Title: Quantum Filtering and Nonlinear Dynamics

Date: May 11, 2005 01:40 PM

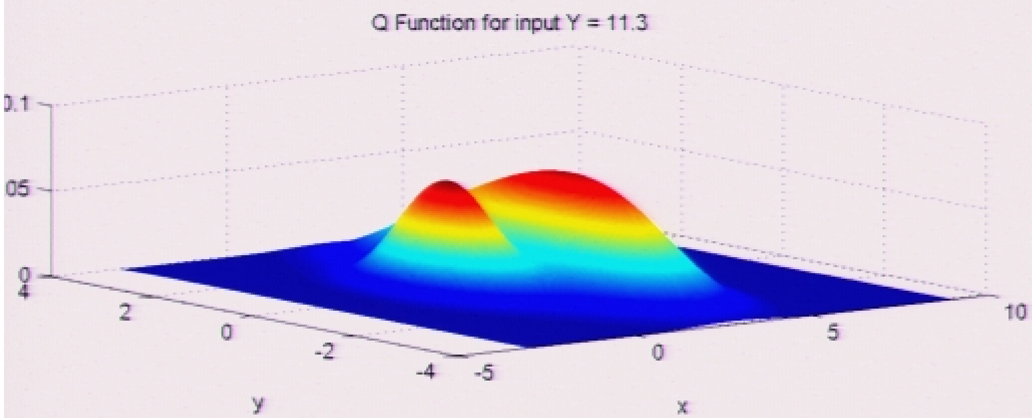
URL: http://pirsa.org/05050019

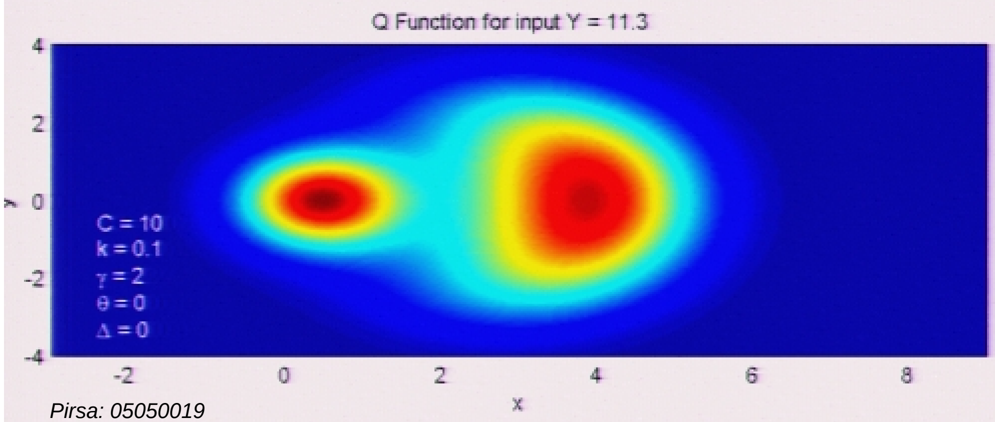
Abstract:

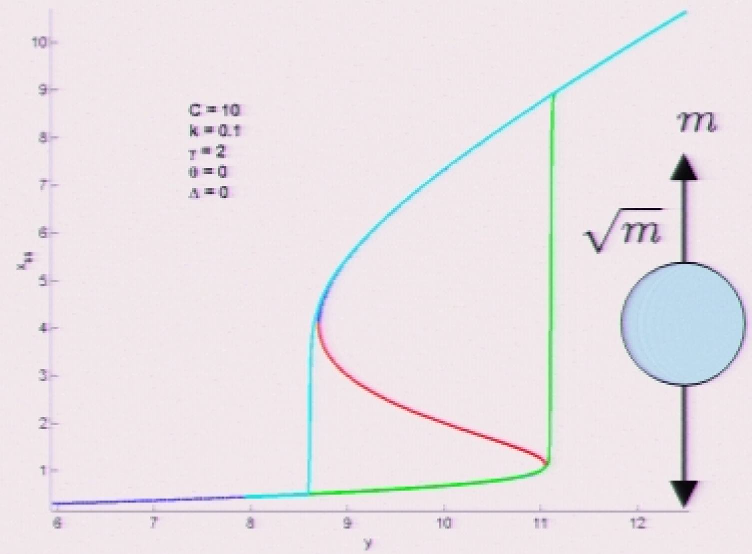
Quantum filtering and nonlinear dynamics

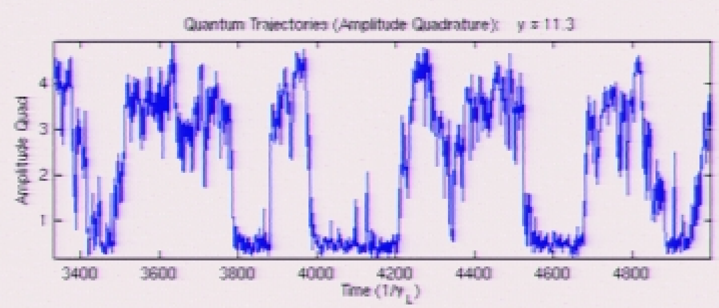
Ramon van Handel, Mike Armen, and Hideo Mabuchi

Physics and Control & Dynamical Systems, California Institute of Technology



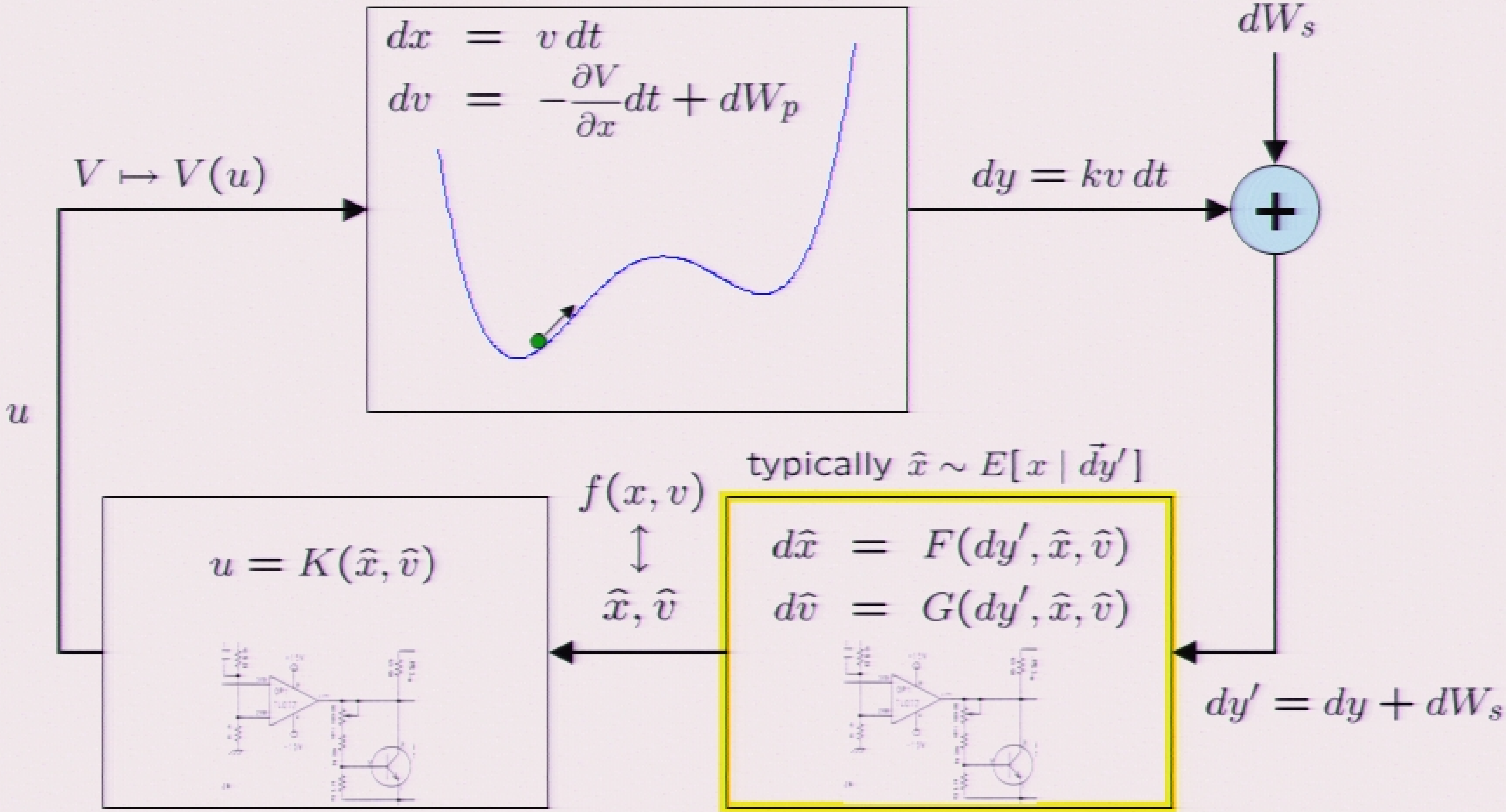






Classical filtering: concepts and applications

(introductory texts in applied mathematics or control theory)



- **applications**: target tracking, mathematical finance, real-time feedback control
- **tools**: Kushner-Stratonovich equation, Kalman filter, robust filtering, filter reduction

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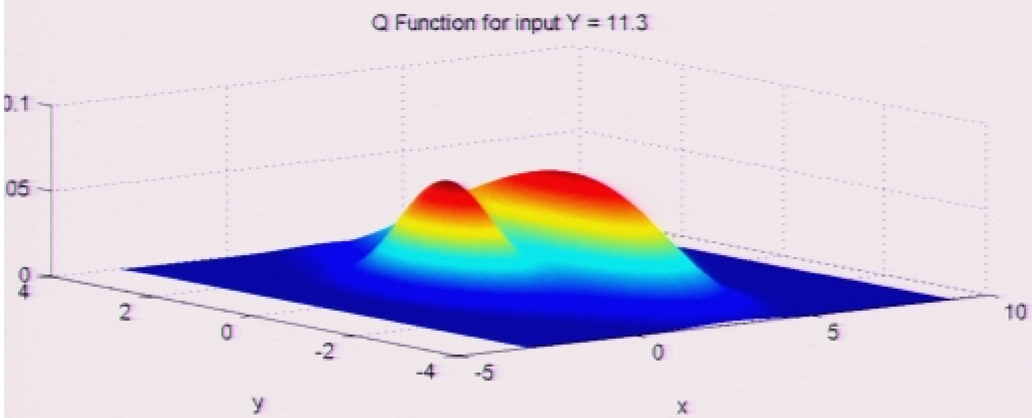

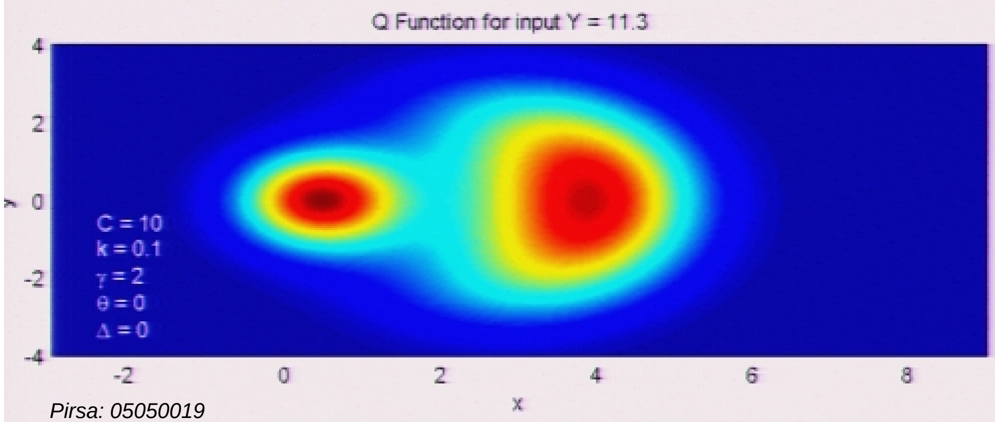

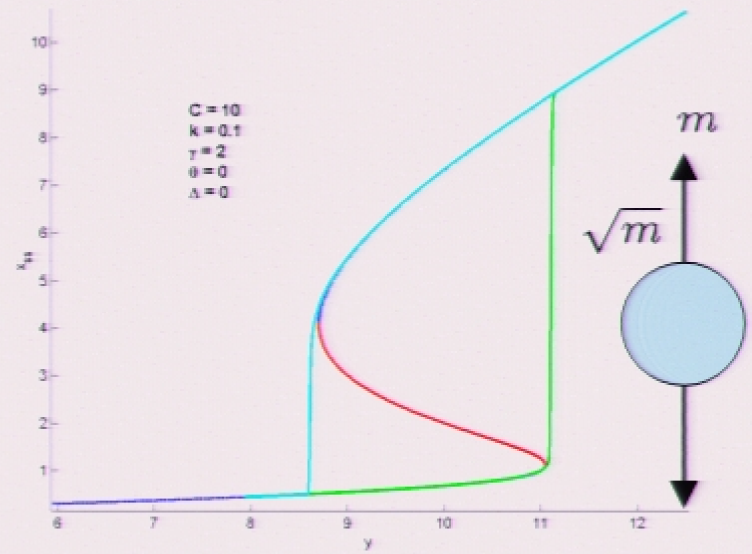

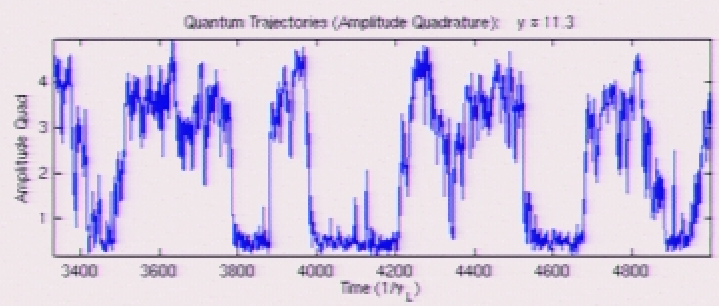

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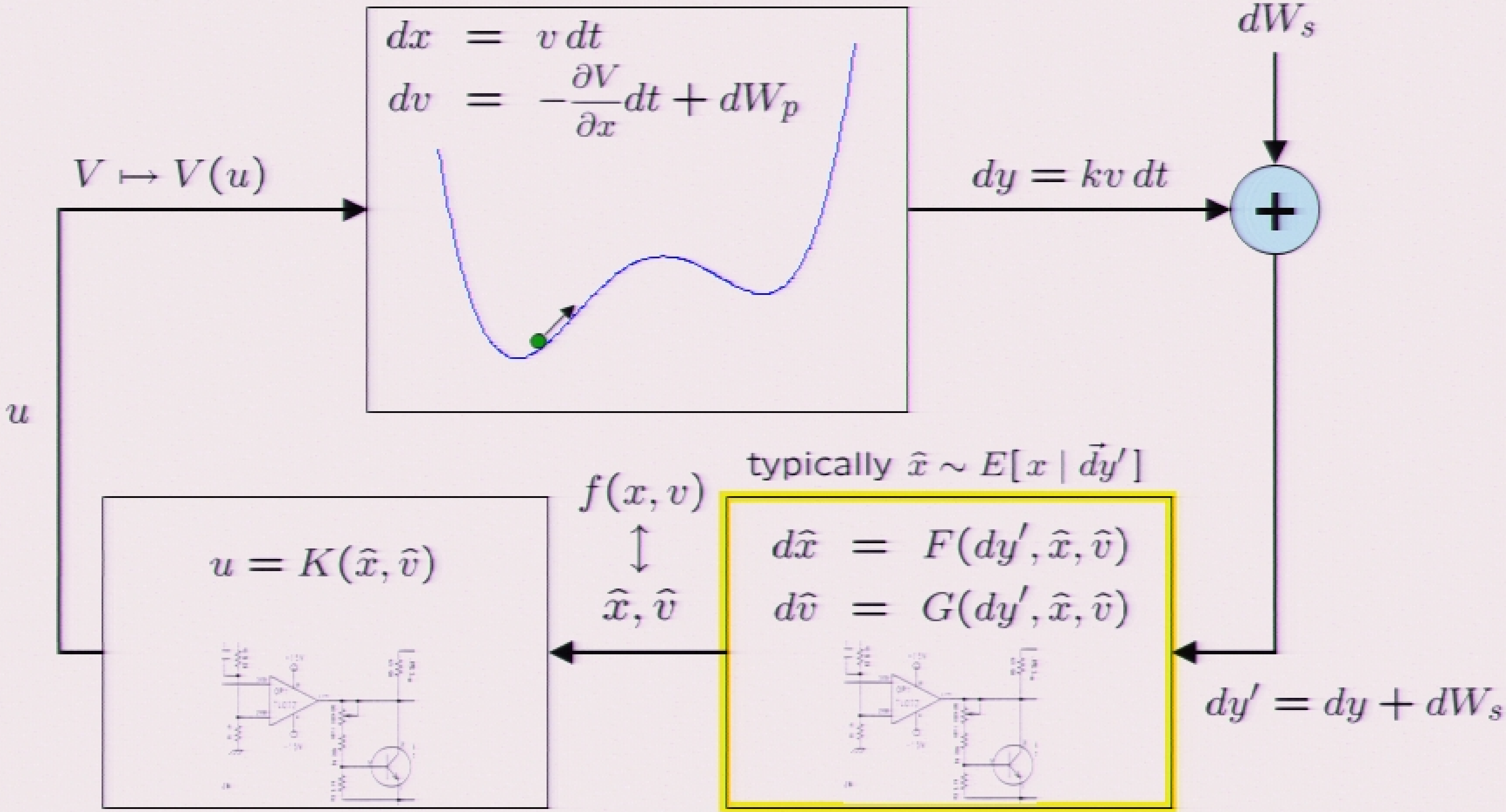

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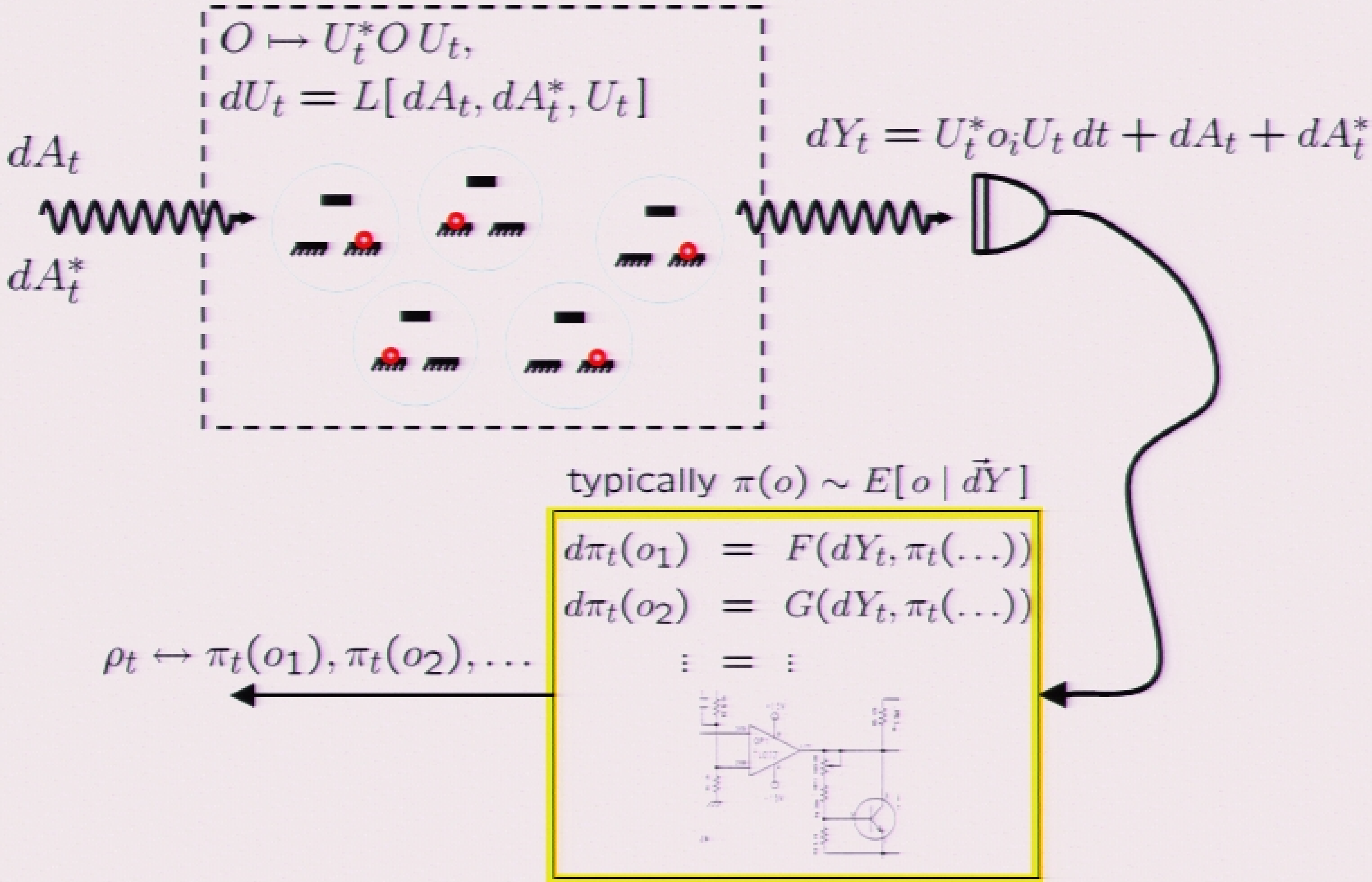
Open quantum systems: from scattering to filtering

Ramon van Handel, John Stockton and HM (to appear, 2005) http://minty.caltech.edu



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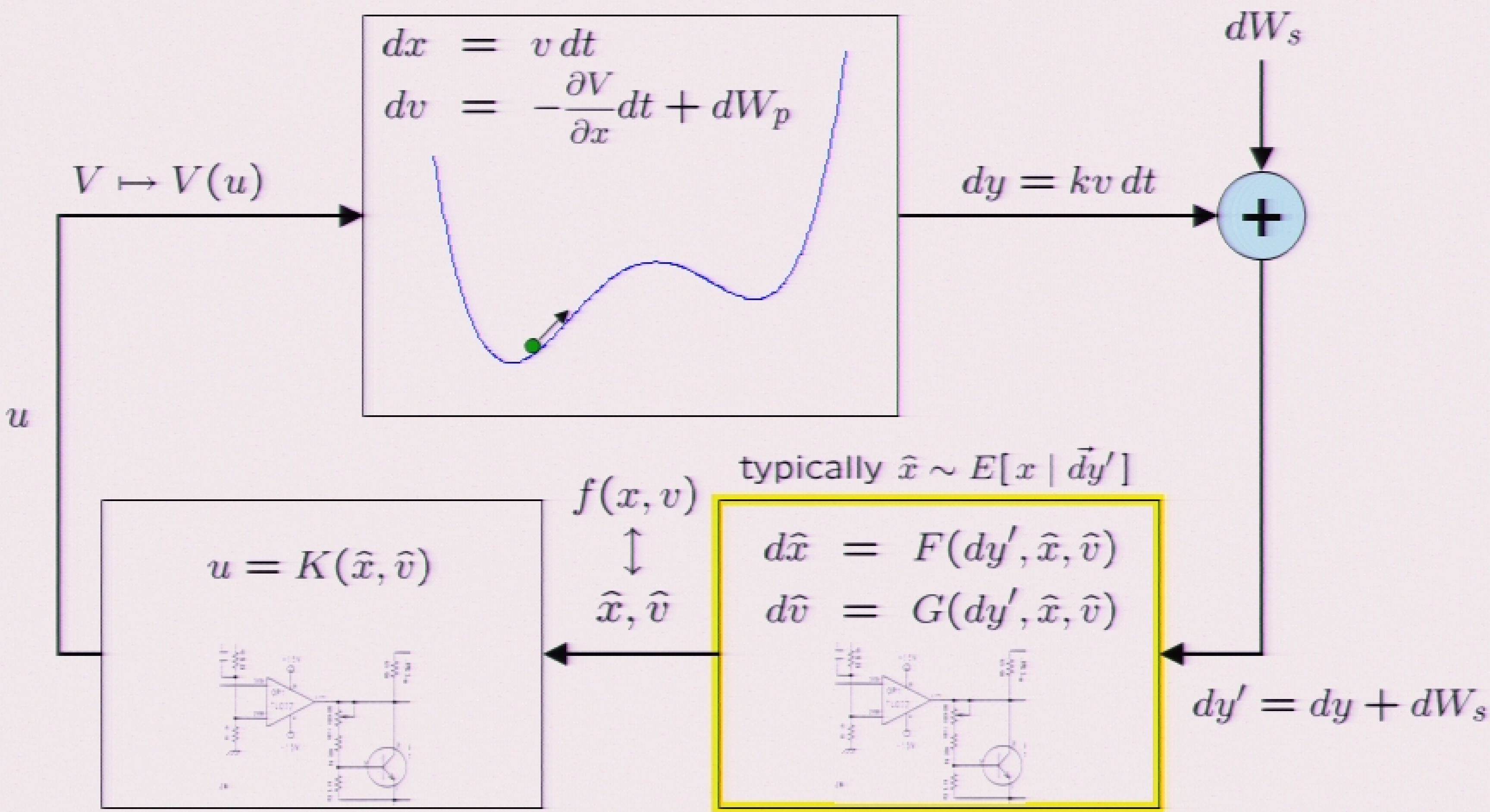

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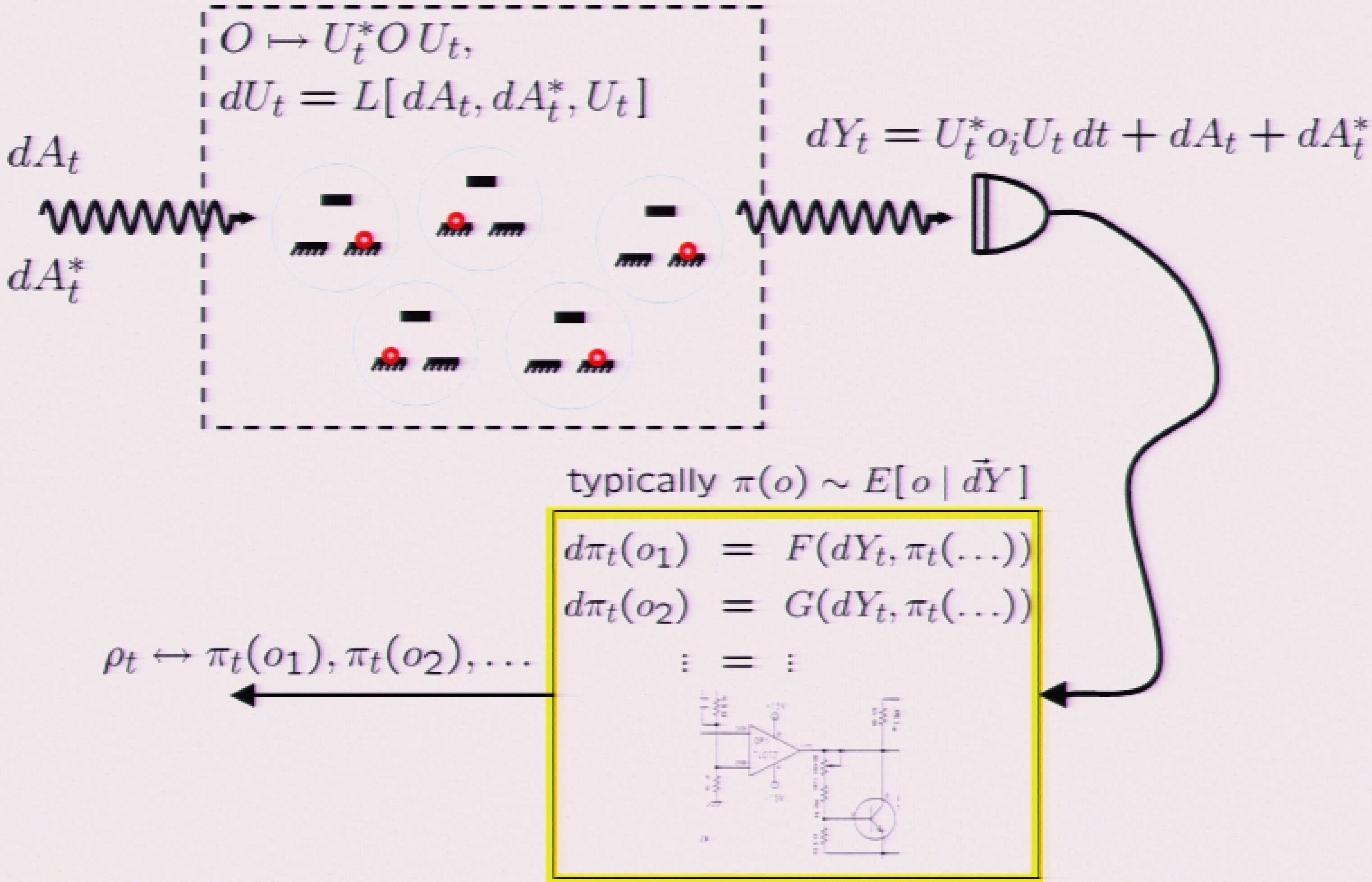

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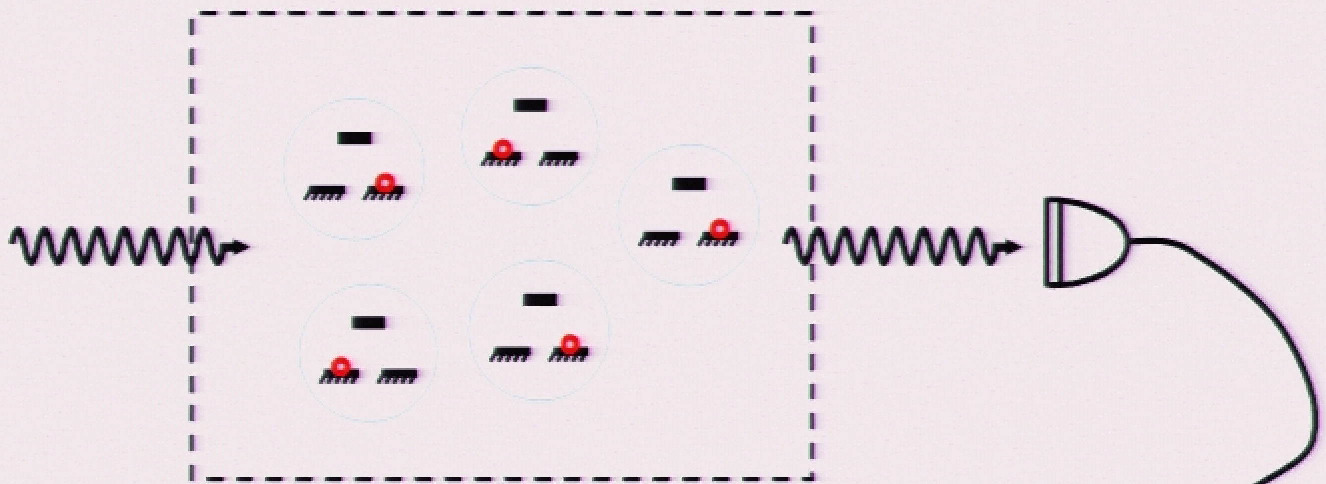
Quantum filtering and quantum trajectories

(Carmichael, Wiseman and Milburn, ...)



Master Equation: unconditional evolution of an open system

$$d\rho_t \;=\; -ih(t)[F_y, \rho_t]\,dt + M\mathcal{D}[F_z]\rho_t\,dt$$

Stochastic Master Equation: conditional ("selective") evolution of an open system

$$d\rho_t \;=\; -ih(t)[F_y, \rho_t]\,dt + M\mathcal{D}[F_z]\rho_t\,dt + \sqrt{M\eta}\mathcal{H}[F_z]\rho_t\,(dY_t - 2\sqrt{M\eta}\,\mathrm{Tr}[\rho_t\,F_z]\,dt)$$

- dY_t represents a measured signal; it "drives" localization of conditional state
- Interpretations of the SME vary widely; comparison to classical filtering...

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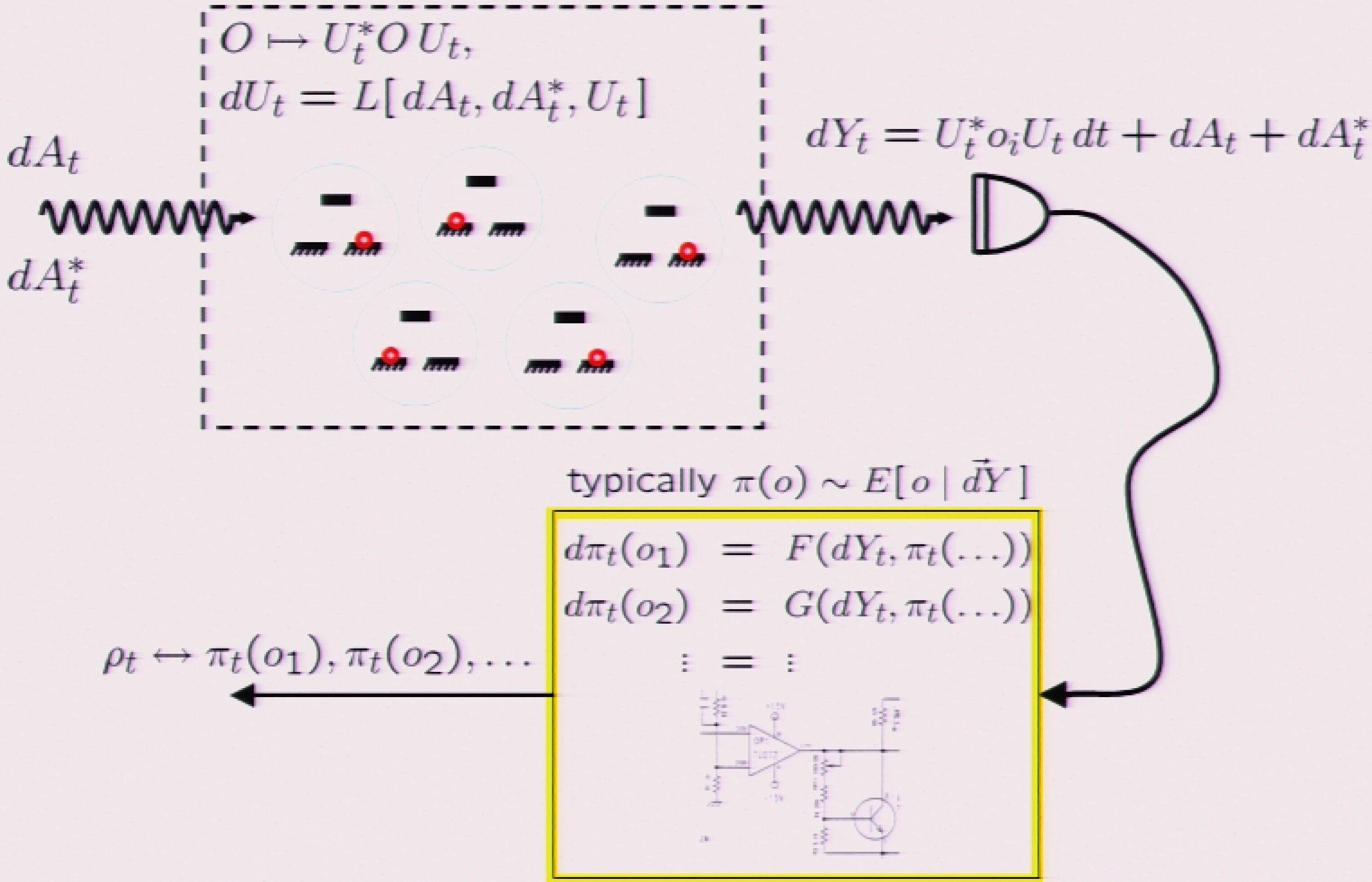

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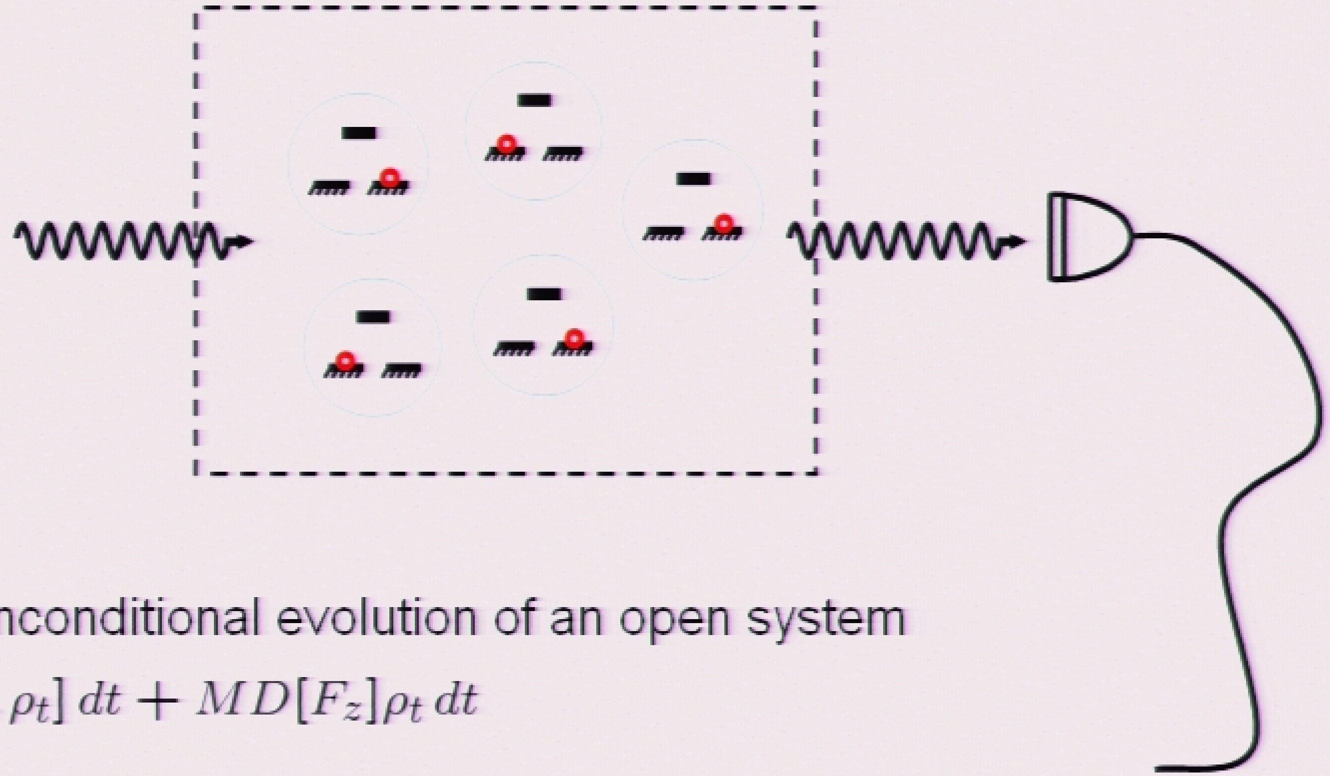

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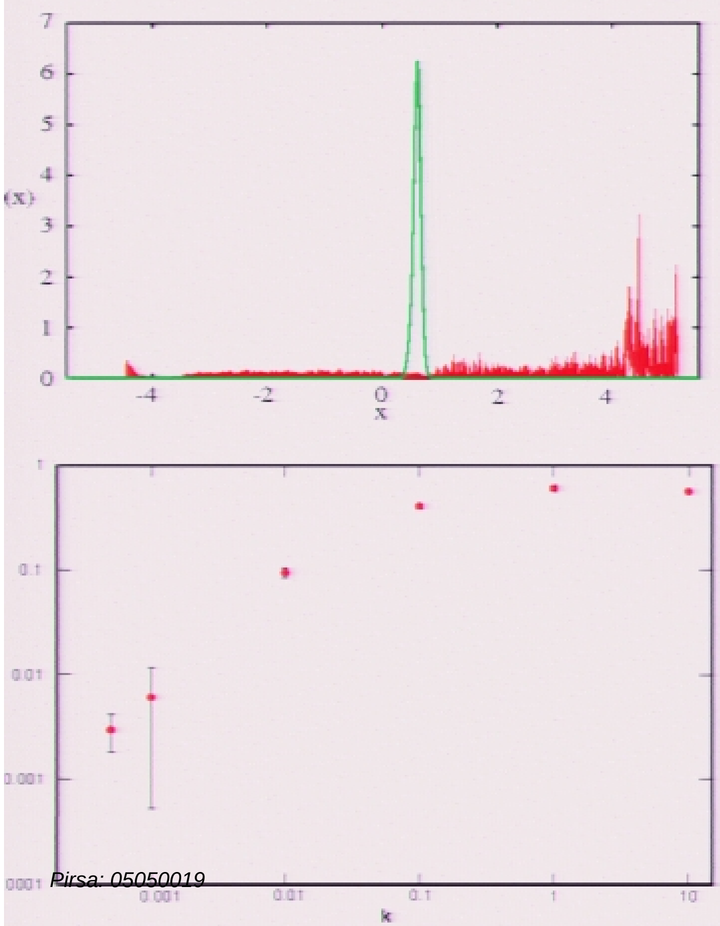
Quantum trajectories reveal nonlinear dynamics?

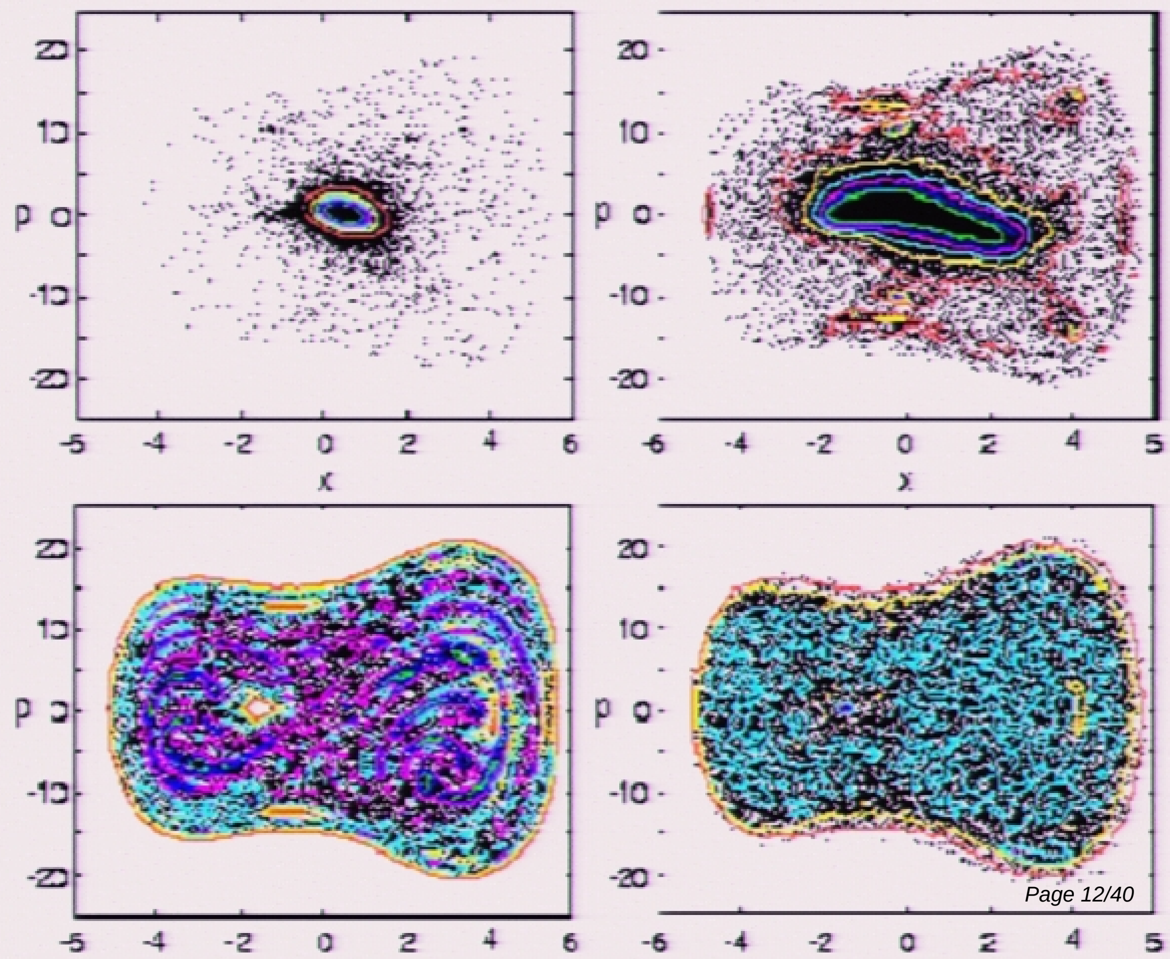
S. Habib, K. Jacobs and K. Shizume, quant-ph/0412159

quantum Duffing oscillator: $H = p^2/wm + Bx^4 - Ax^2 + \Lambda x \cos(\omega t)$

continuous position-measurement SME:

$$d\rho = -\frac{i}{\hbar}[H,\rho]\,dt - k[x,[x,\rho]]\,dt + 4k(x\rho + \rho x - 2\langle x\rangle)(dy - \langle x\rangle\,dt)$$





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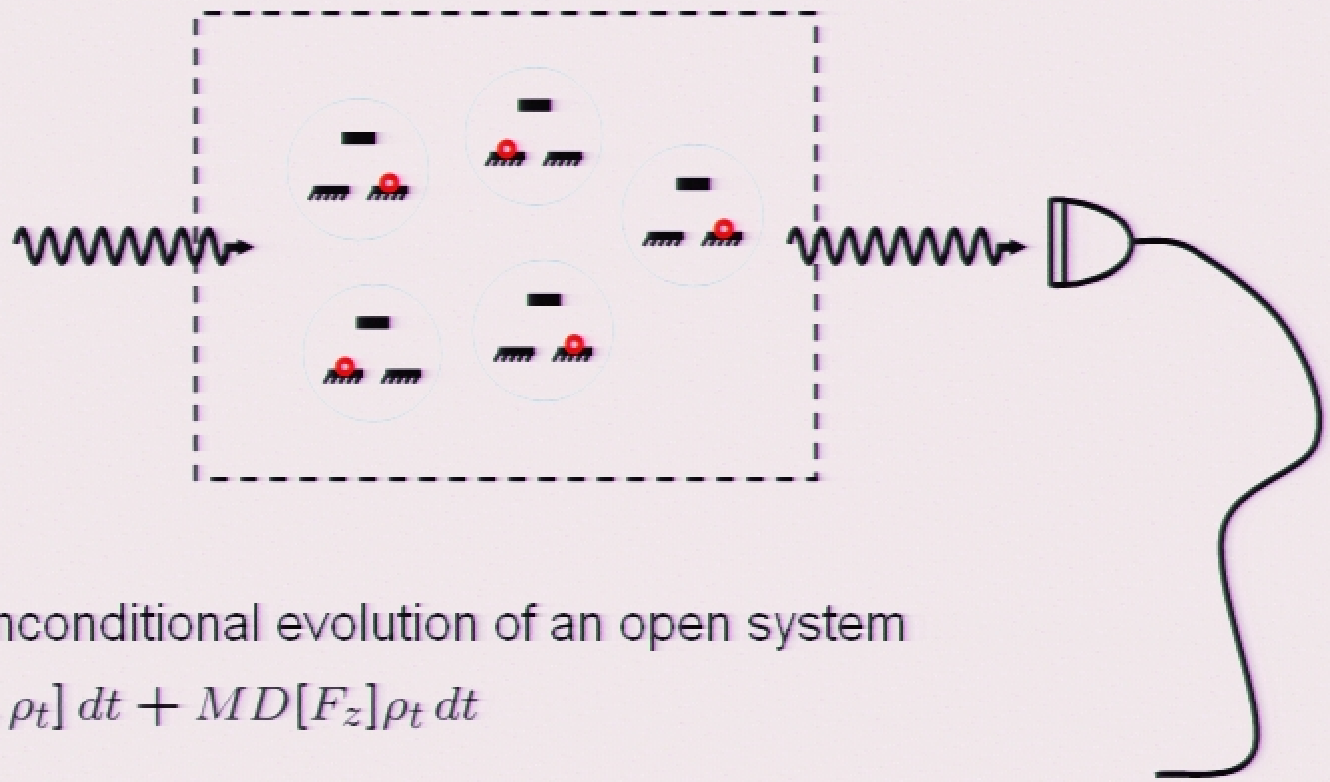

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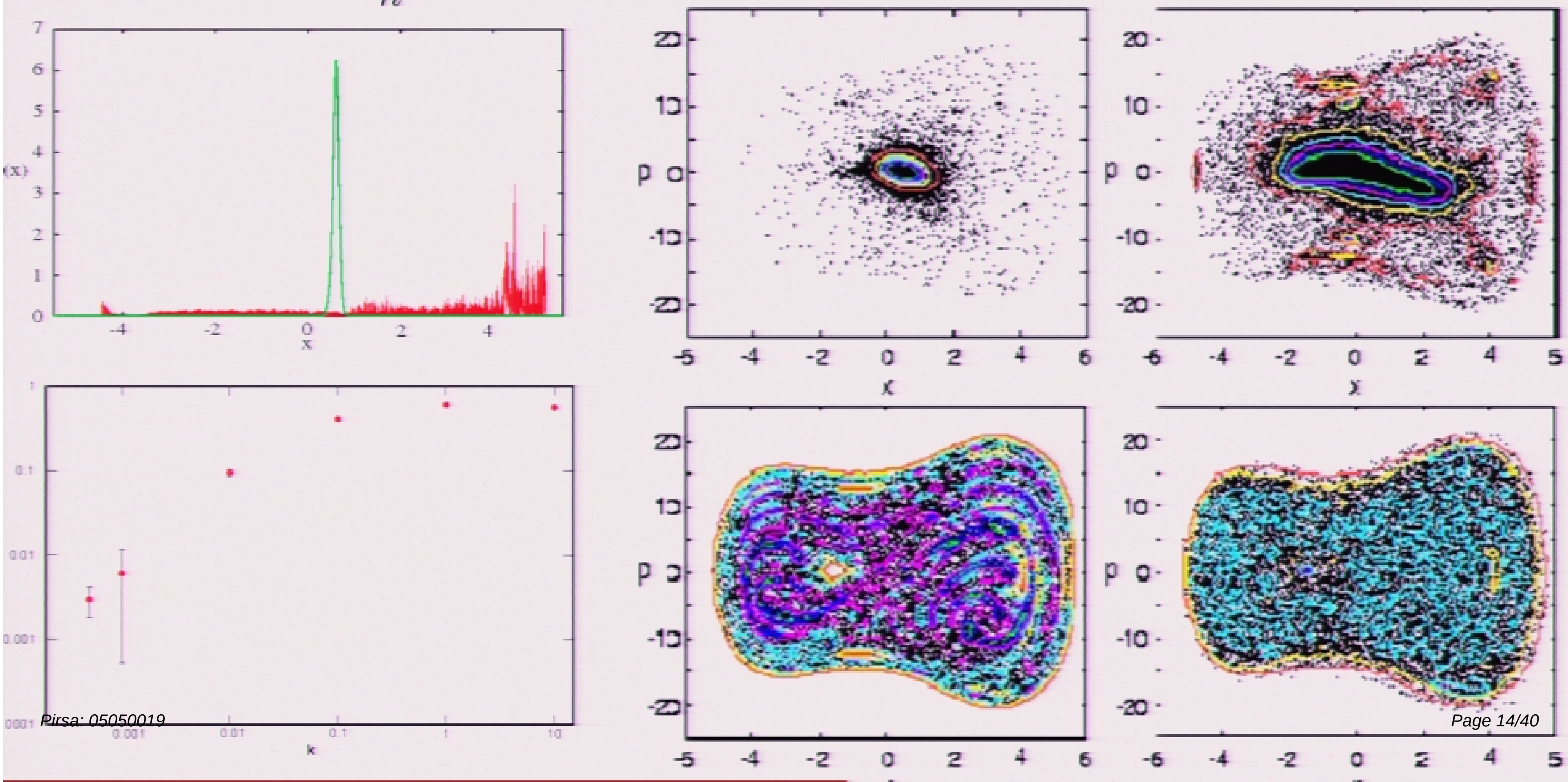
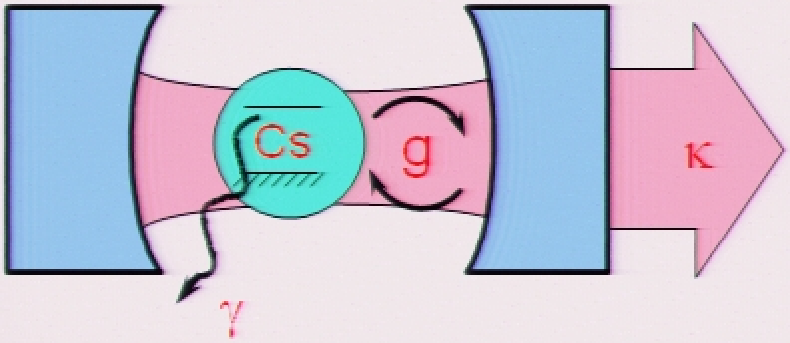


Optical cavity QED with strong coupling

Kimble, Rempe, ...

One atom, strongly coupled



Critical Photon Number

$$m_0 \approx \frac{\gamma_\perp^2}{2g^2} < 1$$

1 photon can induce strong nonlinearity

Critical Atom Number

$$N_0 \approx \frac{2\kappa\gamma_\perp}{g^2} < 1$$

1 atom may be used as an optical switch

The Master Equation for the Jaynes-Cummings Hamiltonian:

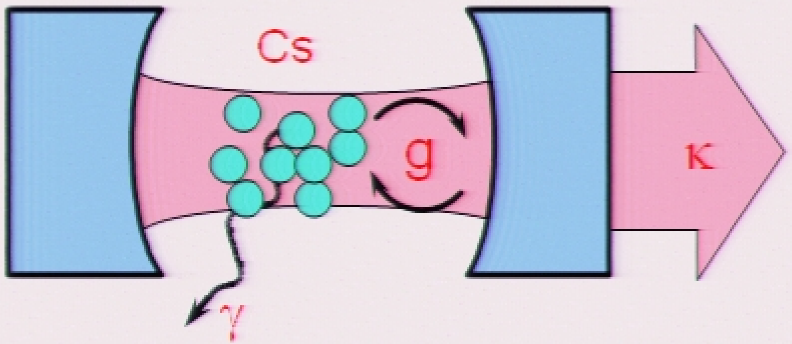
$$\dot{\rho} = \frac{1}{i\hbar}[\, H \,,\, \rho \,] + 2\kappa\mathcal{D}[a]\rho + 2\gamma_\perp\mathcal{D}[\sigma]\rho$$

$$H = \hbar\Delta_c a^\dagger a + \hbar\Delta_a \sigma^\dagger\sigma + i\hbar\mathcal{E}(a^\dagger - a) + i\hbar g(a^\dagger\sigma - a\sigma^\dagger)$$

- Driven, coherent + dissipative linear evolution of the joint state
- Generation of entanglement between atom and cavity field
- Fundamental paradigm for study of open quantum systems

Semi-classical Maxwell-Bloch equations

Many atoms, weakly coupled



Cooperativity Parameter

$$C = \frac{g_{\text{eff}}^2}{2\kappa\gamma_\perp} = \frac{Ng^2}{2\kappa\gamma_\perp}$$

Absorptive Bistability for C > 4

Maxwell-Bloch Equations:

$$\begin{aligned} \dot{x} &= \kappa(1+i\Theta)\,x + (g_{\text{eff}}/2)\,p + \mathcal{E} \\ \dot{p} &= \gamma_\perp(1+i\Delta)\,p + 2g_{\text{eff}}mx \\ \dot{m} &= -\gamma_{||}(m+1) - g_{\text{eff}}(\,x^*p + p^*x\,) \end{aligned}$$

$$\begin{aligned} \sqrt{m_0}\,x &\leftrightarrow \langle a\rangle \\ p &\leftrightarrow 2\langle\sigma\rangle \\ m &\leftrightarrow \langle\sigma_z\rangle = \langle[\sigma^\dagger,\sigma]\rangle \end{aligned}$$

$$\langle a^\dagger\sigma\rangle \rightarrow \langle a^\dagger\rangle\langle\sigma\rangle$$

- Valid in weak-coupling (per atom) regime ⇒ m_0 is relatively large
- Mean-field approximation for cavity field; no atom-field entanglement
- No field fluctuations, but can be added in 2nd-order

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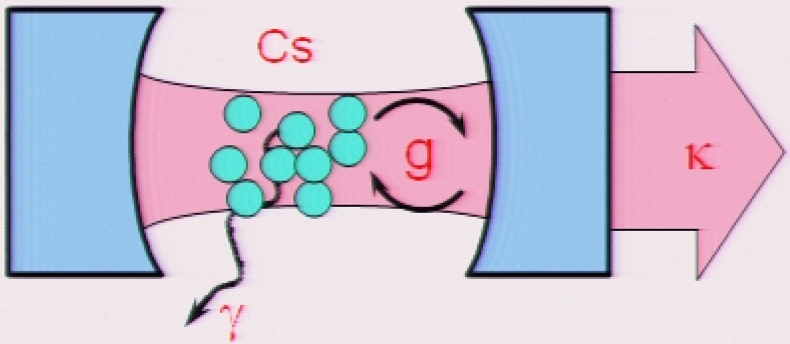

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Bifurcations, instabilities & fluctuations in cavity QED

Kimble, Lugiato, Carmichael, ...

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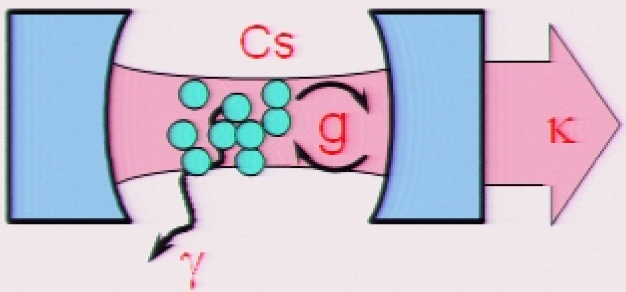
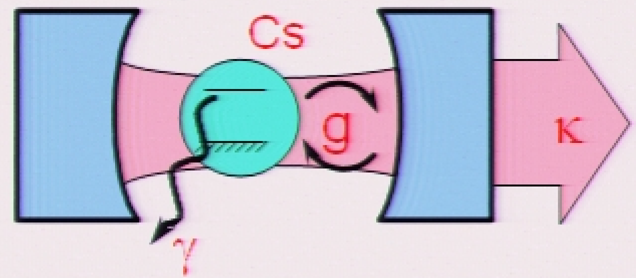
weak coupling

strong coupling





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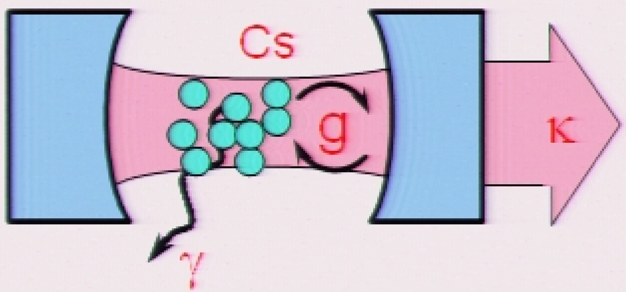

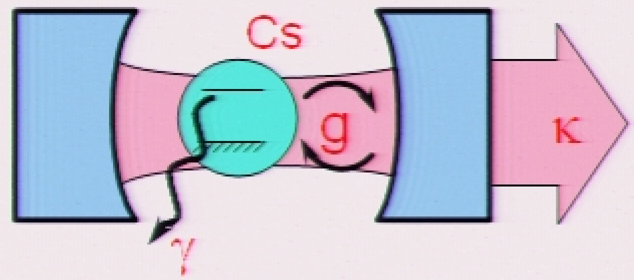

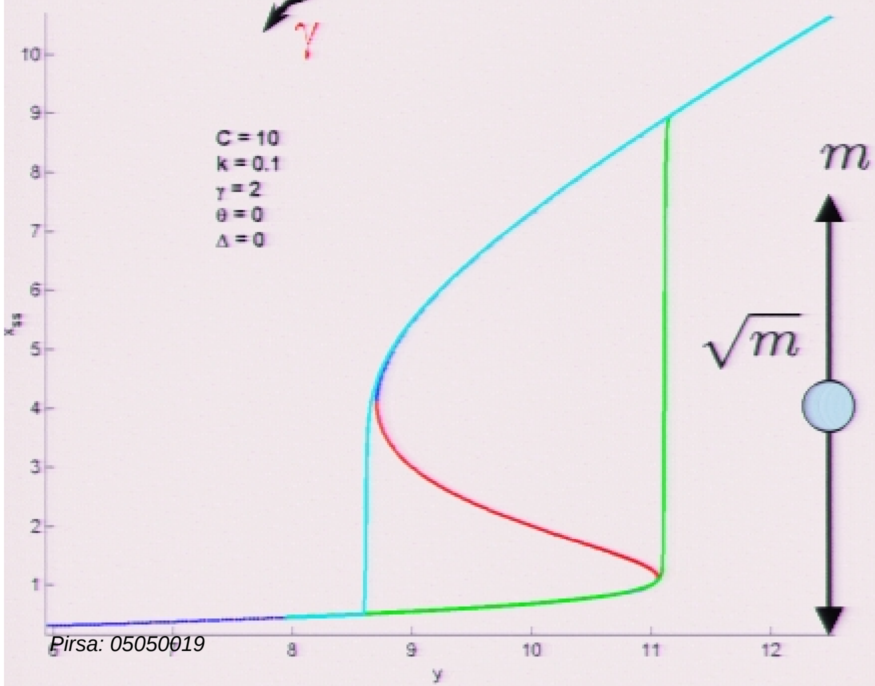

semi-classical nonlinear dynamics

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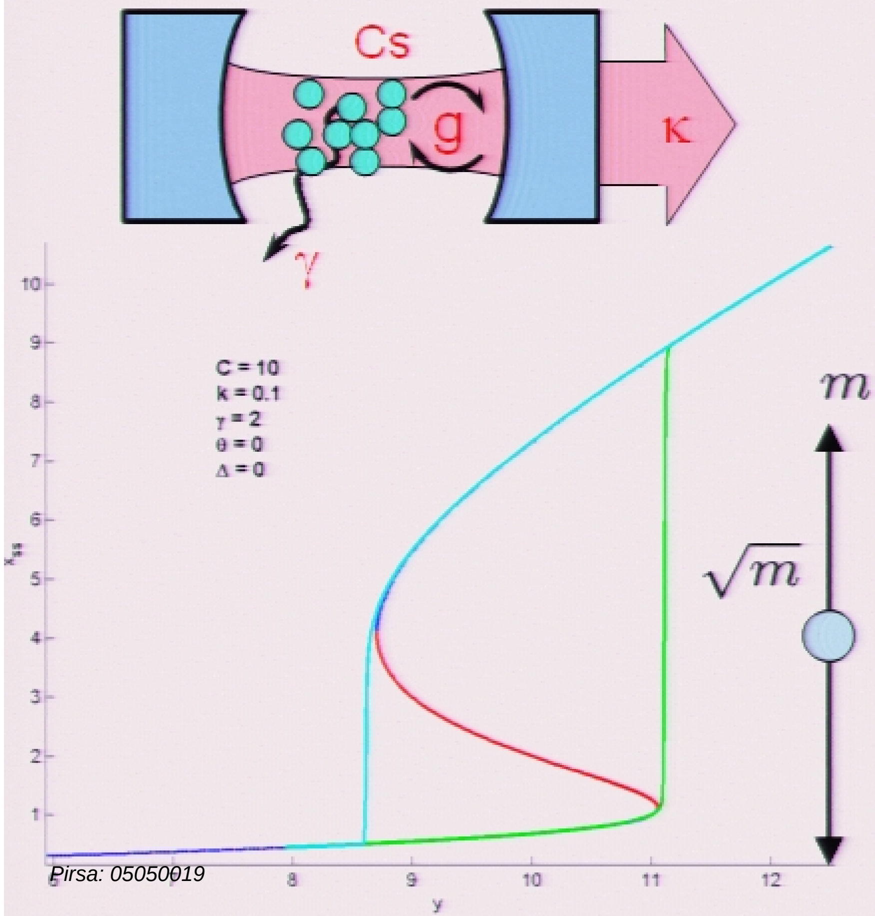
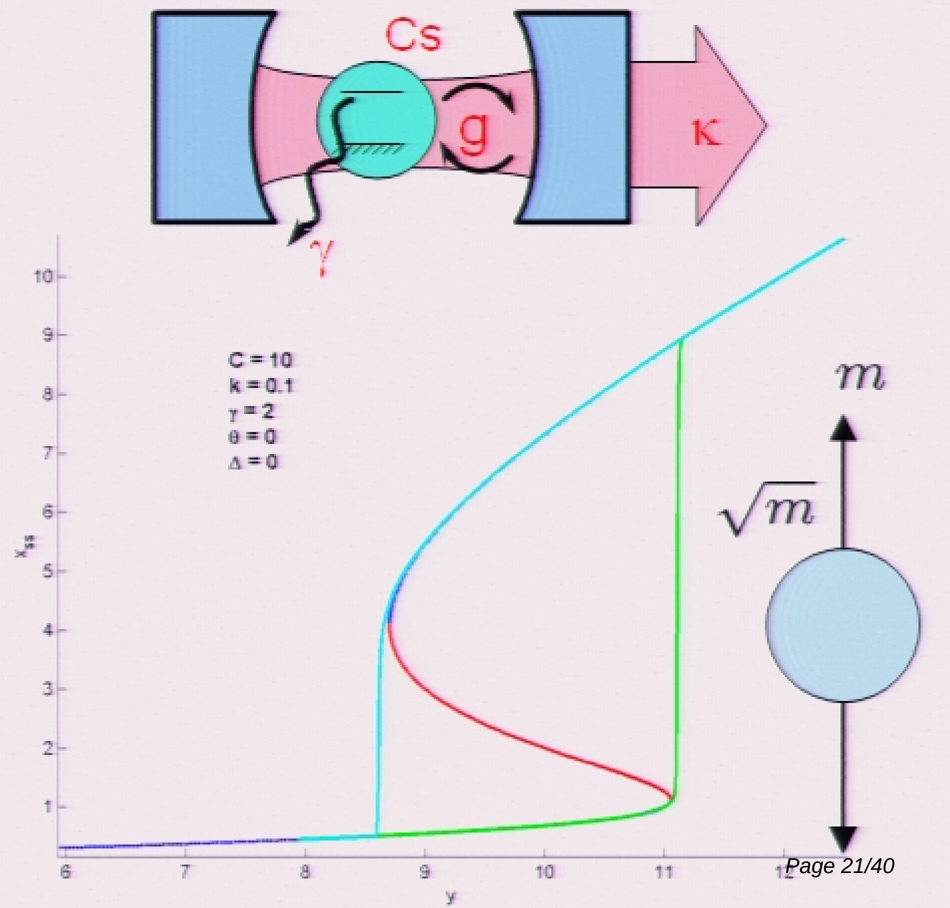
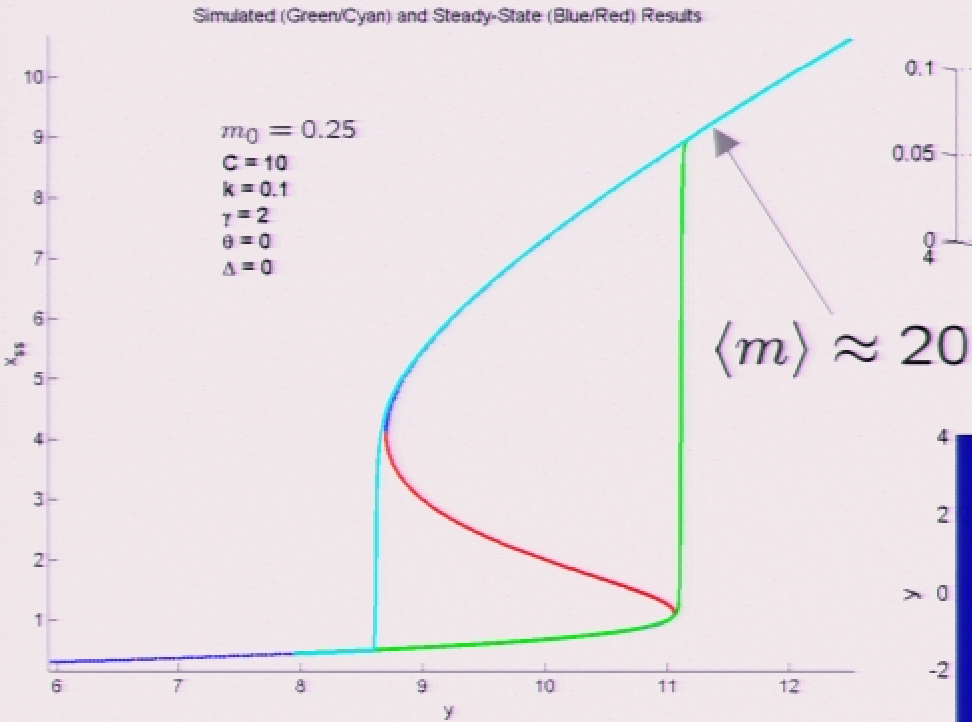


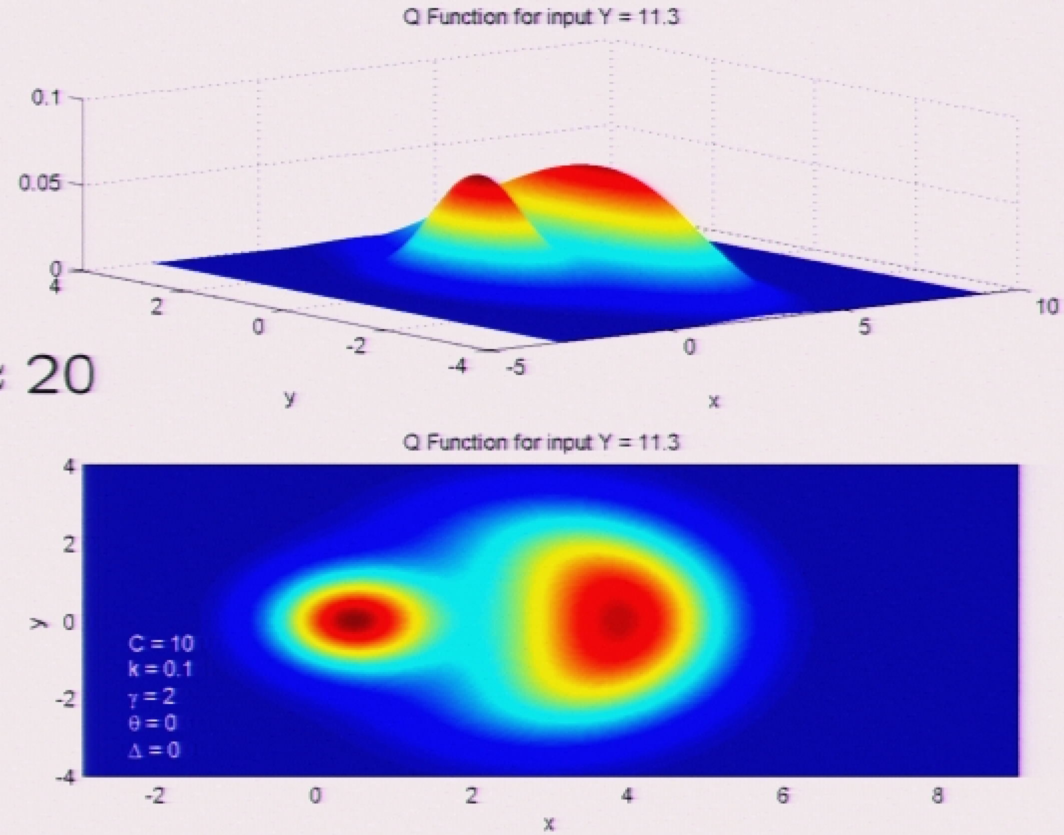


semi-classical nonlinear dynamics | *un-localized in phase portrait*

Single-atom absorptive bistability

Savage and Carmichael, Armen and HM





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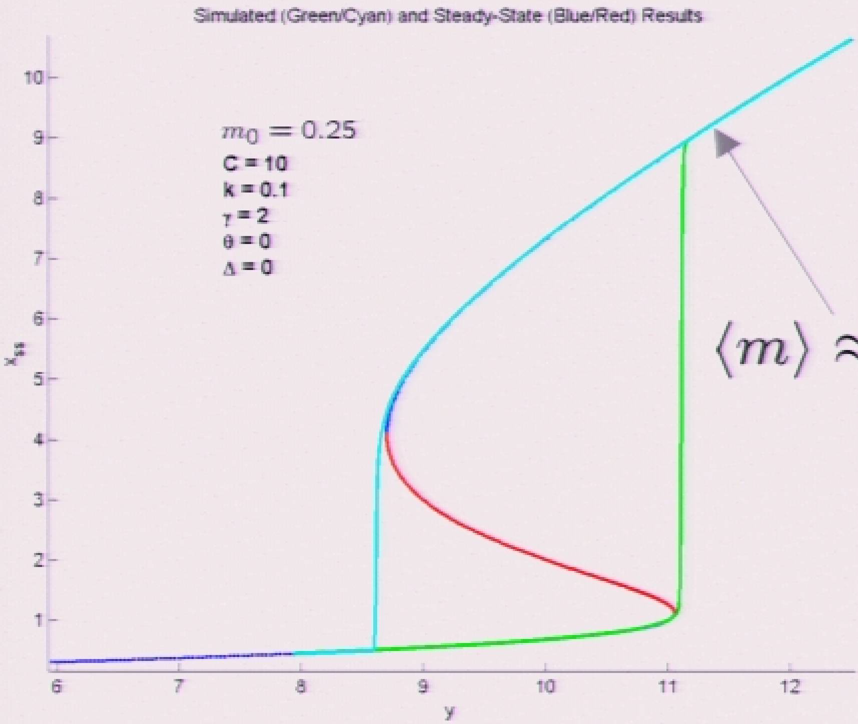
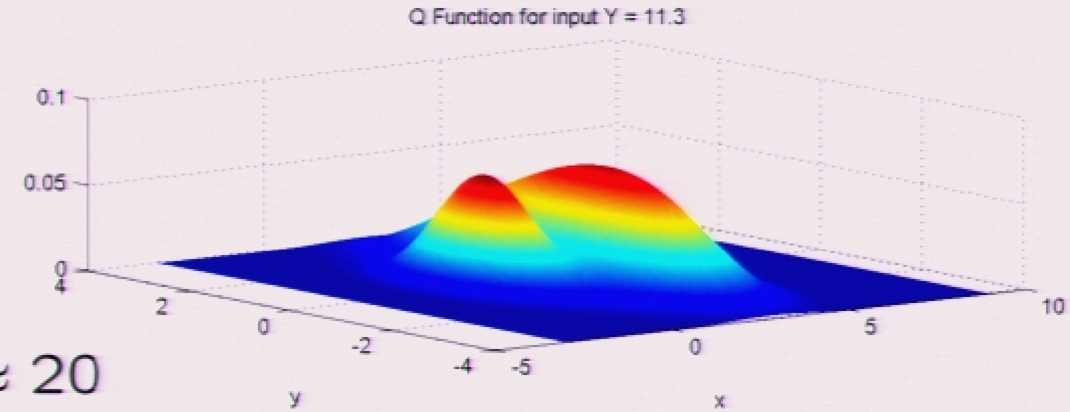
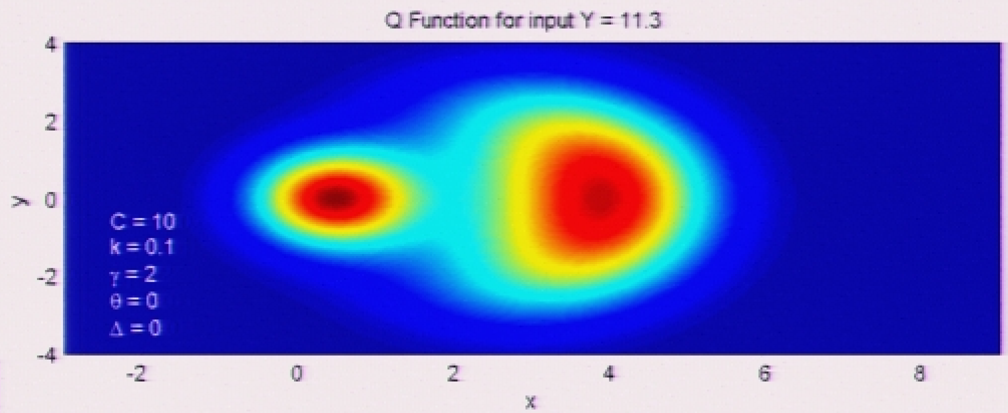
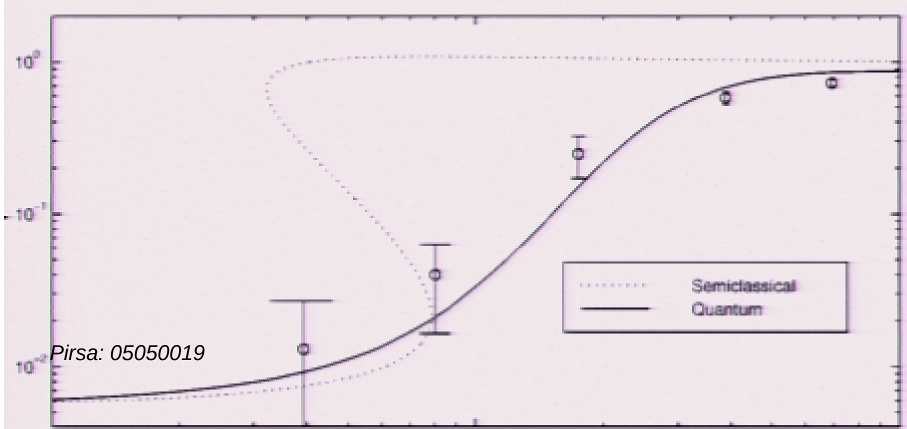


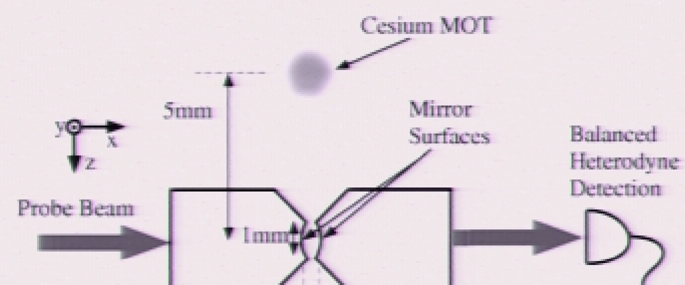
$\langle m \rangle \approx 20$







← *Kimble and co-workers ('98)*



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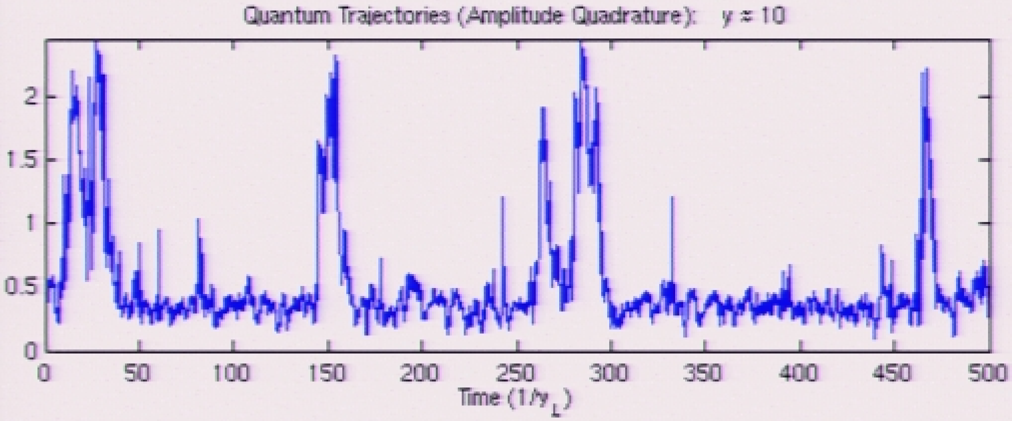

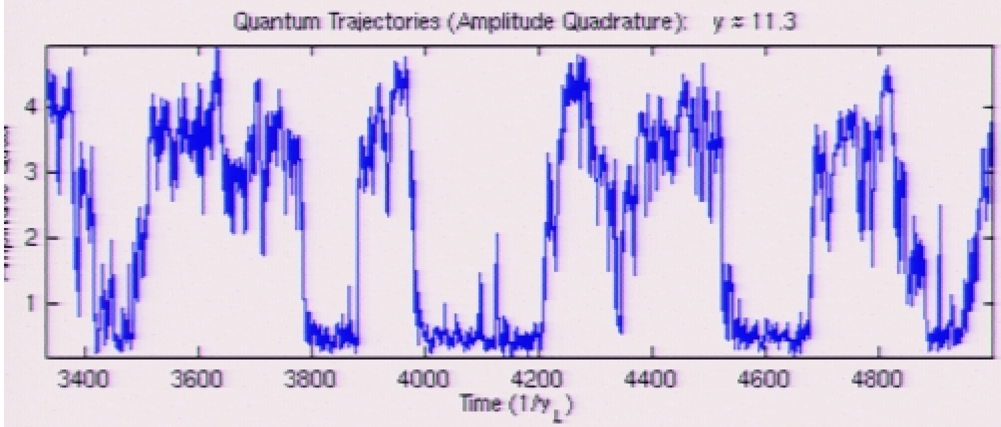

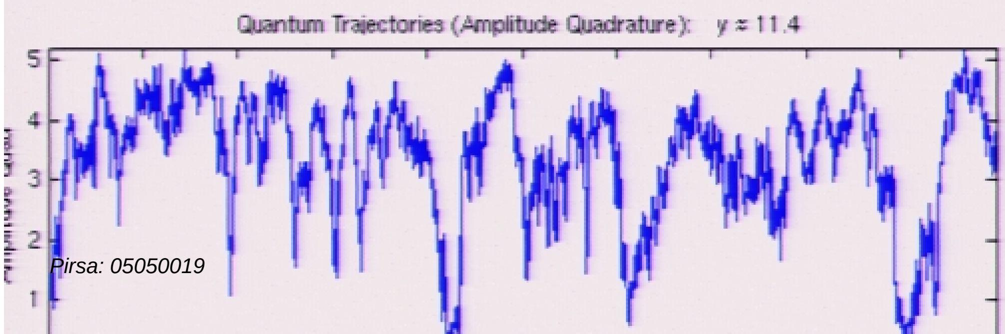

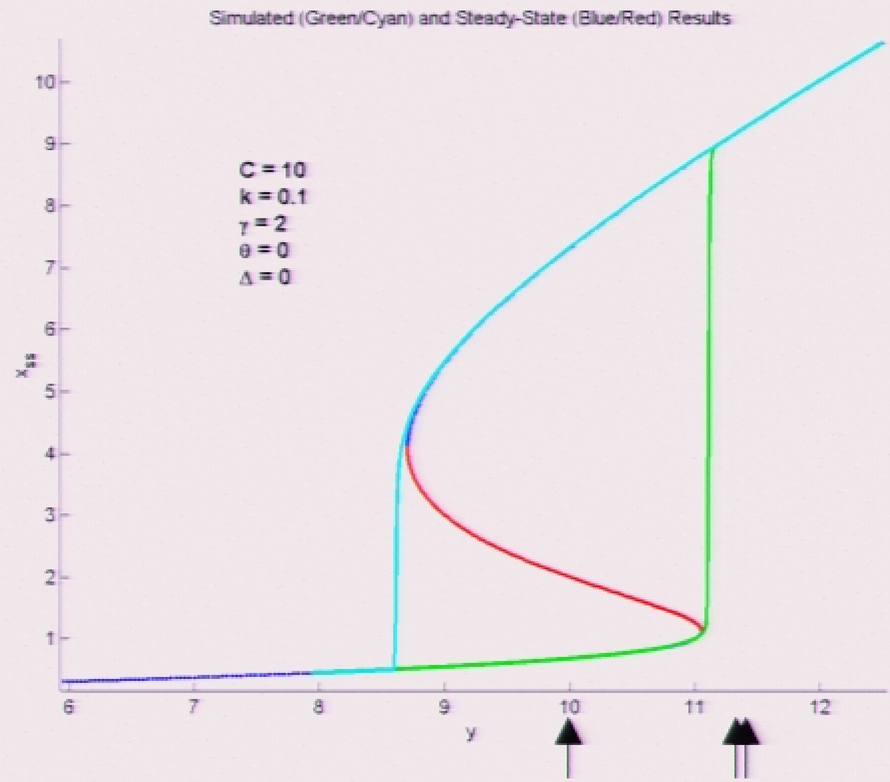

- differential stability of stable nodes?
- fluctuations “explore” phase portrait?

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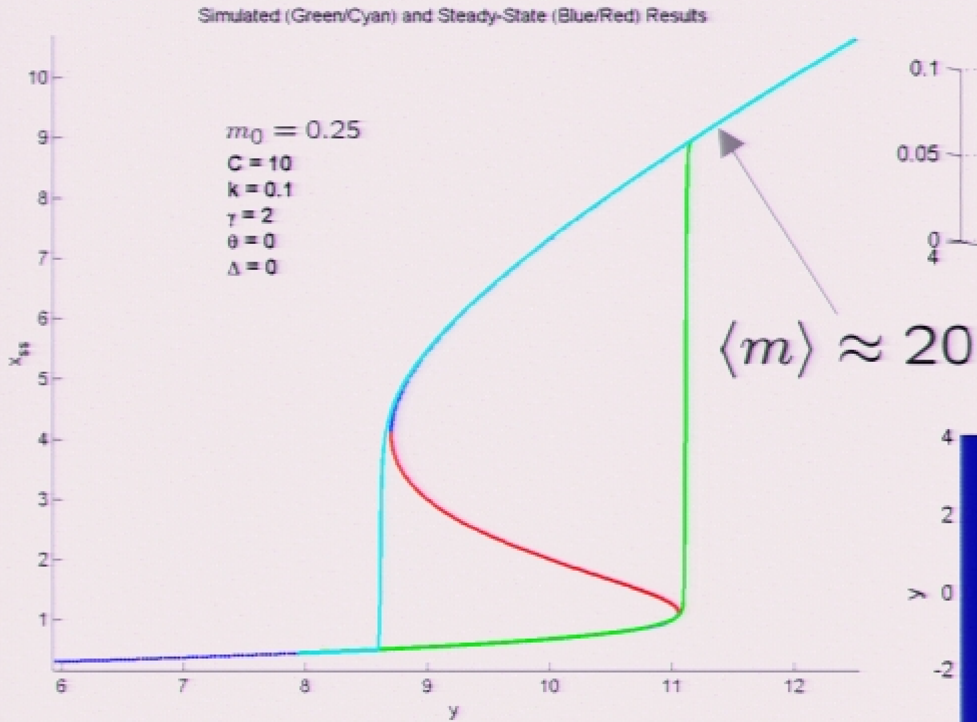
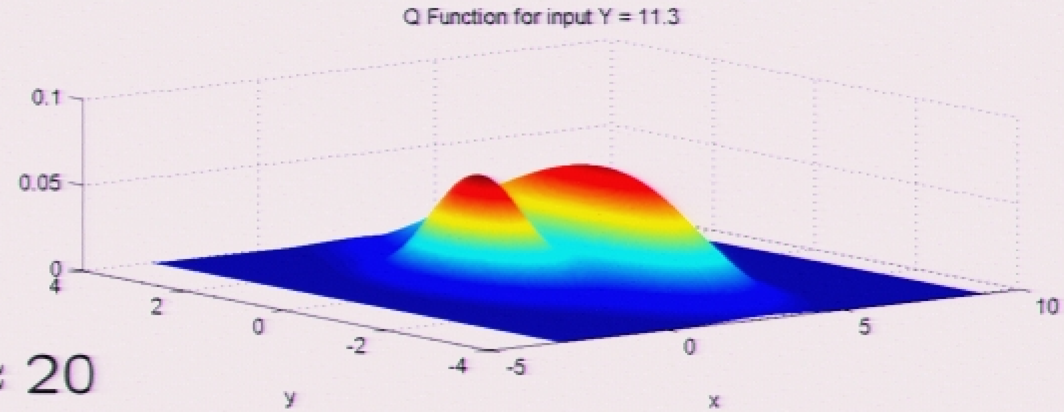
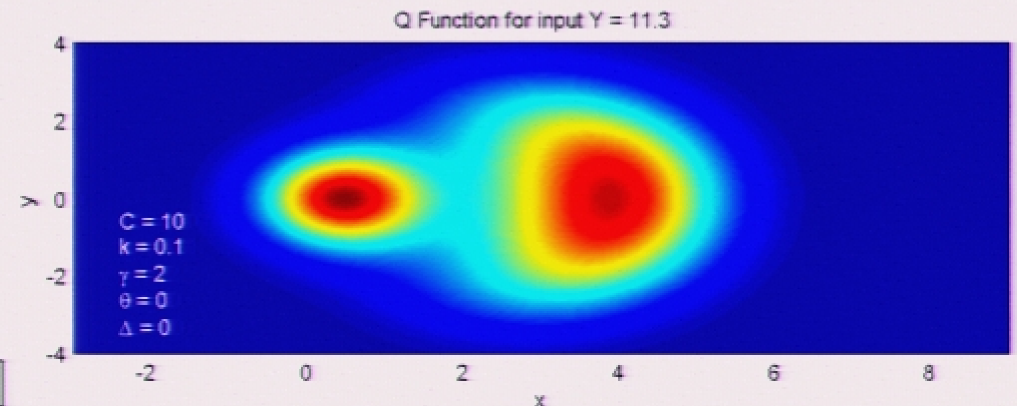
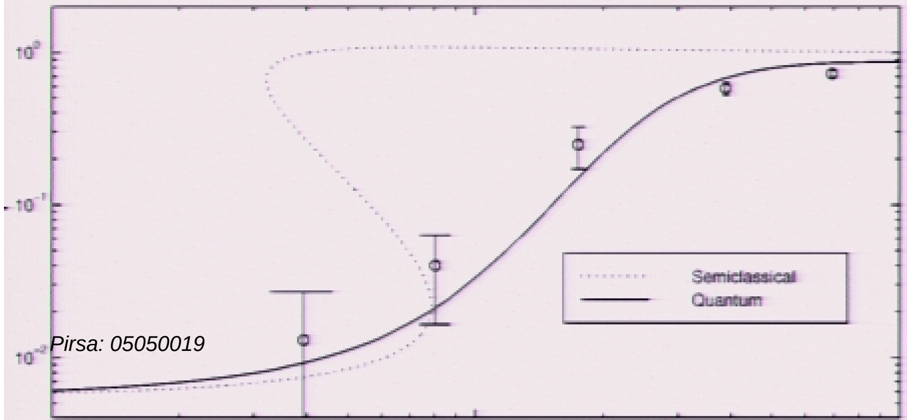
Savage and Carmichael, Armen and HM









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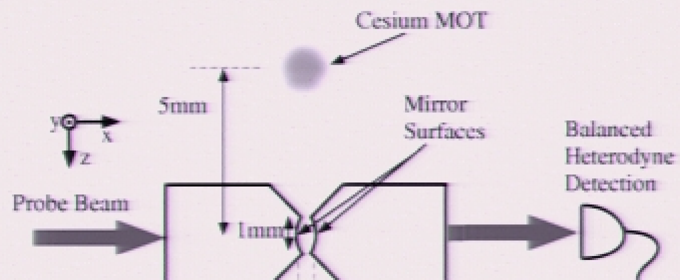
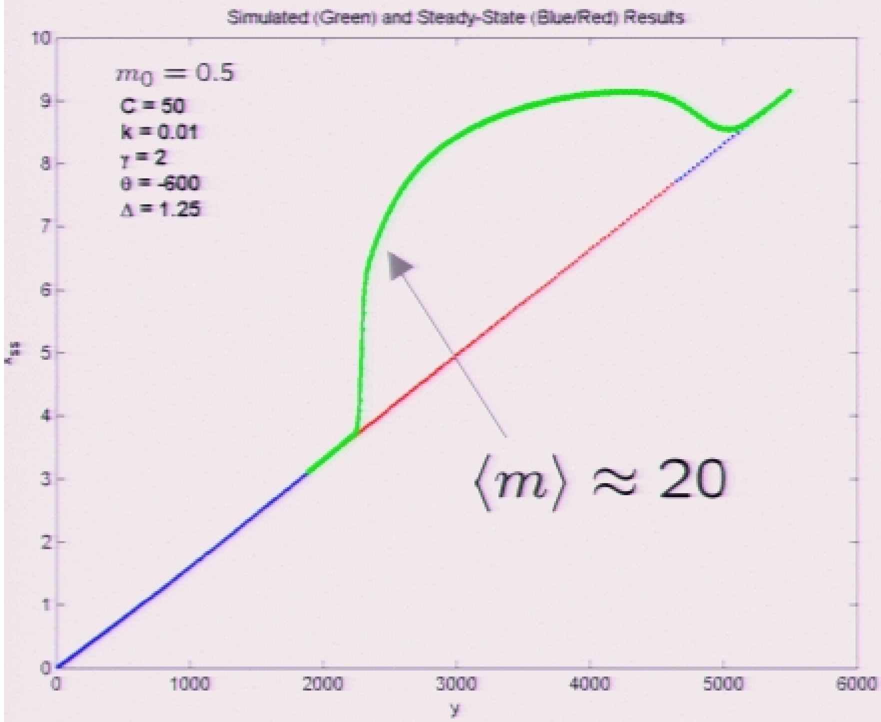
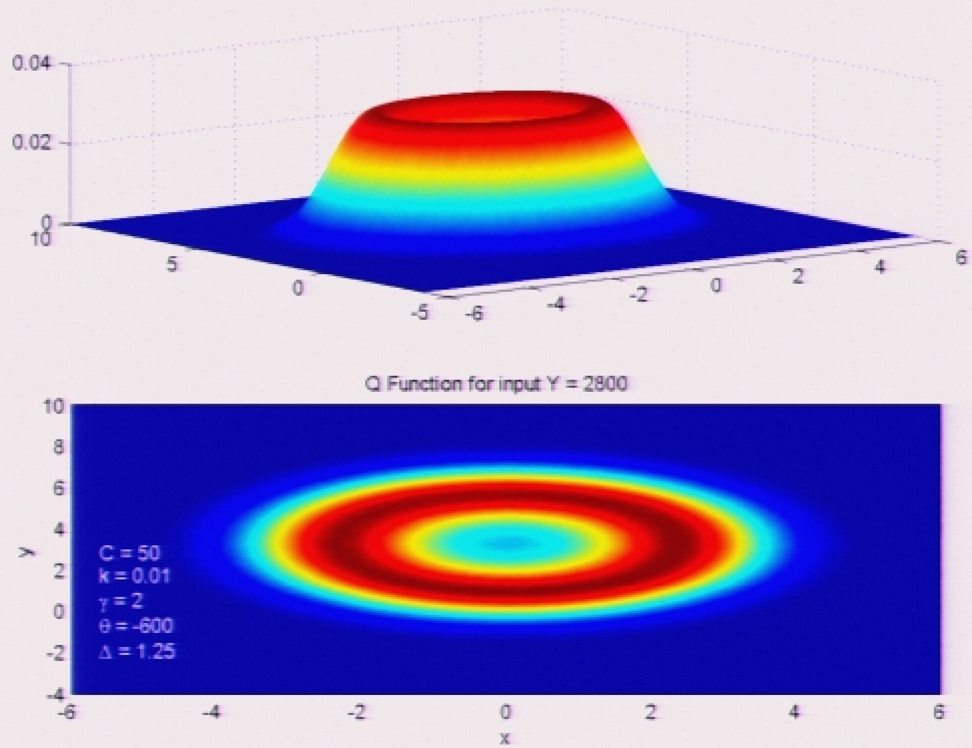
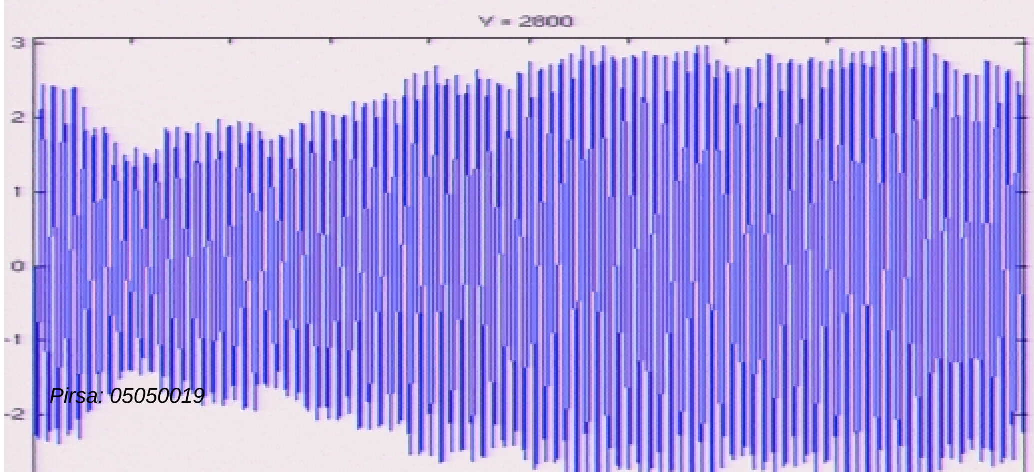


Super-critical Hopf bifurcation: stable limit cycle







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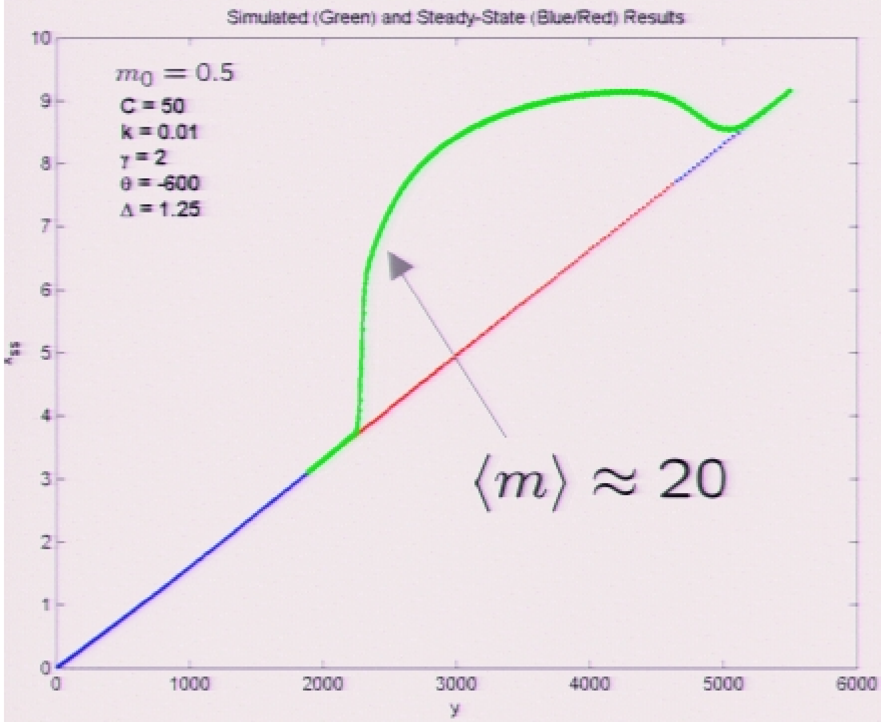
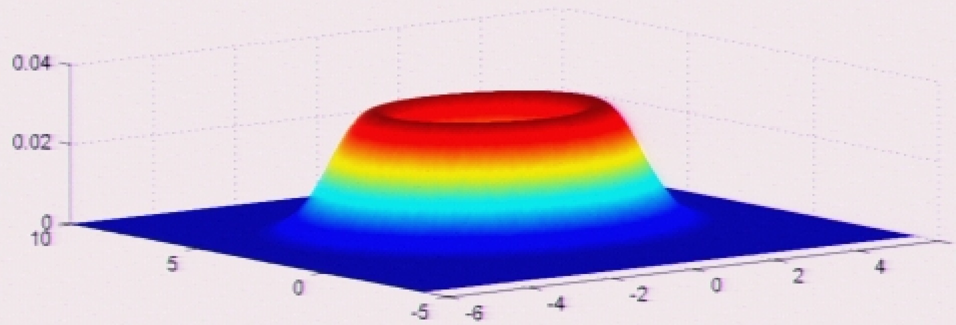
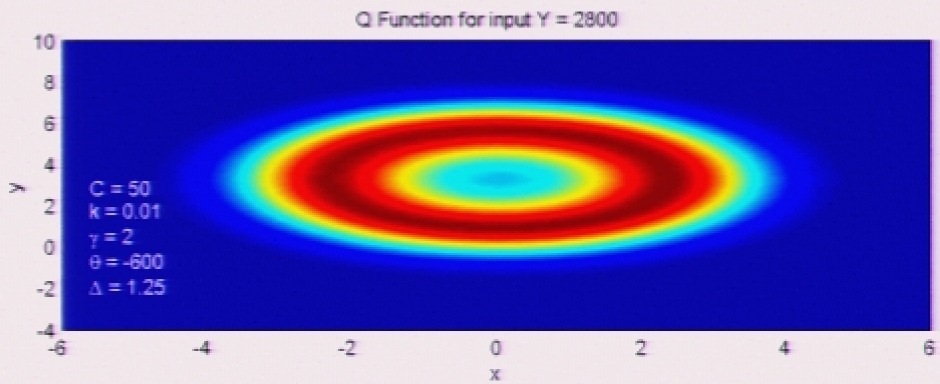
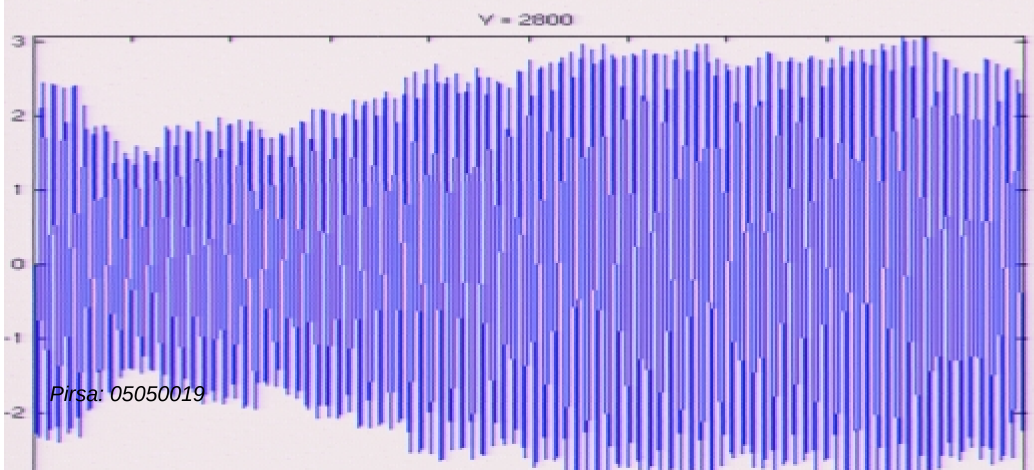
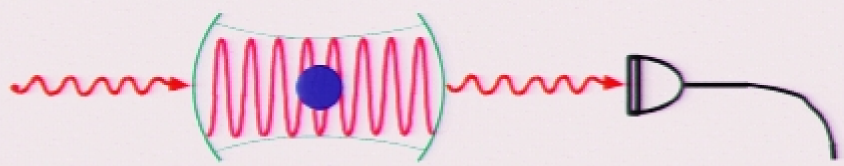




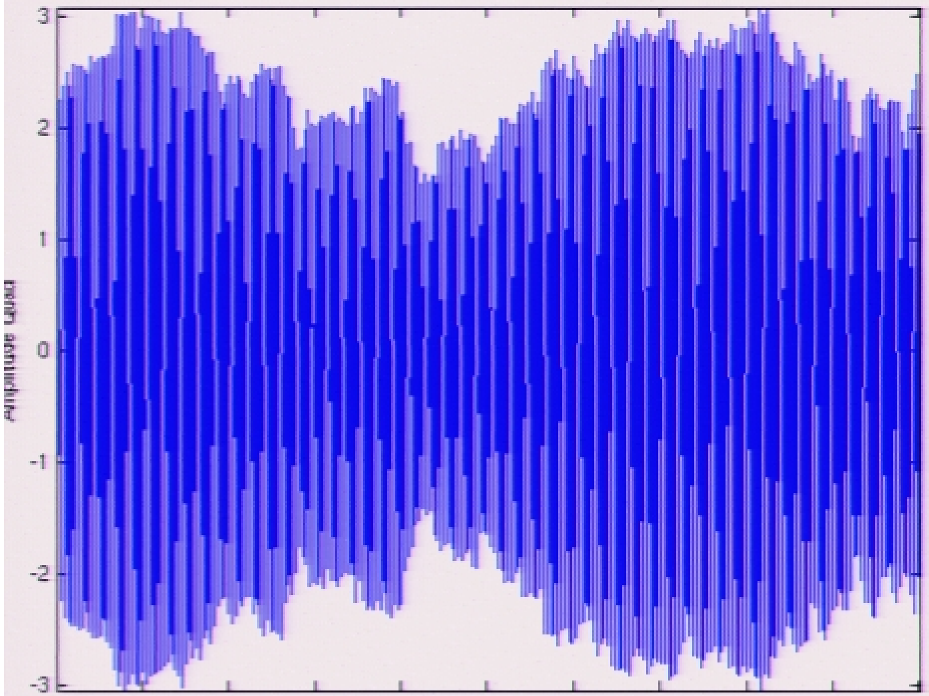


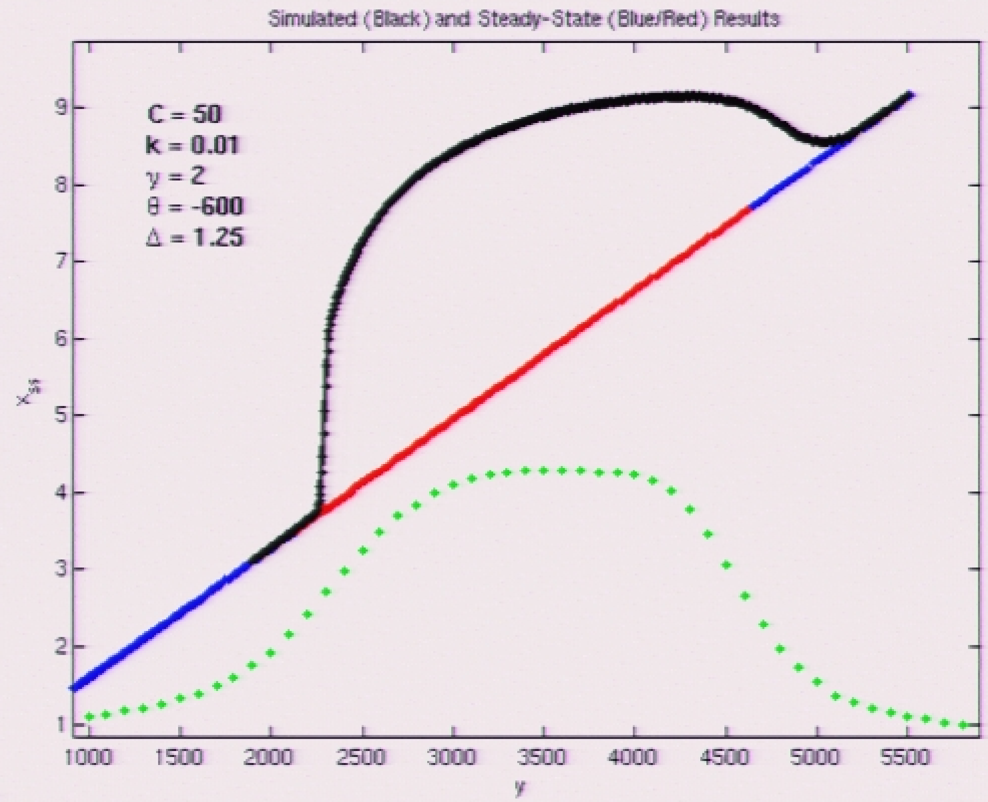


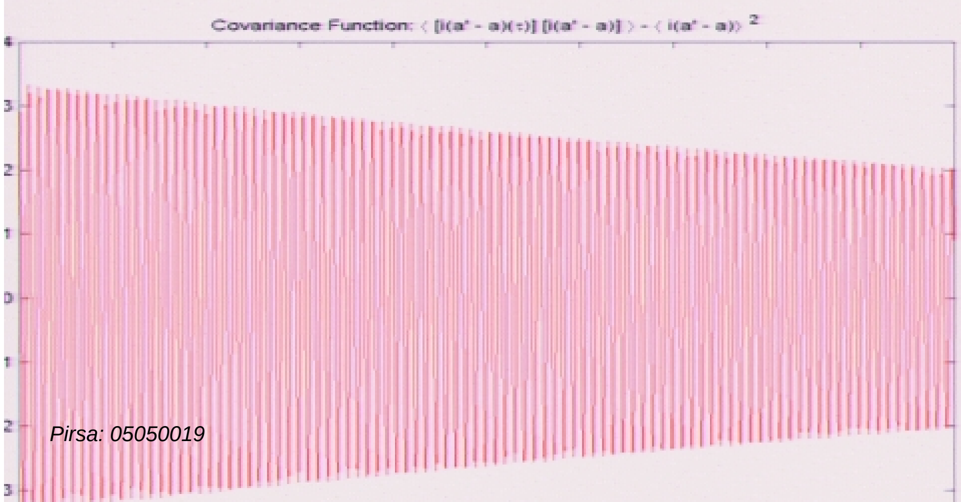


quantum dynamic equilibrium;
realize non-stationary character
via continuous measurement

Super-critical Hopf bifurcation: stable limit cycle

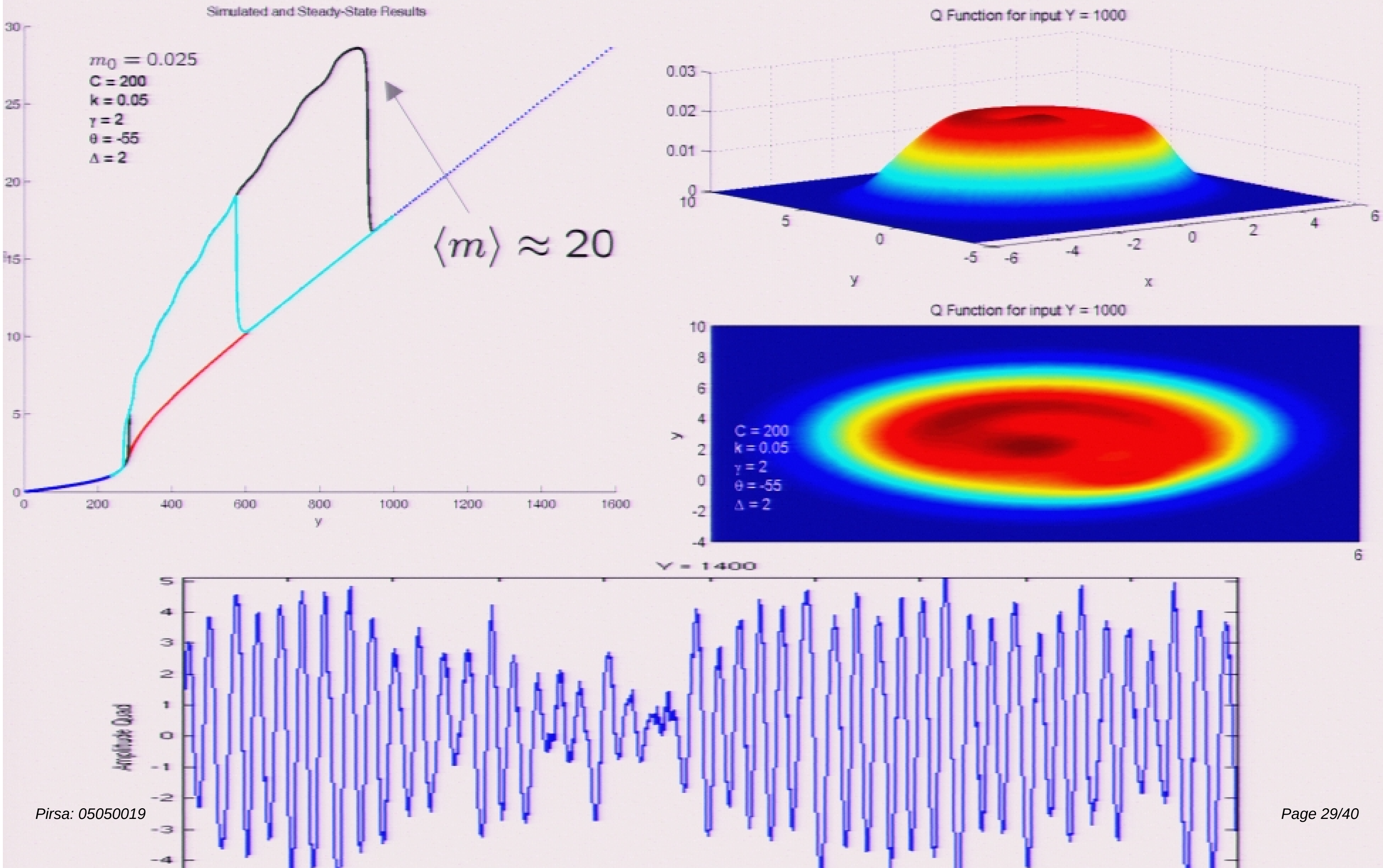




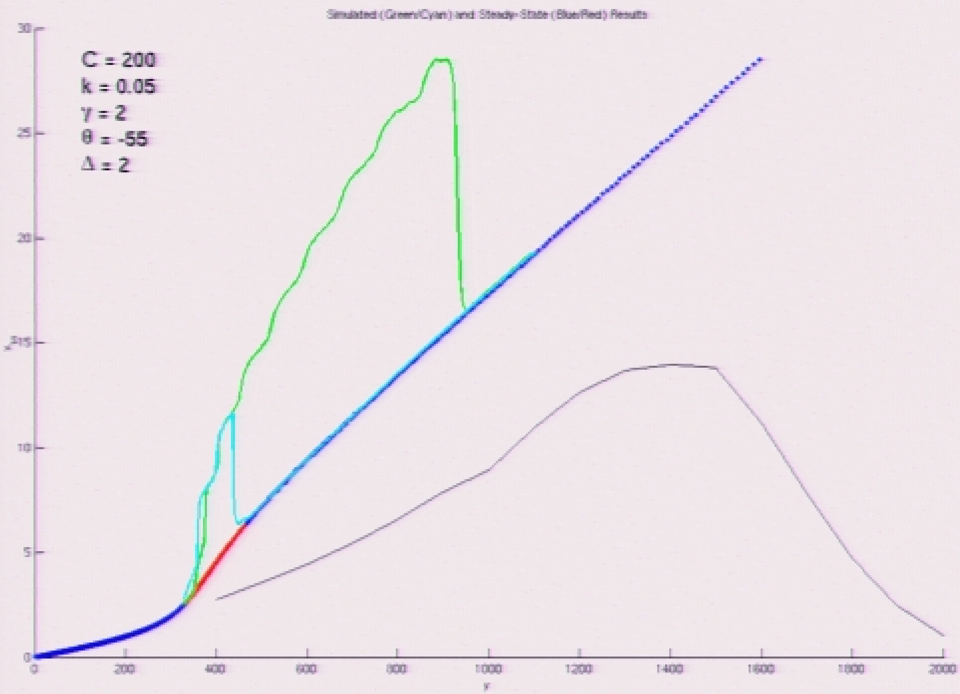


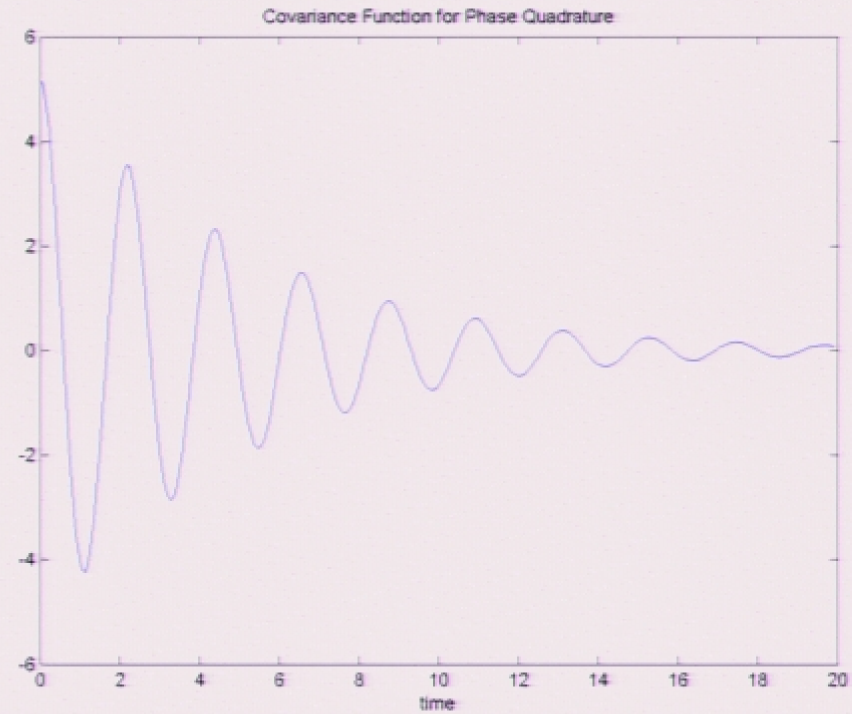
- quantum fluctuations and dephasing?
- fluctuations “explore” phase portrait?

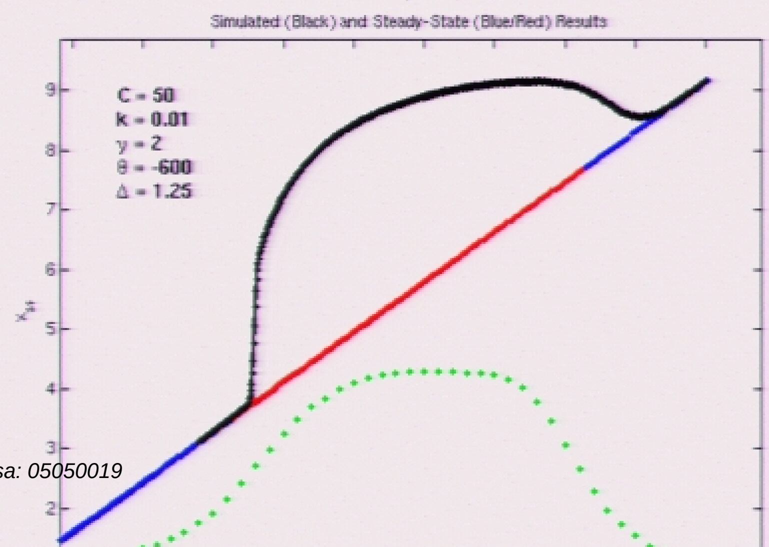
Sub-critical Hopf: fixed point plus limit cycle

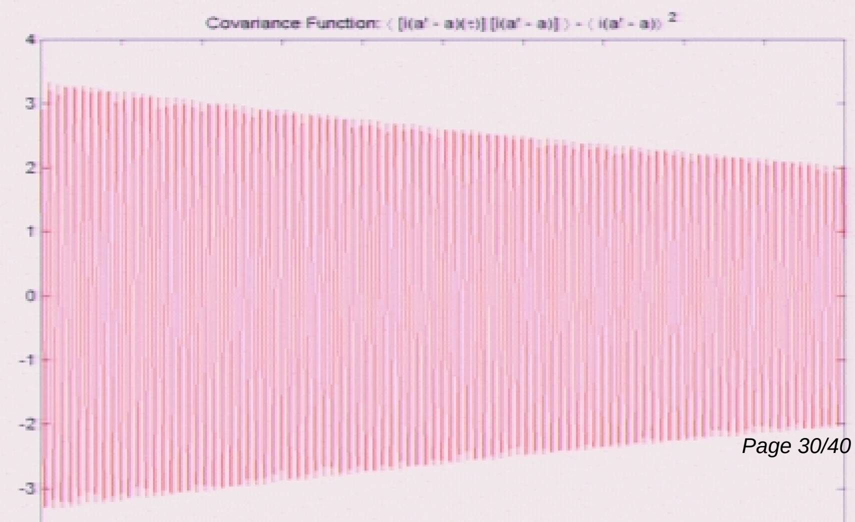


Hopf bifurcations; coherence times

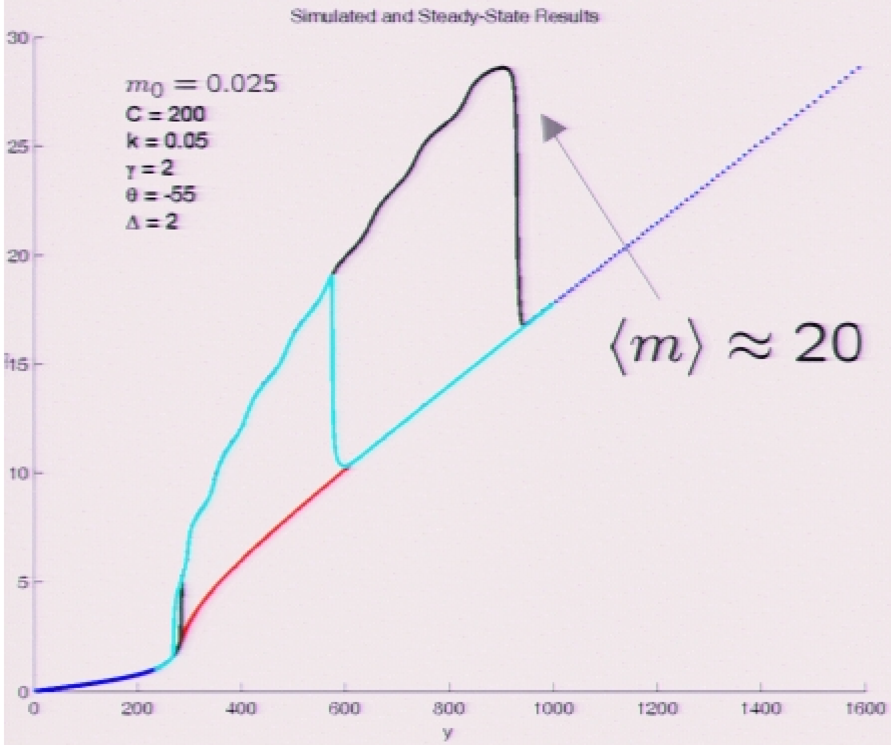


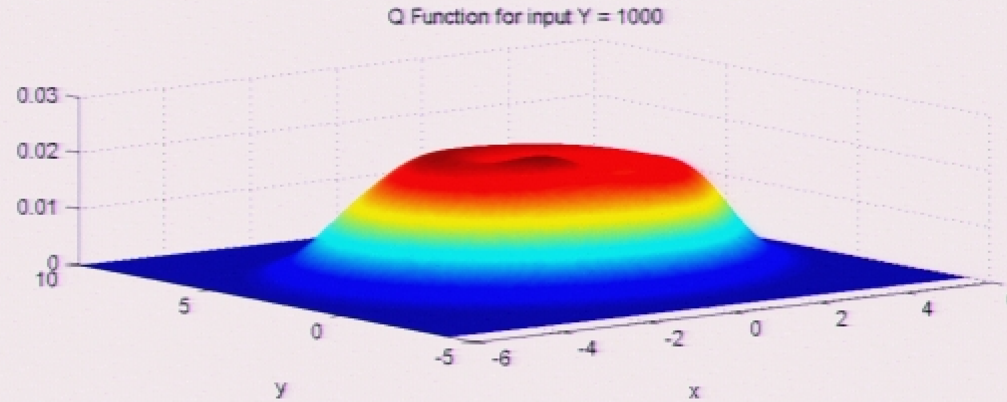


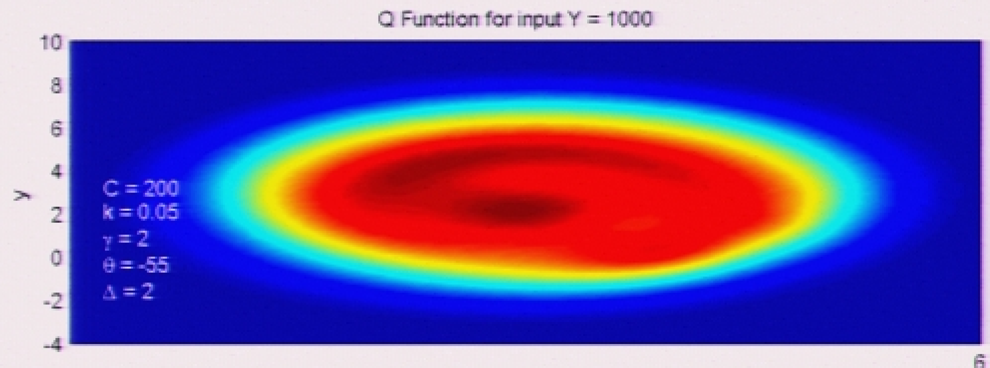


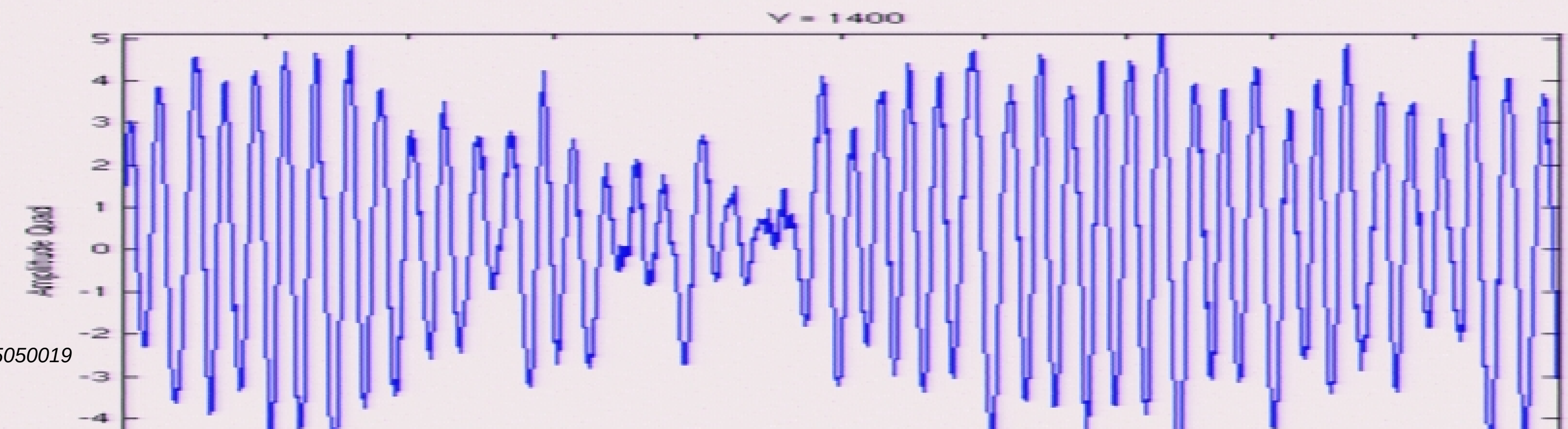


Sub-critical Hopf: fixed point plus limit cycle



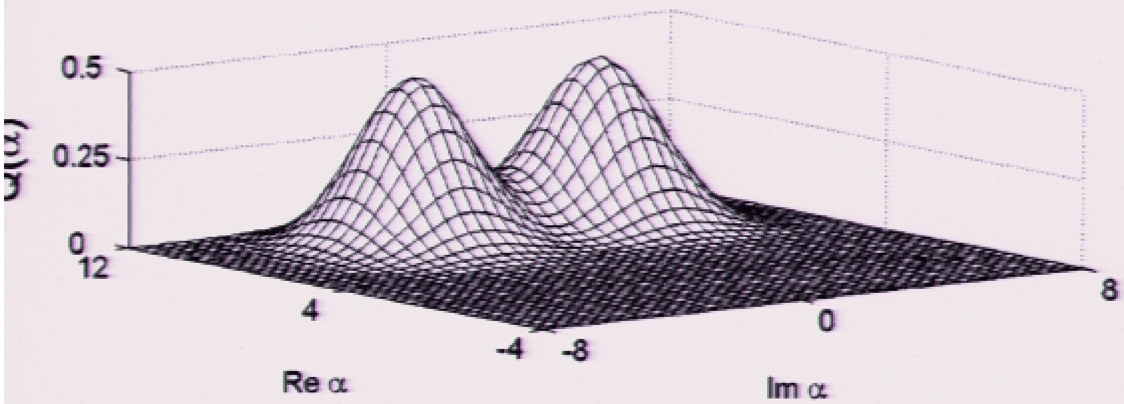


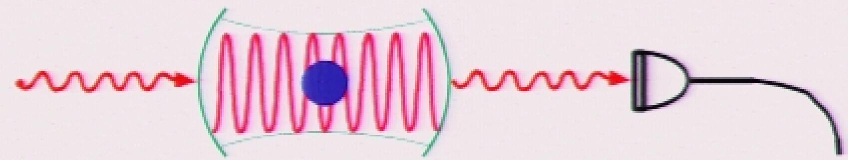




Phase bistability and 'retroactive' quantum jumps

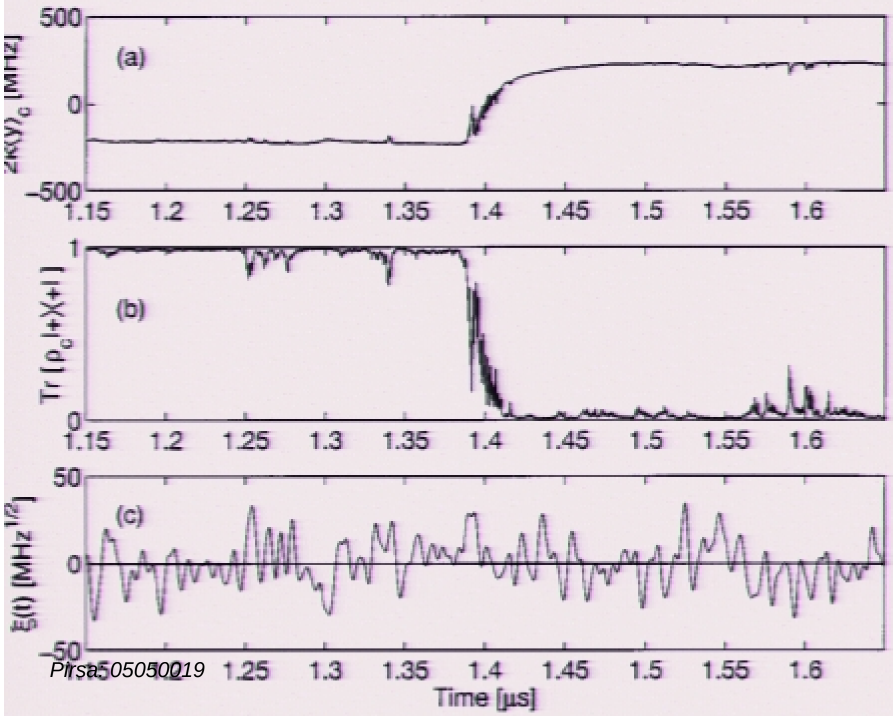
Alsing and Carmichael, Mabuchi and Wiseman

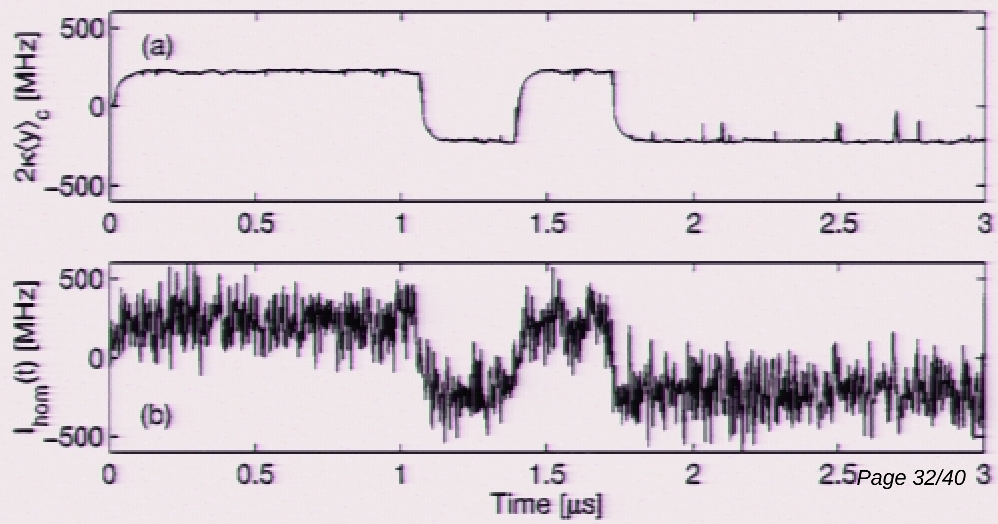




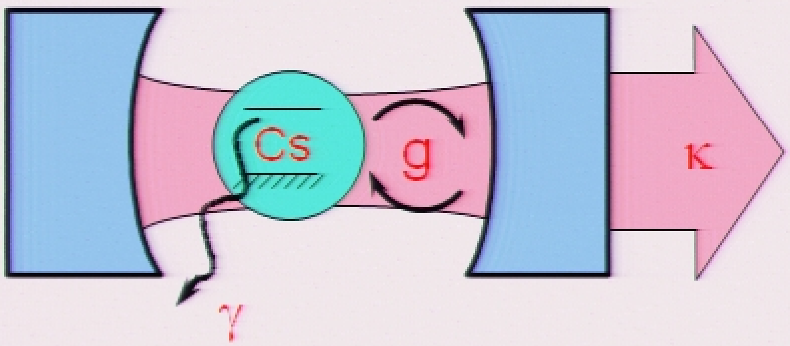
$$d\rho = \mathcal{L}\rho dt + i\sqrt{2\kappa\eta}\left\{a\rho - \rho a^\dagger - \mathrm{Tr}\left[\rho\left(a - a^\dagger\right)\right]\right\} dW_t$$

$$I_{\mathrm{hom}}(t) = 2\eta \mathrm{Tr}\left[(-ia + ia^\dagger)\rho\right] + \sqrt{2\kappa\eta} dW_t/dt$$

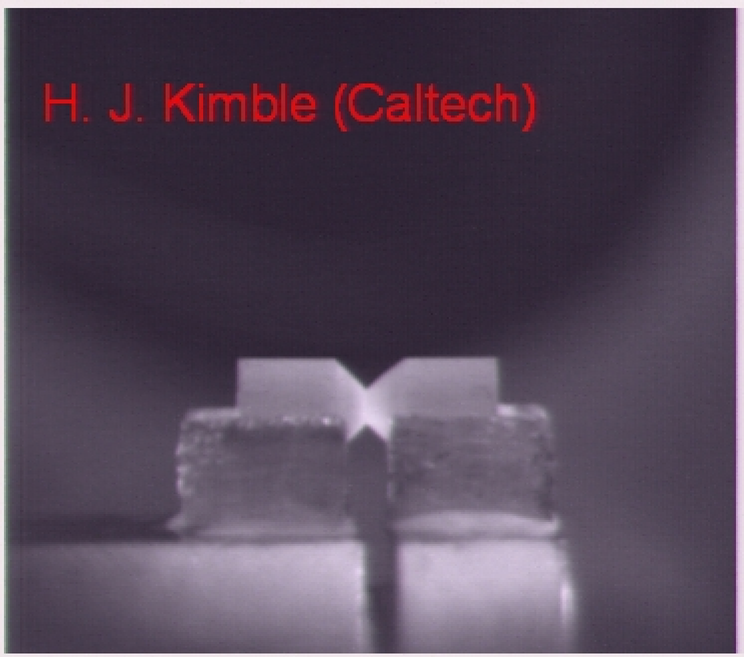




Cavity QED with photonic bandgap structures?



Critical photon number	Critical atom number
$m_0 \approx \dfrac{\gamma^2}{2g^2} < 1$	$N_0 \approx \dfrac{2\gamma\kappa}{g^2} < 1$
Nonlinear optics with *one photon* per mode	*Single-atom* switching of optical cavity response



xperiment: $m_0 \approx 3\times10^{-4}$ $N_0 \approx 6\times10^{-3}$

rojected: $m_0 \approx 6\times10^{-6}$ $N_0 \approx 2\times10^{-4}$

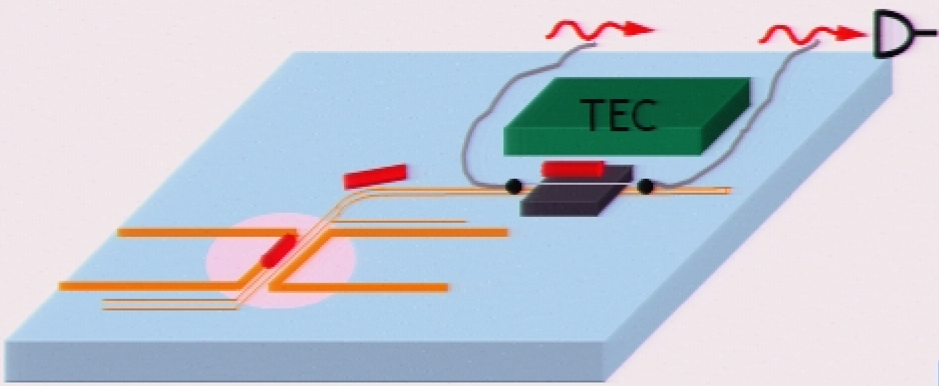
Projected: $m_0 \approx 8\times10^{-9}$ $N_0 \approx 6\times10^{-5}$

¿ surface effects ? ¿ atom localization ?

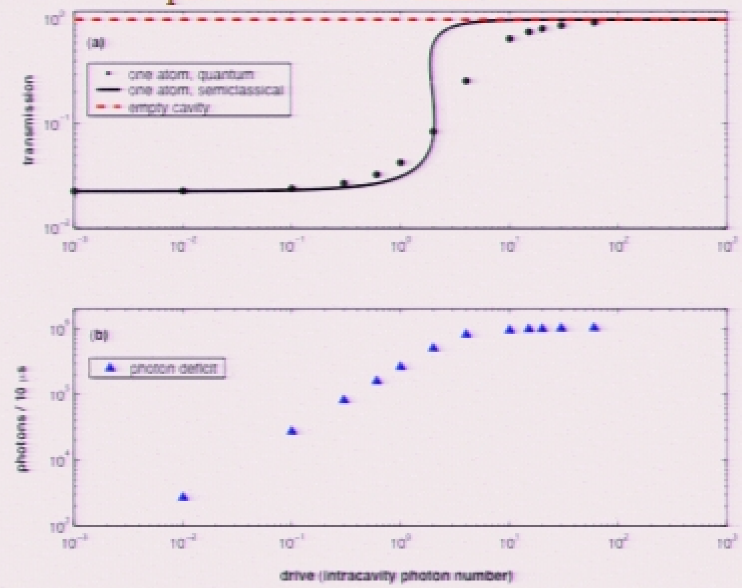
Feasibility of single atom detection using a PBG cavity and magnetic waveguides

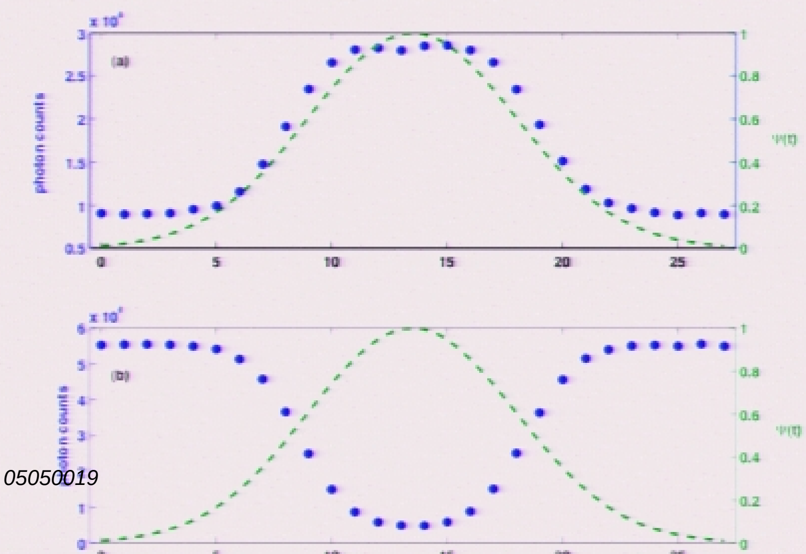
B. Lev, HM, K. Srinivasan, P. Barclay and O. J. Painter

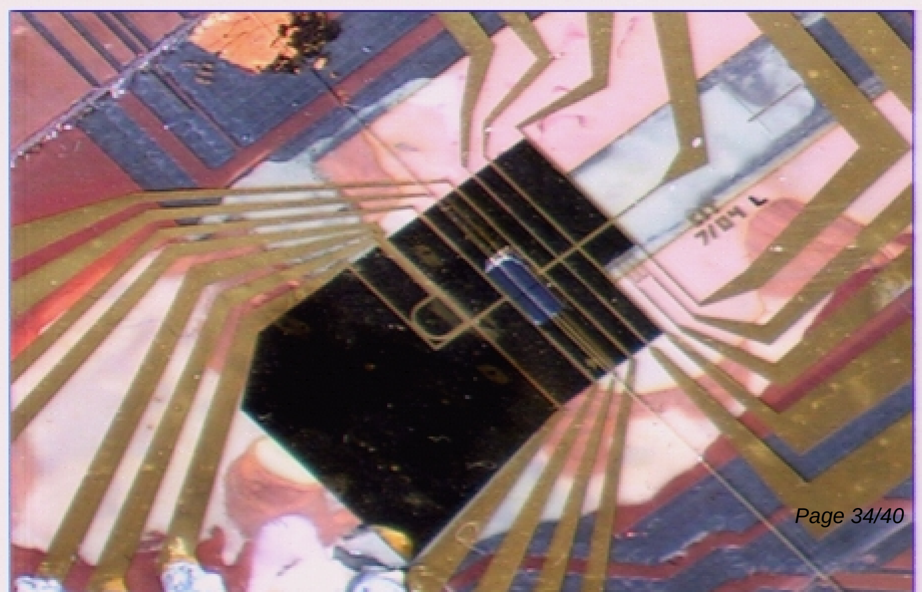
$m_0 = 1.2 \times 10^{-8}$ $\lambda = 2.6$ MHz
$N_0 = 8.4 \times 10^{-5}$ $\kappa = 4.4$ GHz
$g_0 = 17$ GHz

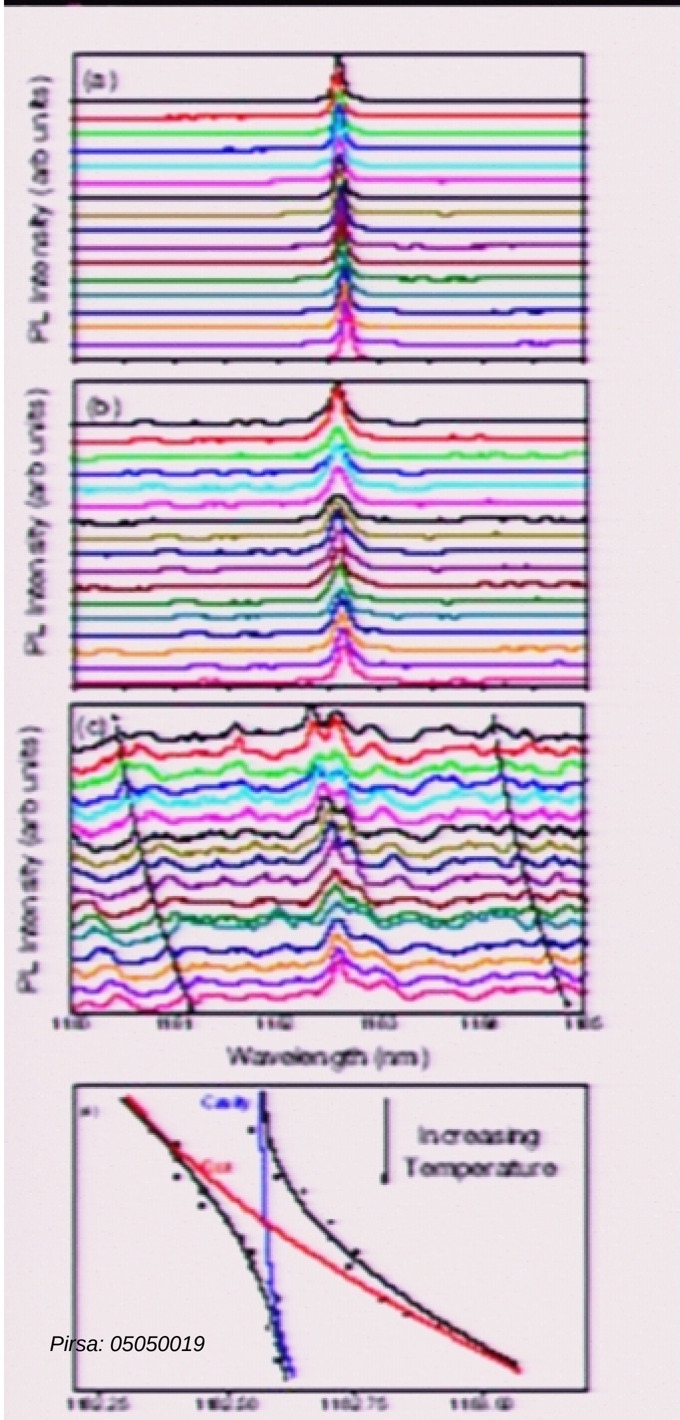


probe laser 10 GHz detuned









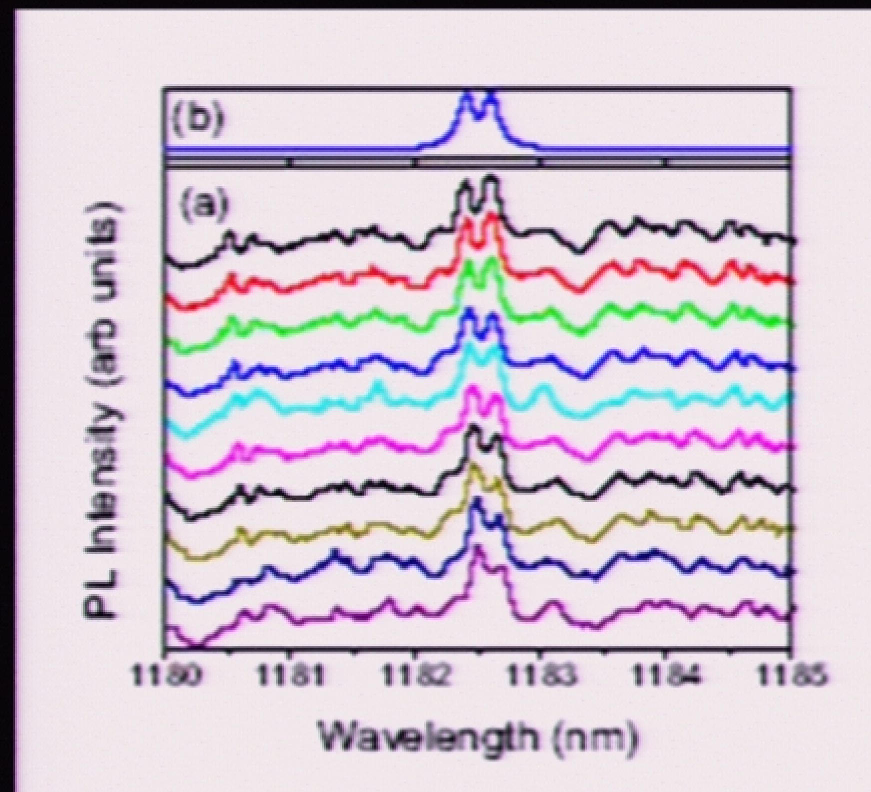
Strong coupling between QD and PC cavities

(Axel Scherer, Caltech)

690 μW
Q~14,000

25 μW
Q~8000

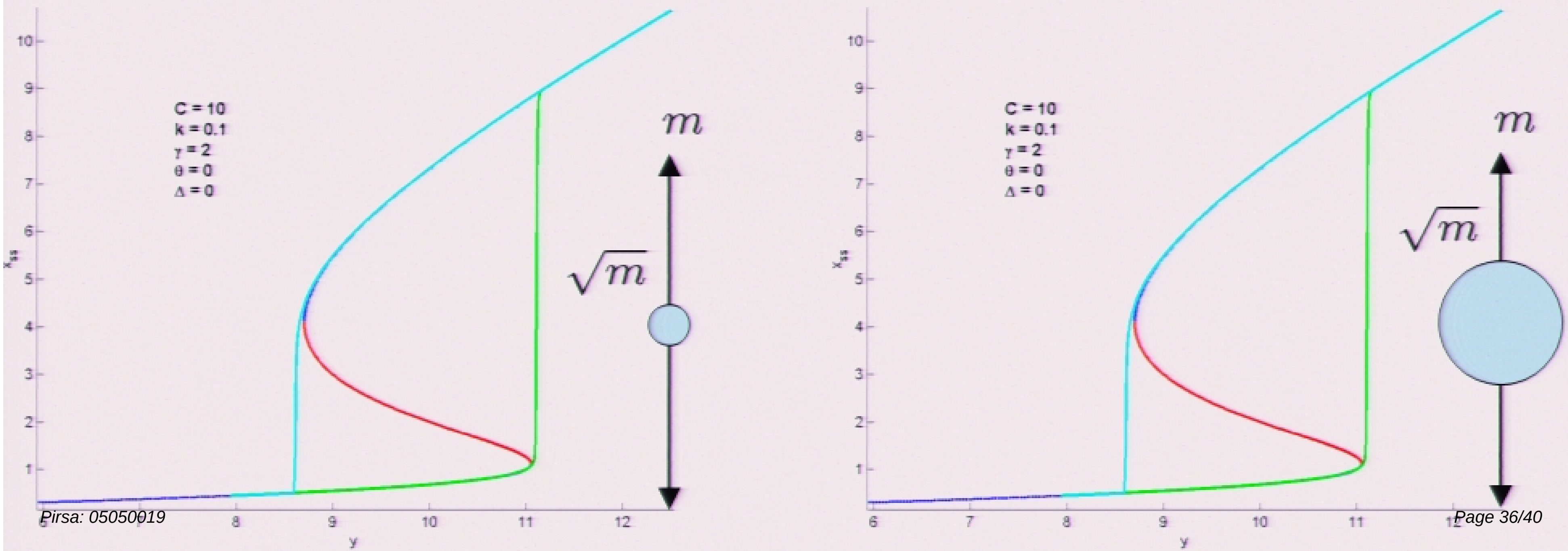
0.78 μW



Strong coupling between quantum dot and high-Q cavity enables both classical and

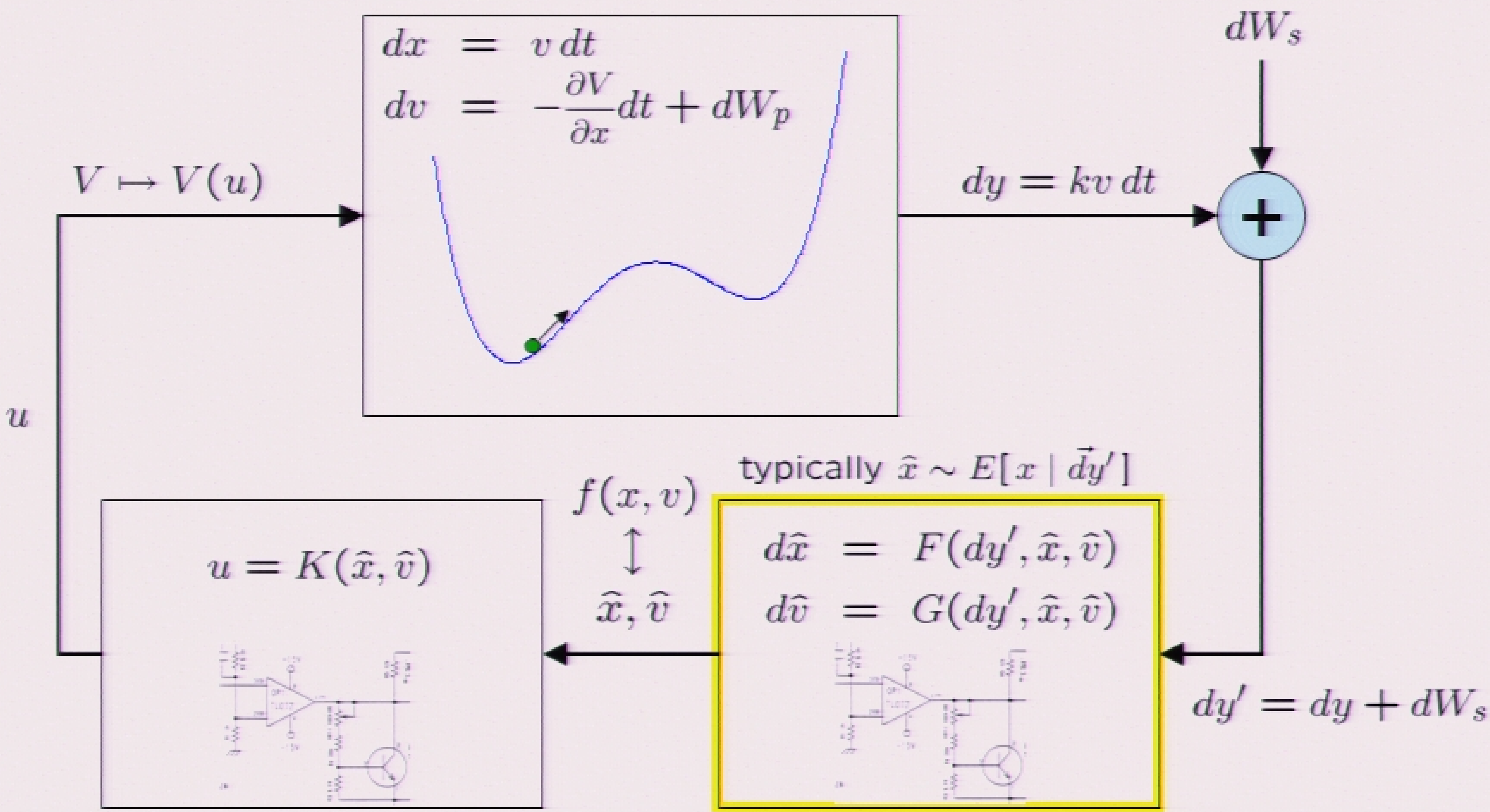
Conclusions

- cavity QED: "nonlinear structures" with $\sim$ few 10^{-18} J switching energy
- photonic crystal resonators could provide $\gtrsim$ GHz strong coupling
- device design requires study of quantum noise + nonlinear dynamics



Classical filtering: concepts and applications

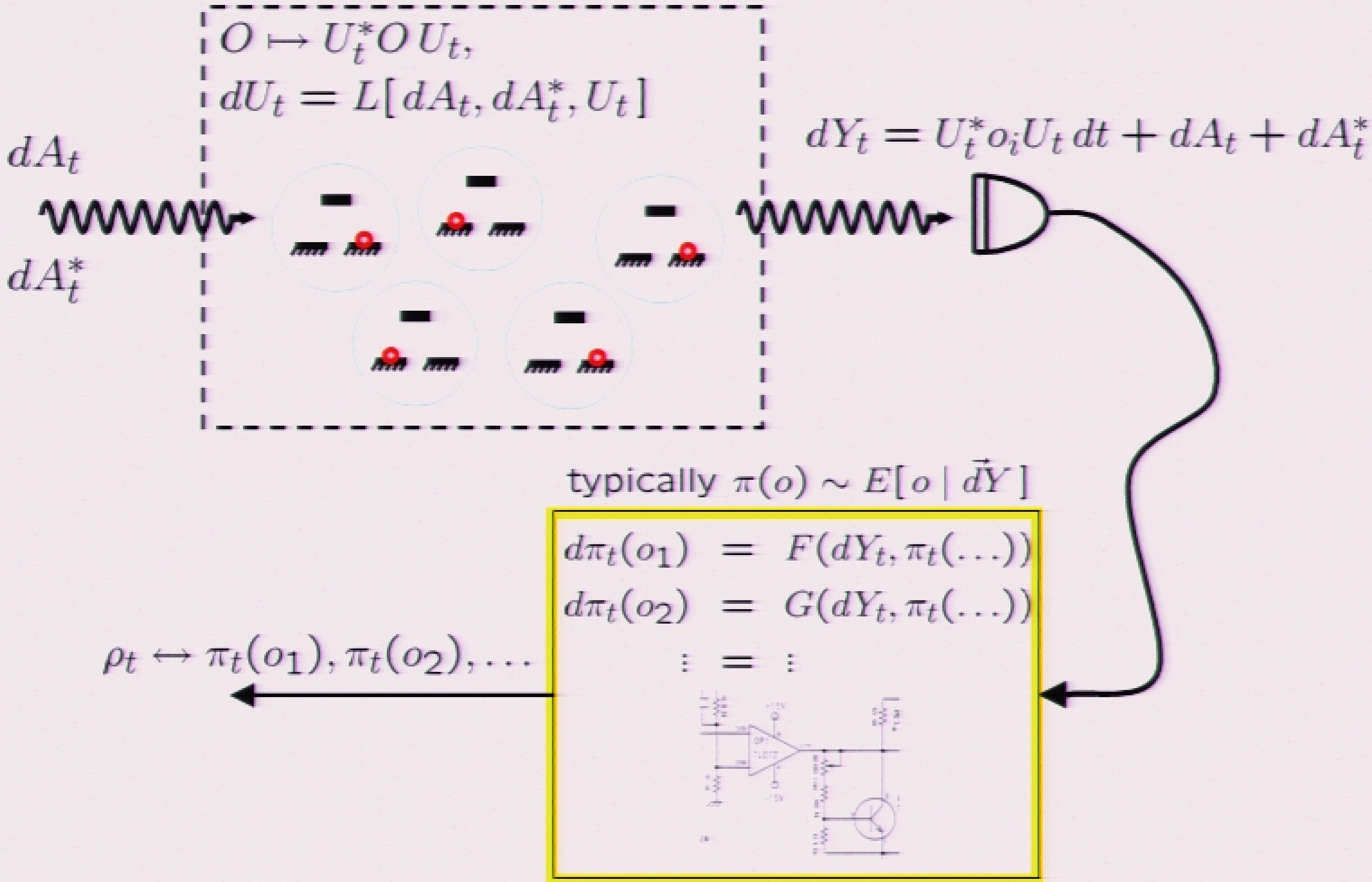
(introductory texts in applied mathematics or control theory)



- **applications**: target tracking, mathematical finance, real-time feedback control
- **tools**: Kushner-Stratonovich equation, Kalman filter, robust filtering, filter reduction

Open quantum systems: from scattering to filtering

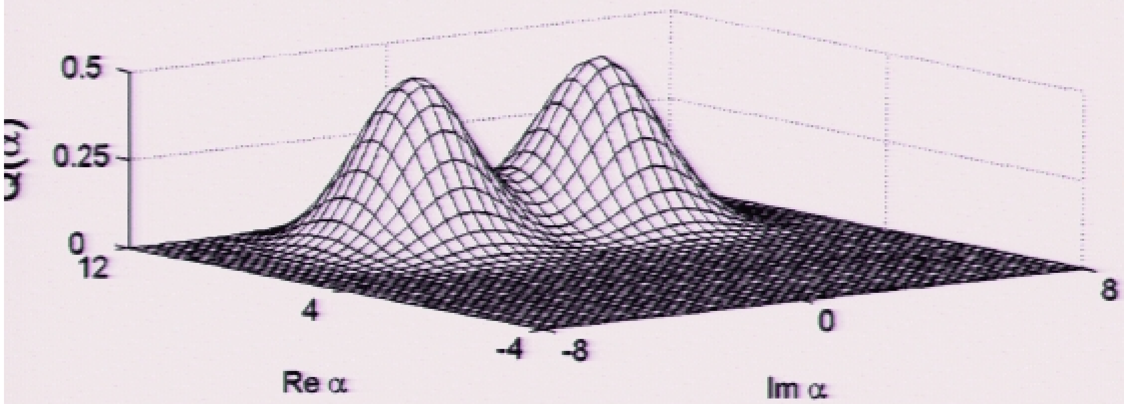
Ramon van Handel, John Stockton and HM (to appear, 2005) http://minty.caltech.edu

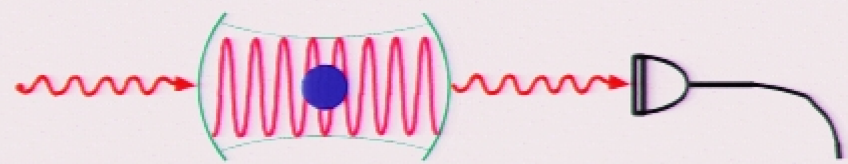


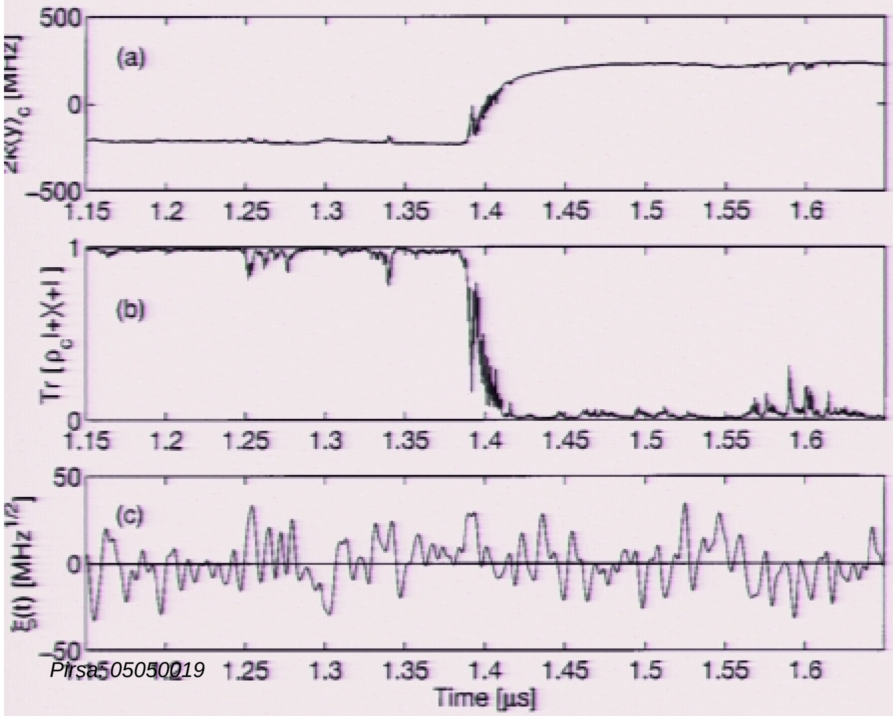
- **applications**: quantum feedback control, precision measurement and sensing
- **tools**: stochastic master (Belavkin) equation, Kalman filter, robust filters, reduction

Phase bistability and ‘retroactive’ quantum jumps

Alsing and Carmichael, Mabuchi and Wiseman







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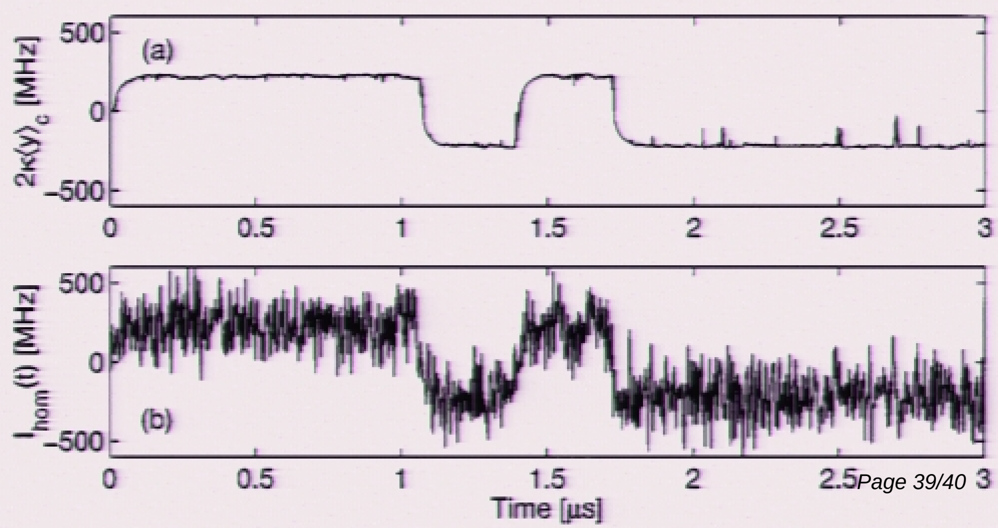

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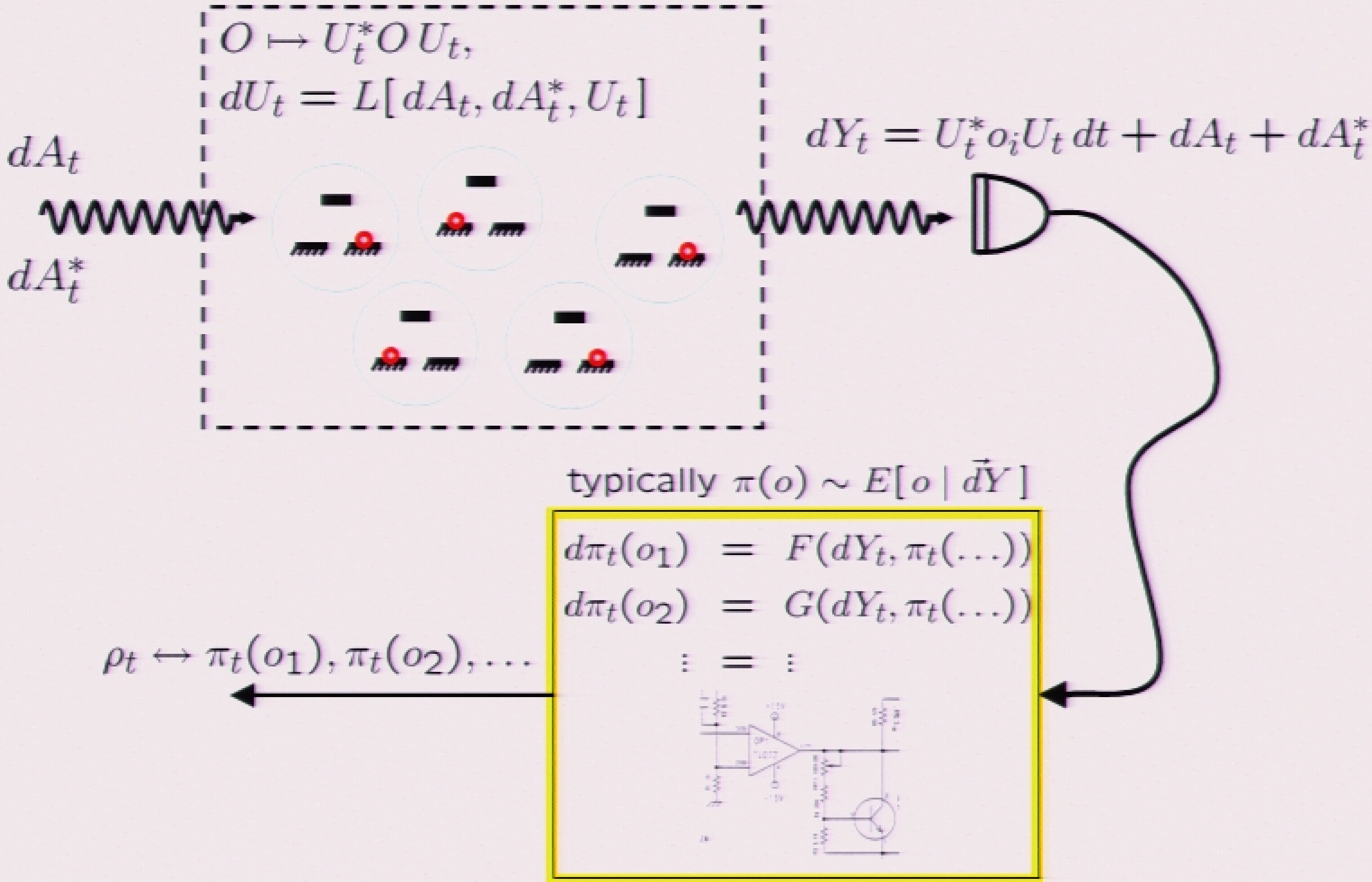

- **applications**: quantum feedback control, precision measurement and sensing
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