of h_{ab}...

$$Q_\xi = \int \sqrt{h}\, T^{ab} \xi_b \, n_a \stackrel{?}{=} H_\xi$$

Hamiltonian Charges

For ξ KVF of h_{ab}...

$$Q_\xi = \int \sqrt{h}\, T^{ab} \xi_b \, n_a \stackrel{?}{\equiv} \underbrace{H_\xi}_{\text{Hamiltonian Charges}}$$

Simple examples:

$$Q_\xi(s) - H_\xi(s) \equiv Q_\xi(\text{pure AdS})$$

$$Q_\xi = \int \sqrt{-h}\, T^{ab} \xi_b \, n_a \stackrel{?}{=} H_\xi$$

Hamiltonian Charges

Simple examples:

$$Q_\xi(s) - H_\xi(s) = Q_\xi(\text{pure AdS})$$

Depends on Ω

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Hamiltonian Charges

Simple Examples:

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For ξ KVF of h_{ab}...

$$Q_\xi = \int \sqrt{h}\, T^{ab} \xi_b \, n_a \stackrel{?}{\equiv} \underbrace{H_\xi}_{\text{Hamiltonian Charges}}$$

Simple examples:

$$Q_\xi(s) - H_\xi(s) = Q_\xi(\text{Pure Ads}) + \text{other Terms}$$

Depends on Ω

?

For ξ KVF of h_{ab}...

$$Q_\xi = \int \sqrt{-h}\, T^{ab} \xi_b \, n_a \stackrel{?}{=} H_\xi$$

Hamiltonian

Charges

Simple examples:

$$Q_\xi(s) - H_\xi(s) = Q_\xi(\text{Pure Ads}) + \text{other Terms}$$

1) Abbott & Deser

2) Gibbons et al.

Depends on Ω

?

For ξ KVF of h_{ab}...

$$Q_\xi = \int \sqrt{-h}\, T^{ab} \xi_b \, n_a \overset{?}{\equiv} H_\xi$$

Hamiltonian Charges

Simple examples:

$$Q_\xi(s) - H_\xi(s) = Q_\xi(\text{Pure Ads}) + \text{other terms}$$

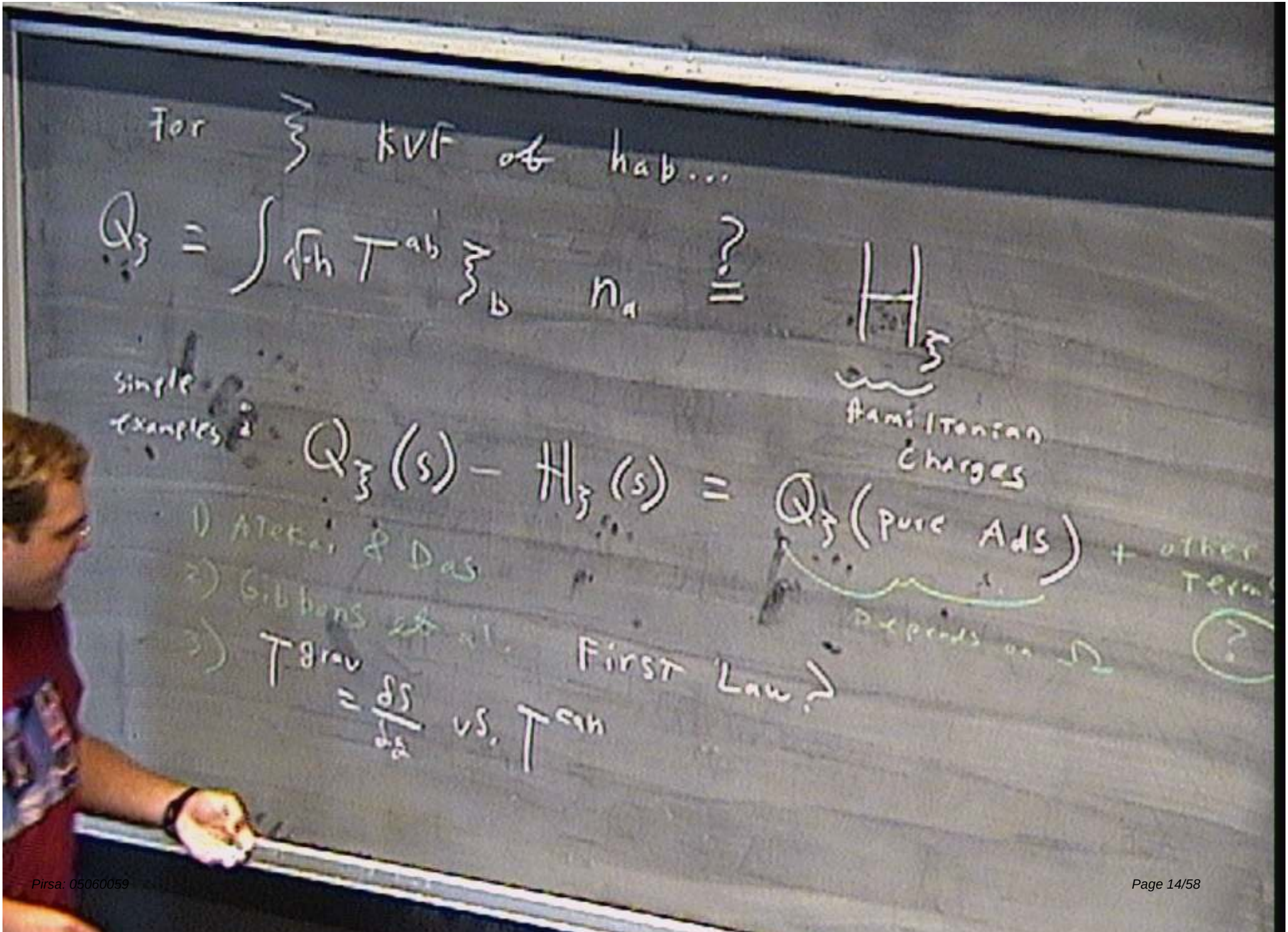
Depends on Ω

?

1) Aliev & Das

2) Gibbons et al.

First Law?


For ξ KVF of h_{ab}...
$Q_\xi = \int \sqrt{-h}\, T^{ab} \xi_b \, n_a \stackrel{?}{=} H_\xi$
Hamiltonian
Charges
Simple examples:
$Q_\xi(s) - H_\xi(s) = Q_\xi(\text{Pure Ads}) + \text{other terms}$
1) Aletsi & Das
2) Gibbons et al.
3) $T^{grav} = \frac{dS}{da}$ vs. T^{can}
First Law?

For ξ KVF of h_{ab}...

$$Q_\xi = \int \sqrt{h}\, T^{ab} \xi_b \, n_a \overset{?}{\equiv} H_\xi$$

Hamiltonian Charges

Simple examples:

$$Q_\xi(s) - H_\xi(s) = Q_\xi(\text{Pure Ads}) + \text{other terms}$$

Depends on Ω

1) Aletsai & Das

2) Gibbons et al.

3) $T^{grav} = \frac{dS}{d\Sigma}$ vs. T^{can} First Law?

For ξ KVF of h_{ab}...

$$Q_\xi = \int \sqrt{h}\, T^{ab} \xi_b \, n_a \stackrel{?}{=} H_\xi$$

Hamiltonian Charges

Simple examples:

$$Q_\xi(s) - H_\xi(s) = Q_\xi(\text{Pure Ads}) + \text{other Terms}$$

1) Abbott & Deser

2) Gibbons et al.

3) T^{grav} First Law?

4) $W = \frac{\partial S}{\partial \xi}$ vs. T^{can}

" (KVF of h_{ab}?

depends on Ω

due to another Error.

For ξ KVF of h_{ab}...

$$Q_\xi = \int \sqrt{-h}\, T^{ab} \xi_b \, n_a \stackrel{?}{\equiv} H_\xi$$

Hamiltonian Charges

AdS_5 or ~/ Trivial Bndy Fields

$$Q_\xi(s) - H_\xi(s) = Q_\xi(\text{Pure Ads})$$

Depends on Ω

1) Ateka & Das

2) Gibbons et al.

3) $T^{grav} = \frac{\delta S}{\delta g}$ First Law?

4) vs. T^{can}

ξ ∈ KVF of h_{ab}?

For ξ KVF of h_{ab}...

$$Q_\xi = \int \sqrt{-h}\, T^{ab} \xi_b \, n_a \overset{?}{\equiv} \underbrace{H_\xi}_{\text{Hamiltonian Charges}}$$

AdS_5 or ... w/ Trivial Bndy Fields

$$Q_\xi(s) - H_\xi(s) = Q_\xi(\text{Pure Ads})$$

Depends on Ω

1) Ashtekar & Das
2) Gibbons et al.
3) T^{grav} First Law?
4) $\xi = \frac{\partial}{\partial t}$ vs. T^{can}

< KVF of h_{ab}?

$$\{Q_s - H_s, A\} = 0 \Leftrightarrow \{Q_s, A\} = \{H_s, A\} = \delta_s A$$

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I) [illegible]

II) Peierls Bracket

$$\{Q_s - H_s, A\} = 0 \Leftrightarrow \{Q_s, A\} = \{H_s, A\} = \delta_s A$$

I) [illegible] done

II.) Peierls Bracket

III) The Regularized S_{Grav} & Q_s

IV) Main Argument

I) [illegible] done

II) Peierls Bracket $\Rightarrow T^{can}$ vs. T^{gr}

III) The Regularized S_{Grav} & Q_ξ

IV) Main Argument

Free Scalar QFT.

$$[\phi(x), \phi(y)] = i\left(G^{+}(x,y) - G^{-}(x,y)\right)$$

Free Scalar QFT

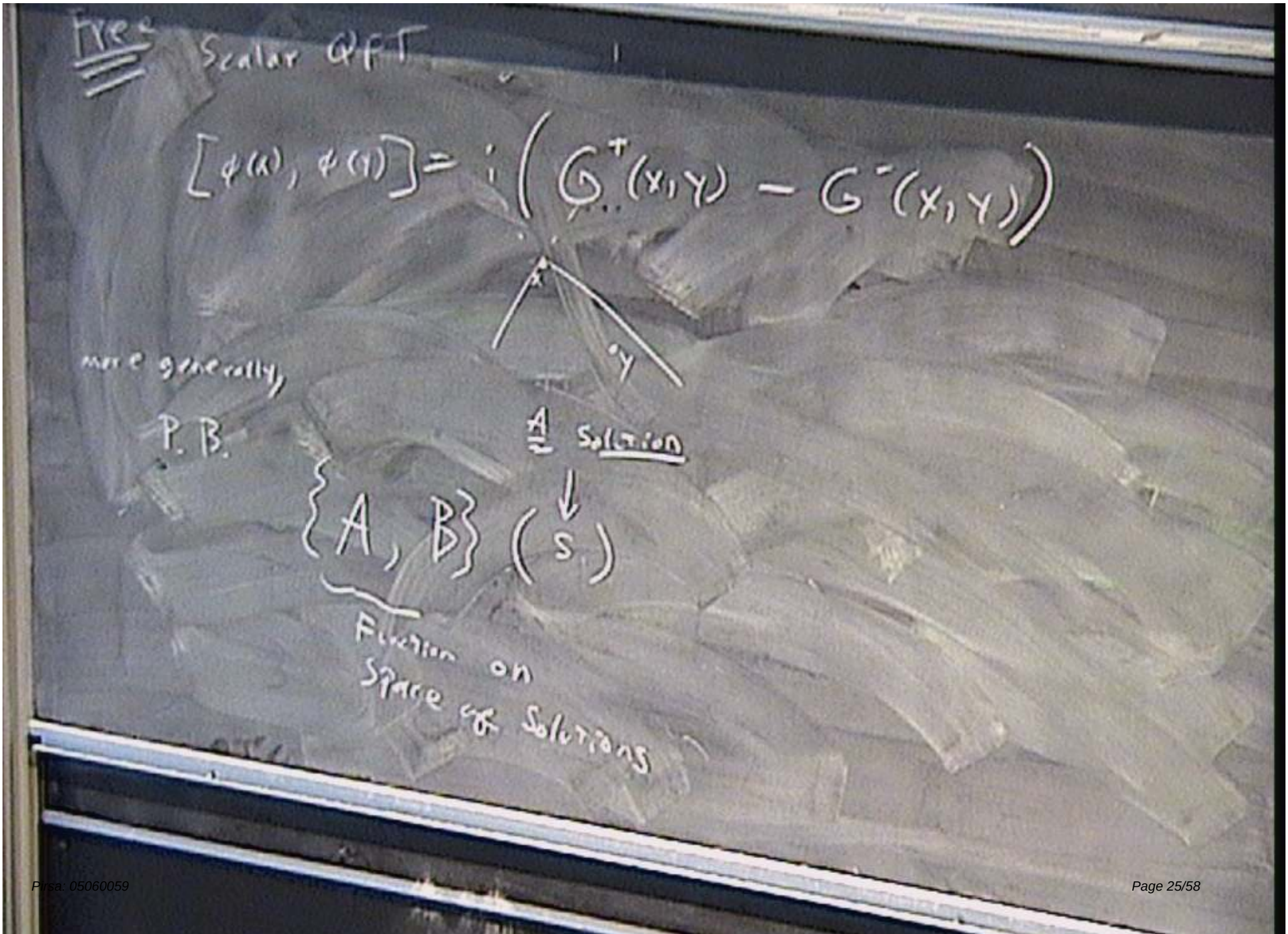
$$[\varphi(x), \varphi(y)] = i\left(G^{+}(x,y) - G^{-}(x,y)\right)$$

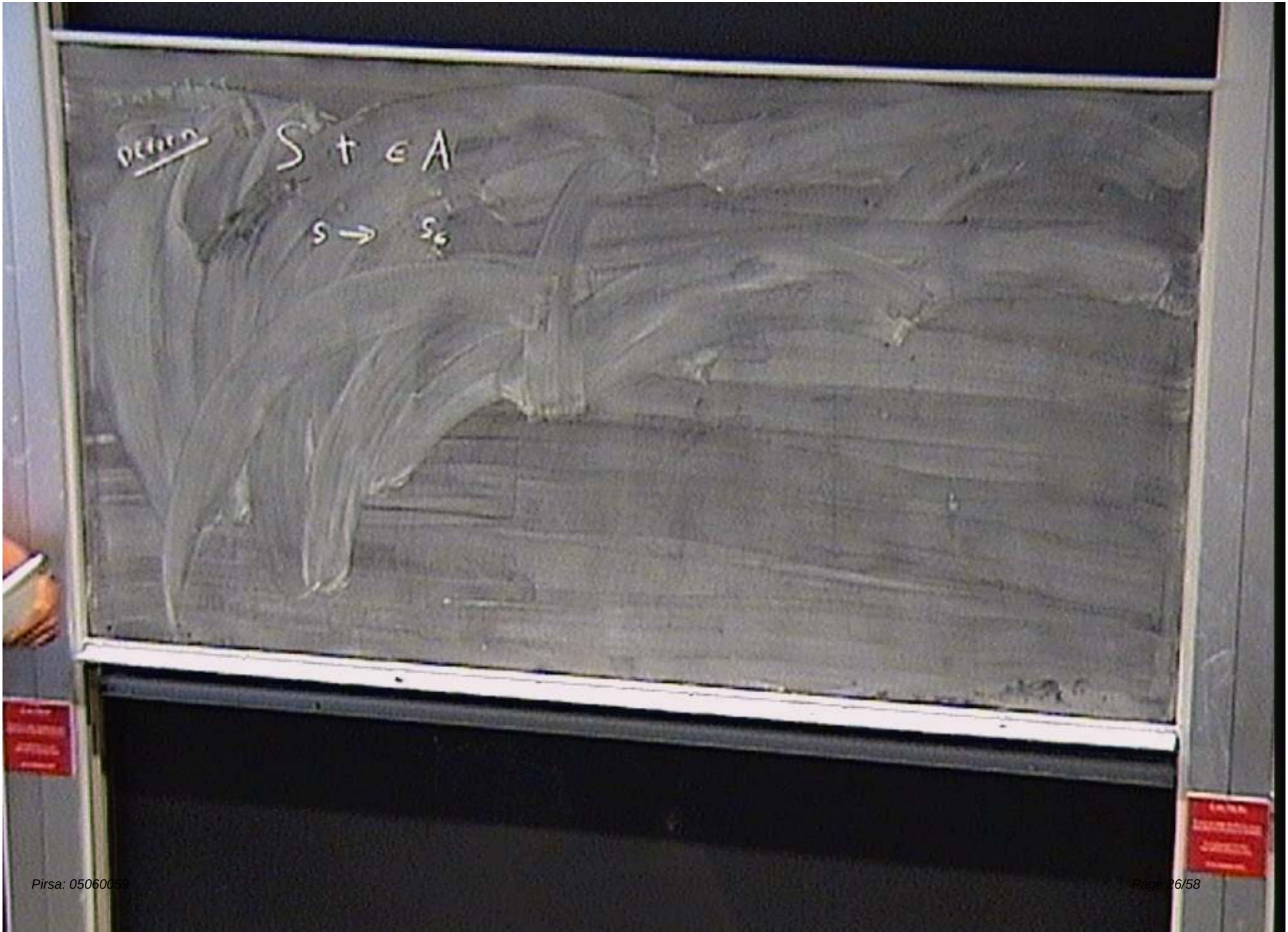
more generally,

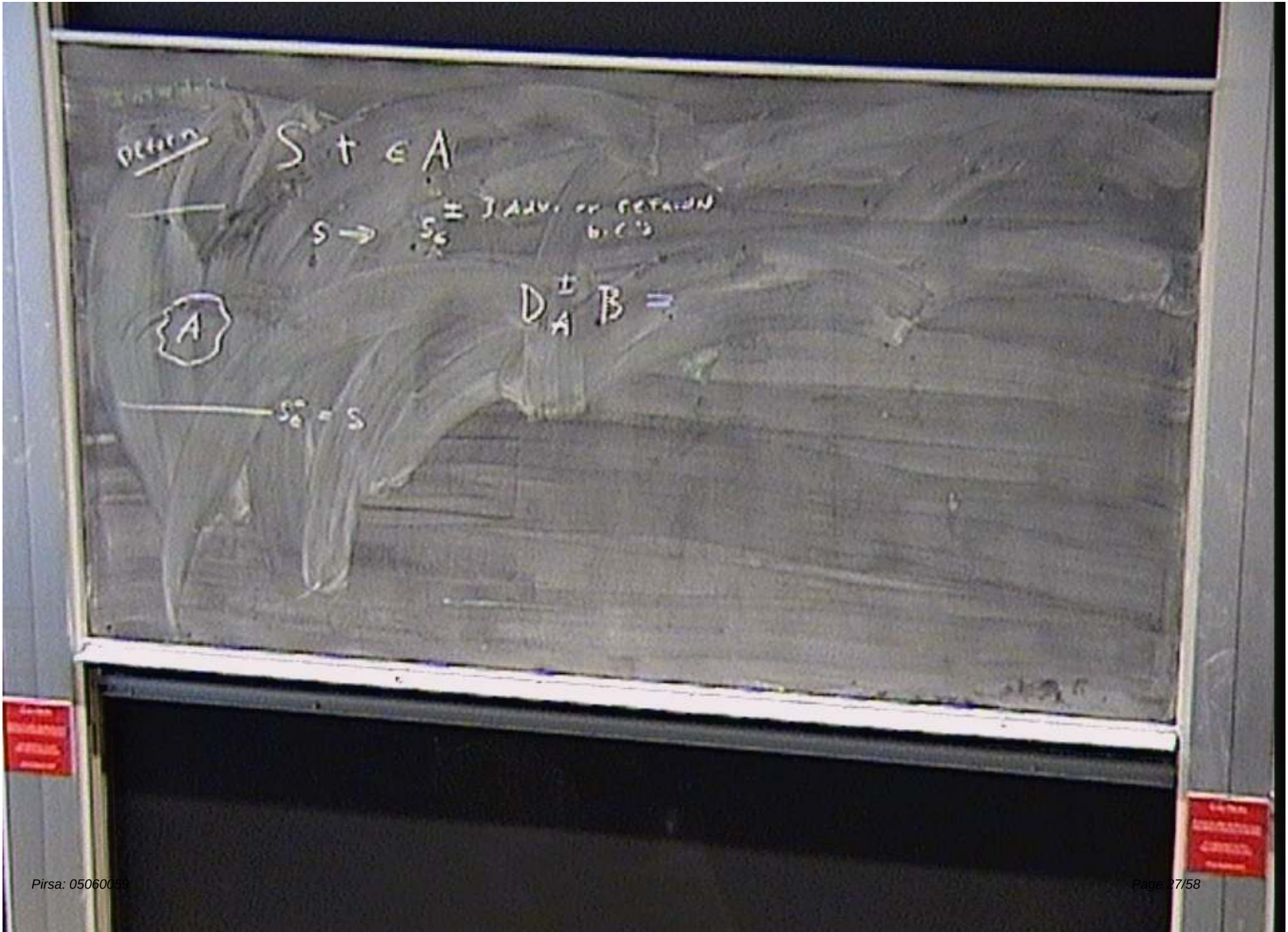
P.B.

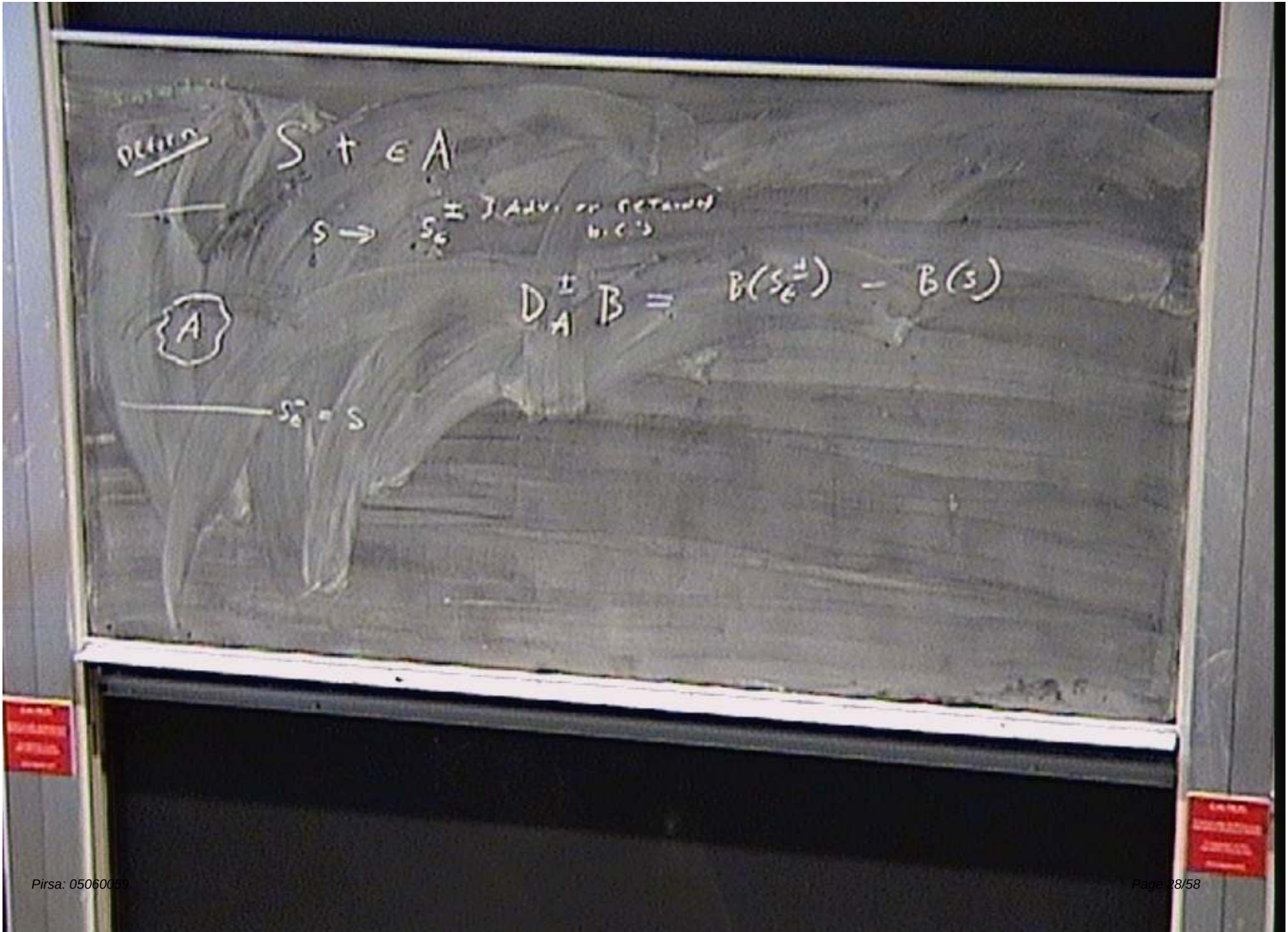
$$\{A, B\}$$

Function on Space of Solutions


Free Scalar QFT
$[\phi(x), \phi(y)] = i\left(G^{+}(x,y) - G^{-}(x,y)\right)$
x
y
more generally,
P.B.
A Solution
$\{A, B\}$ (S)
Function on
Space of Solutions


S t ∈ A
S → S_t


$S + \in A$
$S \to S_\varepsilon^{\pm}$ + ADV. OR RETAIN b.c.'s
$D_A^{\pm} B =$
A
$S_\varepsilon^{\pm} = S$


DEFN
$S + \in A$
$S \to S_\epsilon^{\pm}$ = ADV. OR RETARDED W.R.T. S
$D_A^{\pm} B = B(S_\epsilon^{\pm}) - B(S)$
A
$S_\epsilon^{\pm} \in S$

Free Scalar QFT

$\exp(-S)$

$\exp(iS) \simeq Z_{AdS}\left(\begin{array}{c}\text{asymptotic}\\ \text{values}\\ \text{of Fields}\end{array}\right) = Z_{CFT}\left(\begin{array}{c}\text{Sources}\\ \text{background}\\ \text{Fields}\end{array}\right)$

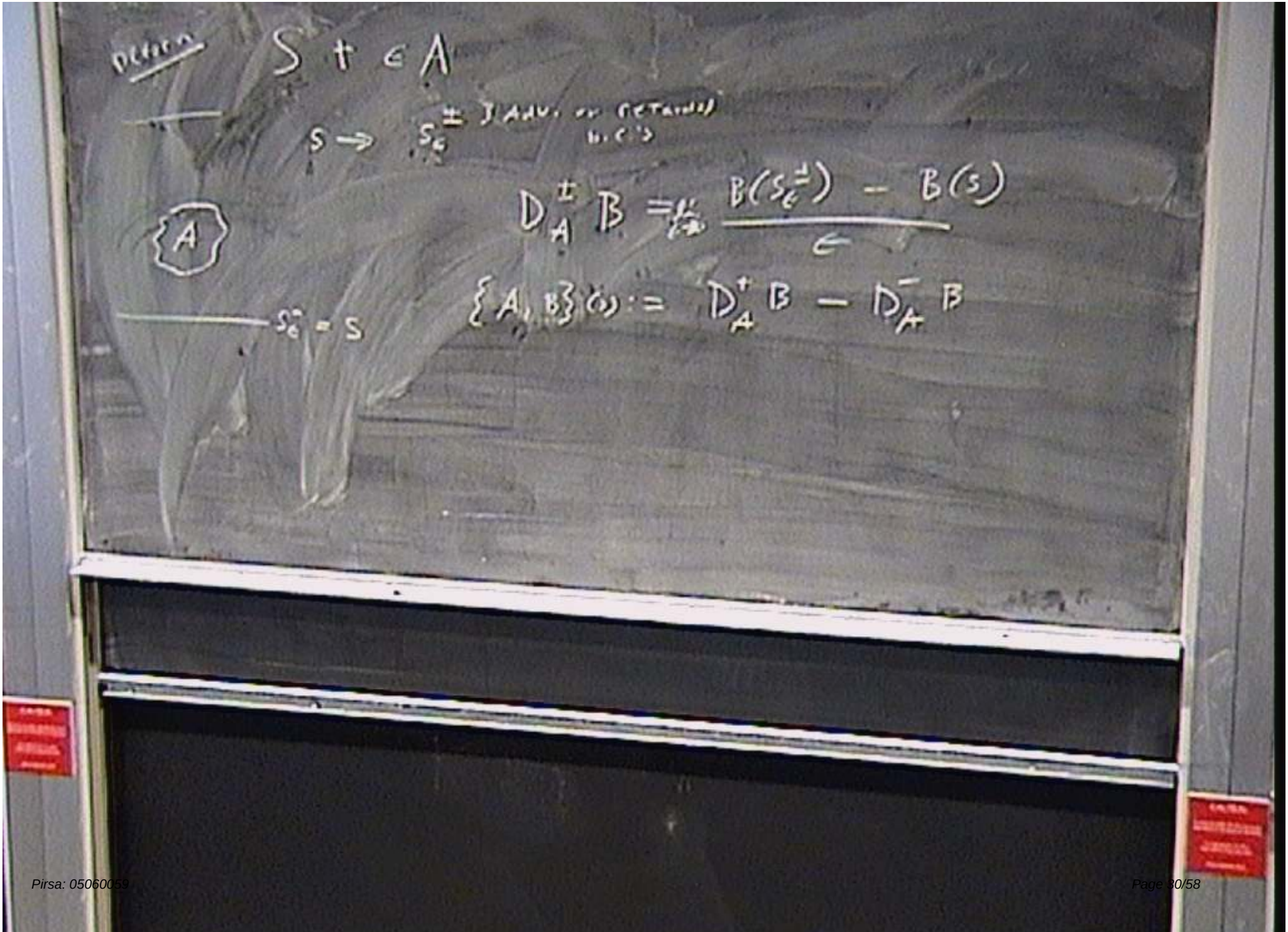
$\Omega = 0$

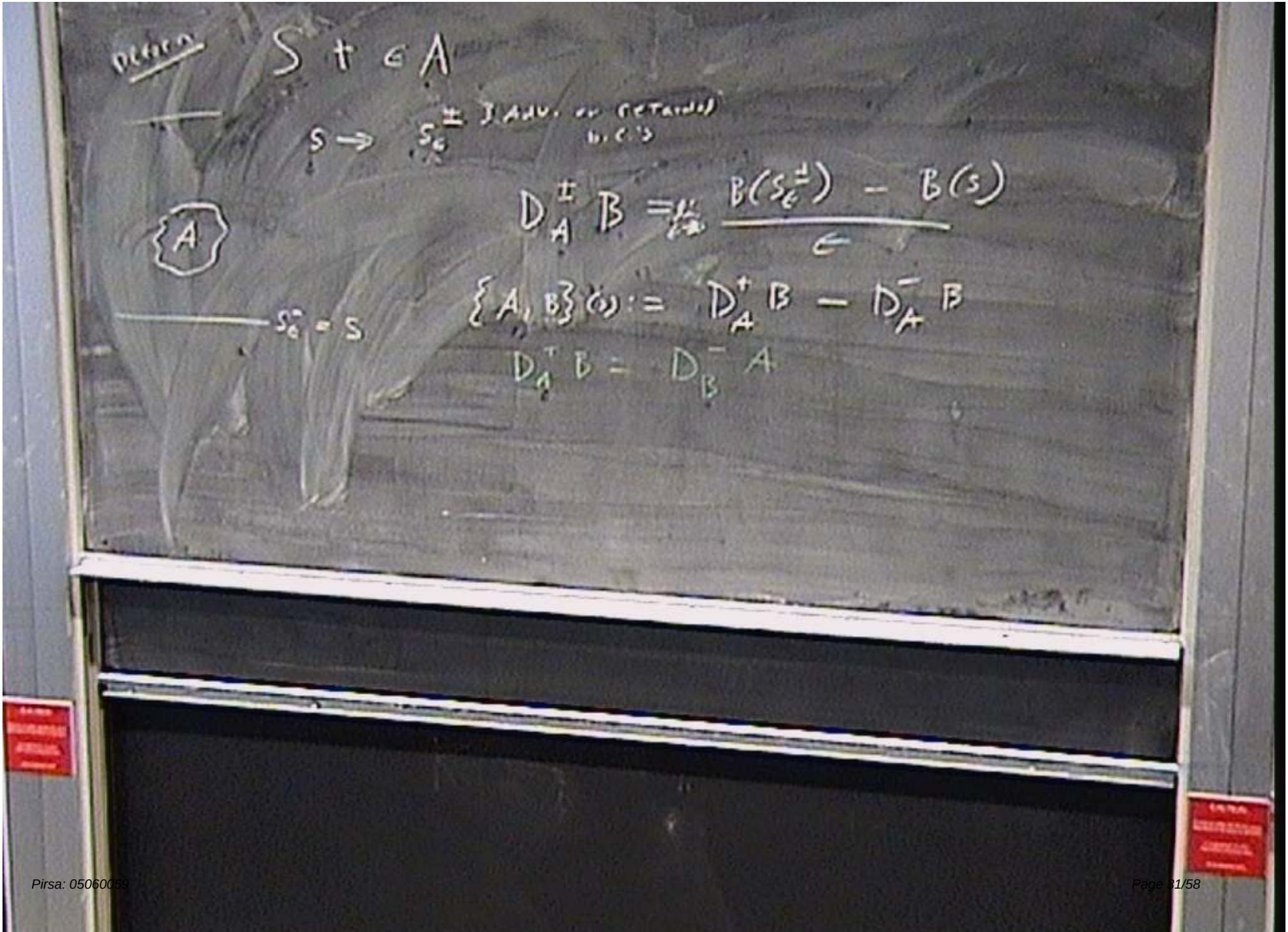
$\Omega^2 g_{ab} = h_{ab}$

Pick Ω

$T^{ab}_{\text{holographic}} := \dfrac{\delta S_{AdS}}{\delta \, h_{ab}}$ ← well-defined "on-shell"

Depends on S


Def
$S + \in A$
$S \to S_\epsilon^{\pm}$ (advanced or retarded b.c.'s)
$D_A^{\pm} B = \lim_{\epsilon \to 0} \frac{B(S_\epsilon^{\pm}) - B(S)}{\epsilon}$
A
$S_\epsilon^{-} = S$
$\{A, B\}(S) := D_A^{+} B - D_A^{-} B$


S + ∈ A
S → S_ε^± (Adv. or Retarded) b. c.'s
D_A^± B = lim B(S_ε^±) − B(S) / ε
A
S_ε^- = S
{A, B}(s) := D_A^+ B − D_A^- B
D_A^+ B = D_B^- A

$S + \in A$

$S \to S_\epsilon^{\pm}$ ∃ Adv. or retarded b.c.'s

$$D_A^{\pm} B = \lim_{\epsilon \to 0} \frac{B(S_\epsilon^{\pm 1}) - B(S)}{\epsilon}$$

4

$$\{A, B\}(S) := D_A^+ B - D_A^- B$$

$S_\epsilon^- = S$

$$D_A^+ B = D_B^- A$$

$\Rightarrow$ Jacobi

$\{Eom, B\} = 0$

$=$

$S + \epsilon B$

$0 = \frac{\delta}{\delta \phi^i}(S + \epsilon B) = \frac{\delta S}{\delta \phi^i} + \epsilon \frac{\delta B}{\delta \phi^i}, \Rightarrow S_\epsilon^\pm$

$Eom = \frac{\delta S}{\delta \phi^i}(S_\epsilon^\pm) = -\epsilon \frac{\delta B}{\delta \phi^i}(S_\epsilon^\pm)$

$\{ \text{Eom}, B \} = 0$

$=$

$S + \epsilon B$

$0 = \frac{\delta}{\delta \phi^i}(S + \epsilon B) = \frac{\delta S}{\delta \phi^i} + \epsilon \frac{\delta B}{\delta \phi^i}, \quad \Rightarrow S_\epsilon^\pm$

$\text{Eom} = \frac{\delta S}{\delta \phi^i}(S_\epsilon^\pm) = -\epsilon \frac{\delta B}{\delta \phi^i}(S_\epsilon^\pm) = -\epsilon \frac{\delta B}{\delta \phi^i}(S) + O(\epsilon^2)$

$D_A^+ \text{Eom} = D_B^- \text{Eom}$

$\{EOM, B\} = 0$

$=$

$S + \epsilon B$

$0 = \frac{\delta}{\delta \phi^i}(S + \epsilon B) = \frac{\delta S}{\delta \phi^i} + \epsilon \frac{\delta B}{\delta \phi^i}, \Rightarrow S_\epsilon^\pm$

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$D_B^+ EOM = D_B^- EOM$

$\{EOM, B\} = 0$

$$S = \int (p_i \dot{q}^i - H)\, dt$$

$$S + \epsilon F = \int (p_i \dot{q}^i - H)\, dt + \epsilon \underbrace{\int f(q, p, t)\, dt}_{F}$$

$$\dot{q}^i = \frac{\partial H}{\partial p_i} - \epsilon \frac{\partial f}{\partial p_i} \quad ; \quad \dot{p}_i = -\frac{\partial H}{\partial q^i} + \epsilon \frac{\partial f}{\partial q^i}$$

Here $f = f(q)\, \delta(t - t_0)$

How do p, q change at $t_0 + \delta$? $(\delta > 0)$

..... $t_0 + \epsilon$ $\Rightarrow$ $D^+_t p(t_0 + \delta) = 0$

t_0 —— Source

$D^+_t q(t_0 + \delta) = 0$

$\cdots\cdots x_0+\delta \Rightarrow D^+_x P(x_0+\delta) = 0$

Source

$D^+_x \xi(x_0+\delta) = 0$

$D^-_{P(x_0+\delta)} = \int_{x_0-\delta}^{x_0+\delta} \delta f_i \, dx = \epsilon \int_{-\infty}^{x_0+\delta} \left(\frac{\partial f}{\partial \xi^i}\right) \delta(x-x_0) \, dx + O(\delta)$

$= \epsilon \left(\frac{\partial f}{\partial \xi^i}\right) + O(\delta)$

$D^-_{\xi^i(x_0+\delta)} = O(\epsilon\delta) \to 0 \quad \text{as } \delta \to 0$

$\{\xi^i(x_0), \xi^j(x_0)\} = 0$

$$D^- p_i(\lambda_0, \varepsilon) = \int_{-\infty}^{\lambda_0+\varepsilon} \delta p_i \, d\lambda = \epsilon \int_{-\infty}^{\lambda_0+\varepsilon} \left(\frac{\partial f}{\partial q^i}\right) \delta(\lambda - \lambda_0) \, d\lambda + o(\varepsilon)$$

$$= \epsilon \left(\frac{\partial f}{\partial q^i}\right) + o(\varepsilon)$$

$$D^- q^i(\lambda_0, \varepsilon) = o(\epsilon\varepsilon) \to 0 \quad \text{as } \varepsilon \to 0$$

$$\Rightarrow \{ q^i(\lambda_0), q^j(\lambda_0) \} = 0$$

$$\{ q^i(\lambda_0), p_j(\lambda_0) \} = \delta^i_j \qquad \text{q.e.d.}$$

Noether's Thm

Suppose $\phi \to \phi + \delta\phi$ is a sym of S.

$$S(\phi + \delta\phi) = S(\phi)$$

Noether's thm

Suppose $\psi \to \psi + \delta\psi$ is a sym of S.

$$S(\psi + \delta\psi) = S(\psi)$$

want $\{H_{\delta}, A\} = \delta A$

Noether's Thm

Suppose $\psi \to \psi + \delta\psi$ is a sym of S.

$$S(\psi + \delta\psi) = S(\psi)$$

Want $\{H_{\delta\psi}, A\} = \delta A$

Choose A_{sym} f. s.t.

$f = 1$

ξ_1

ξ_0

Choose

$f = 0$

Noether's Thm

Suppose $\psi \to \psi + \delta\psi$ is a sym of S.

$$S(\psi + \delta\psi) = S(\psi)$$

Want $\{H_{\delta}, A\} = \delta A$

Choose A_{sym} s.t.

$f = 1$

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Choose

Noether's Thm

Suppose $\varphi \to \varphi + \delta\varphi$ is a sym of S.

$$S(\varphi + \delta\varphi) = S(\varphi)$$

want $\{H_{\delta}, A\} = \delta A$

Choose A_{xx} f. s.t.

$f = 1$

Smooth

Σ_1

Σ_0

$f = 0$

Choose

$\delta_f \phi = f \delta\phi$

$\Rightarrow H_{\delta_f} := \delta_f S$ ← satisfied btwn Σ_0, Σ_1

Noether's thm

Suppose $\psi \to \psi + \delta\psi$ is a sym of S.

$$S(\psi + \delta\psi) = S(\psi)$$

want $\{H_{\delta}, A\} = \delta A$

Choose $\Delta\tau$ f. s.t.

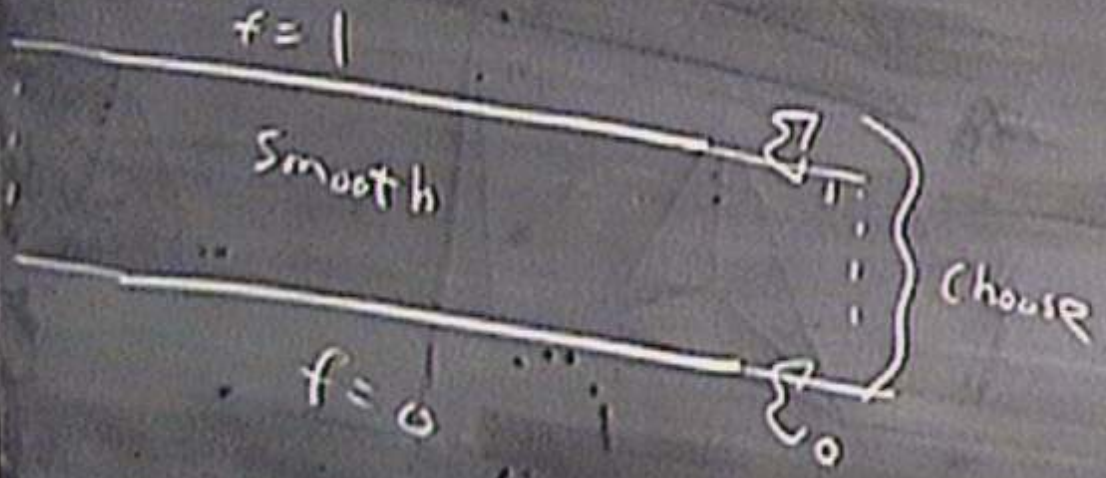
Noether's thm

Suppose $\psi \to \psi + \delta\psi$ is a sym of S.

$$S(\psi + \delta\psi) = S(\psi)$$

want $\{H_{\delta}, A\} = \delta A$

Choose Δm f. s.t.



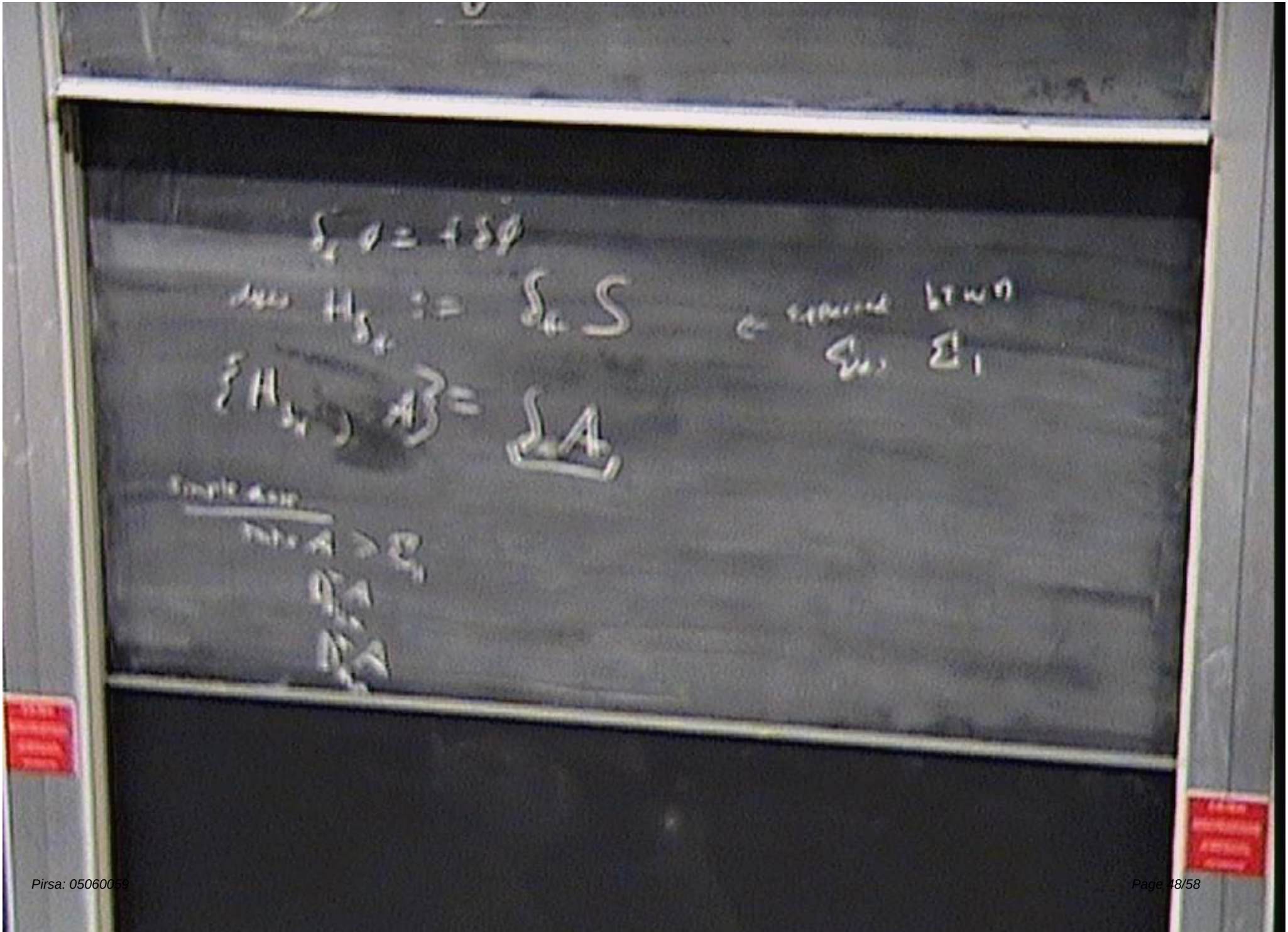
Noether's Thm

Suppose $\psi \to \psi + \delta\psi$ is a sym of S.

$$S(\psi + \delta\psi) = S(\psi)$$

want $\{H_\delta, A\} = \delta A$

Choose $\Delta\psi$ s.t.



Noether's thm

Suppose $\psi \to \psi + \delta\psi$ is a sym of S.

$$S(\psi + \delta\psi) = S(\psi)$$

want $\{H_\delta, A\} = \delta A$

Choose Δ on f. s.t.

$f = 1$ (A)

Smooth $H_{\delta\kappa}$

Choose

$f = 0$

$\delta_\epsilon \phi = \epsilon \delta \phi$

define $H_{\delta_\epsilon} := \delta_\epsilon S$ ← satisfied btwn Σ_0, Σ_1

$\{H_{\delta_\epsilon}, A\} = \delta_\epsilon A$

Simple case

Take $A > \Sigma_1$

$D^-_{H_{\delta_\epsilon}} A$

$D^+_{H_{\delta_\epsilon}} A = 0$

$S + \epsilon H_{\delta_\epsilon} = S + \epsilon \delta_\epsilon S$

$= S(\phi + \epsilon \delta_\epsilon \phi)$

$\delta_\epsilon \phi = \epsilon \delta \phi$

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$\phi \to \phi + \delta_\epsilon \phi$

$\delta_\epsilon \phi = \epsilon \delta \phi$

define $H_{\delta_\epsilon} := \delta_\epsilon S$ ← satisfied btwn Σ_0, Σ_1

$\{H_{\delta_\epsilon}, A\} = \delta_\epsilon A$

Simple case

take $A > \Sigma_1$

$D^{\Gamma}_{H_{\delta_\epsilon}} A = \delta_\epsilon A$

$D^{\Gamma}_{H_{\delta_\epsilon}} A = 0$

$S + \epsilon H_{\delta_\epsilon} = S + \epsilon \delta_\epsilon S$

$= S(\phi + \epsilon \delta_\epsilon \phi)$

$\phi \to \phi + \delta_\epsilon \phi$

$\delta_\epsilon \phi = \epsilon \delta \phi$

defines $H_{\delta_\epsilon} := \delta_\epsilon S$ ← satisfied btwn Σ_0, Σ_1

$\{H_{\delta_\epsilon}, A\} = \underline{\delta_\epsilon A}$

Simple case

Take $A > \Sigma_1$

$D^{\Gamma}_{\Sigma_1} A = \delta_\epsilon A \Rightarrow \{H_{\delta_\epsilon}, A\} = \delta A$

$D^{\Gamma}_{\Sigma_0} A = 0$

$S + \epsilon H_{\delta_\epsilon} = S + \epsilon \delta_\epsilon S$

$= S(\phi + \epsilon \delta_\epsilon \phi)$

$\phi \to \phi + \delta_\epsilon \phi$

$\delta_\epsilon \phi = \epsilon \delta \phi$

define $H_{\delta_\epsilon} := \delta_\epsilon S$ ← satisfied btwn Σ_0, Σ_1

$\{H_{\delta_\epsilon}, A\} = \delta_\epsilon A$

Simple Case

Take $A > \Sigma_1$

$D^+_{H_{\delta_\epsilon}} A = \delta_\epsilon A$

$D^-_{H_{\delta_\epsilon}} A = 0$

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$S + \epsilon H_{\delta_\epsilon} = S + \epsilon \delta_\epsilon S$

$= S(\phi + \epsilon \delta_\epsilon \phi)$

$\phi \to \phi + \delta_\epsilon \phi$

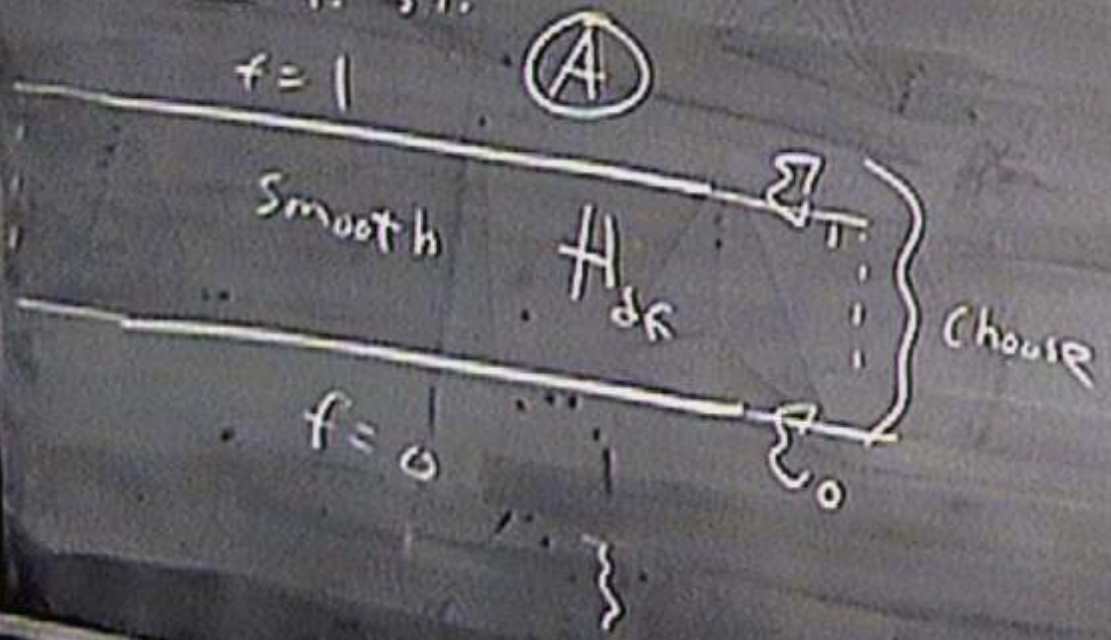
Noether's thm

Suppose $\psi \to \psi + \delta\psi$ is a sym of S.

$$S(\psi + \delta\psi) = S(\psi)$$

want $\{H_\delta, A\} = \delta A$

Choose $\Delta\tau$ f. s.t.



$\delta_\epsilon \phi = \epsilon \delta \phi$

does $H_{\delta_\epsilon} := \delta_\epsilon S$ ← satisfied btwn Σ_0, Σ_1

$\{H_{\delta_\epsilon}, A\} = \delta_\epsilon A$

Simple Rule

Take $A > \Sigma_1$

$D_{\Sigma_1} A = \delta_\epsilon A$

$D_{\Sigma_0} A = 0$

$\Rightarrow \{H_{\delta_\epsilon}, A\} = \delta A$

$S + \epsilon H_{\delta_\epsilon} = S + \epsilon \delta_\epsilon S$

$= S(\phi + \epsilon \delta_\epsilon \phi)$

$\phi \to \phi + \delta_\epsilon \phi$

$$\mathcal{O}_\Sigma = \int T^{ab} \xi_b n_a \sqrt{-h}$$

$$\text{w/ } T^{ab} \cong \frac{\delta S}{\delta h_{ab}} \cdots$$

$$P_i(\lambda_0)\} = \delta_i \quad \xi^a\{P, \cdot\} = 0$$

$$Q_\Sigma = \int T^{ab}{}_{;b}\, n_a \sqrt{-h}$$

+ other terms from other Background Fields ...

$$\sim T^{ab} \cong \frac{\delta S}{\delta h_{ab}} \cdots$$

$$\Gamma[\{x_{ab}\}] = \int \mathcal{L}\{\phi, \partial\phi\} = 0$$