Title: Worldsheet RG and target space time revolution

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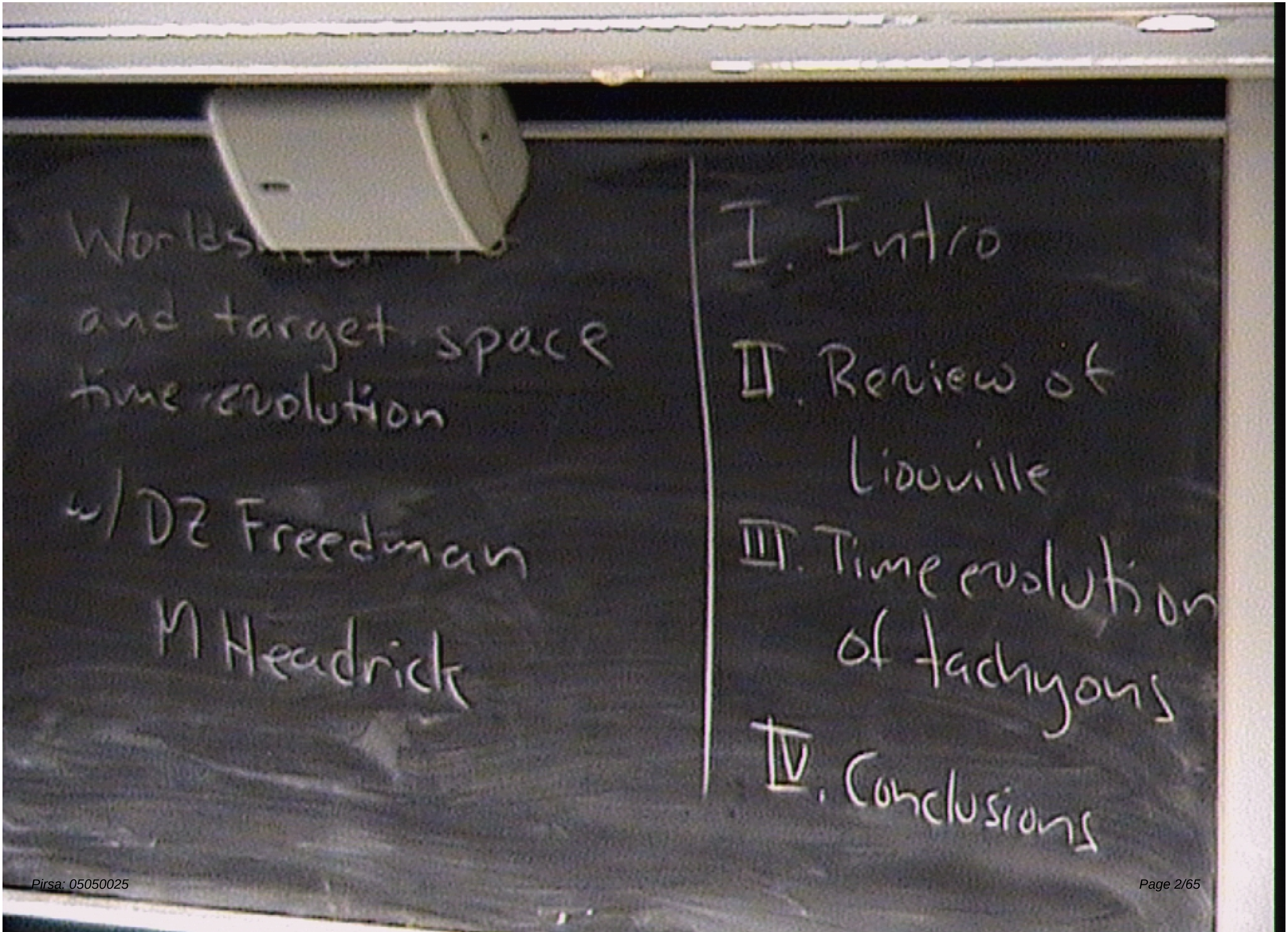
Abstract:



Worlds
and target space
time evolution

w/ DZ Freedman
M Headrick

I. Intro

II. Review of
Liouville

III. Time evolution
of tachyons

IV. Conclusions

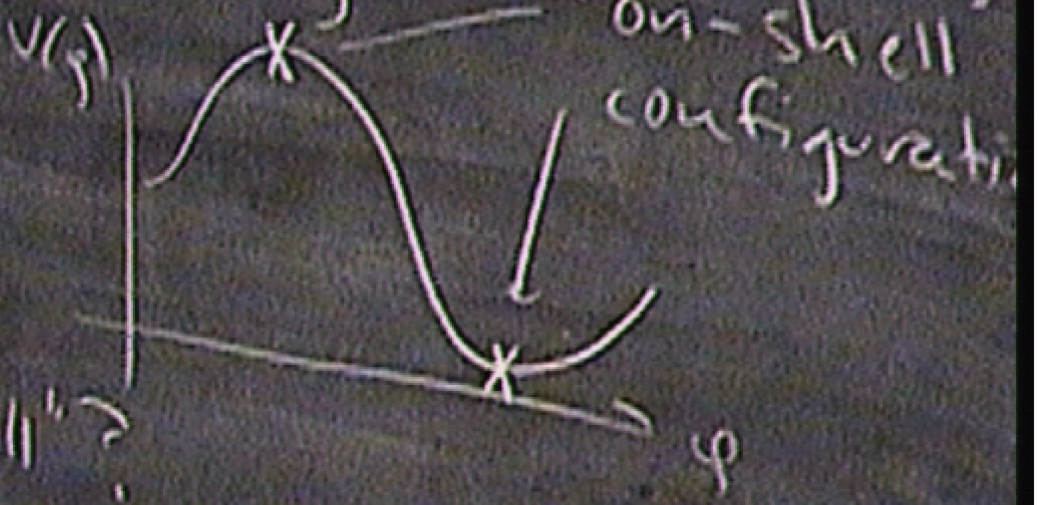
I. Introduction

(1) Consistent classical string backgrounds

$\wr$

2d CFTs

how to go "off-shell"?



I. Introduction

(1) Consistent classical string backgrounds

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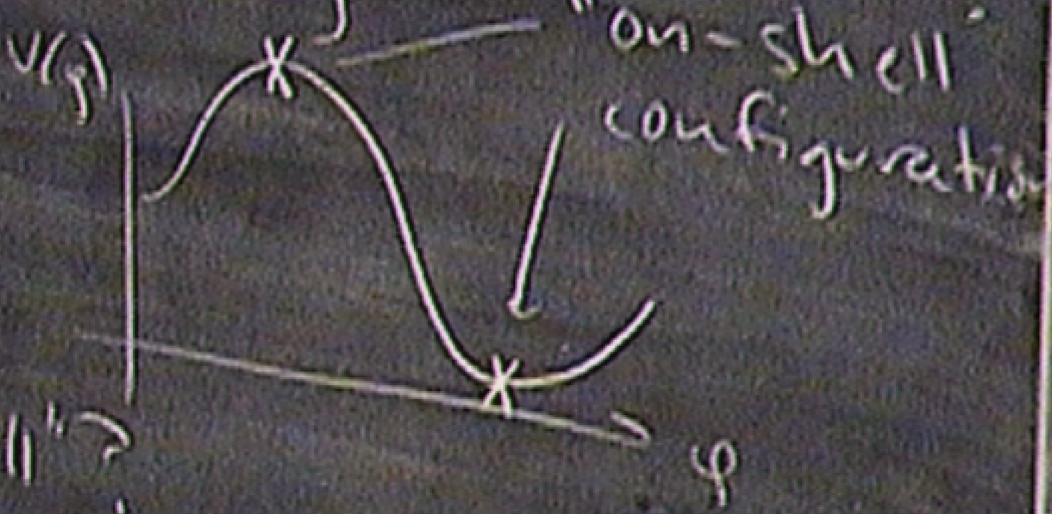
2d CFTs

how to go "off-shell"?



(a) Scattering amplitudes of φ

$$V \sim \sum_n \lambda_n \varphi^n$$

$$\lambda_n \sim \langle \overbrace{V_\varphi \cdots V_\varphi}^{n \text{ times}} \rangle_{CFT}$$

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(2) Look for solutions which interpolate between vacua

(1) Scattering amplitudes of φ

$$V \sim \sum_n \lambda_n \varphi^n$$

$$\lambda_n \sim \langle V_\varphi \cdots V_\varphi \rangle_{CFT}$$

(2) Look for solutions which interpolate between vacua → in space (domain walls,

in time

(2) Scattering amplitudes

<u>ex</u> string theory on $\mathbb{R}^4 \times M_6$

<u>Scalar</u> vertex ops: $\Phi(z,\bar{z})$: $\underset{\mathbb{R}^4}{\underset{\uparrow}{e^{ik\cdot X}}} \quad \underset{M}{\underset{\uparrow}{\mathcal{O}(z,\bar{z})}}$

(2) Scattering amplitudes

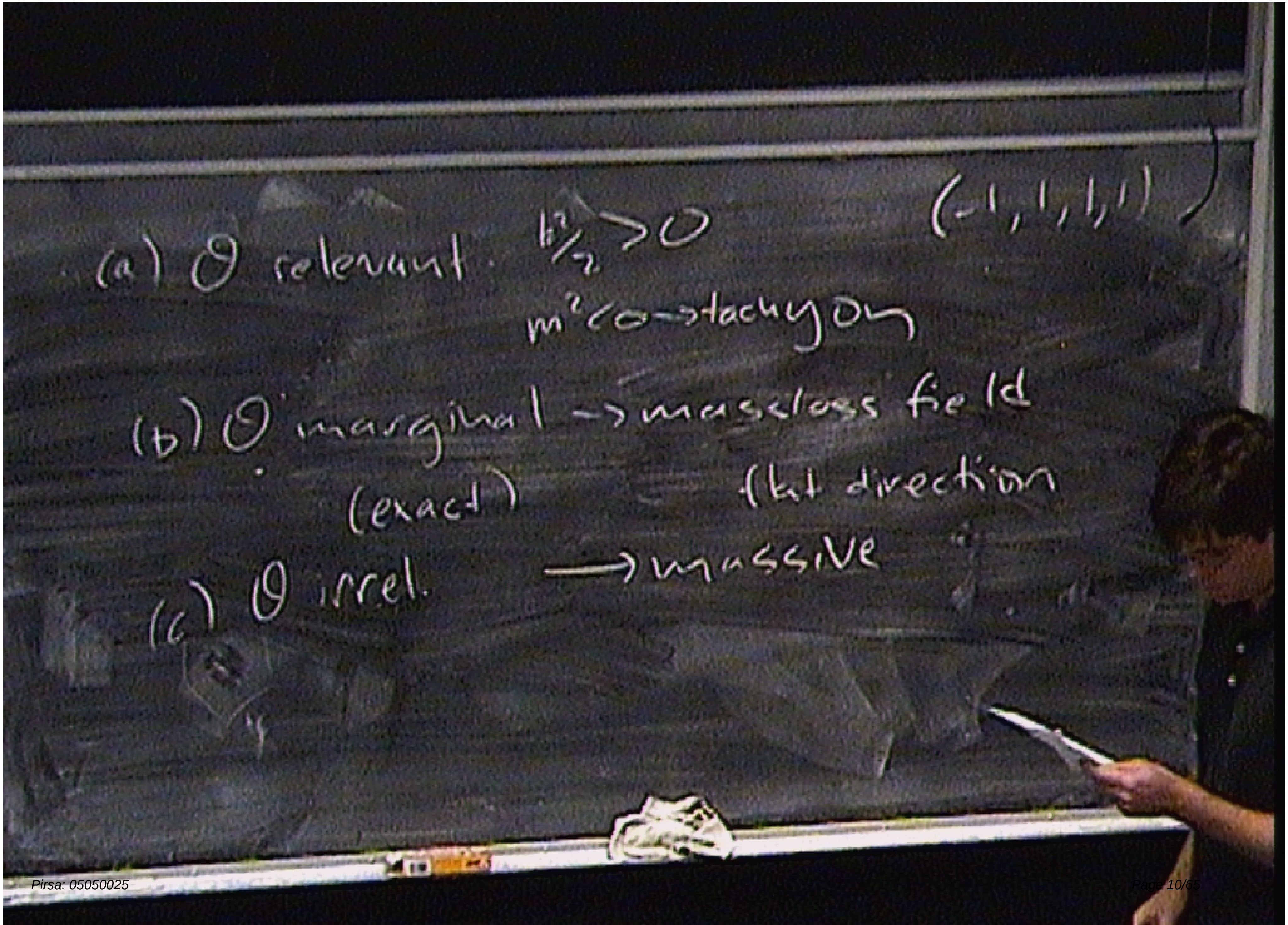
ex string theory on $\mathbb{R}^4 \times M_6$

scalar vertex ops. $\Phi(z,\bar{z}) = \underset{\underset{\mathbb{R}^4}{\uparrow}}{e^{ik\cdot X}} \; \underset{\underset{M}{\uparrow}}{\mathcal{O}(z,\bar{z})}$

this needs to be marginal

$\Delta(e^{ik\cdot X}) = k^2/2$

$\Delta(\mathcal{O}) = \left(1 - k^2/2,\ 1 - k^2/2\right)$



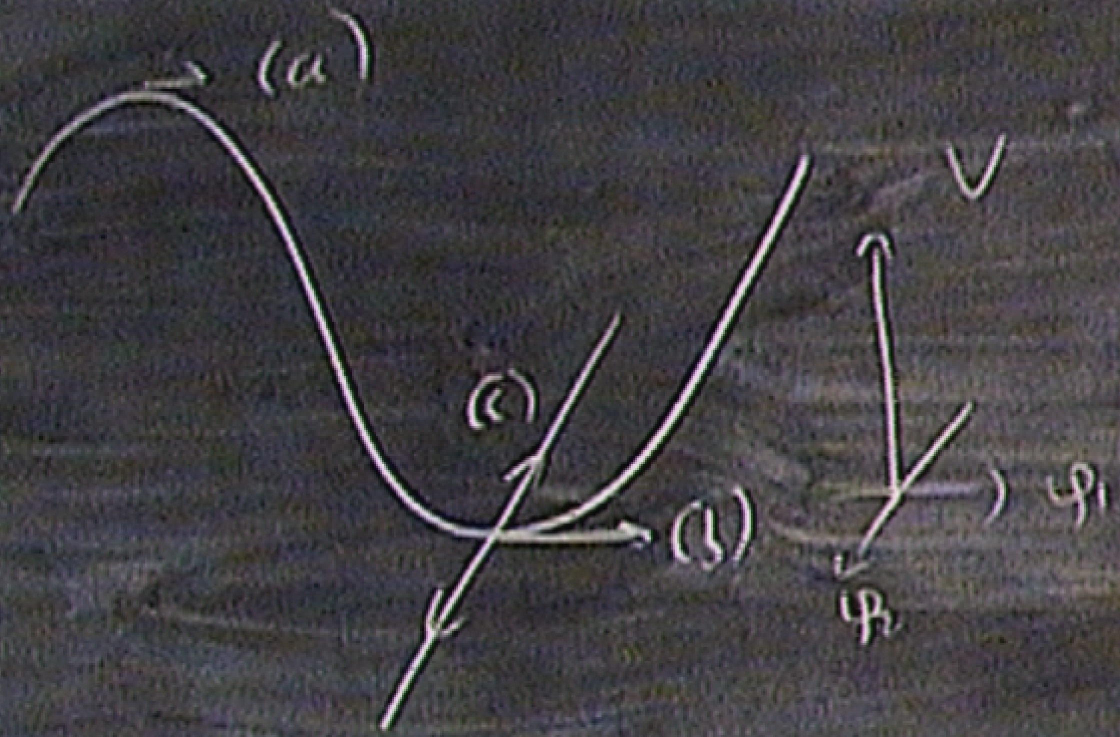
(a) $\mathcal{O}$ relevant. $k^2/2 > 0$ $(-1,1,1,1)$

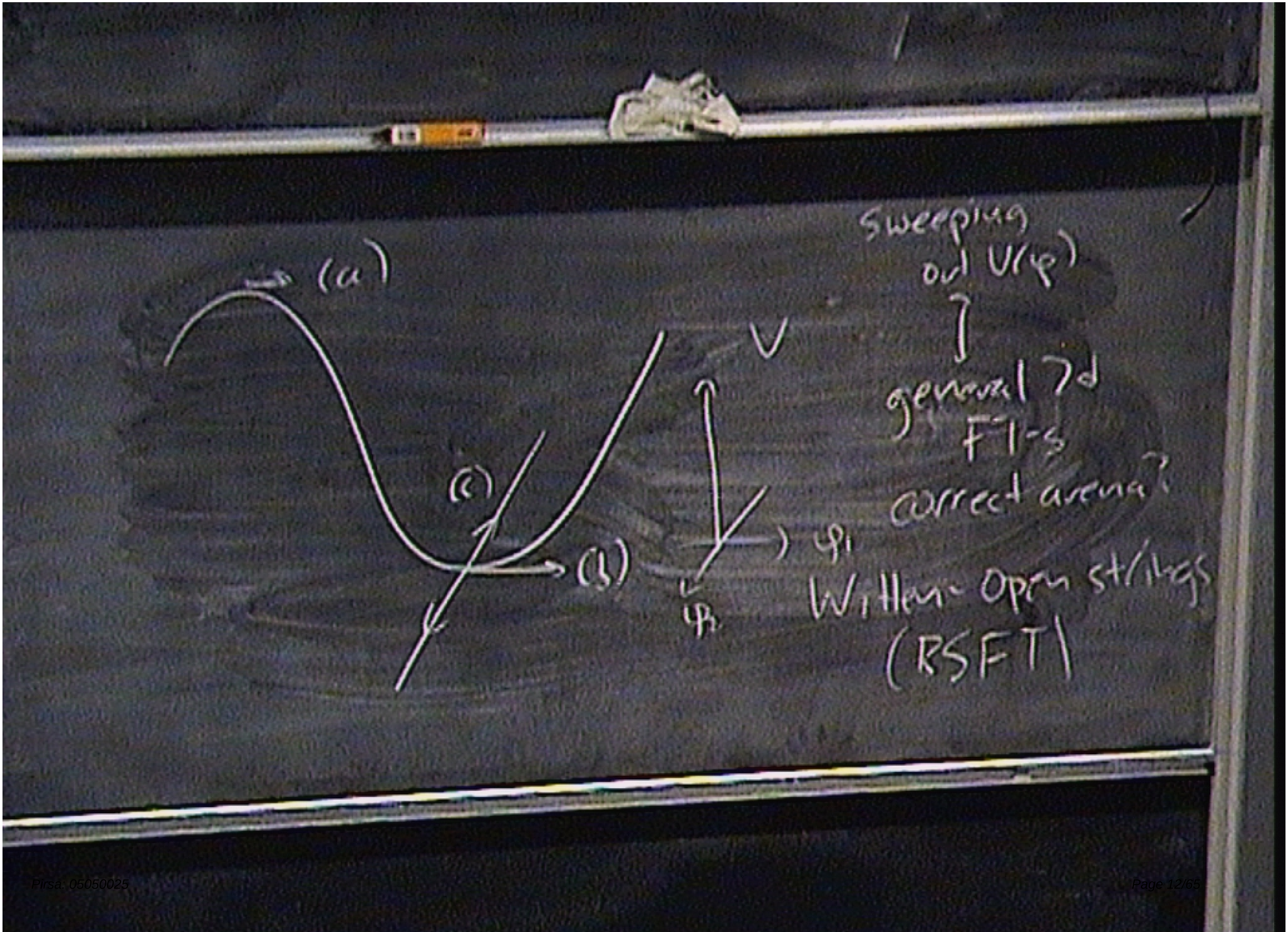
$m^2 < 0 \to$ tachyon

(b) $\mathcal{O}$ marginal $\to$ massless field

(exact) flat direction

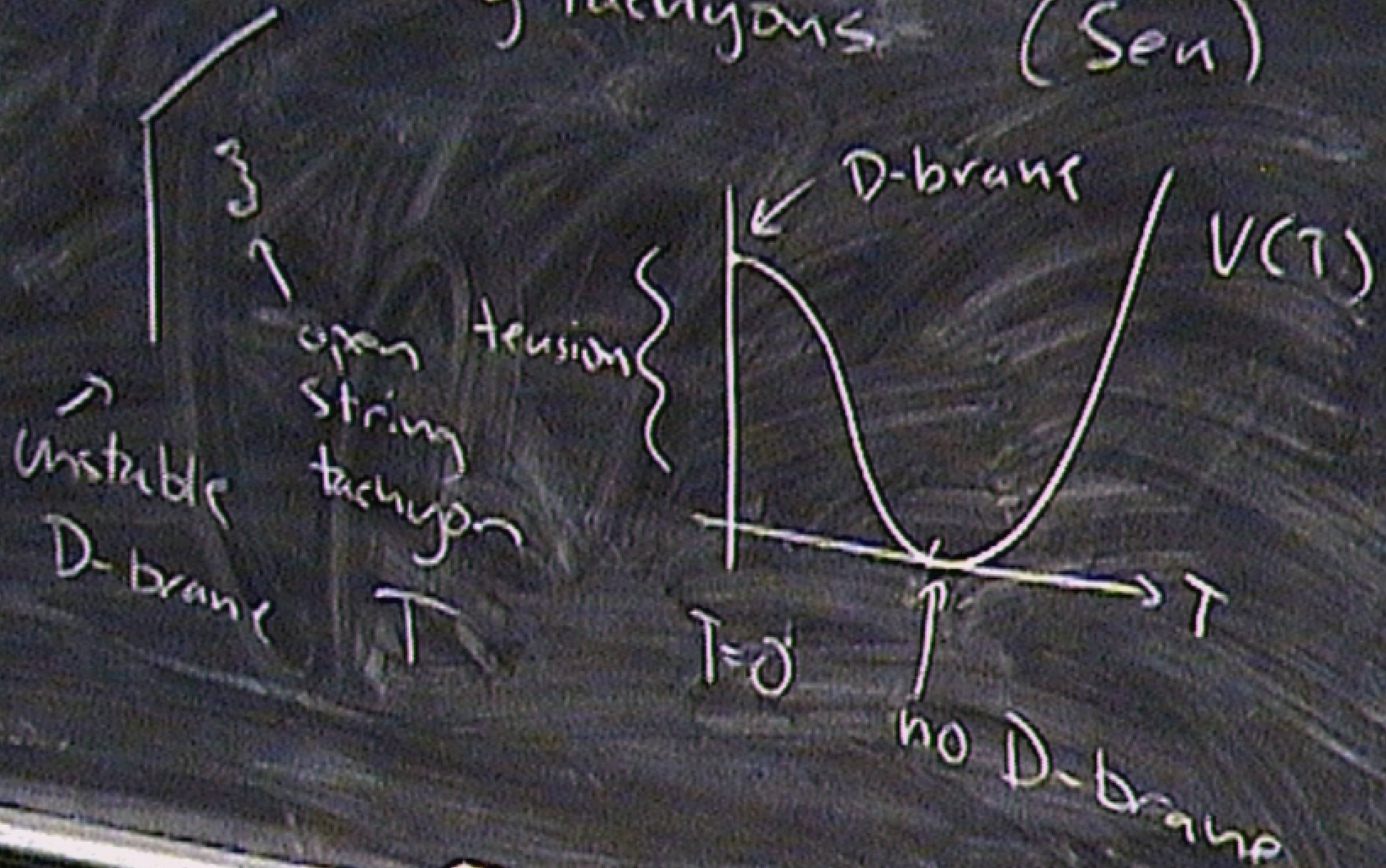
(c) $\mathcal{O}$ irrel. $\longrightarrow$ massive


(a)
V
(c)
(b)
φ_1
φ_2



Sweeping
out V(φ)
⇕
general 2d
FT-s
correct arena?
Witten- Open strings
(BSFT)

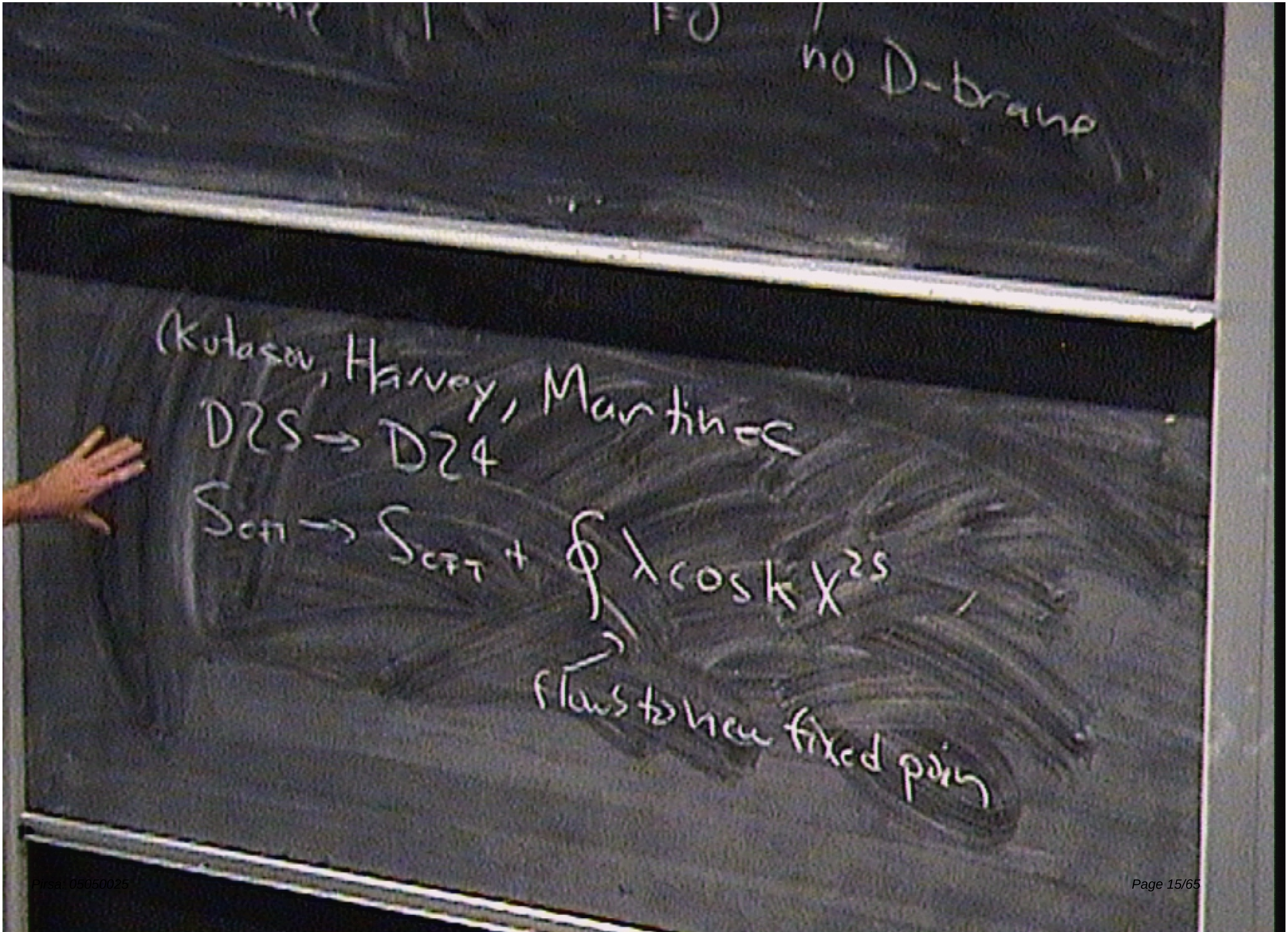
(3) (Open) string tachyons (Sen)



no D-brane

(Kutasov, Harvey, Martinec

D25 → D24



no D-brane

(Kutasov, Harvey, Martinec

D25 → D24

$S_{CFT} \to S_{CFT} + \oint \lambda \cos k X^{25}$

flows to new fixed point

no D-brane

(Kutasov, Harvey, Martinec

Pilch?

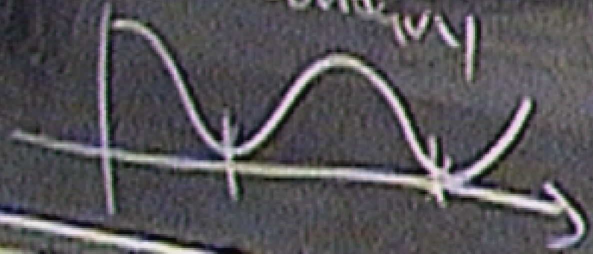
D25 → D24

$S_{CFT} \to S_{CFT} + \oint \lambda \cos k X^{25}$

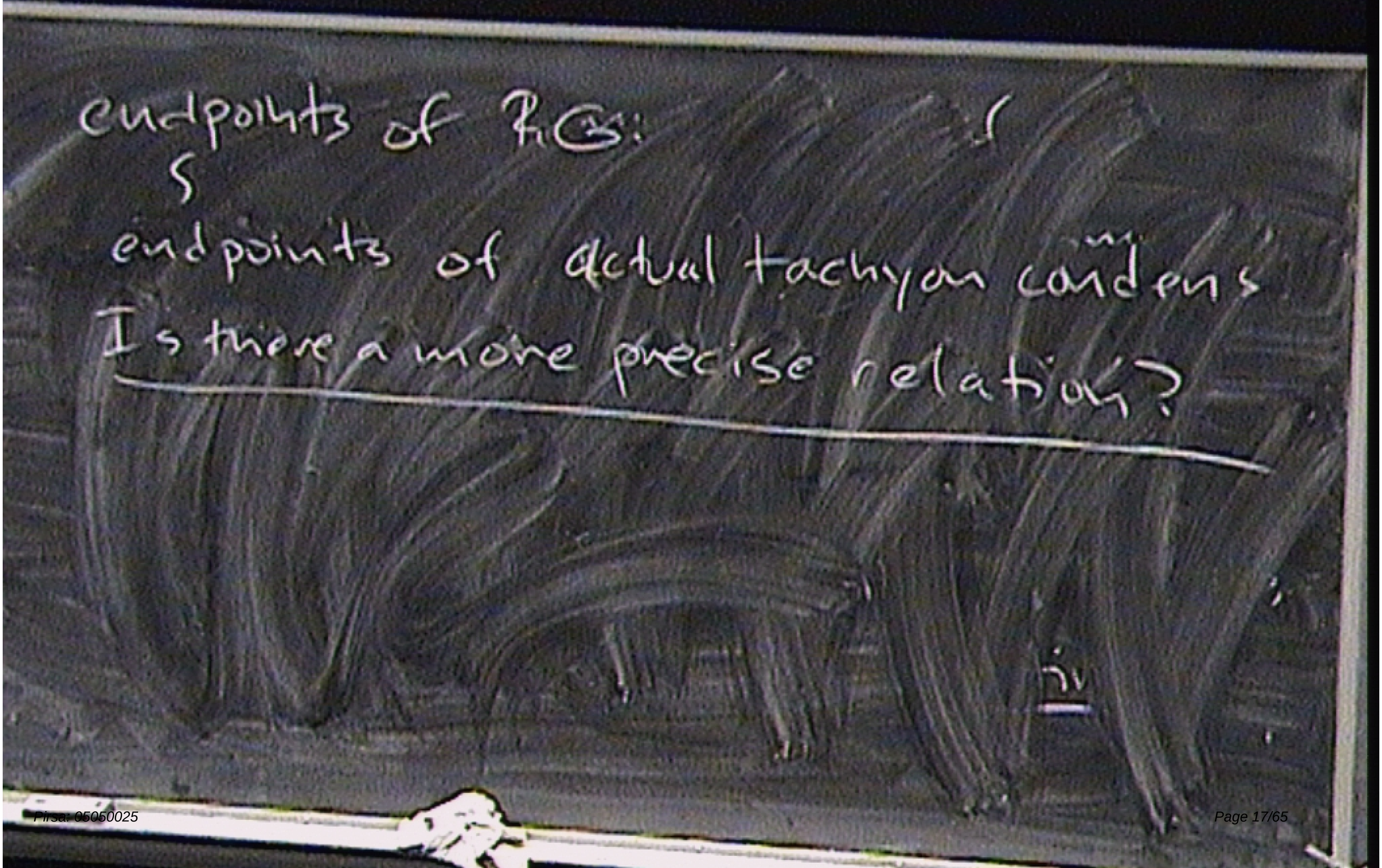
(Saleur Warner)

induces potential on boundary

flows to new fixed point



X^{25} $\lambda \to \infty$

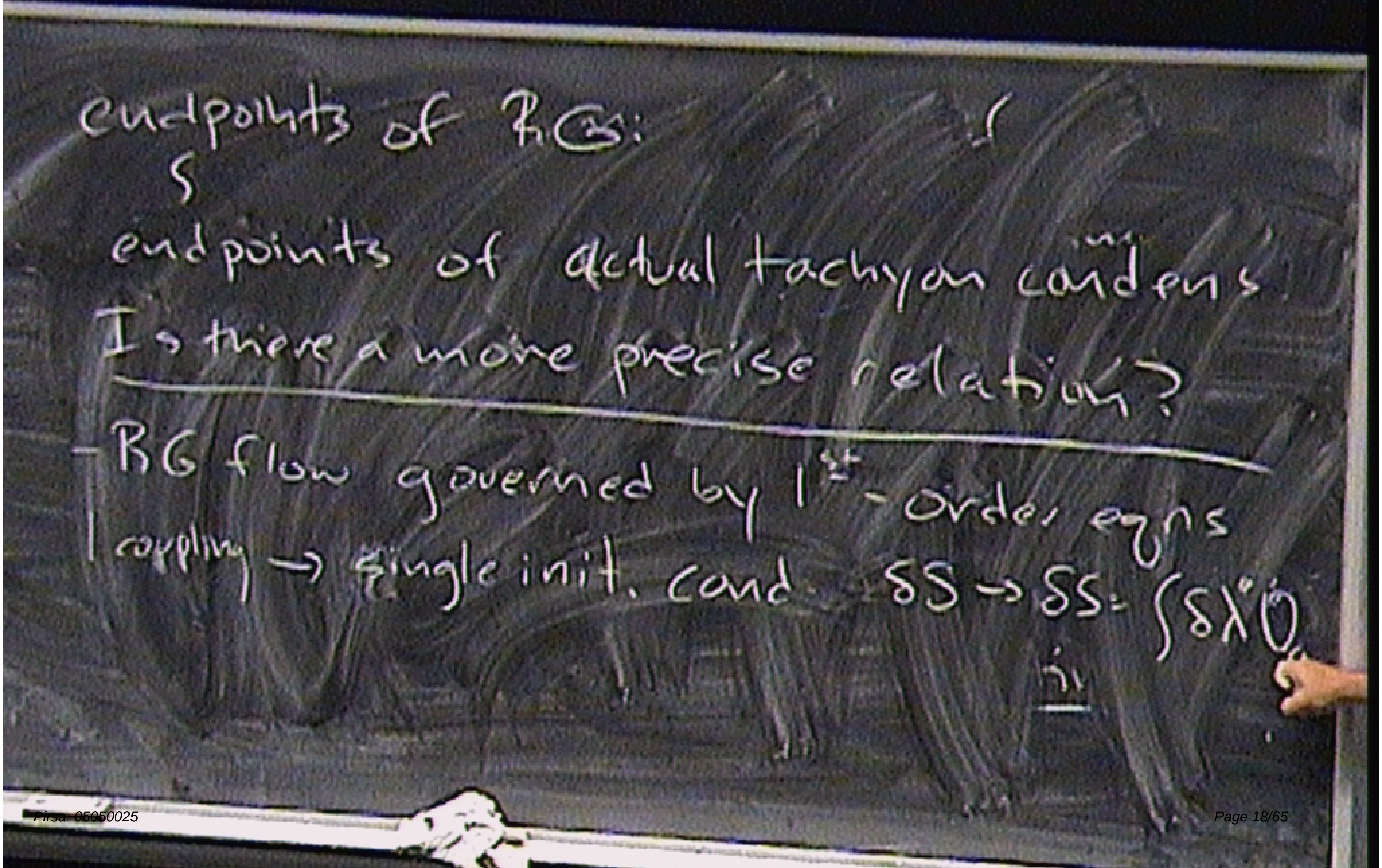


endpoints of RGs:

S

endpoints of actual tachyon condens

Is there a more precise relation?



endpoints of RGs:

$\wr$

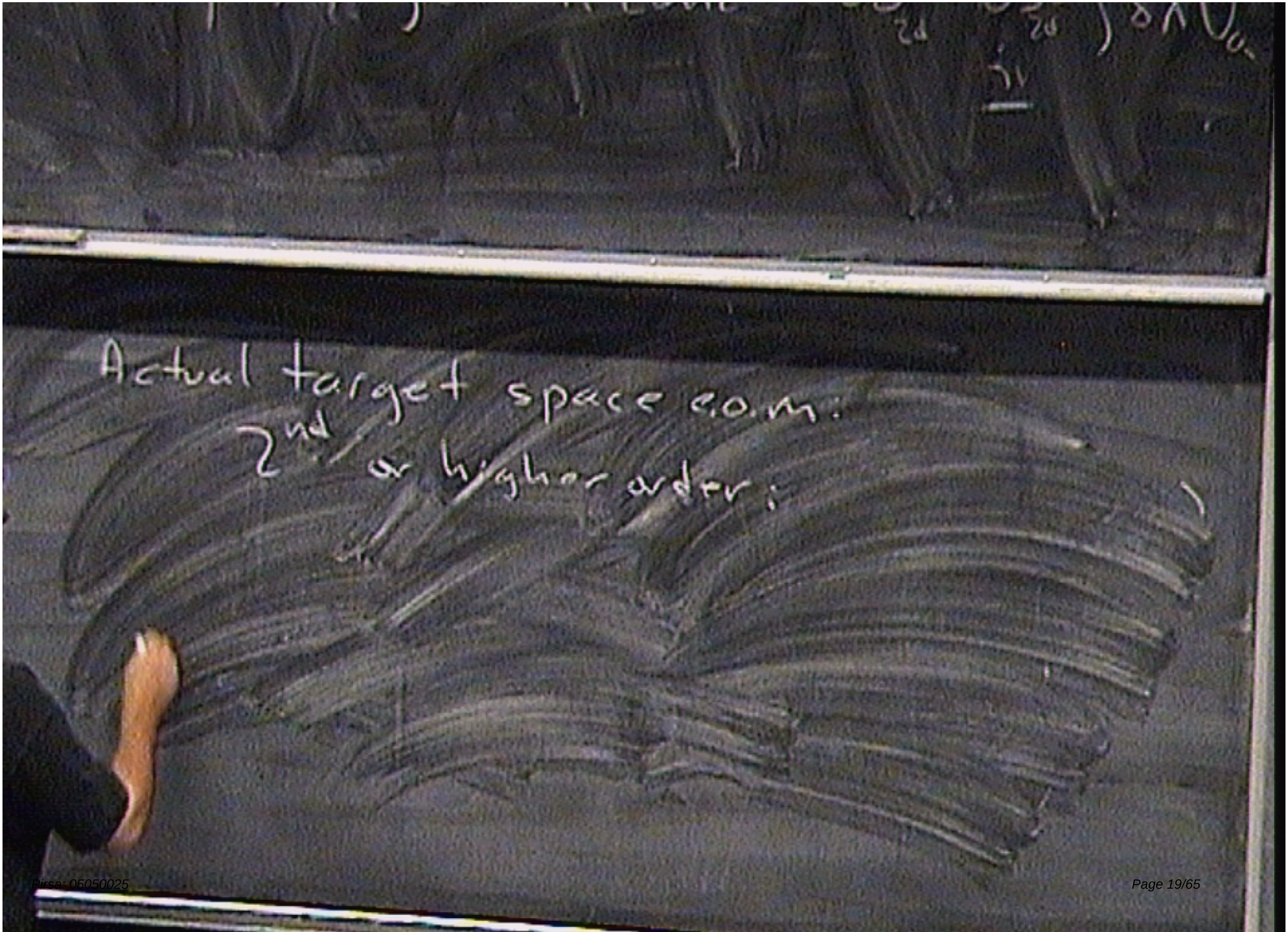
endpoints of actual tachyon cond^{ini}ens

Is there a more precise relation?

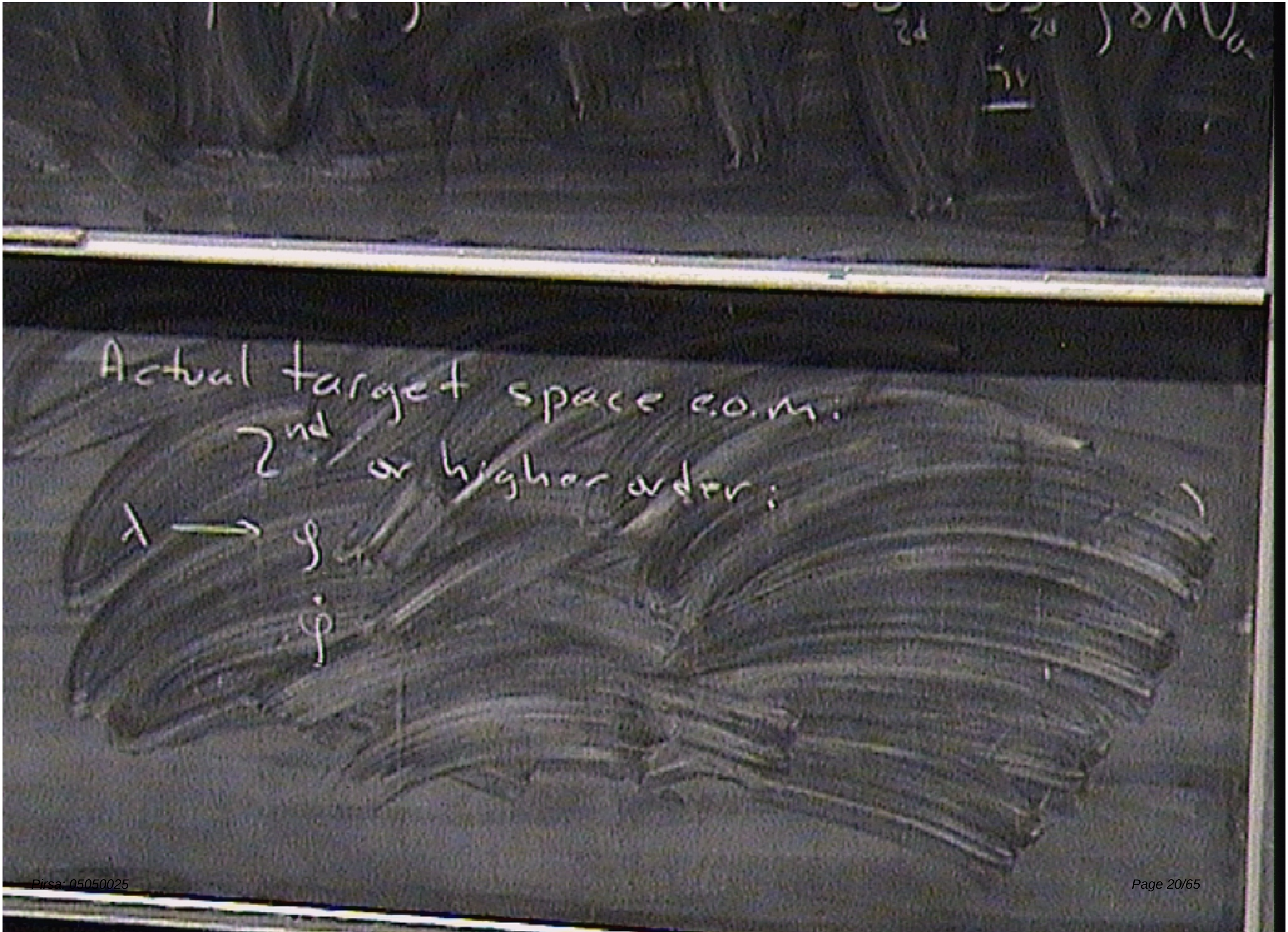
- RG flow governed by 1st-order eqns

| coupling → single init. cond. $\delta S \to \delta S = \int \delta\lambda^i \mathcal{O}_i$

$\underline{i_i}$



Actual target space e.o.m.:
2nd or higher order;

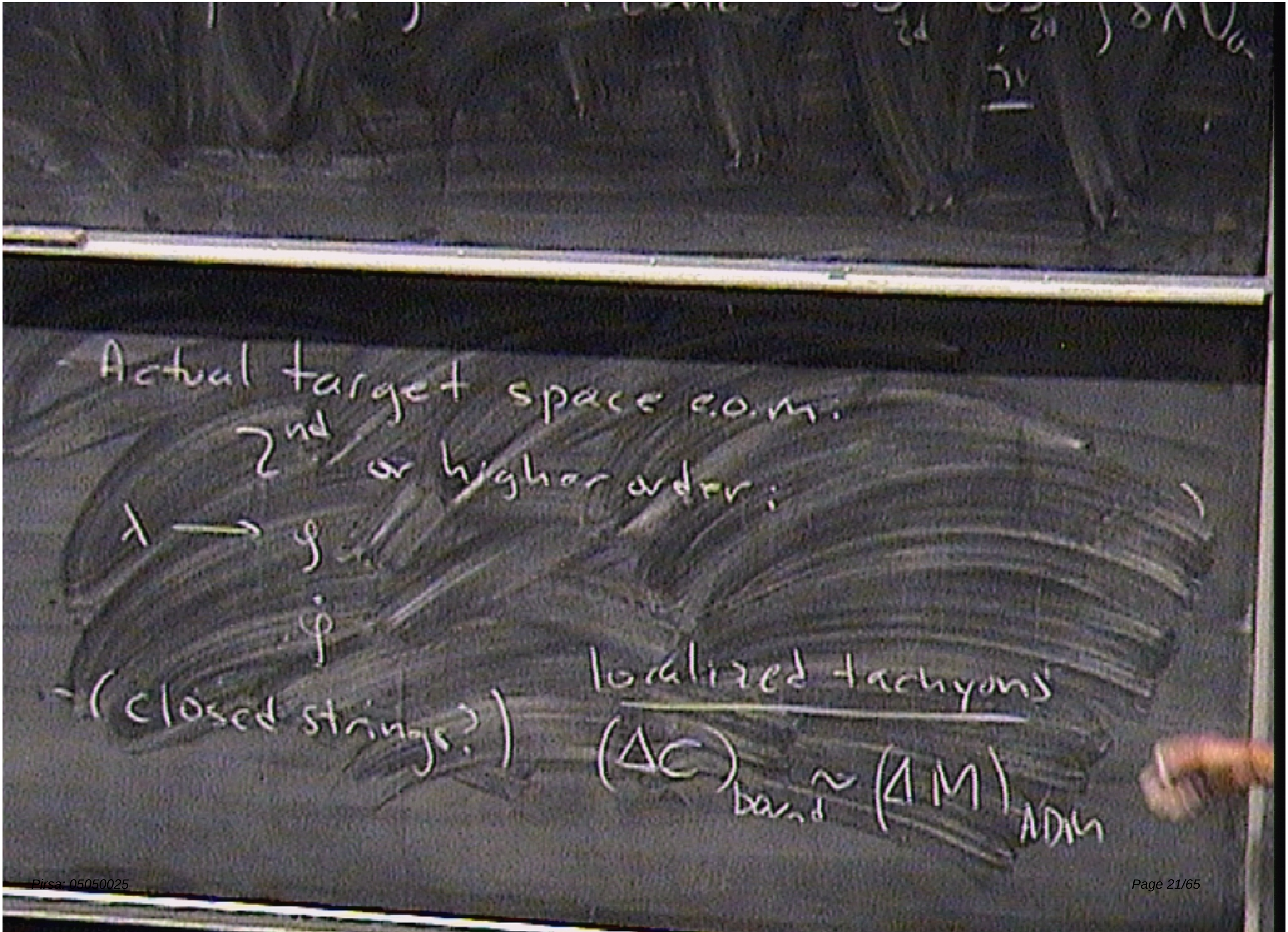


Actual target space e.o.m.:

2nd or higher order:

$\lambda \longrightarrow \varphi$

$\dot{\varphi}$



Actual target space e.o.m.:

2^{nd} or higher order:

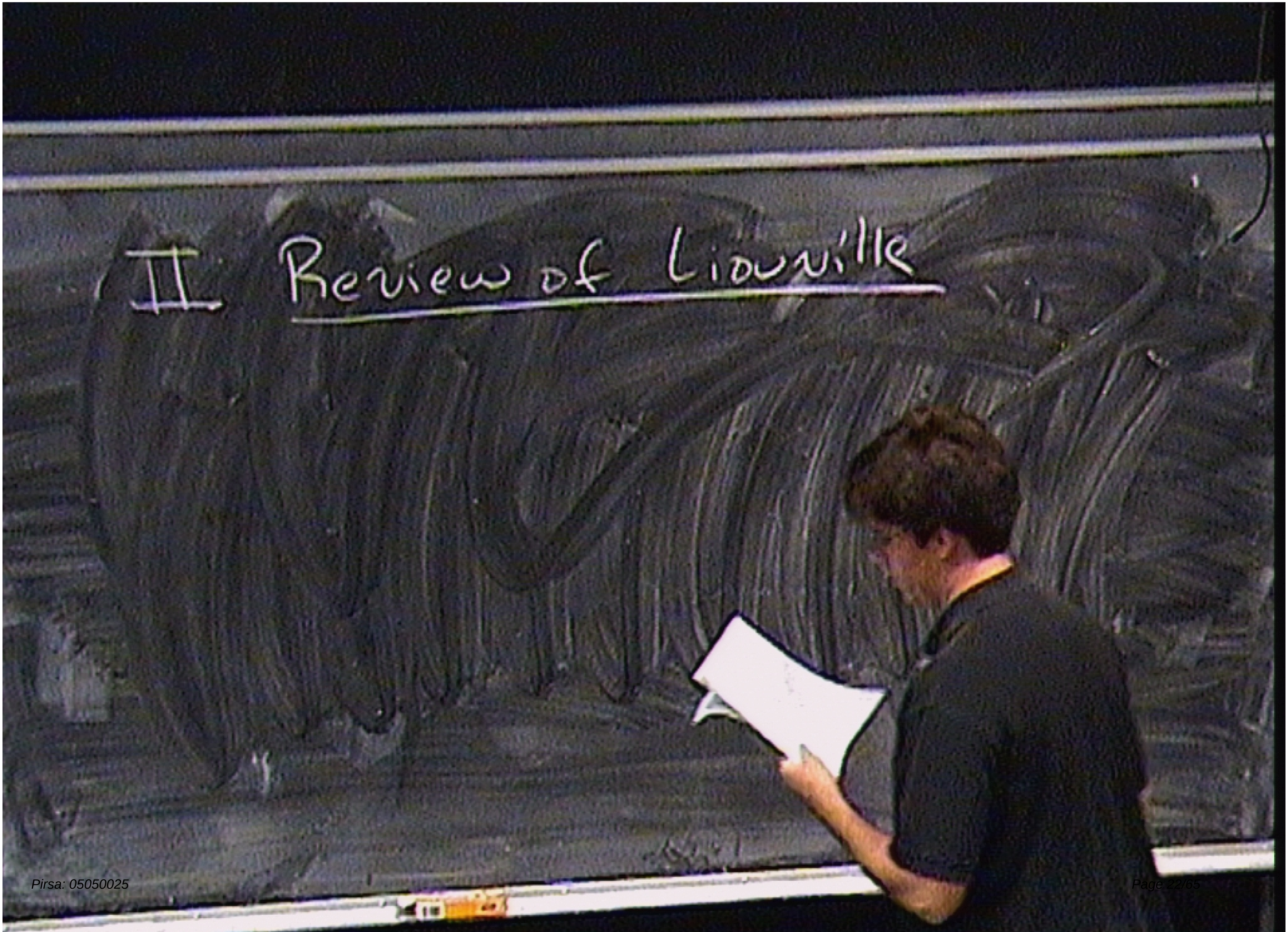
$\lambda \longrightarrow \varphi$

$\dot{\varphi}$

~(closed strings?)

localized tachyons

$(\Delta C)_{band} \sim (\Delta M)_{ADM}$



II Review of Liouville

II. Review of Liouville

(1) Polyakov (80s)

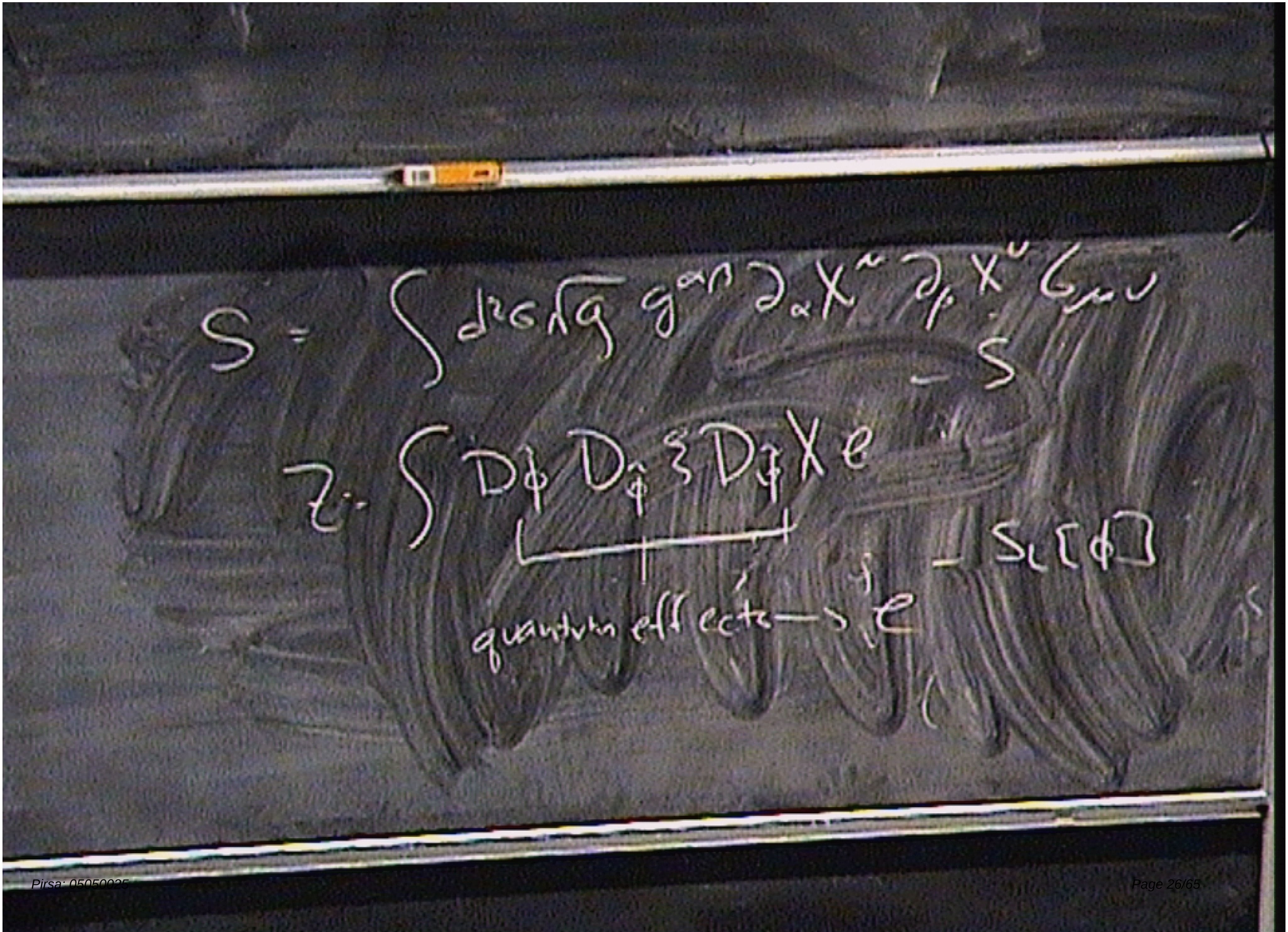
2d CFT + 2d gravity in conformal gauge

$$g_{\alpha\beta} = e^{2\phi}\eta_{\alpha\beta}$$

$$S = \int d^2\sigma \sqrt{g}\, g^{\alpha\beta} \partial_\alpha X^\mu \partial_\beta X^\nu G_{\mu\nu}$$

$$S = \int d^2\sigma \sqrt{g}\, g^{\alpha\beta} \partial_\alpha X^\mu \partial_\beta X^\nu G_{\mu\nu}$$

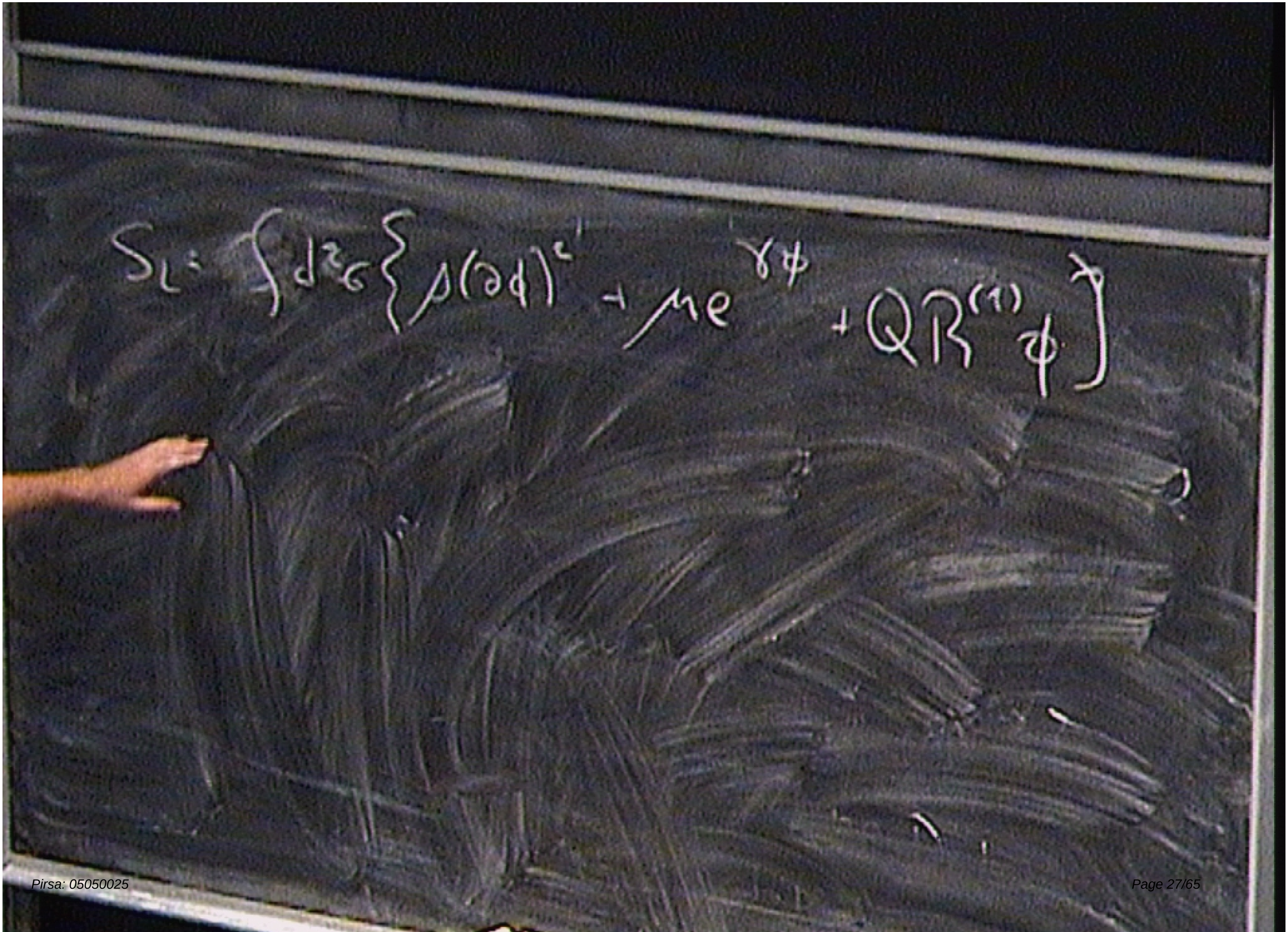
$$Z = \int \underbrace{D\hat{\phi}\, D_{\hat{\phi}}\, \xi\, D_{\hat{\phi}}}_{} X\, e^{-S}$$



$$S = \int d^2\sigma \sqrt{g}\, g^{\alpha\beta}\, \partial_\alpha X^\mu\, \partial_\beta X^\nu\, G_{\mu\nu}$$

$$Z = \int \underbrace{D\phi\, D_{\hat\phi}\xi\, D_{\hat\phi}X}\, e^{-S}$$

quantum effects → $e^{-S_L[\phi]}$



$$S_L = \int d^2\sigma \left\{ \rho(\partial\phi)^2 + \mu e^{\gamma\phi} + Q R^{(2)} \phi \right\}$$

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$$\rho = -1 \qquad c_m \geq 25$$

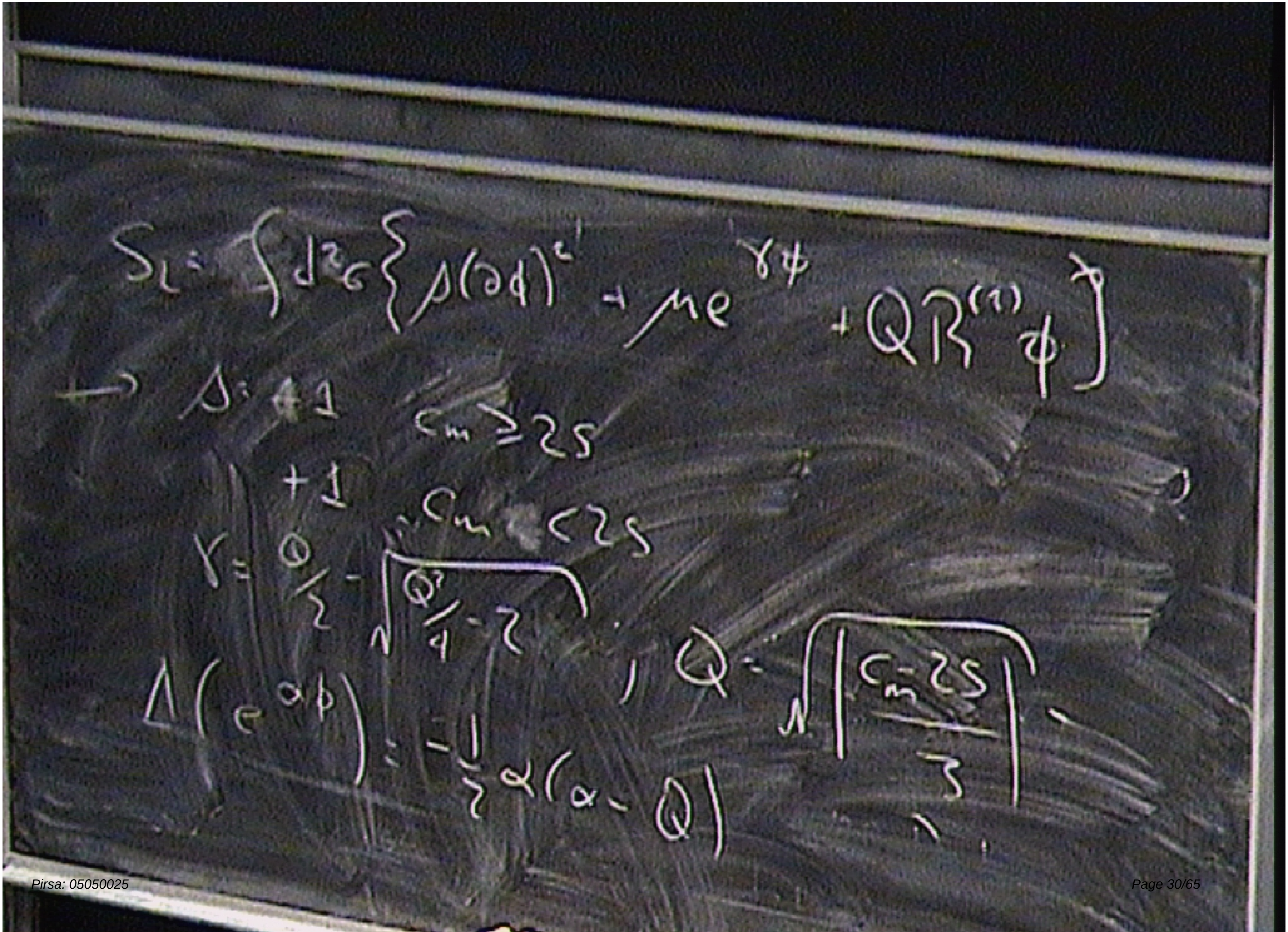
$$+1 \qquad c_m << 25$$

$$S_L = \int d^2\sigma \left\{ \Delta (\partial\phi)^2 + \mu e^{\gamma\phi} + Q R^{(2)} \phi \right\}$$

$$\longrightarrow \Delta = -1 \qquad c_m \geq 25$$

$$+1 \qquad c_m < 25$$

$$\gamma = \frac{Q}{2} - \sqrt{\frac{Q^2}{4} - 2} \,, \quad Q = \sqrt{\left| \frac{c_m - 25}{3} \right|}$$



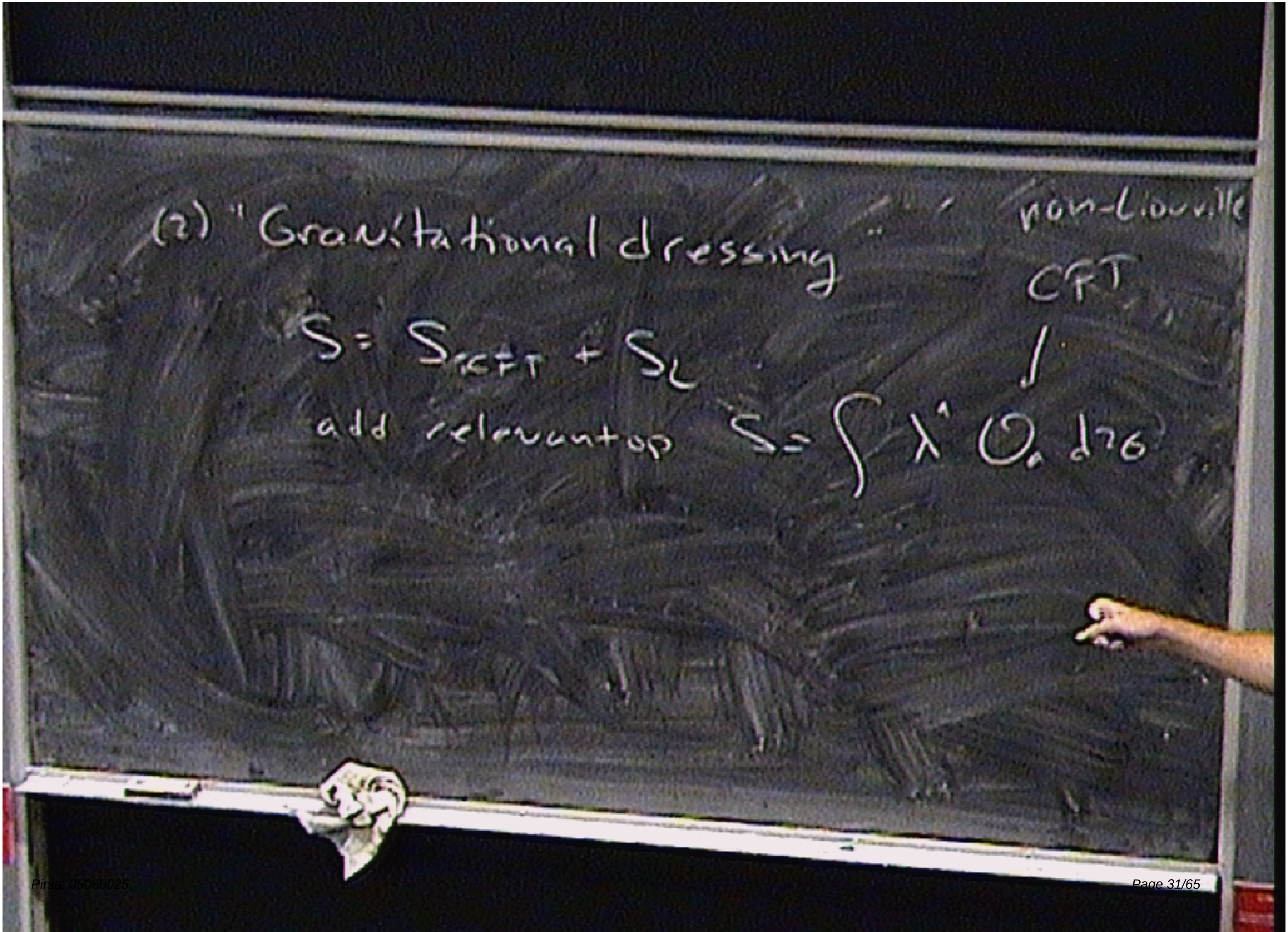
$$S_L = \int d^2\sigma \left\{ \Delta (\partial\phi)^2 + \mu e^{\gamma\phi} + Q R^{(2)} \phi \right\}$$

$$\rightarrow \Delta = -1 \quad c_m \geq 25$$

$$+1 \quad c_m < 25$$

$$\gamma = \frac{Q}{2} - \sqrt{\frac{Q^2}{4} - 2}$$

$$\Delta\left(e^{\alpha\phi}\right) = -\frac{1}{2}\alpha(\alpha - Q), \quad Q = \sqrt{\frac{|c_m - 25|}{3}}$$

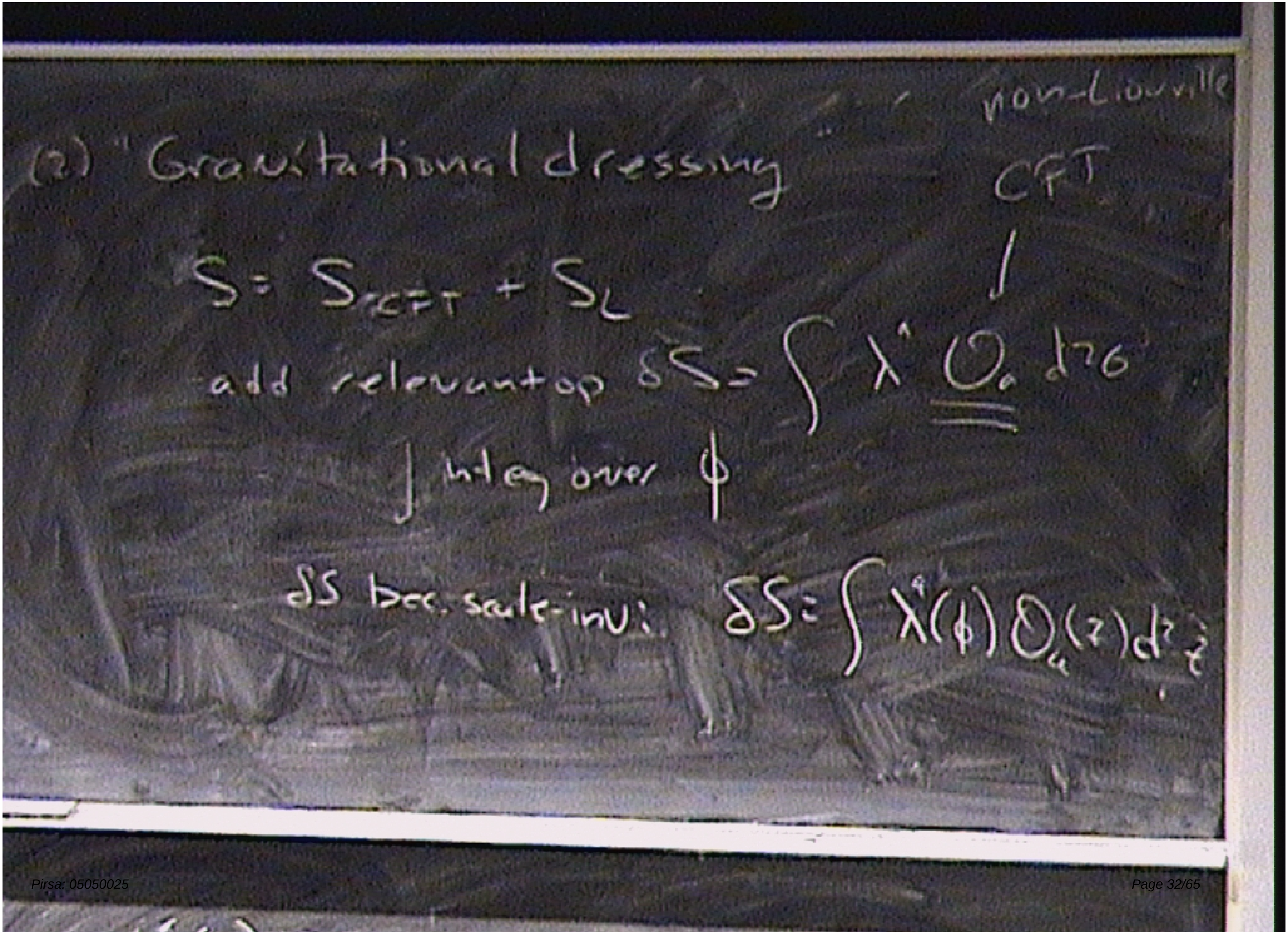


(2) "Gravitational dressing"

non-Liouville CFT

$$S = S_{CFT} + S_L$$

add relevant op $S = \int \lambda^a \mathcal{O}_a \, d^2\sigma$



(2) "Gravitational dressing"

non-Liouville CFT

$$S = S_{CFT} + S_L$$

add relevant op $\delta S = \int \lambda^a \underline{\underline{\mathcal{O}_a}}\, d^2\sigma$

$\downarrow$ integ over ϕ

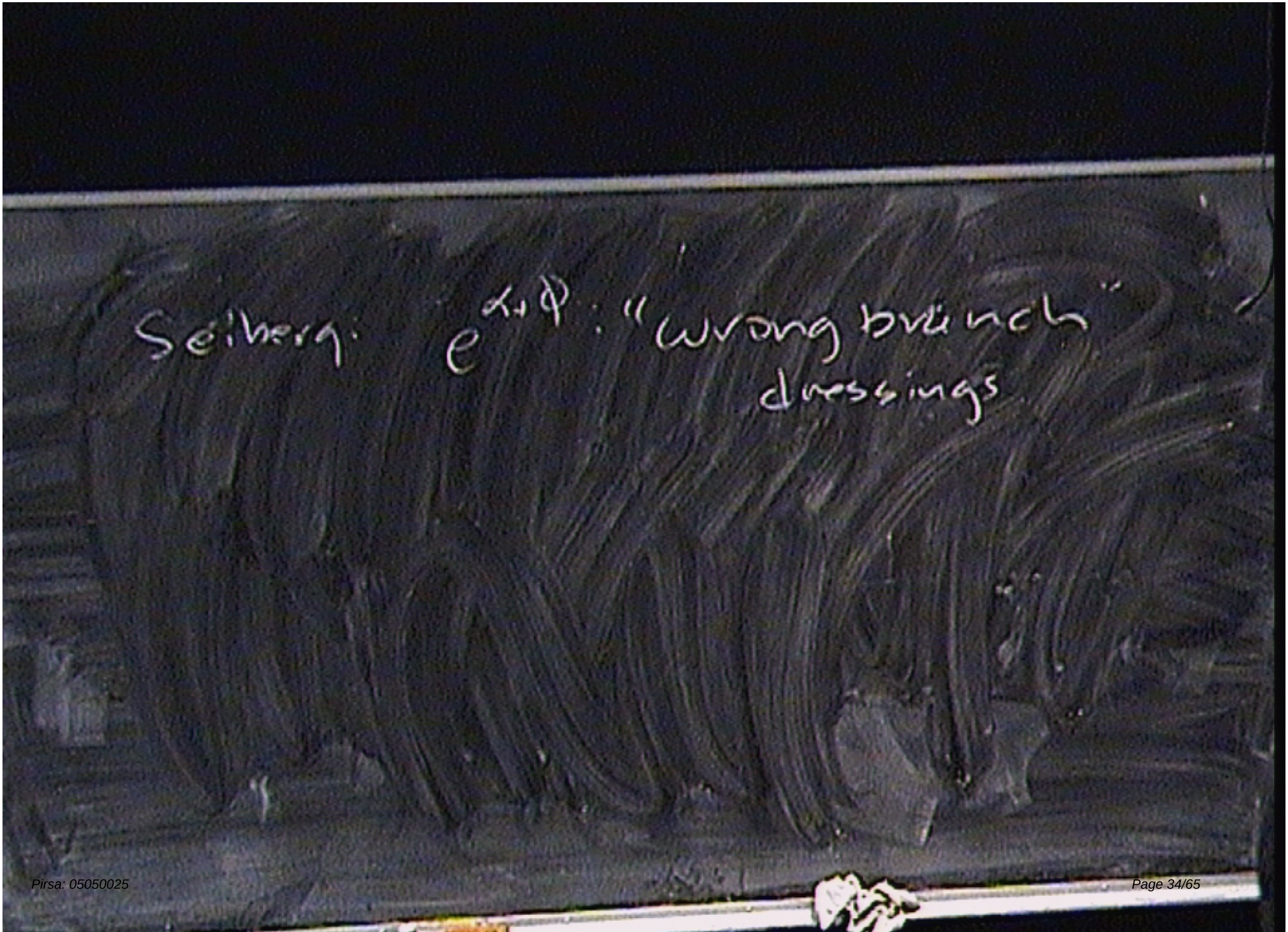
δS bec. scale-inv: $\delta S = \int \lambda^a(\phi)\, \mathcal{O}_a(z)\, d^2 z$

$\lambda'(\phi)\underbrace{O_a}_{1}$ must be dim^{n}: $(1,1)$

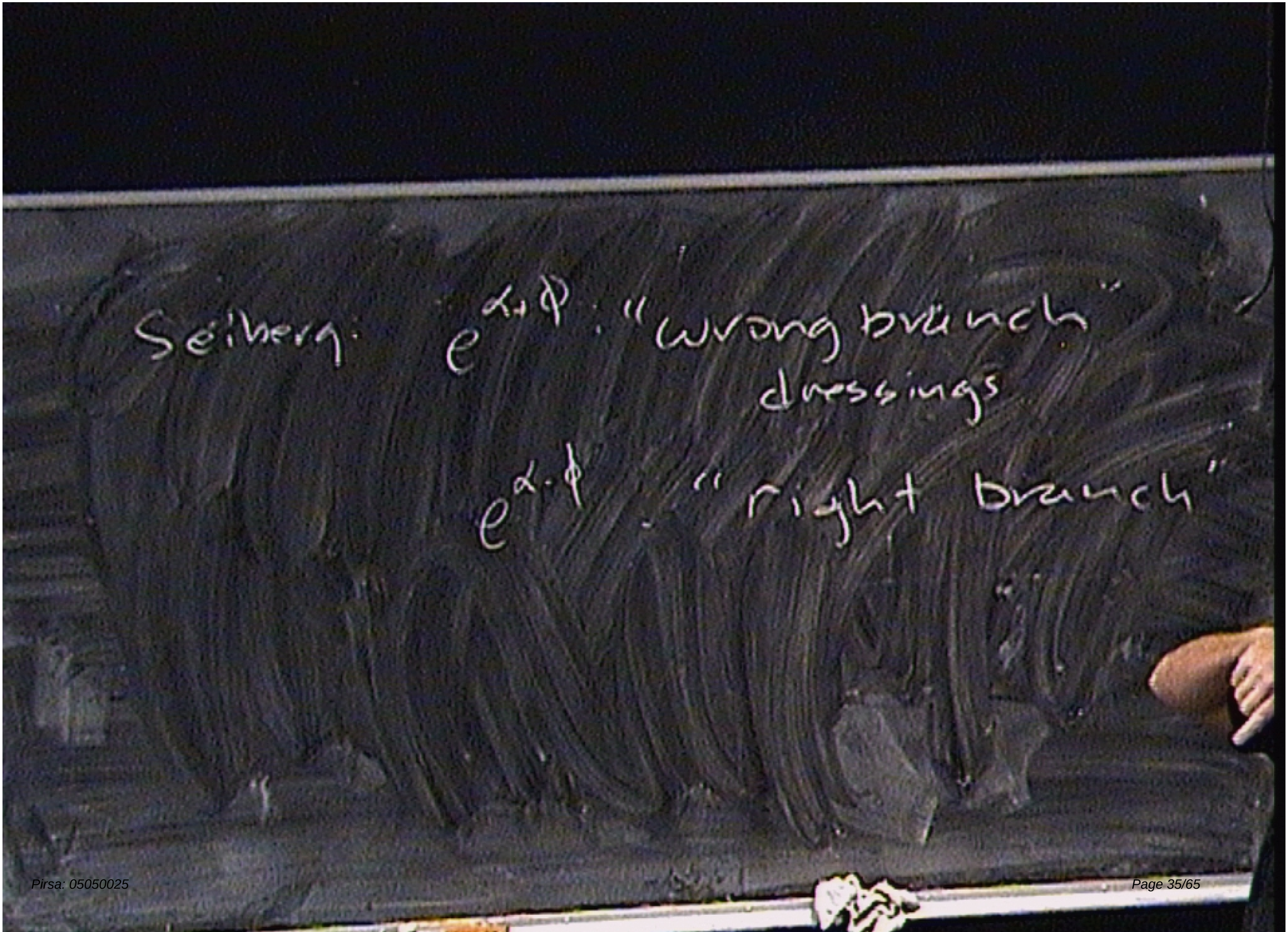
$\big\downarrow$ dim (Δ,Δ)

$(1-\Delta, 1-\Delta)$. $\lambda = e^{\alpha\phi}$ $-\frac{1}{2}\alpha(\alpha-Q) = 1-\Delta$

$$\alpha_{\pm} = \frac{Q}{2} \pm \sqrt{\frac{Q^2}{4} - 2(1-\Delta)}$$



Seiberg: $e^{\alpha_+ \phi}$: "wrong branch" dressings



Seiberg: $e^{\alpha_+ \phi}$: "wrong branch" dressings

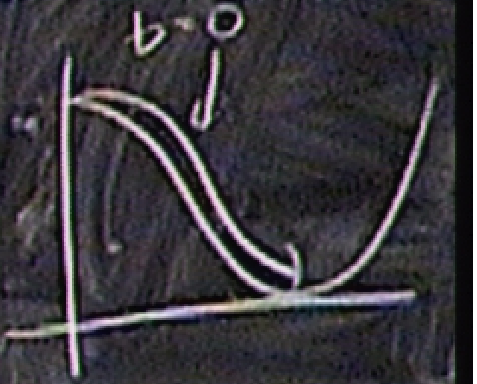
$e^{\alpha_- \phi}$. "right branch"

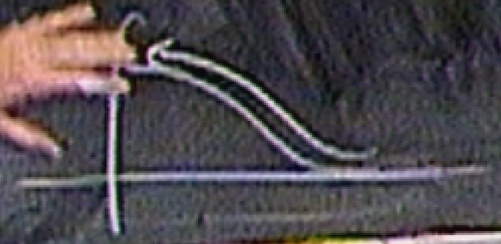
Seiberg: $e^{\alpha_+\phi}$: "wrong branch" dressings

$e^{\alpha_-\phi}$: "right branch"

Sem. <u>open</u> strings.

$$\delta S = \int \lambda(id) \to \oint (\tau e^{\phi} + b e^{-\phi}) \, d\sigma$$



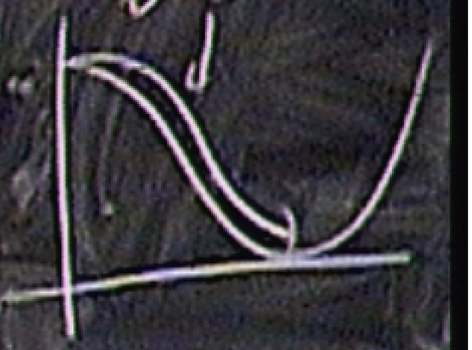


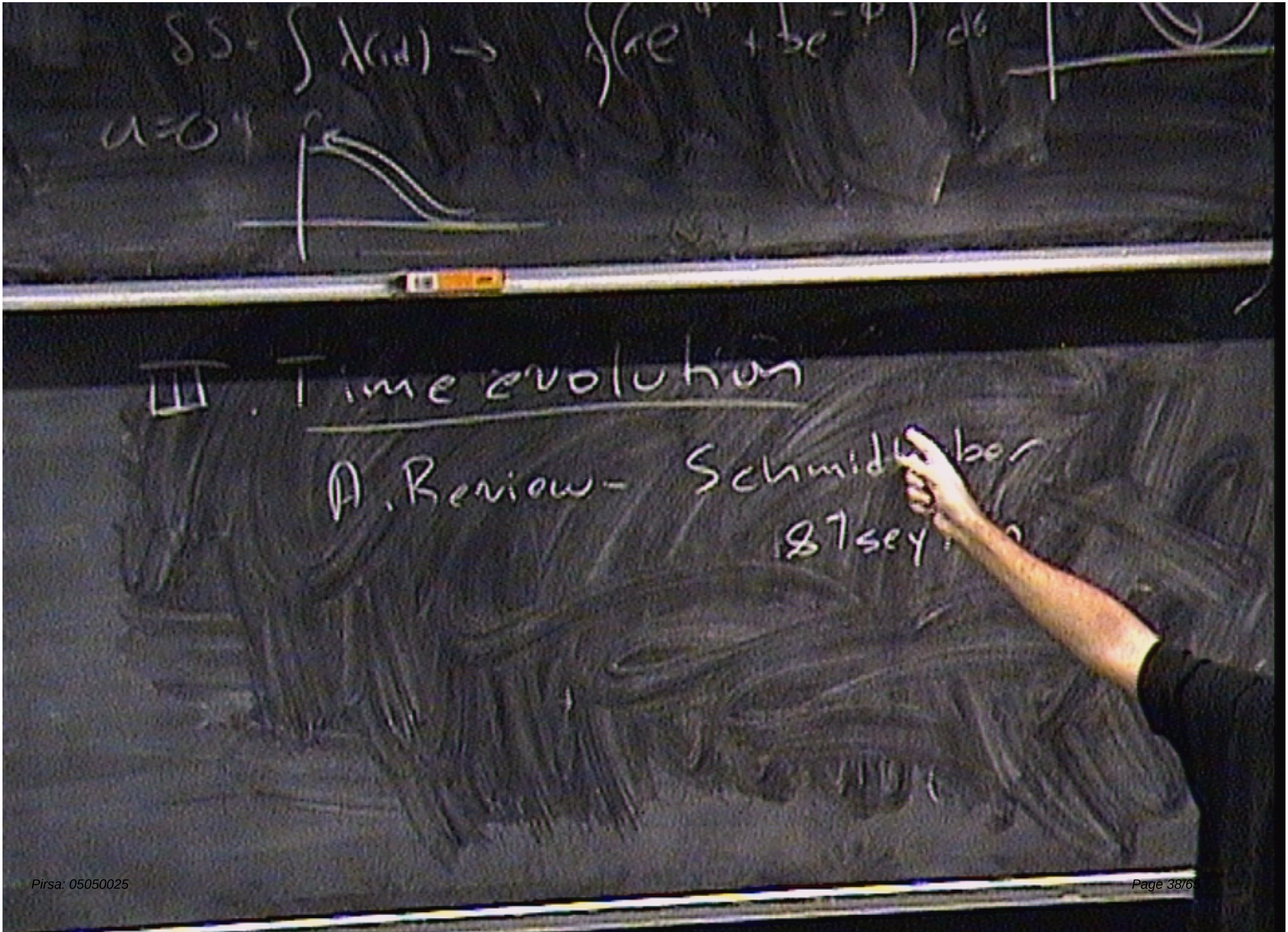
Seiberg: $e^{\alpha_+\phi}$: "wrong branch" dressings

$e^{\alpha_-\phi}$: "right branch"

Sen: open strings.

$$\delta S = \int \lambda(\sigma,\phi) \to \oint (re^{\phi} + be^{-\phi})\, d\sigma$$





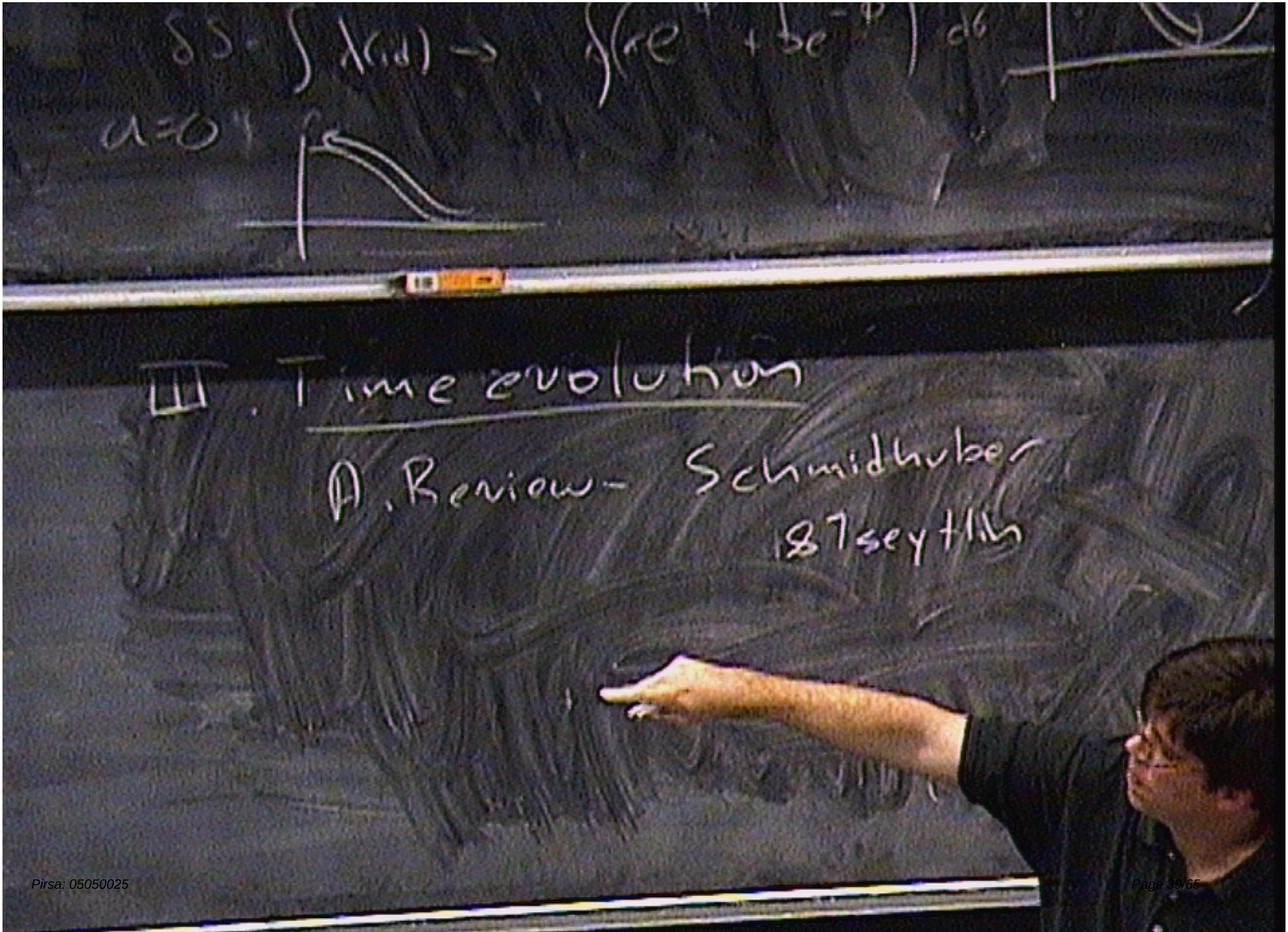
$\delta S = \int \lambda(id) \to \int (ae^{\phi} + be^{-\phi}) d\phi$

$a=0$:

III. Time evolution

A. Review - Schmidhuber

87sey

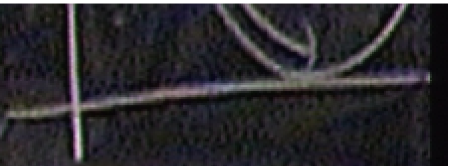


III. Time evolution

A. Review – Schmidhuber

87sey tlh

$\delta S = \int \lambda(u) \to \int (a e^{\phi} + b e^{-\phi})\, d\phi$



$a = 0$

III. Time evolution

A. Review - Schmidhuber 87 sey thh

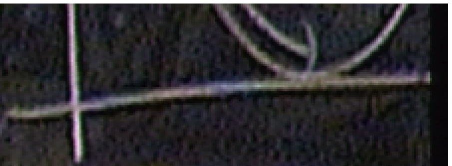
gen 2d

$n \geq 25$ scalar fields

G mod

$$S = \frac{1}{?}\int G_{\mu\nu}\partial X^{\mu}\partial X^{\nu}$$

$\delta S = \int \lambda(\phi) \to \int (a e^{\phi} + b e^{-\phi}) \, d\phi$



$a = 0$

III. Time evolution

A. Review – Schmidhuber gen 87 seytlin

2d σ mod

$n \geq 25$ scalar fields

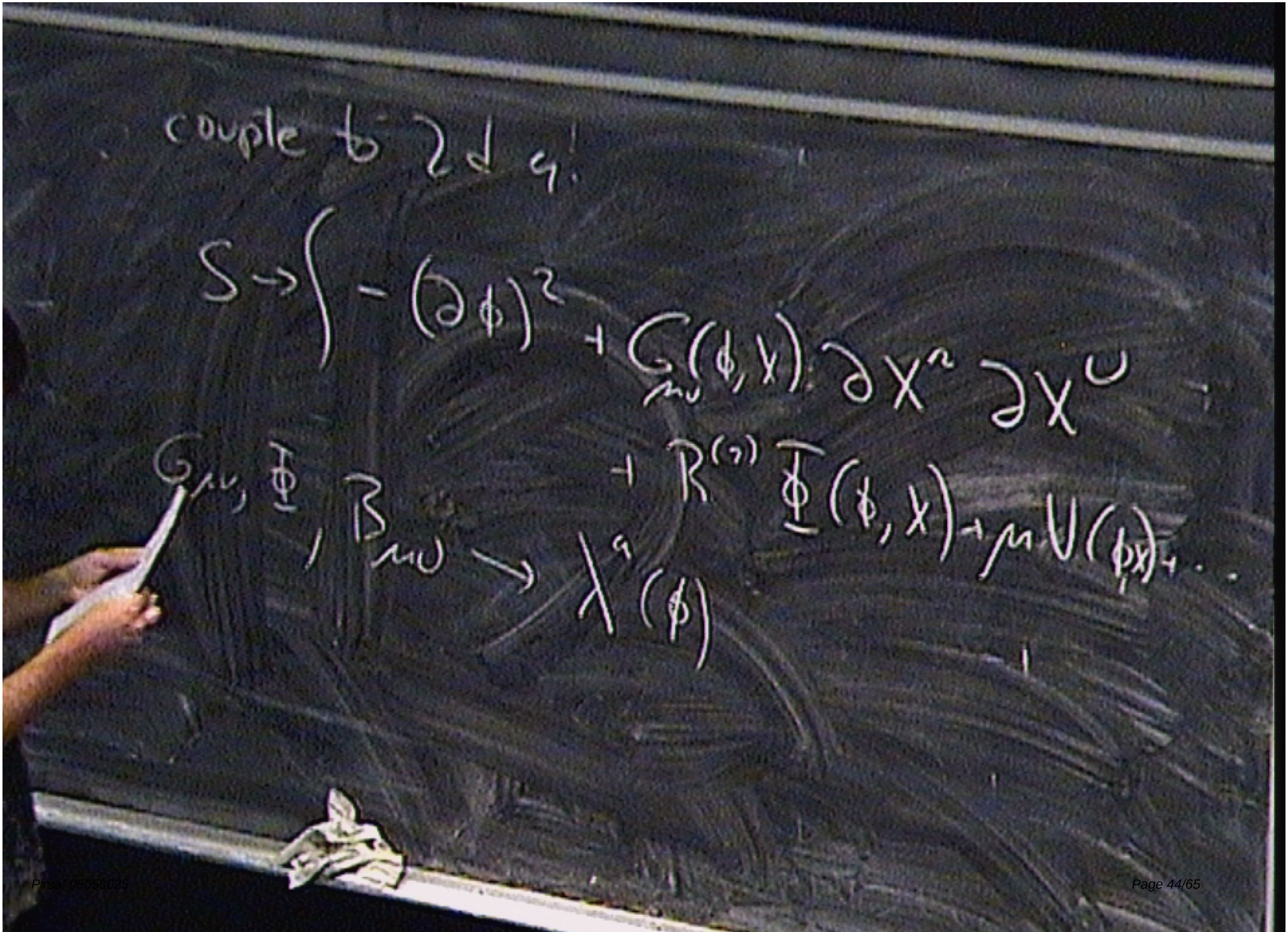
$$S = \frac{1}{2} \int G_{\mu\nu}(X) \, \partial X^{\mu} \partial X^{\nu}$$

couple to 2d g:

$$S \to \int -(\partial\phi)^2 + G_{\mu\nu}(\phi, X)\,\partial X^\mu \partial X^\nu$$

couple to 2d g:

$$S \to \int -(\partial\phi)^2 + G_{\mu\nu}(\phi, X)\,\partial X^\mu \partial X^\nu + R^{(2)}\,\Phi(\phi, X) + \mu V(\phi, X) + \cdots$$



couple to 2d g!

$$S \to \int -(\partial\phi)^2 + G_{\mu\nu}(\phi,X)\,\partial X^\mu \partial X^\nu + R^{(2)}\,\Phi(\phi,X) + \mu V(\phi,X) + \dots$$

$$G_{\mu\nu}, \Phi, B_{\mu\nu} \to \lambda^a(\phi)$$

couple to 2d g!

$$S \to \int -(\partial\phi)^2 + G_{\mu\nu}(\phi,X)\,\partial X^\mu\,\partial X^\nu + R^{(2)}\,\Phi(\phi,X) + \mu V(\phi,X) + \cdots$$

$G_{\mu\nu}, \Phi, B_{\mu\nu} \longrightarrow \lambda^a(\phi)$

computed 2d β-fns.

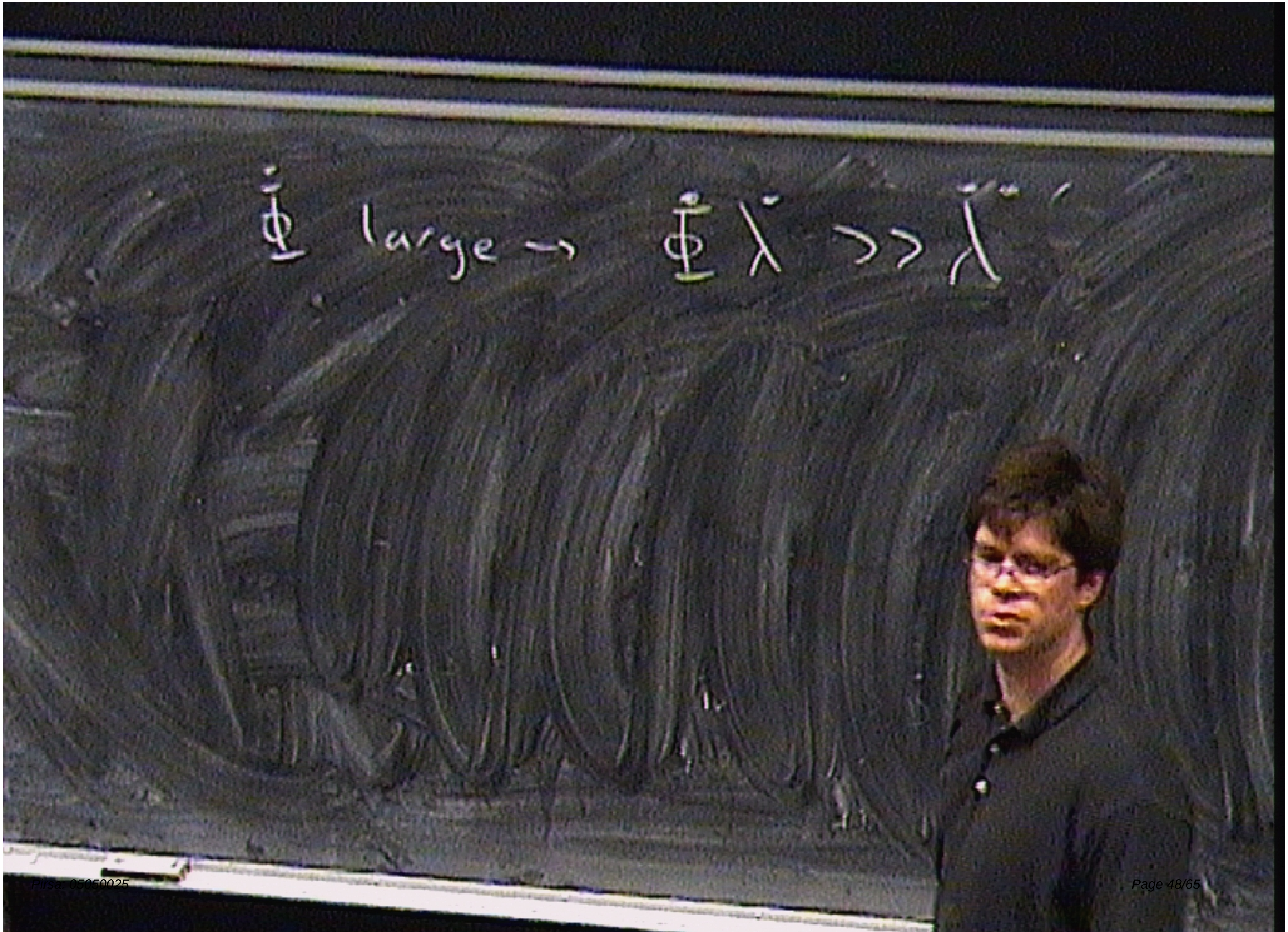
$\bar{\beta}^a(\lambda) \to \beta$-fns for non-grav. 2d ft.

$$\beta^{tot} = 0 \Rightarrow \boxed{\ddot{\lambda}^a - \dot{\Phi}\dot{\lambda}^a + \bar{\beta}^a(\lambda) + O(\dot{\lambda}^2) = 0}$$

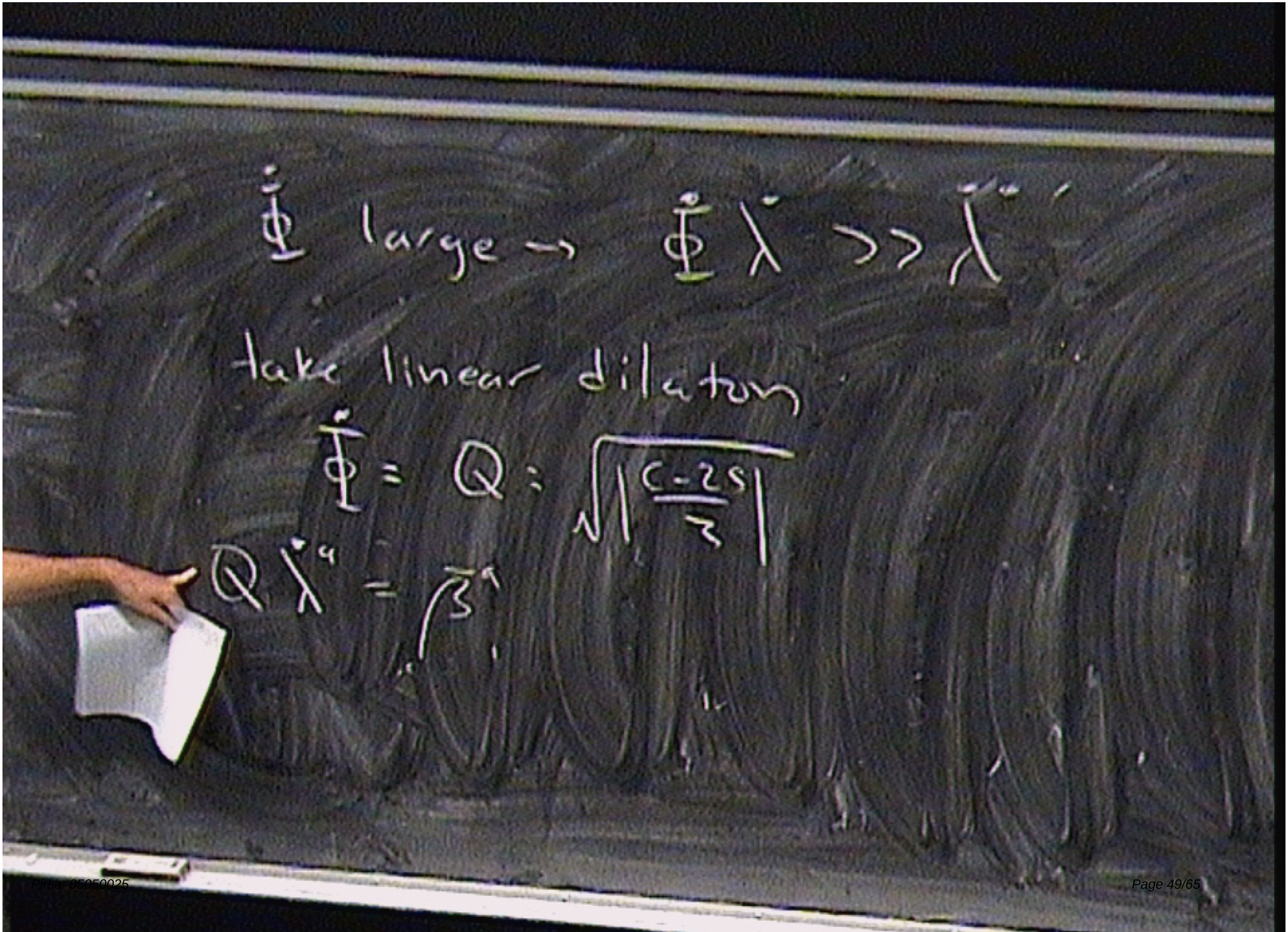
$\bar{\beta}^a(\lambda) \to$ β-fns for non-grav.

$\beta^{tot} = 0 \Rightarrow \boxed{\ddot{\lambda}^a - \dot{\Phi}\dot{\lambda}^a + \bar{\beta}^a(\lambda) = 0 + O(\dot{\lambda}^2)}$ 2d f.t.

$\dot{\lambda}^a = \frac{\partial}{\partial\phi}\lambda^a$



$\dot{\Phi}$ large $\rightarrow$ $\dot{\Phi}\lambda' \gg \lambda''$



$\dot{\Phi}$ large → $\dot{\Phi}\dot{\lambda} >> \ddot{\lambda}$

take linear dilaton

$$\dot{\Phi} = Q = \sqrt{\left|\frac{c-25}{3}\right|}$$

$$Q\dot{\lambda}^4 = \sqrt{3}$$

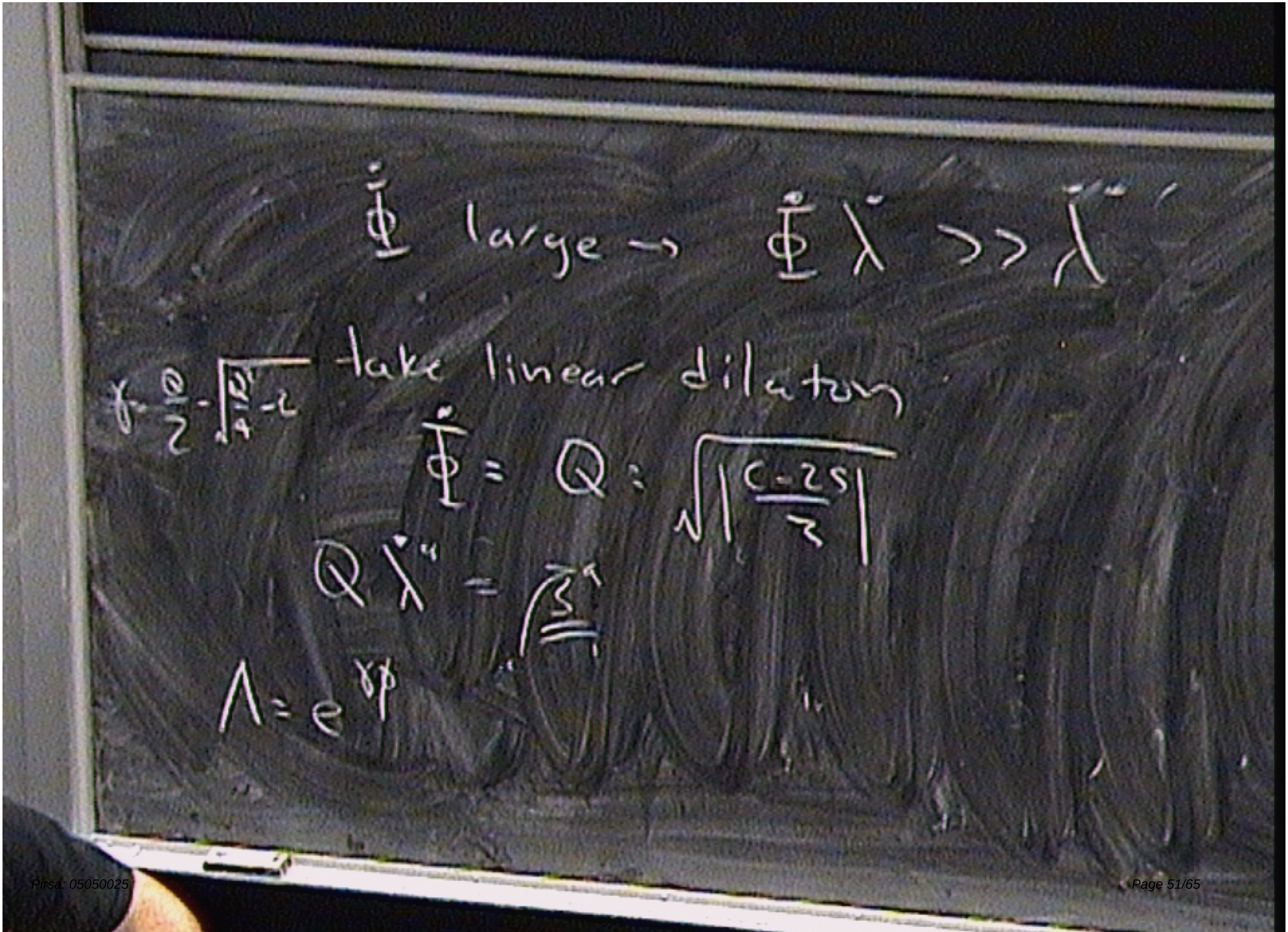
$\dot{\Phi}$ large → $\dot{\Phi}\dot{\lambda} \gg \ddot{\lambda}$

take linear dilaton

$\dot{\Phi} = Q = \sqrt{\left|\frac{c-25}{2}\right|}$

$Q\dot{\lambda}^a = \xi^a$

$\Lambda = e^{\gamma\phi}$



$\dot{\Phi}$ large → $\dot{\Phi}\,\ddot{\lambda} >> \ddot{\lambda}$

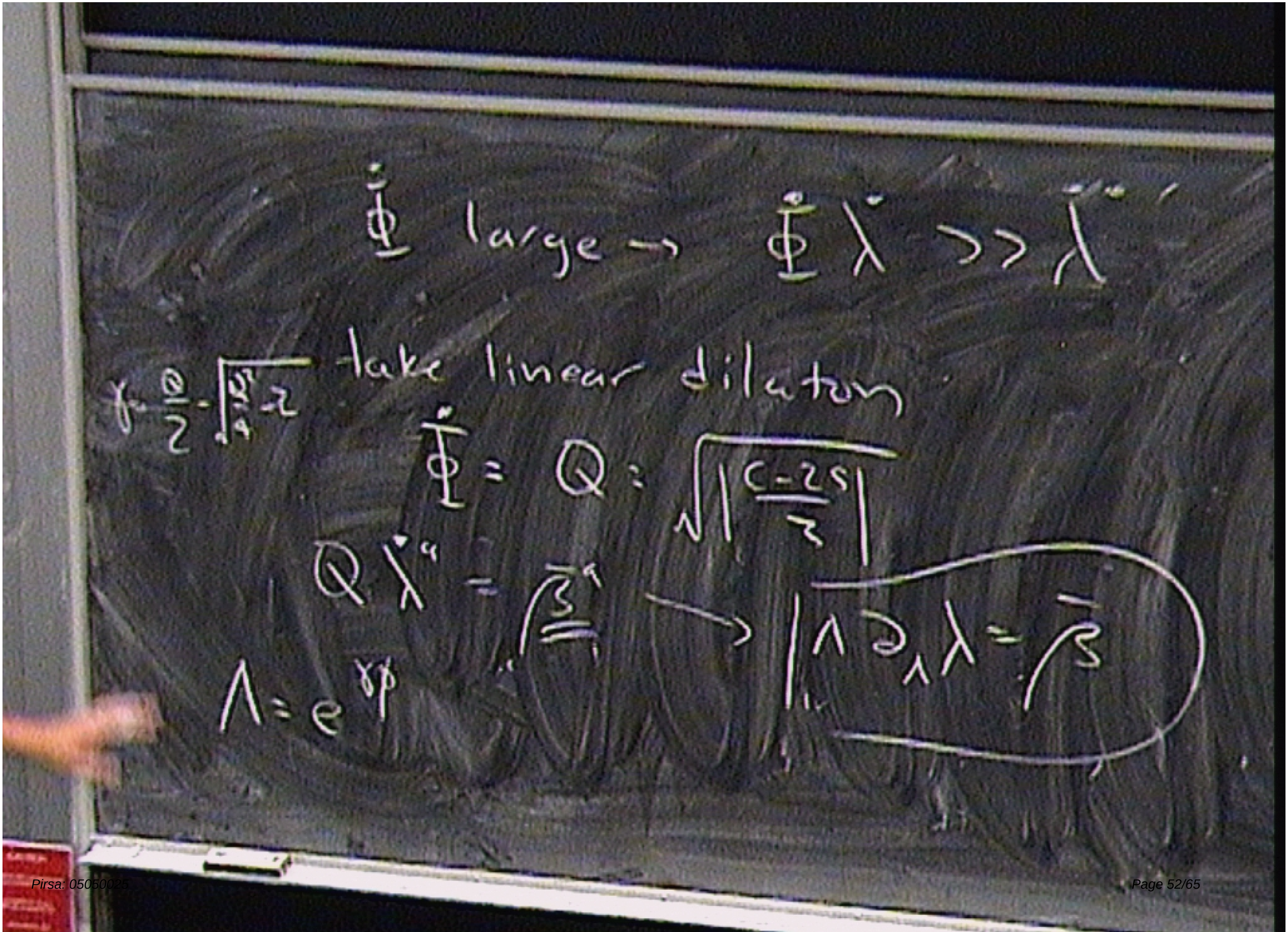
$\gamma = \frac{Q}{2} - \sqrt{\frac{Q^2}{4} - 2}$

take linear dilaton

$\dot{\Phi} = Q = \sqrt{\left|\frac{c-25}{3}\right|}$

$Q\,\check{\lambda}^{4} =$ [illegible]

$\Lambda = e^{\gamma\phi}$



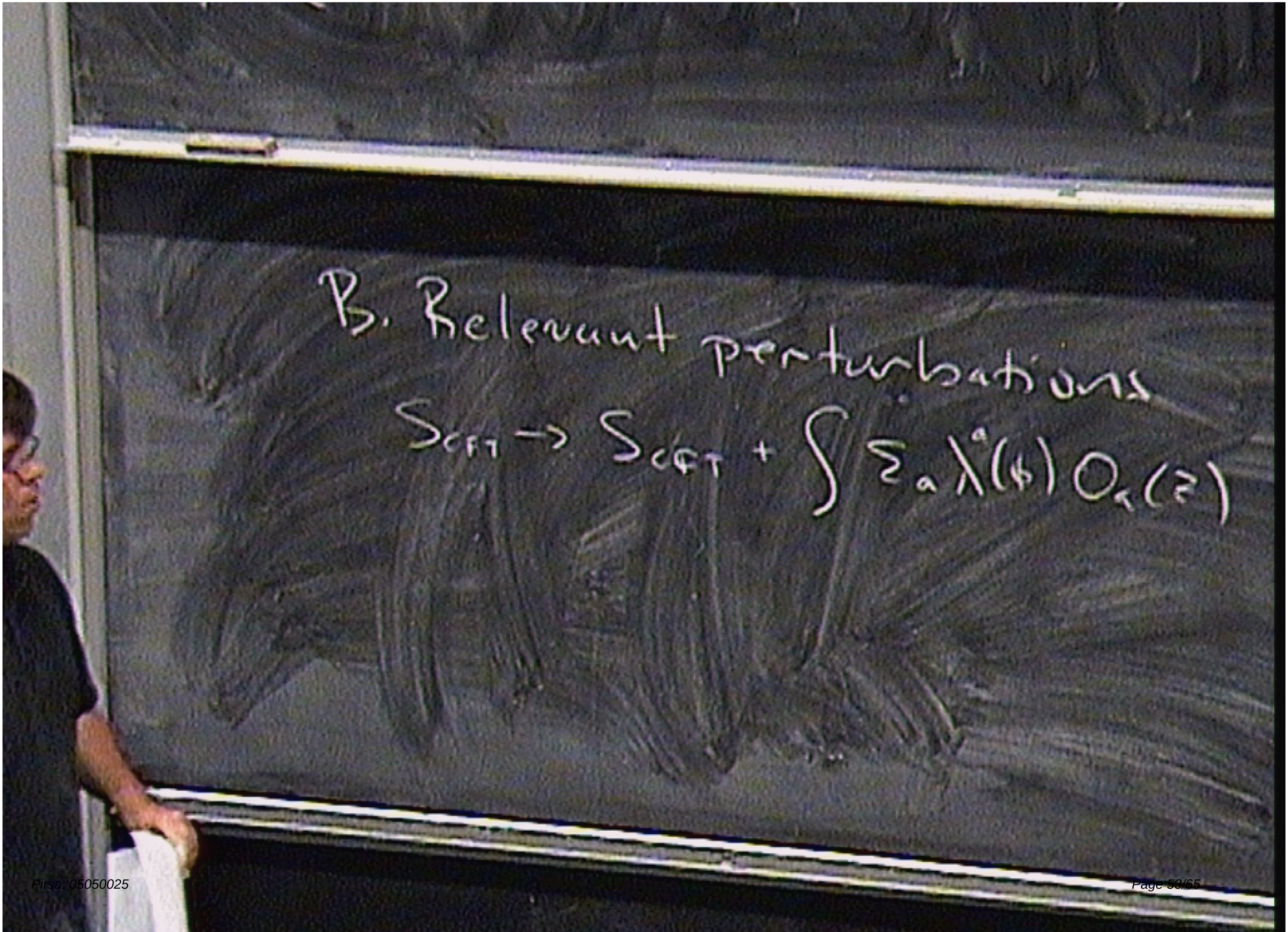
$\dot{\Phi}$ large → $\dot{\Phi}\dot{\lambda} \gg \ddot{\lambda}$

$\gamma = \frac{Q}{2} - \sqrt{\frac{Q^2}{4} - 2}$ take linear dilaton

$$\dot{\Phi} = Q = \sqrt{\left|\frac{c-25}{3}\right|}$$

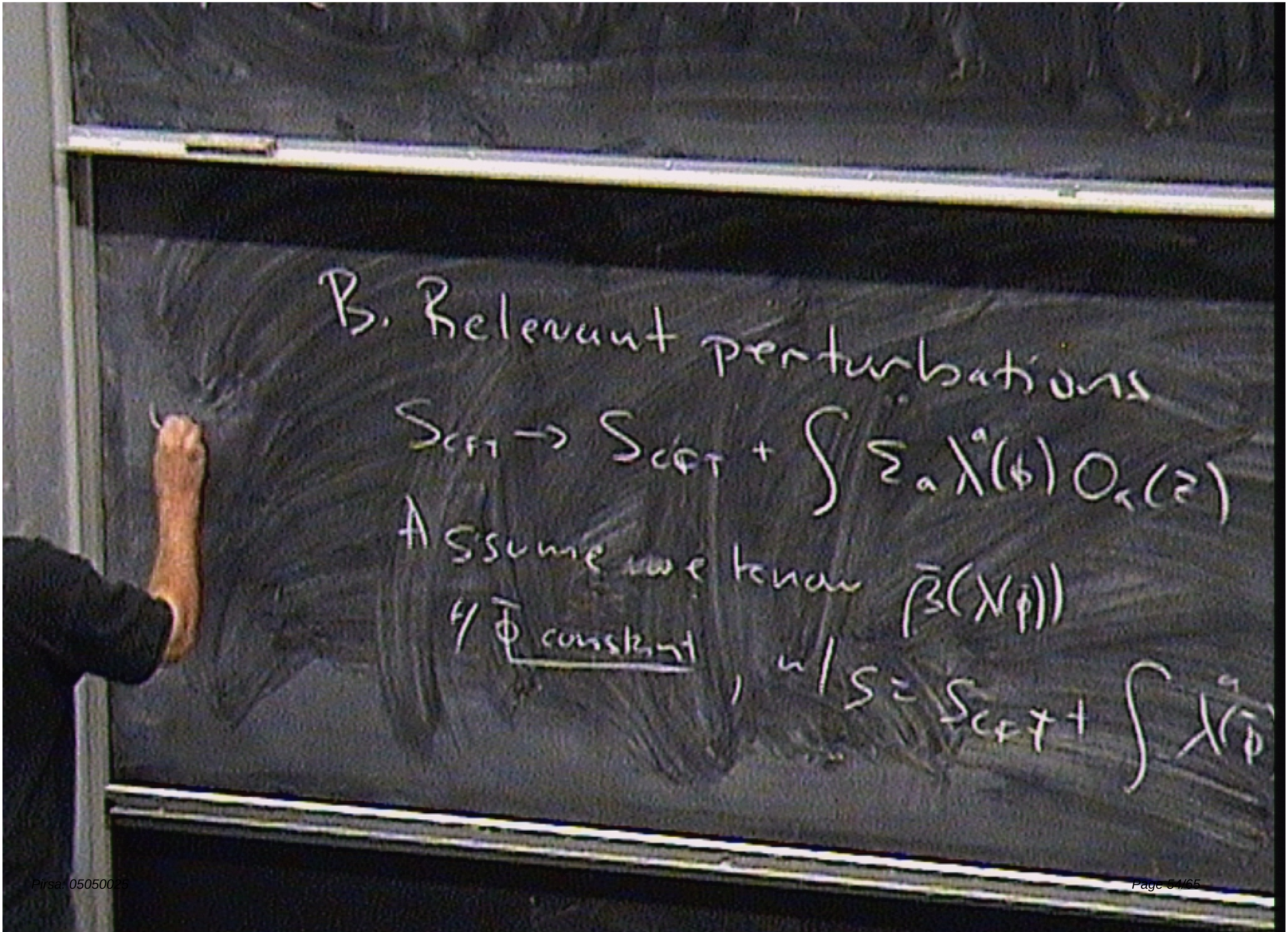
$$Q\dot{\lambda}^a = \bar{\beta}^a \rightarrow \boxed{\Lambda \partial_\Lambda \lambda = \bar{\beta}}$$

$$\Lambda = e^{\gamma\phi}$$



B. Relevant perturbations

$$S_{CFT} \to S_{CFT} + \int \Sigma_a \lambda^a(t)\, \mathcal{O}_a(z)$$



B. Relevant perturbations

$S_{CFT} \to S_{CFT} + \int \Sigma_a \lambda^a(\phi)\, O_a(z)$

Assume we know $\bar\beta(\lambda(\bar\phi))$

f/ $\underline{\bar\Phi \text{ constant}}$, w/ $S = S_{CFT} + \int \lambda^a(\bar\phi$

$\Lambda = e^{\delta\phi}$

B. Relevant perturbations

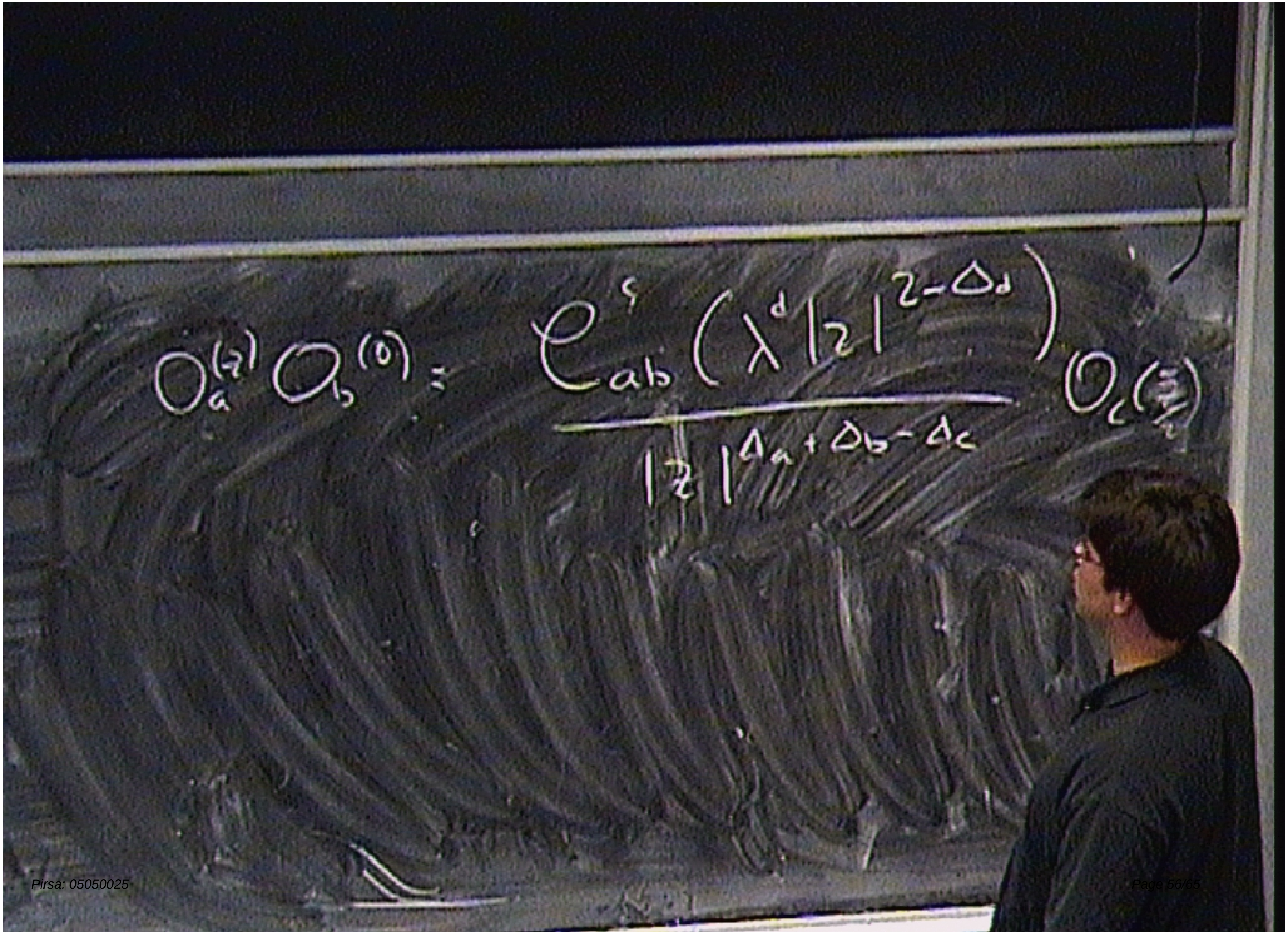
$\phi = \bar{\phi} + \xi(z)$ $\quad S_{CFT} \to S_{CFT} + \int \Sigma_a \lambda^a(\phi)\, O_a(z)$

$\uparrow$

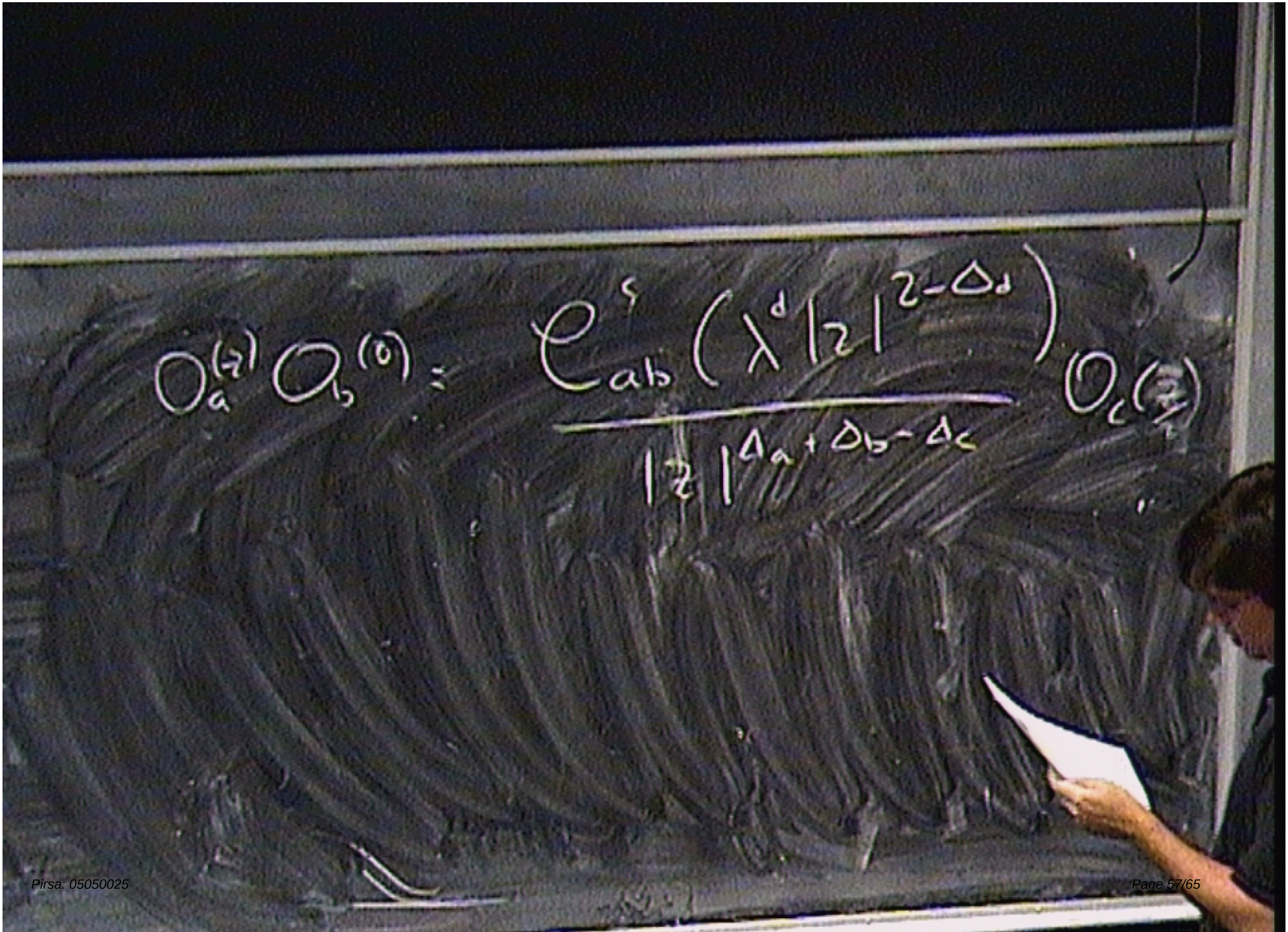
const.

Assume we know $\bar{\beta}(\lambda(\bar{\phi}))$

w/ $\underline{\bar{\Phi} \text{ constant}}$, w/ $S = S_{CFT} + \int \lambda^a(\bar{\phi})\, O_a$



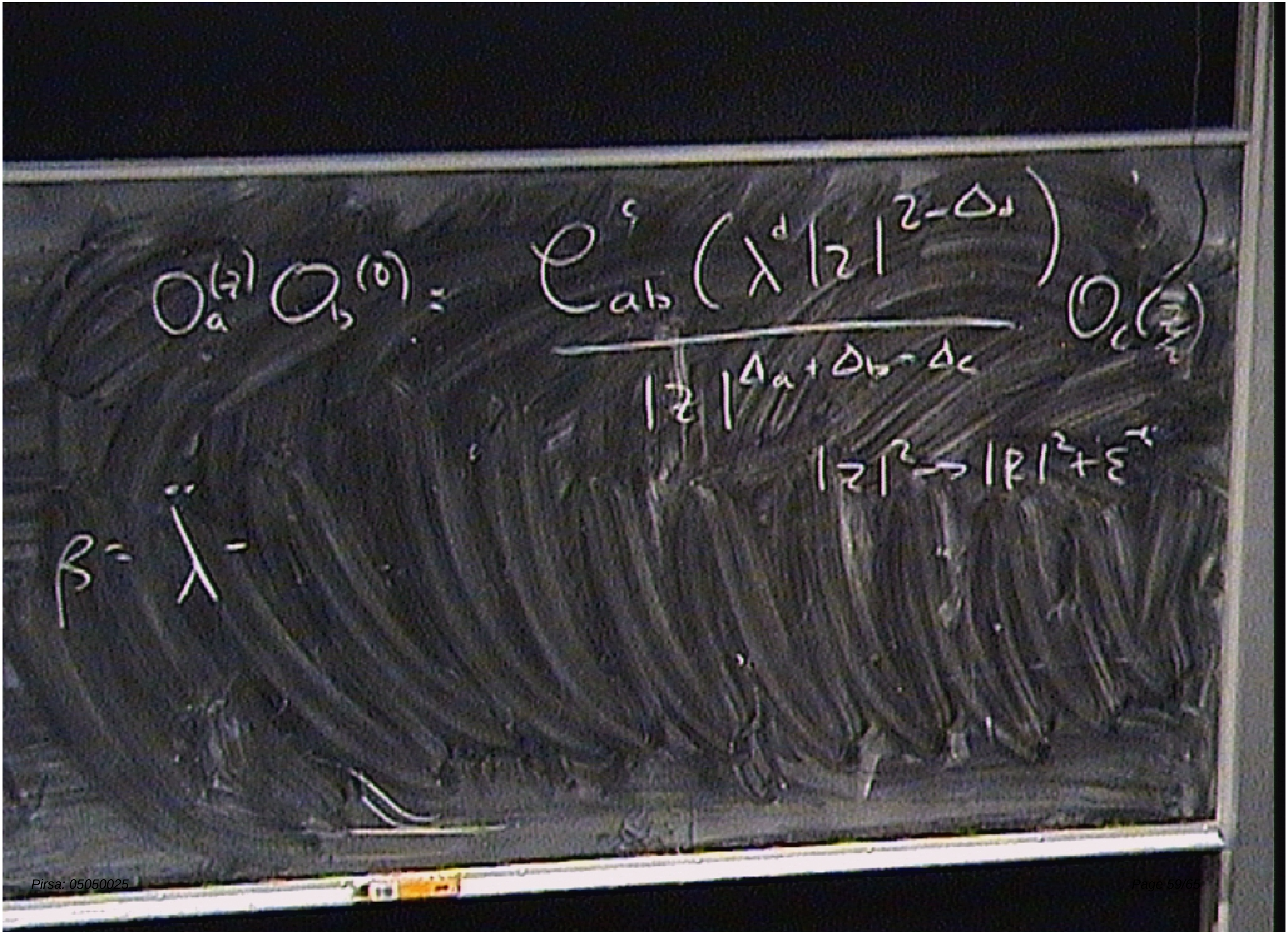
$$O_a^{(z)} O_b^{(0)} = \frac{C_{ab}^{c}\left(\lambda^d |z|^{2-\Delta_d}\right)}{|z|^{\Delta_a+\Delta_b-\Delta_c}} O_c\left(\frac{z}{2}\right)$$



$$\mathcal{O}_a(z)\,\mathcal{O}_b(0) = \frac{C_{ab}^{c}\left(\lambda^d |z|^{2-\Delta_d}\right)}{|z|^{\Delta_a+\Delta_b-\Delta_c}}\,\mathcal{O}_c\left(\tfrac{z}{2}\right)$$

$$O_a^{(z)} O_b^{(0)} = \frac{C_{ab}^{c}\left(\lambda^d |z|^{2-\Delta_d}\right)}{|z|^{\Delta_a+\Delta_b-\Delta_c}} O_c\left(\frac{z}{2}\right)$$

$$\beta = \ddot{\lambda} -$$



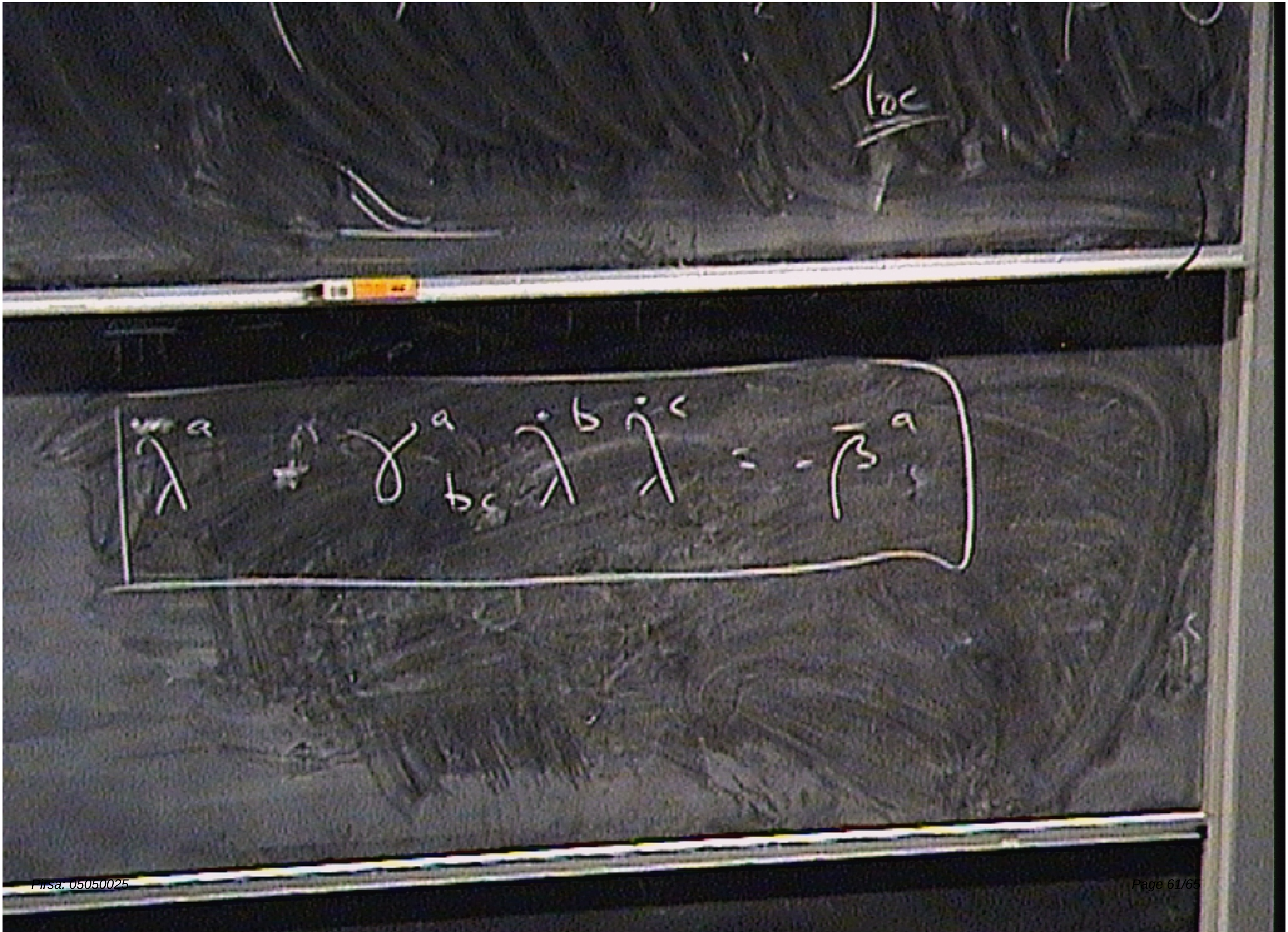
$$O_a^{(z)} O_b^{(0)} = \frac{C_{ab}^{c}\left(\lambda^d |z|^{2-\Delta_d}\right)}{|z|^{\Delta_a+\Delta_b-\Delta_c}} O_c\left(\frac{z}{2}\right)$$

$$|z|^2 \to |z|^2 + \varepsilon^2$$

$$\beta = \ddot{\lambda}^{-}$$

$$O_a^{(z)}\, O_b^{(0)} = \frac{C_{ab}^{c}\left(\lambda^{d}|z|^{2-\Delta_d}\right)}{|z|^{\Delta_a+\Delta_b-\Delta_c}}\, O_c\left(\frac{z}{2}\right)$$

$$\beta^{a} = \lambda^{a} - \left(\epsilon\frac{\partial}{\partial\epsilon}\int d^2z\, \frac{C^{a}_{bc}\left(\lambda^{d}(|z|^2+\epsilon^2)^{\frac{2-\Delta_d}{2}}\right)\Big|_{|z|^2\to|z|^2+\epsilon^2}}{(|z|^2+\epsilon^2)^{\frac{\Delta_b+\Delta_c-\Delta_a}{2}}}\right)\lambda^{b}\lambda^{c} + \beta^{a} = 0$$



bc

$$\boxed{\ddot{\lambda}^a + \gamma^a_{bc}\dot{\lambda}^b\dot{\lambda}^c = -\bar{\beta}^a}$$

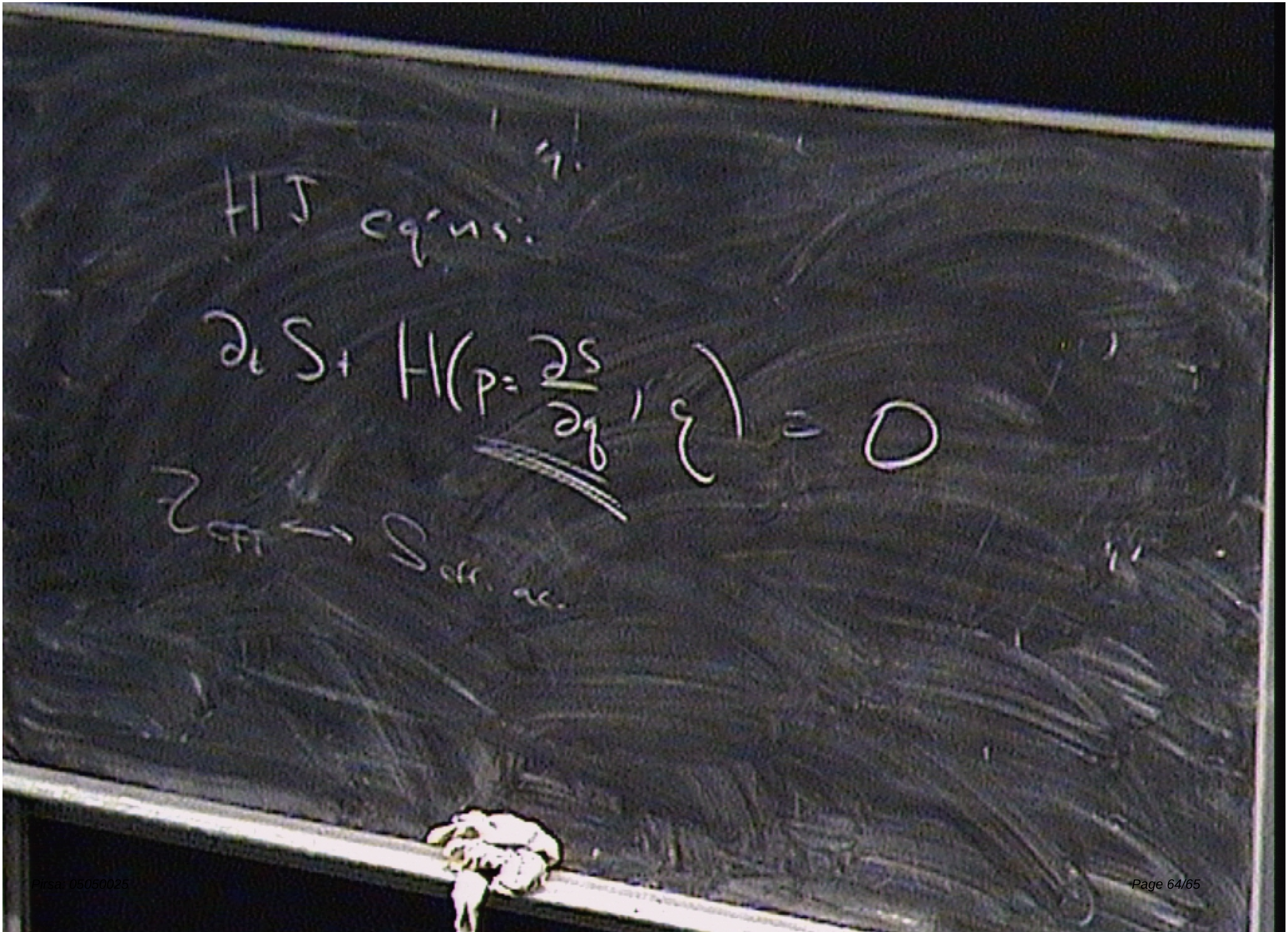
$$\boxed{\ddot{\lambda}^a + \gamma^a_{bc}\dot{\lambda}^b\dot{\lambda}^c = -\bar{\beta}^a}$$

for $S = \int\left(Z_{ab}\dot{\lambda}^a\dot{\lambda}^b - V\right)$

$\bar{\beta}^a = Z^{ab}\partial_b V$

HJ eq'ns:

$$\partial_t S + H\left(p = \frac{\partial S}{\partial q}, \xi\right) = 0$$



HJ eq'ns:

$$\partial_t S + H\left(p = \underline{\frac{\partial S}{\partial q}}, q\right) = 0$$

$Z_{CFT} \rightarrow S_{eff, ac.}$

HJ eq'ns:

$$\partial_t S + H\Big(p = \underline{\frac{\partial S}{\partial q}}, q\Big) = 0$$

$Z_{eff} \to S_{eff.\,ac.}$; $q \to \lambda$

$$p = \partial S / \partial q = \partial_\lambda Z = \langle O \rangle$$