Title: QCD and a Holographic Model of Hadrons

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Abstract:

QCD

and a holographic model of hadrons

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QCD :

- is the correct theory of strong interactions
- asymptotically free
  - $\alpha_S \ll 1$ when $Q^2 \gg \Lambda_{QCD}^2 \sim (200\ \mathrm{MeV})^2$
    $\alpha_S \sim 1$ $Q^2 \sim \Lambda_{QCD}^2$
- Lagrangian written in terms of quarks & gluons
  observed particles are hadrons

$\Rightarrow$ problem of confinement

A string-theory solution of QCD?

Historically, string theory started as an attempt to describe strong interactions

- Regge trajectories $J \sim \alpha' m^2$

$\alpha' \sim (1\,\mathrm{GeV})^{-2}$

More recently: Maldacena's conjecture revives hopes

$\mathcal{N}=4$ SYM $\longleftrightarrow$ type IIB string theory on $AdS_5 \times S^5$

$\uparrow$

a conformal field theory:

no asymptotic freedom

no confinement

Approach to QCD

- top-down : start from $\mathcal{N}=4$ SYM

  adding perturbations to break supersymmetry, conformal invariance ...

  $\Rightarrow$ theories which sometimes have features similar to QCD

- Bottom-up :

  Start from what we know from phenomenology

  Build a model based on the qualitative features

We will see that the simplest model works better than one has any right to expect.

HADRON PHENOMENOLOGY :

seems like a mess at first sight,
but a closer look reveals some structures:

- Resonances are relatively narrow

  e.g. $m_\rho = 770$ MeV
  $\Gamma_\rho = 150$ MeV

  - consequence of large $N_c = 3$

- Chiral symmetry breaking $SU(N_f) \times SU(N_f) \to SU(N_f)$

  - light pseudo-Goldstone boson $\pi(140)$
  - mass splitting between $SU(N_f)$ multiplets
  with opposite parities $m_\rho = 770$ MeV
  $m_{a_1} = 1230$ MeV

- quark model works
  Light mesons correspond to operators $q = \binom{u}{d}$

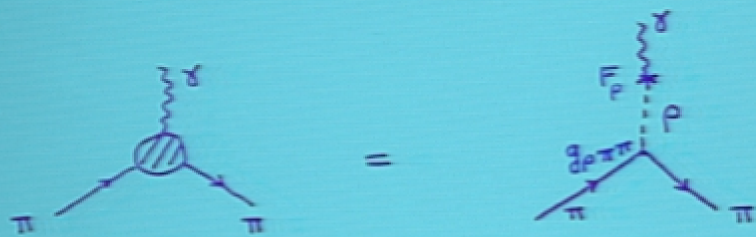
  $\pi^a \sim \bar{q} \gamma^5 \tau^a q$

  $\rho^a \sim \bar{q} \gamma^\mu \frac{\tau^a}{2} q \qquad a_1 \sim \bar{q} \gamma^\mu \gamma^5 \frac{\tau^a}{2} q$

  no exotics, no glueballs (with reservations)

- Vector meson dominance

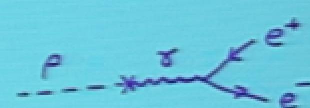
coupling to γ is dominated by intermediate vector meson



taking photon momentum $\rightarrow 0$

$$1 = \frac{F_\rho \, g_{\rho\pi\pi}}{m_\rho^2}$$

F_ρ measured from



$$\Gamma_{\rho \to e^+e^-} \approx 6 \text{ keV} \Rightarrow F_\rho = (345 \text{ MeV})^2$$

$g_{\rho\pi\pi} \approx 6$ from $\Gamma_{\rho \to \pi\pi} = 150$ MeV

We want a holographic model which exhibits some properties of QCD

For lack of imagination, we take the metric to be AdS_5

$$ds^2 = \frac{R^2}{z^2}\left(dt^2 - d\vec{x}^2 - dz^2\right)$$

will set $R=1$

"conformal window"

Confinement: spacetime ends at some radius

will truncate AdS_5 at some $z = z_m$

and impose some boundary conditions (e.g., Neuman) at $z = z_m$

- Chiral symmetry: two sets of conserved currents in QCD

$$J^\mu_L = \bar{q}\gamma^\mu \frac{1-\gamma^5}{2} q , \qquad J^\mu_R = \bar{q}\gamma^\mu \frac{1+\gamma^5}{2} q$$

$\Rightarrow$ two bulk 5D gauge field.

$$A^L_\mu \qquad A^R_\mu$$

(recall AdS/CFT: operator in 4D theory corresponds to field in 5D theory)

conserved currents $\longleftrightarrow$ massless gauge fields

- Chiral symmetry breaking: in QCD characterized by the chiral condensate $\langle \bar{q}_R q_L \rangle$

$$\langle \bar{q}_R^{\alpha} q_L^{\beta} \rangle \sim \delta^{\alpha\beta} \qquad \alpha, \beta = 1 \ldots N_f$$

$\Rightarrow$ in 5D we introduce bulk scalar $X^{\alpha\beta}$.

bifundamental with respect to $SU(N_f) \times SU(N_f)$

Mass of X: $\quad \Delta(\Delta - 4) = m_{5D}^2 \qquad R = 1$

In QCD dimension of $\bar{q}q$ is $\Delta = 3$

ignore anomalous dimension

$$\Rightarrow m_X^2 = -3$$

$\chi SB \Rightarrow$ non zero vev for X. $\quad \langle X \rangle = X_0(z)$

Small z: $\quad X_0^{\alpha\beta}(z) = \frac{1}{2}\left(m_q z + \sigma z^3\right) \delta^{\alpha\beta}$

explicit symmetry breaking by quark masses

spontaneous χSB by chiral condensate

The model:

$$S = \int d^5x \sqrt{g} \; \mathrm{Tr}\left[|D_\mu X|^2 + 3|X|^2 - \frac{1}{4g_5^2}(F_L^2 + F_R^2) \right]$$

$$D_\mu X = \partial_\mu X - i A_\mu^L X + i X A_\mu^R \qquad F_L = dA_L + A_L^2$$
$$F_R = dA_R + A_R^2$$

on truncated AdS $\qquad ds^2 = \frac{1}{z}(dt^2 - d\vec{x}^2 - dz^2)$

$$0 \leq z \leq z_m$$

The X background should arise dynamically, but we simply take

$$X = \frac{1}{2}(m_q z + \sigma z^3)\, \mathbb{1}$$

Impose b.c. on $z = z_m$ ("infrared brane")

$F_{z\mu} = 0 \qquad z = z_m$

$D_z X = 0$ (in practice $X = X_0 \Sigma \quad \Sigma \in SU(2)$
$D_z \Sigma = 0$)

4 free parameters $m_q \quad \sigma \quad z_m \quad g_5$

cf. 3 in QCD $\quad m_q, \Lambda_{QCD}, N_c$

OPE matching :

compute correlator of 2 vector currents $J_\mu = J_\mu^L + J_\mu^R$

According to AdS/CFT: solve classical field equation

$$\partial_z \left(\frac{1}{z} V_\perp \right) + \frac{q^2}{z} V_\perp = 0$$

$$V_\mu(z,x) = A_\mu^L + A_\mu^R = \int \frac{d^4 q}{(2\pi)^4} e^{iq\cdot x} V_\mu(z,q)$$

with b.c.

$$V_\perp^\mu = V_0^\mu \quad \text{at} \quad z = 0$$

$$q_\mu V_\perp^\mu = 0$$

then differentiate the action

$$S = -\frac{1}{2 g_5^2} \int d^4x \; \frac{1}{z} V_\mu^a \partial_z V^{a\mu} \Big|_{z = \varepsilon \to 0}$$

twice with respect to the boundary value V_0^μ

Result :

$$\int d^4x \, e^{iq\cdot x} \langle J_\mu^a(x) J_\nu^b(0) \rangle = \delta^{ab} (q_\mu q_\nu - q^2 g_{\mu\nu}) \Pi_V(Q^2)$$

$Q^2 = -q^2$

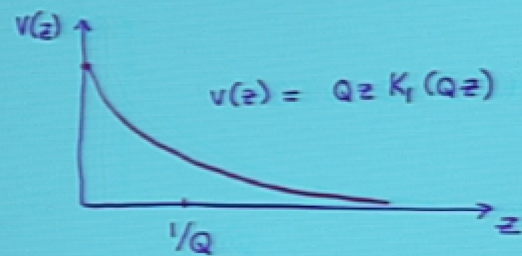
$$\Pi_V(Q^2) = -\frac{1}{g_5^2 Q^2} \frac{\partial_z V(q,z)}{z} \Big|_{z=\varepsilon}$$

$V(q,z)$ solves eq $\left(\frac{V'}{z}\right)' + \frac{q^2}{z} V = 0$

with b.c. $V(q,0) = 1$

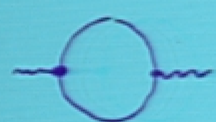
OPE matching (continued)

For large Euclidean Q^2 the solution is insensitive to the boundary condition at $z = z_m$



$$\Rightarrow \quad \Pi_V(Q^2) = -\frac{1}{2g_5^2} \ln Q^2 \qquad Q^2 \gg \frac{1}{z_m^2}$$

On the other hand in QCD



$$\Pi_V(Q^2) = -\frac{N_c}{24\pi^2} \ln Q^2$$

$$\Rightarrow \quad g_5^2 = \frac{12\pi^2}{N_c}$$

meson coupling down by $1/N_c$ as expected

Vector mesons

We identify the ρ meson and its higher excitations
with normalizable modes:

$$\psi_n(z): \qquad \left(\frac{\psi_n'}{z}\right)' + \frac{m_n^2}{z}\psi_n = 0 \qquad \begin{array}{l} \psi_n(0) = 0 \\ \psi_n'(z_m) = 0 \end{array}$$

$$\int \frac{dz}{z} |\psi_n(z)|^2 = 1$$

m_n^2 runs discrete values $n = 0, 1, 2 \ldots$

(roots of $J_0 \times \frac{1}{z_m}$)

Bulk-to-bulk propagator

$$G(q; z, z') = \sum_n \frac{\psi_n(z)\psi_n(z')}{q^2 - m_n^2}$$

$$V(q, z') = -\frac{1}{z}\partial_z G(q; z, z')\Big|_{z\to 0}$$

$$\Rightarrow \Pi_V(Q^2) = -\frac{1}{g_5^2}\sum_n \frac{[\psi_n'(\epsilon)/\epsilon]^2}{m_n^2(q^2 - m_n^2)} = -\sum_n \frac{F_n^2}{m_n^2(q^2 - m_n^2)}$$

decay constants $\quad \langle 0 | J^\mu | \rho_n, \epsilon \rangle = \epsilon^\mu F_n$

$$\Rightarrow \boxed{F_n^2 = \frac{1}{g_5^2}\frac{\psi_n'(\epsilon)}{\epsilon}}$$

Try for the lowest ρ meson :

$$m_\rho = \frac{2.405}{z_m} = 776 \text{ MeV} \Rightarrow z_m = \frac{1}{323 \text{ MeV}}$$

$$\psi_\rho = m_\rho z \, J_1(m_\rho z) \times \text{normalization factor}$$

$$F_\rho^2 = \frac{1}{g_5^2} \frac{\psi_\rho'(\epsilon)}{\epsilon} = (329 \text{ MeV})^2$$

$$g_5^2 = \frac{N_c}{12\pi^2}$$

Experimentally F_ρ can be extracted from $\rho \to e^+ e^-$

$$F_\rho^{exp} = 345 \text{ MeV}$$

agreement better than one has any right to expect

Axial sector

$$S = \int d^5x \left[-\frac{1}{4g_5^2 z} F_A^a F_A^a + \frac{v^2(z)}{2z^3} (\partial\pi^a - A^a)^2 \right]$$

$$A_\mu = \frac{A_\mu^L - A_\mu^R}{2} \qquad X = X_0(z)\, e^{i\tau^a\pi^a} \qquad v(z) = m_q z + \sigma z^3$$

Field equations $\quad A_\mu = A_{\mu\perp} + \partial_\mu \varphi$

$$\partial_z\left(\frac{1}{z}\partial_z A_\perp\right) + \frac{q^2}{z} A_\perp - \frac{g_5^2 v^2}{z^3} A_\perp = 0 \quad \Leftarrow \quad a_1 \text{ tower}$$

$$\left.\begin{aligned} &\partial_z\left(\frac{1}{z}\partial_z \varphi^a\right) + \frac{g_5^2 v^2}{z^3}(\pi^a - \varphi^a) = 0 \\ &-q^2 \partial_z \varphi^a + \frac{g_5^2 v^2}{z^2}\partial_z \pi^a = 0 \end{aligned}\right\} \Leftarrow \pi \text{ tower}$$

From these equations one find

$$m_{a_1}, \quad f_{a_1}, \quad m_\pi, \quad f_\pi$$

$$\langle 0 | \bar{q}\gamma^\mu \frac{\tau^a}{2} q | \pi^b \rangle = \delta^{ab} f_\pi p^\mu$$

One can also prove the Gell-Mann – Oakes – Renner relation $\quad f_\pi^2 m_\pi^2 = 2 m_q \sigma + O(m_q^2)$

Meson couplings: given in general by overlap integrals, e.g.

$$S_{\rho\pi} = \int d^5x \; \frac{v^2(z)}{2z^3} \left(\partial\pi^a - A^a + \varepsilon^{abc} V^b \pi^c\right)^2$$

$$g_{\rho\pi\pi} = g_5 \int dz \; \frac{v^2(z)}{z^3} \underbrace{(\pi-\varphi)\,\pi\,\psi_\rho}_{\text{normalizable modes}}$$

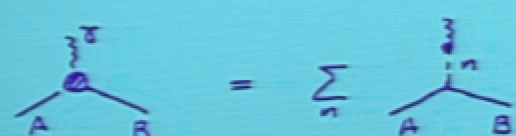
Origin of vector meson dominance:

$$g_{\gamma AB} = \int \underbrace{\psi_\gamma}_{\text{non-normalizable}}(z)\,\psi_A(z)\,\psi_B(z)$$

$$g_{\rho_n AB} \sim \int \underbrace{\psi_{\rho_n}}_{\text{normalizable}}(z)\,\psi_A(z)\,\psi_B(z)$$

Non-normalizable mode can be expanded in series over normalizable modes

$$\psi_\gamma(z) = \sum_n \frac{F_n}{q^2 - m_n^2}\,\psi_n(z)$$



dominated by first few terms

Result:

	Exp.	Model	
m_π	139.6	139.6*	140.1
m_ρ	775.8	775.8*	772.3
m_{a_1}	1230	1363	1278
f_π	92.4	92.4*	87.7
$F_\rho^{1/2}$	345	329	328
$F_{a_1}^{1/2}$	433	452	457
$g_{\rho\pi\pi}$	6.03	6.63	6.79

More things one can do:

- Strange mesons $N_f = 2 \rightarrow N_f = 3$
- Chiral anomaly
- Baryons : skyrmions
- inclusion of glueballs ...

What did we learn ?

- Even the simplest 5D model works surprisingly well,

- chiral symmetry breaking important

- OPE matching ($g_5^2 = \frac{12\pi^2}{N_c}$) needed : can it be extended beyond leading logarithm ?

$$\Pi_V(Q^2) = \frac{N_c}{24\pi^2} \ln Q^2 + \# \frac{\langle G_{\mu\nu} G^{\mu\nu} \rangle}{Q^4} + \frac{\# \langle \bar{q} q \rangle^2}{Q^6} + \ldots$$

$$= \text{sum over hadron resonances}$$

previously employed by QCD sum rules

Shifman, Vainshtein, Zakharov 1979

- Beyond field theory : strings ?

unexplained features of QCD :

- intercept of ρ trajectory close to 1/2
- $m_\rho^2 - m_\pi^2 \approx m_{K^*}^2 - m_K^2 \approx m_{D^*}^2 - m_D^2 \approx m_{B^*}^2 - m_B^2$
- suppression of higher twist effects in DIS
- .....