Title: Coxeter Lecture II - Stabilization of moduli in string theory I

Date: May 10, 2005 02:00 PM

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Abstract:

Outline of Lecture I

1. Cosmological Concordance Model and Problems of M/String Theory in Explaining the Observations.

2. Flux Compactification and Stabilization of Moduli, Metastable de Sitter Space in String Theory

3. Ghost-Free de Sitter Supergravities as Consistent Reductions of String and M-theory: collapsing universe

4. Landscape of String Theory, Statistics of Flux Vacua, CC problem

5. Inflation in String Theory, Cosmic Strings, Scale of SUSY breaking

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Our Universe is an Ultimate Test of Fundamental Physics

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- High-energy accelerators will probe the scale of energies way below GUT scales

- **Cosmology and astrophysics are sources of data in the gravitational sector of the fundamental physics (above GUT, near Planck scale)**

In view of the recent cosmological observations supporting dark energy and inflation it is fair to say that we do not really know what is "fundamental physics"

" Most embarrassing observation in physics – that's the only quick thing I can say about dark energy that's also true." -- **Edward Witten**

What is so embarrassing about it?

Two general problems:

What is so embarrassing about it?

Two general problems:

- Why is the cosmological constant so small, $\Lambda < 10^{-120}$ in Planck density units ?

- Why $\Lambda \sim \rho_{\mathrm{matter}}$?
  Coincidence problem.

addressed by anthropic principle, Weinberg 1987

The third problem:

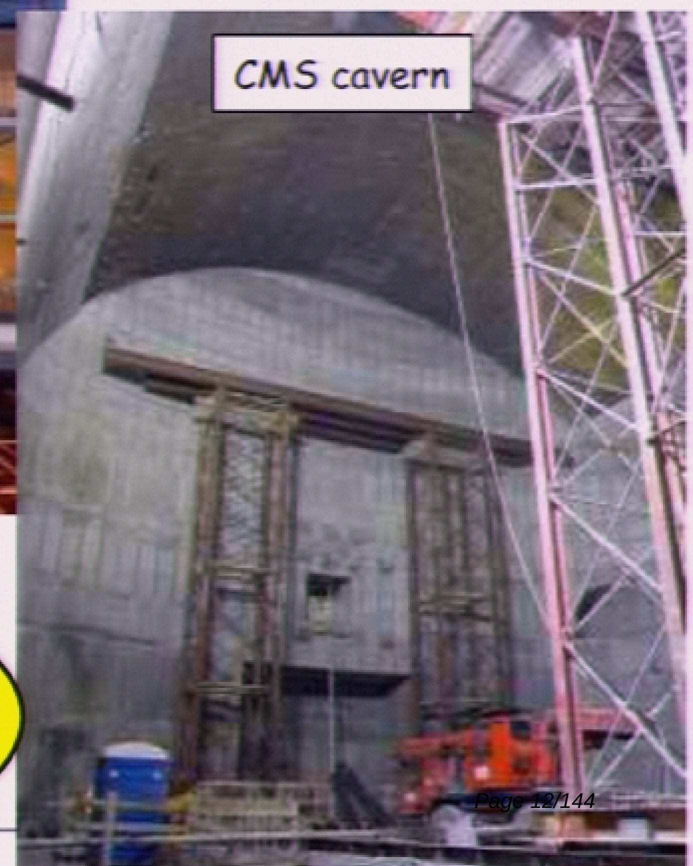
Two years ago it was not clear how one could possibly incorporate a positive cosmological constant **in string theory**

This was **the main reason of embarrassment for string theorists**, because of the cosmological data suggesting that $\Lambda > 0$

One can argue that M/String theory is fundamental

Physics beyond the Standard Model at LHC

Start : summer 2007



Higgs, Standard Model Supersymmetry,
SPLIT SUPERSYMMETRY

F. Giano...

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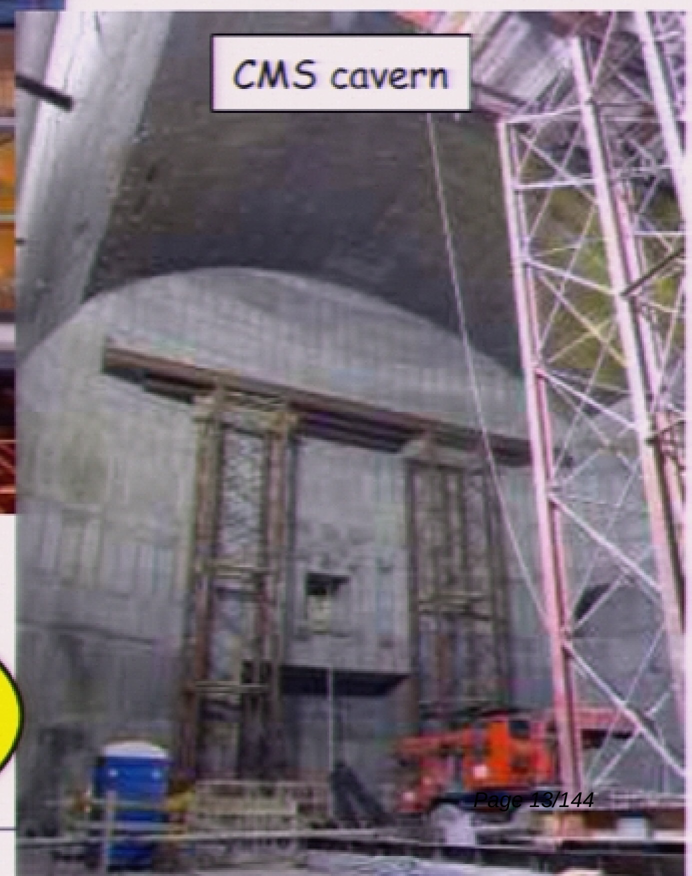

Higgs, Standard Model Supersymmetry,
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F. Giano

LHC discovery reach

Time	reach in squark/gluino mass
1 month at 10^{33}	~ 1.3 TeV
1 year at 10^{33}	~ 1.8 TeV
1 year at 10^{34}	~ 2.5 TeV
ultimate (300 fb^{-1})	up to ~ 3 TeV

LHC should add many crucial pieces to our knowledge of fundamental physics

→ huge impact also on astroparticle physics and cosmology ?

→ in ~ 3 years particle physics may enter the most glorious epoch of its history ...

IF SUSY IS THERE

The significance of discovery of supersymmetry in nature,

(which will manifests itself via existence of supersymmetric particles)

is the discovery of the **fermionic dimensions of spacetime**.

It will be the most fundamental discovery in physics after Einstein's relativity

$$(t, \vec{x}) \to (t', \vec{x}') \quad \Rightarrow \quad x^\mu \to (x^\mu)'$$

SUPERSYMMETRY

$$(x^\mu, \theta_\alpha) \to ((x^\mu)', \theta'_\alpha)$$

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Gravity → **Supergravity/String theory**

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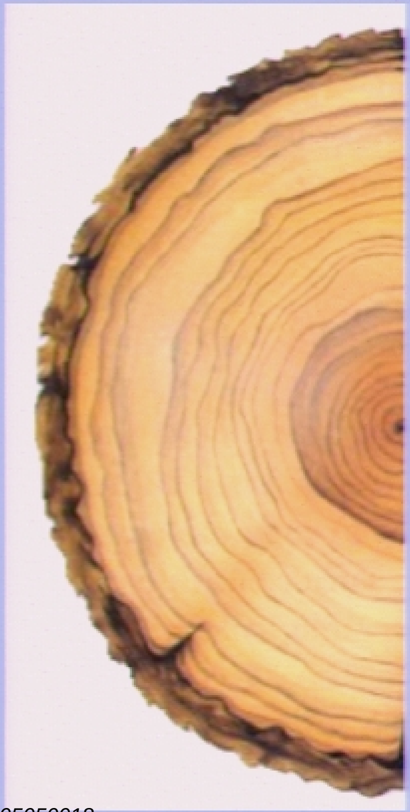
Gravity → **Supergravity/String theory**

Fundamental Physics

Astrophysics → Cosmology → Field Theory

SN, CMB, LSS → a(t) → Equation of state w(z) → V(ϕ)

V (ϕ(a(t)))

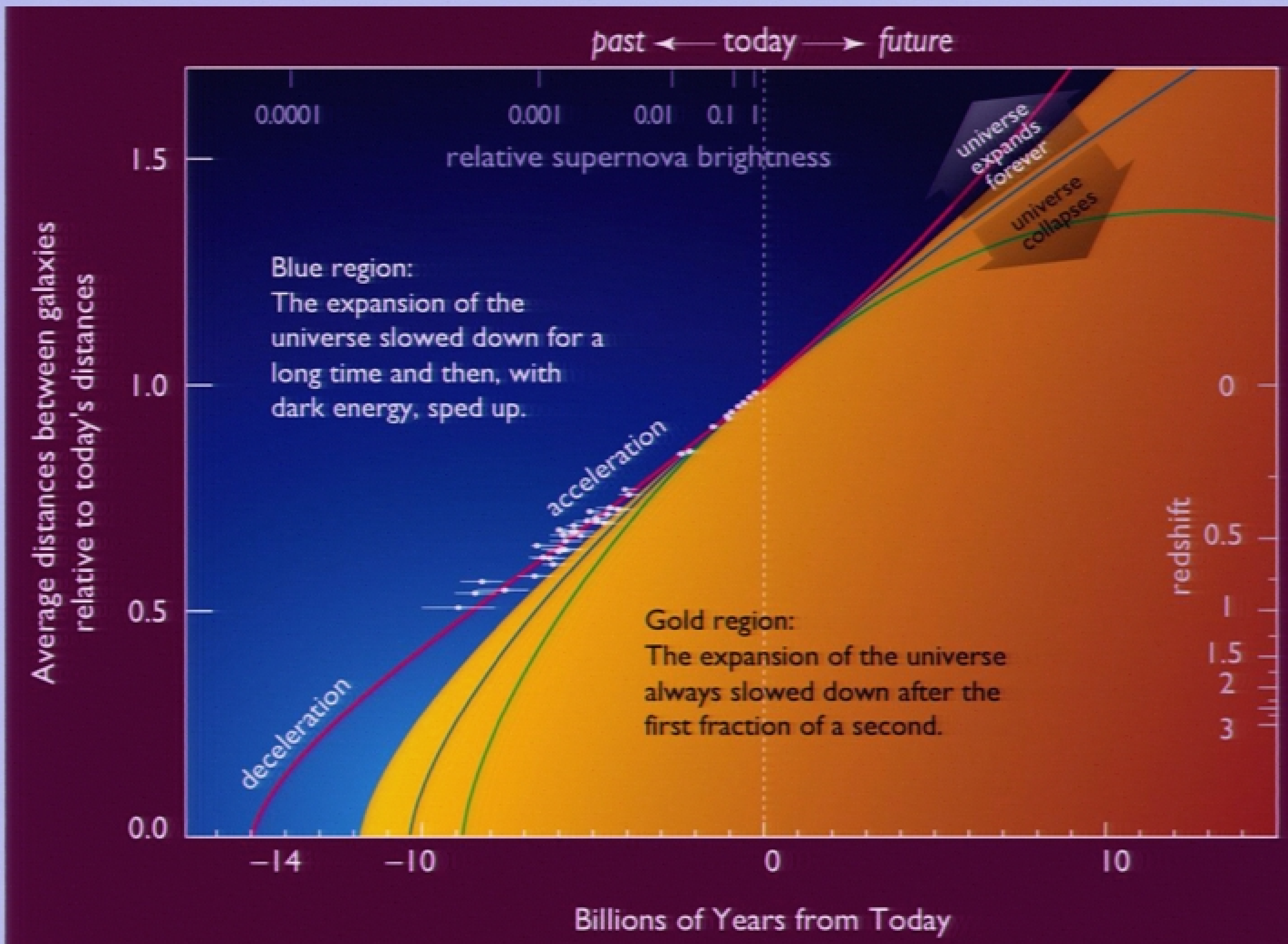


The subtle slowing and growth of scales with time – a(t) – map out the cosmic history like tree rings map out the Earth's climate history.

STScI

Map the expansion history of the universe

Discovery! Acceleration



Exploding stars – supernovae – are bright beacons that allow us to measure precisely the expansion over the last 10 billion years

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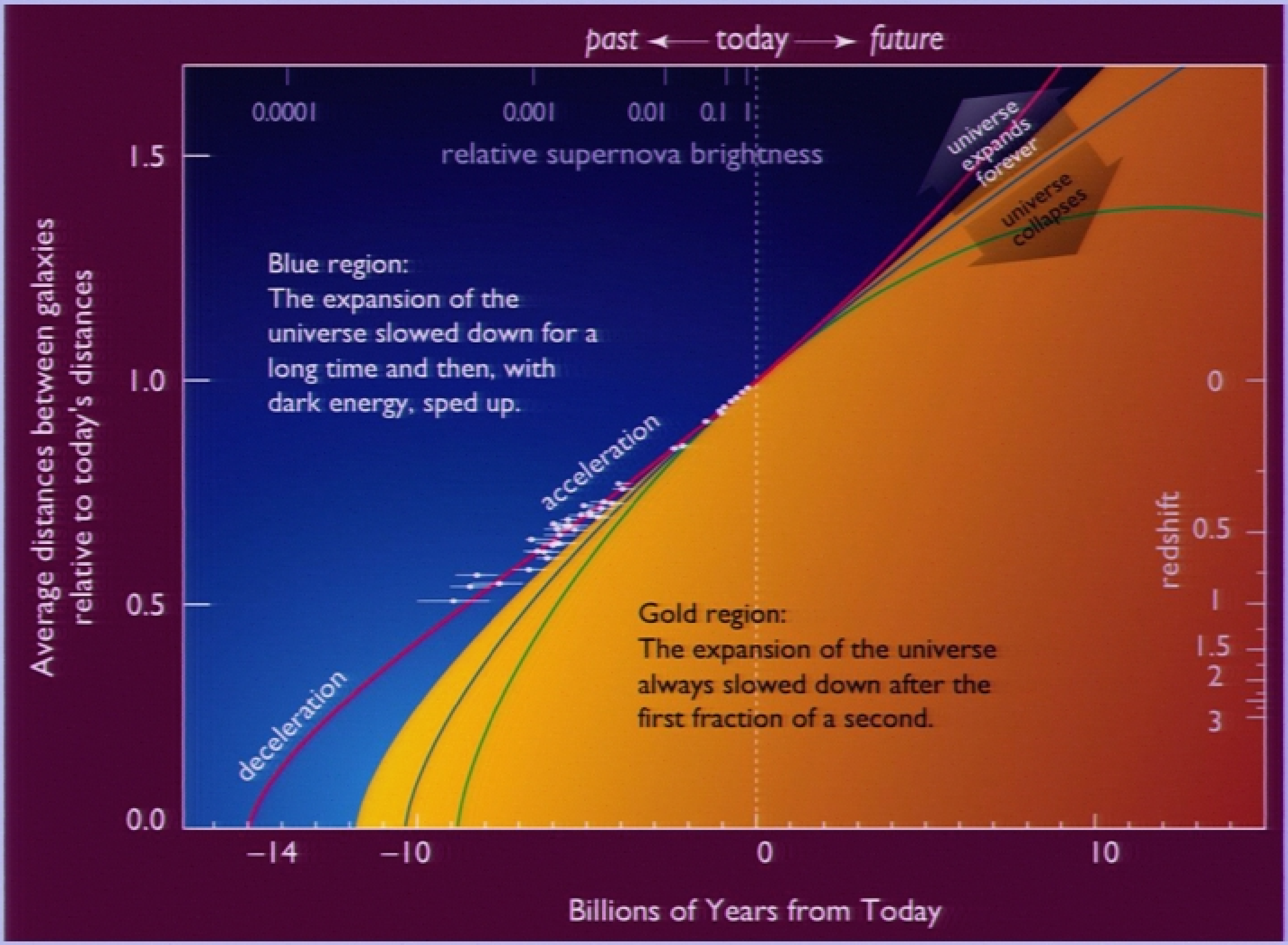

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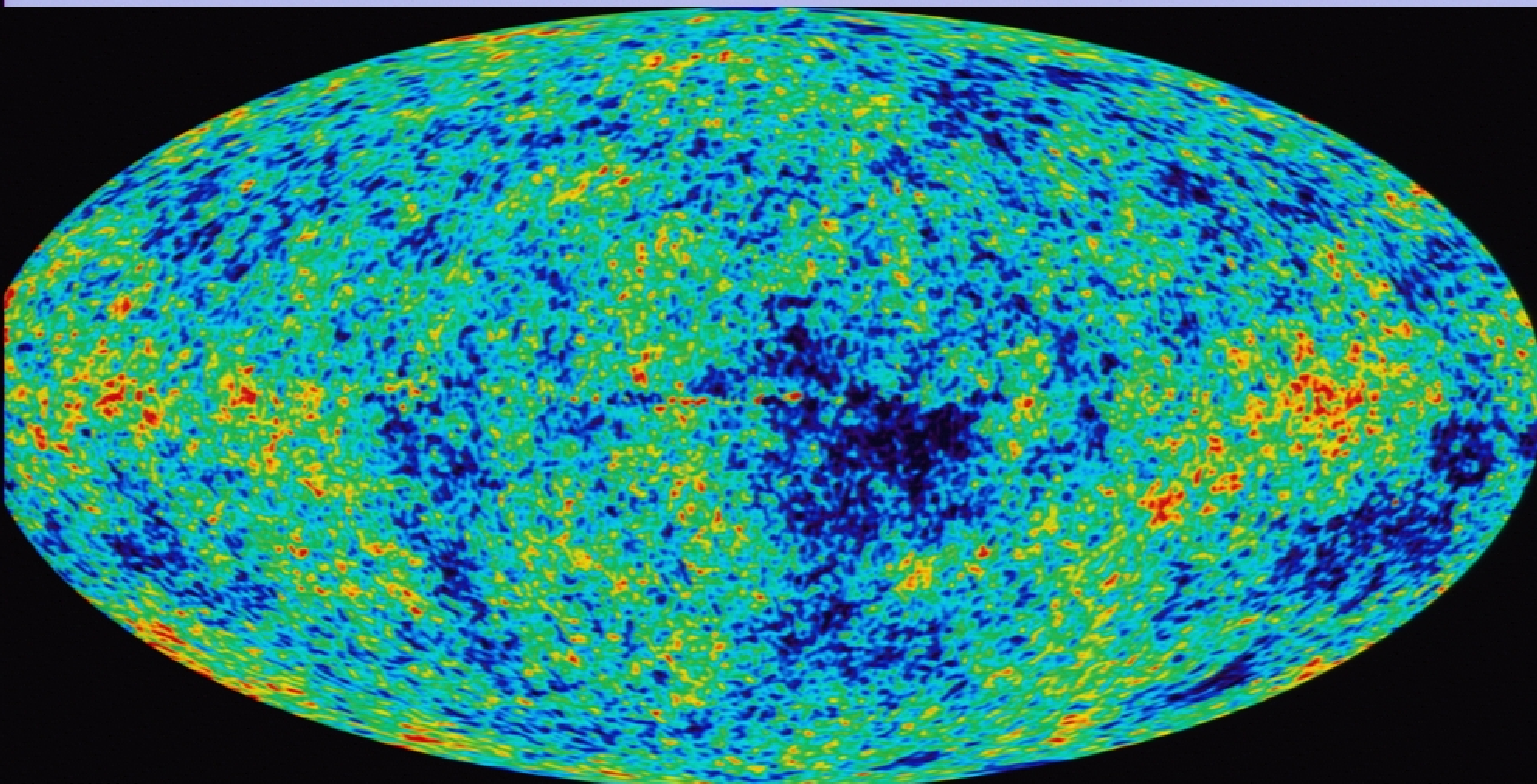
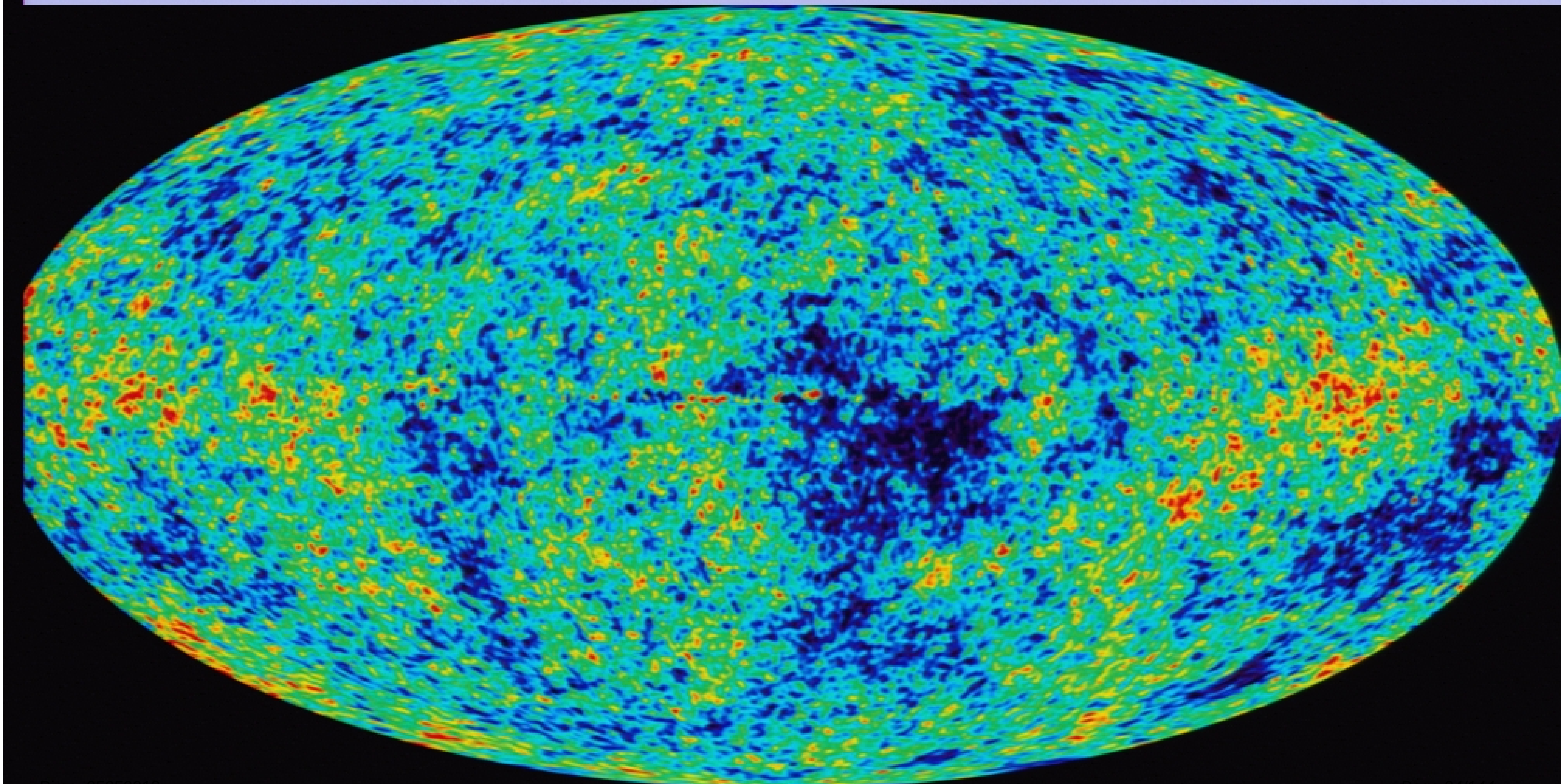
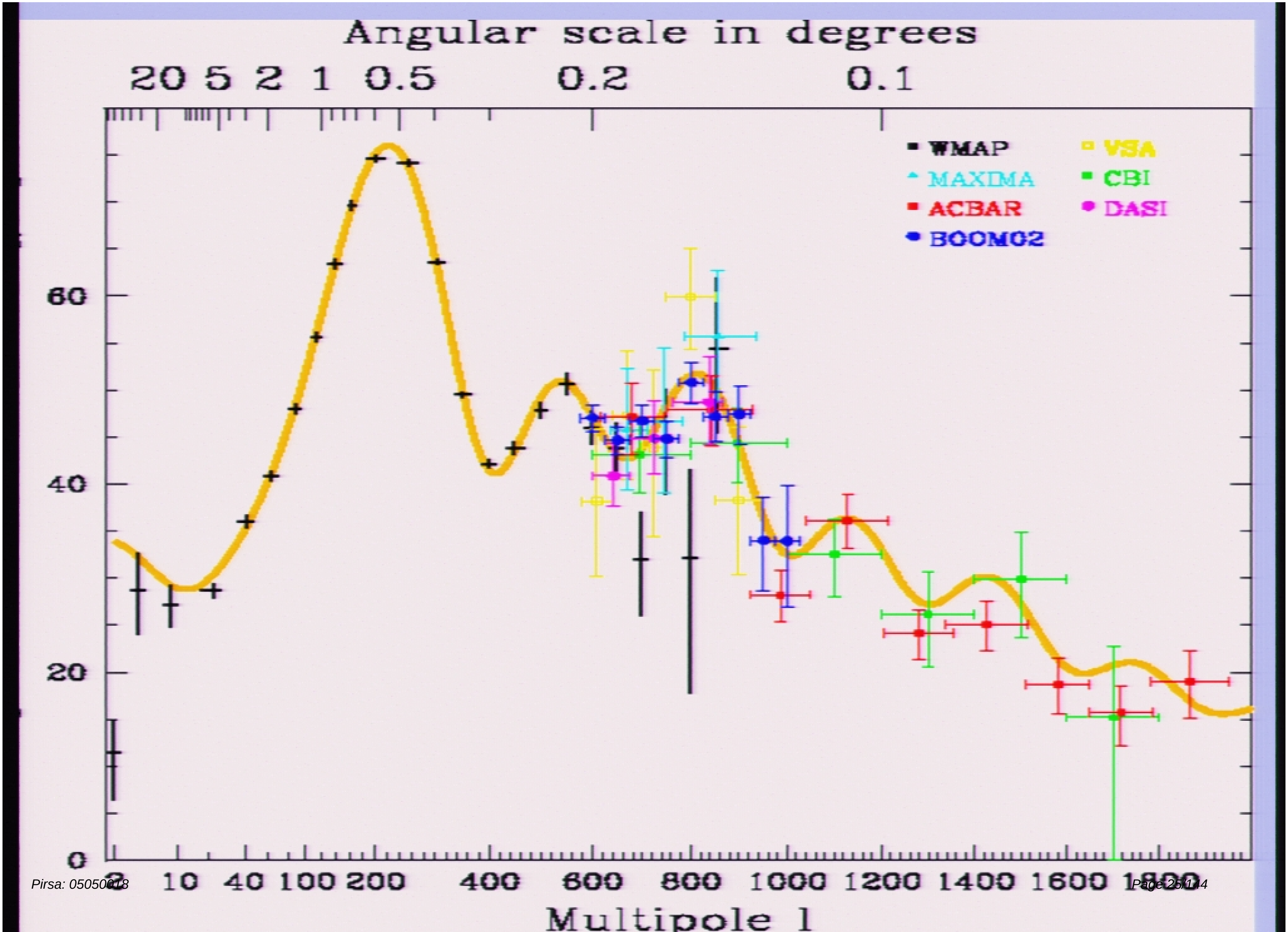
WMAP and the temperature of the sky



WMAP and the temperature of the sky




Angular scale in degrees
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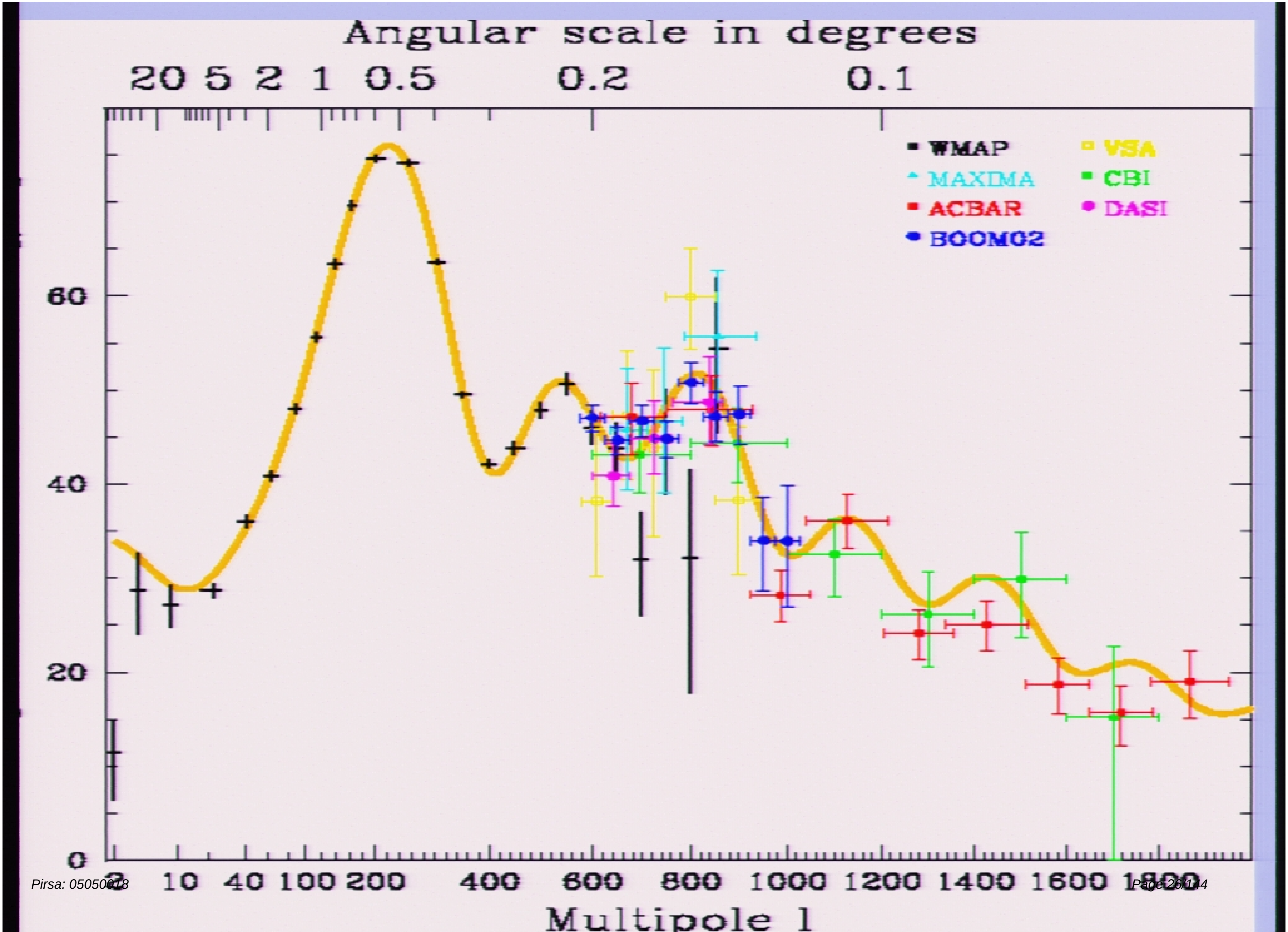
Multipole l



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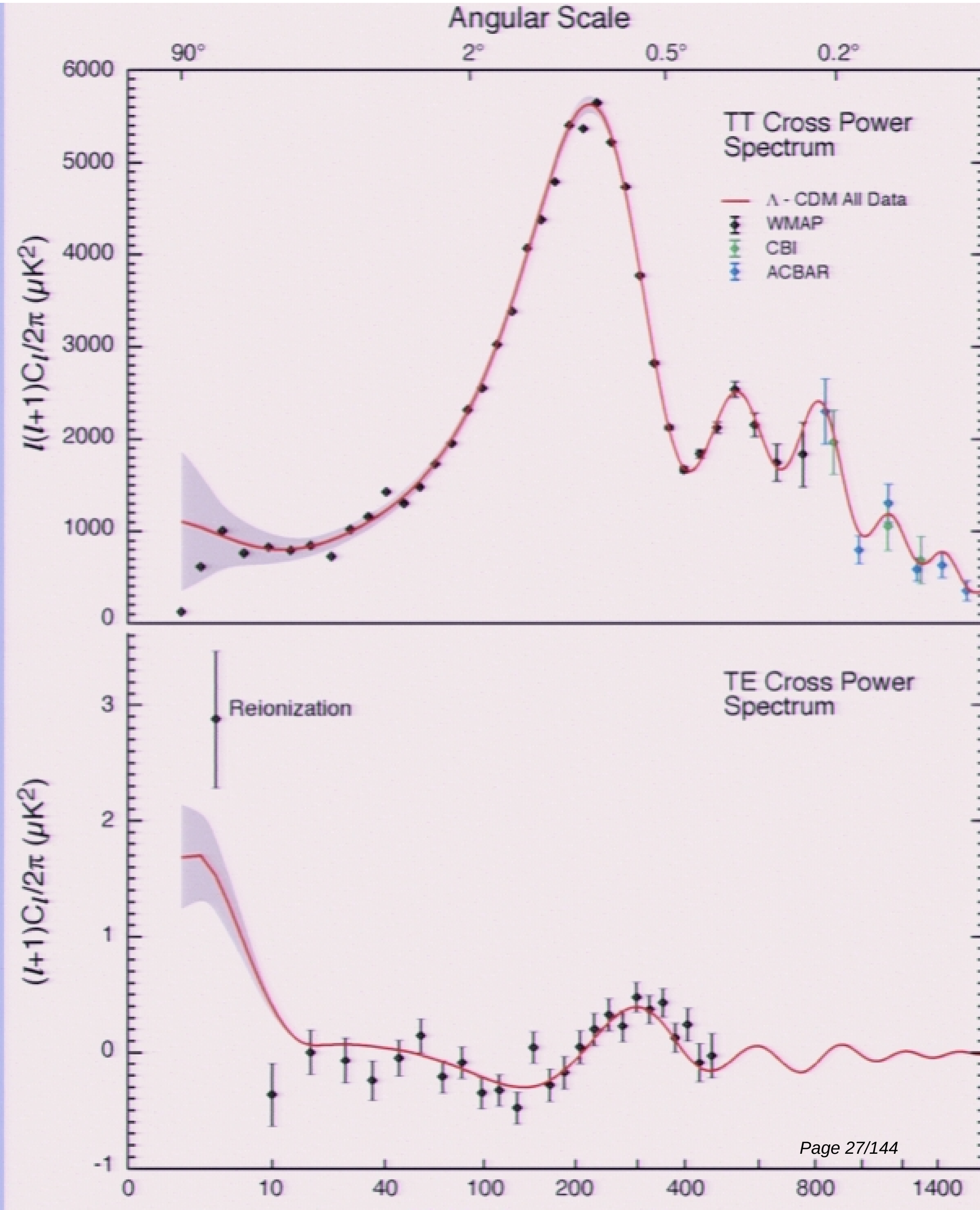
WMAP

and spectrum of the microwave background anisotropy



WMAP

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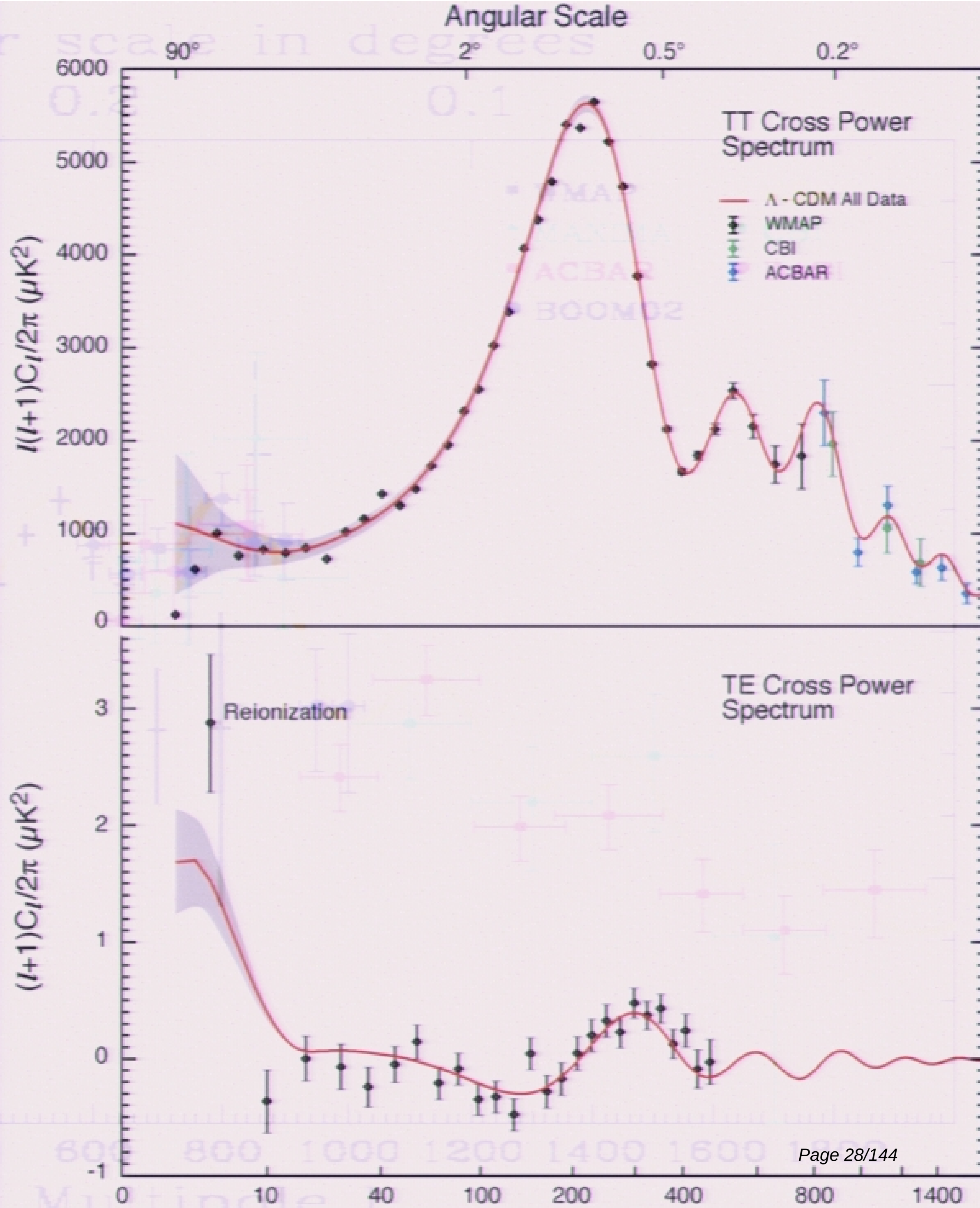

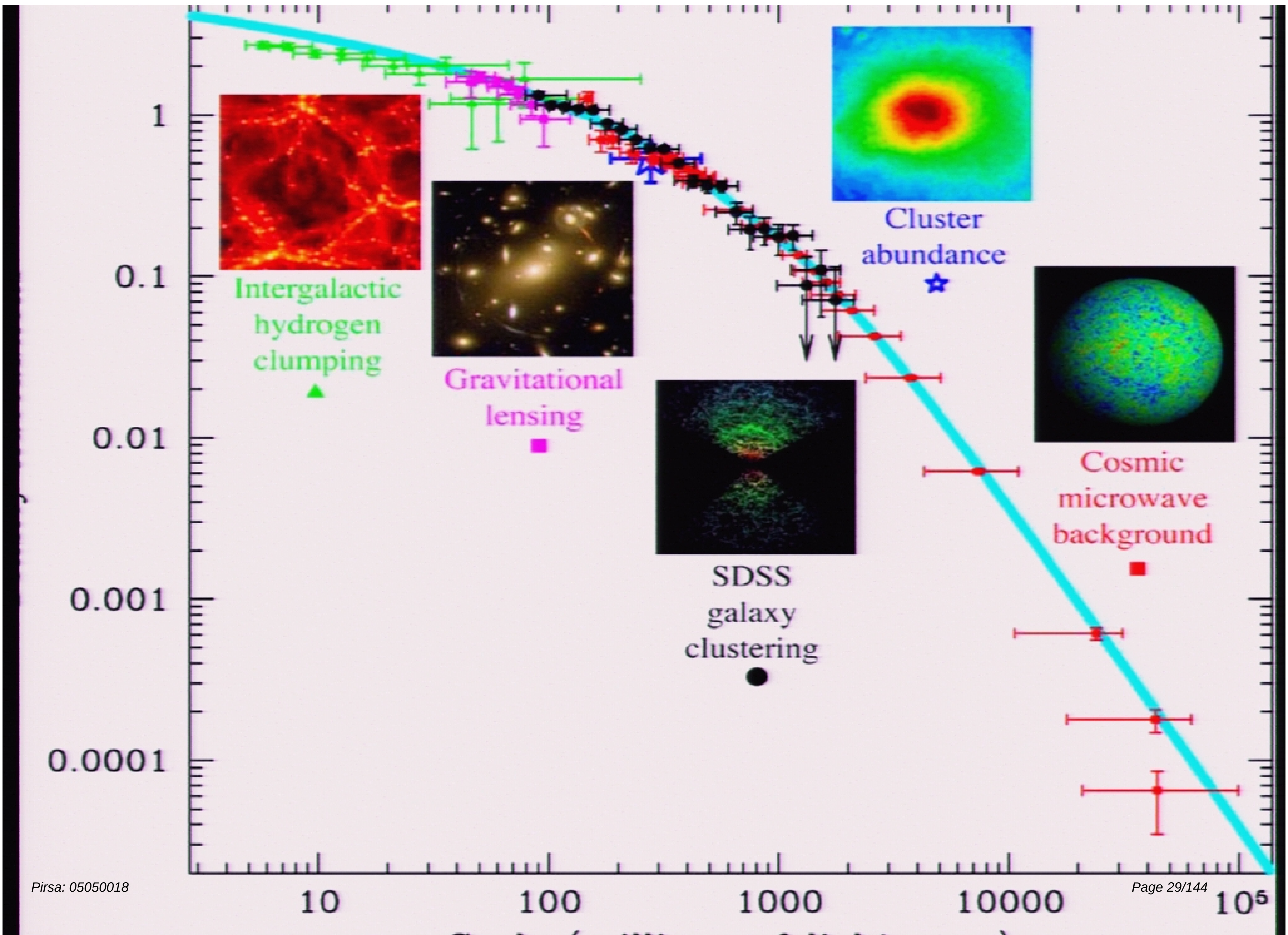

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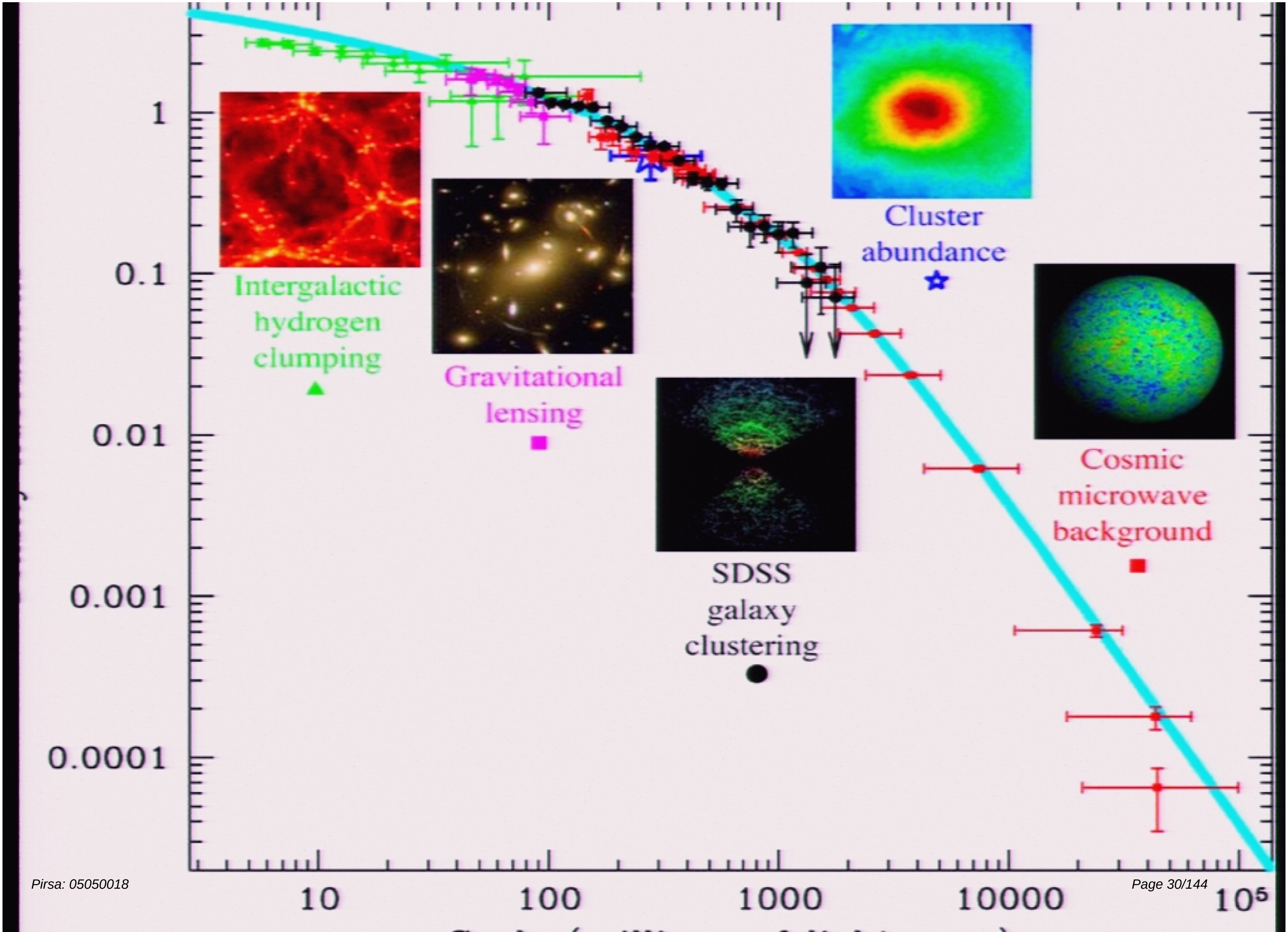
Intergalactic hydrogen clumping
Gravitational lensing
SDSS galaxy clustering
Cluster abundance
Cosmic microwave background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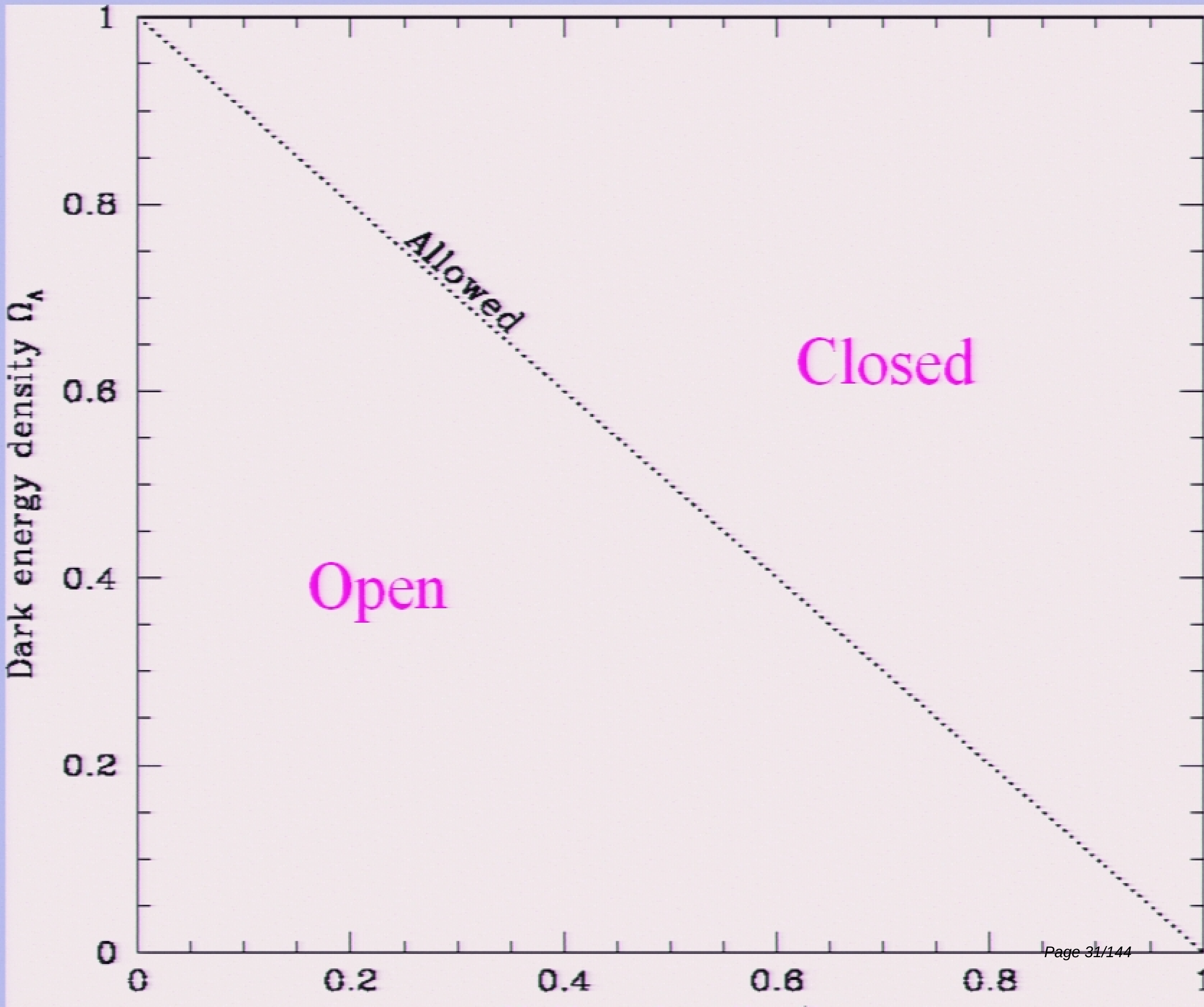
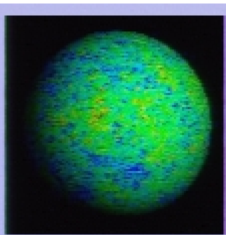
How much dark energy is there?





CMB

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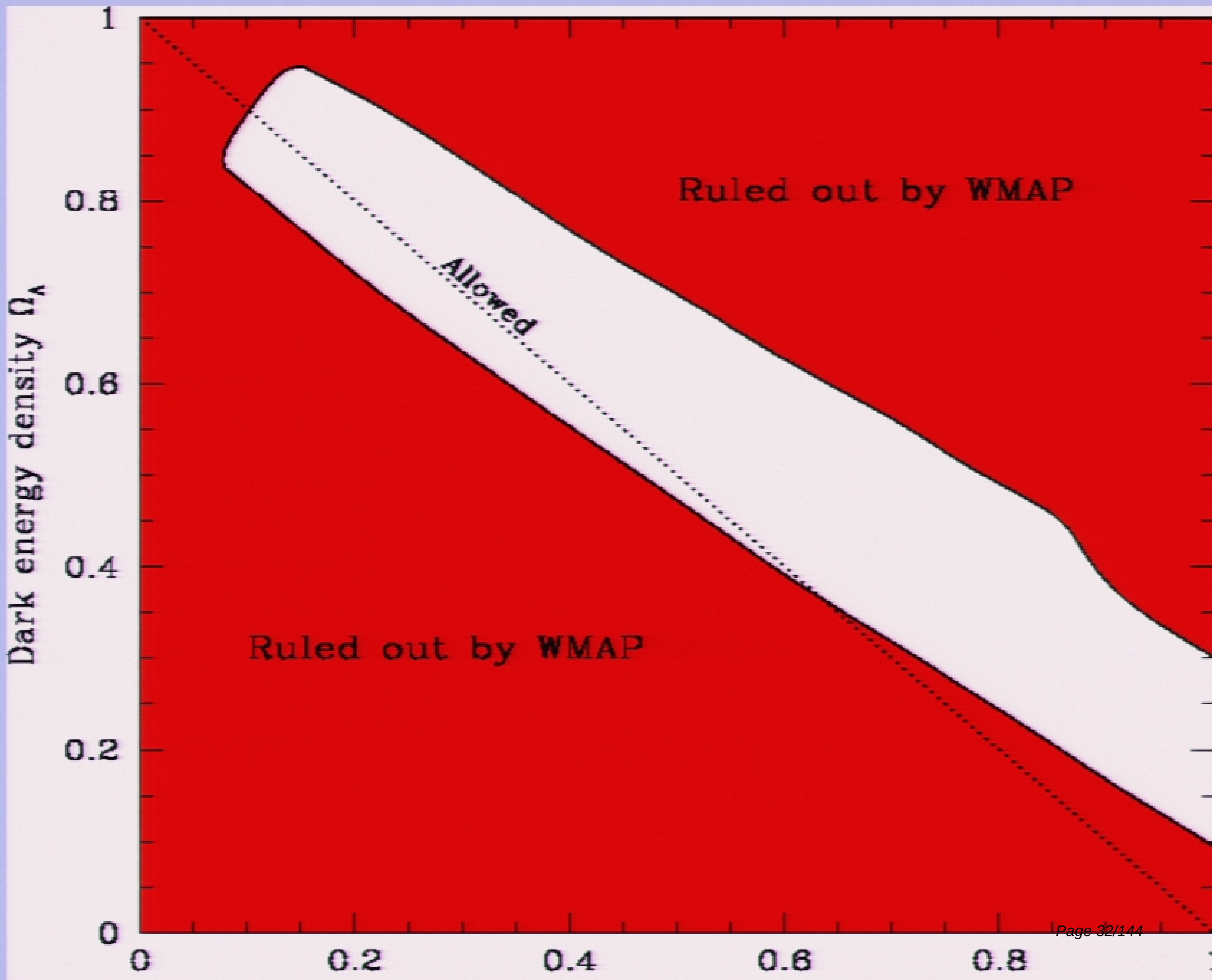

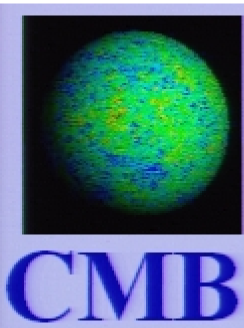

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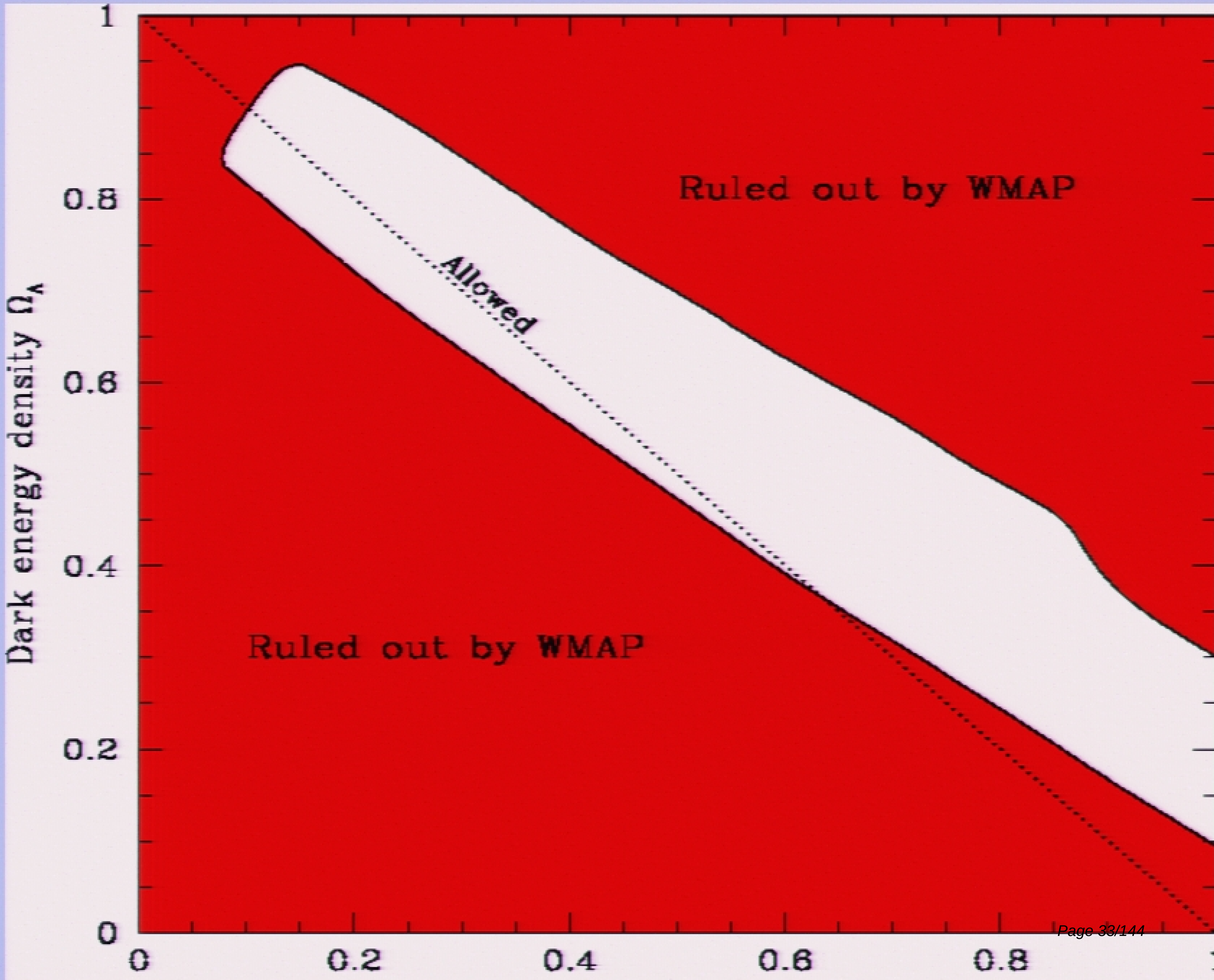

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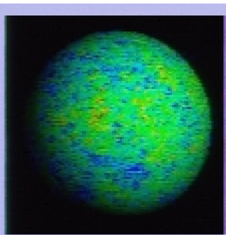
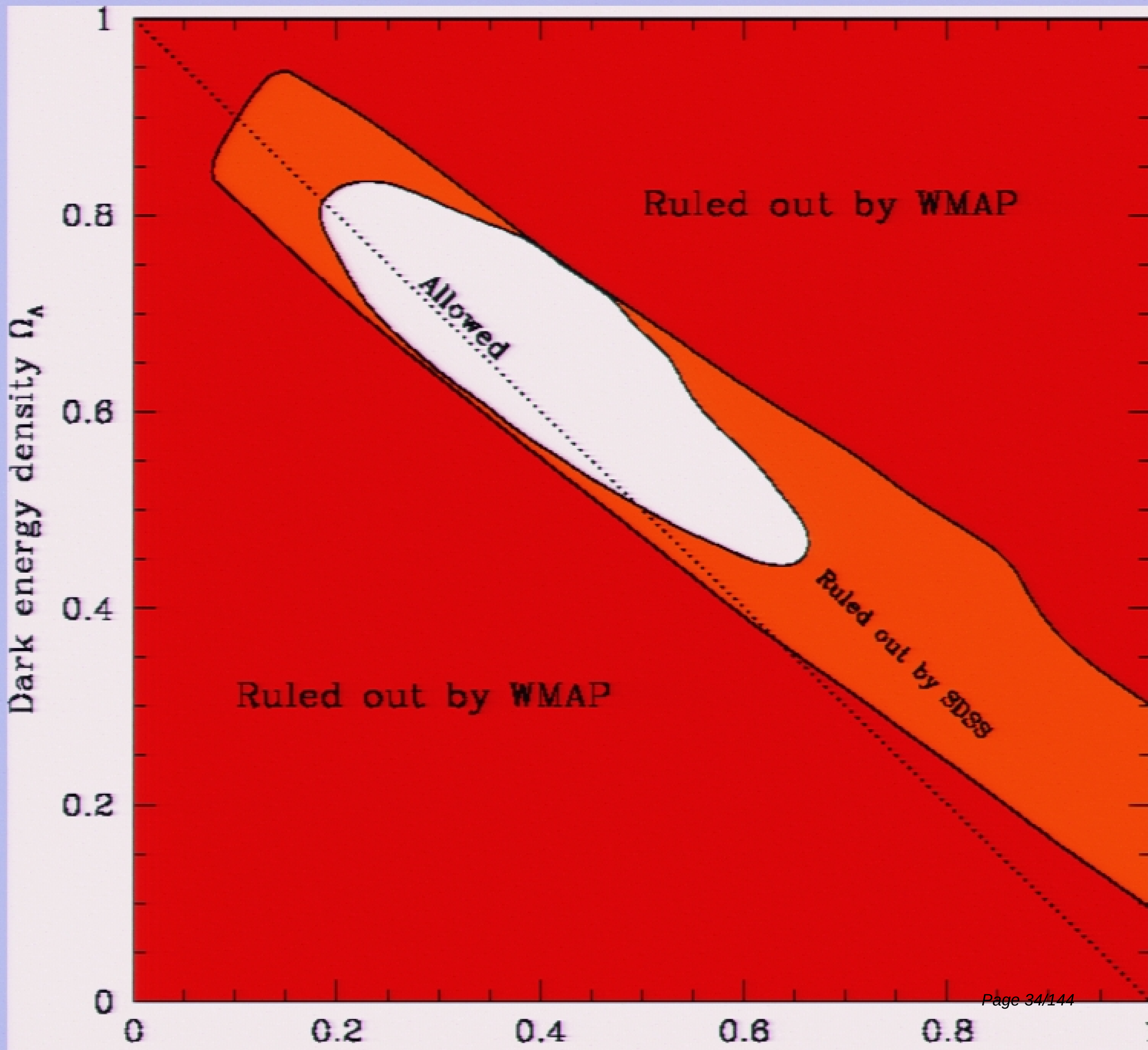


CMB

+

LSS



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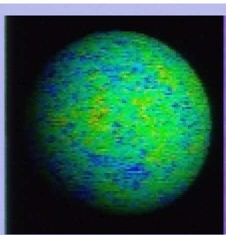

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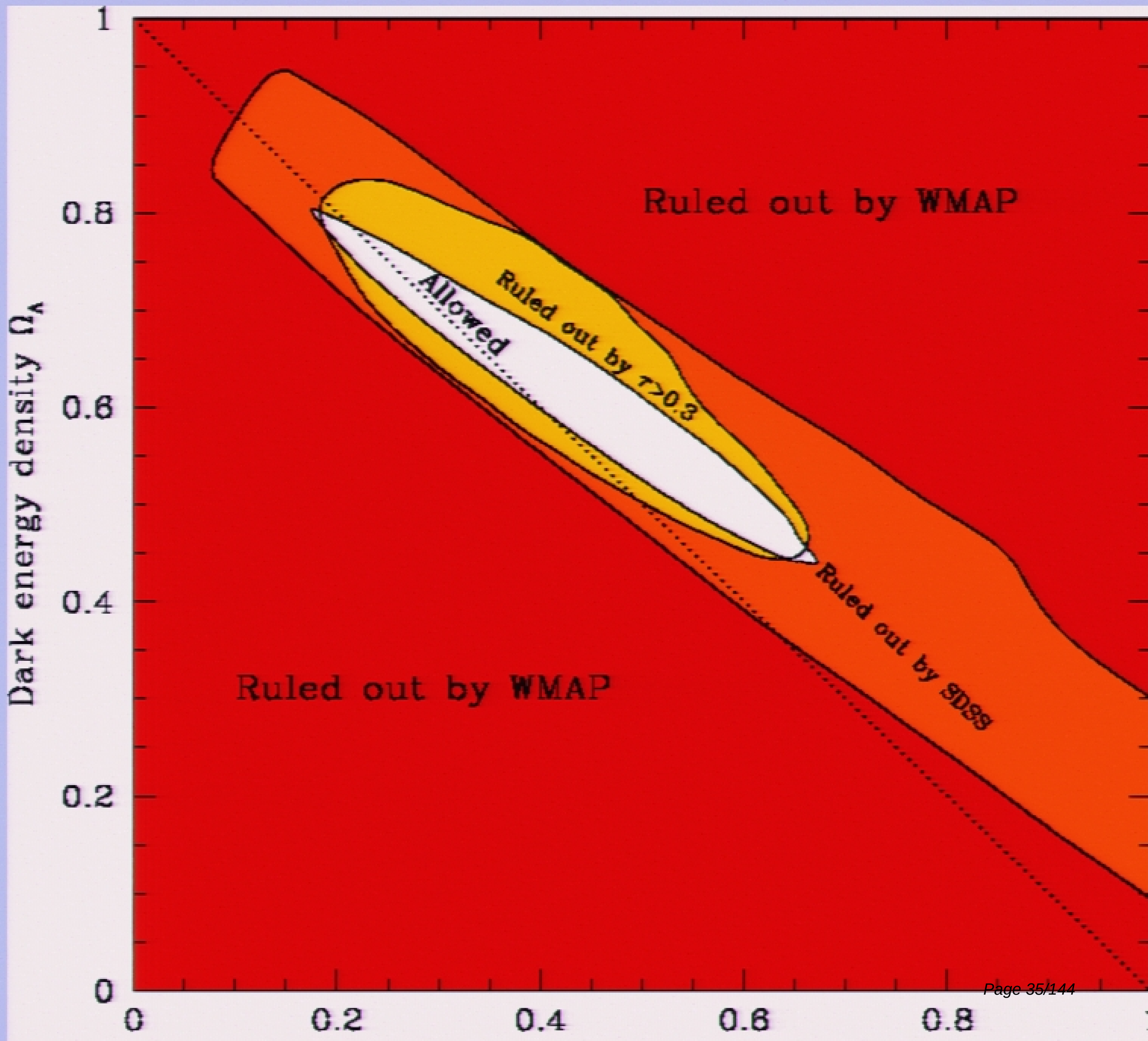
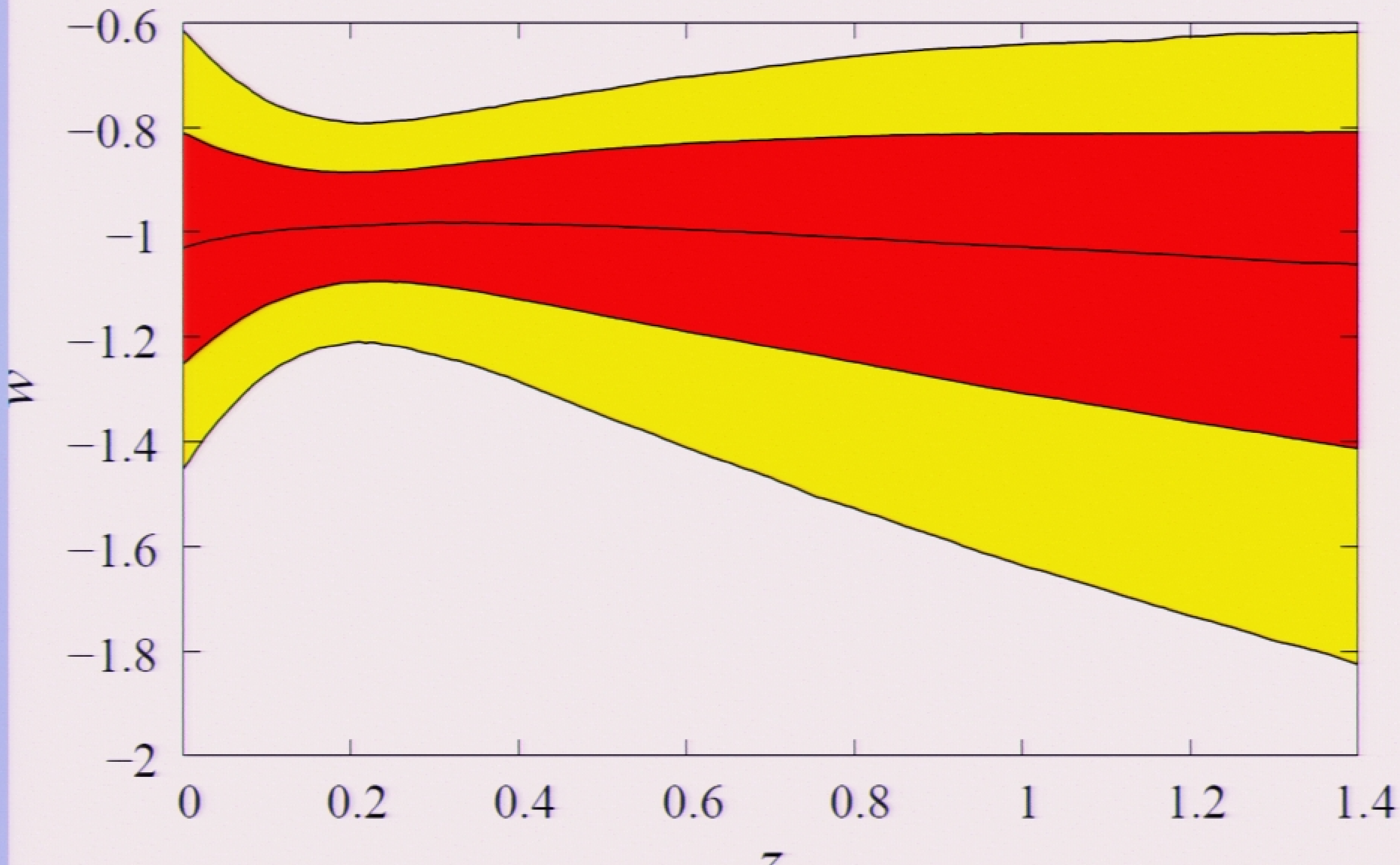


DARK ENERGY

Total energy in 3d flat FRW universe

$$\Omega_D \sim 0.7$$

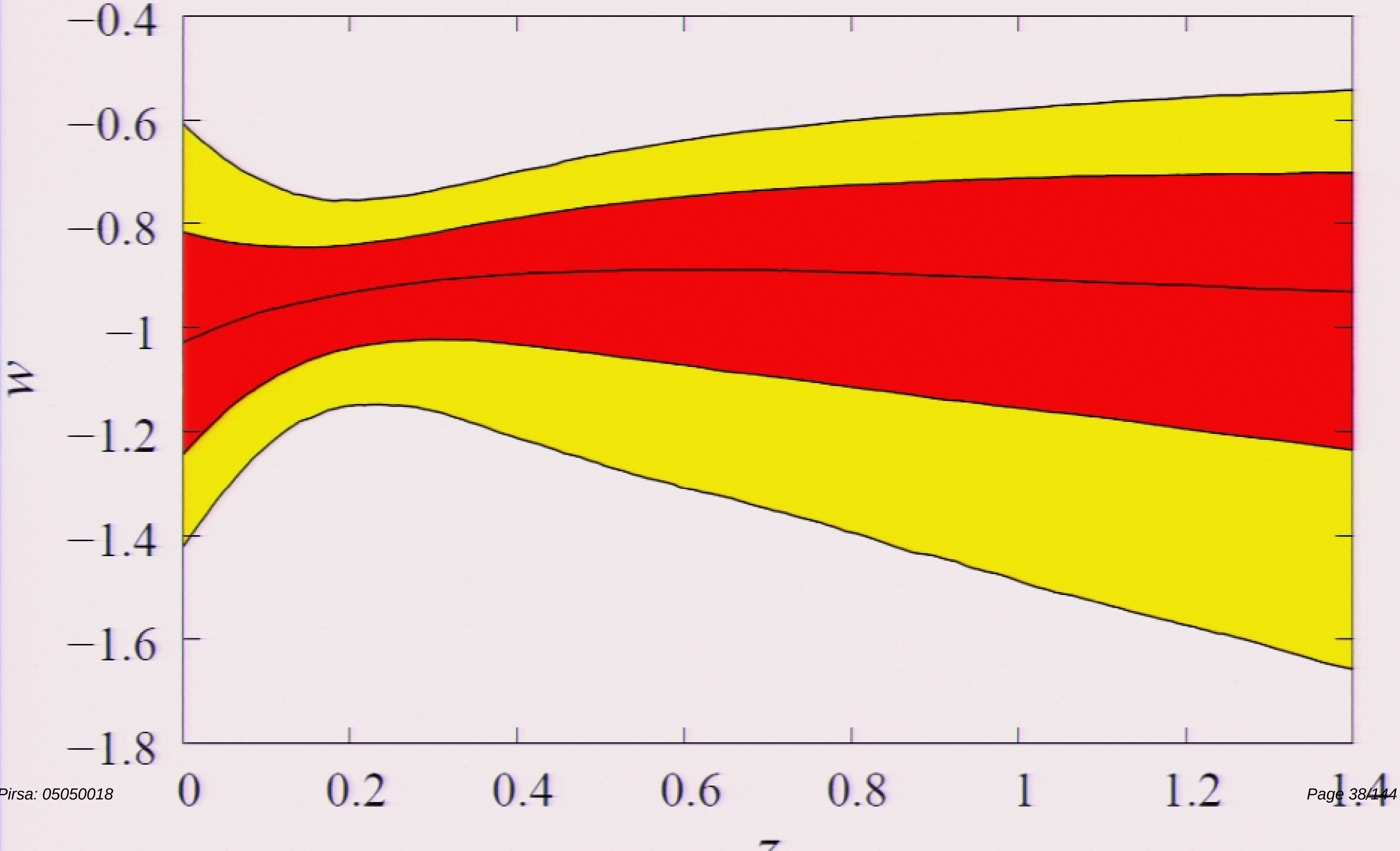
$$\Omega_T = \Omega_D + \Omega_M = 1$$



astro-ph/0407372

FIG. 10: Median (central line), 68% (inner, red) and 95% (outer, yellow) intervals of w(z) using all the data in the chains

constraints are reasonably model independent as long as w is a smooth function of redshift. We find that the simplest solution, w = −1, fits the data at all redshifts.



Constraining Dark Energy with X-ray Galaxy Clusters, Supernovae and the Cosmic Microwave Background

David Rapetti[1,2,3⋆], Steven W. Allen[1,3] and Jochen Weller[1,4,5]

Spring 2005

sets. We examine a series of dark energy models with up to three free parameters: the current dark energy equation of state w_0, the early time equation of state $w_{\rm et}$ and the scale factor at transition, $a_{\rm t}$. From a combined analysis of all three data sets, assuming a constant equation of state and that the Universe is flat, we measure $w_0 = -1.05^{+0.10}_{-0.12}$. Including $w_{\rm et}$ as a free parameter and allowing the transition scale factor to vary over the range $0.5 < a_{\rm t} < 0.95$ where the data sets have discriminating power, we measure $w_0 = -1.27^{+0.33}_{-0.39}$ and $w_{\rm et} = -0.66^{+0.44}_{-0.62}$. We find no significant evidence for evolution in the dark energy equation of state parameter with redshift. Marginal hints of evolution in the supernovae data become less significant when the cluster constraints are also included in the analysis. The complementary nature of the data sets leads to a tight constraint on the mean matter density, $\Omega_{\rm m}$ and alleviates a number of other parameter degeneracies, including that between the scalar spectral index $n_{\rm s}$, the physical baryon density $\Omega_{\rm b}h^2$ and the optical depth τ. This complementary nature also allows us to ex-

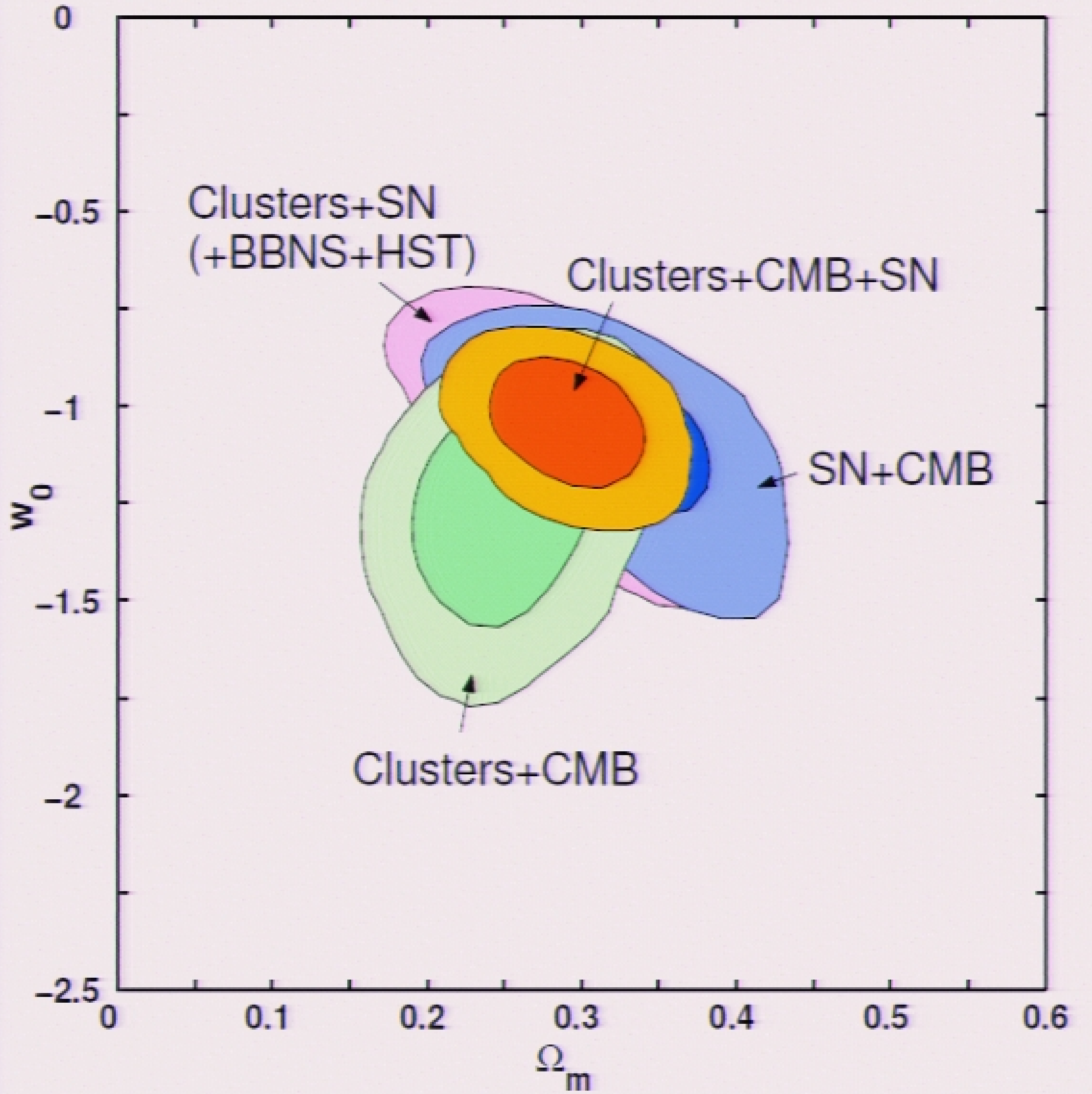


Figure 1. The 68.3 and 95.4 per cent confidence limits in the $(\Omega_{\rm m}, w_0)$ plane for the various pairs of data sets and for all three data sets combined. A constant dark energy equation of state parameter is assumed.

New data

New data

Boomerang,…., WMAP, 2005 ???

Planck, SNAP, LSST ..., 2010-2012

Cosmological Concordance Model

- Early Universe Inflation
- Near de Sitter space
- 13.7 billion years ago
- During 10^{-35} sec

- Current Acceleration
- Near de Sitter space
- Now
- During few billion years

$$\frac{\dot{a}}{a} = H \approx \text{const}$$

$$V \sim H^2 M_P^2 \qquad V \sim H^2 M_P^2$$

$$H_{infl} \leq 10^{-5} M_p \qquad H_{accel} \sim 10^{-60} M_P$$

$$\frac{\ddot{a}}{a} > 0$$

String Theory and Cosmology

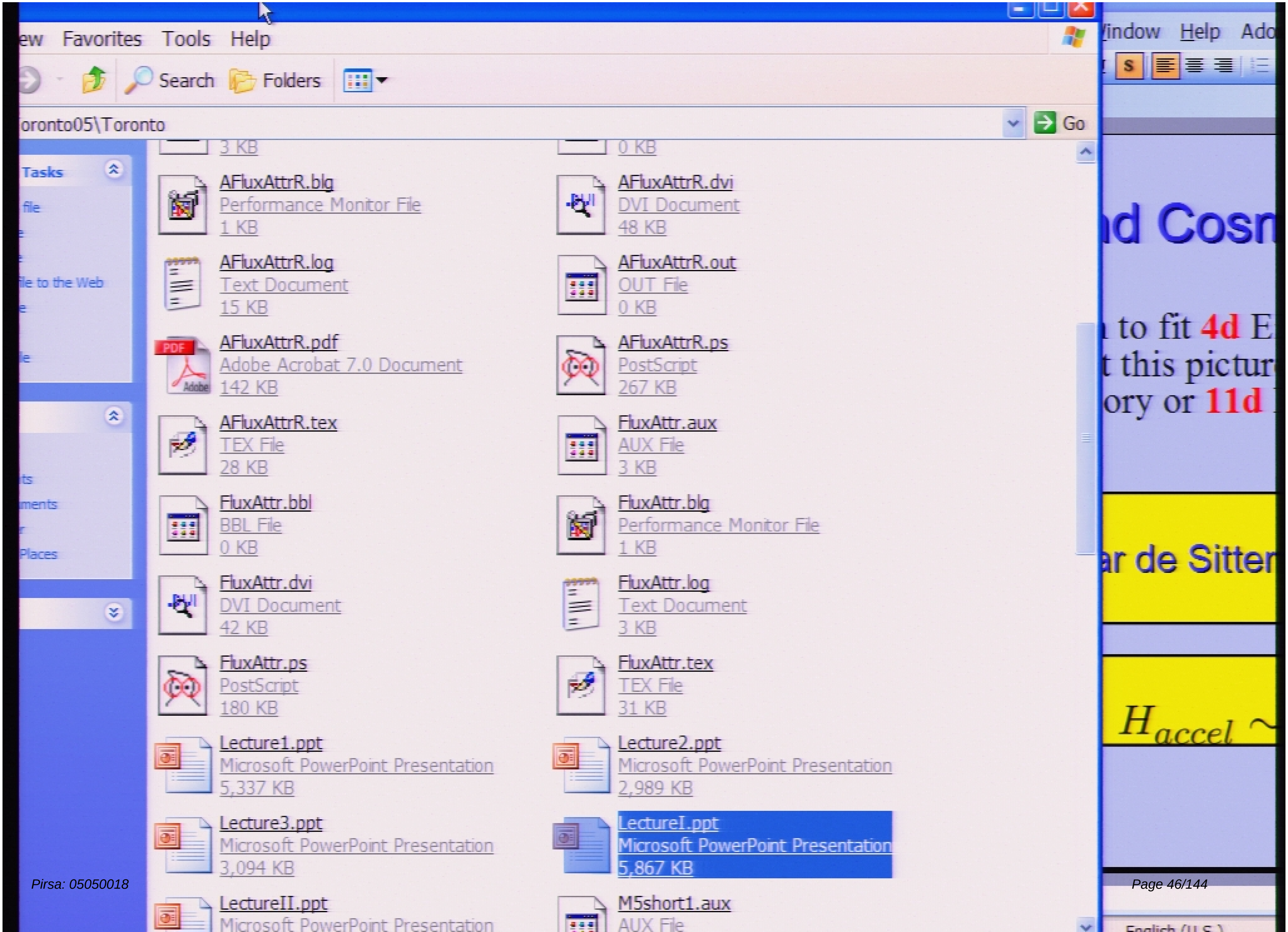
All observations so far seem to fit **4d** Einstein GR. We need to know how to get this picture from the compactified **10d** string theory or **11d** M-theory and supergravity

String Theory and Cosmology

All observations so far seem to fit **4d** Einstein GR. We need to know how to get this picture from the compactified **10d** string theory or **11d** M-theory and supergravity

How to get de Sitter or near de Sitter 4d space?

$$H_{infl} \leq 10^{-5} M_p \qquad H_{accel} \sim 10^{-60} M_P$$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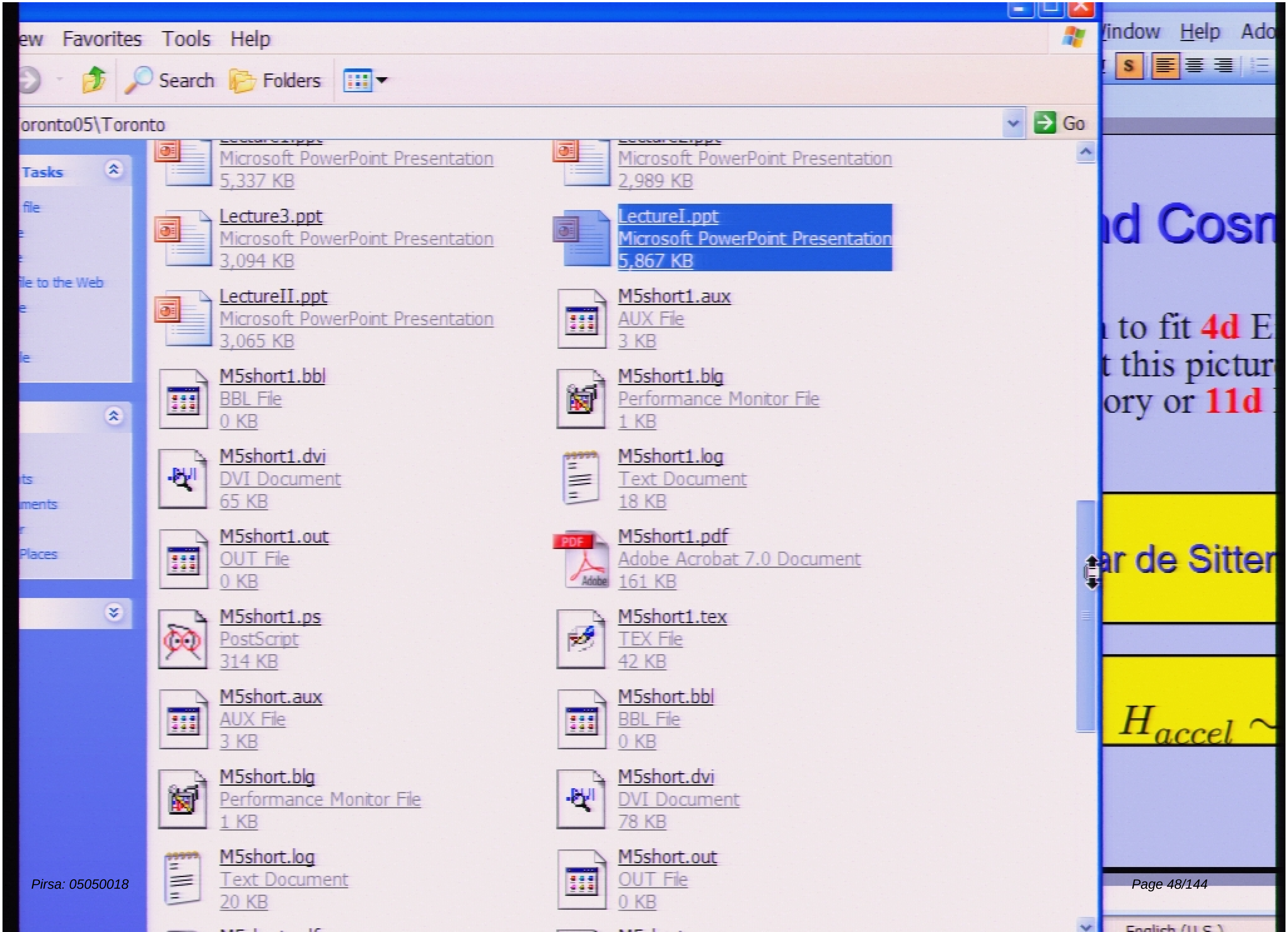
H_{accel} ~

II. Stabilization of moduli in string theory I

- Recent developments in fixing moduli near black hole horizon and black hole attractors
- 1) A striking role of stringy corrections converting a classical singularity into a regular black hole with the singularity covered by the horizon.
- 2) An emergent relation between black hole attractors and cosmology with regard to moduli stabilization. Explicitly attractive K3.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A Simple Example of Moduli Fixing

Aspinwall, R.K.

We analyze M-theory compactified on K3xK3 with fluxes and its F-theory limit, which is dual to an orientifold of the type IIB string on $K3 \times T^2/Z_2$

We argue that instanton effects will generically fix all of the moduli.

Before branes are introduced

Moduli space is no more

Cosmology, Supersymmetry and Special Geometry

Cosmology, Supersymmetry and Special Geometry

In familiar case of **Near Extremal Black Holes**

DUALITY SYMMETRY protects exact entropy formula from large quantum corrections

Cosmology, Supersymmetry and Special Geometry

In familiar case of **Near Extremal Black Holes**

DUALITY SYMMETRY protects exact entropy formula from large quantum corrections

DUALITY SYMMETRY (shift symmetry)

protects the **flatness of the potential**

in D3/D7 inflation model from large quantum corrections

Shift Symmetry of $\mathcal{G}$

Shift Symmetry of $\mathcal{G}$

- Flatness of the effective supergravity inflaton potential follows from the shift symmetry of

Shift Symmetry of $\mathcal{G}$

- Flatness of the effective supergravity inflaton potential follows from the shift symmetry of $\mathcal{G} \equiv K + \ln|W|^2$

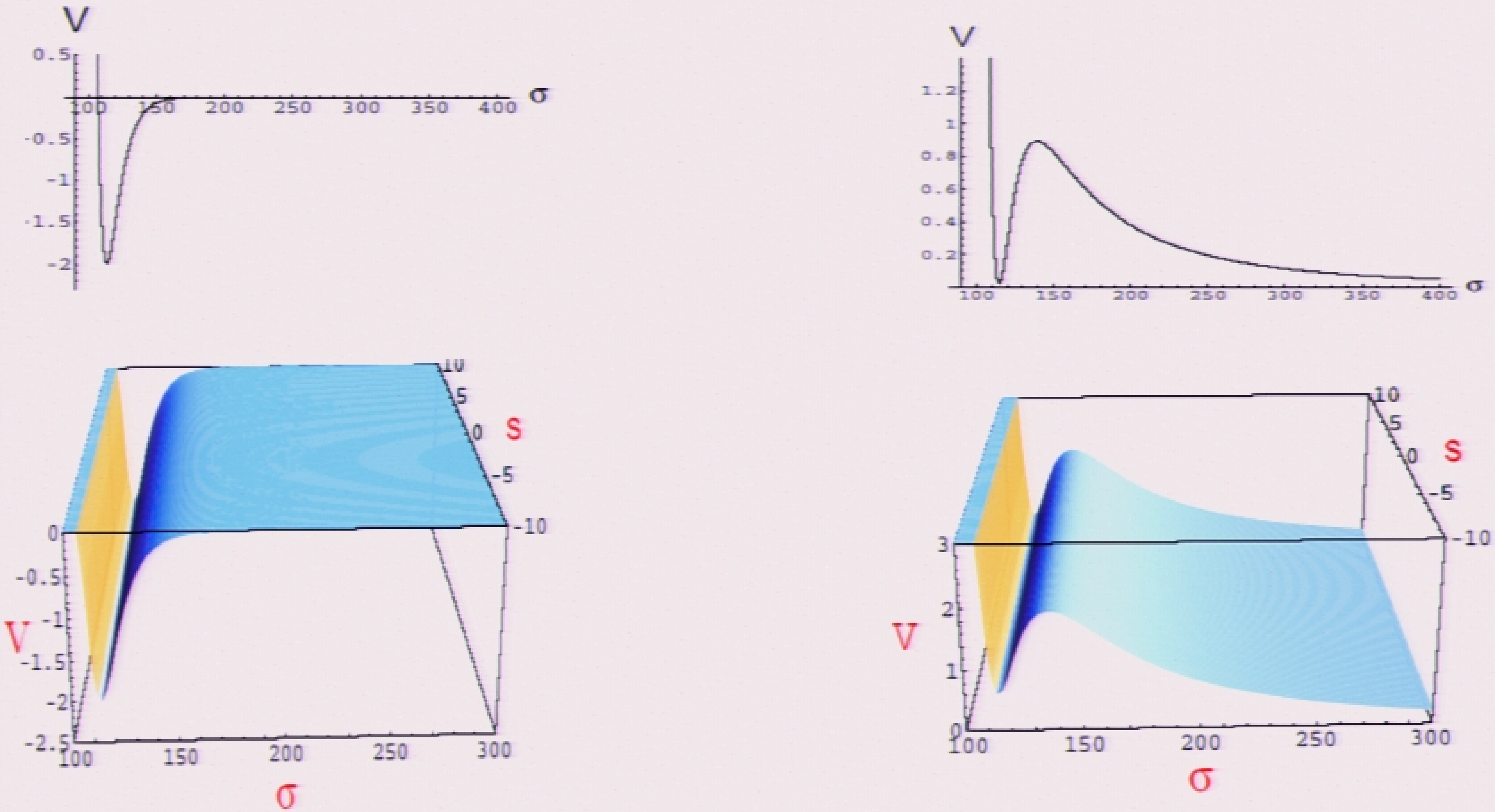
$$V = e^{\mathcal{G}}[|\mathcal{G}_{,z}|^2 - 3]$$

We need models where the position of the D3 brane after stabilization of the volume is still a modulus

SHIFT SYMMETRY

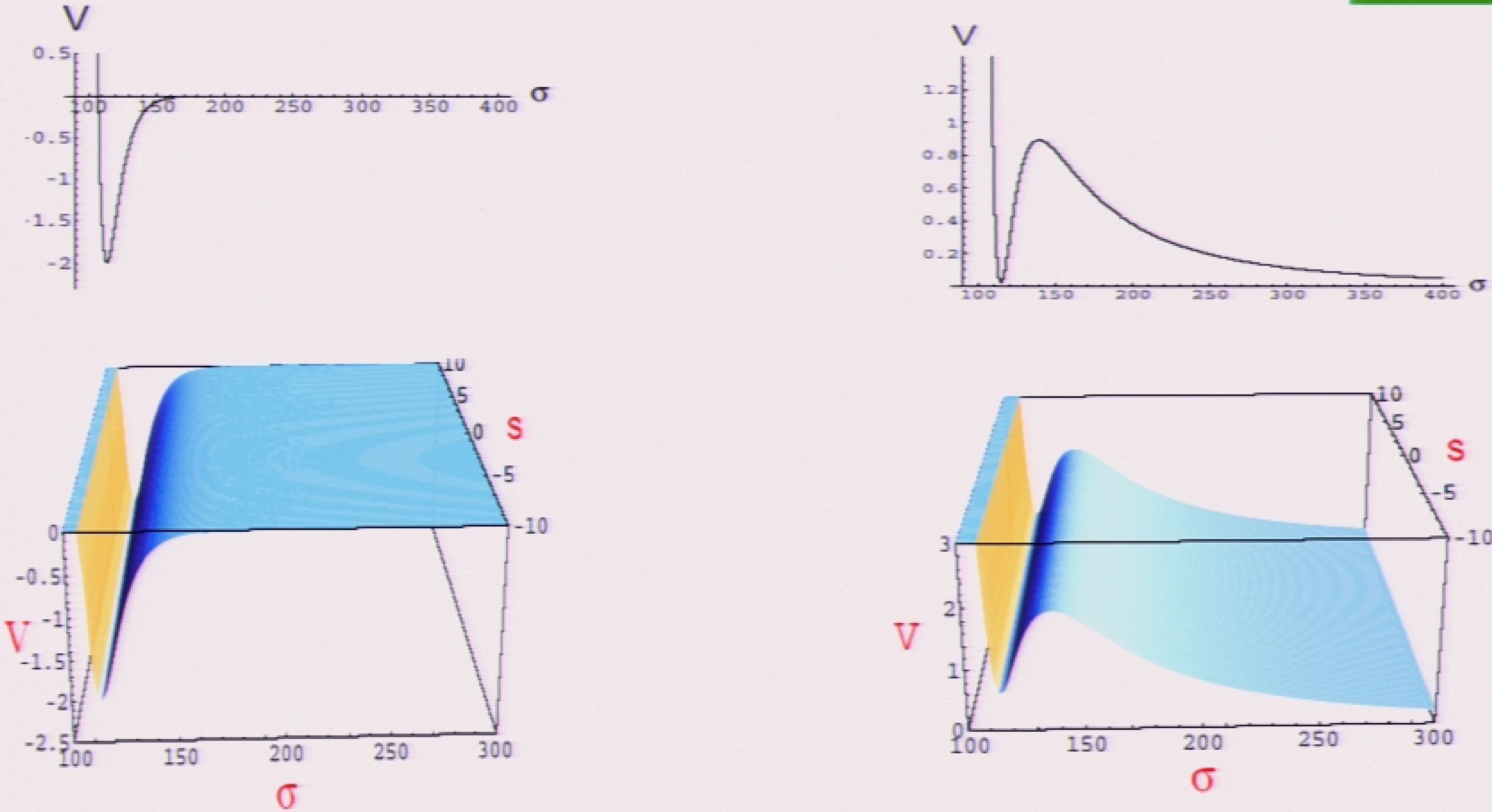
and volume stabilization

Inflaton Trench



Inflaton Trench

Hsu, R.K., Prokushkin

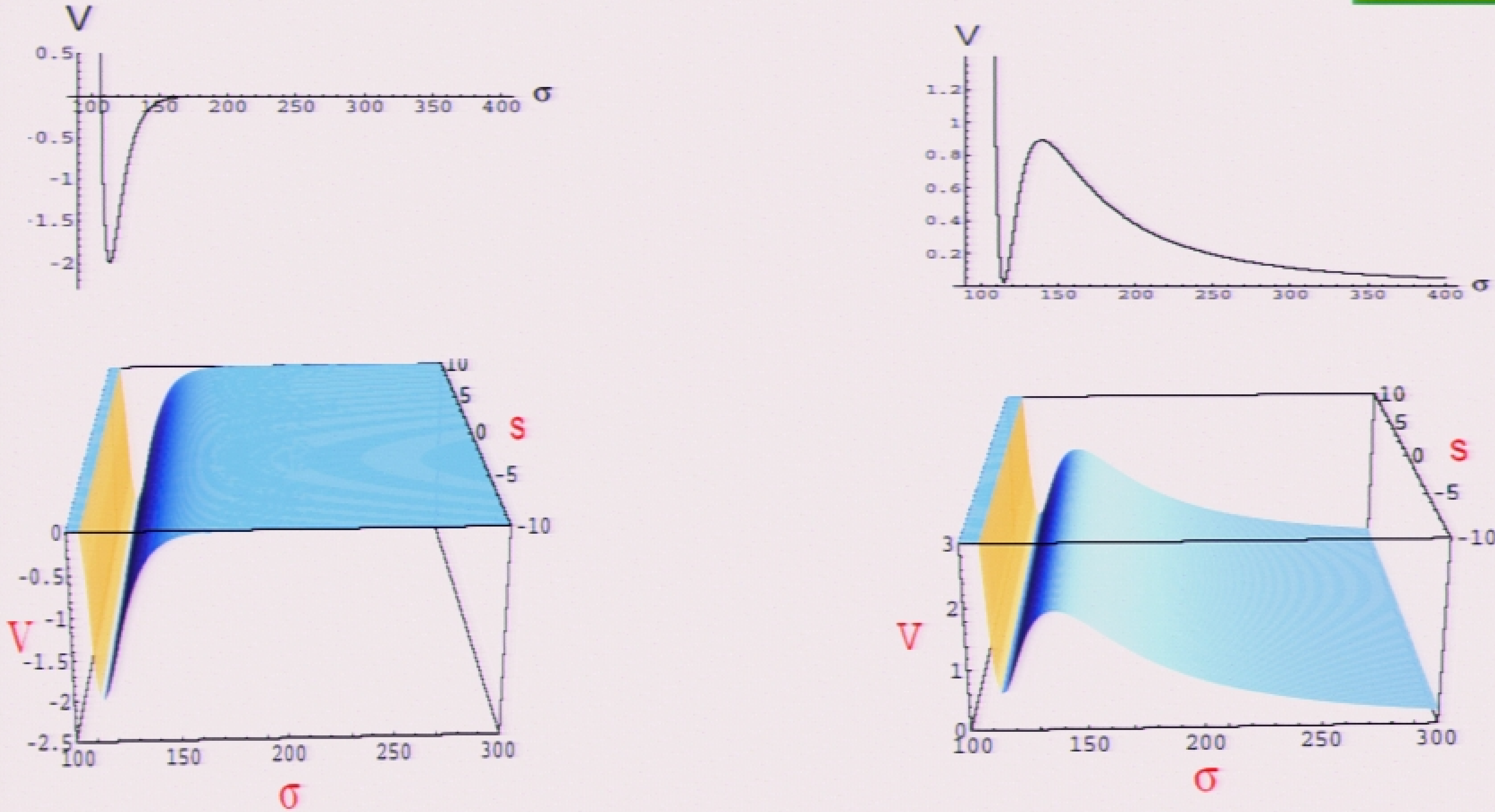


- Supersymmetric Ground State of Branes in Stabilized Volume

SHIFT SYMMETRY

Inflaton Trench

Hsu, R.K., Prokushkin



- Supersymmetric Ground State of Branes in Stabilized Volume

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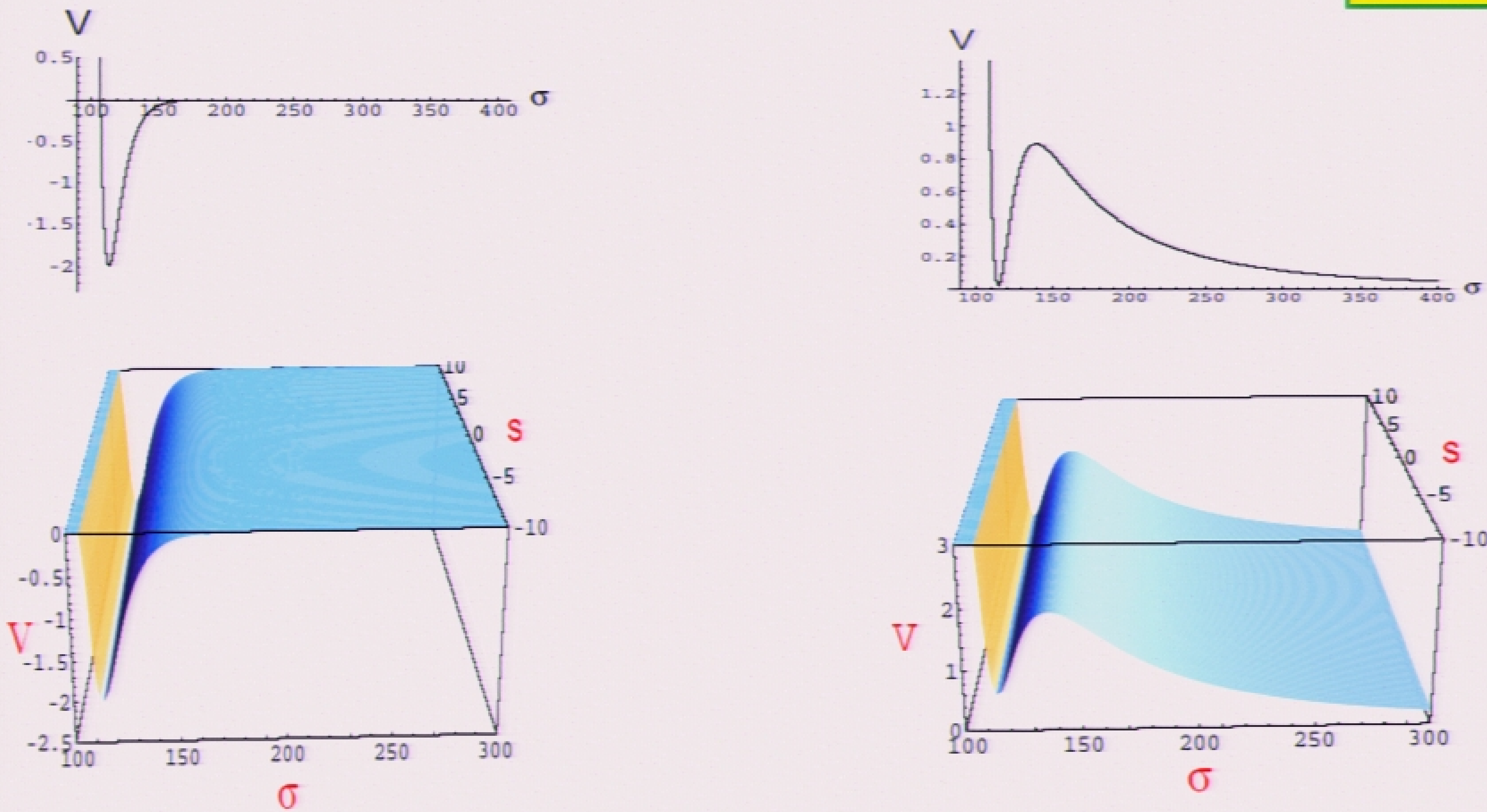

- Supersymmetric Ground State of Branes in Stabilized Volume

SHIFT SYMMETRY

The motion of branes does not destabilize the volume

String Theory and
N=2 Special Geometry

String Theory and N=2 Special Geometry

Angelantonj, D'Auria, Ferrara and Trigiante

- Type IIB string theory compactified on

String Theory and N=2 Special Geometry

Angelantonj, D'Auria, Ferrara and Trigiante

- Type IIB string theory compactified on

$$K3 \times \frac{T^2}{Z^2}$$

- orientifold with fluxes, mobile D3 branes and heavy D7 branes

String Theory and N=2 Special Geometry

Angelantonj, D'Auria, Ferrara and Trigiante

- Type IIB string theory compactified on

$$K3 \times \frac{T^2}{Z^2}$$

- orientifold with fluxes, mobile D3 branes and heavy D7 branes

Coset Space

String Theory and N=2 Special Geometry

Angelantonj, D'Auria, Ferrara and Trigiante

- Type IIB string theory compactified on

$$K3 \times \frac{T^2}{Z^2}$$

- orientifold with fluxes, mobile D3 branes and heavy D7 branes

Coset Space $\frac{SU(1,1)}{U(1)} \times \frac{SO(2,2+n_3)}{SO(2)\times SO(2+n_3)}$

Duality

String Theory and N=2 Special Geometry

Angelantonj, D'Auria, Ferrara and Trigiante

- Type IIB string theory compactified on

$$K3 \times \frac{T^2}{Z^2}$$

- orientifold with fluxes, mobile D3 branes and heavy D7 branes

Isometry of the compactified space provides shift symmetry slightly broken by quantum corrections

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Special Kähler geometry

N=2 supergravity with vector multiplets

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de Wit, Van Proeyen, 1984

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- **Supersymmetric Black Hole Entropy**

Symplectic Invariant

Duality and symplectic transformations

$$\mathcal{L}_1 = \frac{1}{4}(\text{Im}\ \mathcal{N}_{\Lambda\Sigma})\mathcal{F}^{\Lambda}_{\mu\nu}\mathcal{F}^{\mu\nu\Sigma} - \frac{i}{8}(\text{Re}\ \mathcal{N}_{\Lambda\Sigma})\varepsilon^{\mu\nu\rho\sigma}\mathcal{F}^{\Lambda}_{\mu\nu}\mathcal{F}^{\Sigma}_{\rho\sigma}$$

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Ferrara, R. K., Strominger, 1996

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Inflaton Shift is a Duality

$$(y^r)' = y^r + \beta^r \qquad s' = s \qquad t' = t \qquad u' = u$$

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Conclusion: $\mathcal{G}(\rho, \bar{\rho}; \phi - \bar{\phi})$

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A Stringy Cloak for a Null Singularity

Dabholkar, R. K., Maloney
hep-th/0410076

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$$S_{cl} = \frac{1}{4}A_{cl} = 0 \qquad S = \frac{1}{2}A = 4\pi\sqrt{\frac{c_2\,p\,q}{24}}$$

C_2 depends on topology of Calabi-Yau (second Chern-class coefficient)

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N=2 BPS mass formula, M=|Z|

- The BPS mass is equal to the central charge, which depends on moduli and charges: symplectic invariant

$$M_{BPS}^2 = |Z|^2 = |\langle Q, V\rangle|^2 = e^K |q_I X^I(z) - p^I F_I(z)|^2$$

- The ADM mass of the black hole is equal to the value of the central charge when moduli are at infinity

$$M_{ADM}^2 = |Z(p, q, z_\infty, \bar{z}_\infty|^2$$

Attractor equations

- Introduce a symplectic vector

$$\Pi = \begin{pmatrix} Y^I \\ F_J(Y) \end{pmatrix} \qquad \text{where } \; Y^I \equiv \bar{Z}\, X^I \,.$$

- At the attractor point there is an algebraic relation between the fixed values of moduli and charges

$$Y^I - \bar{Y}^I = ip^I \,, \qquad F_I(Y) - \bar{F}_I(\bar{Y}) = iq_I$$

Calabi-Yau black holes

$$S_{cl} = \frac{A_{cl}}{4} = 2\pi \sqrt{\hat{q}_0 D_{ABC} p^A p^B p^C}$$

- Classical area=0 if $D_{ABC} p^A p^B p^C = 0$

- Quantum corrected entropy and area

$$A = 4\pi |Z|^2_{r=0} = 8\pi \sqrt{\frac{c_{2A}\, p^A |\hat{q}_0|}{24}}$$

$$S = 4\pi \sqrt{\frac{c_{2A}\, p^A |\hat{q}_0|}{24}}$$

Stabilization of moduli via instantons: breaking the isometries of the manifold

- When is this possible?

- **Can we use fluxes and instanton corrections to fix all moduli but the inflaton?**

Flux vacua and supersymmetric attractors

We propose a universal formulation of supersymmetric attractor equation: It is valid for the flux vacua or for BPS black holes, depending on the choic of the components of flux either in the compact space or in 4d space. As a example, we define flux vacua with a rigid *explicitly attractive K3 surface* wher a class of moduli are fixed by fluxes. The explicit values of complex structure are extracted from the previously known solution of the attractor equation fc the black holes with the same symmetry.

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There is a direct relation between the mathematical structure, established for supersymmetric black hole attractors and flux vacua. These flux vacua of string theory are considered quite seriously in the context of string cosmology where a landscape of such vacua seem to exist.

We will show here that the supersymmetric attractor equations studied in the past for black holes give us a new tool in studies of flux vacua. A specific example will show how to use the supersymmetric attractor equations studied for black holes to find in flux vacua an explicit dependence of the complex structure moduli on fluxes.

Attractors and special geometry

We start with a short overview of attractors and special geometry. However, we will present it in a form where the origin of the symplectic flux vector (p^Λ, q_Λ) is not specified. It will be associated later with various fluxes of string theory, those which break Lorentz symmetry of 4d (fluxes with components in 4d) and those which do not break it (fluxes with components only in the compact space).

Special Kähler manifold can be defined by constructing flat symplectic bundle of dimension $2n+2$ over Kähler-Hodge manifold with symplectic section defined as

$$V = (L^\Lambda, M_\Lambda), \qquad \Lambda = 0, 1, ...n \ ,$$

where (L, M) obey the symplectic constraint $i(\bar{L}^\Lambda M_\Lambda - L^\Lambda \bar{M}_\Lambda) = 1$ and $L^\Lambda(z, \bar{z})$ and $M_\Lambda(z, \bar{z})$ depend on scalar fields $z, \bar{z}$ which are

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$$D_{\bar{k}} L^\Lambda = (\partial_{\bar{k}} - \frac{1}{2} K_{\bar{k}}) L^\Lambda = 0 \ ,$$

where K is the Kähler potential. Symplectic invariant form of the Kähler potential can be found from this equation by introducing the *holomorphic section* (X^Λ, F_Λ):

$$L^\Lambda = e^{K/2} X^\Lambda \ , \qquad M_\Lambda = e^{K/2} F_\Lambda \ , \qquad (\partial_{\bar{k}} X^\Lambda = \partial_{\bar{k}} F_\Lambda = 0) \ .$$

The Kähler potential is $K = -\ln i(\bar{X}^\Lambda F_\Lambda - X^\Lambda \bar{F}_\Lambda)$. The Kähler metric is given by $g_{k\bar{k}} = \partial_k \partial_{\bar{k}} K$. Finally from special geometry one finds that there exists a complex symmetric $(n+1) \times (n+1)$ matrix

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One can introduce a symplectic vector charge which in the future will be either an electric-magnetic charge of a 4d black hole or a symplectic flux in a compactified Calabi-Yau manifold.

$$\begin{pmatrix} p^\Lambda \\ q_\Lambda \end{pmatrix} = \begin{pmatrix} \int \mathcal{F}^\Lambda \\ \int \mathcal{G}_\Lambda \end{pmatrix} .$$

In the generic point of the moduli space there are **two symplectic invariants** homogeneous of degree 2 in electric and magnetic charges:

$$I_1 = I_1(p, q, z, \bar{z}) = -\frac{1}{2} P^t \mathcal{M}(\mathcal{N}) P \,,$$

$$I_2 = I_2(p, q, z, \bar{z}) = -\frac{1}{2} P^t \mathcal{M}(\mathcal{F}) P \,.$$

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Here $P = (p, q)$ and $\mathcal{M}(\mathcal{N})$ is the real symplectic $(2n+2) \times (2n+2)$ matrix

$$\begin{pmatrix} A B \\ C D \end{pmatrix}$$

where

$$A = \mathrm{Im}\mathcal{N} + \mathrm{Re}\mathcal{N}\,\mathrm{Im}\mathcal{N}^{-1}\mathrm{Re}\mathcal{N} \ , \qquad B = -\mathrm{Re}\mathcal{N}\,\mathrm{Im}\mathcal{N}^{-1}$$

$$C = -\mathrm{Im}\mathcal{N}^{-1}\mathrm{Re}\mathcal{N} \ , \qquad D = \mathrm{Im}\mathcal{N}^{-1} \ .$$

Note that one can rewrite these two invariants as follows

$$I_1 = |Z|^2 + |D_i Z|^2 \ ,$$

$$I_2 = |Z|^2 - |D_i Z|^2$$

Here we have defined a *covariantly holomorphic central change*

$$Z(z,\bar{z},q,p) \equiv (L^\Lambda q_\Lambda - M_\Lambda p^\Lambda) \ .$$

where $D_{\bar{i}} Z \equiv (\partial_{\bar{i}} - \frac{1}{2} K_{\bar{i}}) Z = 0$ and $D_i \bar{Z} \equiv (\partial_i - \frac{1}{2} K_i) \bar{Z} = 0$.

One could also use a *holomorphic central charge* defined as

$$W = e^{-K(z,\bar{z})/2} Z(z,\bar{z},q,p) \equiv (X^\Lambda q_\Lambda - F_\Lambda p^\Lambda) \ , \qquad \partial_{\bar{i}} W = 0 \ .$$

and the anti-holomorphic one,

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This holomorphic charge W may be associated with the superpotential in N=1 supersymmetric theory, however, here it is used in the context of symplectic structure of special geometry which is a

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This holomorphic charge W may be associated with the superpotential in N=1 supersymmetric theory, however, here it is used in the context of symplectic structure of special geometry which is a property of N=2 theories. It is not present in a generic N=1 theory, only in the ones which originates from N=2.

A minimization condition for both symplectic invariants which specifies the values of moduli in terms of charges is given by

$$D_i Z(z, \bar{z}, q, p) \equiv (\partial_i + \frac{1}{2} K_i) Z = 0,$$

$$D_{\bar{i}} \bar{Z} \equiv (\partial_{\bar{i}} + \frac{1}{2} K_{\bar{i}}) \bar{Z}(z, \bar{z}, q, p) = 0 \ .$$

It is also a requirement of an unbroken supersymmetry. In terms of the holomorphic charge W the minimization condition is the familiar one

$$D_i W(z, q, p) \equiv (\partial_i + K_i) W = 0,$$

$$D_{\bar{i}} \bar{W}(\bar{z}, q, p) \equiv (\partial_{\bar{i}} + K_{\bar{i}}) \bar{W}(\bar{z}, q, p) = 0 \ .$$

Note that $X^\Lambda(z)$ are subject to holomorphic redefinitions (sec-

tions of a holomorphic line bundle):

$$X^\Lambda(z) \to X^\Lambda(z)\, e^{-f(z)} \ ,$$

so that

$$L^\Lambda(z) \to L^\Lambda(z)\, e^{\frac{\bar{f}(z)-f(z)}{2}} \ .$$

This occurs because $L^\Lambda = e^{K/2}X^\Lambda$ and $K \to K + f + \bar{f}$ under Kähler transformations, so that

$$Z(q,p,z) \to Z(q,p,z)\, e^{\frac{\bar{f}(z)-f(z)}{2}} \ .$$

However, $|Z|$ is both symplectic and Kähler gauge invariant, this is why the connection drops and $D_i Z = 0$ ($D_{\bar{i}} Z = 0$) entails $\partial_i |Z| = 0$.

Thus $|Z|^2 = e^K |W|^2$ is the object in our construction which may be related to supersymmetric physical observables due to both

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This occurs because $L^\Lambda = e^{K/2} X^\Lambda$ and $K \to K + f + \bar{f}$ under Kähler transformations, so that

$$Z(q,p,z) \to Z(q,p,z)\, e^{\frac{\bar{f}(z)-f(z)}{2}} .$$

However, $|Z|$ is both symplectic and Kähler gauge invariant, this is why the connection drops and $D_i Z = 0$ ($D_{\bar{i}} Z = 0$) entails $\partial_i |Z| = 0$.

Thus $|Z|^2 = e^K |W|^2$ is the object in our construction which may be related to supersymmetric physical observables due to both

tions of a holomorphic line bundle):

$$X^\Lambda(z) \to X^\Lambda(z)\, e^{-f(z)} ,$$

so that

$$L^\Lambda(z) \to L^\Lambda(z)\, e^{\frac{\bar f(z) - f(z)}{2}} .$$

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symplectic and Kähler invariance: at the attractor point it does not depend on moduli, only on fluxes:

$$\partial_i |Z|^2 = \partial_{\bar{i}} |Z|^2 = 0$$

For the superpotential the minimization condition means that

$$\partial_i \left(e^K |W|^2\right) = \partial_{\bar{i}} \left(e^K |W|^2\right) = 0$$

At the minimum both invariants I_1 and I_2 are equal to each other. This minimization condition can be also presented in the form of the **attractor equations**

$$\begin{pmatrix} p^\Lambda \\ q_\Lambda \end{pmatrix} = i \begin{pmatrix} \bar{Z} L^\Lambda - Z \bar{L}^\Lambda \\ \bar{Z} M_\Lambda - Z \bar{M}_\Lambda \end{pmatrix} ,$$

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This algebraic equation relates the values of charges to some functions of moduli. Many solutions of these equations are known in the form where moduli are functions of fluxes.

$$z_{fix} = z_{fix}(q,p) \qquad \bar{z}_{fix} = \bar{z}_{fix}(q,p)$$

These solutions are known as "fixed moduli". Also at the attractor point the square of the central charge is moduli independent

$$|Z|^2_{fix} = |Z|^2_{fix}(p,q) \qquad |W|^2_{fix} = |W|^2_{fix}(p,q)$$

This explains why the covariantly holomorphic central charge was used in most applications of supersymmetric attractors.

So far we have intentionally not given any interpretation to the symplectic invariants, charges etc. In this general form the attractor equations can be understood either in the context of black hole or in the context of flux vacua.

Black hole attractors

When we study 4d black hole attractors in N=2 ungauged supergravity we are interested in a 4d geometry with vector fields. The kinetic term of gauge fields is defined by the period matrix $\mathcal{N}_{\Lambda\Sigma}$ which depends only on scalar fields of the vector multiplets z^i. The vector field action can be also rewritten as $(\mathrm{Im}\,\mathcal{N}_{\Lambda\Sigma}\mathcal{F}^\Lambda\mathcal{F}^\Sigma + \mathrm{Re}\,\mathcal{N}_{\Lambda\Sigma}\mathcal{F}^{\Lambda *}\mathcal{F}^\Sigma) = \mathcal{F}^{\Lambda *}\mathcal{G}_\Lambda$, where $\mathcal{G}_\Lambda = \mathrm{Re}\,\mathcal{N}_{\Lambda\Sigma}\mathcal{F}^\Sigma - \mathrm{Im}\,\mathcal{N}_{\Lambda\Sigma}{}^*\mathcal{F}^\Sigma$. The symplectic structure of equation of motion is manifest in terms of the Sp($2n_v$ +2) symplectic vector field strength $(\mathcal{F}^\Lambda, \mathcal{G}_\Lambda)$. These vector fields in the symplectic basis decompose in the susy basis into the vector field of the gravitational multiplet (graviphoton) and the vector fields of the vector multiplets. The graviphoton is given by the following symplectic invariant combination of the vector fields in the action

$$T = M_\Lambda \mathcal{F}^\Lambda - L^\Lambda \mathcal{G}_\Lambda$$

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$$T = M_\Lambda \mathcal{F}^\Lambda - L^\Lambda \mathcal{G}_\Lambda \, .$$

The central charge for the general N=2 theories is a charge of the graviphoton

$$Z(z,\bar{z},q,p) = e^{\frac{K(z,\bar{z})}{2}}(X^\Lambda(z)q_\Lambda - F_\Lambda(z)\,p^\Lambda) = (L^\Lambda q_\Lambda - M_\Lambda p^\Lambda)\ .$$

Here the symplectic covariant charges are electric and magnetic charges of the vector fields in 4d defined as follows

$$\begin{pmatrix} p^\Lambda \\ q_\Lambda \end{pmatrix} = \begin{pmatrix} \int_{S^2} \mathcal{F}^\Lambda \\ \int_{S^2} \mathcal{G}_\Lambda \end{pmatrix}.$$

where the integration is performed over some S^2 in 4d. These electric and magnetic charges break 4d Lorentz symmetry and therefore black hole attractors were for a long time not clearly related to the flux vacua in string theory where in 4d the vacua of interest for cosmology preserve 4d Lorentz symmetry

The black hole attractor equations here are given by

$$\begin{pmatrix} p^\Lambda \\ q_\Lambda \end{pmatrix} = \mathrm{Re} \begin{pmatrix} 2i\bar{Z}L^\Lambda \\ 2i\bar{Z}M_\Lambda \end{pmatrix} ,$$

where Z is the graviphoton charge depending on moduli and on conserved charges (p^Λ, q_Λ) and (L^Λ, M_Λ) are covariantly holomorphic sections depending on moduli. The mass of the BPS black hole at the arbitrary point in the moduli space is given by

$$M^2_{\text{blackhole}}(z, \bar{z}; p, q) = |Z|^2 + |DZ|^2 .$$

Near the horizon where the moduli are attracted to the fixed point the graviphoton charge has to be covariantly constant

$$D_i Z(z, \bar{z}, q, p) \equiv (\partial_i + \frac{1}{2} K_i) Z = 0 , \quad D_{\bar{i}} \bar{Z} \equiv (\partial_{\bar{i}} + \frac{1}{2} K_{\bar{i}}) \bar{Z} = 0 .$$

The minimal value of the black hole mass as a function in the moduli space defines the area of the horizon A and the entropy S of the classical black hole solution.

$$M^2_{\rm min}\Big(z_{fix}(p,q), \bar z_{fix}(p,q); p, q\Big) = |Z|^2_{DZ=0} = \frac{A(p,q)}{4\pi} = \pi S(p,q),$$

Another interpretation of these equations comes from introducing a concept of the **double-extreme black holes** which have the extremal ADM mass for a given set of (p,q) charges. These black holes have the values of moduli fixed to their extremal values everywhere, not only at the horizon. For such black holes it is sufficient to solve the attractor equations to find the solution everywhere since the moduli are frozen and are defined by the values of charges.

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Flux vacua

Now we would like to reinterpret the attractor equations as defining the flux vacua in string theory compactified in Calabi-Yau space. A significant part of this interpretation is well known as it was developed by *Strominger, Moore, Ooguri, Vafa, Verlinde*. It was proposed that black hole attractors may be related to GKP-KKLT model of vacuum stabilization.

Here we would like to stress a particularly important part of the relation between supersymmetric attractor equations, flux vacua and BPS black holes. To use the attractor equations for the flux vacua all we have to do is to relate the symplectic charge vector to fluxes in internal space. Consider, for example a compactification of M theory on some CY 4-fold. We choose a holomorphic symplectic

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basis such that

$$X^\Lambda(z) = \int_{A^\Lambda} \Omega \qquad F_\Lambda(z) = \int_{B_\Lambda} \Omega$$

Here Ω is a holomorphic 4-form on a Calabi-Yau space and

$$K = -i \int \Omega \wedge \bar{\Omega}$$

The 4-form flux has a (0,4) and a (4,0) components. This also means that the symplectic flux vector is related to 4-form fluxes in an internal space as follows

$$q_\Lambda = \int_{A_\Lambda} \mathcal{F} \qquad p^\Lambda = \int_{B^\Lambda} \mathcal{G}$$

Therefore, as different from the 4d black holes where electric and magnetic charges break Lorentz symmetry (fluxes have components in 4d space), here fluxes have only non-vanishing components in the compact space, and therefore the minimization of symplectic invariants leads to flux vacua which are Lorentz covariant in 4d. *The formal structure of the attractor equations when we use the symplectic charge vector* (q_Λ, p^Λ) *is the same in both cases.*

Indeed, let us introduce the relevant holomorphic central charge W as follows

$$W = \int F_4 \wedge \Omega = X^\Lambda q_\Lambda - p^\Lambda F_\Lambda$$

and also a corresponding covariantly holomorphic central charge

$$Z = L^\Lambda q_\Lambda - p^\Lambda M_\Lambda = e^{K/2} W$$

Now again we can construct 2 symplectic invariants, I_1 and I_2 which depend on moduli and fluxes. The requirement of minimization of

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symplectic invariants is

$$D_i W(z,q,p) \equiv (\partial_i + K_i)\left(\int F_4 \wedge \Omega\right) = 0$$

This condition is often imposed to define a supersymmetric flux vacua and it is equivalent to the requirement that the covariantly holomorphic central charge is covariantly constant, i. e.

$$D_i Z(z,\bar{z};q,p) \equiv (\partial_i + \frac{1}{2}K_i)\left(\int F_4 \wedge e^{K/2}\Omega\right) = 0$$

Since we have now established one-to-one correspondence with the supersymmetric attractor equations for black holes, it follows that the attractor equations derived there are valid for flux vacua. The symplectic charge vector (p,q) is defined as an integral of the symplectic 4-form flux in CY space. Flux vacua are now defined by the

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supersymmetric attractor equations

$$D_i W(z,q,p) = 0 \quad \Rightarrow \quad D_i Z(z,\bar{z};q,p) = 0$$

$$\Rightarrow \quad \begin{pmatrix} p^\Lambda \\ q_\Lambda \end{pmatrix} = \mathrm{Re} \begin{pmatrix} 2ie^K \bar{W} X^\Lambda \\ 2ie^K \bar{W} F_\Lambda \end{pmatrix}$$

Here we have presented the attractor equations via the superpotential $\bar{W}$ and Kähler potential K as well as holomorphic section (X,F). However, they are equivalent to the one expressed via the central charge $\bar{Z}$ and covariantly holomorphic section (L,M). At the attractor point the Kähler potential and the superpotential depend only on fluxes.

$$(|Z|^2)_{fix}(p,q) = (e^K|W|^2)_{fix}(p,q)$$

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Explicitly attractive K3 surfaces

Attractive K3 surfaces are always rigid - any infinitesimal deformation of complex structure would always decrease the rank of the Picard lattice on K3, which is equal to 20. Torelli's theorem defines the complex structure of the attractive K3 surface via $\Omega^{(2.0)} = q - \tau p$.

However, we may use the solutions of the complete set of attractor equations and give the explicit answer for all moduli in terms of fluxes. Thus we want to find the moduli on $\frac{SU(1,1)}{U(1)} \times \frac{SO(2,n)}{SO(2)\times SO(n)}$ manifold.

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Part of these attractor equations were already used in the definition of the attractive K3 surface by Moore. Moreover, he has proposed the interpretation of the attractor value of the

$$|Z|^2_{fix} = (p^2q^2 - (p\cdot q)^2)^{1/2} = \frac{A}{4\pi}\,,$$

as an area of the unit cell in the transcendental lattice T_S of the K3 surface. He also gives the fixed value of the modulus on $\frac{SU(1,1)}{U(1)}$.

$$\bar\tau = \frac{p\cdot q}{p^2} - i\frac{(p^2q^2 - (p\cdot q)^2)^{1/2}}{p^2}\,.$$

We will find an explicit definition of the attractive K3 surface, where the axion-dilaton as well as 20 complex structure moduli are given at the fixed points as functions of all fluxes.

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Fixing moduli on ST[2,n] manifold

Consider an ST[2,n] manifold which is an $\frac{SU(1,1)}{U(1)} \times \frac{SO(2,n)}{SO(2)\times SO(n)}$ symmetric manifold. A stabilization of moduli on this manifold was established by solving explicitly the supersymmetric attractor equations in this theory.

$$\begin{pmatrix} p^\Lambda \\ q_\Lambda \end{pmatrix} = \mathrm{Re} \begin{pmatrix} 2i\bar{Z}L^\Lambda \\ 2i\bar{Z}M_\Lambda \end{pmatrix}$$

where Z is the central charge depending on moduli and on conserved charges (p^Λ, q_Λ) and (L^Λ, M_Λ) are covariantly holomorphic sections depending on moduli. Here we would like to use this known solution of the attractor equations to show how fluxes stabilize all moduli describing this manifold

Our starting point is

$$\begin{pmatrix} X^\Lambda \\ F_\Lambda \end{pmatrix} = \begin{pmatrix} X^\Lambda \\ \tau\, X_\Lambda \end{pmatrix}, \qquad X^\Lambda X_\Lambda \equiv X \cdot X = 0\,, \quad \Lambda = 0, 1, \ldots n+1.$$

The metric $\eta_{\Lambda\Sigma} = \mathrm{diag}(+, +, -, \ldots, -)$ is used for changing the position of the indices: $X_\Lambda = \eta_{\Lambda\Sigma} X^\Sigma$. Note that X^Λ are not independent and satisfy the constraint $X \cdot X = 0$.

Using

$$F_\Lambda = \tau X_\Lambda$$

in the stabilization and we can bring them to the following form

$$p^\Lambda = i\bar{Z} e^{K/2} X^\Lambda - i Z e^{K/2} \overline{X}^\Lambda \,, q_\Lambda = i\bar{Z} e^{K/2} \tau X_\Lambda - i Z e^{K/2} \bar{\tau}\, \overline{X}_\Lambda \,.$$

These equations can be massaged further by contracting them with X using the constraint $X \cdot X = 0$. After quite a bit of algebra one

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finds

$$\tau\bar{\tau} = \frac{q^2}{p^2}\,, \qquad \tau + \bar{\tau} = \frac{2p\cdot q}{p^2}.$$

From the above two equations we can obtain the fixed value of the axion-dilaton, which is the moduli on $\frac{SU(1,1)}{U(1)}$

$$\tau = \frac{p\cdot q}{p^2} + i\frac{(p^2q^2 - (p\cdot q)^2)^{1/2}}{p^2},$$

The fixed value of the central charge $|Z|$ follows

$$|Z|^2_{fix} = (p^2q^2 - (p\cdot q)^2)^{1/2}\,,$$

Translating black hole attractor solutions into K3

So far these attractor equations were already used in the definition of the attractive K3 surface by Moore. At this point he suggested to rely on Torelli's theorem which defines the complex structure of the attractive K3 surface via $\Omega^{(2.0)} = q - \tau p$. Alternatively, we may use the solutions of the complete set of attractor equations and give the answer for all moduli in terms of fluxes.

Thus we want to find the moduli on $\frac{SO(2,n)}{SO(2)\times SO(n)}$ manifold. For this purpose use the attractor eqs. and find that

$$\bar\tau p^\Lambda - q^\Lambda = i\bar Z e^{K/2}(\bar\tau - \tau)\, X^\Lambda,$$

which leads to a beautiful equation:

$$\frac{X^\Lambda}{X^\Sigma} = \frac{\bar\tau p^\Lambda - q^\Lambda}{\bar\tau p^\Sigma - q^\Sigma}\,.$$

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$$\frac{X^\Lambda}{X^\Sigma} = \frac{\bar{\tau} p^\Lambda - q^\Lambda}{\bar{\tau} p^\Sigma - q^\Sigma} .$$

Note that the ratio $\frac{X^\Lambda}{X^\Sigma}$ does not give us yet the moduli since $X \cdot X = 0$. This constraint can be solved, in particular using a generalized Calabi-Vesentini coordinates:

$$X^0 = -\frac{1}{2}(1 - \eta_{ij}\, t^i t^j)\ ,$$
$$X^{i-1} = t^i\ , \qquad\qquad i, j = 2, \dots, n+1\ ,$$
$$X^{n+1} = \frac{1}{2}(1 + \eta_{ij}\, t^i t^j)\ .$$

The solution for moduli is

$$t^i = \frac{X^{i-1}}{X^{n+1} - X^0} = \frac{\overline{\tau} p^{i-1} - q^{i-1}}{\overline{\tau}(p^{n+1} - p^0) - (q^{n+1} - q^0)}\ .$$

Thus on one hand we have learned that the double-extreme black hole of N=2 supergravity with $\frac{SU(1,1)}{U(1)} \times \frac{SO(2,n)}{SO(2)\times SO(n)}$ symmetry has

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Thus on one hand we have learned that the double-extreme black hole of N=2 supergravity with $\frac{SU(1,1)}{U(1)} \times \frac{SO(2,n)}{SO(2)\times SO(n)}$ symmetry has the following properties. The $n+1$ complex vector multiplet moduli are functions of charges:

$$t^1 = \tau = \frac{p \cdot q}{p^2} - i \frac{(p^2 q^2 - (p \cdot q)^2)^{1/2}}{p^2}, \quad t^i = \frac{\overline{\tau} p^{i-1} - q^{i-1}}{\overline{\tau}(p^{n+1} - p^0) - (q^{n+1} - q^0)},$$

On the other hand the solutions of the same attractor equations gives an explicit definition of the attractive K3 surface, where the axion-dilaton and all 20 complex structure moduli are given at the fixed points as functions of all fluxes. All we have to do to specify the case of *explicitly attractive K3 surface* is to take $n = 20$ case of the general solution of the attractor equation.

quod erat demonstrandum

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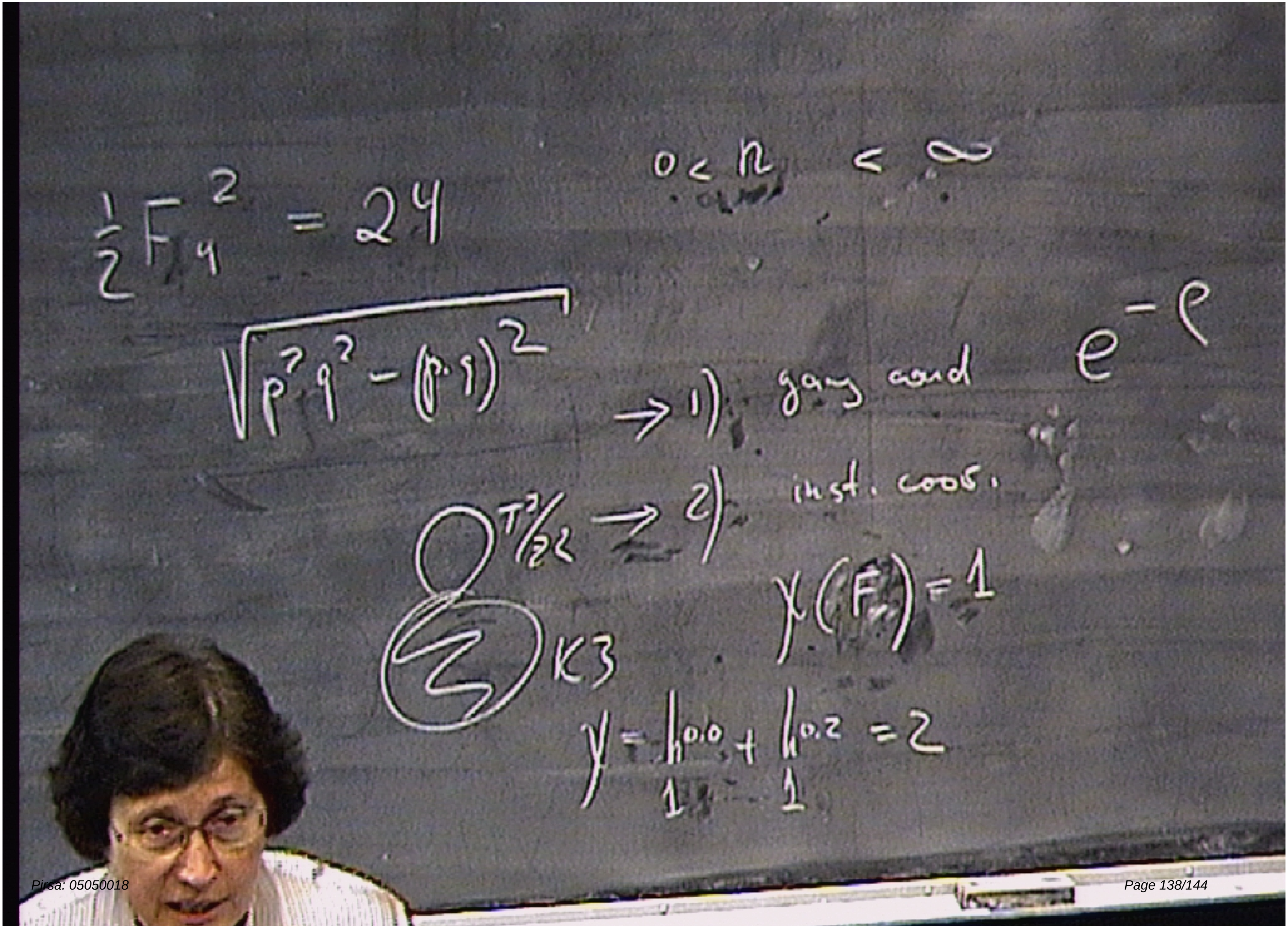

$\frac{1}{2} F_4^2 = 24$
$0 < n < \infty$
$\sqrt{p^2 q^2 - (p \cdot q)^2}$
$e^{-\rho}$
→ 1) gaug cond
→ 2) inst. coor.
T^7/Z_2
K3
$\chi(F) = 1$
$y = h^{0,0} + h^{0,2} = 2$
1 1

$\frac{1}{2} F_4^{\,2} = 24$

$0 < R < \infty$

$\sqrt{p^2 q^2 - (p \cdot q)^2}$

$\rightarrow$ 1) gauge cond

$e^{-\rho}$

$T^2/\mathbb{Z}_2 \rightarrow$ 2) inst. coor.

$h^{0,}$

$\chi(F) = 1$

$\chi = 1$

K3

$\chi = h^{0,0} + h^{0,2} = 2$

$y = (-1)^{\rho} h(\rho,\rho)$

$0 < n < \infty$

$\chi_4^2 = 24$

$\sqrt{p^2 q^2 - (p \cdot q)^2}$

$\rightarrow$ 1) gary cond $\quad e^{-\rho}$

$T^2/\mathbb{Z}_2 \rightarrow$ 2) inst. coor. $\quad h^{0,0}$

$\chi(F) = 1 \quad \chi = 1$

K3