Title: Quiver gauge theories and dimer models

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Abstract:

Dimer models and quiver gauge theories

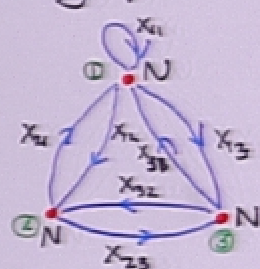
A. Hanany & K.K hep-th/0503149
S. Franco & A. Hanany & K.K. & D. Vegh & B. Wecht
hep-th/0504110

- Discovered connection between mathematics of dimer models and physics of quiver gauge theories
- Rich & well-studied mathematical structure is leading to new insights
- OUTLINE
  - review of (toric) quiver gauge theories
  - the planar quiver construction
  - physical realisation in string theory
  - application of dimer models
  - simplifications & new insights

What is a quiver gauge theory?

- $d=4$, $\mathcal{N}=1$ SUSY
- Product gauge group $\prod_i SU(N_i)$
- chiral matter in the bifundamental $(N_i, \bar{N}_j)$
  and adjoint $(N_i, \bar{N}_i)$

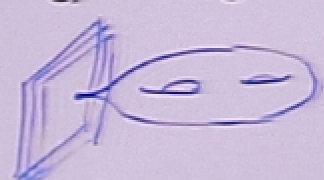
Depicted by quiver diagram + superpotential



$$W = X_{21}X_{12}X_{23}X_{32} - X_{32}X_{23}X_{31}X_{13} + X_{13}X_{31}X_{11} - X_{12}X_{21}X_{11}$$

Moduli space given by D- and F-terms in usual way.

- They arise in string theory on world-volume of D3 probing a singular CY_3



$\mathbb{R}^{3,1} \times CY_3$

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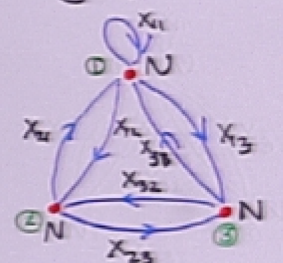

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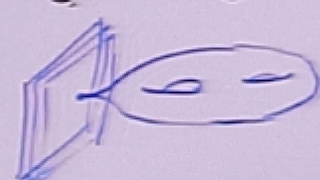

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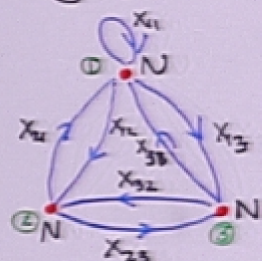

$$
\begin{aligned}
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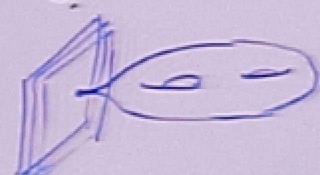

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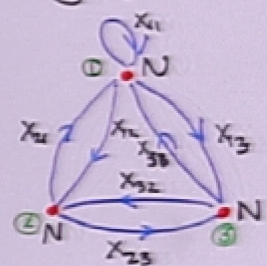

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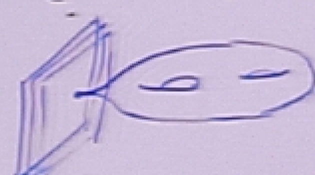

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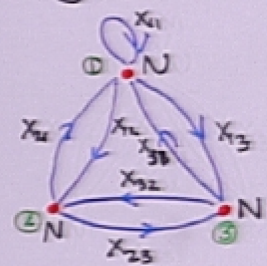

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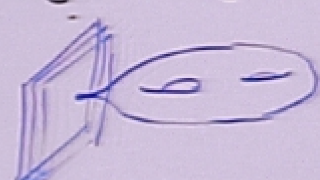

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A toric quiver theory has all gauge groups of equal rank, and corresponds to D3-branes on a non-compact toric 3-fold.

How are the toric quiver theories and toric geometries related?

Much previous work, systematized by Feng, Hanany and He in 0003085 & followup.

$$\begin{tikzcd} \text{Toric quiver} \arrow[r, bend left, "\text{Forward algorithm}"] & \text{Toric Moduli space} \arrow[l, bend left, "\text{Inverse algorithm}"] \end{tikzcd}$$

non-unique: multiple quivers related by Seiberg duality

Computationally intensive algorithms.

Our results make use of extra structure present in the toric quiver theories.

- by writing the quiver and superpotential separately this structure is not manifest

Properties of toric quivers (previously observed but not exploited)

- each field appears precisely twice in W, with opposite signs
- in all known examples the quiver obeys the relation

$$\underbrace{N_g}_{\#\text{gauge groups}} + \underbrace{N_W}_{\#\text{terms in } W} - \underbrace{N_f}_{\#\text{fields}} = 0$$

We showed that toric quivers can always be written as planar graphs on T^2, such that terms in the superpotential correspond to faces of the graph

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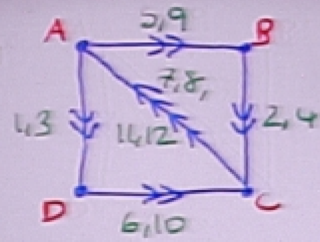
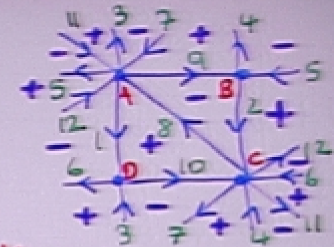


QUIVER + SUPERPOTENTIAL



PLANAR QUIVER

$$W = X_1 X_{10} X_8 - X_3 X_{10} X_7 - X_2 X_8 X_9$$
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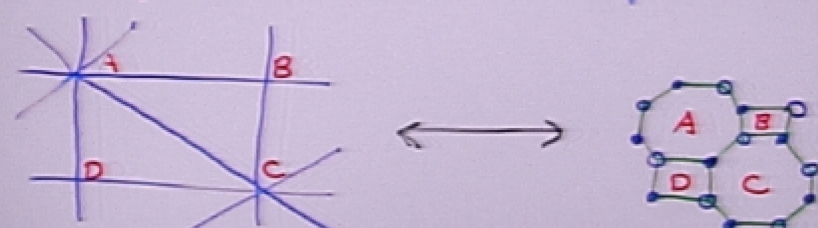
$$+ X_2 X_{12} X_5 - X_4 X_{11} X_5$$

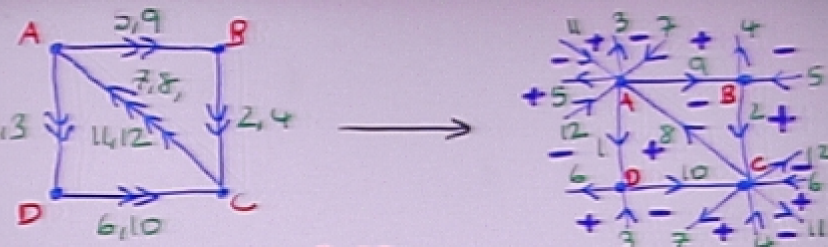
- We used the toric condition to glue together two faces along every edge → tiling of 2D surface w/o boundary
- Euler relation $N_g - N_W + N_f = 0$ ← proof using R-symmetry of W
- Notice: signs of adjacent faces are always opposite!
- Combine quiver + W into a single object, the planar quiver; well-studied in planar dual guise



2-colouring of faces ⟷ bipartite graph

5

e.g.



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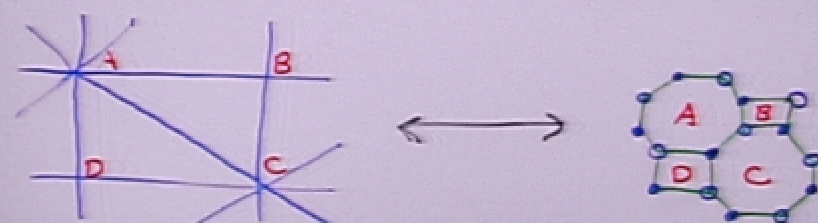

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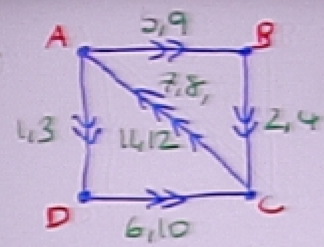

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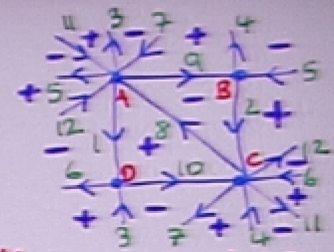

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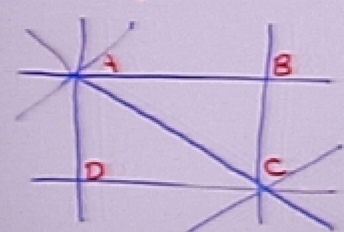

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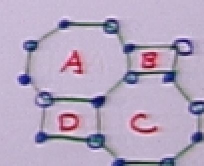

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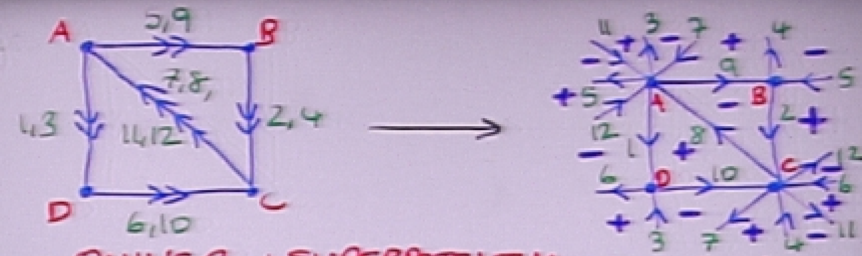

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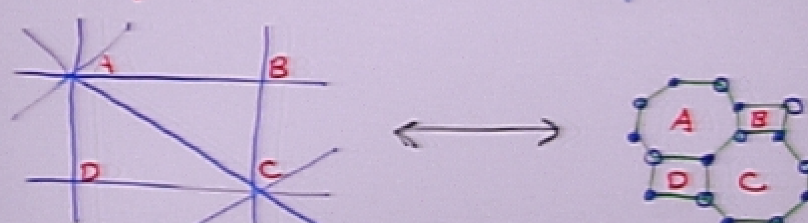


2-colouring of faces ⟷ bipartite graph

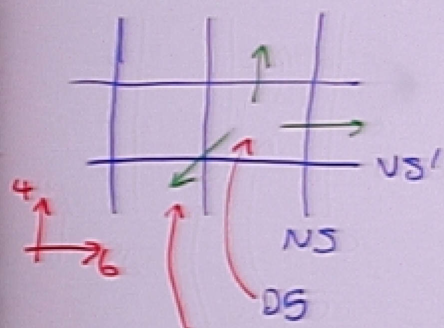
- This dual graph is <u>physical</u> – exists in string theory
- Combinatorics (/stat. mech.) of bipartite graphs is well-developed; powerful calculational tools.

1) Engineering of quiver theory using branes

Previous constructions

"Brane boxes" 9801134, 9805139, ...

"Brane diamonds" 9903093, ...



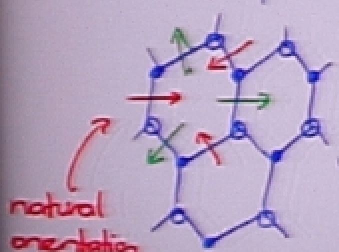
NS5-branes along 012345

NS' along 0123 67

D5-branes along 01234 6

bifundamentals between two gauge groups

- rules for constructing quiver theories, but ad hoc.
- dual of planar quiver gives concrete construction



Graph → NS5 rotated in 46 plane

Faces → D5's (gauge groups)

(dual) Edges → stretched strings

Vertices → local interaction of multiple strings

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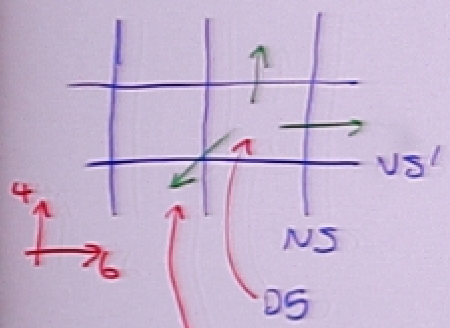

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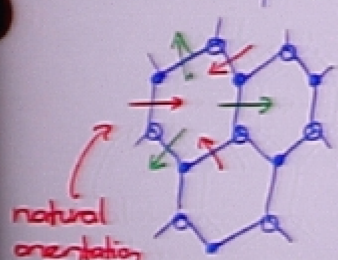

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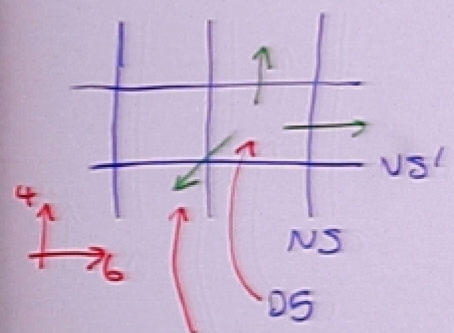

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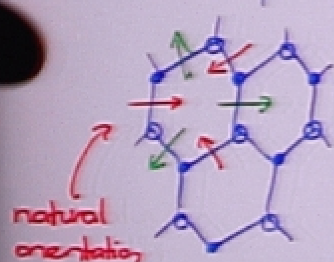

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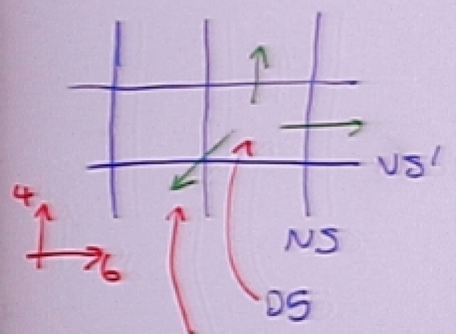

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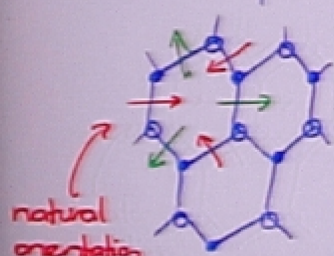

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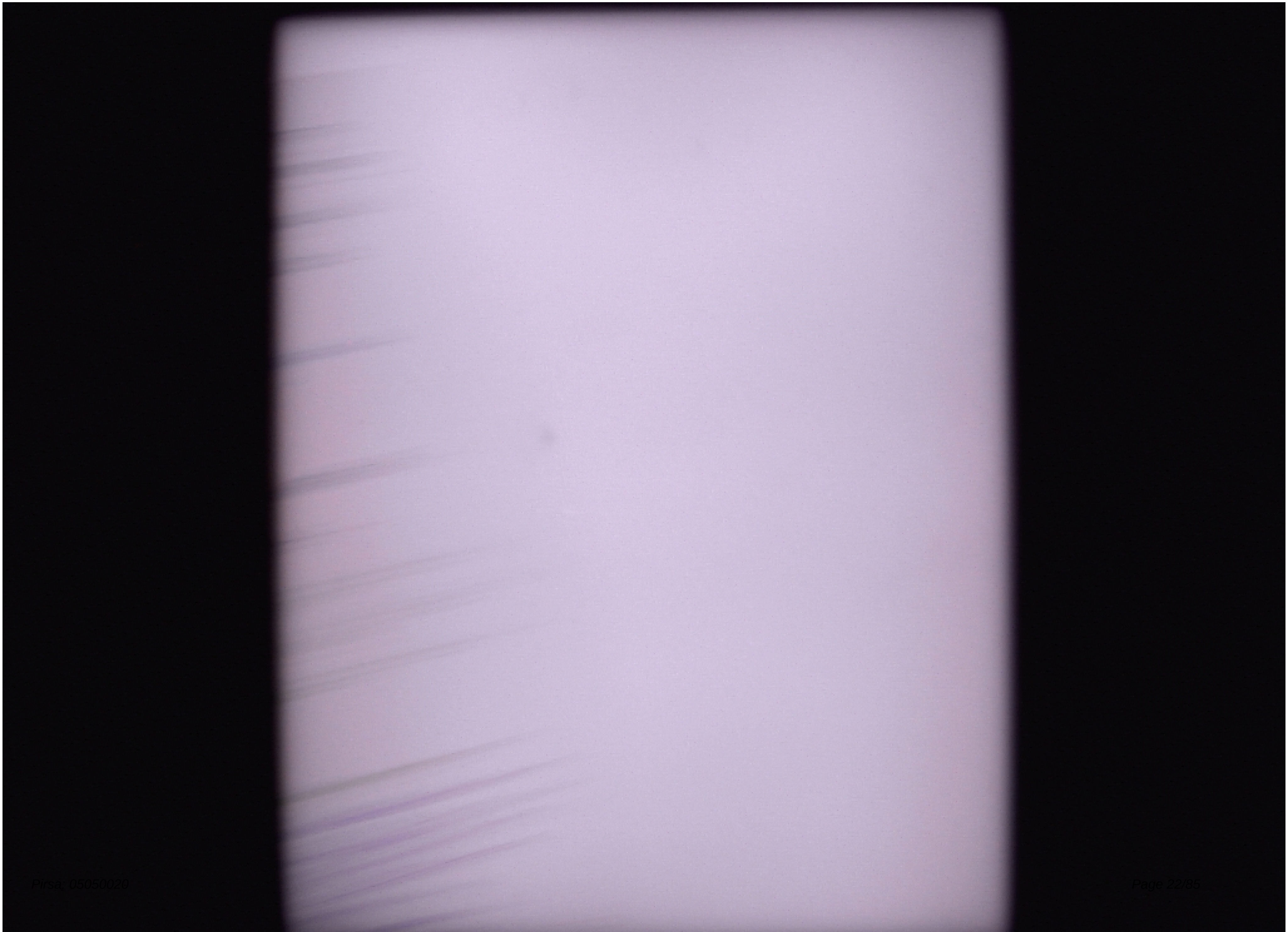


Using this construction it is easy to write down such "brane tilings" for <u>any</u> toric quiver and superpotential

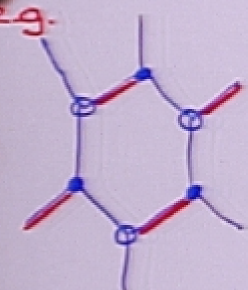
glue together even-sided polygons according to quiver data

- straightforward construction of toric quiver theories in string theory, which naturally encodes both matter content and interactions
- We can use <u>dimer models</u> to study their properties

→ combinatorics of random <u>perfect matchings</u> of a periodic bipartite graph

↑ collection of edges incident to each vertex exactly once

e.g.



- Kasteleyn (1960s)
- Recent progress: Kenyon, Okounkov, Sheffield math-ph/0311005

Main tool: Kasteleyn matrix

- weighted, signed adjacency matrix of graph

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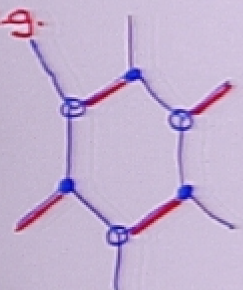

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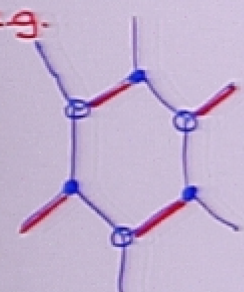

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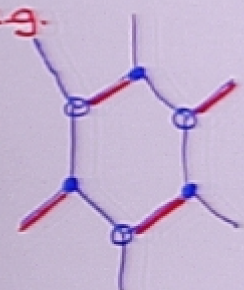

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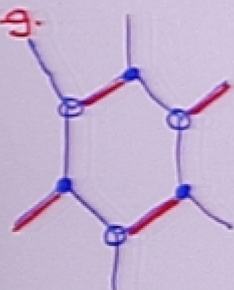

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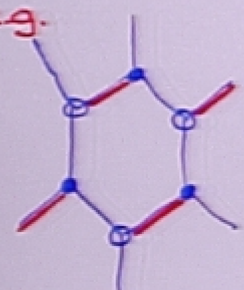


- Kasteleyn (1960s)
- Recent progress: Kenyon, Okounkov, Sheffield math-ph/0311005

Main tool: Kasteleyn matrix

- weighted, signed adjacency matrix of graph

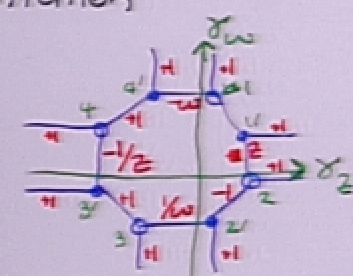
7

- take all edge weights = 1 in bare graph to make contact with previous results; in general arbitrary c-numbers

- sign rule: assign +/− to edges s.t. ∀ faces

$$\mathrm{sign}\Big(\prod_{i\in \text{face}} e_i\Big) = \begin{cases} +1 & \#\text{edges} = 2 \bmod 4 \\ -1 & \#\text{edges} = 0 \bmod 4 \end{cases}$$

- construct paths γ_z, γ_w in dual graph that wind the two cycles of T^2; when γ crosses an edge multiply the weight by $z, 1/z$ $(w, 1/w)$ according to orientation



$$K = \begin{array}{c c} & \begin{array}{cccc} 1' & 2' & 3' & 4' \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \end{array} & \begin{pmatrix} 1 & 1 & 0 & -1/w \\ 1/z & -1 & 1 & 0 \\ 0 & w & 1 & 1 \\ 1 & 0 & -z & 1 \end{pmatrix} \end{array}$$

$$\det K = \frac{-1}{w} - w - \frac{1}{z} - z - 5$$

- In general

$$P(z,w) \equiv \det K = z^{h_{z_0}} w^{h_{w_0}} \sum_M{}' c_{h_z,h_w} (-1)^{h_x+h_y+h_x h_y} z^{h_x} w^{h_y}$$

where c_{h_z,h_w} counts certain closed curves on the graph

- take all edge weights = 1 in bare graph to make contact with previous results; in general arbitrary $\mathbb{C}$-numbers

- sign rule: assign +/- to edges s.t. $\forall$ faces

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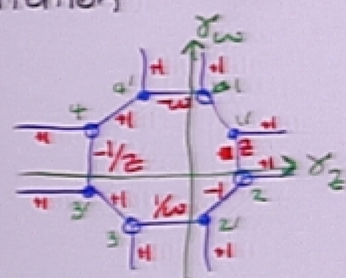

$$K = \begin{array}{c|cccc} & 1' & 2' & 3' & 4' \\ \hline 1 & 1 & 1 & 0 & -1/w \\ 2 & 1/z & -1 & 1 & 0 \\ 3 & 0 & w & 1 & 1 \\ 4 & 1 & 0 & -z & 1 \end{array}$$

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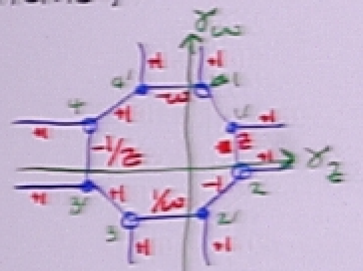

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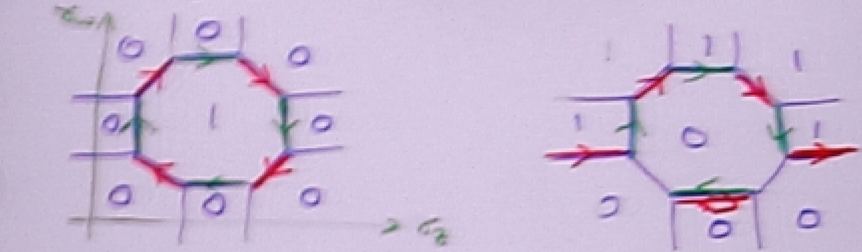
8

Choose a fixed "reference" matching M_0.
Then for every matching M, $M - M_0$ defines a set of oriented closed curves

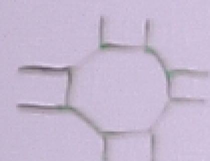


$h_z = 0,\ h_w = 0$ $\qquad$ $h_z = 0,\ h_w = 1$

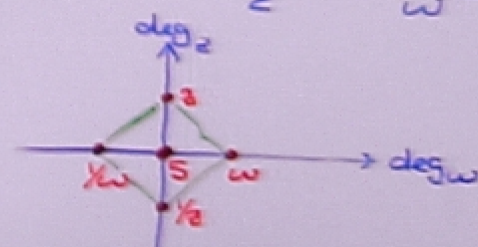
Curves are counted with their <u>height change</u> along γ_z, γ_w: height function h_z, h_w changes by ± 1 upon crossing a curve with +/- orientation

Writing the monomials in $\det K$ as points in $\mathbb{Z}^2$ graded by h gives a convex polygon

eg.



$$\det K = -\frac{1}{z} \mp z - \frac{1}{w} - w - 5$$



This is the toric diagram of the CY space that is the moduli space of the toric quiver gauge theory! 9

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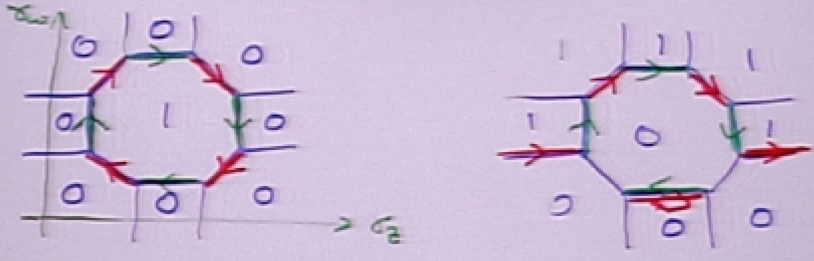

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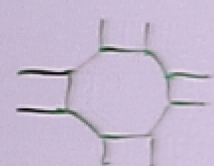

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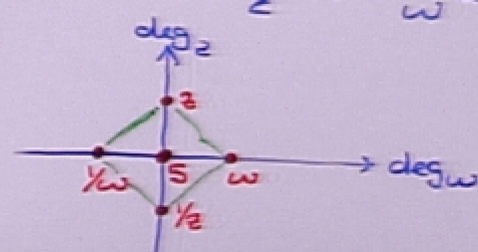

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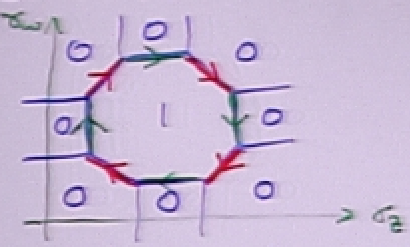
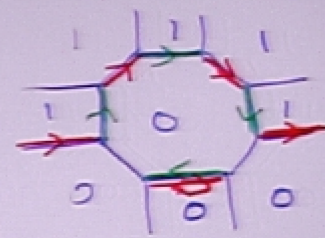


$h_z = 0,\ h_w = 0$



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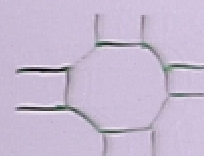

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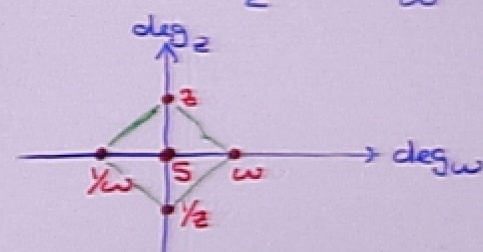

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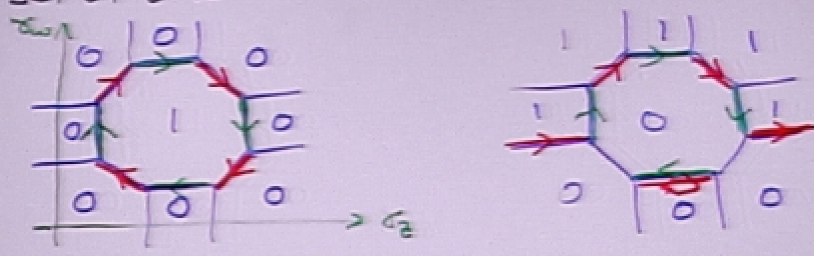
Then for every matching M, $M - M_0$ defines a set of oriented closed curves



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Writing the monomials in $\det K$ as points in $\mathbb{Z}^2$ graded by h gives a convex polygon

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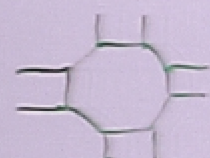

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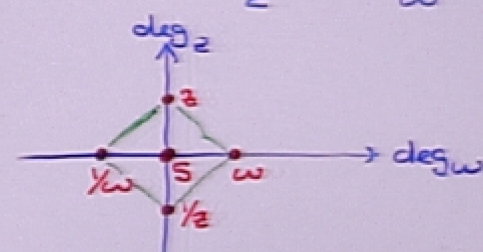

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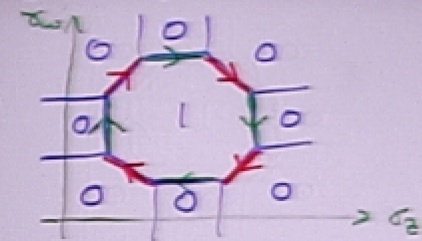
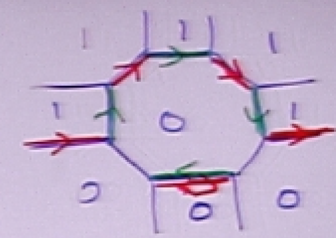
a set of oriented closed curves



$h_z = 0,\ h_w = 0$



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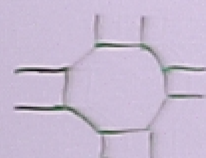

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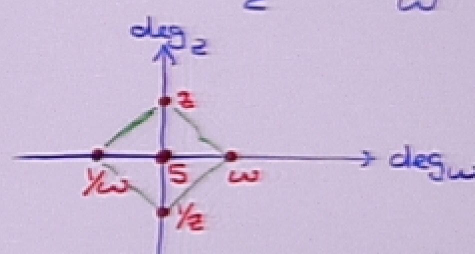

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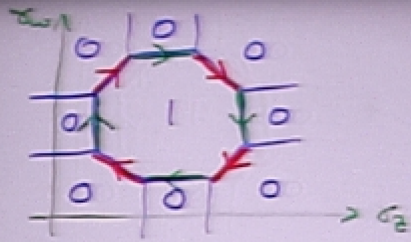

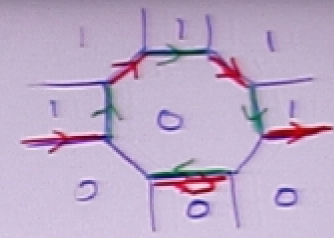

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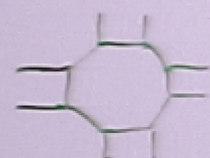

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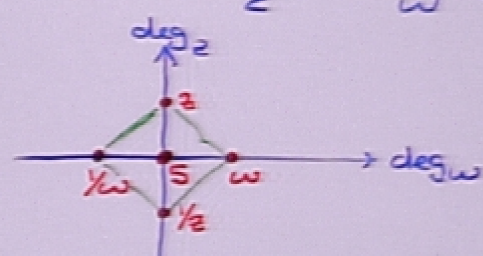

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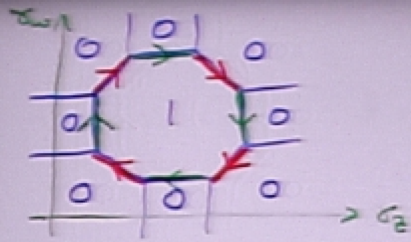

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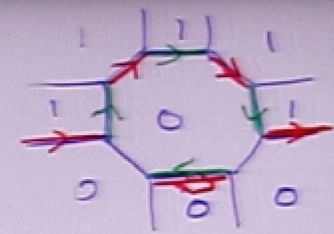

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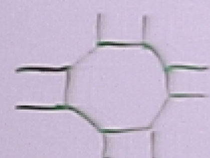

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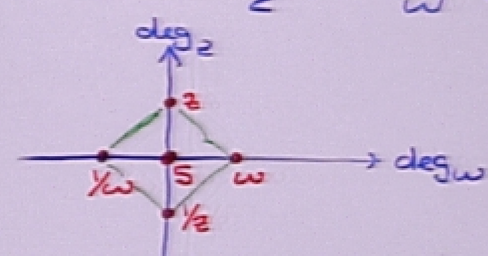

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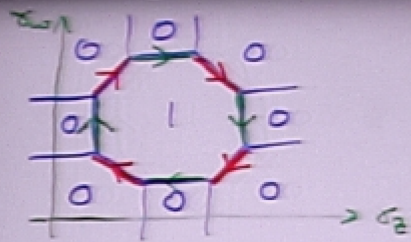

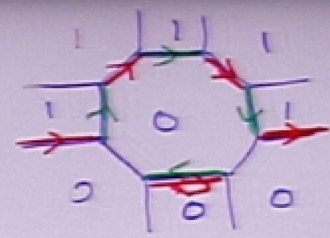

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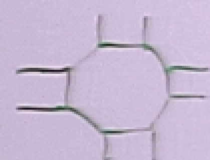

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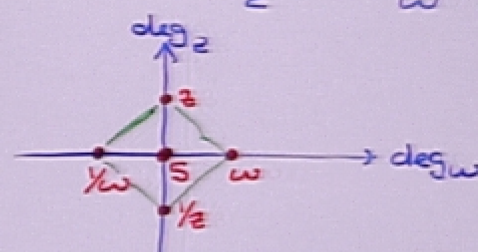

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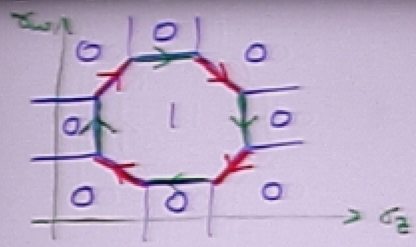

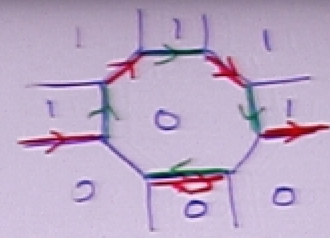

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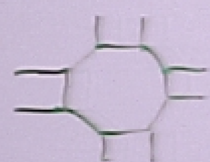

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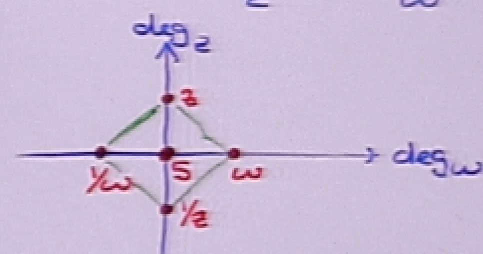

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Choose a fixed "reference" matching M_0.
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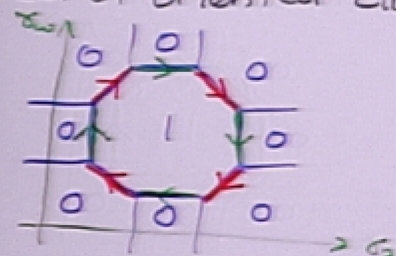

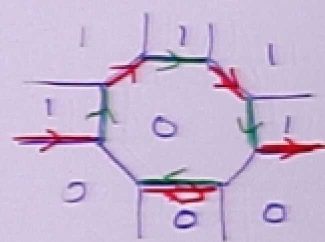

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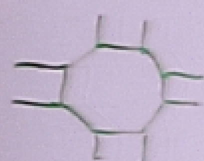

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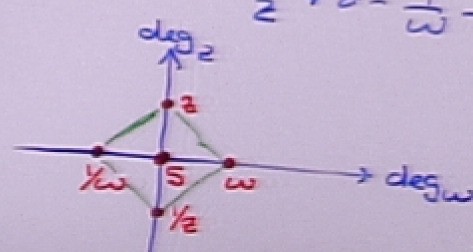

This is the toric diagram of the CY space that ...
the moduli space ...

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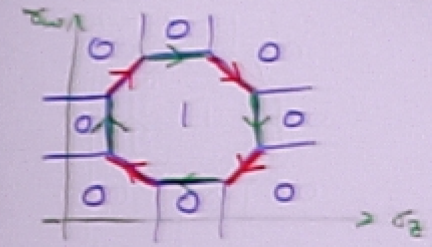

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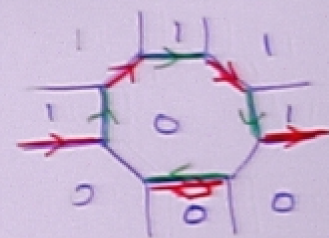

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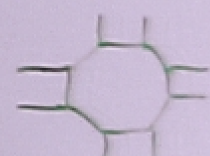

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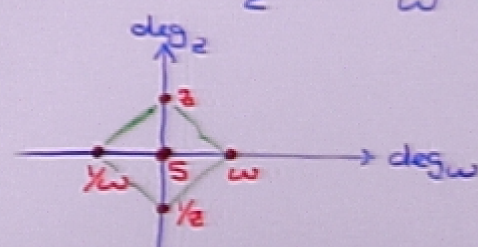

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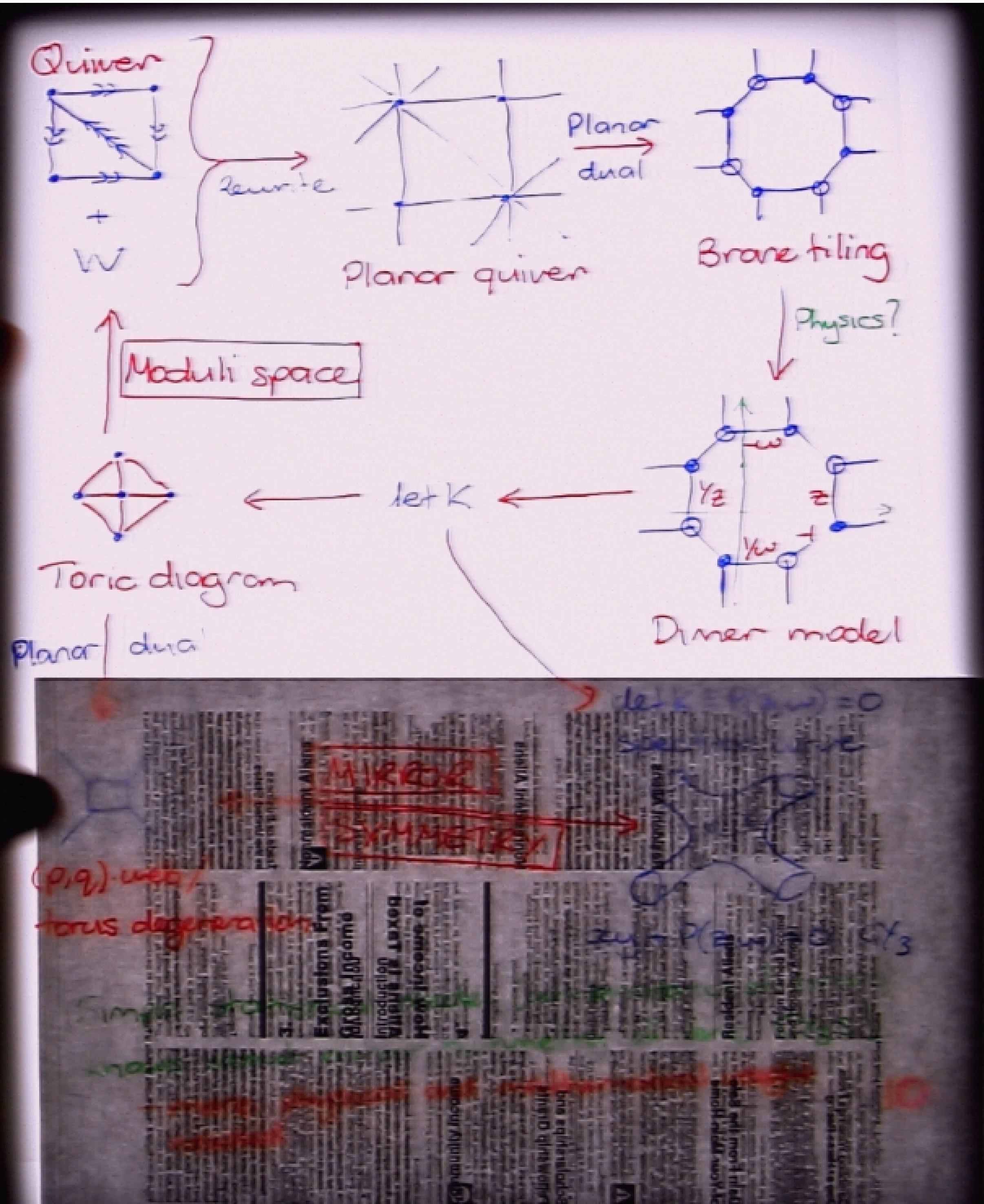
Quiver
+
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Planar quiver
Planar dual
Brane tiling
Physics?
Diner model
det K
Toric diagram
Moduli space
Planar dual
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torus degeneration
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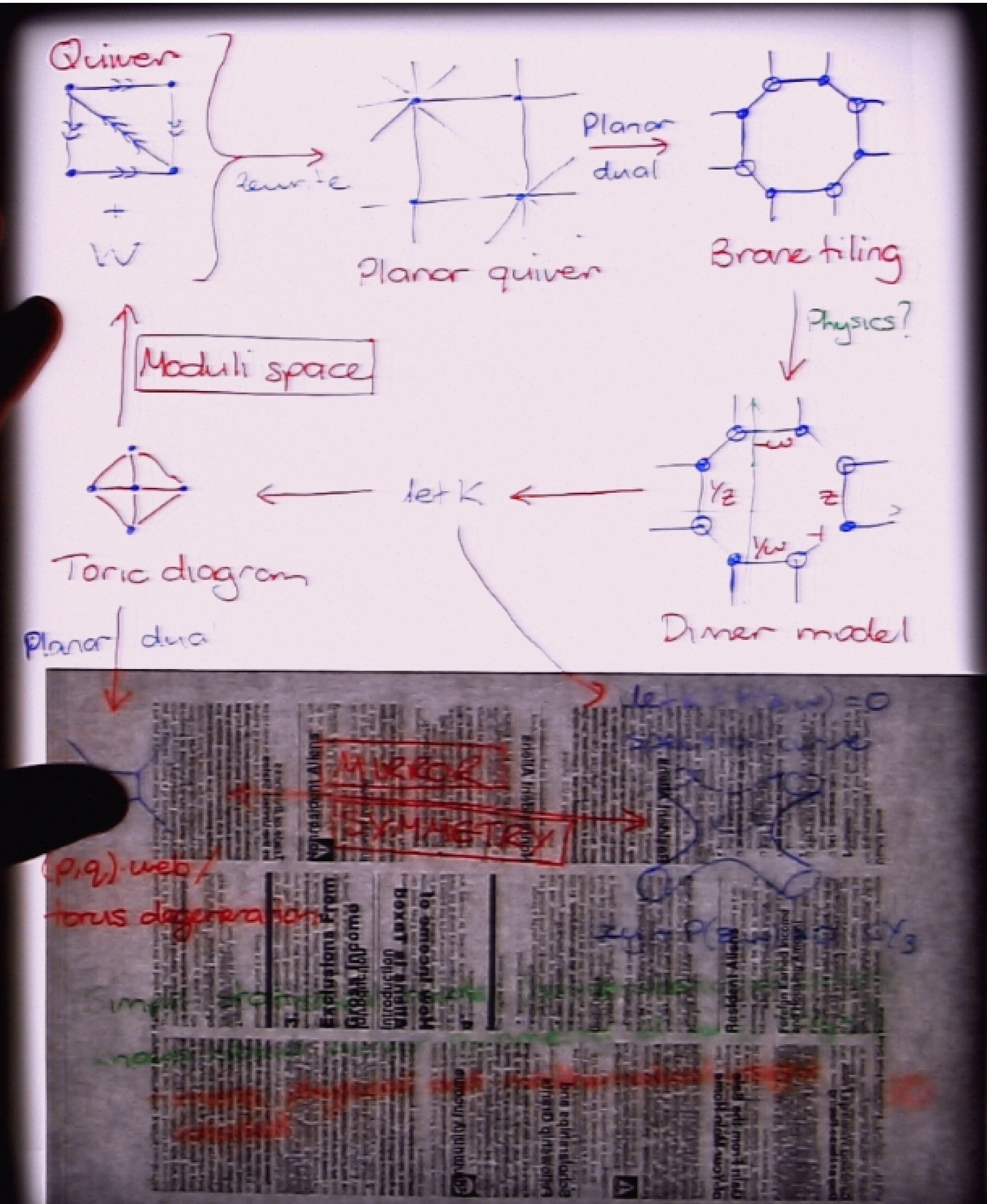
SYMMETRY
det K = P(z,w) = 0
zw = P(z,w) = 0 CY3



Quiver
+
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Physics?
Moduli space
Toric diagram
det K
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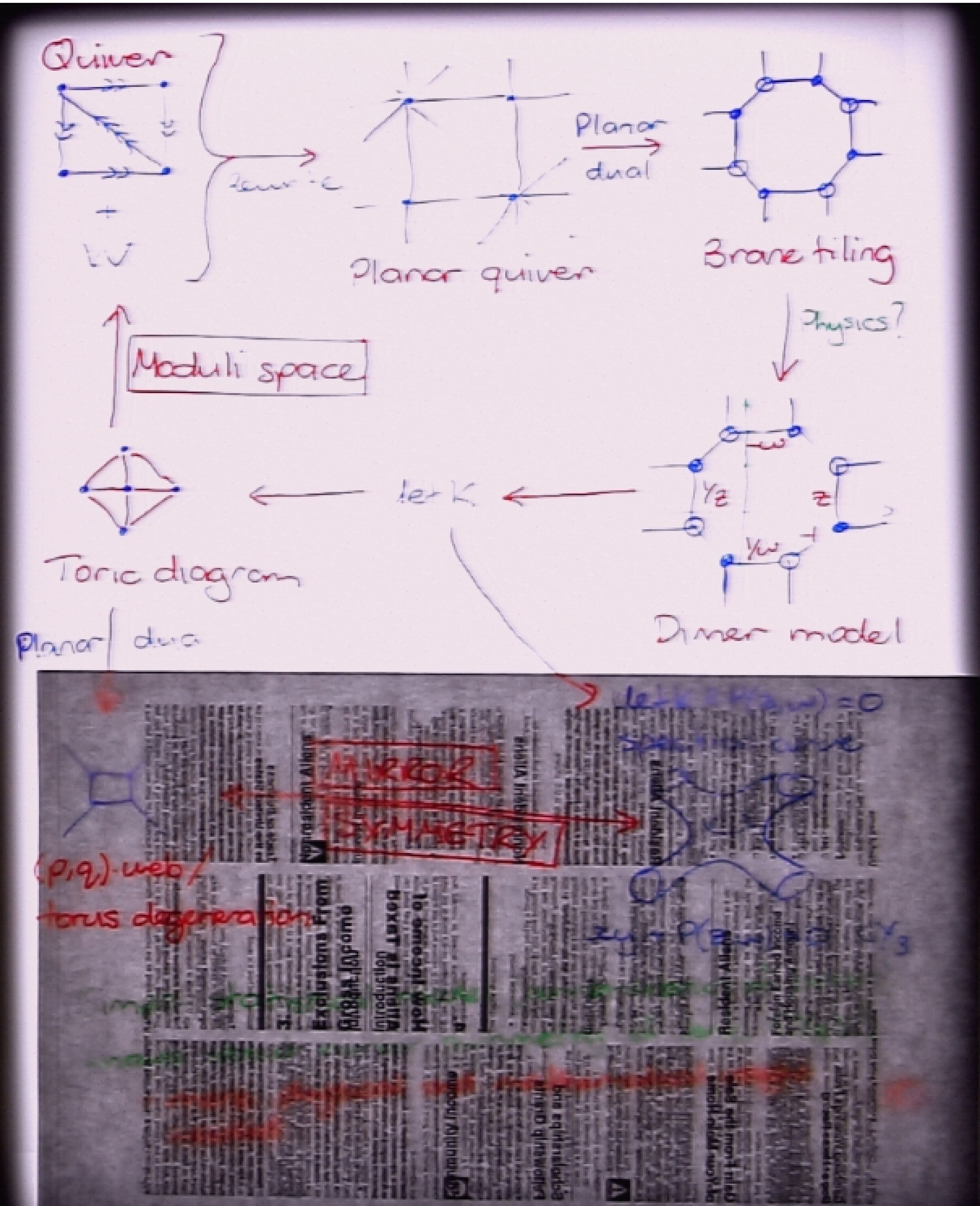
1/w
Dimer model
Planar dual
MIRROR
SYMMETRY
(p,q)-web/
torus degeneration



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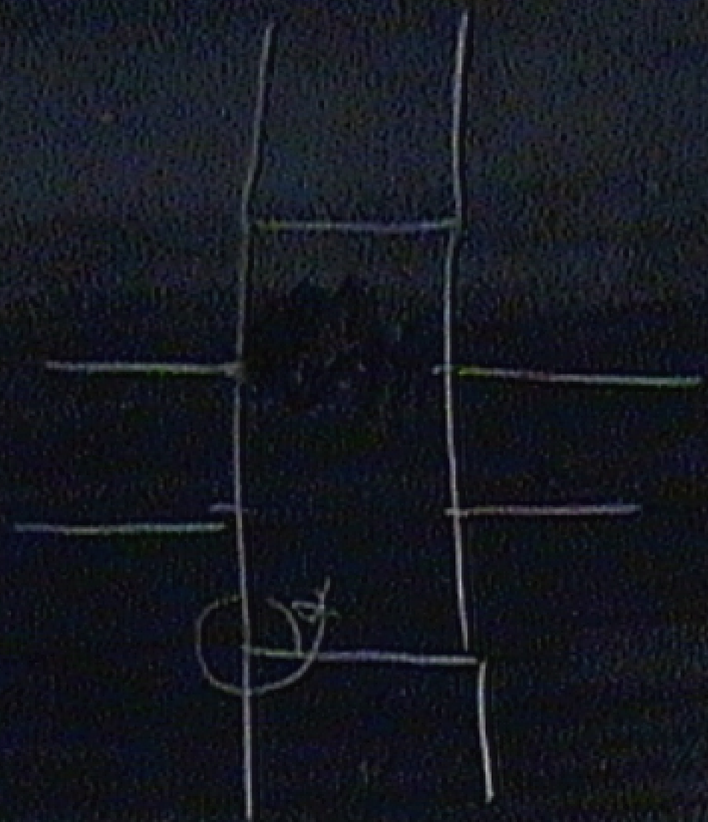


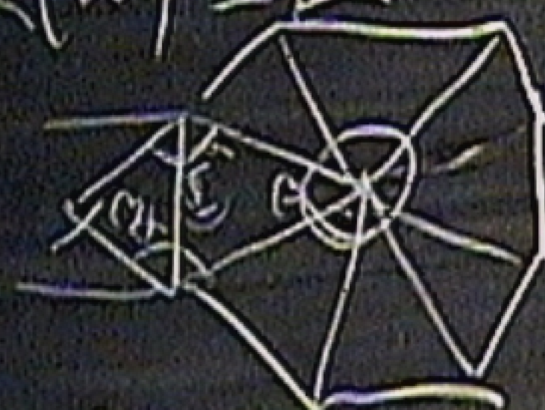
Wecht & Intriligator

a-max

$$\beta(g_c) = 0$$

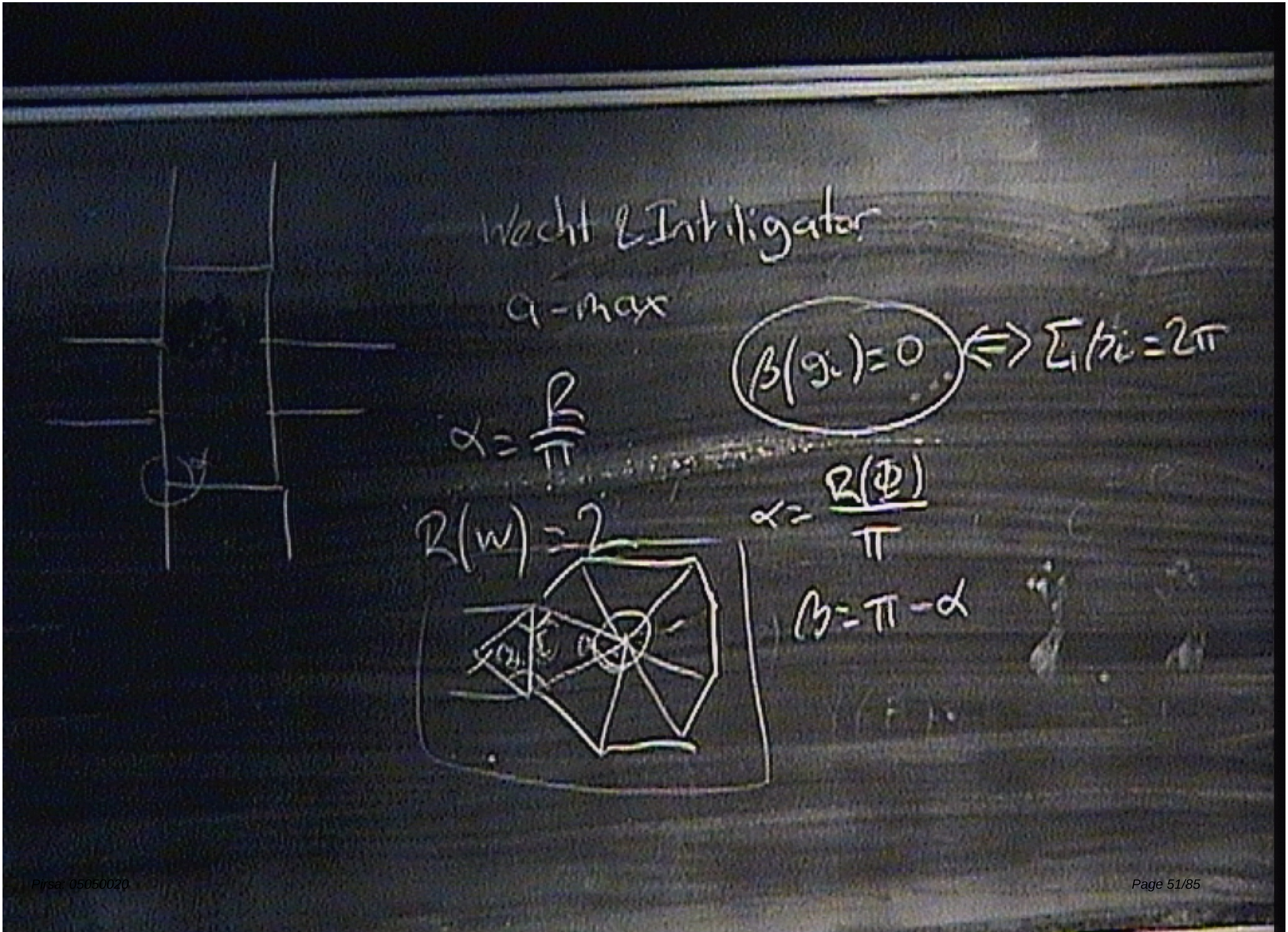
$$\alpha = \frac{R}{\pi}$$

$$R(W) = 2$$



$$\alpha = \frac{R(\Phi)}{\pi}$$

$$\beta = \pi - \alpha$$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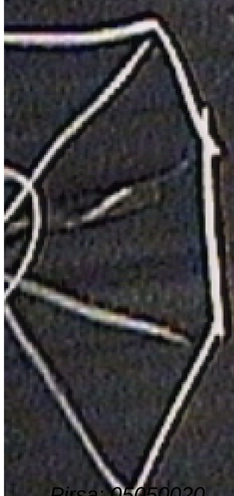
deform α, β

$\Rightarrow U(1)$ R-symm

mixes w/ $U(1)$



Intriligator

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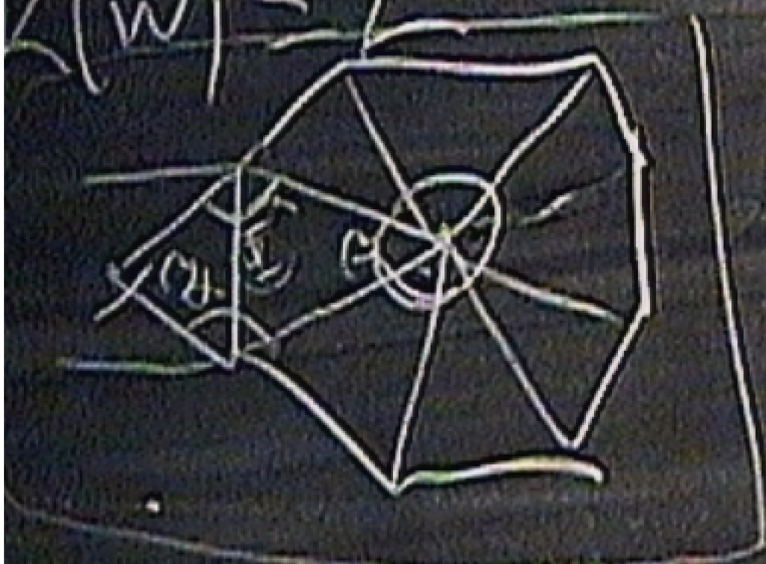

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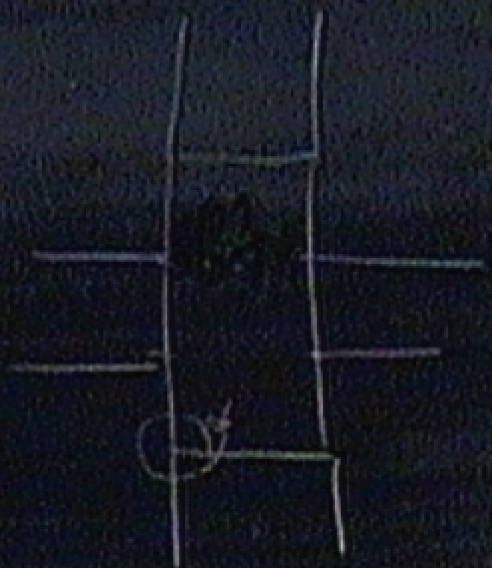
Preferred embedding from a-max,

$\cancel{TrR} - \underline{TrR^3}$



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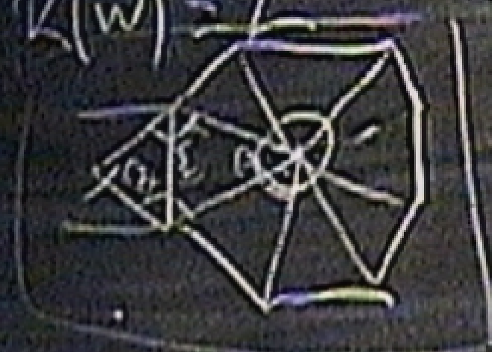

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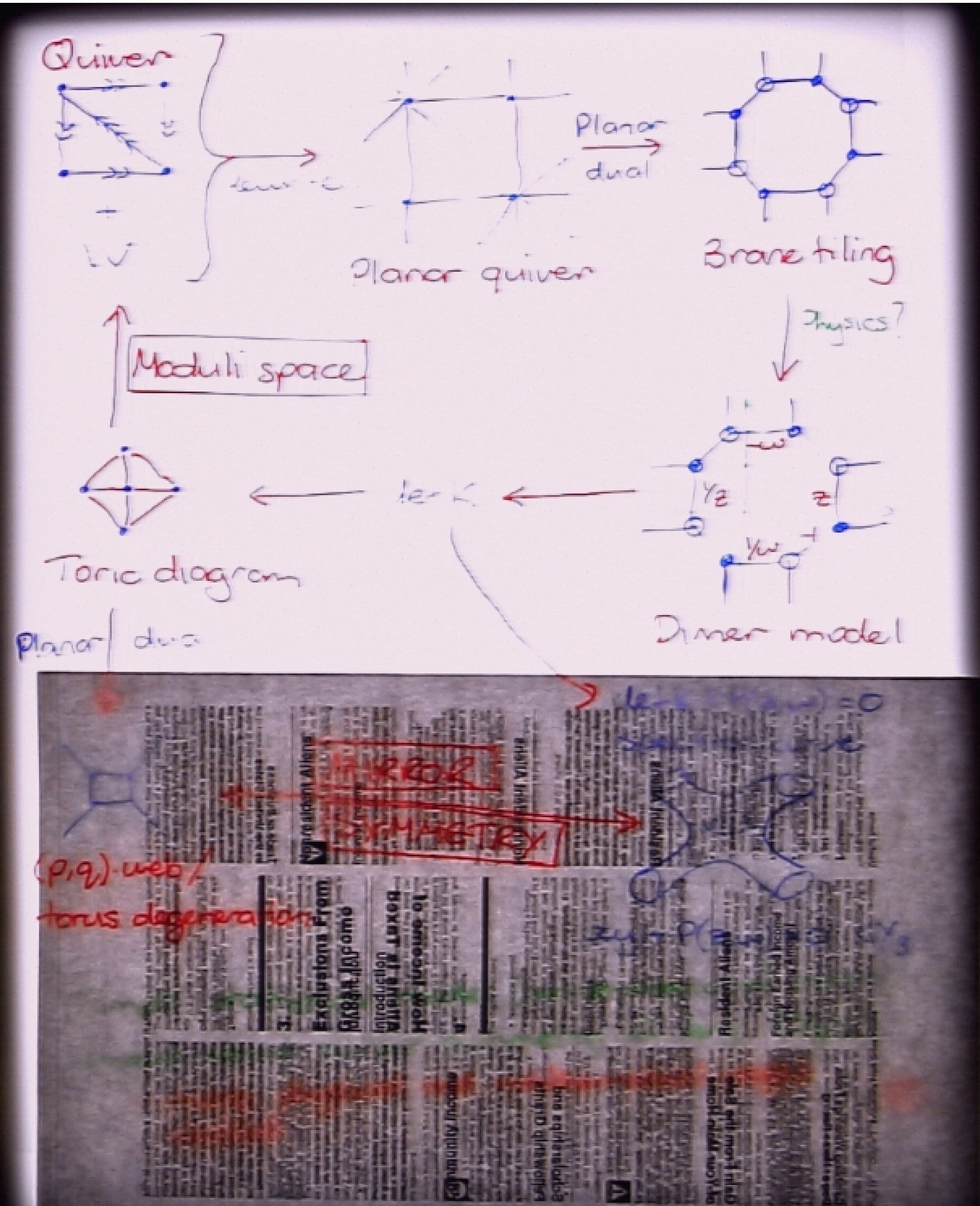
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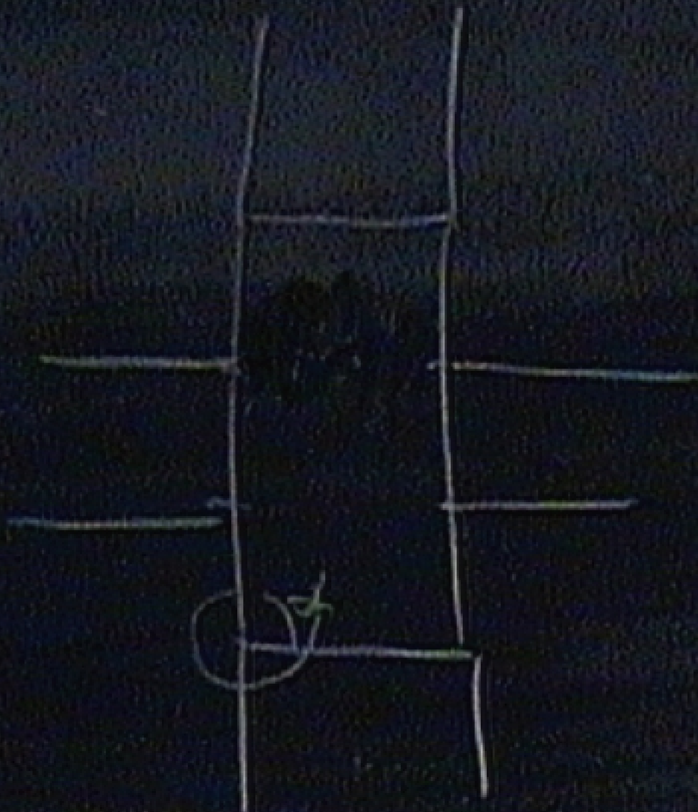

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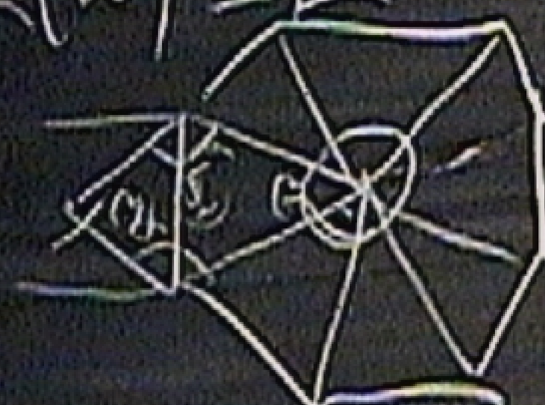

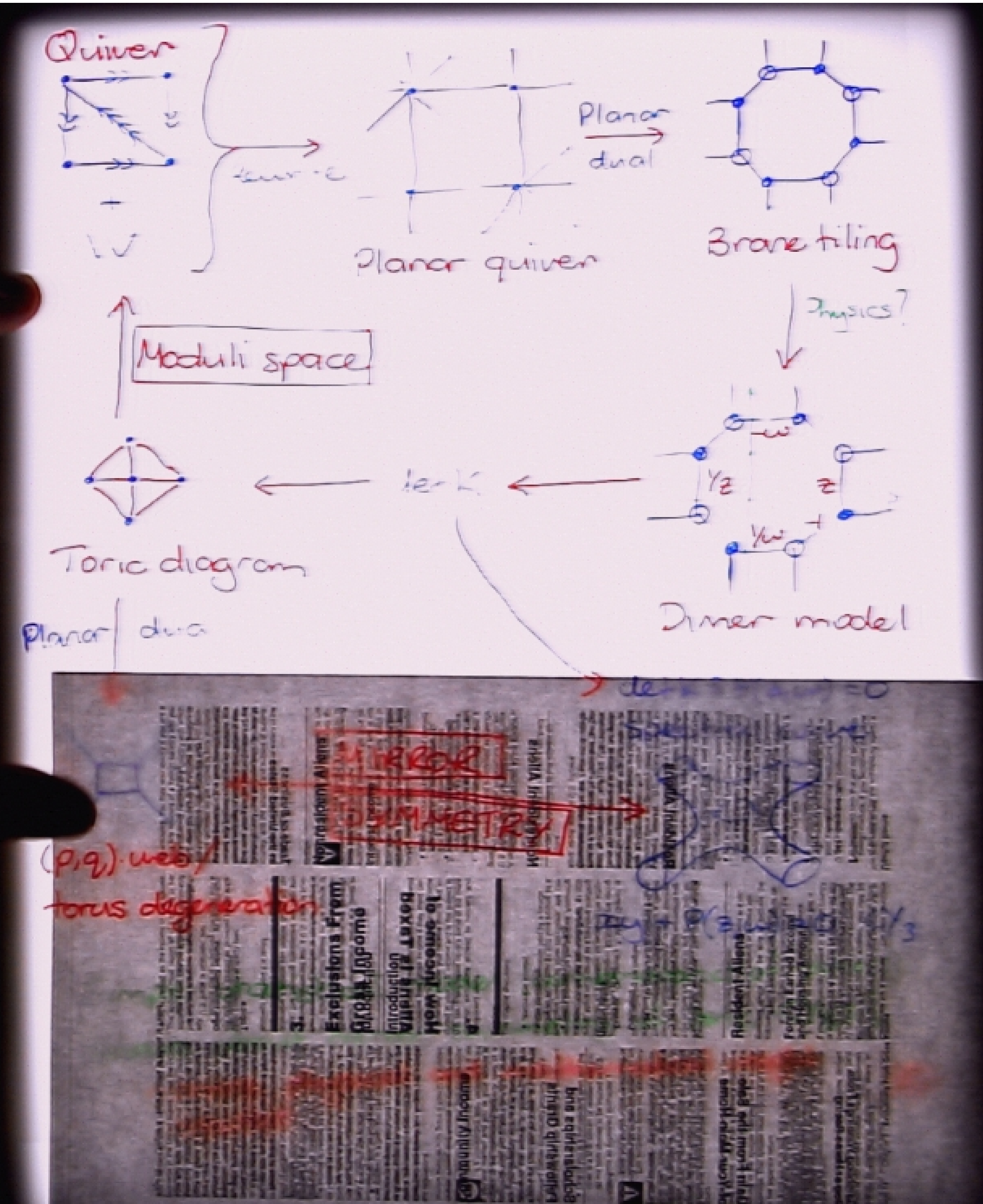

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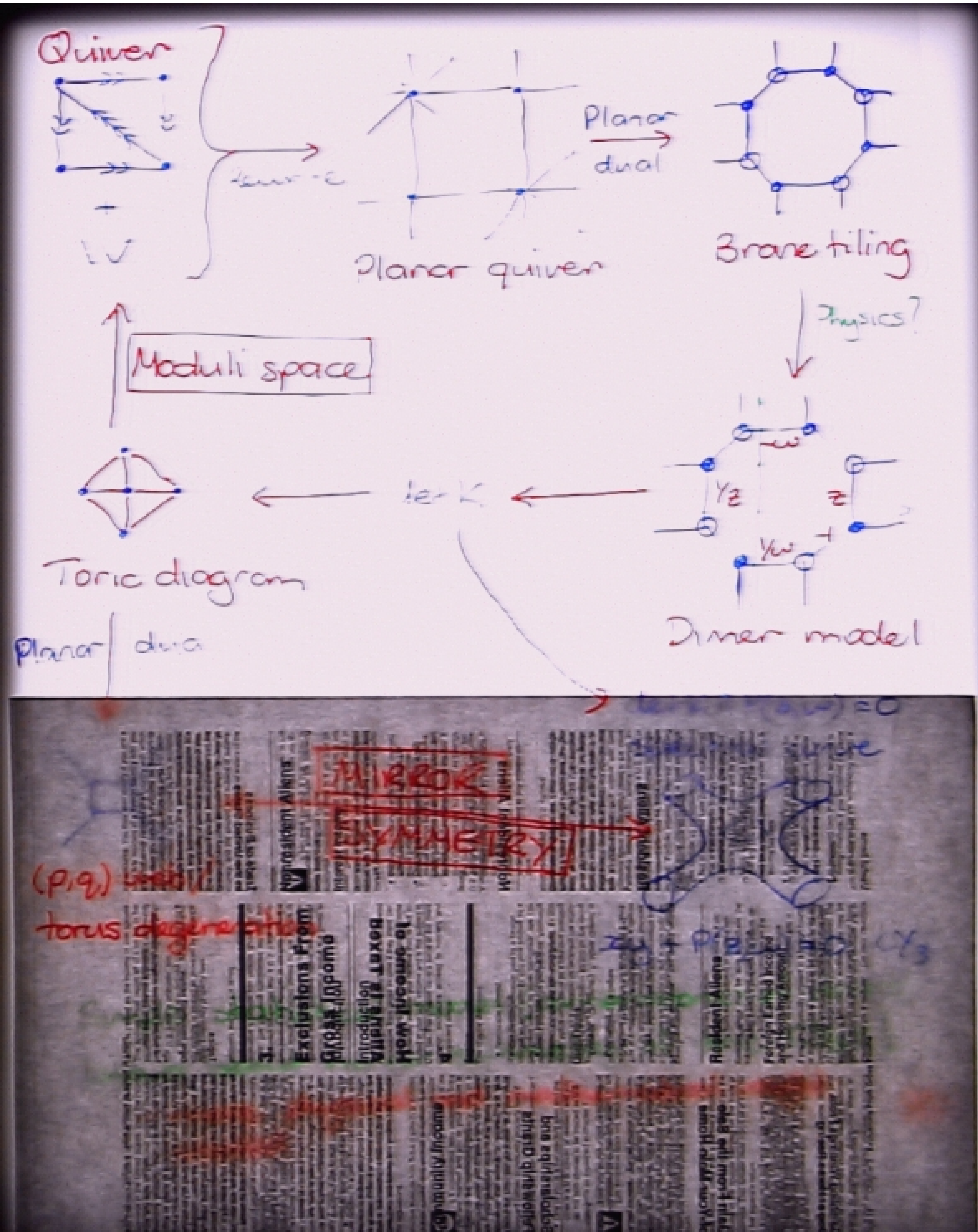
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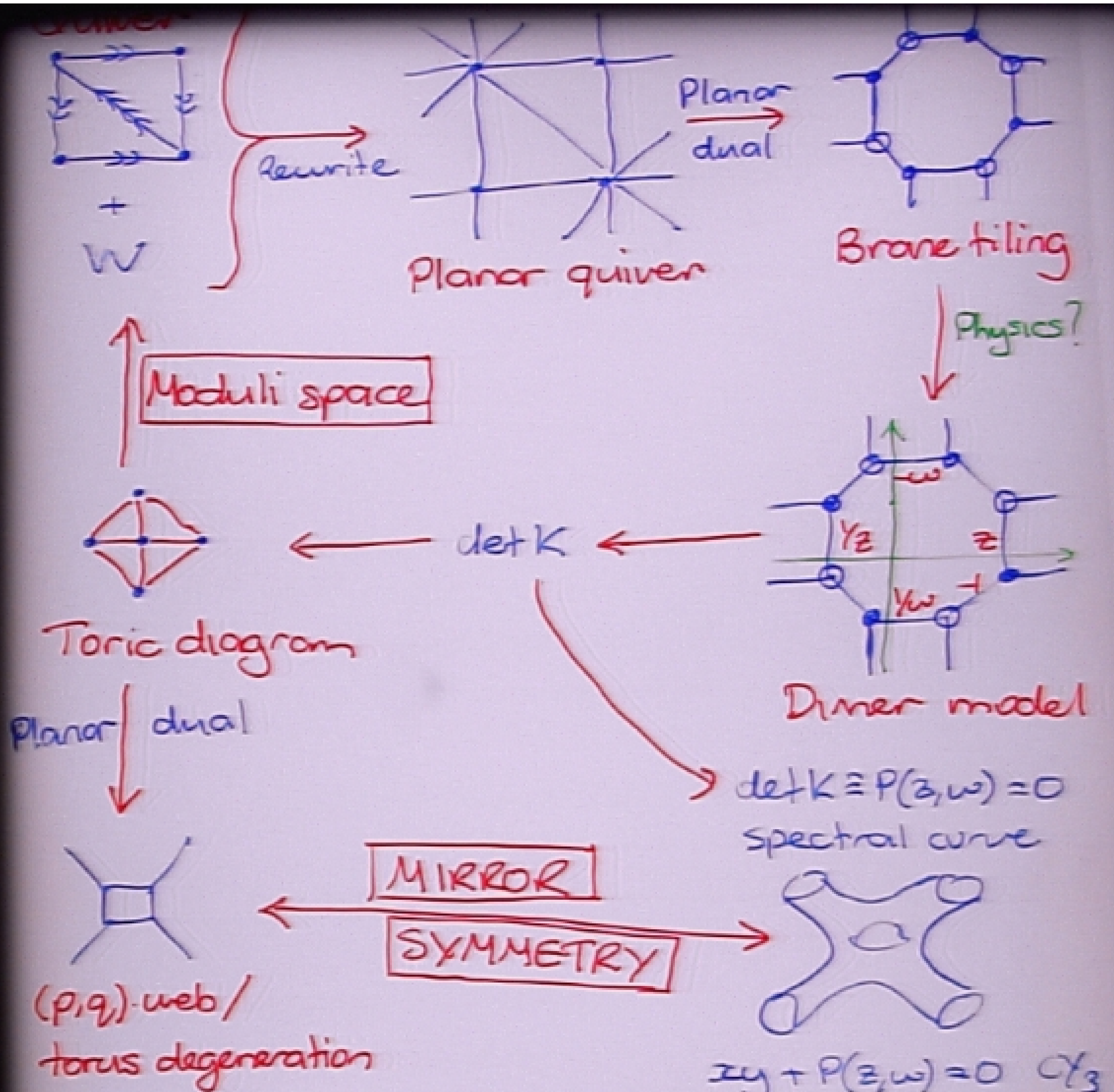

Simple statistical model (combinatorics of dimers)
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– more physical and mathematical insight needed

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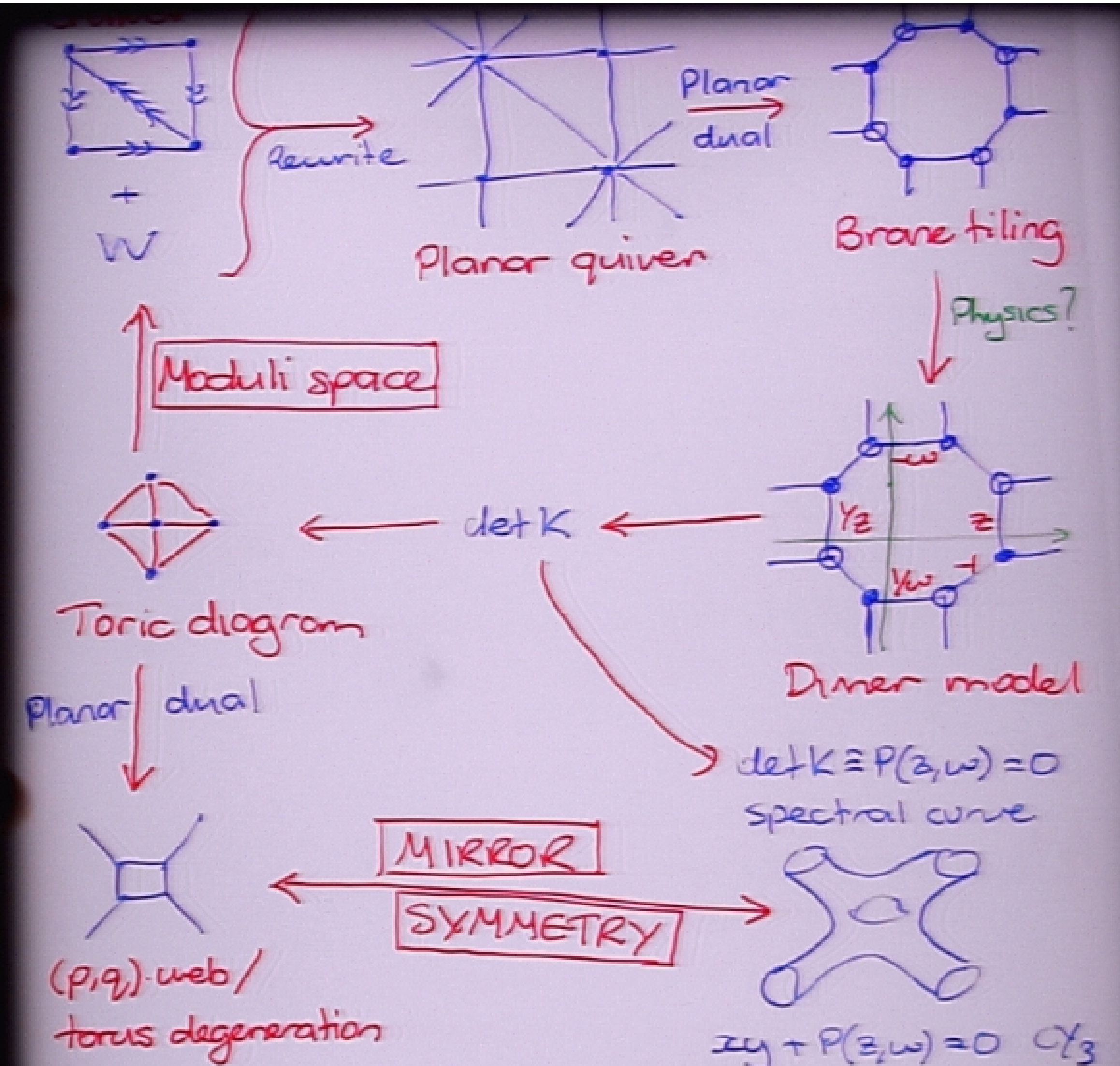

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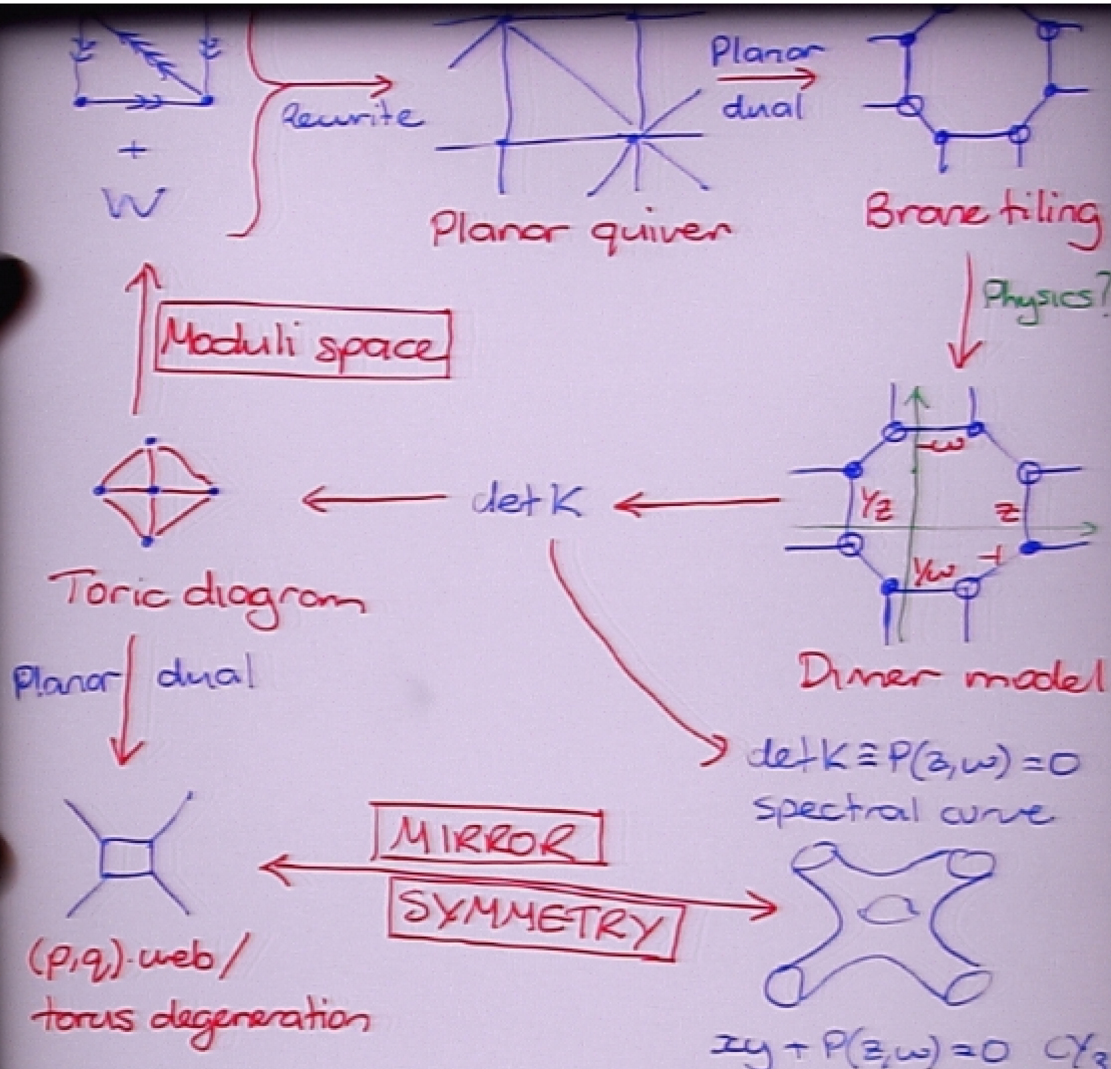

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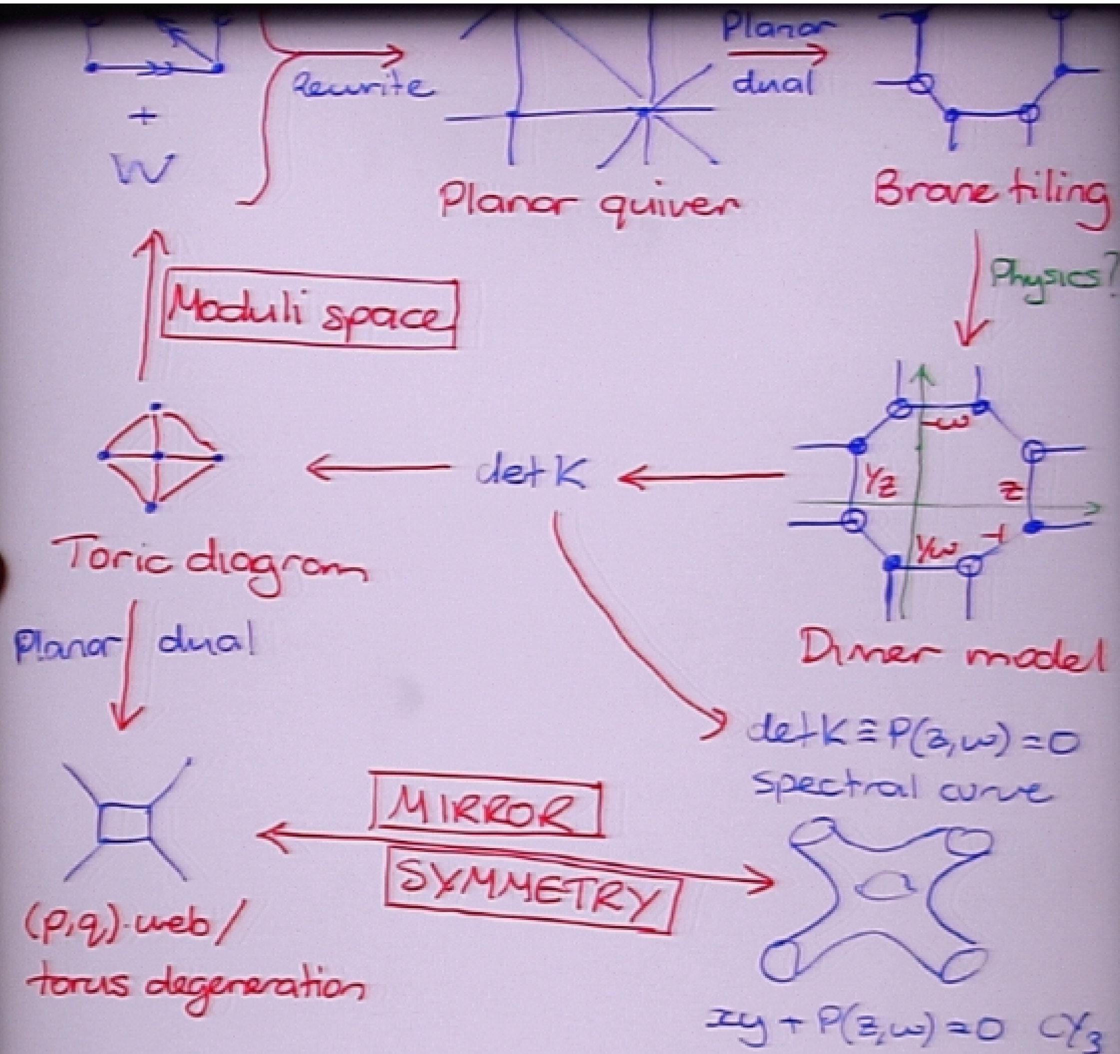

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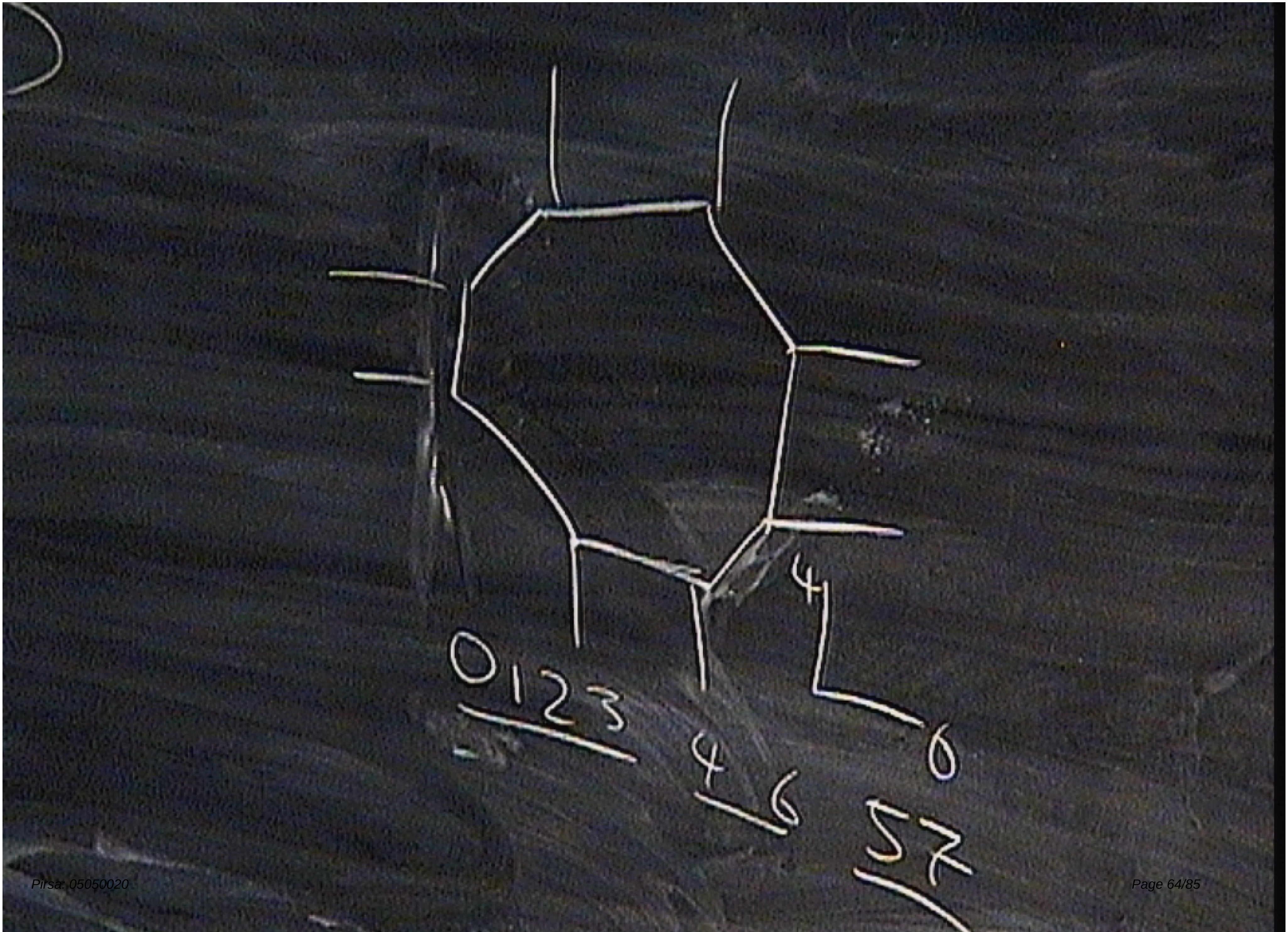
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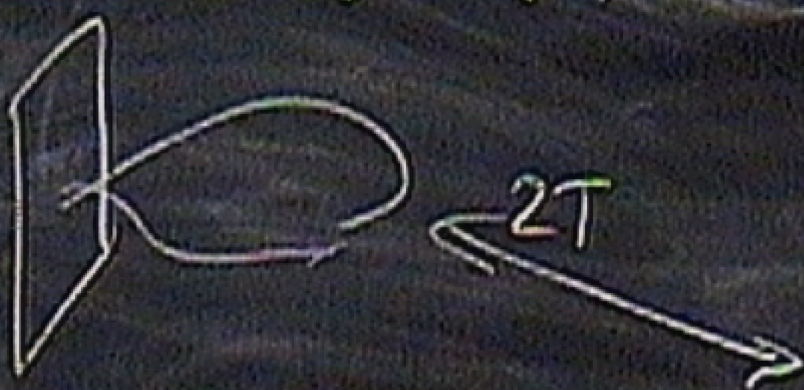
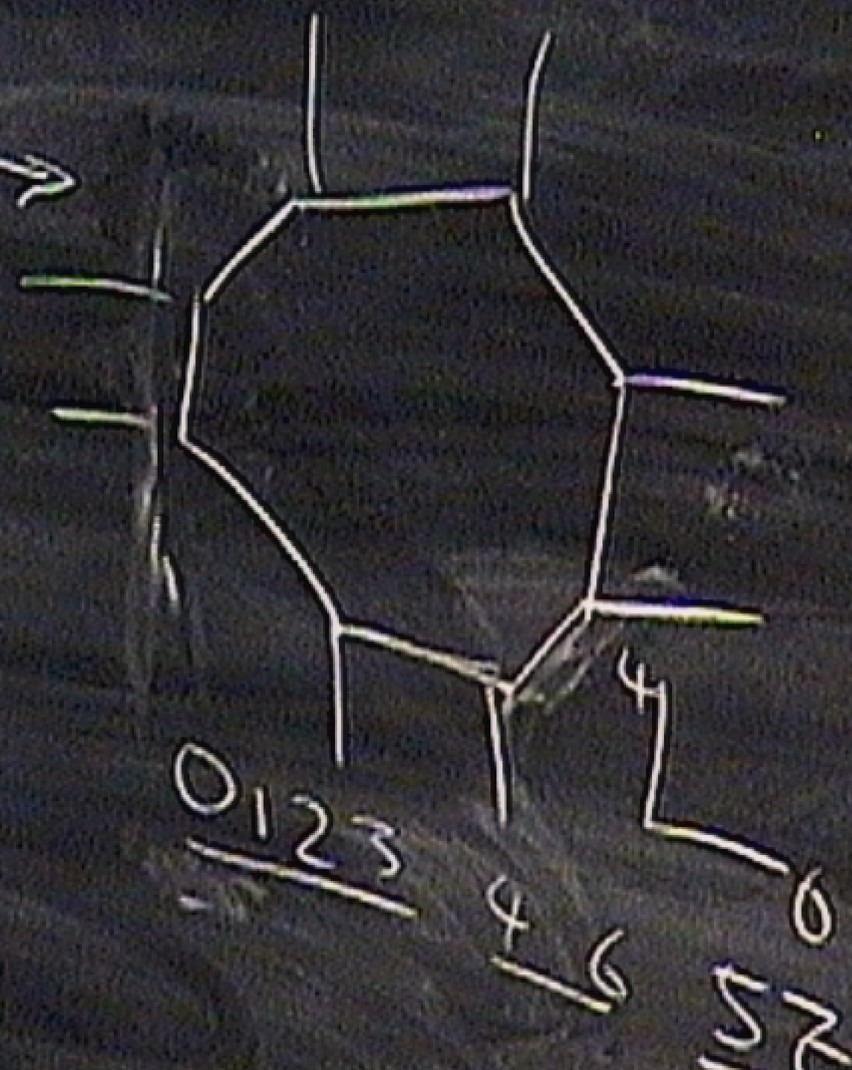
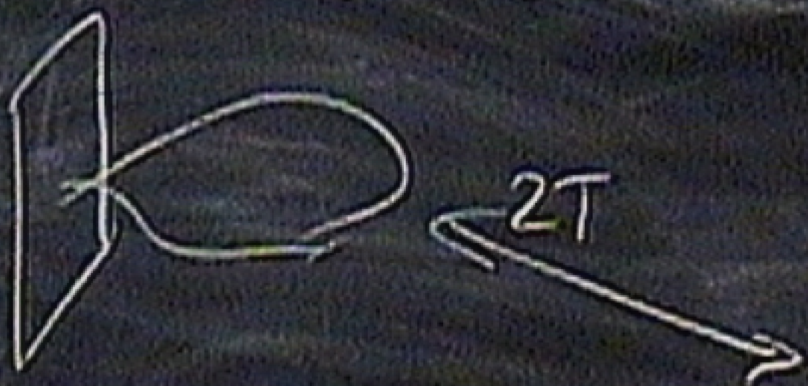
Aganagic, Karch, Lüst, Miemec



D3 at singular pt in CY_3



Aganagic, Karch, Lüst, Miemiec



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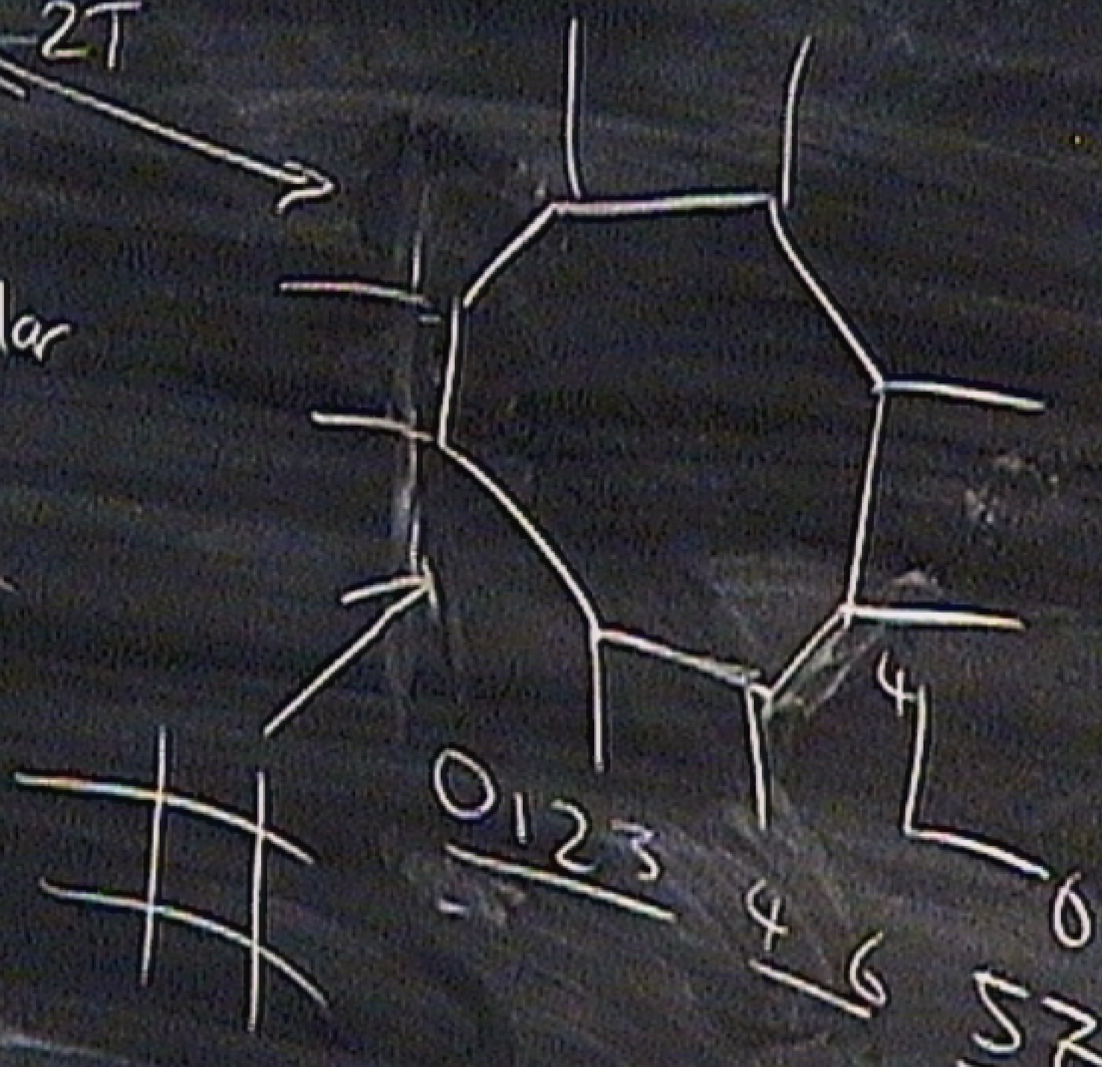

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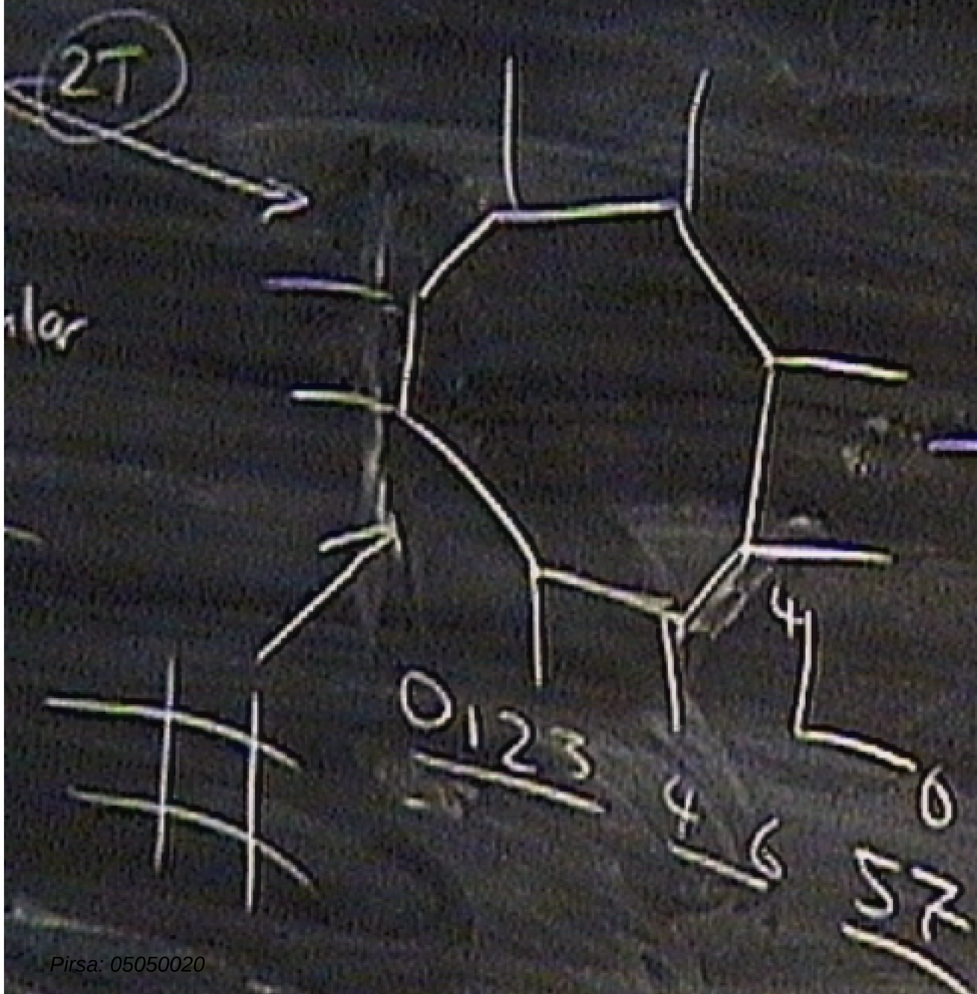
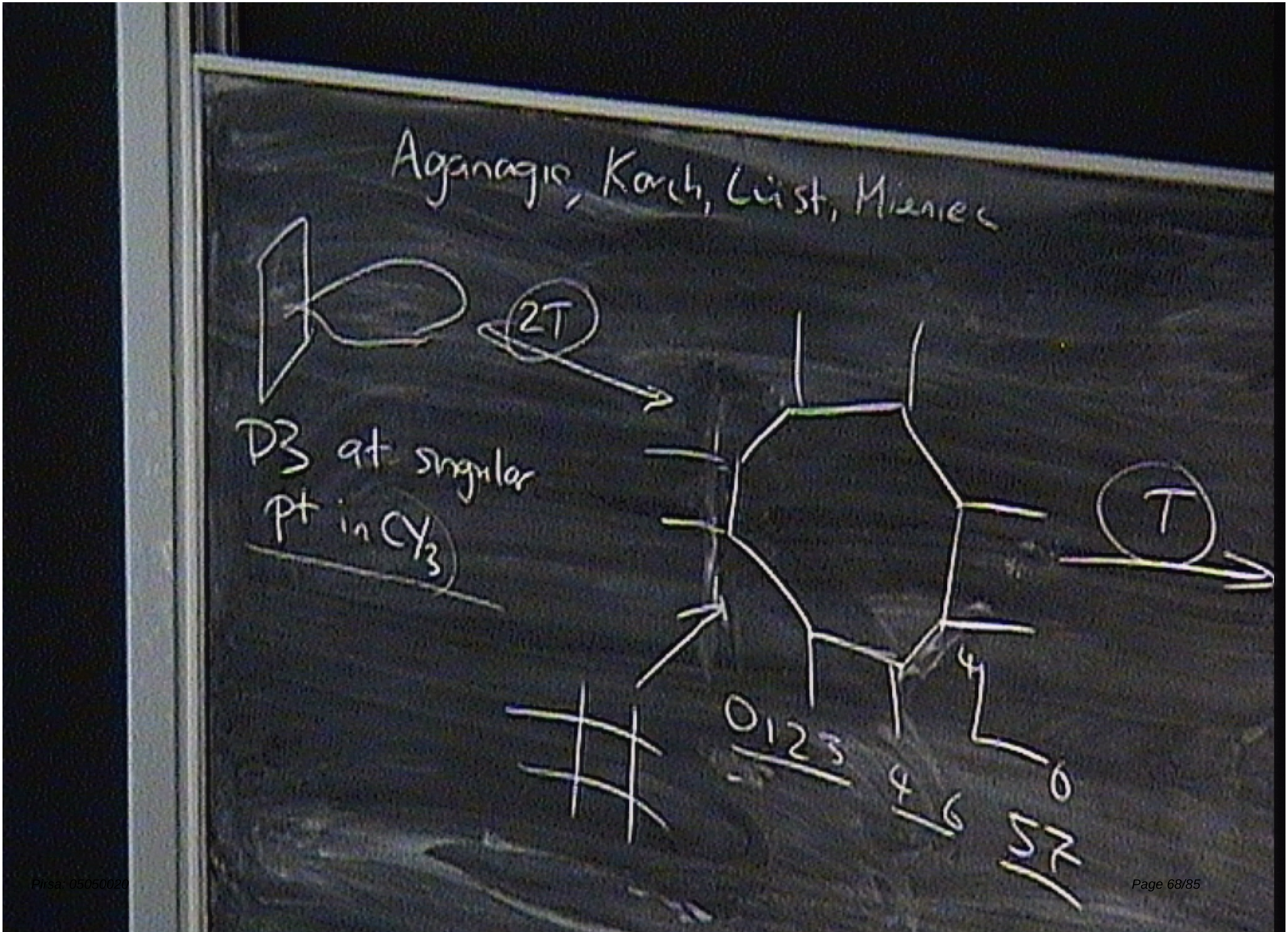


...ular

D6 wrapping
3 cycle
in $xy + P(z,w) = 0$


Aganagic, Karch, Lüst, Miemiec
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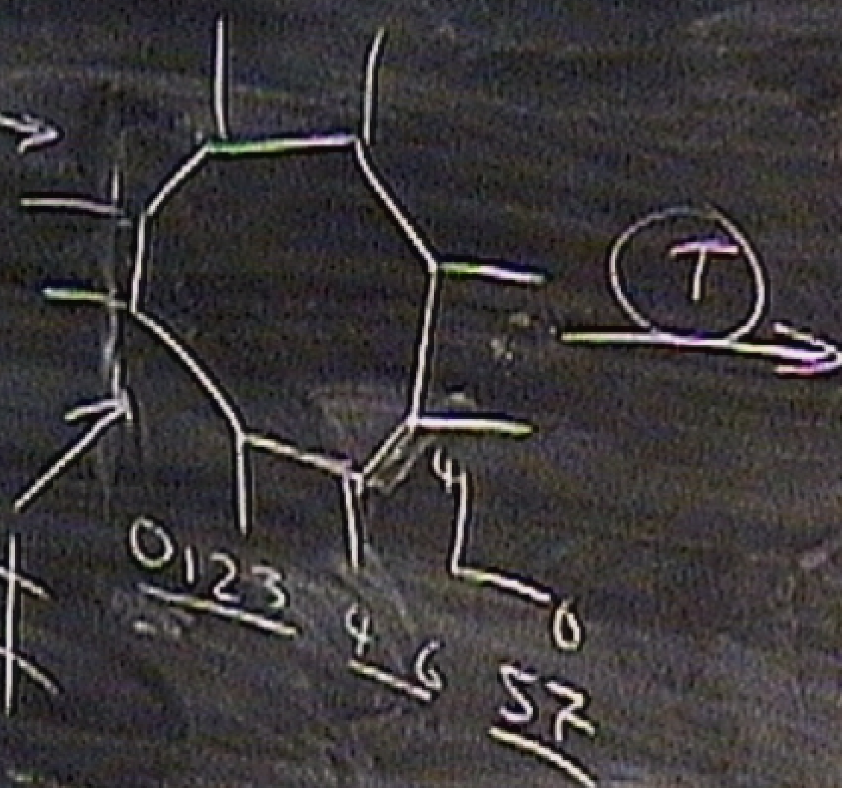
Aganagic, Karch, Lüst, Miemiec

D3 at singular pt in CY_3

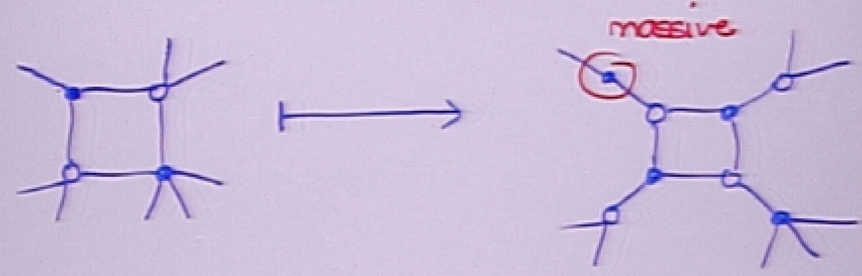
Rank, fc conds, hol curves



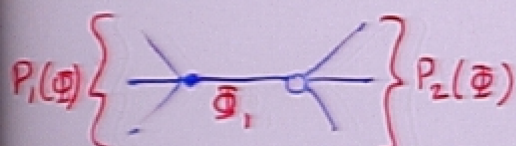
D6 wrapping 3 cycle in $zy + P(z,w) = 0$

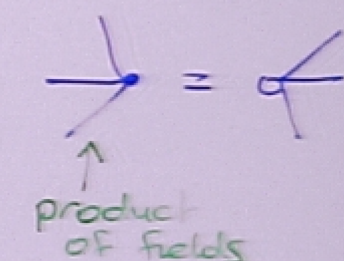
• Insight into properties of gauge theory from brane tiling/dimer model

• phases of theory under Seiberg duality



• F-term constraints





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• Proof of equivalence of dimer results with previous Forward Algorithm for moduli space

→ "Fast Forward Algorithm"

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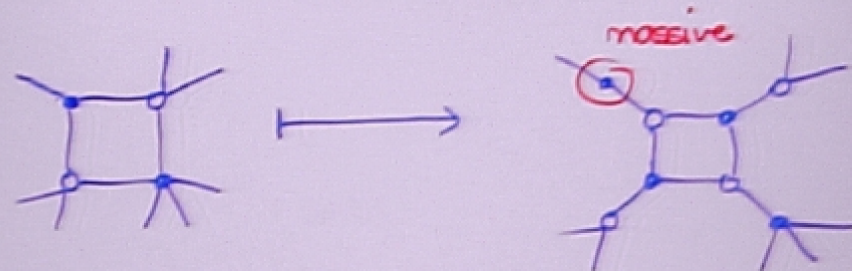

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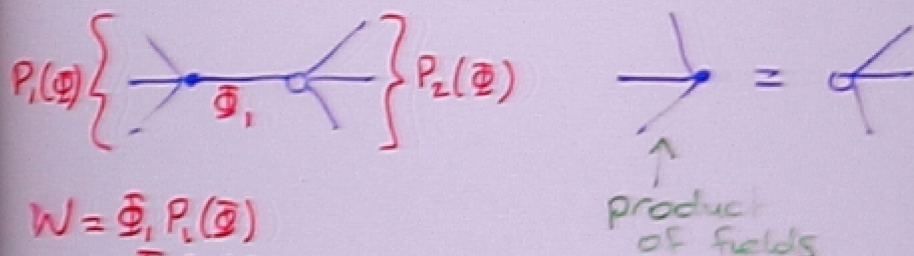

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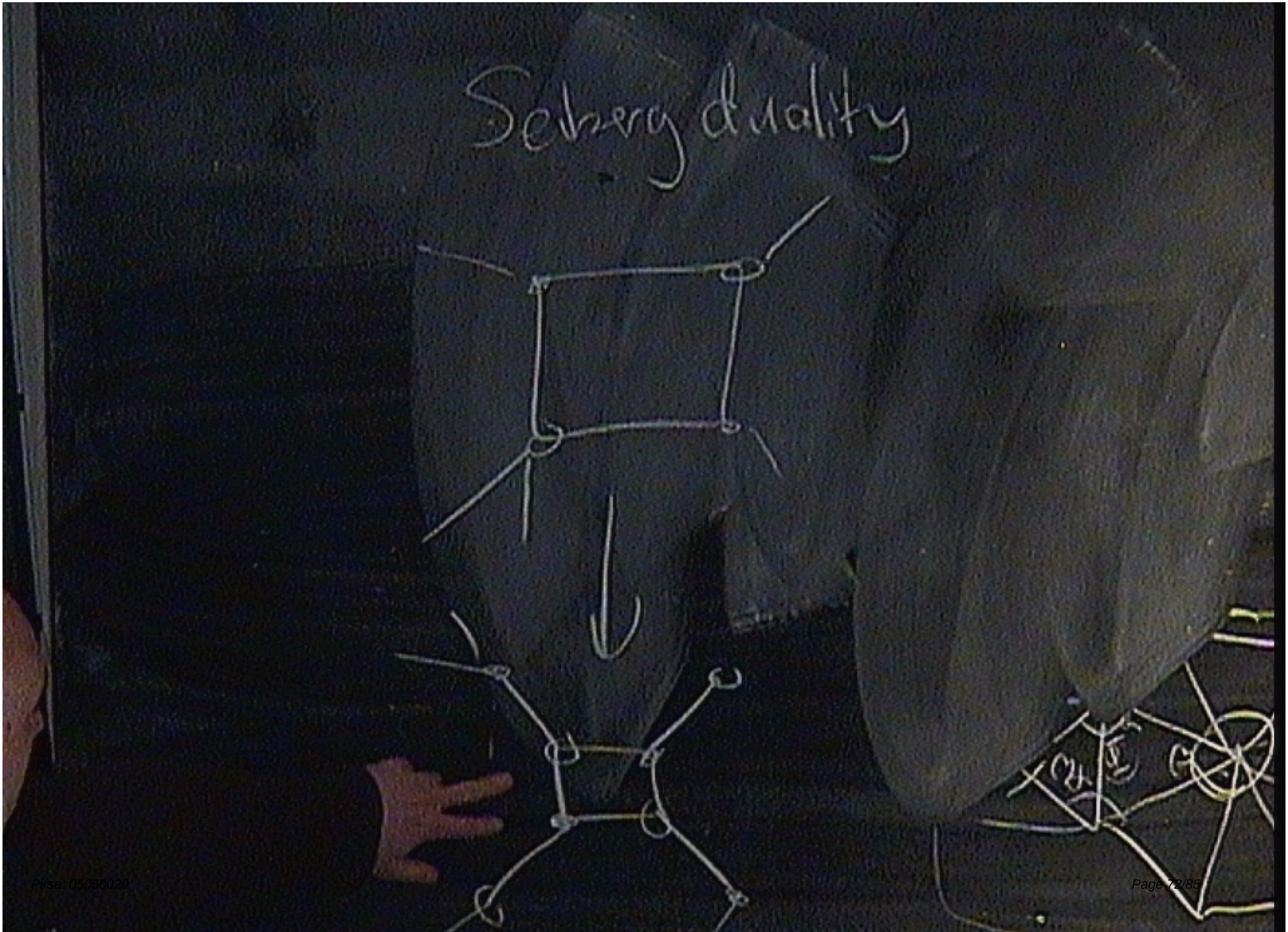
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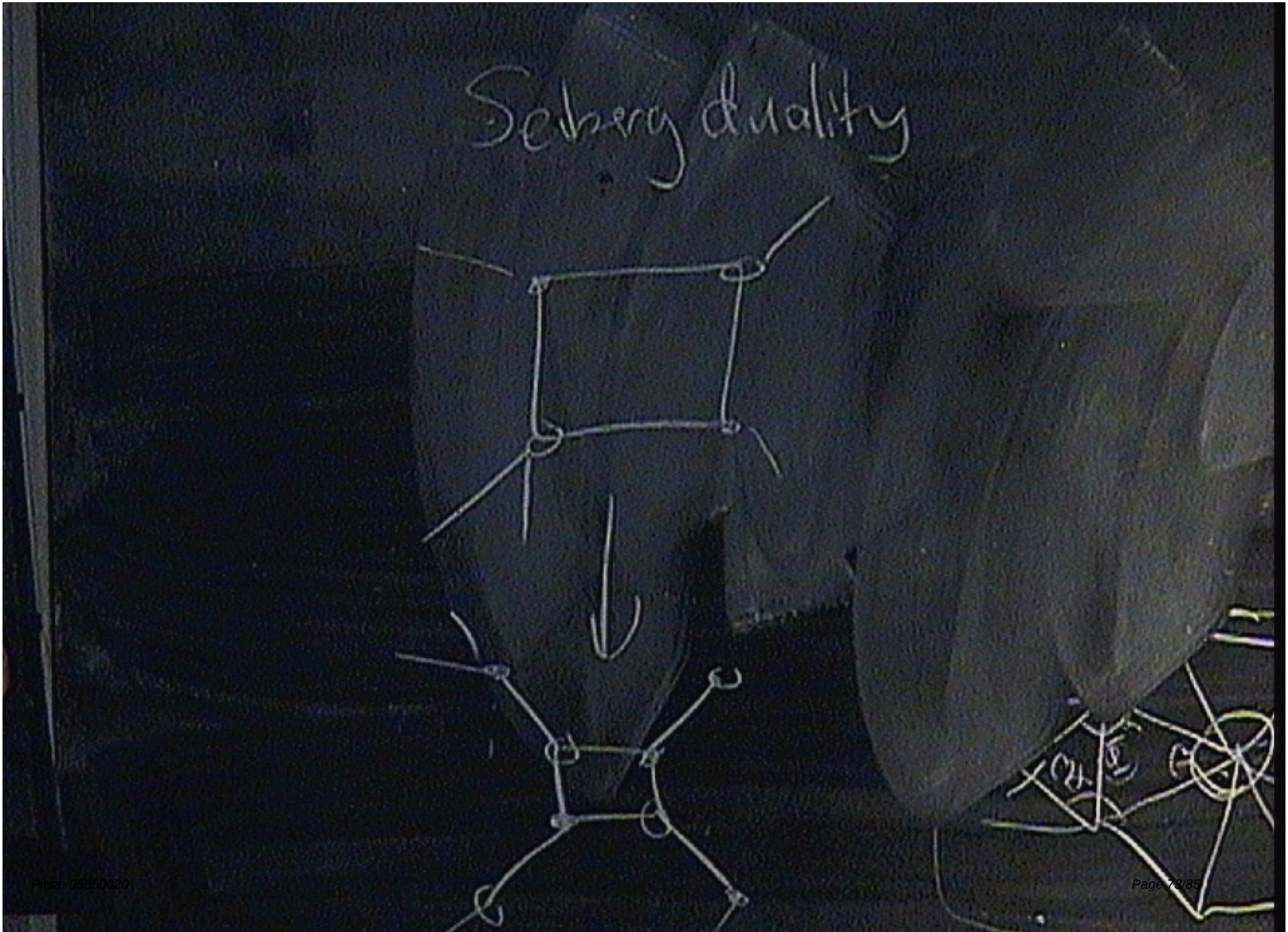
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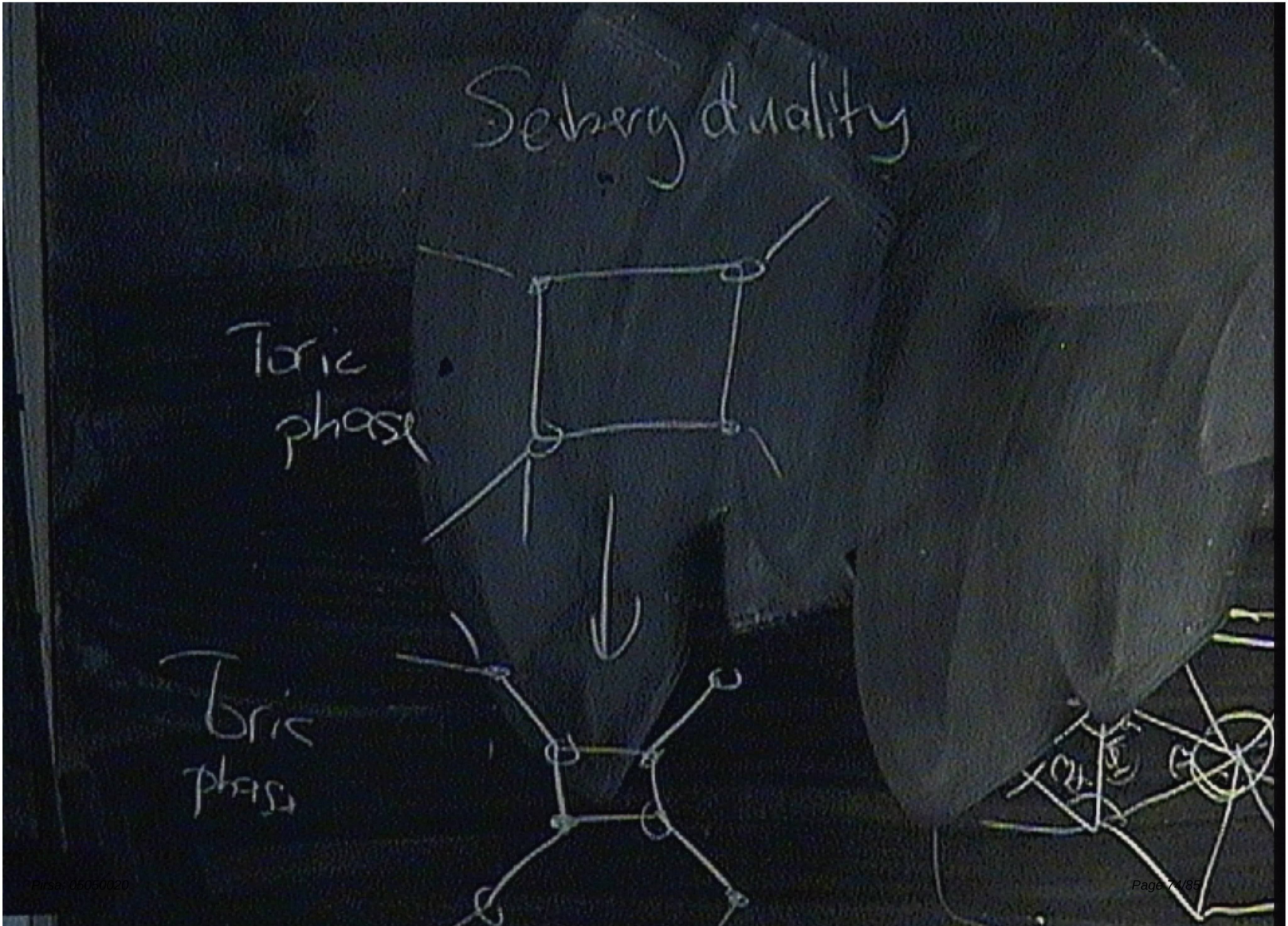
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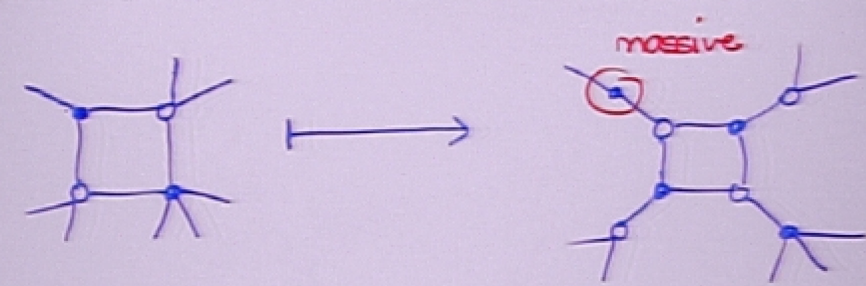

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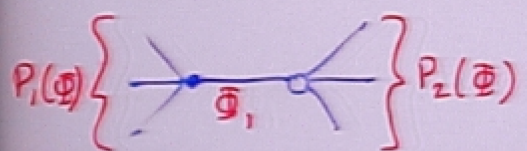

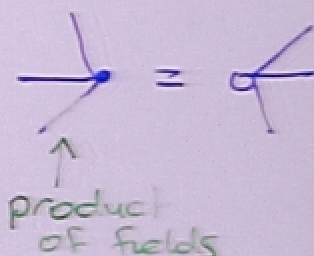

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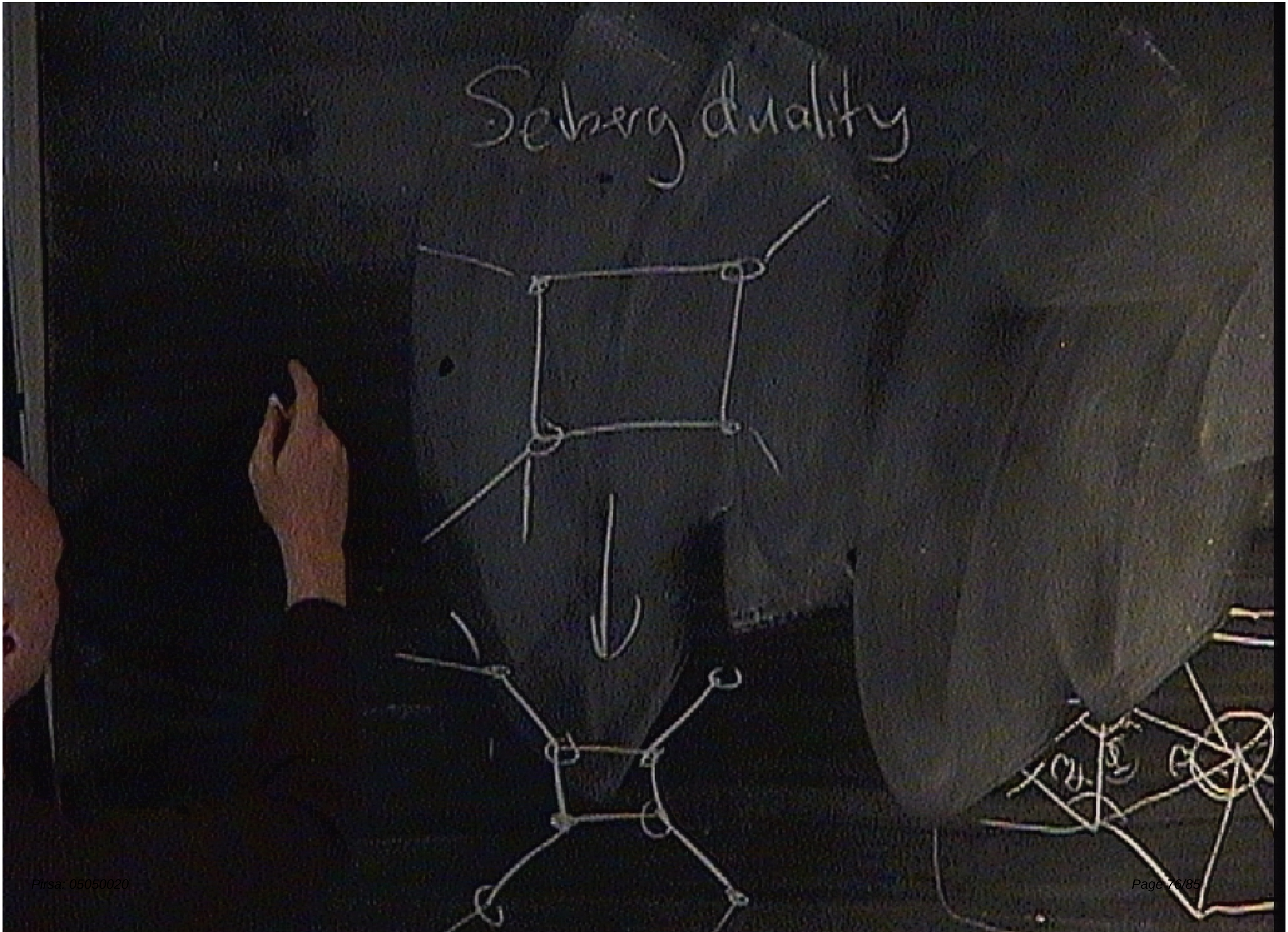
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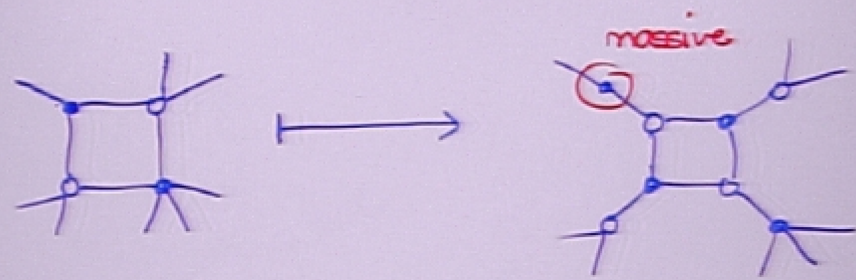

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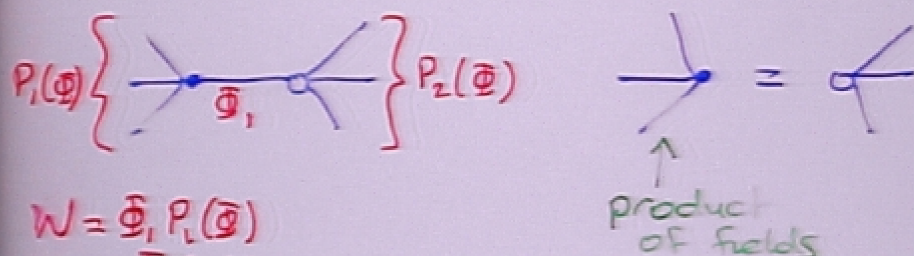

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91

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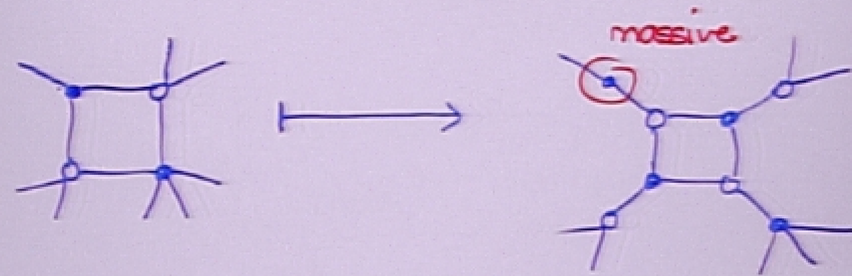

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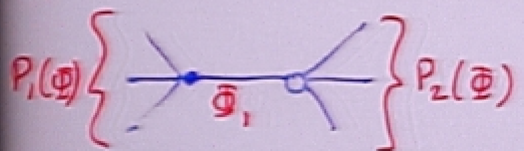

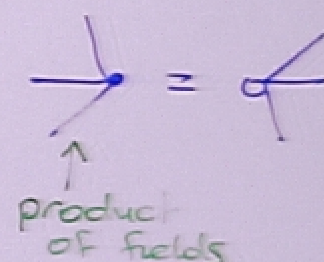

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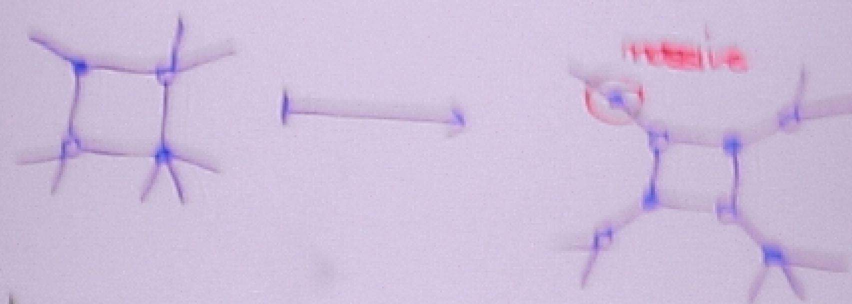

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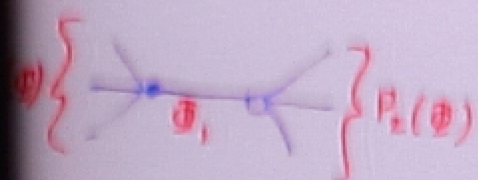

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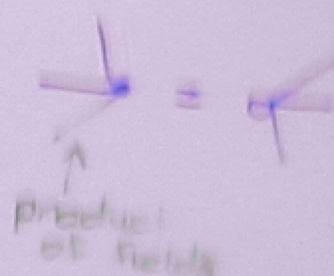

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SUMMARY

- Combining the toric quiver + super-potential into the periodic quiver exploits the hidden structure of these theories
  - better representation of gauge theory data; matter content and interactions
- dual graph (brane tiling) is physical - describes system of intersecting NS5 and D5 that engineers the quiver theory
- playground for dimer models; provides powerful tools for computing properties of gauge theory (eg. moduli space)
  - proof of results, but lacking physical understanding
- dimer models have many other magical properties that should be studied from physical and mathematical point of view!
- Supersymmetry of brane tilings
- a-maximization
- mirror symmetry

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