Title: How famous is a scientist? Theory, society, ergodicity and communications in topologies

Date: Jun 02, 2005 02:15 PM

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Abstract: In the first part of the talk, we will discuss our recent paper, "How Famous is a Scientist? Famous to Those Who Know Us". Our findings show that fame and merit in science are linearly related, and that the probability distribution for a certain level of fame falls off exponentially. This is in sharp contrast with more Â“popularly famousÂ” groups of people, for which fame is exponentially related to merit (number of downed planes), and the probability of fame decays in power-law fashion. We will define fame in terms of the type of popularity growth model as a rich-get-richer scheme which leads to a scale-free graph. We will discuss the statistics and ergodicity properties of cycles in the topology of a large scale graph, and likewise the roles of communities and subcommunities to understanding the large scale graphs. In, "Statistics of Cycles: How Loopy is your Network?" we study the distribution of cycles of length h in large networks (of size N>>1) and find it to be an excellent ergodic estimator, even in the extreme inhomogeneous case of scale-free networks. The distribution is sharply peaked around a characteristic cycle length, h ~NÃ¡. Finally, we will analyze Communication and Synchronization in Disconnected Networks with Dynamic Topology: Moving Neighborhood Networks.

Theory, Society, Ergodicity and Communications in Network Topologies.

How famous is a Scientist?

Erik Bollt, Clarkson University
bolltem@clarkson.edu
http://www.clarkson.edu/~bolltem

Dani ben-Avraham, Clarkson U.
Joe Kirk, Clarkson U.
Hernan Rozenfeld, Clarkson U.
Jim Bagrow, Clarkson U.
Joe Skufca, USNA, U. MD, Clarkson U.
Dan Stilwell, Virginia Tech.
D. Gray Roberson, Virginia Tech.

Some Graph Theory

G_A -Graph generated by adjacency matrix $A>0$

$G_A = (E,V)$ $\quad V=\{v_i\}_{i=1}^{N}$ -vertices labeling $\{B_i\}$

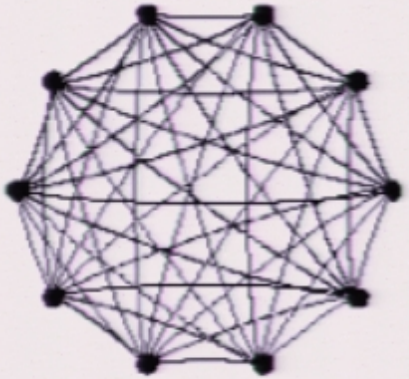
$E=\{(v_i, v_j): i \leq i,j \leq N,\ A_{i,j}>0\}$ -set of edges

Taxonomy of Popular Random Graph Communities: Global Topology

Which Best Describes the World-Wide Web?

All-All

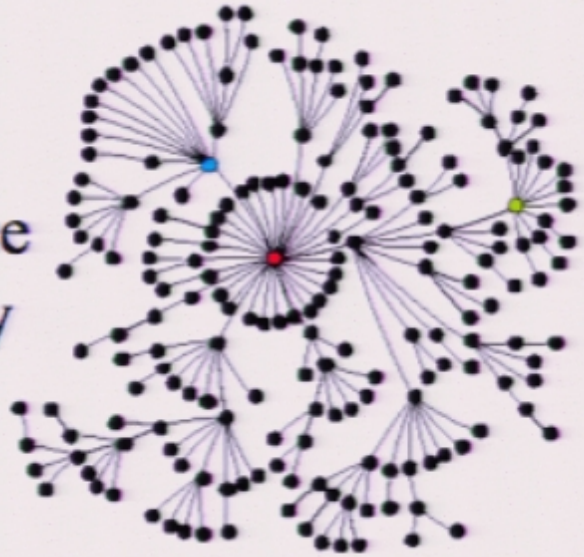
E=10,N=10



Scale-Free

- E=199,N=200
- Red->k=33 degree
- $\phi(k)$ - probability

$\phi(k) \sim k^{-\lambda}$



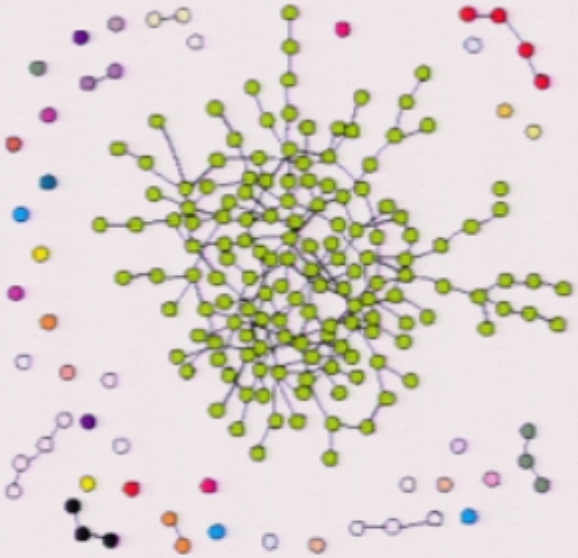
ER Random

E=193, N=200

Giant Component

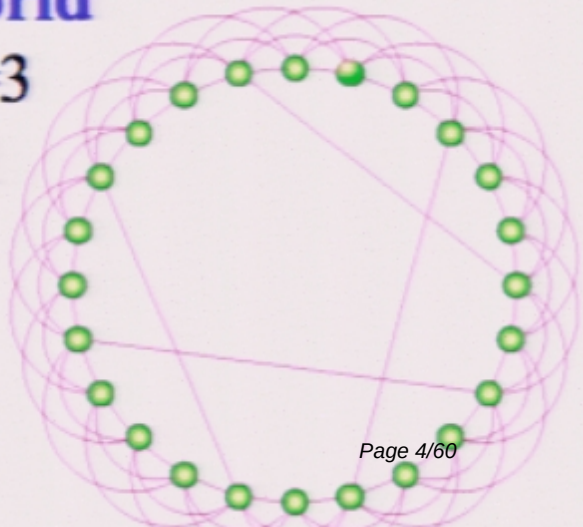
Expect if E>N/2

No Central Hub



Small-World

- N=24, k=3
- $\phi(k)$ add



tragatz Nature 2001

Who is famous?

What is fame?

Is fame linked to Merit?

V. Simkin og V.P. Roychowdhury; Theory of Aces: Fame by chance or merit?; Condensed Matter; arXiv.org; 2. oktober 2003

mes Bagrow, Hernan Rozenfeld, Erik Bollt, and Daniel ben-Avraham, "How Famous is a Scientist? ---
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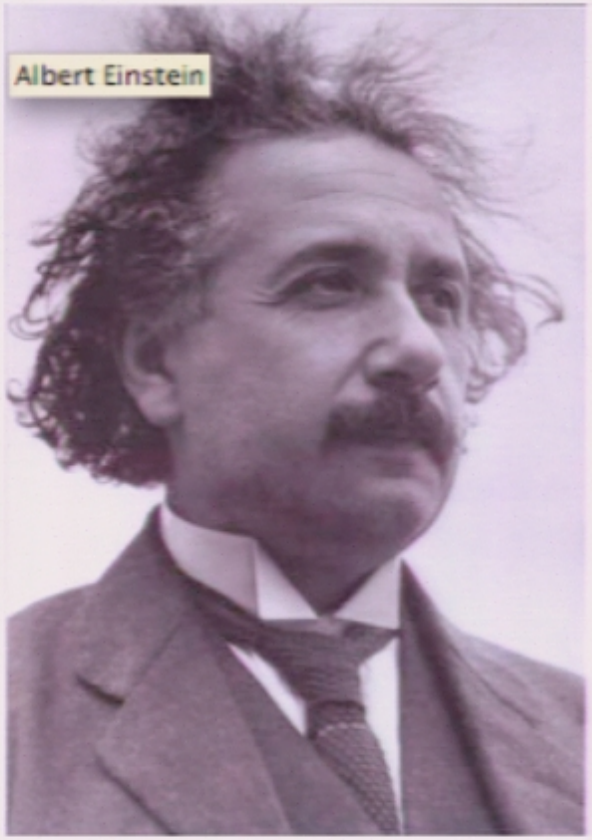
How "Famous" are these folks?

Guess Who

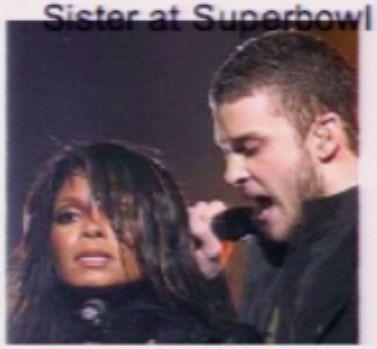


4,710,000

Brother

7,010,000

Sister at Superbowl



1,500,000

Michael Jordan

1,560,000

Tom Cruise

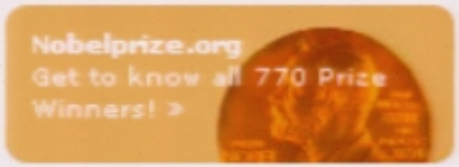
3,510,000

Isaac Newton

Me

~660-9730

An "anonymous" Nobel winner from 1990's



59,900

My favorite Fields Winner

Wayne Gretzky

442,000

I challenge you

1. Name 10 Nobel prize winners from the 1980's-Today.

2. Name 10 "Famous" athletes.

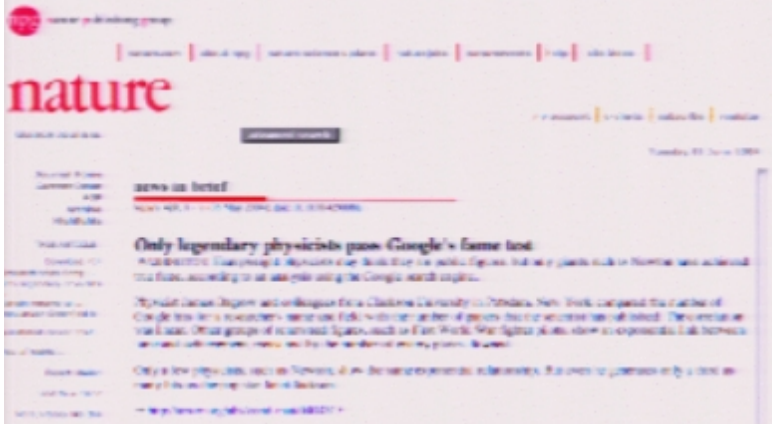
3. Name 10 "Famous" movie actors.

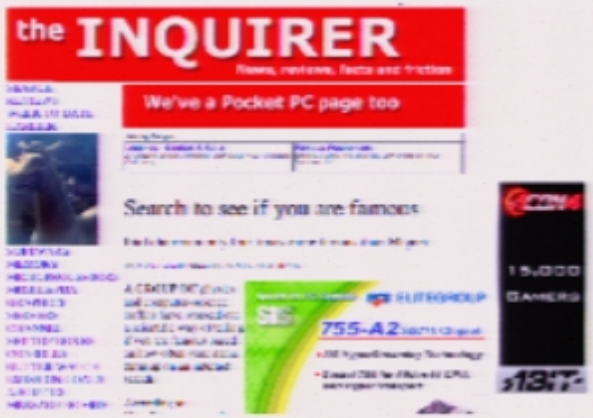
4. Name 10 "Famous" (economists/biologists/ Engineers/…not in your (sub)field).

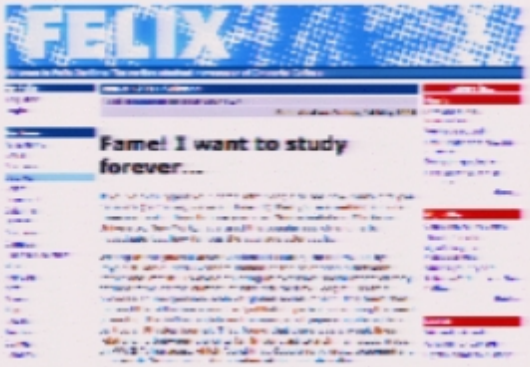
Don't tell anyone: I don't even like football or hockey!

Assert: Fame is a mechanism

==>







Theory of Aces

Let F Fame be ~number of times cited by google

Let A Merit be ~number of enemv aircraft shot down

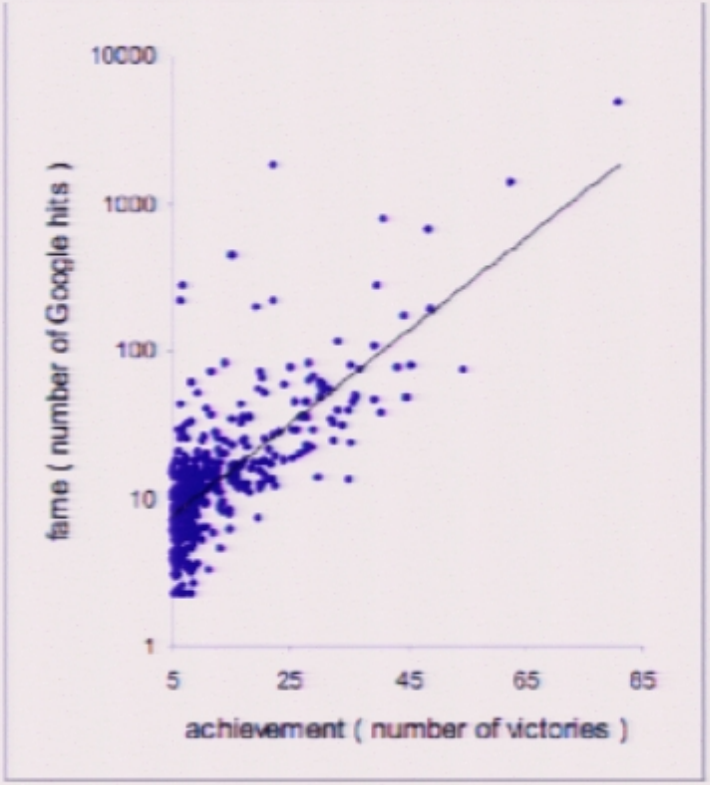


Figure 1. A scatter plot[3] of fame versus achievement for 393 German WWI aces. The correlation coefficient of 0.72 suggests that $0.72^2 \cong 52\%$ of the variation in fame is explained by the variation in achievement. The straight line is the fit using Eq.1 with $\beta \cong 0.074$.

Assert: Fame is a mechanism

How famous is a scientist? —Famous to those who know us

==>

nature

Only legendary physicists pass Google's fame test

the INQUIRER

We've a Pocket PC page too

Search to see if you are famous

FELIX

Fame! I want to study forever...

NEWSFACTOR TECHNOLOGY NEWS

E-Commerce

Thin & Light

physicsweb

Physics and fame

Theory of Aces

Let F Fame be ~number of times cited by google

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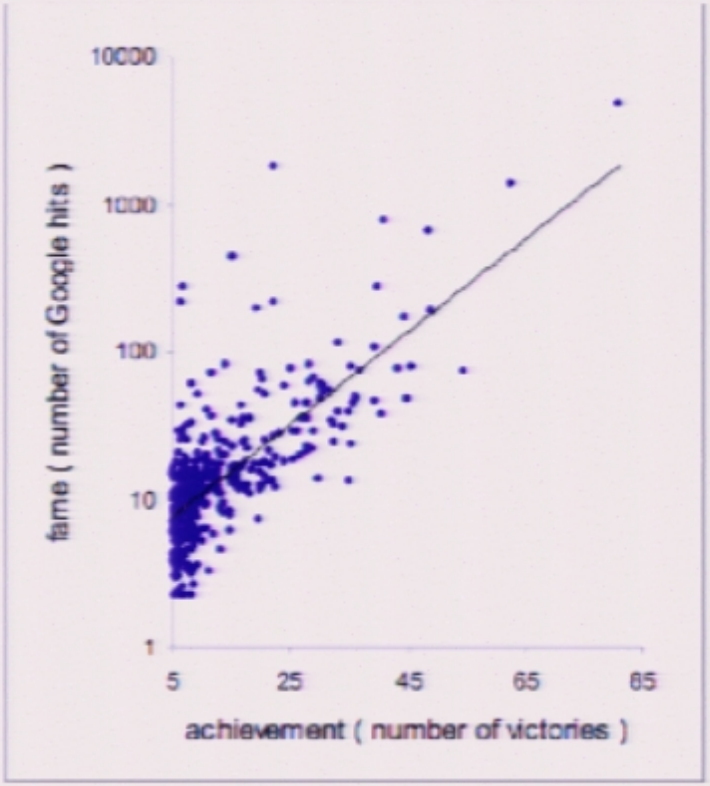

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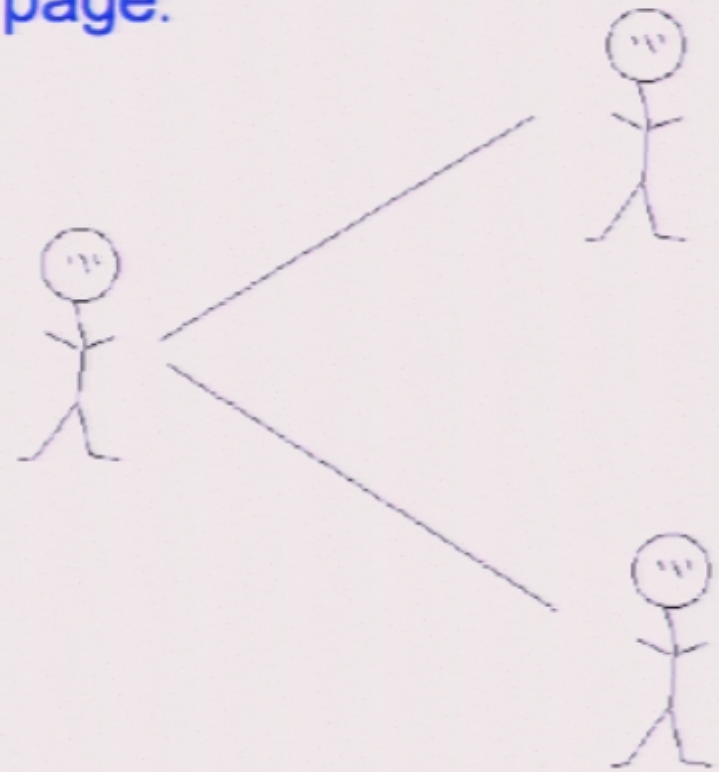
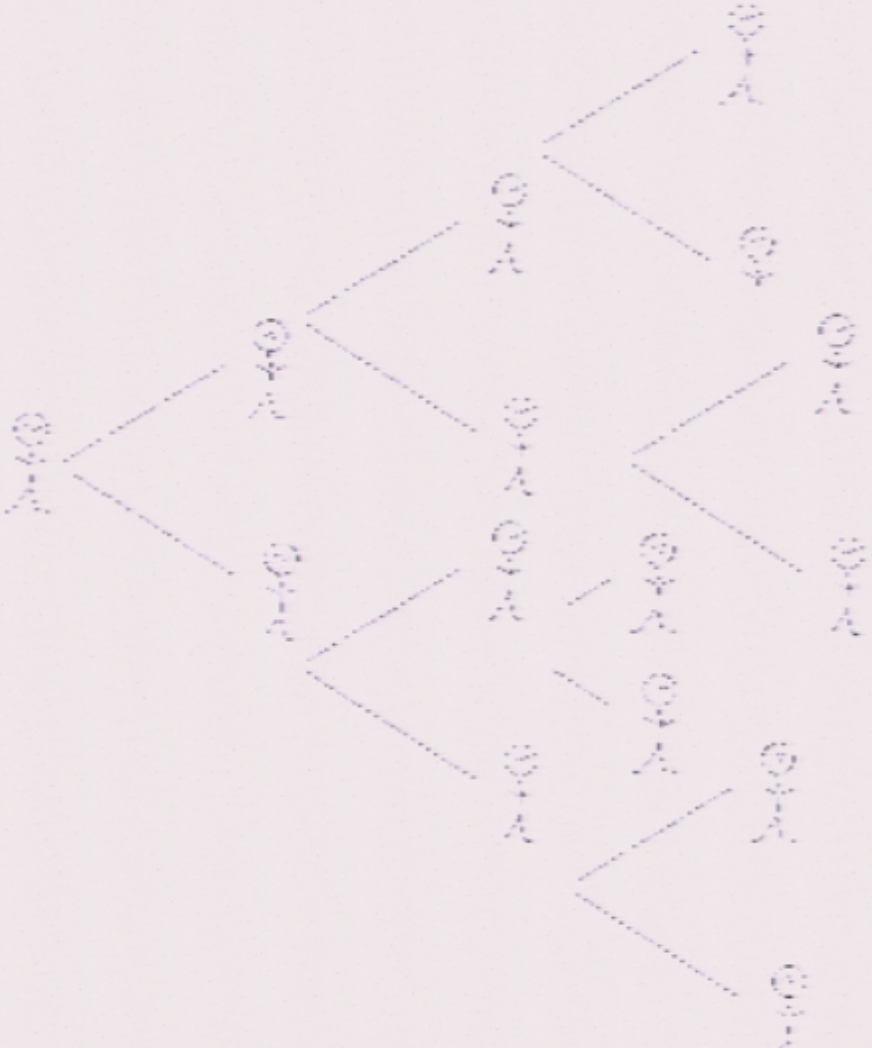
A stochastic model of multiplicative fitness.
In the beginning, there was one web page.

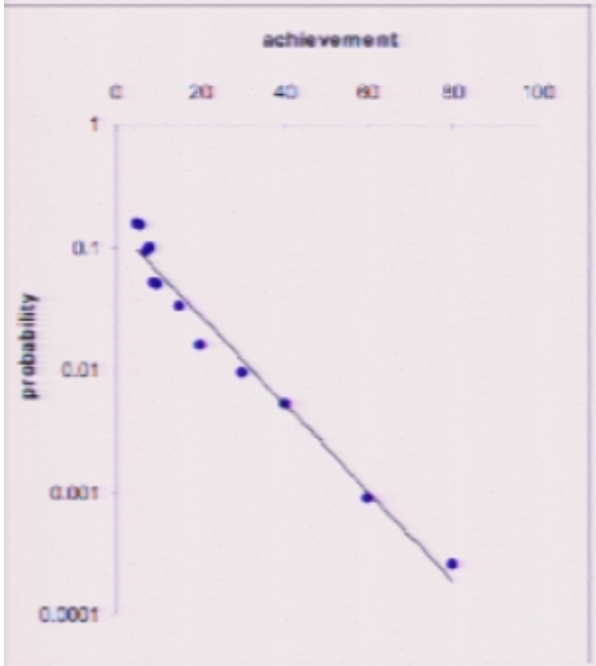
They Told Two Friends



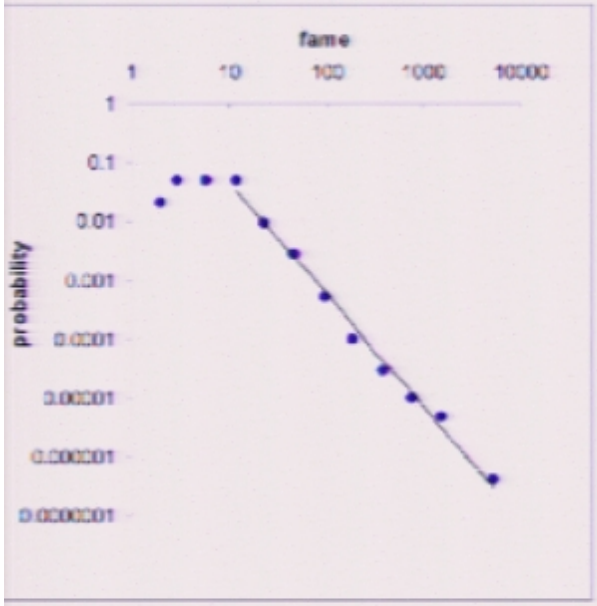
G. Bianconi and A.-L. Barabási, "Competition and multiscaling in evolving networks", cond-mat/0011029; Erophys. Lett. **54**, 436 (2001).

And so on, and so on, and so on….





igure 2. The distribution of achievement (number of victories) btained using a sample of 393 German WWI aces. The straight ne is the fit using Eq.2 with $\alpha \cong 0.083$.



igure 3. The distribution of fame (number of Google hits) omputed using a sample of 393 German WWI aces. The straight ne is the fit $p(F) \propto F^{-\gamma}$ with $\gamma \cong 1.9$.

A rate equation for number of pages F_k which refer to ace-k is proportional to A_k, achievement, and N is total number of webpages about aces.

$$\frac{dF_k}{dN} = A_k F_k \Big/ \sum_{i=1}^{M} A_i F_i$$

Leads asymptotically to solution distributed solution:

$$p(F) \propto F^{-\gamma}\,;\gamma = 2 + \frac{M}{N-M}$$

But…that assumes unlimited resource.
Consider x'=ax versus x'=ax(1-rx)

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low shall we measure Fame: Number of hits on Google with the following .exicon:

Author's name" AND "condensed matter" OR "statistical physics"
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form of lexicon,
is Google PageRank okay to measure fame,
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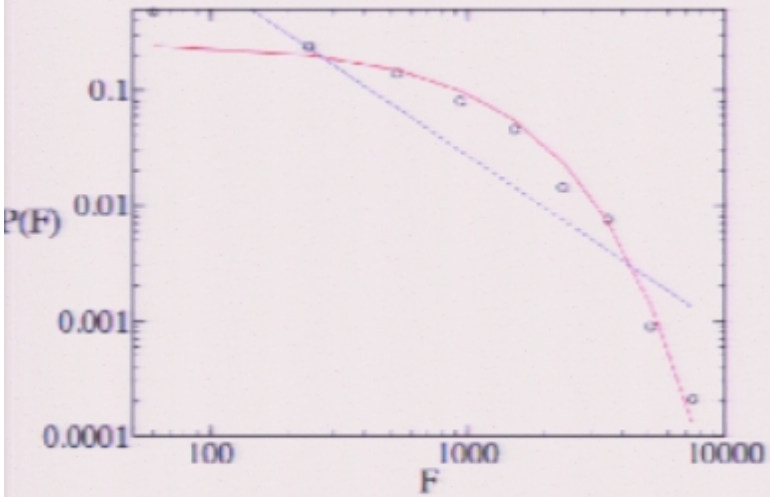
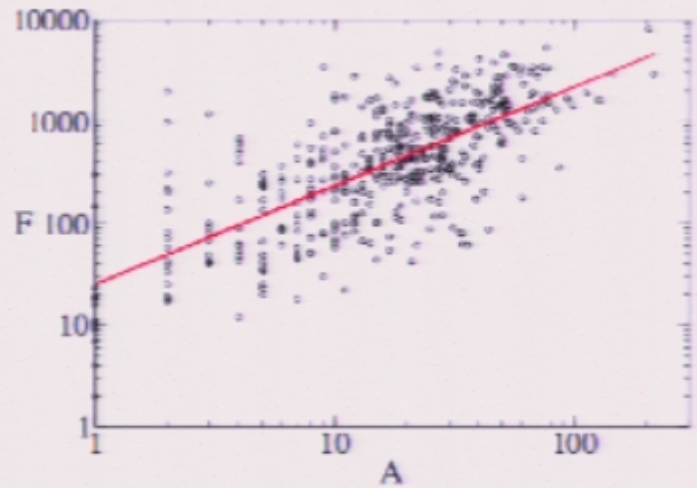
what if I am a mediocre scientist, but a terrific hockey player?
sample of scientists, etc, etc.

Good sociological data is hard and expensive to come by.

What is different about scientists?





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Aces, and other "popular groups"

$$P(F) \sim F^{-\gamma}, \quad \gamma \approx 2$$

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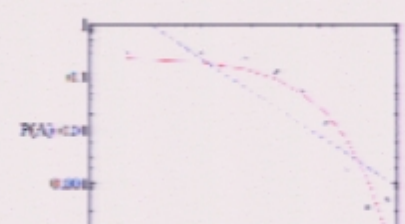

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Finite small group of people might read out papers...and only p% do...

I am FAMOUS! I am known internationally around the world (to a few dozen people) and I am known well (to those studying hemi-semi-demi-groups),

True Fame*: Let the mechanism of exponential growth be the definition of fame. *Popular fame.

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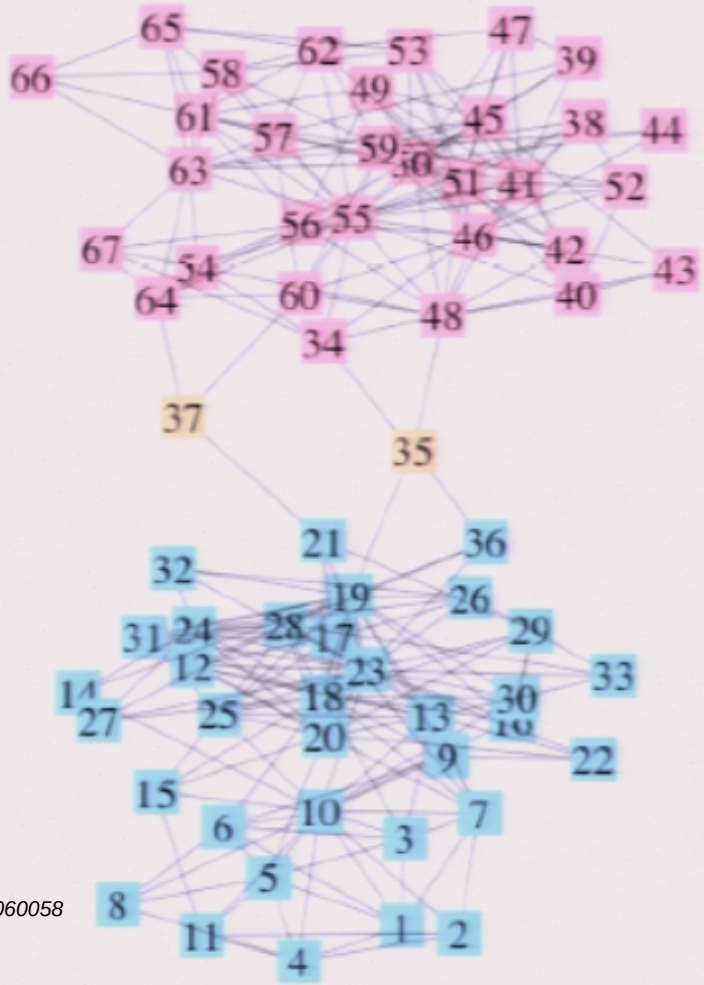
What are Communities?



- Groups of vertices that are densely connected amongst themselves while being loosely connected to the rest of the graph
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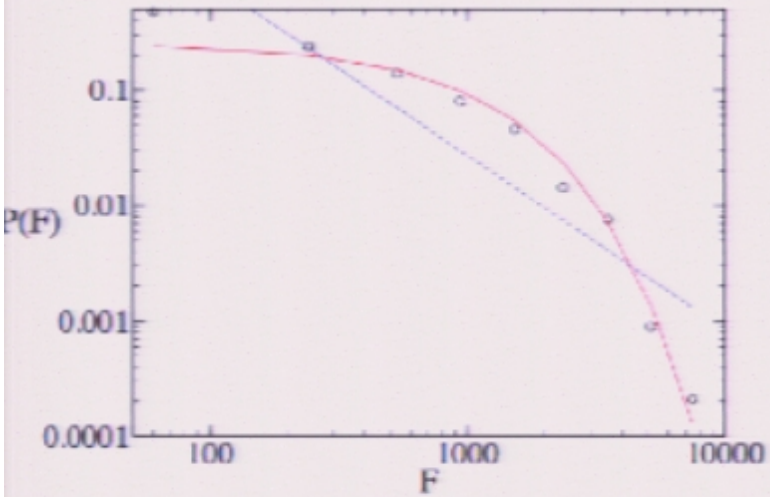

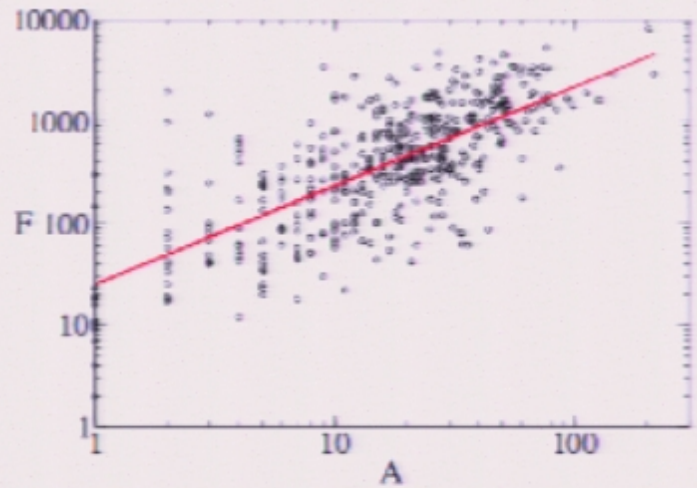

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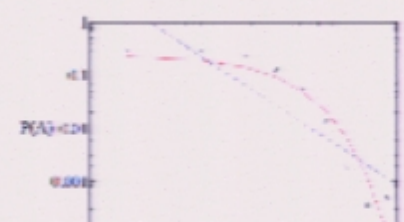

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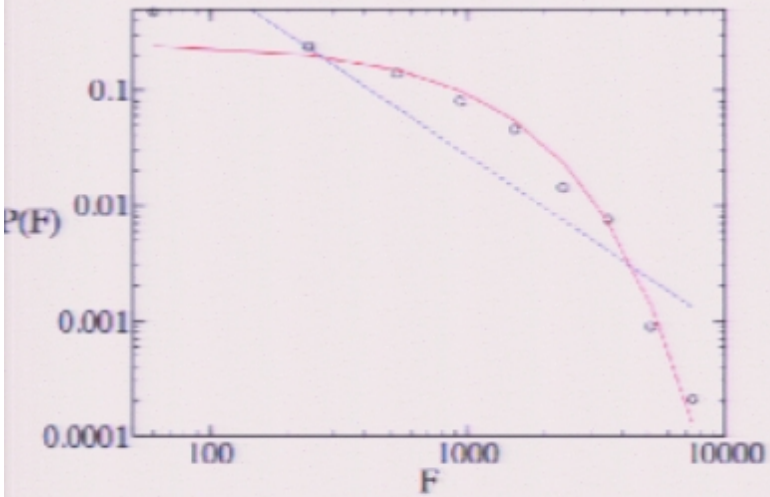

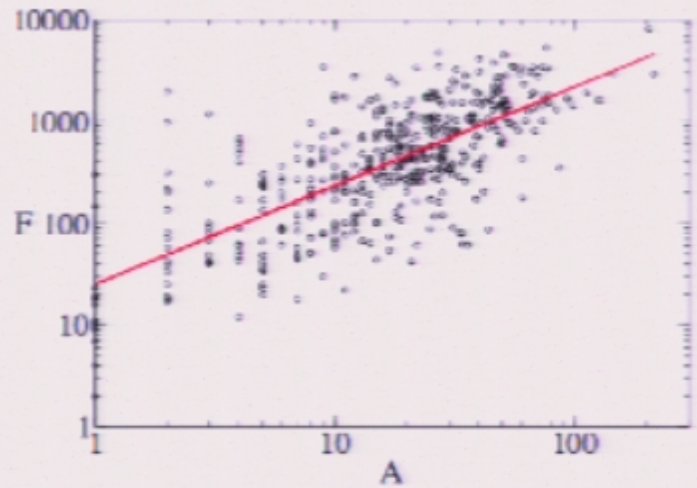

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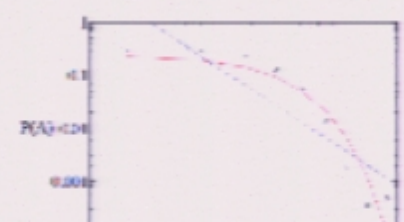

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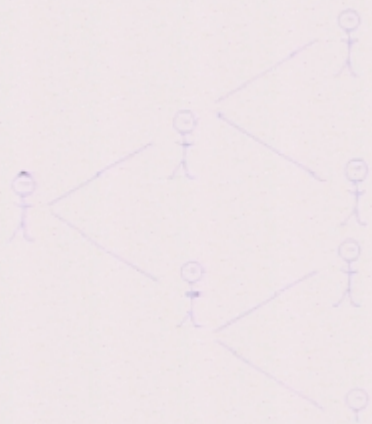

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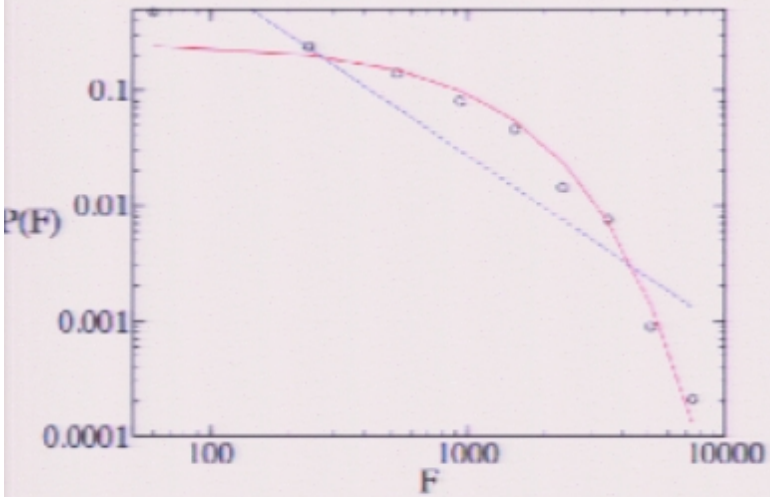

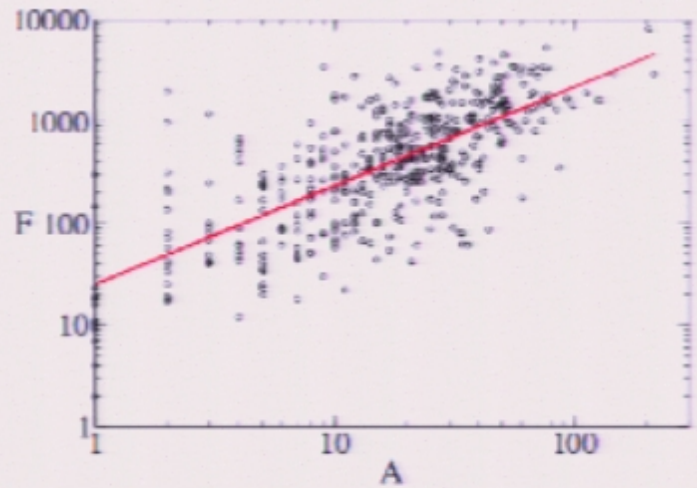

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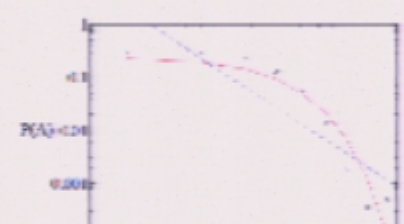
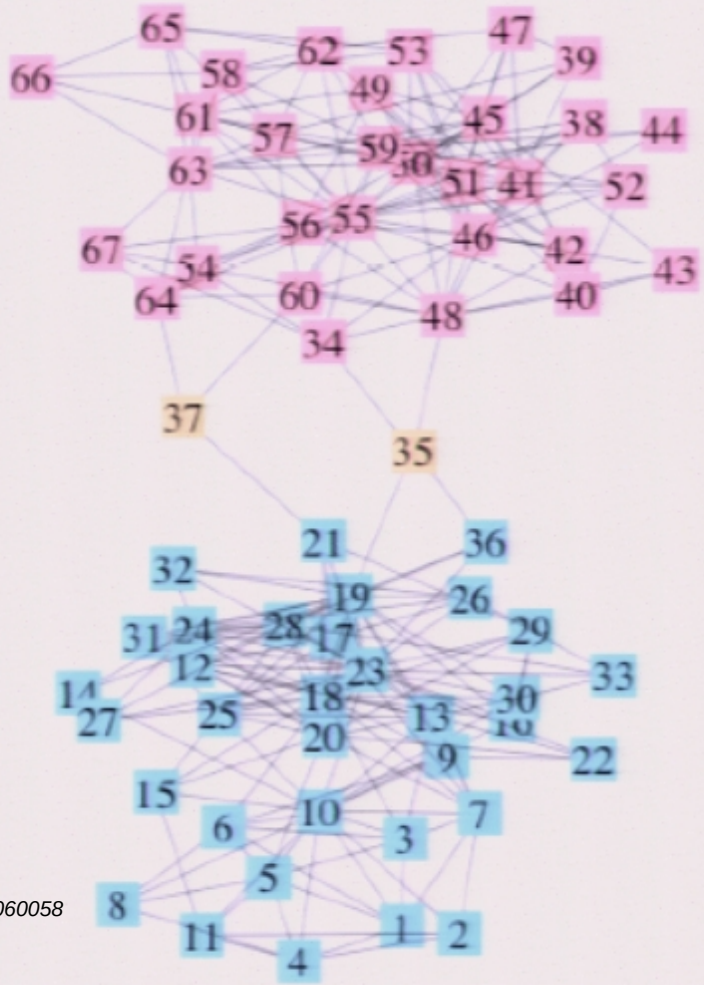
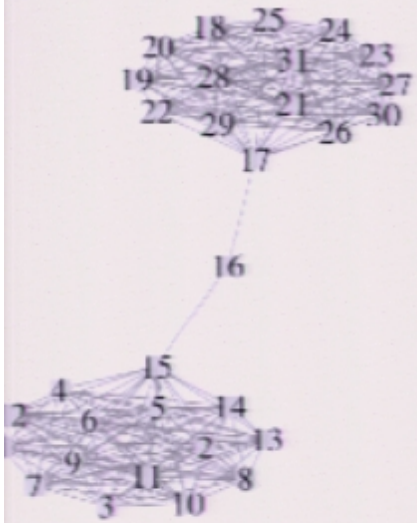


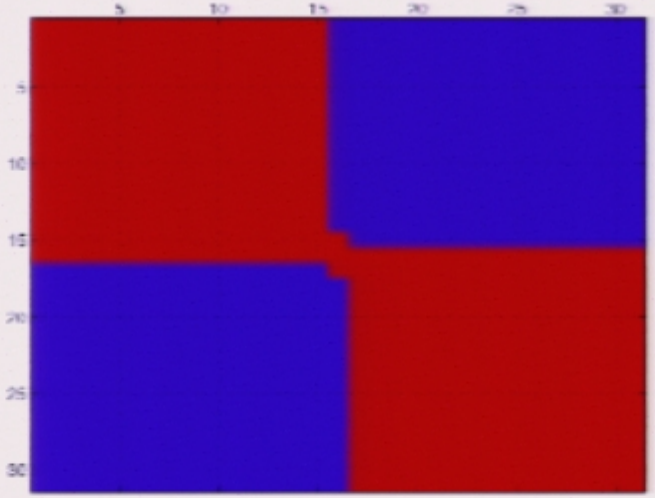
What are Communities?

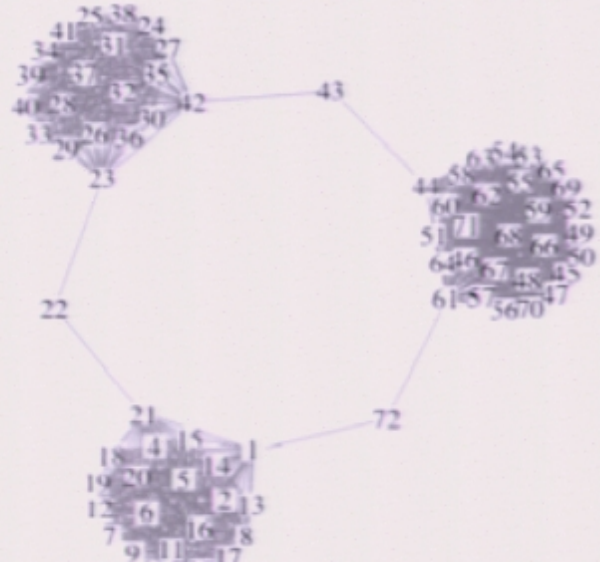


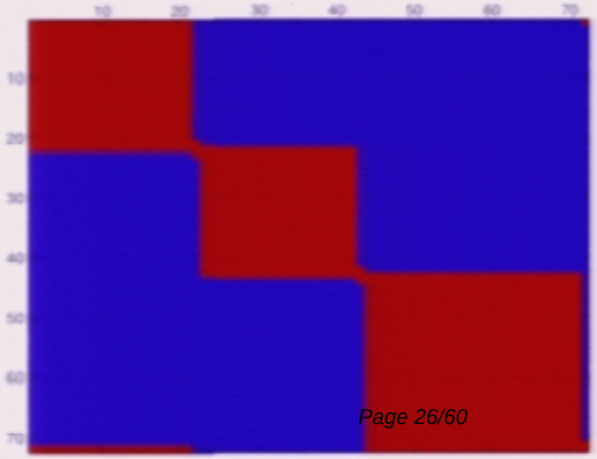
- Groups of vertices that are densely connected amongst themselves while being loosely connected to the rest of the graph
- Detection can be difficult (expensive)

Some example M's









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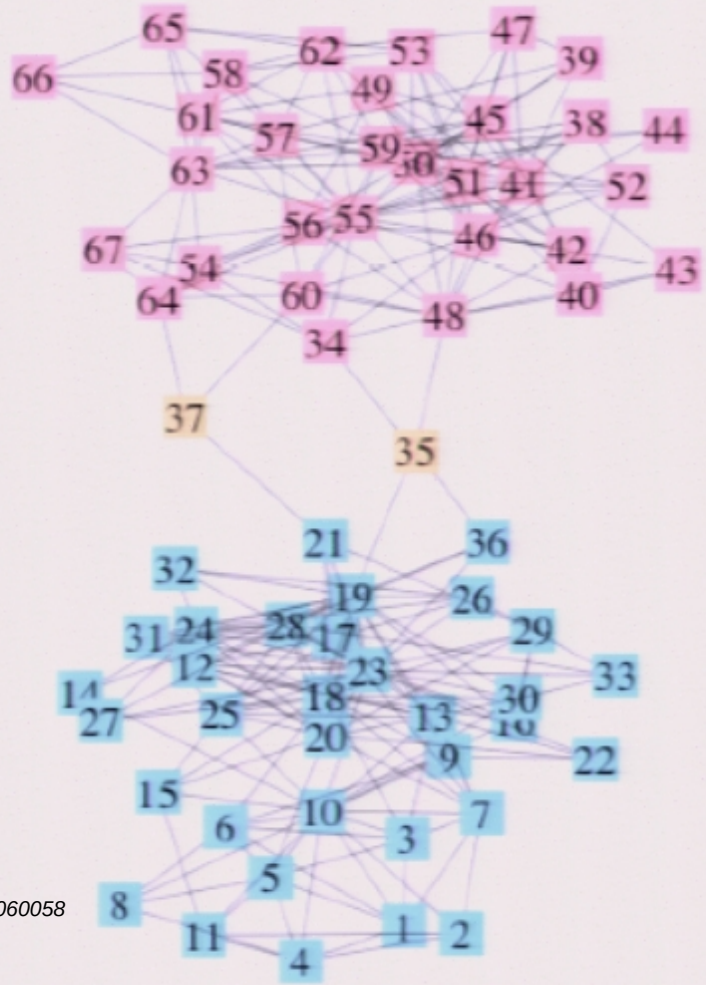

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Why Find Communities?

- Social networks
- Parallel computing optimization
- Drug dependency in protein-protein interaction networks
- etc

Detection Methods

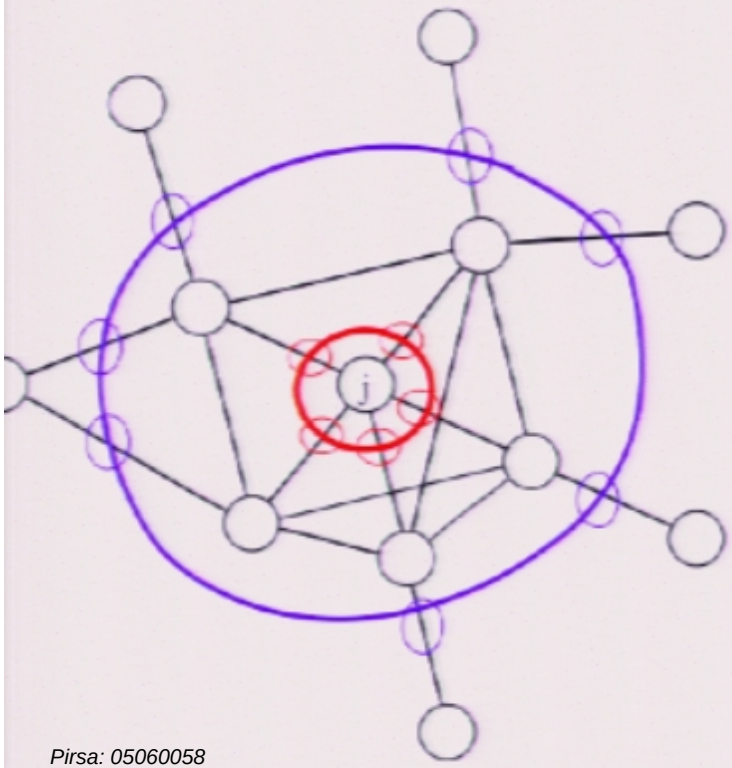
- Most are either agglomerative or divisive
  - rewiring the network from scratch while optimizing some parameter
  - selectively removing edges, splitting the network apart
- The method shown here is unlike these
  - allows for detection with only local information

Our local method

- Choose a starting vertex
- Spread outward until some criteria has been met, then stop
- Does not require knowledge of graph beyond spread

Notation

$K_j^l \equiv$ Total Emerging Degree of an l-shell starting from vertex j



- $K_j^0 = 5$
- $K_j^1 = 7$
- $K_j^2 = 0$

$$\Delta K_j^l = \frac{K_j^l}{K_j^{l-1}}$$

$$\Delta K_j^1 = \frac{7}{5}$$

$$\Delta K_j^2 = \frac{0}{7}$$

The Algorithm

- Choose a starting vertex j
- Center an l-shell around j (l=0) and spread outward
  - compute the total emerging degree at each depth of the l-shell
- Continue until the current Change in Total Emerging Degree crosses some threshold
- All vertices covered by the last l-shell created are then members of vertex j's community

Stopping Criteria

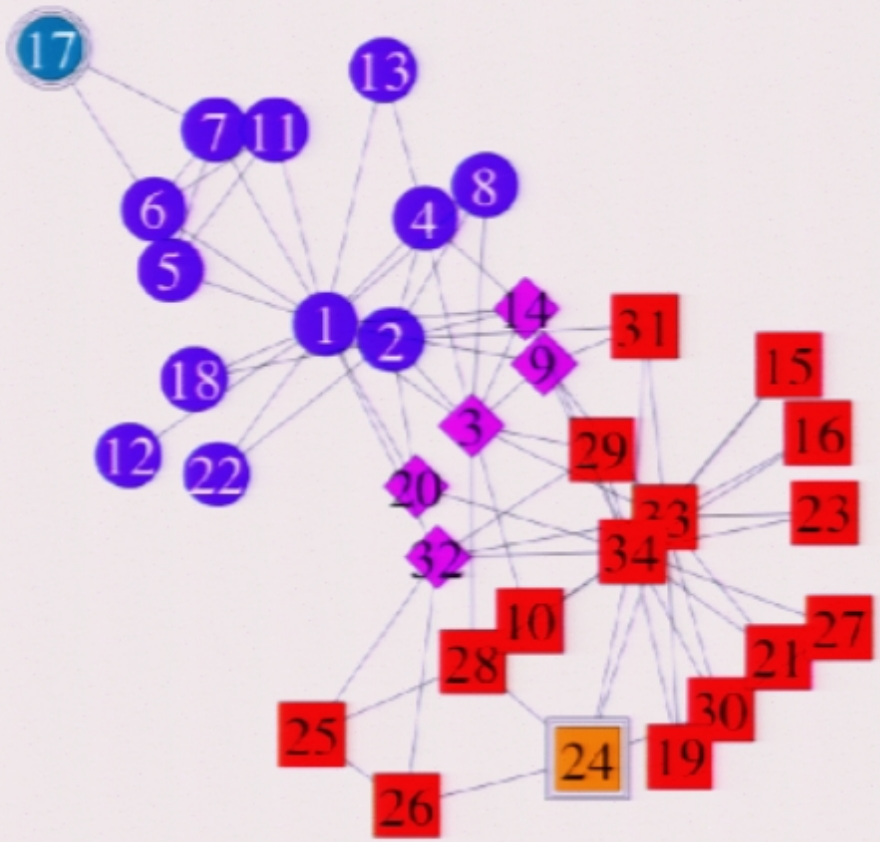
Stop when:

$$\Delta K_j^l < \alpha$$

where α is some arbitrary threshold, > 0

All vertices covered by shells of size l or less are then members of j's community

Result: Karate Club

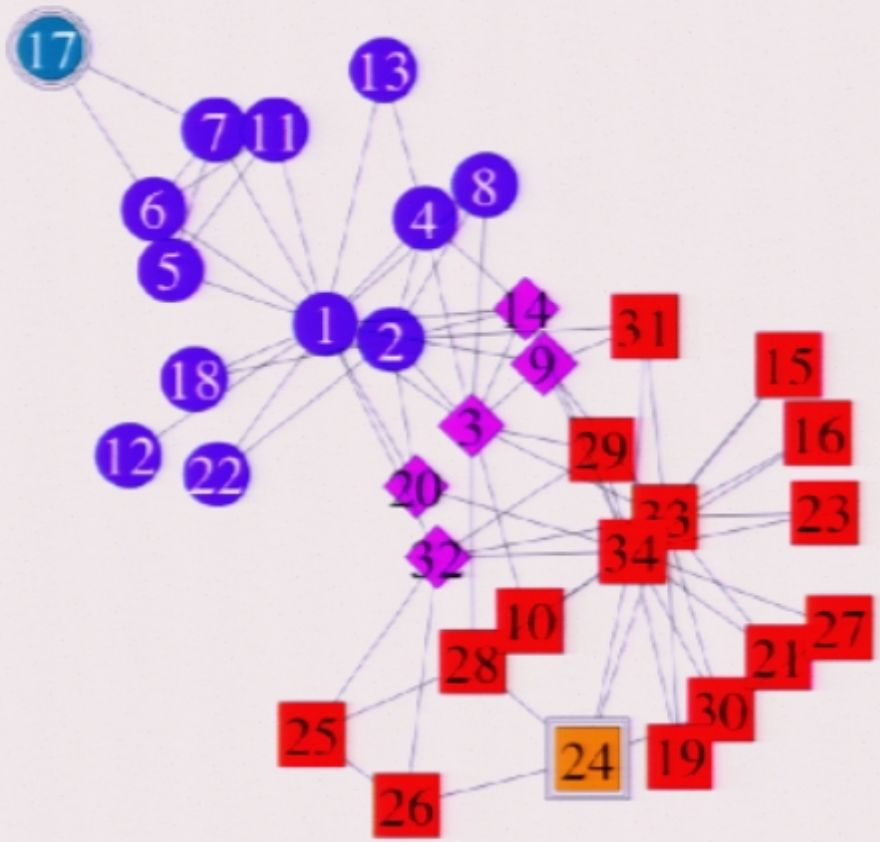


- Two Runs
  - Starting from vertices 17 and 24
- α = 1.9
- Indicates overlap vertices

A Global Application

- Can start the algorithm from all n vertices
- Need some way to relate results from disparate runs
- Define Membership Matrix, M
  - M_{ij} = 1 if vertex j is a member of i's community
  - 0 otherwise
  - M is of size n x n for a network of n vertices

Result: Karate Club



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A Global Application

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  - 0 otherwise
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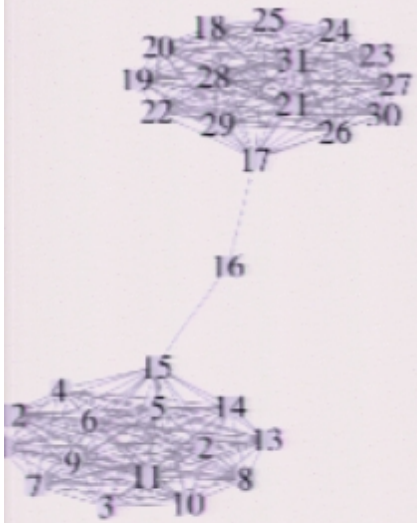
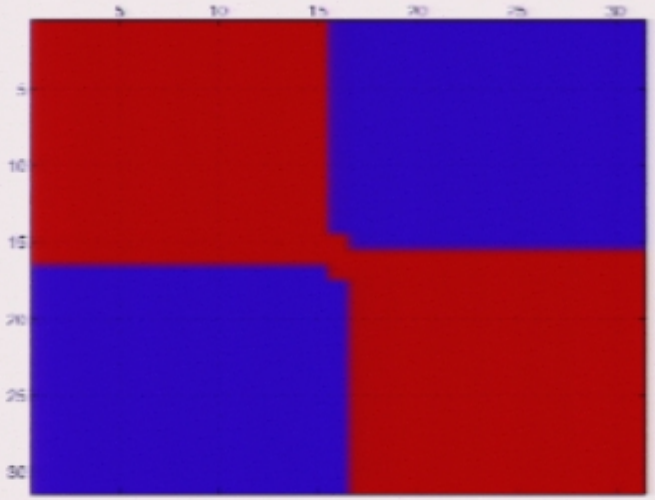
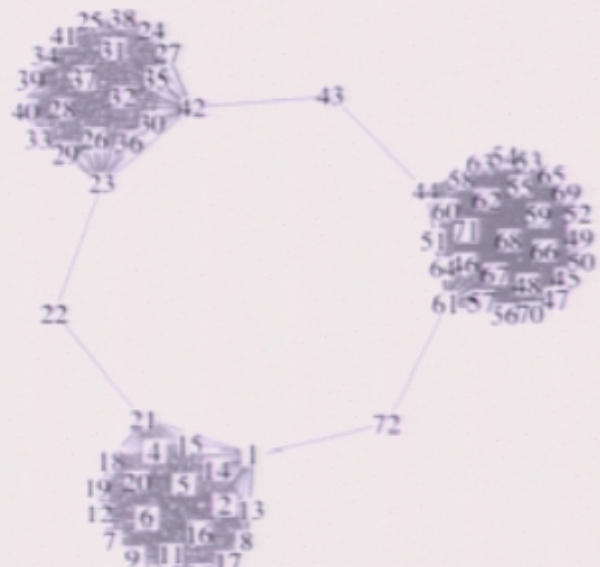
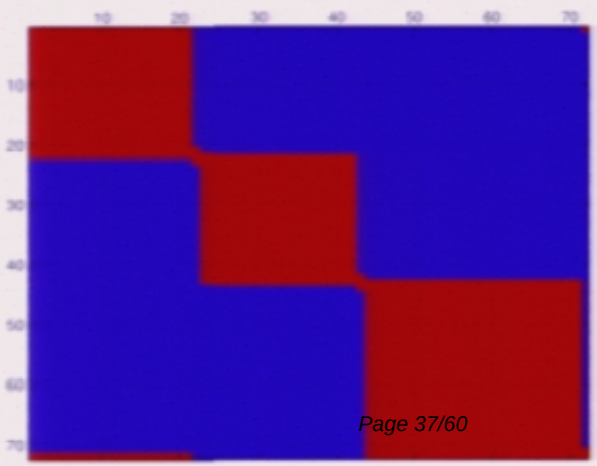
Some example M's



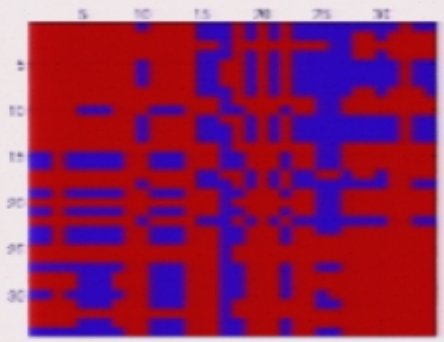






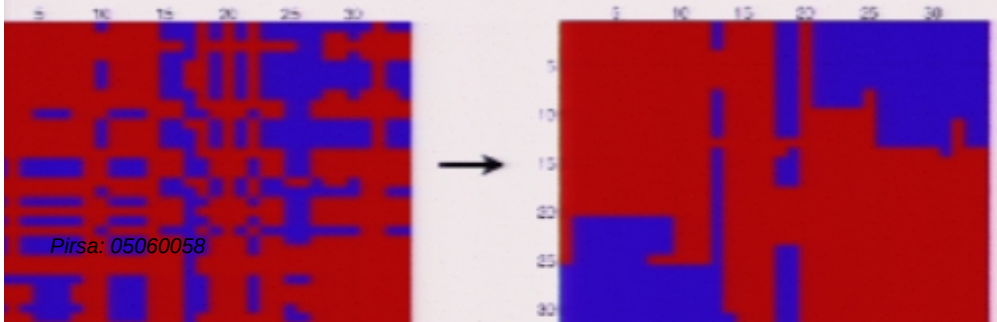
Problem with M

- These last results are trivial
  - the vertices are already numbered consecutively by community
- The Karate Club is not:



$\alpha = 1.2$

Sorting Example



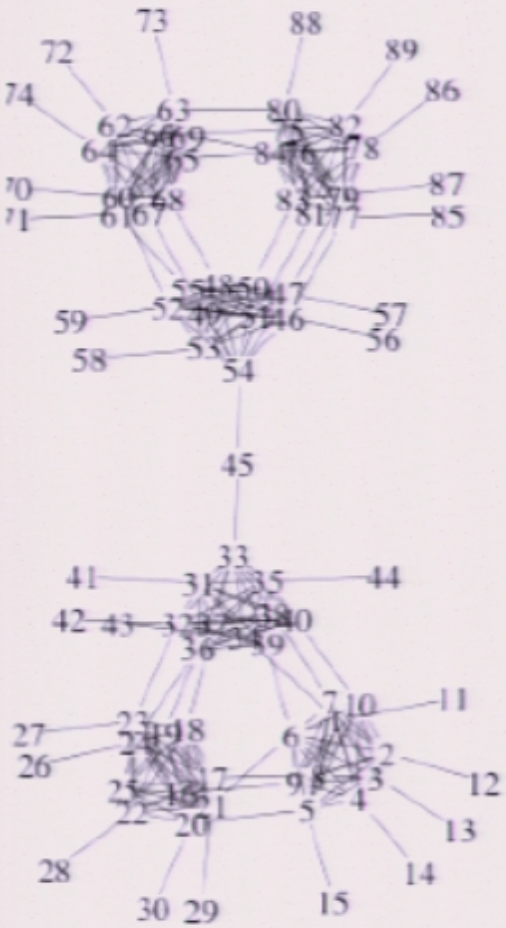
Solution: Sort M

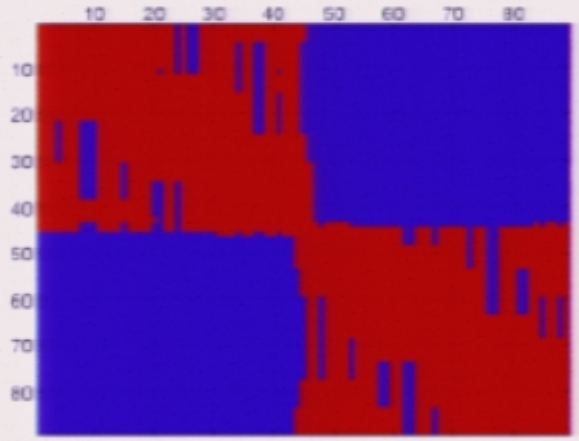
Define the distance between rows:

$$Distance(i,j) = n - \sum_{k=1}^{n} \delta(M_{ik}, M_{jk})$$

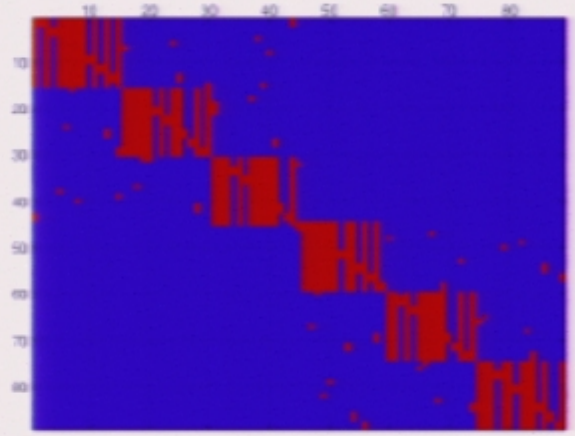
- To sort row i
  - Find Distance(i,j) for all j>i
  - Choose "closest" row, call it k, and interchange rows (and columns) i+1 and k
- Repeat for i+1, etc.

Results

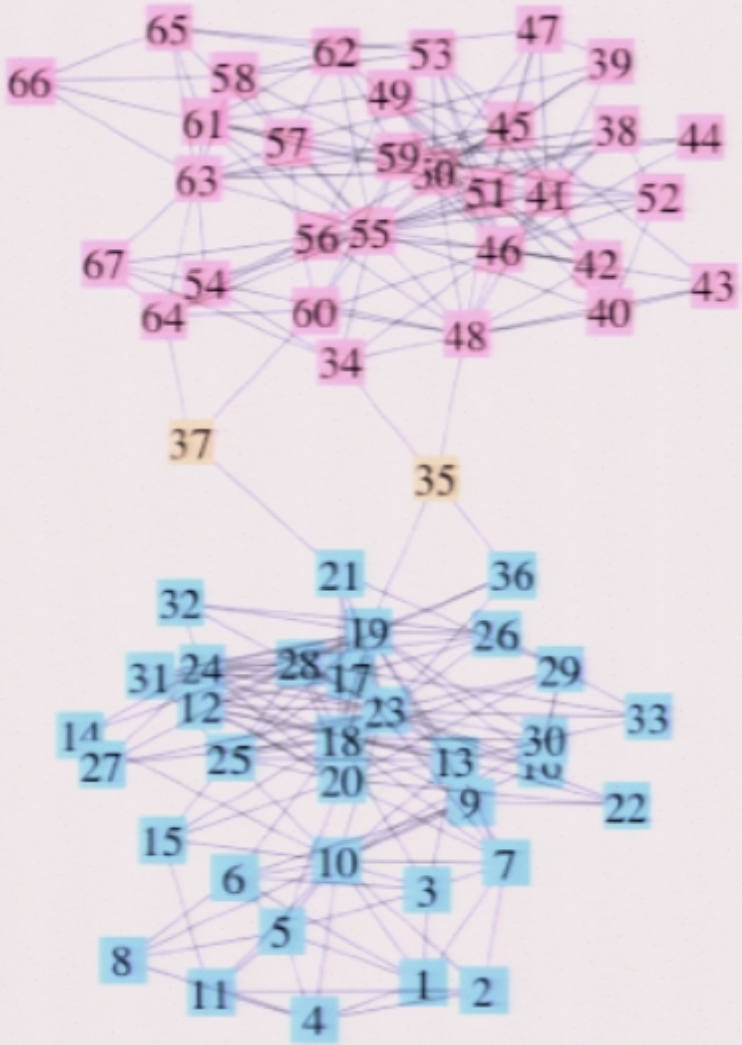




$\alpha = 0.87$



$\alpha = 3$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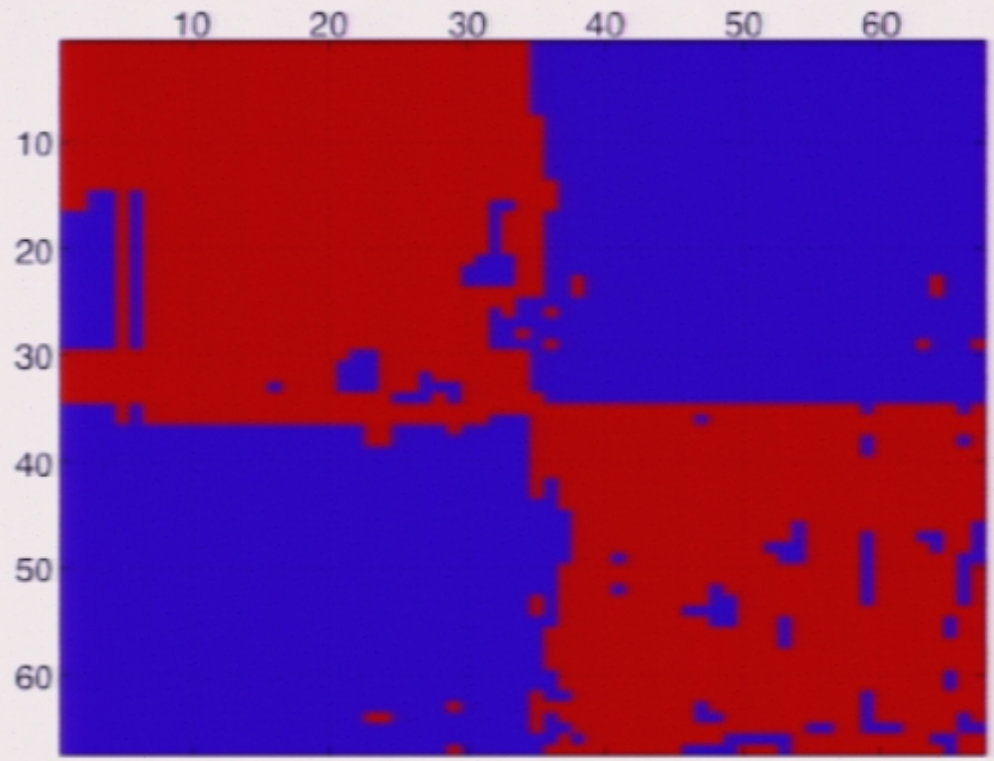
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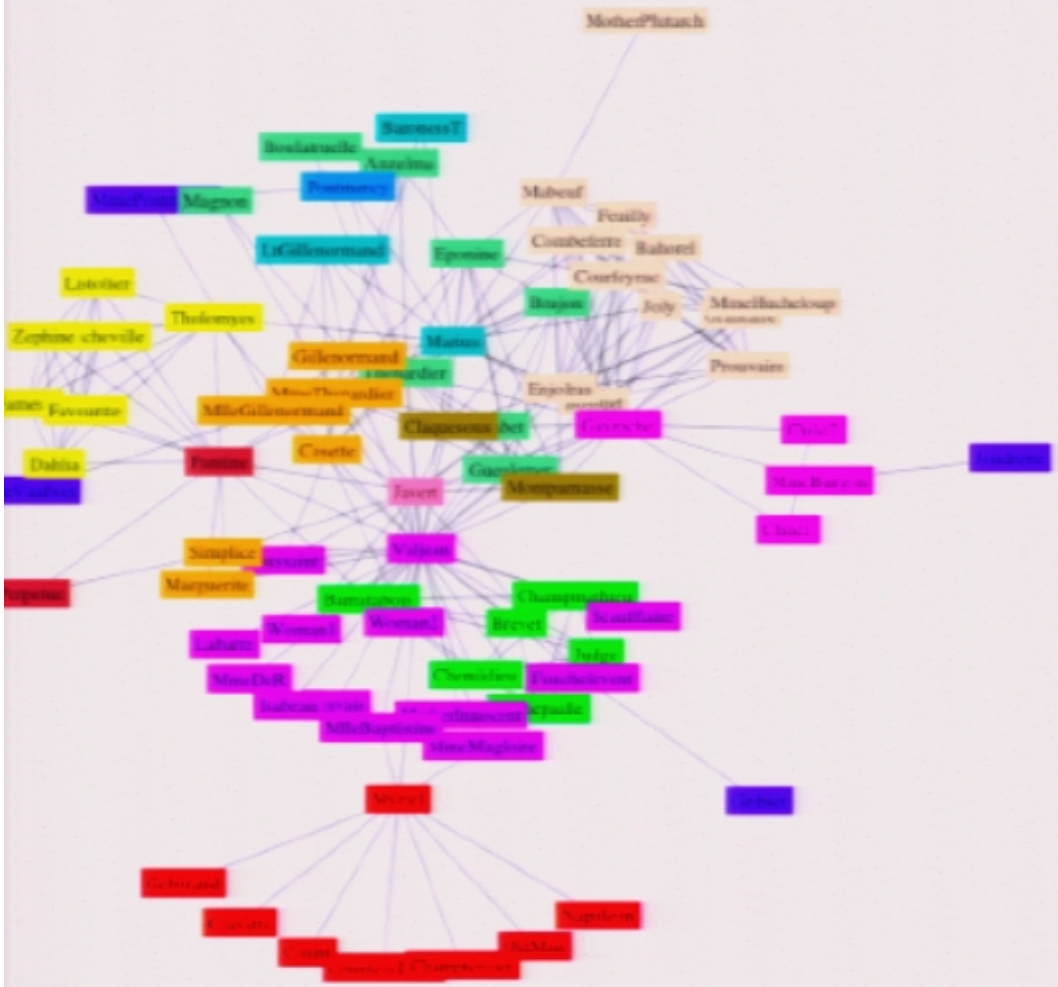
$$\alpha = 1.2$$


MotherPlutarch
Mabeuf
Feuilly
Courfeyrac
Bahorel
Bossuet
Joly
Mme Hucheloup
Prouvaire
Enjolras
Gillenormand
Marius
Cosette
Fantine
Tholomyes
Listolier
Zephine cheville
Favourite
Dahlia
Mme Thenardier
Mlle Gillenormand
Eponine
Magnon
Pontmercy
Anzelma
Boulatruelle
Babet
Montparnasse
Gavroche
Gueulemer
Claquesous
Javert
Valjean
Simplice
Marguerite
Brevet
Chenildieu
Cochepaille
Judge
Fauchelevent
Jacquelin
Mme Magloire
Mme Dor
Lebaupin
Woman 1
Woman 2
Child
Myriel
Napoleon
Champmathieu
Tholomyes
Listolier
Fameuil
Blacheville
Favourite
Dahlia
Zephine
Mme Thenardier
Mlle Gillenormand
Mme Pontmercy
Mme Magloire
Mme Hucheloup
Mme Dor
Mme Bonaparte
Child
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Pontmercy
Napoleon
Myriel
Champmathieu
Brevet
Chenildieu
Judge
Cochepaille
Fauchelevent
Jacquelin
Woman 1
Woman 2
Lebaupin
Simplice
Marguerite
Valjean
Javert
Cosette
Fantine
Dahlia
Zephine
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Gueulemer
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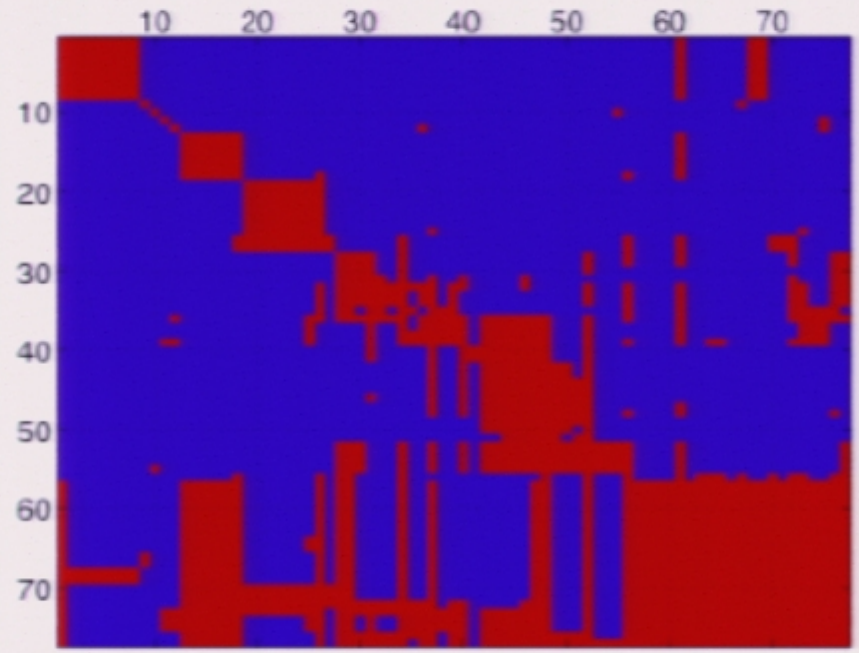
Mabeuf
Prouvaire
Mme Hucheloup
Child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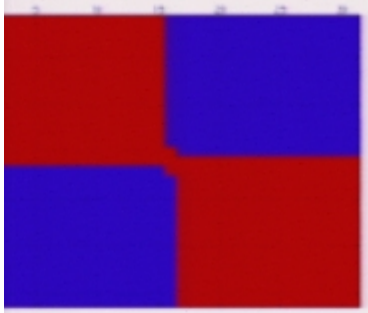
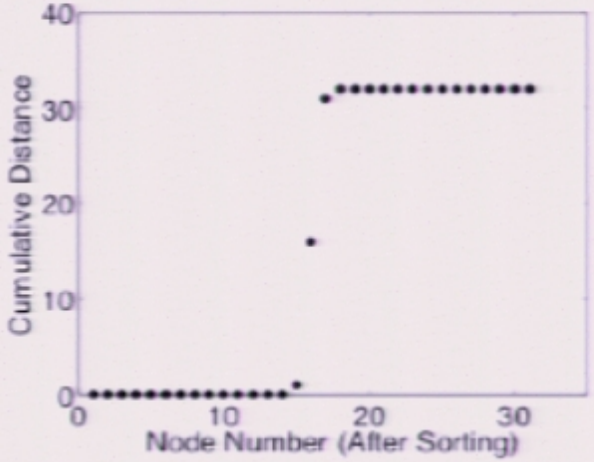
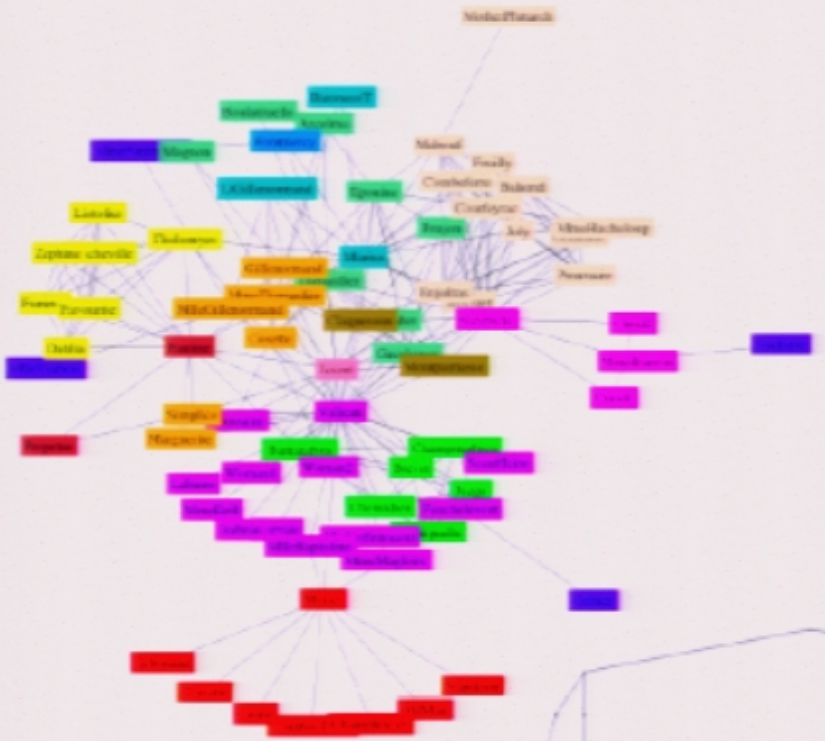
$$\alpha = 6.9$$

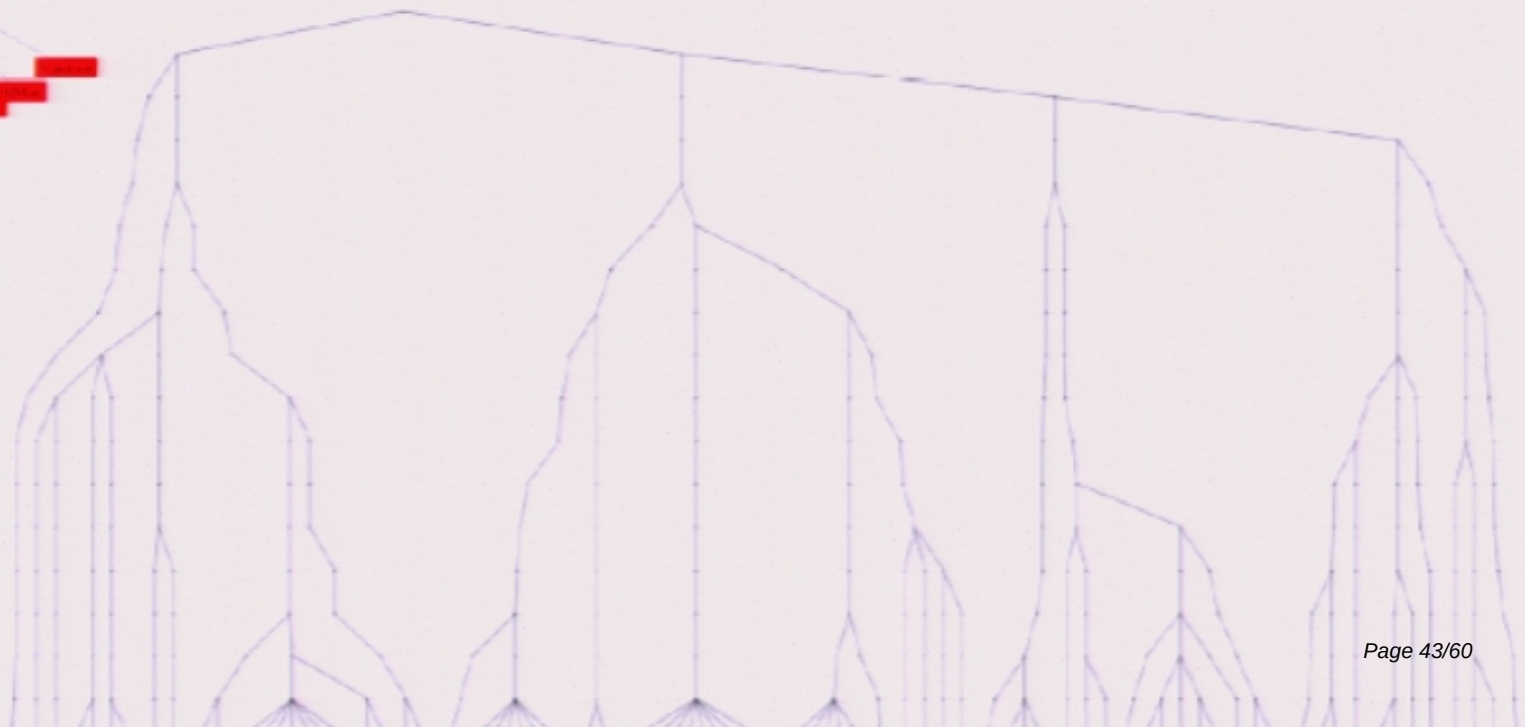
- Choose shortest difference between adjacent points
  - cluster all nodes closer than that distance
  - choose next shortest difference





Example Dendrogram





- J. P. Bagrow and E. M. Bollt. A local method for detecting communities. arXiv/cond-mat:0412482, 2004.
- M. Girvan and M.E.J. Newman, Community structure in social and biological networks. Proc. Natl. Acad. Sci. USA 99, 7821-7826 (2002).
- M.E.J. Newman and M. Girvan, Finding and evaluating community structure in networks, Phys. Rev. E 69, 026113 (2004).
- V. Krebs, Political Patterns on the WWW – Divided We Stand. Home page: http://www.orgnet.com/divided2.html

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Statistics of Cycles: How Loopy is your Network?

Hernán D. Rozenfeld*
Joseph E. Kirk**
Erik M. Bollt**
Daniel ben-Avraham*

Clarkson University
Department of Physics*
Department of Math and Computer Science**
Potsdam, NY, 13699.

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Overview

- **Introduction**
  - Motivation
  - Networks and Cycles
- **Results**
  - Cycles on a Deterministic Scale-Free Network
  - Cycles on a Lattice, Cycles on a Complete Graph.
  - Cycles on Random Scale-Free Network. Sampling.
- **Conclusions**

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Motivation

In order to characterize Networks we need to study their properties. Finding global characteristics and properties of Networks.

•Degree Distribution: Probability to find a node with degree k.

Delta Distribution: $P(k)=\delta_{k,k'}$ (lattices or Regular Graph).
Power-law Distribution: $P(k)\sim k^{-\lambda}$ (Barabasi – Albert).

•Average Clustering Coefficient.

•Diameter.

•*Complete Distribution of Cycles:*

- Important for propagation along the network.
- Understanding the structure and topology of the network.
- Characterization of networks.

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Networks and Cycles

h-cycle: closed path through h connected links that is self-avoiding.

N_h: total number of distinct h-cycles in the network.

h_*: size of typical cycle in the network.

Finding cycles in Networks

↓

Depth-First-Search Algorithm

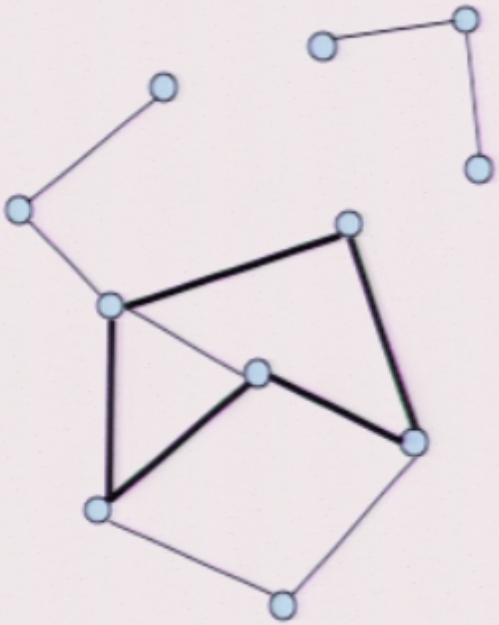
↓

Full distribution of cycles of all sizes

↓

Extremely expensive:

N_h grows exponentially with the cycle size!



Networks and Cycles

This is the first time the complete distribution of cycles and the typical cycle size h_* are studied. As will be shown this quantity describes the distribution of cycles in the thermodynamic limit.

This topic is a very active field of research.
Some recent work on cycles:

1. G. Bianconi, G. Caldarelli, A. Capocci. arXiv:cond-mat/0408349.
2. S. N. Dorogovtsev, J.F.F Mendes. arXiv:cond-mat/0404593.
3. E. Bollt, D. ben-Avraham. arXiv:cond-mat/0409465.
4. E. Marinari, R. Monasso. arXiv:cond-mat/0407253.
5. D. Sergi. arXiv:cond-mat/0412472.
6. A. Vazquez, J.G. Oliveira, A-L Barabasi. arXiv:cond-mat/0501399.
7. E. Ben-Naim, P.L. krapivsky. arXiv:cond-mat/0408620.
8. G. Bianconi, M. Marsili. arXiv:cond-mat/0502552.
9. S. N. Dorogovtsev, J.F.F. Mendes, J.G. Oliveira. arXiv:cond-mat/0411526.
10. H-J. Kim, J.M. Kim, S. N. Dorogovtsev, Mendes, J.F.F. arXiv:physics/0503168.
11. P.G. Lind, M.C. Gonzalez, H.J. Herrmann. arXiv:cond-mat/0504241.

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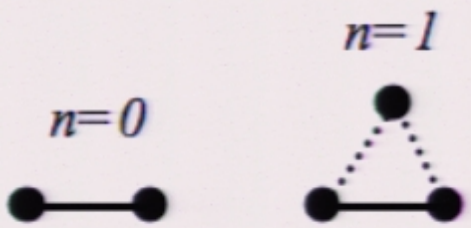
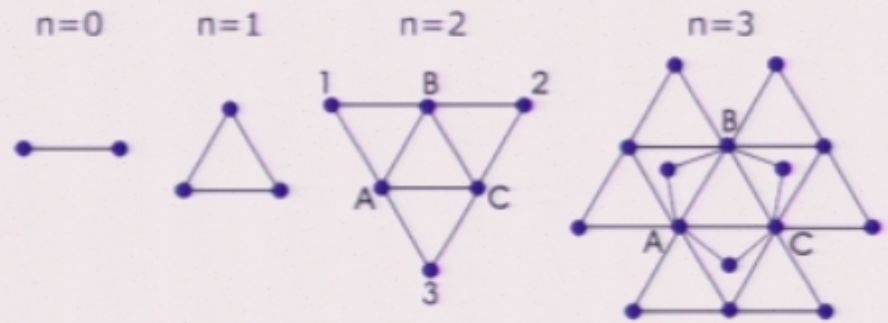
Cycles on a Deterministic Scale-Free Network





S.N. Dorogovtsev, A.V. Goltsev, and J.F.F. Mendes, Phys. Rev. E 65, 066122 (2002).

$$P(k) \sim k^{-\lambda}, \qquad \lambda = 1 + \frac{\ln 2}{\ln 3}$$

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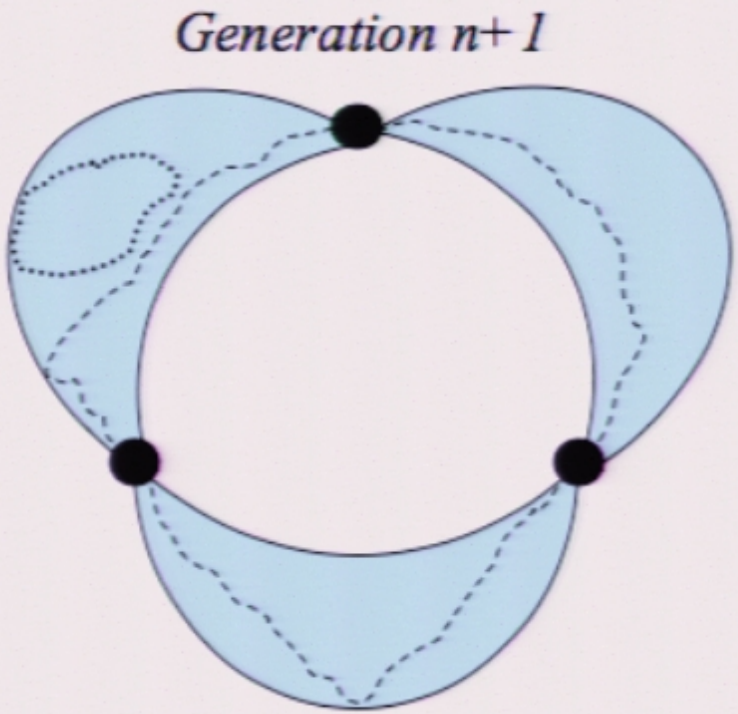
Cycles on a Deterministic Scale-Free Network



- $N_h(n)$: number of cycles of size h in generation n.
- $L_h(n)$: number of paths of size h between two hubs in generation n.

$$N_h(n+1) = 3N_h(n) + \sum_{\substack{h_1,h_2,h_3 \\ h_1+h_2+h_3=h}} L_{h_1}(n)L_{h_2}(n)L_{h_3}(n)$$

$$L_h(n+1) = L_h(n) + \sum_{\substack{h_1,h_2 \\ h_1+h_2=h}} L_{h_1}(n)L_{h_2}(n)$$

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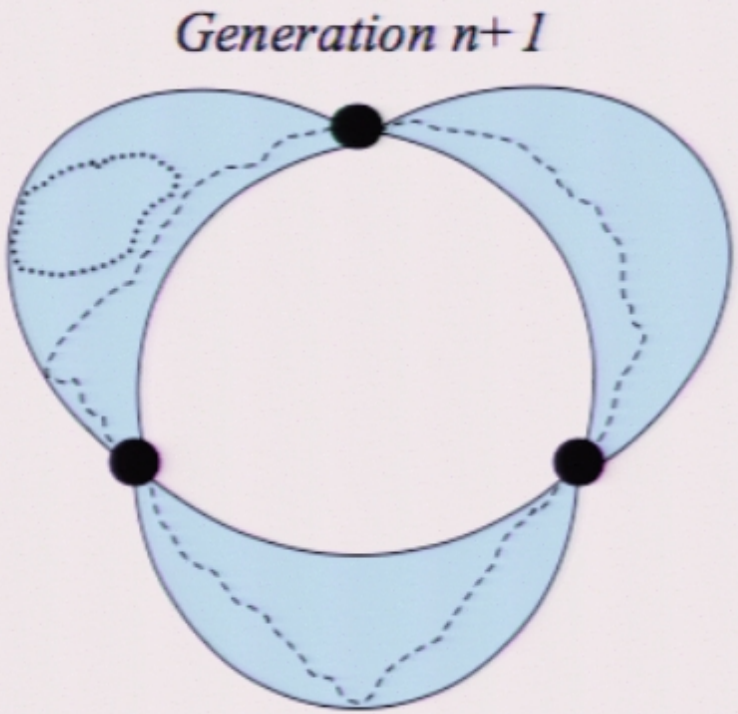
Cycles on a Deterministic Scale-Free Network



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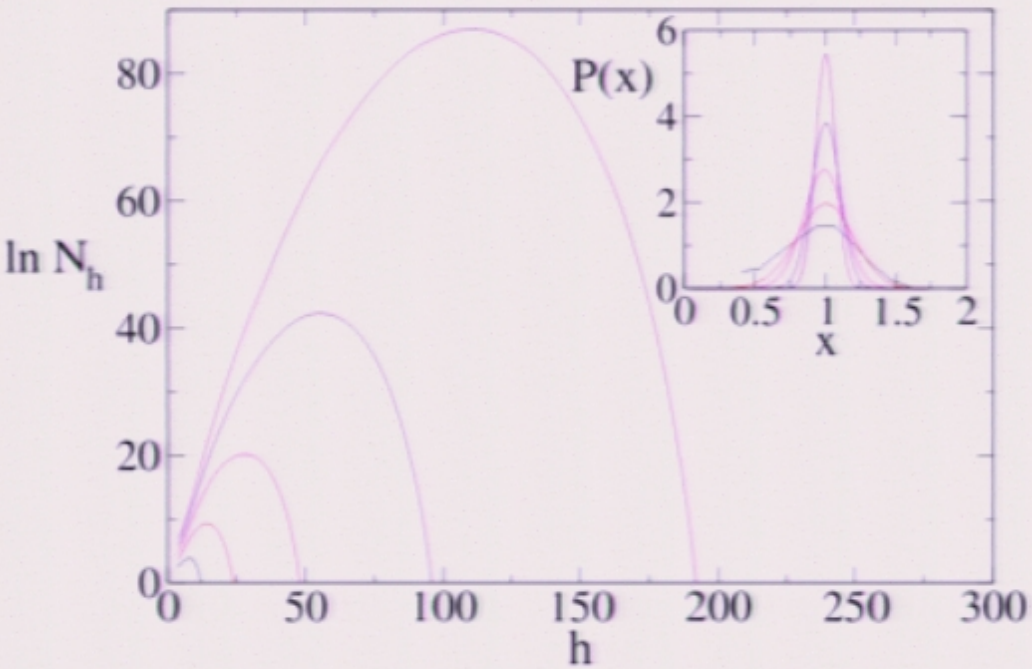
Cycles on a Deterministic Scale-Free Network



- $f(x)$ is well approximated by a parabola about $x=1$ or $h=h_*$.
- The distribution of cycles is nearly a Gaussian of width $1/h_*$
- In the thermodynamic limit $h_* \to \infty$ or $N \to \infty$ it converges to a delta function.

$$\ln N_h = h_* f\left(\frac{h}{h_*}\right), \qquad h_* \sim 2^n$$

$$\downarrow$$

$$N(n) = \frac{3^n + 3}{2} \longrightarrow \boxed{h_* \sim N^{\alpha} \quad , \alpha = \frac{\ln 2}{\ln 3}}$$

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Cycles on a Lattice, Fractal, Complete Graph.

•It was found by Jensen and Guttman that the typical cycle length in a lattice (regular graph) with $N \times N$ sites follows $h_* \sim N$ or α=1.

•For the Sierpinsky Gasket, a fractal lattice, all nodes have the same degree k=4 (except for three nodes). Again $h_* \sim N$ or α=1.

•For the Complete Graph K_N the distribution of cycles can N_h be obtained analytically

$$N_h = \frac{N!}{2h(N-h)!}$$

so $h_* \sim N$ or α=1.

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Statistics of Cycles: How Loopy is your Network?

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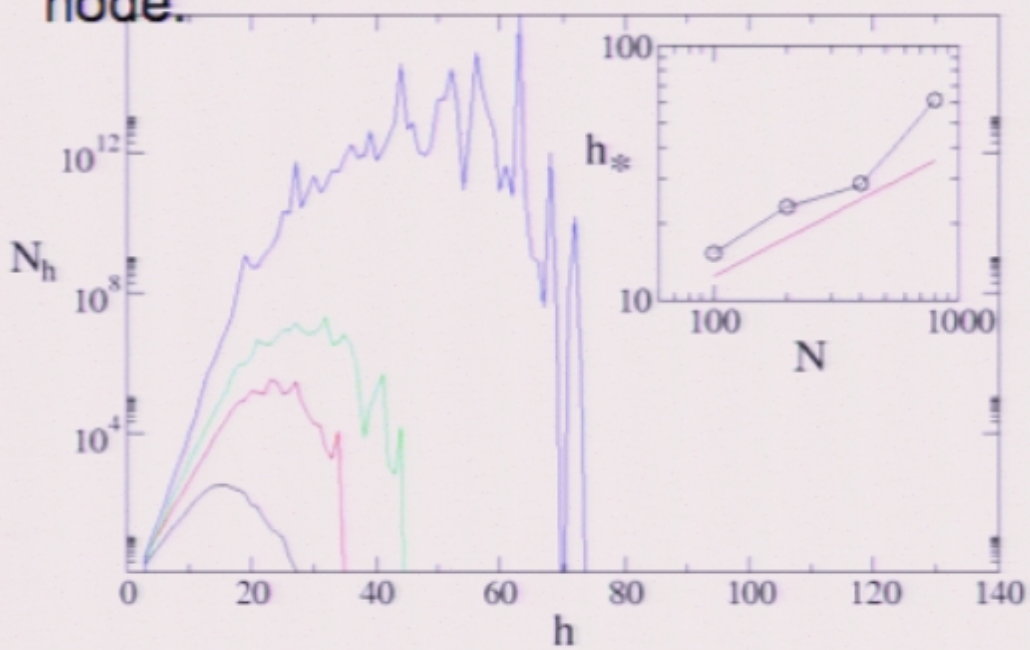
Cycles on Random Scale-Free Networks

Exact counting is not possible in random Networks.

Sampling Algorithm: self-avoiding walk. The walk is terminated when it comes back to the initial node.



$$P(k) \sim k^{-\lambda}$$

$N = 100, 200, 400, 800.$

$\lambda = 3.$

$$h_* \sim N^{\alpha} \qquad , \alpha = \frac{1}{2}$$

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Cycles on Random Scale-Free Networks

We applied the sampling algorithm to Scale-Free Networks with degree exponent λ=3.

For Small Scale Free Networks $h_* \sim N^{\alpha}$ $,\alpha = \frac{1}{\lambda - 1}$.
For large Networks h_* increases.

Why?

For $N \approx 100$ most cycles are formed between the hub and nodes in the first shell. The likely cycle length is then proportional to the degree of the hub.

$$h_* \sim K \sim N^{\frac{1}{\lambda-1}} \Rightarrow \alpha = \frac{1}{\lambda - 1}$$

As N grows larger, nodes in higher shells form part of interconnected cycles and alpha increases.

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Conclusions and open questions

•Cycles of small length have been studied before.
•Our study indicates that the full distribution of cycles, of all possible lengths, displays additional useful properties:

- For large nets, the distribution resembles a delta function that peaks about a typical cycle size, $h \sim N^{\alpha}$.
- The exponent α serves as a single figure of merit that characterizes the "loopiness" of the net in question.
- For regular lattices and fractals, and for complete graphs and ER above criticality α=1.
- For small random scale-free nets $\alpha = 1/(\lambda-1)$, but increases as the nets become larger.
- It remains an open question whether the loopy exponent saturates at $\alpha = 1$ as $N \rightarrow \infty$.
- Is there a more efficient sampling algorithm?
- Ergodicity: the distribution of cycles that pass through a node is similar for most nodes of the net.

Cycles are important: Cycles are the backbone:

Cycles find communities.

PART 4: …Networks that change dynamically

Identical oscillators properly communicating can synchronize