Title: Aspects of c < 1 String with Time-like Linear Dilaton Matter

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Abstract:

Aspects of $c < 1$ String with Time-like Linear Dilaton Matter

Tadashi Takayanagi (Harvard U.)

Based on
hep-th/0411019
hep-th/0503184 with Seiji Terashima (Rutgers)
hep-th/0503237

①Introduction

2D String Theory is the simplest example of quantum gravity which is

(1) Exactly Solvable (via c=1 Matrix Model)

(2) Dynamical (∃ massless scalar field)

(3) Non-perturbatively well-defined
(Especially in 2D type0 string)

2D (Bosonic) String Theory

We consider (1+1) dim. spacetime: (X^0, ϕ).

On the world-sheet, there exist

X^0 : Time-like free boson (c=1 matter)

ϕ : Liouville field (c=25)

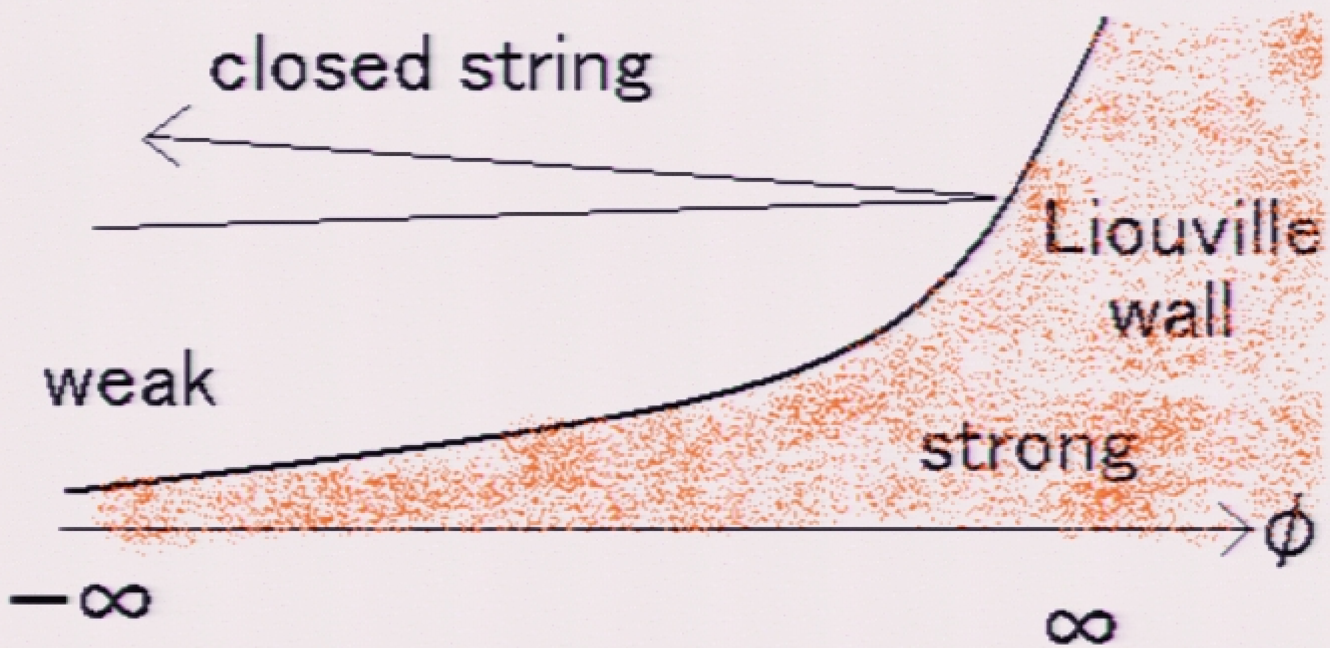
with the linear dilaton

s.t. $g_s = e^{2\phi}$.

→ We regulate the strongly coupled region

by the Liouville term $\mu \int dz^2 e^{2\phi}$.

2D String Theory



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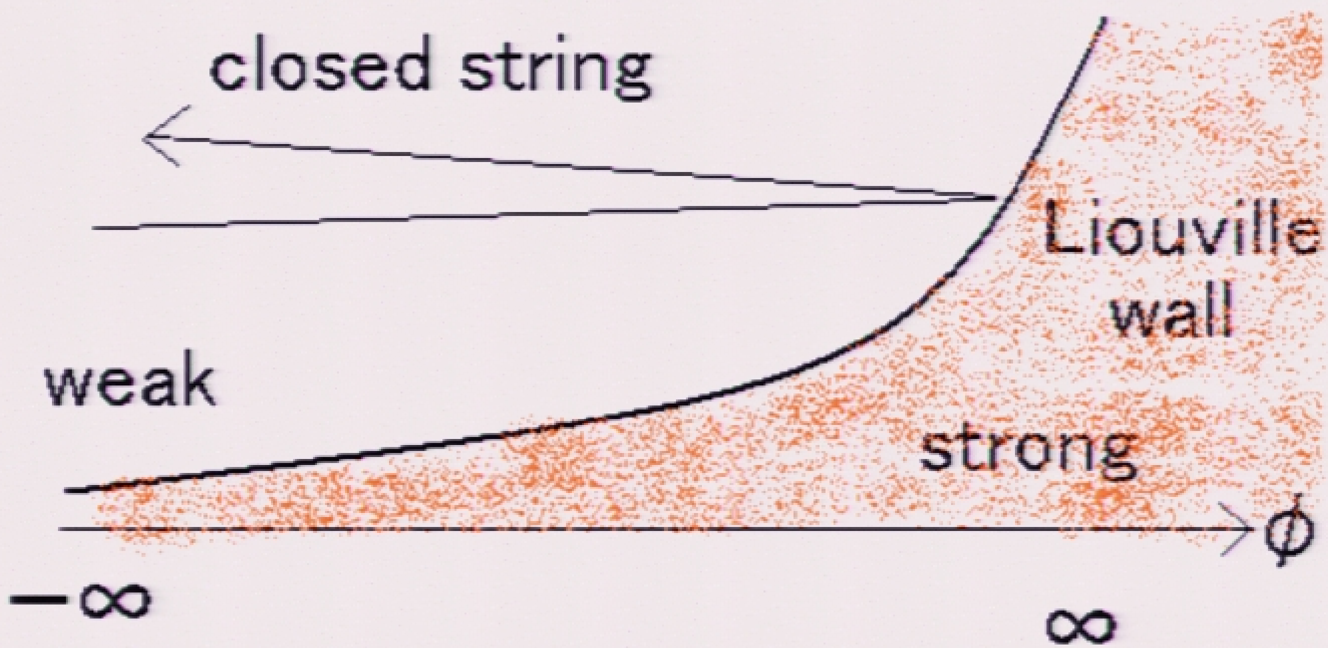

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C=1 Matrix Model

2D string theory has a non-perturbative dual description known as the c=1 matrix model.

$$S_{c=1} = \int dt\, \mathrm{Tr}[(\mathrm{D}_0\Phi)^2 + \Phi^2]$$

covariant derivative $\mathrm{D}_0\Phi = \partial_t\Phi + i[A_0, \Phi]$

$\Phi : N \times N$ Hermitian matrix,

$U(N)$ gauge sym. : $\Phi \to g\Phi g^{-1}$

Note: We always consider the matrix model after the double scaling limit.

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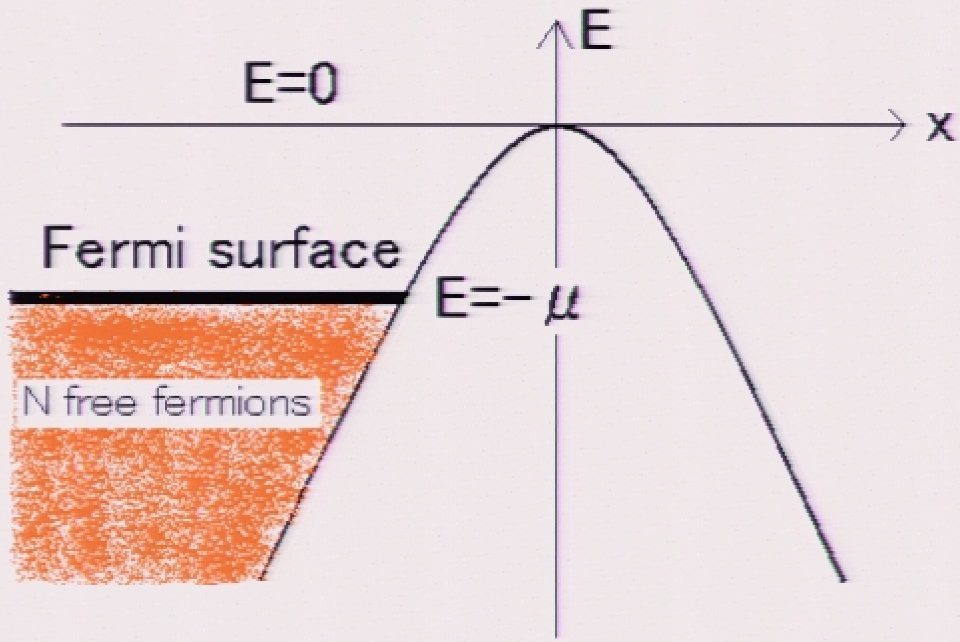
By using the gauge sym. we can **diagonalize** the matrix into N eigenvalues X_i (i=1,2,....,N) .

They exactly behave like **N free fermions** in the inverse harmonic potential $U(\Phi) = -\Phi^2$.

The Fermi level is given by $p^2 - x^2 = -\mu.$



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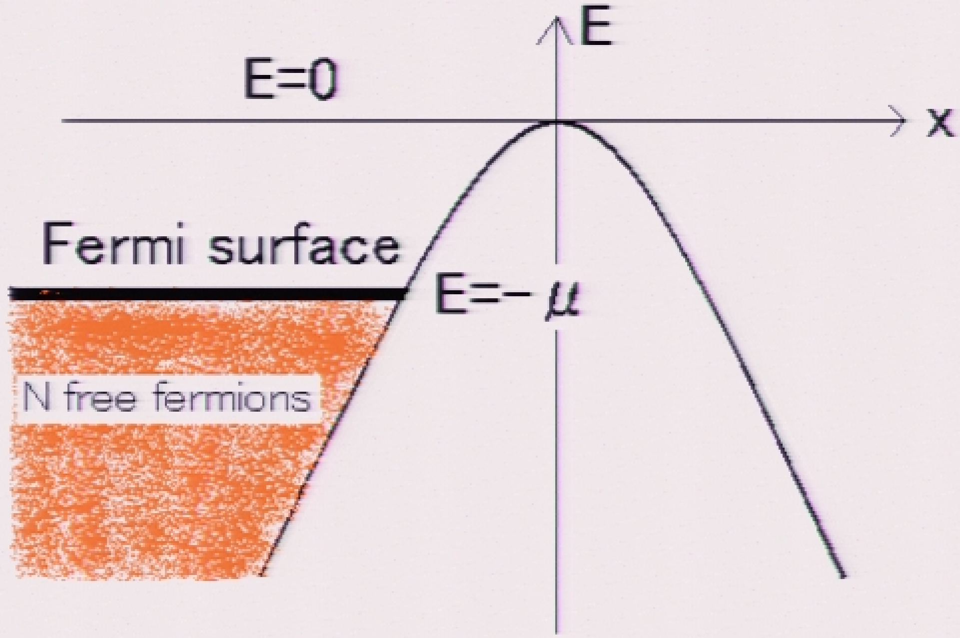

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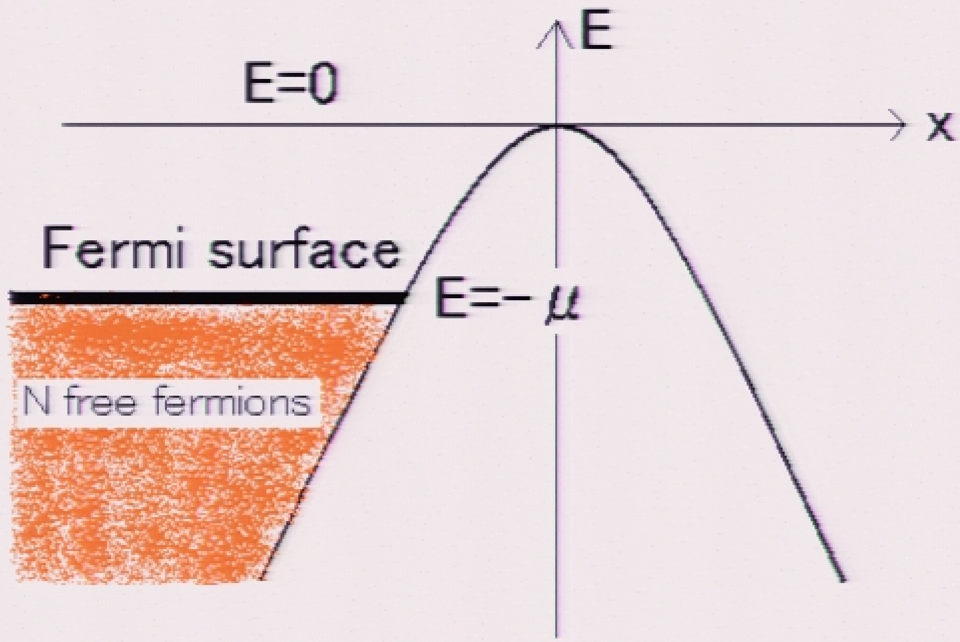

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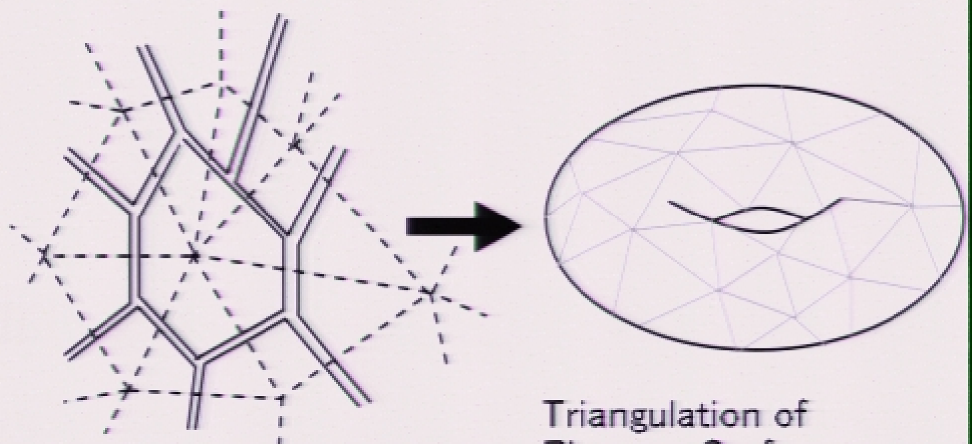
=Genus (g) expansion



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However, recently, its more physical understanding was proposed from the viewpoint of open-closed duality (or holography).

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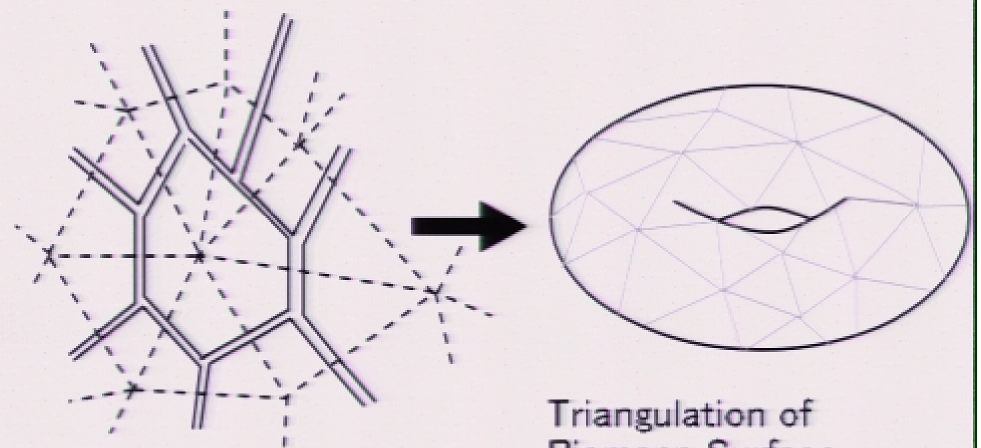

Feynmann Diagram of Matrix Model

Triangulation of Riemann Surface

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Open-Closed Duality in 2D String

c=1 matrix model = open string theory of
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Roughly, we can identify the matrix Φ with the open-string tachyon field T on the D0-branes.

One may be puzzled because

open string field theory actions generally include many complicated interaction terms of higher powers of T, while the matrix model is the free quadratic action.

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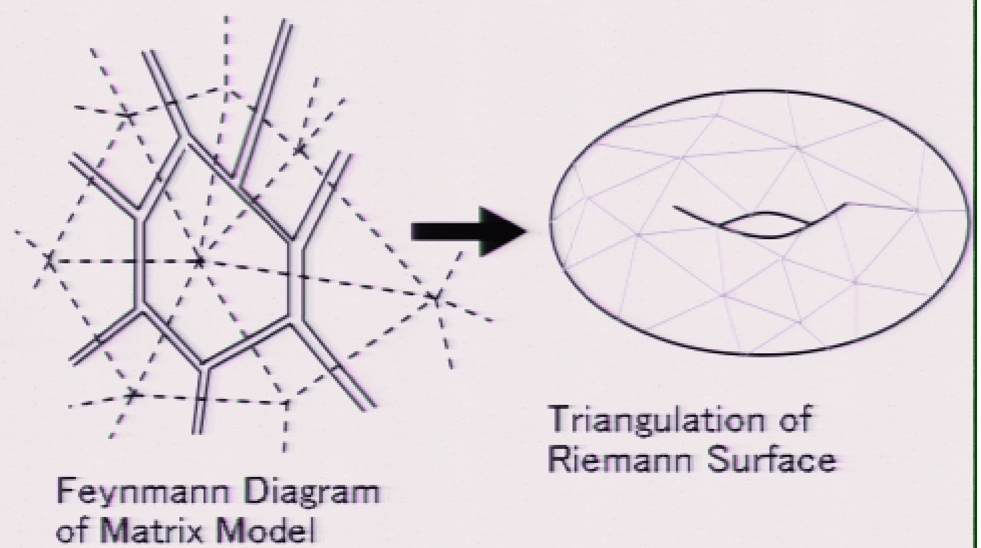

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Actually, such a drastic simplification of open string theory does occur in 2D string theory.

Takayanagi-Terashima

We can show that any on-shell scattering amplitudes of open strings on D0-branes are trivial in 2D string.

Furthermore, with respect to the off-shell interactions, we can claim:

$$S_{c=1}(\Phi) \cong S_{BSFT}(T)$$

Boundary string field theory for 2D string is equivalent to the c=1 matrix model (at least classically) up to a smooth field redefinition.

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open string field theory actions generally include many complicated interaction terms of higher powers of T, while the matrix model is the free quadratic action.

In this way, 2D string theory (c=1 string) is an instructive and intriguing toy example of open-closed duality in string theory.

In this talk, we will discuss a simple but non-trivial (time-dependent) deformation of c=1 string, that is

c<1 string with time-like linear dilaton matter

(i) This theory has its matrix model dual.
(ii) It is also equivalent to a topological string when compactified.

Takayanagi

(Notice: It is different from the minimal c<1 string.)

Contents

② c<1 String with Time-like Linear Dilaton Matter

Definition on the world-sheet

Time (matter) X^0 : time-like linear dilaton CFT

$c_m = 1 - 6q^2$

Space (Liouville) ϕ : space-like linear dilaton CFT

$c_L = 1 + 6Q^2$

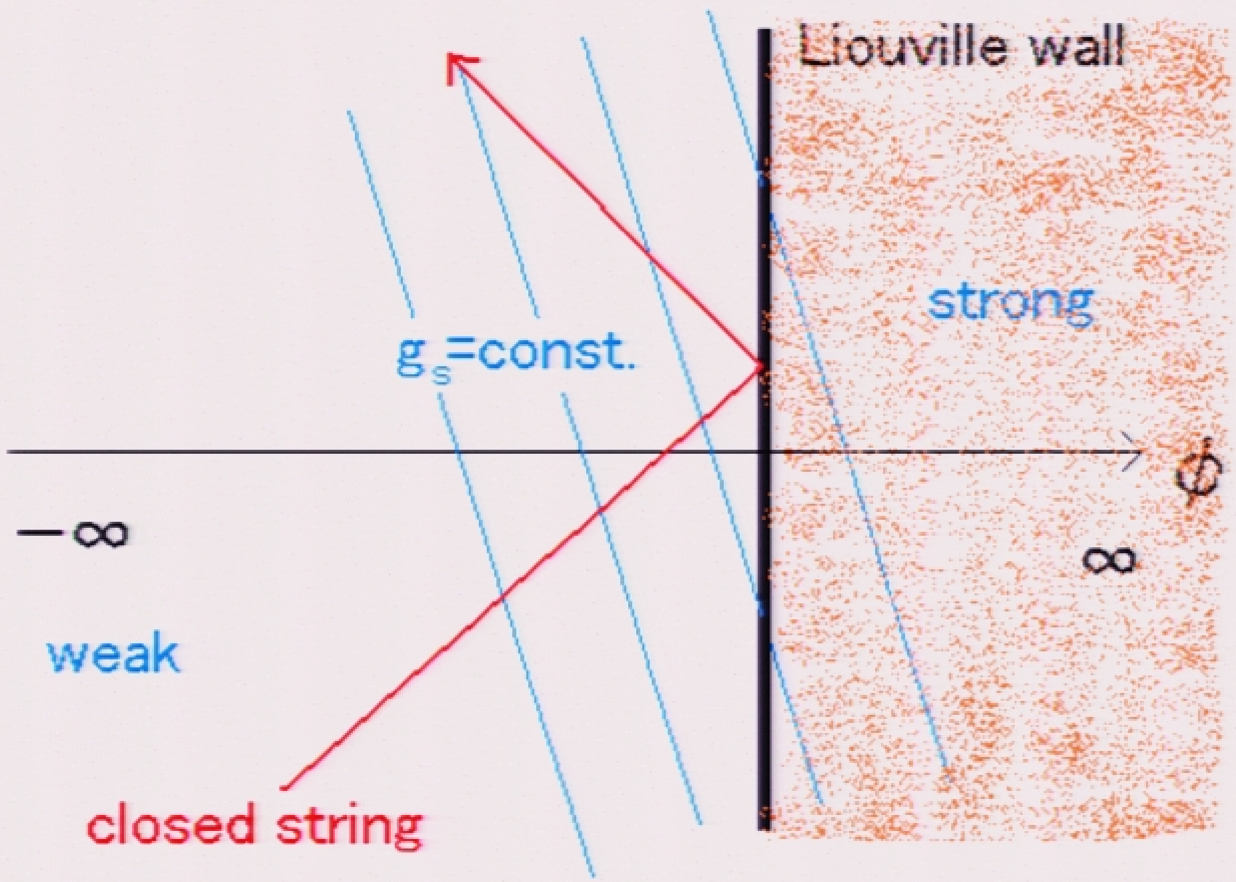
$$c_m + c_L = 26 \rightarrow \quad Q = b + \frac{1}{b}, \quad q = \frac{1}{b} - b \qquad (0 < b \leq 1)$$

String coupling: $g_s = e^{qX^0 + Q\phi}$

Liouville term : $\mu \int dz^2 \, e^{2b\phi}$

Closed String Scattering off the Wall

Scattered closed strings will not go into the strongly coupled region.



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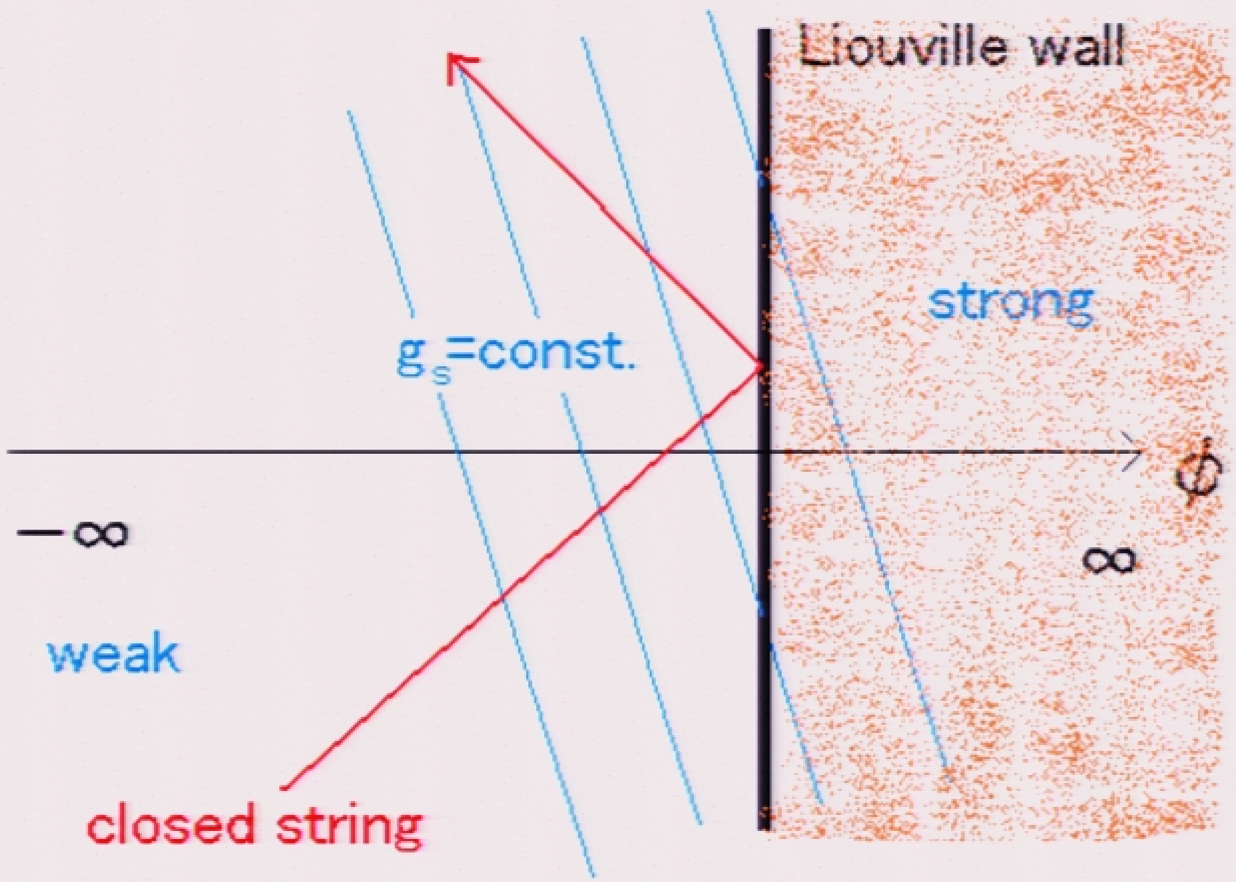

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Description as a background in 2D string

We can regard the c<1 string as a time-dependent background in the c=1 string, i.e. 2D string theory.

To see this, we consider the Lorentz boost

$$\tilde{X}^0 = \frac{Q}{2}X^0 + \frac{q}{2}\phi \ , \quad \tilde{\phi} = \frac{q}{2}X^0 + \frac{Q}{2}\phi$$

Then we find the conventional string coupling

$$g_s = e^{2\tilde{\phi}} .$$

But, the Liouville term becomes time-dependent

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In this way, we have obtained a one-parameter time-dependent background in 2D string.
This theory can be solvable on the world-sheet as is obvious in the c<1 string description.

The next question will be its matrix model dual.
There are two different approaches to this problem.

(i) To directly construct the matrix model by applying the open-closed duality
(ii) To find the corresponding time-dependent background in c=1 matrix model.

③Matrix Model Dual and Holography

(3-1) Direct Construction

Consider N unstable D0-branes in the c<1 string.

$$\text{Boundary State}: \ |D0\rangle = |Neumann\rangle_{X^0} \otimes |ZZ\rangle_{\phi}$$

As in the c=1 string case, the naïve guess leads to

$$S_{c<1} = \int dt\, e^{-qt}\ \mathrm{Tr}[(D_0\Phi)^2 + \Phi^2]$$

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Check of the quadratic action

We can check the action by computing the open string tachyon scattering amplitudes.

On-shell vertex: $V_+ = e^{X^0/b}, \quad V_- = e^{-bX^0}$

Actually, the momentum conservation is very restrictive. For the amplitude like $\left\langle (V_+)^{N_+} (V_-)^{N_-} \right\rangle$ we find

$$-N_- b + \frac{N_+}{b} = \left(\frac{1}{b} - b \right) \chi$$

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Then it turns out that any non-trivial amplitudes are vanishing when b^2 is irrational. This is because

$$N_+ - \chi = (N_- - \chi) b^2 \quad \rightarrow \quad N_\pm = \chi \leq 1$$

Since the physical properties should be continuous with respect to b^2 , we can conclude that the open string S-matrices are trivial in any $c<1$ string.

Thus this shows the action is given by the free quadratic one which we proposed!

This also strongly supports that the $c=1$ matrix model action is quadratic by taking the $b=1$ limit.

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Relation to c=1 matrix model action

After we diagonalize the matrix Φ, we obtain

$$S_{c<1} = \int dt\, e^{-qt} \sum_{i=1}^{N} [(\dot{y}_i)^2 + (y_i)^2].$$

If we define new eigenvalues $x_i(t) \equiv e^{-qt/2} y_i(t)$,

then it becomes equivalent to the c=1 matrix model

$$S_{c<1} = \int dt \sum_{i=1}^{N} \left[(\dot{x}_i)^2 + \frac{Q^2}{4} (x_i)^2 \right] .$$

This is natural since we know that the c<1 string is identical to a background in 2D string.

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(3-2) Description in c=1 Matrix Model

On the other hand, we can regard the c<1 string as a time-dependent bg. in 2D string theory.

Generally speaking, any time-dependence in 2D string corresponds to some time-dependent motion of the fermi surface in c=1 matrix model.

Polchinski, Moore-Plesser,.......
Alexandrov-Kazakov-Kostov,
Karczmarek-Strominger,......

In our case, we can determine the form of fermi surface explicitly by comparing the matrix model results with the string theory ones.

Matrix Model Dual of c<1 String

We argue that the matrix model dual is given by the following fermi surface in the c=1 matrix model

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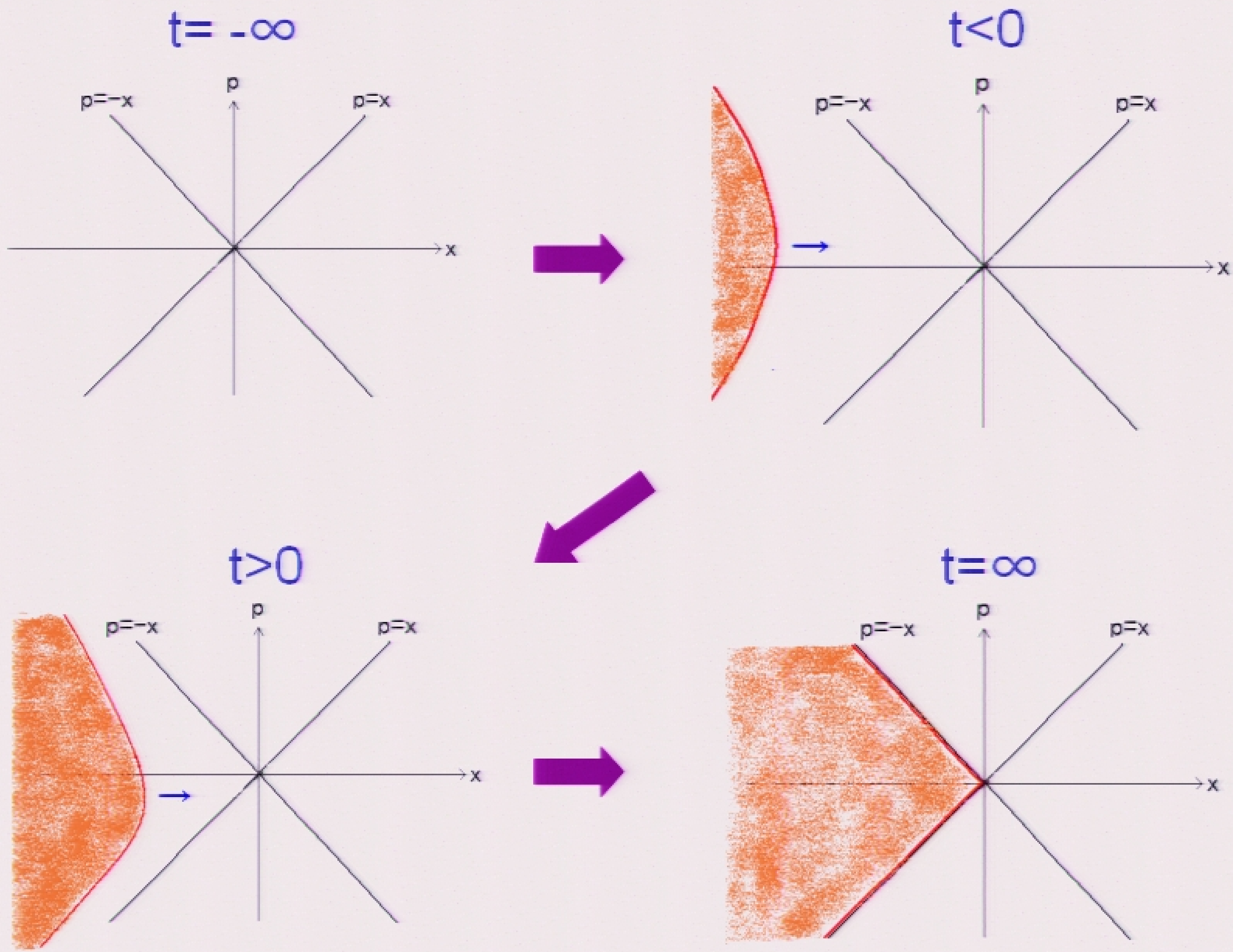

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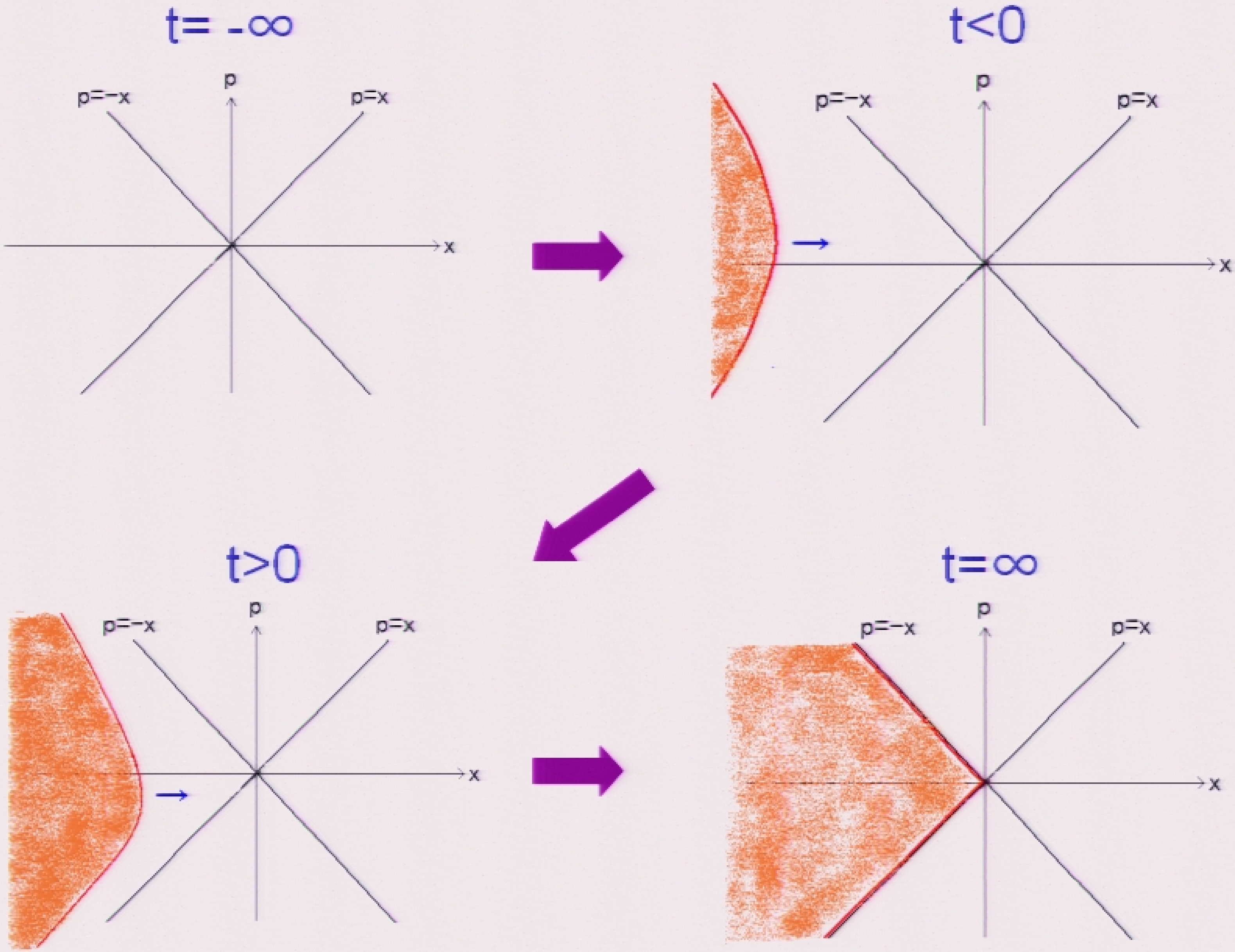

(i) At t=-∞: The fermi surface is pushed into infinity

→ No spacetime.

(ii) At t=∞: The fermi surface reaches at μ =0.

→ Linear dilaton vacuum.

This time-dependent behavior is consistent with the Liouville term (=closed string tachyon condensation)

$$T_{closed} = \mu\, e^{(b^2-1)\tilde{X}^0 + (b^2+1)\tilde{\phi}} \quad ,$$

because T_{closed} becomes ∞ at t=-∞ and 0 at t=∞.

(Remember that we are assuming $0 < b \leq 1$.)

Asymptotic Behavior

The deviation of the fermi surface from the lines $p=\pm x$ is identified with the closed string field in the asymptotic region

$$p_\pm(t,x) = \mp x \pm \frac{\partial_\pm \eta(t,x)}{x} \quad (|x| \to \infty).$$

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The eigenvalue direction x can be identified with the space coordinate as $|x| \approx e^{-\tilde{\phi}}$.

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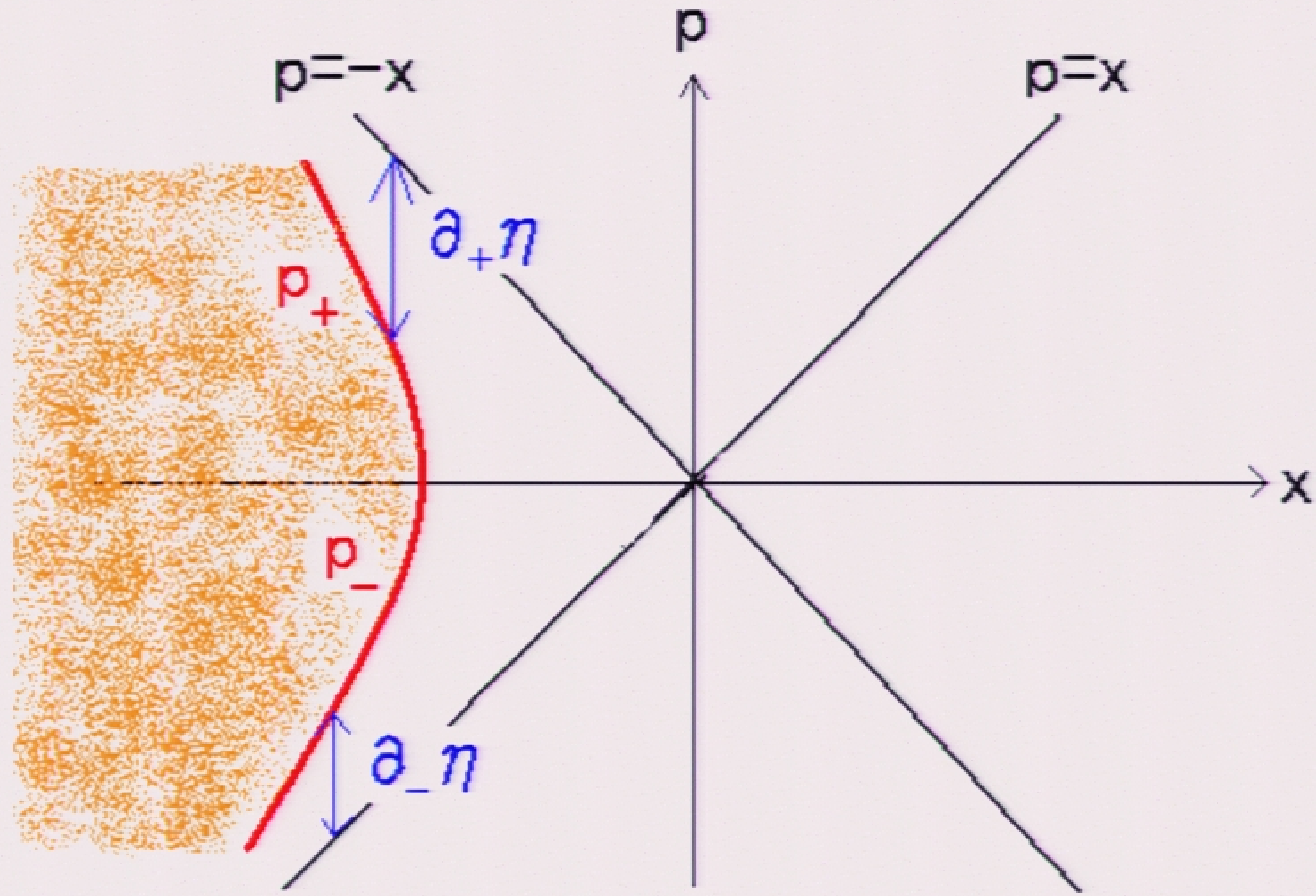

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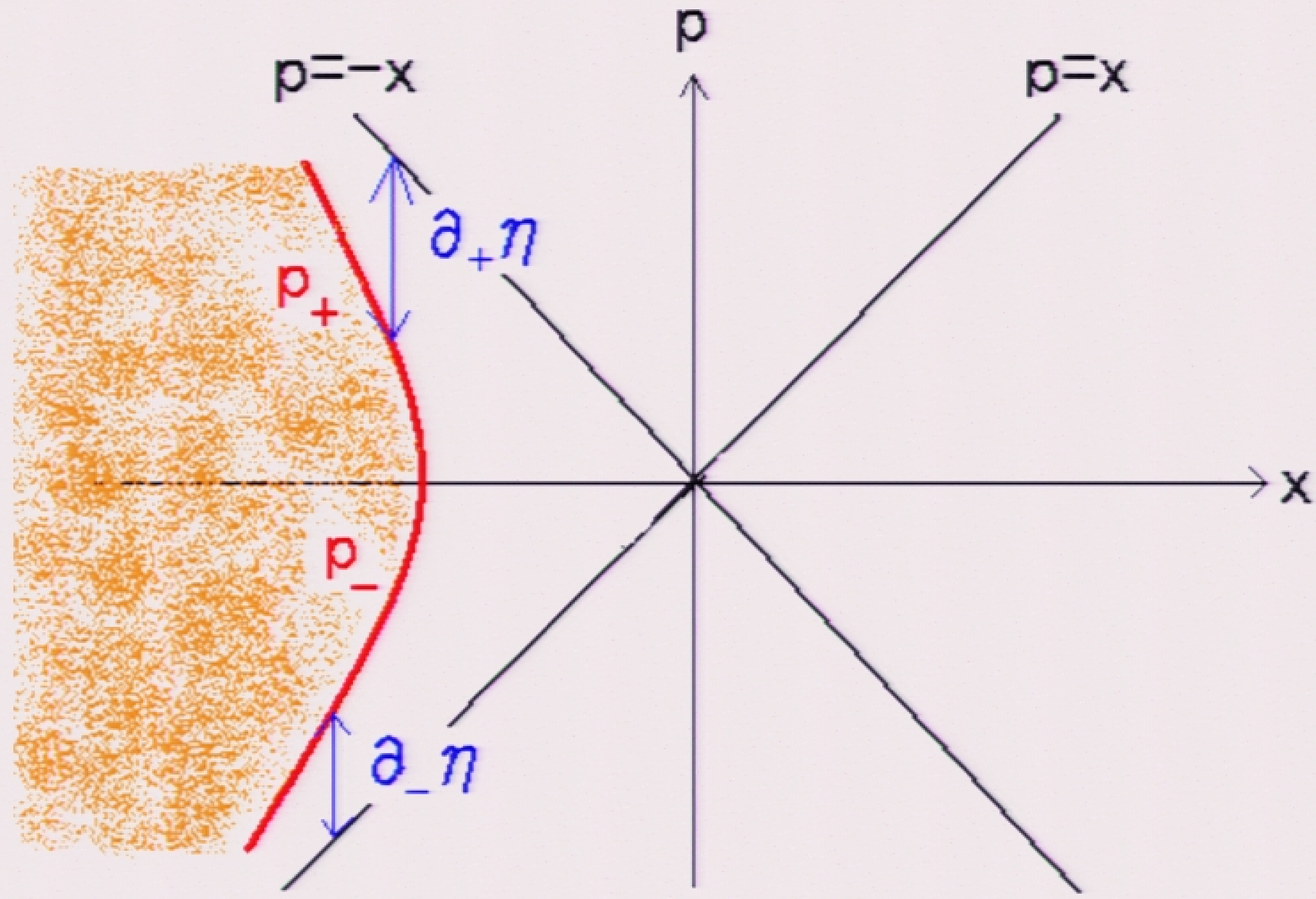

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Since the incoming waves just become the outgoing ones, we find the Polchinski's scattering equation

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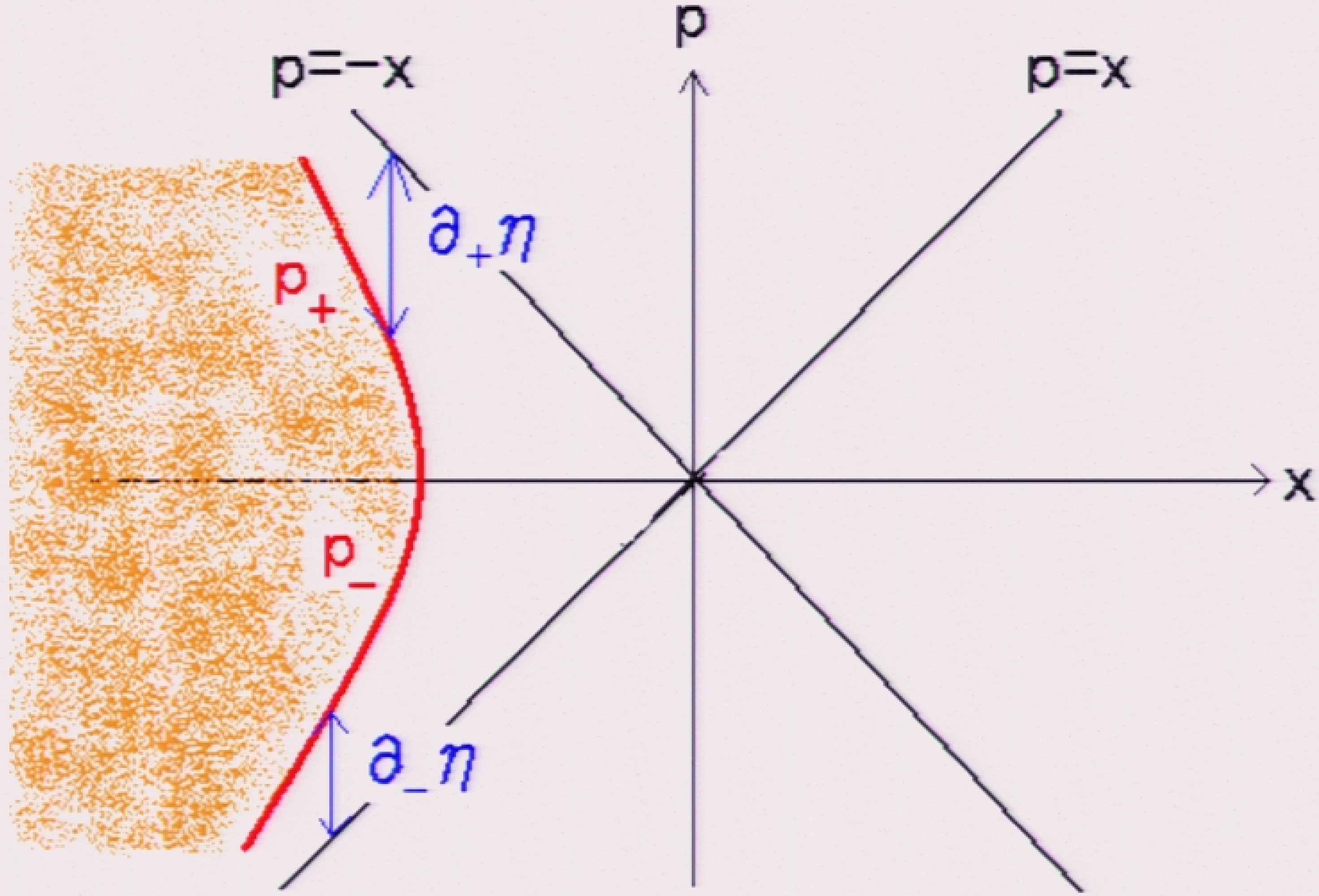

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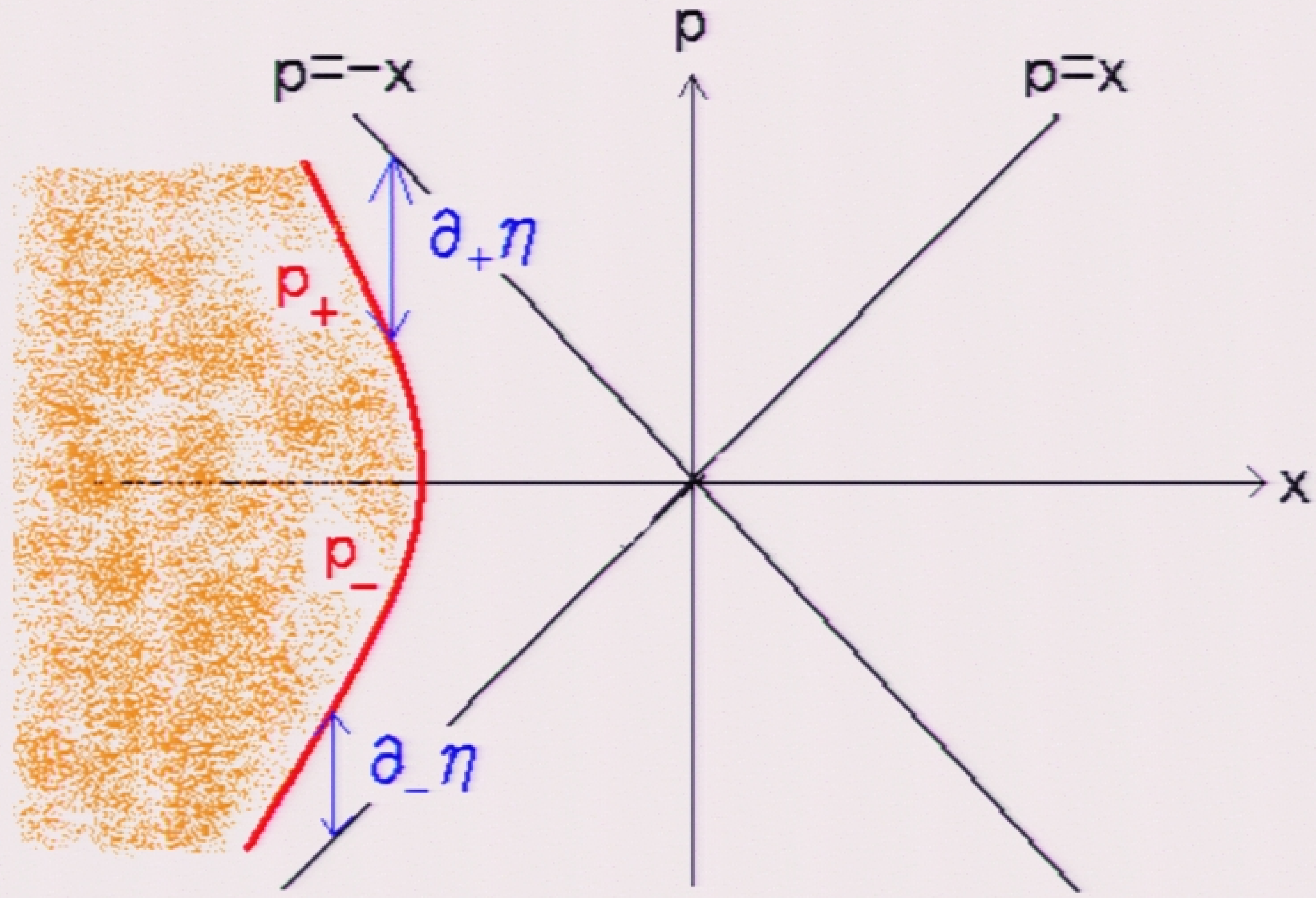

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By applying this to our time-dependent model, we can obtain the tree level 1→n scattering

$$\partial_+\eta(y) = \sum_{n=1}^{\infty} \frac{(b\mu^{-1/b^2})^{n-1}}{2^{n-1}\cdot n!} \cdot \frac{\Gamma(-b^{-1}\partial_y + 1)}{\Gamma(-b^{-1}\partial_y + 2 - n)} \cdot \left(e^{(n-1)qy}(\partial_-\eta(y))^n\right).$$

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Klebanov-Maldacena-Seiberg

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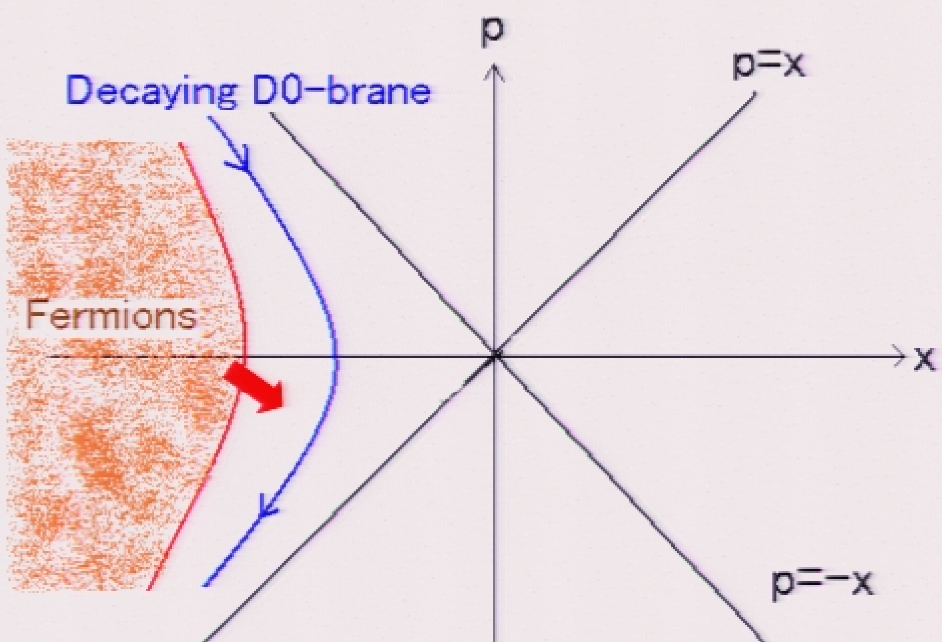

Prediction of new rolling tachyon boundary states in the time-like linear dilaton CFT.

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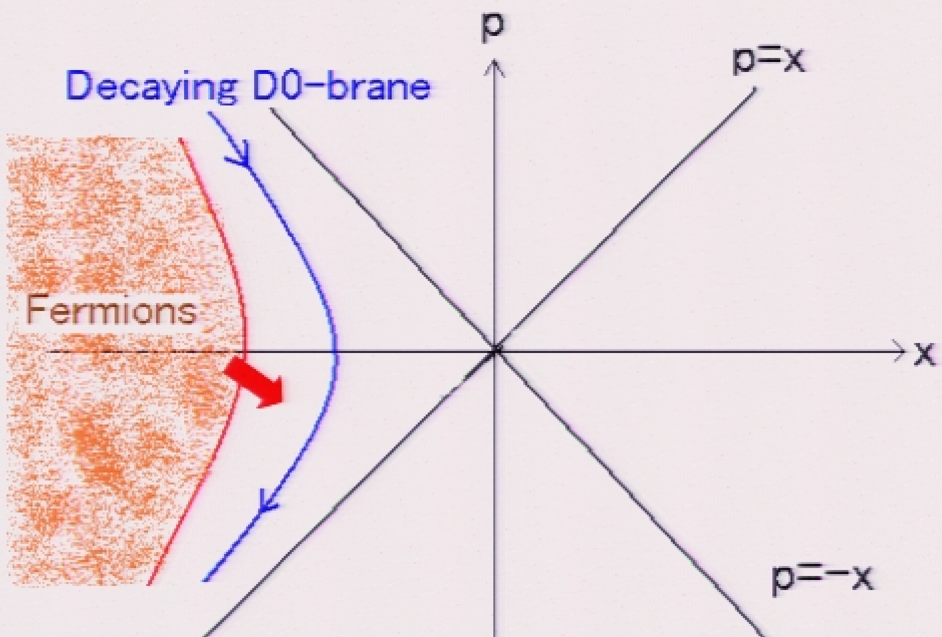

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④ c<1 String from 2D Black holes

Noncritical string theories like 2D string theory or minimal strings often turn out to be equivalent to the topological models such as topological strings.

Examples

(1) c=1 string at self-dual radius

= topological string on conifold Ghoshal-Vafa

= Twisted N=2 SL(2,R)/U(1) at the level 3

Muhki-Vafa

(2) (1,n) minimal string (c<1 minimal string)

= Twisted N=2 SU(2)/U(1) at the level n Li,…

= topological string on $x+y^n+zw=0$

Aganagic-Dijkgraaf-Klemm-Marino-Vafa

Such a kind of equivalence is very helpful for further understandings of the string theory from topological or geometrical viewpoints.

Below we would like to claim that
our non-minimal c<1 string also have a topological String description:

$$c = 1 - 6(n-1)^2/n \ \text{String at the radius } R = \sqrt{\alpha' n}$$

$$\cong \text{ Twisted } N=2 \left(\frac{SL(2,R)_{2+n}}{U(1)} \right) \Big/ Z_n \text{ Model}$$

$$\cong \text{ Topological LG model } W = X^{-n}$$

see also Aharony-Ganor-Sonnenschein-Yankielowicz-Sochen, …,Ooguri-Vafa, …

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Remarks

- We have assumed that n is a positive integer.
  For general values of n, many results suggest

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  For example, we can find for n=1/2

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see also Aharony-Ganor-Sonnenschein-Yankielowicz-Sochen, …,Ooguri-Vafa, …

Remarks

- We have assumed that n is a positive integer.
  For general values of n, many results suggest

$$c = 1 - 6(n-1)^2 / n \ \text{String at the radius } R = \sqrt{\frac{\alpha'}{n}}$$

$$\cong \text{Twisted } \ N = 2 \ SL(2,R)_{2+n} / U(1) \ \text{Model}$$

- For a rational value n=p/q, its central charge
  is the same as that of (p,q) minimal model.
  For example, we can find for n=1/2

$$\underbrace{\text{non-minimal } (1,2)}_{\text{c=-2 string}} \text{ string at the radius } R = \sqrt{2\alpha'}$$

$$\cong \text{ topological twist of } \ \text{2D black hole in type 0 string}$$

Evidences for the Equivalence

(1) Physical Spectrum in Free Field Representation

$$N=2\ \ SL(2,R)/U(1)\ \text{Kazama - Suzuki Model}$$

$$\cong \underset{c=1+6/n}{\phi} \oplus \underset{c=2}{(\beta,\gamma)} \oplus \underset{c=1}{X} \oplus \underset{c=-2}{(\eta,\xi)} \oplus \underset{c=1}{(\psi,\overline{\psi})}$$

⬇ Topological Twist $T \to T + \frac{1}{2}\partial J$

$$\underset{c=1+6(n+1)^2/n}{\phi} \oplus \cancel{\underset{c=2}{(\beta,\gamma)}} \oplus \underset{c=1-6(n-1)^2/n}{X} \oplus \cancel{\underset{c=-2}{(\eta,\xi)}} \oplus \underset{c=-26}{(\psi,\overline{\psi})}$$

Liouville field — Time (Matter) — (b,c) ghost

$$\cong\ c = 1-6(n-1)^2/n\ \ \text{noncritical string}$$

Tachyon States

For example, we can find the following physical state in the twisted theory.

$$V_j = \bar{\psi}\exp\left[\sqrt{2/n}\left(i(j+n/2)X+(j+n/2+1)\phi\right)\right]$$

This corresponds to the tachyon state in the c<1 string with the momentum $p=\sqrt{2/n}(j+1/2)$.

Other physical states also match with the c<1 string.

(2) Three Point Function of tachyons

→ Leg Factors can be reproduced.

(3) Topological LG model

→ General N particle scattering amplitudes agree.

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⑤ Conclusion

- We have defined the (non-minimal) $c<1$ string and constructed its matrix model dual. This is the simplest time-dependent bg. in 2D string.

- Open string theory on unstable D0-branes in this model have the quadratic action (i.e. matrix model) because S-matrix is trivial.

- The compactified $c<1$ strings are argued to be equivalent to the twisted N=2 SL(2,R)/U(1) at general values of the level k.

Future Problems

- Matrix model analysis beyond the tree level
- D-brane interpretation of the loop operator
- Construction of boundary states for the decaying D-branes in the time-like linear dilaton CFT

:

- Geometrical picture of the twisted model
- Type 0 non-critical string = `N=2 twisted SCFT'?
- Non-critical N=2 String and matrix model

:

:

:

④ c<1 String from 2D Black holes

Noncritical string theories like 2D string theory or minimal strings often turn out to be equivalent to the topological models such as topological strings.

Examples

(1) c=1 string at self-dual radius

= topological string on conifold   Ghoshal-Vafa

= Twisted N=2 SL(2,R)/U(1) at the level 3

Muhki-Vafa

(2) (1,n) minimal string (c<1 minimal string)

= Twisted N=2 SU(2)/U(1) at the level n Li,…

= topological string on $x+y^n+zw=0$

Aganagic-Dijkgraaf-Klemm-Marino-Vafa