

Title: Relaxation and unitarity in eternal black hole

Date: May 20, 2005 02:00 PM

URL: <http://pirsa.org/05050023>

Abstract:

Hawking (1976) : information is lost in BH

Maldacena (2001) : Topological diversity of
grav. instantons restores unitarity

Hawking (2003) : information is not lost in BH

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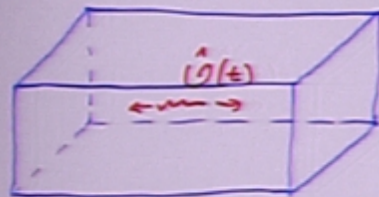
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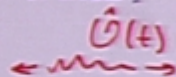
1.
Relaxation in unitary theory (at temperature $T = \beta^{-1}$)
 Finite volume: $(L_1, L_2, L_3, \dots)$



$t > 0$
 $\langle \hat{O}(t) \hat{O}(0) \rangle_\beta \sim \sum_i A_i e^{i\omega_i t}$
 discrete set of frequencies $\{\omega_i\}$
 $\omega_i \sim \frac{1}{L_i}$

Relaxation is quasi-periodic;
 Information is never lost

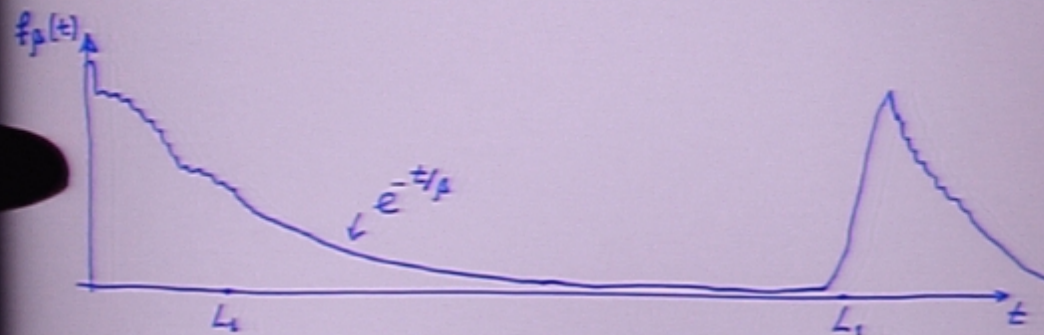
Infinite volume:



$\langle \hat{O}(t) \hat{O}(0) \rangle_\beta \sim e^{-t/\beta}$

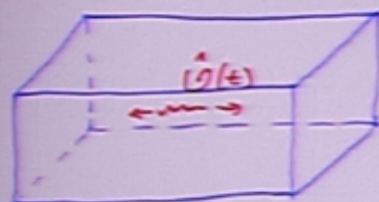
Exponential decay; spectrum is continuous

Limit $L_1 \rightarrow \infty$ $(L_2, L_3, \dots)$ finite; $L_1 \gg L_2, L_3, \dots$



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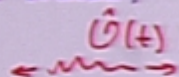
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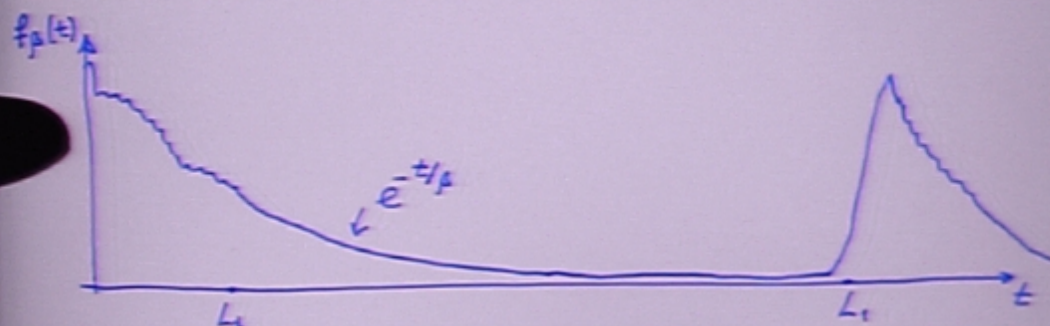
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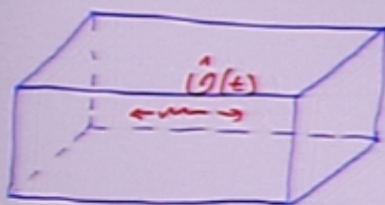
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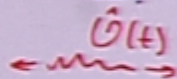
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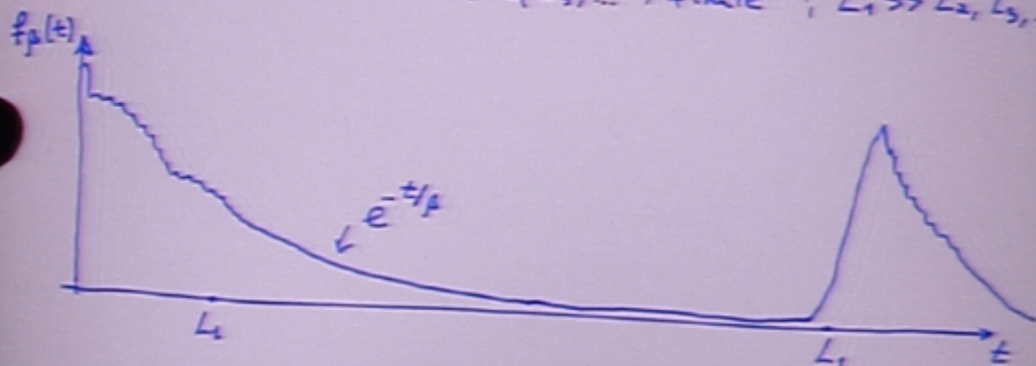
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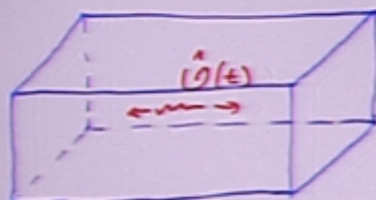
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Finite volume: $(L_1, L_2, L_3, \dots)$



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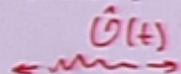
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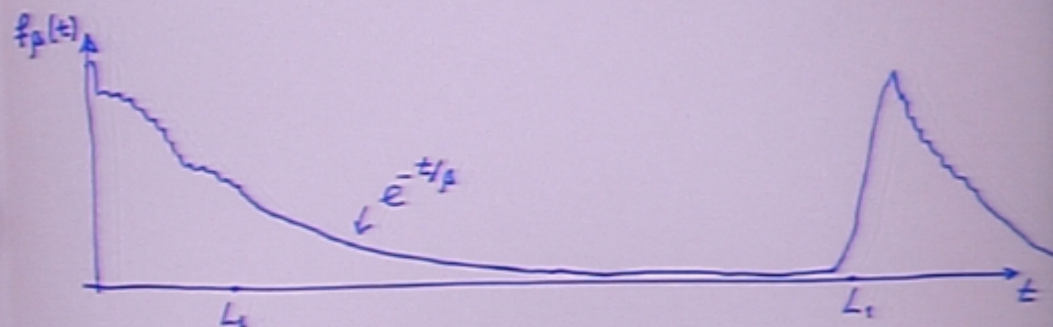
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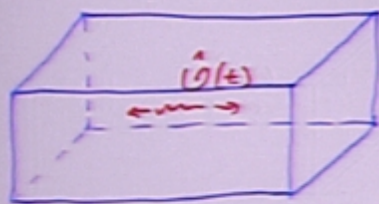
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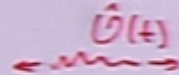
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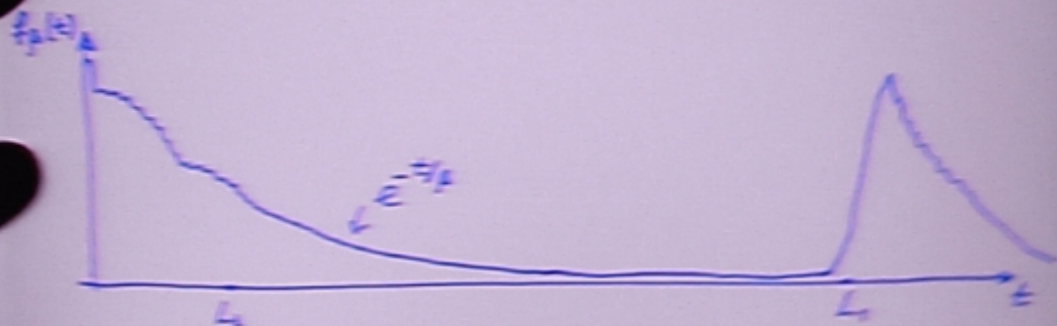
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Calculus of variations (at minimum point)
 First order conditions



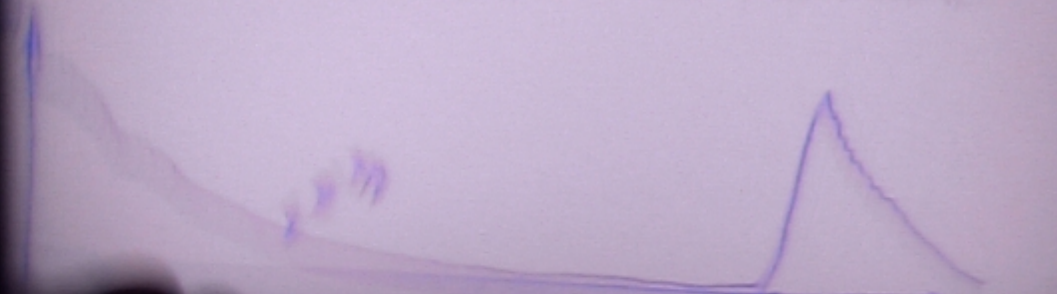
First order conditions
 Second order conditions

First order conditions
 Second order conditions

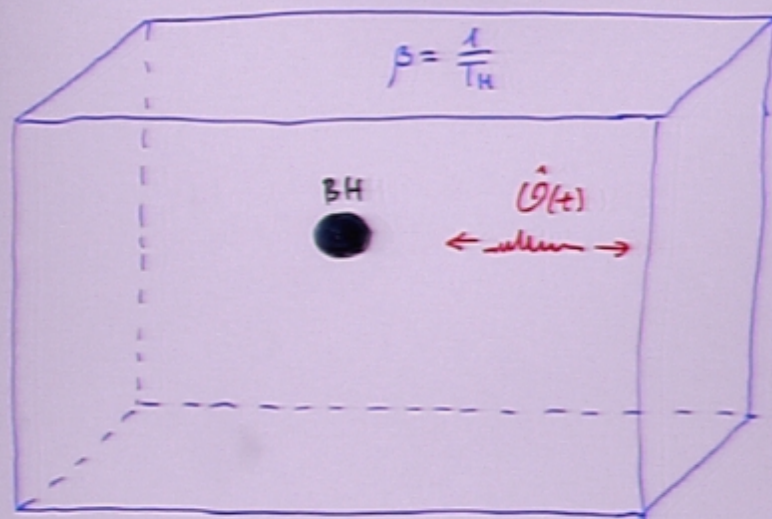


First order conditions
 Second order conditions

First order conditions
 Second order conditions



Asymptotically AdS black hole (in finite volume)



Perturbation falls inside BH horizon

$$\langle \hat{\mathcal{O}}(t) \hat{\mathcal{O}}(0) \rangle_p \sim \sum_n e^{-i\omega_n t} B_n$$

$$\omega_n = \omega + i\beta_n, \quad \beta_n < 0 \quad n=0, 1, 2, \dots$$

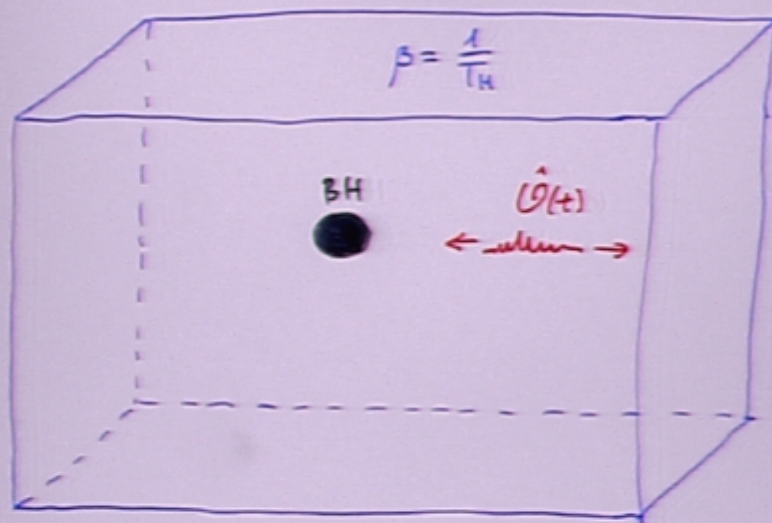
- discrete set of complex modes $\{\omega_n\}$
- Quasi-Normal Modes

- exponential decay, $\langle \hat{\mathcal{O}}(t) \hat{\mathcal{O}}(0) \rangle \sim e^{-\beta_0 t}, \quad t \gg T_H^{-1}$

- effective infinite volume $L_{\text{opt}} \sim \int_{x_H} \frac{dx}{g(x)} = \infty$

- clear manifestation of loss of information

Asymptotically AdS black hole (in finite volume)



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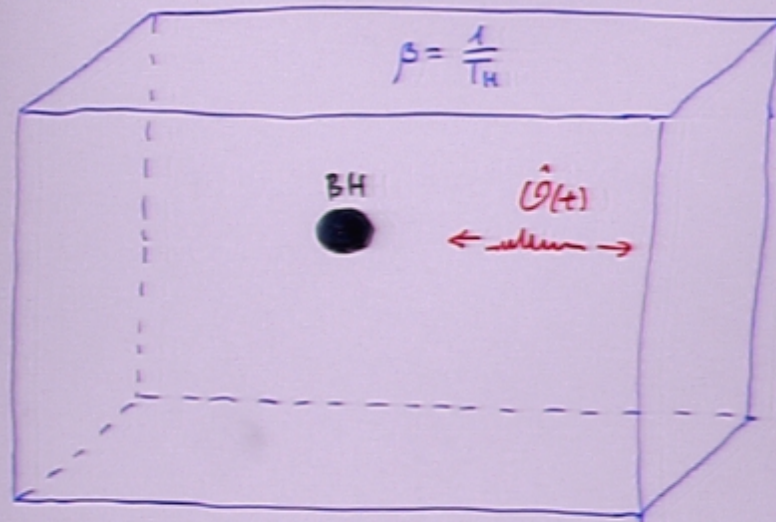
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Asymptotically AdS black hole (in finite volume)



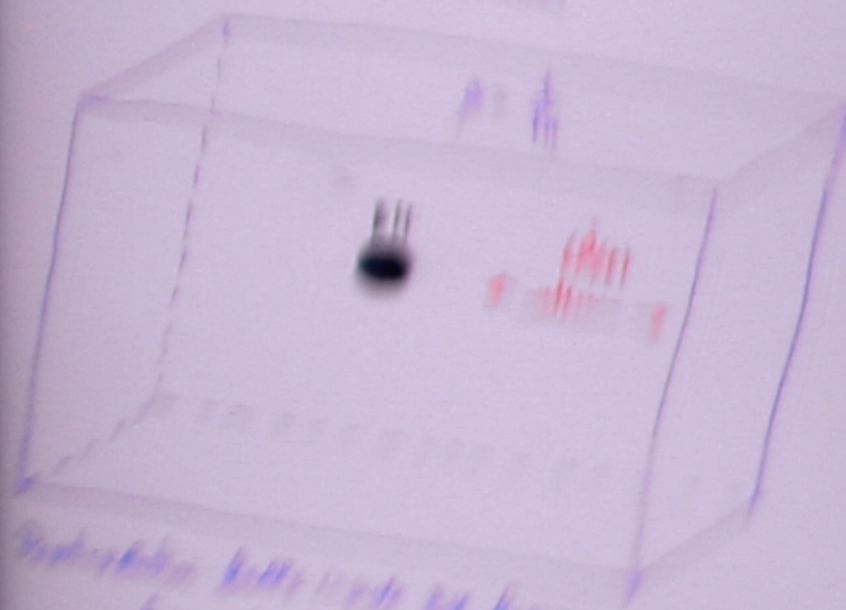
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- discrete set of complex modes $\{\omega_n\}$
 - Quasi-Normal Modes
- exponential decay, $\langle \hat{\mathcal{O}}(t) \hat{\mathcal{O}}(0) \rangle \sim e^{-\beta_0 t}, \propto T_H^{-1}$
- effective infinite volume $L_{\text{eff}} \sim \int_{x_H} \frac{dx}{g(x)} = \infty$
- clear manifestation of loss of information

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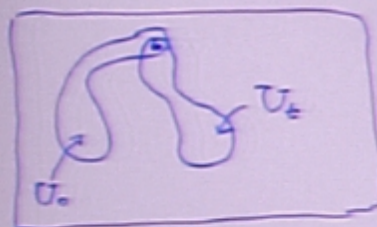


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Recurrence Time (Poincaré)

3.

Classical system: phase space has finite volume



Liouville evolution

For some finite t

$$U_0 \cap U_t \neq \emptyset$$

(eternal return)

Quantum system:

- unitary evolution

- phase space is compact $\rightarrow$ discrete spectrum $\{\omega_i\}$
under energy shell

Hierarchy of time scales: Srednicki, Suskind (02), Breiten-Rohrman (03)

$$\text{Heisenberg time } t_H \sim \frac{1}{\langle \delta\omega \rangle}; \quad \langle \delta\omega \rangle \approx \langle \omega_i - \omega_j \rangle$$

reflects discreteness of spectrum

For $t \ll t_H$ spectrum $\approx$ continuous

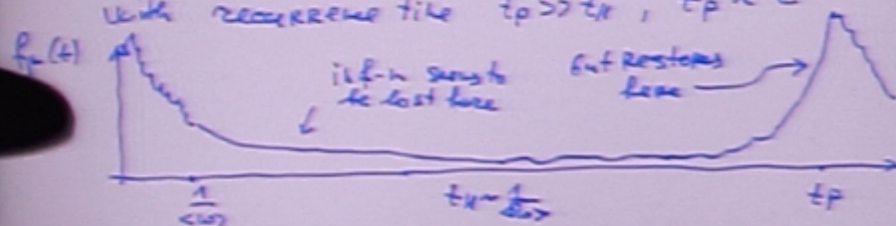
System may look dissipative for $\frac{1}{\langle \omega \rangle} \ll t \ll t_H$
(exp. decay)

For $t \gg t_H$ discreteness of spectrum becomes important

corr. functs start to show large time periodicity

(and come arbitrarily close to initial value)

With recurrence time $t_P \gg t_H$, $t_P \sim e^{S(t)}$ entropy of system

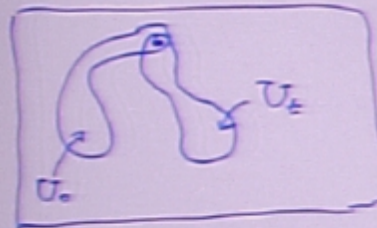


- information completely restored at $t \sim t_P$

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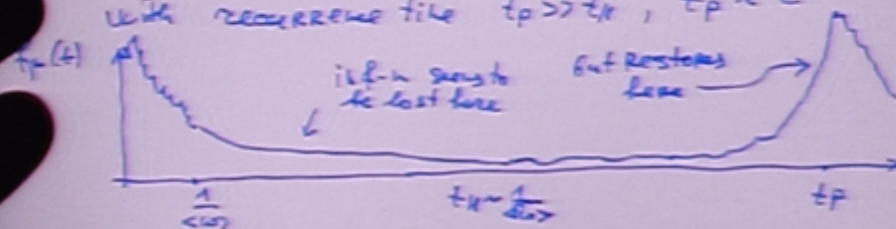
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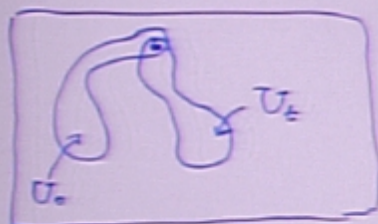


- information completely destroyed at $t \sim t_p$

Recurrence Time (Poincaré)

3.

Classical system: phase space has finite volume



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reflects discreteness of spectrum

For $t < t_h$ spectrum $\approx$ continuous

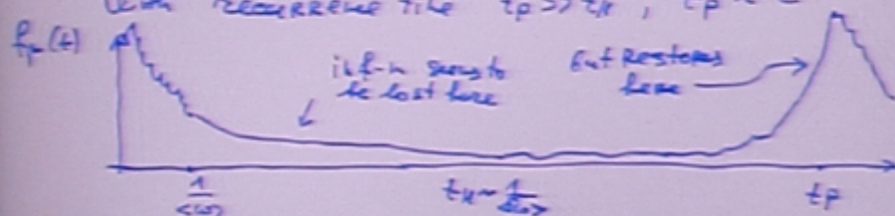
system may look dissipative for $\frac{1}{\langle \omega \rangle} \ll t \ll t_h$
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For $t \gg t_h$ discreteness of spectrum becomes important

corr. functs start to show large time periodicity

(and come arbitrarily close to initial value)

with recurrence time $t_p \gg t_h$, $t_p \sim e^{\gamma t_h}$ entropy of system



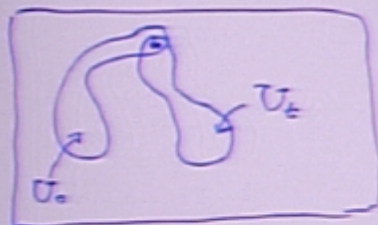
- information completely restored only at $t \sim t_p$

Recurrence Time (Poincaré)

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Classical system:

phase space has finite volume



Liouville evolution

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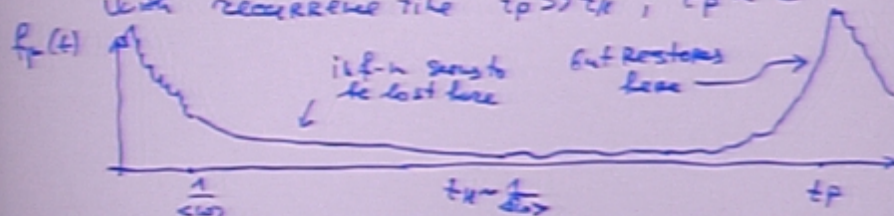
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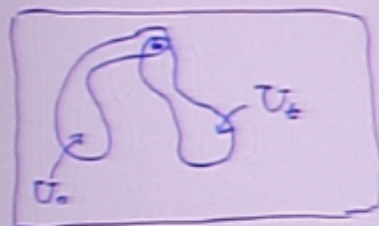


- information completely reconstructed at $t \sim t_P$

Recurrence Time (Poincaré)

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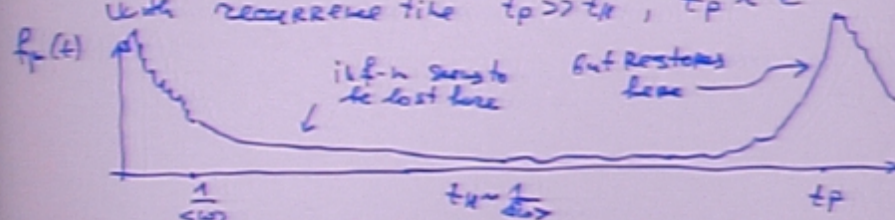
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With recurrence time $t_P \gg t_H$, $t_P \sim e^{S_{\text{extropy of system}}}$



- information completely restoring at $t \sim t_P$

Vertebrate Tax. (Continued)

Amphibian Cycle:

plus open for fish culture



Amphibian cycle
for open fish & H
H1 H2 H3 H4
(Amphibian culture)

Amphibian Cycle:

Amphibian cycle

plus open for fish culture
Amphibian cycle

Amphibian cycle: Amphibian, Amphibian, Amphibian, Amphibian

Amphibian cycle: Amphibian, Amphibian, Amphibian, Amphibian

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Semiclassical BH

- no recurrences!
- information is lost!

How to cure?

- find a way to assign a discrete spectrum $\{\omega_i\}$ of real frequencies with BH
 - Spectrum should be non-trivial enough to reproduce complex time evolution:
 - exp. decay for $t \rightarrow t_H (t_p)$
 - restoration of unitarity at $t \sim t_p e^{\frac{t}{t_H}}$
 - more complicated than simple equidistant spectrum
- BH is not a clock!

Semiclassical BH

- no recurrences!
- information is lost!

How to cure?

- find a way to assign a discrete spectrum $\{\omega_i\}$ of real frequencies with BH
- spectrum should be non-trivial enough to reproduce complex time evolution:
 - exp. decay for $t > t_H (t_p)$
 - restoration of unitarity at $t \sim t_p \sim e^{\frac{A}{4}}$
- more complicated than simple equidistant spectrum

BH is not a clock!

Semiclassical BH

- no recurrences!
- information is lost!

How to cure?

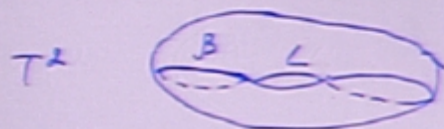
- find a way to assign a discrete spectrum $\{\omega_i\}$ of real frequencies with BH
- spectrum should be non-trivial enough to reproduce complex time evolution:
 - exp. decay for $t \rightarrow t_0$ (t_0)
 - restoration of unitarity at $t \rightarrow t_0 + e^{\frac{1}{H}}$
- more complicated than simple equidistant spectrum

BH is not a clock!

AdS₃/CFT₂

5.

AdS₃ spaces asymptote to $T^2 = (t_E, \phi)$, periods $\beta = \frac{1}{T}$ and L



They are quotients of hyperbolic space H^3

$$ds^2 = \frac{e^2}{y^2} (dx d\bar{x} + dy^2), \quad y > 0 \quad (l=c)$$

Two obvious spaces:

BTZ black hole: $w = \phi + i t_E$; $ds^2 = L^2 \left[d\left(\frac{\phi}{L}\right) + \tau d\left(\frac{t_E}{\beta}\right) \right]^2$
modular parameter $\tau_{BTZ} = \frac{i\beta}{L}$

Thermal AdS: $w = t_E + i\phi$; $\tau_{AdS} = \frac{iL}{\beta} = -\frac{1}{\tau_{BTZ}}$

Two spaces are T-dual to each other: $t_E \leftrightarrow \phi$, $\beta \leftrightarrow L$

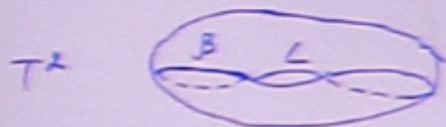
AdS/CFT: Maldacena (97)

gravity with negative CC $\leftrightarrow$ large N CFT on T^2
 $\Lambda \sim -\frac{1}{e^2}$ central charge $C = \frac{3}{2} \frac{e}{G}$
 $k = \frac{e}{G}$ plays role of N

turn on a field $\varphi_{(n,s)}$ $\leftrightarrow$ conformal operator
in bulk $\mathcal{O}_{(h,\bar{h})}$ on T^2

AdS₃/CFT₂

AdS₃ spaces asymptotic to $T^2 = (t_E, \phi)$, periods $\beta = \frac{1}{T}$ and L



They are quotients of hyperbolic space $\mathbb{H}^3$
 $ds^2 = \frac{L^2}{y^2} (dx^2 + dy^2 + dz^2)$, $y > 0$ ($L=1$)

Two obvious spaces:

BTZ black hole: w/o ϕ + t_E ; $ds_{BTZ}^2 = L^2 \left[d\left(\frac{\phi}{L}\right) + T d\left(\frac{t_E}{L}\right) \right]^2$
modular parameter $\tau_{BTZ} = \frac{i\beta}{L}$

Thermal AdS: w/o t_E + $i\phi$; $\tau_{AdS} = \frac{iL}{\beta} = -\frac{1}{\tau_{BTZ}}$

Two spaces are T-dual to each other: $t_E \leftrightarrow \phi$, $\beta \leftrightarrow L$

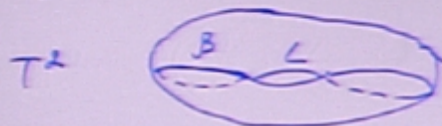
AdS/CFT: Maldacena (97)

gravity with negative CC $\leftrightarrow$ large N CFT on T^2
 $\Lambda \sim -\frac{1}{\ell^2}$ central charge $C \sim \frac{3}{2} \frac{E}{G}$
 $k = \frac{C}{6}$ plays role of N

turn on a field $\varphi_{(n,s)}$ $\leftrightarrow$ conformal operator
in bulk $\mathcal{O}_{(n,s)}$ on T^2

AdS₃/CFT₂

AdS₃ spaces asymptote to $T^2 = (t_E, \phi)$, periods $\beta = \frac{1}{T}$ and L



They are quotients of hyperbolic space H^3
 $ds^2 = \frac{e^2}{y^2} (dx^2 + dy^2)$, $y > 0$ ($\ell=1$)

Two obvious spaces:

BTZ black hole: $w = \phi + i t_E$; $ds^2 = L^2 \left[d\left(\frac{\phi}{L}\right)^2 + T d\left(\frac{t_E}{\beta}\right)^2 \right]$
 modular parameter $\tau_{BTZ} = \frac{i\beta}{L}$

Thermal AdS: $w = t_E + i\phi$; $\tau_{AdS} = \frac{iL}{\beta} = -\frac{1}{\tau_{BTZ}}$

Two spaces are T-dual to each other: $t_E \leftrightarrow \phi$, $\beta \leftrightarrow L$

AdS/CFT: Maldacena (97)

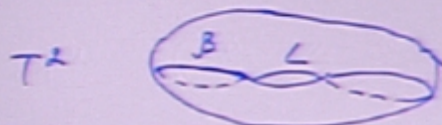
gravity with negative CC $\leftrightarrow$ large N CFT on T^2
 $\Lambda \sim -\frac{1}{\ell^2}$ central charge $C = \frac{2}{\pi} \frac{e}{G}$
 $k = \frac{C}{6}$ plays role of N

take a field $\mathcal{O}_{(h,s)}$ in bulk $\leftrightarrow$ conformal operator $\mathcal{O}_{(h,\bar{s})}$ on T^2

AdS₃/CFT₂

5.

AdS₃ spaces asymptotic to $T^2 = (t_E, \phi)$, periods $\beta = \frac{1}{T}$ and L



They are quotients of hyperbolic space H^3

$$ds^2 = \frac{e^2}{y^2} (dx d\bar{x} + dy^2), \quad y > 0 \quad (d=1)$$

Two obvious spaces:

BTZ black hole: $w = \phi + i t_E$; $ds^2 = L^2 \left[d\left(\frac{\phi}{L}\right)^2 + L d\left(\frac{t_E}{\beta}\right)^2 \right]$
modular parameter $\tau_{BTZ} = \frac{i\beta}{L}$

Thermal AdS: $w = t_E + i\phi$; $\tau_{AdS} = \frac{iL}{\beta} = -\frac{1}{\tau_{BTZ}}$

Two spaces are T-dual to each other: $t_E \leftrightarrow \phi$, $\beta \leftrightarrow L$

AdS/CFT: Maldacena (97)

gravity with negative CC $\leftrightarrow$ large N CFT on T^2
 $\Delta \sim -\frac{1}{e^2}$ central charge $C = \frac{2}{\pi} \frac{e}{G}$
 $k = \frac{e}{6}$ plays role of N

turn on a field $\varphi_{(h,s)}$ in bulk $\leftrightarrow$ conformal operator $\mathcal{O}_{(h,s)}$ on T^2

Boundary (real-time) CORR. functions

Birmingham, Sach, SS
(2003)

Two contributions:

$$\langle \mathcal{O}(t, t) \mathcal{O}(0, 0) \rangle = e^{-\tilde{S}_{BTZ}} \langle \mathcal{O} \mathcal{O} \rangle_{BTZ} + e^{-\tilde{S}_{AdS}} \langle \mathcal{O} \mathcal{O} \rangle_{AdS}$$

$$\tilde{S}_{BTZ} = -k \cdot \pi \frac{L \cdot T}{2}; \quad \tilde{S}_{AdS} = -\frac{k \pi}{2L \cdot T}$$

(euclidean actions)

Chakraborty (99)
Koski-Mukku (98)

$$\langle \mathcal{O} \mathcal{O} \rangle_{BTZ} = \sum_n \frac{1}{\left[\sin \frac{\pi}{p} (\phi + i n L) \right]^{\Delta}} \left(\frac{1}{\phi + i n L} \right) \sim e^{-\frac{2\pi \Delta t}{p}} \quad (\text{exp. decays})$$

$$\langle \mathcal{O} \mathcal{O} \rangle_{AdS} = \sum_n \frac{1}{\left[\sin \frac{\pi}{2} (\phi + i n \pi) \right]^{\Delta}} \left(\frac{1}{\phi + i n \pi} \right)$$

oscillates

For large T - BTZ-dominates

For small T - AdS-dominates

the transition occurs at $1/T = L$ (Hawking-Page transition)

Sharp transition for $k \rightarrow \infty$

In momentum representation:

poles of $\langle \rangle_{BTZ}$ are the complex QNR!

CORR. Function in boundary CFT decays!

$\Rightarrow$ AdS/CFT does not seem to solve
unitarity problem (?)

Boundary (real-time) CORR. functions

B. P. van Nieuwenhuizen, Sachdev, SS (2003)

Two contributions:

$$\langle \mathcal{O}(\phi, t) \mathcal{O}(0, 0) \rangle = e^{-S_{\text{BTZ}}} \langle \mathcal{O} \mathcal{O} \rangle_{\text{BTZ}} + e^{-S_{\text{AdS}}} \langle \mathcal{O} \mathcal{O} \rangle_{\text{AdS}}$$

$$S_{\text{BTZ}} = -k \cdot \pi \frac{L \cdot T}{2}; \quad S_{\text{AdS}} = -\frac{k \pi}{2L \cdot T}$$

(euclidean actions)

$$\langle \mathcal{O} \mathcal{O} \rangle_{\text{BTZ}} = \sum_n \frac{1}{\left[\frac{\sinh \frac{\pi}{p} (\phi + t + iL)}{p} \right]^\Delta} \sim e^{-\frac{\pi \phi}{p} t}$$

Chakraborty (98)
Mukhi-Venkatesh (98)
(exp. decays)

$$\langle \mathcal{O} \mathcal{O} \rangle_{\text{AdS}} = \sum_n \frac{1}{\left[\sin \frac{\pi}{L} (\phi + t + i\eta \beta) \right]^\Delta} \sim \frac{1}{(\phi + t)}$$

oscillates

For large T - BTZ-dominates

For small T - AdS-dominates

the transition occurs at $1/T = L$ (Hawking-Page transition)

Sharp transition for $k \rightarrow \infty$

In momentum representation:

poles of $\langle \rangle_{\text{BTZ}}$ are the complex QNM!

CORR. Function in boundary CFT decays!

$\Rightarrow$ AdS/CFT does not seem to solve
unitarity problem (?)

Boundary (real-time) CORR. function

G.
Barnington, Sach, SS
(2003)

Two contributions:

$$\langle \mathcal{O}(t, t) \mathcal{O}(0, 0) \rangle = e^{-S_{\text{BTZ}}} \langle \mathcal{O} \mathcal{O} \rangle_{\text{BTZ}} + e^{-S_{\text{AdS}}} \langle \mathcal{O} \mathcal{O} \rangle_{\text{AdS}}$$

$$S_{\text{BTZ}} = -k \cdot \pi \frac{L \cdot T}{2}; \quad S_{\text{AdS}} = -\frac{k \pi}{2L \cdot T}$$

(euclidean actions)

Chakraborty (98)
Koschmieder (98)

$$\langle \mathcal{O} \mathcal{O} \rangle_{\text{BTZ}} = \sum_n \frac{1}{\left[\sinh \frac{\pi}{p} (t + i\epsilon) \right]^\Delta} \frac{1}{(t + i\epsilon)} \sim e^{-\frac{2\pi \Delta t}{p}} \quad (\text{exp. decays})$$

$$\langle \mathcal{O} \mathcal{O} \rangle_{\text{AdS}} = \sum_n \frac{1}{\left[\sinh \frac{\pi}{2} (t + i\epsilon) \right]^\Delta} \frac{1}{(t + i\epsilon)}$$

oscillates

For large T - BTZ-dominates

For small T - AdS-dominates

the transition occurs at $1/T = L$ (Hawking-Page transition)

Sharp transition for $k \rightarrow \infty$

In momentum representation:

poles of $\langle \rangle_{\text{BTZ}}$ are the complex QNM!

CORR. Function in boundary CFT decays!

$\Rightarrow$ AdS/CFT does not seem to solve
unitarity problem (?)

Boundary (real-time) corr. functions

Witten, Susskind, SS (2001)

Free calculations:

$$\langle \psi(t, L) \psi(0, 0) \rangle = e^{-S_{BTZ}} \langle \psi \psi \rangle_{BTZ} + e^{-S_{AdS}} \langle \psi \psi \rangle_{AdS}$$

$$S_{BTZ} = -k \cdot \pi \frac{L \cdot T}{2}; \quad S_{AdS} = -\frac{k \pi}{2L \cdot T}$$

(euclidean actions)

$$\langle \psi \psi \rangle_{BTZ} = \sum_n \frac{1}{\left[\frac{\pi k \pi}{P} (L + i\epsilon) \right]^{\Delta} \left(\frac{1}{L + i\epsilon} \right)} \sim e^{-\frac{\pi \Delta}{P} \epsilon t}$$

(exp. decays)

$$\langle \psi \psi \rangle_{AdS} = \sum_n \frac{1}{\left[\sin \frac{\pi}{2} (L + i\epsilon) \right]^{\Delta} \left(\frac{1}{L + i\epsilon} \right)}$$

oscillator

For large T - BTZ-dominates

For small T - AdS-dominates

the transition occurs at $1/T = L$ (Hawking-Page transition)

Sharp transition for $k \rightarrow \infty$

In momentum representation:

peaks at ω_{BTZ} and ω_{AdS} are the angular QFT!

$SL(2, \mathbb{Z})$ family

$SL(2, \mathbb{Z})$ modular transformations

$$\tau \rightarrow \tau' = \frac{a\tau + b}{c\tau + d} \quad ; \quad w (= \phi + it_E) \rightarrow w' = \frac{w}{c\tau + d}, \quad ad - bc = 1$$

a, b, c, d are integers

$SL(2, \mathbb{Z}) / \mathbb{Z}$ - modular parabolic group
 $(a, b) \rightarrow (a+c, b+d)$
 $\uparrow$
 completely determined by pairs (c, d)

$c = 0$ - thermal IdS

$c \neq 0$ - black holes

Action:
$$S(\tau') = \frac{\pi k i}{2} \left[\frac{a\tau + b}{c\tau + d} - \frac{a\bar{\tau} + b}{c\bar{\tau} + d} \right] = -\frac{\pi k \cdot \beta L}{c^2 \beta^2 + d^2 L^2}$$

These are Maldacena's
 topologically diverse instantons

$SL(2, \mathbb{Z})$ family

$SL(2, \mathbb{Z})$ modular transformations

$$\tau \rightarrow \tau' = \frac{a\tau + b}{c\tau + d} \quad ; \quad w (= \phi + it_E) \rightarrow w' = \frac{w}{c\tau + d}, \quad ad - bc = 1$$

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$SL(2, \mathbb{Z})$ family

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$$\tau \rightarrow \tau' = \frac{a\tau + b}{c\tau + d} \quad ; \quad w (= \phi + it_E) \rightarrow w' = \frac{w}{c\tau + d}, \quad ad - bc = 1$$

a, b, c, d are integers

$SL(2, \mathbb{Z}) / \mathbb{Z}$ - modulo parabolic group

$(a, b) \rightarrow (a+c, b+d)$
 ↑
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$c = 0$ - thermal IdS

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Action:
$$S(\tau') = \frac{\pi k i}{2} \left[\frac{a\tau + b}{c\tau + d} - \frac{a\bar{\tau} + b}{c\bar{\tau} + d} \right] = -\frac{\pi k \cdot \beta L}{c^2 \beta^2 + d^2 \cdot L^2}$$

These are Maldacena's
 topologically diverse instantons

$SL(2, \mathbb{Z})$ family

$SL(2, \mathbb{Z})$ modular transformations

$$t \rightarrow t' = \frac{at+b}{ct+d}, \quad w (=t+it\epsilon) \rightarrow w' = \frac{w}{ct+d}, \quad ad-bc=1$$

a, b, c, d are integers

$SL(2, \mathbb{Z})/\mathbb{Z}$ - modular parabolic group
 $(a, b) \rightarrow (a+c, b+d)$
↑
completely determined by pairs (c, d)

$c=0$ - Hecke id

$c \neq 0$ - black holes

Hecke:
$$S(t') = \frac{\pi k_i}{3} \left[\frac{at+b}{ct+d} - \frac{a\bar{t}+b}{c\bar{t}+d} \right] = -\frac{\pi k_i p_L}{c^2 p^2 + d^2 k_i^2}$$

these are Maldacena's
topologically distinct instantons

$SL(2, \mathbb{Z})$ family

$SL(2, \mathbb{Z})$ modular transformations

$$\tau \rightarrow \tau' = \frac{a\tau + b}{c\tau + d} \quad ; \quad w (= \phi + it_E) \rightarrow w' = \frac{w}{c\tau + d}, \quad ad - bc = 1$$

a, b, c, d are integers

$SL(2, \mathbb{Z}) / \mathbb{Z}$ - modulo parabolic group

$\uparrow$
completely determined by pairs (c, d)

$c = 0$ - thermal AdS

$c \neq 0$ - black holes

Action:
$$S(\tau') = \frac{\pi k i}{2} \left[\frac{a\tau + b}{c\tau + d} - \frac{a\bar{\tau} + b}{c\bar{\tau} + d} \right] = -\frac{\pi k \cdot \beta L}{c^2 \rho^2 + d^2 L^2}$$

These are Maldacena's
topologically diverse instantons

Can sum over $S^1 L(2,2)$ family
produce quasi-periodic result?

8.

Eisenstein c.o.p.r. fun.

$$\langle \mathcal{O}(w, \bar{w}) \mathcal{O}(q, 0) \rangle_{S^1 L(2,2)} = \sum_{(c,d)} \sum_{h \in \mathbb{Z}} \frac{e^{-S(c')}}{|c\tau + d|^{2\Delta}} \frac{(\pi/L)^{2\Delta}}{|\sinh \pi(\frac{w'}{L} + h\tau)|^{2\Delta}}$$

appropriate weight

Kleban, Parnitz, Radchen (04)

Real-time: $w \rightarrow \phi - t$, $\bar{w} \rightarrow \phi + t$

Large t limit:

$$\langle \mathcal{O}(t, \phi) \mathcal{O}(q, 0) \rangle_{S^1 L(2,2)} \approx \sum_{(c,d)} e^{\frac{\pi k p L}{c^2 p^2 + d^2 L^2}} \left[\frac{\pi^2}{c^2 p^2 + d^2 L^2} \right]^{\Delta} e^{-\frac{2\Delta \pi |t|}{c^2 p^2 + d^2 L^2}}$$

each individual term
exp. decays

How about the sum?

An example: $I(t) = \sum_{k=1}^{\infty} \frac{1}{k^2} e^{a/k^2} e^{-t/k}$

characteristic decay time $\sim k \rightarrow$ the sum decays
slower than $e^{-\lambda t}$

Indeed, $I(t) \sim \frac{a^2}{t} + \mathcal{O}\left(\frac{a^2 e^{a^2} e^{-t}}{t}\right)$

In our case: ($\Delta=2$)

$$\langle \mathcal{O}(t) \mathcal{O}(q) \rangle_{S^1 L(2,2)} \sim \frac{1}{t} \frac{1}{pL \sqrt{p^2 + L^2}} e^{\frac{\pi k p L}{p^2 + L^2}}$$

power law!

- better than exp. decay but still

not quasi-periodic!

Can sum over $SL(2, \mathbb{Z})$ family
produce quasi-periodic result?

8.

Euclidean cor. fun.

$$\langle \mathcal{O}(w, \bar{w}) \mathcal{O}(0, 0) \rangle_{SL(2, \mathbb{Z})} = \sum_{(c, d)} \sum_{n \in \mathbb{Z}} \frac{e^{-S(z')}}{|c\tau + d|^{2\Delta}} \frac{(\pi/L)^{2\Delta}}{|\sinh \pi(\frac{w'}{L} + n\tau')|^{2\Delta}}$$

Kleban, Peneder, Rabinov (04)

appropriate weight

Real-time: $w \rightarrow \phi - t, \bar{w} \rightarrow \phi + t$

Large t limit:

$$\langle \mathcal{O}(t, \phi) \mathcal{O}(0, 0) \rangle_{SL(2, \mathbb{Z})} = \sum_{(c, d)} e^{\frac{\pi k p L}{c^2 p^2 + d^2 L^2}} \left[\frac{\pi^2}{c^2 p^2 + d^2 L^2} \right]^{\Delta} e^{-\frac{2\Delta p \bar{n} |t|}{c^2 p^2 + d^2 L^2}}$$

each individual term exp. decays

How about the sum?

Example: $I(t) = \sum_{n=1}^{\infty} \frac{1}{n^2} e^{a/n^2} e^{-t/n}$

characteristic decay time $\sim n \rightarrow$ the sum decays slower than e^{-t}

Indeed, $I(t) \sim \frac{a^2}{t} + \mathcal{O}\left(\frac{a^2 e^{-a^2/t}}{t}\right)$

In our case: ($\Delta=2$)

$$\langle \mathcal{O}(t) \mathcal{O}(0) \rangle_{SL(2, \mathbb{Z})} \sim \frac{1}{t} \frac{1}{pL \sqrt{p^2 + L^2}} e^{\frac{\pi k p L}{p^2 + L^2}}$$

power law!

- better than exp. decay but still

not quasi-periodic!

Can sum over $S^1 L(2,2)$ family
produce quasi-periodic result?

8.

Euclidean cor. fun.

$$\langle \mathcal{O}(w, \bar{w}) \mathcal{O}(0,0) \rangle_{S^1 L(2,2)} = \sum_{(c,d)} \sum_{h \in \mathbb{Z}} \frac{e^{-S(c)}}{|c\tau+d|^{2\Delta}} \frac{(\pi/L)^{2\Delta}}{|\sinh \pi(\frac{w'}{L} + h\tau)|^{2\Delta}}$$

Klein, Poincaré, Rademacher (04)

appropriate weight

Real-time: $w \rightarrow \phi - t$, $\bar{w} \rightarrow \phi + t$

Large t limit:

$$\langle \mathcal{O}(t, \phi) \mathcal{O}(0,0) \rangle_{S^1 L(2,2)} = \sum_{(c,d)} e^{\frac{\pi k p L}{c^2 \tau^2 + d^2 L^2}} \left[\frac{\pi^2}{c^2 \tau^2 + d^2 L^2} \right]^{\Delta} e^{-\frac{2\Delta \pi h t}{c^2 \tau^2 + d^2 L^2}}$$

each individual term
exp. decays

How about the sum?

Example: $I(t) = \sum_{n=1}^{\infty} \frac{1}{n^2} e^{a/n^2} e^{-t/n}$

characteristic decay time $\sim n \rightarrow$ the sum decays
slower than e^{-t}

Indeed, $I(t) \sim \frac{a^2}{t} + \mathcal{O}\left(\frac{a^2 e^{a^2} e^{-t}}{t}\right)$

In our case: ($\Delta=2$)

$$\langle \mathcal{O}(t) \mathcal{O}(0) \rangle_{S^1 L(2,2)} \sim \frac{1}{t} \frac{1}{pL \sqrt{\tau^2 + L^2}} e^{\pi k p L / (\tau^2 + L^2)}$$

power law!

- better than exp. decay but still

not quasi-periodic!

Produce quasi-periodic result?

Eulerian corr. fun.

$$\langle \mathcal{O}(w, \bar{w}) \mathcal{O}(0, 0) \rangle_{\text{SC}(2, 2)} = \sum_{(c, d)} \sum_{h \in \mathbb{Z}} \frac{e^{-S(c, d)}}{|c\tau + d|^{2\Delta}} \frac{(\bar{\pi}/L)^{2\Delta}}{|\text{Sh} \pi(\frac{w'}{L} + h\tau)|^{2\Delta}}$$

appropriate weight

Klein, Pappas, Rindler (04)

Real-time: $w \rightarrow \phi - t, \bar{w} \rightarrow \phi + t$

Large t limit:

$$\langle \mathcal{O}(t, \phi) \mathcal{O}(0, 0) \rangle_{\text{SC}(2, 2)} \approx \sum_{(c, d)} e^{\frac{\pi k p L}{c^2 p^2 + d^2 L^2}} \left[\frac{\pi^2}{c^2 p^2 + d^2 L^2} \right]^{\Delta} e^{-\frac{2\Delta p \bar{p} t}{c^2 p^2 + d^2 L^2}}$$

each individual term
exp. decays

How about the sum?

An example: $I(t) = \sum_{n=1}^{\infty} \frac{1}{n^2} e^{a^2/n^2} e^{-t/n}$

characteristic decay time $\sim n \rightarrow$ the sum decays
slower than $e^{-\lambda t}$

Indeed, $I(t) \sim \frac{a^2}{t} + \mathcal{O}\left(\frac{a^2 e^{-t/a^2}}{t}\right)$

In our case: ($\Delta=2$)

$$\langle \mathcal{O}(t) \mathcal{O}(0) \rangle_{\text{SC}(2, 2)} \sim \frac{1}{t} \frac{1}{pL \sqrt{p^2 + L^2}} e^{\frac{\pi k p L}{p^2 + L^2}}$$

power law!

- better than exp. decay but still

not quasi-periodic!

Can sum over $\sum_{\mathbf{L}} L(2, 2)$ find

Produce quasi-periodic result?

Qualitative exp. for

$$\langle \psi(\mathbf{r}, t) | \psi(\mathbf{r}, t) \rangle_{\text{exp}} = \sum_{(\mathbf{L}, t)} \sum_{n \in \mathbb{Z}} \frac{e^{-\frac{1}{2} \pi n^2}}{|\mathbf{L} + t|^{1/2}} \frac{(\pi/L)^{1/2}}{|\sin \pi(\mathbf{L}' + t \pi)|^{1/2}}$$

For Periodic, Reduction (exp)

appropriate weight

and time: $\mathbf{r} \rightarrow \mathbf{r} + t, \bar{\mathbf{r}} \rightarrow \mathbf{r} + t$

age & limit:

$$\langle \psi(\mathbf{r}, t) | \psi(\mathbf{r}, t) \rangle_{\text{exp}} = \sum_{(\mathbf{L}, t)} \frac{\pi k p L}{\pi^2 p^2 + t^2} \left[\frac{\pi^2}{L^2 p^2 + t^2} \right]^A e^{-\frac{1}{2} \pi p^2 t^2}$$

each individual term
exp. decay

how about the sum?

$$\sum_{n \in \mathbb{Z}} \frac{1}{n^2} e^{-\frac{1}{2} \pi n^2} e^{-t/n}$$

characteristic decay time $\sim 1/n \rightarrow$ the sum decays
Indeed, $\sum_{n \in \mathbb{Z}} \frac{1}{n^2} e^{-\frac{1}{2} \pi n^2} e^{-t/n} \sim \frac{1}{t} + O\left(\frac{e^{-t}}{t}\right)$ forget the exp

no exp. (exp)

$$\langle \psi(\mathbf{r}, t) | \psi(\mathbf{r}, t) \rangle_{\text{exp}} \sim \frac{1}{t} + \frac{1}{2\pi i} \frac{\pi k p L}{\pi^2 p^2 + t^2} e^{-\frac{1}{2} \pi p^2 t^2}$$

Can sum over $SL(2, \mathbb{Z})$ family
produce quasi-periodic result?

Eisenstein c.o.p.r. fun.

$$\langle \mathcal{O}(w, \bar{w}) \mathcal{O}(0, 0) \rangle_{SL(2, \mathbb{Z})} = \sum_{(c, d)} \sum_{k \in \mathbb{Z}} \frac{e^{-S(c')}}{|c\tau + d|^{2\Delta}} \frac{(\pi/L)^{2\Delta}}{|\sinh \pi(\frac{w'}{L} + k\tau)|^{2\Delta}}$$

appropriate weight

(Klein, Poincaré, Rademacher (co))

Real-time: $w \rightarrow \phi + i\tau$, $\bar{w} \rightarrow \phi + i\tau$

Large t limit:

$$\langle \mathcal{O}(t, \phi) \mathcal{O}(0, 0) \rangle_{SL(2, \mathbb{Z})} \sim \sum_{(c, d)} e^{\frac{\pi k p L}{c^2 p^2 + d^2 L^2}} \left[\frac{\pi^2}{c^2 p^2 + d^2 L^2} \right]^{\Delta} e^{-\frac{2\Delta p \pi |t|}{c^2 p^2 + d^2 L^2}}$$

each individual term
exp. decays

How about the sum?

An example: $I(t) = \sum_{k=1}^{\infty} \frac{1}{k^2} e^{a^2/k^2} e^{-t/k}$

characteristic decay time $\sim k \rightarrow$ the sum decays
slower than $e^{-\lambda t}$

Indeed, $I(t) \sim \frac{a^2}{t} + \mathcal{O}\left(\frac{a^2 e^{a^2/t}}{t}\right)$

In our case: $(\Delta=2)$

$$\langle \mathcal{O}(t) \mathcal{O}(0) \rangle_{SL(2, \mathbb{Z})} \sim \frac{1}{t} \frac{1}{pL \sqrt{p^2 + L^2}} e^{\frac{\pi k p L}{p^2 + L^2}}$$

power law!

- better than exp. decay but still

not quasi-periodic!

Can sum over $SL(2, \mathbb{Z})$ family
produce quasi-periodic result?

8.

Euclidean corr. fun.

$$\langle \mathcal{O}(w, \bar{w}) \mathcal{O}(0, 0) \rangle_{SL(2, \mathbb{Z})} = \sum_{(c, d)} \sum_{n \in \mathbb{Z}} \frac{e^{-S(\tau')}}{|c\tau + d|^{2\Delta}} \frac{(\pi/L)^{2\Delta}}{|\sinh \pi(\frac{w'}{L} + n\tau')|^{2\Delta}}$$

Klein, Poincaré, Rademacher (04)

appropriate weight

Real-time: $w \rightarrow p-t, \bar{w} \rightarrow p+t$

Large t limit:

$$\langle \mathcal{O}(t, p) \mathcal{O}(0, 0) \rangle_{SL(2, \mathbb{Z})} = \sum_{(c, d)} e^{\frac{\pi k p L}{c^2 p^2 + d^2 L^2}} \left[\frac{\pi^2}{c^2 p^2 + d^2 L^2} \right]^{\Delta} e^{-\frac{2\Delta \pi |d| t}{c^2 p^2 + d^2 L^2}}$$

each individual term
exp. decays

How about the sum?

An example: $I(t) = \sum_{n=1}^{\infty} \frac{1}{n^2} e^{a^2/n^2} e^{-t/n}$

characteristic decay time $\sim n \rightarrow$ the sum decays
slower than $e^{-\lambda t}$

Indeed, $I(t) \sim \frac{a^4}{t} + \mathcal{O}\left(\frac{a^4 e^{-t}}{t}\right)$

In our case: ($\Delta=2$)

$$\pi k p L / p^2 + L^2$$

$$\langle \mathcal{O}(t) \mathcal{O}(0) \rangle_{SL(2, \mathbb{Z})} \sim \frac{1}{t} \frac{1}{p L \sqrt{p^2 + L^2}} e$$

power law!

- better than exp. decay but still

not quasi-periodic!

Can sum over $SL(2, \mathbb{Z})$ finally
produce quasi-periodic result?

8.

Euclidean cor. fun.

$$\langle \mathcal{O}(w, \bar{w}) \mathcal{O}(0, 0) \rangle_{SL(2, \mathbb{Z})} = \sum_{(c, d)} \sum_{n \in \mathbb{Z}} \frac{e^{-S(\tau')}}{|c\tau + d|^{2\Delta}} \frac{(\pi/L)^{2\Delta}}{|\text{Sinh} \pi(\frac{w'}{L} + n\tau')|^{2\Delta}}$$

Klein, Poincaré, Rademacher (ca)

appropriate weight

Real-time: $w \rightarrow p-t, \bar{w} \rightarrow \bar{p}+t$

Large t limit:

$$\langle \mathcal{O}(t, \phi) \mathcal{O}(0, 0) \rangle_{SL(2, \mathbb{Z})} = \sum_{(c, d)} e^{\frac{\pi k p L}{c^2 p^2 + d^2 L^2}} \left[\frac{\pi^2}{c^2 p^2 + d^2 L^2} \right]^{\Delta} \cdot e^{-\frac{2\Delta p \bar{p} |t|}{c^2 p^2 + d^2 L^2}}$$

each individual term
exp. decays

How about the sum?

Example: $I(t) = \sum_{n=1}^{\infty} \frac{1}{n^2} e^{a^2/n^2} e^{-t/n}$

characteristic decay time $\sim n \rightarrow$ the sum decays
slower than e^{-t}

Indeed, $I(t) \sim \frac{a^2}{t} + \mathcal{O}\left(\frac{e^{a^2} \cdot e^{-t}}{t}\right)$

In our case: ($\Delta=2$)

$$\pi k p / (p^2 + L^2)$$

$$\langle \mathcal{O}(t) \mathcal{O}(0) \rangle_{SL(2, \mathbb{Z})} \sim \frac{1}{t} \frac{1}{p L \sqrt{p^2 + L^2}} e$$

power law!

- better than exp. decay but still

not quasi-periodic!

Can sum over $SL(2, \mathbb{Z})$ finally
produce quasi-periodic result?

8.

Euclidean e.o.m. fun.

$$\langle \mathcal{O}(w, \bar{w}) \mathcal{O}(0, 0) \rangle_{SL(2, \mathbb{Z})} = \sum_{(c, d)} \sum_{n \in \mathbb{Z}} \frac{e^{-S(\tau')}}{|c\tau + d|^{2\Delta}} \frac{(\pi/L)^{2\Delta}}{|\sinh \pi(\frac{w'}{L} + n\tau')|^{2\Delta}}$$

Klein, Poincaré, Rademacher (et)

appropriate weight

Real-time: $w \rightarrow \phi - t$, $\bar{w} \rightarrow \phi + t$

Large t limit:

$$\langle \mathcal{O}(t, \phi) \mathcal{O}(0, 0) \rangle_{SL(2, \mathbb{Z})} \approx \sum_{(c, d)} e^{\frac{\pi k p L}{c^2 p^2 + d^2 L^2}} \left[\frac{\pi^2}{c^2 p^2 + d^2 L^2} \right]^{\Delta} e^{-\frac{2\Delta p \bar{p} |t|}{c^2 p^2 + d^2 L^2}}$$

each individual term
exp. decays

How about the sum?

Example: $I(t) = \sum_{n=1}^{\infty} \frac{1}{n^2} e^{a^2/n^2} e^{-t/n}$

characteristic decay time $\sim n \rightarrow$ the sum decays
slower than e^{-t}

Indeed, $I(t) \sim \frac{a^2}{t} + \mathcal{O}\left(\frac{a^2 e^{-t}}{t}\right)$

In our case: ($\Delta=2$)

$$\frac{\pi k p L}{p^2 L^2 + c^2}$$

$$\langle \mathcal{O}(t) \mathcal{O}(0) \rangle_{SL(2, \mathbb{Z})} \sim \frac{1}{t} \frac{1}{p L \sqrt{p^2 + c^2}} e$$

power law!

- better than exp. decay but still

not quasi-periodic!

Can sum over $SL(2, \mathbb{Z})$ family
produce quasi-periodic result?

8.

Euclidean e.o.m. sum

$$\langle \mathcal{O}(w, \bar{w}) \mathcal{O}(0, 0) \rangle_{SL(2, \mathbb{Z})} = \sum_{(c, d)} \sum_{n \in \mathbb{Z}} \frac{e^{-S(\tau')}}{|c\tau + d|^{2\Delta}} \frac{(\pi/L)^{2\Delta}}{|\sinh \pi(\frac{w'}{L} + n\tau')|^{2\Delta}}$$

Kleban, Peneder, Rabinov (04)

appropriate weight

Real-time: $w \rightarrow \phi - t$, $\bar{w} \rightarrow \phi + t$

Large t limit:

$$\langle \mathcal{O}(t, \phi) \mathcal{O}(0, 0) \rangle_{SL(2, \mathbb{Z})} = \sum_{(c, d)} e^{\frac{\pi k p L}{c^2 p^2 + d^2 L^2}} \left[\frac{\pi^2}{c^2 p^2 + d^2 L^2} \right]^{\Delta} e^{-\frac{2\Delta p \bar{p} k t}{c^2 p^2 + d^2 L^2}}$$

each individual term
exp. decays

How about the sum?

Example: $I(t) = \sum_{n=1}^{\infty} \frac{1}{n^2} e^{\frac{a^2}{4n^2}} e^{-t/n}$

characteristic decay time $\sim n \rightarrow$ the sum decays
slower than e^{-t}

Indeed, $I(t) \sim \frac{a^2}{t} + \mathcal{O}\left(\frac{e^{a^2/4t}}{t}\right)$

In our case: ($\Delta=2$)

$$\pi k p / (p^2 + L^2)$$

$$\langle \mathcal{O}(t) \mathcal{O}(0) \rangle_{SL(2, \mathbb{Z})} \sim \frac{1}{t} \frac{1}{p L \sqrt{p^2 + L^2}} e$$

power law!

- better than exp. decay but still

not quasi-periodic!

Sum over $S_L(z, z)$ family
 produce quasi-periodic result?

corr. fun.

$$\langle \psi(q_0) \rangle_{S_L(z, z)} = \sum_{(c, d)} \sum_{N \in \mathbb{Z}} \frac{e^{-S(c')}}{|c\tau + d|^{2\Delta}} \frac{(\pi/L)^{\Delta}}{|\sin \pi(\frac{1}{L} + \frac{c\tau}{L})|^{2\Delta}}$$

appropriate weight

Refined (06)

$\tau \rightarrow \tau + 1, \bar{\tau} \rightarrow \bar{\tau} + 1$

Unit:

$$\langle \psi(q_0) \rangle_{S_L(z, z)} = \sum_{(c, d)} e^{\frac{\pi k p L}{c^2 p d^2 L^2}} \left[\frac{\tau^2}{c^2 p d^2 L^2} \right]^{\Delta} e^{-\frac{L \pi \sqrt{H+1}}{c^2 p d^2 L^2}}$$

each individual term
 decays

What the sum?

$$I(t) = \sum_{n=1}^{\infty} \frac{1}{n^2} e^{\frac{2\pi i n t}{L}} e^{-\frac{2\pi n^2 t}{L}}$$

dominant during time $n \sim \sqrt{t} \rightarrow$ the sum decays

$$I(t) \sim \frac{1}{\sqrt{t}} \sim O\left(\frac{1}{\sqrt{t}}\right)$$

[10-11]

Thyff

Why AdS/CFT produces decaying Result?

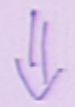
If Boundary CFT is unitary
CORR. functions should be quasi-periodic

Resolution: (Bianchi, Suss, S.S.)

additionally to β and L there should be
another scale in the game

$$L_{eff} \sim k-L$$

- multiply wound strings see this larger volume; $L_{eff} = \infty$ in the limit $k \rightarrow \infty$
(where AdS/CFT is supposed to work)



in order to solve unitarity problem
have to go to finite k

Q: what is the gravity counterpart
of the duality at finite k ?

Whatever it is it likely does not have horizon

Why MS/CFT boundary decaying to 0?

If boundary CFT is unitary
CFT functions should be quasi-periodic

Definition (Brylinski-Lieberman)

additionally to β and t there should be
another τ in the group
 $f_{off} \sim f \cdot t$

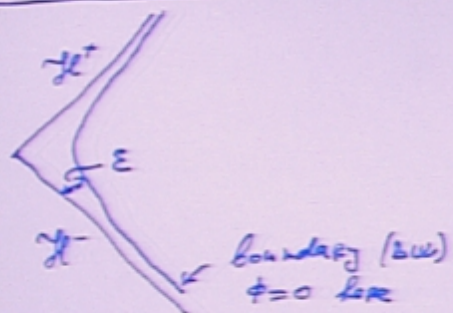
- multiply around strings for this group
where $f_{off} = -$ is for $f = -$
(where MS/CFT is supposed to unit)

in order to solve unitarity problem
have to go to finite k

what is the gravity counterpart
of the duality at finite k ?

because it is it probably doesn't have a proper

Brick wall model ('t Hooft, 85)



1. complex BNM are how gone!

2. discrete spectrum

$$\omega_n \sim \pi n / L_{opt}, \quad n \in \mathbb{Z}$$

$$L_{opt} = \int_{\epsilon} \frac{dx}{g(x)} \sim \ln \frac{1}{\epsilon}$$

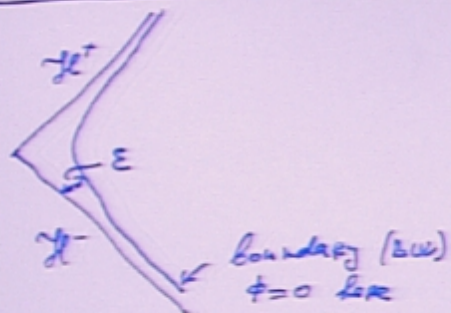
3. finite entropy, $S \sim \frac{A_{wall}}{\epsilon^2}$

4. non-smooth!

a rather crude way to model

(unknown) non-perturbative Planckian physics

Brick wall model ('t Hooft, 85)



1. complex QNM are how gone!

2. discrete spectrum

$$\omega_n \sim \pi n / L_{\text{opt}}, \quad n \in \mathbb{Z}$$

$$L_{\text{opt}} = \int_{\epsilon} \frac{dx}{g(x)} \sim \ln \frac{1}{\epsilon}$$

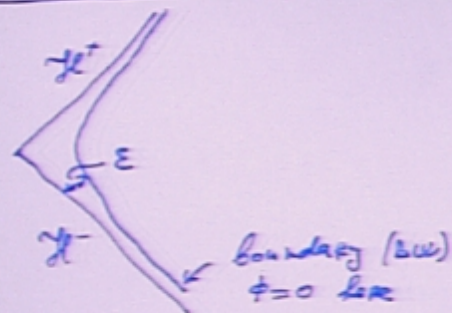
3. finite entropy, $S \sim \frac{A_{\text{hor}}}{\epsilon^2}$

4. non-smooth!

a rather crude way to model

(unknown) non-perturbative Planckian physics

Brick wall model ('t Hooft, 85)



1. complex B.M are how good!

2. discrete spectrum

$$\omega_n \sim \pi n / L_{opt}, \quad n \in \mathbb{Z}$$

$$L_{opt} = \int_E \frac{dx}{g(x)} \sim \ln \frac{1}{E}$$

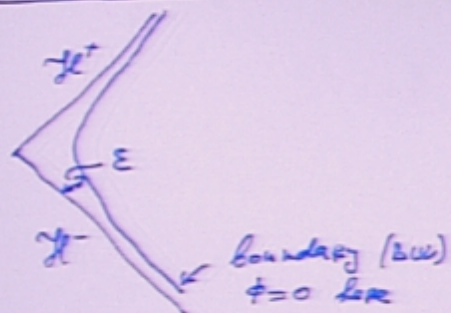
3. finite entropy, $S \sim \frac{A_{max}}{E^2}$

4. non-smooth!

a. rather crude way to model

(unknown) non-perturbative Planckian physics

10.
Brick wall model ('t Hooft, 85)



1. complex BVM are how gone!

2. discrete spectrum

$$\omega_n \sim \pi n / L_{\text{opt}}, \quad n \in \mathbb{Z}$$

$$L_{\text{opt}} = \int_{\epsilon} \frac{dx}{g(x)} \sim \ln \frac{1}{\epsilon}$$

3. finite entropy, $S \sim \frac{A_{\text{wall}}}{\epsilon^2}$

4. non-smooth!

5. rather crude way to model

(unknown) non-perturbative Planckian physics

The Proposal (S.S. (2004)) S.S. (1997)

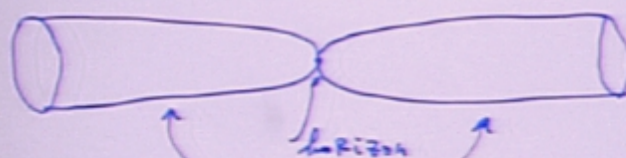
11.

Deformation of BTZ metric

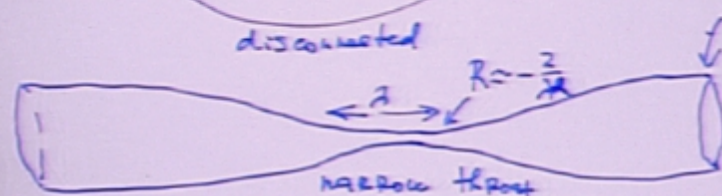
$$ds^2 = -(\sinh^2 y + \lambda^2(k)) dt^2 + dy^2 + \cosh y d\phi^2 \quad (\text{BTZ}_k)$$

deformation parameter $\lambda(k)$ is some function of
large N parameter k
(when $k \rightarrow \infty, \lambda \rightarrow 0$)

The Euclidean Geometry (t_E compactified)

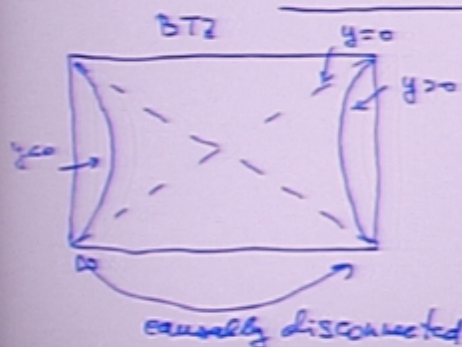


classical BTZ

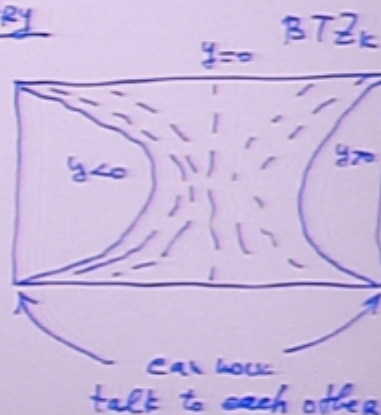


modified BTZ
(BTZ_k)

Lorentzian Geometry



causally disconnected



can now
talk to each other

The Proposal (S.S. (2004) S.S. (1997))

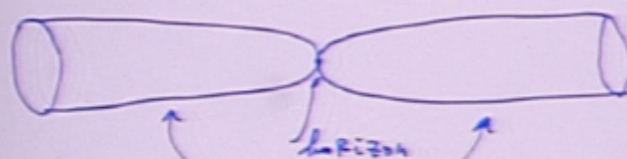
11.

Deformation of BTZ metric

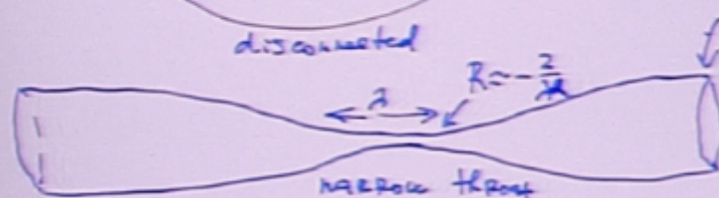
$$ds^2 = -(\sinh^2 y + \lambda^2(k)) dt^2 + dy^2 + \cosh^2 y d\phi^2 \quad (BTZ_k)$$

deformation parameter $\lambda(k)$ is some function of
large N parameter k
(under $k \rightarrow \infty, \lambda \rightarrow 0$)

The Euclidean Geometry (t_E compactified)

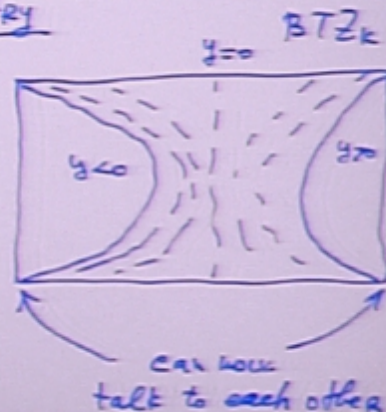
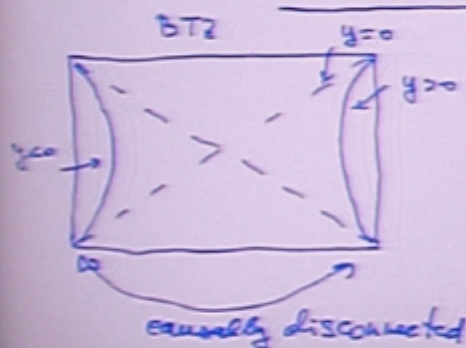


classical BTZ



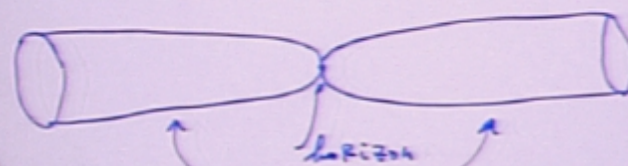
modified BTZ
(BTZ_k)

Lorentzian Geometry

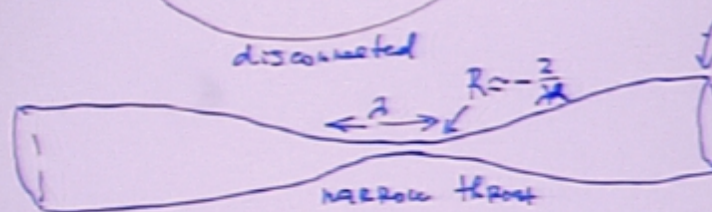


Deformation of BTZ metric
 $ds^2 = -(\sinh^2 y + \lambda^2(k)) dt^2 + dy^2 + \cosh y d\phi^2$ (BTZ_k)
 deformation parameter $\lambda(k)$ is some function of
 large N parameter k
 (when $k \rightarrow \infty$, $\lambda \rightarrow 0$)

The Euclidean Geometry (t_E compactified)

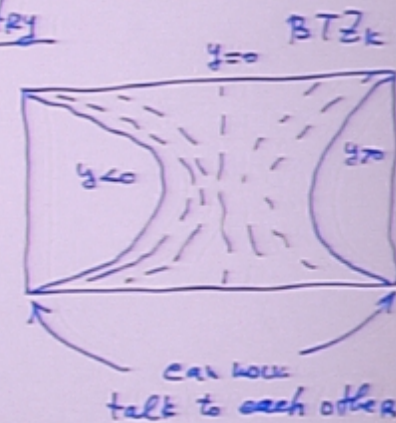
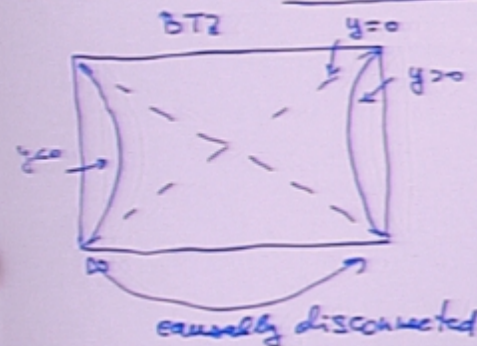


classical BTZ



modified BTZ
(BTZ_L)

Lorentzian Geometry



The Proposal (S.S. (2004)
S.S. (1997))

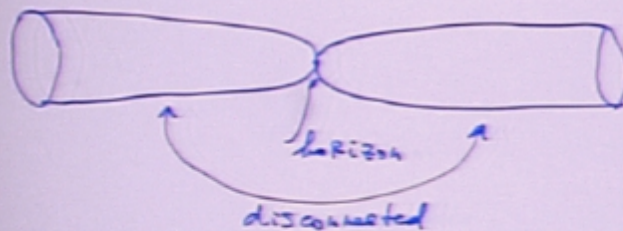
11.

Deformation of BTZ metric

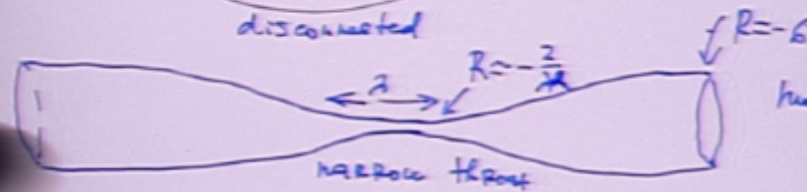
$$ds^2 = -(\sinh^2 y + \lambda^2(k)) dt^2 + dy^2 + \cosh^2 y d\phi^2 \quad (\text{BTZ}_k)$$

deformation parameter $\lambda(k)$ is some function of
large N parameter k
(when $k \rightarrow \infty$, $\lambda \rightarrow 0$)

The Euclidean Geometry (t_E compactified)

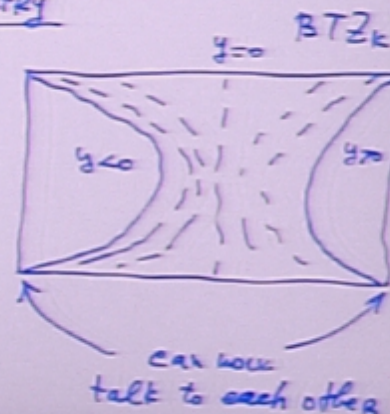
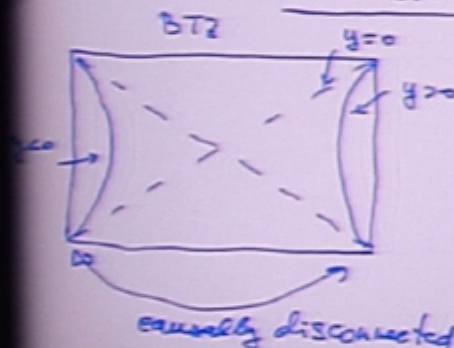


classical BTZ



modified BTZ
(BTZ_k)

Lorentzian Geometry



The Proposal (S.S. (2004) S.S. (1997))

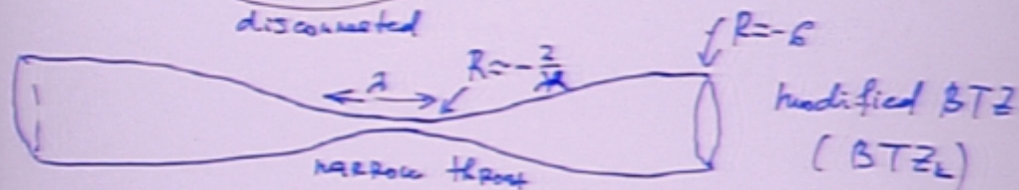
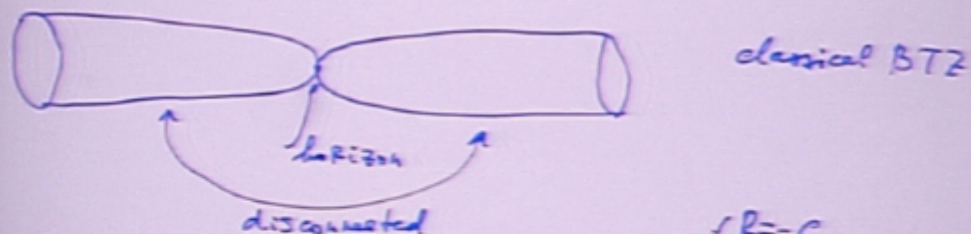
11.

Deformation of BTZ metric

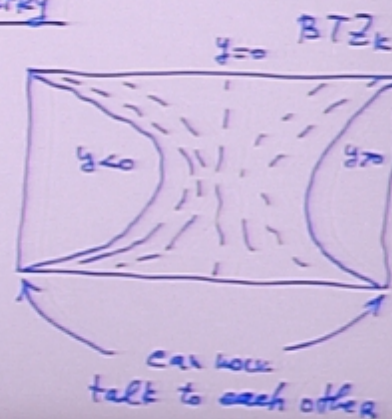
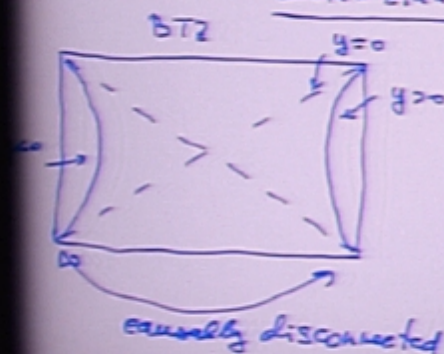
$$ds^2 = -(\sinh^2 y + \lambda^2(k)) dt^2 + dy^2 + \cosh^2 y d\phi^2 \quad (BTZ_k)$$

deformation parameter $\lambda(k)$ is some function of
large N parameter k
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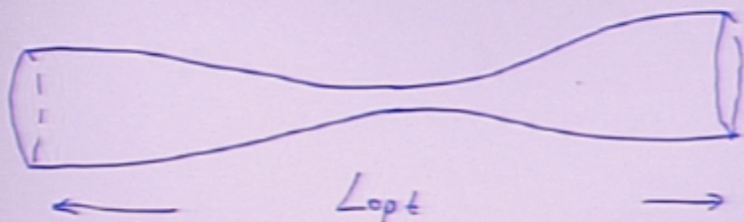
The Euclidean Geometry (t_E compactified)



Lorentzian Geometry



Some consequences



1. L_{opt} is finite, $L_{opt} \approx 2 \ln \frac{4}{\lambda}$
2. entropy is finite, $S \sim \frac{Area}{\lambda(k)} \cdot N$
 $\lambda(k)$ ↑ number of species of particles

3. time in the throat
 $t_{th} \sim \lambda \cdot t$
 λ ↑ time at ∞

flows slow for a distant observer

and is practically frozen (as measured by clock at ∞)

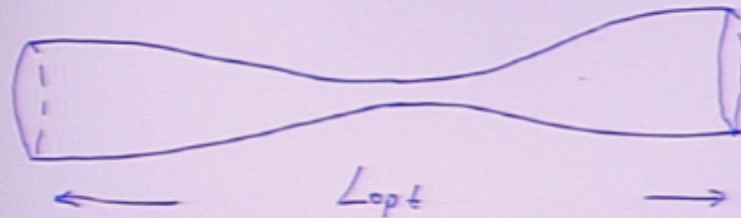
Of course, this is what we expect at horizon

But now it is not infinitely slow!

4. If $L_{eff} \sim L_{opt} \Rightarrow \lambda(k) \sim e^{-L \cdot k} \sim e^{-S_{BH}}$
 entropy $S_{BH} \sim L \cdot k$

Some consequences

12-



1. L_{opt} is finite, $L_{opt} \approx 2 \ln \frac{4}{\lambda}$
2. entropy is finite, $S \sim \frac{Area}{\lambda(k)} \cdot N$
 $\lambda(k)$ number of species of particles
3. time in the throat
 $t_{throat} \sim \lambda \cdot t$
 λ time at ∞

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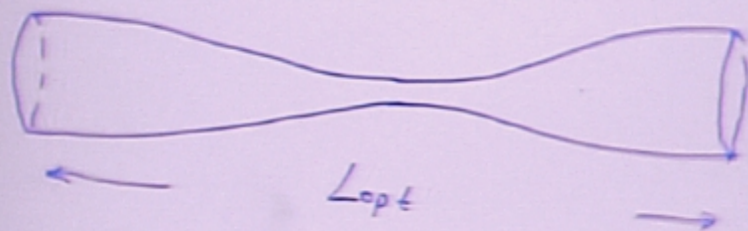
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4. If $L_{eff} \sim L_{opt} \Rightarrow \lambda(k) \sim e^{-L \cdot k} \sim e^{-S_{BH}}$
 entropy $S_{BH} \sim L \cdot k$

Some consequences

12.



1. L_{opt} is finite, $L_{opt} \approx 2 \ln \frac{L}{\lambda}$
2. entropy is finite, $S \sim \frac{A_{max}}{\lambda(k)} N$
? number of species of particles
3. time in the throat

$$t_{throat} \sim \lambda \frac{1}{c}$$

time at ∞

flow slow for a distant observer
 and is practically frozen (as measured by distant observer)
 Of course, there is what we expect at horizon
 But now it is not infinitely slow!

The Spectrum

13.

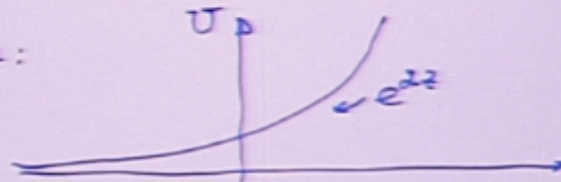
Turn on a bulk field $\phi \sim e^{-i\omega t} e^{iL_0 \phi} \frac{1}{(cky)^{1/2}} \psi(y)$

$$Z = \int \frac{dy}{\sqrt{4\pi h^2 y + \lambda^2}}$$

Effective Schrodinger eq-n:

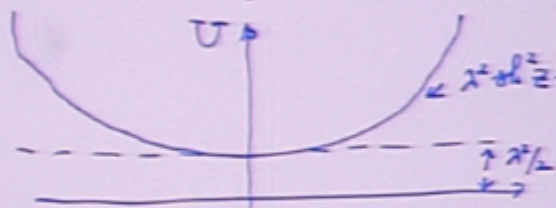
$$\partial_z^2 \psi(z) + (\omega^2 - U(z)) \psi = 0$$

Classical BTZ:



continuous spectrum

BTZ_k:



discrete spectrum

Spectrum: ($\lambda < 1$)

$$\omega_n^2 = n^2 + p_n^2, \quad n \in \mathbb{Z} \quad (\text{prediction for } CFT_k)$$

$$n^2 = \frac{\lambda^2}{2}; \quad p_n = \frac{\pi}{2L_0 \frac{\lambda}{2}} \left(n + \frac{1}{2}\right)$$

Two time scales: $t_k \sim \frac{1}{\langle \delta\omega \rangle} \sim L_0 \frac{1}{\lambda} \sim L-k$

$$t_p \sim \frac{1}{m} \sim 1/\lambda \sim e^{L/k} \sim e^{S_{BH}}, \quad t_p \gg t_k$$

Long-time correlators

$$\langle \mathcal{O}(t) \mathcal{O}(0) \rangle \sim \frac{1}{m} e^{-i\omega t}, \quad \text{periodicity with } t_p \sim \frac{1}{m} \sim e^{L/k}$$

(Poincaré recurrence)

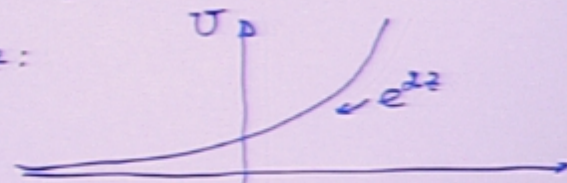
Turn on a bulk field $\phi \approx e^{-m|y|} e^{-\frac{1}{(dy)^{1/2}} \psi(y)}$

$$z = \int \frac{dy}{\sqrt{\sinh^2 y + \lambda^2}}$$

Effective Schrödinger eq-n:

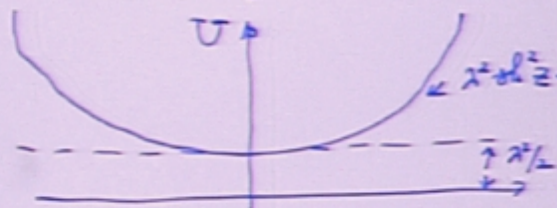
$$\partial_z^2 \psi(z) + (\omega^2 - U(z)) \psi = 0$$

Classical BTZ:



continuous spectrum

BTZ_k:



discrete spectrum

Spectrum: ($\lambda \ll 1$)

$$\omega_n^2 = m^2 + p_n^2, \quad n \in \mathbb{Z} \quad (\text{prediction for } \text{CFT}_k)$$

$$m^2 = \frac{\lambda^2}{2}; \quad p_n = \frac{\pi}{2L_n \frac{L}{\lambda}} \left(n + \frac{1}{2}\right)$$

Two time scales: $t_k \sim \frac{1}{\omega_k} \sim L_n \frac{1}{\lambda} \sim L \cdot k$

$$t_p \sim \frac{1}{m} \sim 1/\lambda \sim e^{Lk} \sim e^{S_{\text{BH}}}, \quad t_p \gg t_k$$

Long-time exp-s

$$\langle \mathcal{O}(t) \mathcal{O}(0) \rangle \sim \frac{1}{mL} e^{-ikt}, \quad \text{periodicity with } t_p \sim \frac{1}{m} \sim e^{Lk}$$

(Poincaré recurrence)

The restoration of unitarity in BH (a scenario)

$$t_H \sim L \cdot k, \quad t_P \sim e^{L \cdot k}$$

$$L \ll t_H \ll t_P$$

- For $t \ll t_H$ spectrum is $\approx$ continuous
 $\Rightarrow$ usual non-unitary thermodynamic behavior
 of semiclassical B.H. (BTZ_k looks like BTZ)
- For $t \sim t_H$ discreteness of spectrum
 shows up in some periodicities
 (Spectrum $\approx$ equidistant on this time scale)
 but not all inf-n restored yet - some hidden in long-time oscillations
- For $t \sim t_P \gg t_H$
 Spectrum deviates from equidistant
 mass $m \sim \mu$ plays the role
 System shows long-time correlations
 with period $t_P \sim e^{\Delta_{BH}}$
 all inf-n available in the system (black hole)
 released

[Faint, illegible handwriting on lined paper, possibly bleed-through from the reverse side.]

B O U R X
T L N I Z
Z V I T Z
K E T Y L
S S A P E

STEVE SOLVES UNITAPITY PUZZLE

BOWRA
TENZ
ZVITZ
KETYL
SSAPE

82K SOLVES UNITARITY PUZZLE

Conclusion

BOURUTLNIEBVTIEKETYLSSAPE

B	O	U	R	U
T	L	N	I	E
E	V	I	T	E
K	E	T	Y	L
S	S	A	P	E

BTZK SOLVES UNITARITY PUZZLE

Conclusion

BOURUTLNIZZVITZKETYLSSAPE

B	O	U	R	U
T	L	N	I	Z
Z	V	I	T	Z
K	E	T	Y	L
S	S	A	P	E

BTZ_k SOLVES UNITARITY PUZZLE

Conclusion

BOURUTLNIZZVITZKETYLSSAPE

B	O	U	R	U
T	L	N	I	Z
Z	V	I	T	Z
K	E	T	Y	L
S	S	A	P	E

BTZk SOLVES UNITARITY PUZZLE

Conclusion

BOURUTLNIZZVITZKETYLSSAPE

B	O	U	R	U
T	L	N	I	Z
Z	V	I	T	Z
K	E	T	Y	L
S	S	A	P	E

BTZ_k SOLVES UNITARITY PUZZLE

Conclusion

BOURUTLNIZZVITZKETYLSSAPE

B	O	U	R	U
T	L	N	I	Z
Z	V	I	T	Z
K	E	T	Y	L
S	S	A	P	E

BTZ_k SOLVES UNITARITY PUZZLE

Conclusion

BOURUTLNIZZVITZKETYLSSAPE

B	O	U	R	U
T	L	N	I	Z
Z	V	I	T	Z
K	E	T	Y	L
S	S	A	P	E

BTZk SOLVES UNITARITY PUZZLE

Conclusion

BOURUTLNIZZVITZKETYLSSAPE

B	O	U	R	U
T	L	N	I	Z
Z	V	I	T	Z
K	E	T	Y	L
S	S	A	P	E

BTZ_k SOLVES UNITARITY PUZZLE

Conclusion

BOURUTLNIZZVITZKETYLSSAPE

B	O	U	R	U
T	L	N	I	Z
Z	V	I	T	Z
K	E	T	Y	L
S	S	A	P	E

BTZ_k SOLVES UNITARITY PUZZLE

Conclusion

BOURUTLNIZZVITZKETYLSSAPE

B	O	U	R	U
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BTZk SOLVES UNITAPITY PUZZLE

Conclusion

1. The results of the experiment show that the rate of reaction is directly proportional to the concentration of the reactants.

1	2	3	4	5
1	2	3	4	5
1	2	3	4	5
1	2	3	4	5
1	2	3	4	5

2. The rate of reaction is also affected by the temperature of the reactants.