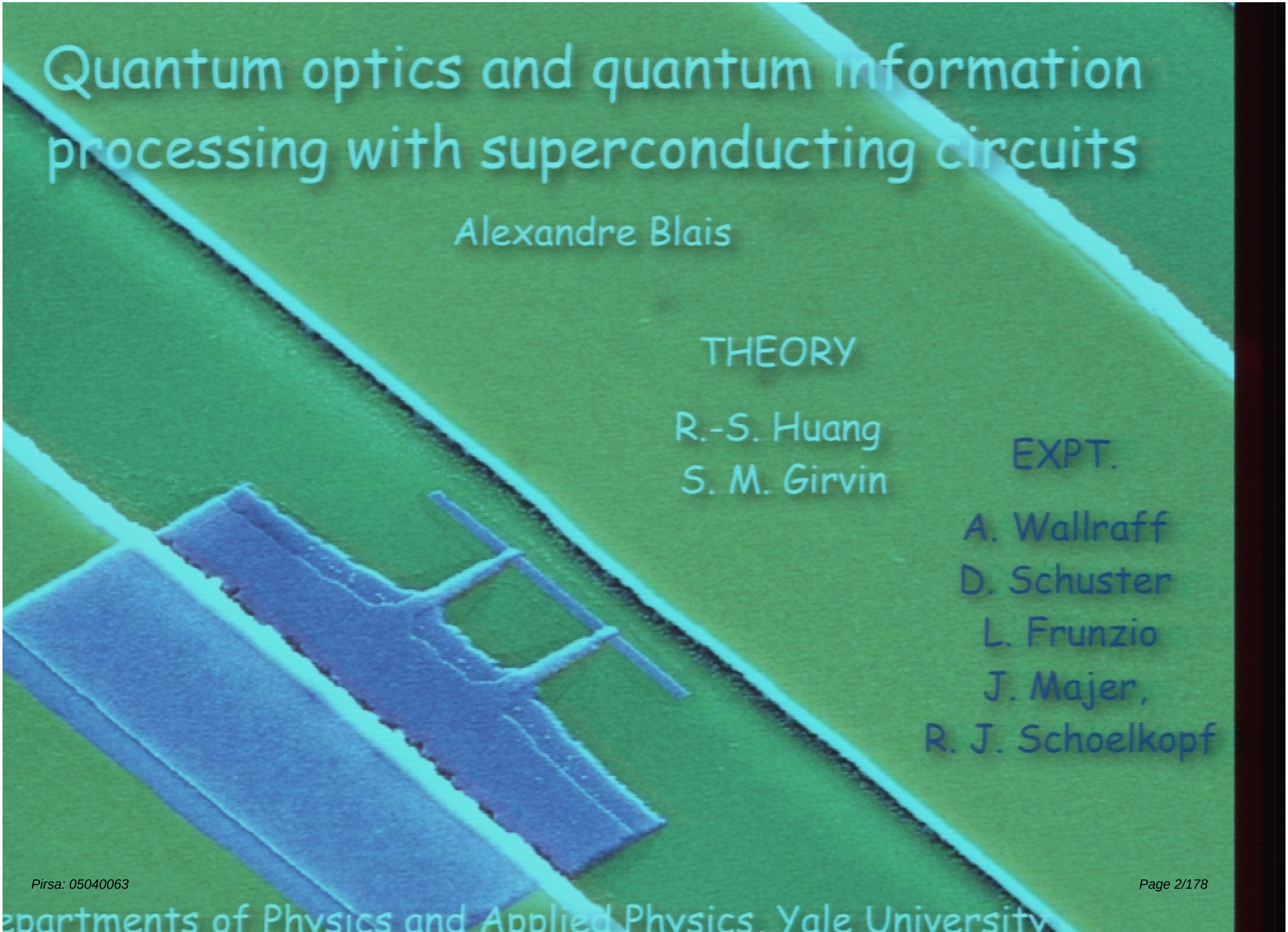
Title: Quantum Optics and Quantum Information Processing with Superconducting Circuits

Date: Apr 20, 2005 02:00 PM

URL: http://pirsa.org/05040063

Abstract: Superconducting circuits based on Josephson junctions are promising candidates for the implementation of solid-state qubits. In most of the recent experiments on these circuits, the qubits are controlled by a classical field containing a large number of photons. The possibility of coherently coupling these systems to a single photon has been recently suggested, opening the possibility to study analogs of quantum optics in condensed matter systems. I will review one of these proposals based on a superconducting charge qubit fabricated inside a high quality transmission line resonator and will describe its recent experimental realization. When the qubit is brought into resonance with the resonator, vacuum Rabi splitting is observed indicating that the regime of strong coupling has been reached. When the qubit is detuned from the cavity, I will explain how quantum non-demolition measurement can be realized. I will discuss how the measurement process can be quantitatively understood in this regime allowing us to explore the effect of measurement back-action on the qubit and to extract, for the first time in superconducting qubits, large visibility in Rabi oscillations.



Quantum optics and quantum information processing with superconducting circuits

Alexandre Blais

THEORY

R.-S. Huang
S. M. Girvin

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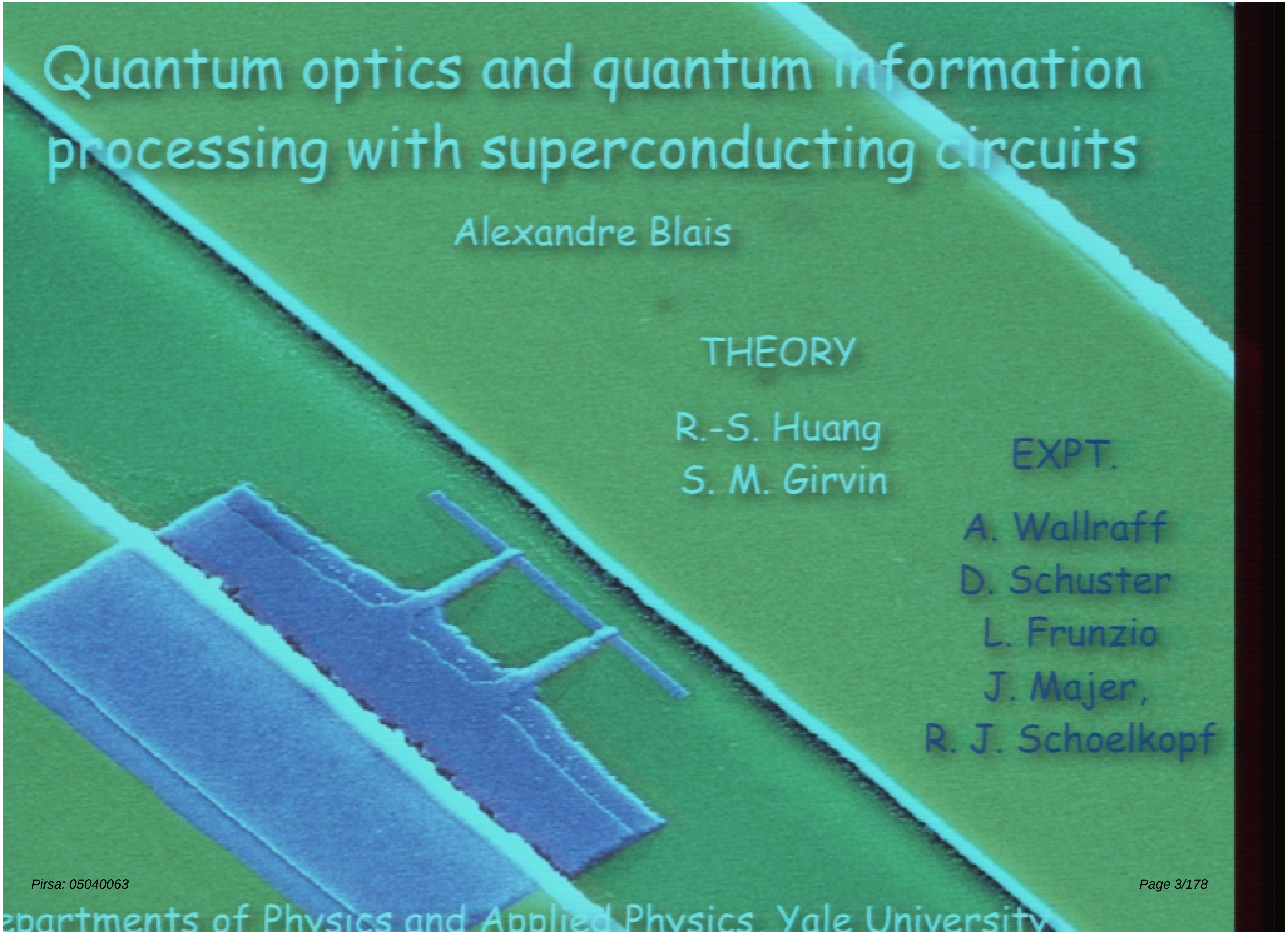
D. Schuster
L. Frunzio
J. Majer,
R. J. Schoelkopf

epartments of Physics and Applied Physics, Yale University



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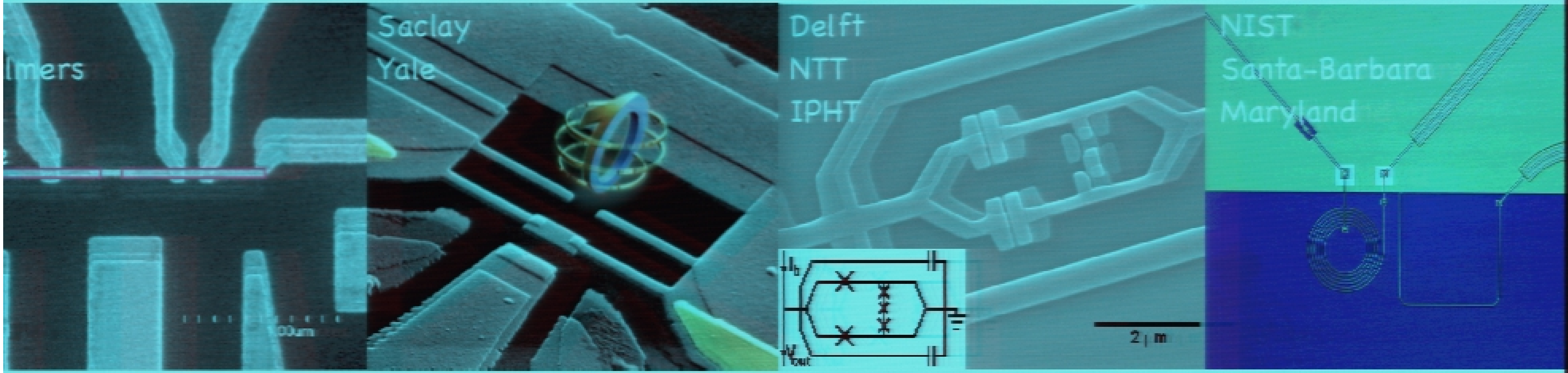
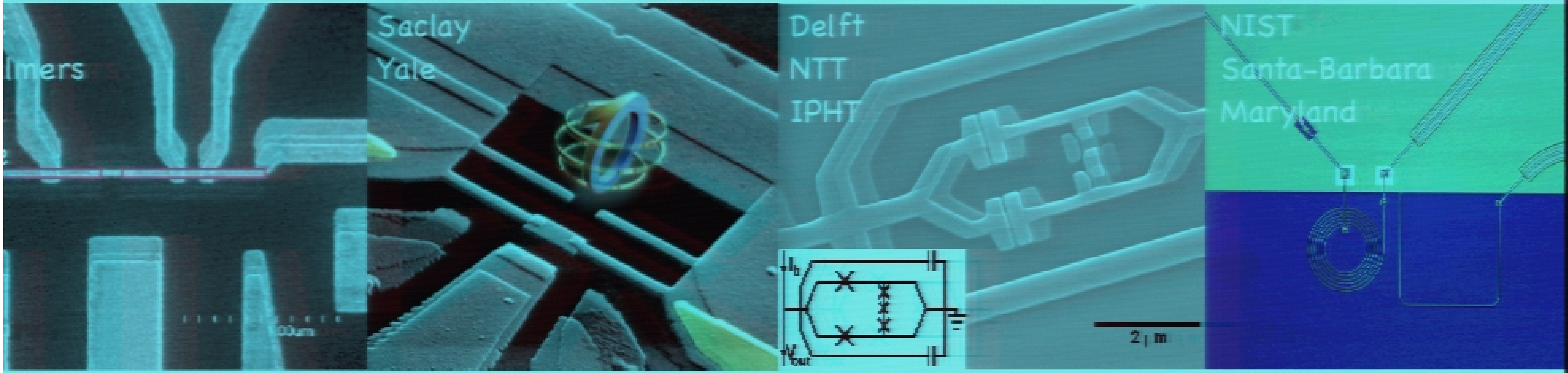
Superconducting quantum electrical circuits



Charge → Phase

Superconducting quantum electrical circuits



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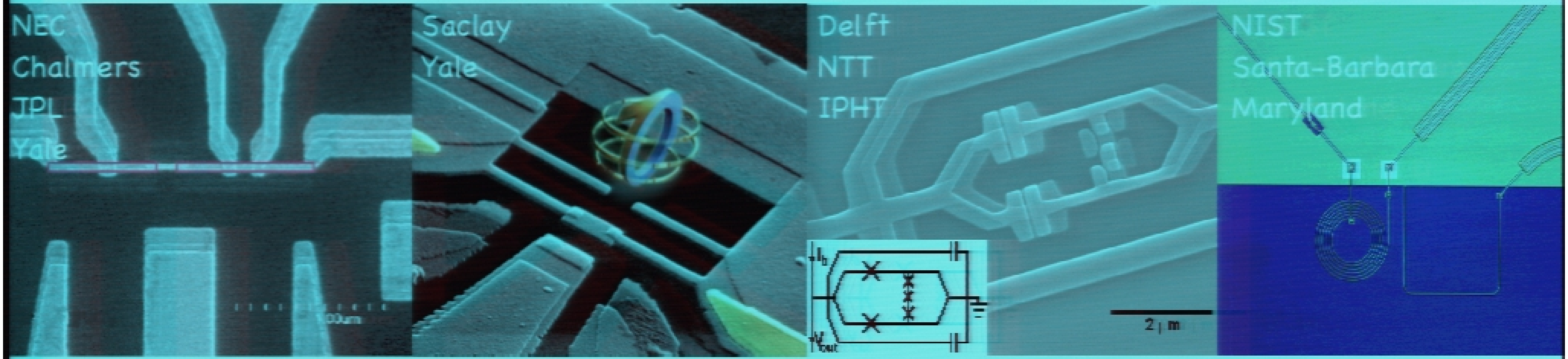

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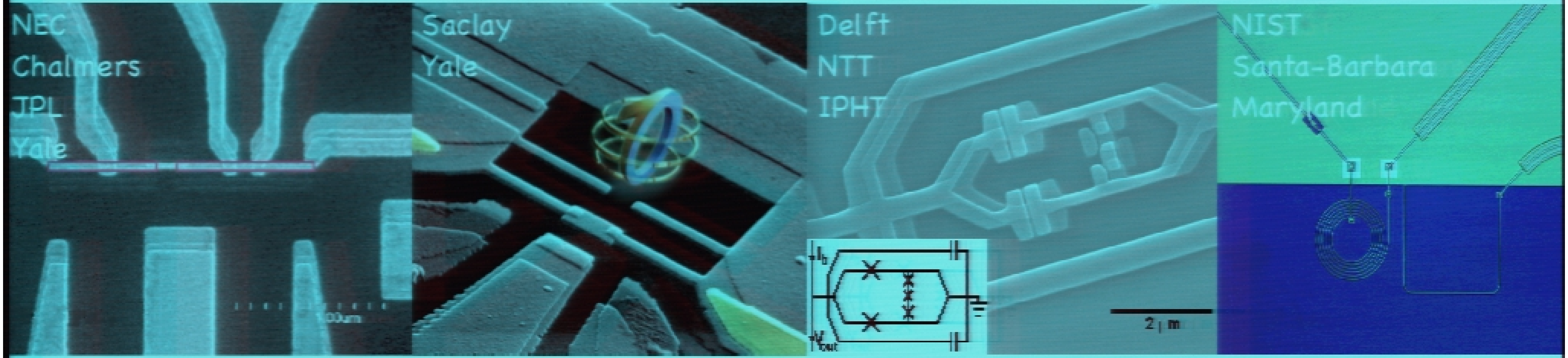

Charge → Phase

1999 → 2004

Coherent control   Coupled qubits   First two-qubit gates

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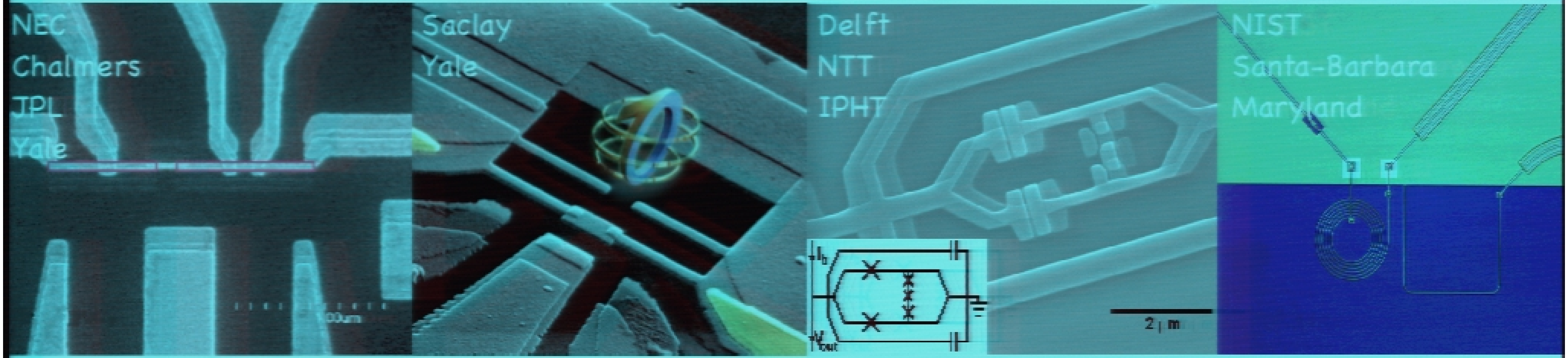

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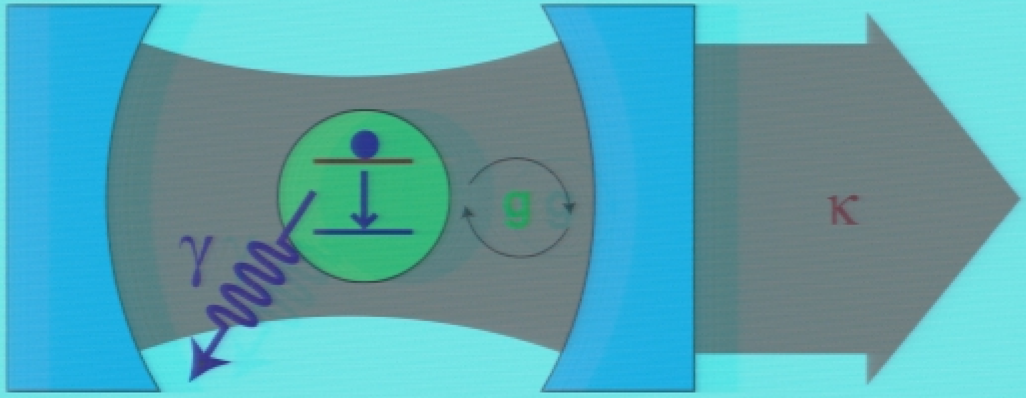
Coupling with quantum field: on-chip quantum optics

Outline

- Cavity Quantum Electrodynamics
- Circuit Quantum Electrodynamics
    - Artificial cavity and atoms
    - Strong coupling regime
- Spectroscopy
    - Ac-Stark shift
    - Measurement-induced dephasing
- Coherent control
    - Approaching unit visibility
- Outlook

Cavity Quantum Electrodynamics

Engineering the vacuum



Mabuchi & Doherty, Science **298**, 1372 (2002)

Cavity Quantum Electrodynamics

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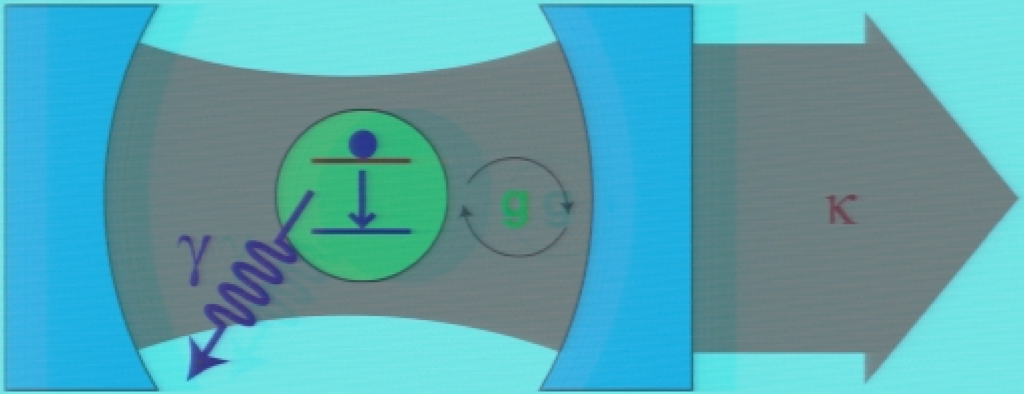

Jaynes–Cummings Hamiltonian:

$$H = \hbar\omega_r a^\dagger a + \hbar\frac{\omega_a}{2}\sigma_z + \hbar g(a^\dagger\sigma_- + a\sigma_+)$$

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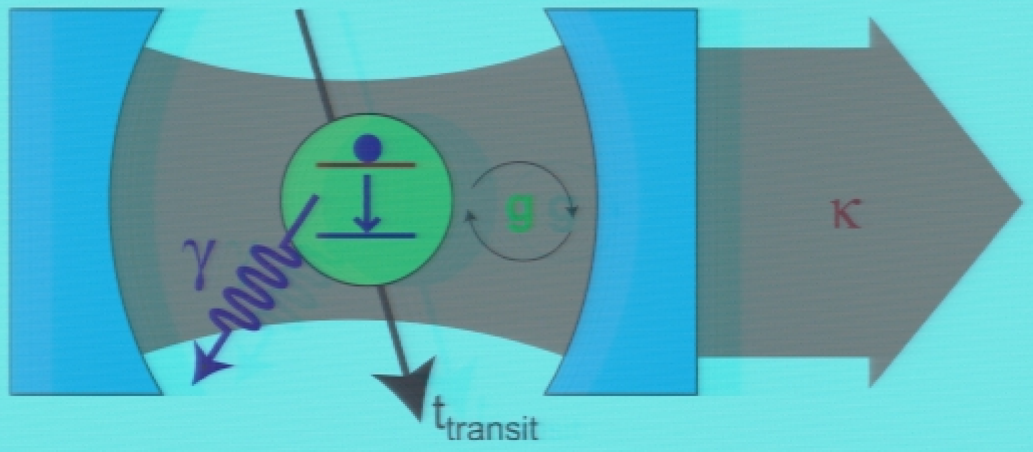

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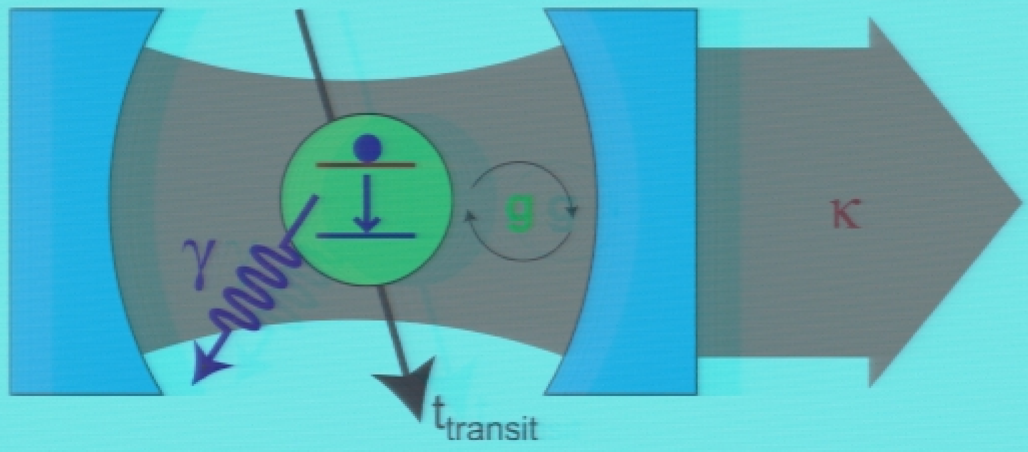

Jaynes–Cummings Hamiltonian:

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Strong coupling: $g \gg \kappa, \gamma, t_{\text{transit}}^{-1}$

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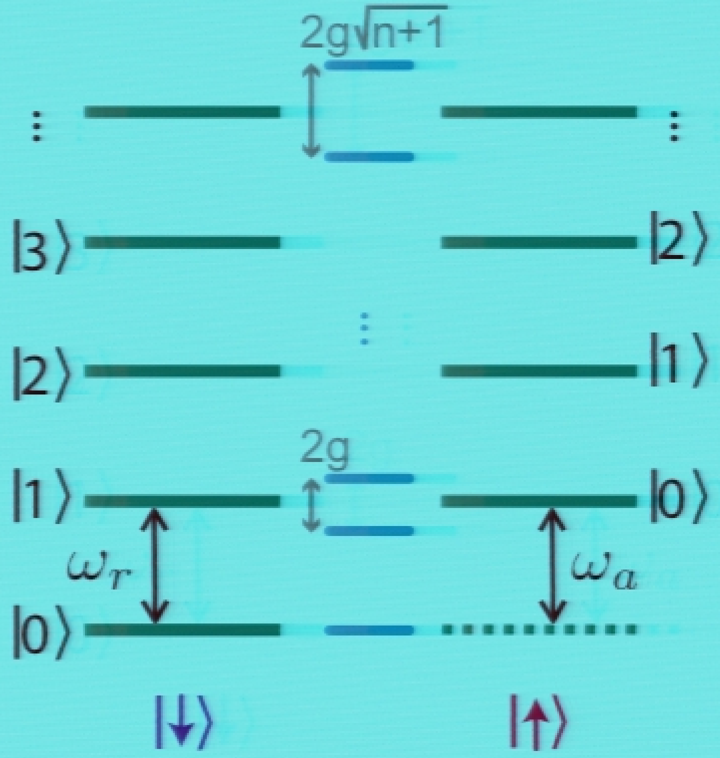
Cavity QED

$$H = \hbar\omega_r a^\dagger a + \hbar\frac{\omega_a}{2}\sigma_z + \hbar g(a^\dagger\sigma_- + a\sigma_+)$$

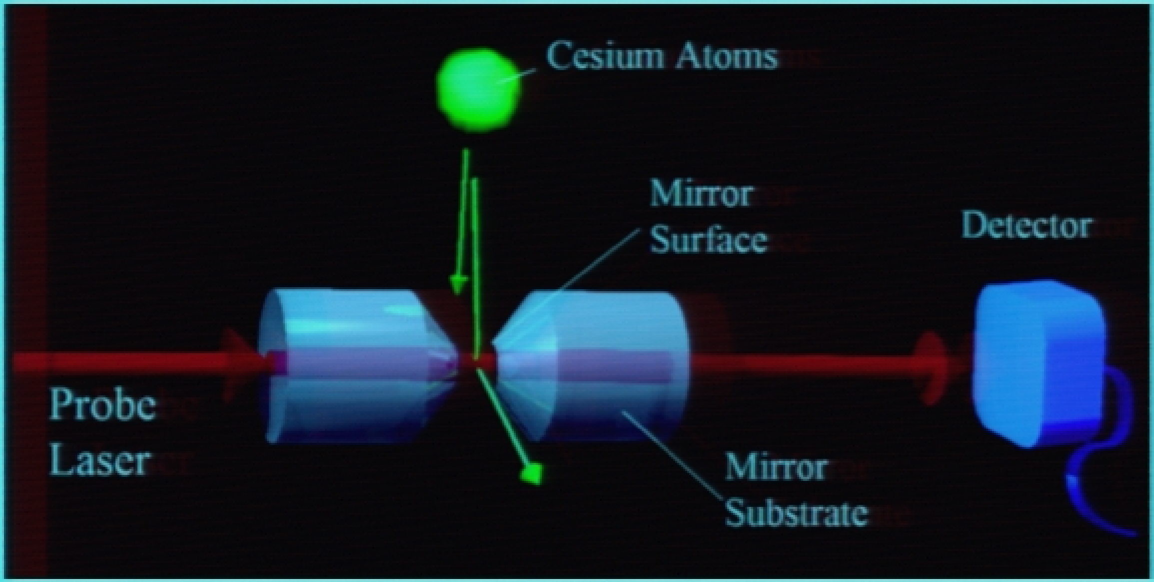
In resonance:

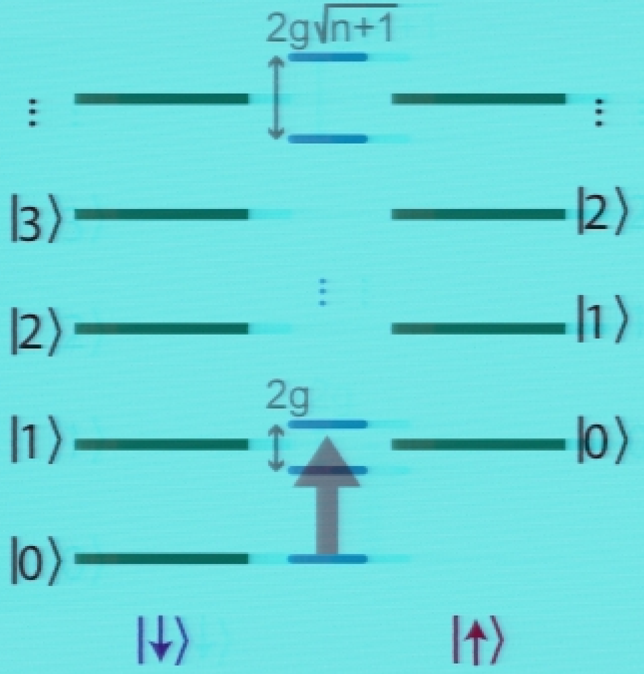
$$\Delta = \omega_a - \omega_r = 0$$



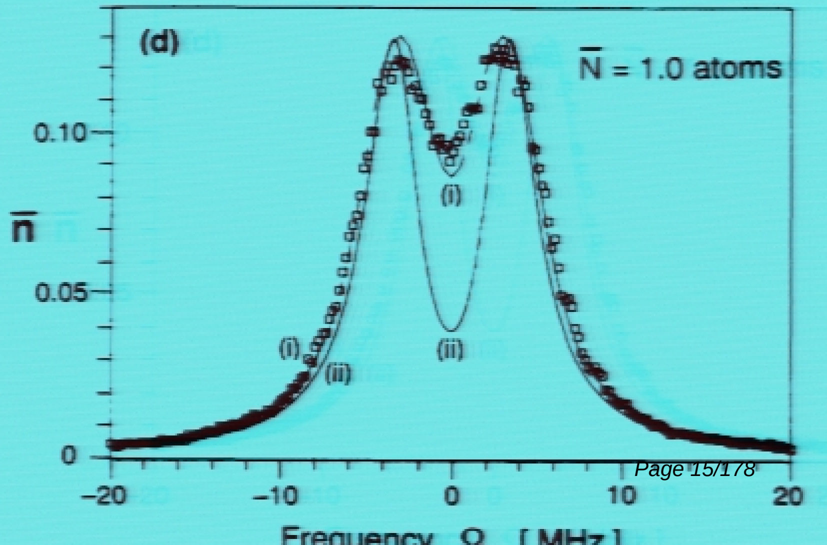
Cavity QED

Optical frequencies, Alkali atoms





Thompson et al., PRL **68**, 1132 (1992)
Boca et al., PRL **93**, 233603 (2004)



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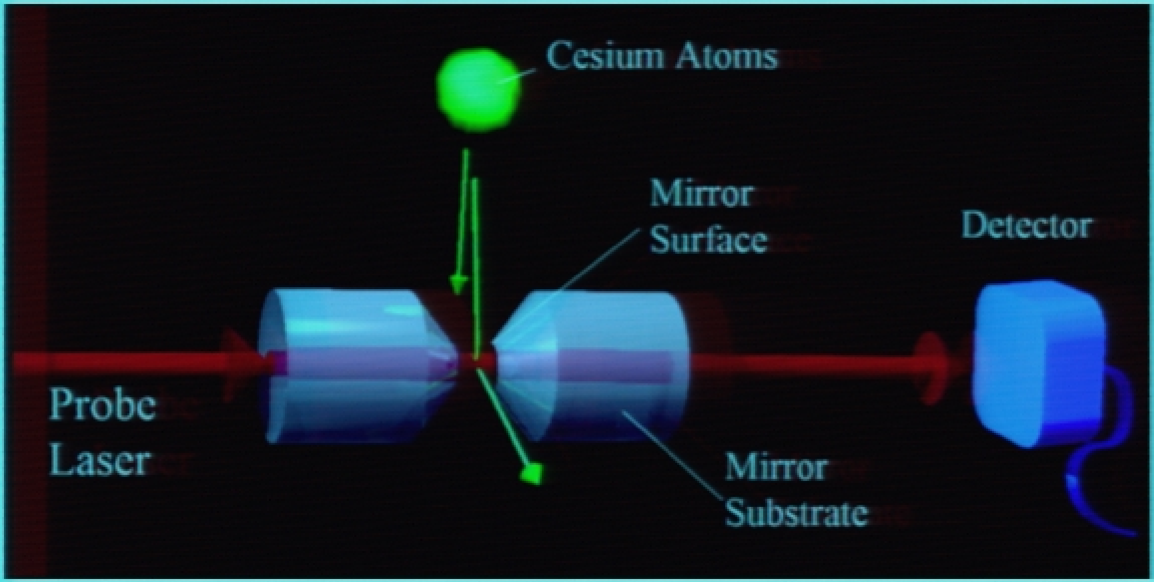

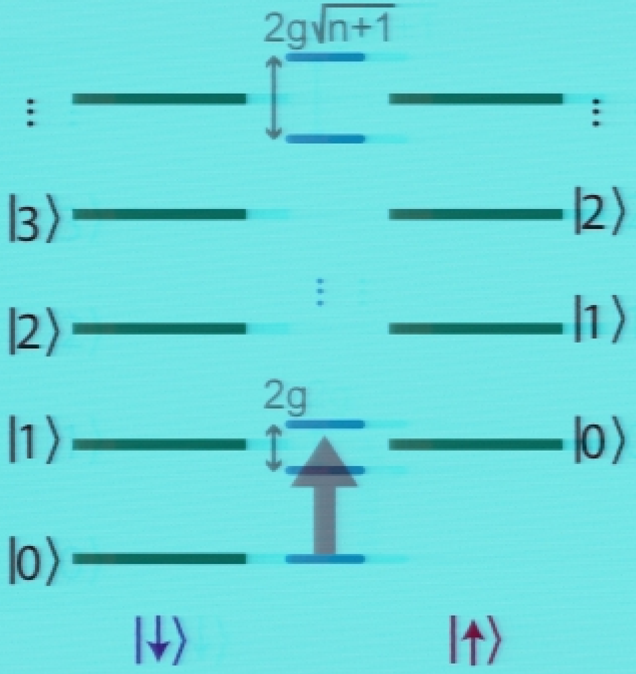

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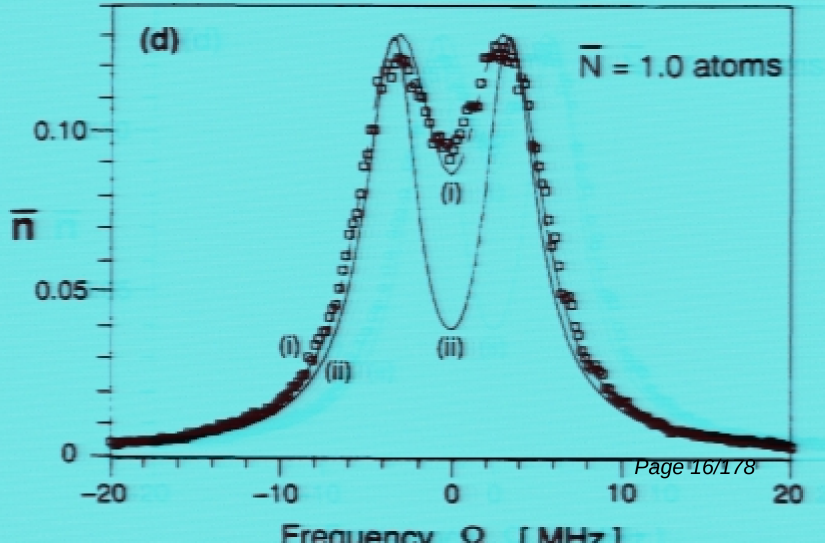

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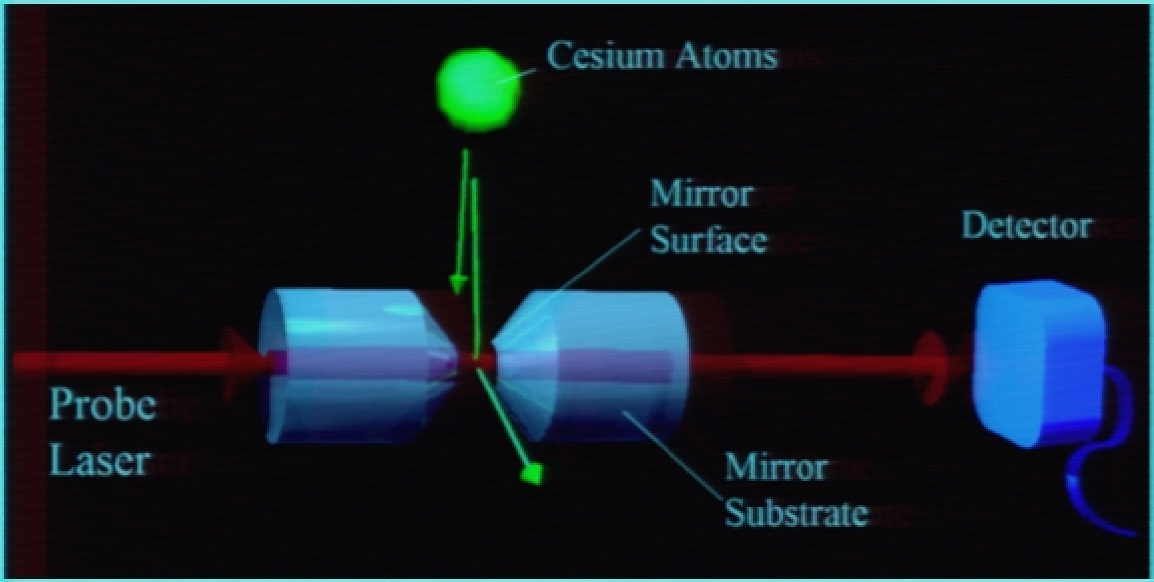

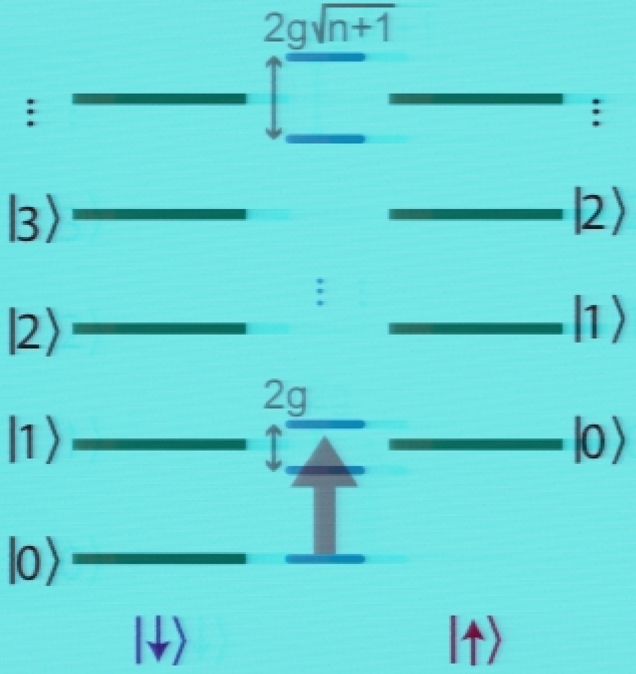

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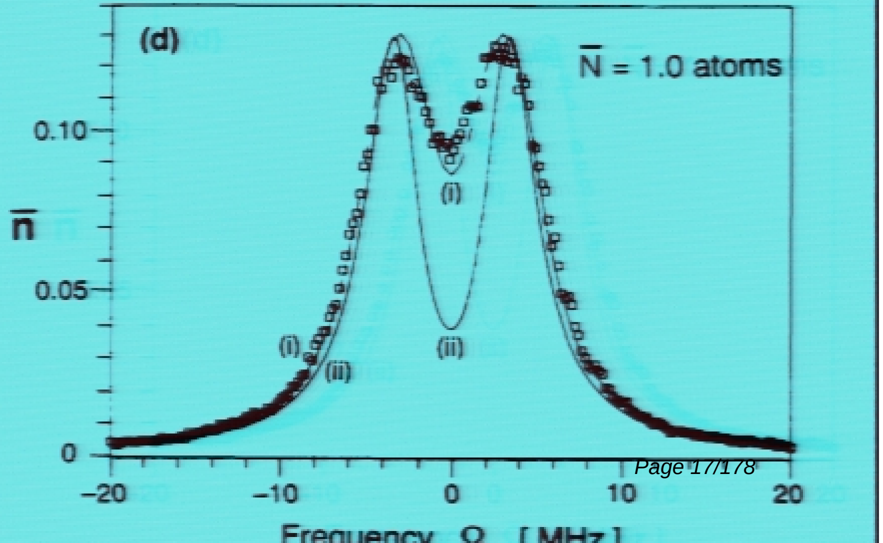

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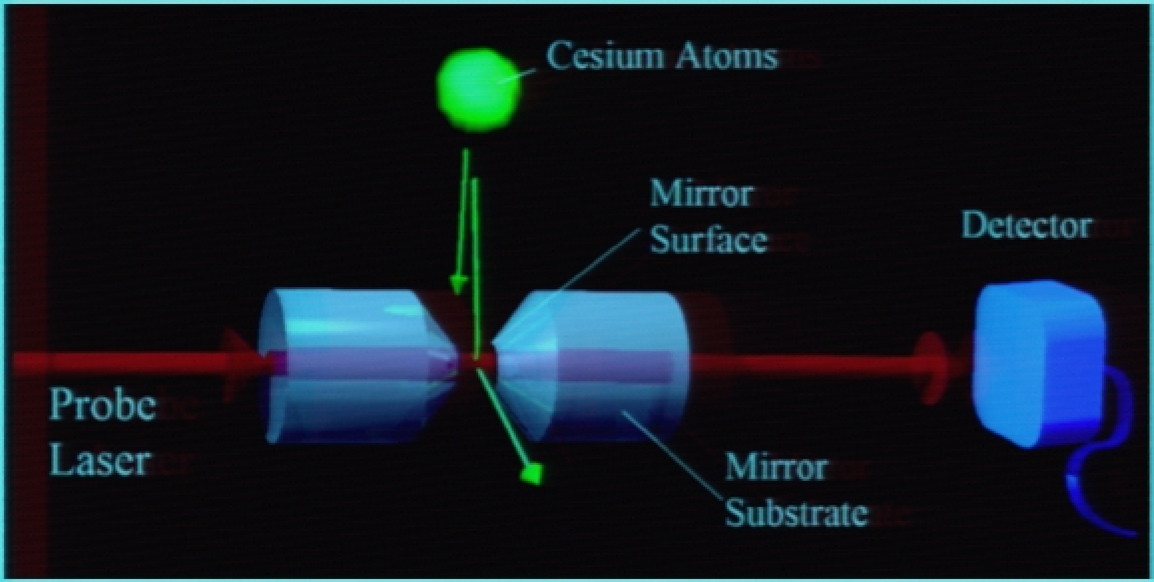

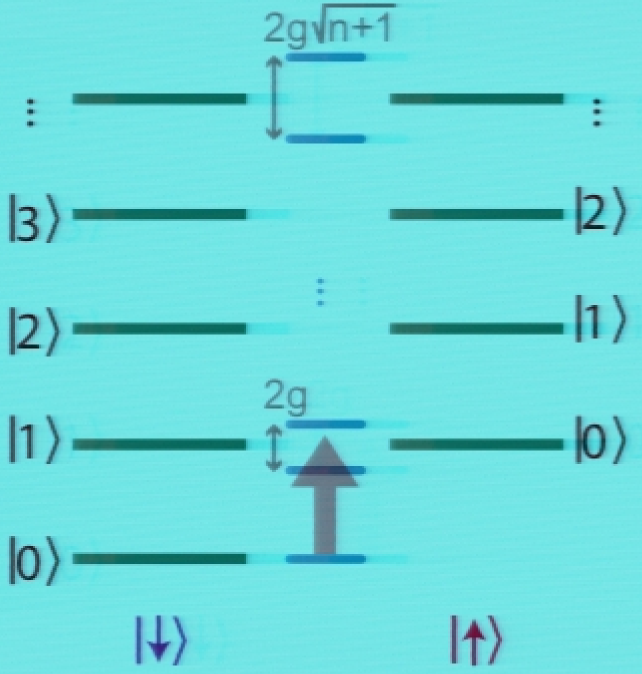

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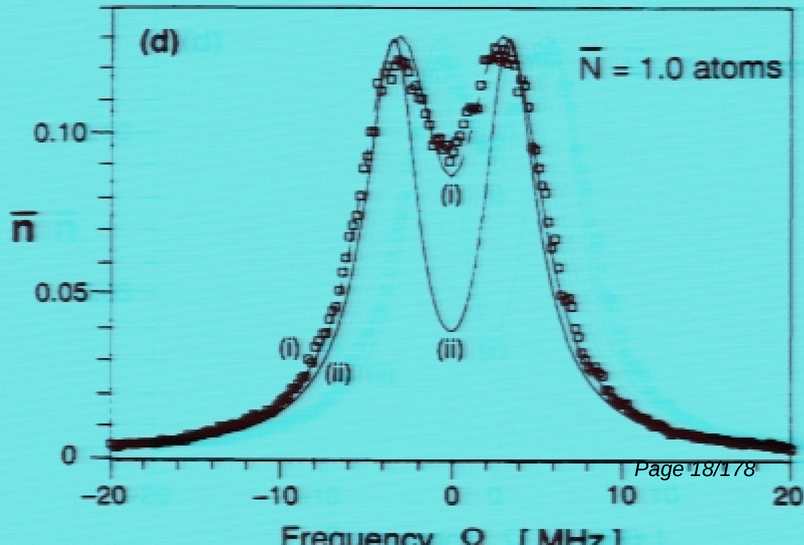
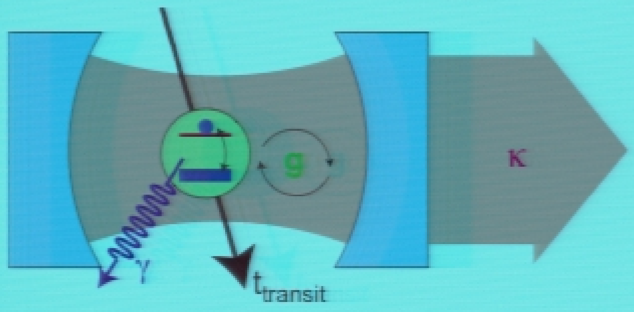
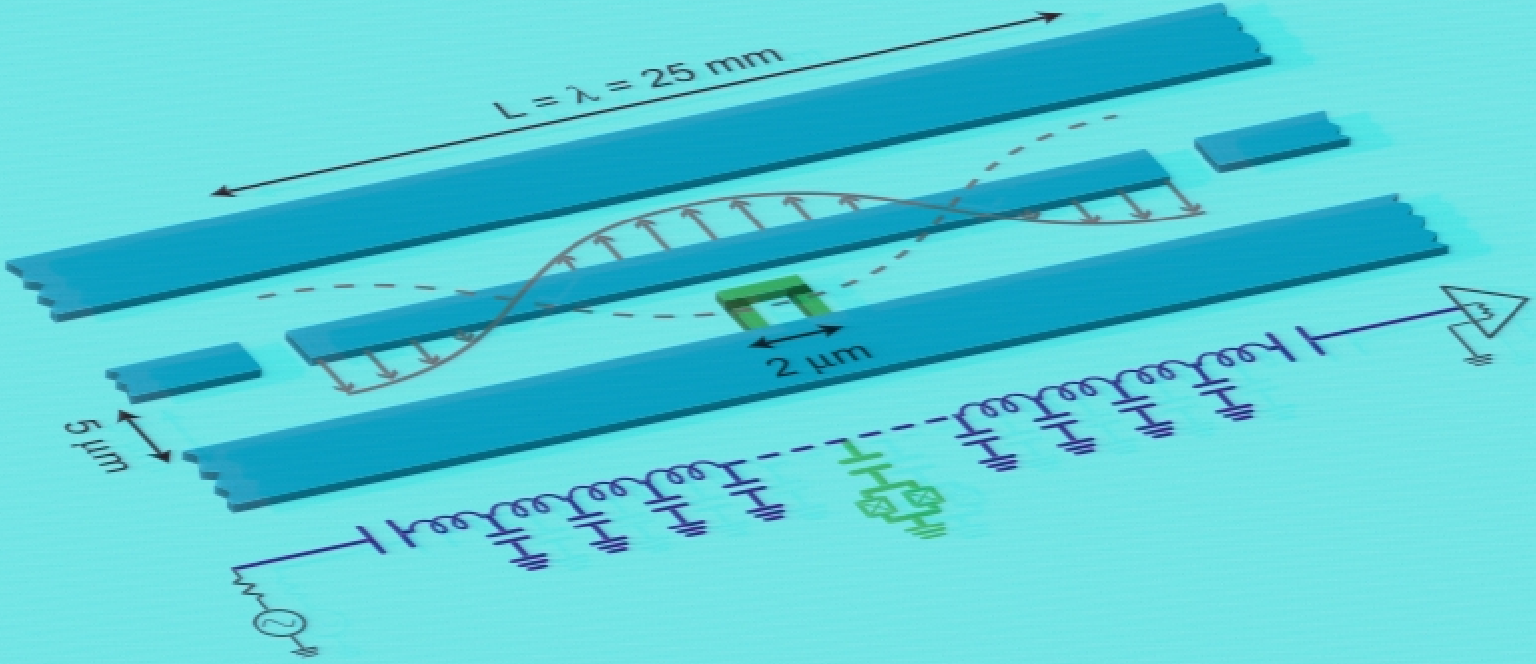


Circuit Quantum Electrodynamics





Cavity: superconducting 1D waveguide resonator

Atom: superconducting charge qubit

AB, Huang, Wallraff, Girvin & Schoelkopf, PRA 69, 062320 (2004)

Cavity QED

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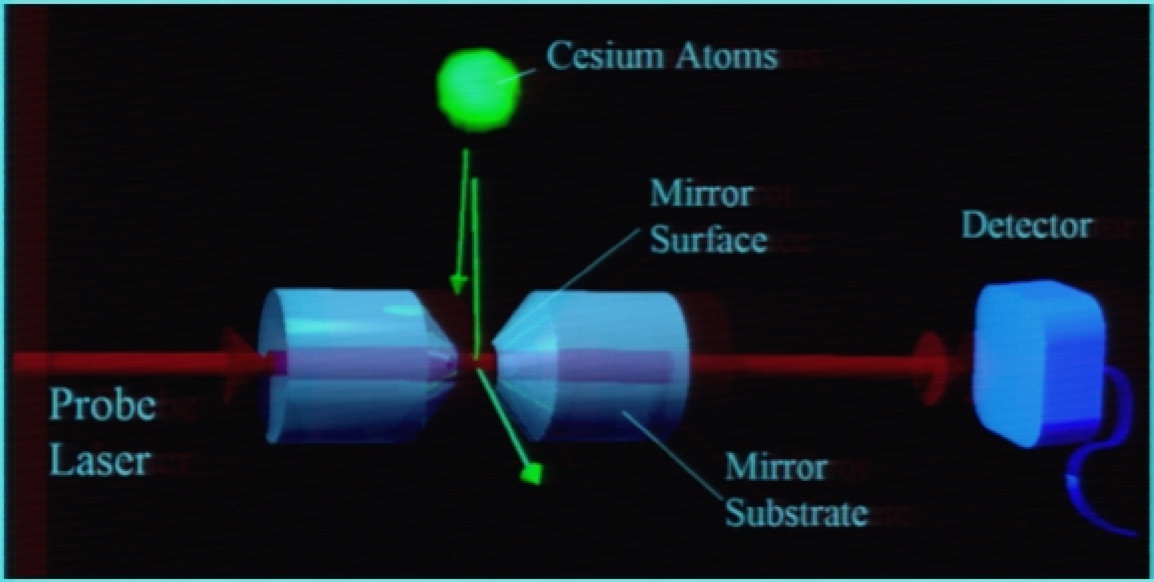

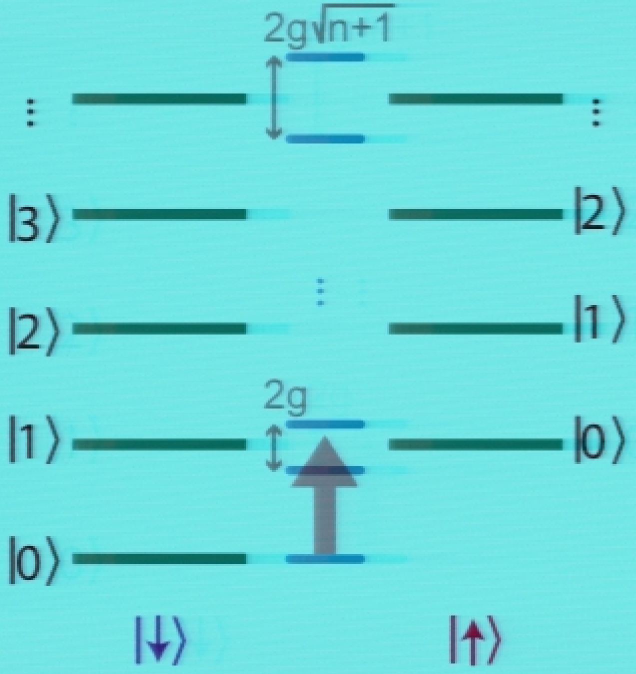

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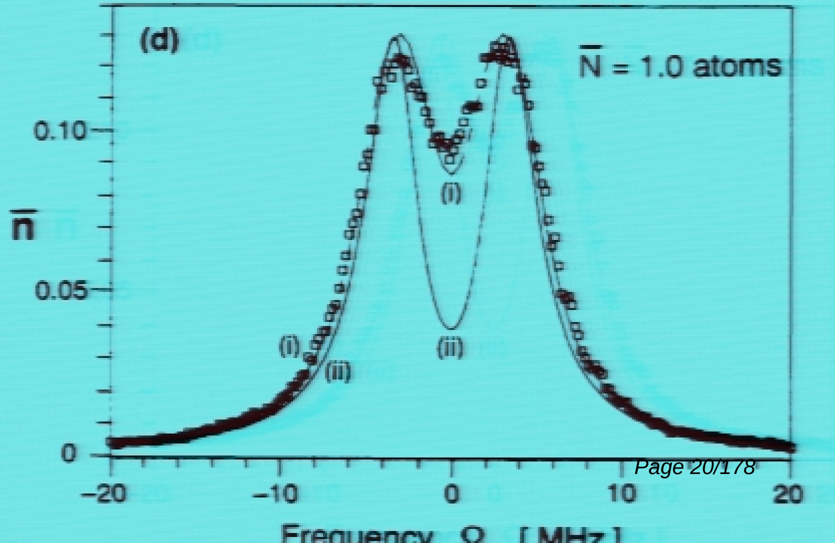

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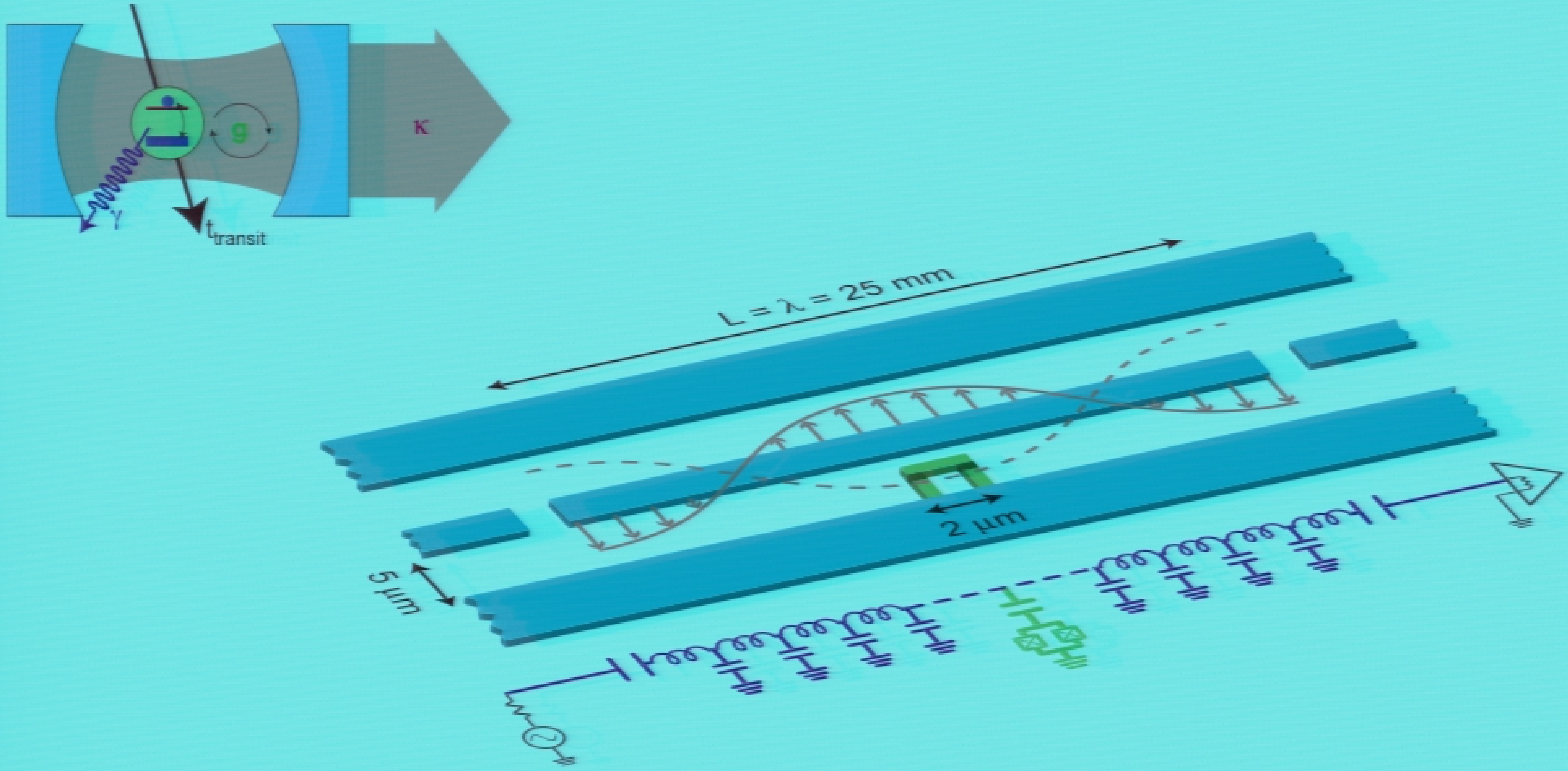

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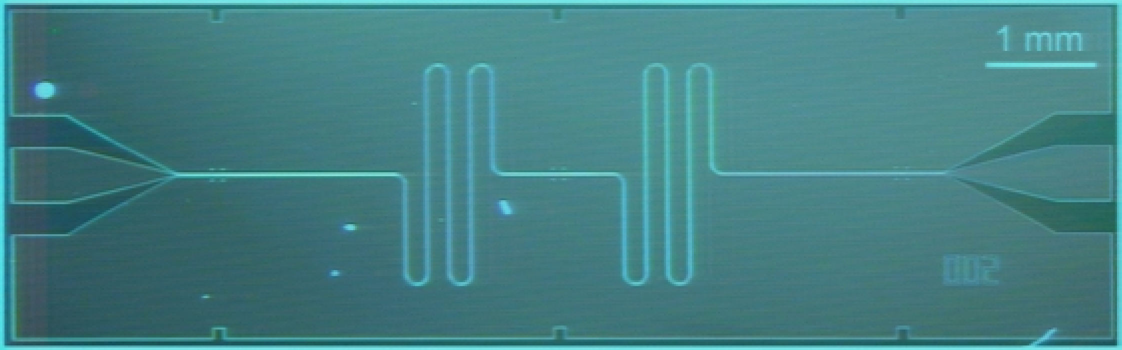

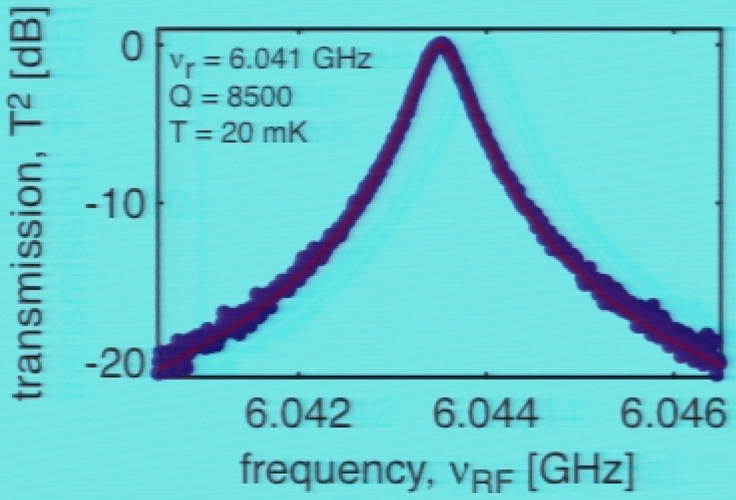

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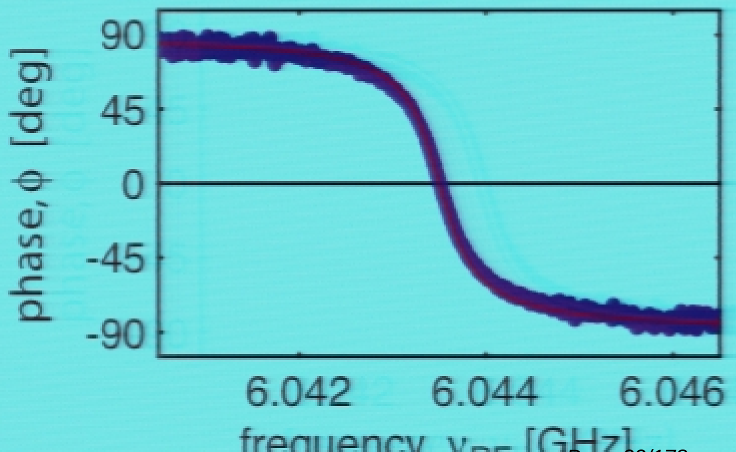
$$T_\kappa = 1/\kappa \approx 224\,\mathrm{ns}$$

$$h\nu_r = 300\,\mathrm{mK}$$

$$\rightarrow \; N_{th} \approx 0.06 \text{ @ } 100\mathrm{mK}$$



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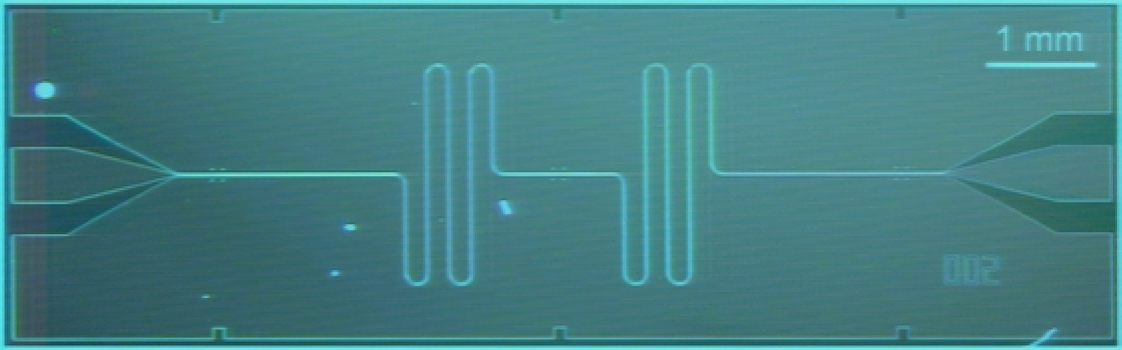

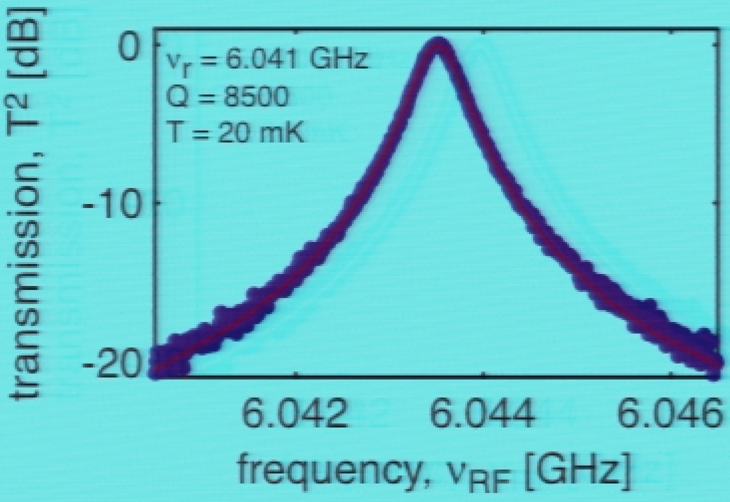

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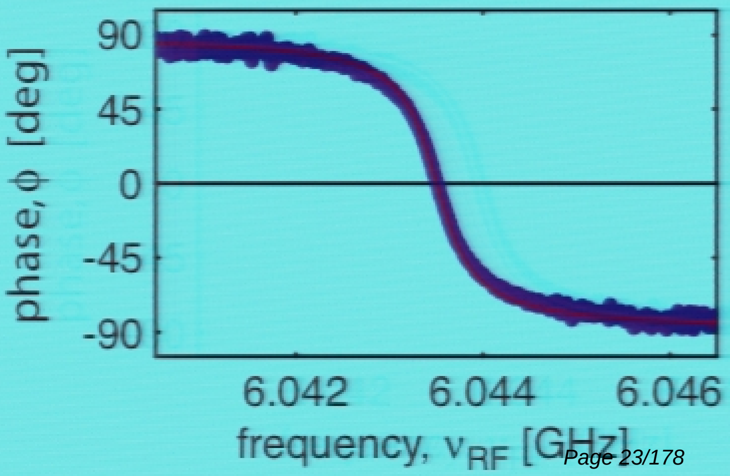

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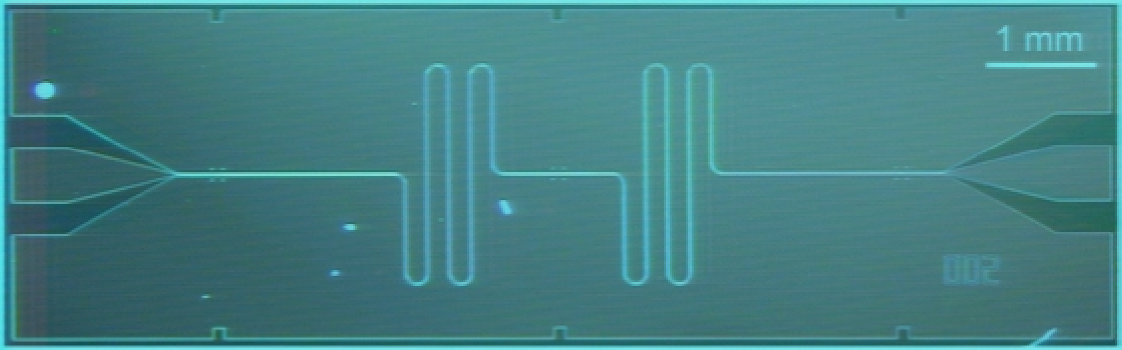

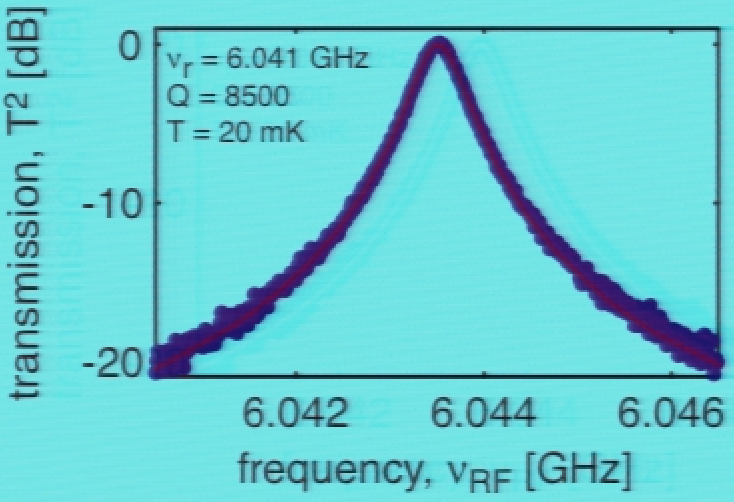

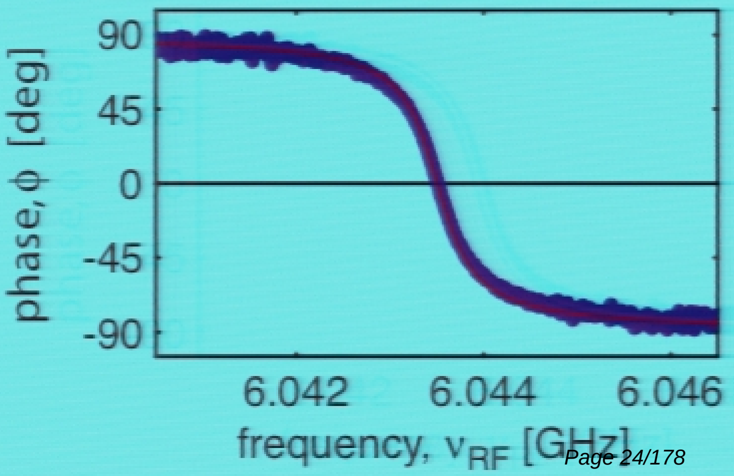

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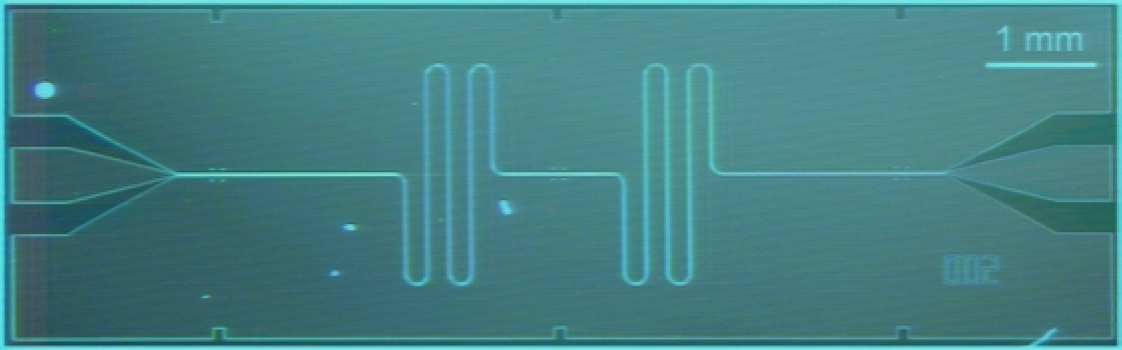

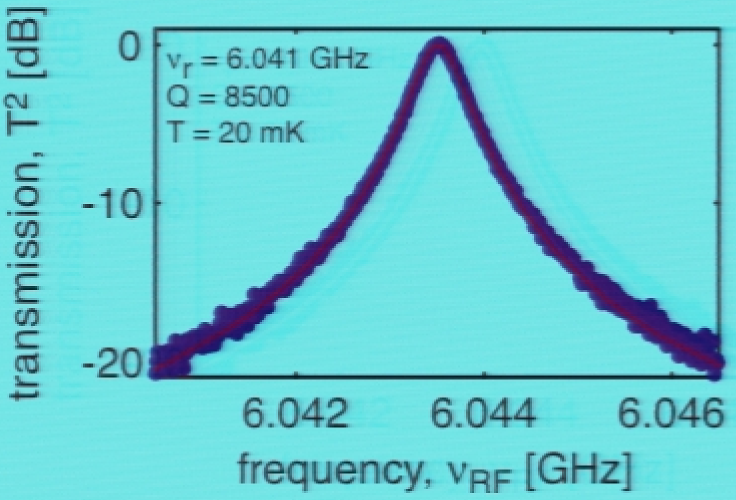

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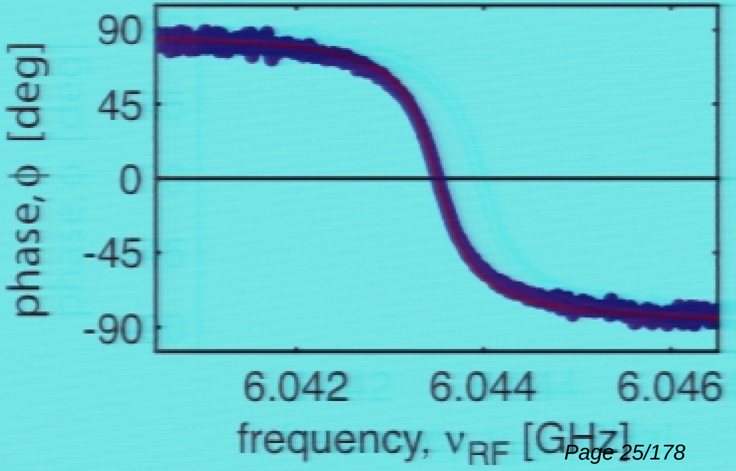

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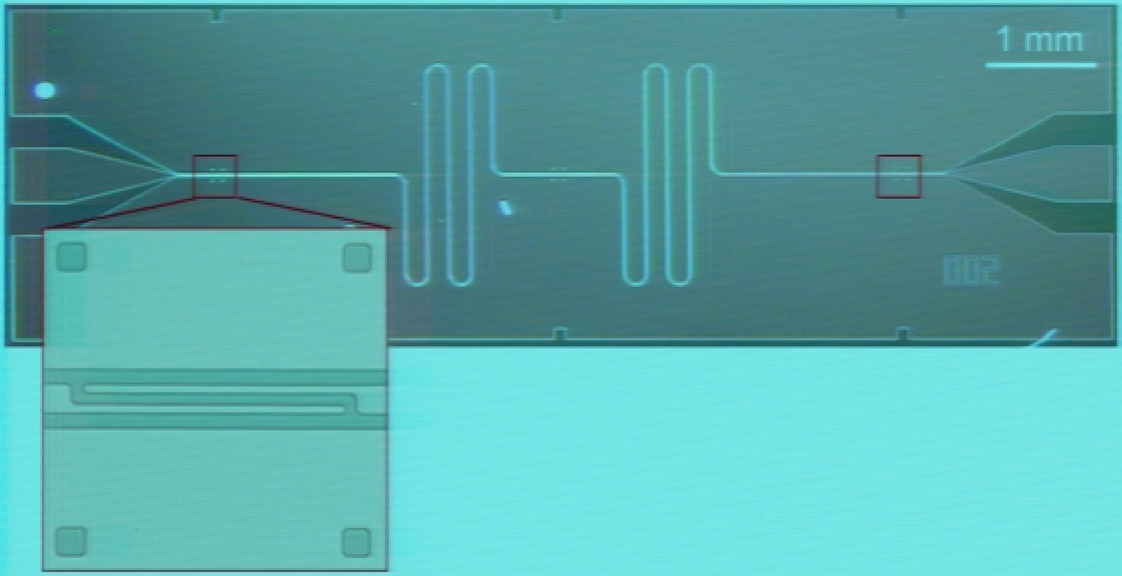

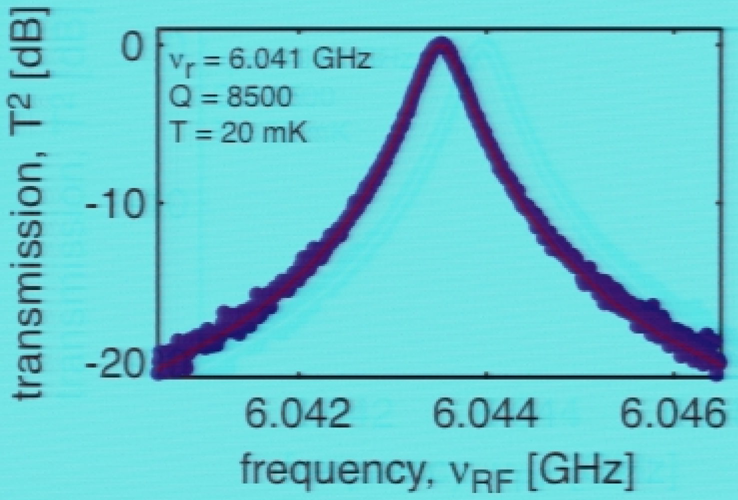

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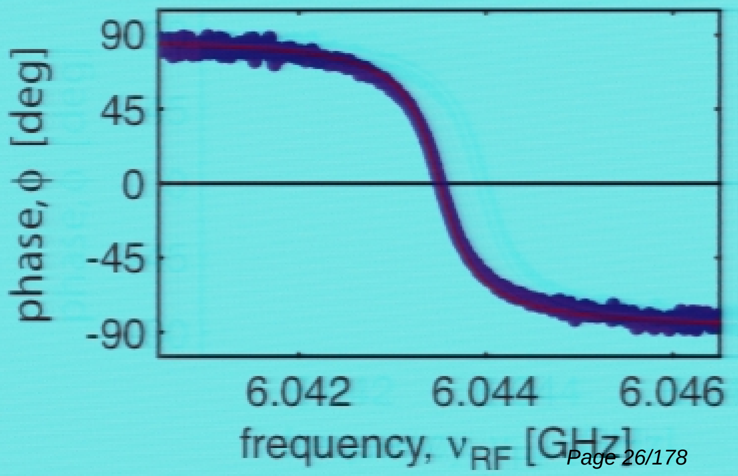

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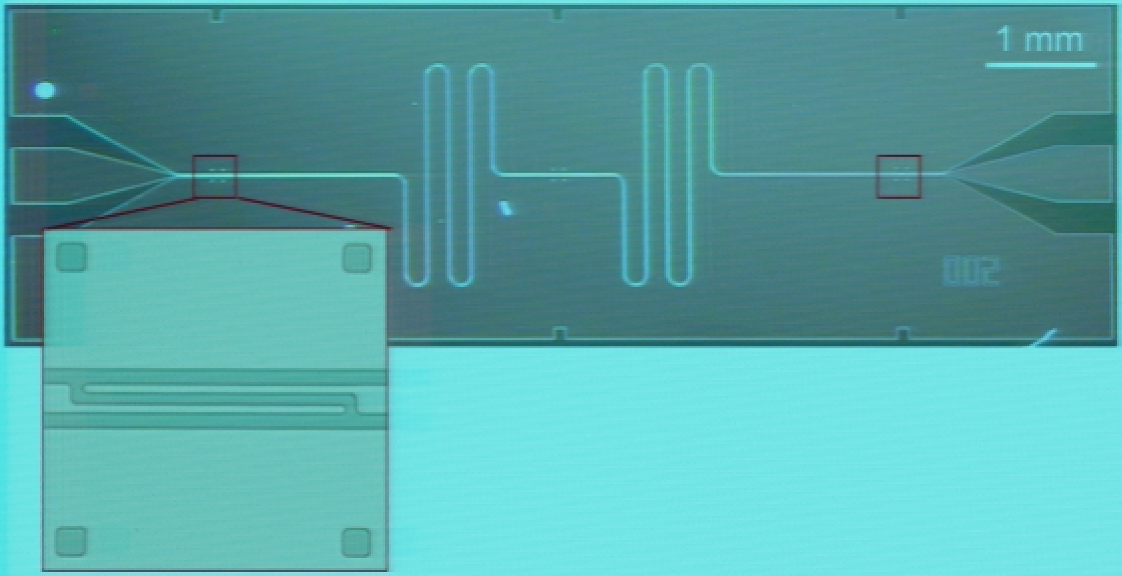

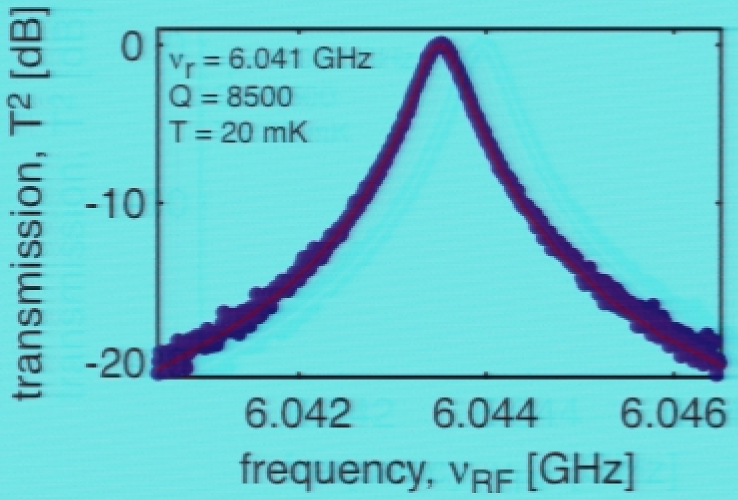

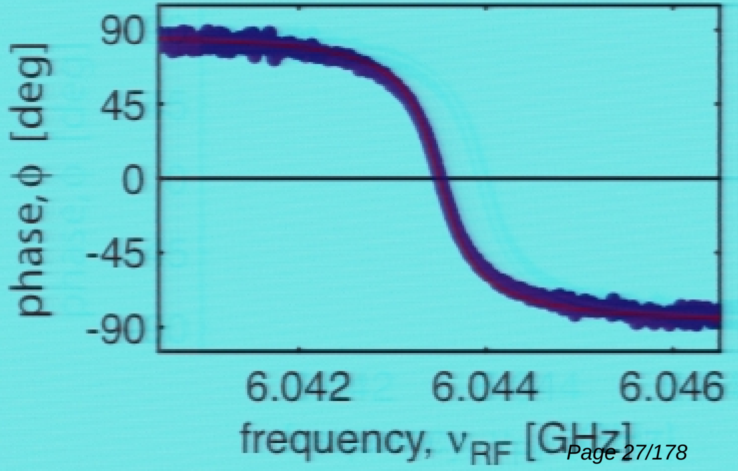

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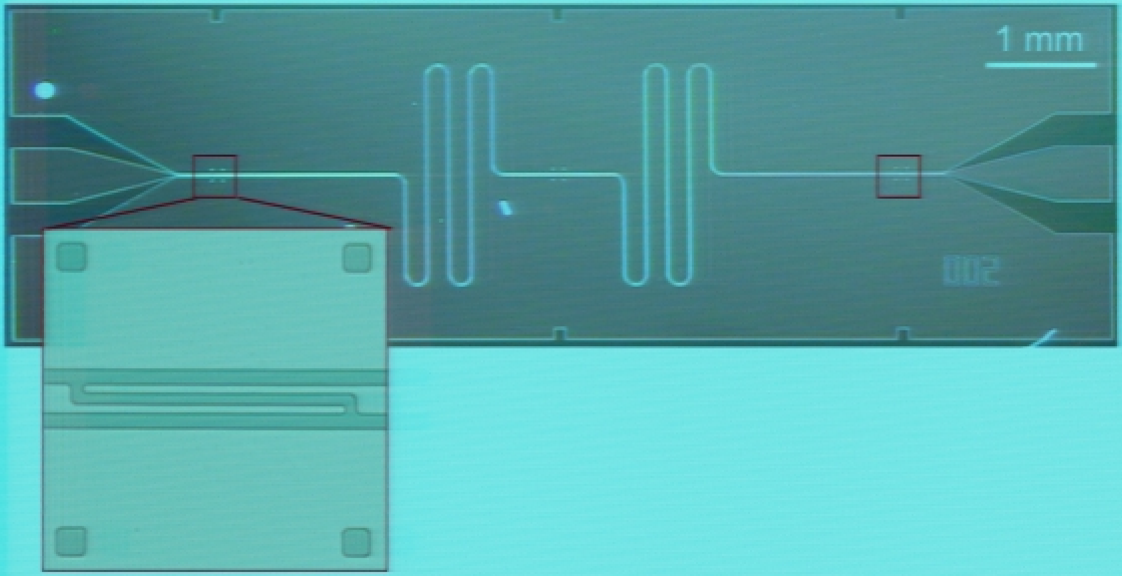

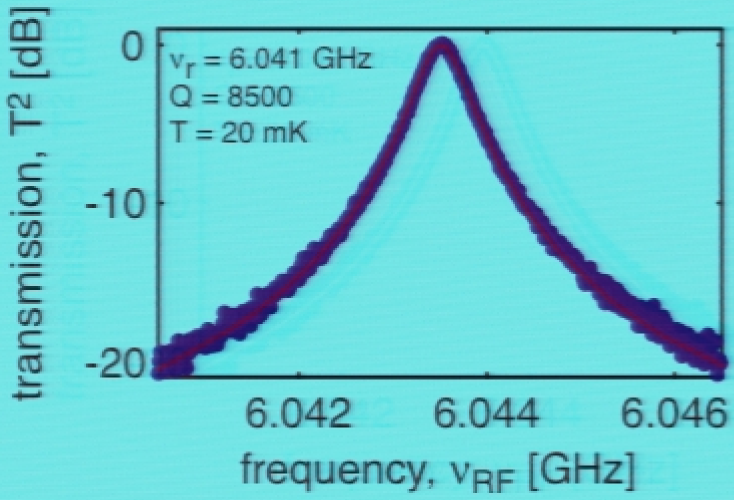

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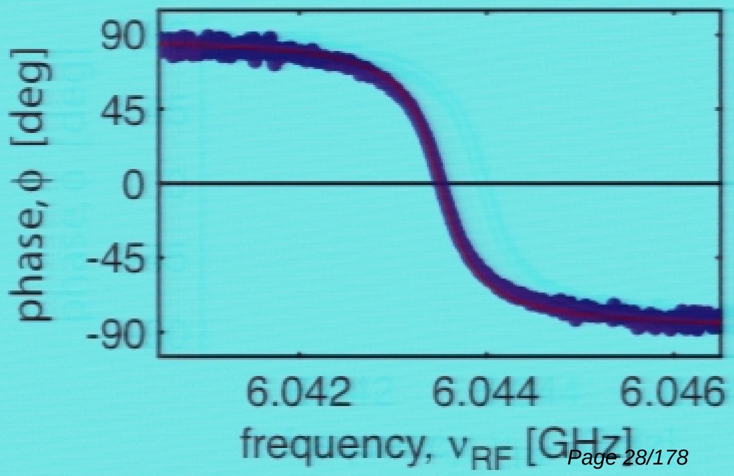

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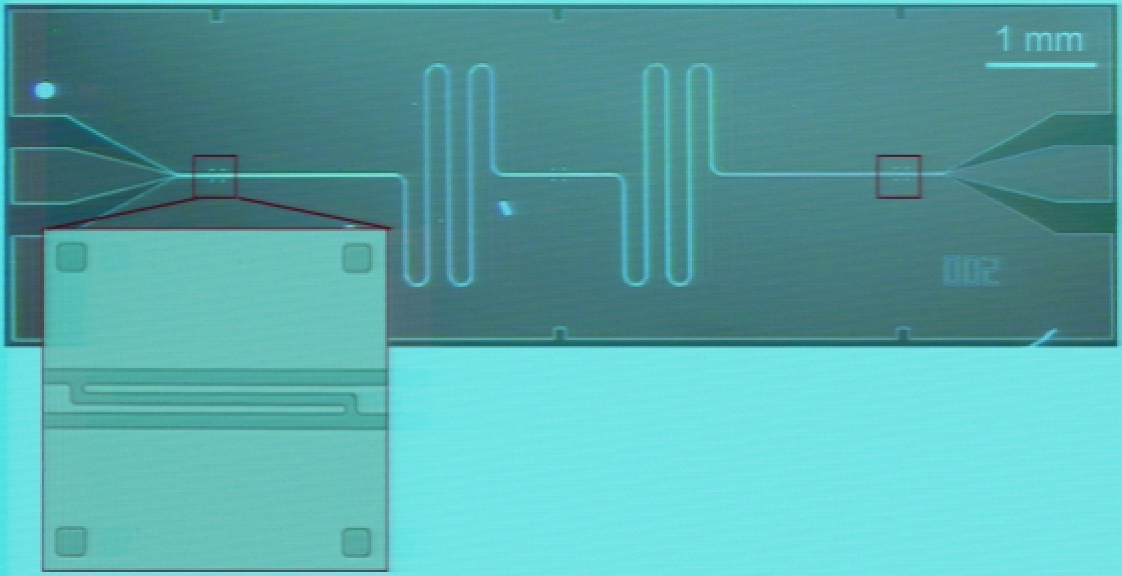

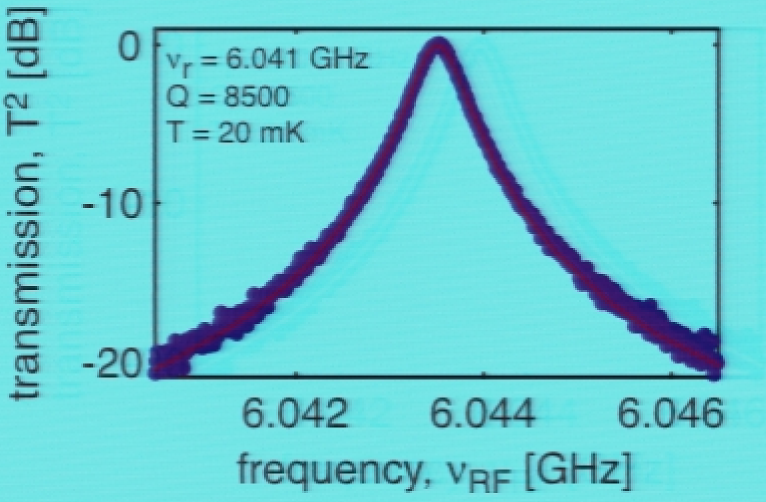

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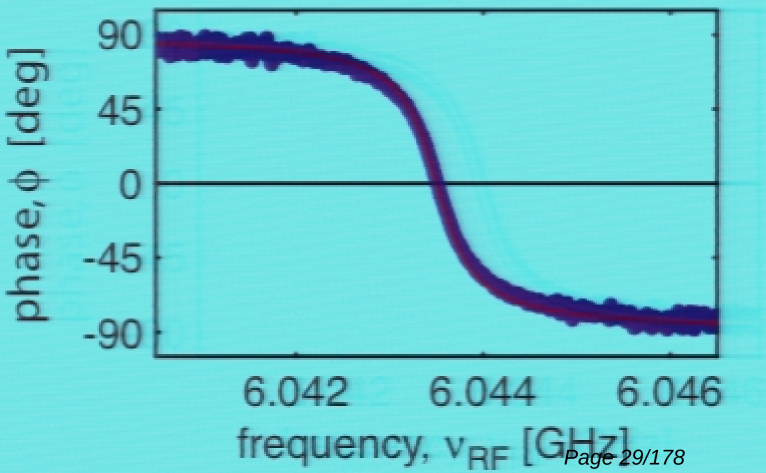

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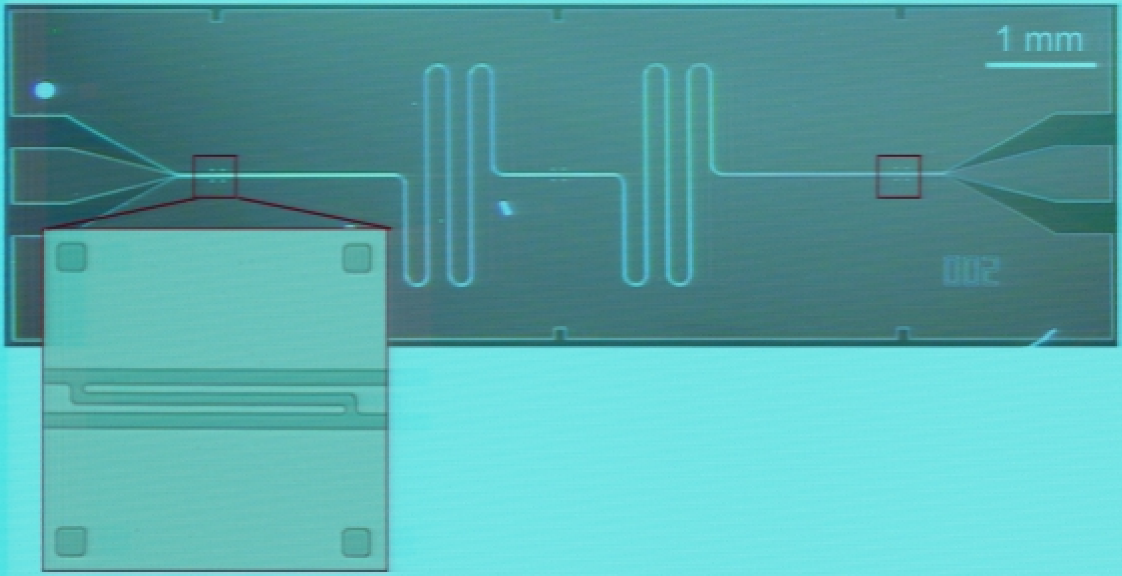

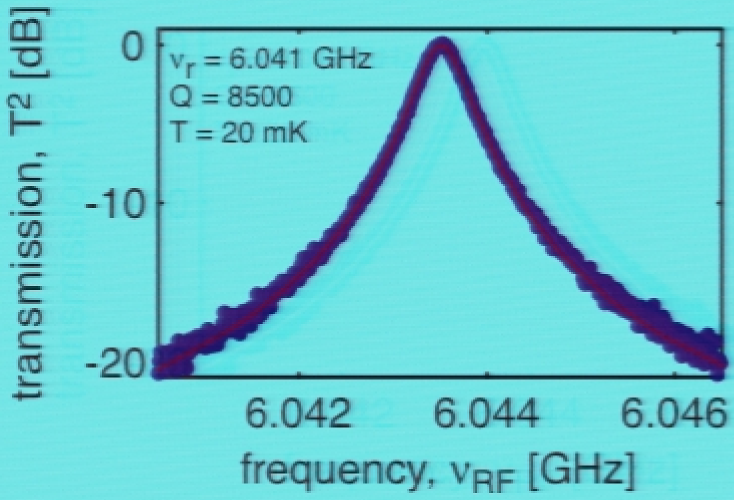

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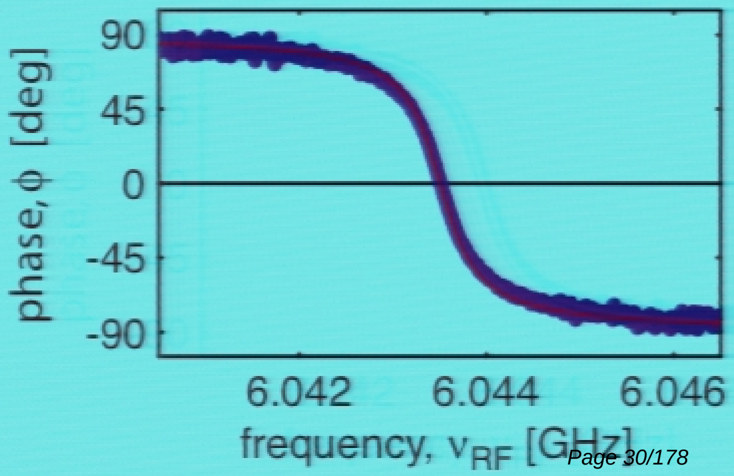
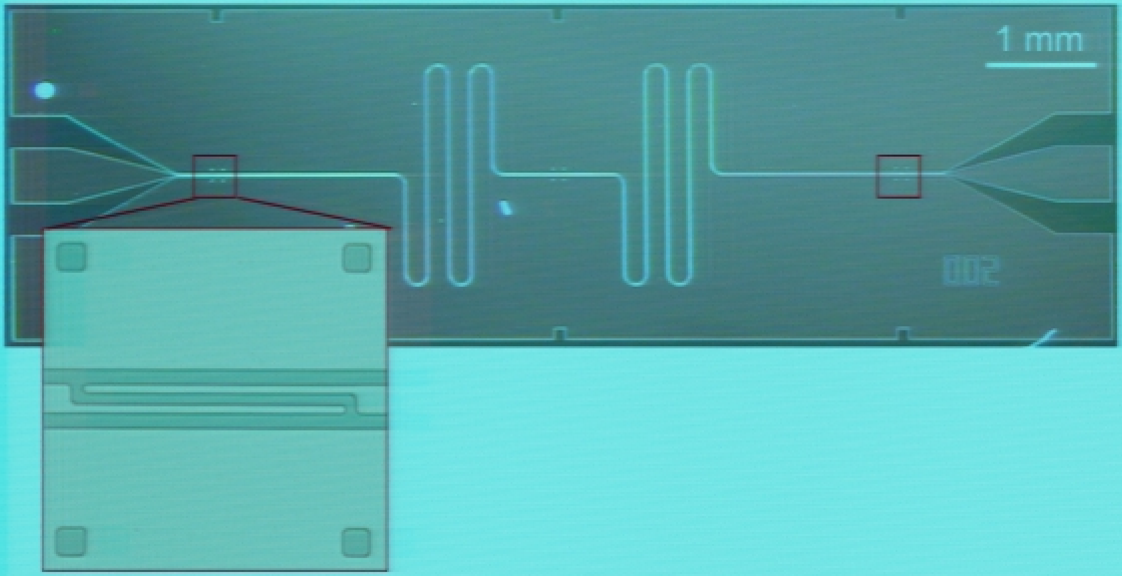


Cavity: superconducting 1D waveguide resonator



$$\kappa/2\pi = \nu_r/Q \approx 0.7\,\mathrm{MHz}$$

$$T_\kappa = 1/\kappa \approx 224\,\mathrm{ns}$$

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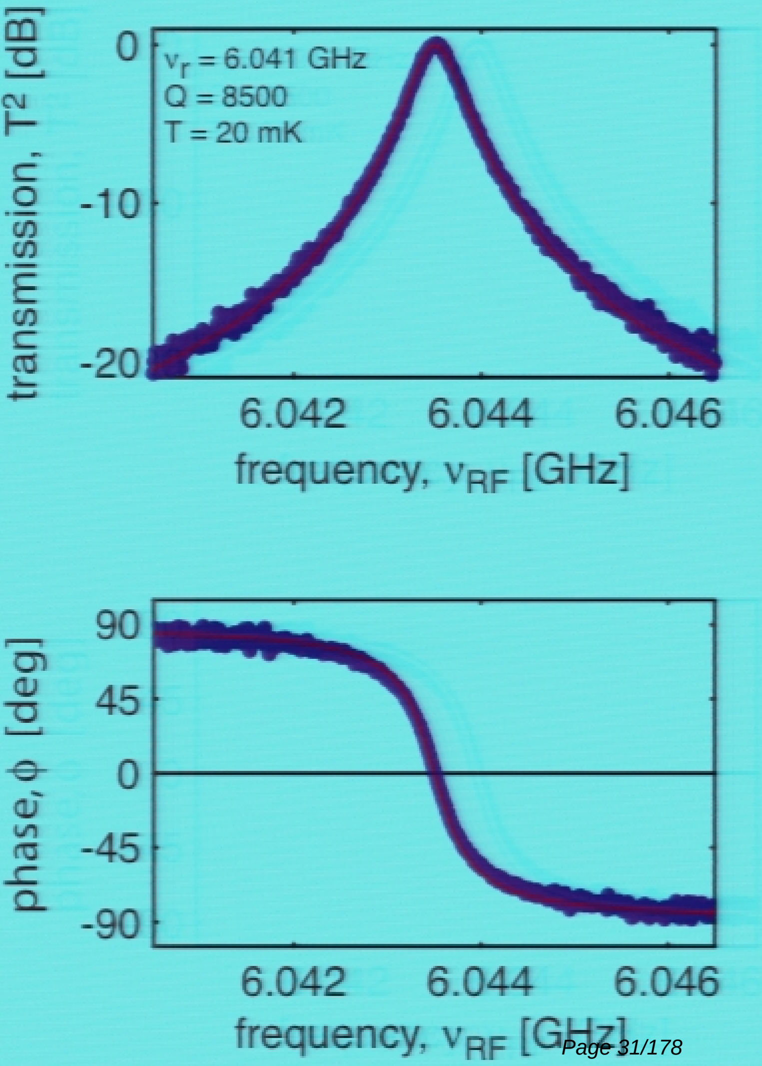

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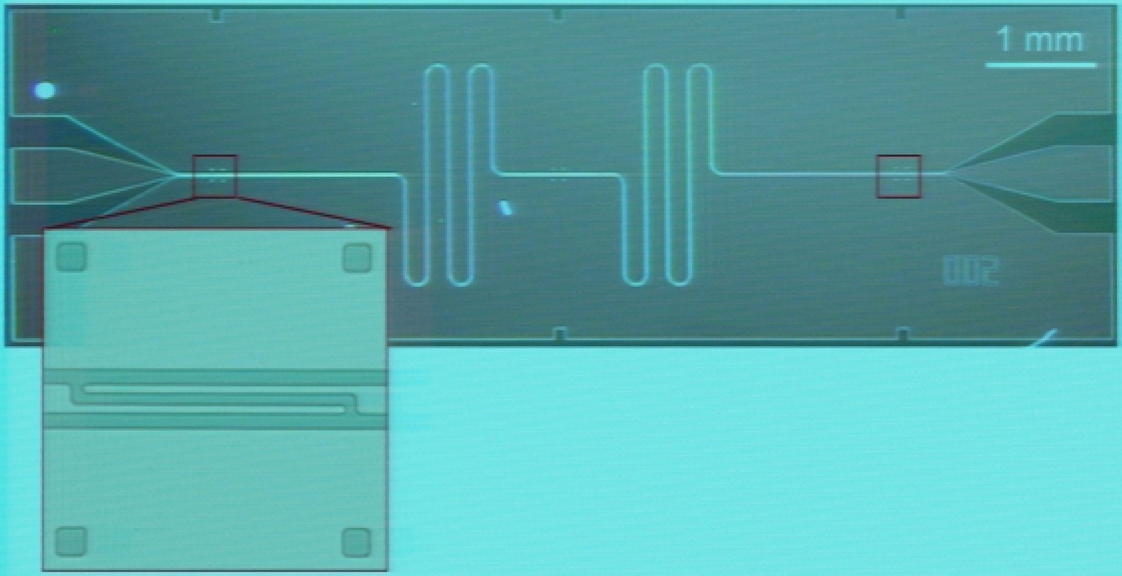

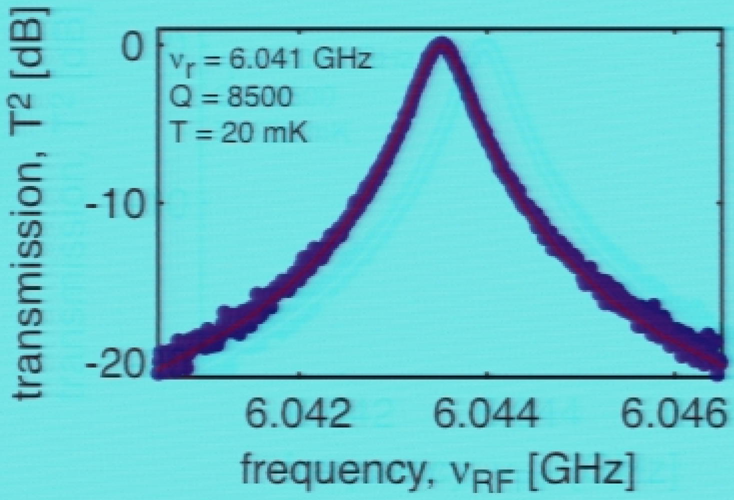

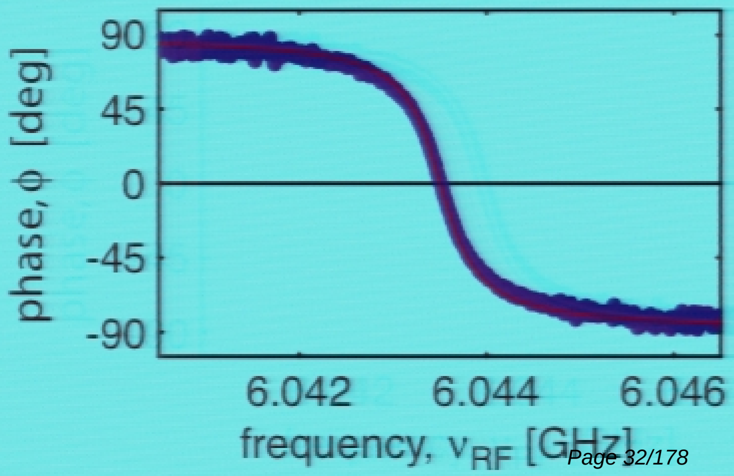

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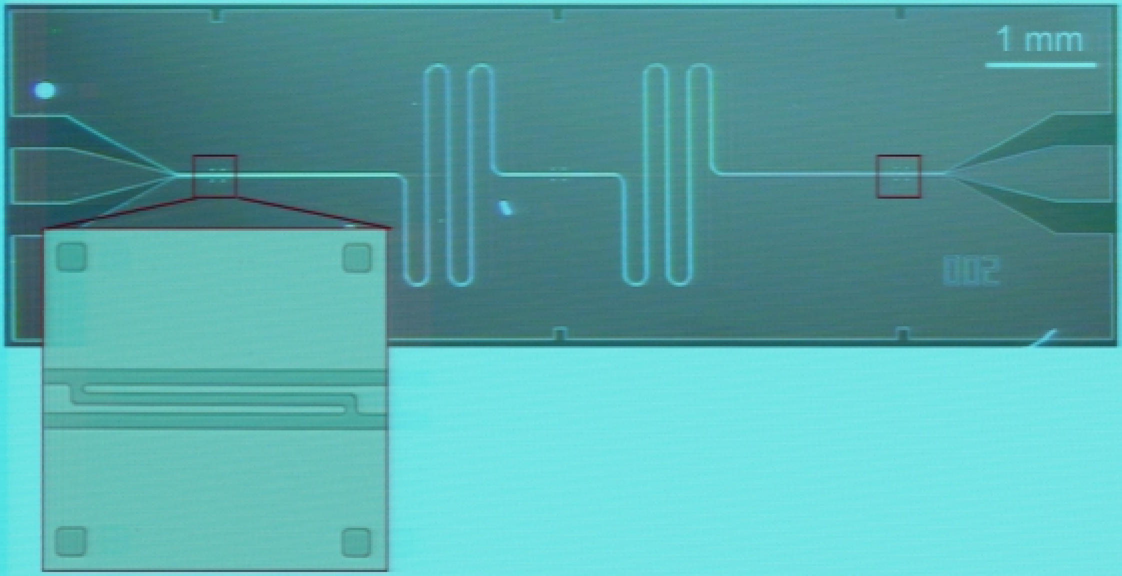

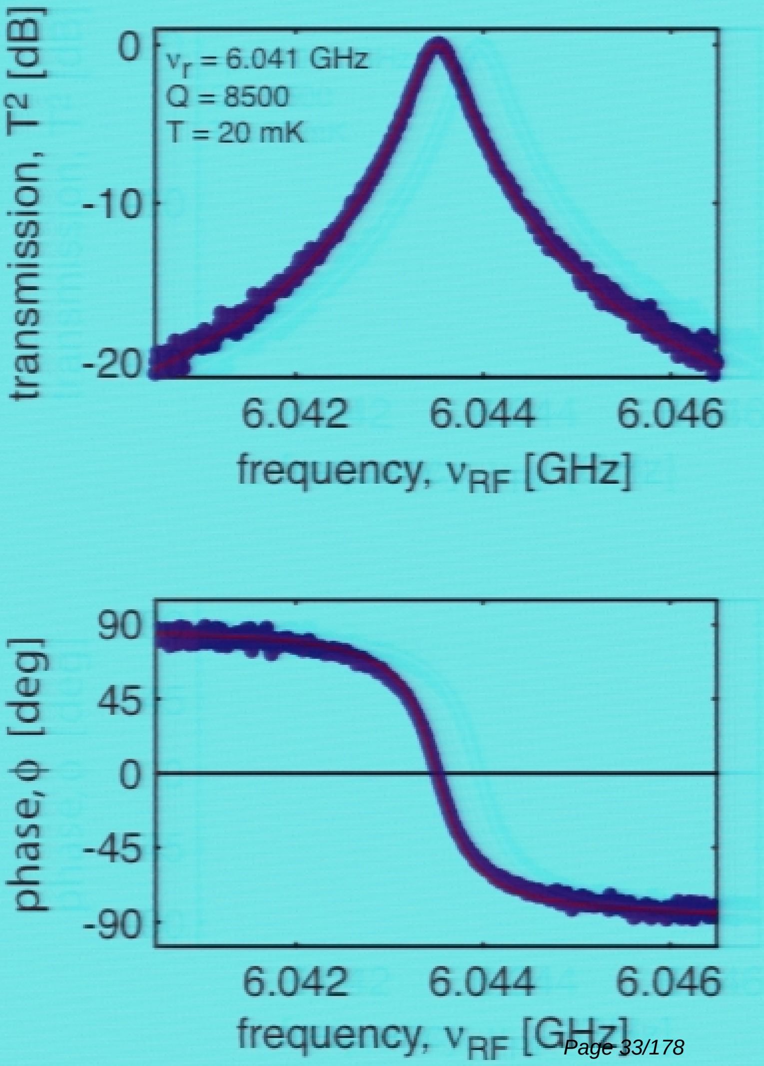

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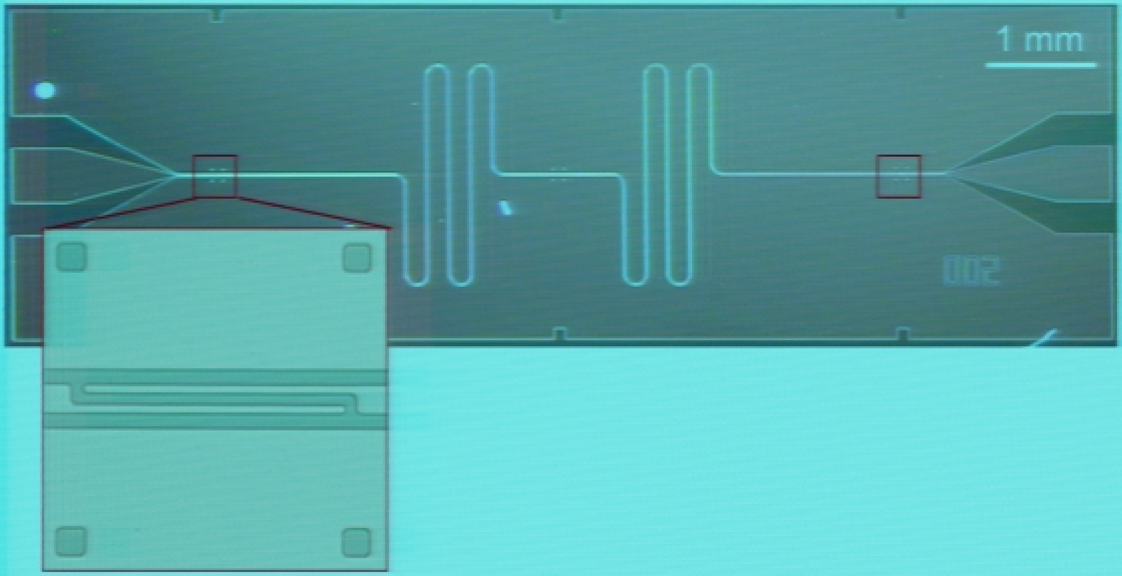

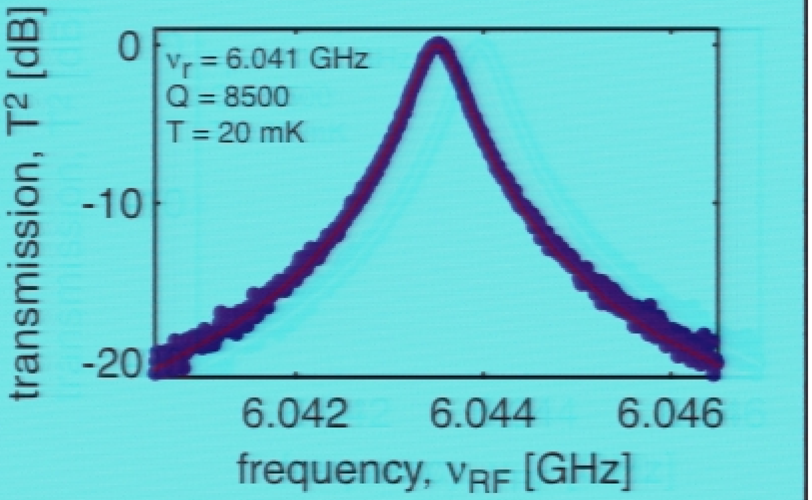

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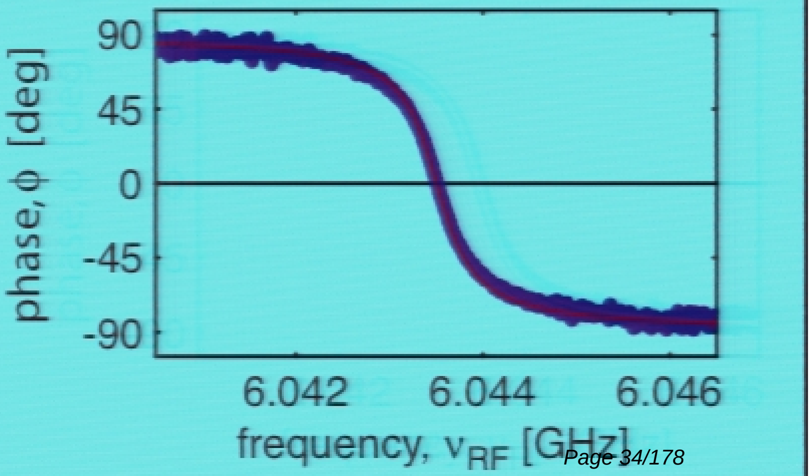

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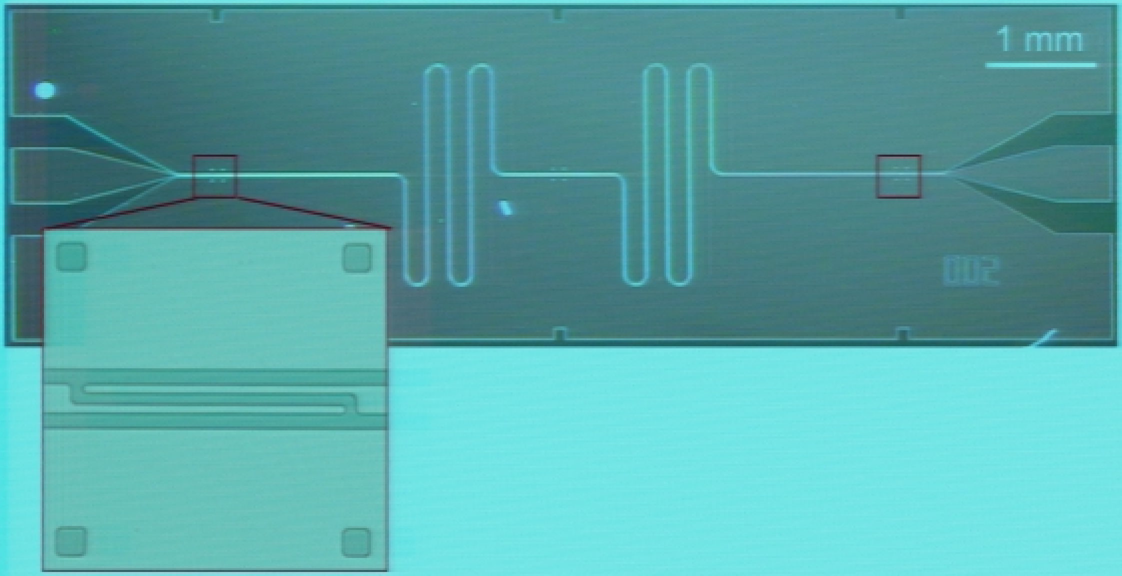

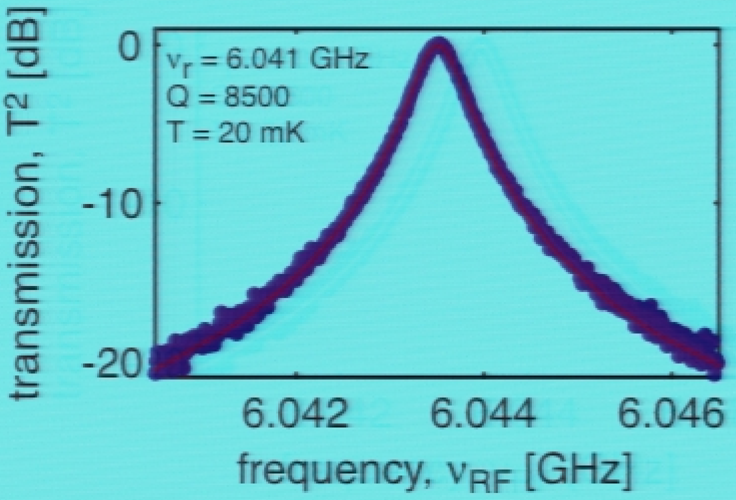

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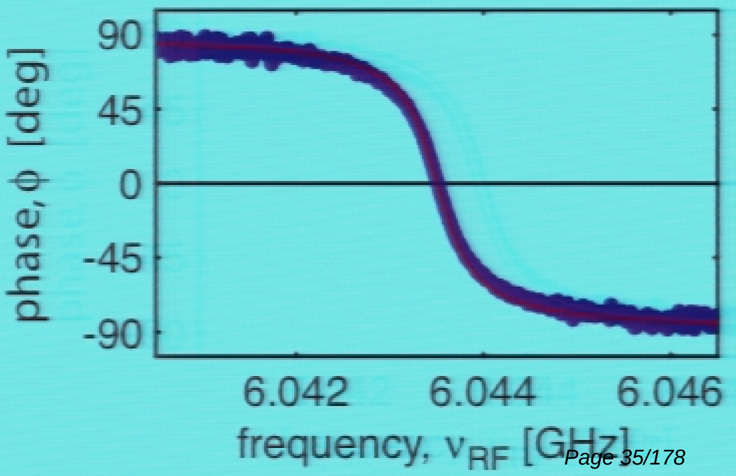

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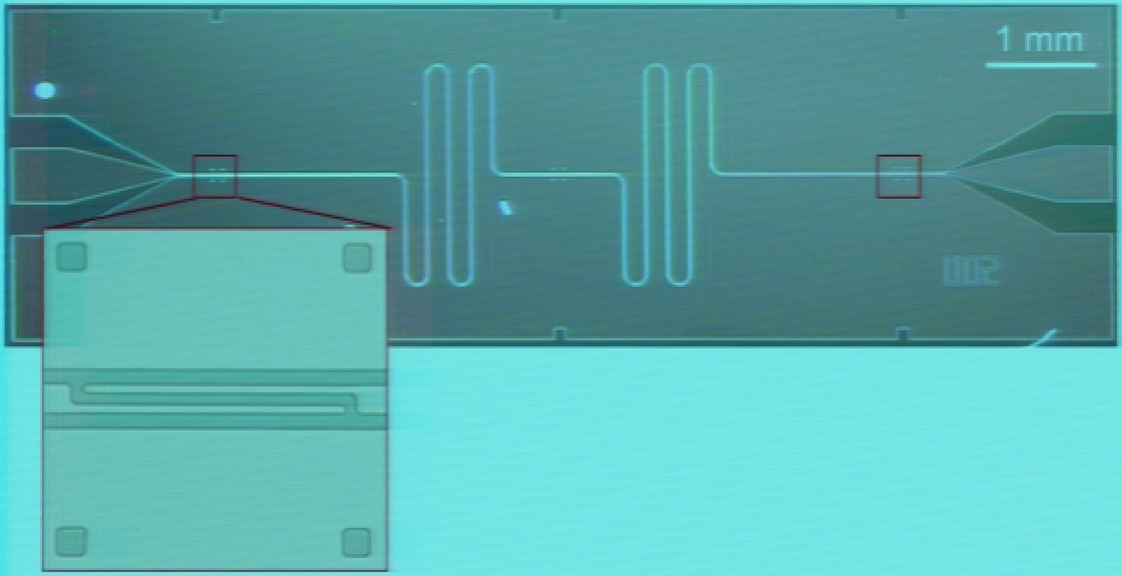

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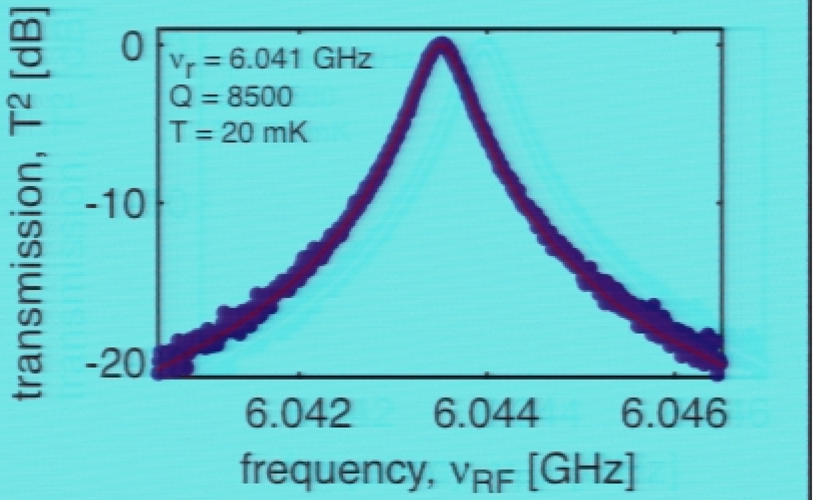

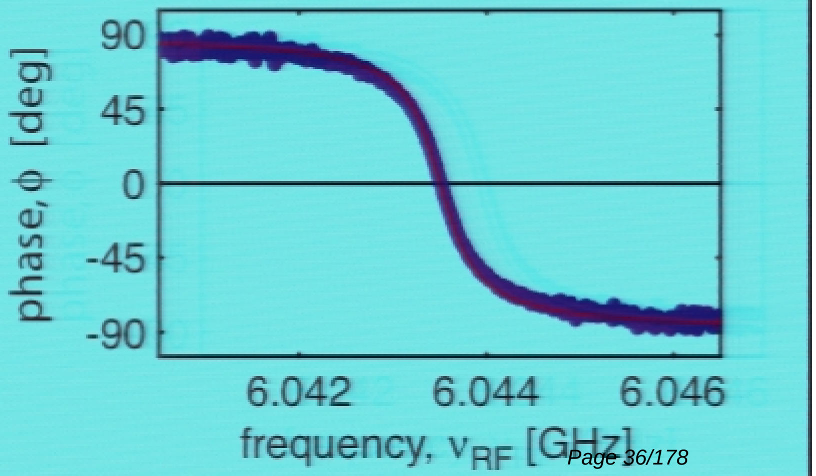

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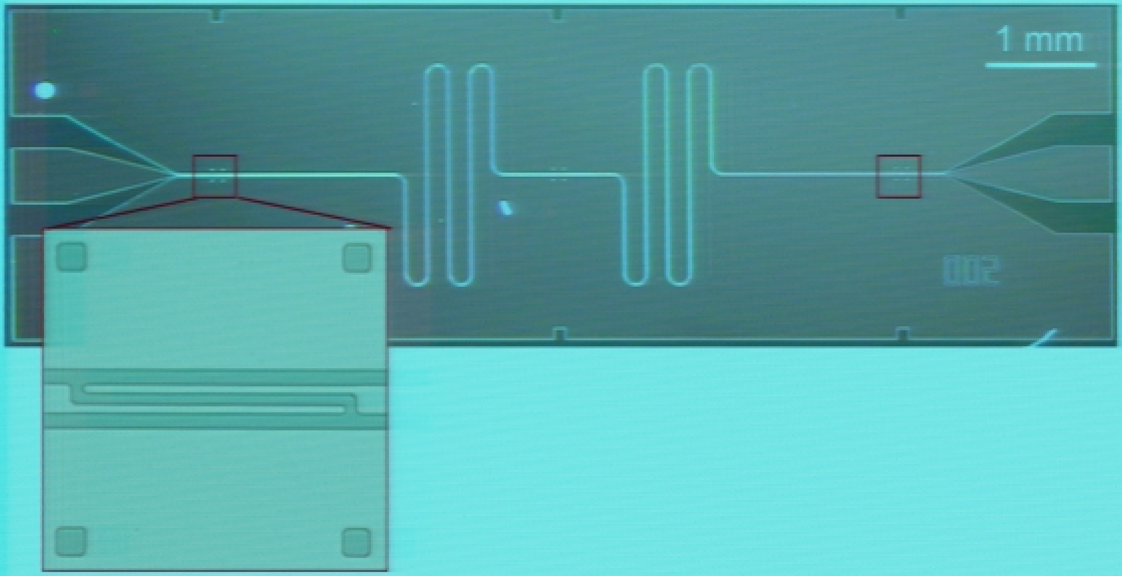

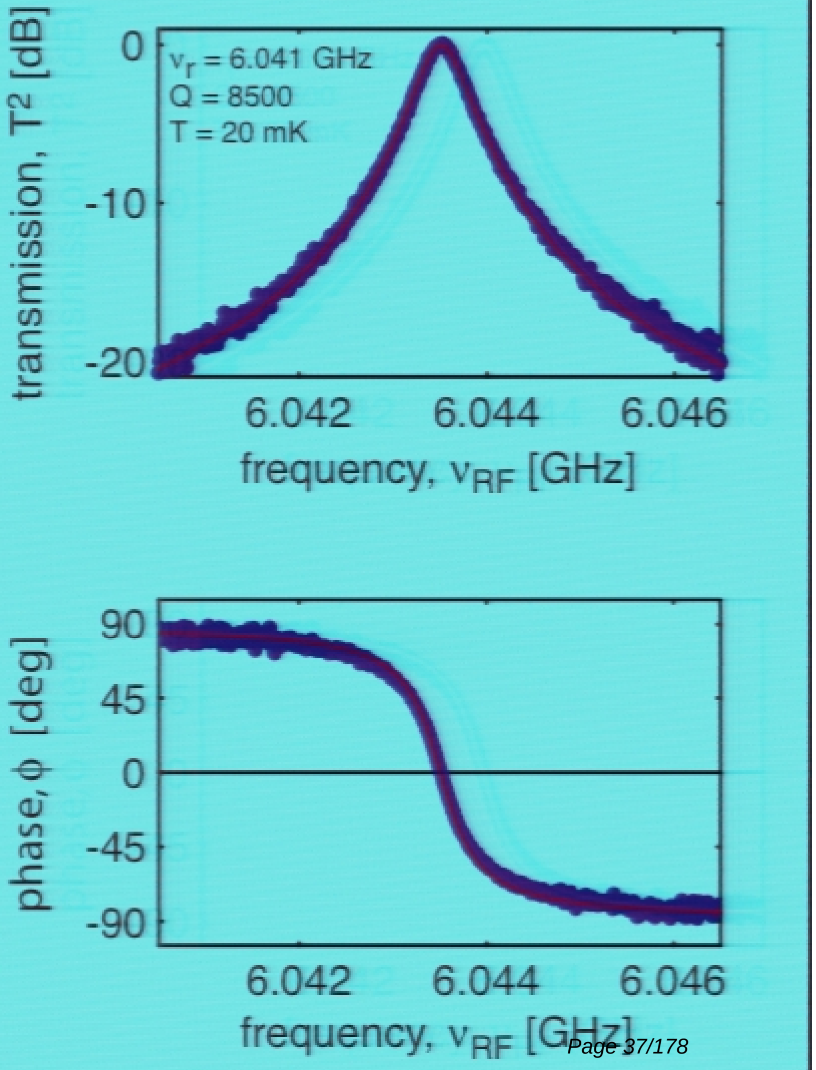

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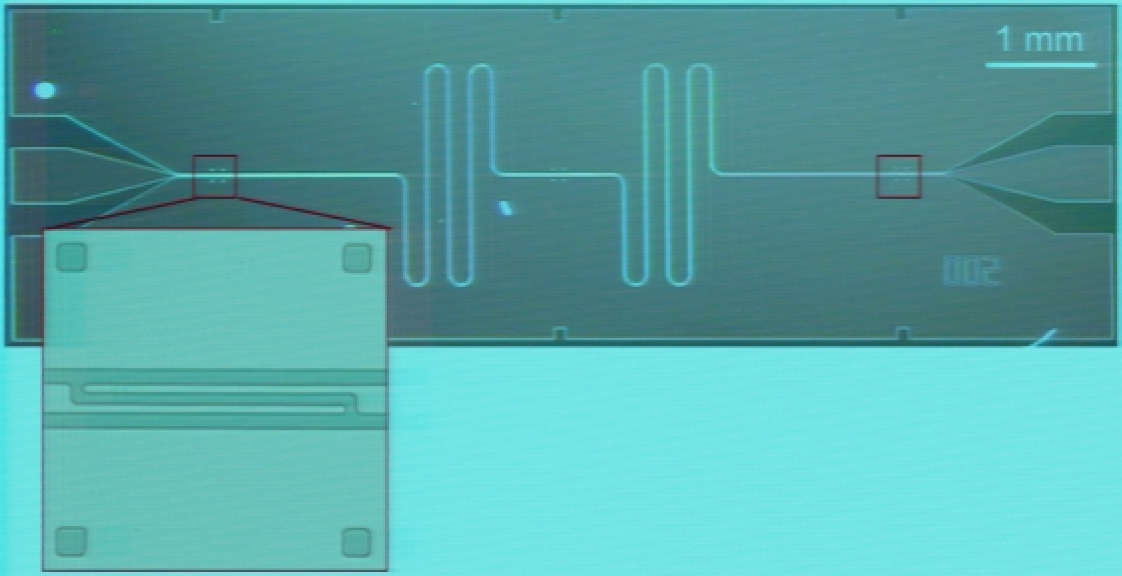

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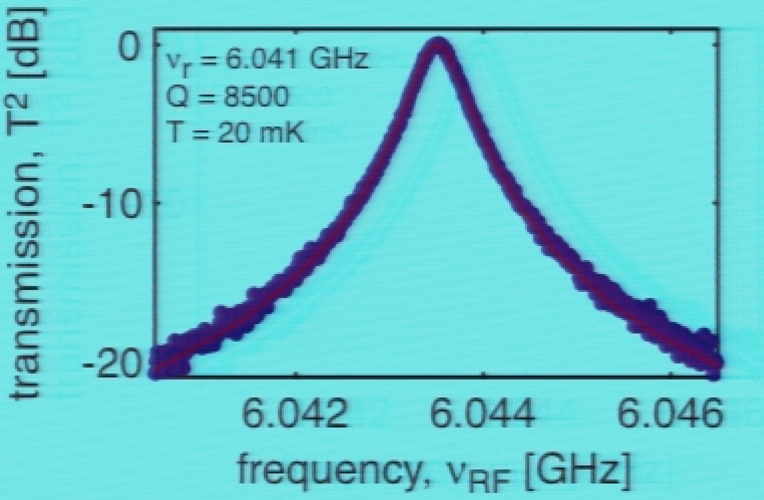

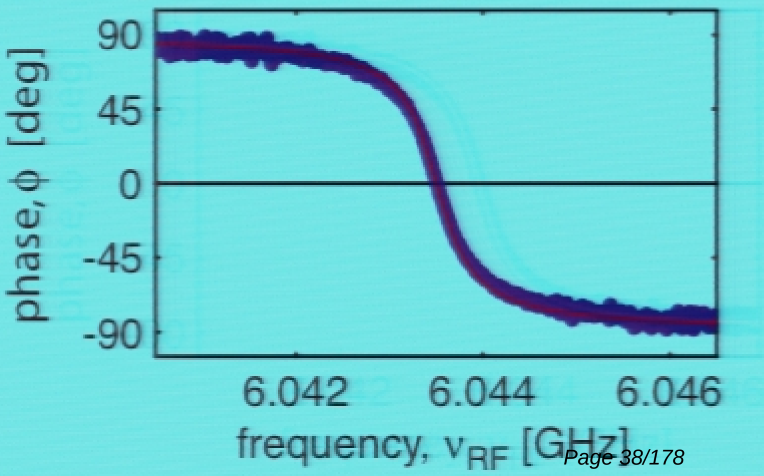

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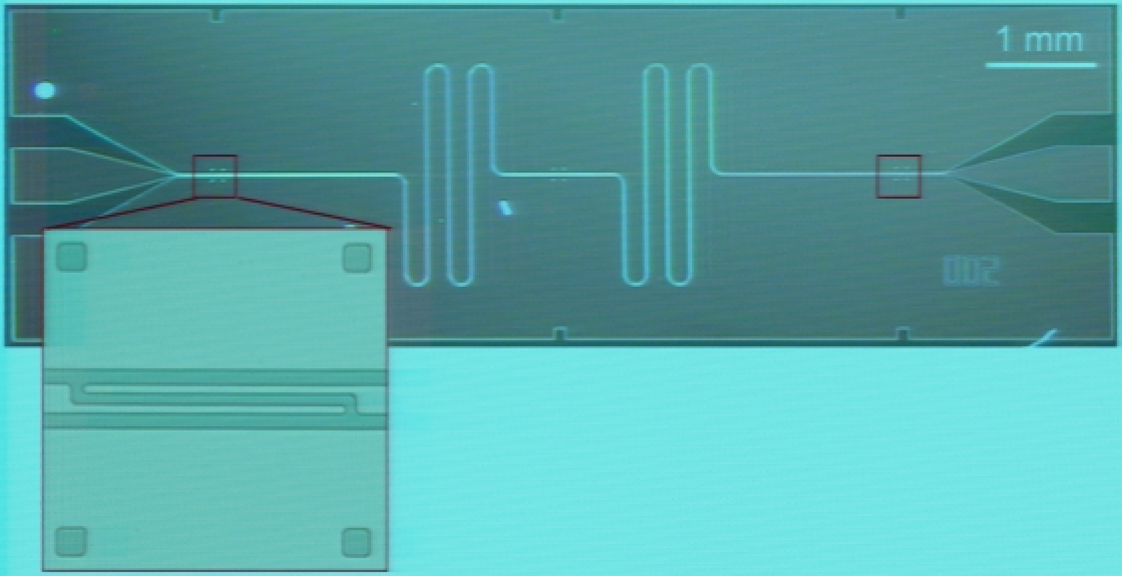

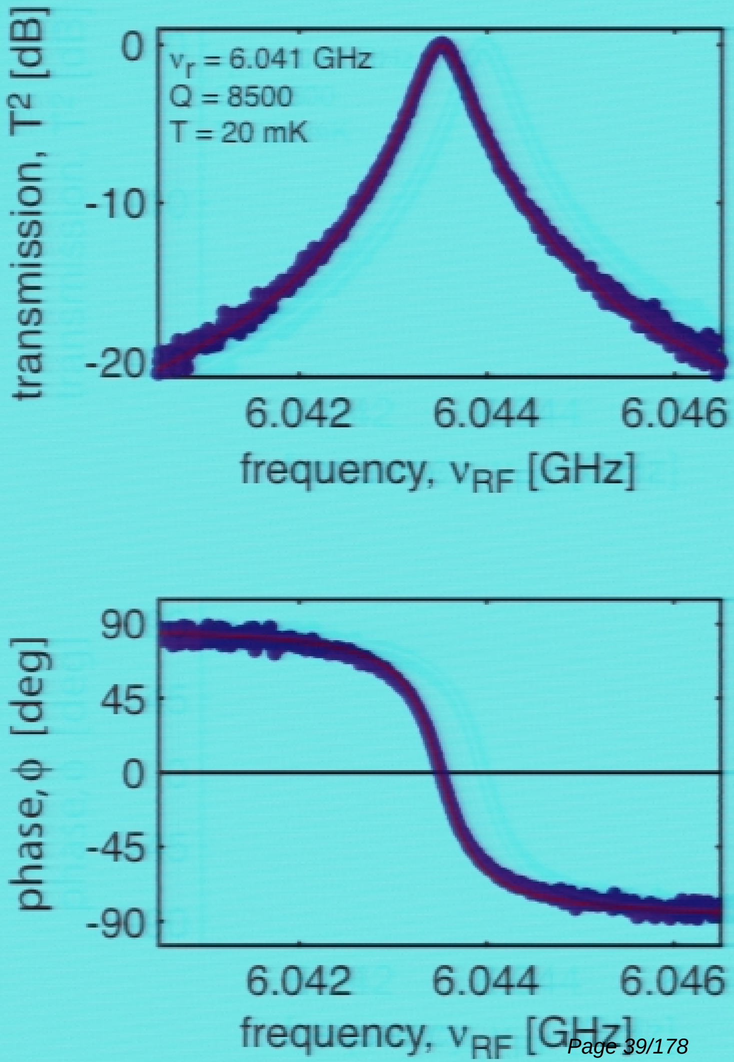

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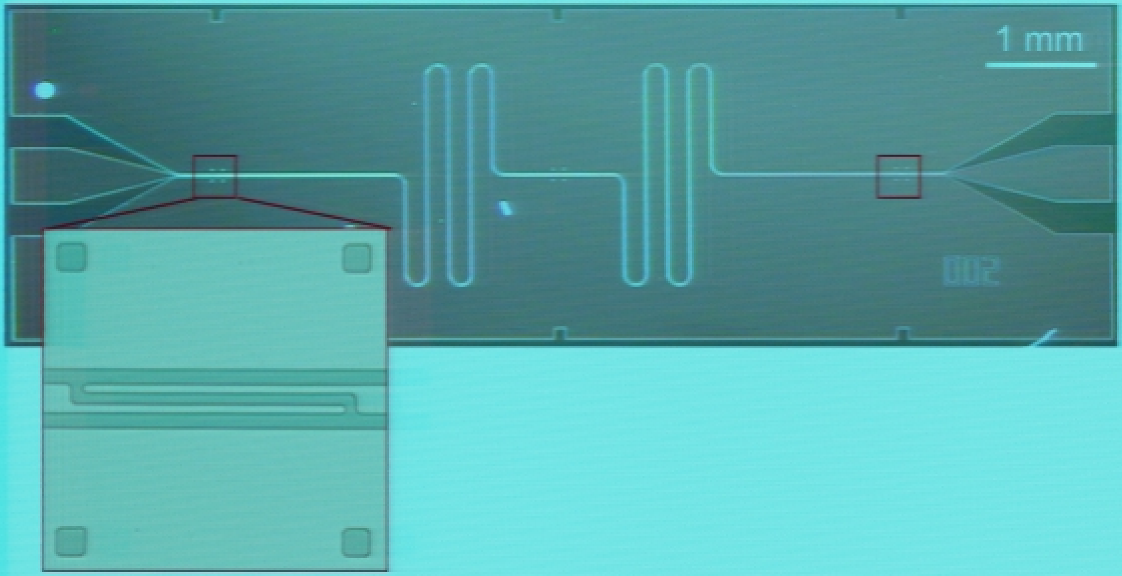

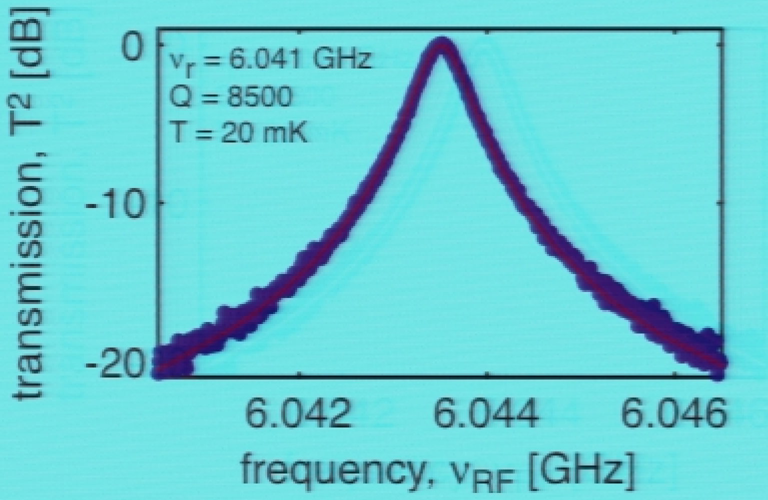

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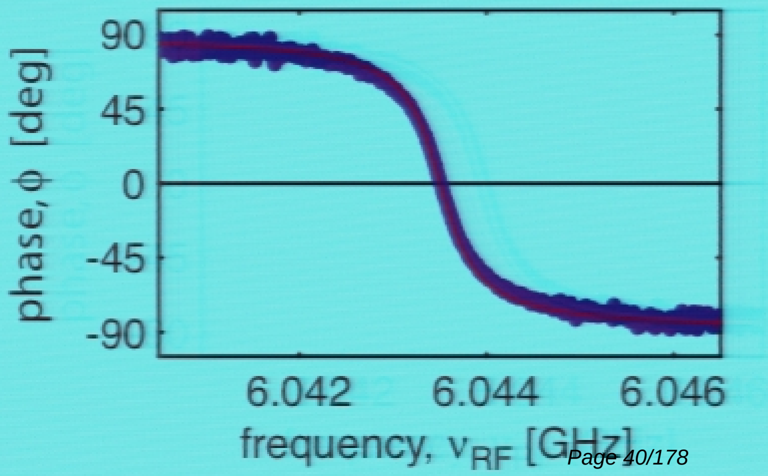

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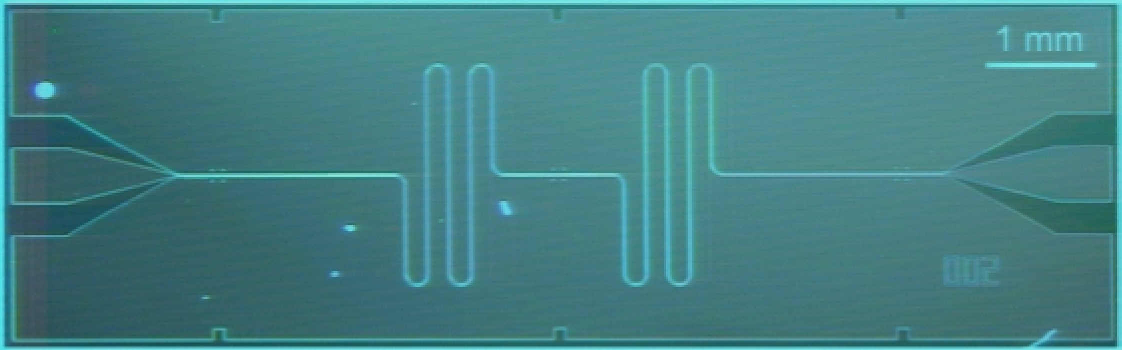

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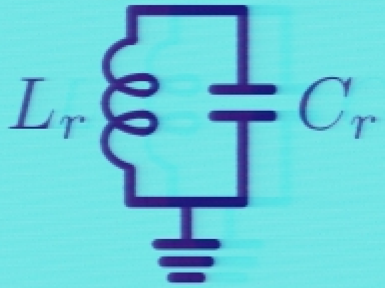

$$V^0_{\mathrm{RMS}} = \sqrt{\frac{\hbar\omega_r}{2C_r}} \sim 1\,\mu\mathrm{V}$$

$$H = \hbar\omega_r \left(a^\dagger a + \frac{1}{2} \right)$$

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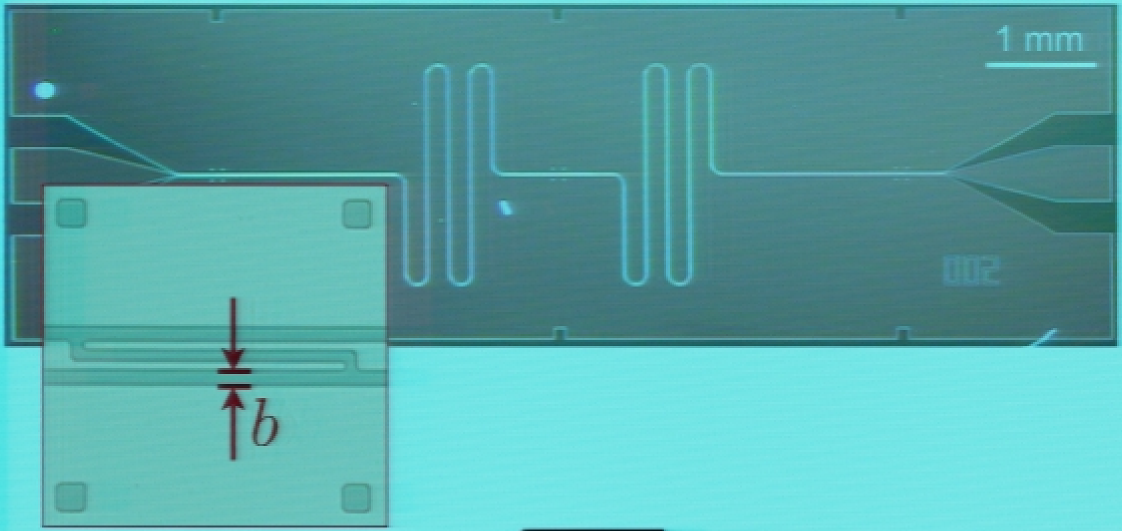

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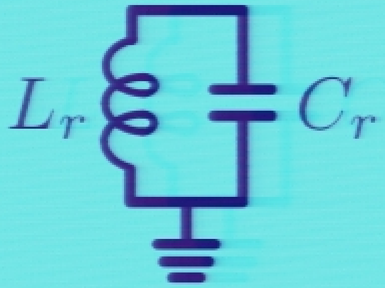

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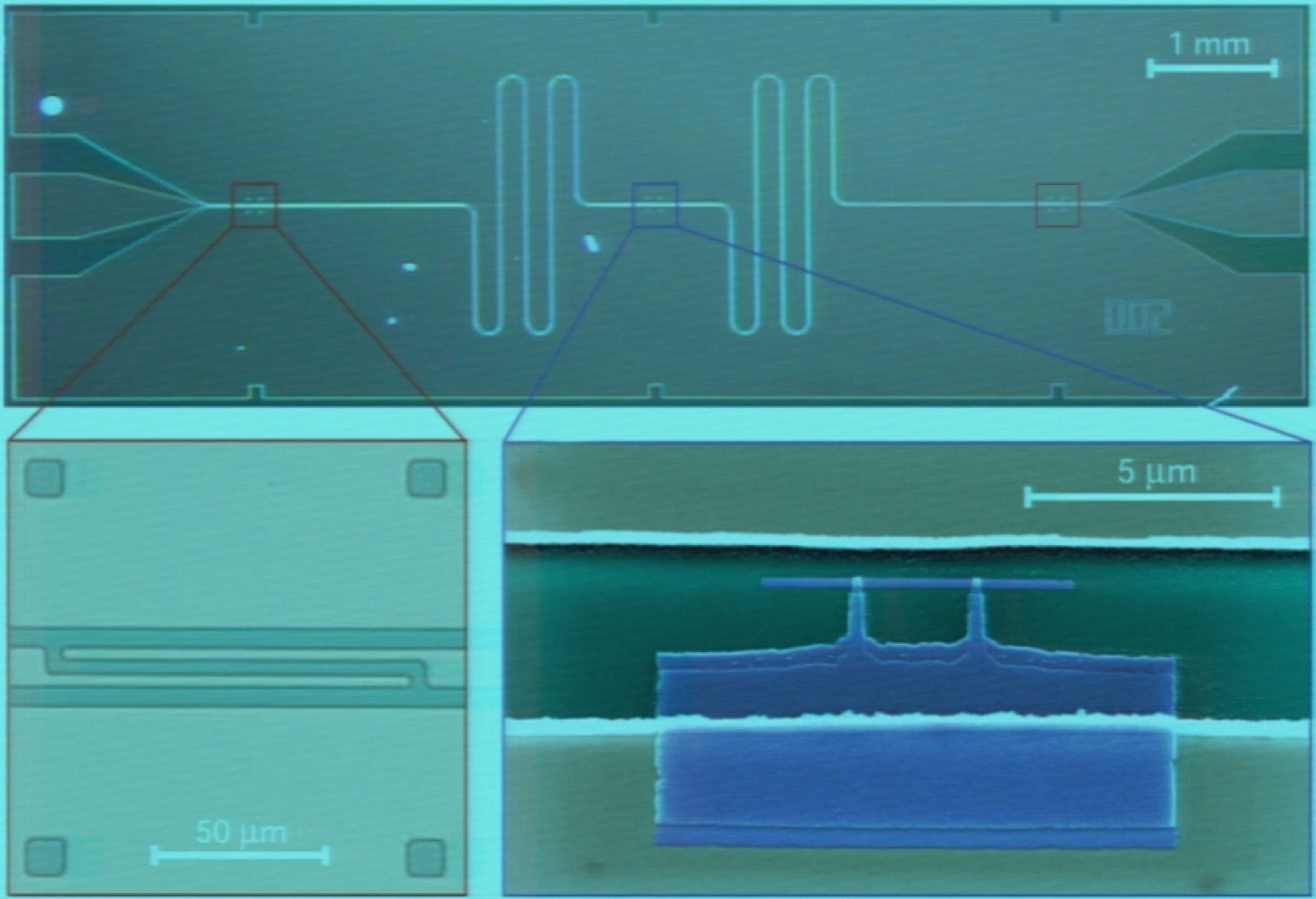
$$\rightarrow E_0 = \frac{V^0_{\mathrm{RMS}}}{b} \sim 0.2\ \mathrm{V/m}$$

100 X more intense than in 3D micro-wave cavities

$\omega_r/2\pi \approx 6$ GHz $(C_r \sim 1$ pF$)$, $b \approx 5\ \mu$m

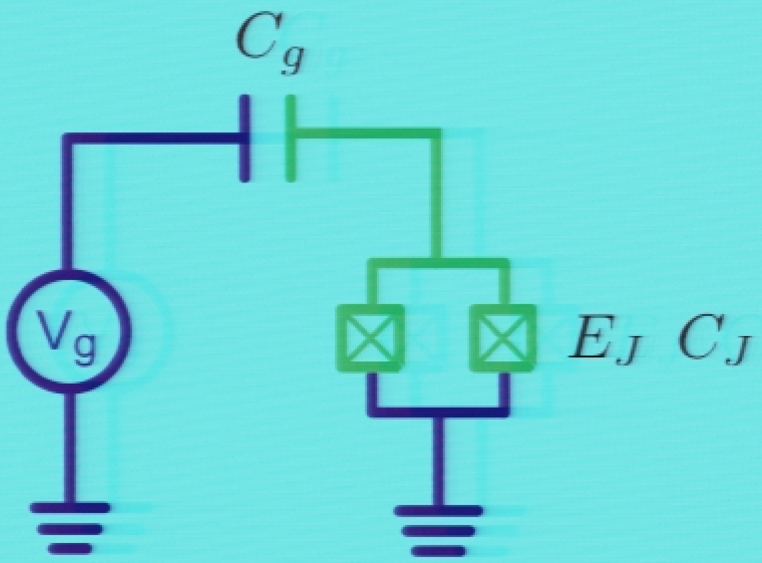
Circuit QED

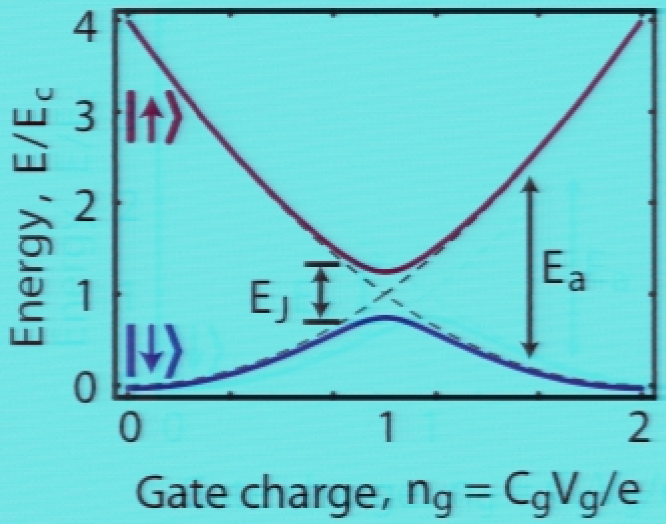
On-chip quantum optic



Wallraff, Schuster, AB, Frunzio, Huang, Majer, Kumar,

Artificial atom: Cooper-pair box





Bouchiat et al. Physica Scripta 176, 165 (1998)

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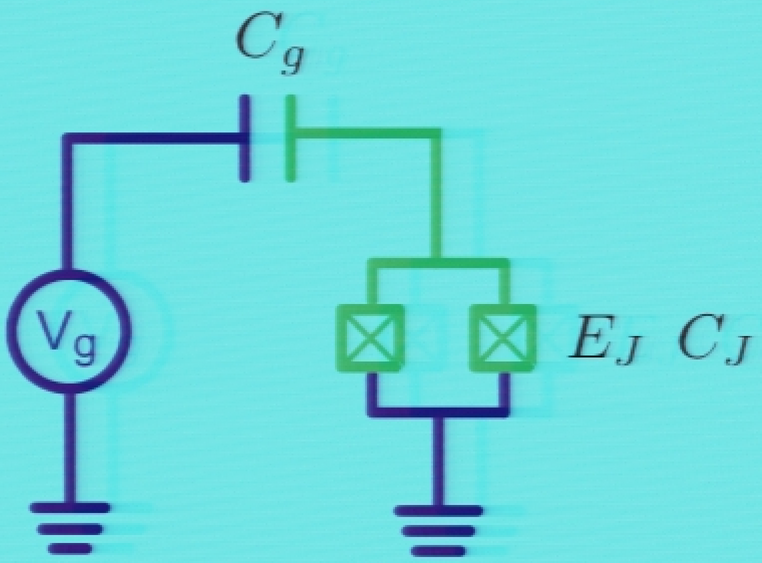

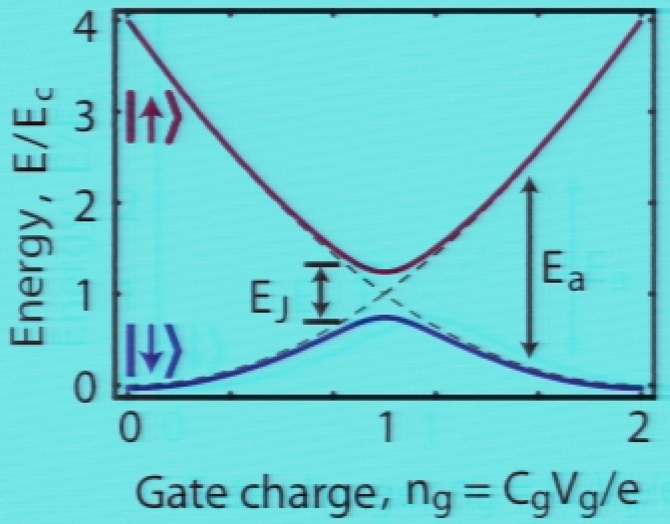

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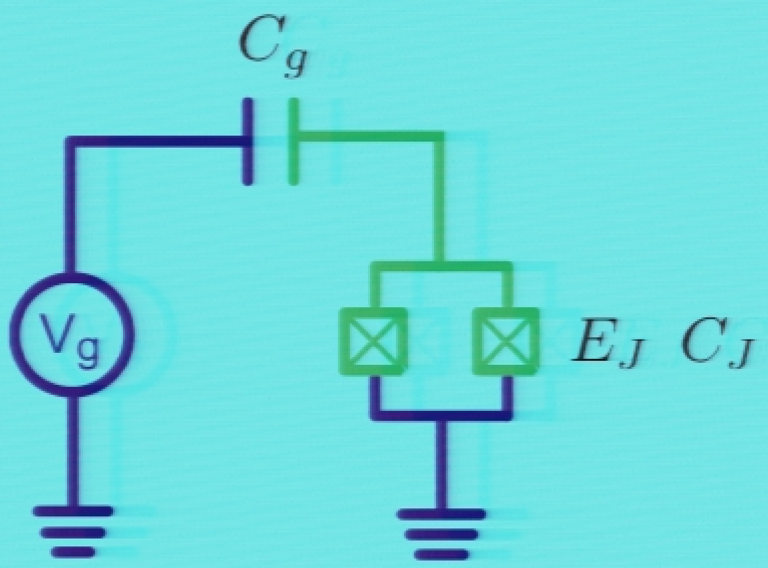

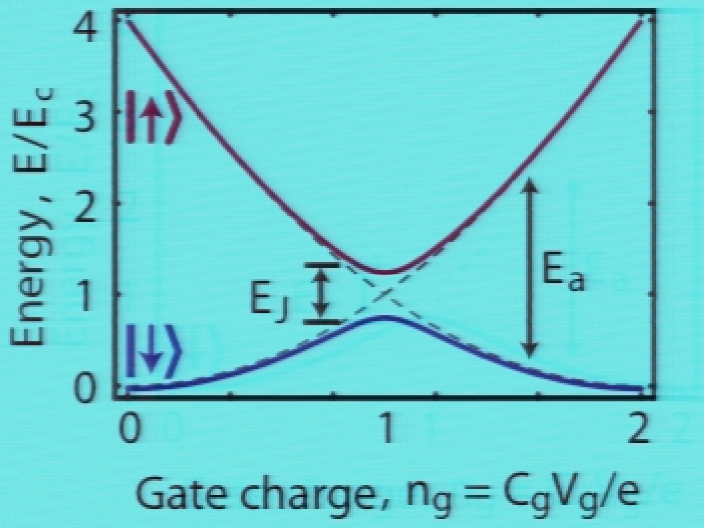

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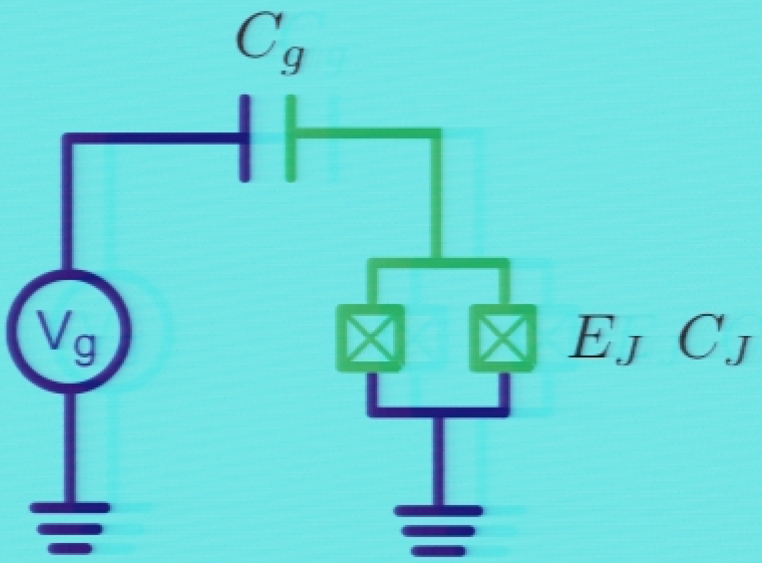

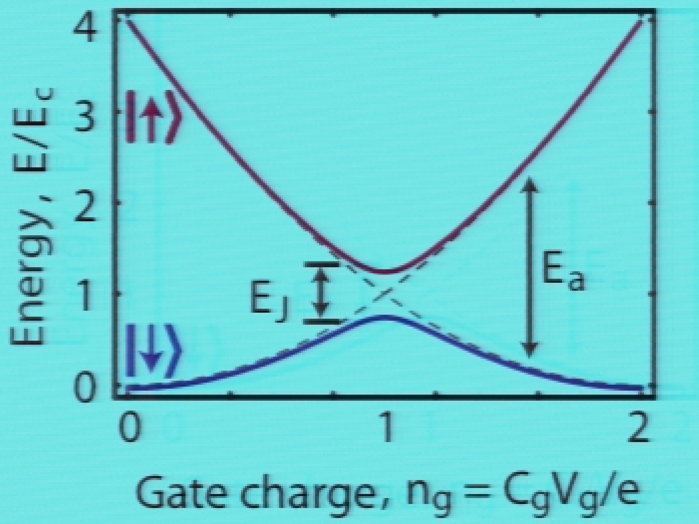

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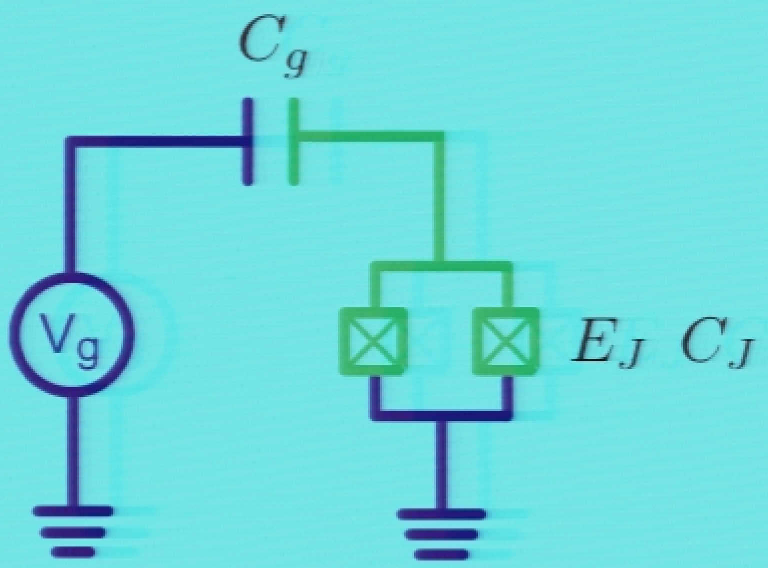

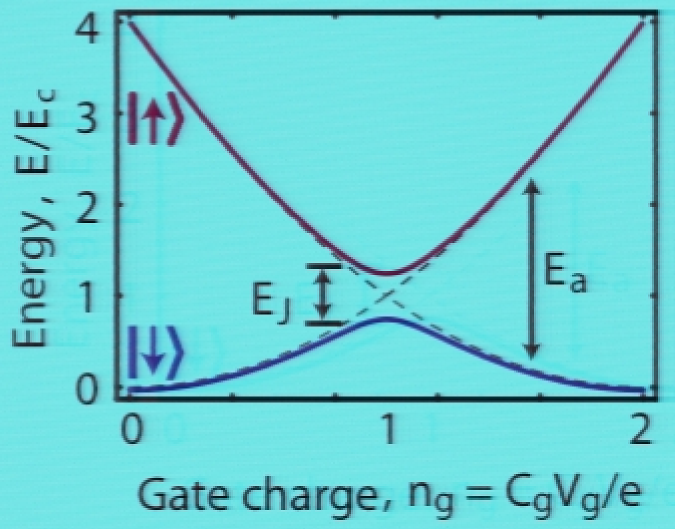

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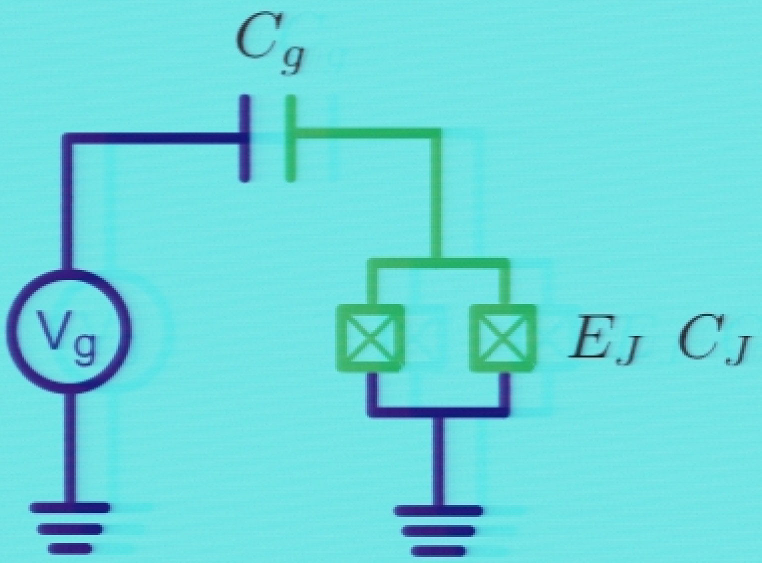

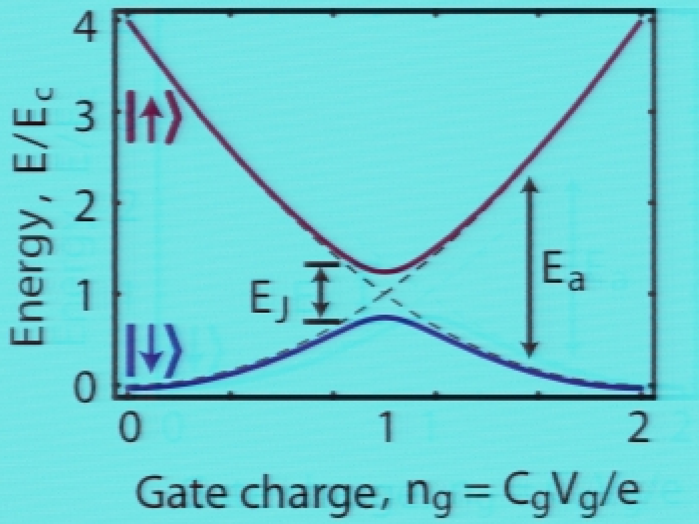

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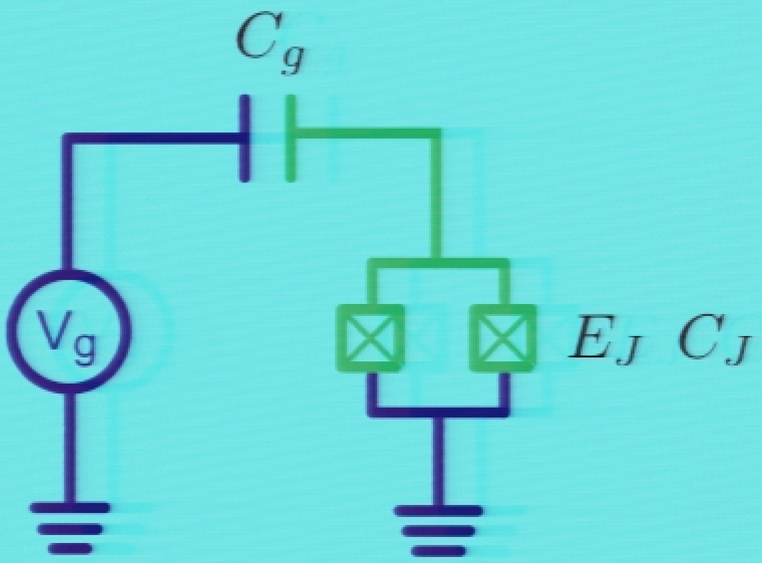

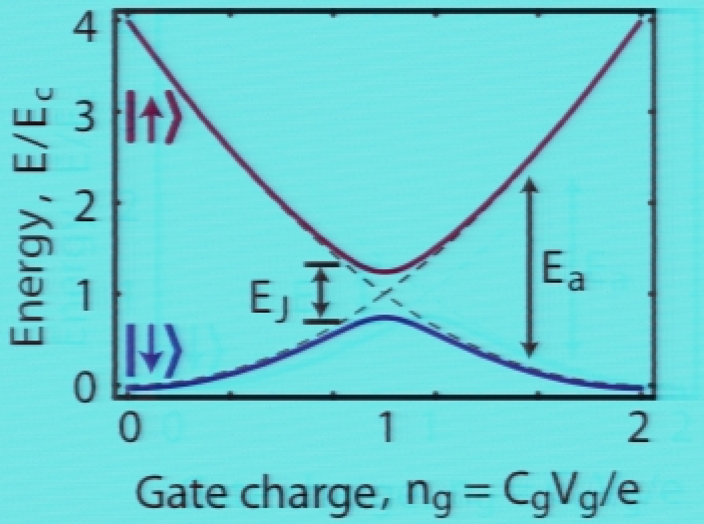

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$$H_Q = -\frac{E_J}{2}\sigma_z - \frac{E_{el}}{2}\sigma_x$$

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Effective fields:

$$E_J = E_{J,\max}\cos\left(\pi\Phi_x/\Phi_0\right)$$

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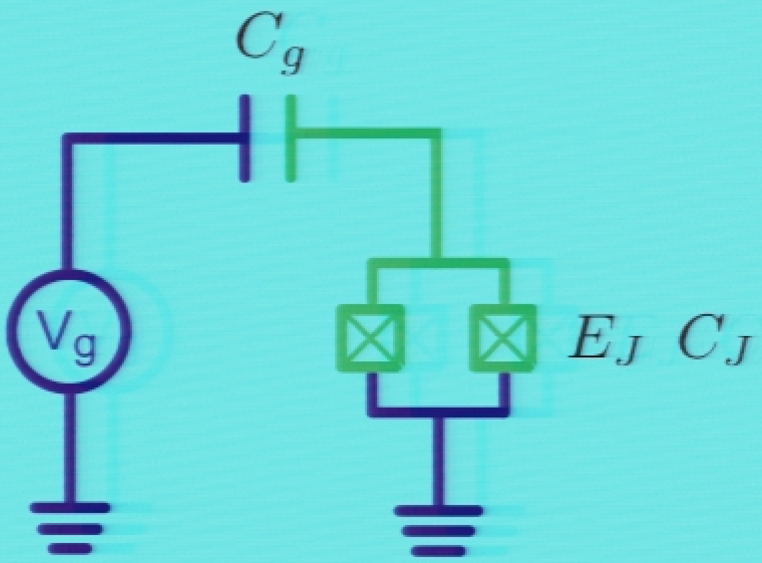

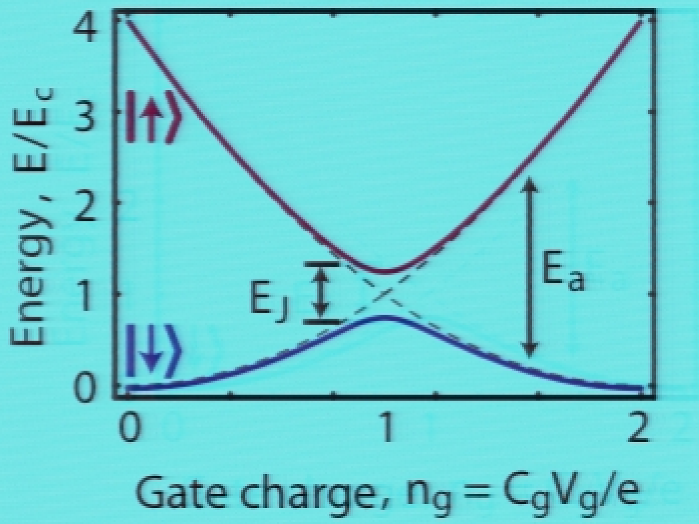

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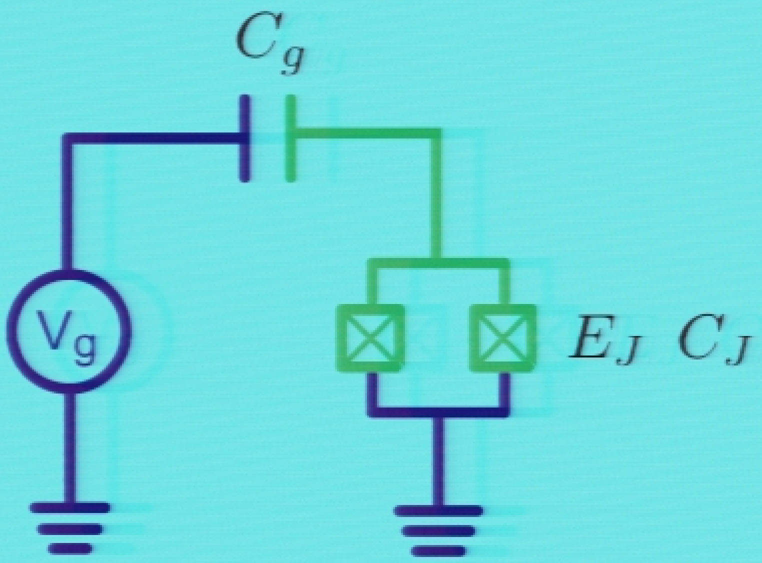

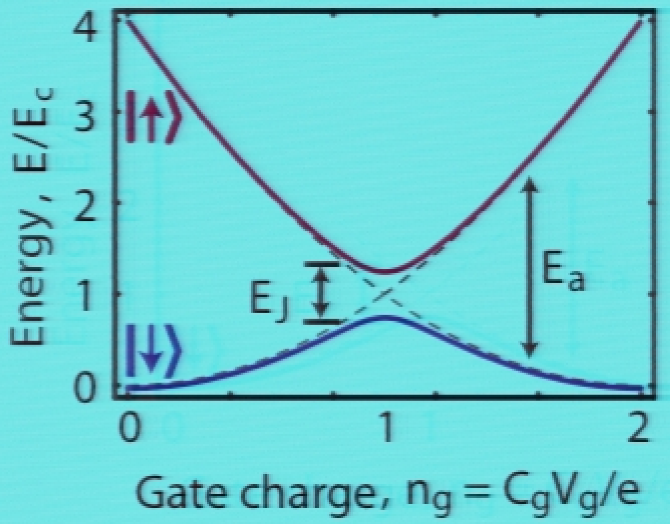

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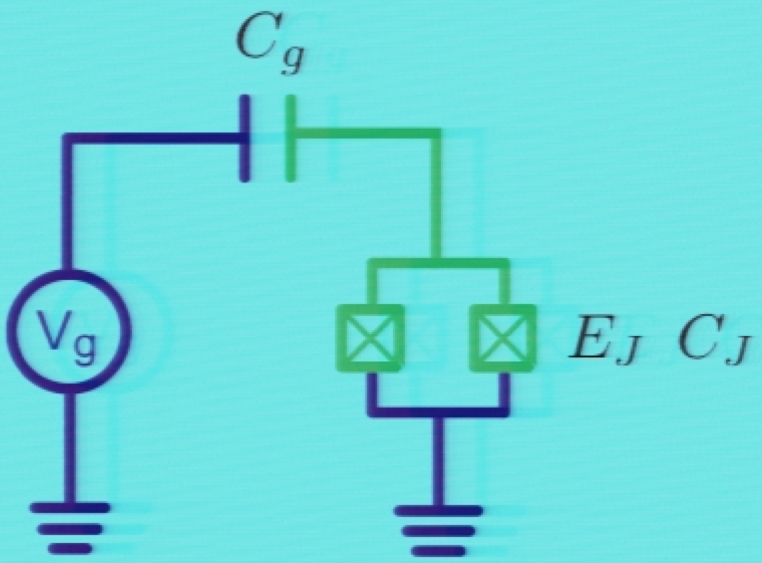

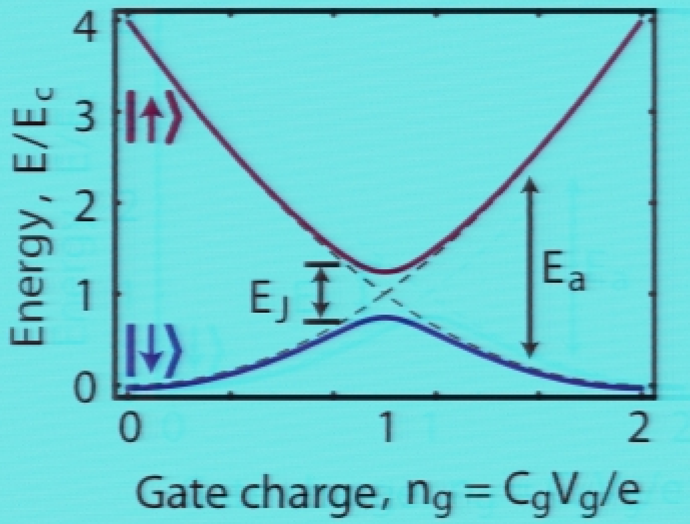

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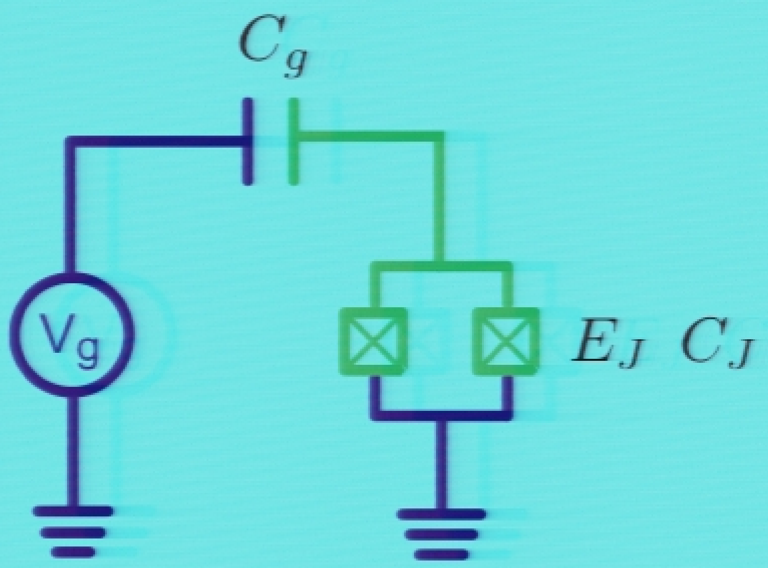

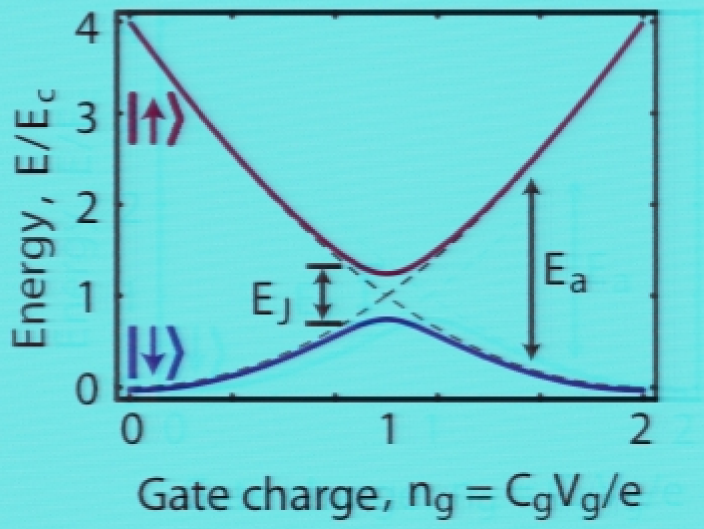

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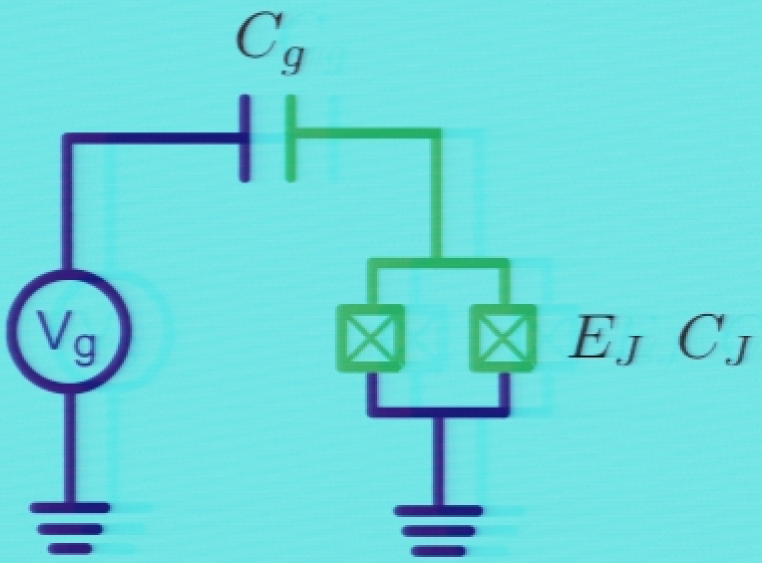

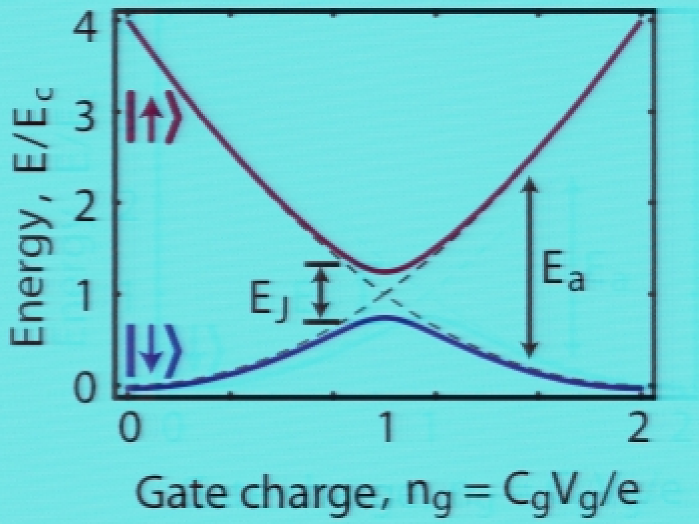

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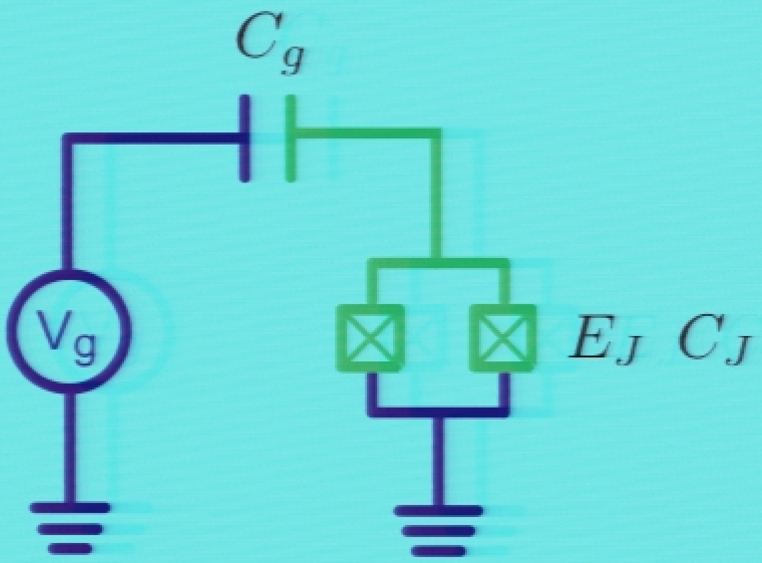
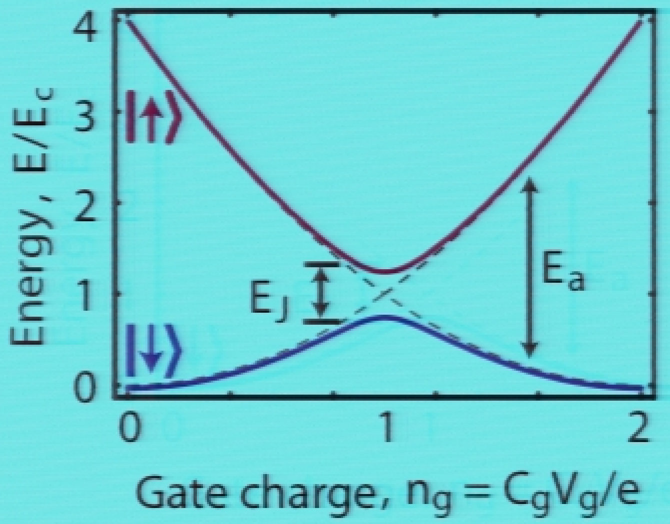
Bouchiat et al. Physica Scripta 176, 165 (1998)

Artificial atom: Cooper-pair box





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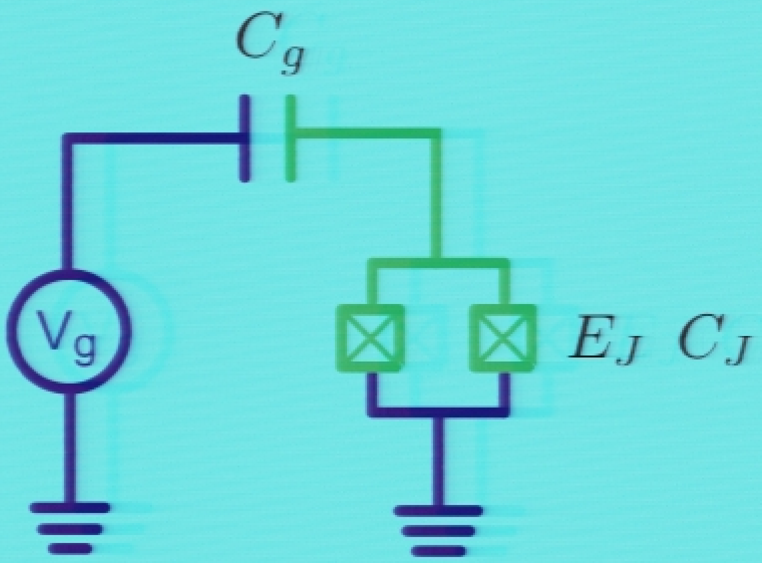

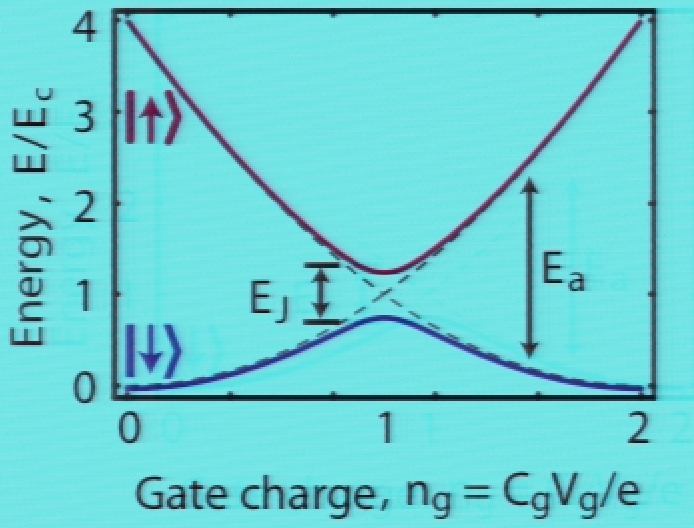

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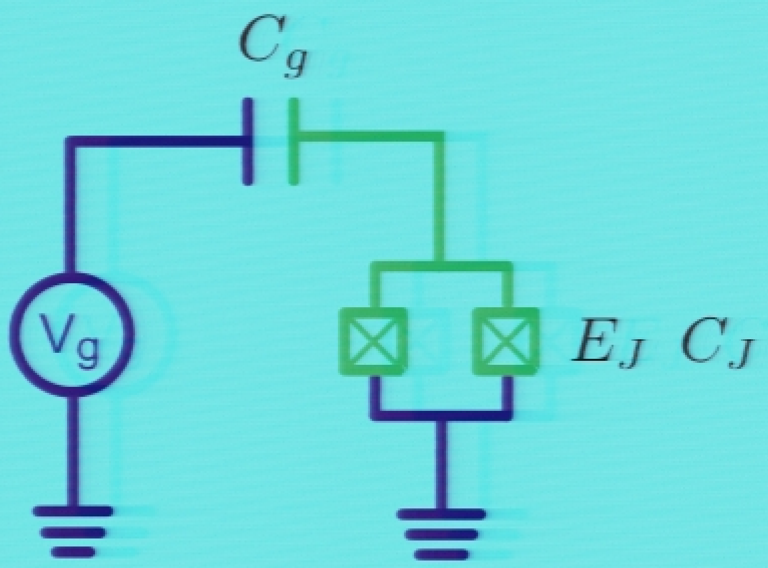

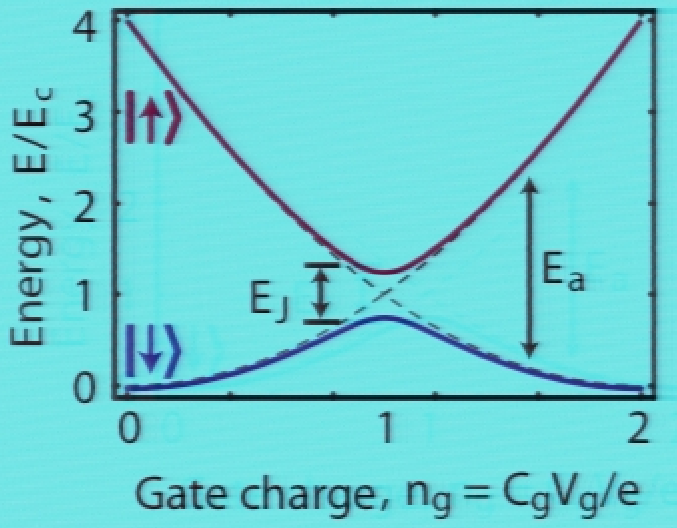

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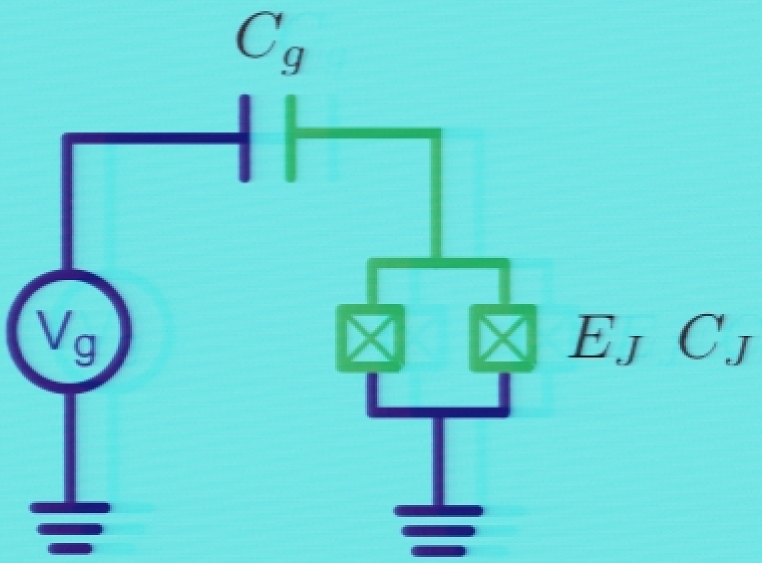

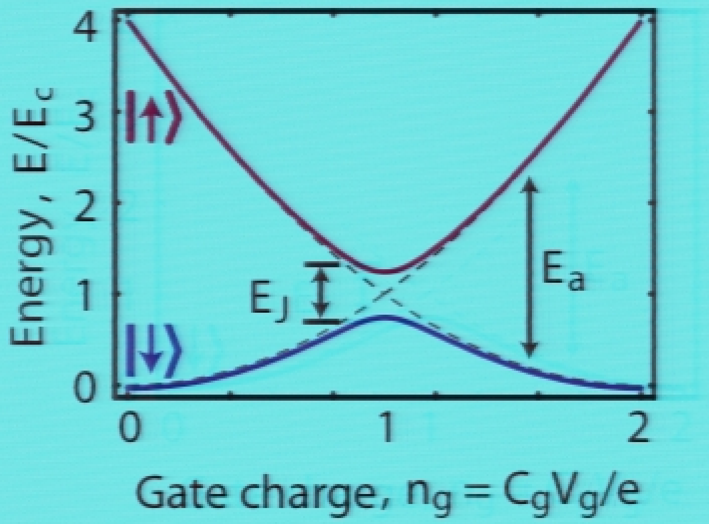

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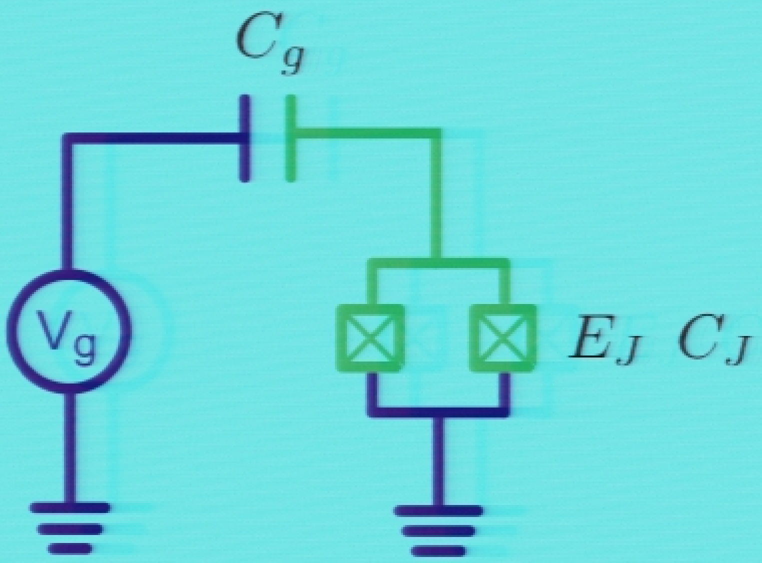

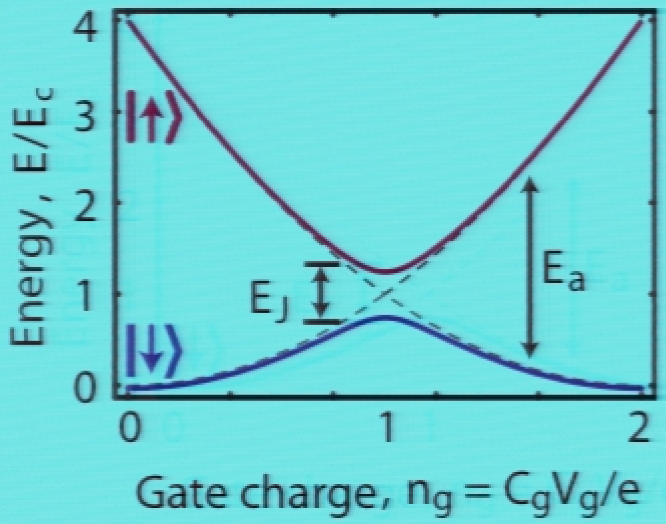

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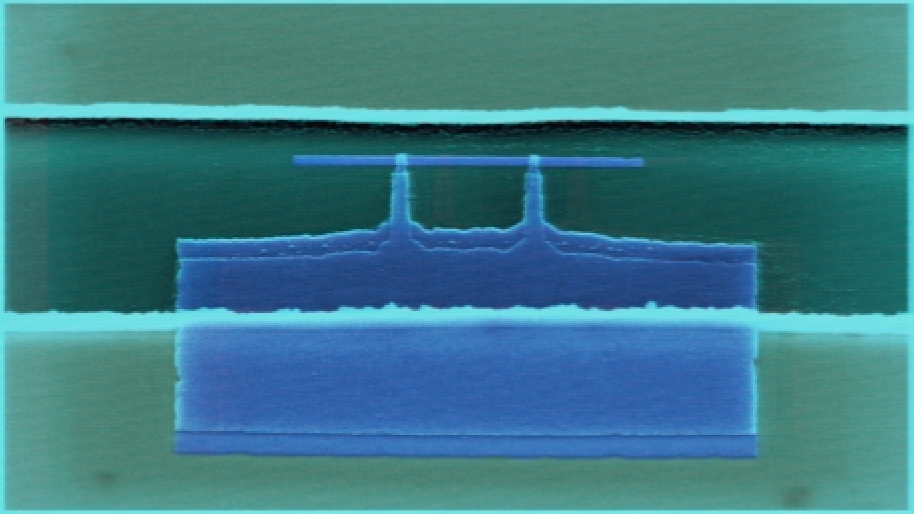
Resonator-qubit coupling

Jaynes-Cummings interaction



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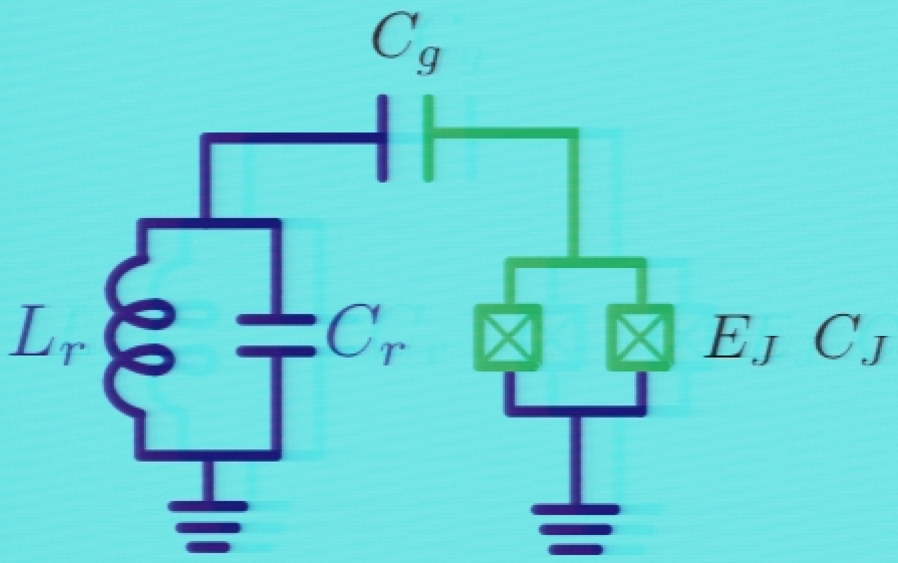

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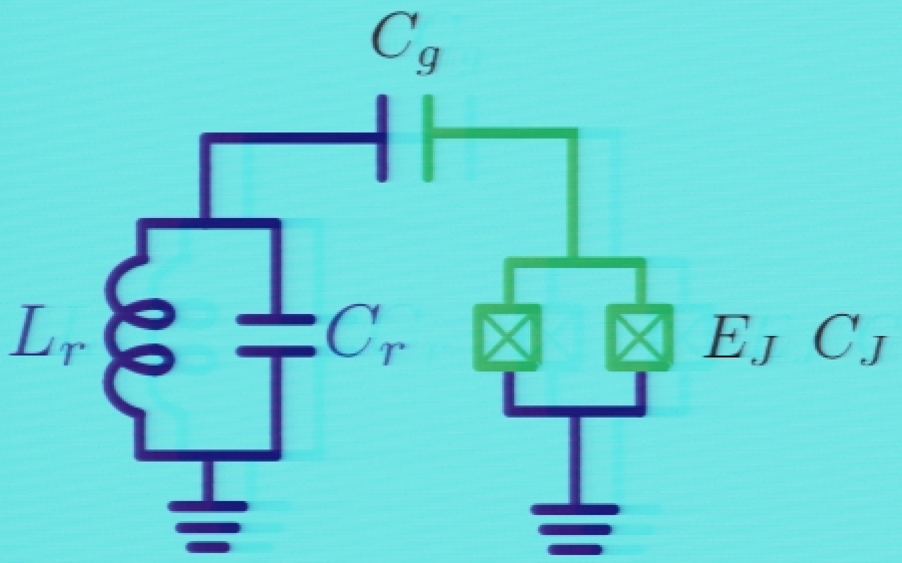

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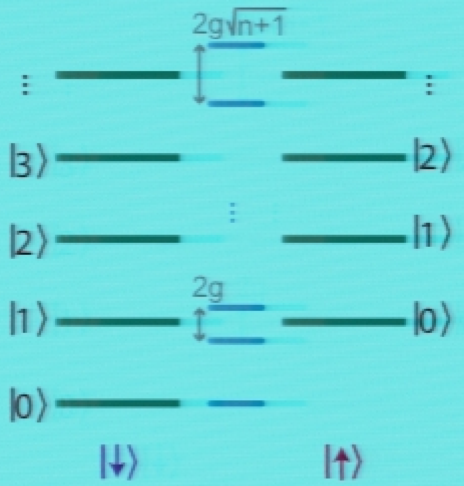
$$H = \hbar\omega_r a^\dagger a + \hbar\frac{\omega_a}{2}\sigma_z + \hbar g(a^\dagger\sigma_- + a\sigma_+)$$

$$\frac{g}{2\pi} = \frac{e}{h}\frac{C_g}{C_\Sigma}V^0_{\mathrm{RMS}} \sim 1 - 100\ \mathrm{MHz}$$

$$t_{\mathrm{transit}} = \infty$$

Vacuum Rabi splitting

First observation in superconducting circuits



Wallraff, Schuster, AB, Frunzio, Huang, Majer,

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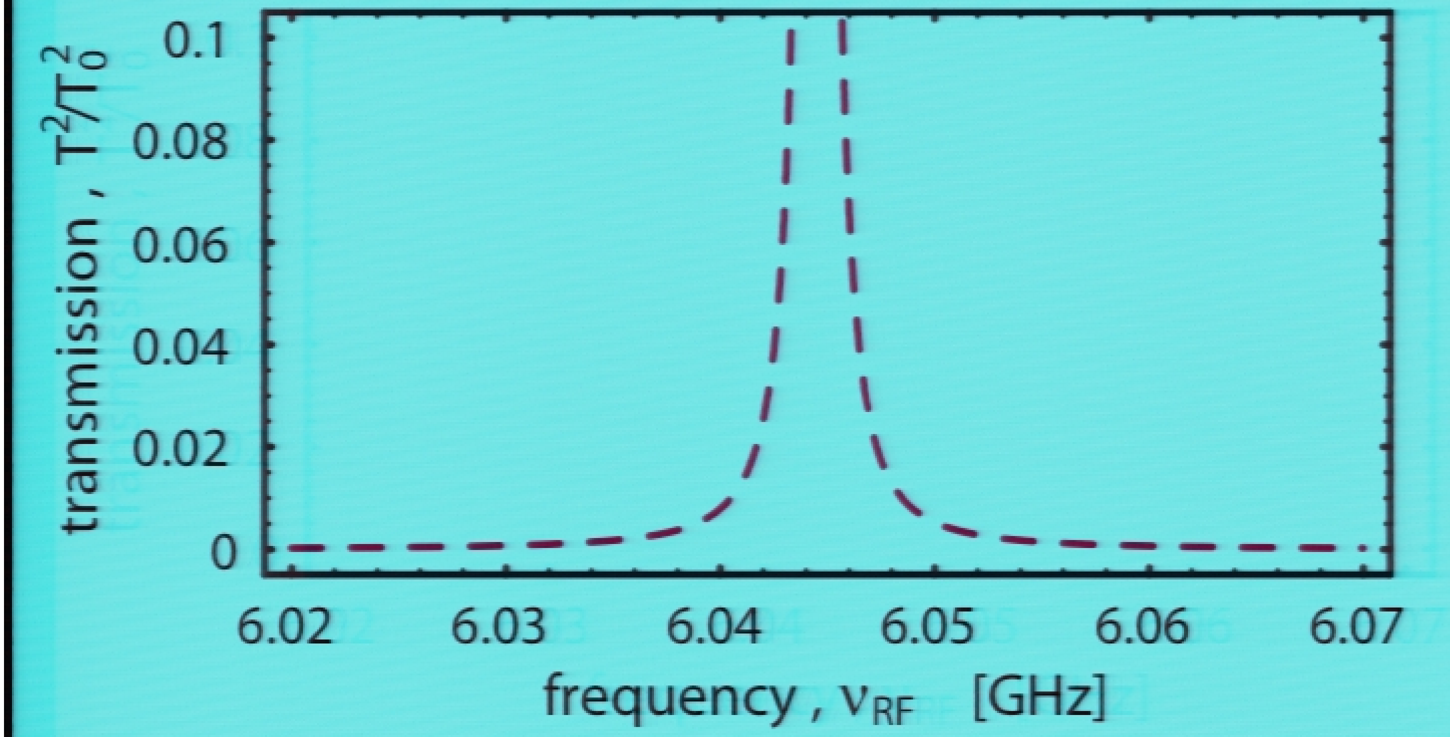

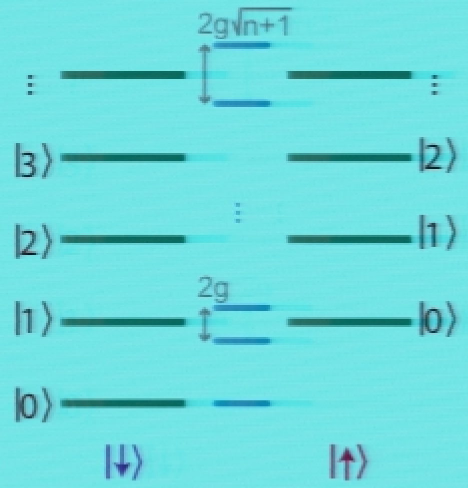

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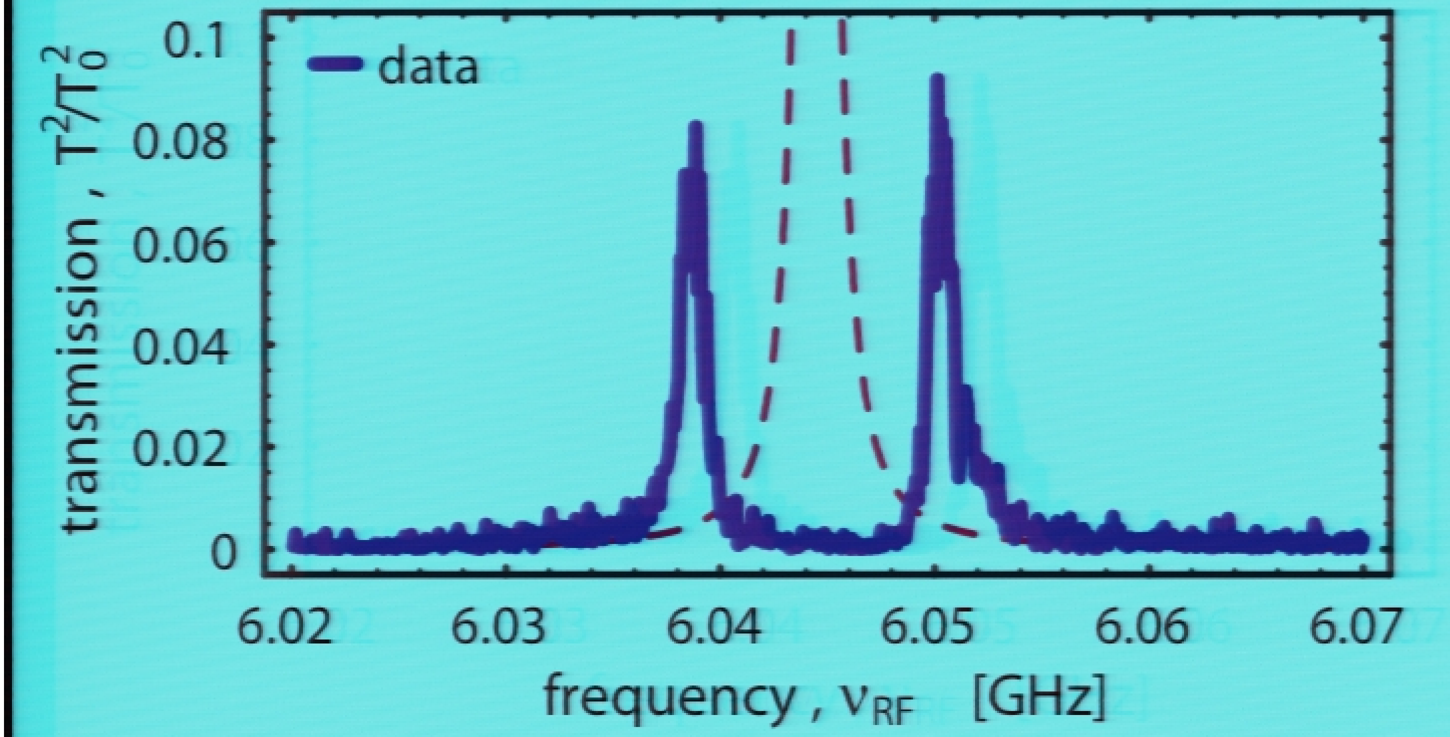

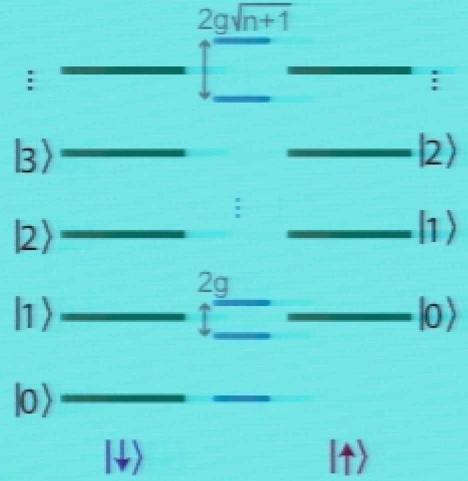

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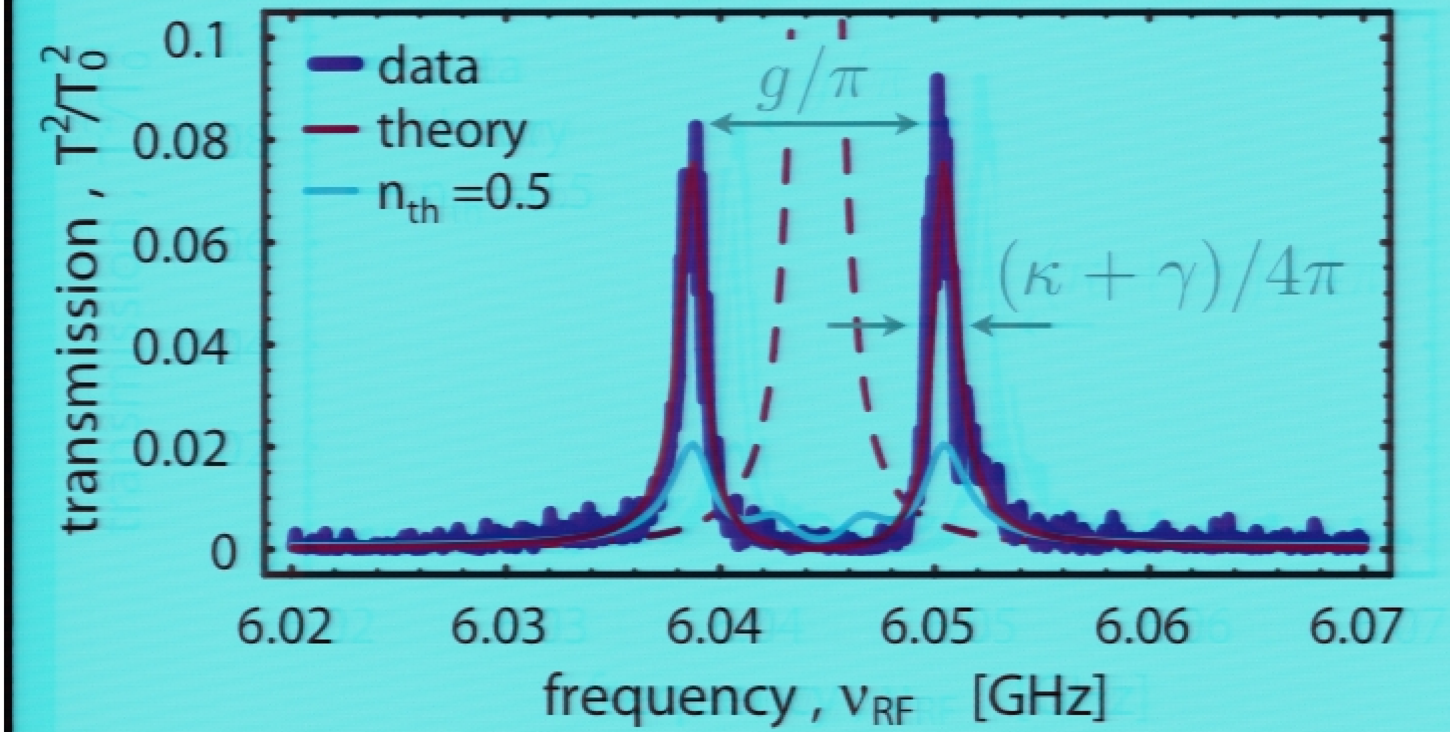

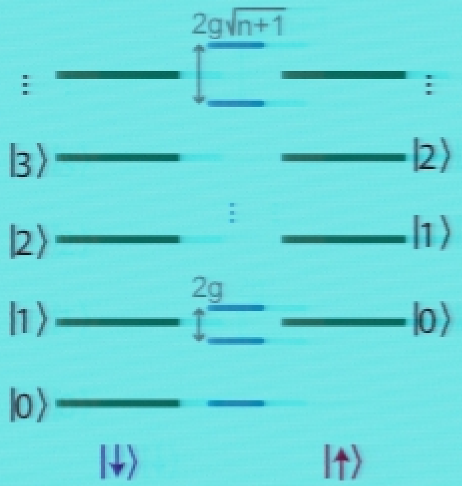

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→ Strong coupling CQED

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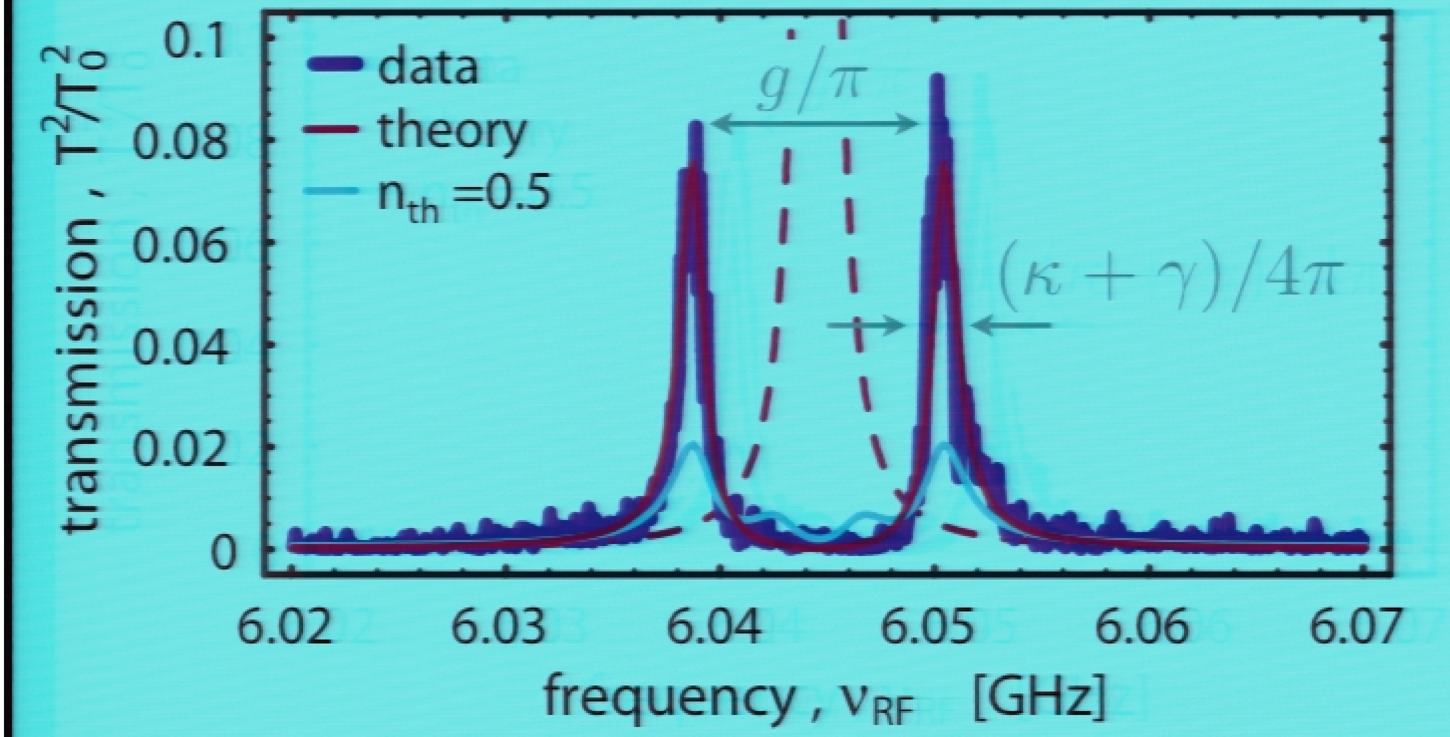

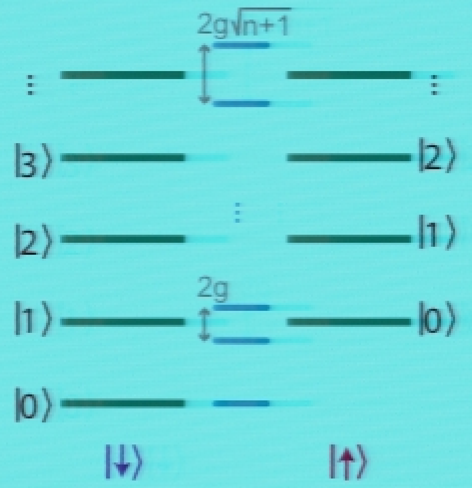

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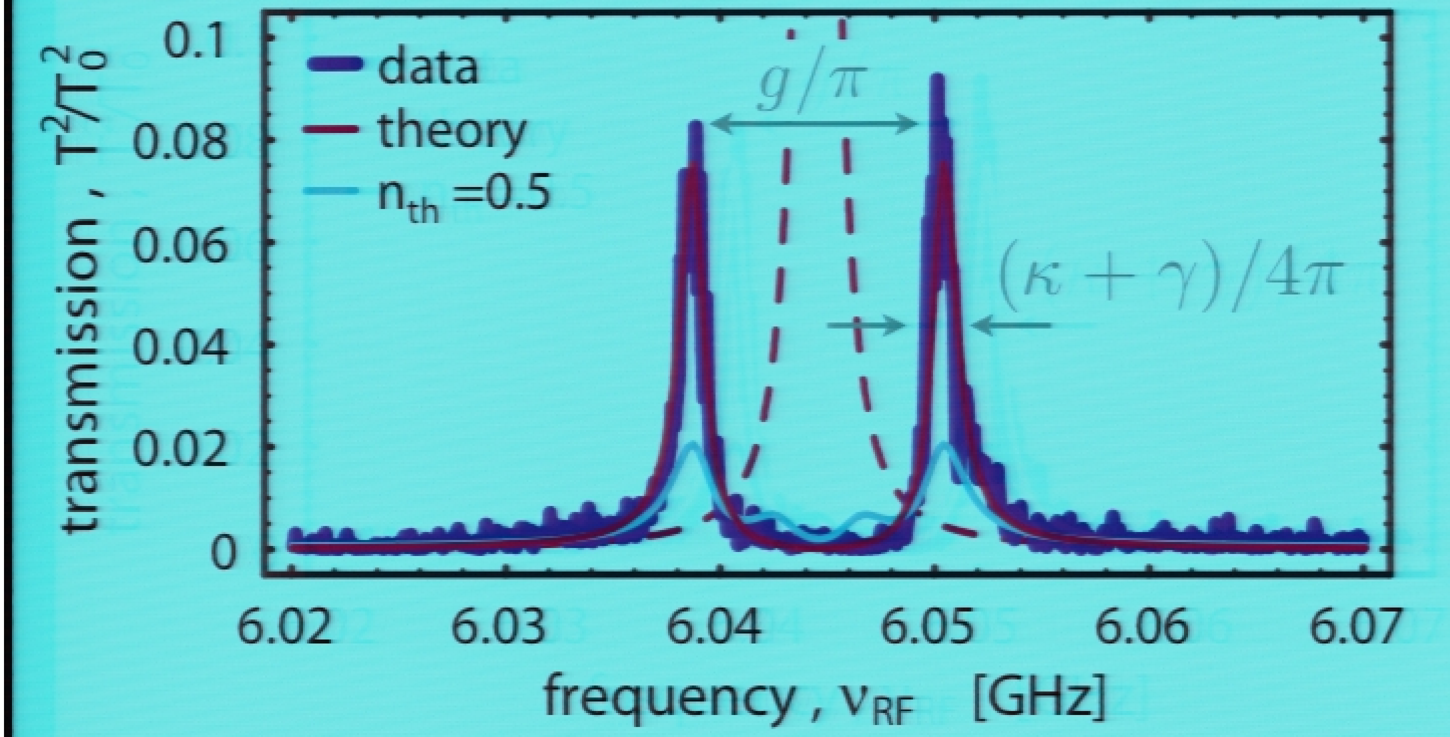

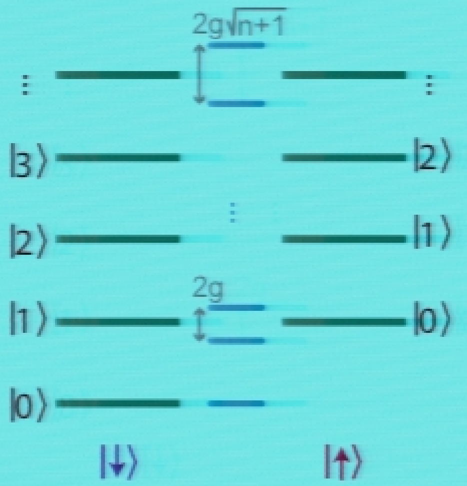

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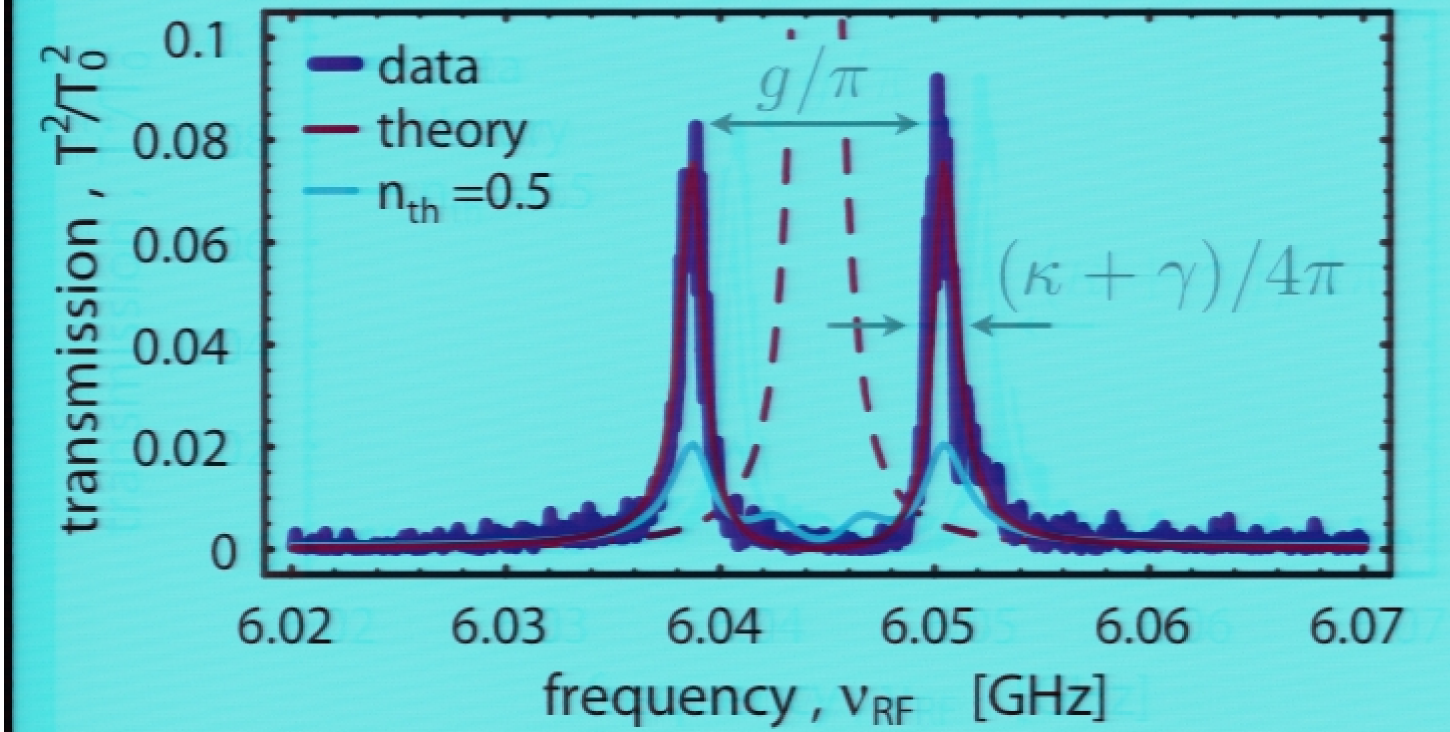

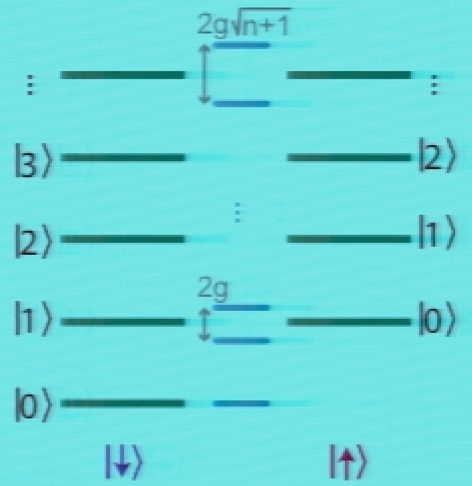

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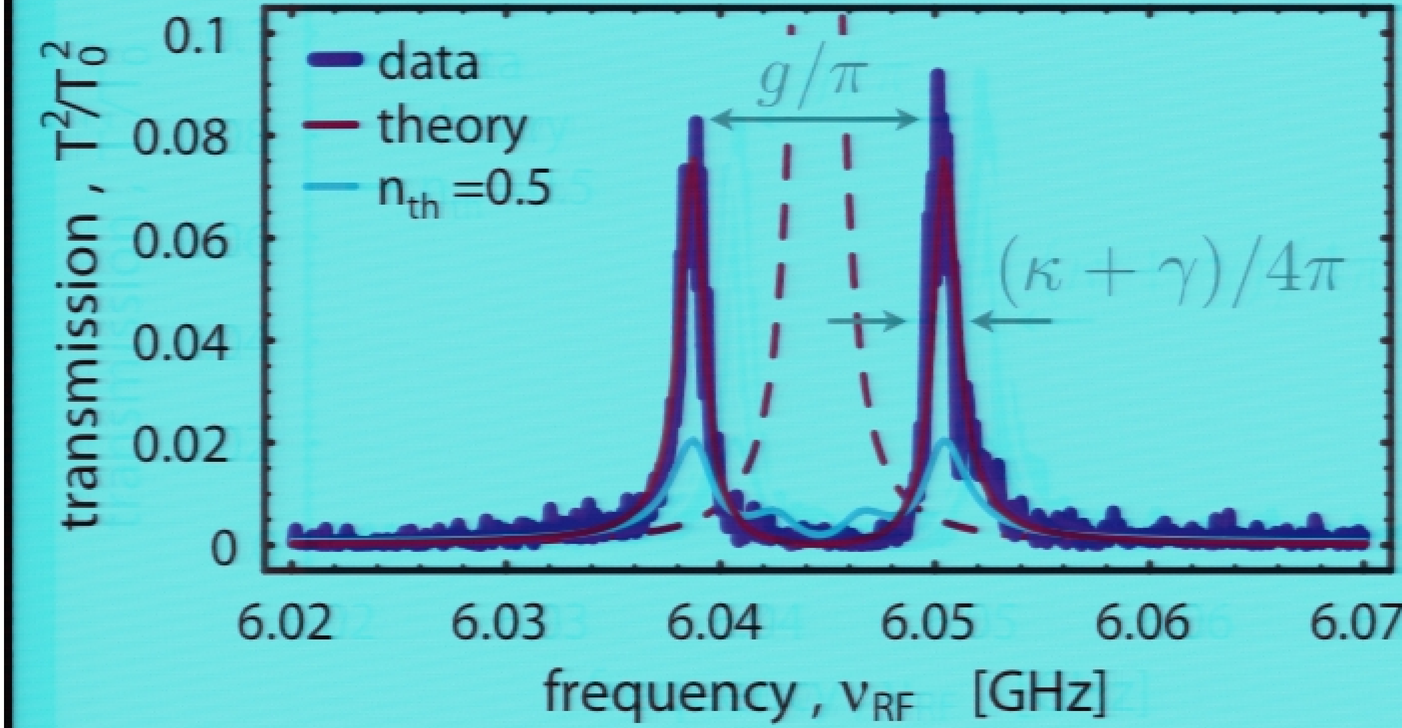

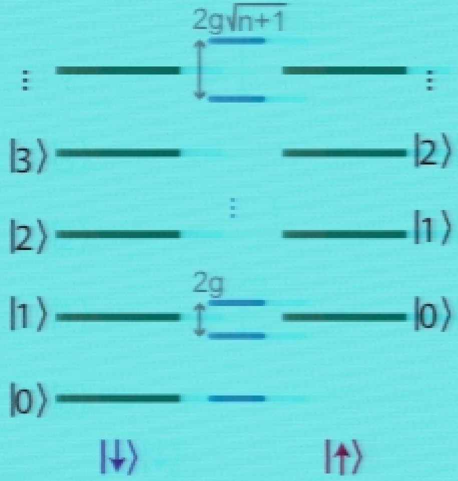

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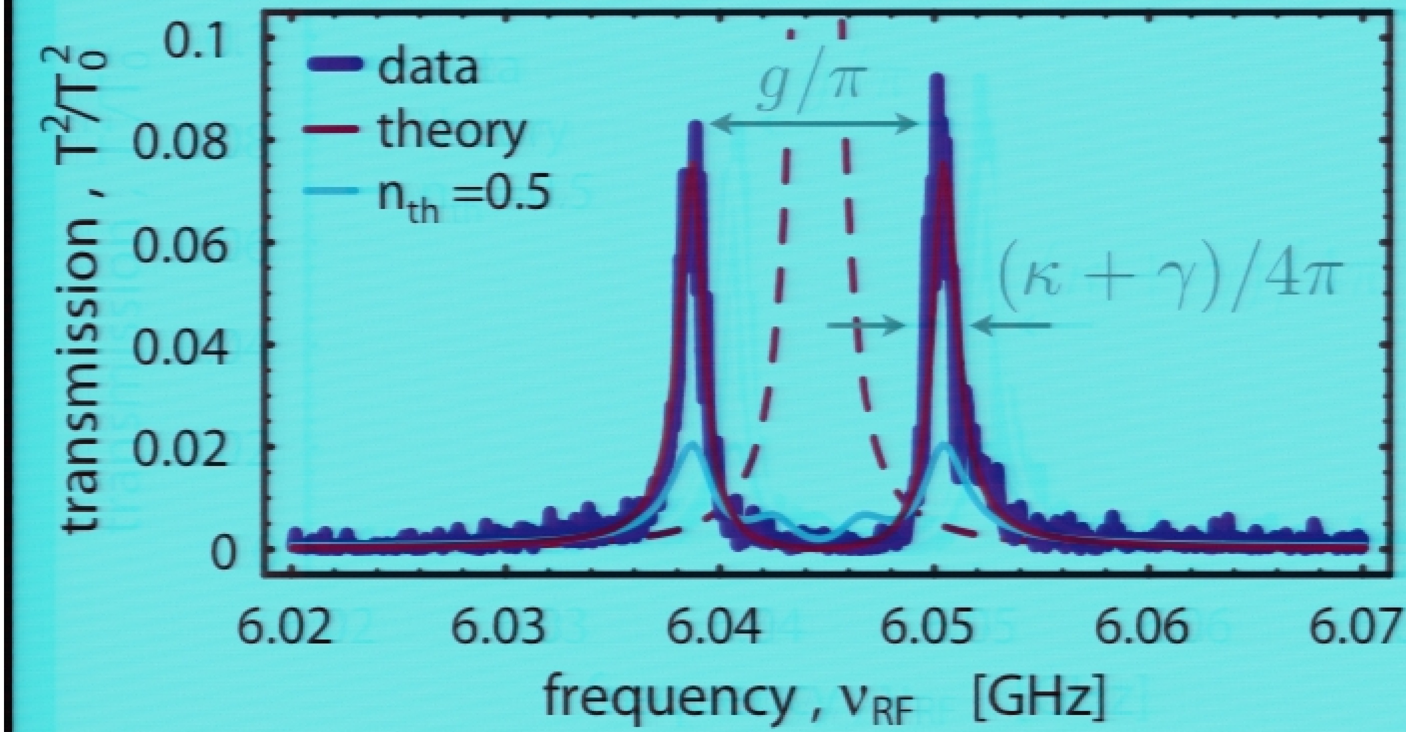

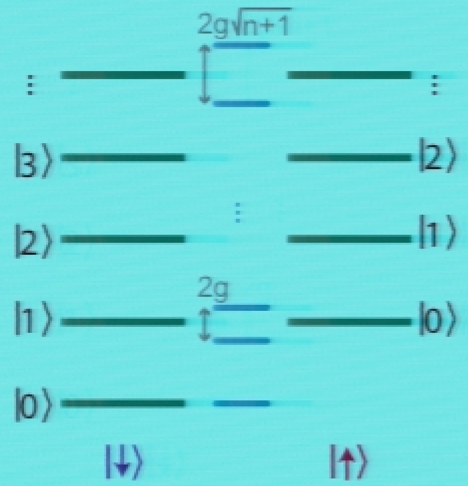

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For $|\Delta| = |\omega_a - \omega_r| \gg g$

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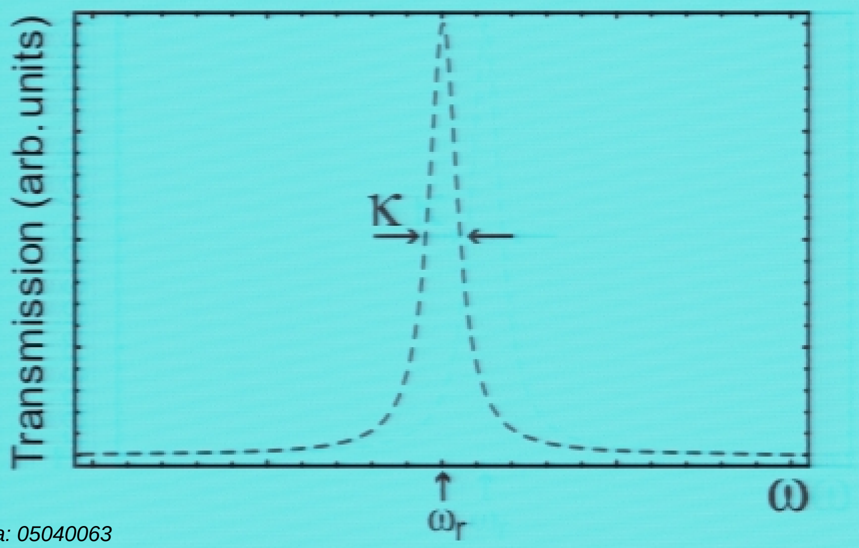

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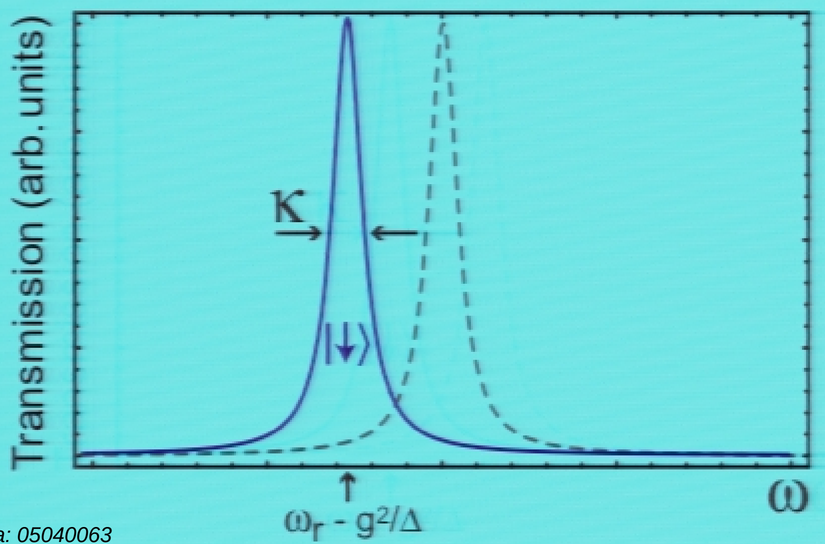

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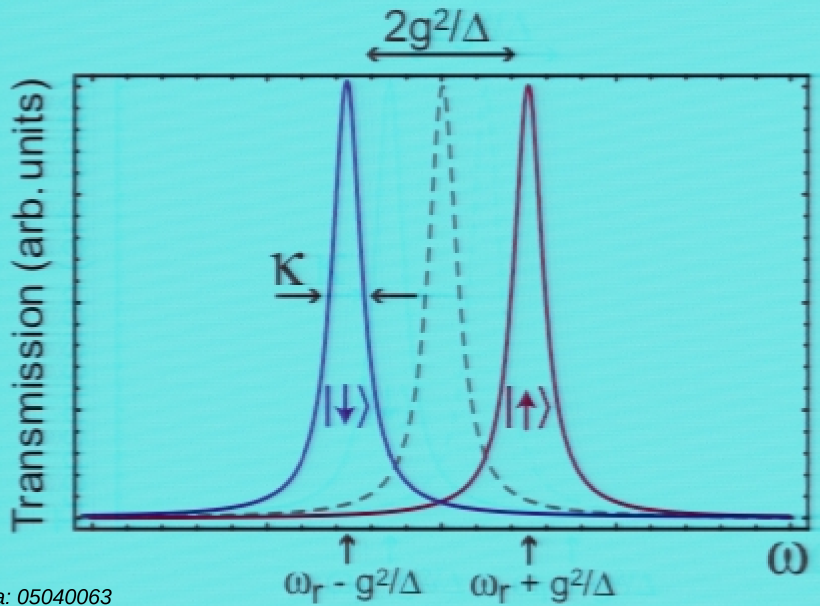

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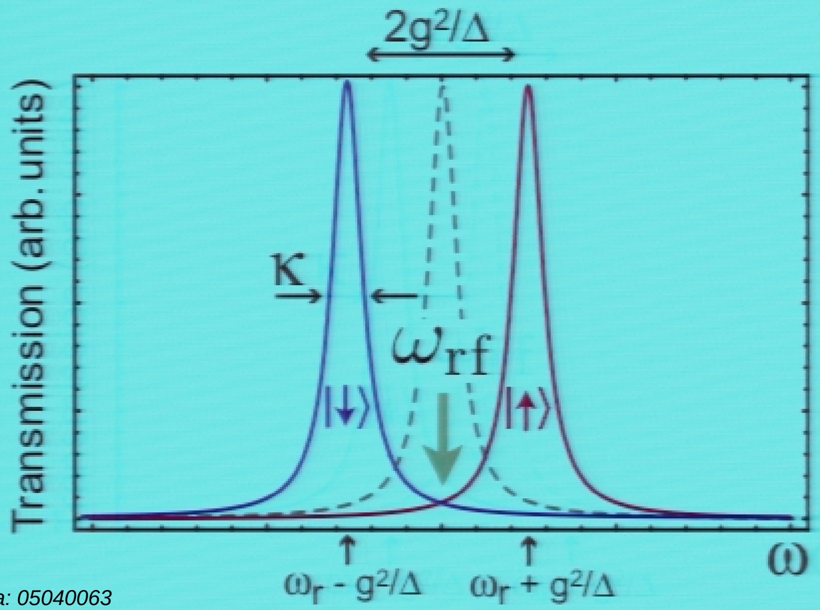

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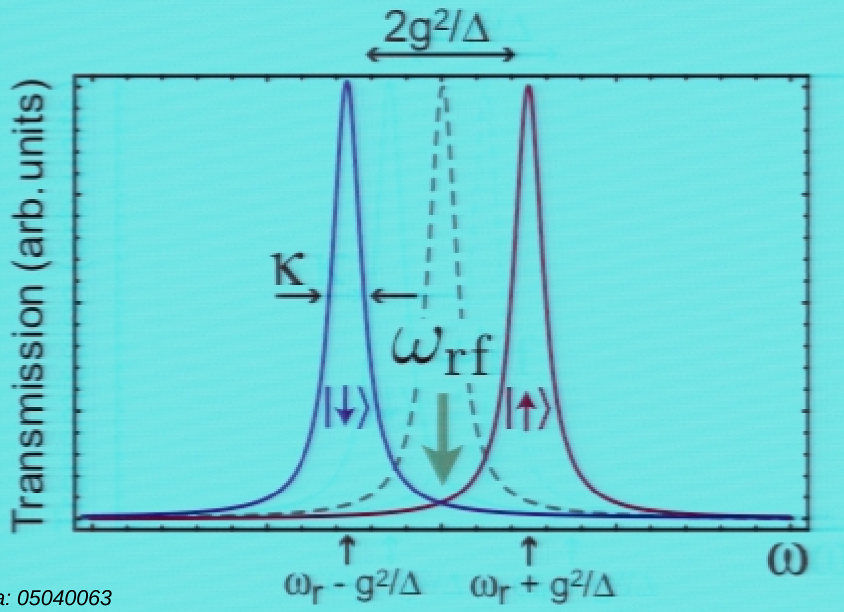

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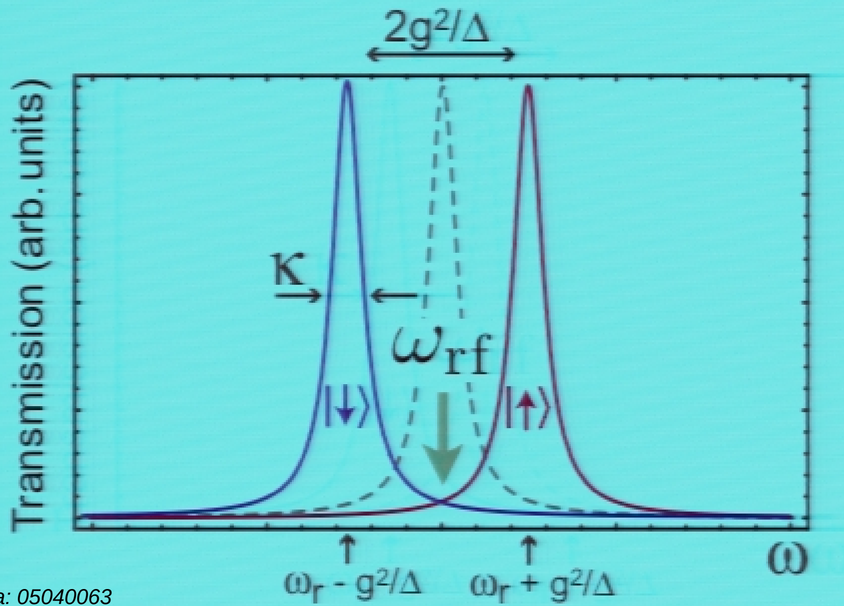

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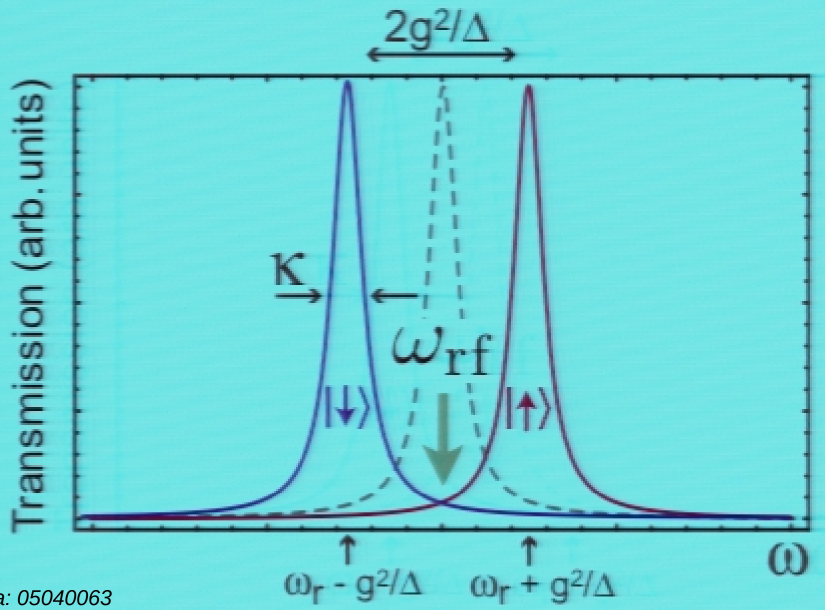

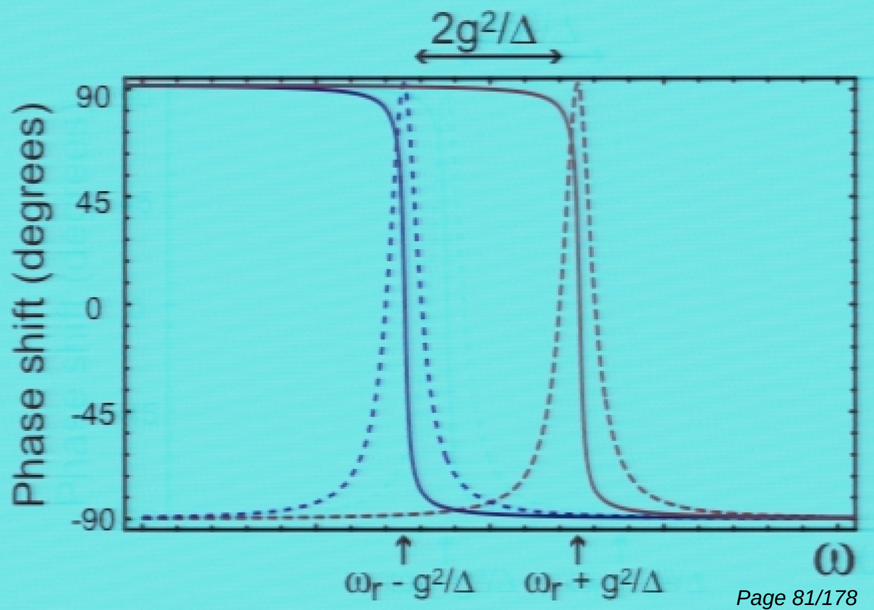

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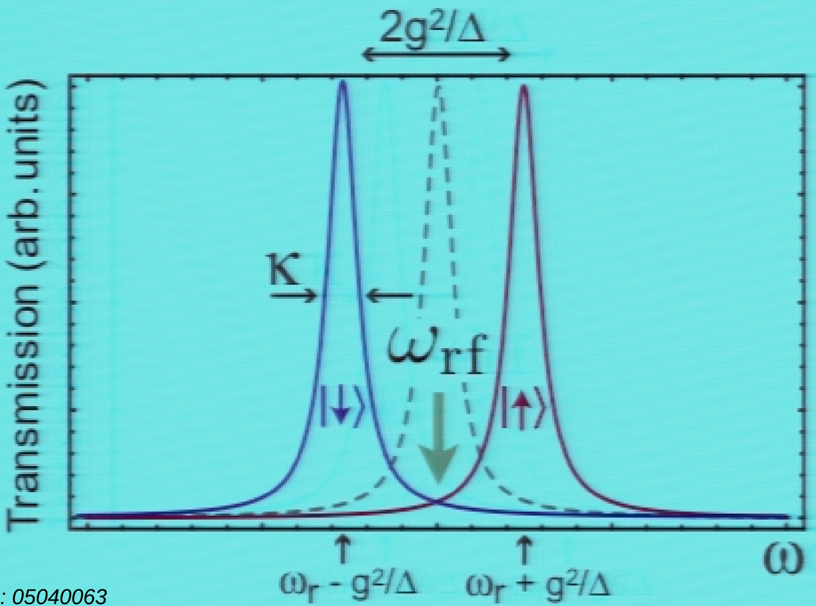

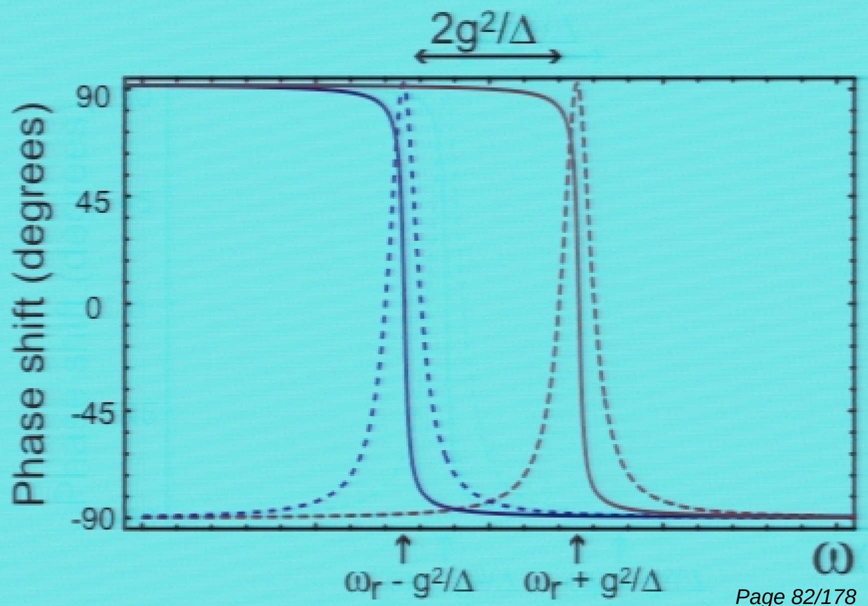

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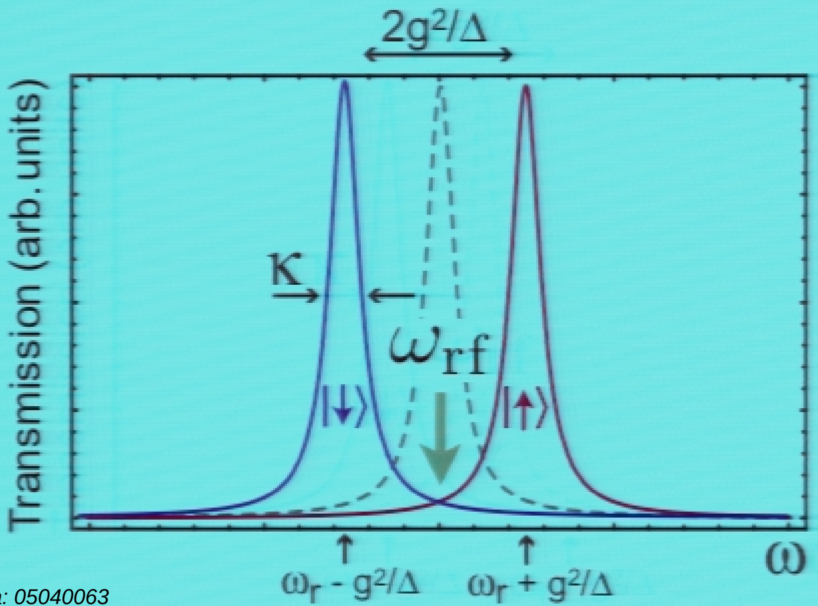

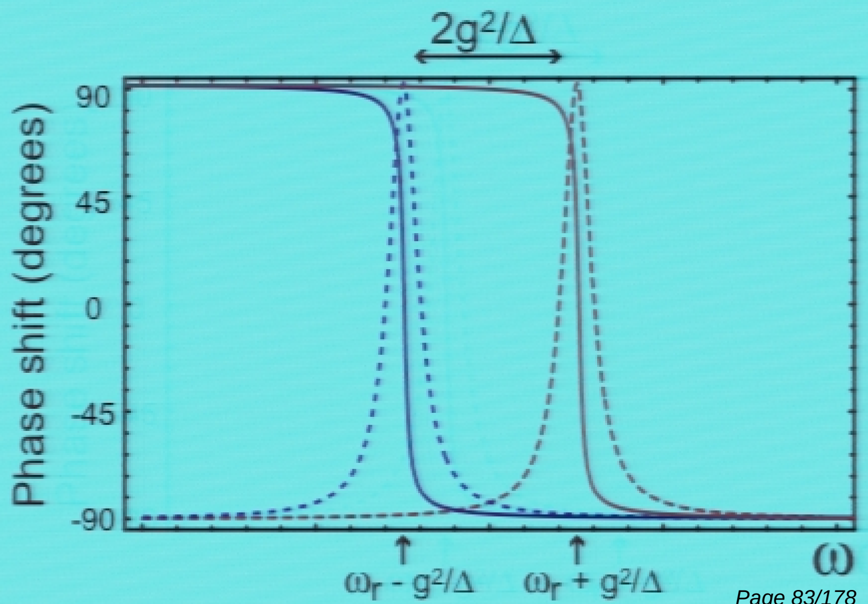

AB, Huang, Wallraff, Girvin & Schoelkopf, PRA 69, 062320 (2004)

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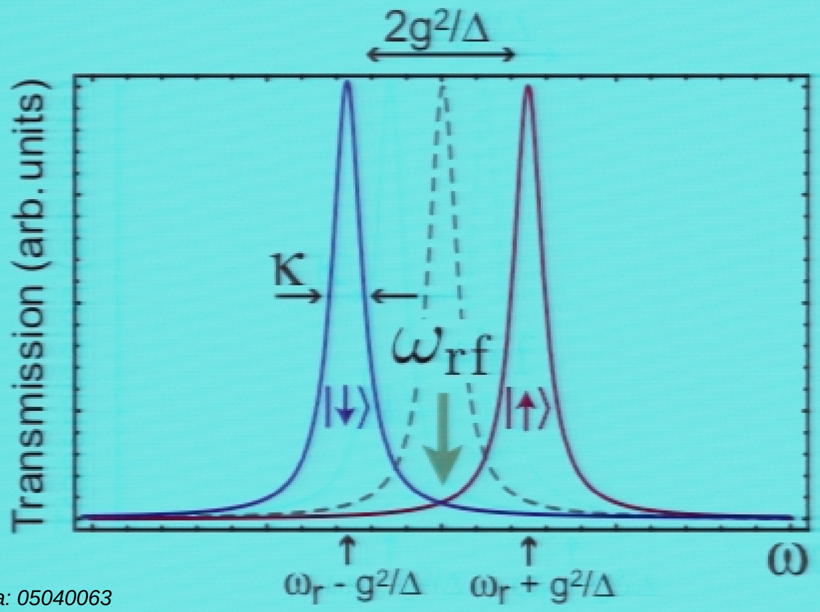

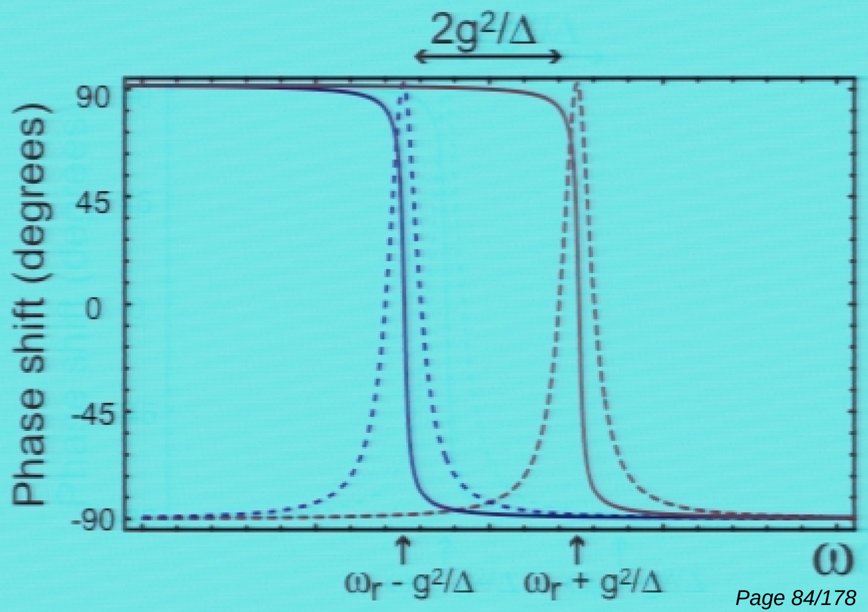

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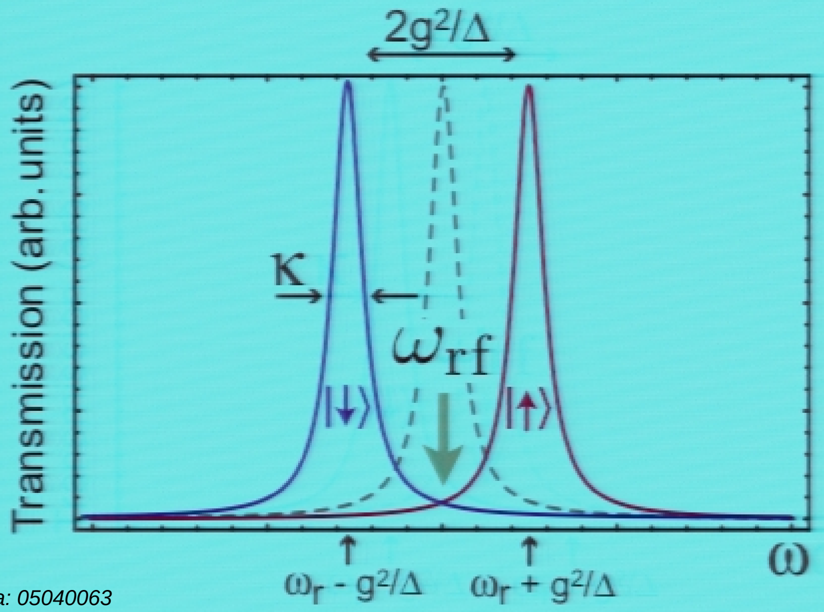

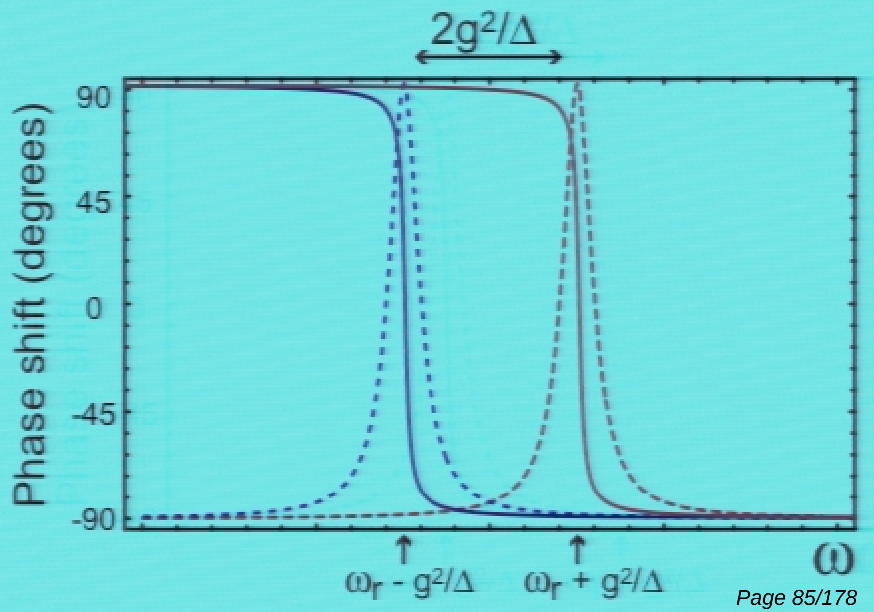

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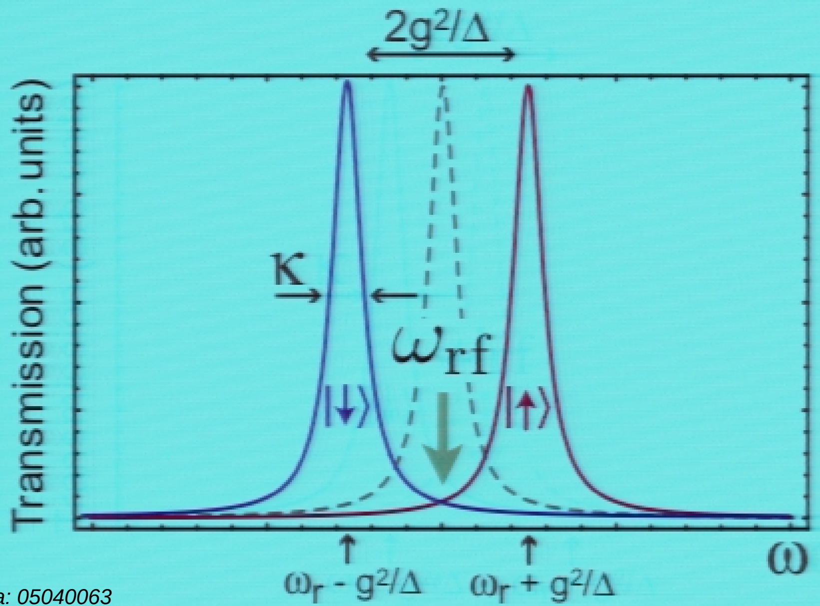

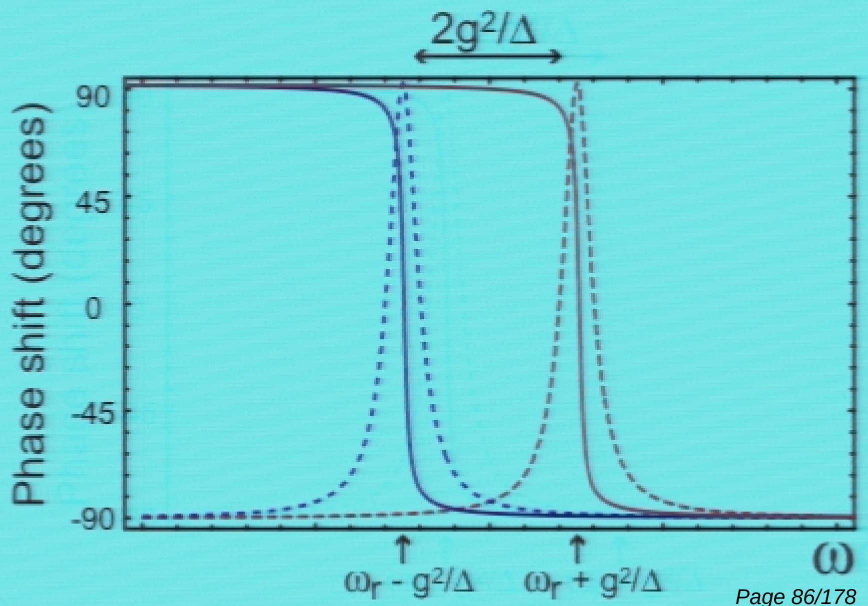

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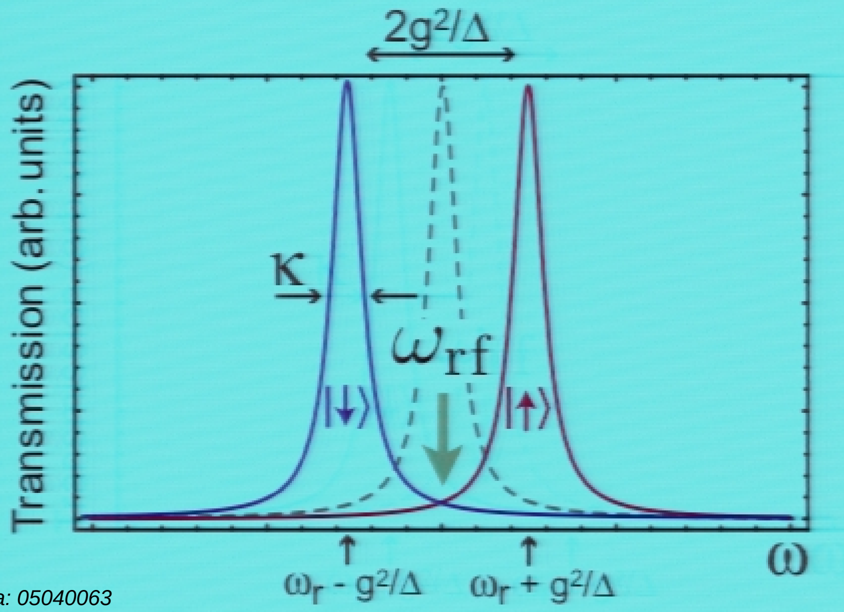

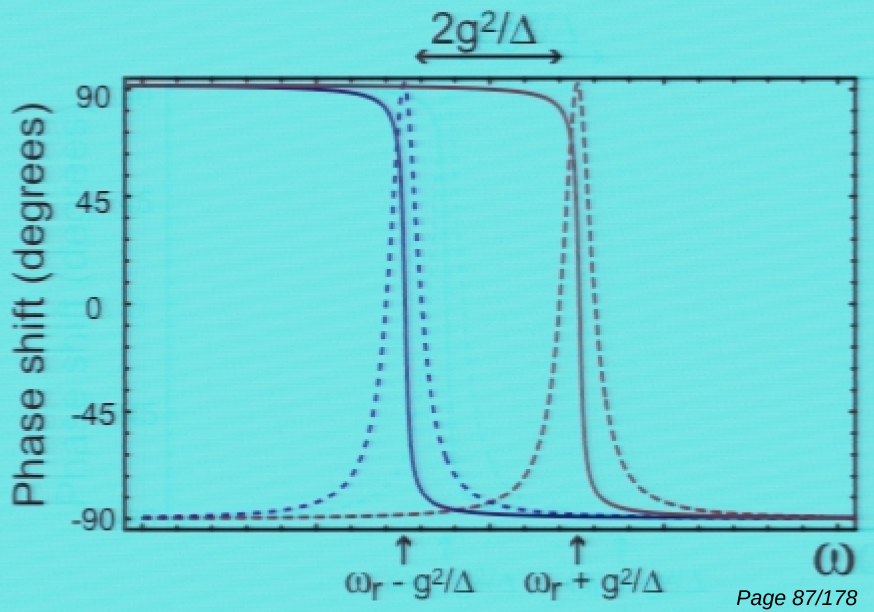

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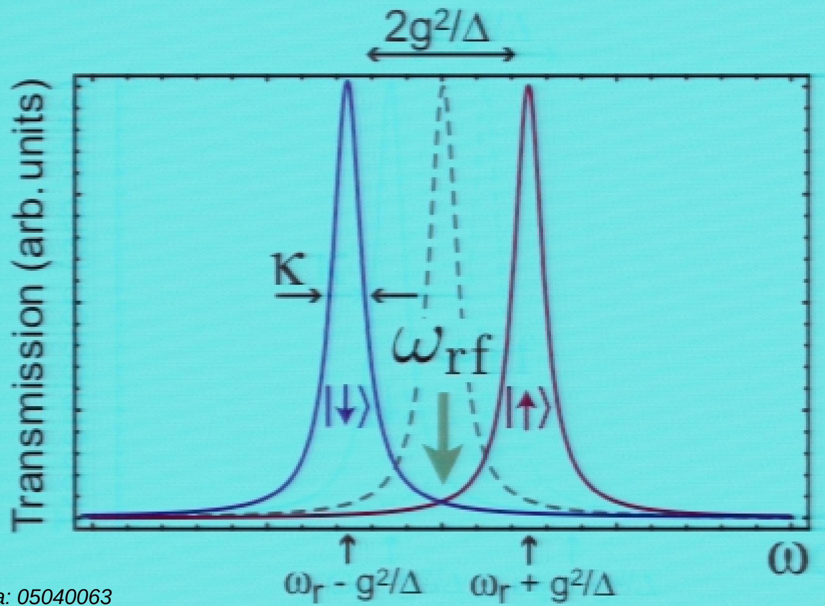

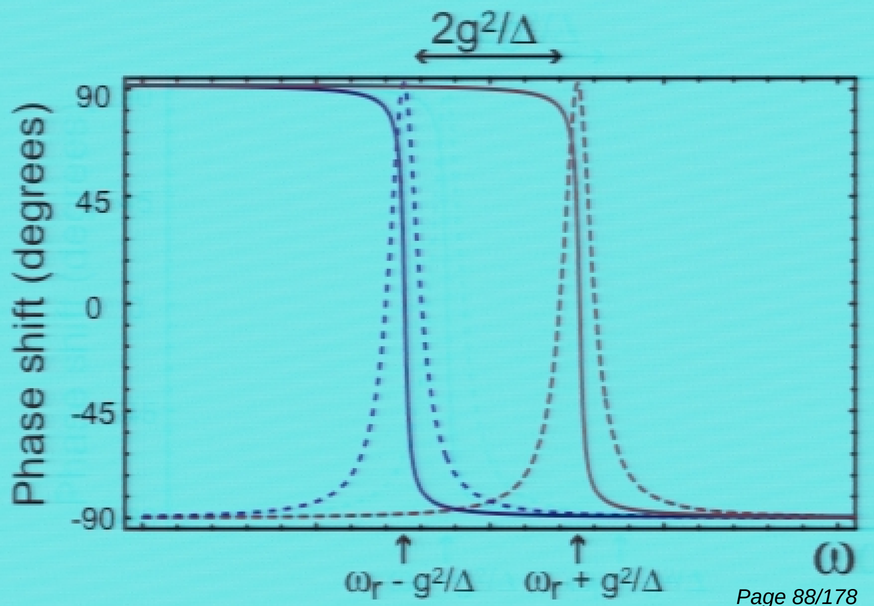

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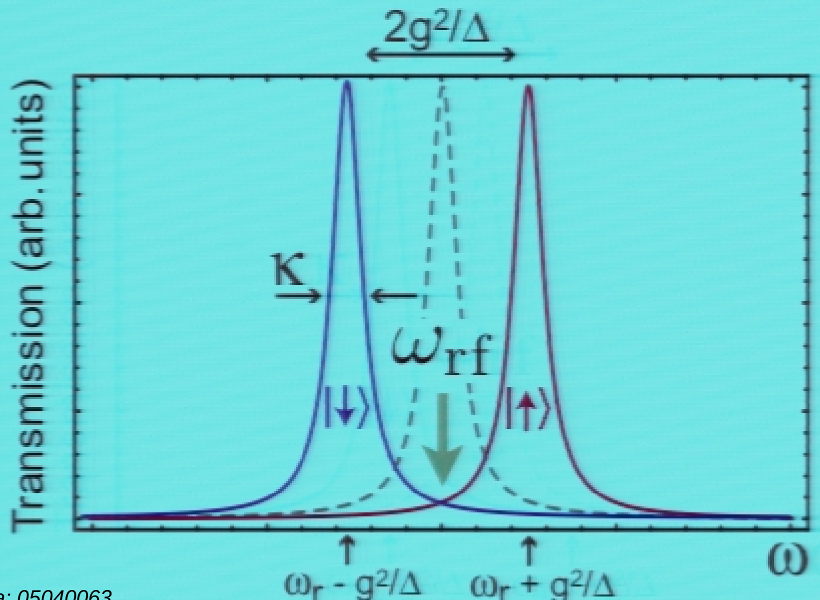

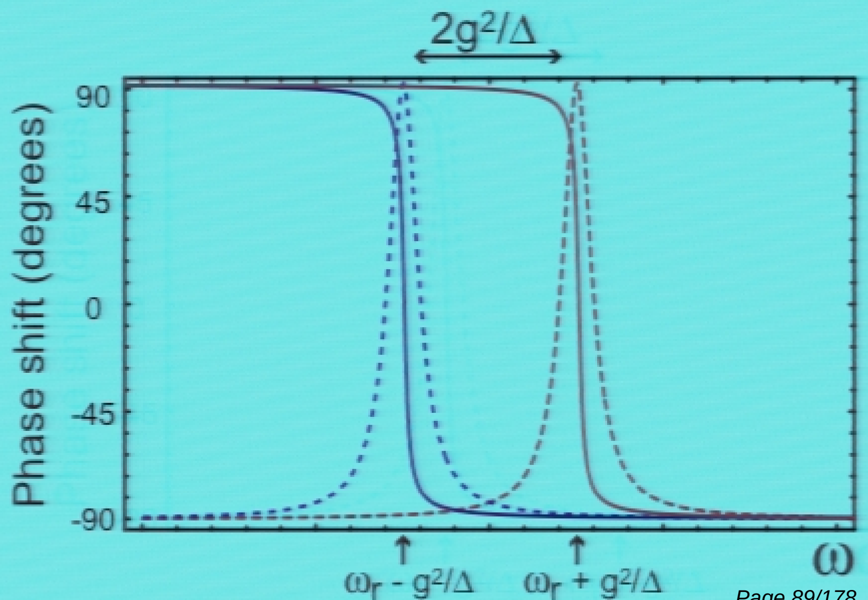

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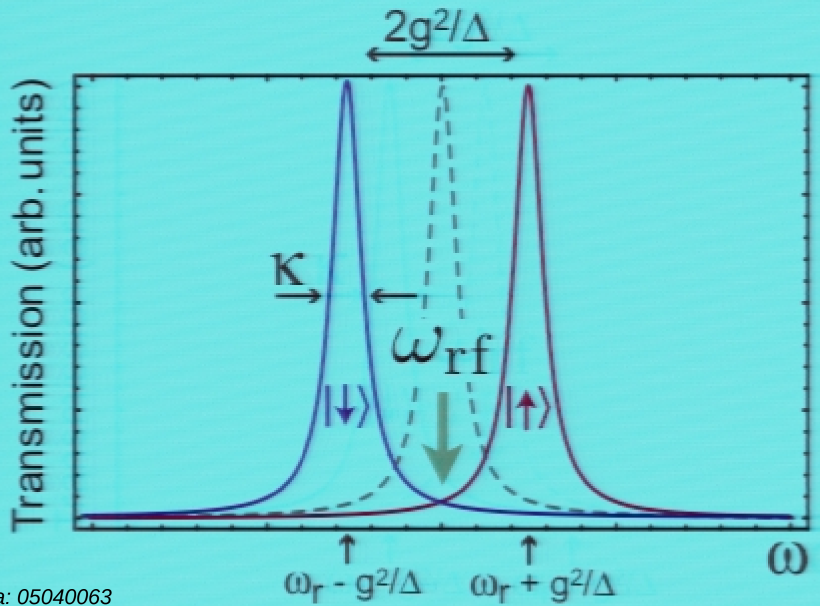

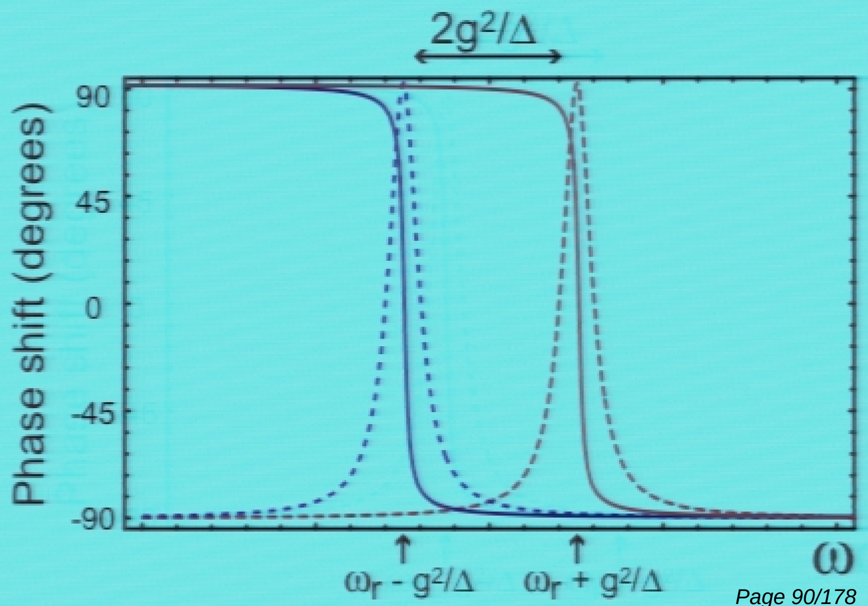

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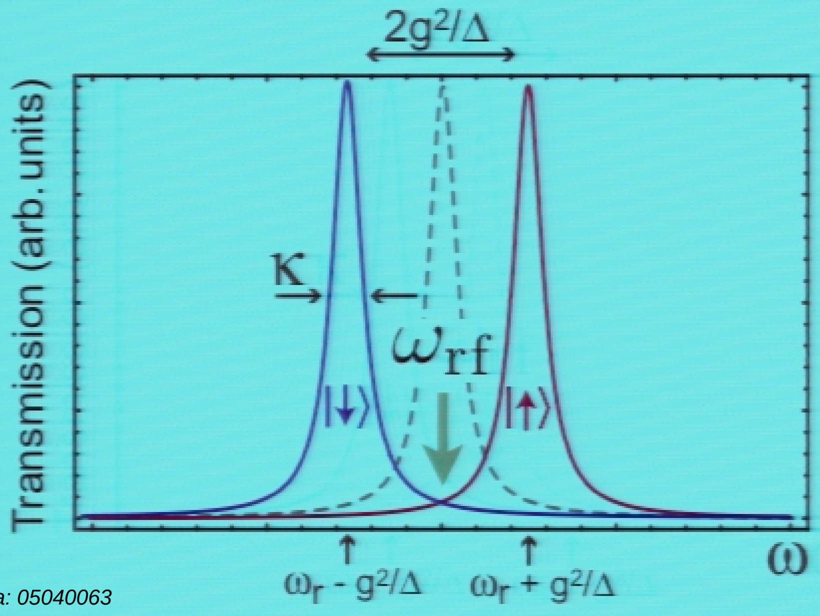

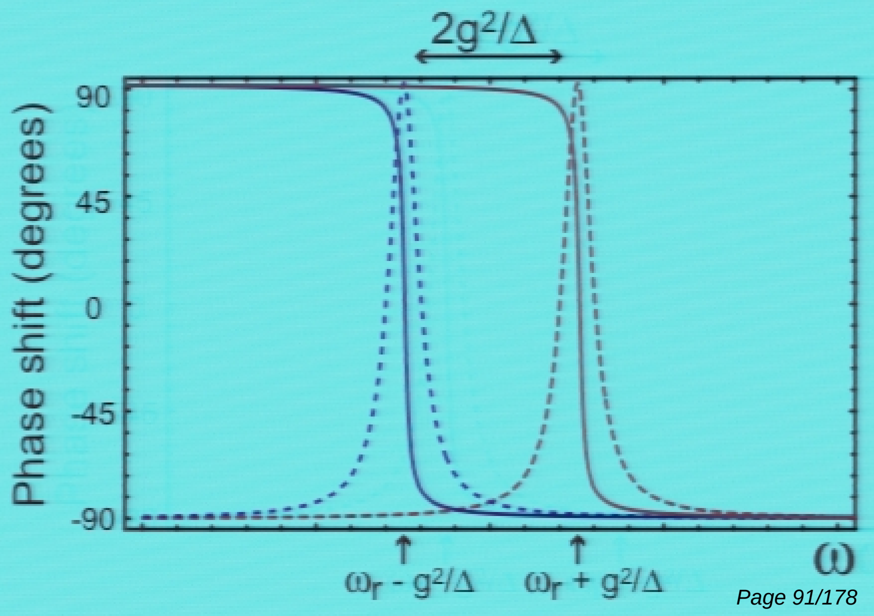

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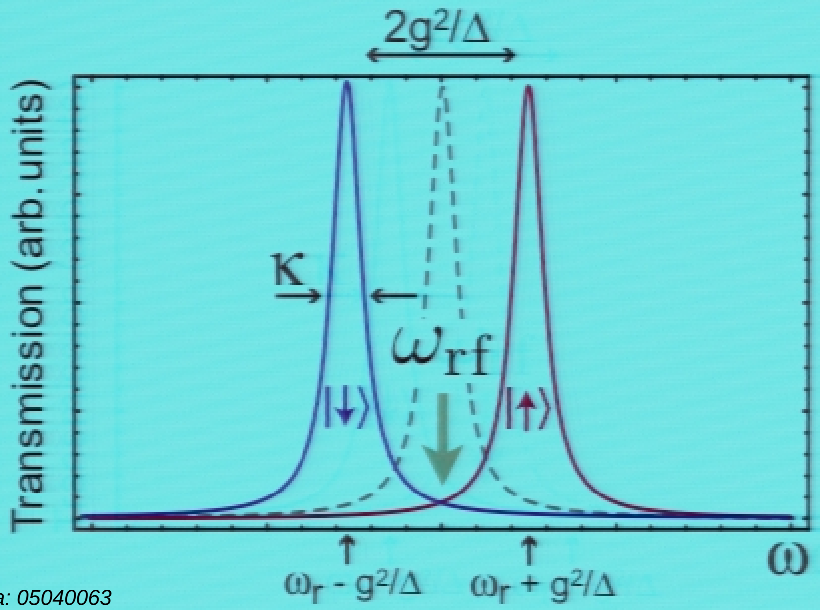

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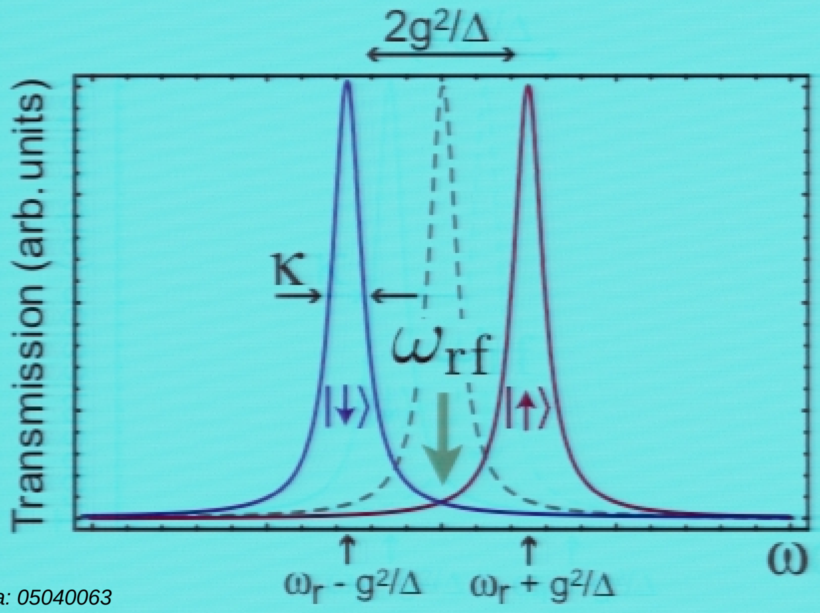

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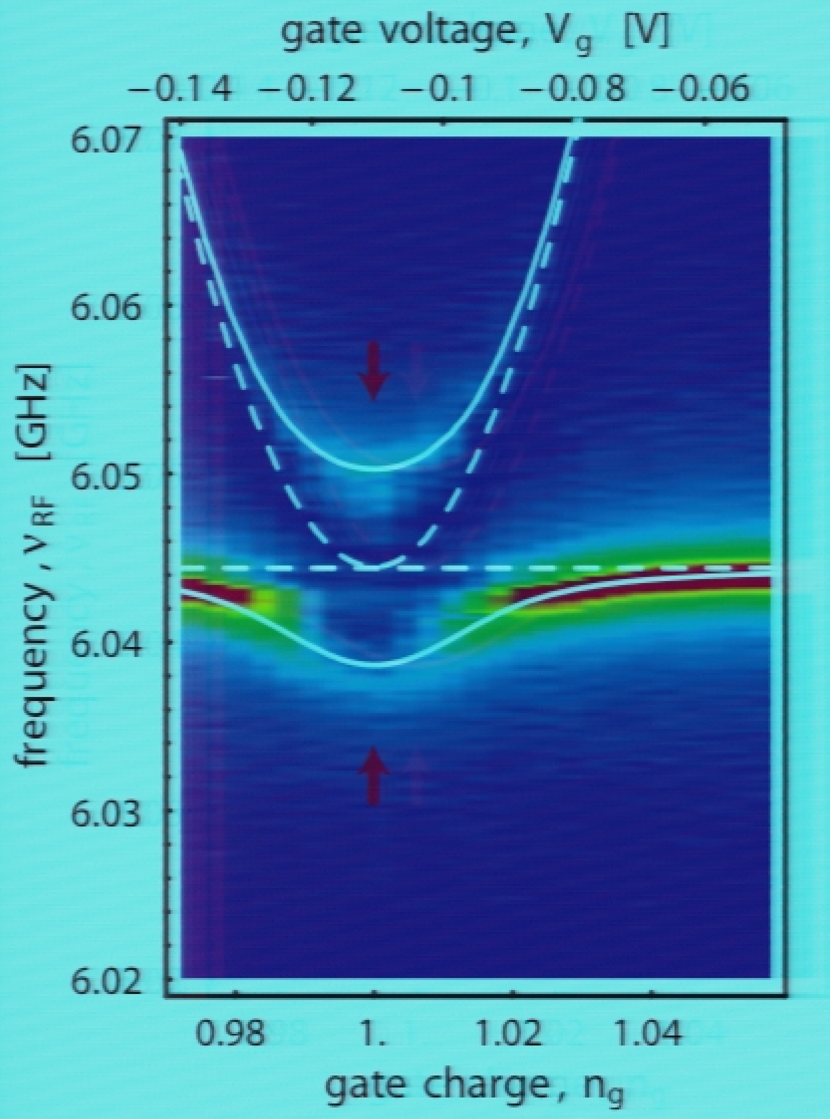
AB, Huang, Wallraff, Girvin & Schoelkopf, PRA 69, 062320 (2004)

Vacuum Rabi splitting

First observation in superconducting circuits



Wallraff, Schuster, AB, Frunzio, Huang, Majer,

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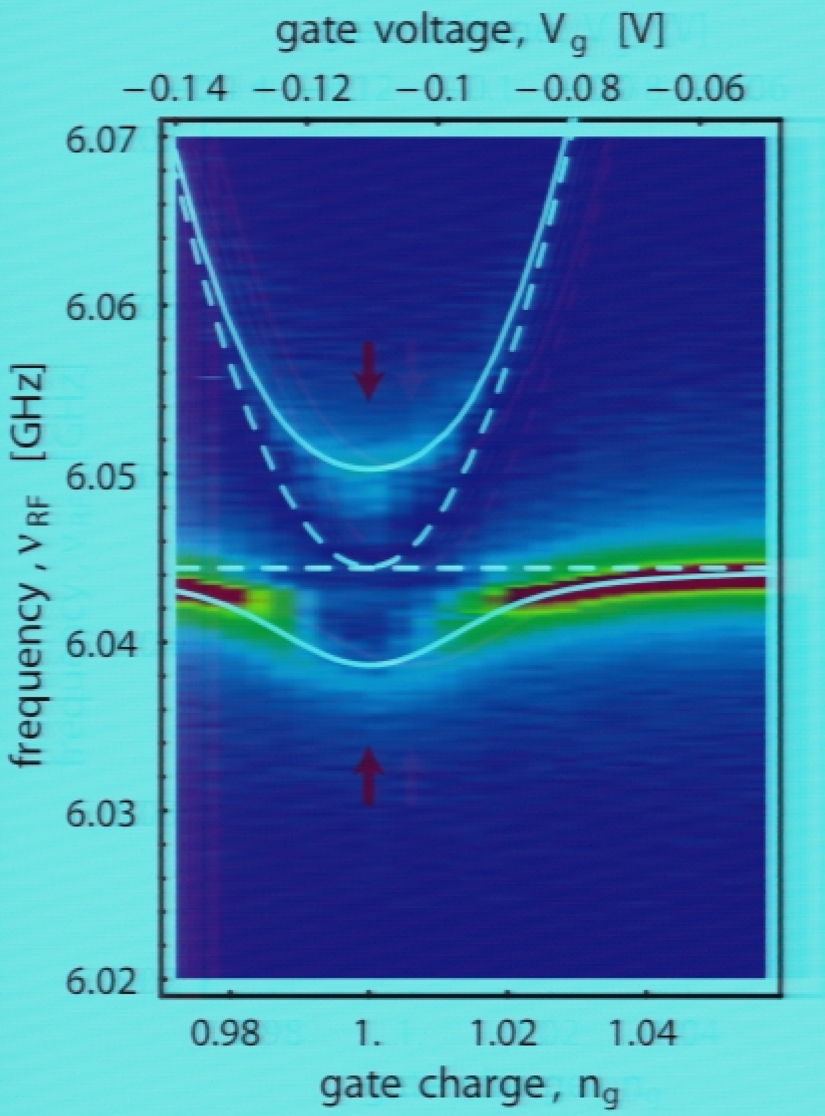

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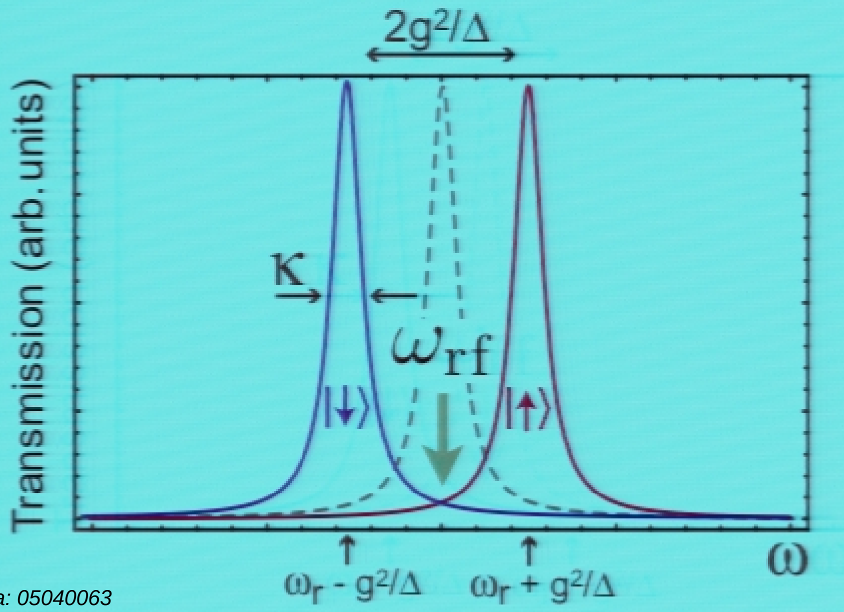

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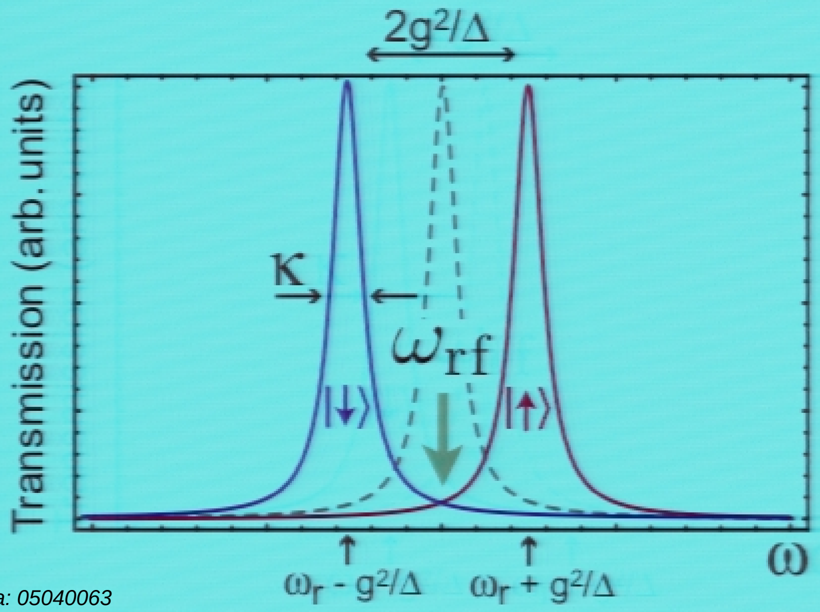

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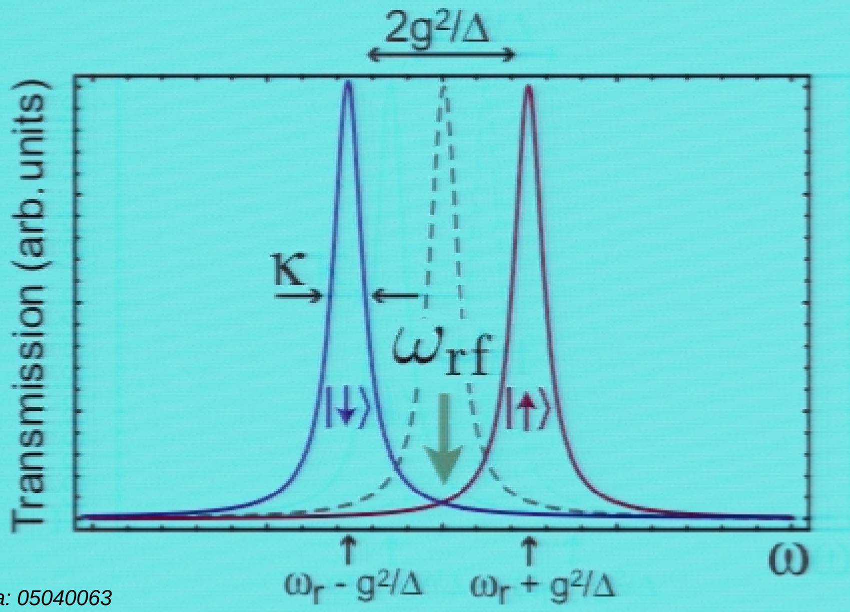

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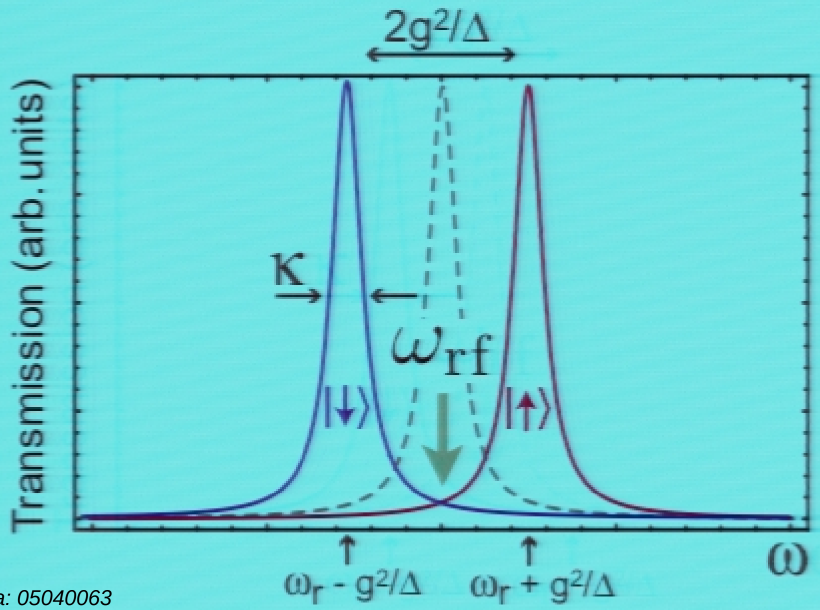

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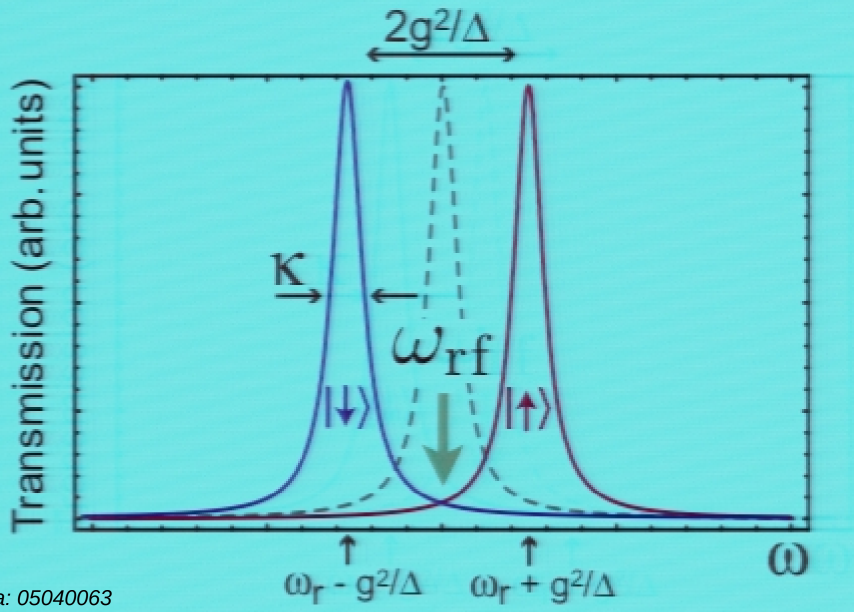

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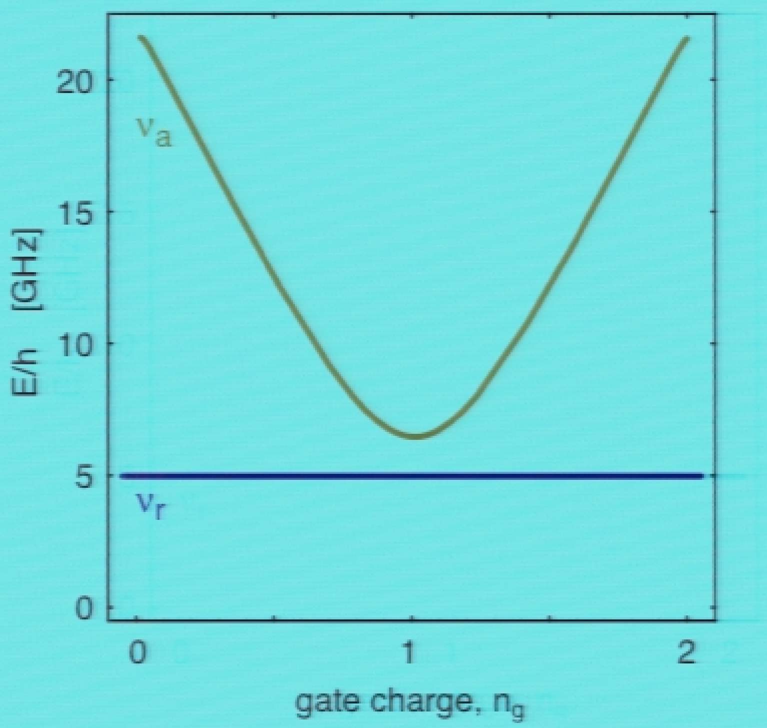

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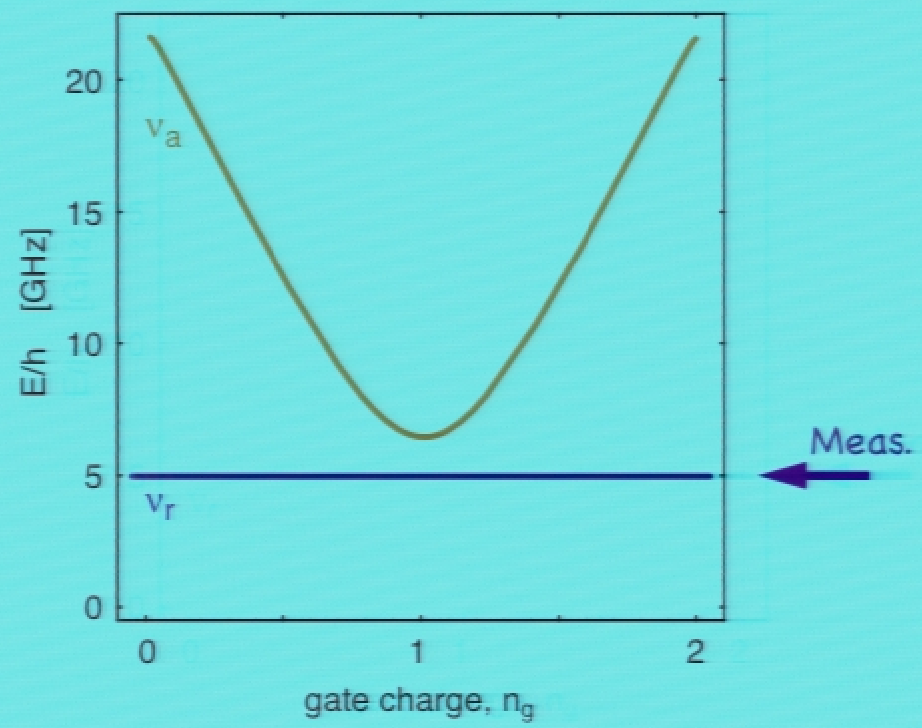

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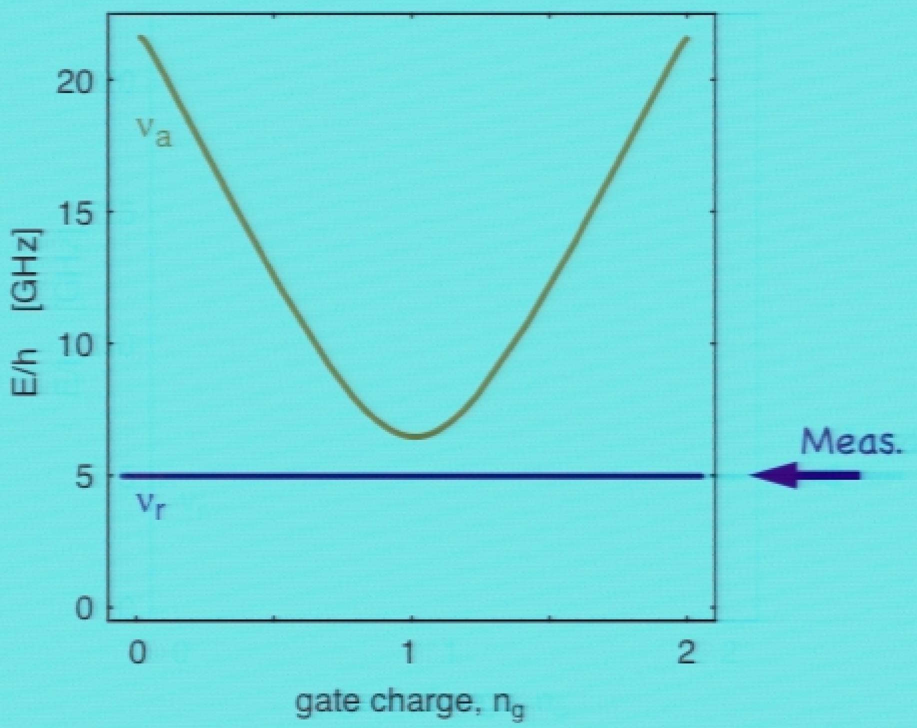

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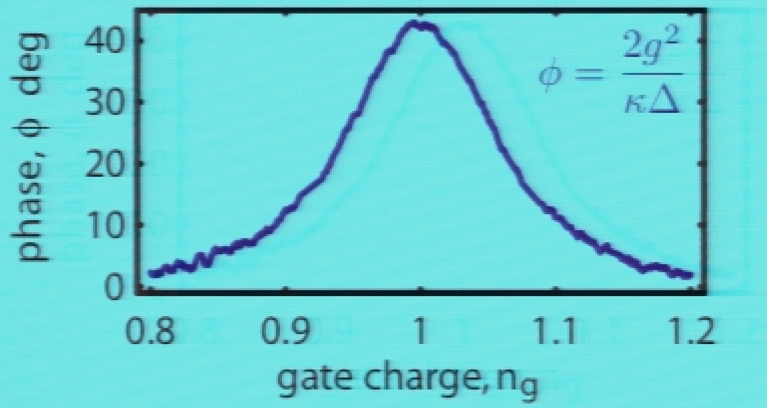

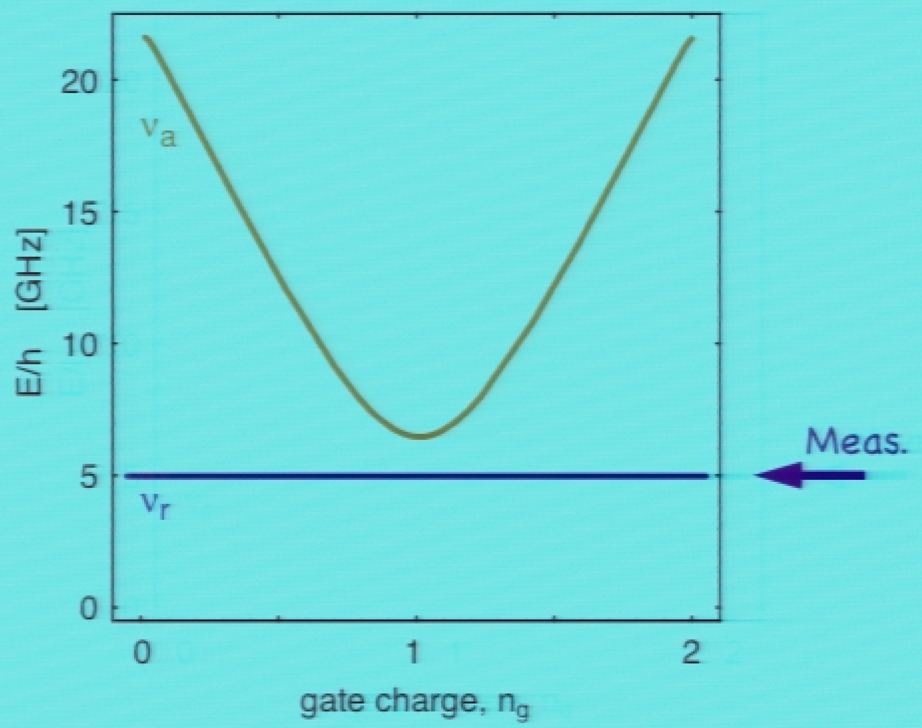

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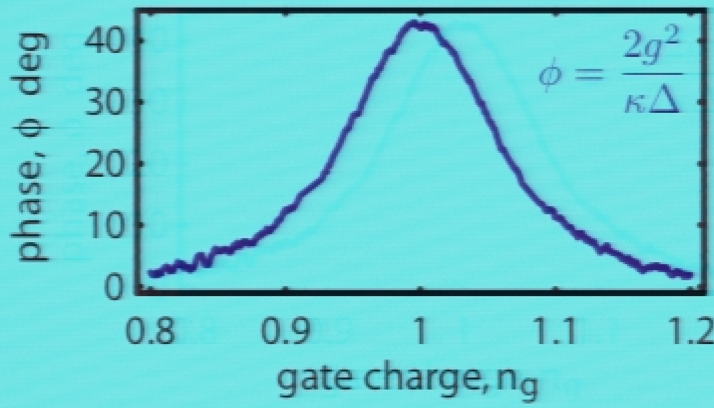

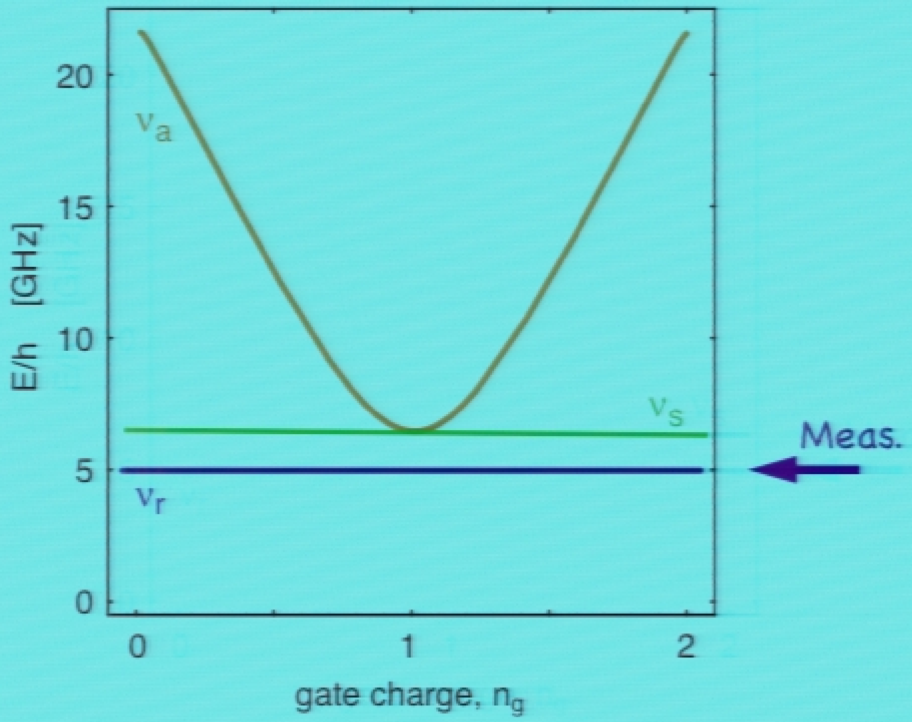

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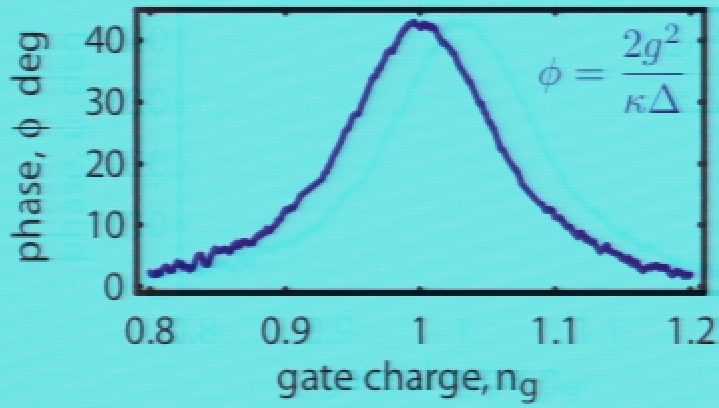

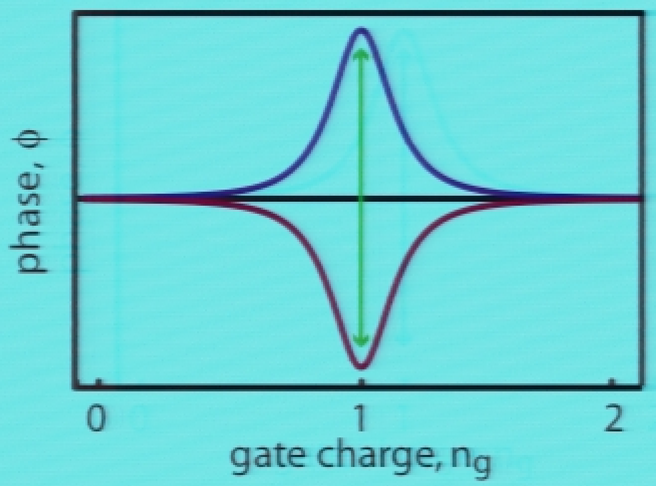

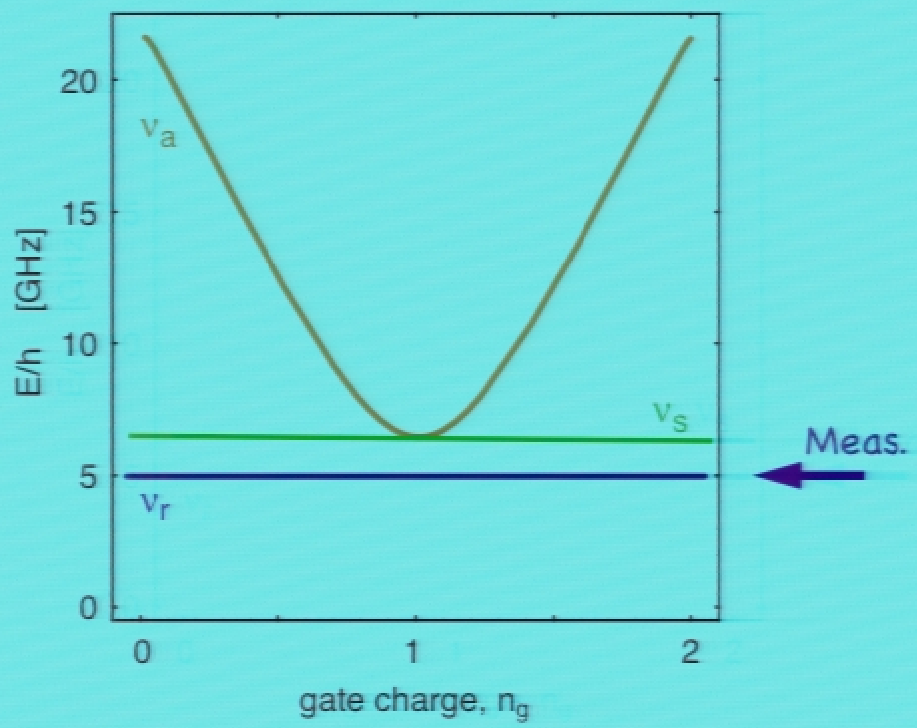

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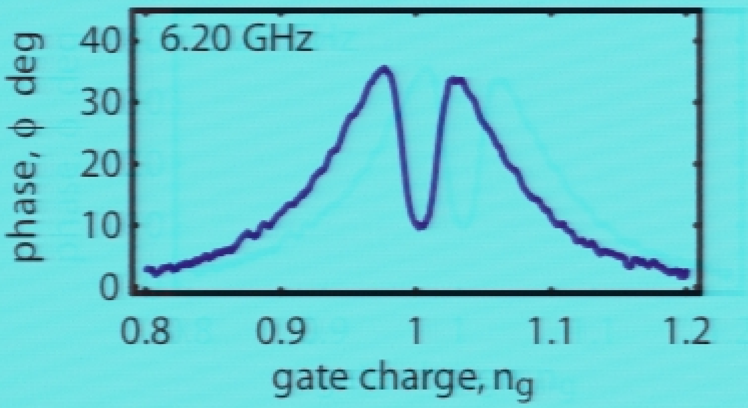
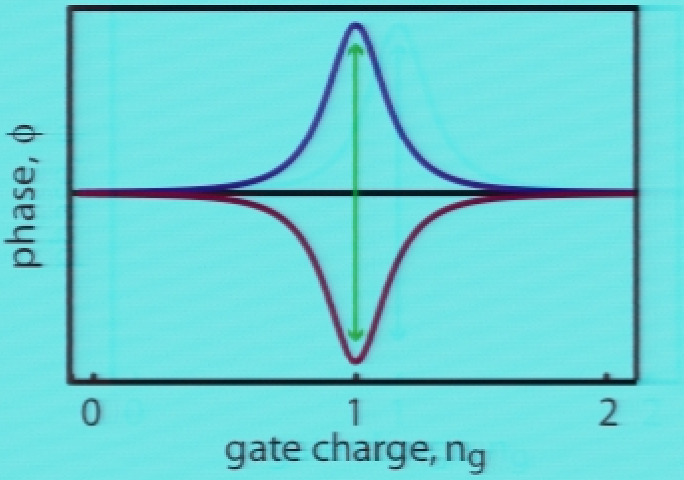
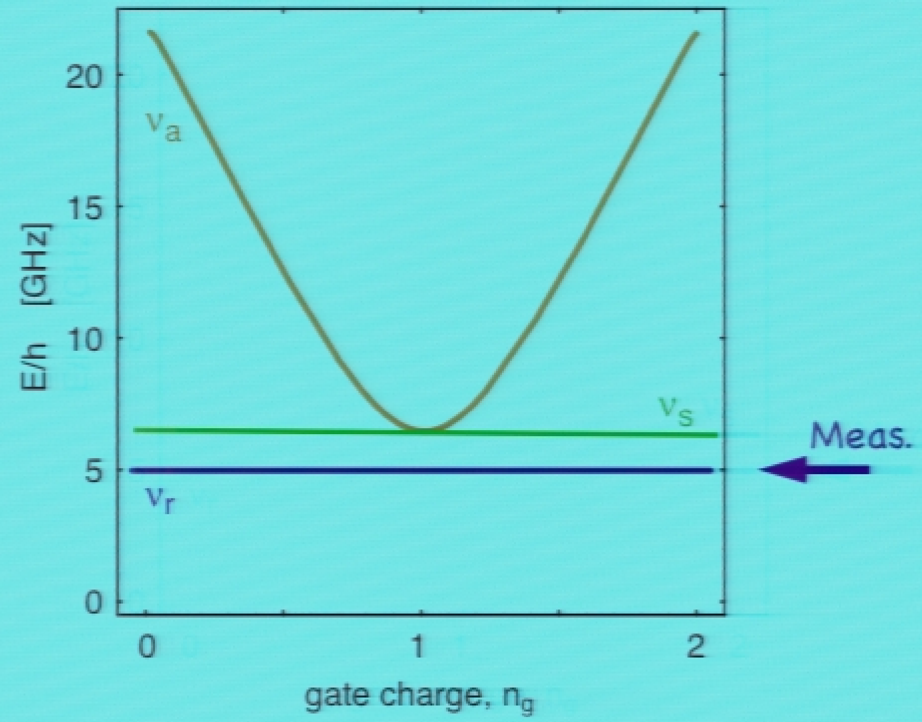
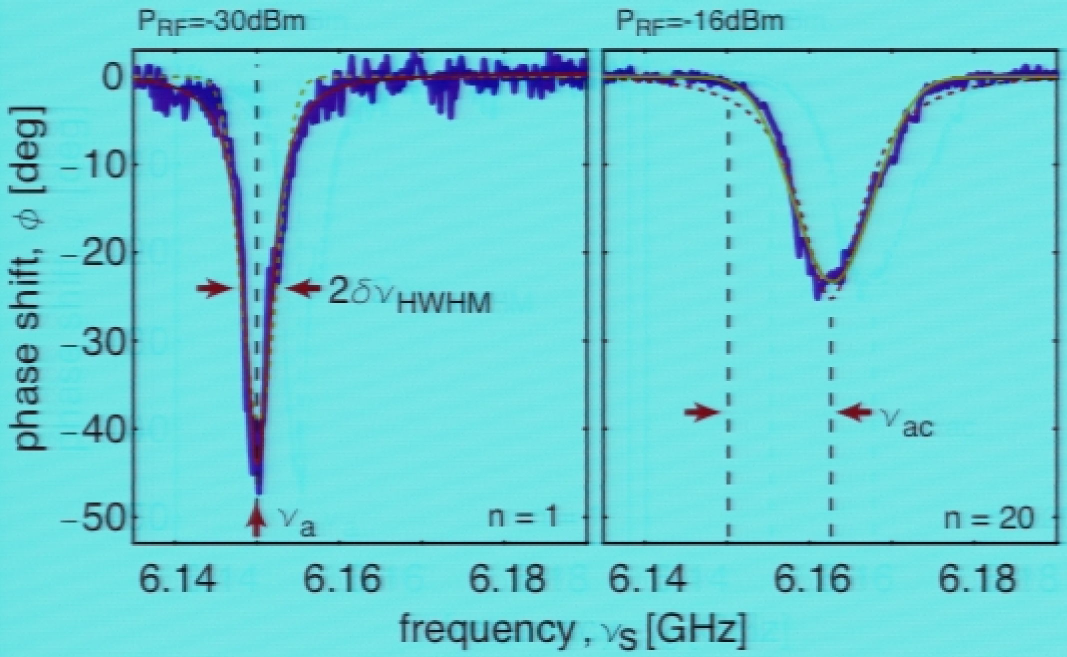






Ac-Stark Shift

- Fixed spectroscopy power
- Change read-out power



Schuster, Wallraff, AB, Girvin & Schoelkopf, PRL 94, 123602 (2005)

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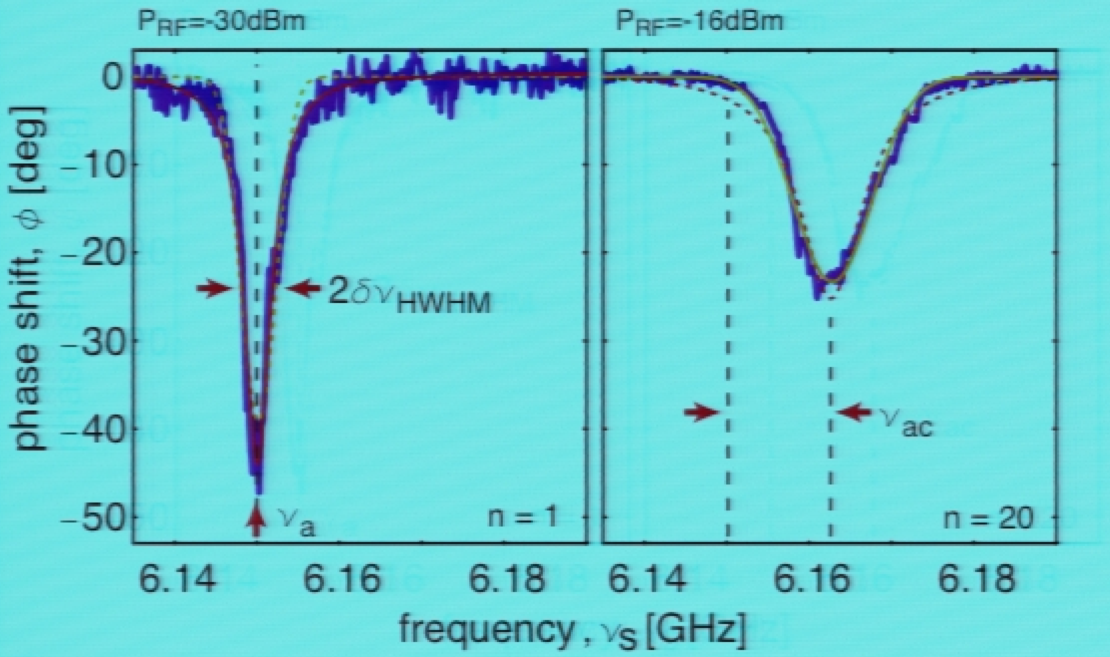

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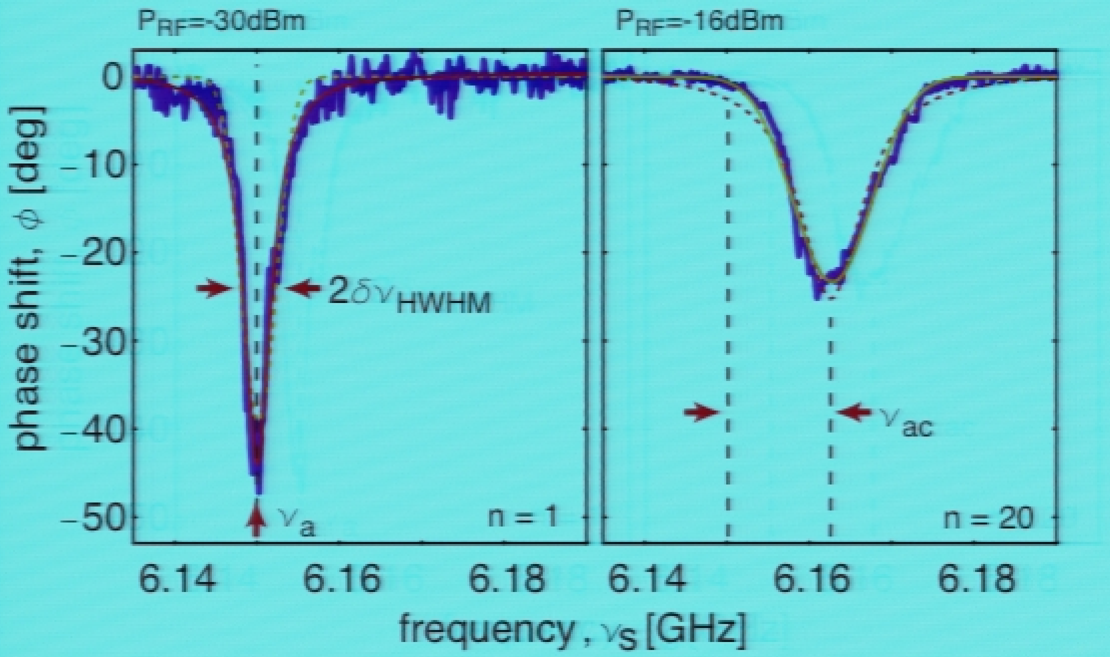

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Schuster, Wallraff, AB Girvin & Schoelkopf, PRL 94, 123602 (2005)

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Schuster, Wallraff, AB Girvin & Schoelkopf, PRL 94, 123602 (2005)

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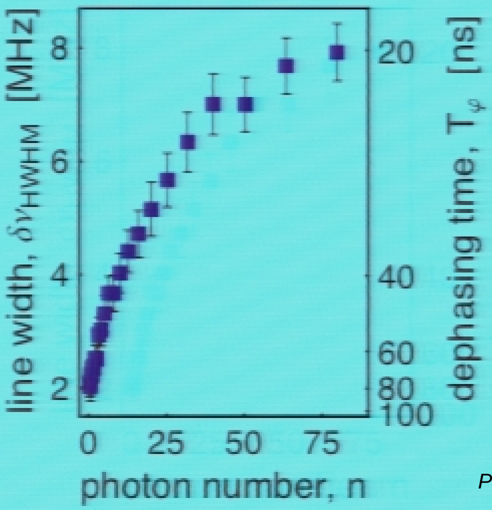

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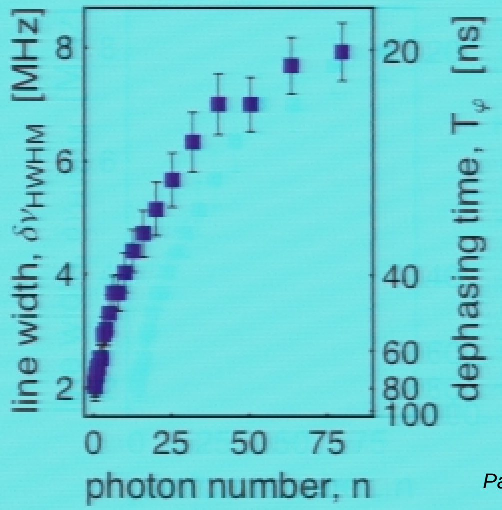

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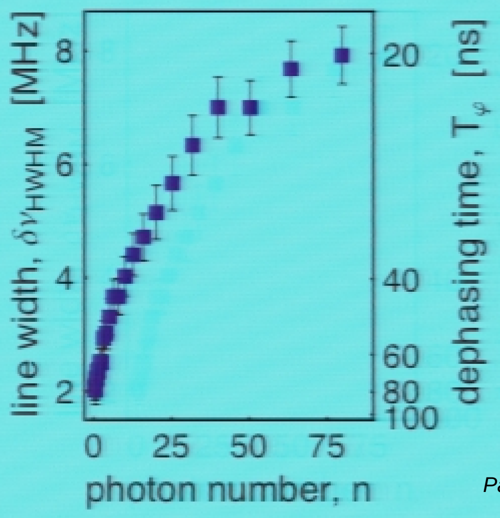

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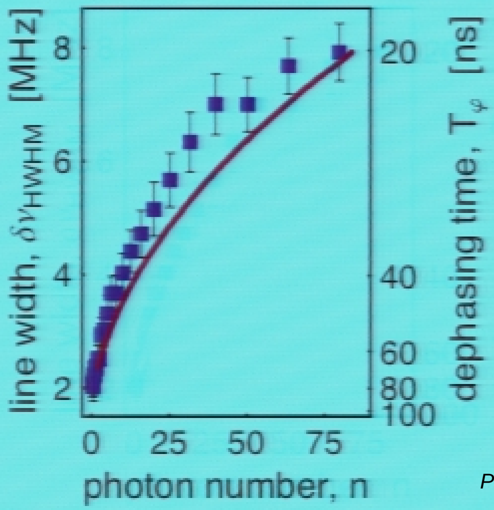

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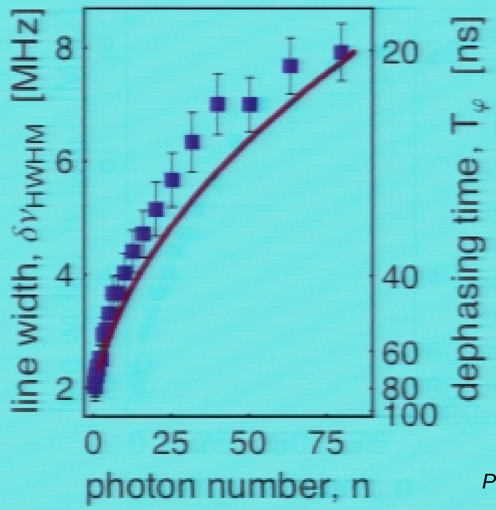

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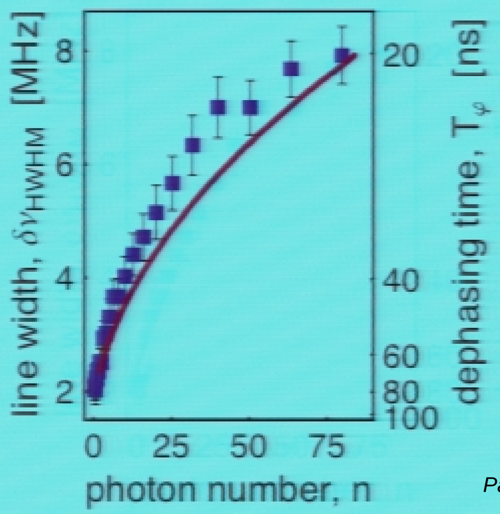

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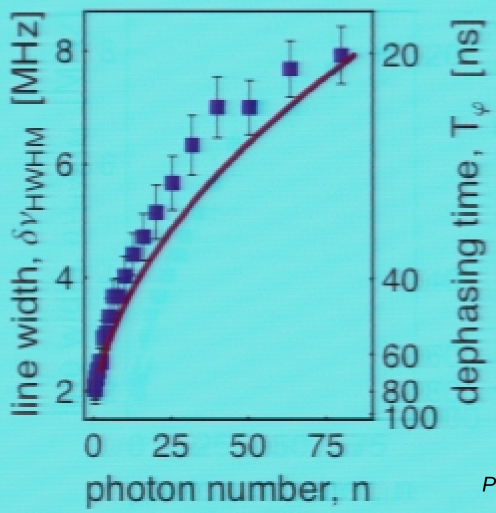

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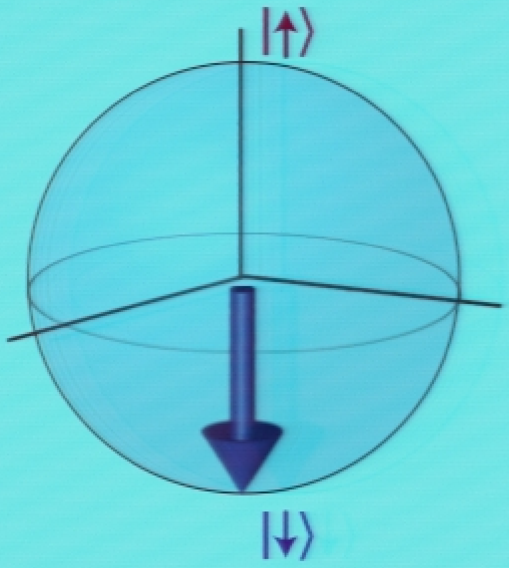
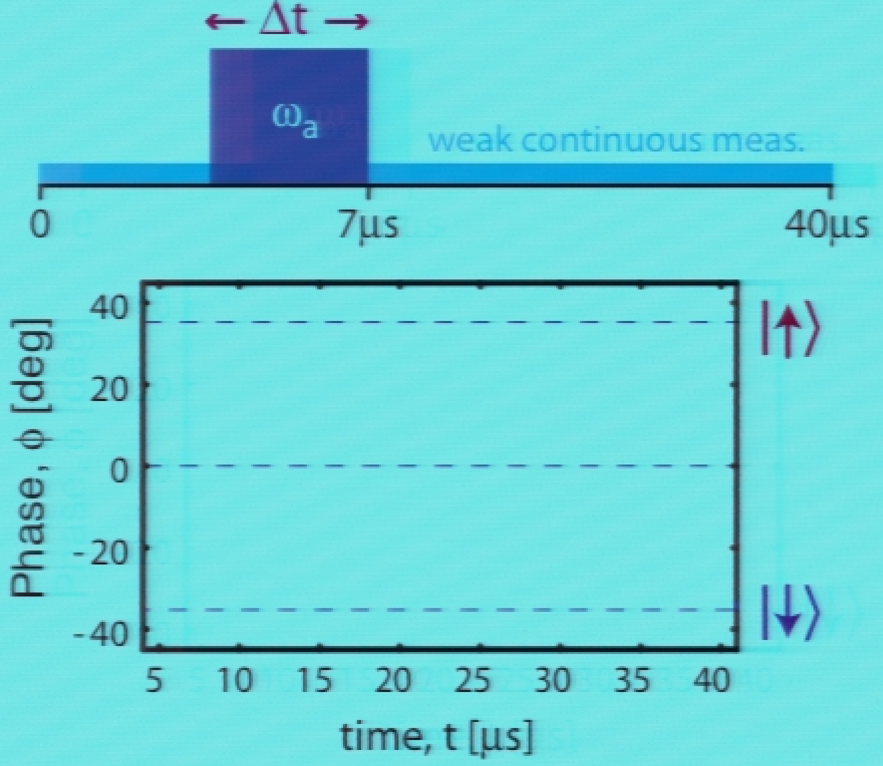
Time-domain control

Coherent control

Temporal response





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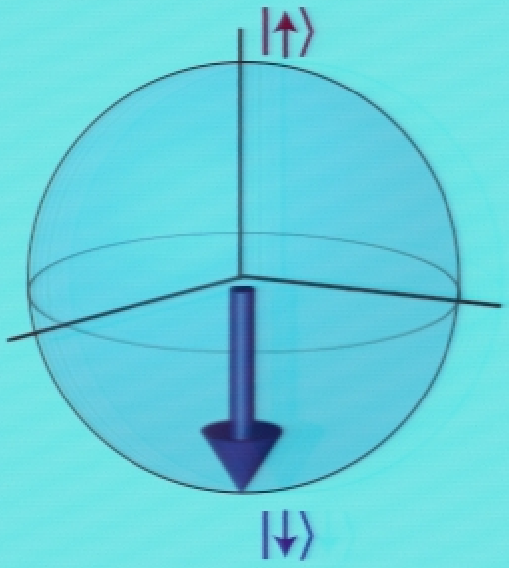

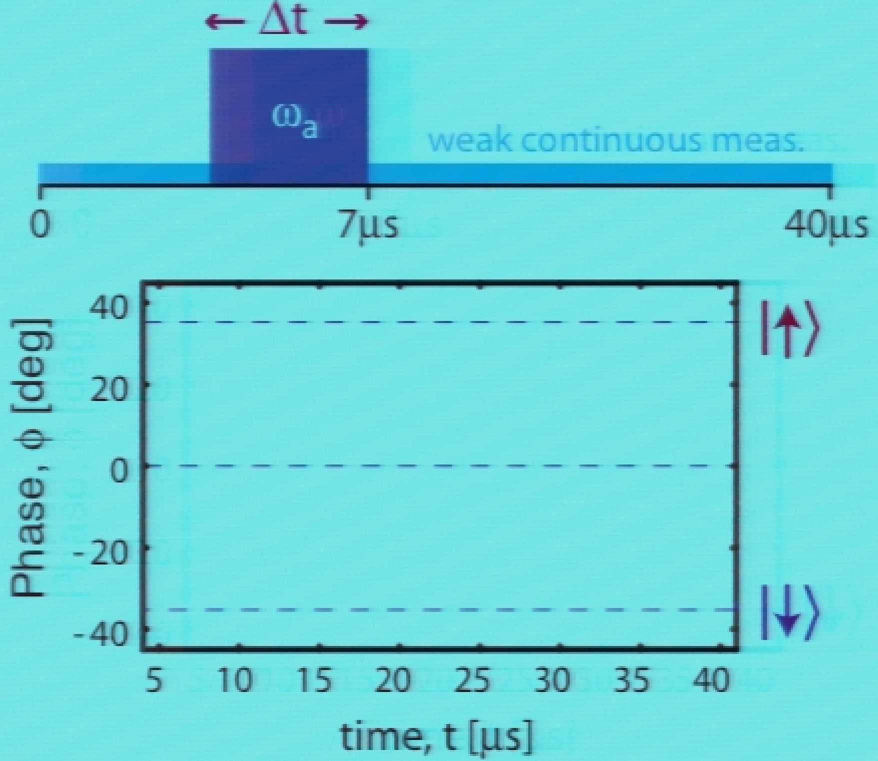

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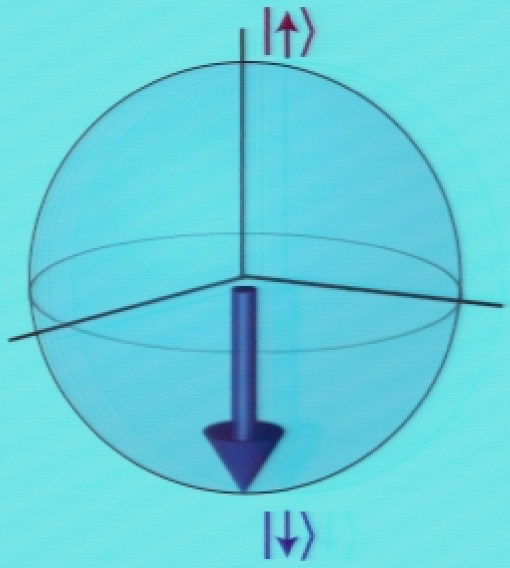

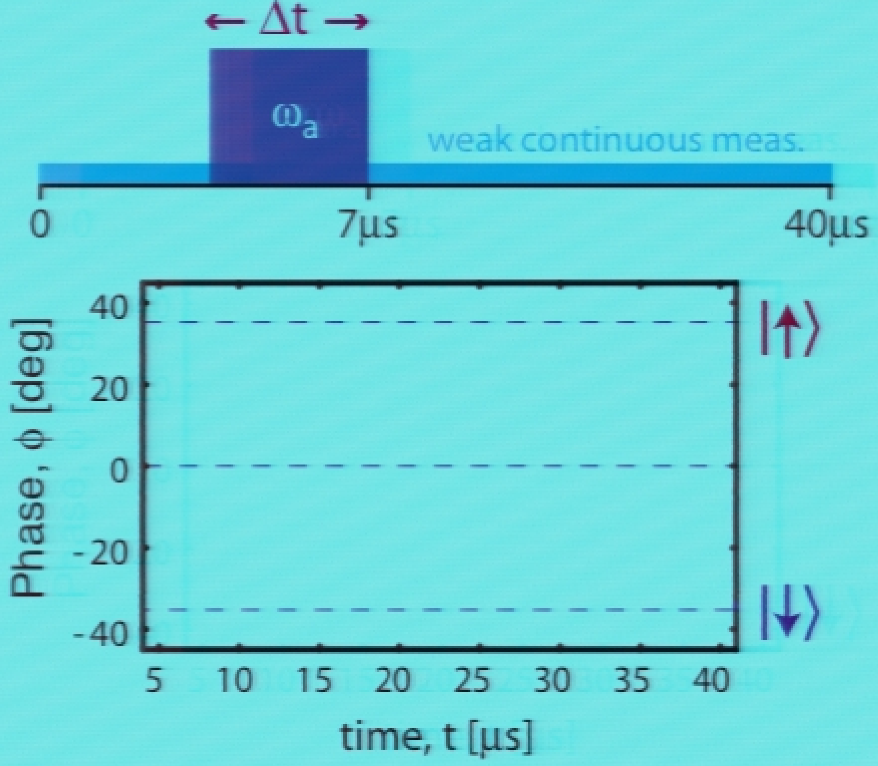

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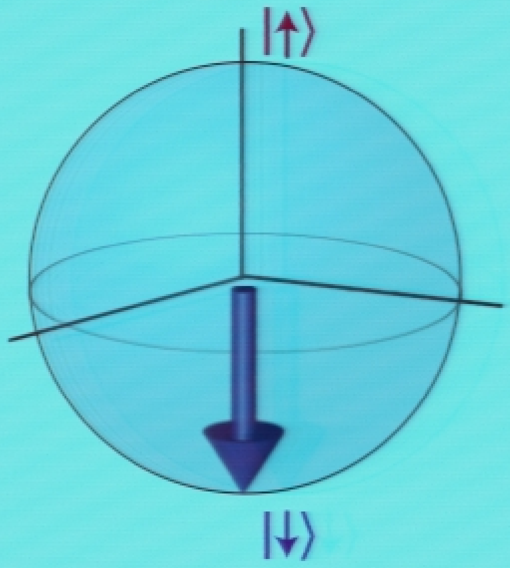

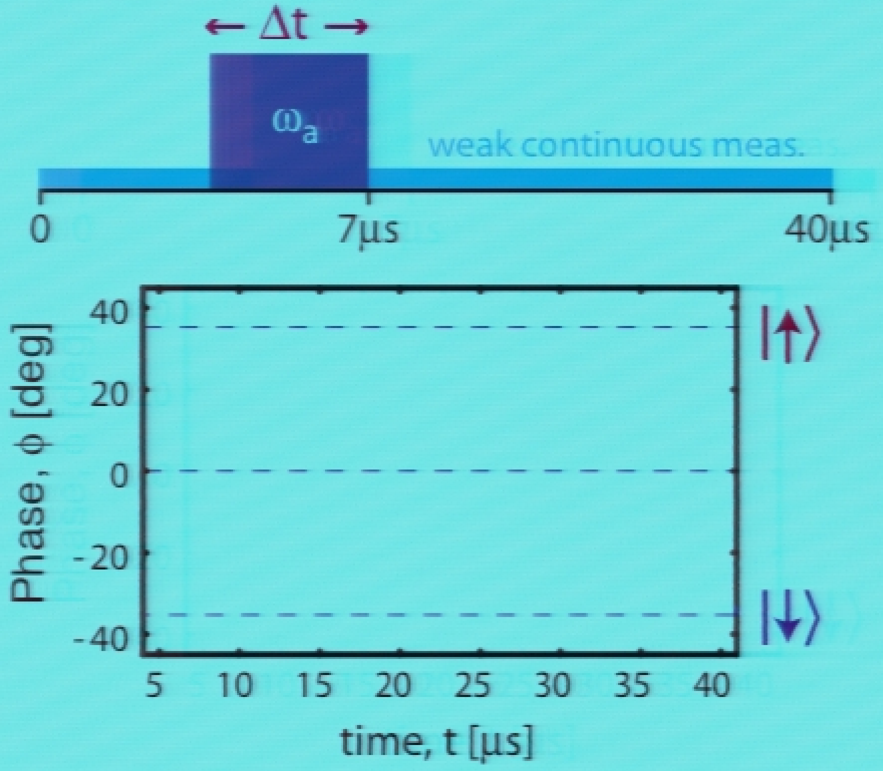

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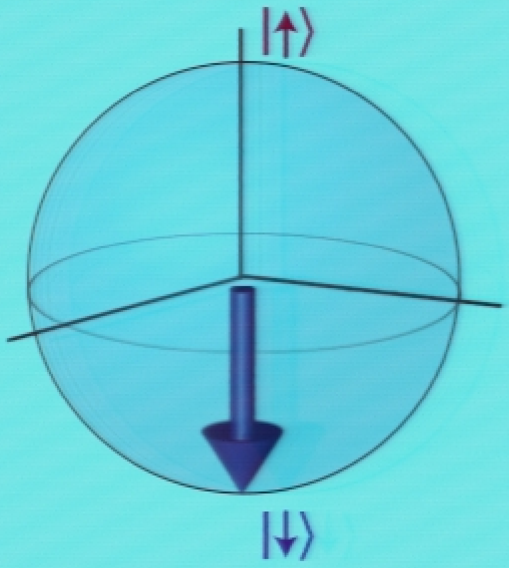

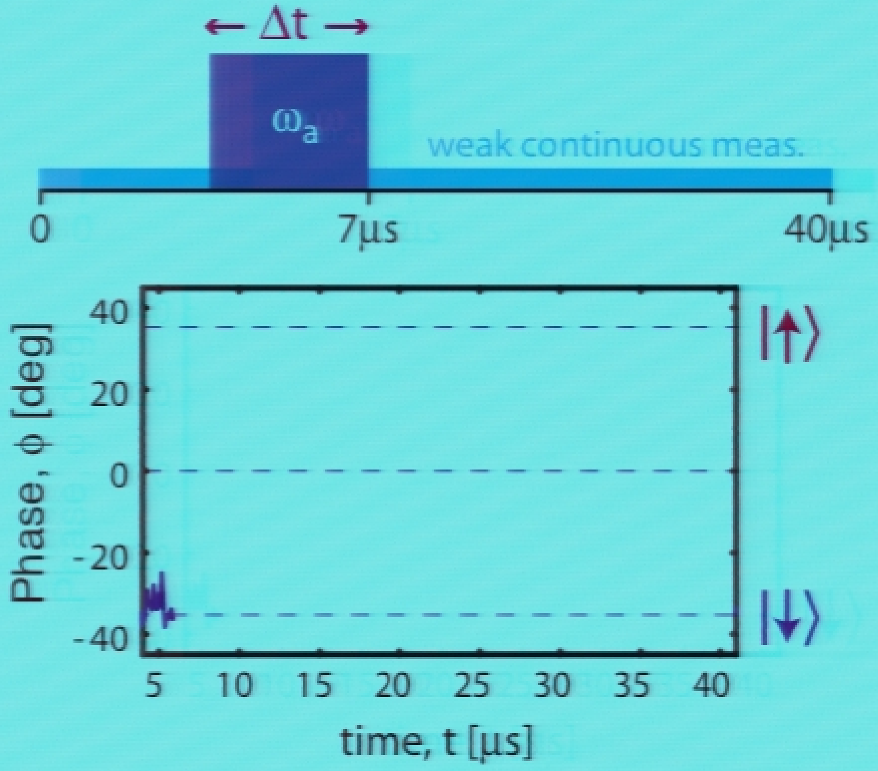

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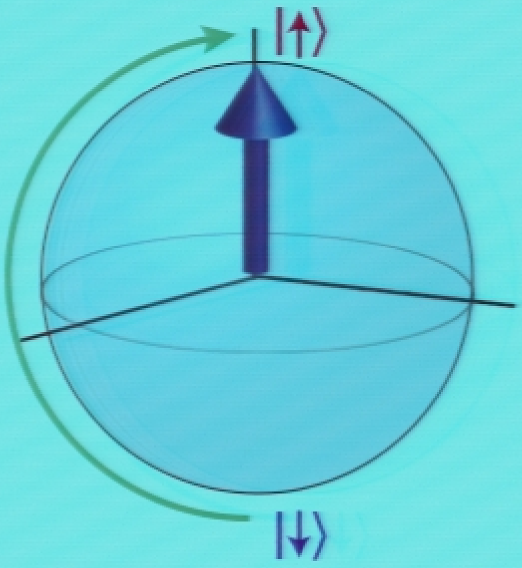
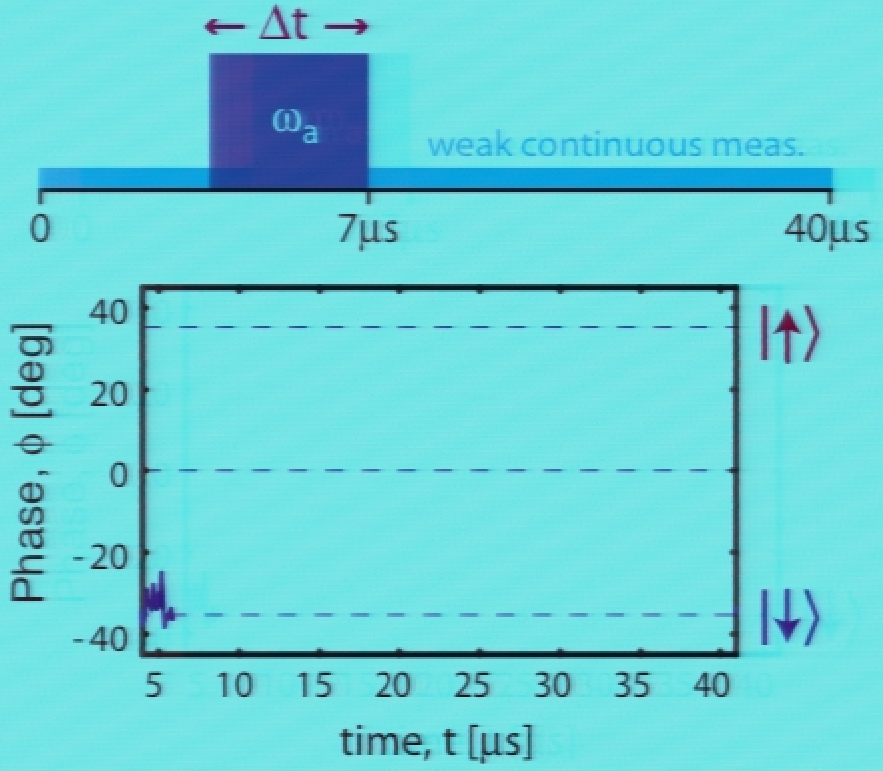
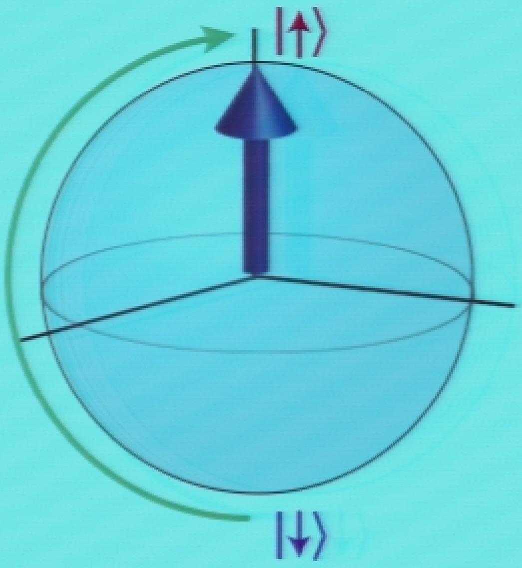
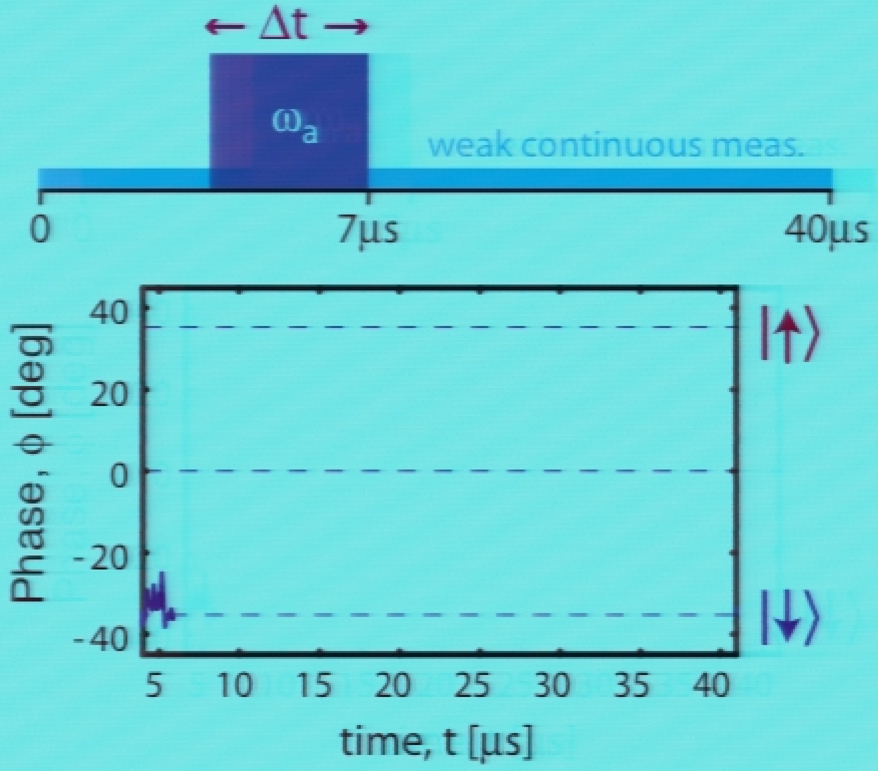




Coherent control

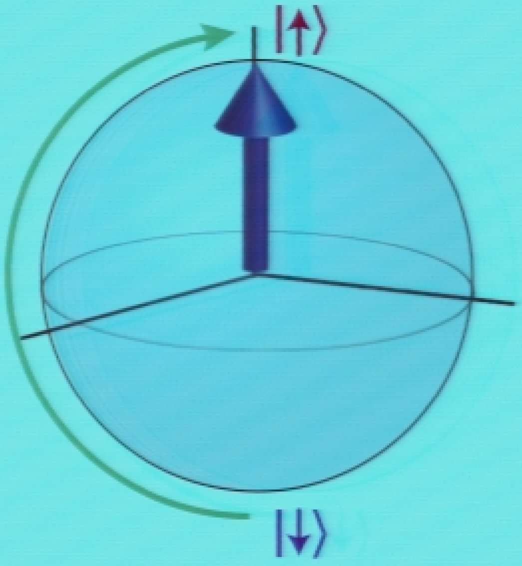
Temporal response

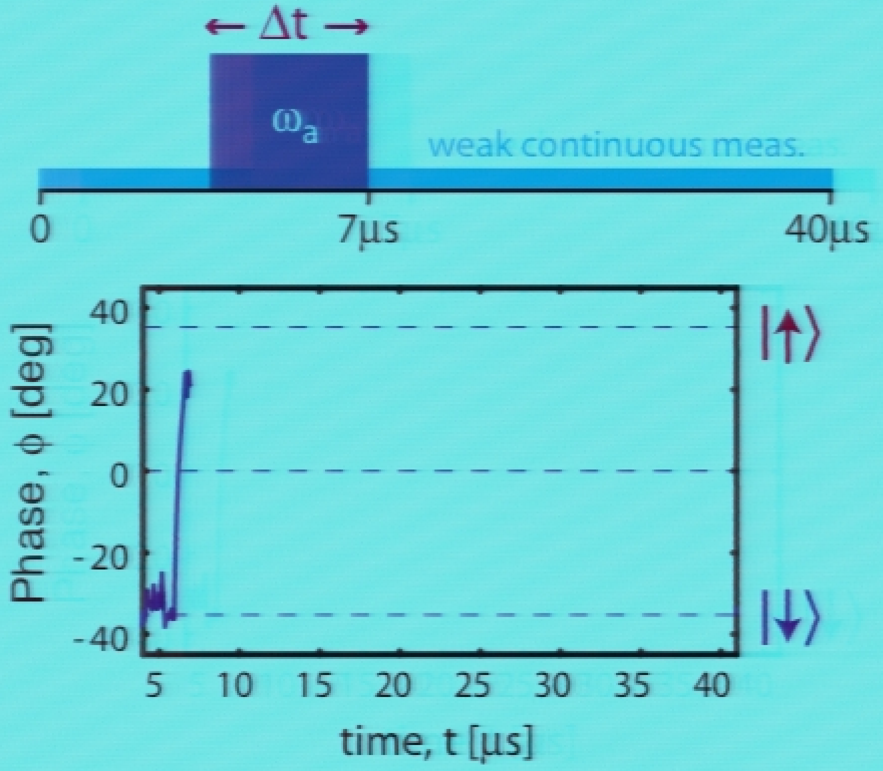




Coherent control

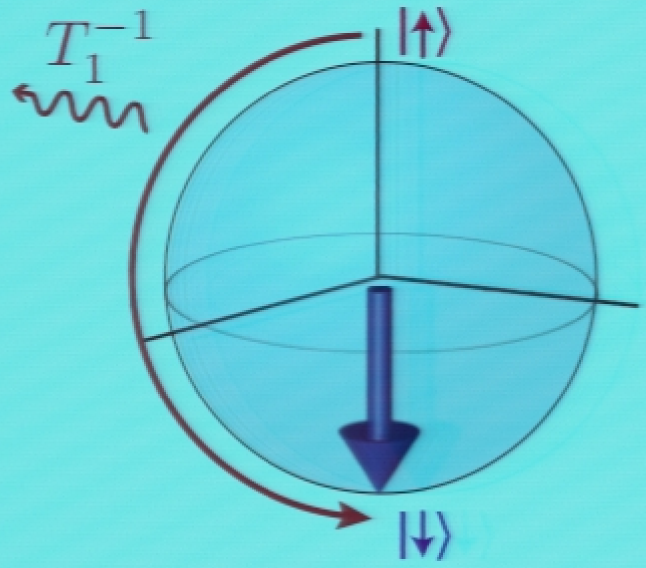
Temporal response

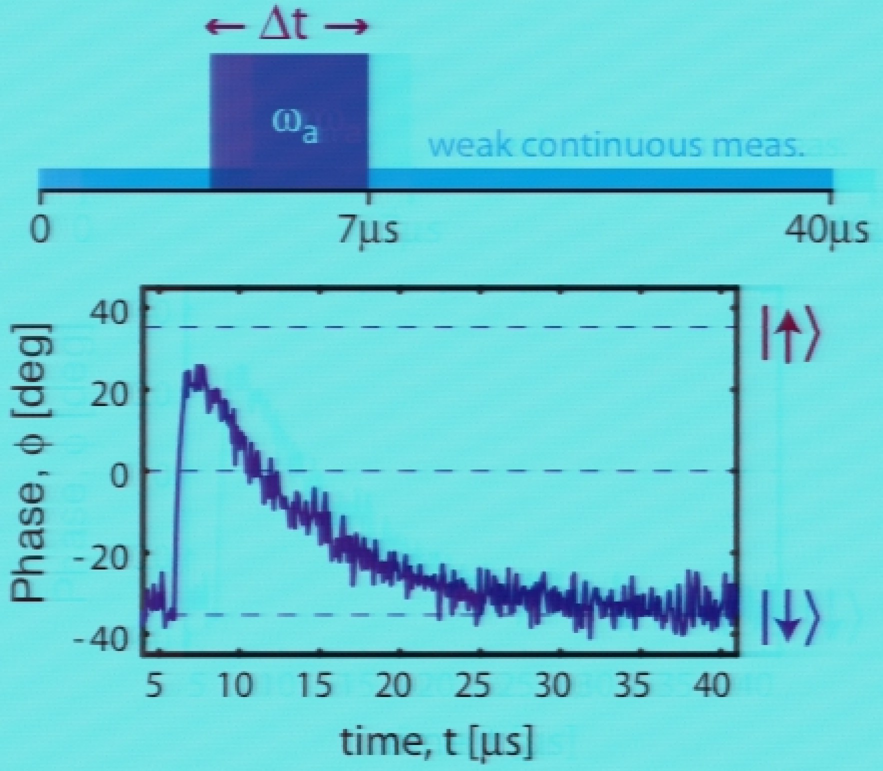




Coherent control

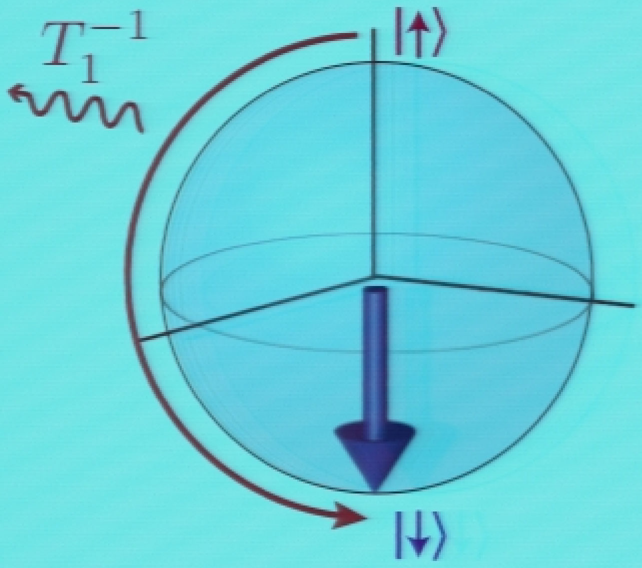
Temporal response

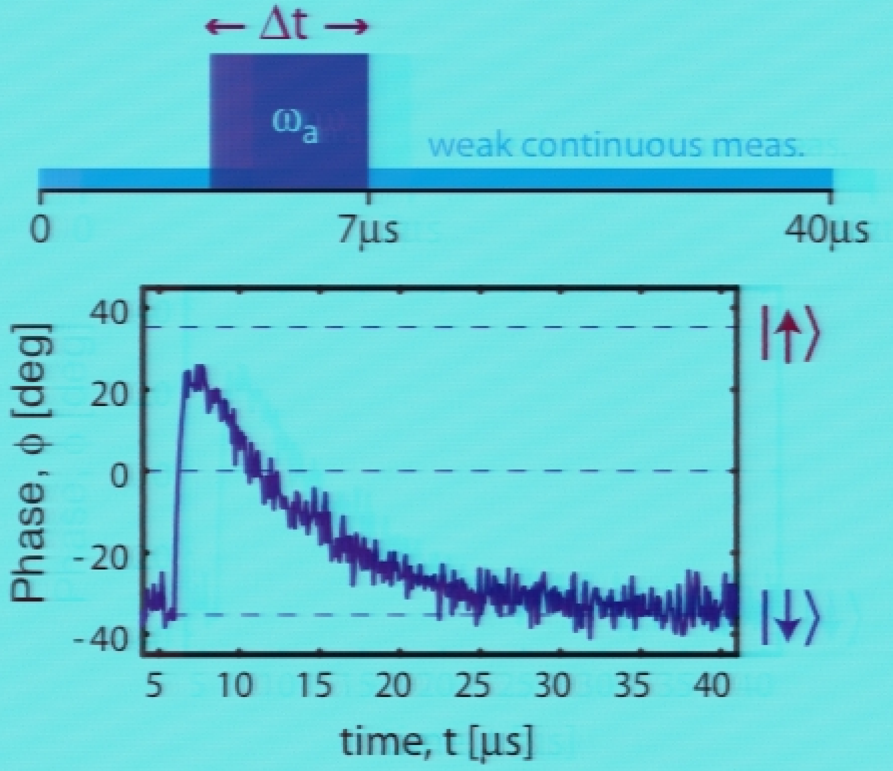




Coherent control

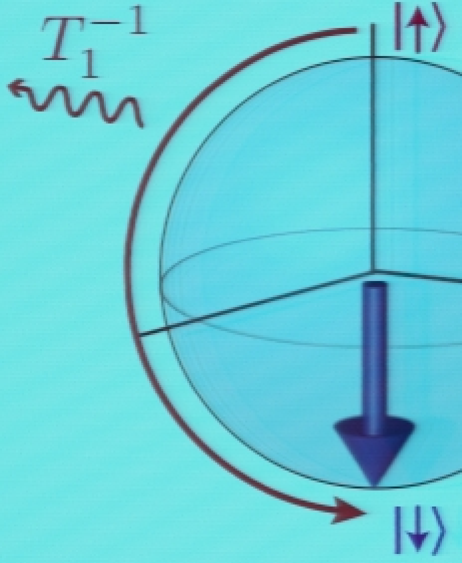
Temporal response

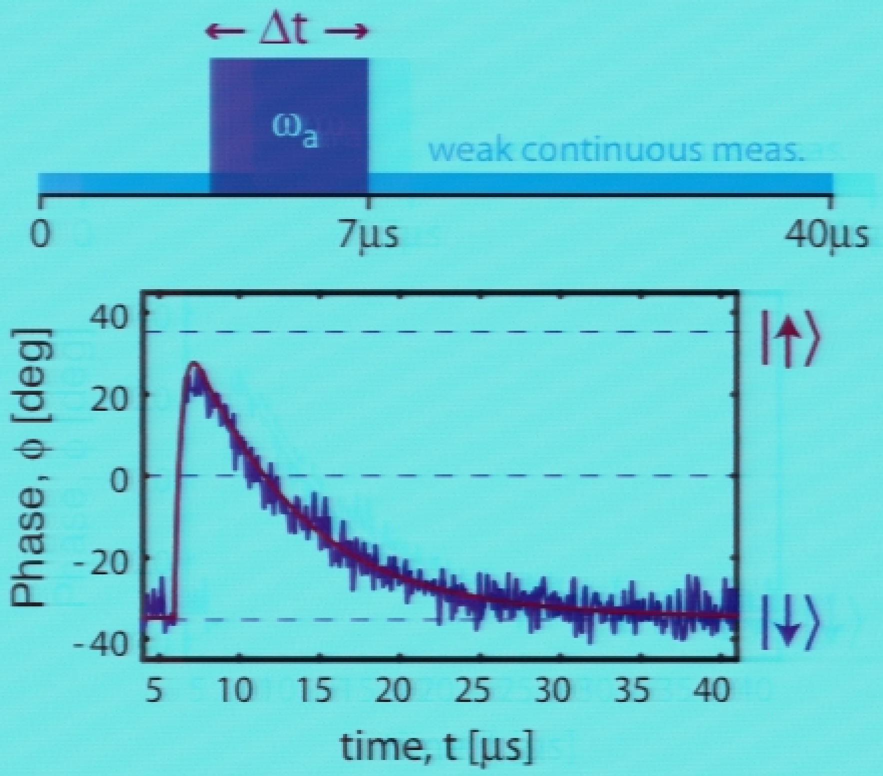




Coherent control

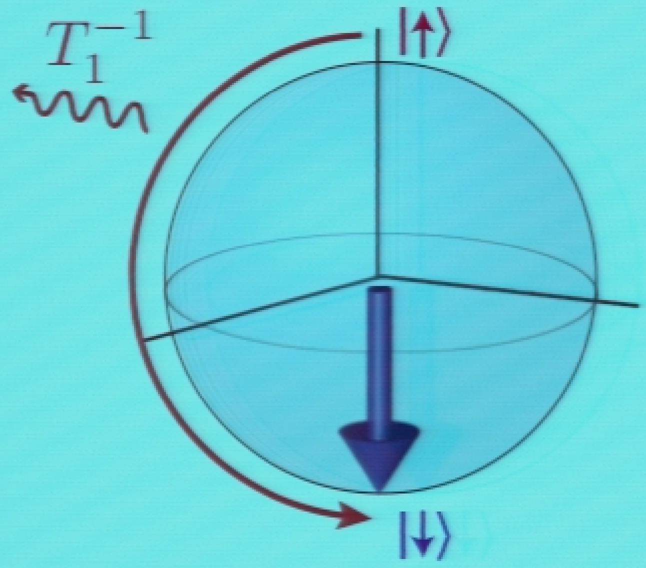
Temporal response

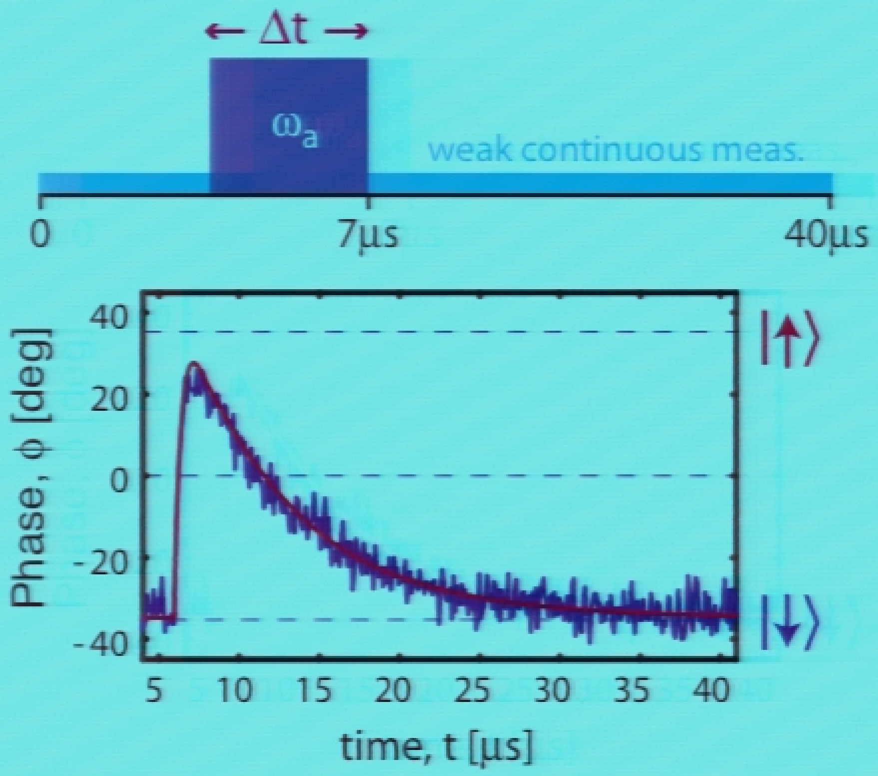




Coherent control

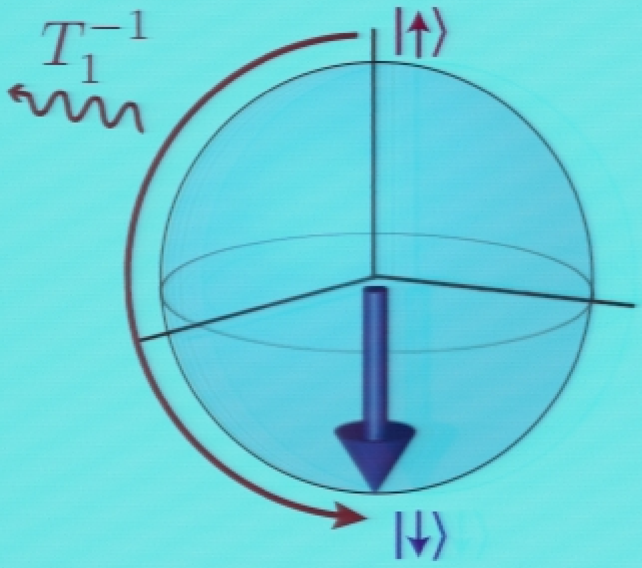
Temporal response

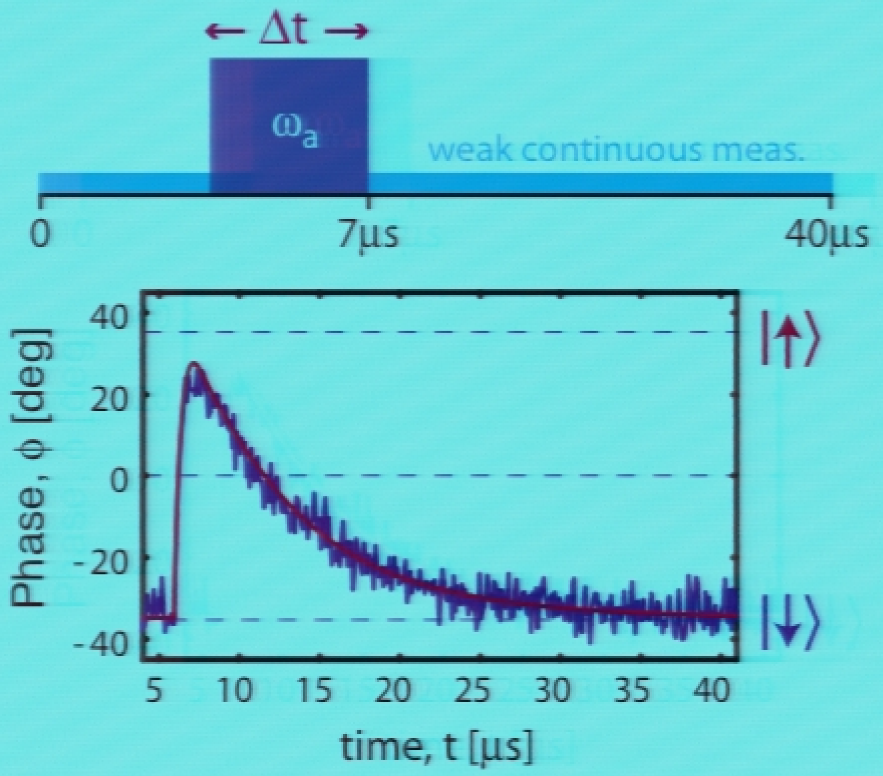




Coherent control

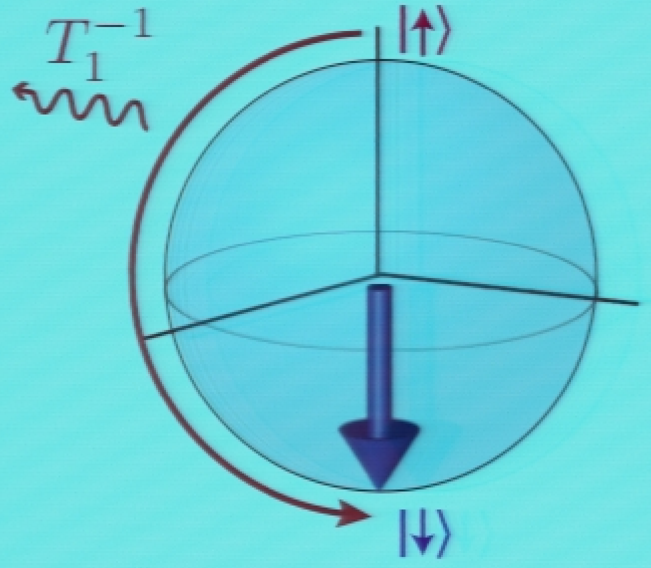
Temporal response

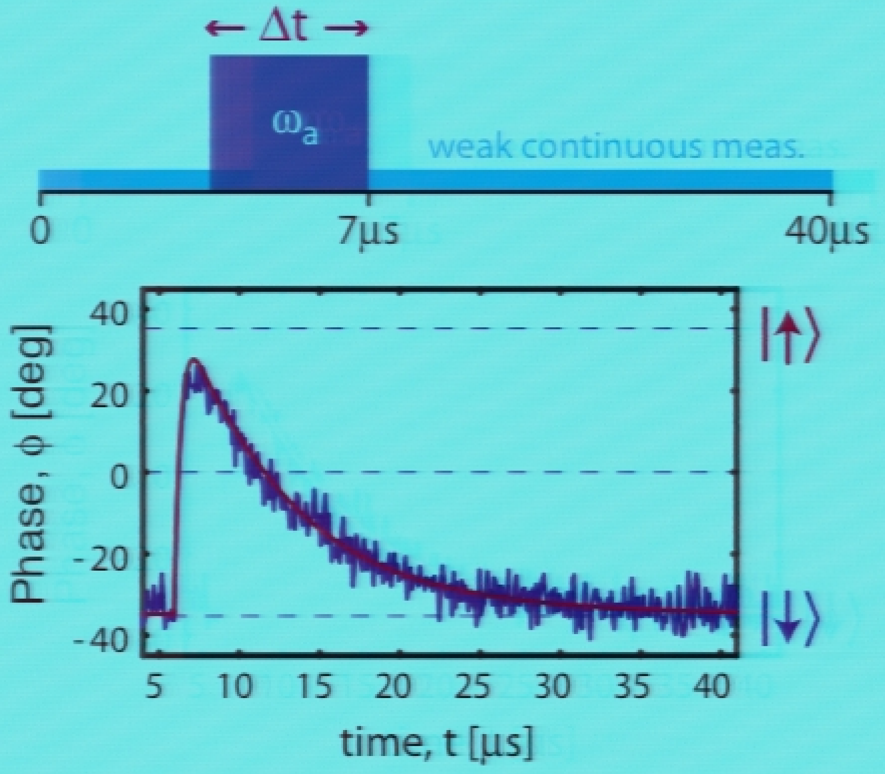




Coherent control

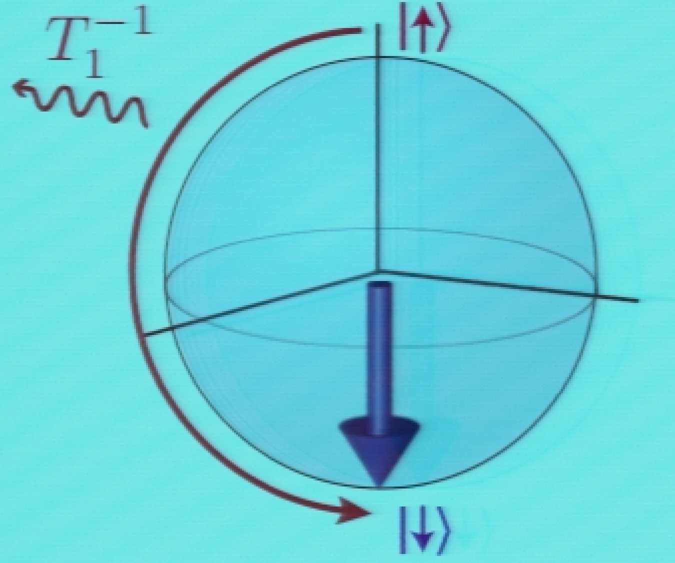
Temporal response

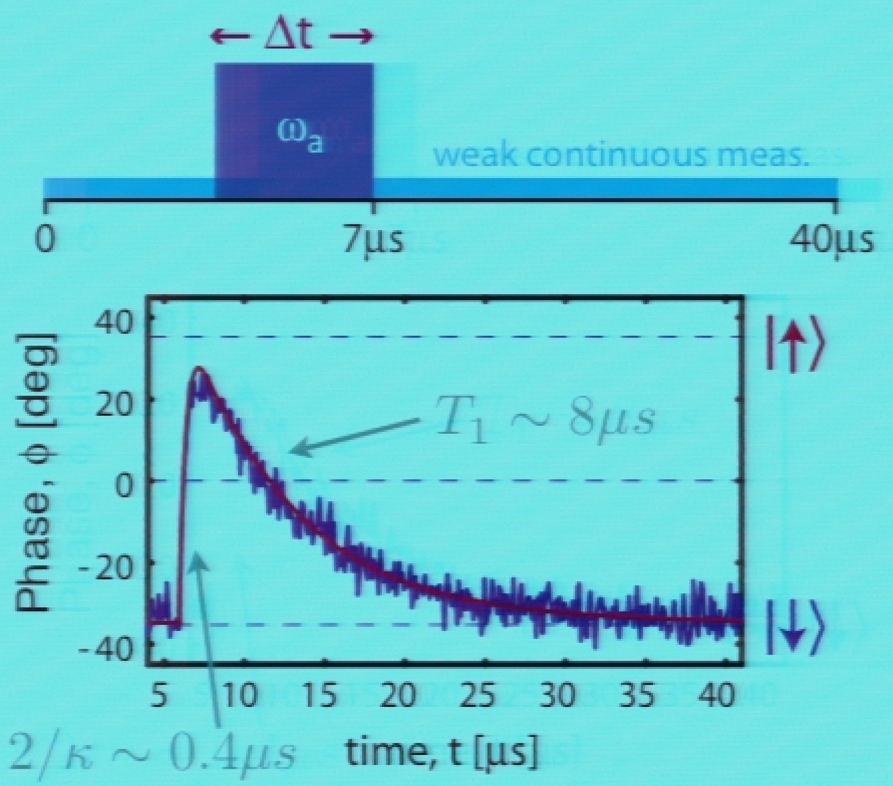




Coherent control

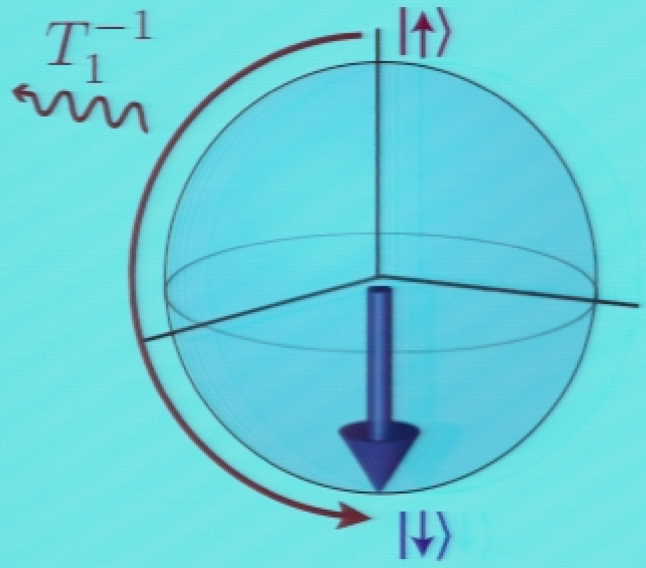
Temporal response

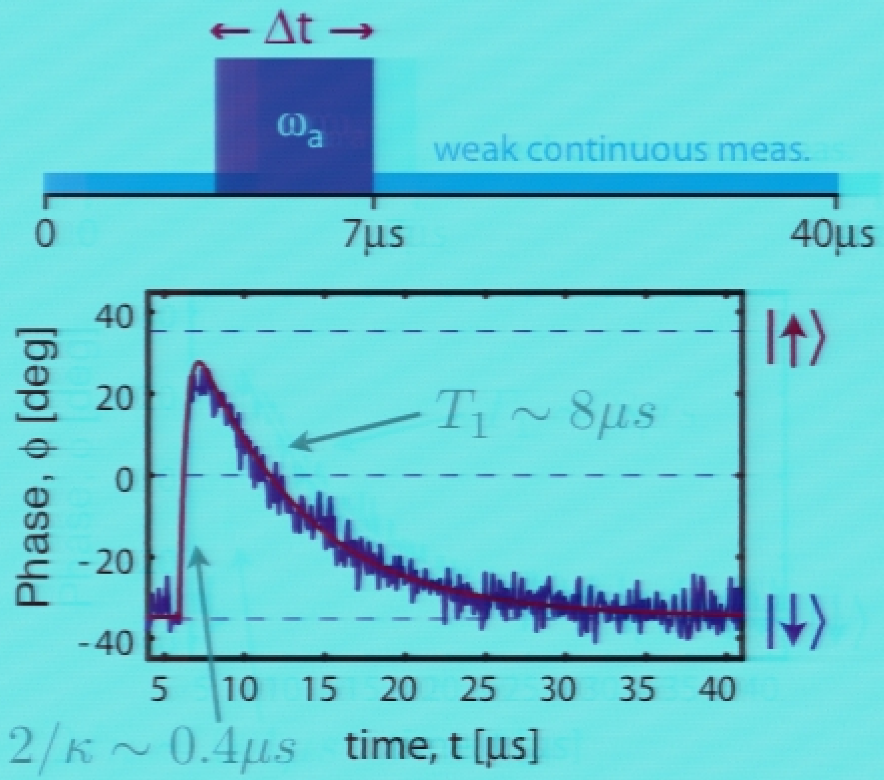




Coherent control

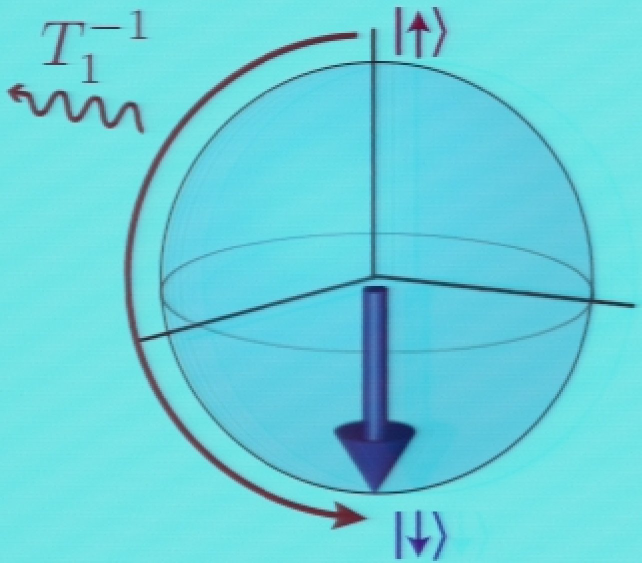
Temporal response

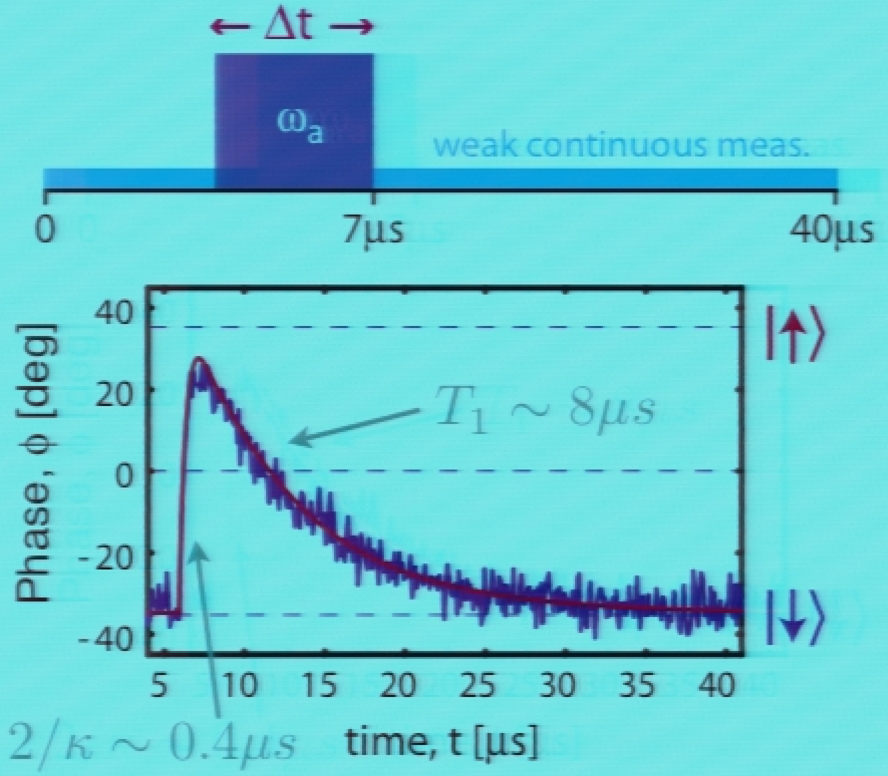




Coherent control

Temporal response



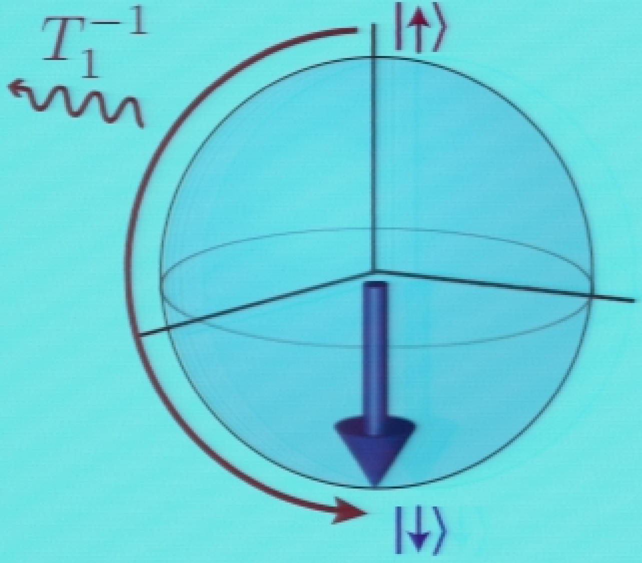


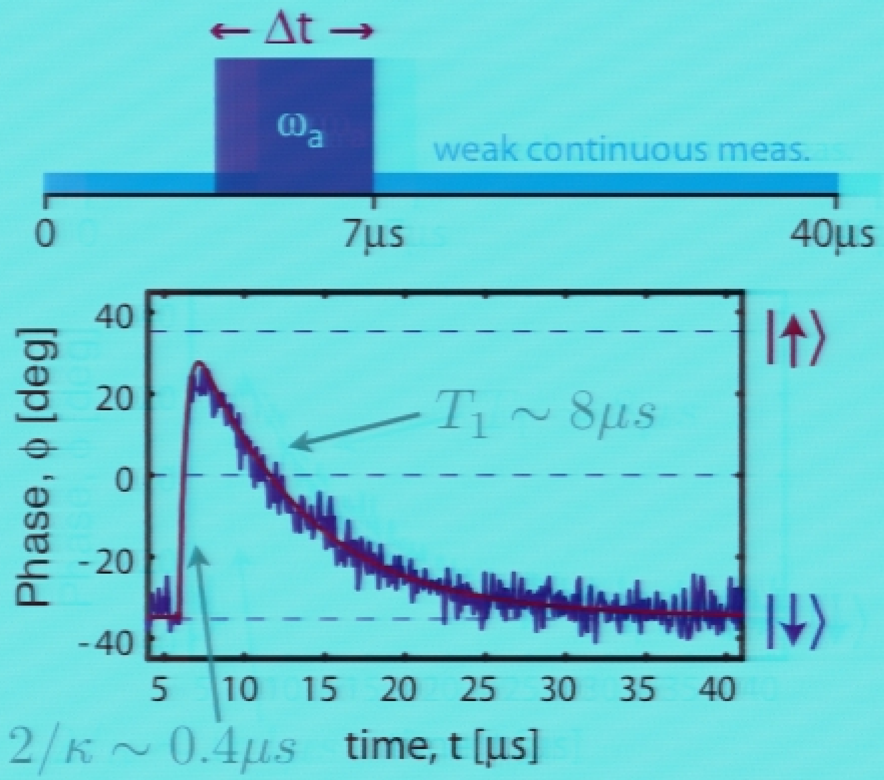
$T_1 \sim 8\mu s$

$2/\kappa \sim 0.4\mu s$

Coherent control

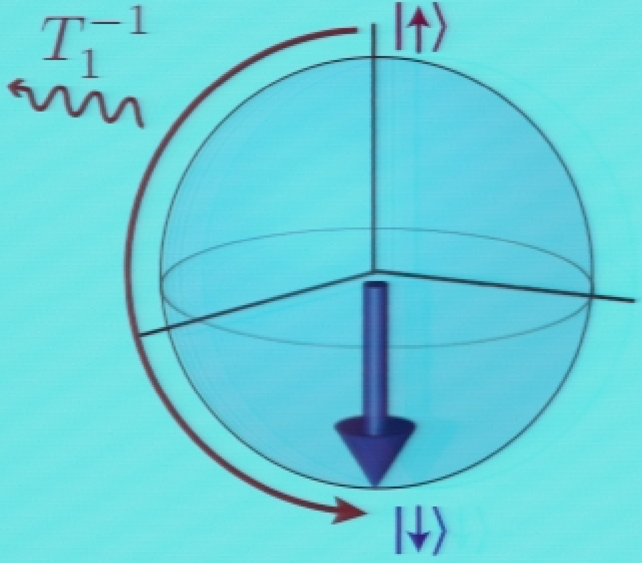
Temporal response

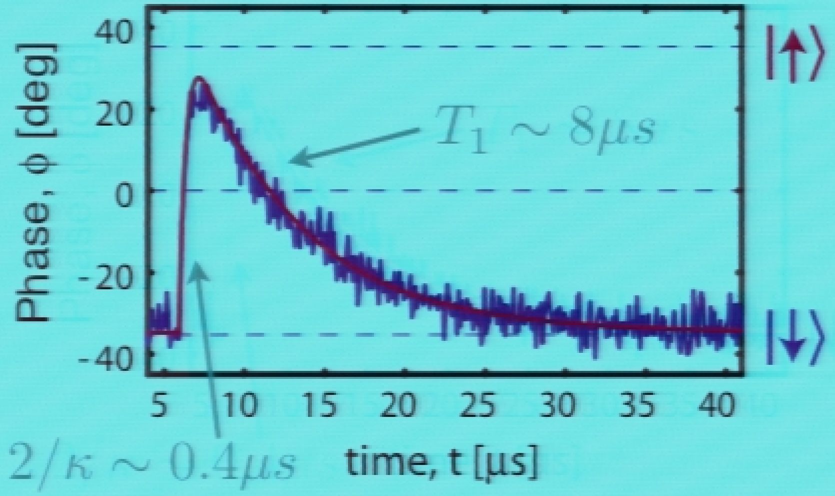




Coherent control

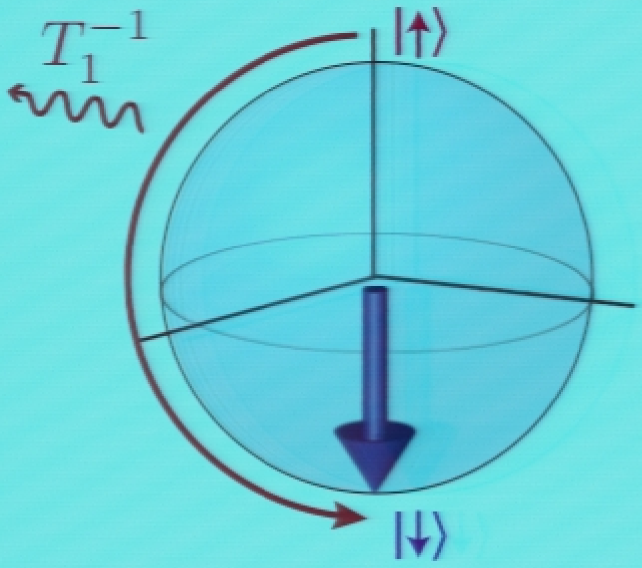
Temporal response

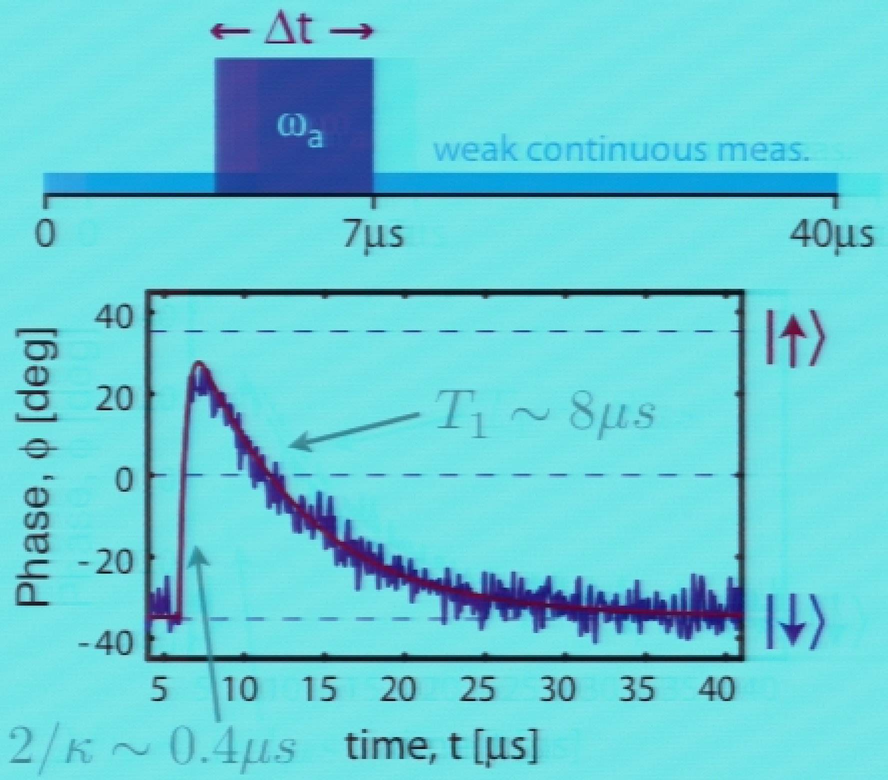




Coherent control

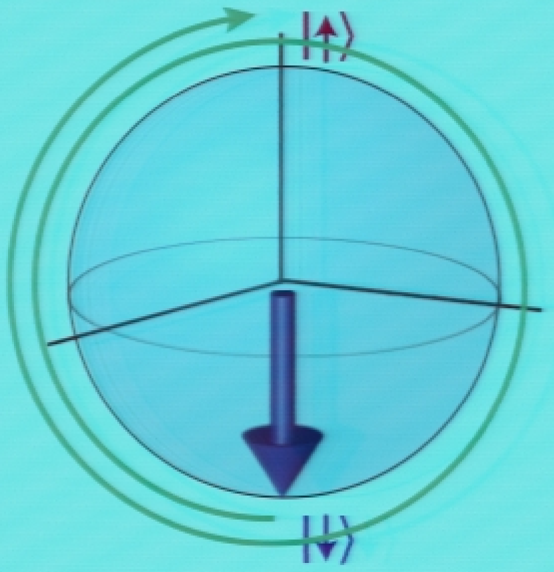
Temporal response

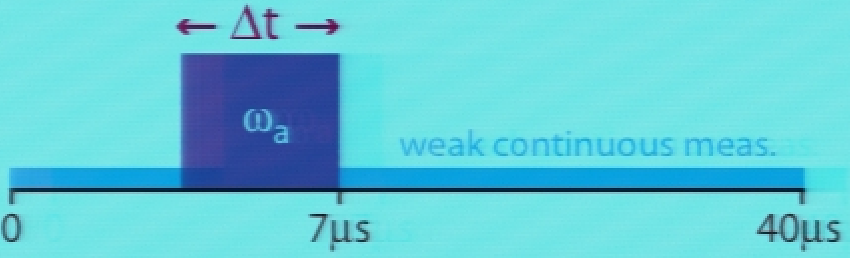


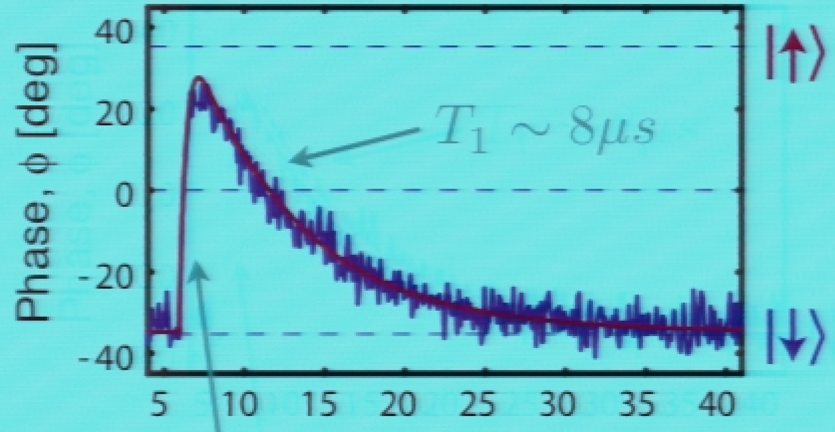


Coherent control

Temporal response

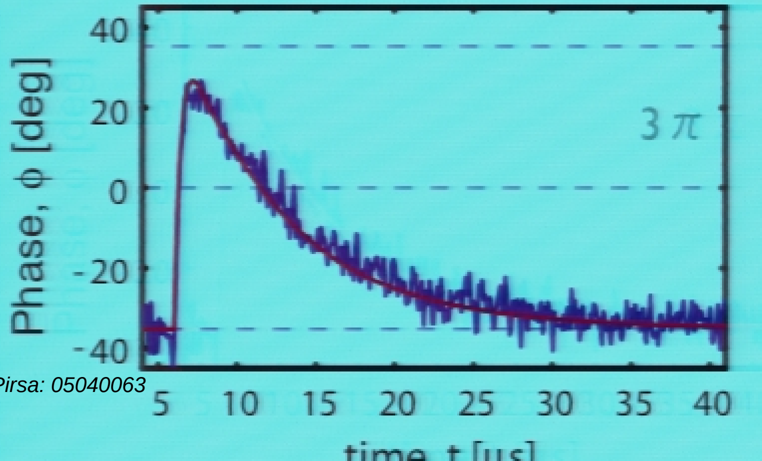






$T_1 \sim 8\mu s$

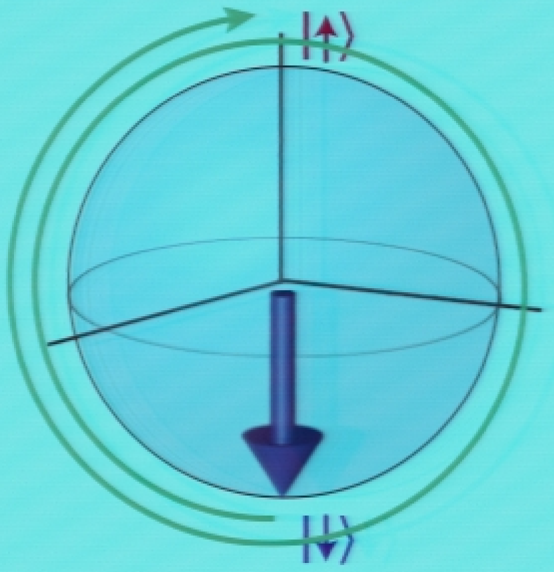
$2/\kappa \sim 0.4\mu s$

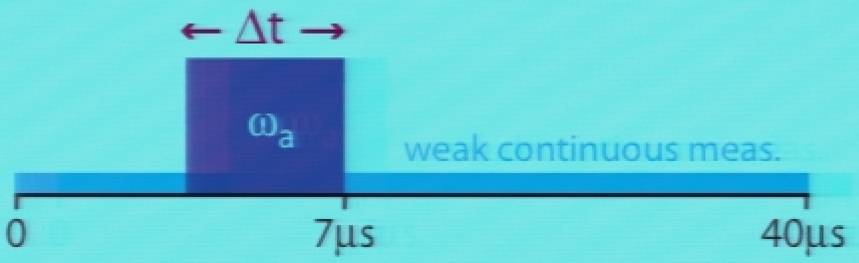


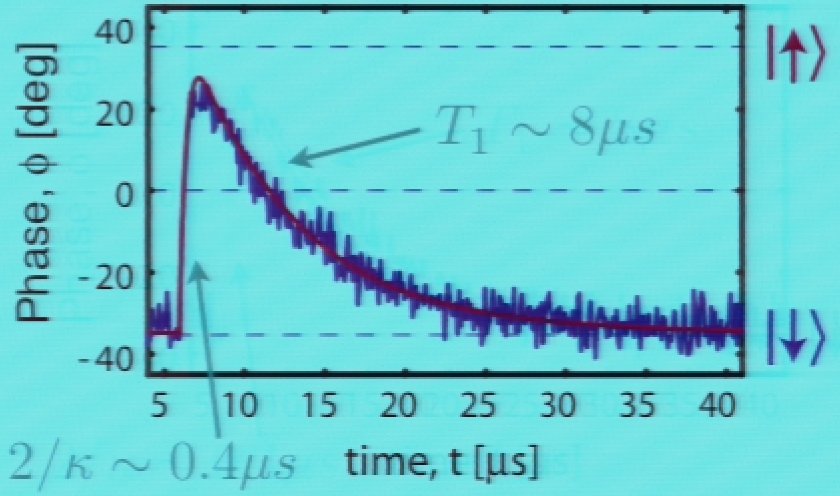
$3\,\pi$

Coherent control

Temporal response







$T_1 \sim 8\mu s$

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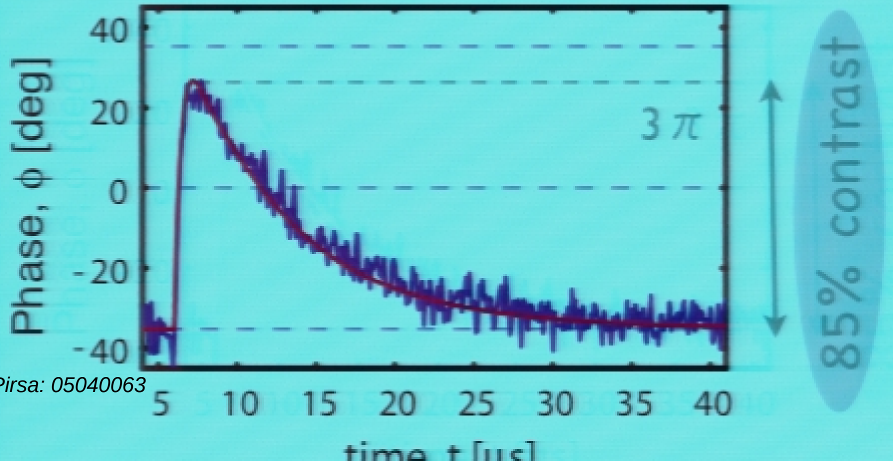
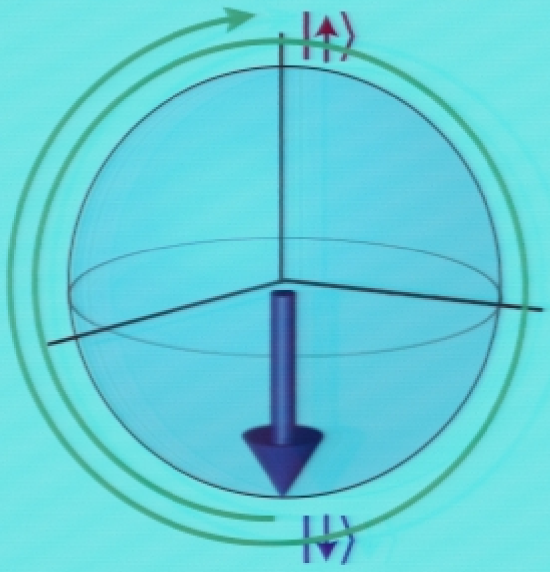
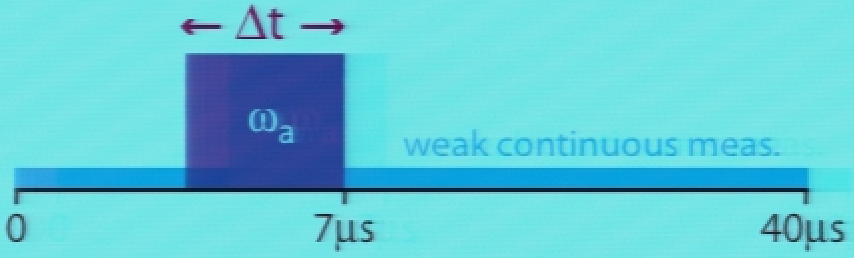
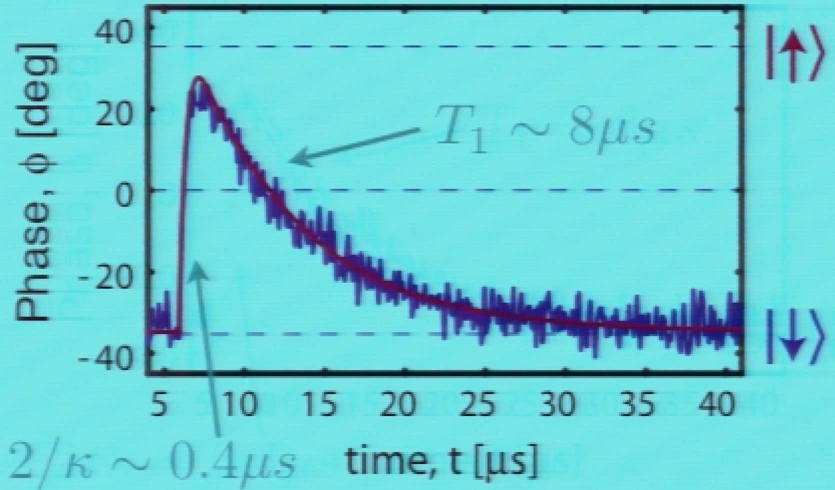


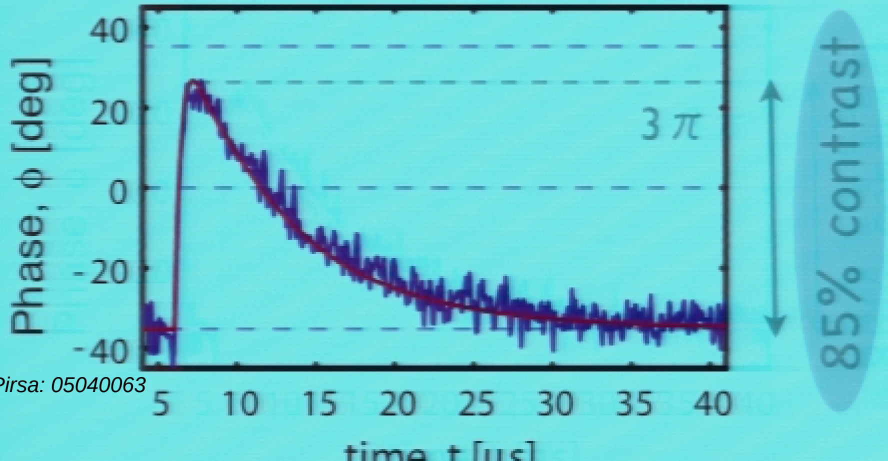
Coherent control

Temporal response



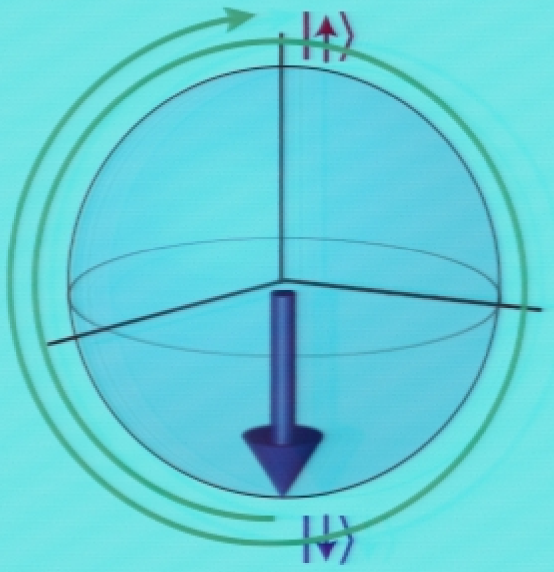


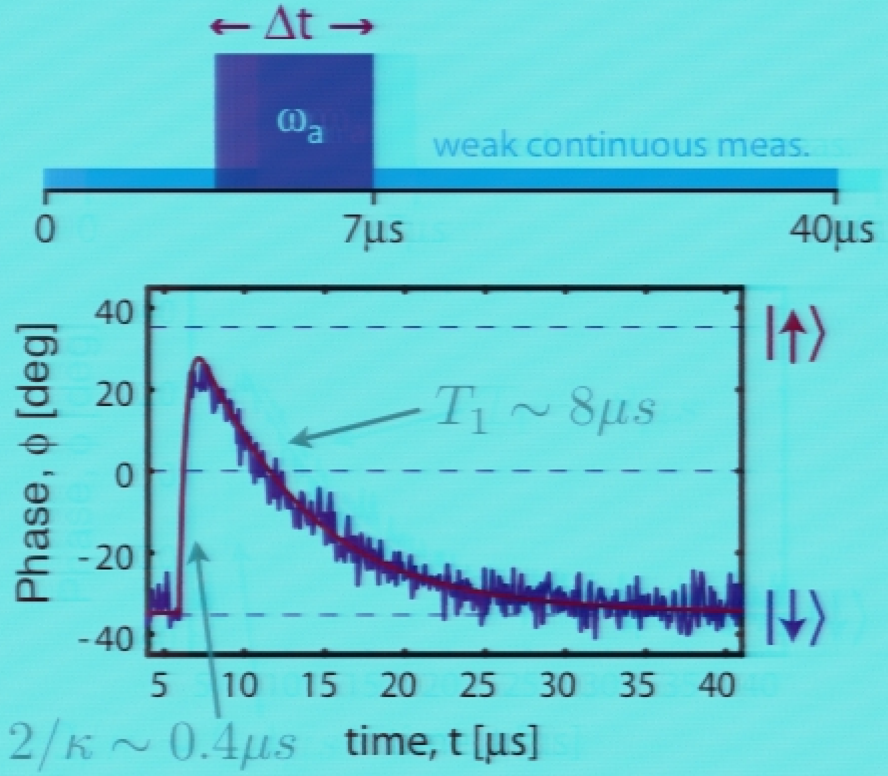


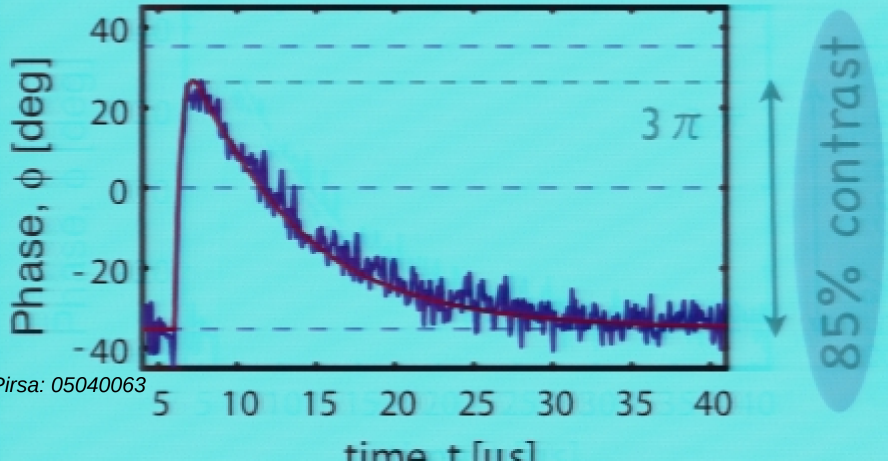


Coherent control

Temporal response

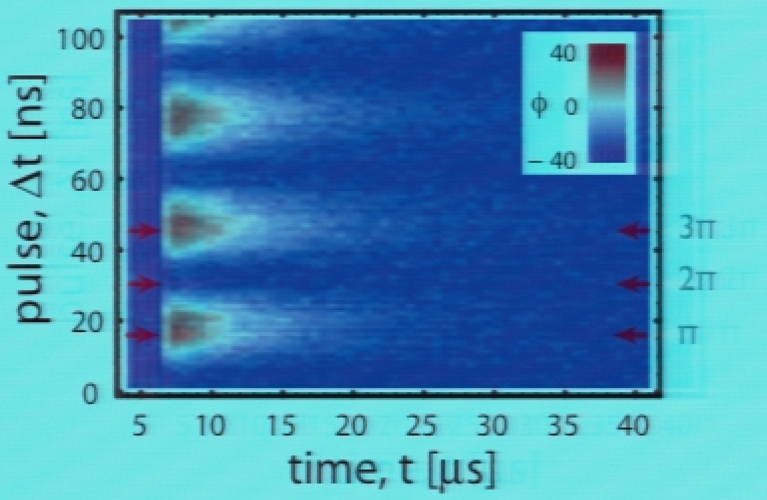


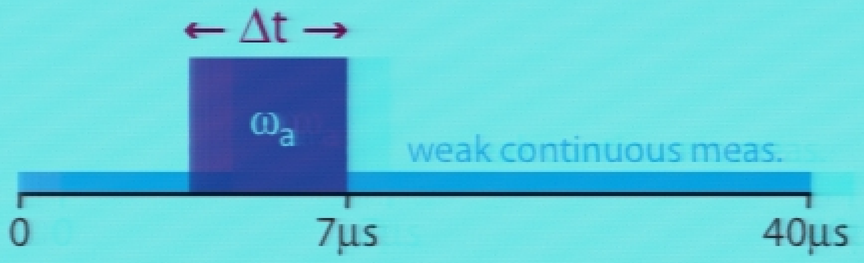




Coherent control

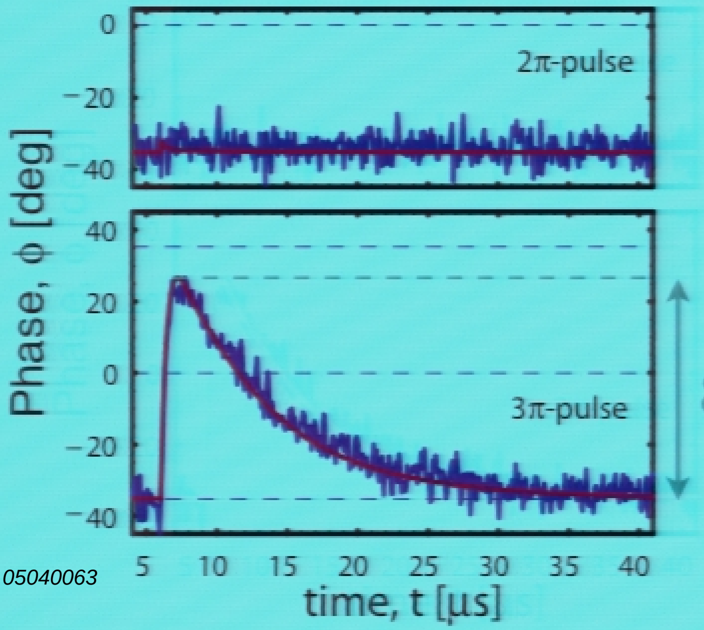
Rabi oscillations





Time scales: $T_1 \sim 8\mu s$

$2/\kappa \sim 0.4\mu s$

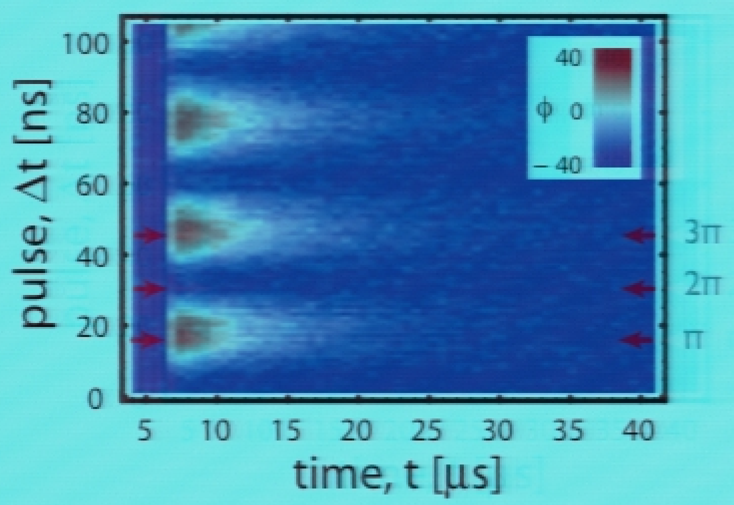


85% contrast

Wallraff, Schuster, AB, ... Girvin & Schoelkopf, cond-mat/0502645

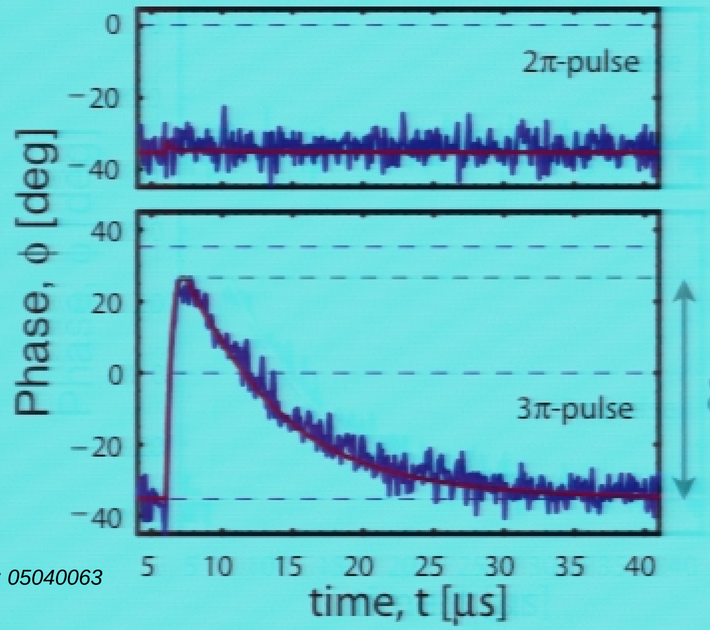
Coherent control

Rabi oscillations



Time scales: $T_1 \sim 8\mu s$

$2/\kappa \sim 0.4\mu s$



85% contrast

Relaxation

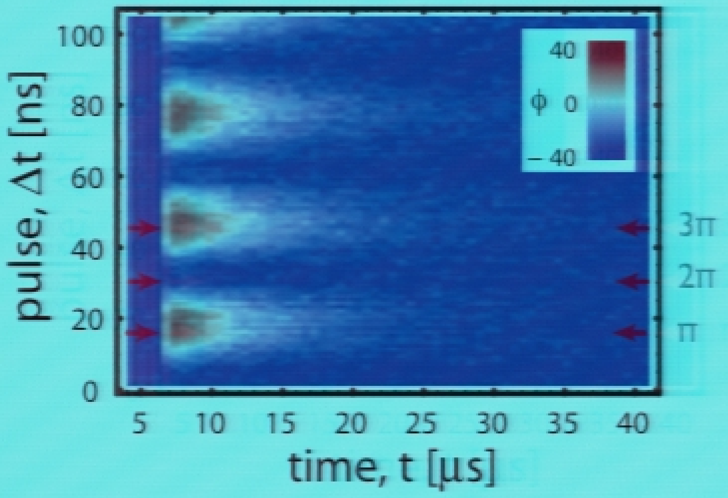
Finite readout time

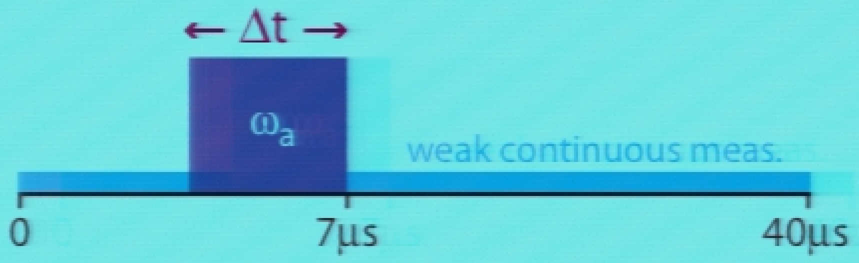
Imperfect pulses

Wallraff, Schuster, AB, ... Girvin & Schoelkopf, cond-mat/0502645

Coherent control

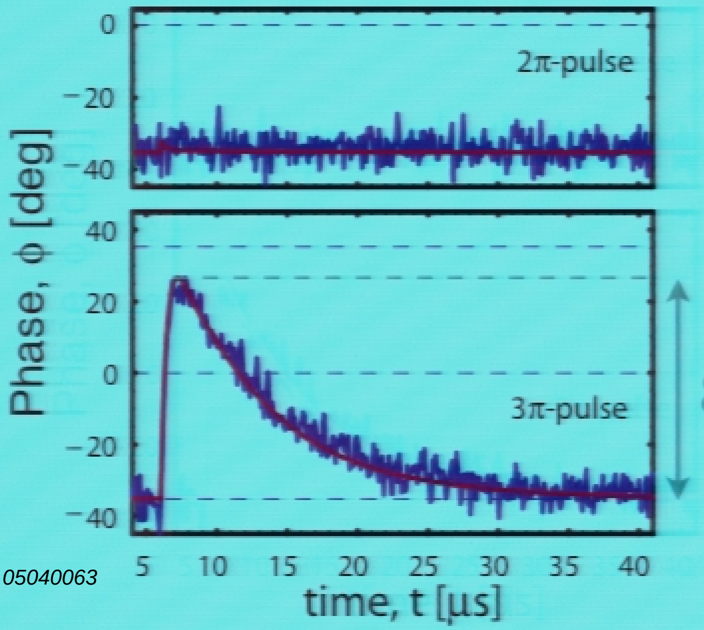
Rabi oscillations





Time scales: $T_1 \sim 8\mu s$

$2/\kappa \sim 0.4\mu s$



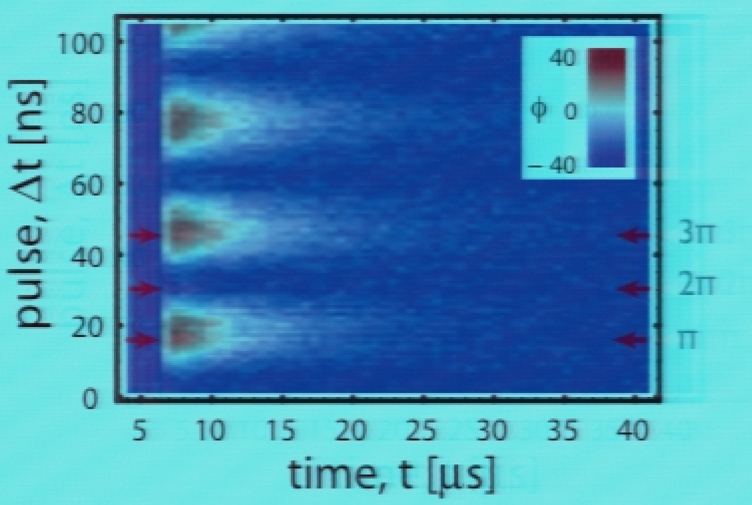
85% contrast

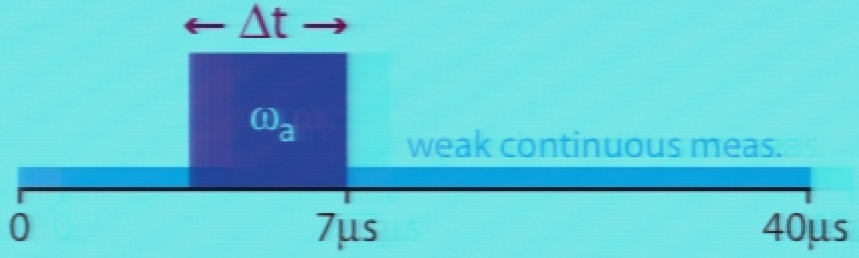
- ✔ Relaxation
- ✔ Finite readout time
- Imperfect pulses

Wallraff, Schuster, AB, Girvin & Schoelkopf, cond-mat/0502645

Coherent control

Rabi oscillations





Time scales: $T_1 \sim 8\mu s$

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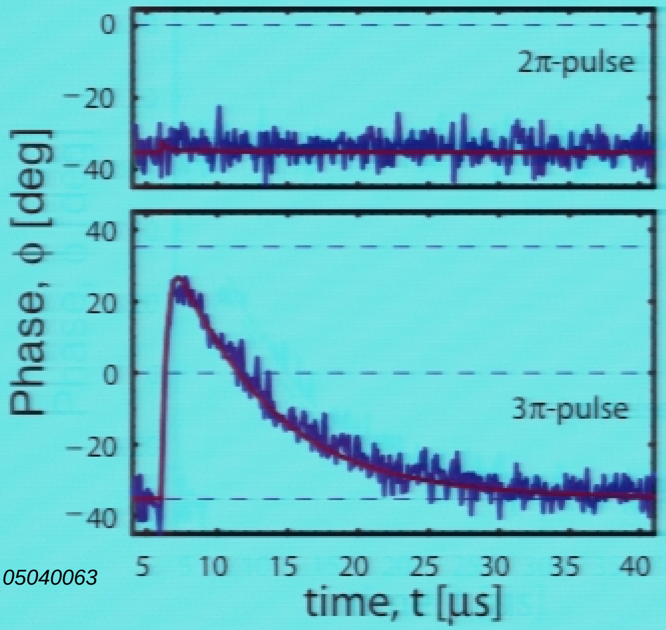
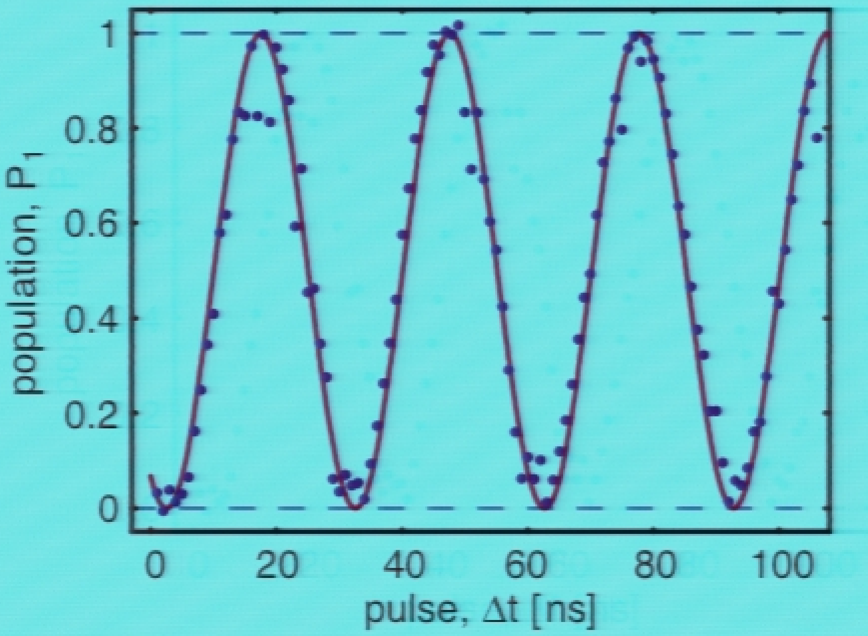
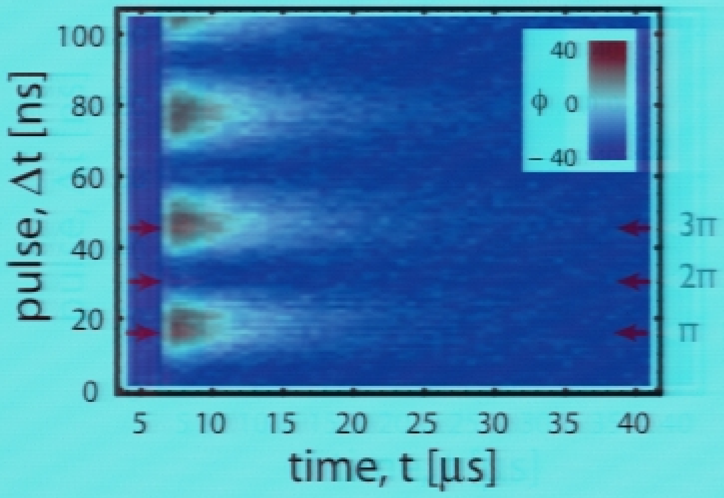




Wallraff, Schuster, AB, Girvin & Schoelkopf, cond-mat/0502645

Coherent control

Rabi oscillations



Time scales: $T_1 \sim 8\mu s$

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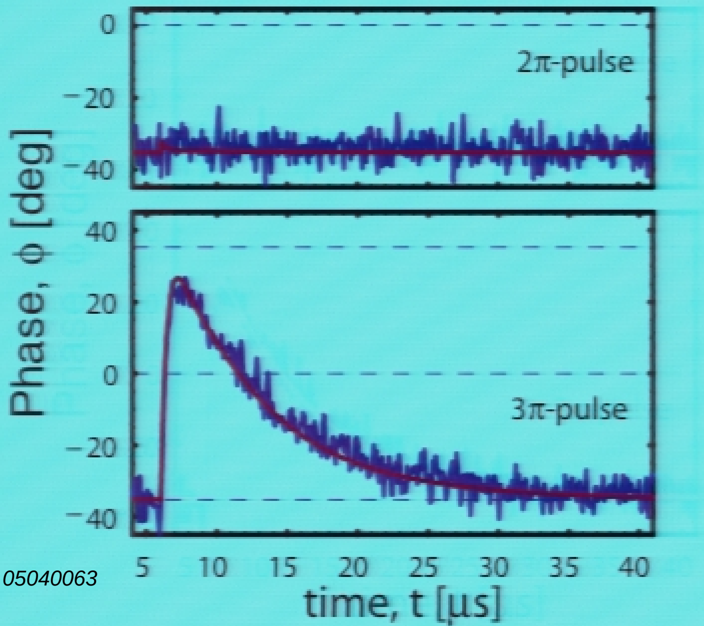
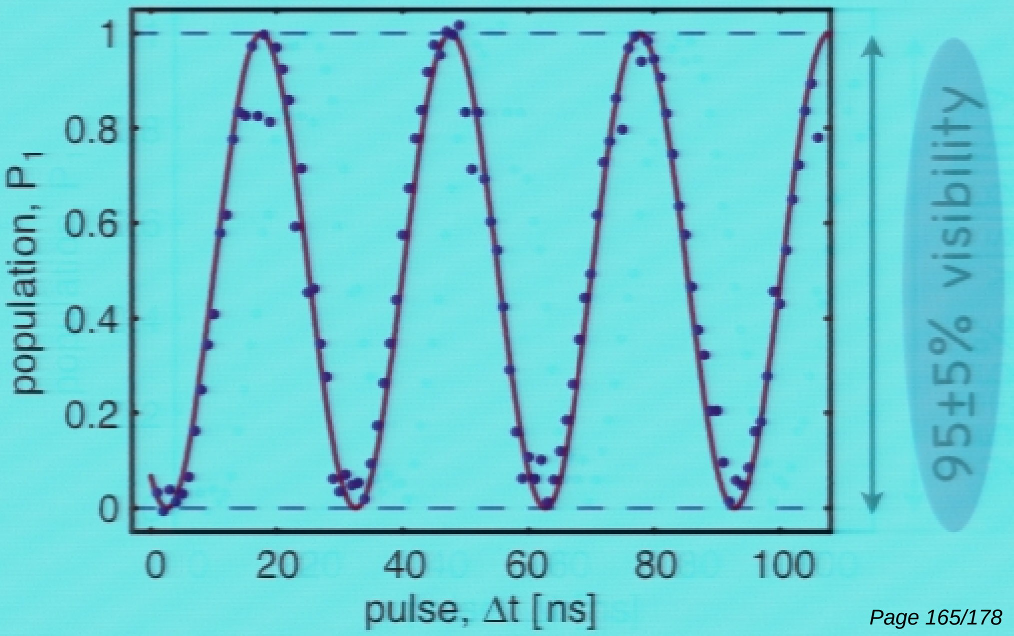
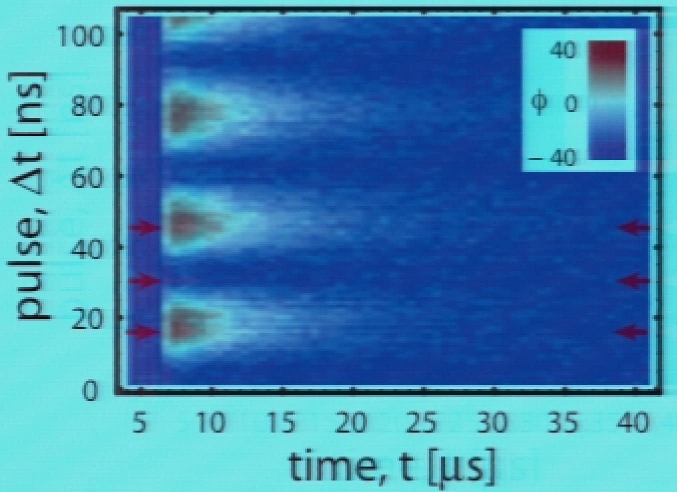


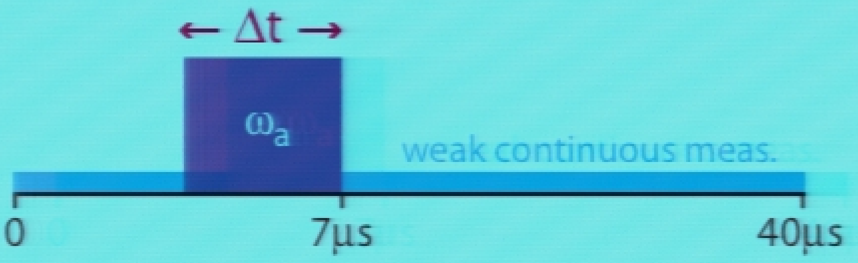


Wallraff, Schuster, AB, Girvin & Schoelkopf, cond-mat/0502645

Coherent control

Rabi oscillations





Time scales: $T_1 \sim 8\mu s$

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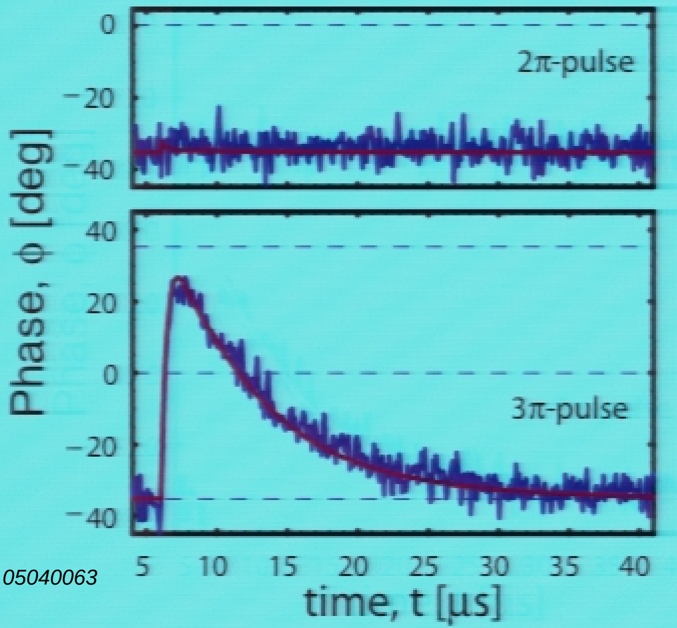
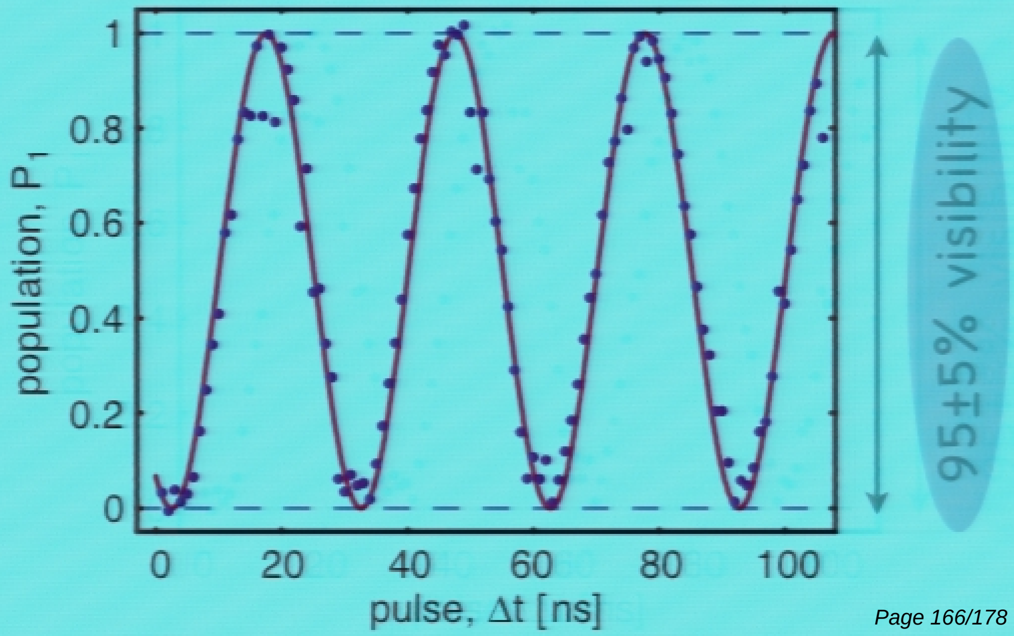
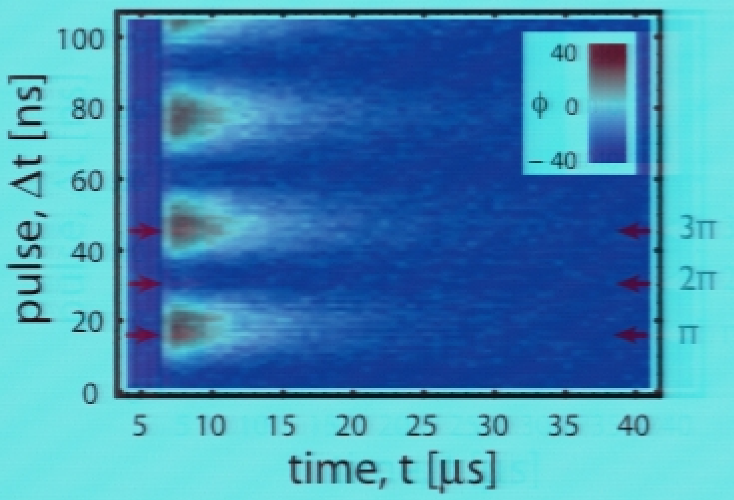


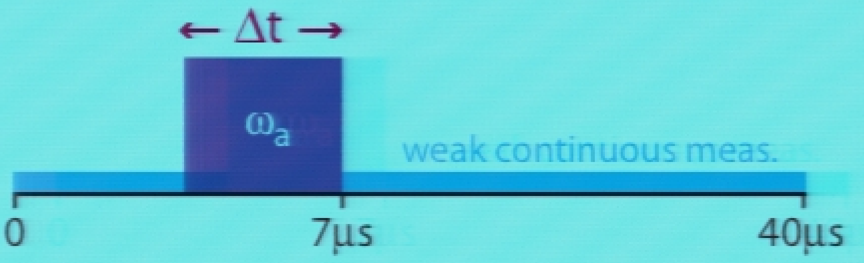


Wallraff, Schuster, AB, … Girvin & Schoelkopf, cond-mat/0502645

Coherent control

Rabi oscillations





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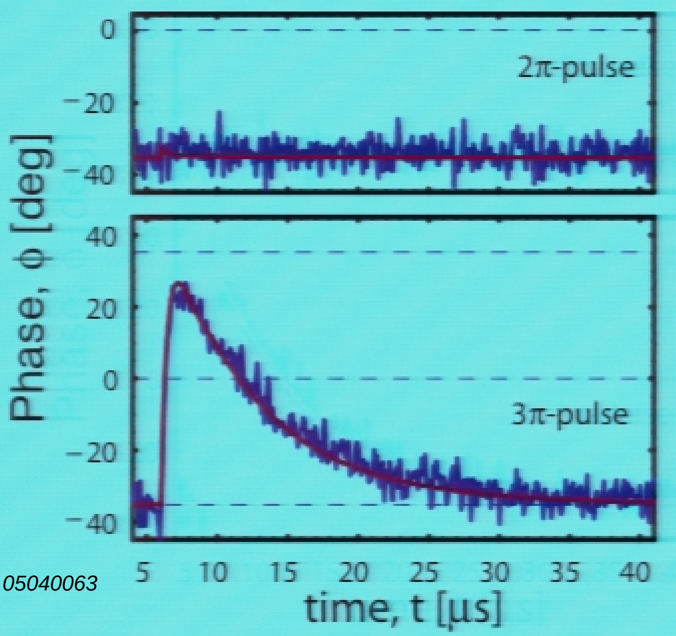
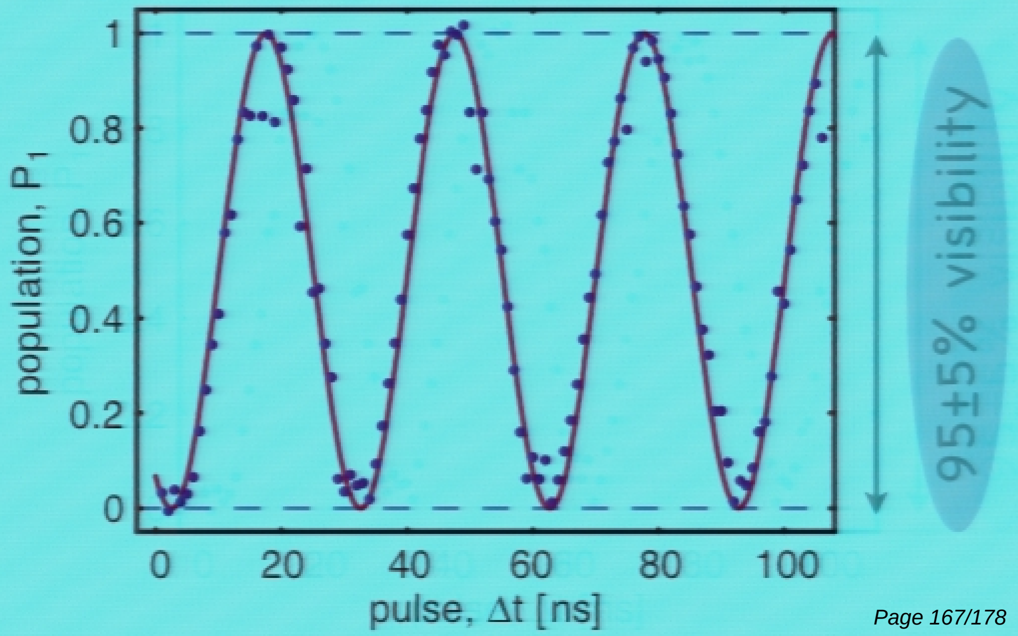
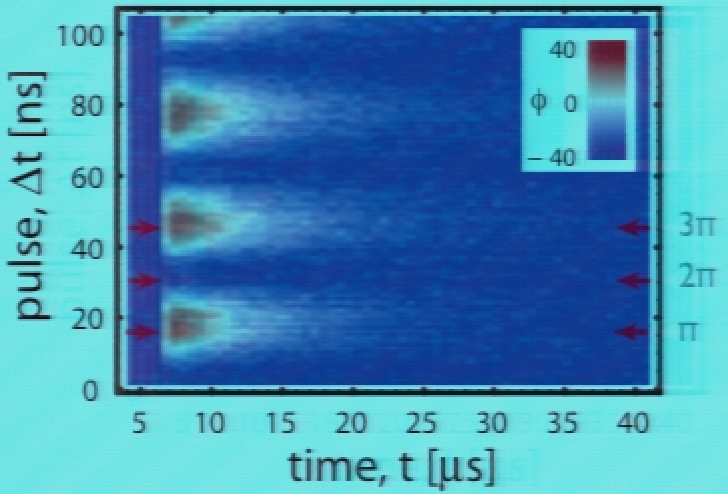


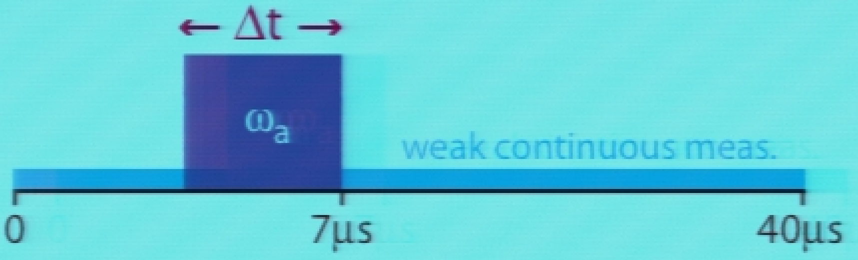


Wallraff, Schuster, AB, Girvin & Schoelkopf, cond-mat/0502645

Coherent control

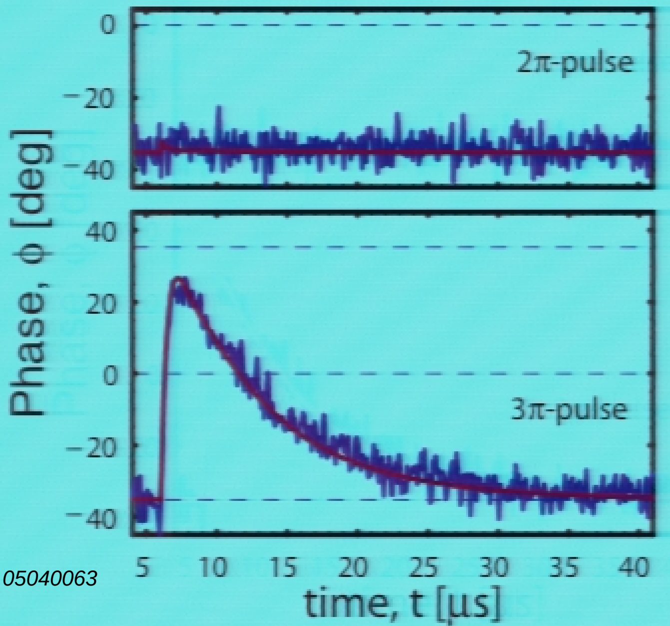
Rabi oscillations

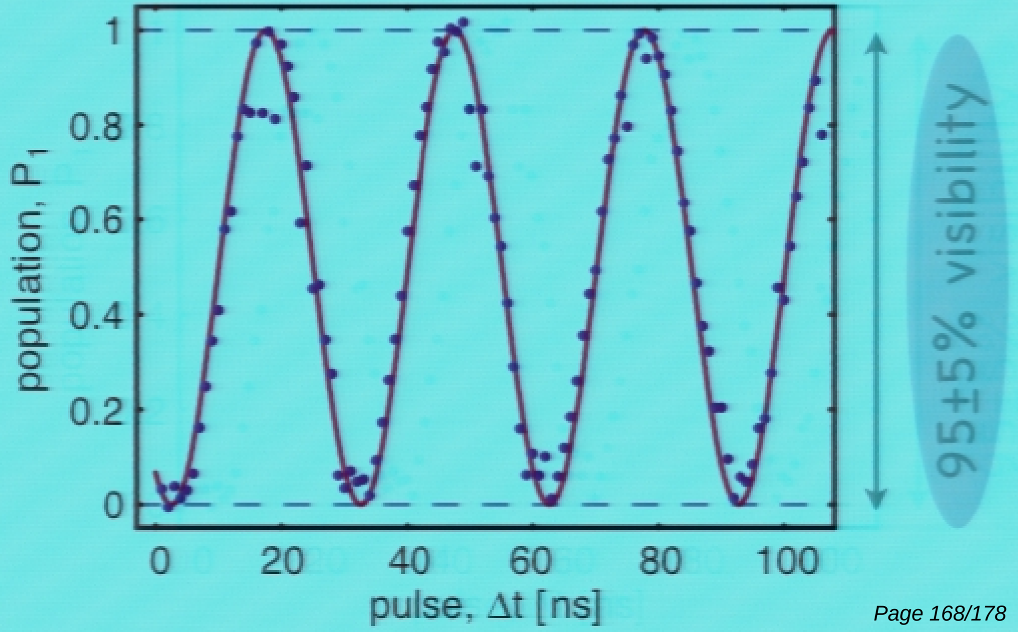




Time scales: $T_1 \sim 8\mu s$

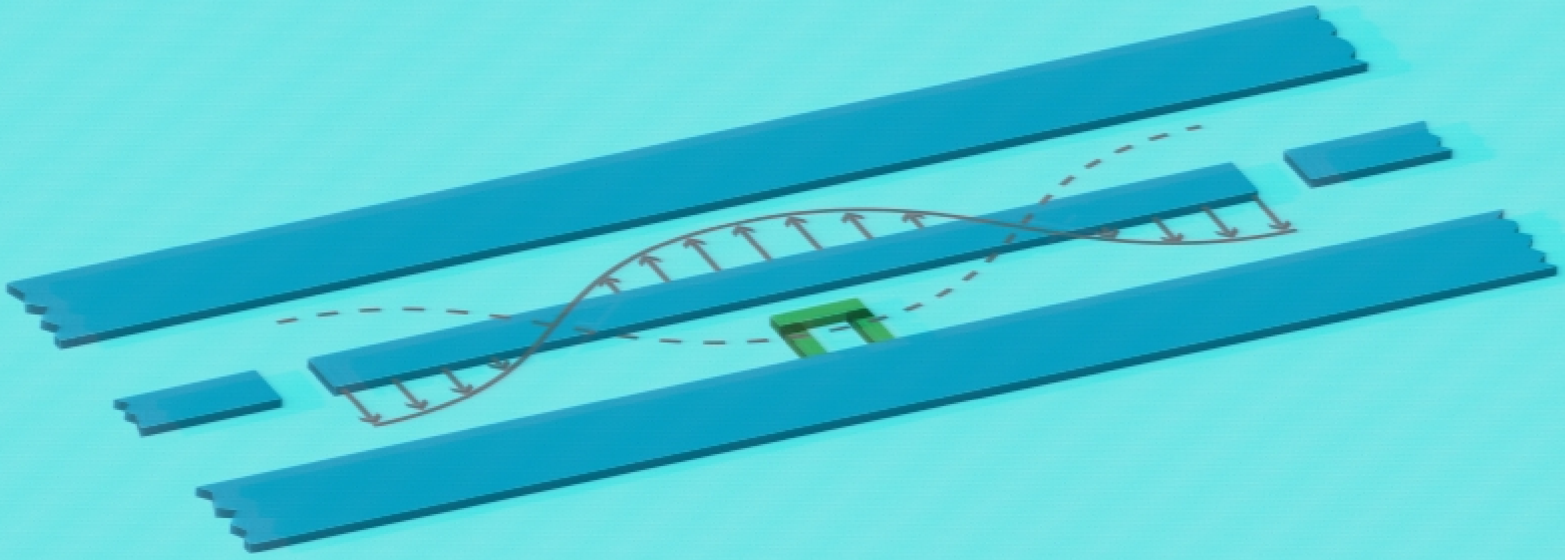
$2/\kappa \sim 0.4\mu s$





Wallraff, Schuster, AB, ... Girvin & Schoelkopf, cond-mat/0502645

Summary



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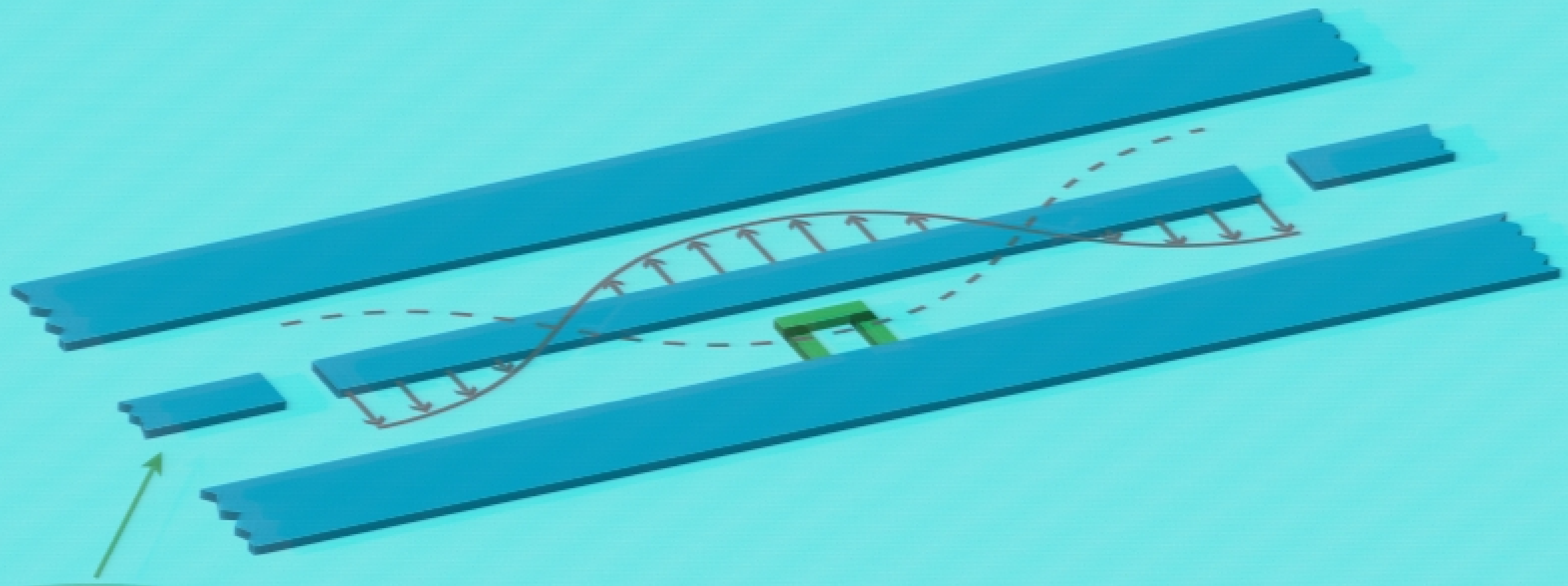
PACKARD FOUNDATION

CRSNG NSERC

ARDA

Summary



Cavity with large vacuum field and long photon lifetime

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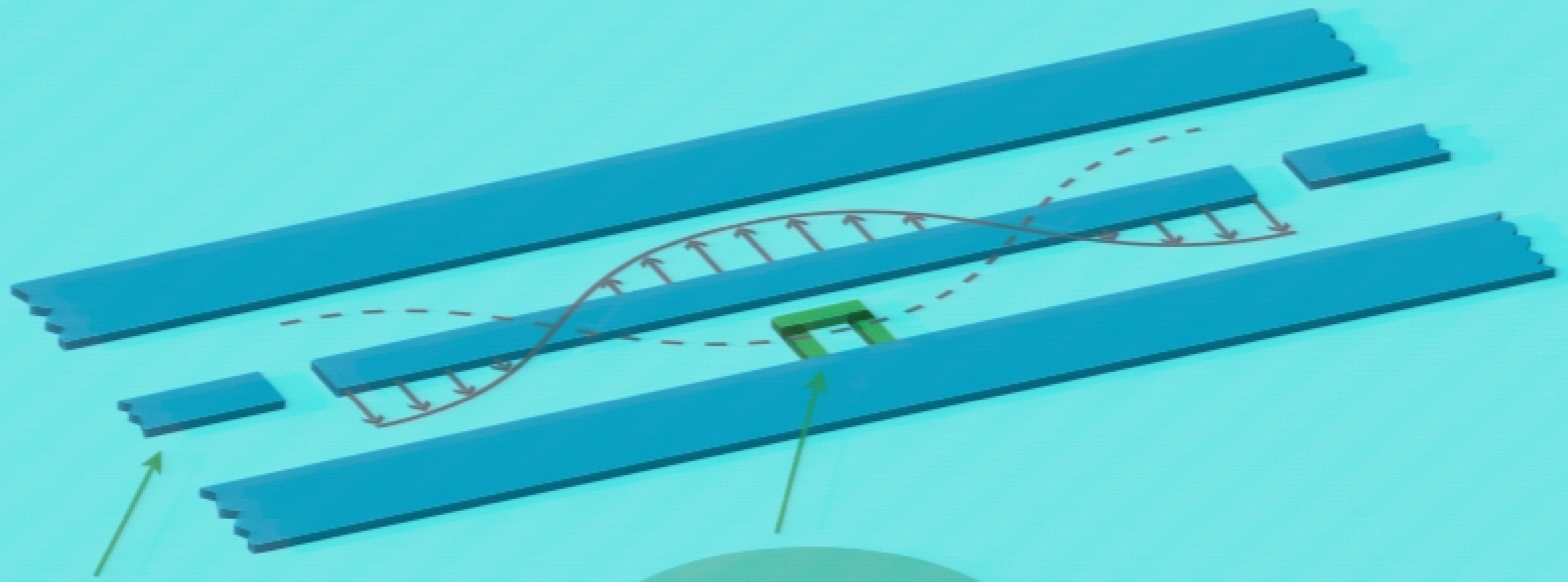
Summary



Cavity with large vacuum field and long photon lifetime

Artificial atom with long coherence time

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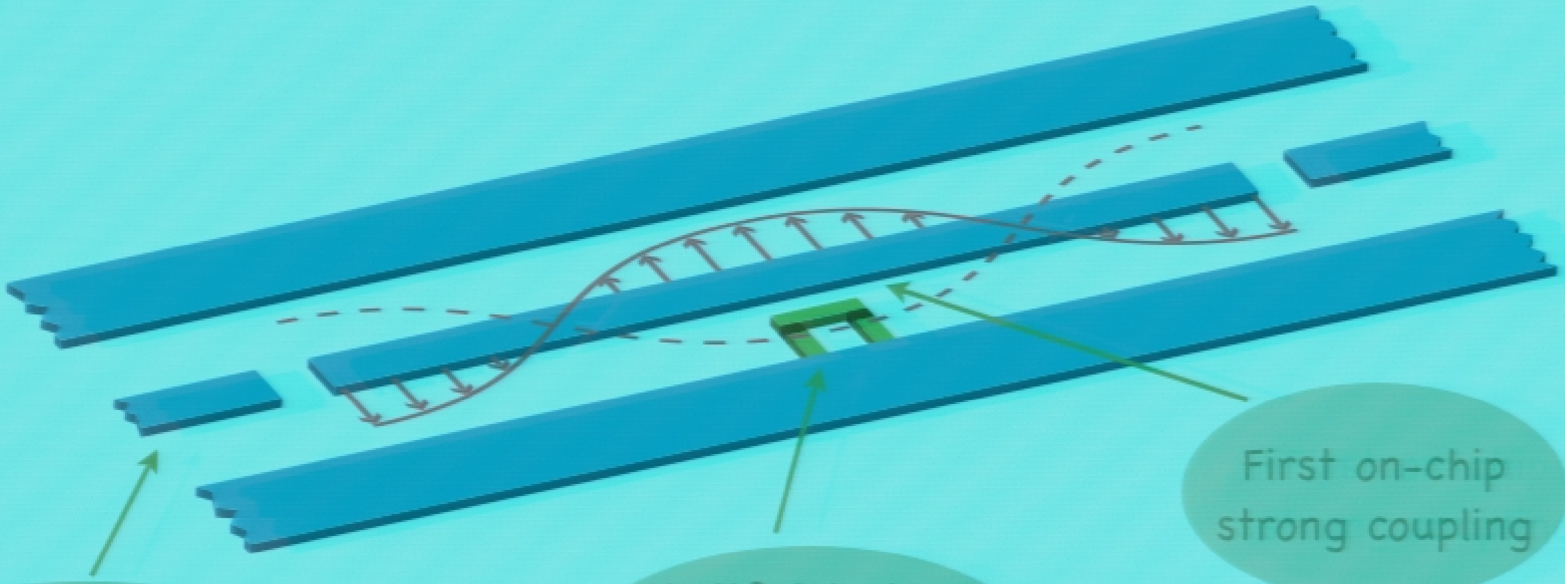

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Artificial atom with long coherence time

First on-chip strong coupling

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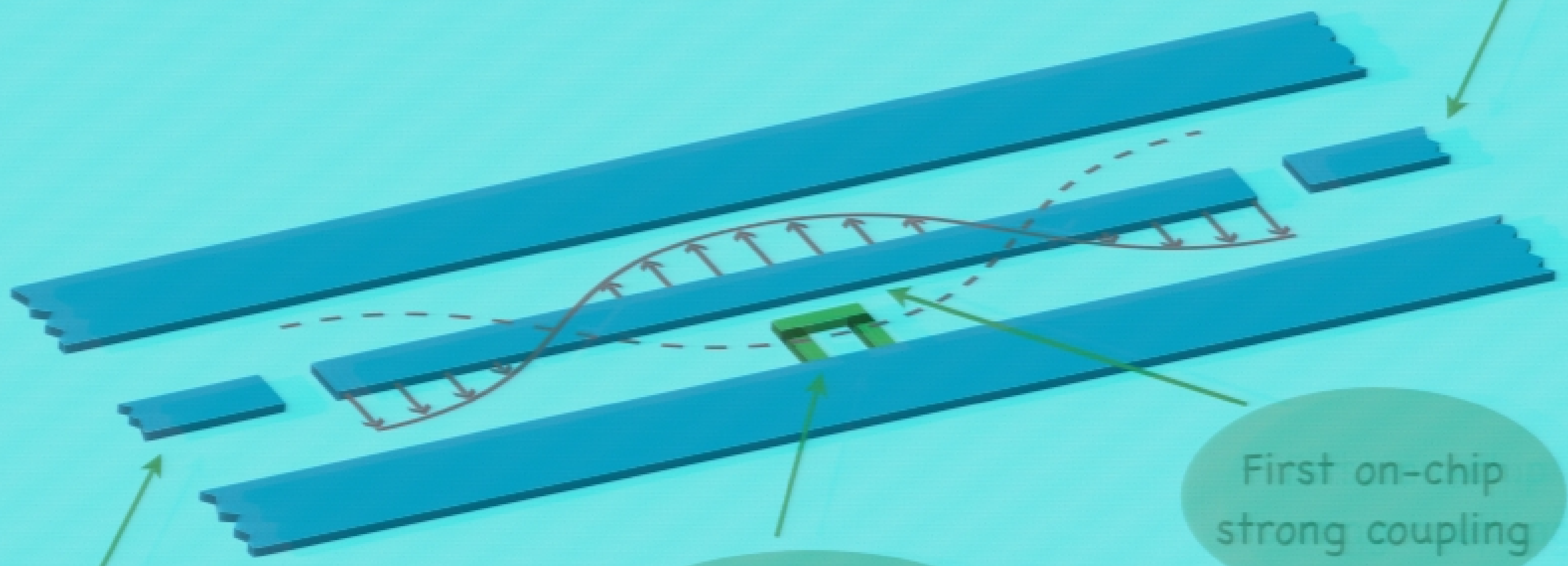

Dispersive measurement well understood, back-action, first large visibly

First on-chip strong coupling

Artificial atom with long coherence time

Cavity with large vacuum field and long photon lifetime

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Summary

What's next:
- Two-qubit gates
- Single-photon source and detection
- …

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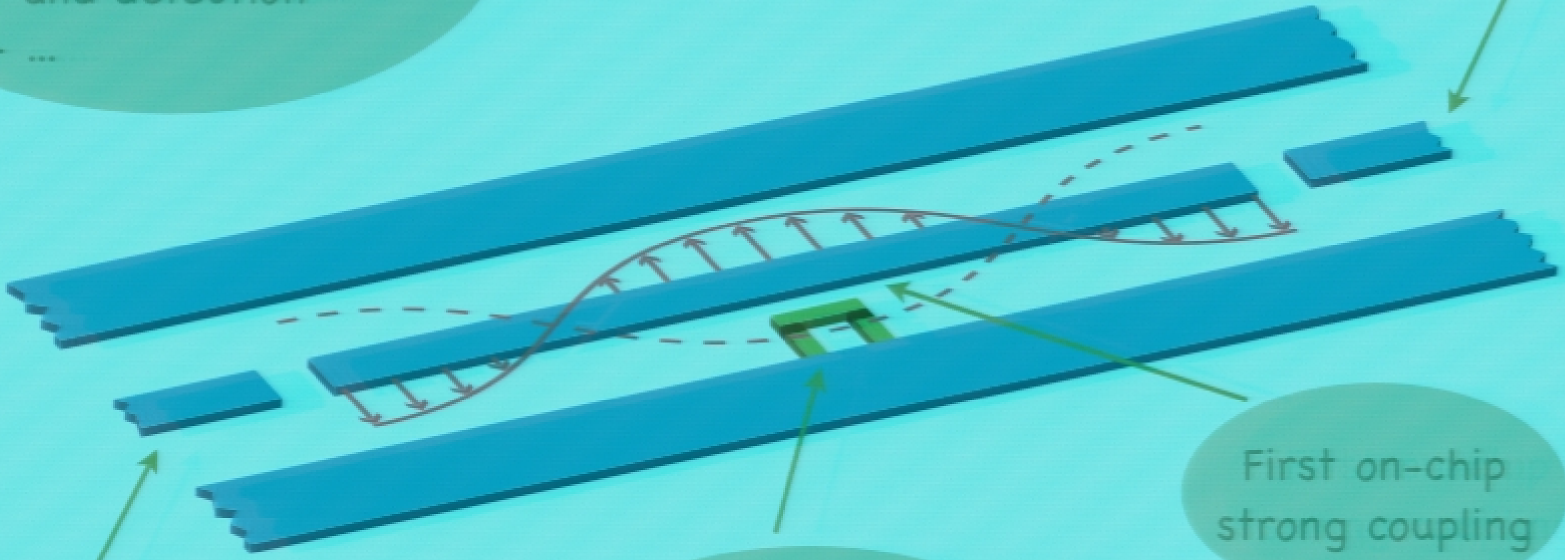

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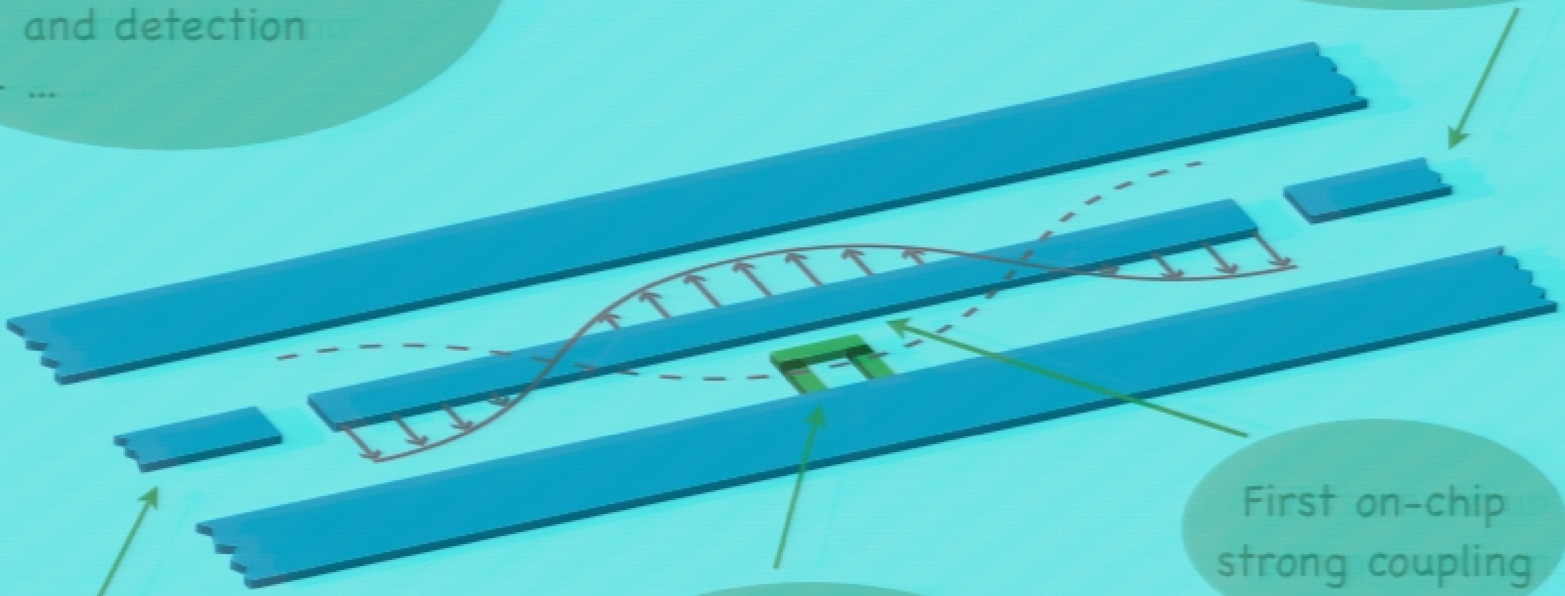

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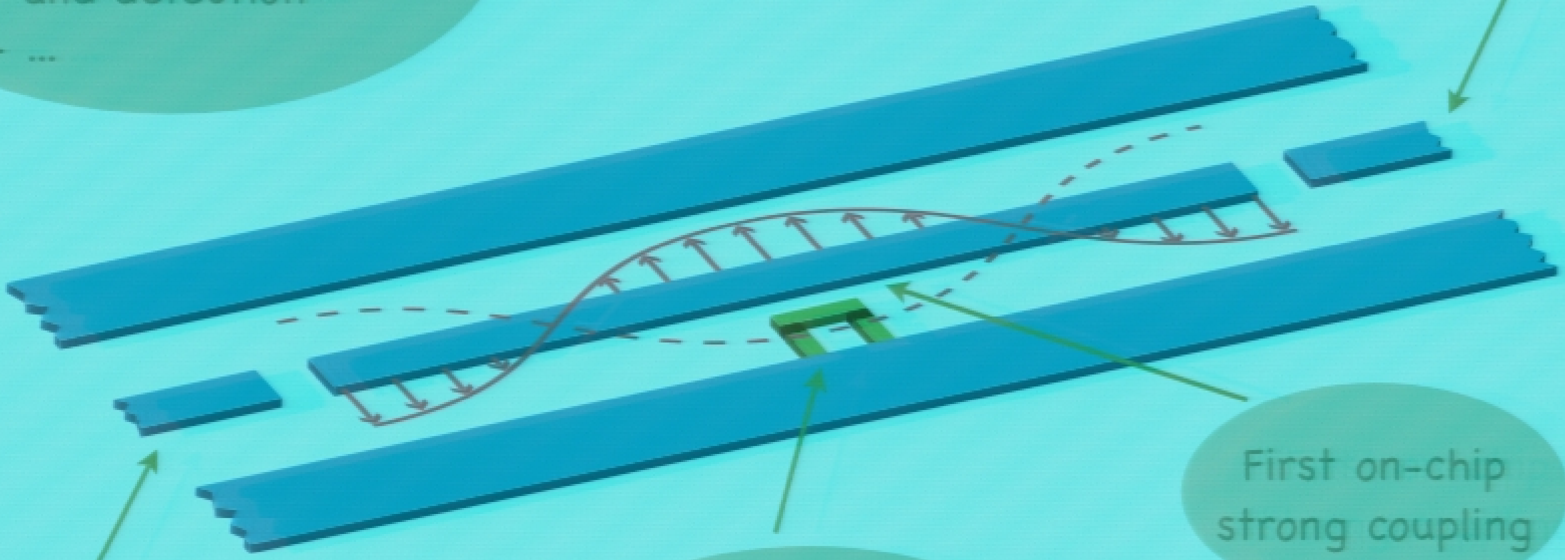

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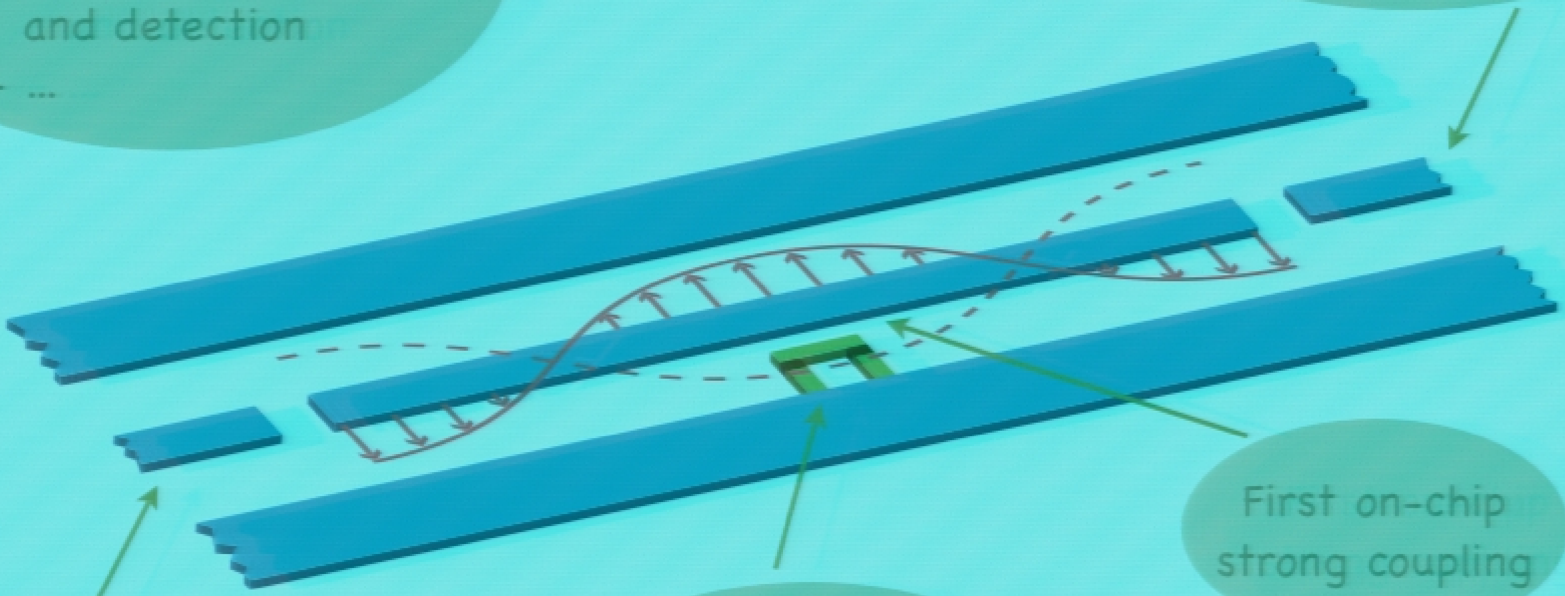

What's next:

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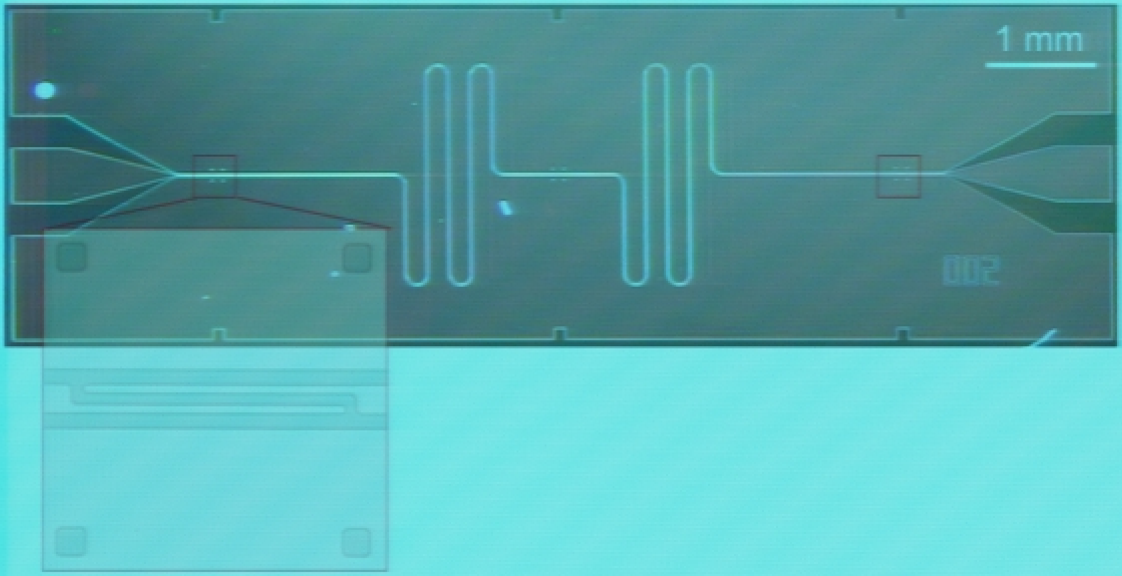
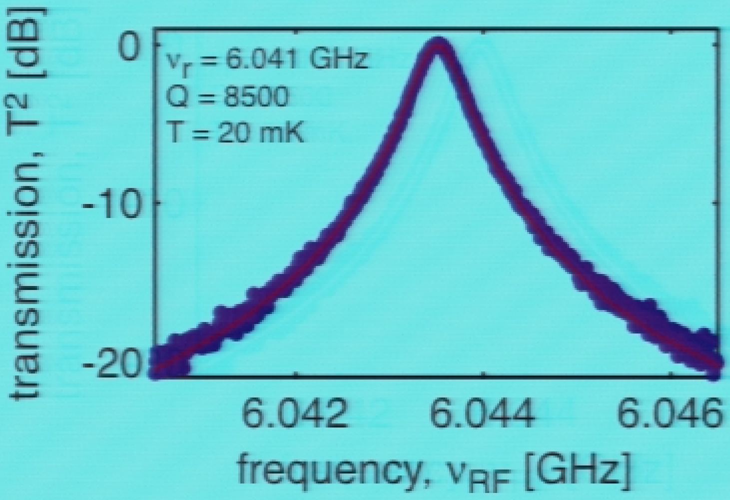
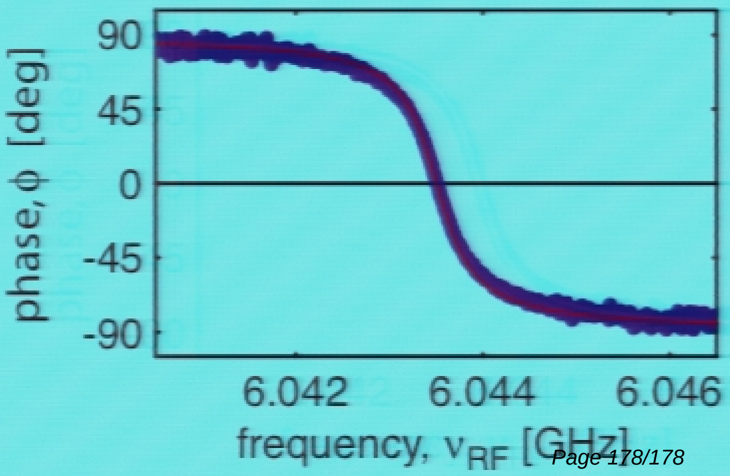
Cavity: superconducting 1D waveguide resonator







$$\kappa/2\pi = \nu_r/Q \approx 0.7\,\mathrm{MHz}$$

$$T_\kappa = 1/\kappa \approx 224\,\mathrm{ns}$$

$$h\nu_r = 300\,\mathrm{mK}$$

$$\rightarrow N_{th} \approx 0.06 \ @\ 100\mathrm{mK}$$