

Title: Clarifying holographic charges

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Abstract:

$$Z_{\text{Ads}} = Z_{\text{CFT}} \left(\begin{array}{c} \text{sources} \\ \text{background} \\ \text{fields} \end{array} \right)$$

$$\frac{\exp(-S)}{\exp(iS)} \simeq Z_{\text{Ads}} \left(\begin{array}{c} \text{asymptotic} \\ \text{values} \\ \text{of Fields} \end{array} \right) = Z_{\text{CFT}} \left(\begin{array}{c} \text{sources} \\ \text{background} \\ \text{Fields} \end{array} \right)$$



$$\lim_{\Omega \rightarrow 0} \int_{\text{Pick}} \Omega^2 g_{ab} = h_{ab}$$

$$\lim_{\Omega \rightarrow 0} \underbrace{\Omega^2 g_{ab}}_{\text{PICK}} = h_{ab}$$

$$T_{\text{collaborative}} := \frac{\delta S_{\text{AAS}}}{\delta h_{ab}}$$

$$\lim_{\Omega \rightarrow 0} \frac{1}{P_{\text{CCR}}} \Omega^2 g_{ab} = h_{ab}$$

$$T_{\text{collapsar}}^{\text{sub}} = \frac{\delta S_{\text{ADS}}}{\delta h_{ab}}$$

← well-defined
"on-shell"

for $\sum$ KVF of $h_{ab} \dots$

$$Q_3 = \int \sqrt{-h} T^{ab} \sum_b n_a$$

for $\sum$ kVF of hab...

$$Q_3 = \int d^3x T^{ab} \sum_b n_a \stackrel{?}{=} H$$

H
Hamiltonian
charges

For $\{$ KVF of $h_{ab} \dots$

$$Q_{\{} = \int \sqrt{-h} T^{ab} \}_{b} n_a \stackrel{?}{=} \underbrace{H_{\{}}_{\text{Hamiltonian Charges}}$$

single
examples:

$$Q_{\{}(s) - H_{\{}(s) = Q_{\{}(\text{pure Ads})$$

$$Q_3 = \int \sqrt{-h} T^{ab} \sum_b n_a \stackrel{?}{=} H_3$$

single
examples:

$$Q_3(s) - H_3(s) = Q_3(\text{pure Ads})$$

Hamiltonian
charges

depends on Ω

$$Q_3 = \int \sqrt{-h} T^{ab} \sum_b n_a \stackrel{?}{=} H_3$$

single
examples:

$$Q_3(s) - H_3(s) = Q_3(\text{pure Ads})$$

Hamiltonian
charges

Depends on Ω

For $\sum$ KVF of hab...

$$Q_{\sum} = \int d^3x T^{ab} \sum_b n_a \stackrel{?}{=} H_{\sum}$$

Simple
examples:

$$Q_{\sum}(s) - H_{\sum}(s) = \underbrace{Q_{\sum}(\text{pure Ads})}_{\text{depends on } \Omega} + \text{other Terms}$$

(?)

For $\{$ KVF of $h_{ab} \dots$

$$Q_{\{} = \int d^3x T^{ab} \}_{b} n_a \stackrel{?}{=} H_{\{}$$

single
examples:

$$Q_{\{}(s) - H_{\{}(s) =$$

Hamiltonian
charges

$$Q_{\{}(\text{pure Ads}) + \text{other terms}$$

1) Atetari & Das

2) Gibbons et al.

Depends on Ω

?

For $\sum$ KVF of hab...

$$Q_{\sum} = \int d^3x T^{ab} \sum_b n_a \stackrel{?}{=} H_{\sum}$$

Simple
examples:

$$Q_{\sum}(s) - H_{\sum}(s) =$$

Hamiltonian
charges

$$Q_{\sum}(\text{pure Ads}) + \text{other terms}$$

1) A. Tekar & Das

2) Gibbons et al.

Depends on Ω

First Law?

For $\sum$ KVF of $h_{ab} \dots$

$$Q_{\sum} = \int \sqrt{-h} T^{ab} \sum_b n_a \stackrel{?}{=} H_{\sum}$$

$H_{\sum}$

Hamiltonian
charges

single
examples:

$$Q_{\sum}(s) - H_{\sum}(s) =$$

$Q_{\sum}(\text{pure Ads})$

+ other
terms

1) Atkeson & Das

2) Gibbons et al.

3) $T^{\text{grav}} = \frac{\delta S}{\delta g} \text{ vs. } T^{\text{em}}$

First Law?

Depends on Ω

?

For $\{$ KVF of hab...

$$Q_{\{} = \int d\mathbf{h} T^{ab} \}_{b} n_a \stackrel{?}{=} H_{\{}$$

single

examples:

$$Q_{\{}(s) - H_{\{}(s) =$$

Hamiltonian
charges

$$Q_{\{}(\text{pure Ads}) + \text{other terms}$$

1) Atetari & Das

2) Gibbons et al.

3) $T^{\text{grav}} = \frac{dS}{ds}$ vs. T^{em}

First Law?

Depends on Ω

?

For $\sum$ KVF of hab...

$$Q_{\sum} = \int \sqrt{-h} T^{ab} \sum_b n_a \stackrel{?}{=} H_{\sum}$$

Simple

examples:

$$Q_{\sum}(s) - H_{\sum}(s) = \underbrace{Q_{\sum}(\text{pure Ads})}_{\text{Hamiltonian Charges}} + \text{other terms}$$

1) Aretakis & Das

2) Gibbons et al.

3) T_{grav}

$$= \frac{\delta S}{\delta g} \text{ vs. } T_{\text{can}}$$

4) $\sum$

" (KVF of hab?)

First Law?

Depends on Ω

← due to another error

For $\sum$ KVF of hab...

$$Q_\Sigma = \int \sqrt{-h} T^{ab} \sum_b n_a \stackrel{?}{=} H_\Sigma$$

H_Σ
Hamiltonian
charges

ADS
w/ Trivial
Bdy Fields

$$Q_\Sigma(s) - H_\Sigma(s) = Q_\Sigma(\text{pure Ads})$$

Depends on Ω

1) Aoki & Das

2) Gibbons et al.

3) T_{grav}

$$= \frac{\delta S}{\delta g} \text{ vs. } T^{\text{can}}$$

4) $\sum$

" (KVF of hab?)

First Law?

← due to
another
error

For $\sum$ KVF of $h_{ab} \dots$

$$Q_{\sum} = \int \sqrt{-h} T^{ab} \sum_b n_a \stackrel{?}{=} H_{\sum}$$

$H_{\sum}$
Hamiltonian
charges

ADS
w/ Trivial
Bdy Fields

$$Q_{\sum}(s) - H_{\sum}(s) = Q_{\sum}(\text{pure Ads})$$

Depends on Ω

1) Atiyah & Das

2) Gibbons et al.

3) T_{grav}

$$= \frac{\delta S}{\delta g}$$

vs. T^{can}

4) $\sum$

" < KVF of h_{ab} ?

First Law?

← due to
another
error

$$\{Q_3 - H_3, A\} = 0 \Leftrightarrow \{Q_3, A\} = \{H_3, A\} = \int_\Sigma A$$

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I) not done

II) Poincaré bracket

$$\{Q_3 - H_3, A\} = 0 \Leftrightarrow \{Q_3, A\} = \{H_3, A\} = \int_\Sigma A$$

I) now done

II) Poincaré bracket

III) The Regulated Series Q_3

IV) Main Argument

I) intro - done

II) Peierls Bracket $\rightarrow T^{\text{can}}$ vs. T^{gr}

III) The Regularized S_{grav} & Q_3

IV) Main Argument

Free Scalar QFT

$$[\phi(x), \phi(y)] = i \left(G^+(x, y) - G^-(x, y) \right)$$

Free Scalar QFT

$$[\phi(x), \phi(y)] = i \left(G^+(x, y) - G^-(x, y) \right)$$



more generally,

P.B.

$\{A, B\}$

Function on
Space of Solutions

Free Scalar QFT

$$[\phi(x), \phi(y)] = i \left(G^+(x, y) - G^-(x, y) \right)$$



more generally,

P.B.

A Solution

$\{A, B\} \left(\begin{smallmatrix} \downarrow \\ s \end{smallmatrix} \right)$

Function on
Space of Solutions

Defn

$S \vdash A$

$S \rightarrow S_0$

DEFIN

$$S \vdash A$$

$$S \rightarrow S_0 \equiv \text{J ADV. OF DEFINITION h.c.'s}$$

$\{A\}$

$$D_A^{\pm} B =$$

$$S_0^* = S$$

Defn

$$S \neq \in A$$

$$S \rightarrow S_0 \pm \text{Adv. or Retard. h.c.'s}$$

$\{A\}$

$$D_A^\pm B = B(S_0^\pm) - B(S)$$

$$S_0^- = S$$

Free Scalar QFT

$$\exp(-S) \approx Z_{\text{Ads}} \left(\begin{smallmatrix} \text{asymptotic} \\ \text{values} \\ \text{of fields} \end{smallmatrix} \right) = Z_{\text{CFT}} \left(\begin{smallmatrix} \text{sources} \\ \text{background} \\ \text{fields} \end{smallmatrix} \right)$$



$$\lim_{\Omega \rightarrow 0} \sqrt{\Omega} g_{\text{AdS}} = h_{\text{g.b.}}$$

Pick

$$T_{\text{Holographic}} = \frac{\delta S_{\text{AdS}}}{\delta h_{\text{g.b.}}}$$

depends on Ω

← well-defined "on-shell"

Defn

$$S \dagger \in A$$

$$S \rightarrow S_e^{\pm} \quad \begin{array}{l} \pm \text{ ADV. or RETARD} \\ \text{w.r.t. } S \end{array}$$

A

$$D_A^{\pm} B = \frac{B(S_e^{\pm}) - B(S)}{\leftarrow}$$

$$S_e^{\pm} = S$$

$$\{A, B\}(s) := D_A^+ B - D_A^- B$$

Defn

$$S \dagger \in A$$

$$S \rightarrow S_{\epsilon}^{\pm} \quad \text{J Adv. or Retardd} \\ \text{b.c.'s}$$

(A)

$$D_A^{\pm} B = \lim_{\epsilon \rightarrow 0} \frac{B(S_{\epsilon}^{\pm}) - B(S)}{\epsilon}$$

$$S_{\epsilon}^{\pm} = S$$

$$\{A, B\}(\omega) := D_A^{+} B - D_A^{-} B$$

$$D_A^{+} B = D_B^{-} A$$

$$S \pm \in A$$

$$S \rightarrow S_{\epsilon}^{\pm} \quad \pm \text{ Adv. or Retard. h.c.}$$

$$D_A^{\pm} B = \frac{B(S_{\epsilon}^{\pm}) - B(S)}{\epsilon}$$

$$\{A, B\}(\omega) := D_A^+ B - D_A^- B$$

$$D_A^+ B = D_B^- A$$

$\Rightarrow$ Jacobi

$$S_{\epsilon}^{\pm} = S$$

$$\{E_{om}, \underline{B}\} = 0$$

$$S + \epsilon B$$

$$0 = \frac{\delta}{\delta \pi^i} (S + \epsilon B) = \frac{\delta S}{\delta \pi^i} + \epsilon \frac{\delta B}{\delta \pi^i} \Rightarrow S_\epsilon^\pm$$

$$E_{om} = \frac{\delta S}{\delta \pi^i} (S_\epsilon^\pm) = -\epsilon \frac{\delta B}{\delta \pi^i} (S_\epsilon^\pm)$$

$$\{E_{OM}, \underline{B}\} = 0$$

$$S + \epsilon B$$

$$0 = \frac{\delta}{\delta \phi^i} (S + \epsilon B) = \frac{\delta S}{\delta \phi^i} + \epsilon \frac{\delta B}{\delta \phi^i} \Rightarrow S_c^\pm$$

$$E_{OM} = \frac{\delta S}{\delta \phi^i} (S_c^\pm) = -\epsilon \frac{\delta B}{\delta \phi^i} (S_c^\pm) = -\epsilon \frac{\delta B}{\delta \phi^i} (S) + O(\epsilon^2)$$

$$D_A^+ E_{OM} = \bar{D}_B E_{OM}$$

$$\{E_{OM}, B\} = 0$$

$$S + \epsilon B$$

$$0 = \frac{\delta}{\delta \varphi^i} (S + \epsilon B) = \frac{\delta S}{\delta \varphi^i} + \epsilon \frac{\delta B}{\delta \varphi^i} \Rightarrow S_c^\pm$$

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$$D_A^+ E_{OM} = D_B^- E_{OM}$$

$$\{E_{OM}, B\} = 0$$

$$S = \int (P_i \dot{q}^i - H) dt$$

$$S + \epsilon F = \int (P_i \dot{q}^i - H) dt + \epsilon \underbrace{\int f(q, x) dx}_F$$

$$\dot{q}^i = \frac{\partial H}{\partial p_i} - \epsilon \frac{\partial f}{\partial p_i} \quad ; \quad \dot{p}_i = -\frac{\partial H}{\partial q^i} + \epsilon \frac{\partial f}{\partial q^i}$$

here $f = f(q) \delta(t - x_0)$

How do p, q change at $x_0 + \epsilon$? ($\epsilon > 0$)

..... $x_0 + \epsilon$ $\Rightarrow$ $D_x^+ p(x_0 + \epsilon) = 0$
 t_0 ——— source $D_x^+ q(x_0 + \epsilon) = 0$

$\dots x_0 + \epsilon$
 $\underbrace{\hspace{2cm}}_{\text{Source}} \Rightarrow D_{\mathbf{x}}^+ p(x_0 + \epsilon) = 0$
 $D_{\mathbf{x}}^+ \varphi(x_0 + \epsilon) = 0$

$$\begin{aligned}
 D^- p(x_0 + \epsilon) &= \int_{-\infty}^{x_0 + \epsilon} \delta f_1 dx = \epsilon \int_{-\infty}^{x_0 + \epsilon} \left(\frac{\partial f}{\partial q_1} \right) \delta \left(\frac{1}{\epsilon} x \right) dx + o(\epsilon) \\
 &= \epsilon \left(\frac{\partial f}{\partial q_1} \right) + o(\epsilon)
 \end{aligned}$$

$$D^- \varphi(x_0 + \epsilon) = o(\epsilon) \rightarrow 0 \text{ as } \epsilon \rightarrow 0$$

$$\{ \varphi'(x_0), \varphi'(x_0) \} = 0$$

$$\begin{aligned} \bar{D} p_i(t_0 + s) &= \int_{-\infty}^{t_0 + s} \delta \dot{p}_i dt = \epsilon \int_{-\infty}^{t_0 + s} \left(\frac{\partial f}{\partial q_i} \right) \delta(q(t) + x_0) dt + o(\epsilon) \\ &= \epsilon \left(\frac{\partial f}{\partial q_i} \right) + o(\epsilon) \end{aligned}$$

$$\bar{D} q(t_0 + s) = o(\epsilon s) \rightarrow 0 \text{ as } s \rightarrow 0$$

$$\Rightarrow \{ \dot{q}(t_0), \dot{q}(t_0) \} = 0;$$

$$\{ \dot{q}(t_0), p_i(t_0) \} = \delta_{ij} \quad \in \{ p, p \}$$

Mach's thm

Suppose $\phi \rightarrow \phi + \delta\phi$ is a sym of S .

$$S(\phi + \delta\phi) = S(\phi)$$

Another's thm

Suppose $\psi \rightarrow \psi + \delta\psi$ is a sym of S .

$$\text{want } \begin{cases} S(\psi + \delta\psi) = S(\psi) \\ \{H_\delta, A\} = \{A\} \end{cases}$$

Mathew's thm

Suppose $\psi \rightarrow \psi + \delta\psi$ is a sym of S .

$$S(\psi + \delta\psi) = S(\psi)$$

want

$$\{H_\delta, A\} = SA$$

choose Any f . s.t.

$$f = 1$$

$$\Sigma_1$$

$$\Sigma_0$$

} choose

$$f = 0$$

Another's thm

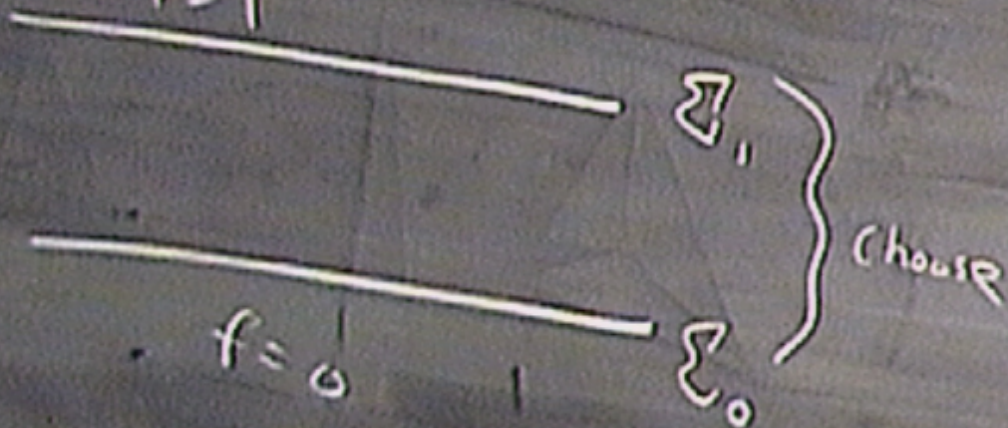
Suppose $\psi \rightarrow \psi + \delta\psi$ is a sym of S .

want
$$S(\psi + \delta\psi) = S(\psi)$$

$$\{H_\delta, A\} = \delta A$$

choose any f s.t.

$f = 1$



Noether's thm

Suppose $\phi \rightarrow \phi + \delta\phi$ is a sym of S .

want
$$S(\phi + \delta\phi) = S(\phi)$$
$$\{H_\delta, A\} = \delta A$$

choose any f on Σ .

$f = 1$

Smooth

$f = 0$

Σ_1

Σ_0

} choose

$$\delta_f \phi = f \delta \phi$$

$$\text{def } H_{\delta_f} := \delta_f S \quad \leftarrow \text{supported b/w } \Sigma_0, \Sigma_1$$

Nether's thm

Suppose $\psi \rightarrow \psi + \delta\psi$ is a sym of S .

$$S(\psi + \delta\psi) = S(\psi)$$

want $\{H_\delta, A\} = SA$

choose A s.t. $f = 1$

$$f = 1$$

Smooth

$$f = 0$$

$$\Sigma_1$$

$$\Sigma_0$$

choose

Mathieu's thm

Suppose $\psi \rightarrow \psi + \delta\psi$ is a sym of S .

$$S(\psi + \delta\psi) = S(\psi)$$

want $\{H_\delta, A\} = SA$

choose any f s.t.

$$f = 1$$

Smooth

$$f = 0$$

Σ_1

Σ_0

choose



Mathew's thm

Suppose $\psi \rightarrow \psi + \delta\psi$ is a sym of S .

$$S(\psi + \delta\psi) = S(\psi)$$

want $\{H_\delta, A\} = \delta A$

choose any f s.t.

$$f = 1$$

Smooth

$$f = 0$$

$$\Sigma_1$$

$$\Sigma_0$$

choose



$$\delta_2 \phi = + \delta \phi$$

$$\text{also } H_{\delta_2} = \sum_{\alpha} S$$

← separated by η
 Σ_1, Σ_1

$$\{H_{\delta_2}, A\} = \underline{\underline{\lambda A}}$$

Simple case

$$\text{Take } A = \mathbb{R}$$

$$\begin{aligned} & \delta_1 A \\ & \delta_2 A \\ & \delta_3 A \end{aligned}$$

Nash's thm

Suppose $\psi \rightarrow \psi + \delta\psi$ is a sym of S .

$$S(\psi + \delta\psi) = S(\psi)$$

want $\{H_\delta, A\} = SA$

choose Δ f. s.t.

$$f=1$$

(A)

Smooth

H_δ

choose

$$f=0$$

Σ_0



$$\delta_f \phi = f \delta \phi$$

$$\text{def } H_{\delta_f} := \delta_f S$$

← supported b/w Σ_0, Σ_1

$$\{H_{\delta_f}, A\} = \underbrace{\delta_f A}$$

Simple case

take $A \in \Sigma_1$

$$D_{H_{\delta_f}}^* A$$

$$D_{H_{\delta_f}}^* A = 0$$

$$\begin{aligned} S + \epsilon H_{\delta_f} &= S + \epsilon \delta_f S \\ &= S(\phi + \epsilon \delta_f \phi) \end{aligned}$$

$$\delta_f \phi = f \delta \phi$$

$$\text{def } H_{\delta_f} := \delta_f S$$

← separated b/w Σ_0, Σ_1

$$\{H_{\delta_f}, A\} = \underbrace{\delta_f A}$$

Simple rule

Take $A \in \Sigma_1$

$$D_{H_{\delta_f}}^* A$$

$$D_{H_{\delta_f}}^* A = 0$$

$$S + \epsilon H_{\delta_f} = S + \epsilon \delta_f S$$

$$= S(\phi + \epsilon \delta_f \phi)$$

$$\phi \rightarrow \phi + \delta_f \phi$$

$$\delta_f \phi = \epsilon \delta \phi$$

$$\text{def } H_{\delta_f} := \delta_f S$$

← supported by Σ_0, Σ_1

$$\{H_{\delta_f}, A\} = \underbrace{\delta_f A}$$

Simple rule

$$\text{Take } A \in \Sigma_1$$

$$D_{H_{\delta_f}}^* A = \delta_f A$$

$$D_{H_{\delta_f}}^* A = 0$$

$$S + \epsilon H_{\delta_f} = S + \epsilon \delta_f S$$

$$= S(\phi + \epsilon \delta_f \phi)$$

$$\phi \rightarrow \phi + \delta_f \phi$$

$$\delta_f \phi = f \delta \phi$$

$$\text{def } H_{\delta_f} := \delta_f S$$

← started with
 Σ_0, Σ_1

$$\{H_{\delta_f}, A\} = \underbrace{\delta A}$$

Simple case

Take $A \in \Sigma_1$

$$D_{H_{\delta_f}}^* A = \delta_f A \Rightarrow \{H_{\delta_f}, A\} = \delta A = \int (\phi + f \delta \phi)$$

$$D_{H_{\delta_f}}^* A = 0$$

$$\phi \rightarrow \phi + \delta_f \phi$$

$$S + \epsilon H_{\delta_f} = S + \epsilon \delta_f S$$

$$\delta_f \phi = f \delta \phi$$

$$\text{def } H_{\delta_f} := \delta_f S$$

← separated b/w Σ_0, Σ_1

$$\{H_{\delta_f}, A\} = \underbrace{\delta A}$$

Simple case

Take $A \in \Sigma_1$

$$D_{H_{\delta_f}}^+ A = \delta_f A$$

$$D_{H_{\delta_f}}^- A = 0$$

$$\Rightarrow \boxed{\{H_{\delta_f}, A\} = \delta A} = \int (\phi + f \delta \phi)$$

$$\phi \rightarrow \phi + \delta_f \phi$$

$$S + \epsilon H_{\delta_f} = S + \epsilon \delta_f S$$

Noether's thm

Suppose $\psi \rightarrow \psi + \delta\psi$ is a sym of S .

$$S(\psi + \delta\psi) = S(\psi)$$

want $\{H_\delta, A\} = 0$

choose A f. s.t.

$$f=1$$

$$\textcircled{A}$$

Smooth

$$H_\delta$$

choose

$$f=0$$

$$\Sigma_1$$

$$\Sigma_0$$



$$\delta_f \phi = f \delta \phi$$

$$\text{def } H_{\delta_f} := \delta_f S$$

← supported by Σ_0, Σ_1

$$\{H_{\delta_f}, A\} = \delta A$$

Simple Rule

Take $A \in \Sigma_1$

$$D_{H_{\delta_f}} A = \delta_f A$$

$$D_{H_{\delta_f}}^* A = 0$$

$$\Rightarrow \boxed{\{H_{\delta_f}, A\} = \delta A} = \int (\phi + f \delta \phi)$$

$$S + \epsilon H_{\delta_f} = S + \epsilon \delta_f S$$

$$\phi \rightarrow \phi + \delta_f \phi$$

$$Q_3 = \int T^{ab} n_a \sqrt{-h}$$

$$\text{w/ } T^{ab} \equiv \frac{\delta S}{\delta h_{ab}} \dots$$

$$\{x, p\} = \delta$$

$$\{P, p\} = 0$$

$$\{P, P\} = 0$$

$$Q_3 = \int T^{ab} n_a \sqrt{-h} + \text{other terms from other Brd fields.}$$

$$\Rightarrow T^{ab} \equiv \frac{\delta S}{\delta h_{ab}}$$

$$\{P, \mathcal{H}\} = \mathcal{H} \quad \{P, P\} = 0$$