Title: Complex Correlations in Self-Organized Critical Phenomena

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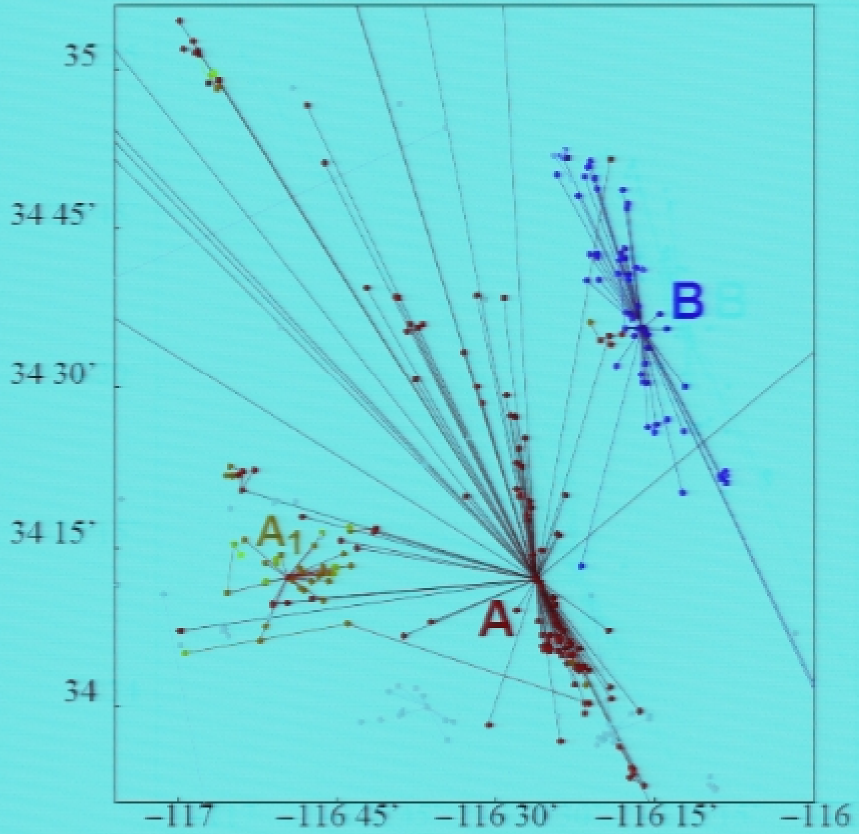
Abstract: Natural critical phenomena are characterized by laminar periods separated by events where bursts of activity take place, and by the interrelated self-similarity of space-time scales and of the event sizes. One example are earthquakes: for this case a new approach to quantify correlations between events reveals new phenomenology. By linking correlated earthquakes one creates a scale-free network of events, which can have applications in hazard assessment. Solar flares are another example of critical phenomenon, where event sizes and time scales are part of a single self-similar scenario: rescaling time by the rate of events with intensity greater than an intensity threshold, the waiting time distributions conform to scaling functions that are independent of the threshold. The concept of self-organized criticality (SOC) is suitable to describe critical phenomena, but we highlight problems with most of the classical models of SOC (usually called sandpiles) to fully capture the space-time complexity of real systems. In order to fix this shortcoming, we put forward a strategy giving good results when applied to the simplest sandpile models.

Complex correlations in self-organized critical phenomena



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C. Vanderzande (K.U.Leuven)

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0: Critical phenomena

1: Earthquakes

* Metric to quantify correlations

* Scale-free network

* Old and new phenomenology

2: Solar flares

Definition by means of thesholds

Waiting times: scaling picture

Superstatistics

3: Self-organized criticality (SOC)

- Features of present models
- Suitable to describe flares/quakes?
- A new model

4: Conclusions

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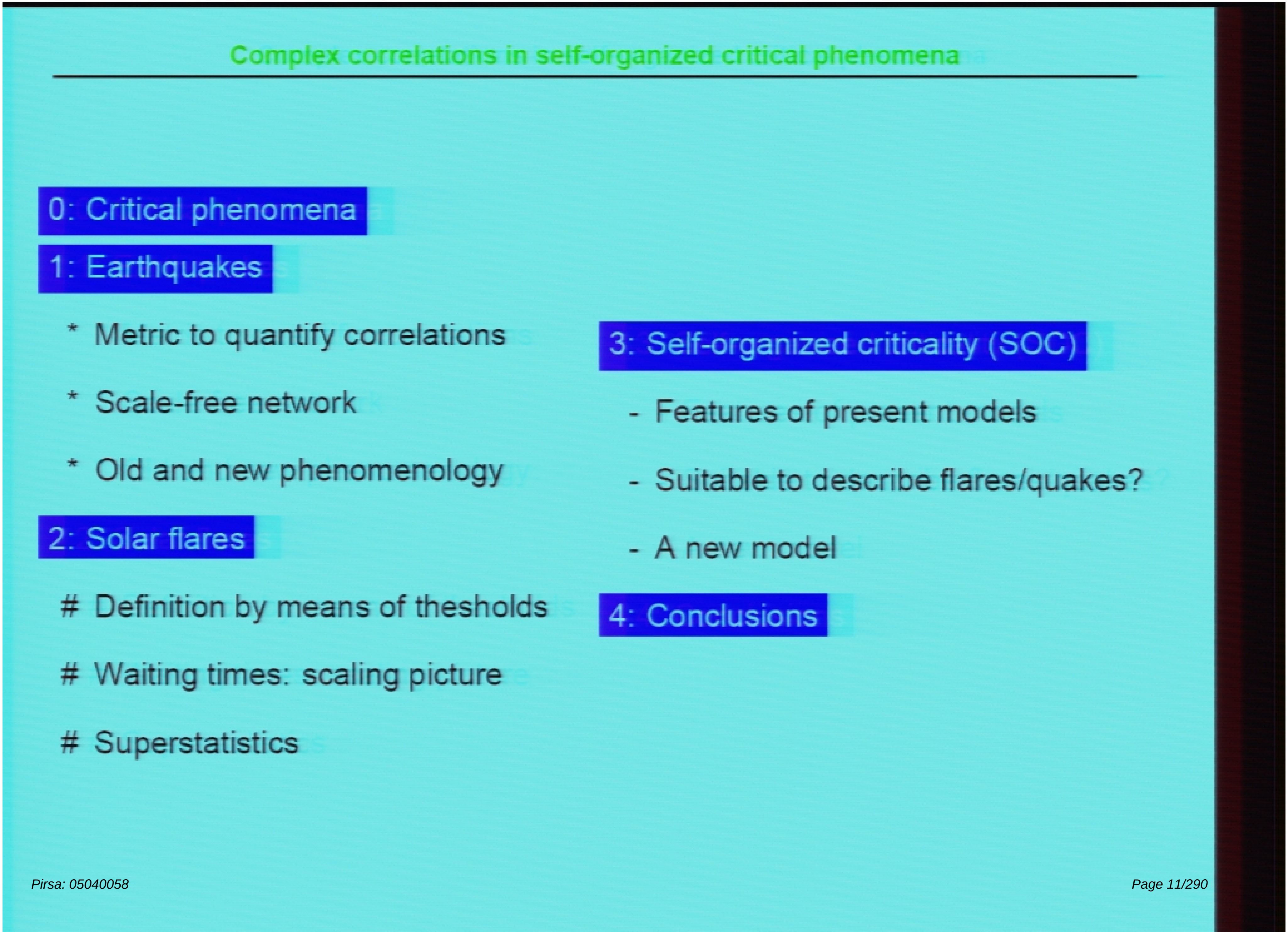

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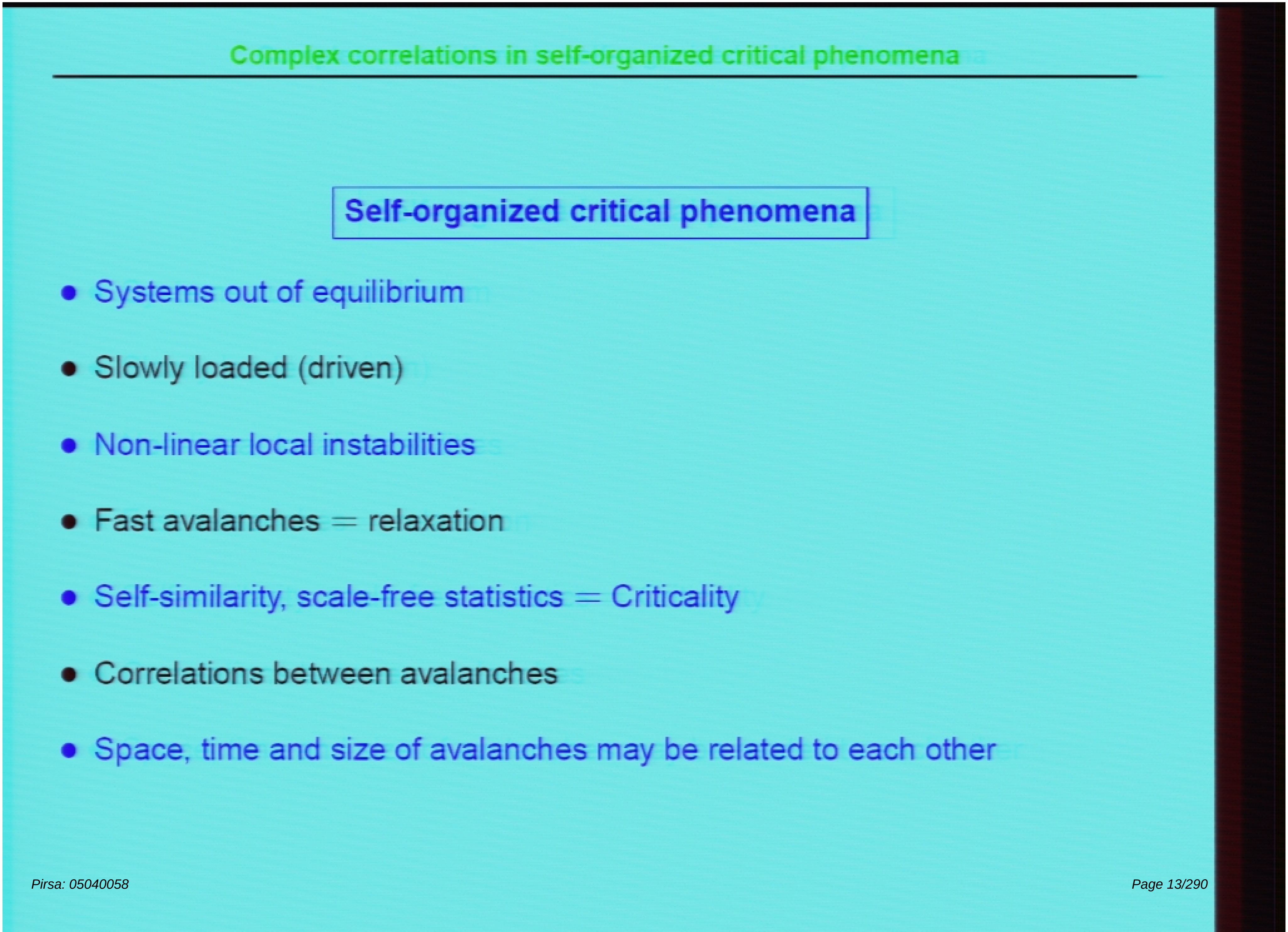

Self-organized critical phenomena

- Systems out of equilibrium
- Slowly loaded (driven)
- Non-linear local instabilities
- Fast avalanches = relaxation
- Self-similarity, scale-free statistics = Criticality
- Correlations between avalanches
- Space, time and size of avalanches may be related to each other

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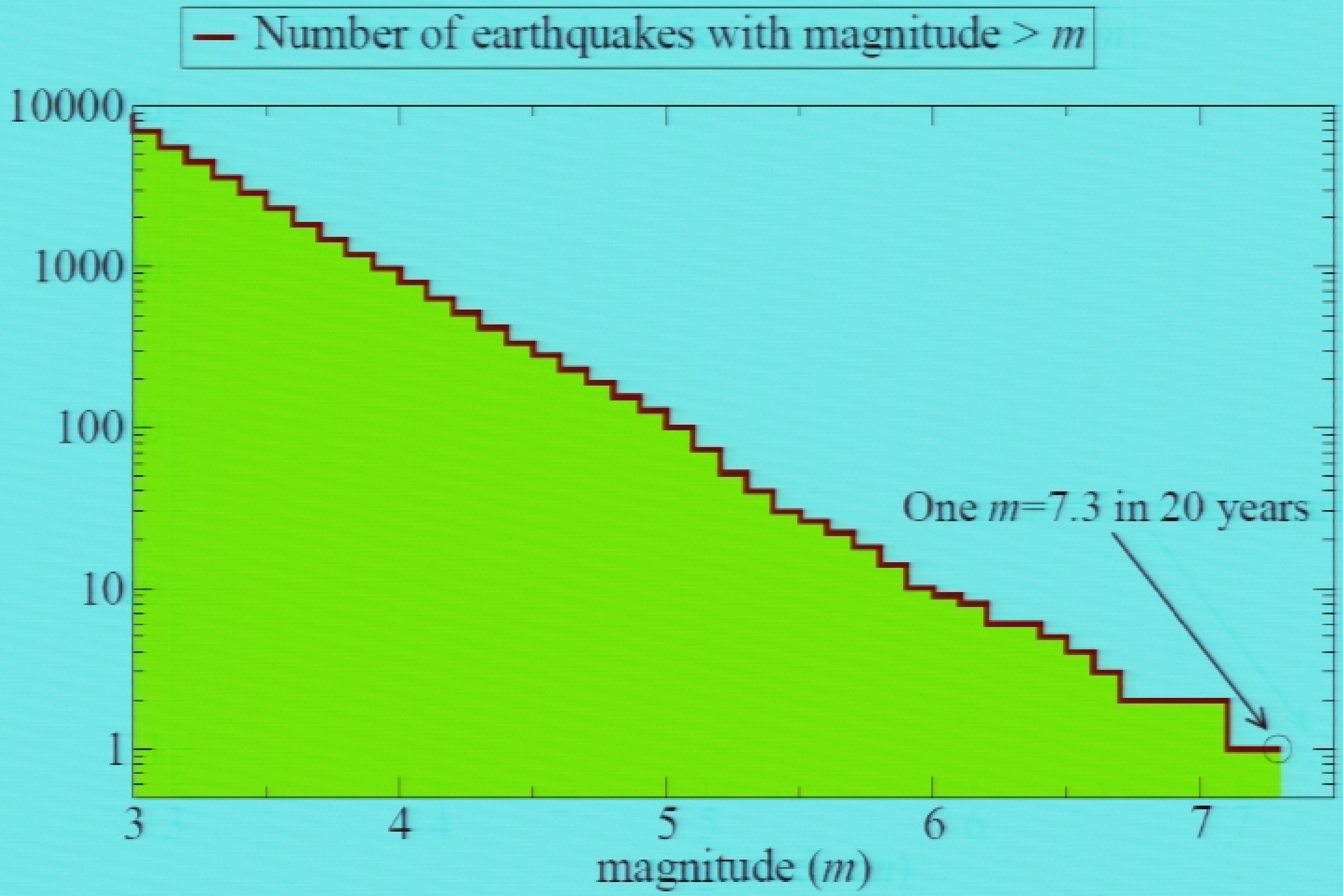

Gutenberg-Richer law: $n(>m) \sim 10^{-bm} \sim (\text{energy})^{-B+1}$,

$b \approx 1$, $B \approx 2b/3$

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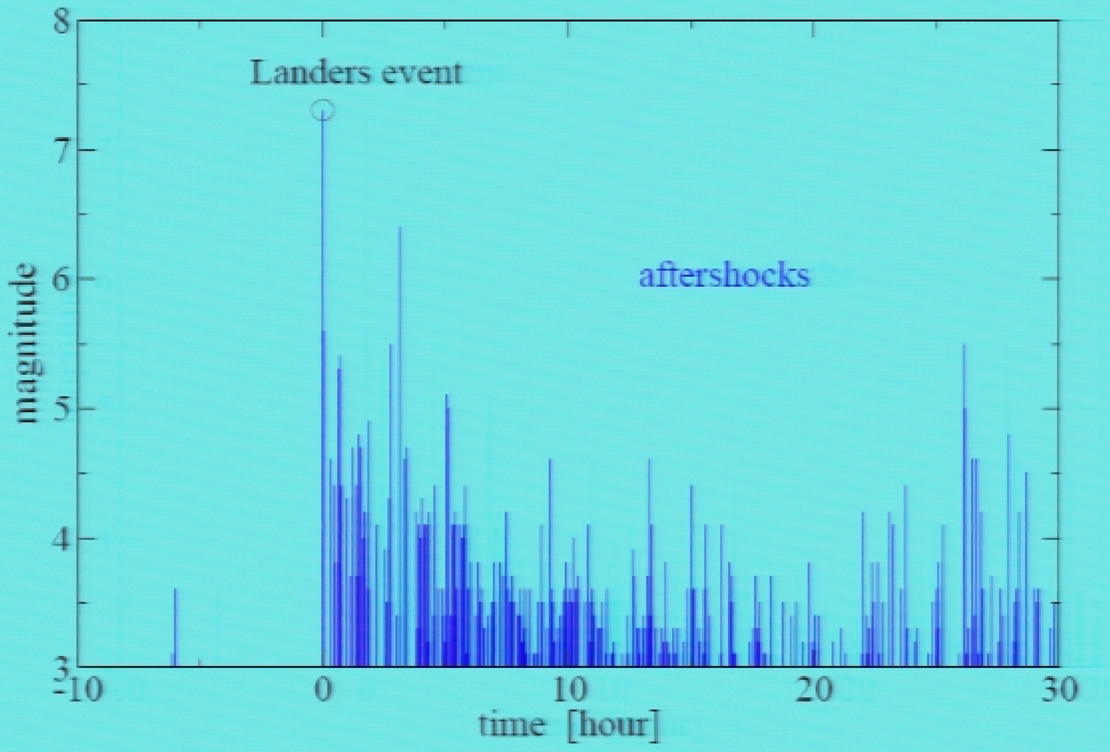

Omori law of the rate of events after a main shock: $\sim \dfrac{1}{c+t}$

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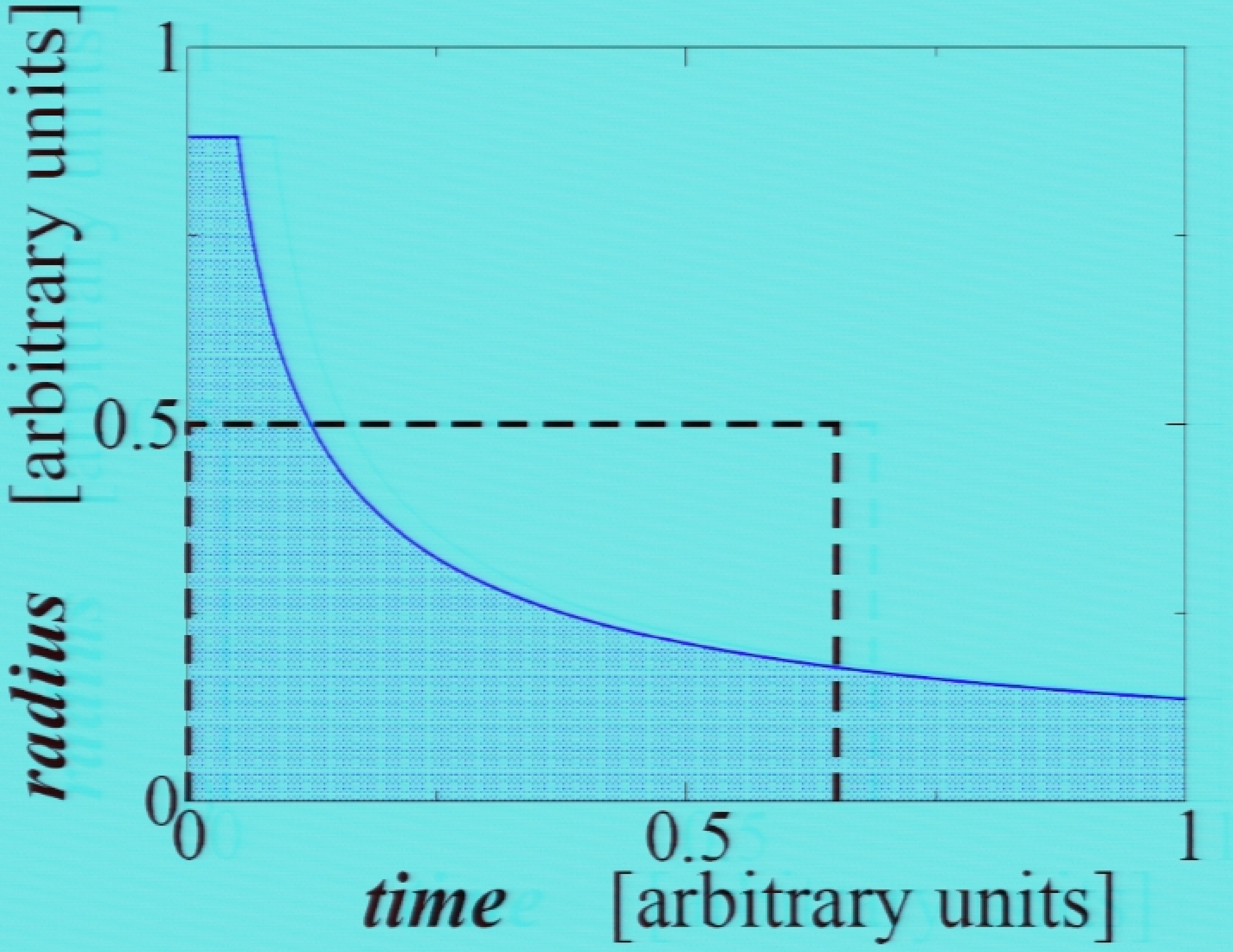
Space-time windows to collect aftershocks



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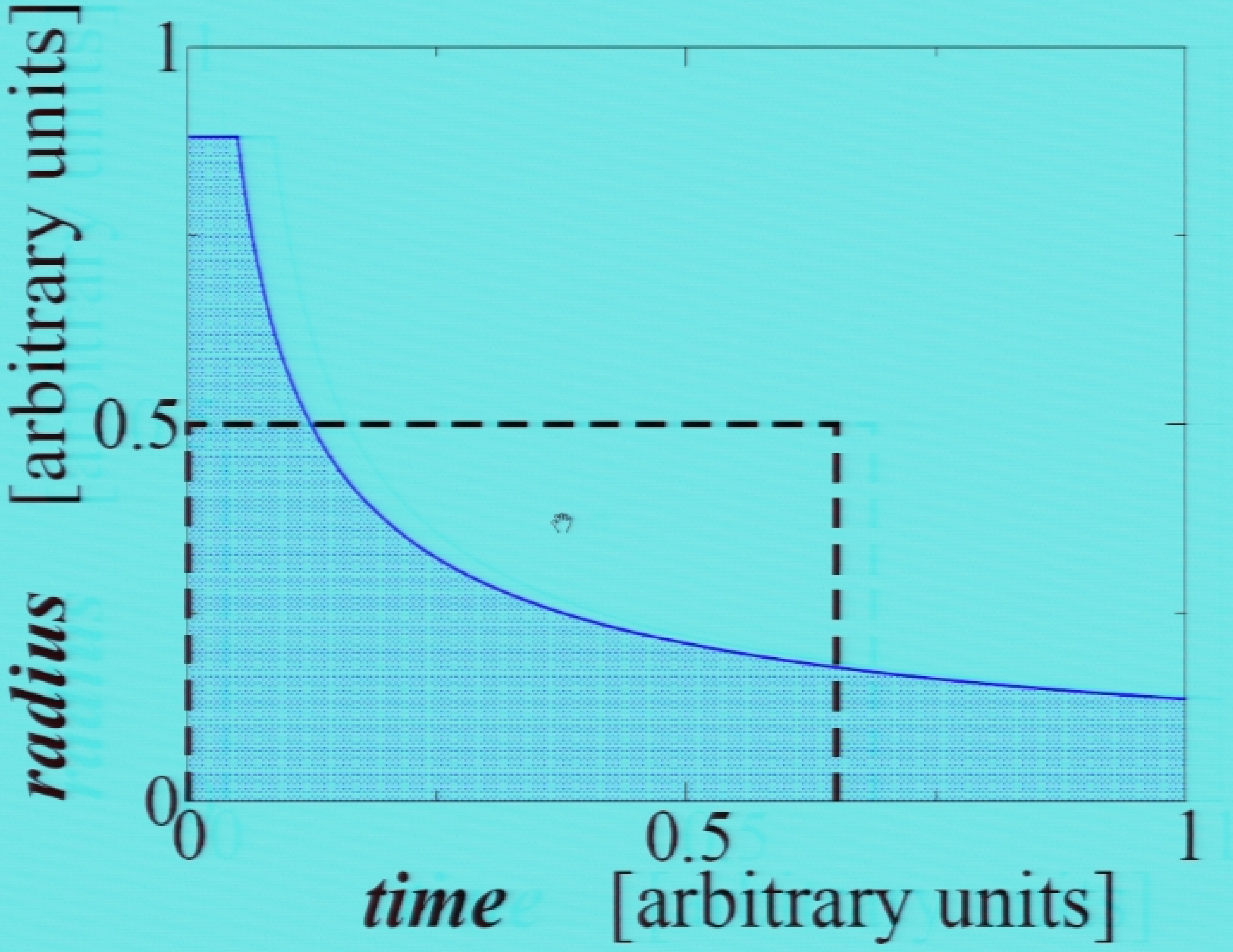

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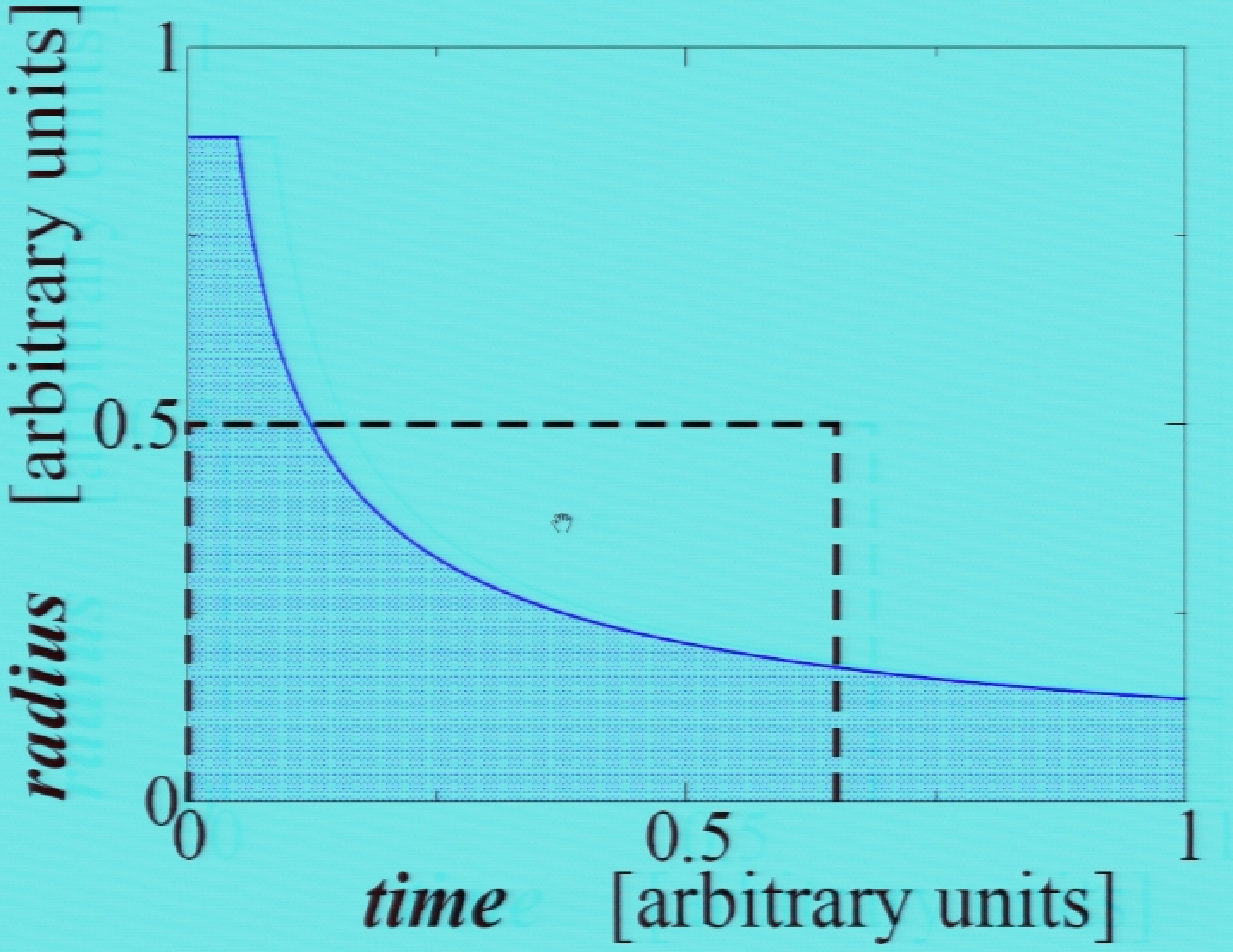

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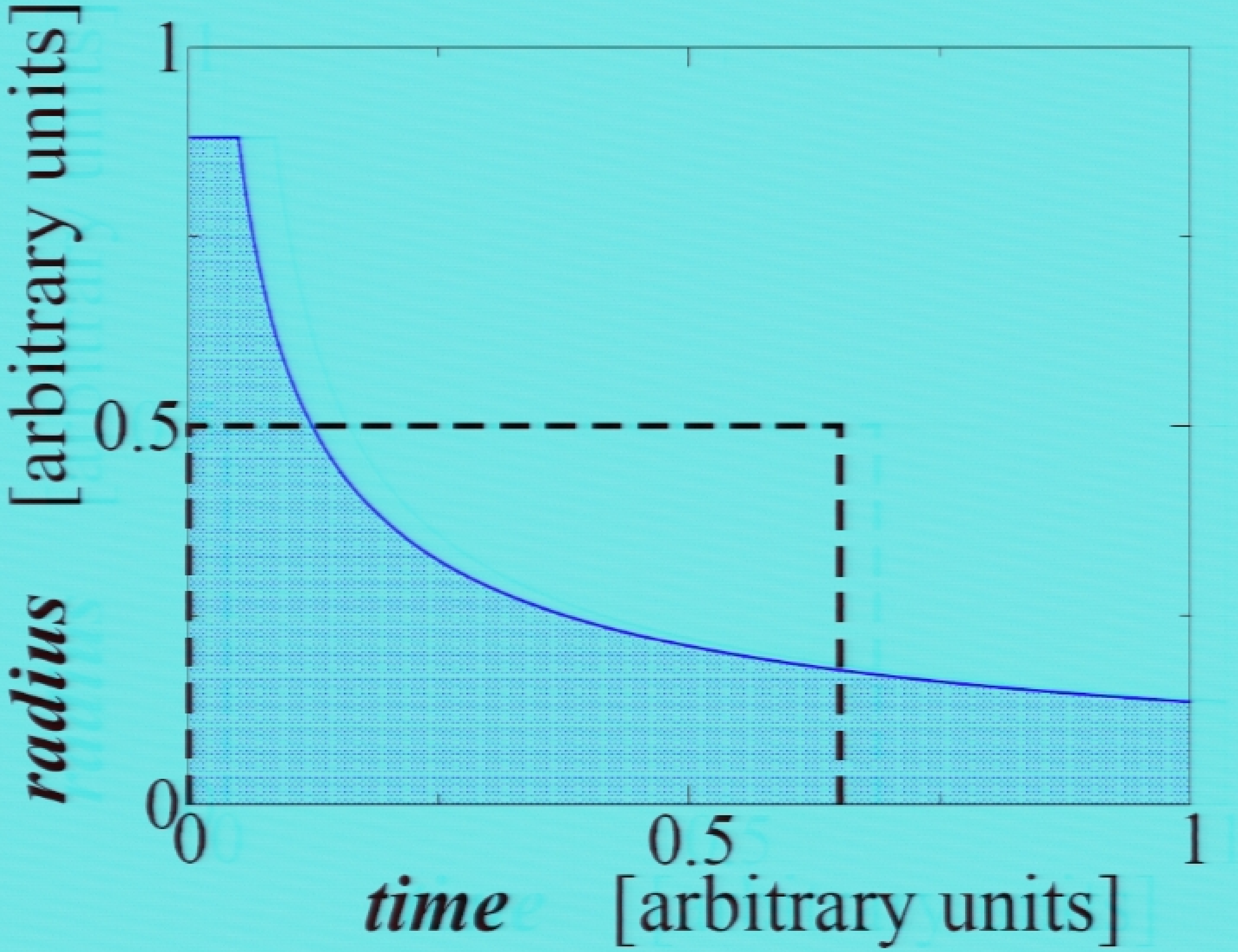

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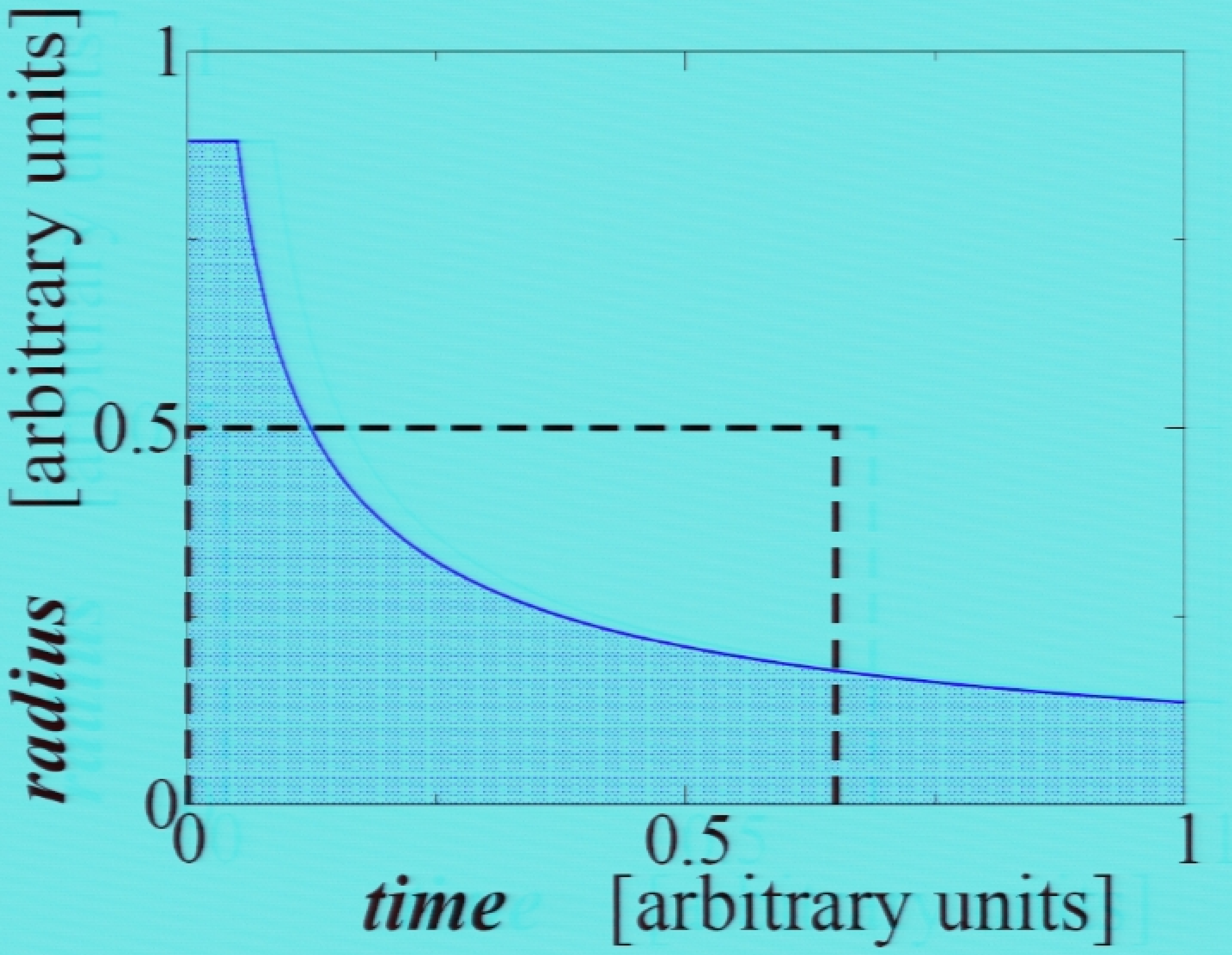

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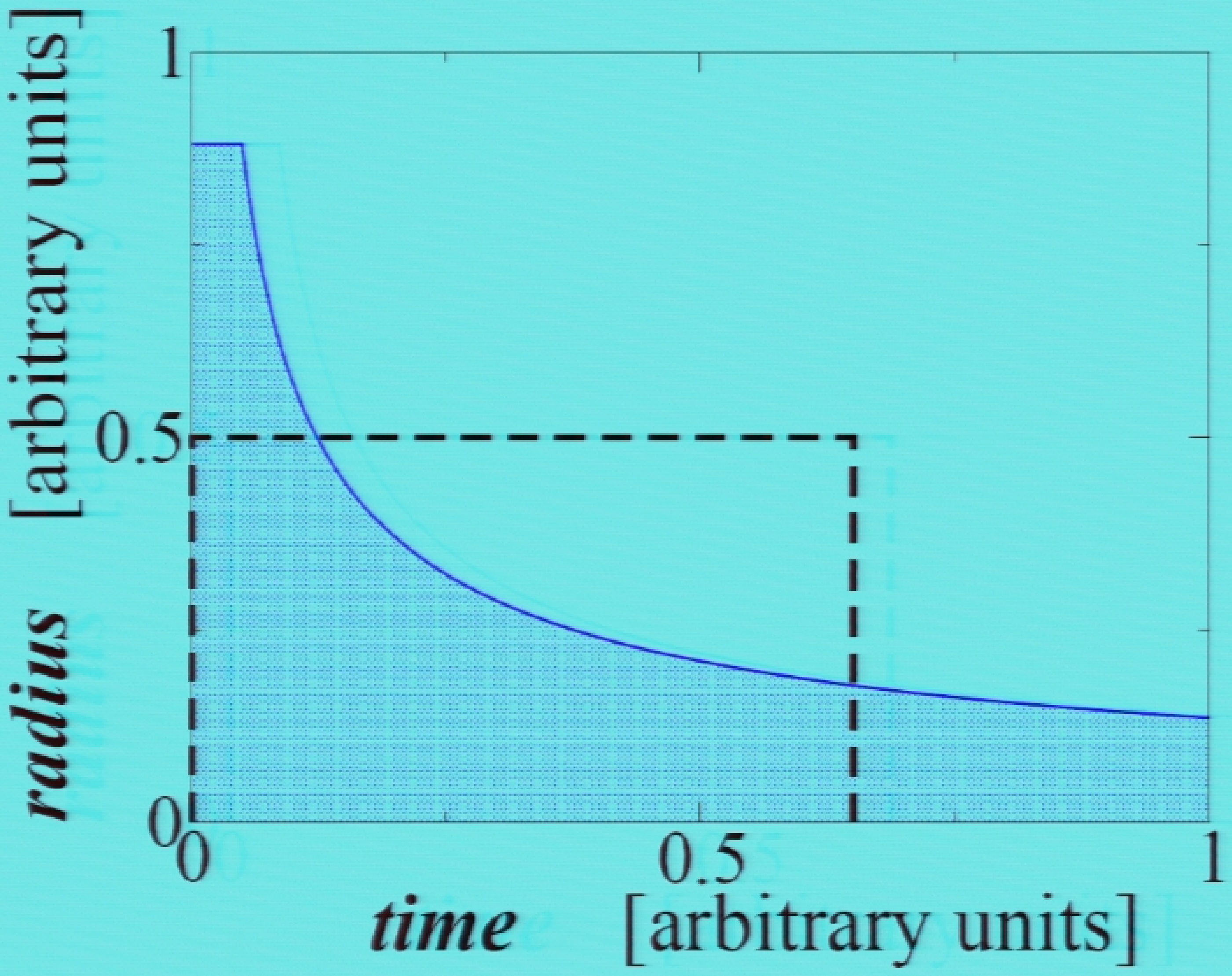

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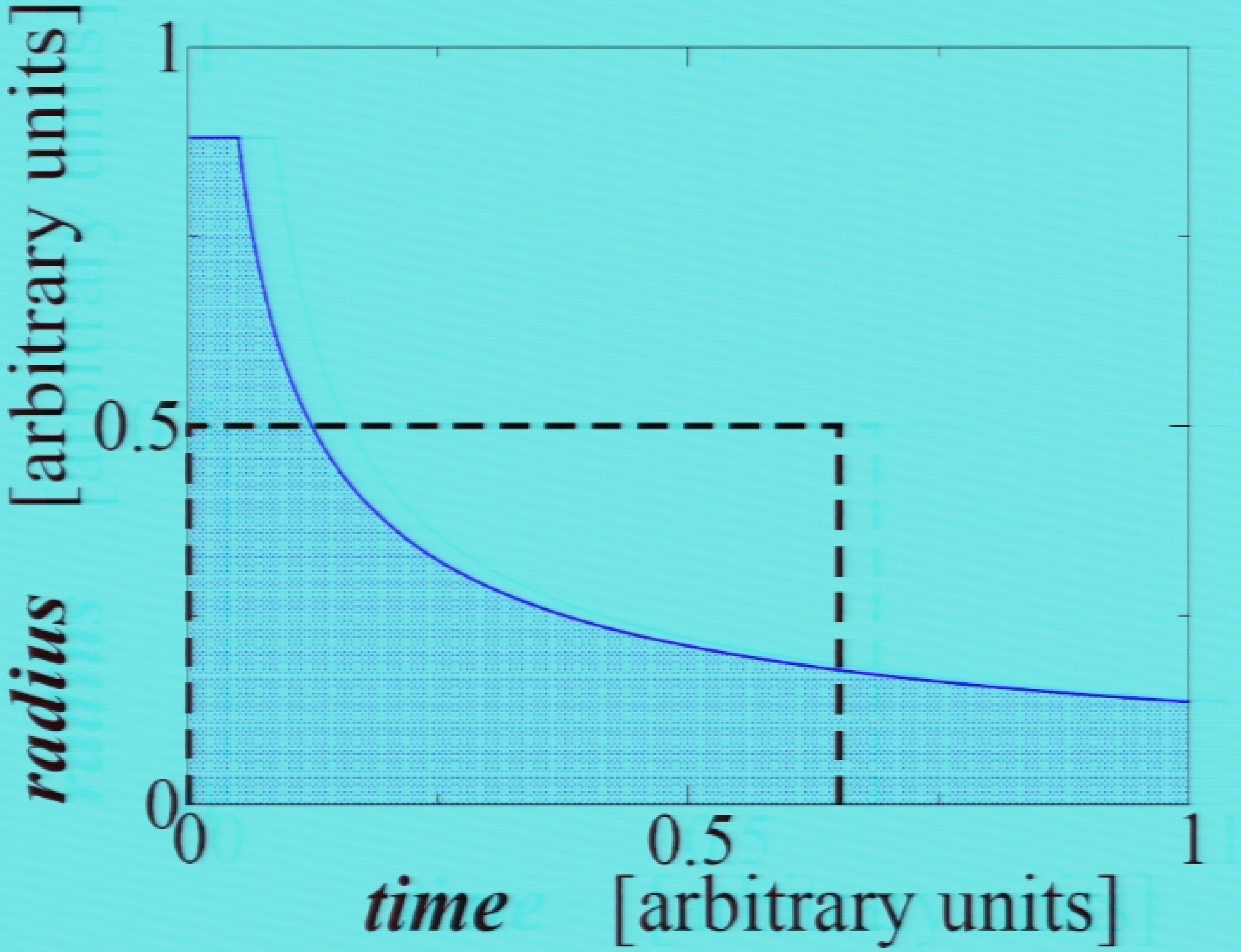

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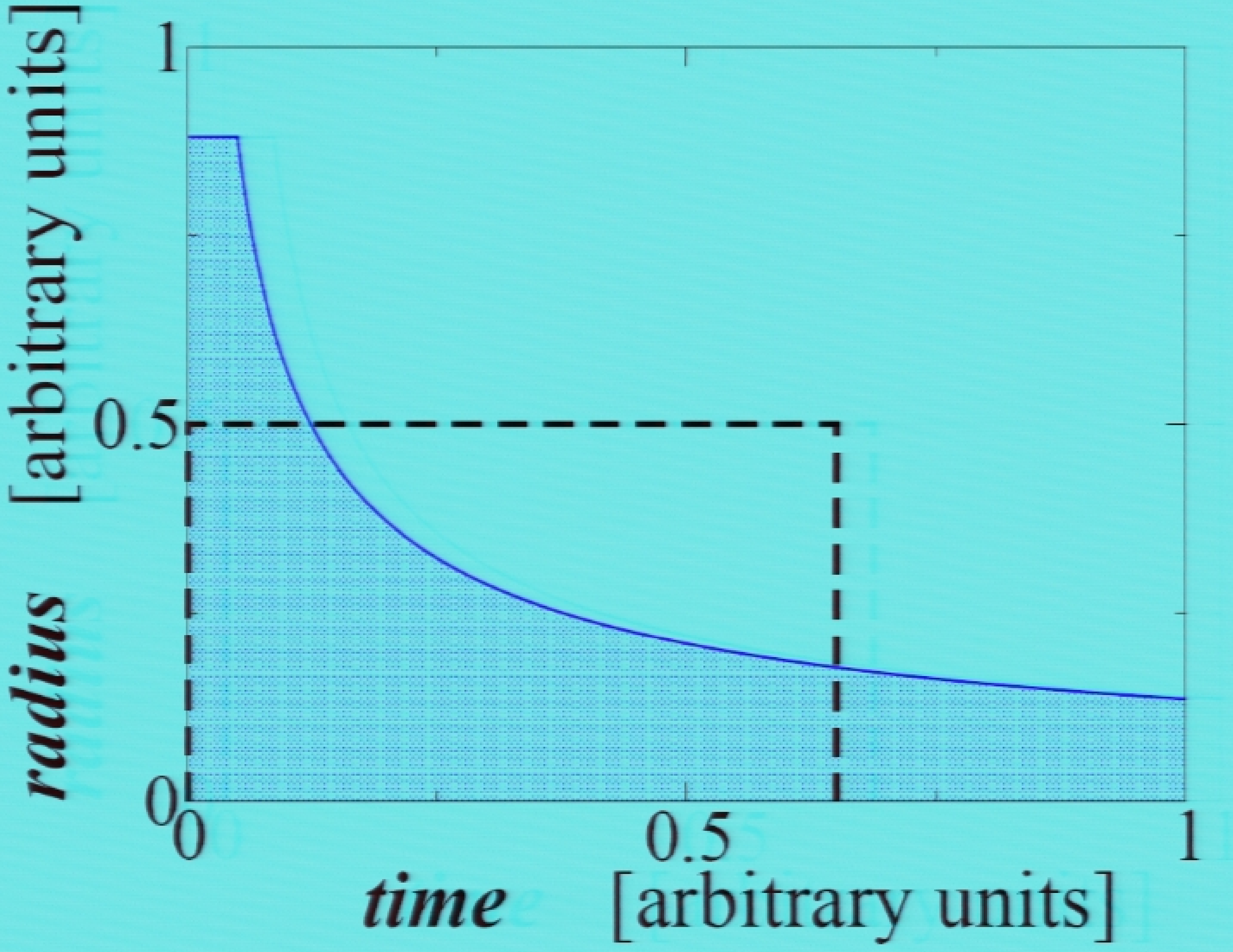

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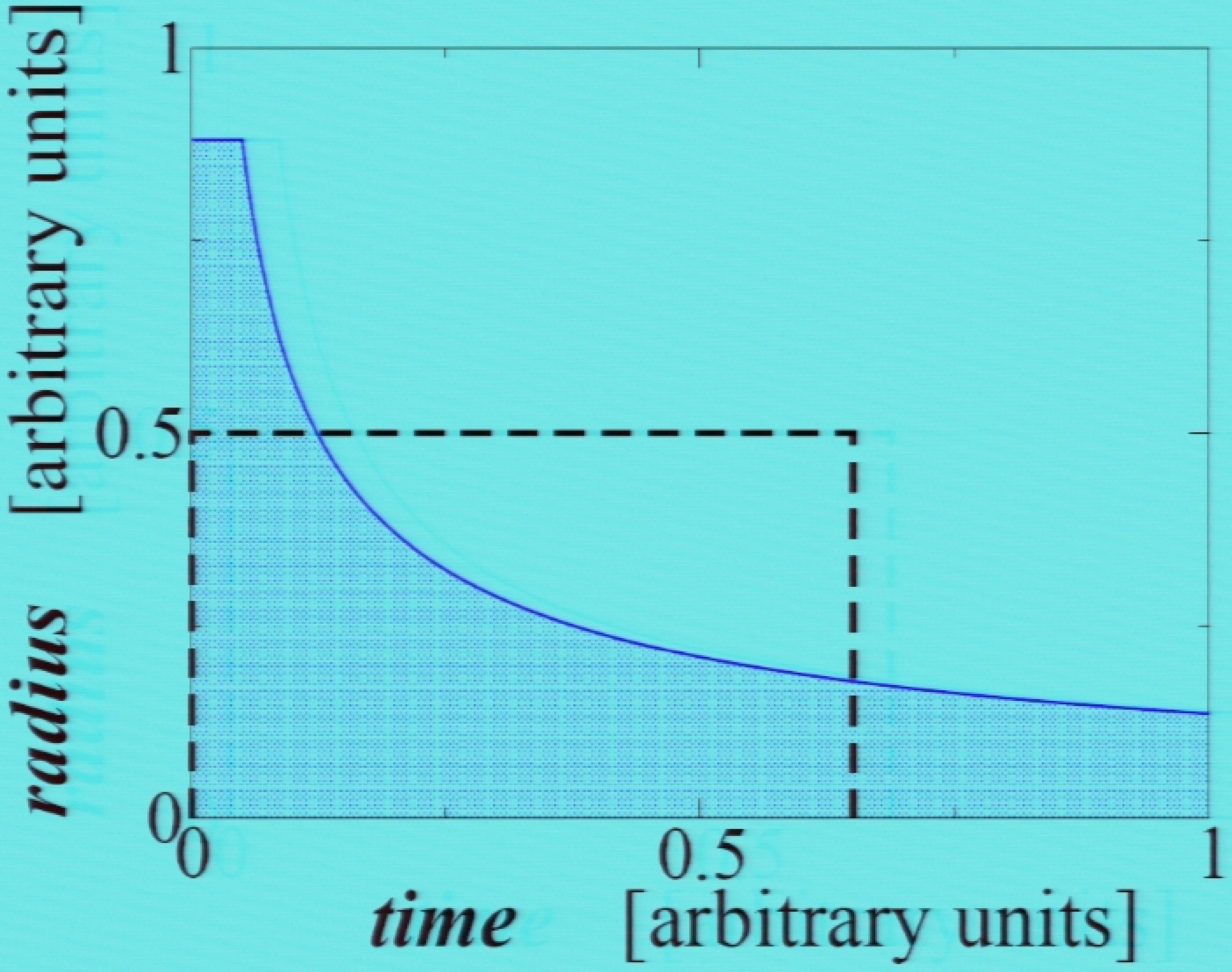

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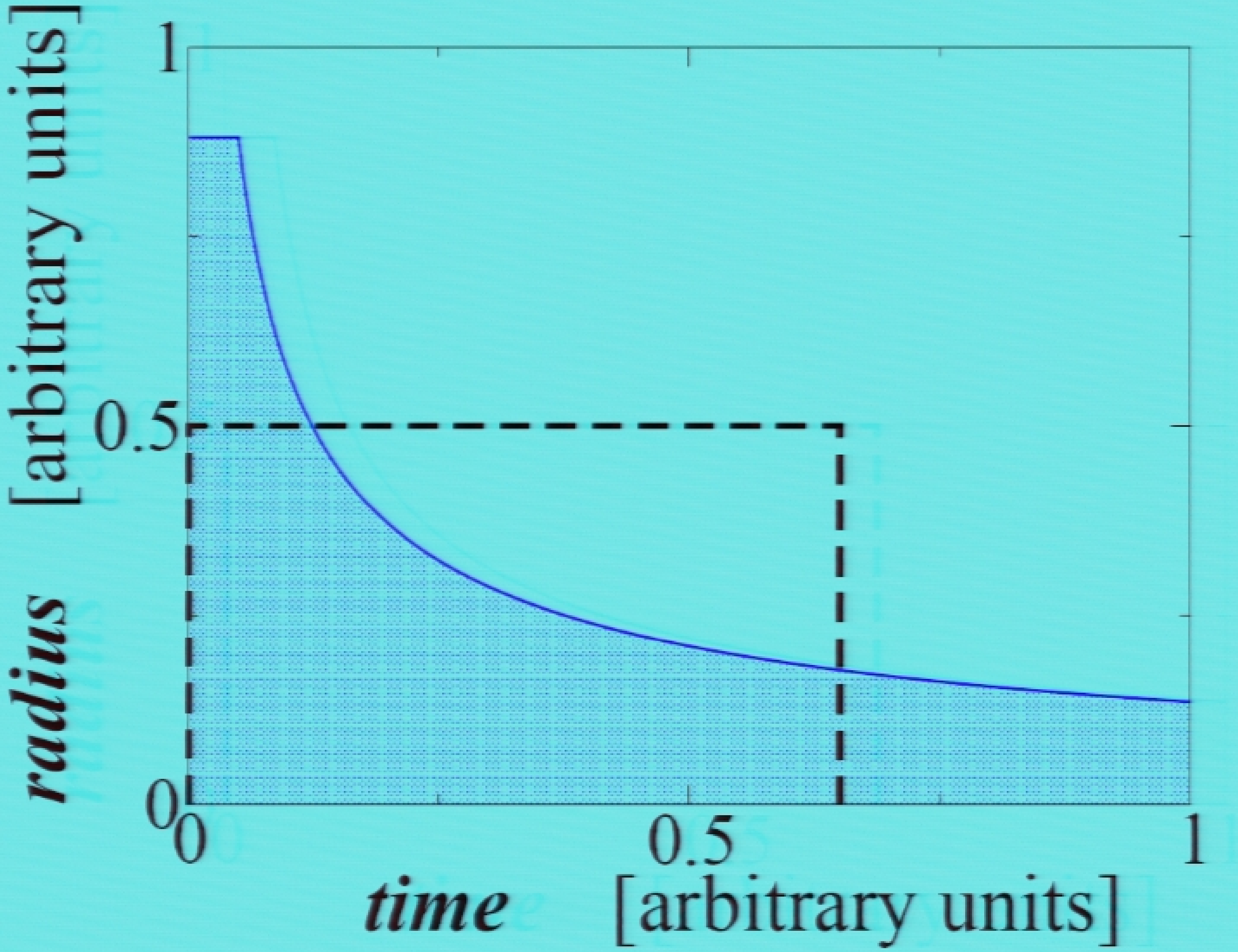

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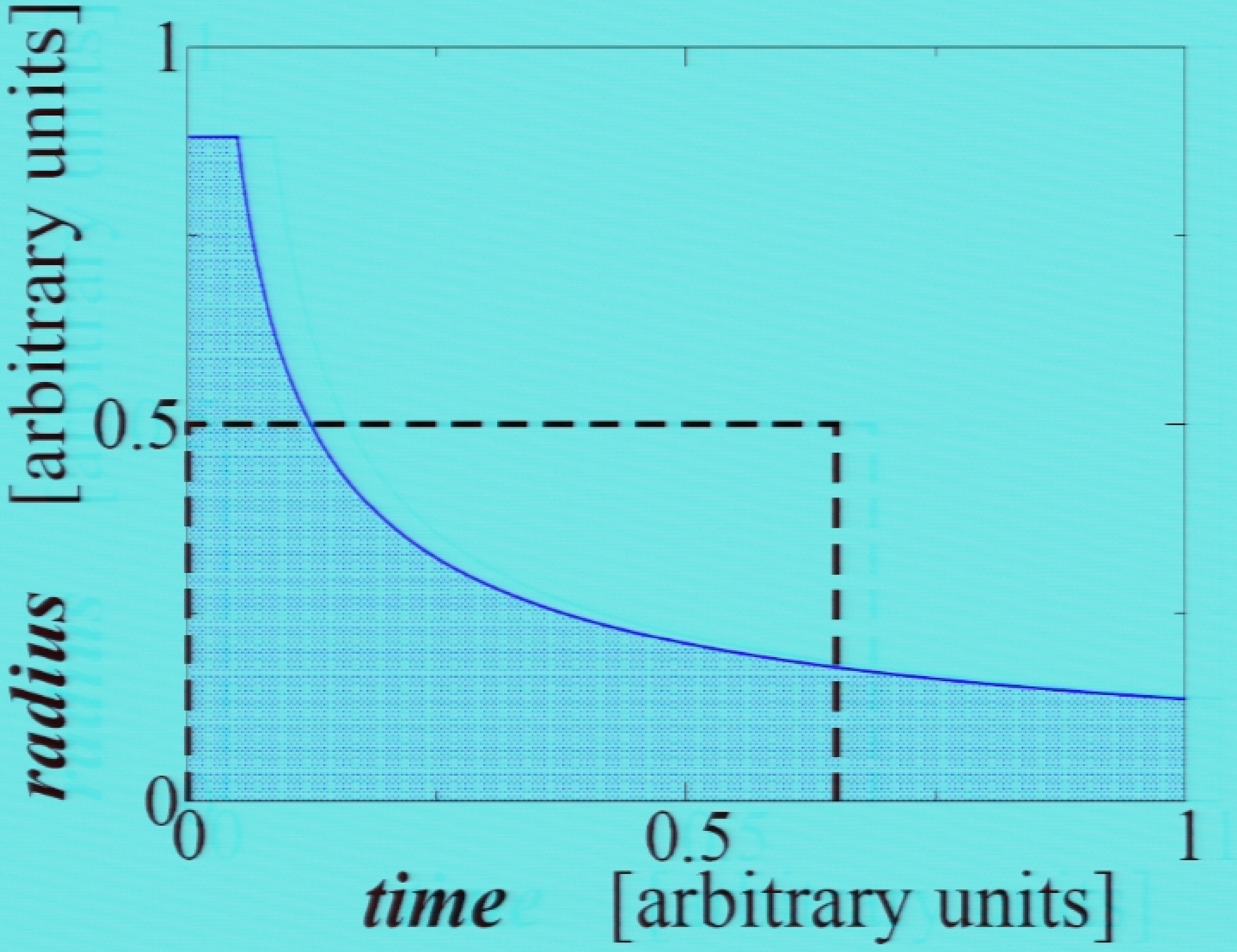

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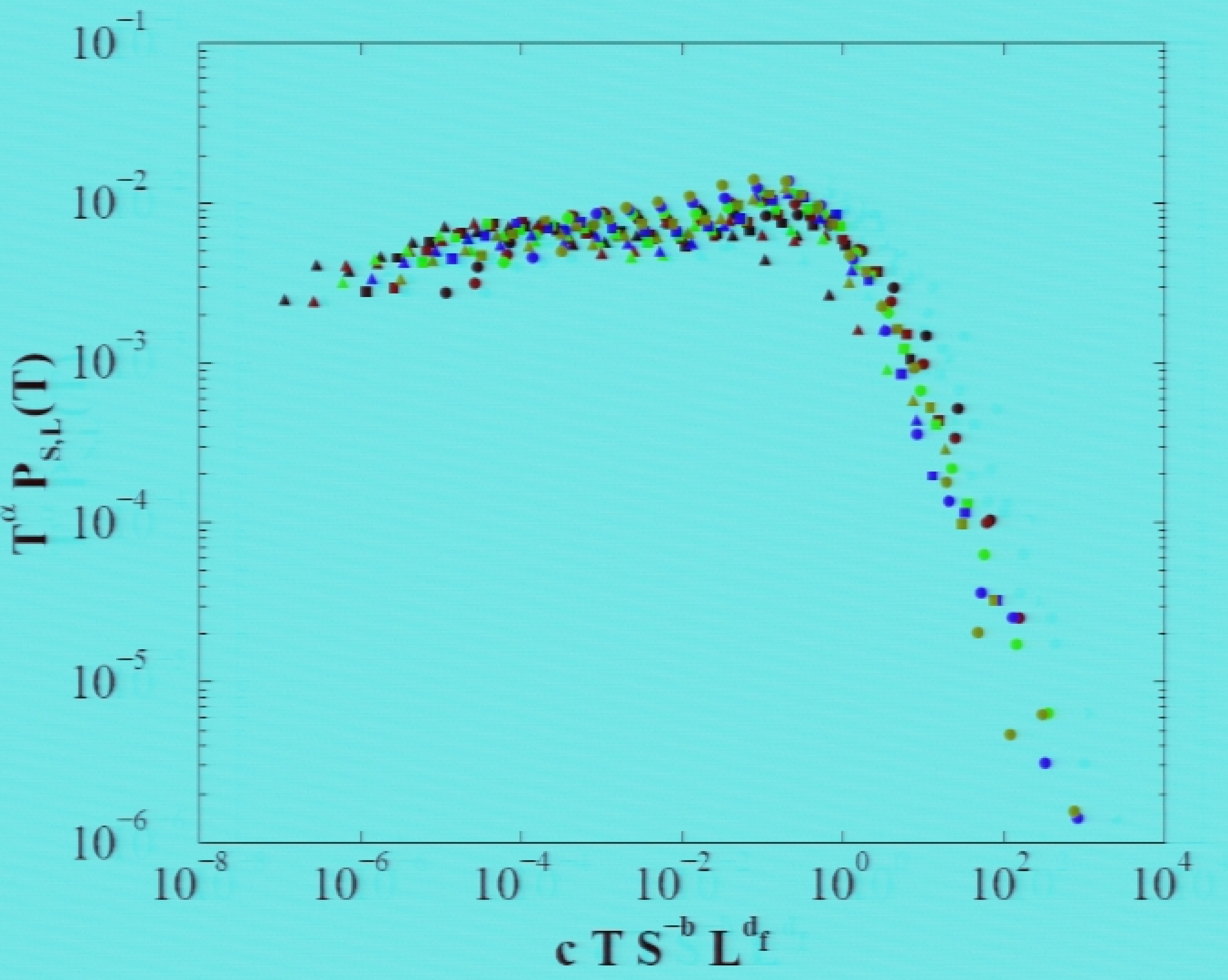
Unified scaling law of waiting times, involving space, time, and magnitude

[Bak, Christensen, Danon & Scanlon, PRL 2002] [Corral, PRE 2003]



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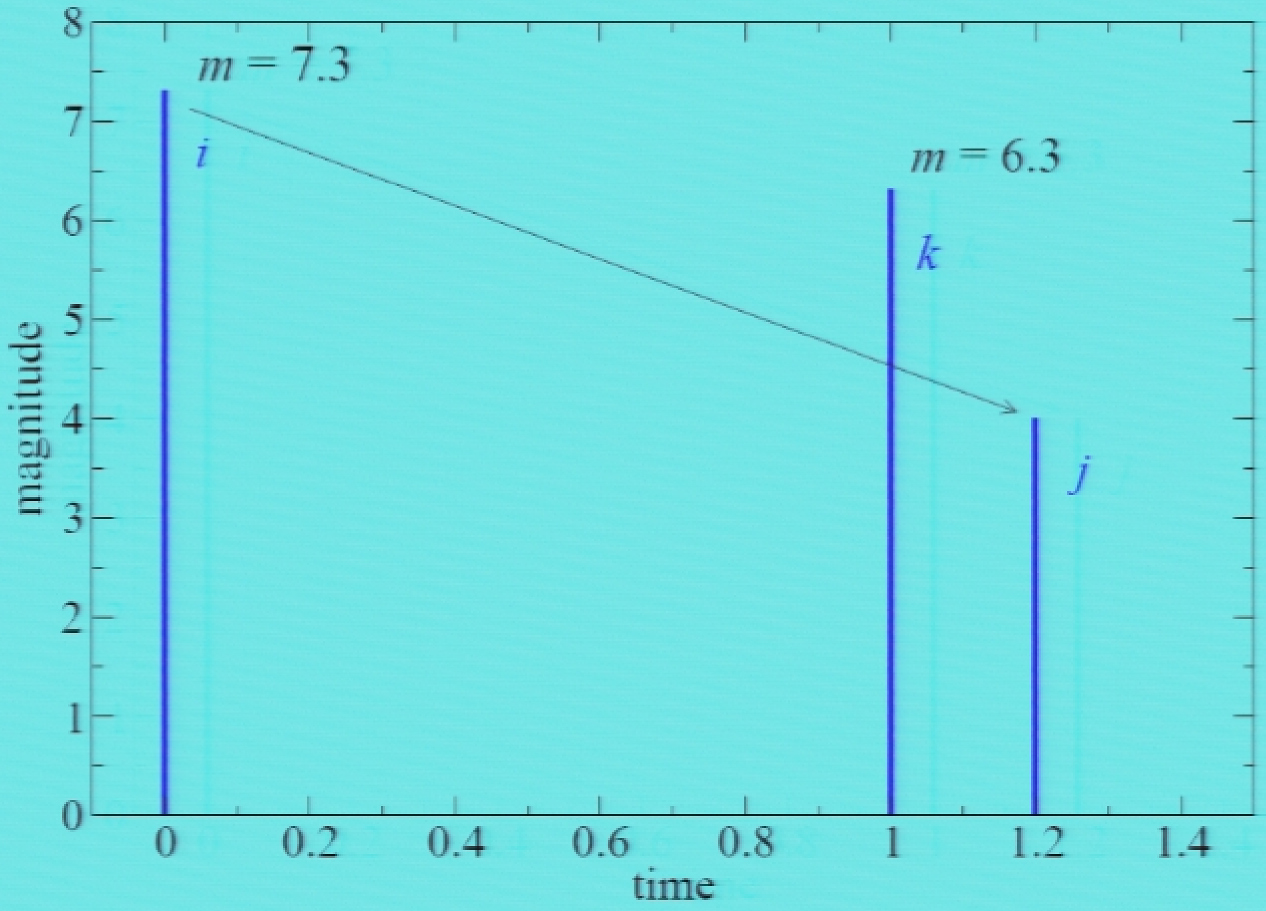

$n(\geq 7.3) \propto 10^{-7.3} \times 1.2$ hour $\sim 6 \times 10^{-8} \rightarrow$ more unusual

$n(\geq 6.3) \propto 10^{-6.3} \times 0.2$ hour $\sim 10^{-7}$

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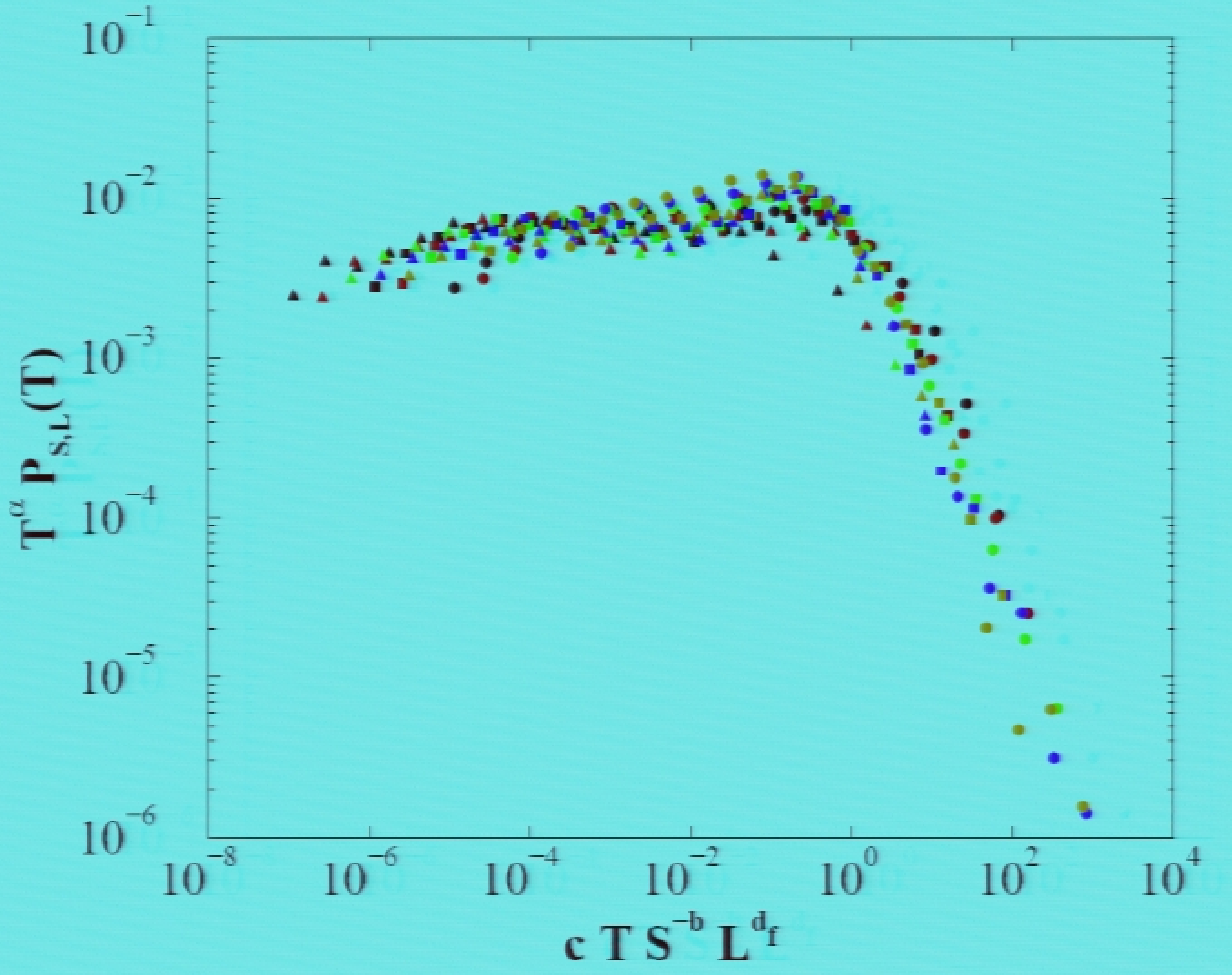

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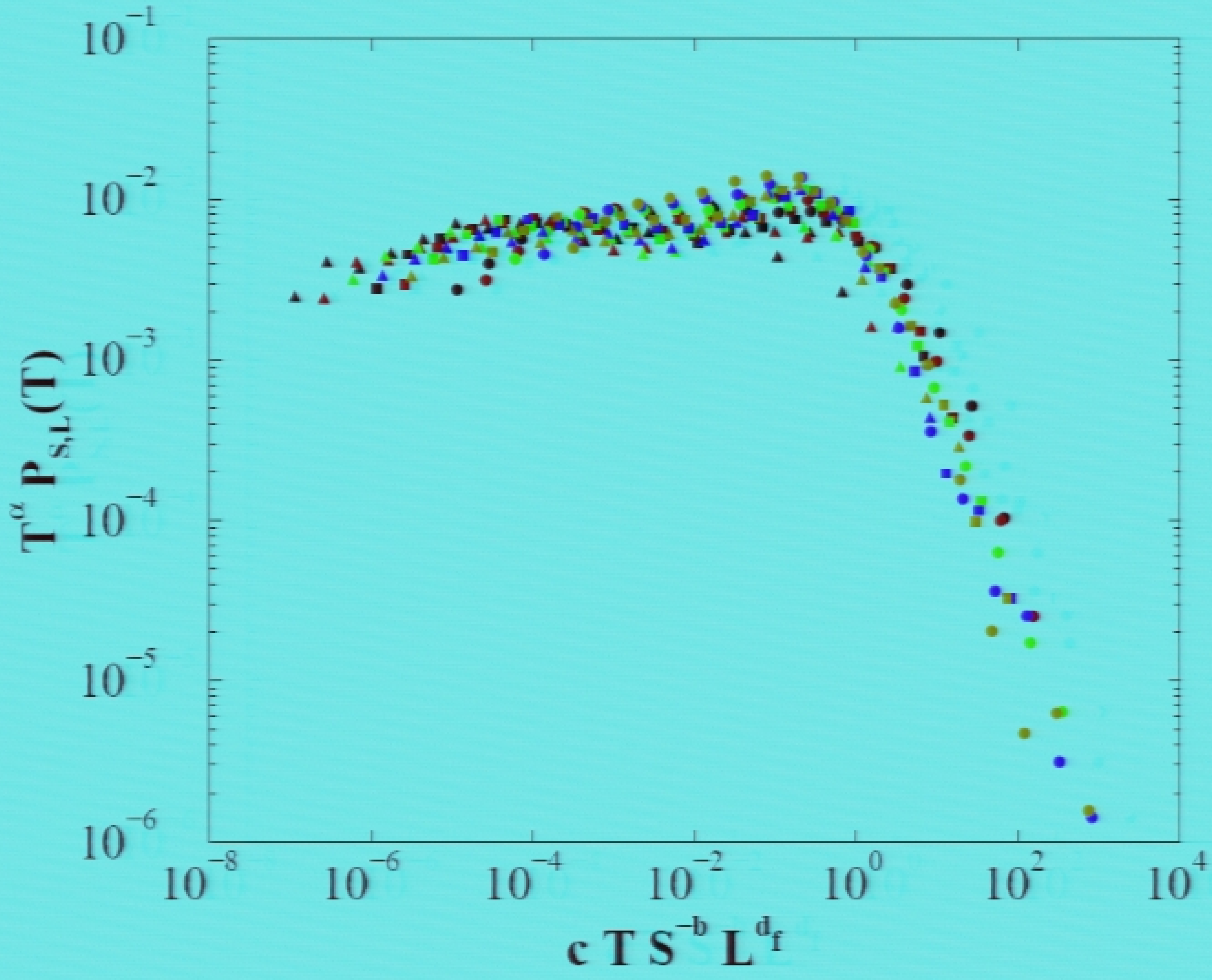

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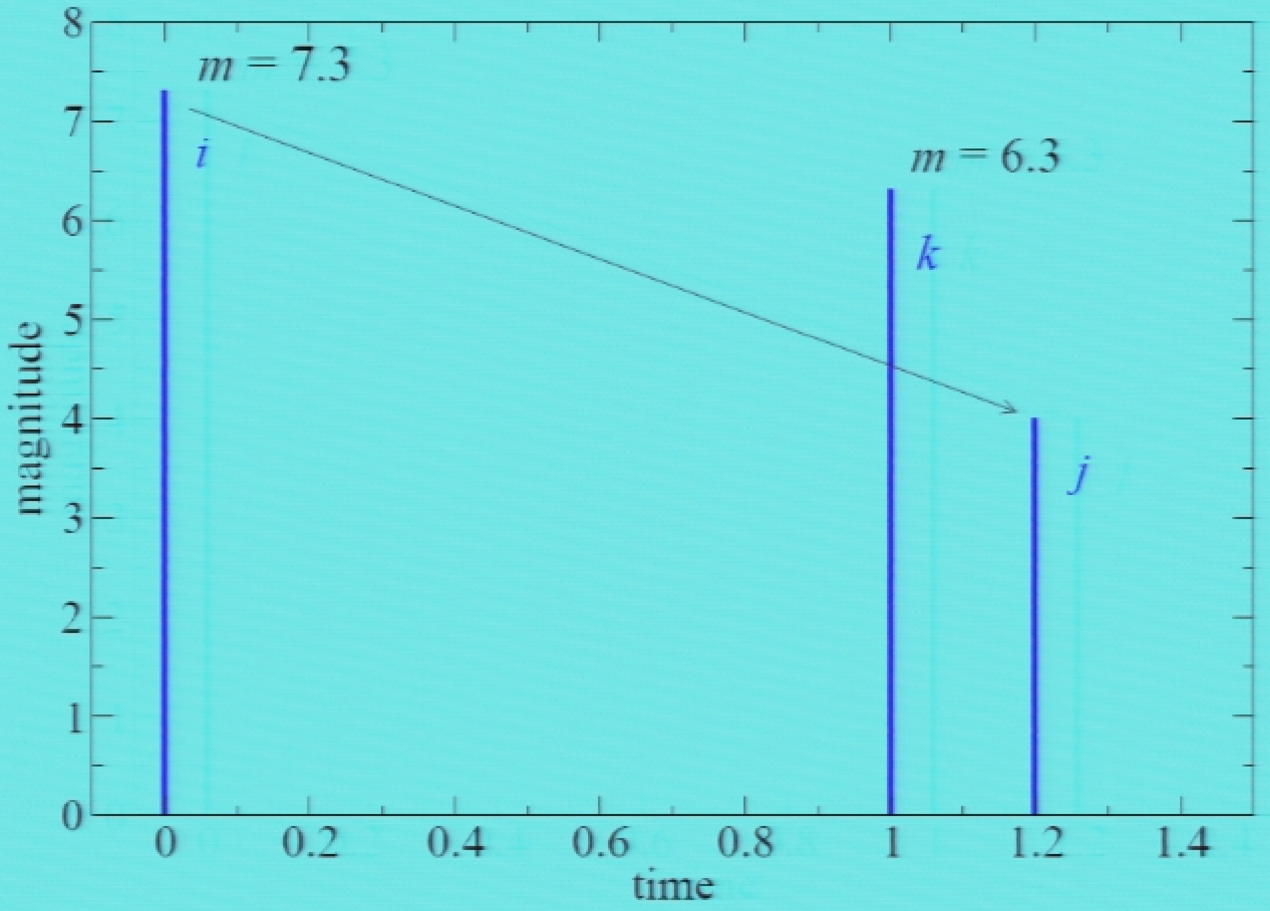

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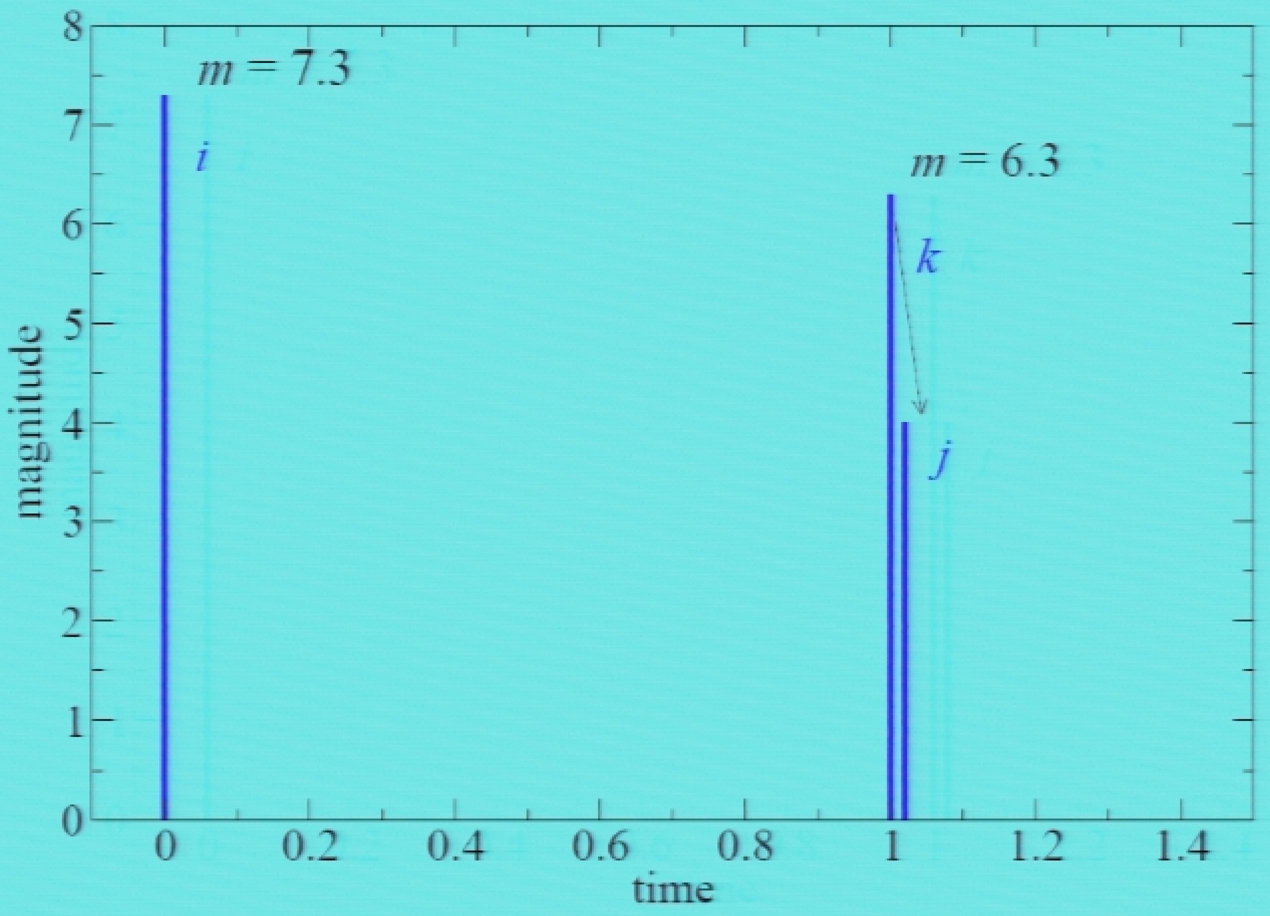

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Measure correlation between events: estimate how much the (patently false) null hypothesis that “earthquakes are uncorrelated” is violated.

mean number of events in volume $(m \pm \Delta m/2, l, t)$

$$n = \underbrace{\boxed{C\,\Delta m}}_{\text{Const.}}\ \underbrace{\boxed{10^{-bm}}}_{\text{Gutenberg-Richter}}\ \underbrace{\boxed{l^{d_f}}}_{\text{(fractal) area}}\ \underbrace{\boxed{t}}_{\text{time interval}}$$

Events i and j: metric

$$n_{ij} = C\,\Delta m\,10^{-bm_i}\,(l_{ij})^{d_f}\,t_{ij}$$

Correlation:

$$c_{ij} = 1/n_{ij}$$

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Const. Gutenberg-Richter (fractal) area time interval

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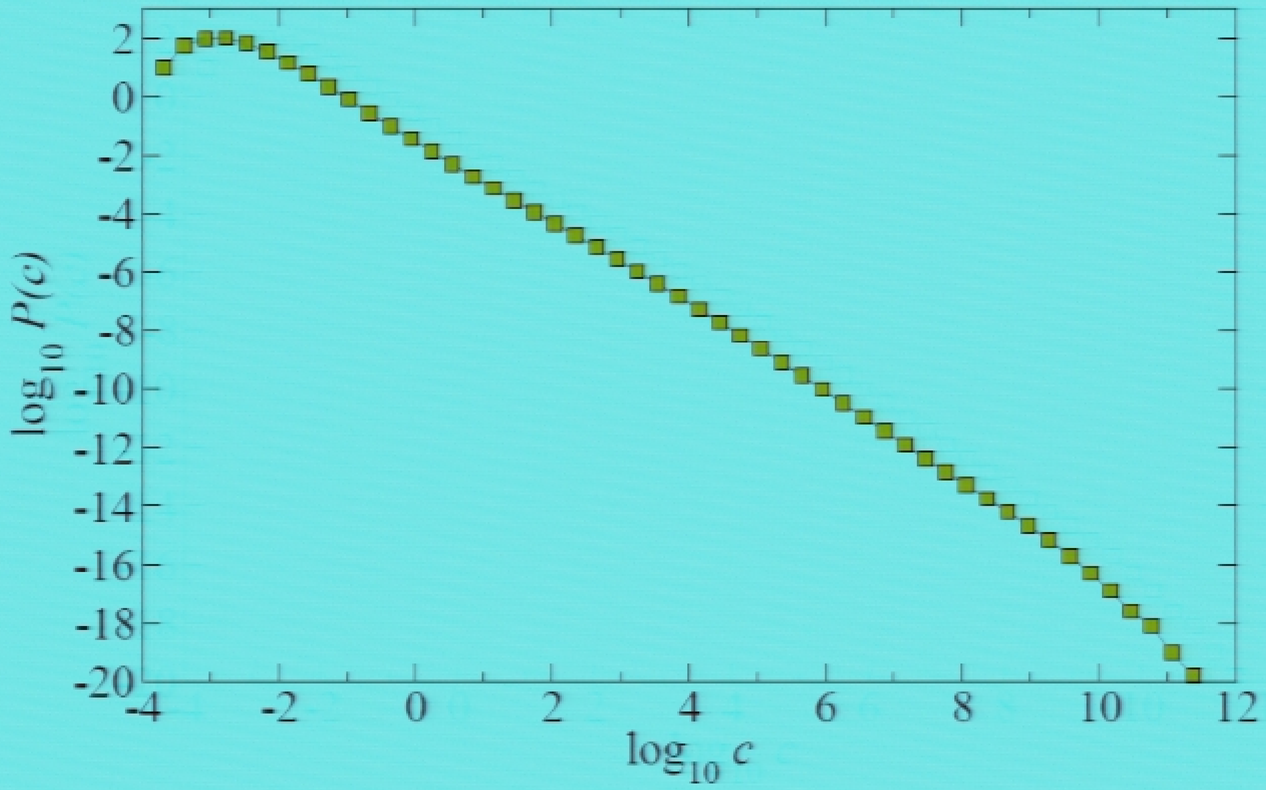

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Draw only one link to a new event j: largest c_{ij} ($=$ or smallest $n_{ij} \equiv n_j^*$)

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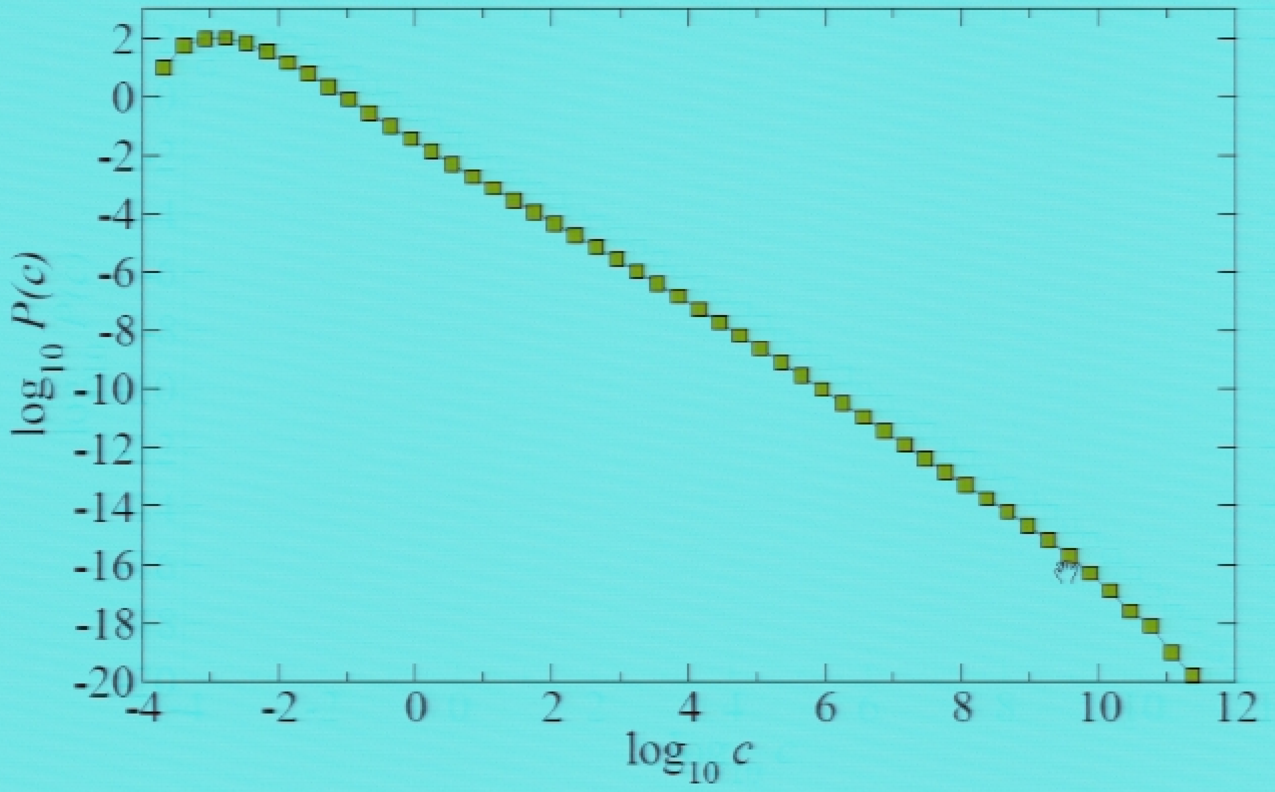

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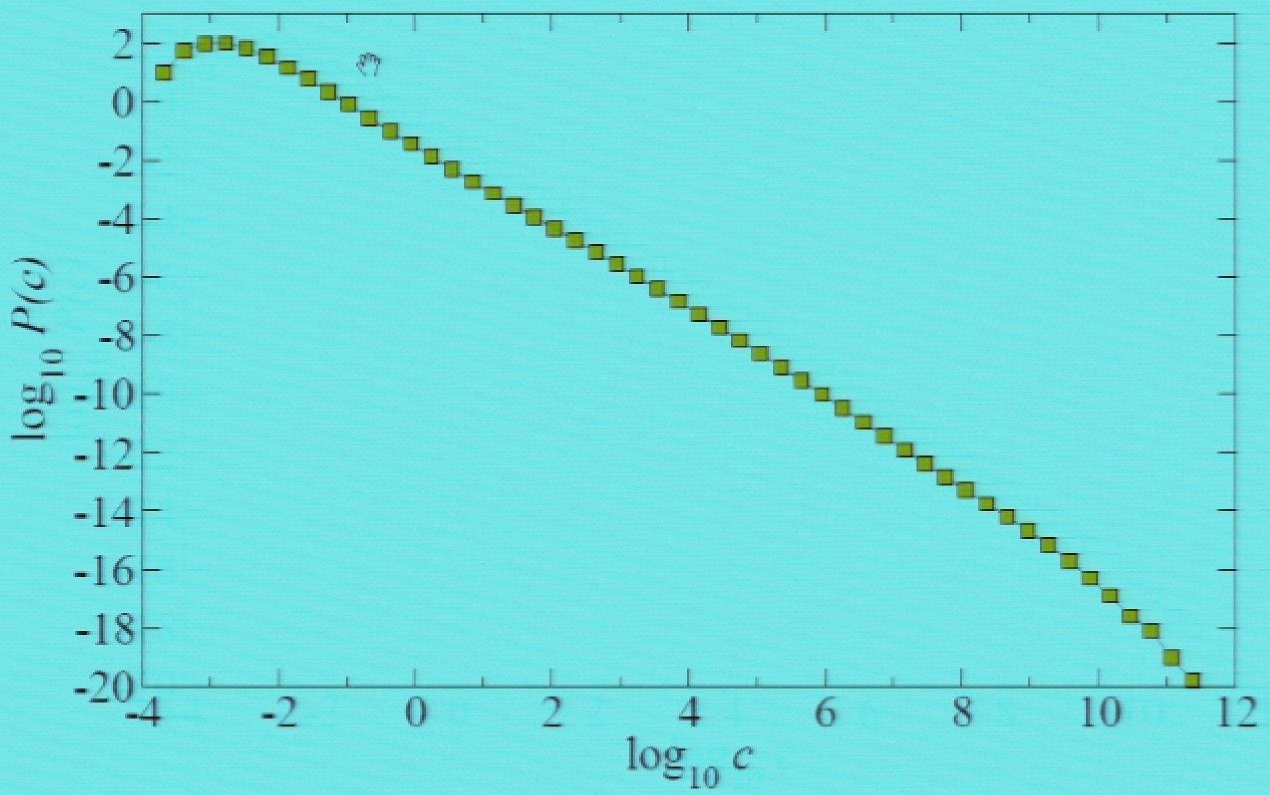

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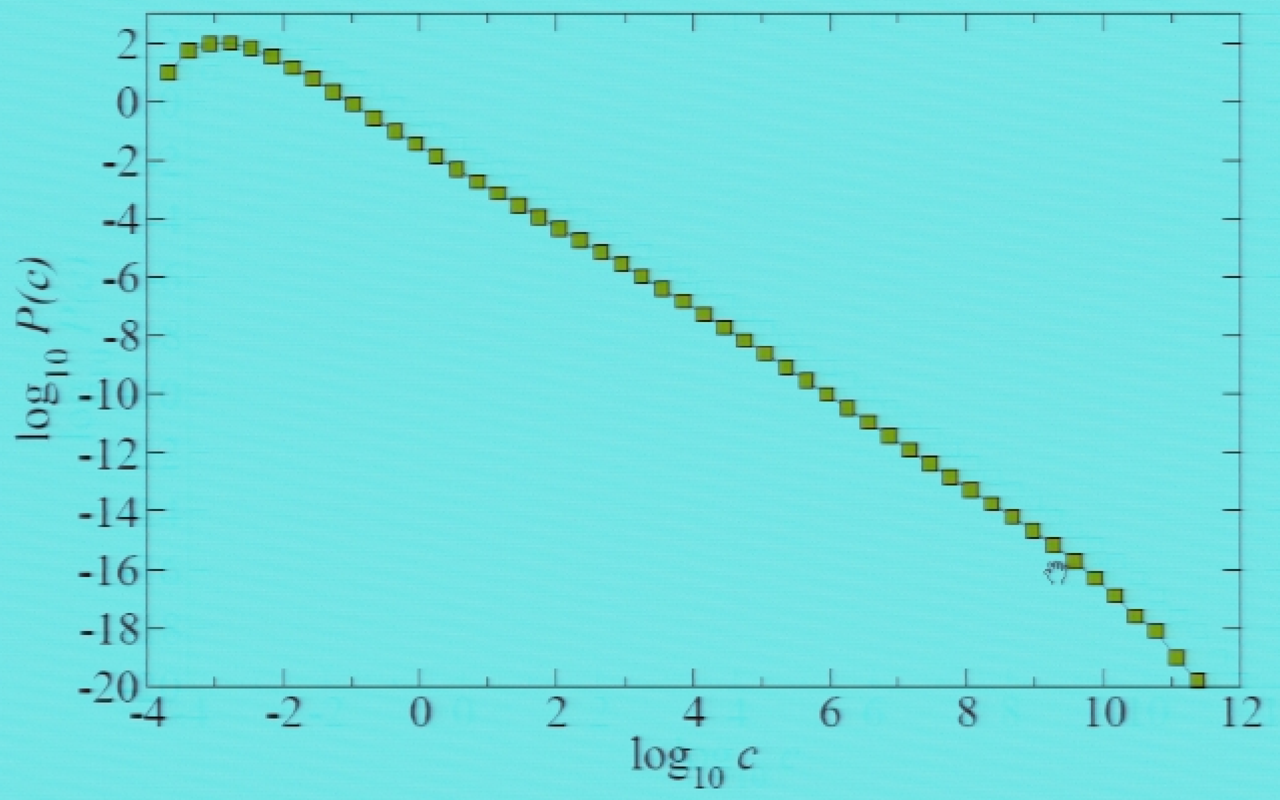

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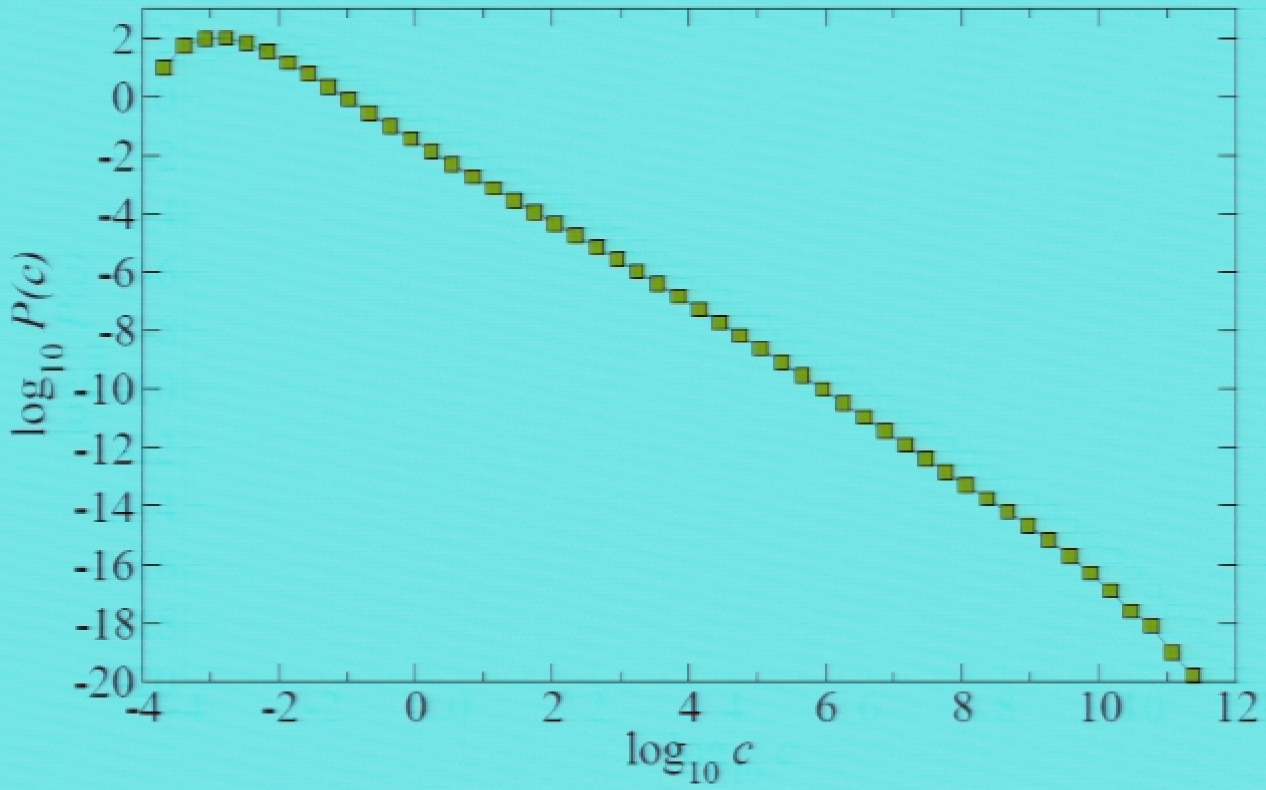

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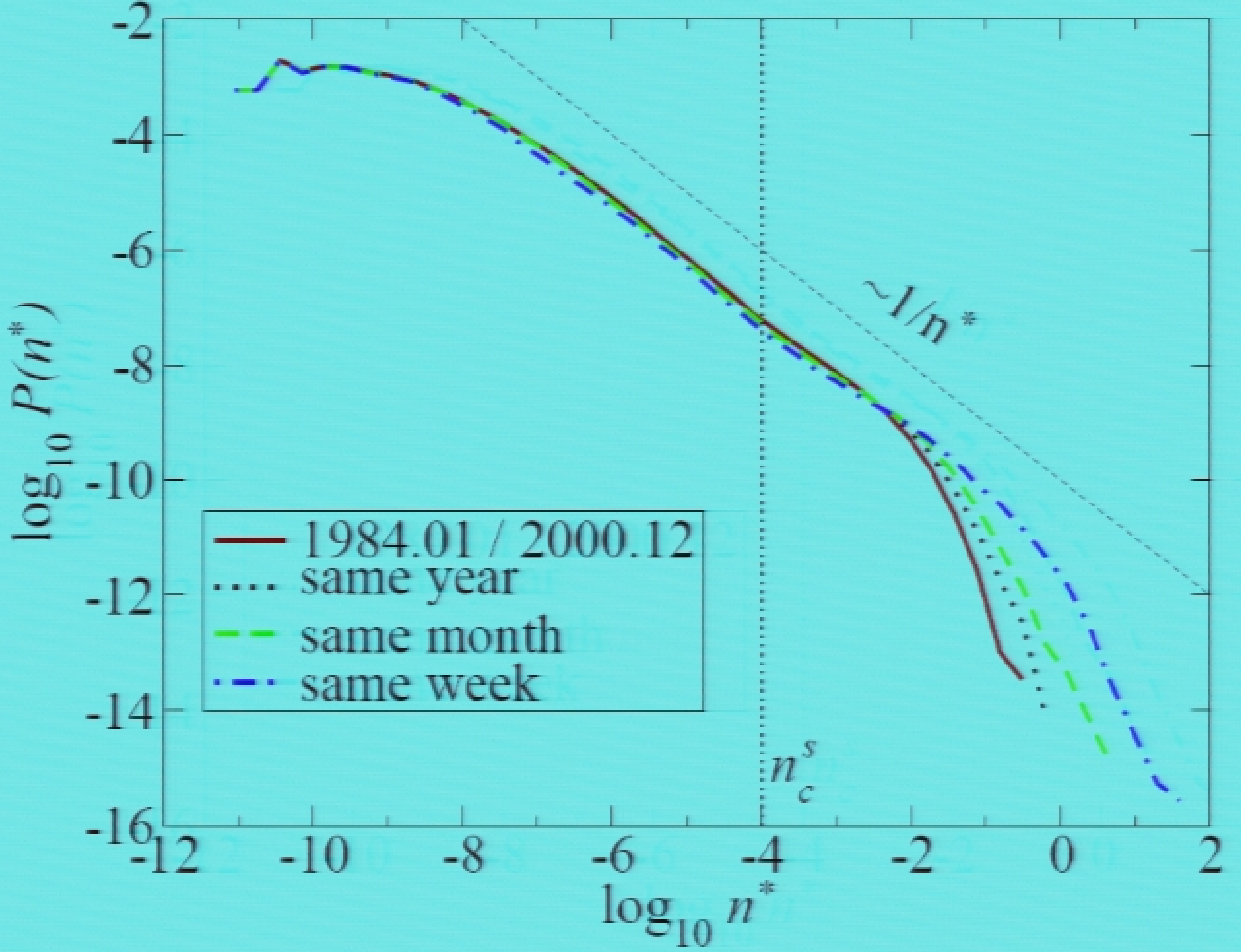

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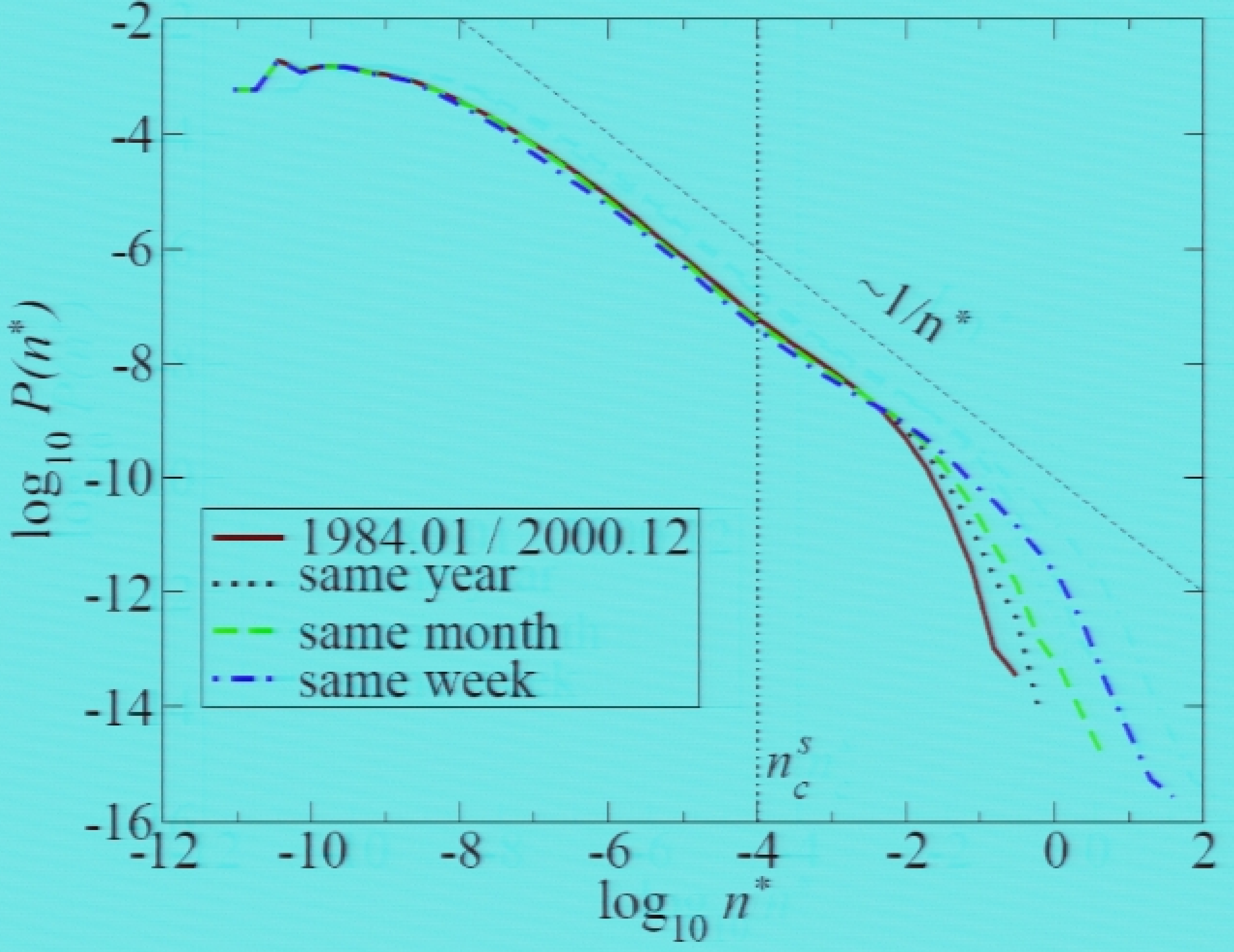

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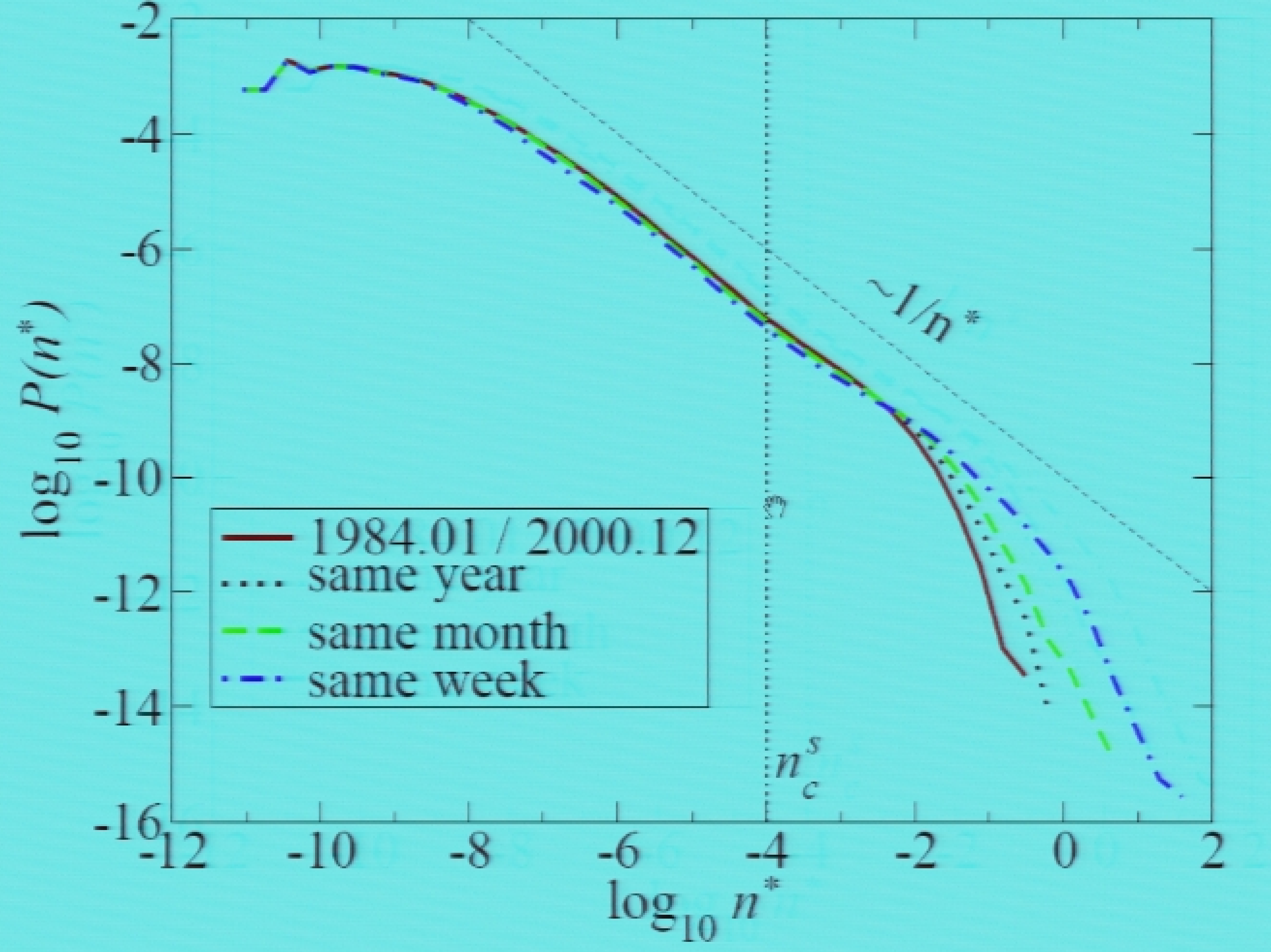

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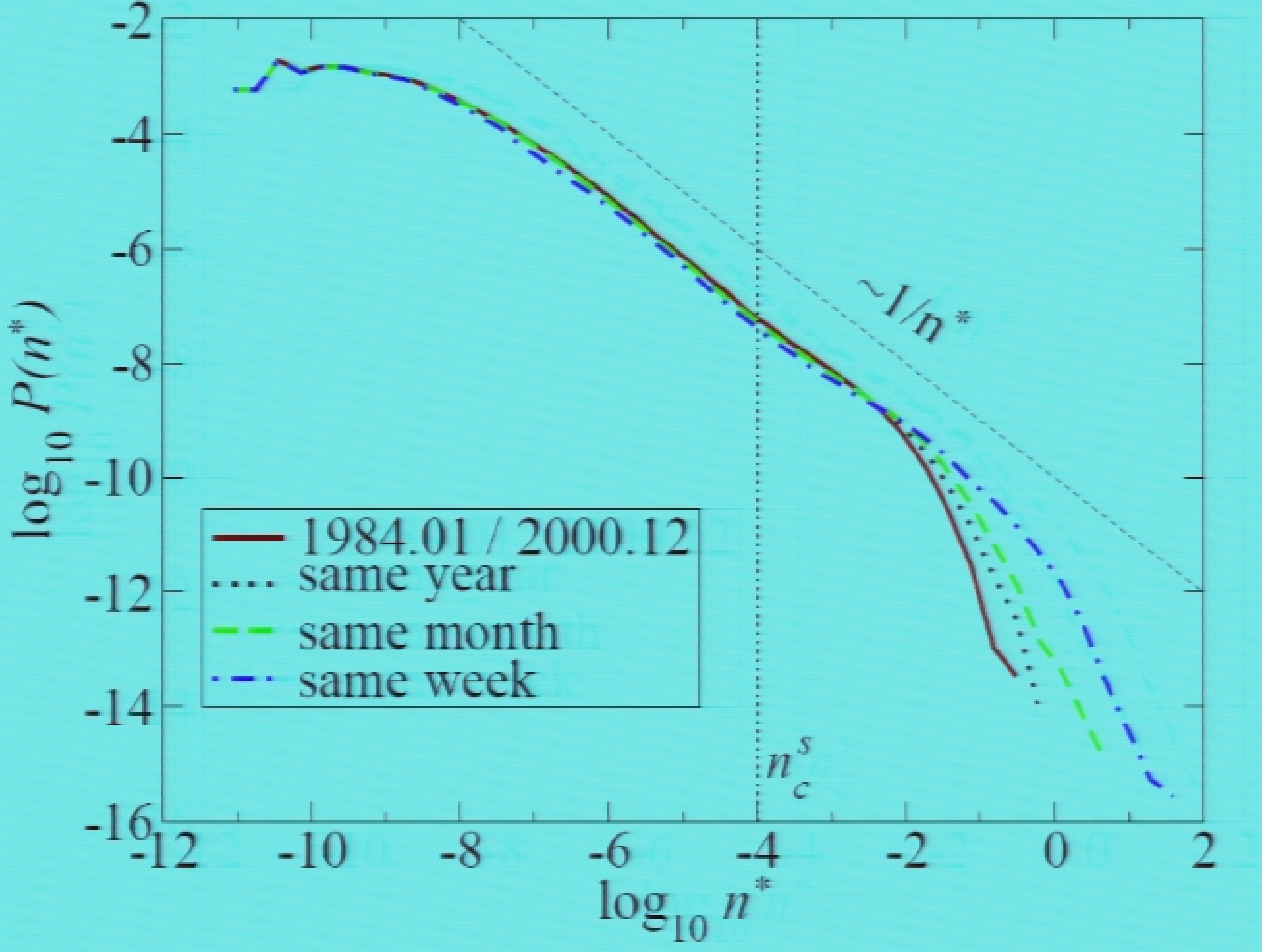

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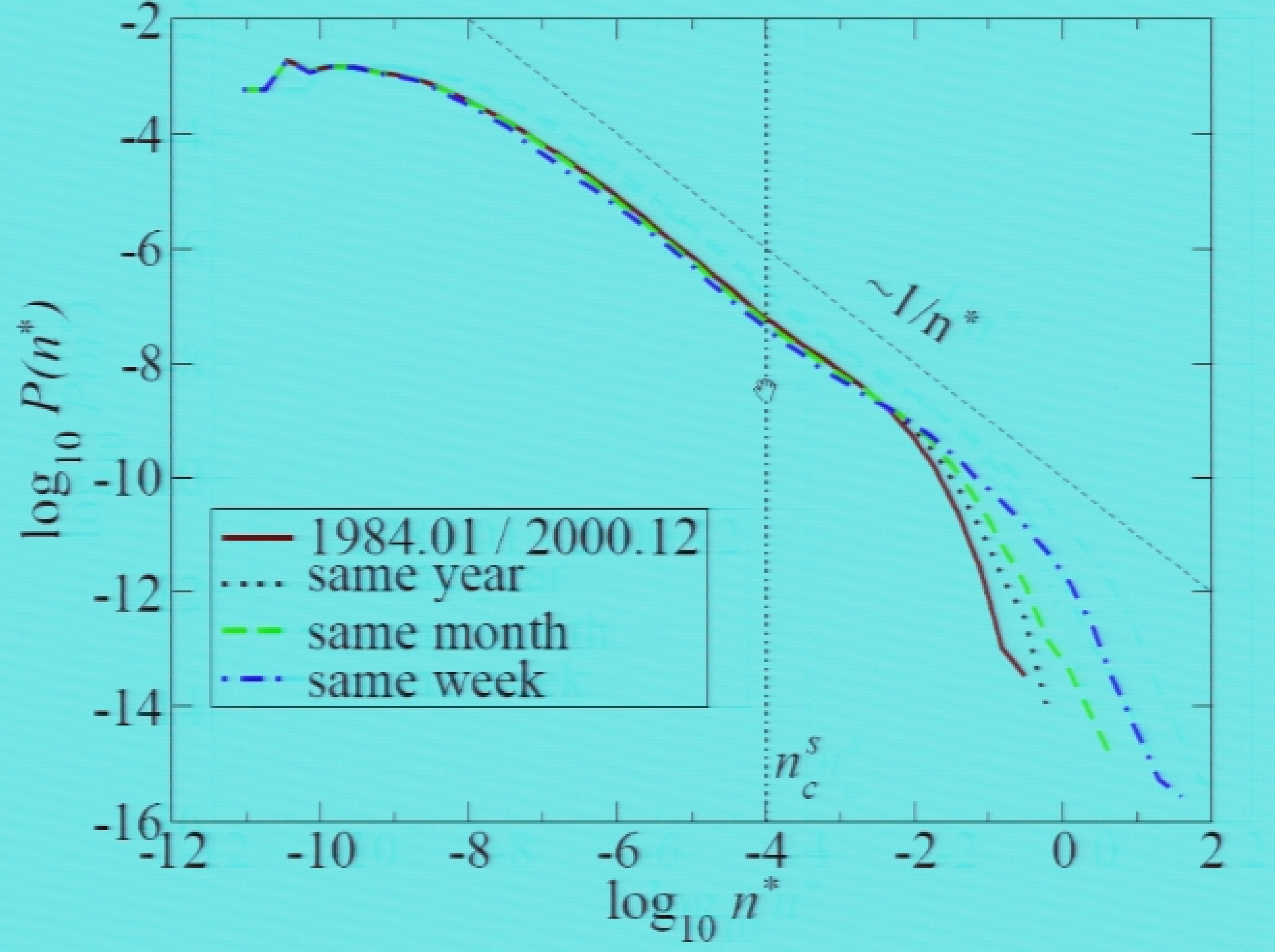

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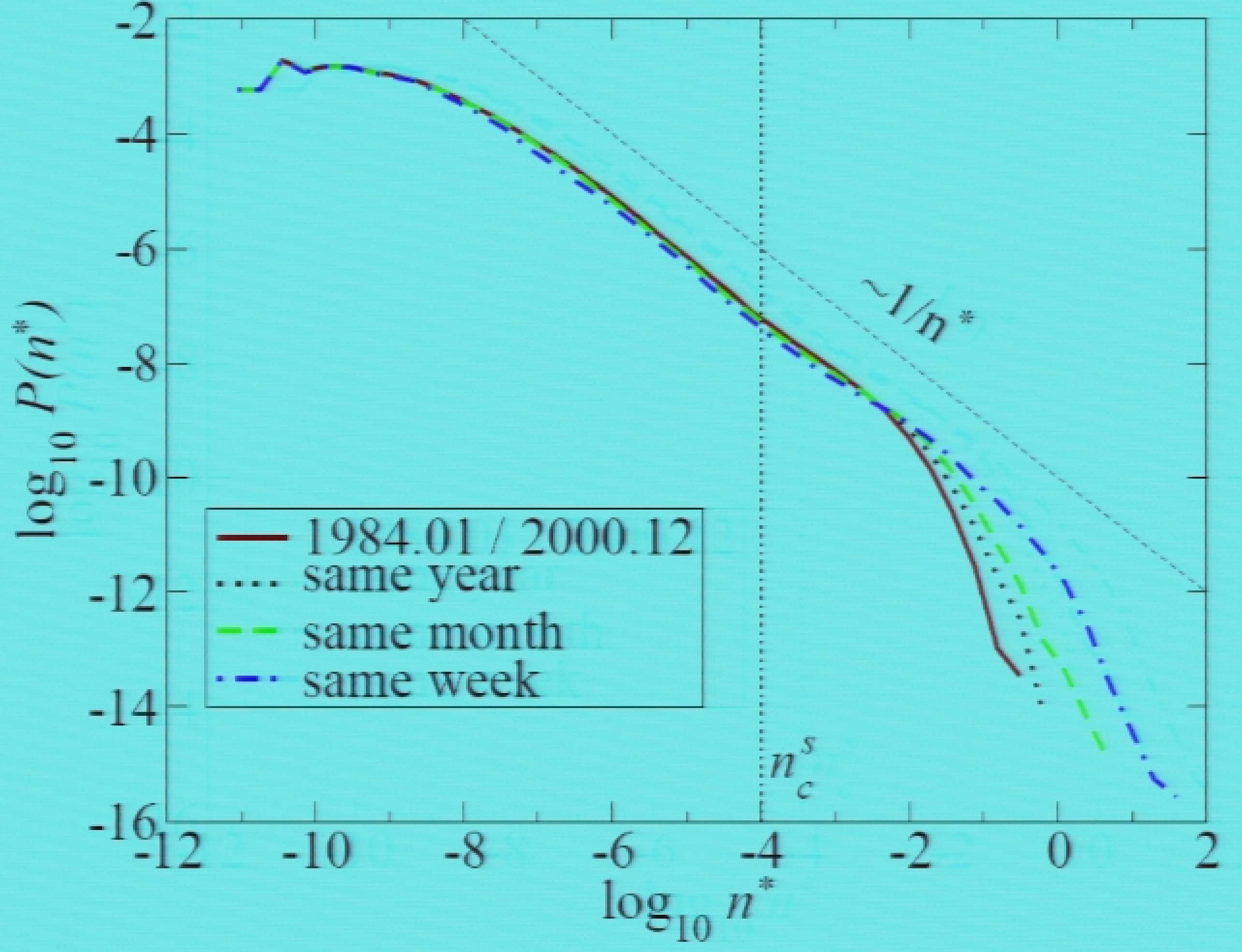

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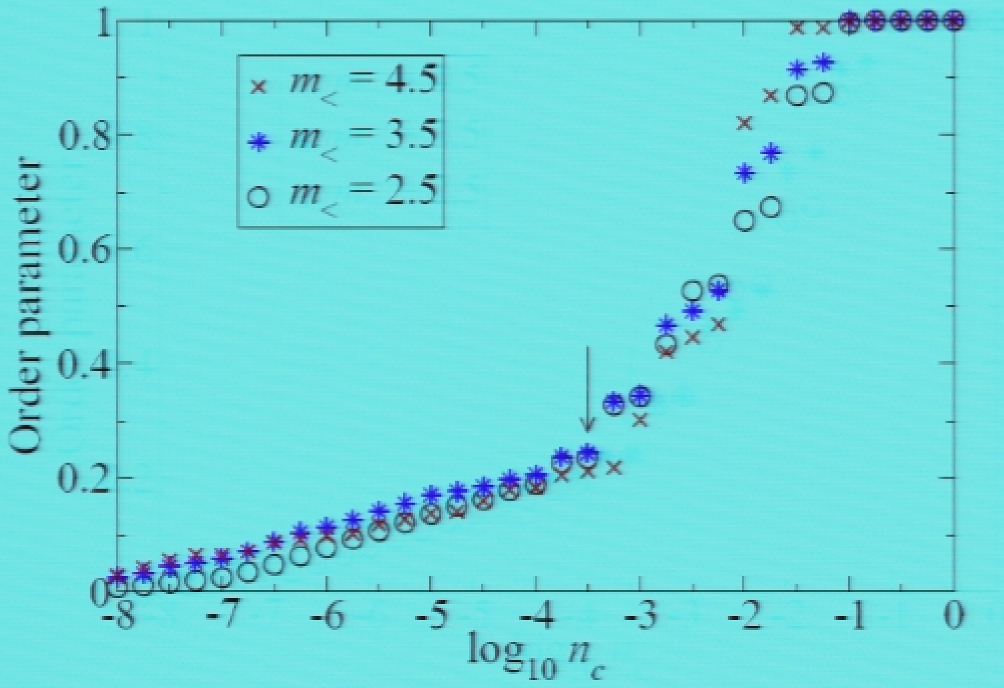
Complex correlations in self-organized critical phenomena

Put a link if $n_{ij} < n_c$

Order parameter:

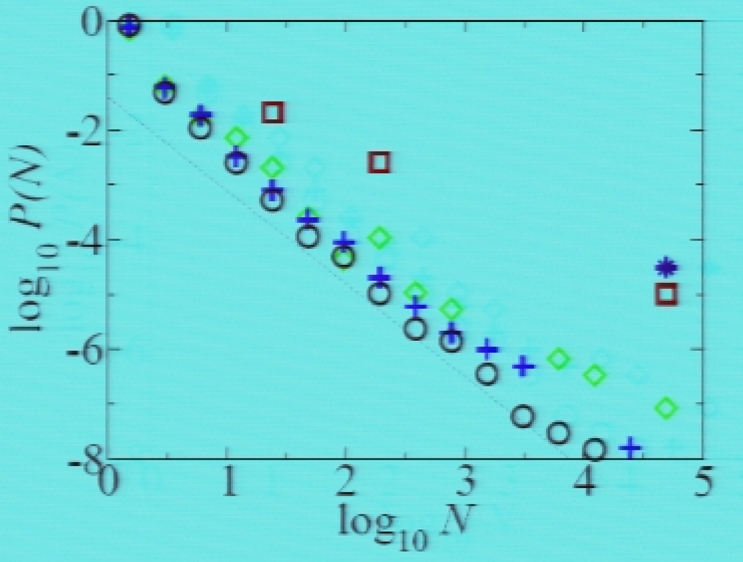
fraction of nodes

in the biggest cluster



Distribution of cluster sizes

$P(N) \sim N^{-1.7}$



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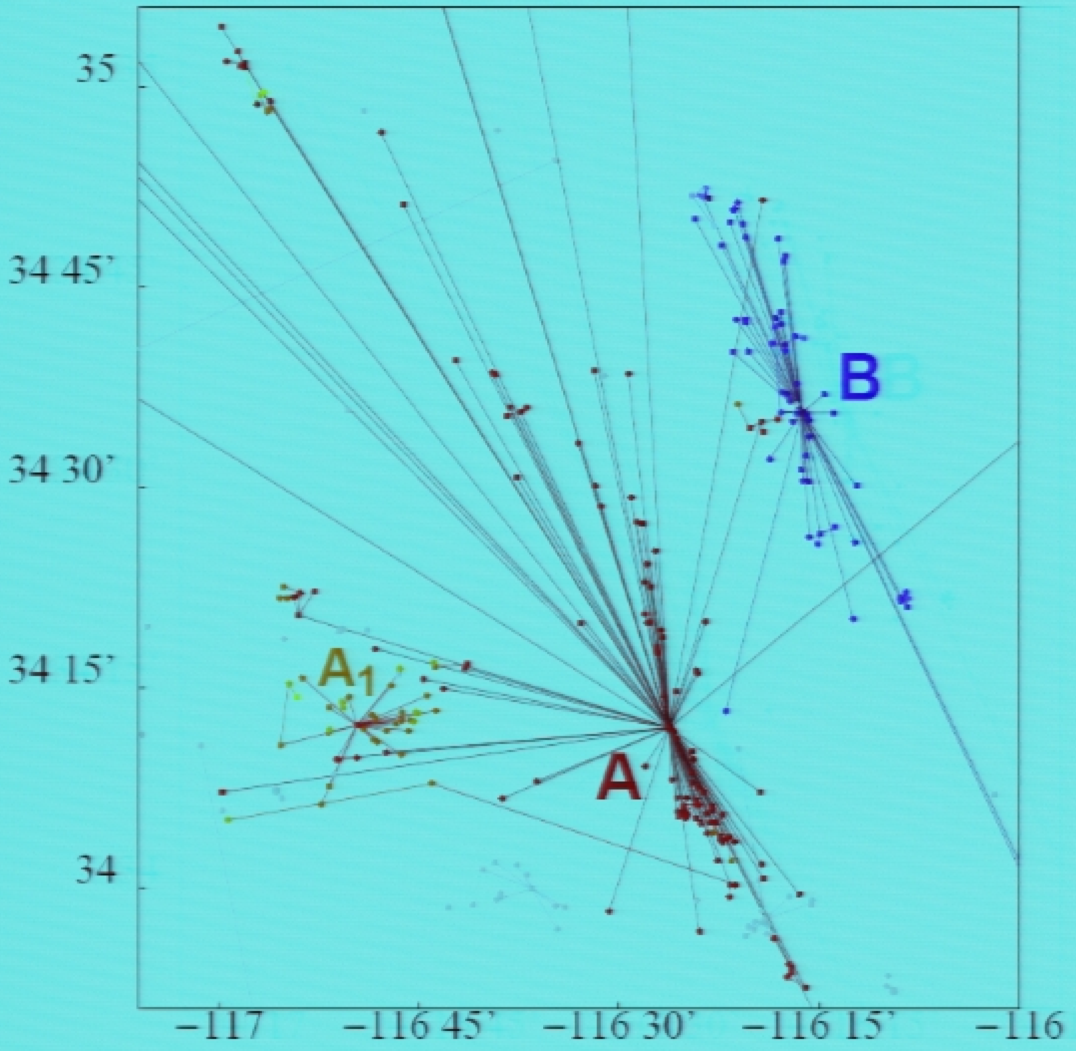


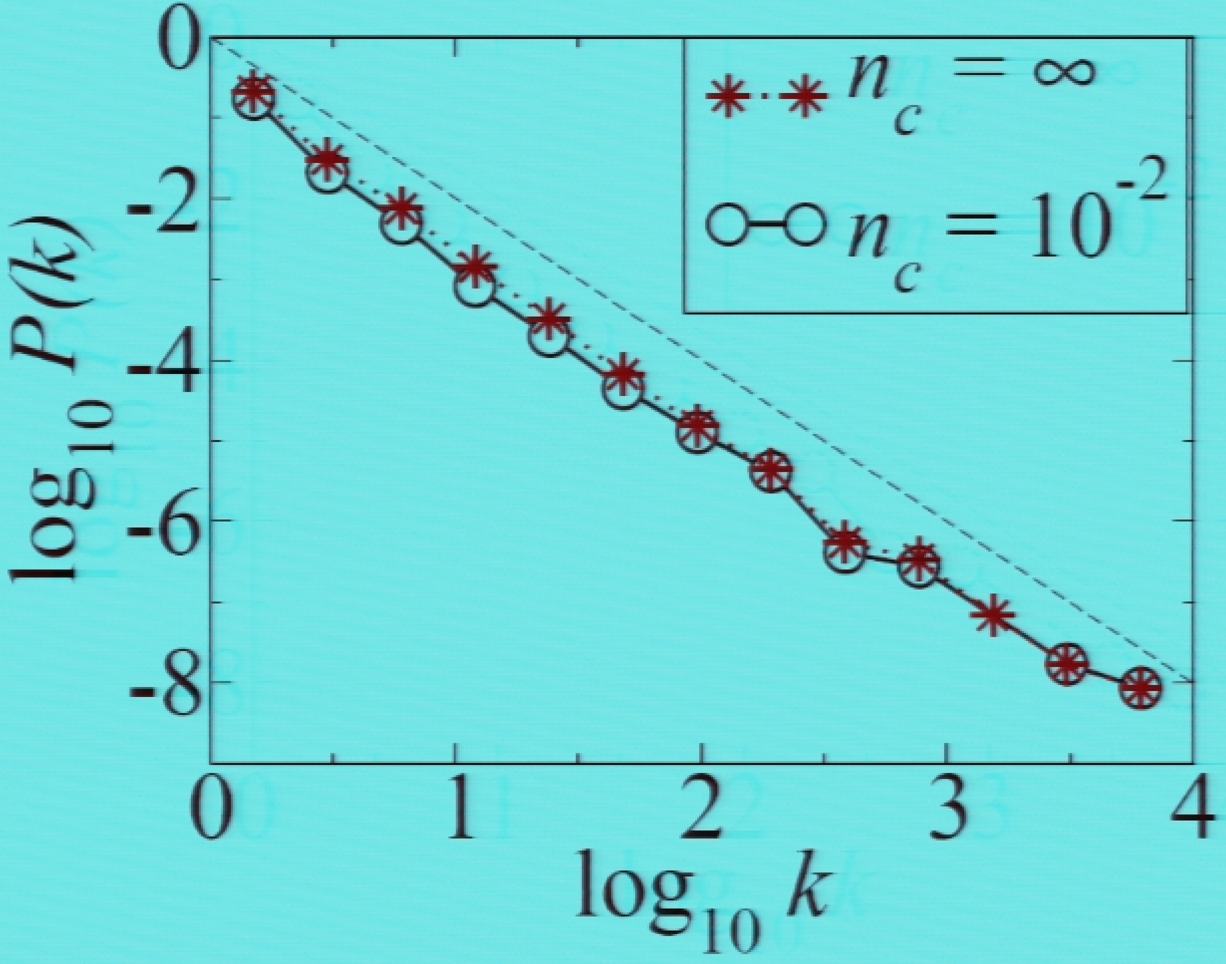
Landers earthquake (**A**)

Hector Mine earthquake (**B**)

Complex correlations in self-organized critical phenomena

Out-degree distribution:

scale free $\sim k^{-2}$



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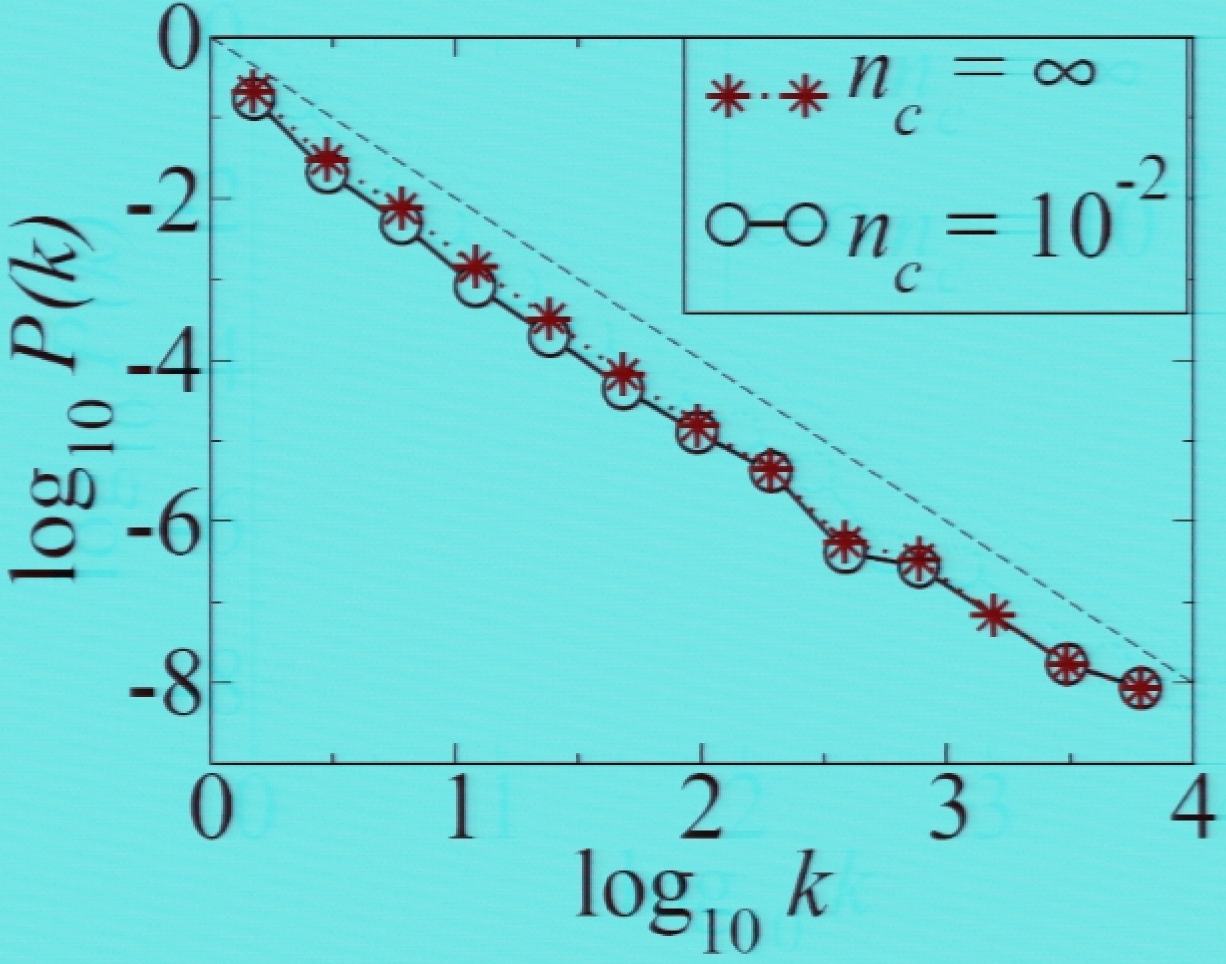

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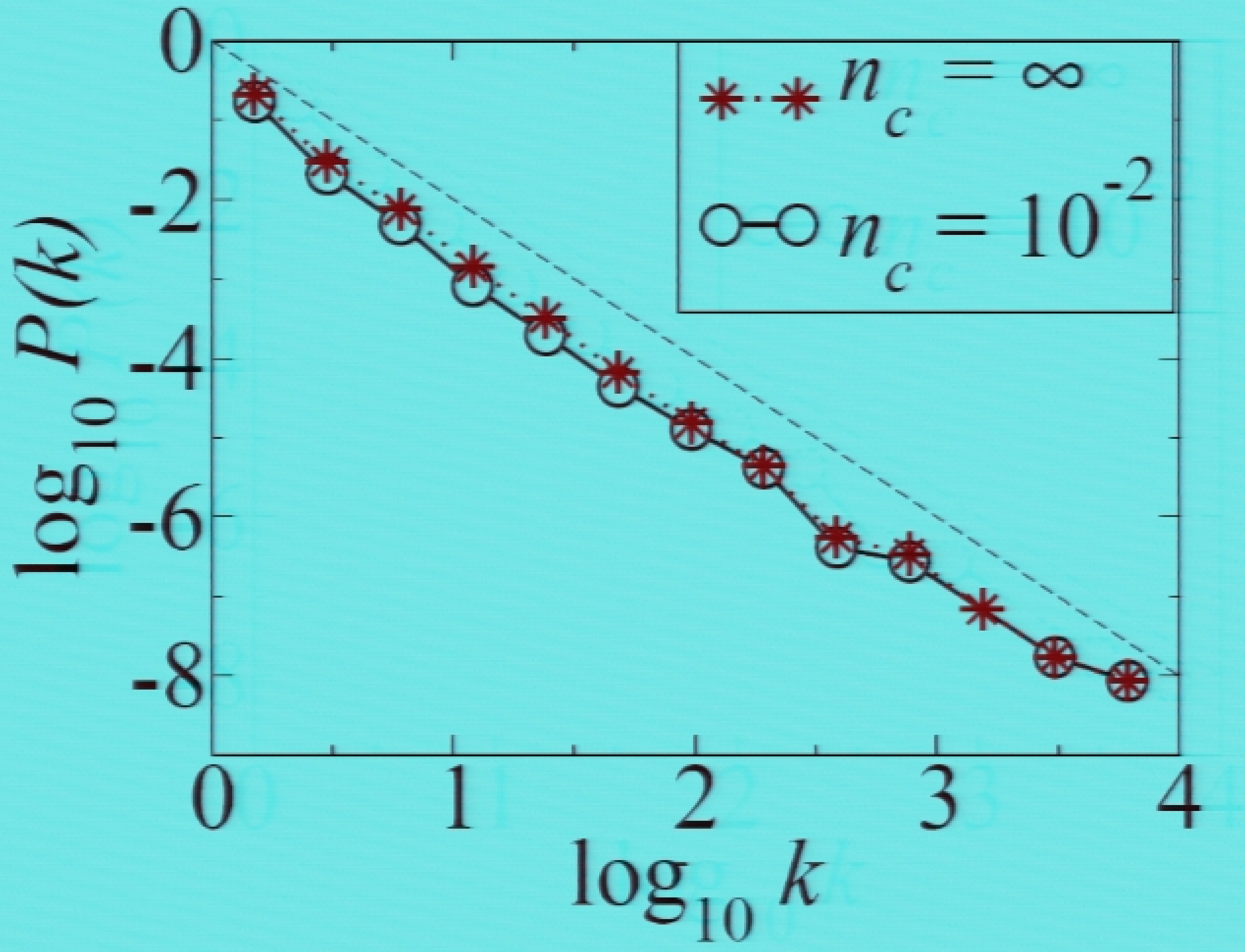

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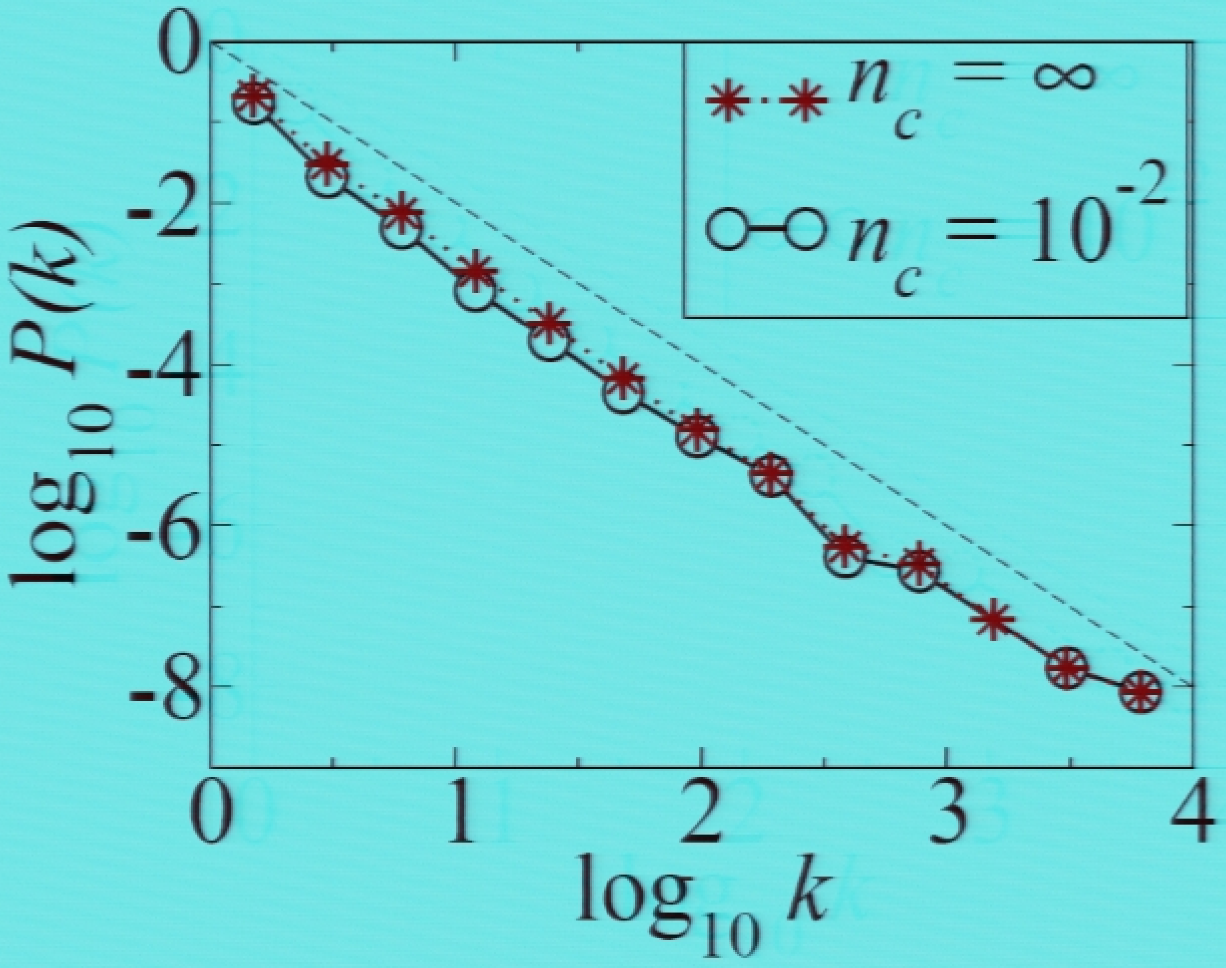
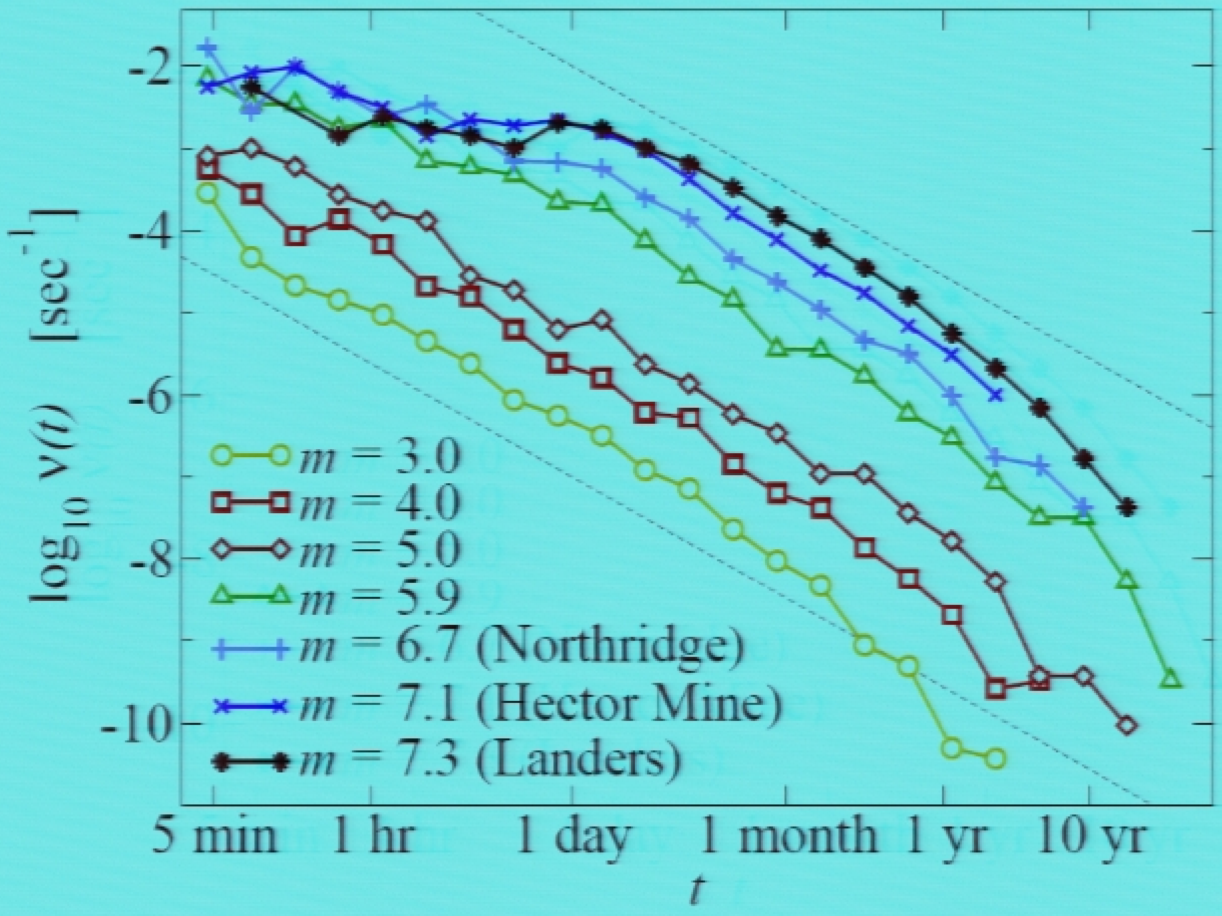


Complex correlations in self-organized critical phenomena

Omori's law: rate of events after an earthquake

$$\nu(t) \sim \frac{1}{c(m)+t}$$



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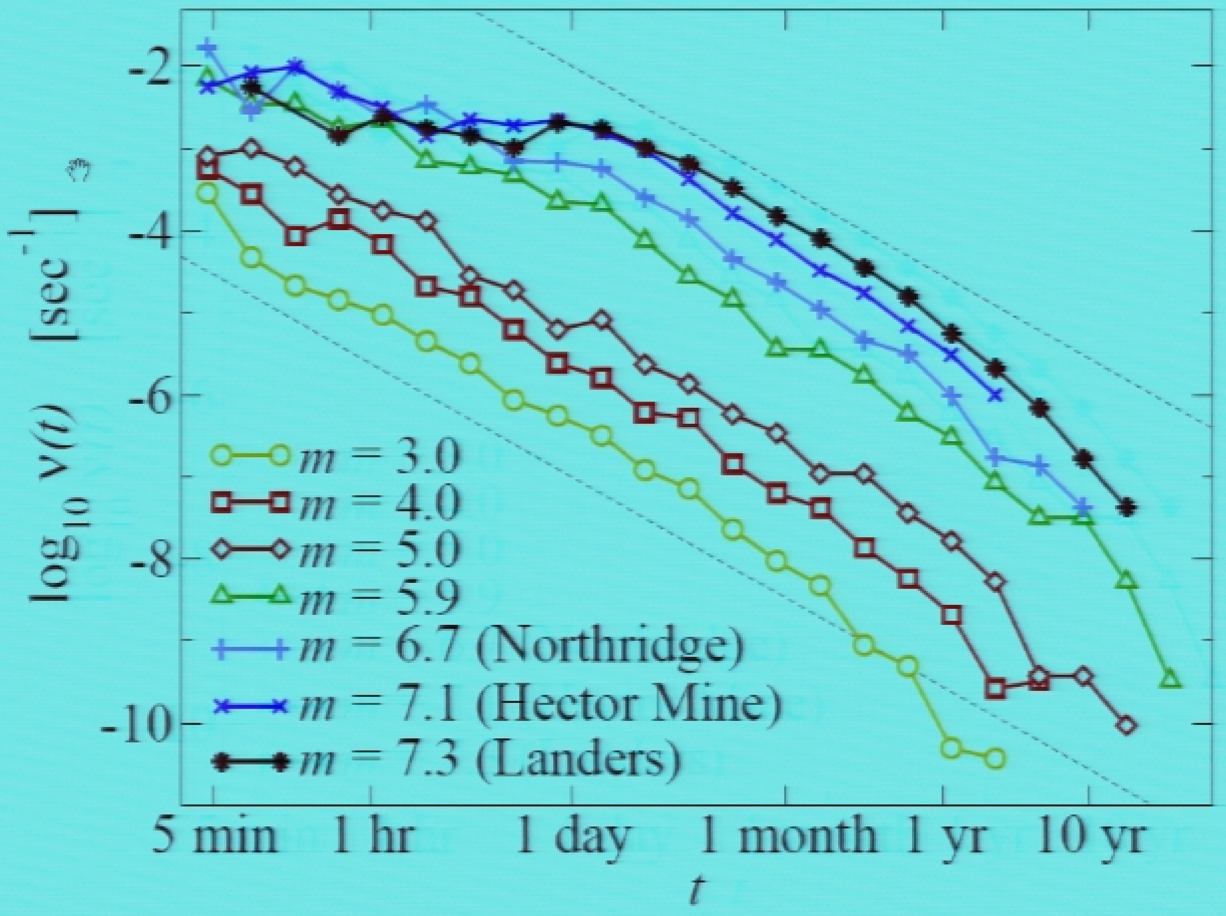

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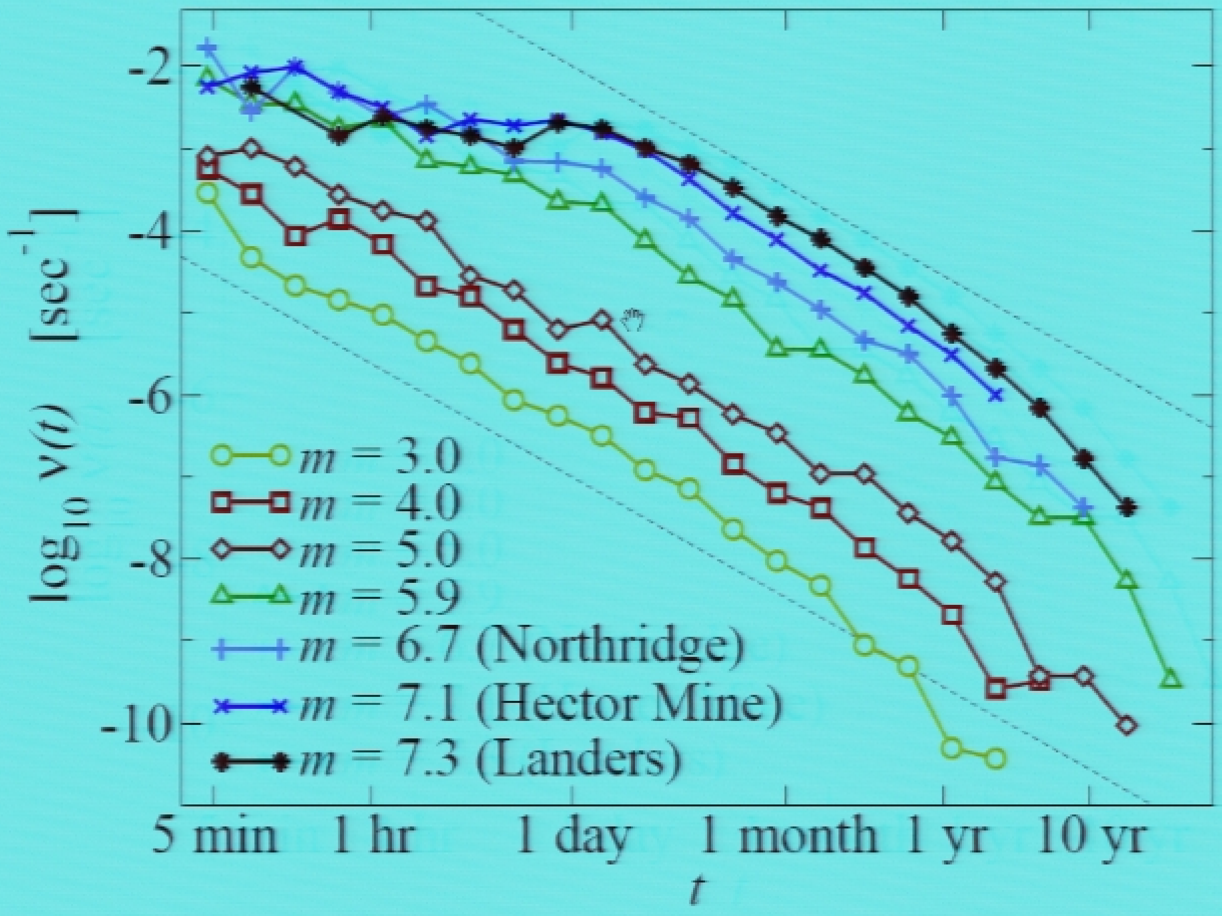

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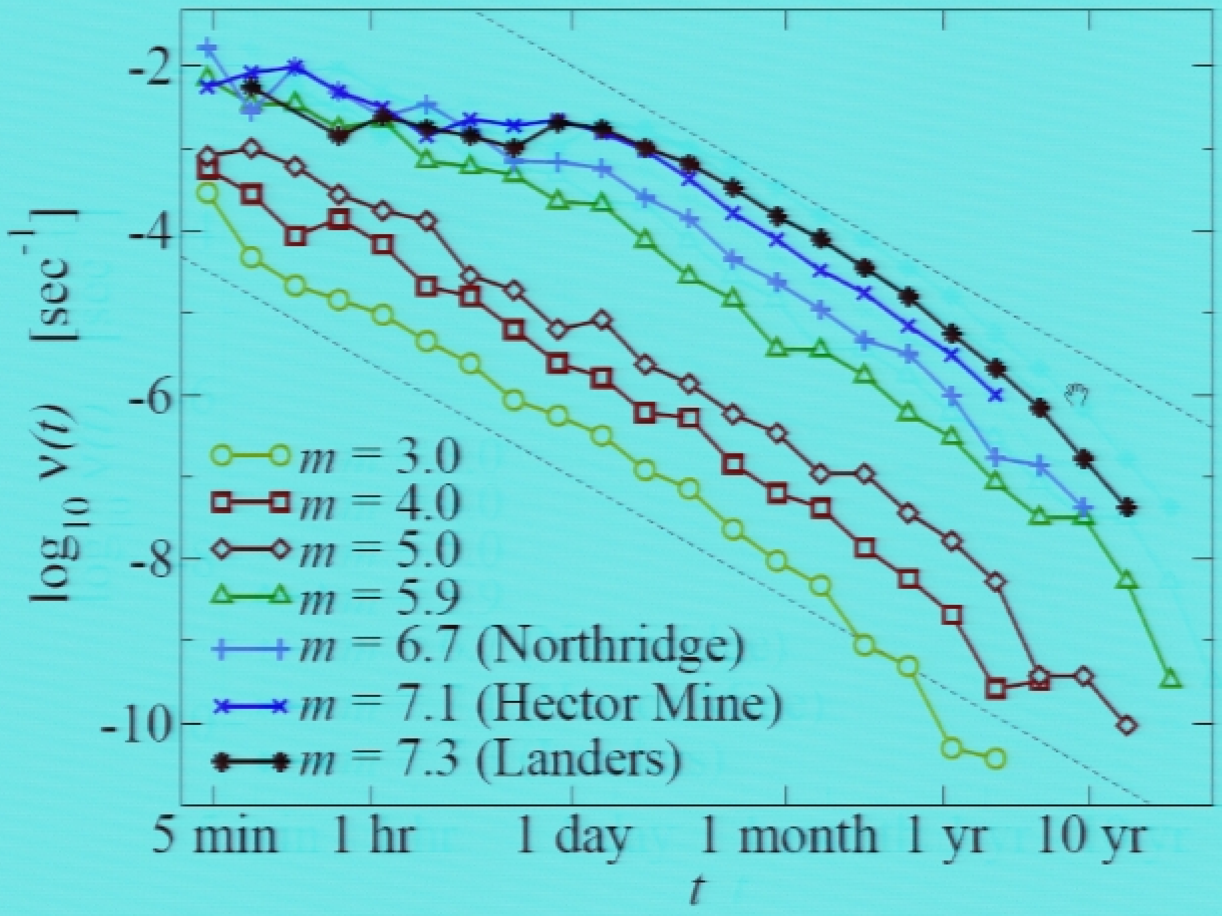

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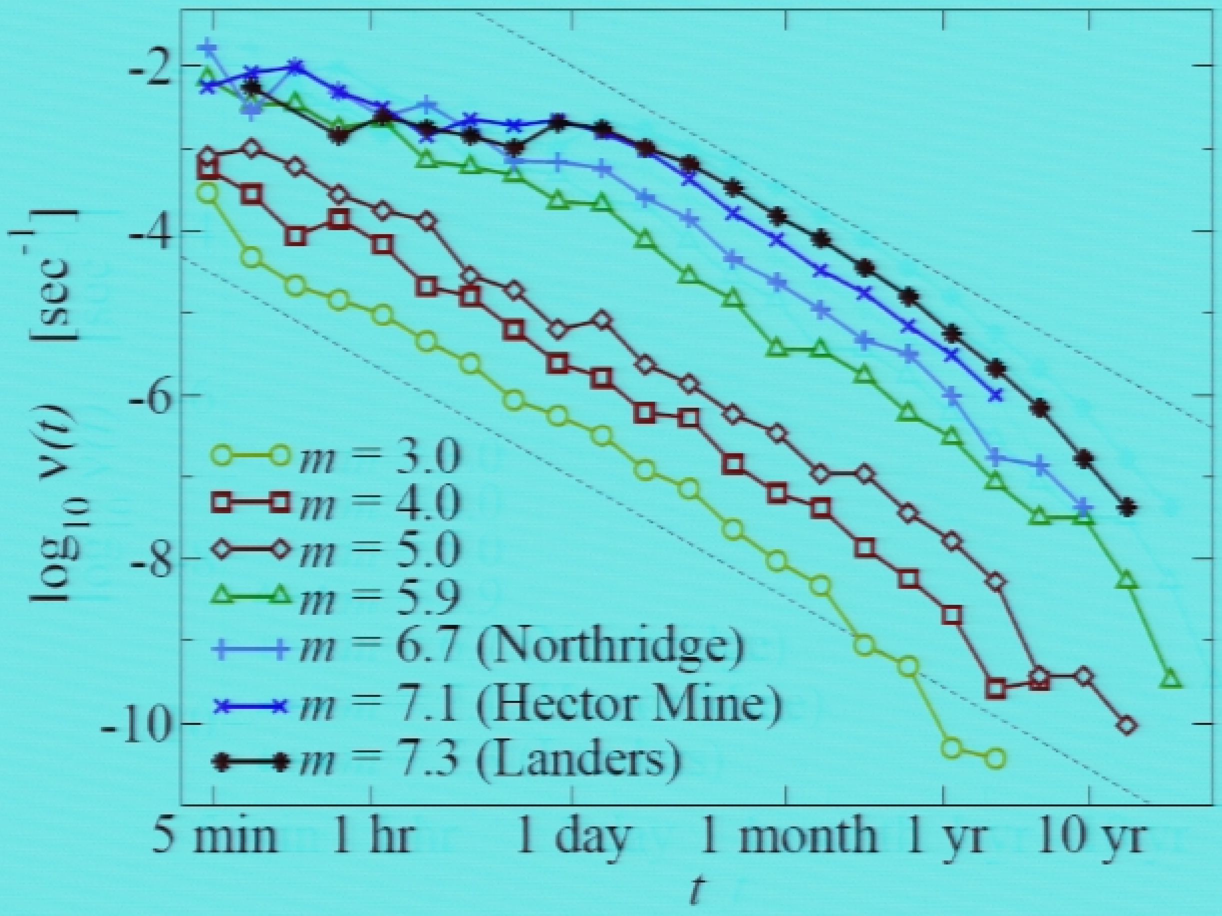

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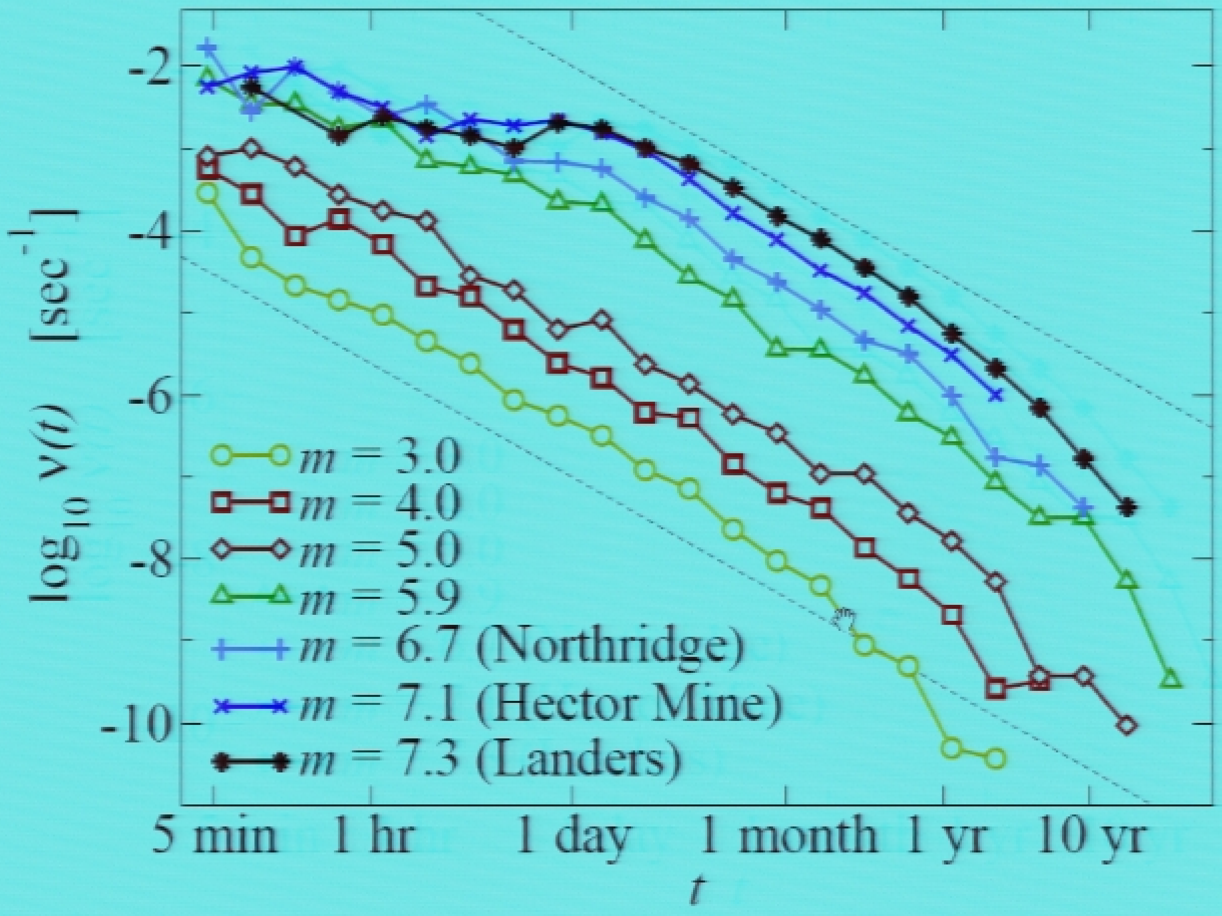

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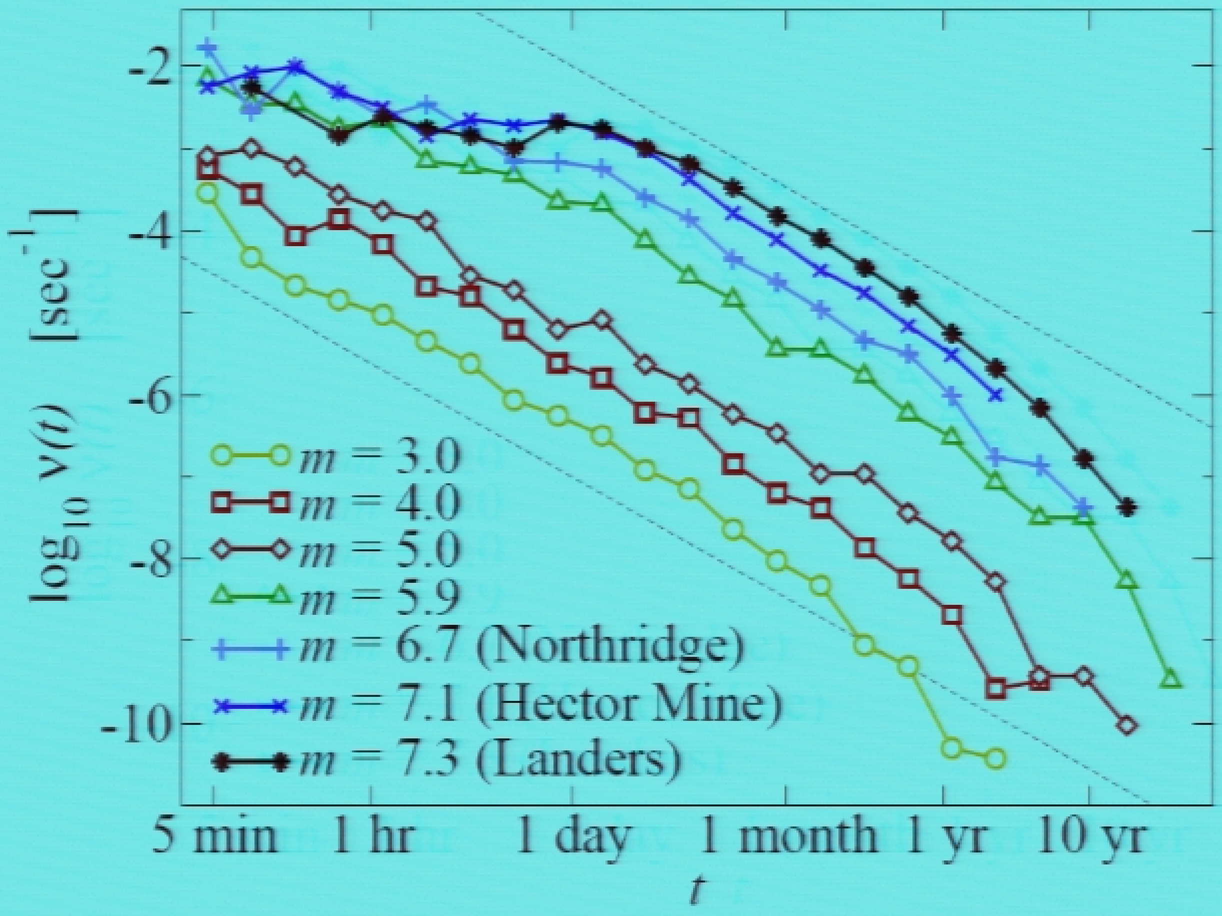
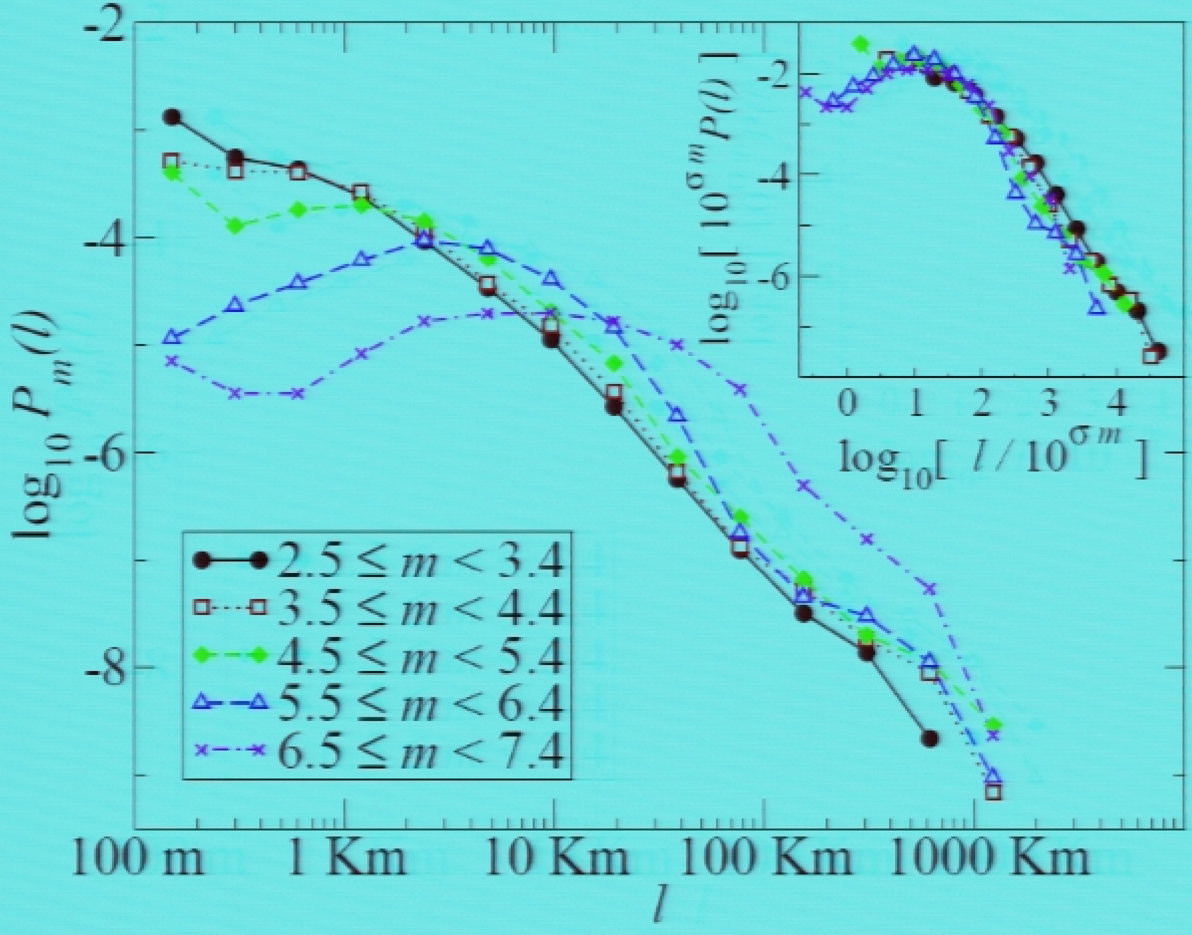


Complex correlations in self-organized critical phenomena

New phenomenology: Distribution of aftershock distances

typical length $\boxed{l_m \sim 10^{\sigma m}}$

$\sigma = 0.3 \div 0.4 \neq$ “classical” $\sigma = 1/2$ of aftershocks within the rupture area



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finite space windows

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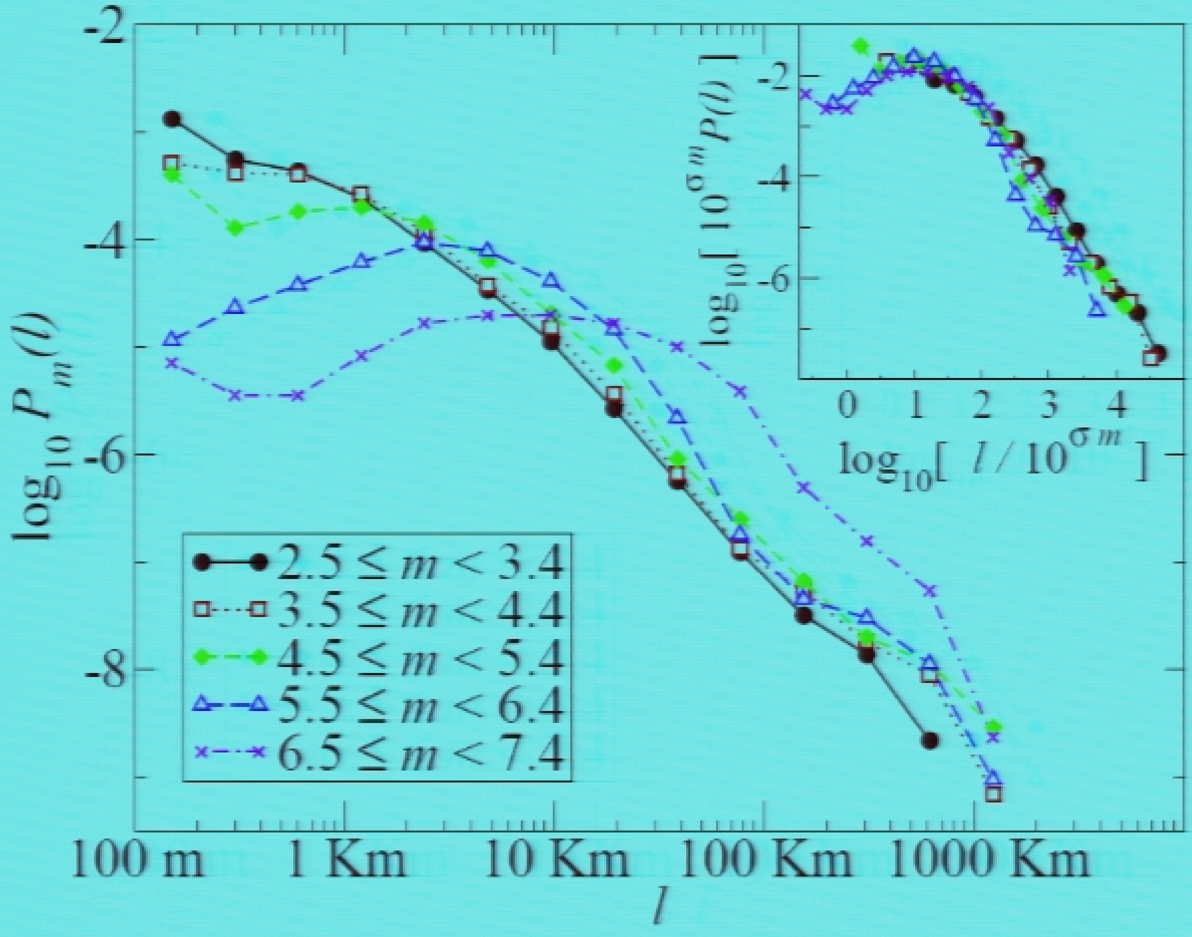

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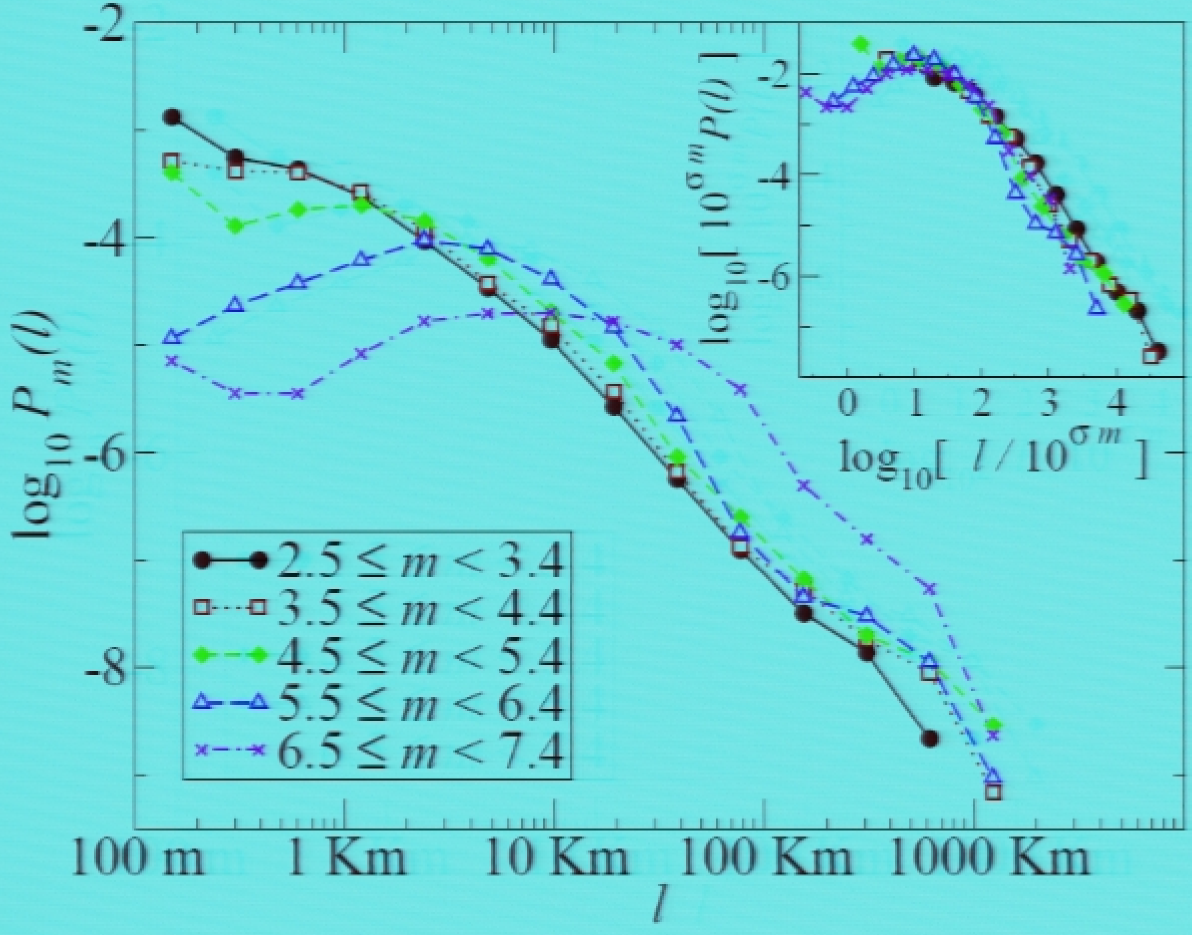

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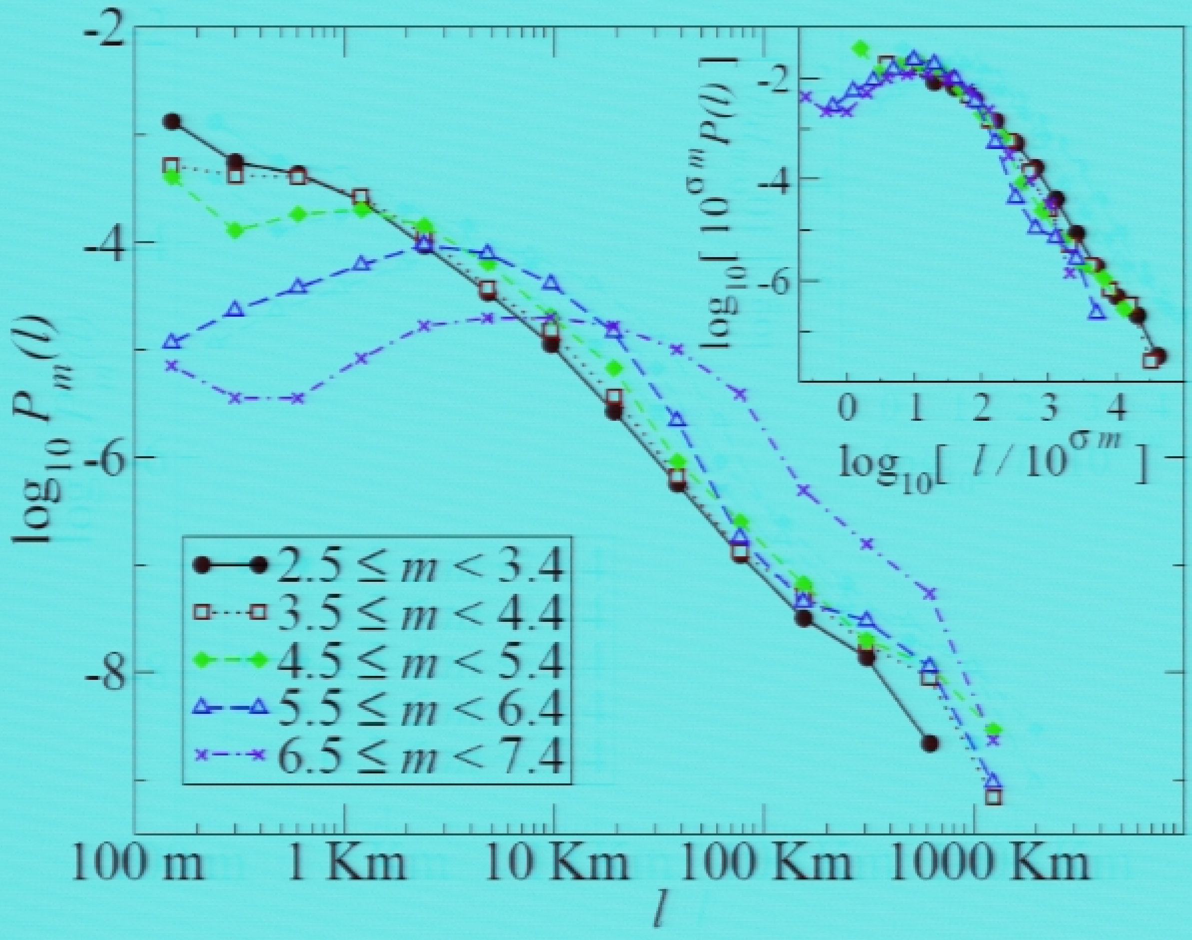

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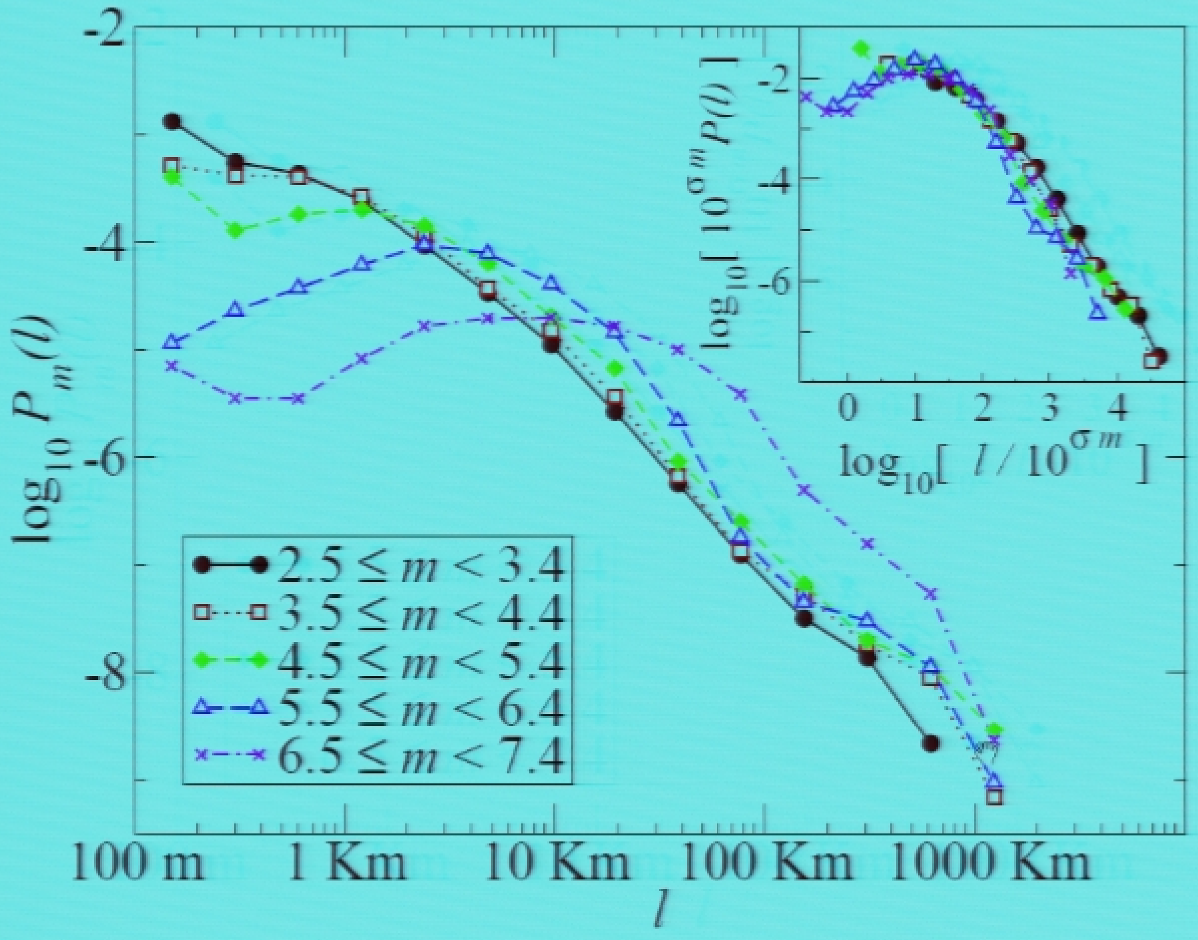

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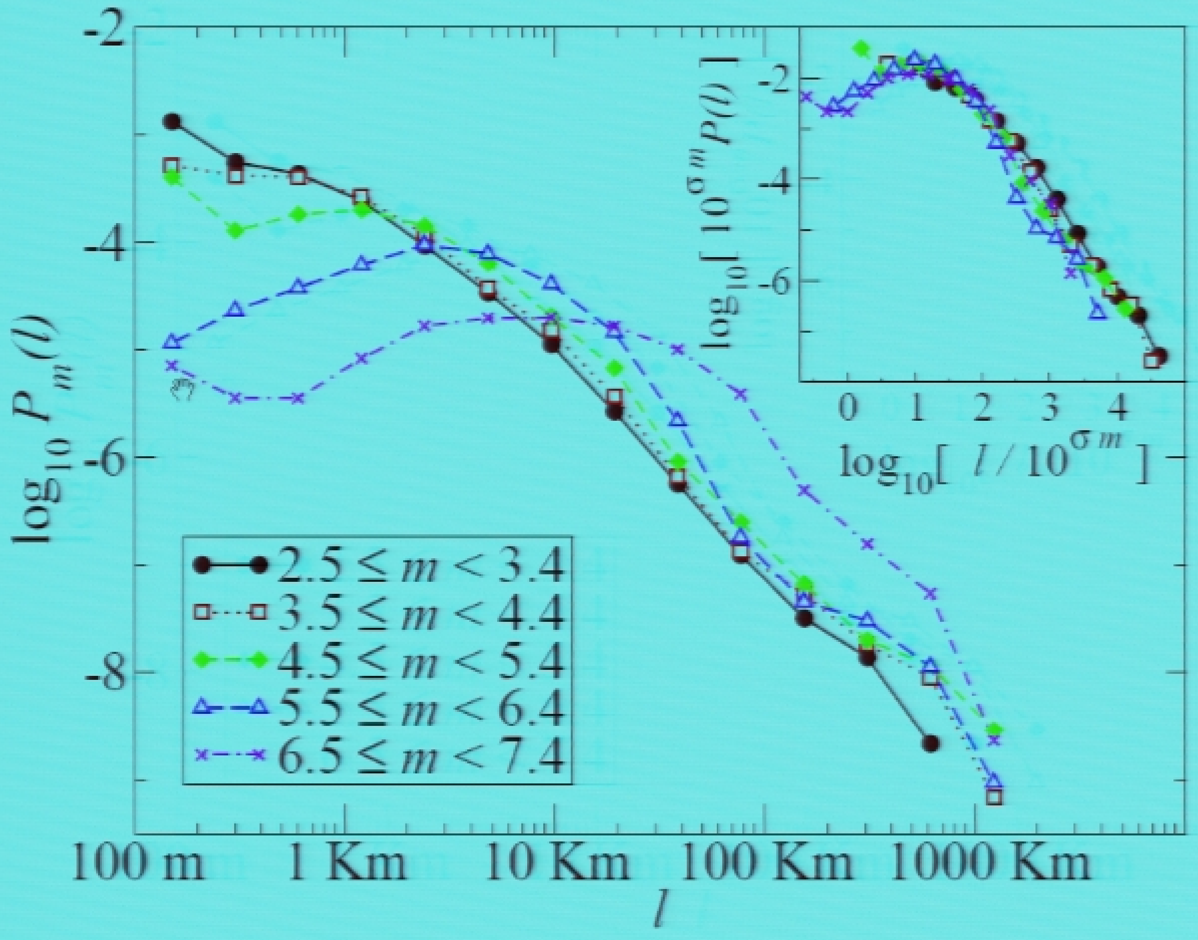

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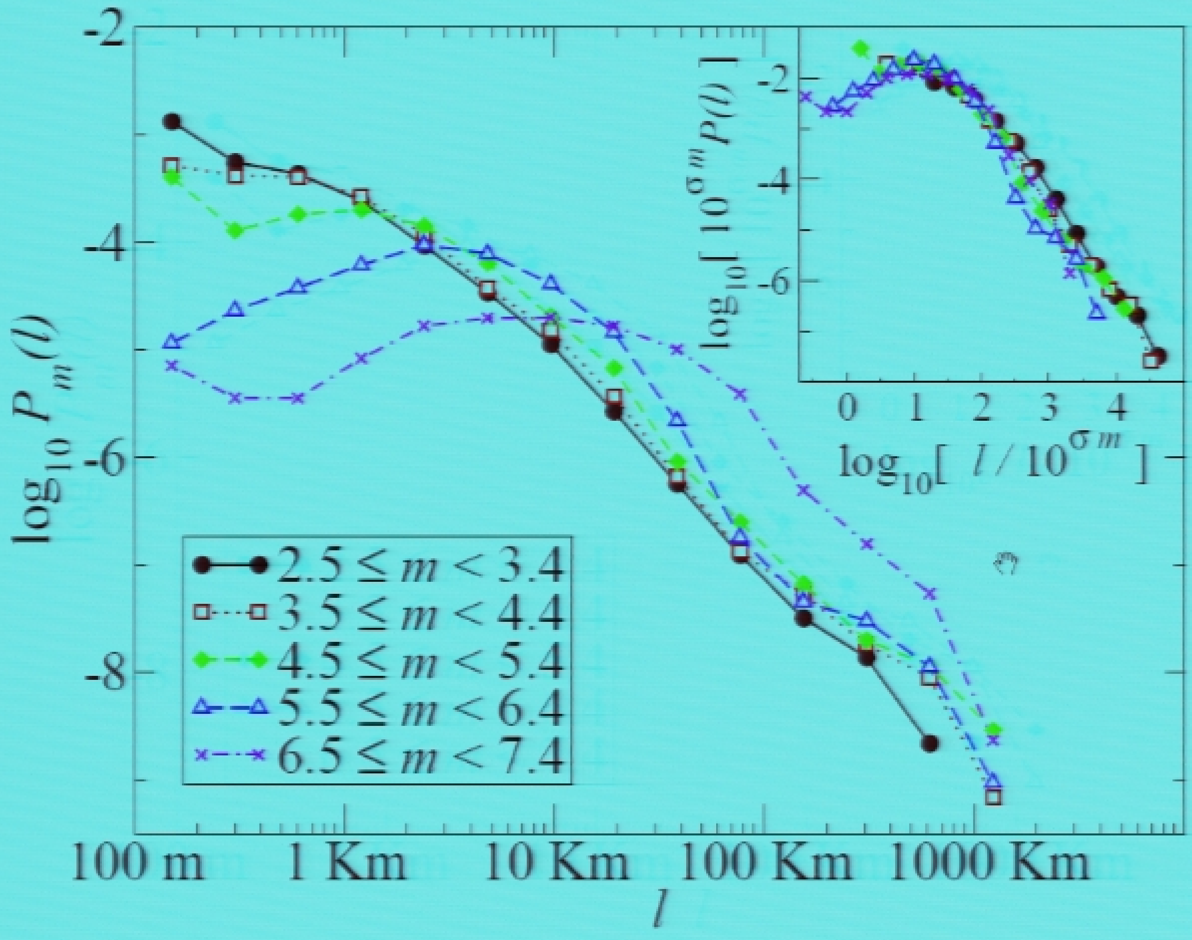

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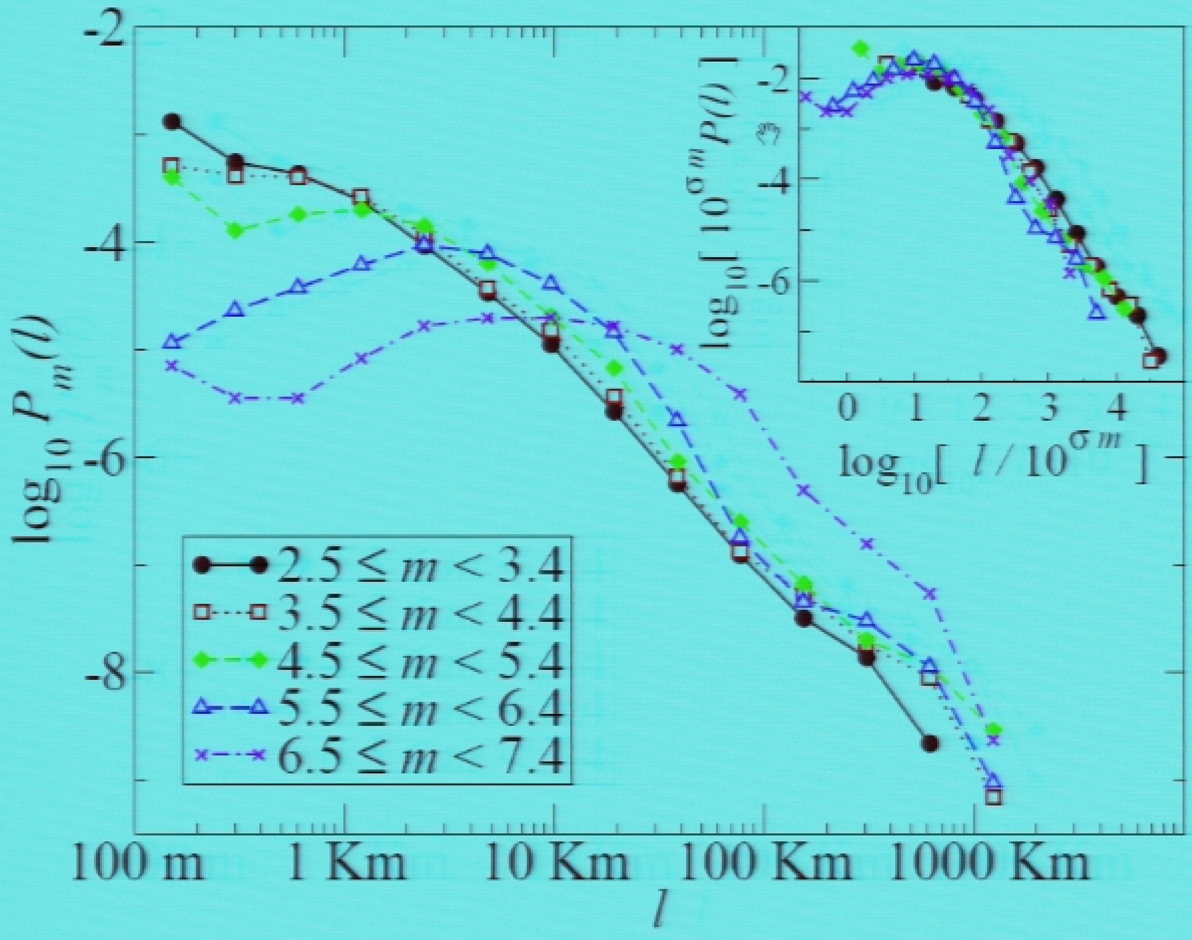

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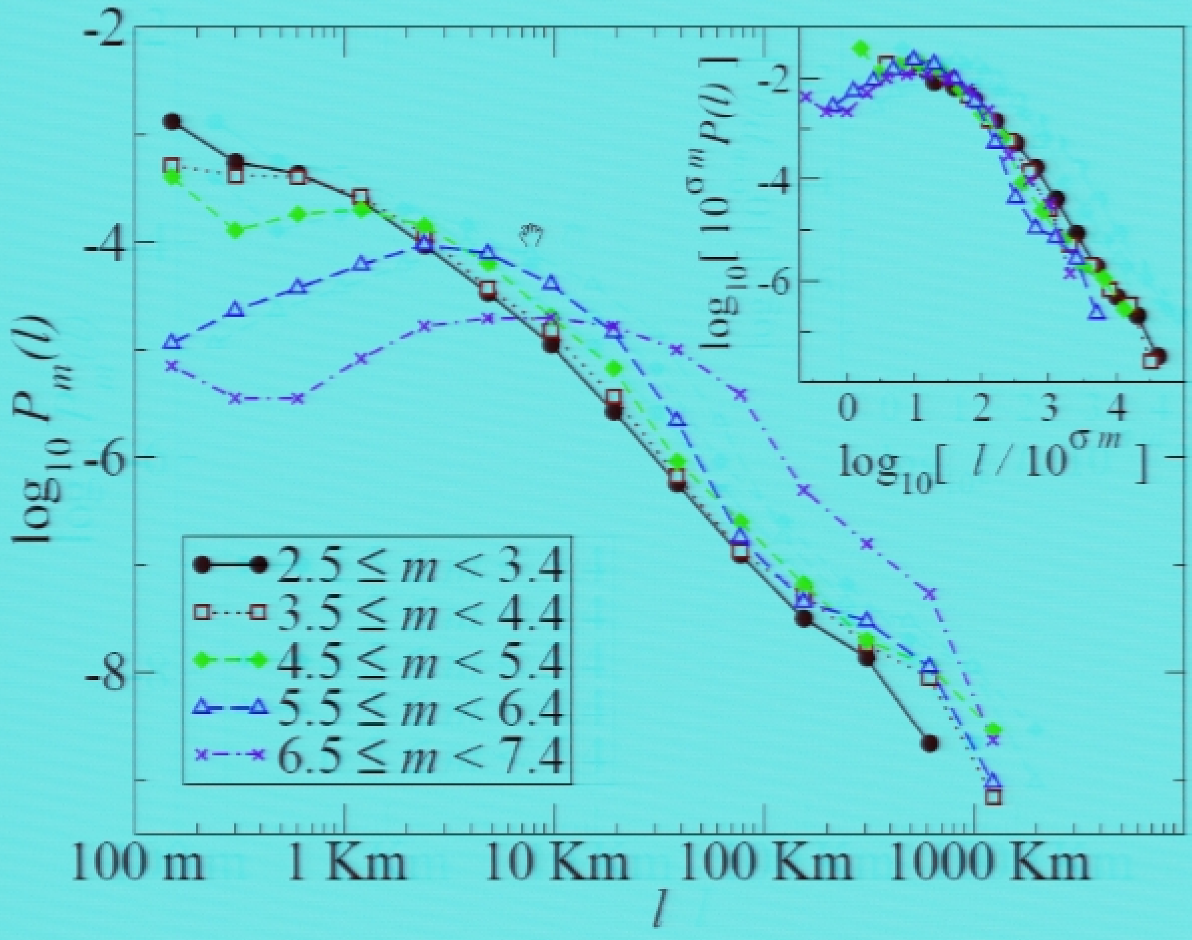

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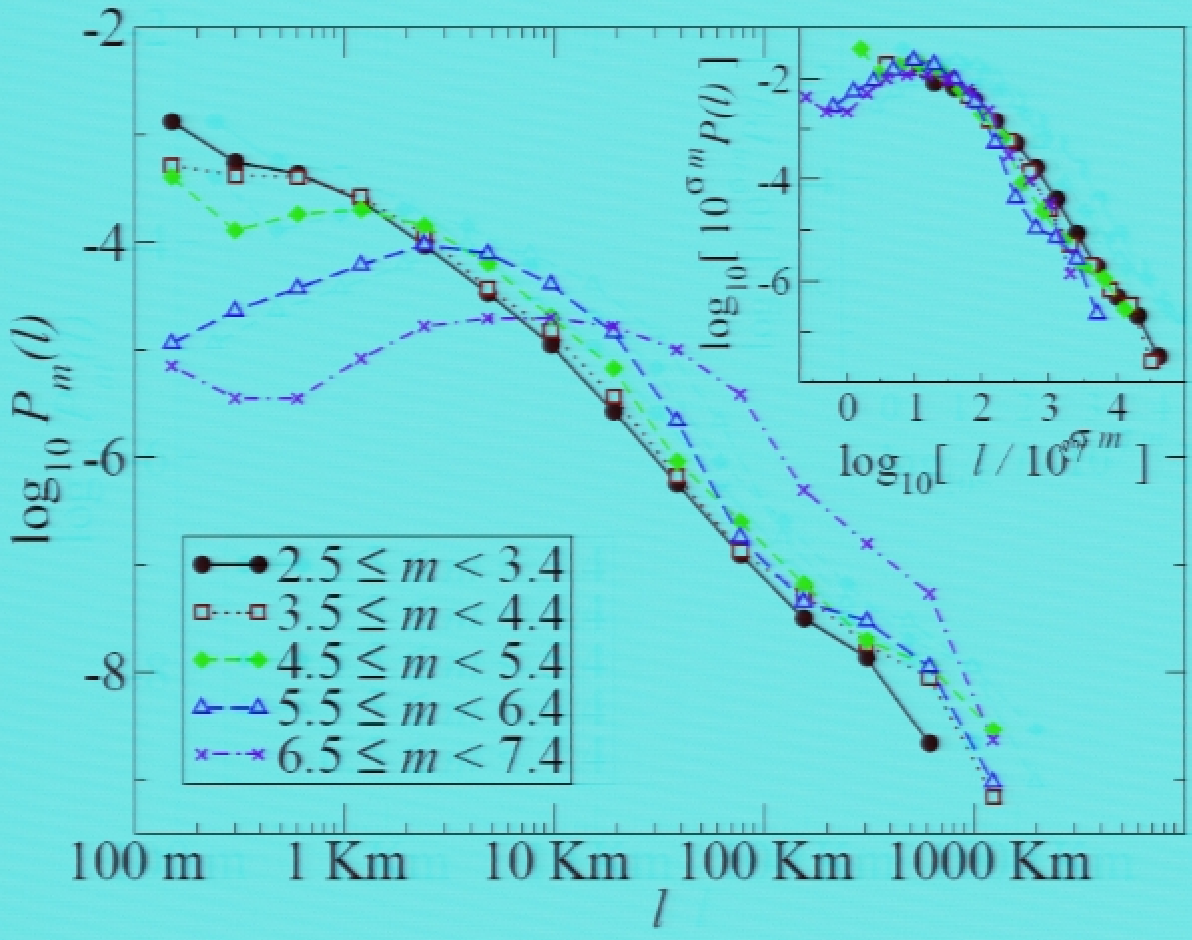

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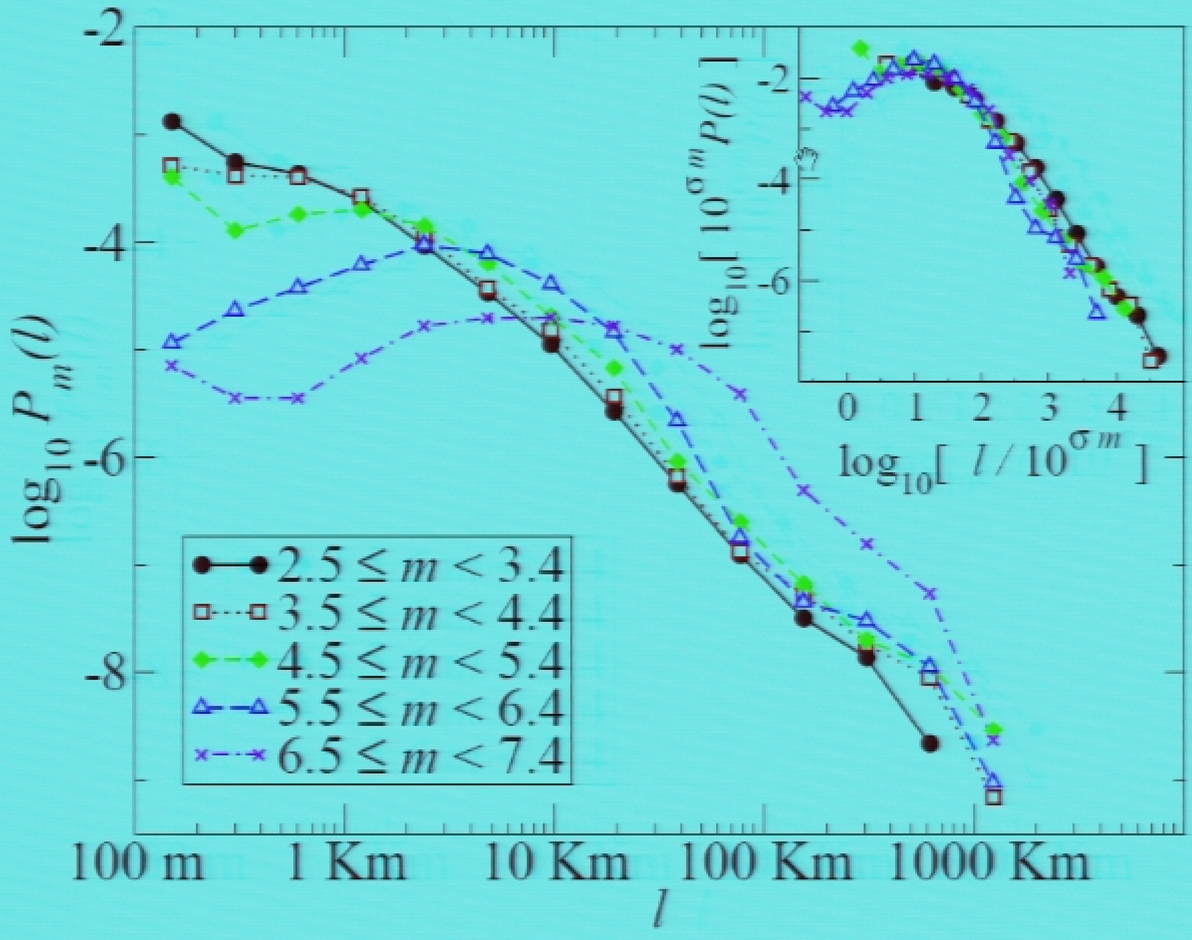

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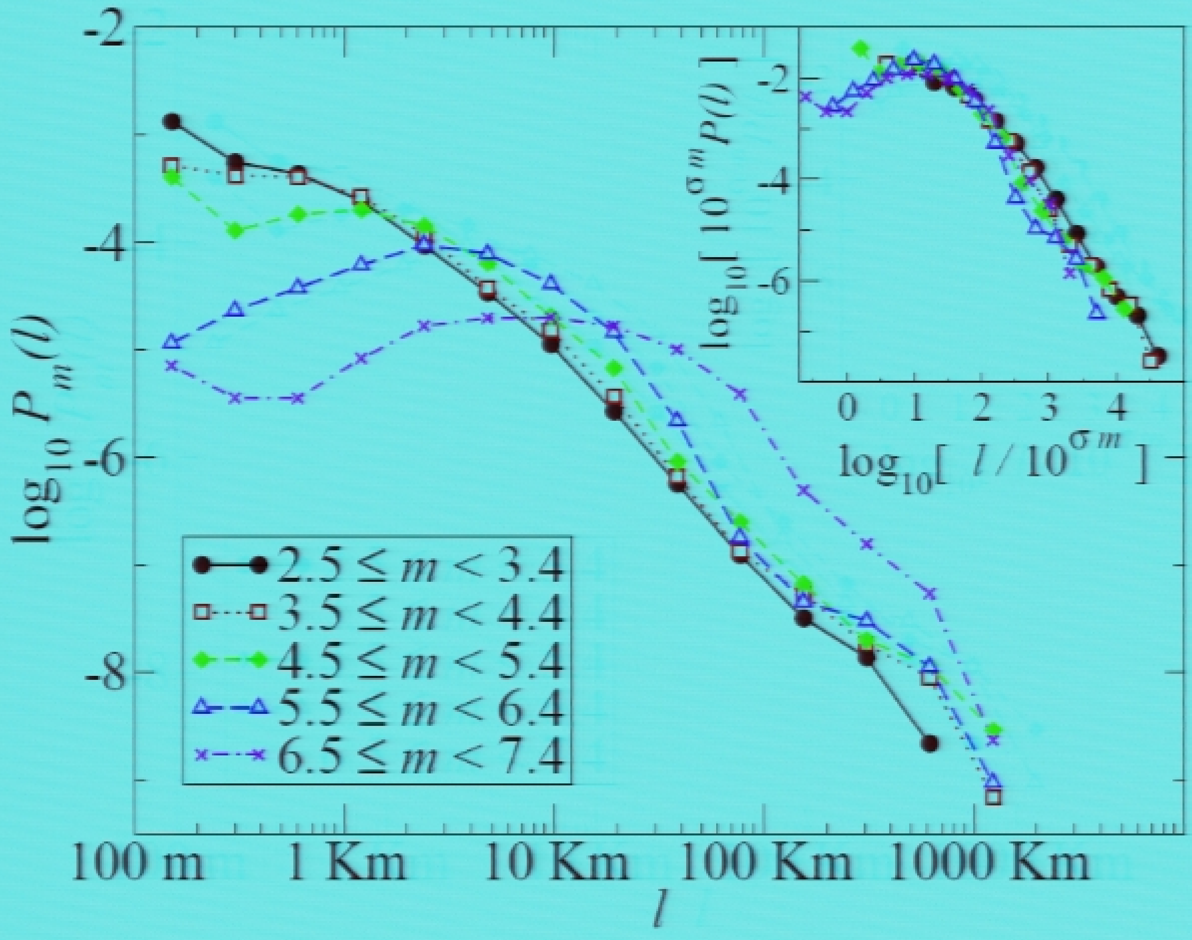

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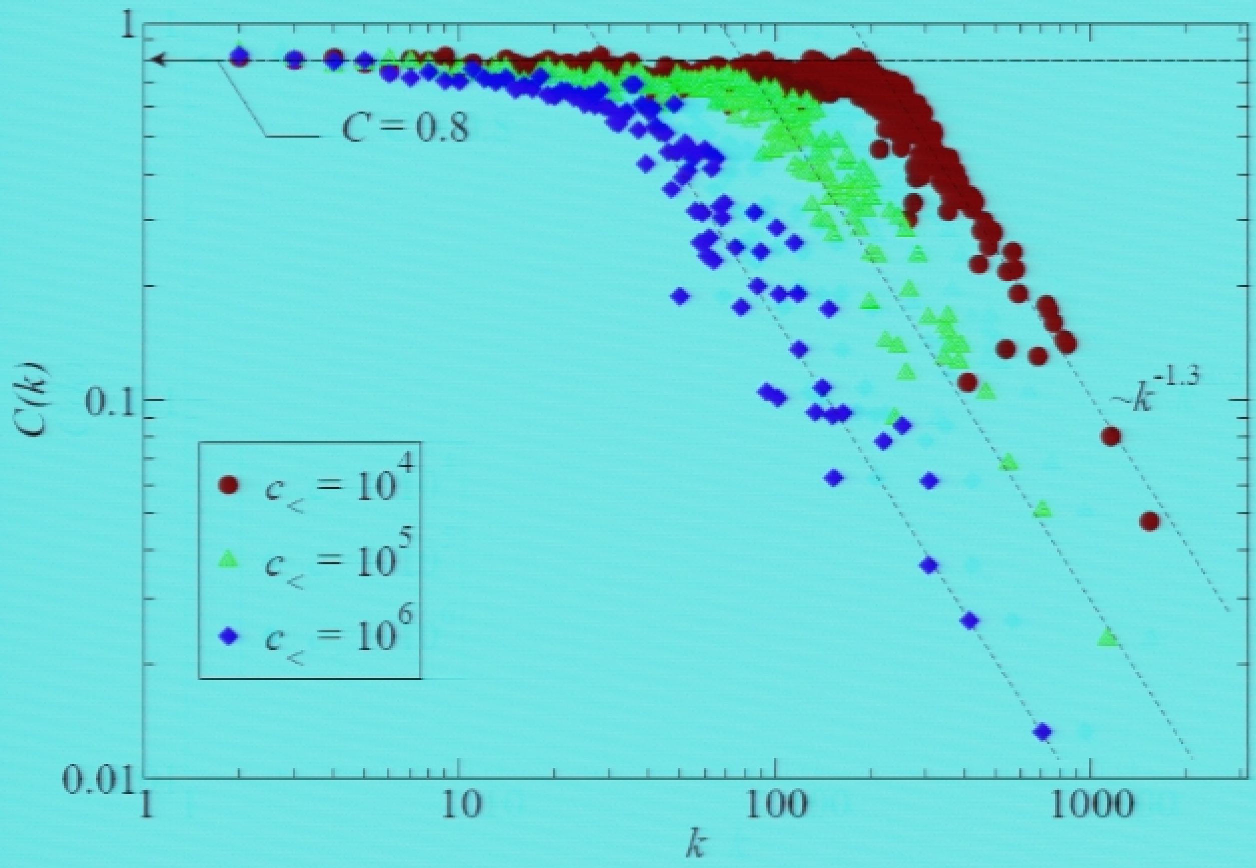
Relax the constraint of only one incoming link: clustering



universal asymptotes:

small $k \rightarrow C = 0.8$

large $k \rightarrow \sim k^{-1.3}$

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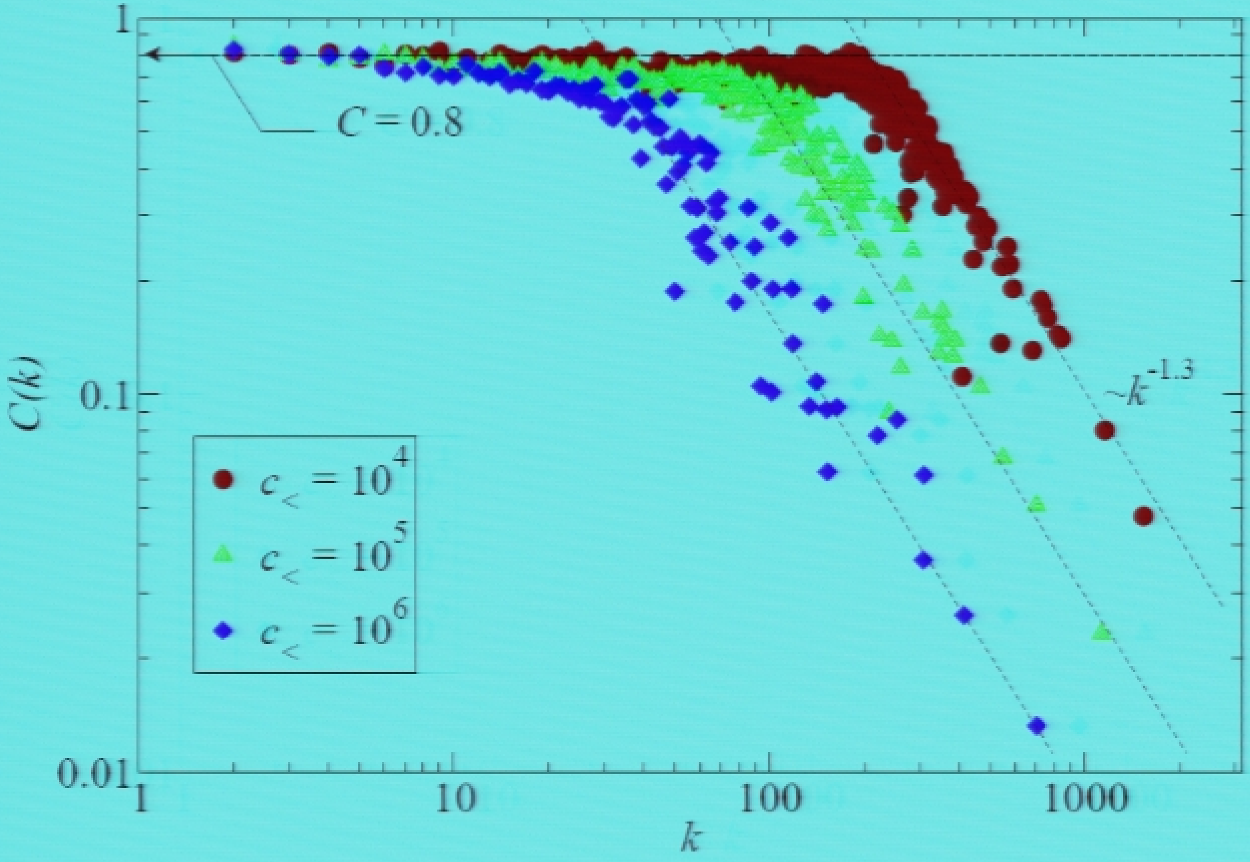

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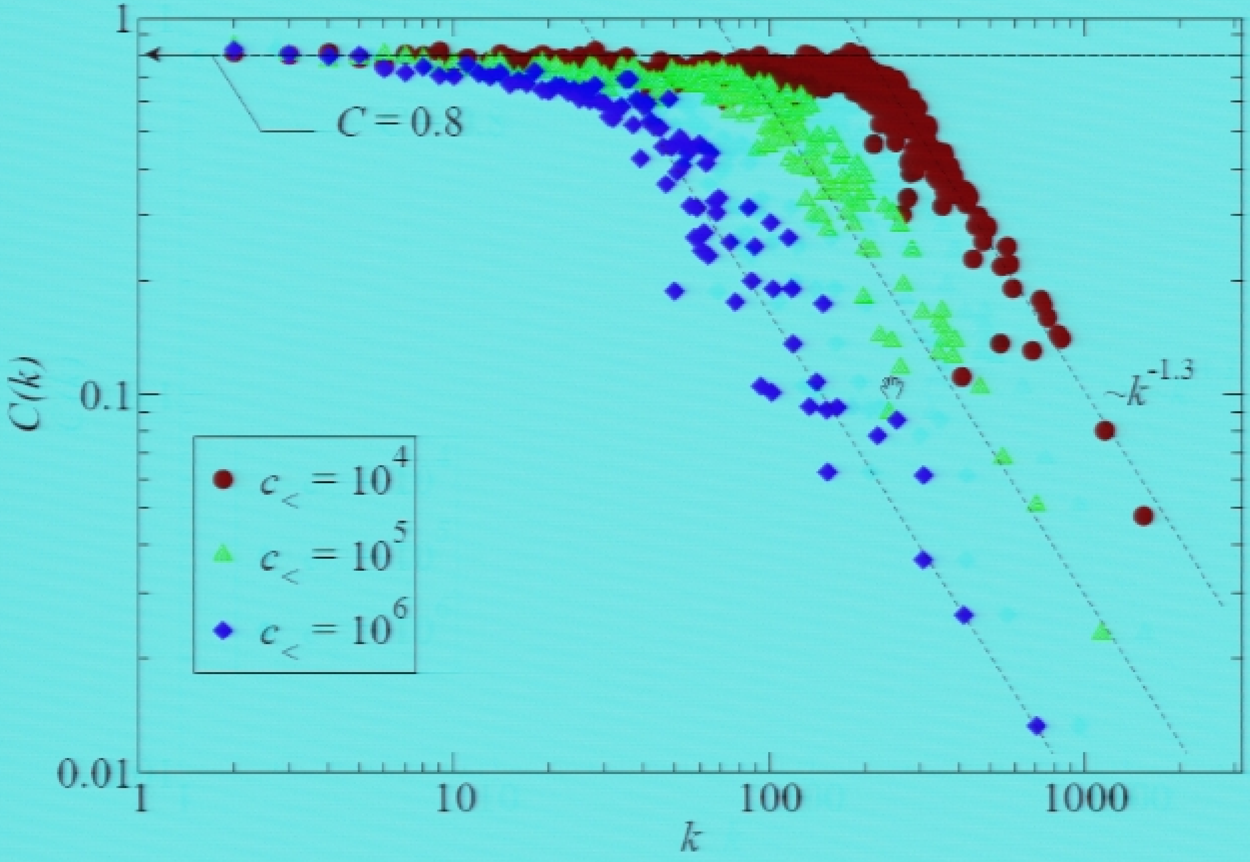

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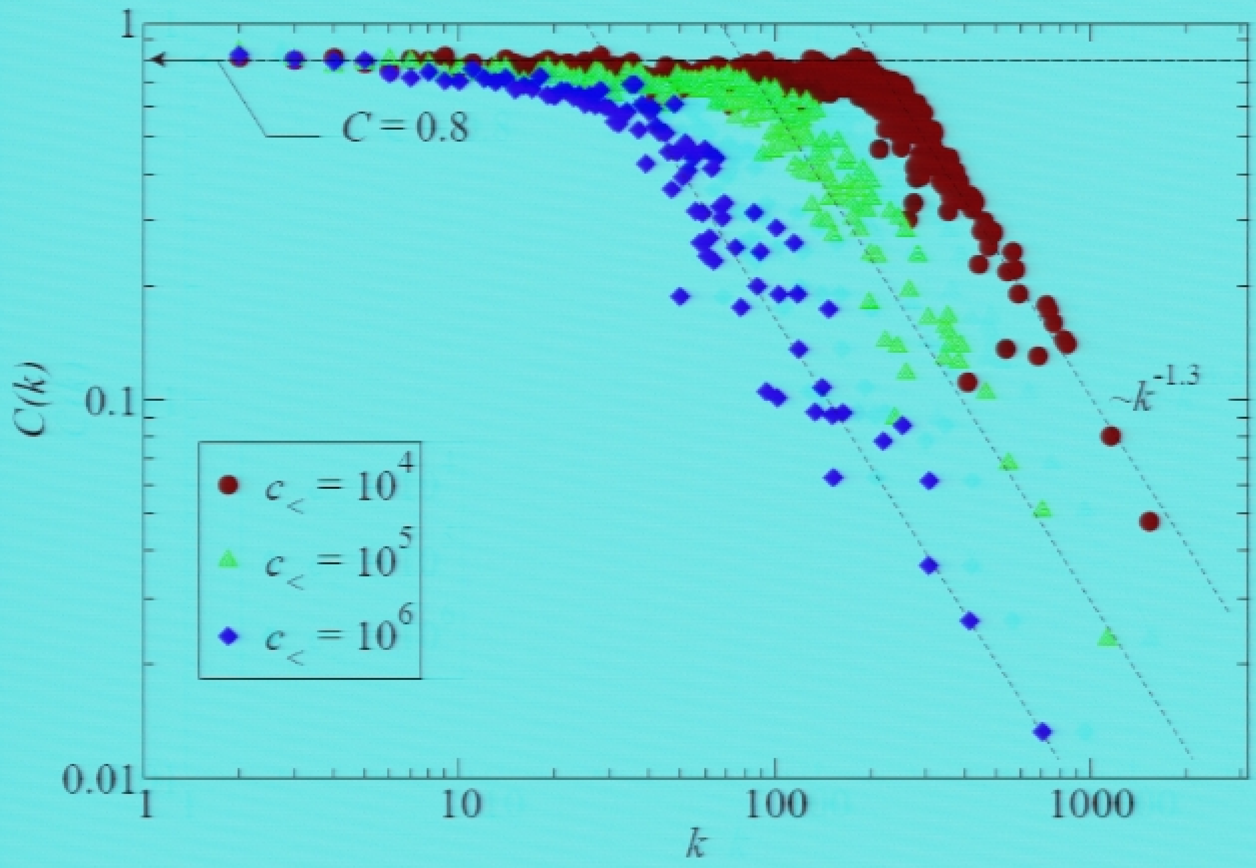

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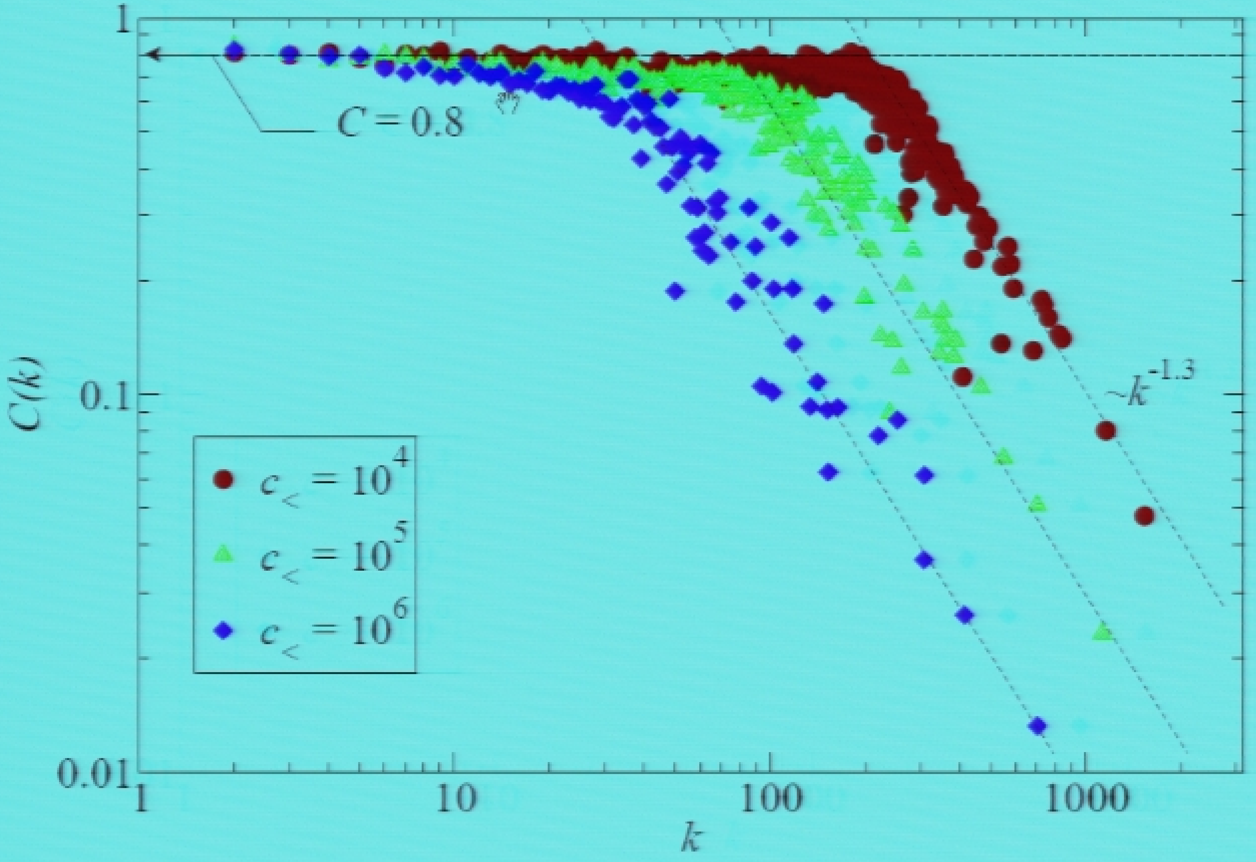

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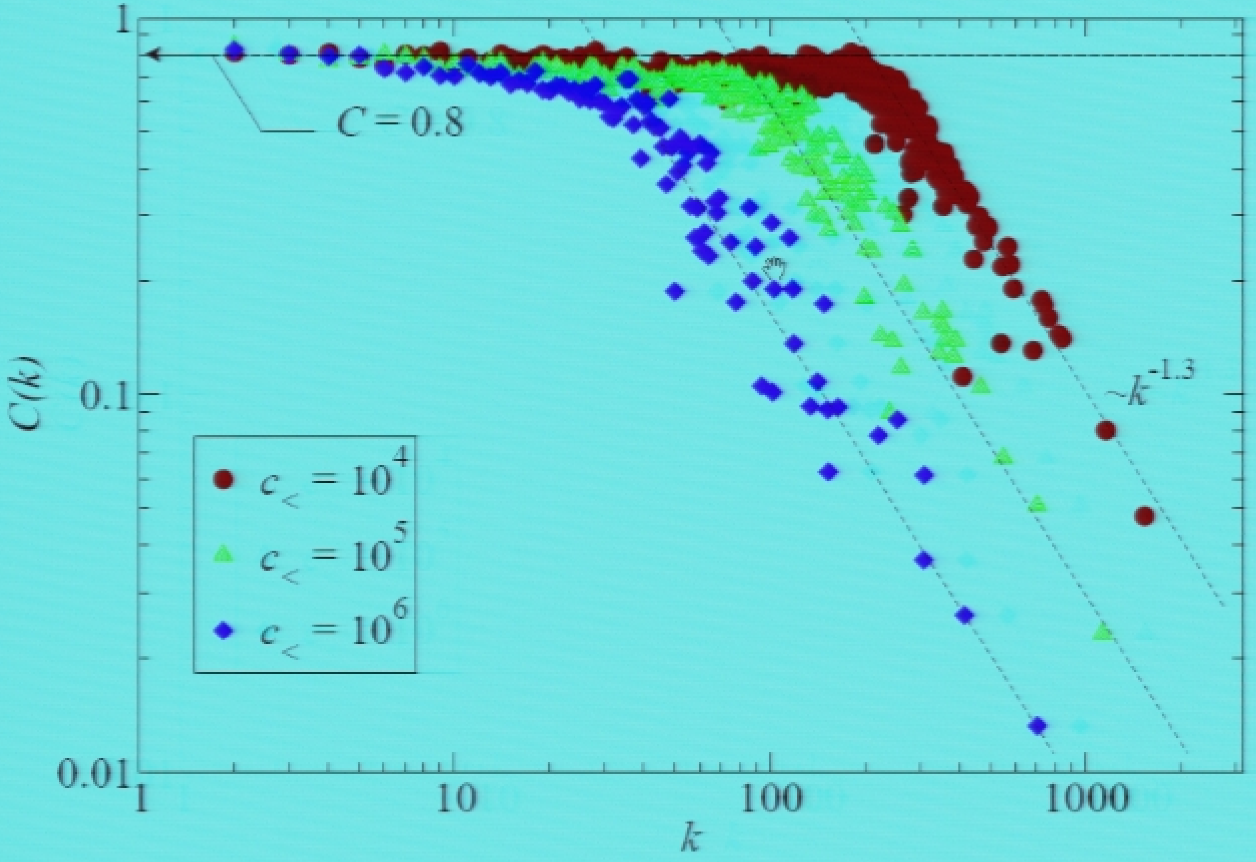

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Complex correlations in self-organized critical phenomena

network motifs ↔ correlations in the stress field

wide aftershock area ↔ high stress

far aftershocks ↔ long correlation length

Critical point scenario

[Zöller, Hainzl and Kurths, J. Geoph. Res. (2001)]

[Jaumé and Sykes, Pure Appl. Geoph. (1999)]

network motifs involving far aftershocks → earthquake precursors?

Earthquakes: summary

- Metric based on violation of a null hypothesis
- Network of earthquakes, scale-free:
  - cluster size
  - out-degree (= number of aftershocks)
  - Omori law, for all magnitudes
  - aftershock distances, for all magnitudes
- Results very stable to variations of parameters
- Possibility of hazard assessment

[Baiesi and Paczuski, Phys. Rev. E 69, 066106 (2004)]
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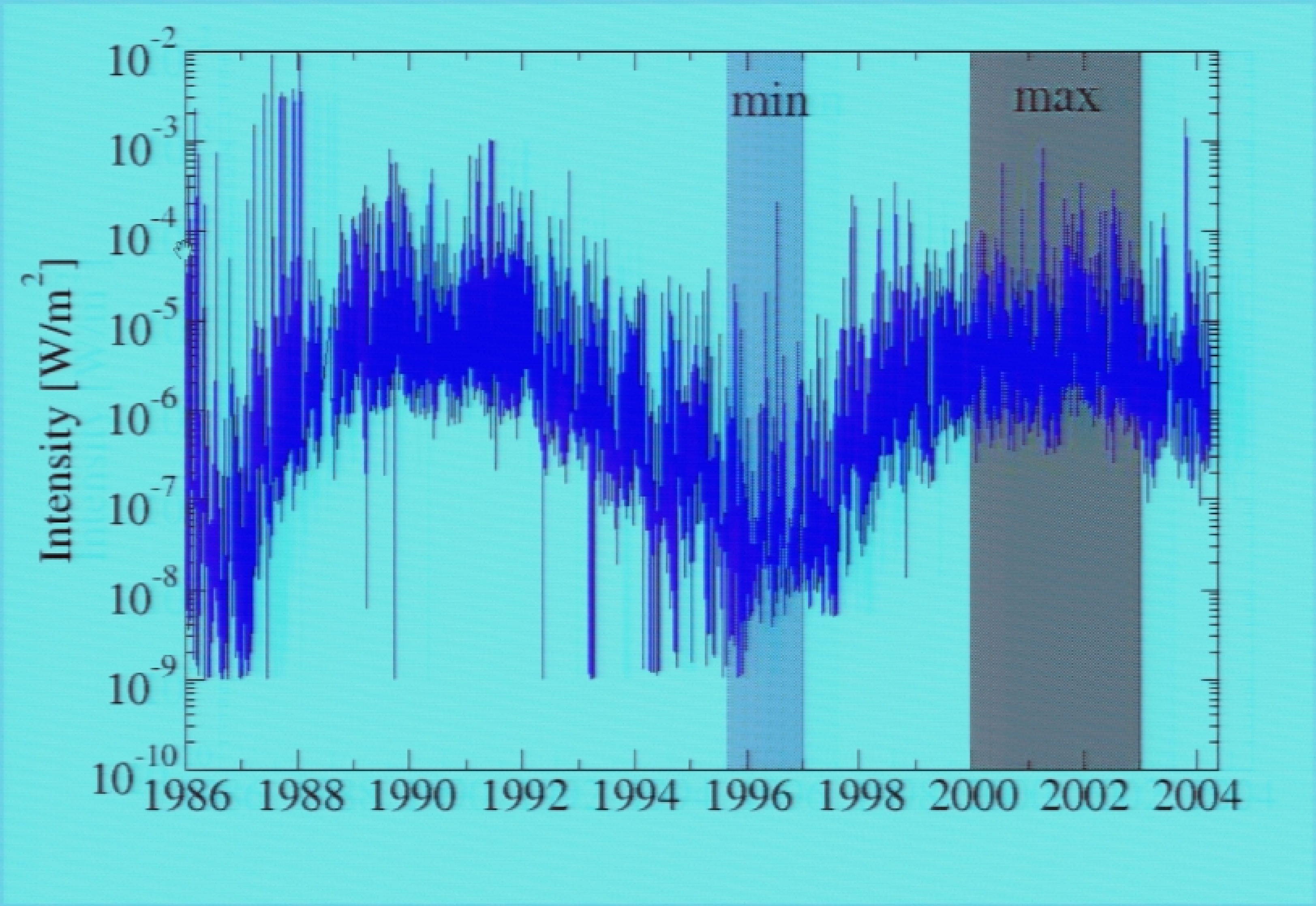
Complex correlations in self-organized critical phenomena

Solar flares: X-ray emissions, recorded by GOES satellites



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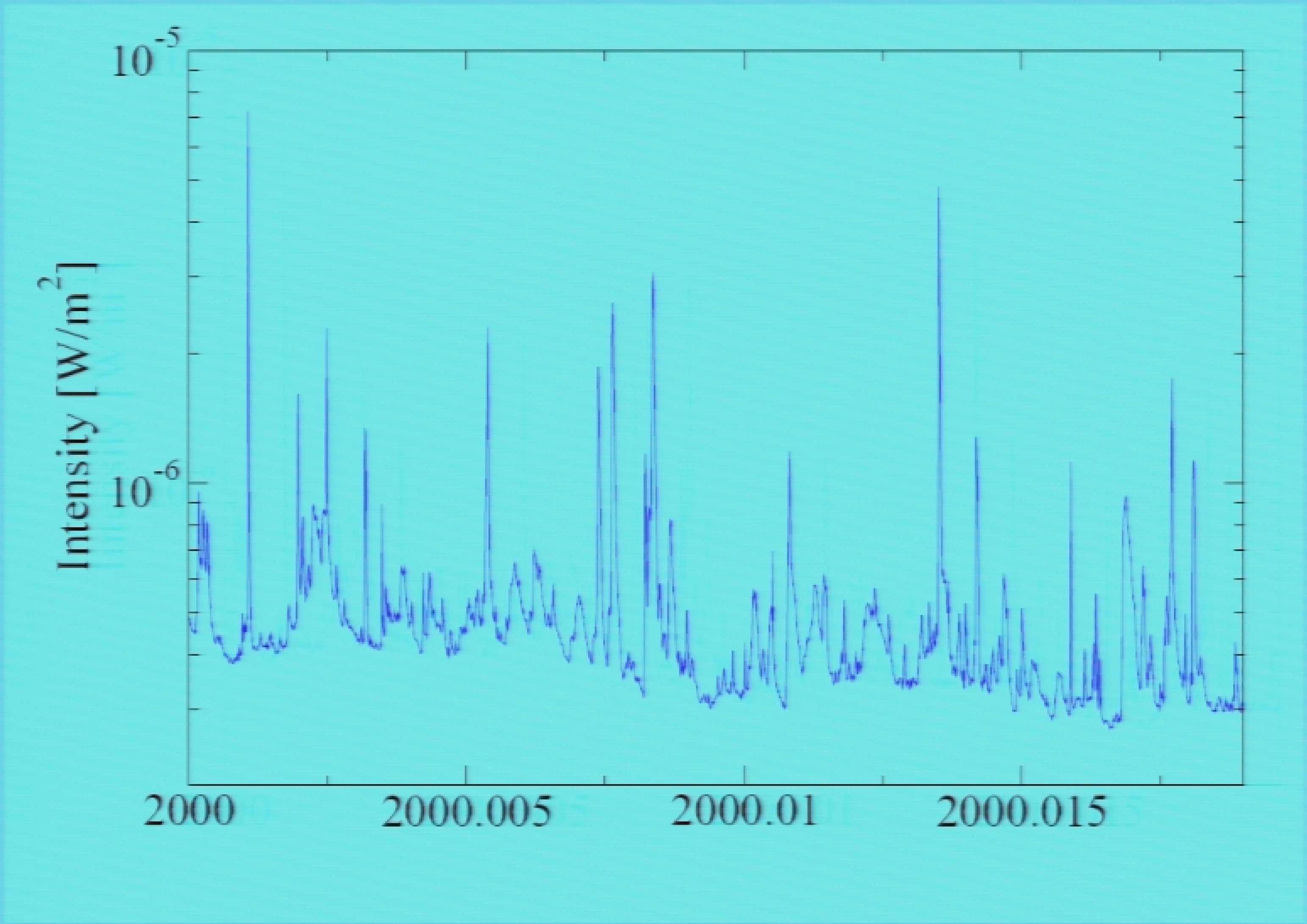
Solar flares: what is an “event”?



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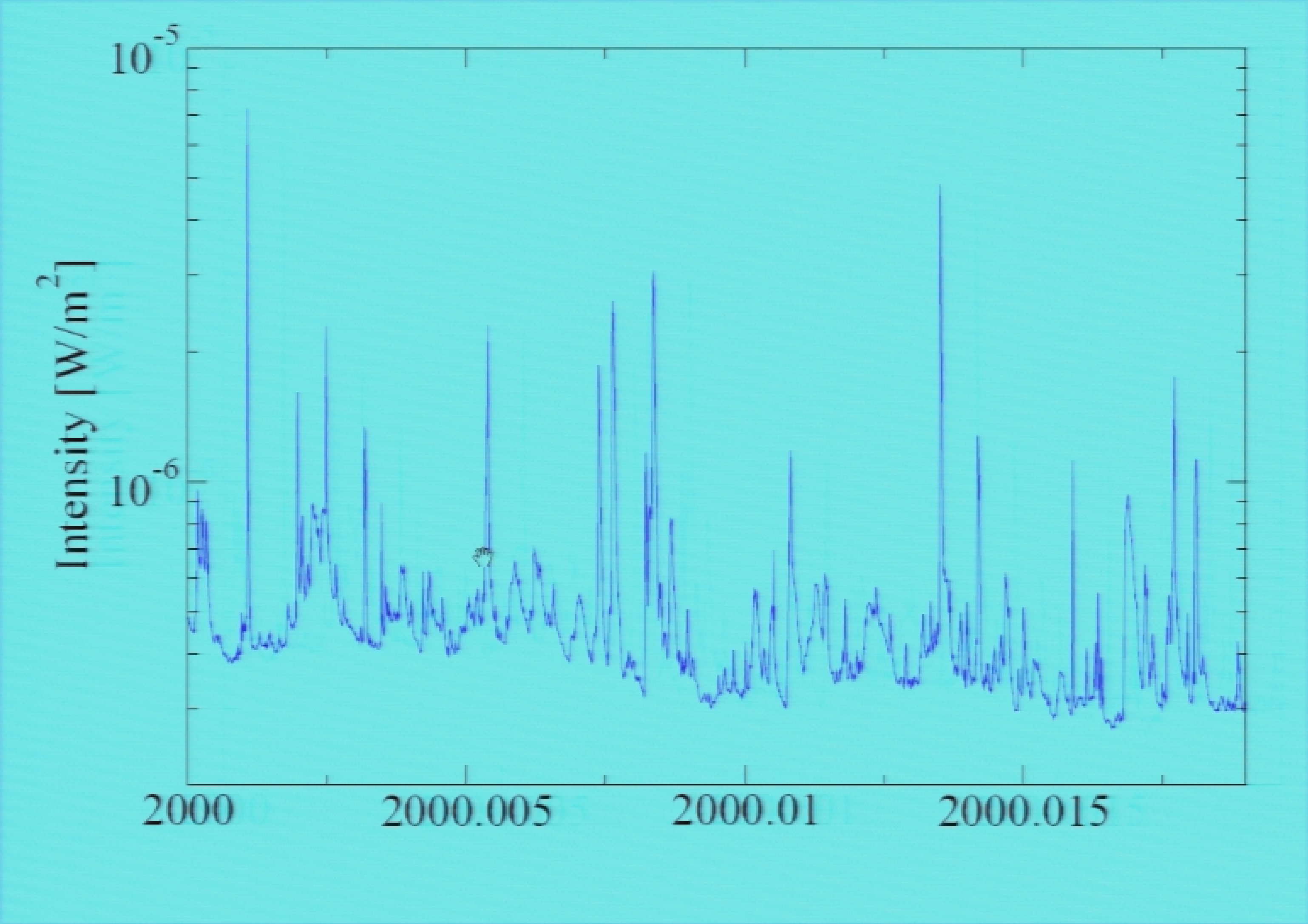

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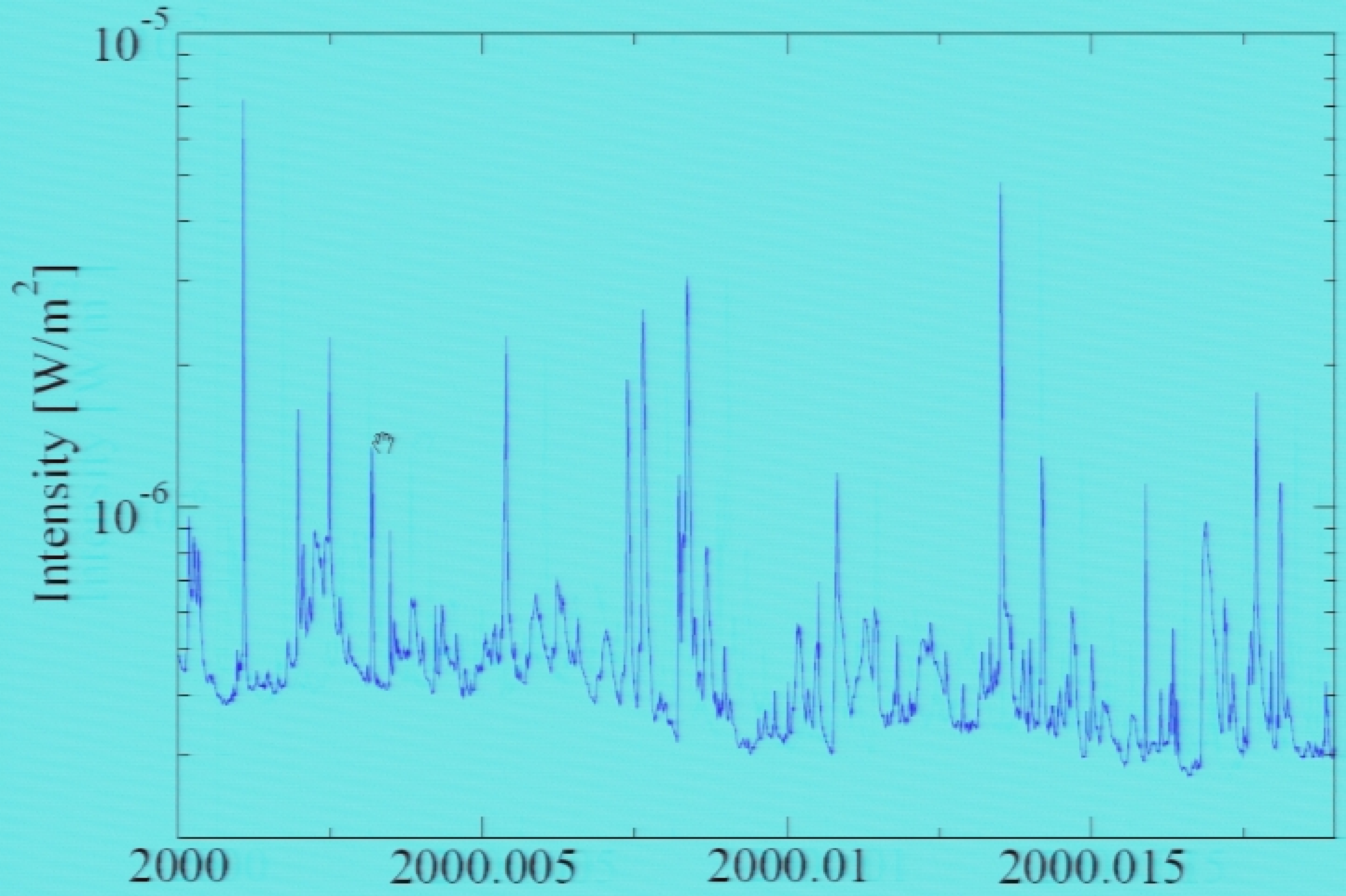

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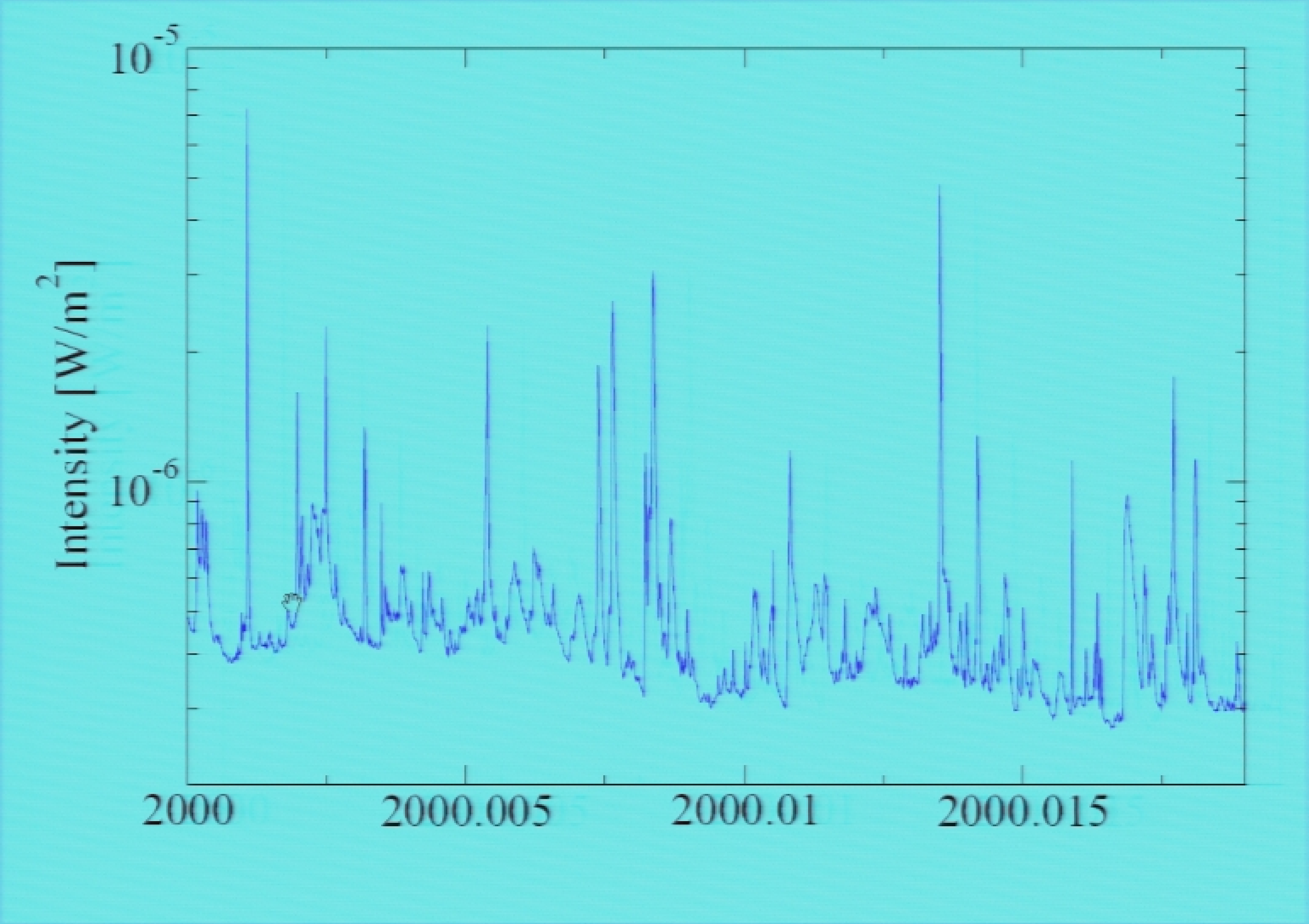

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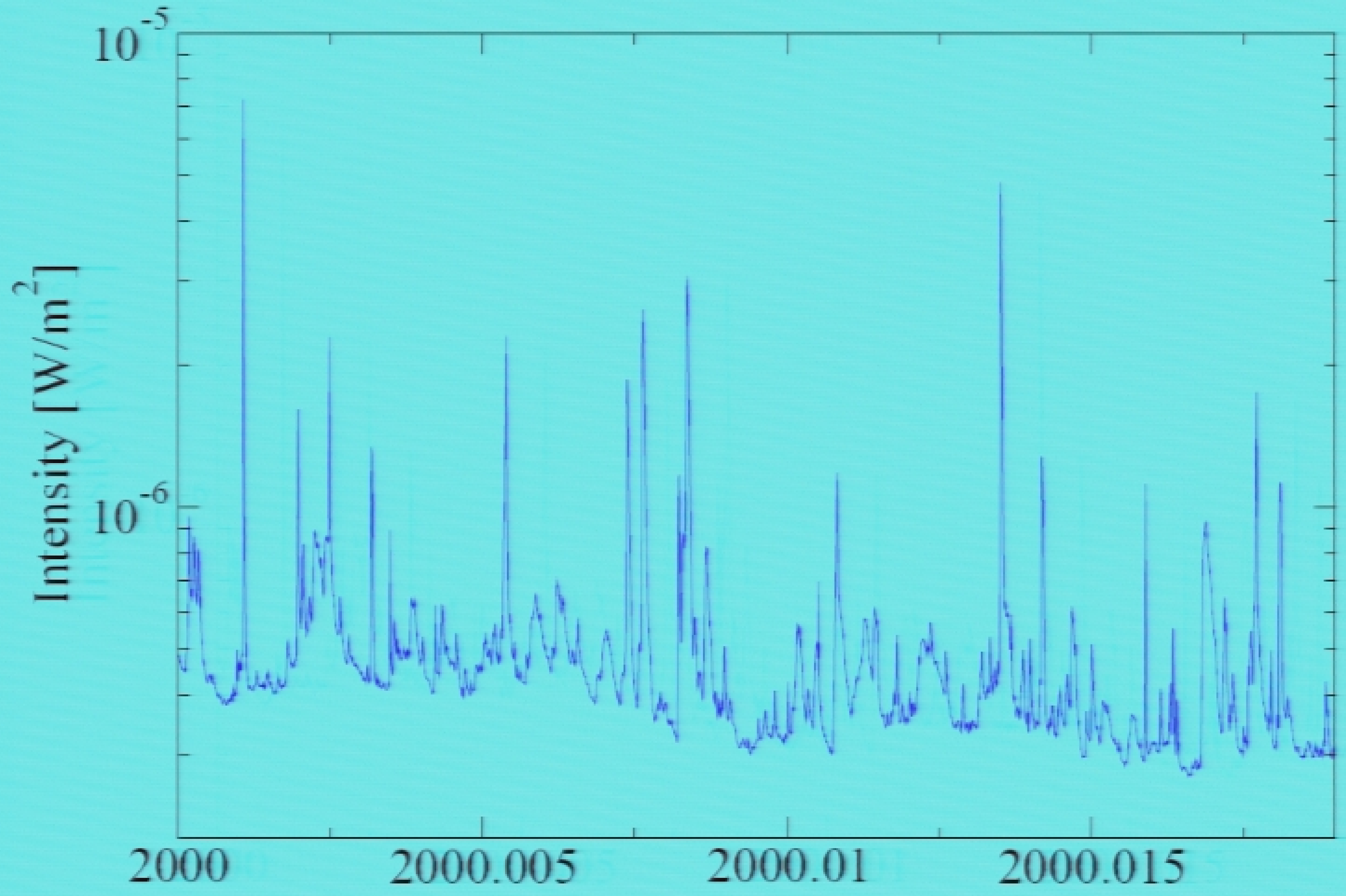

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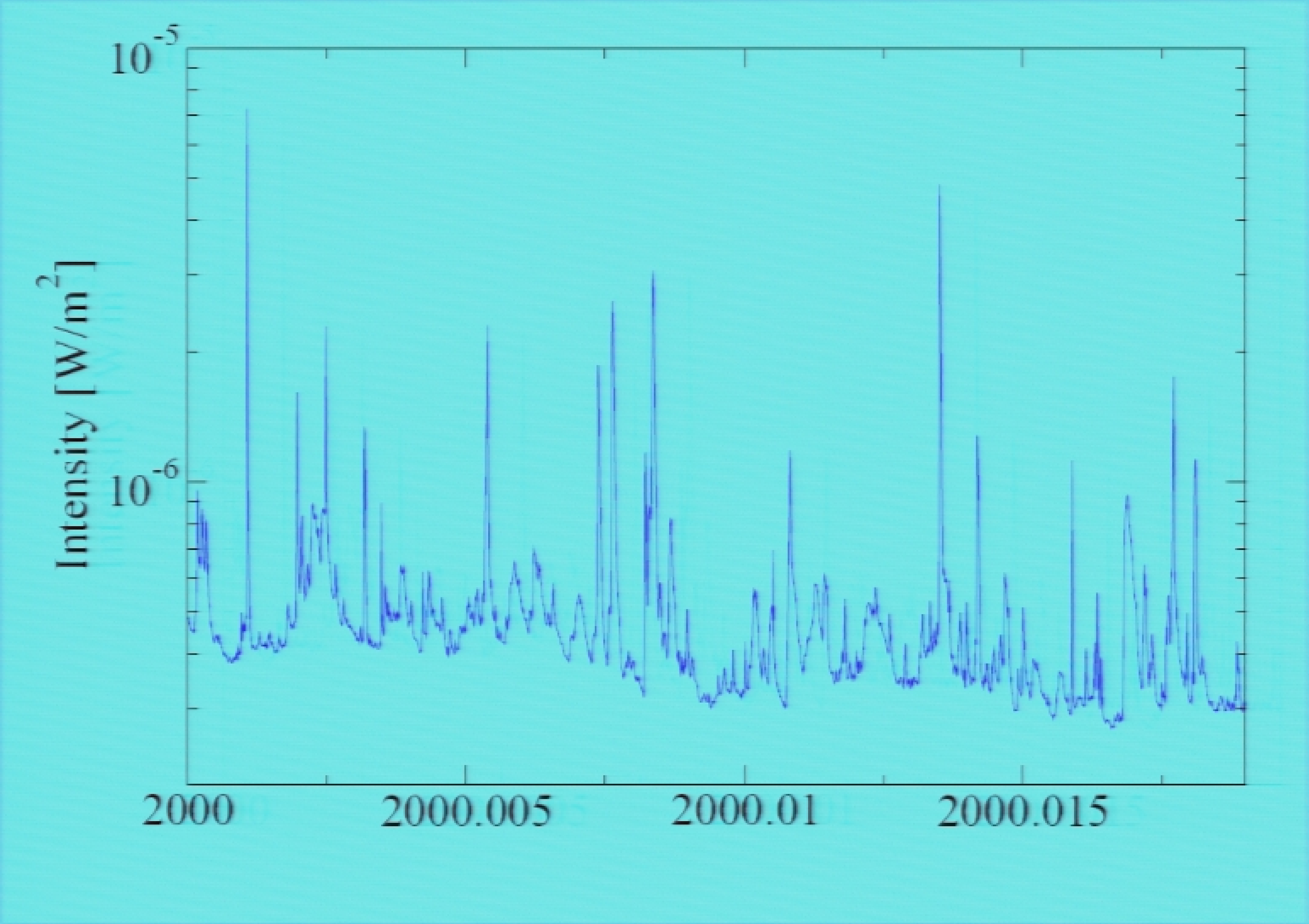

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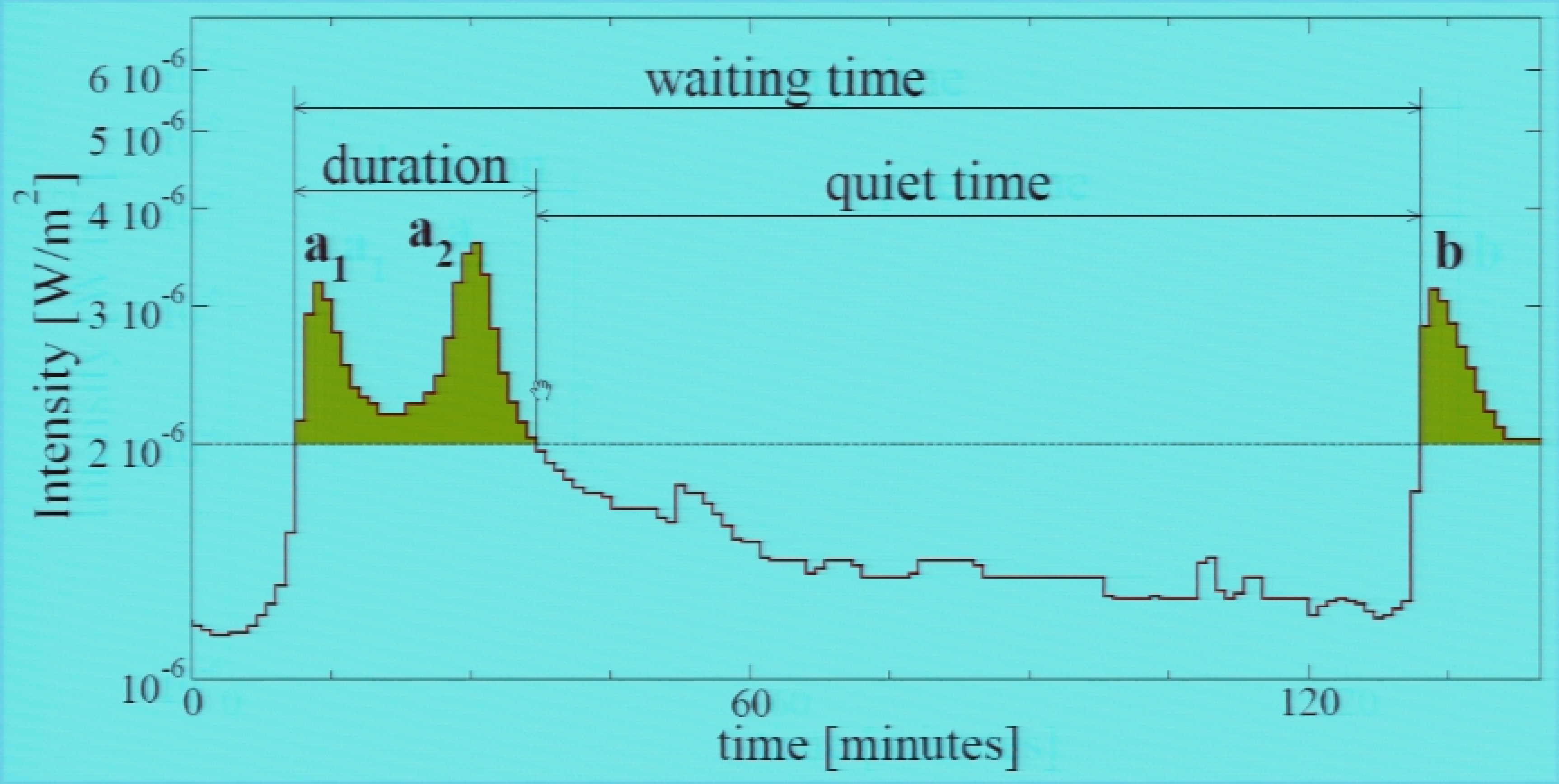

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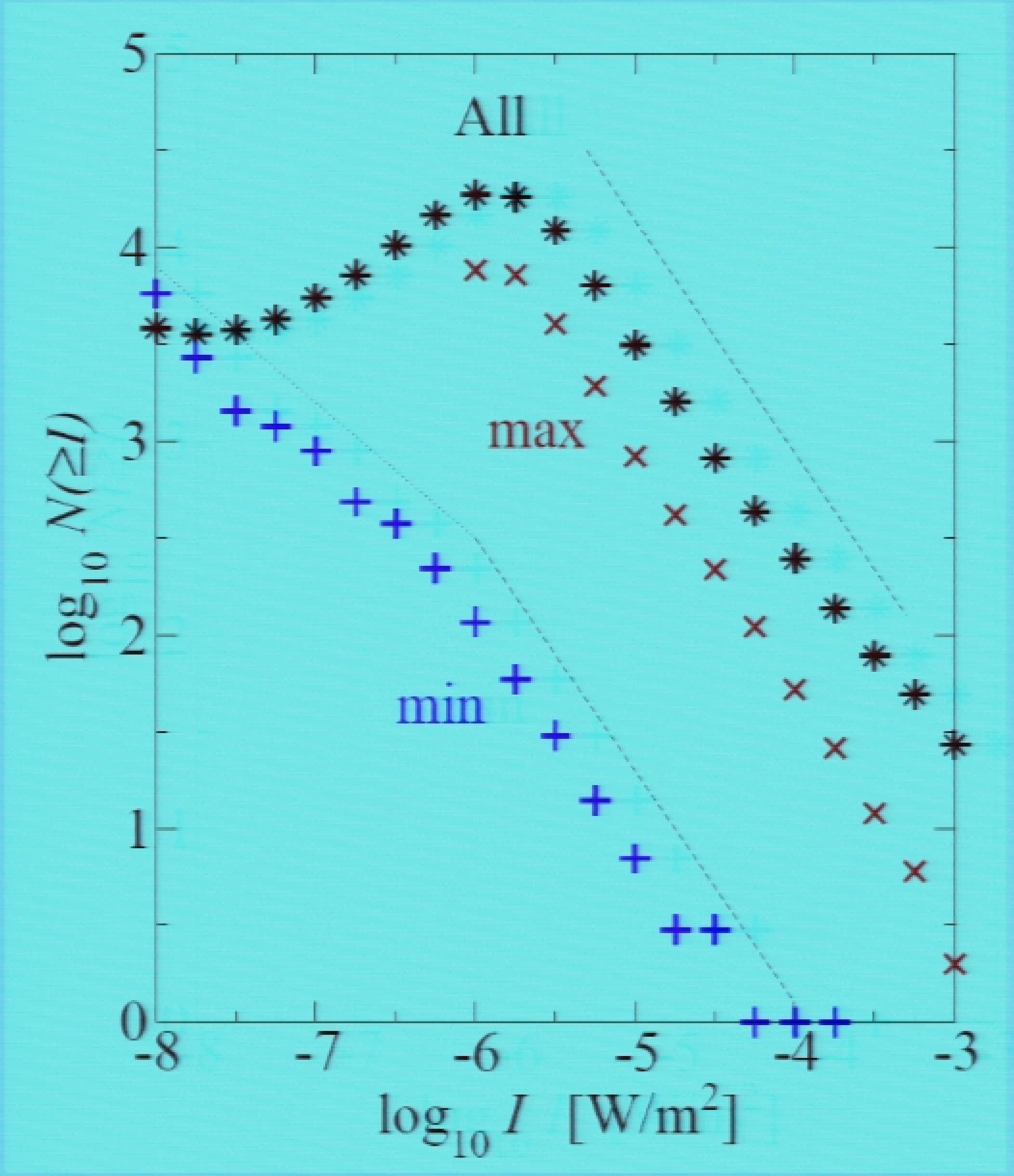
Solar flares:

Number of events with

intensities $\geq I$



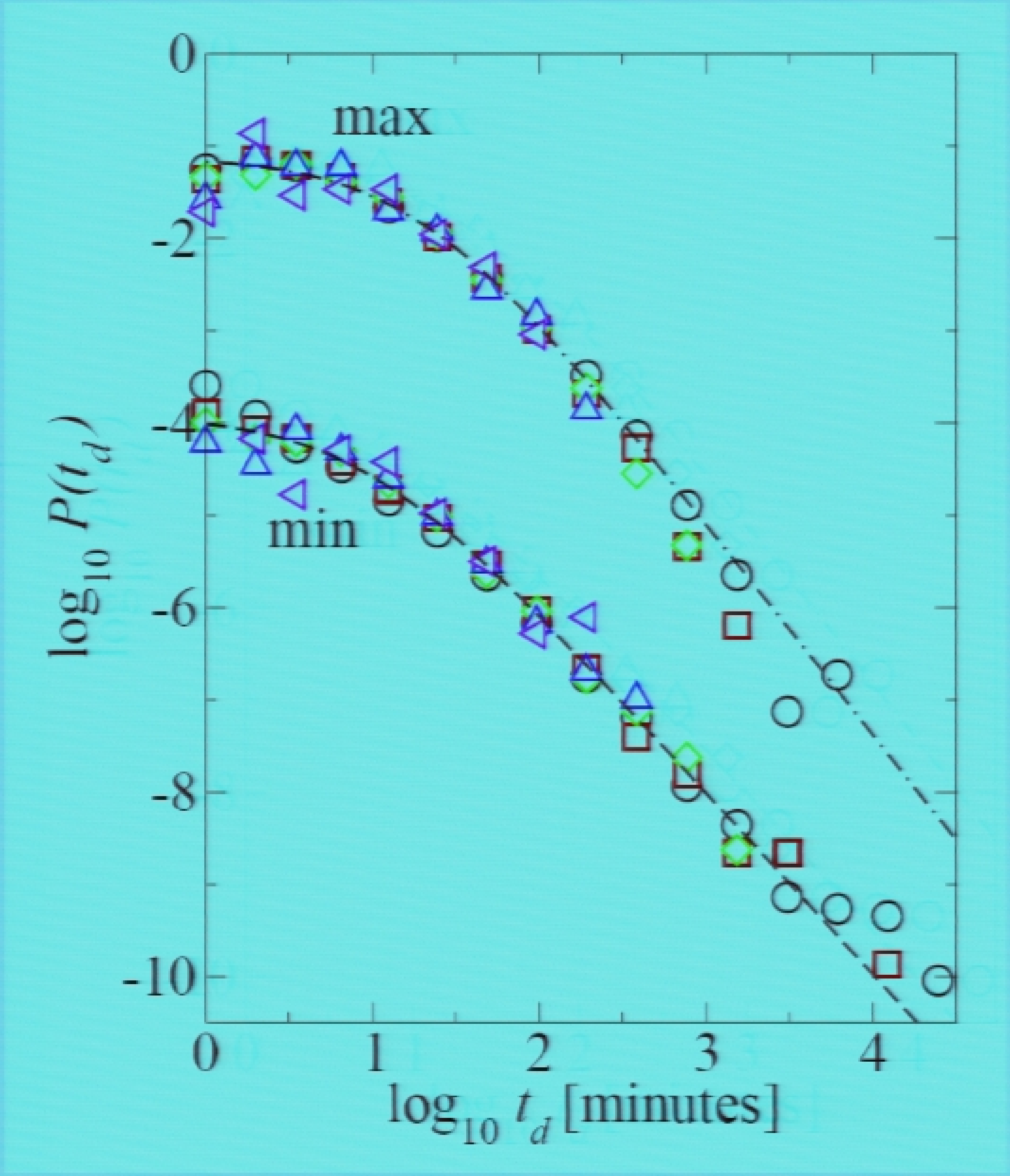
Complex correlations in self-organized critical phenomena

Distribution of duration times

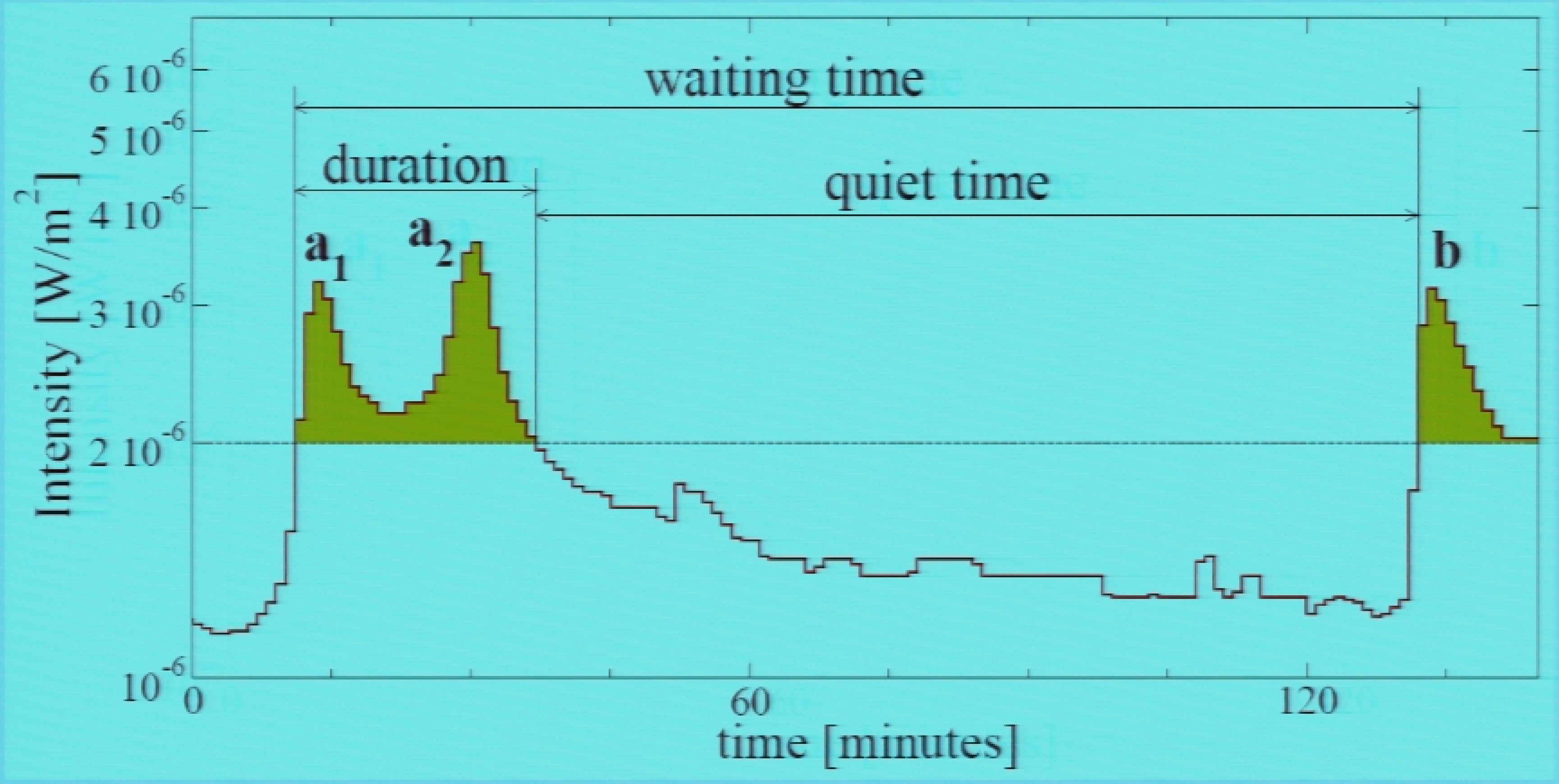
$$P(t_d) \sim \frac{1}{(1 + t_d/t_d^*)^{\gamma_{\rm dur}}}$$

min: $\gamma_{\rm dur} = 2.0(1)$

max: $\gamma_{\rm dur} = 2.3(4)$



Complex correlations in self-organized critical phenomena



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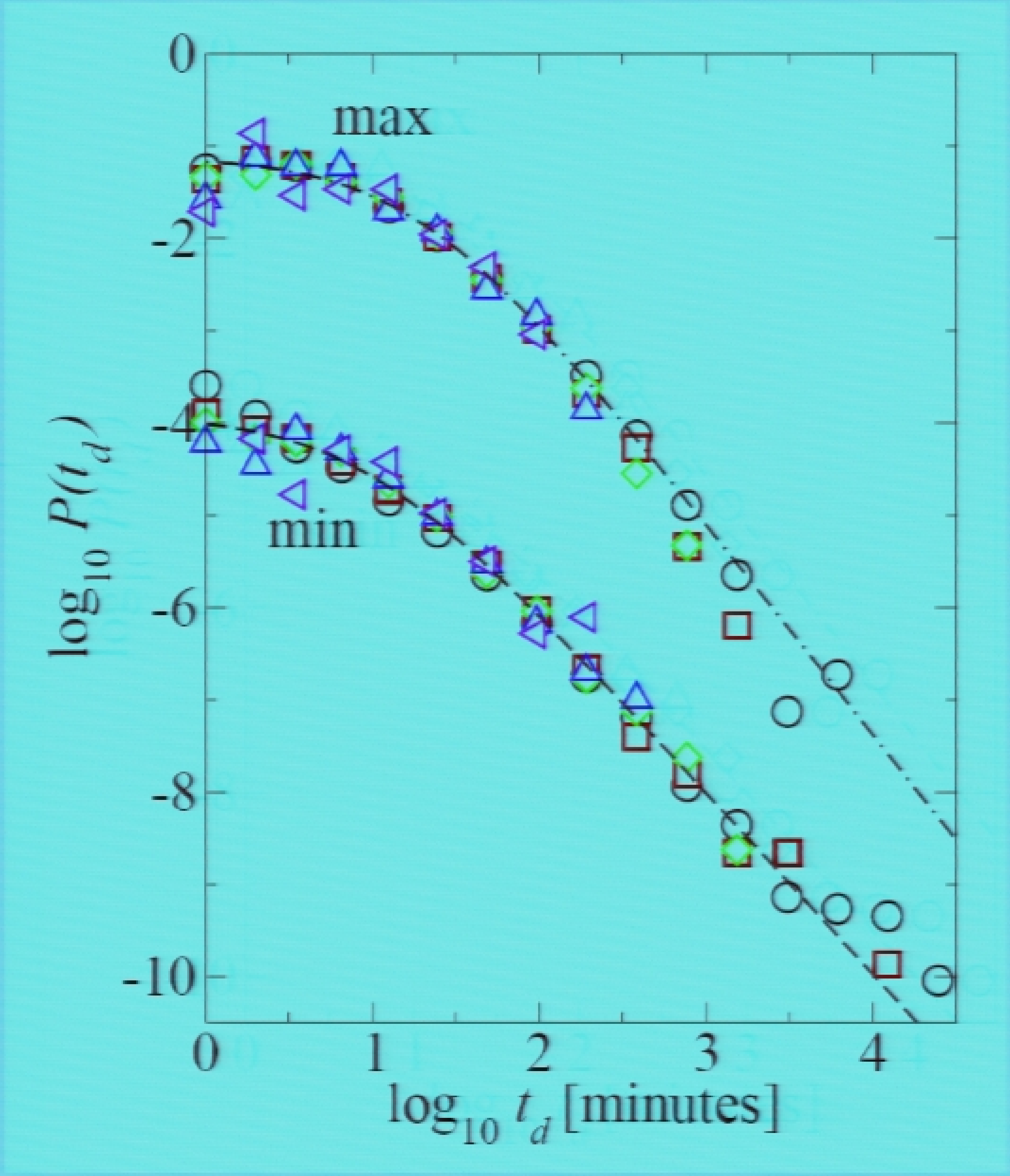

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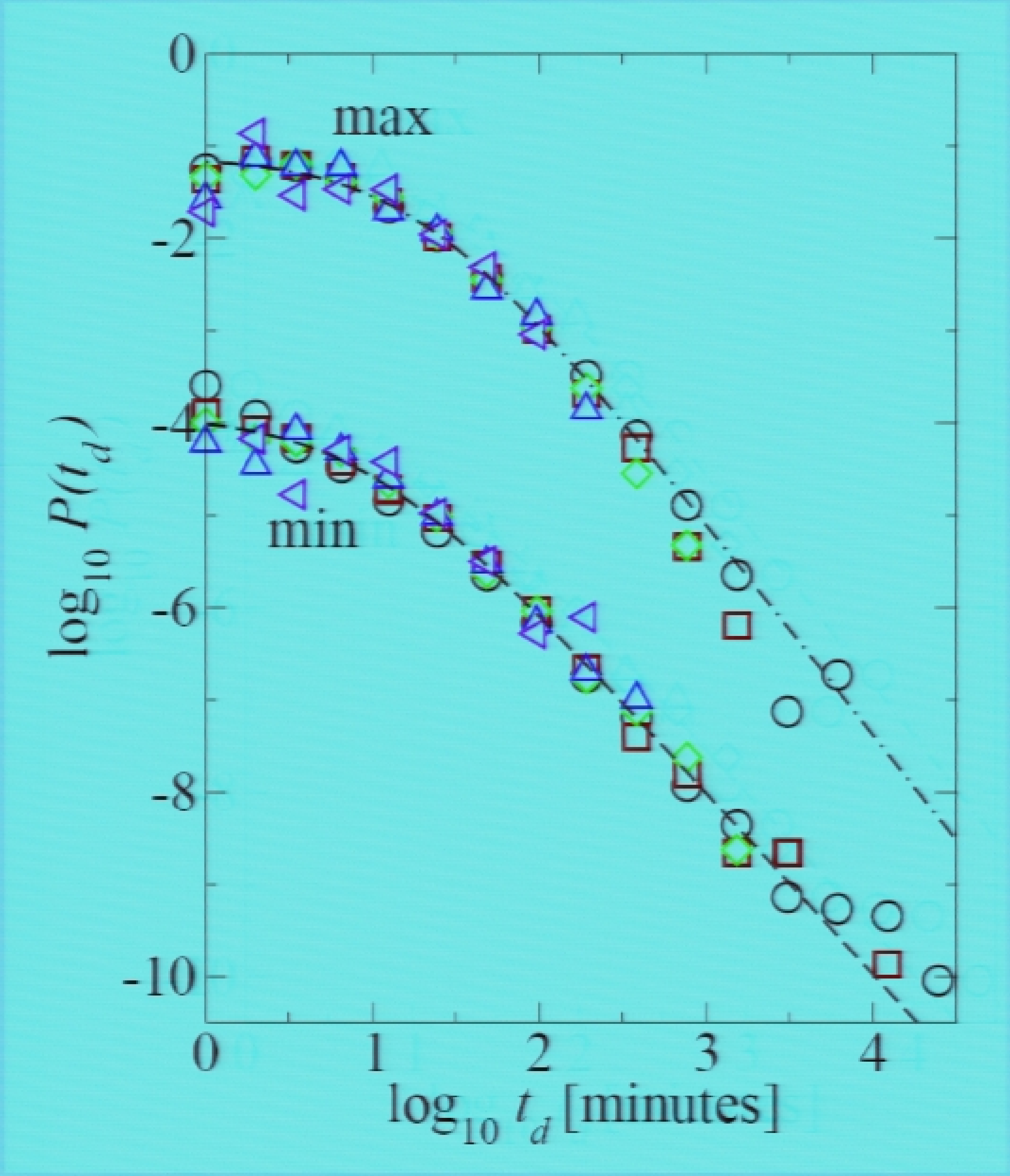

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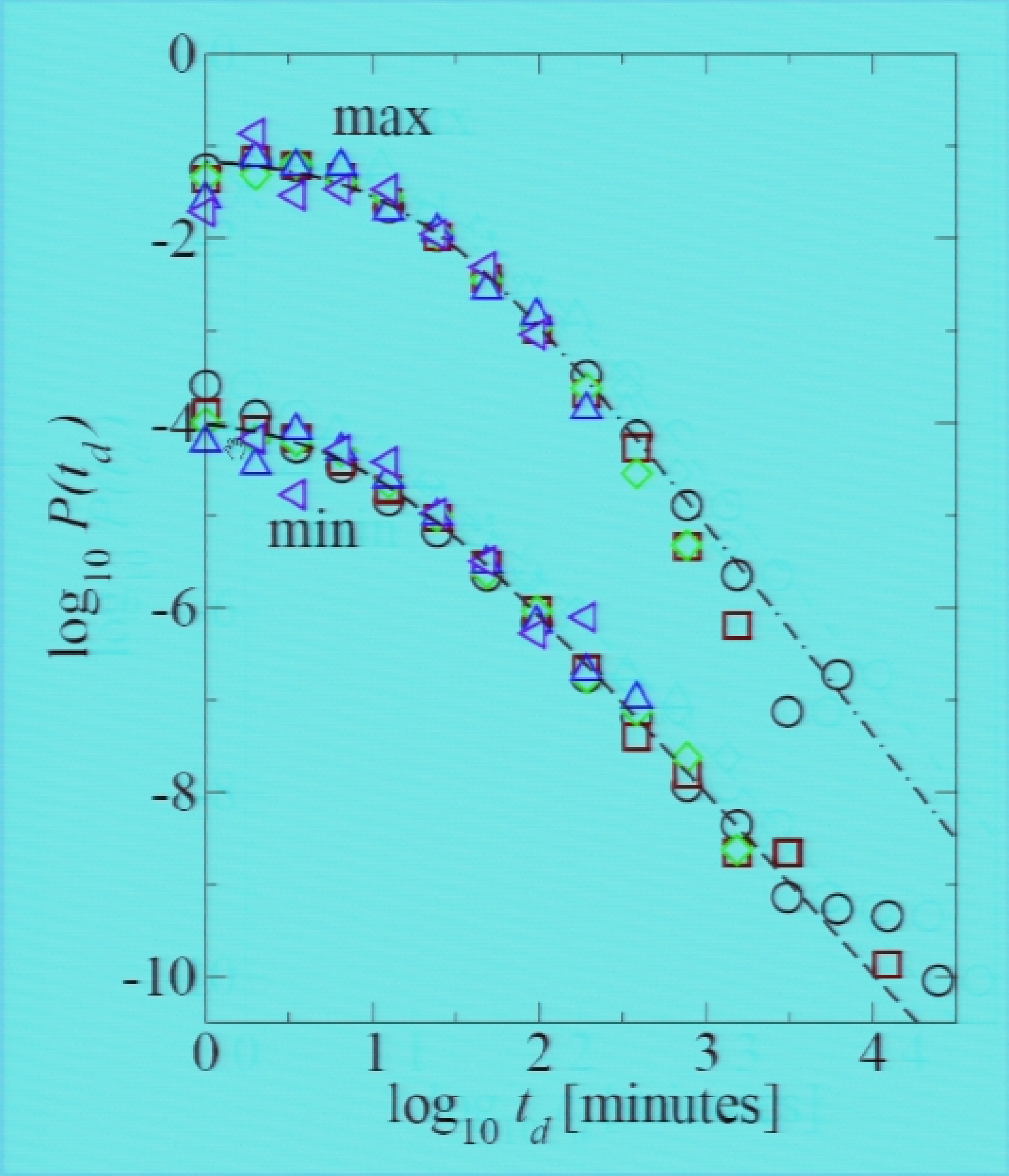

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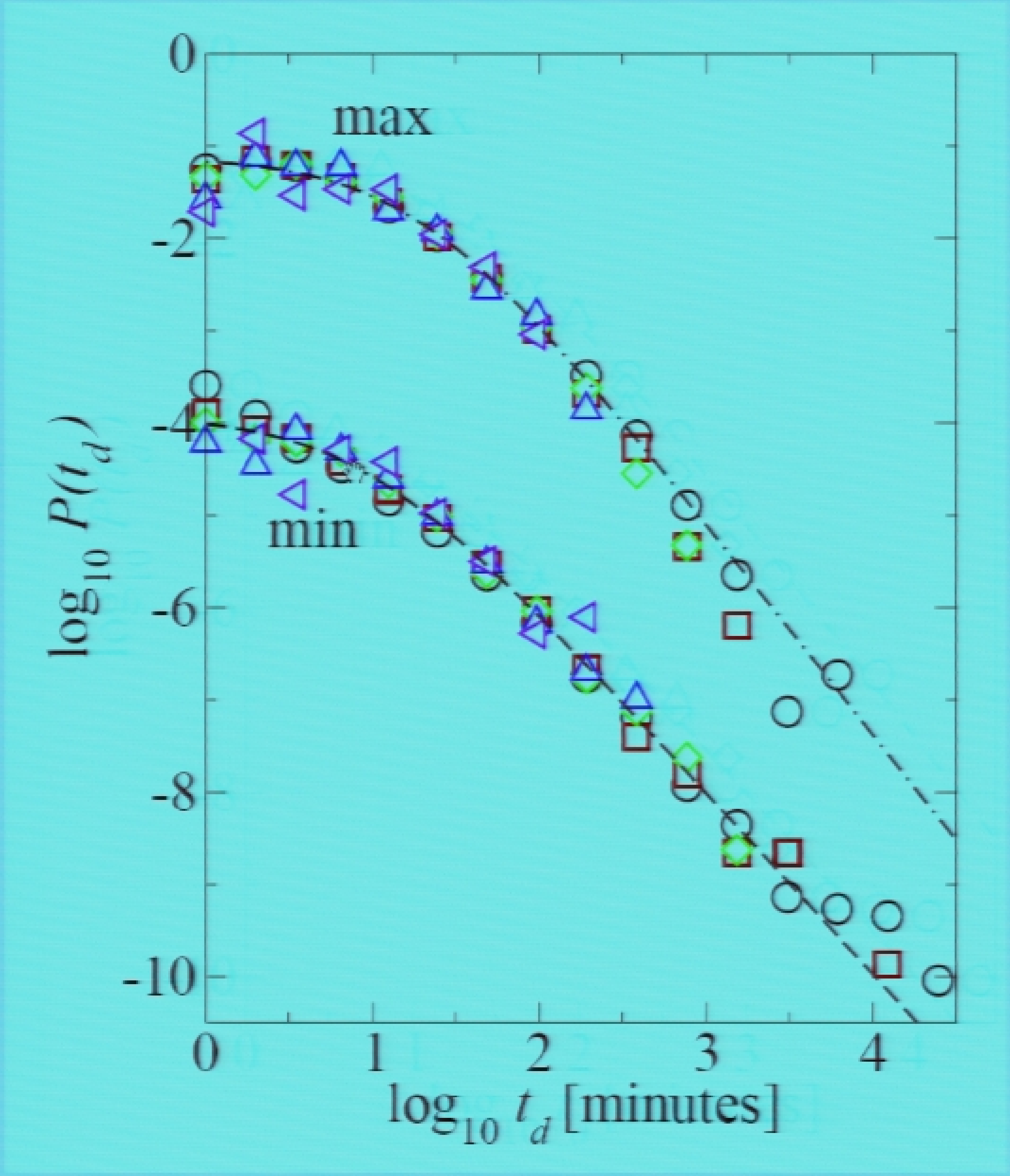
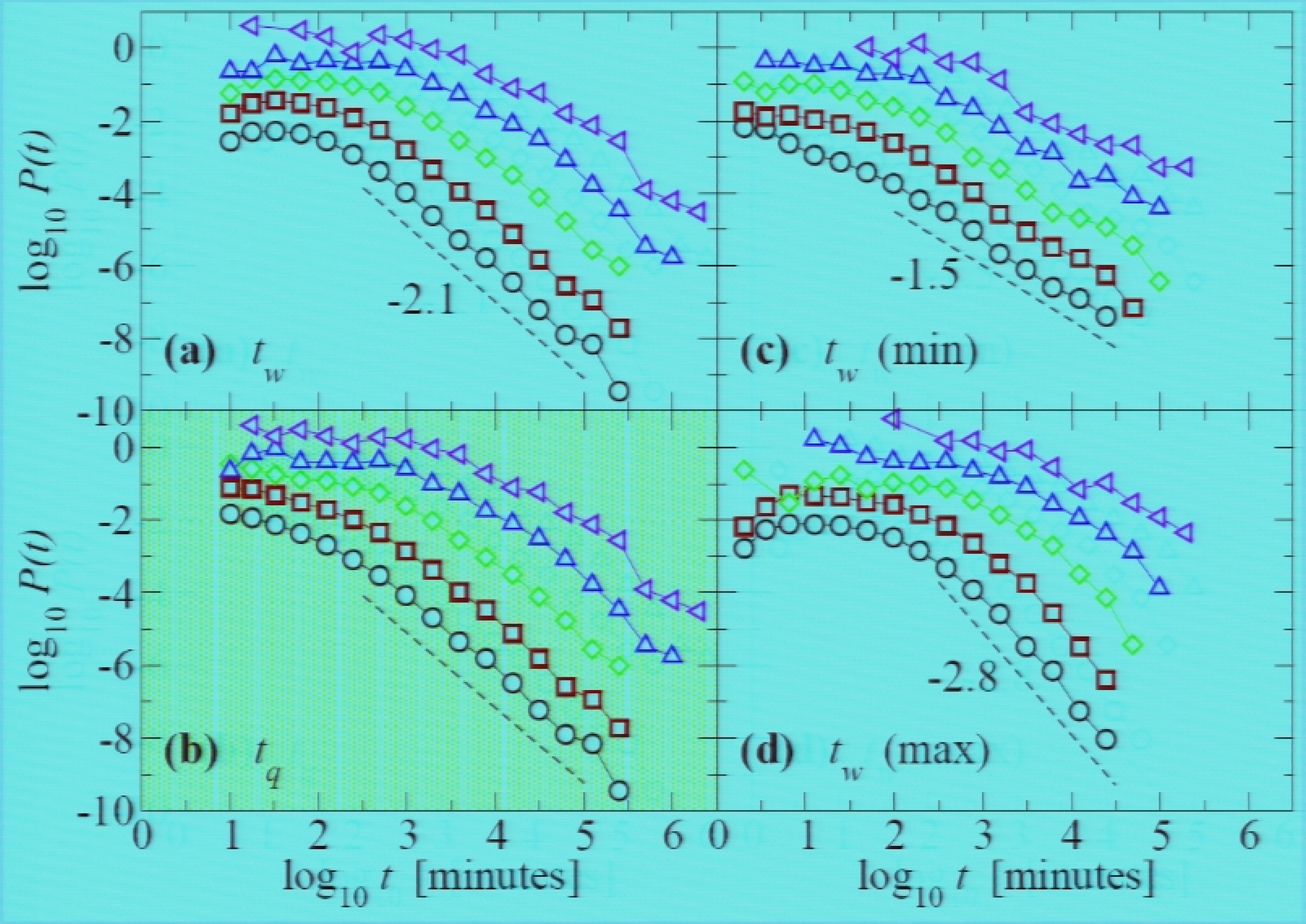


Complex correlations in self-organized critical phenomena

Waiting and quiet times for different thresholds



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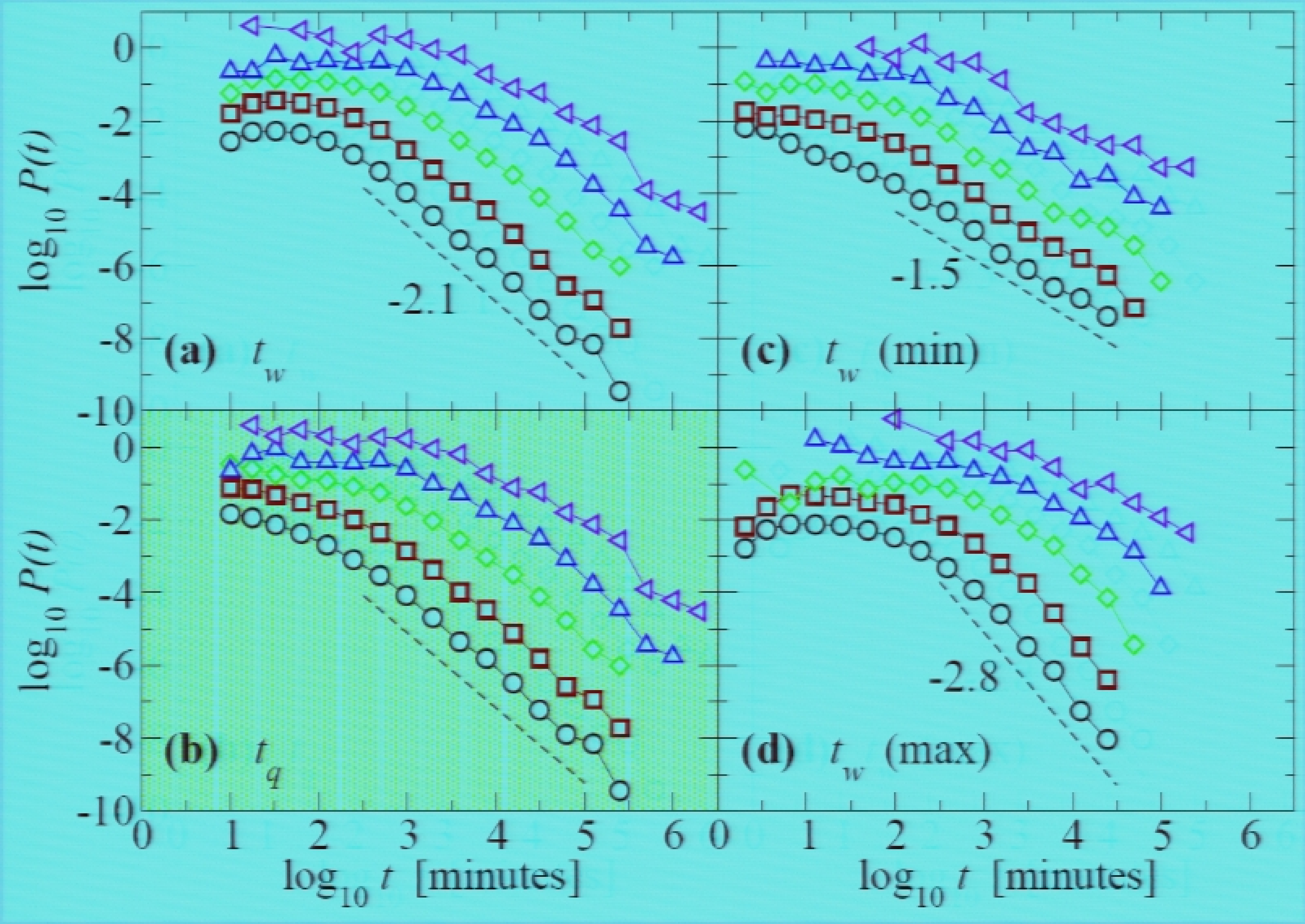

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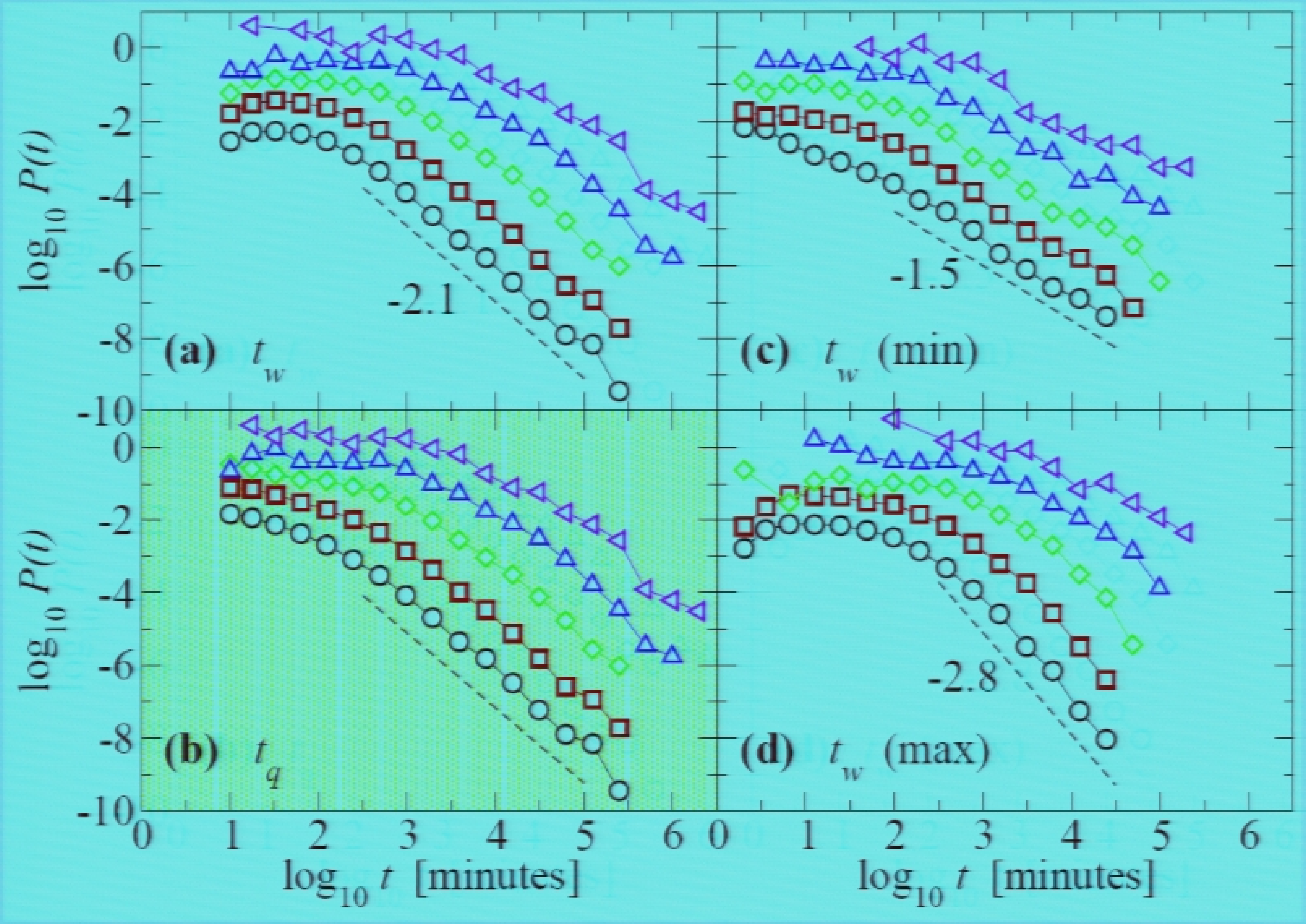

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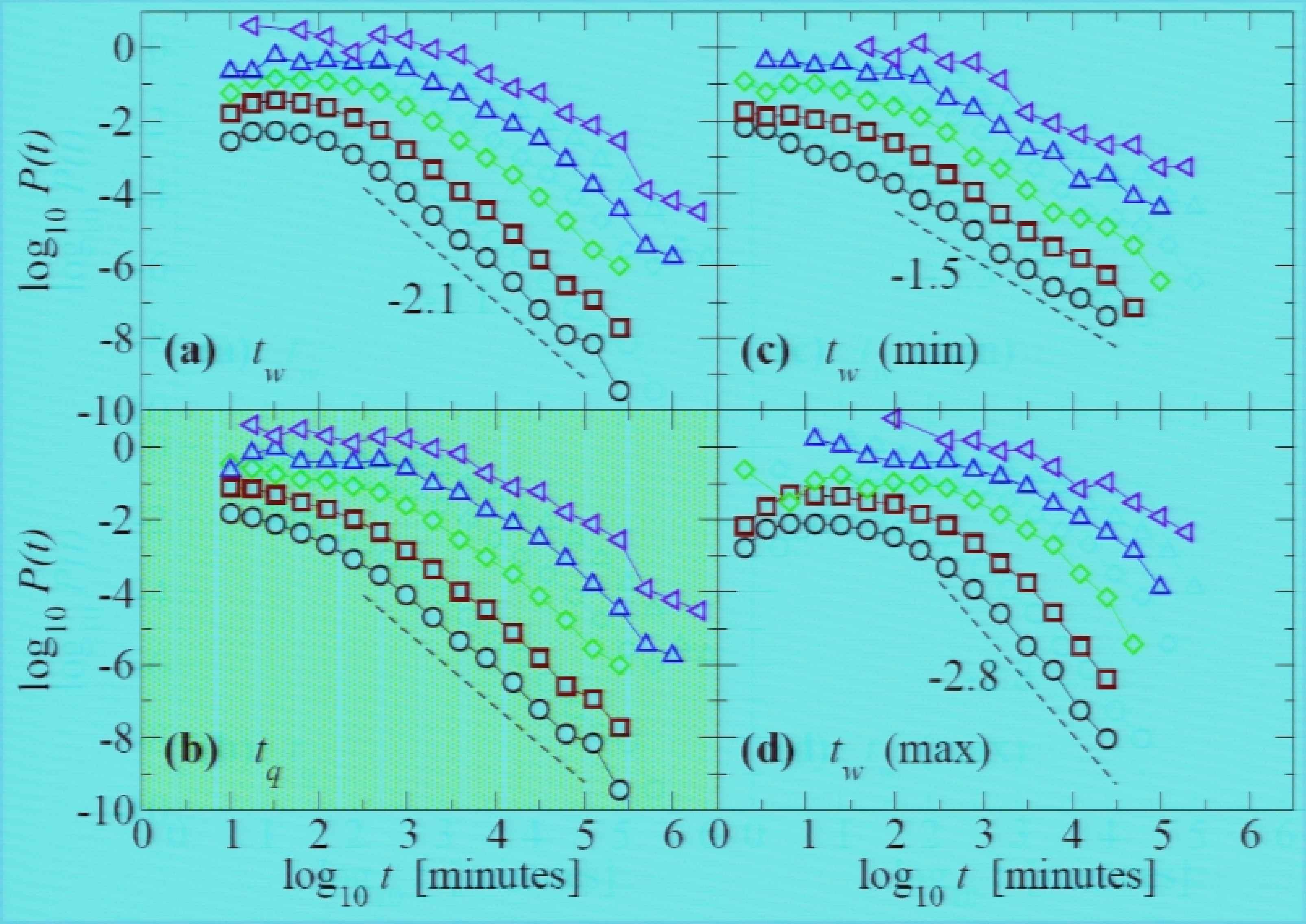

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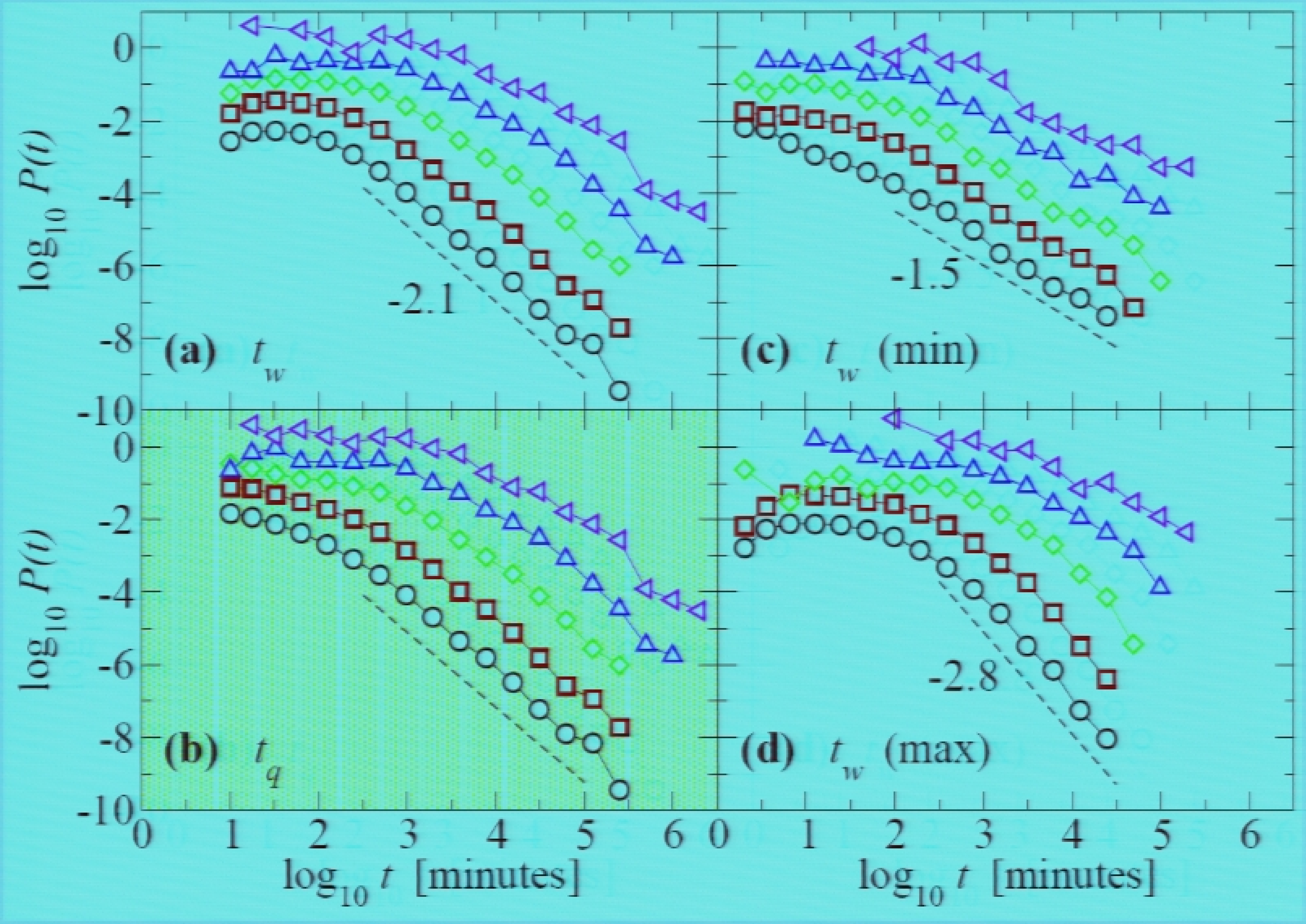

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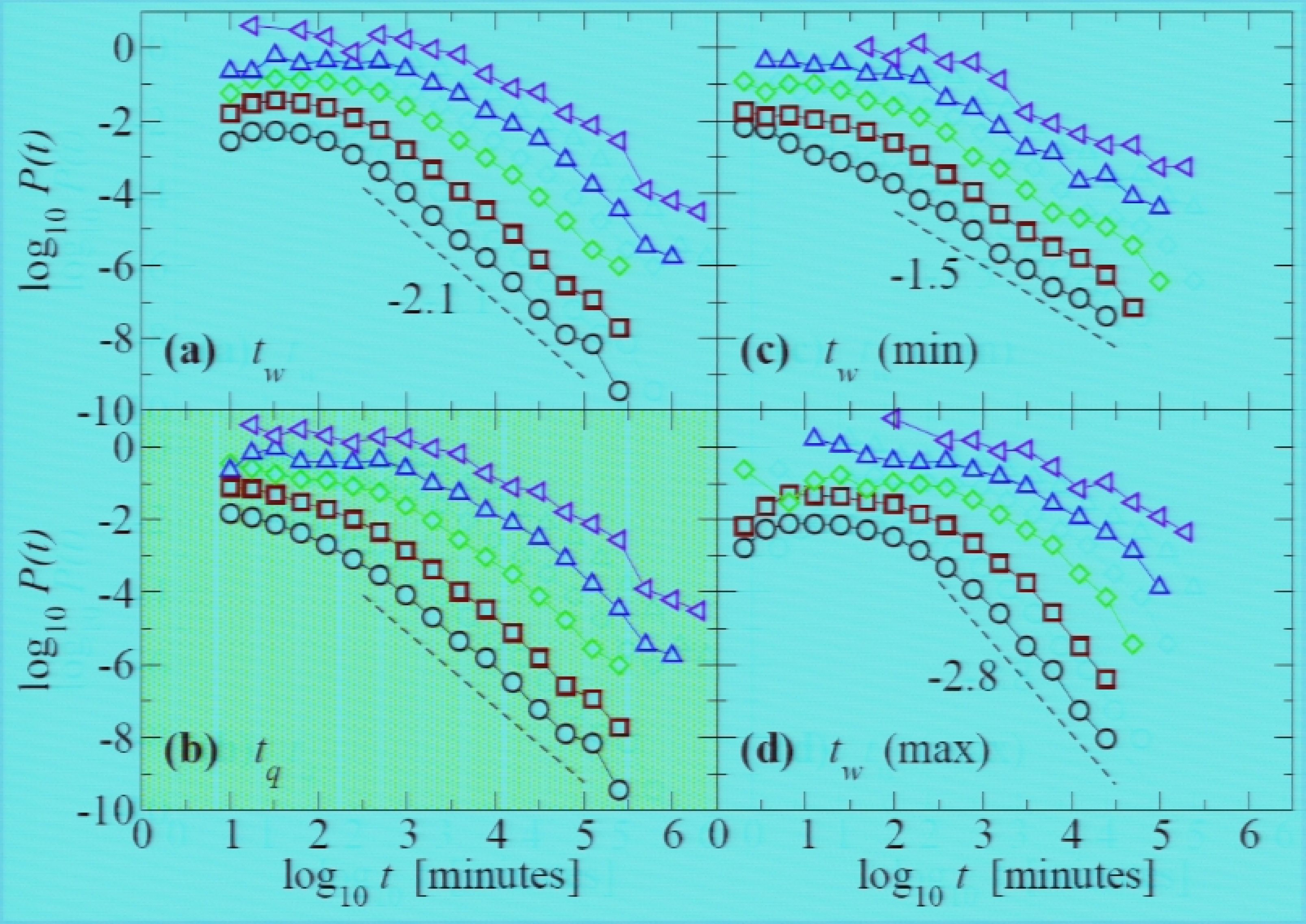

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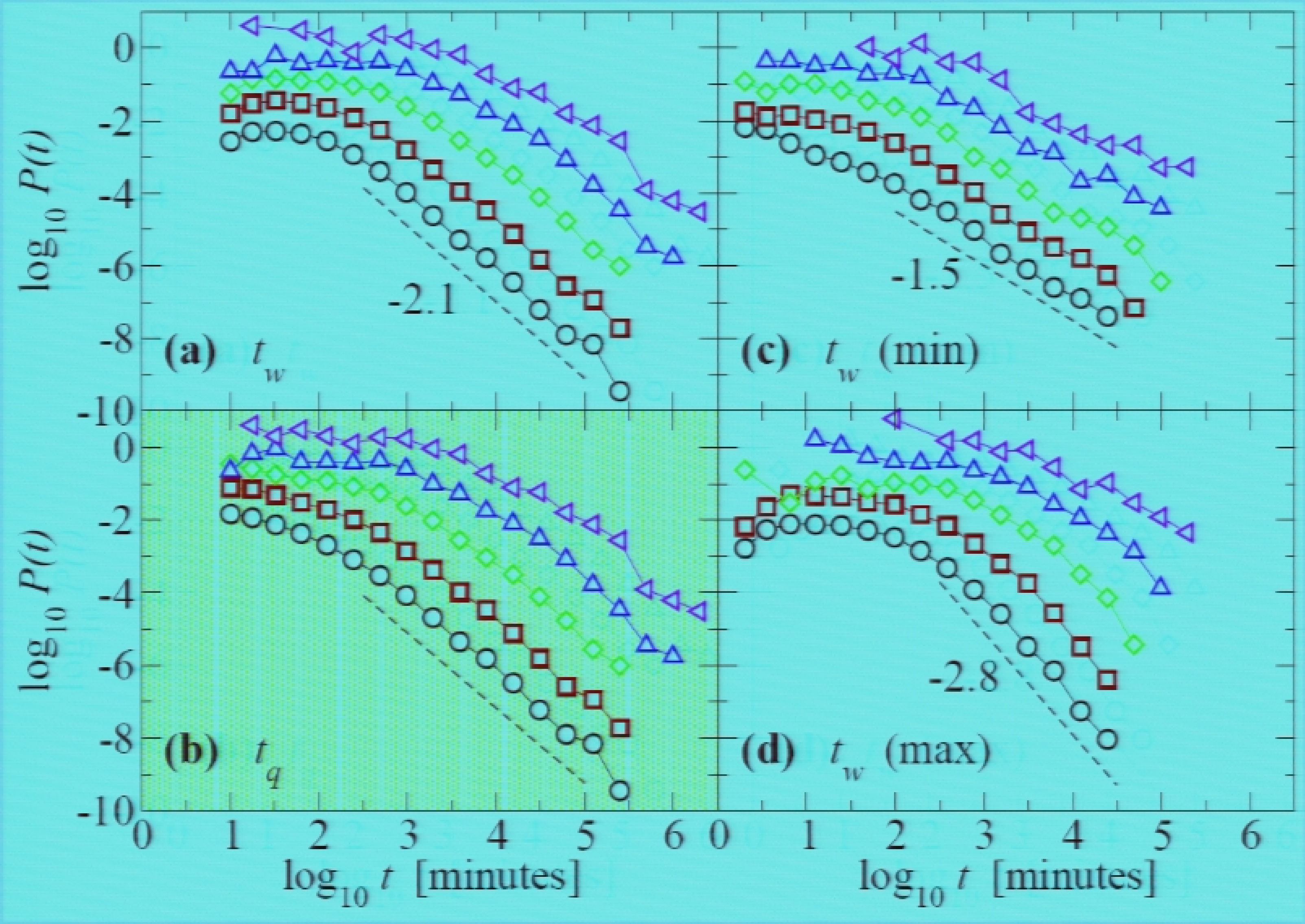

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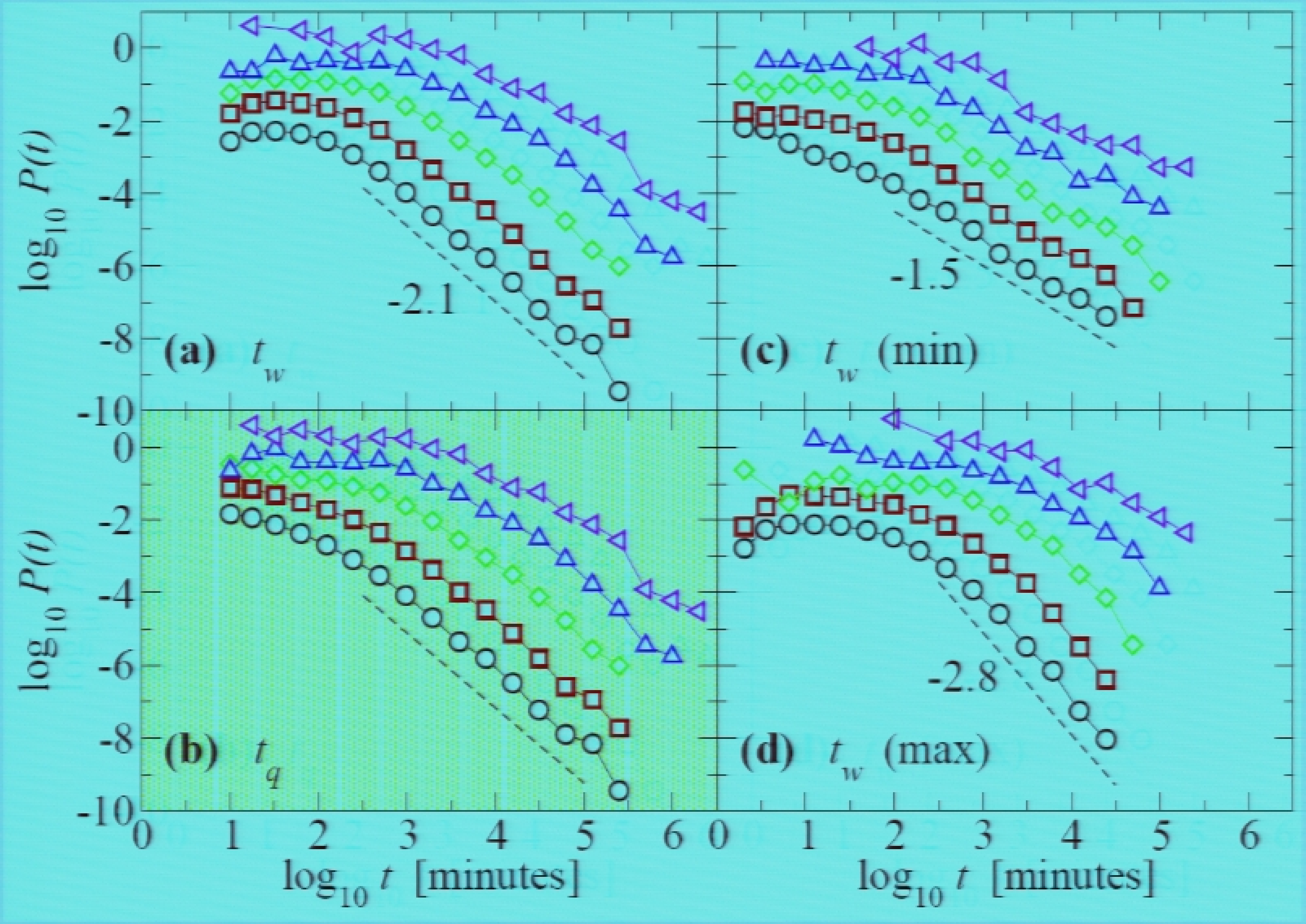

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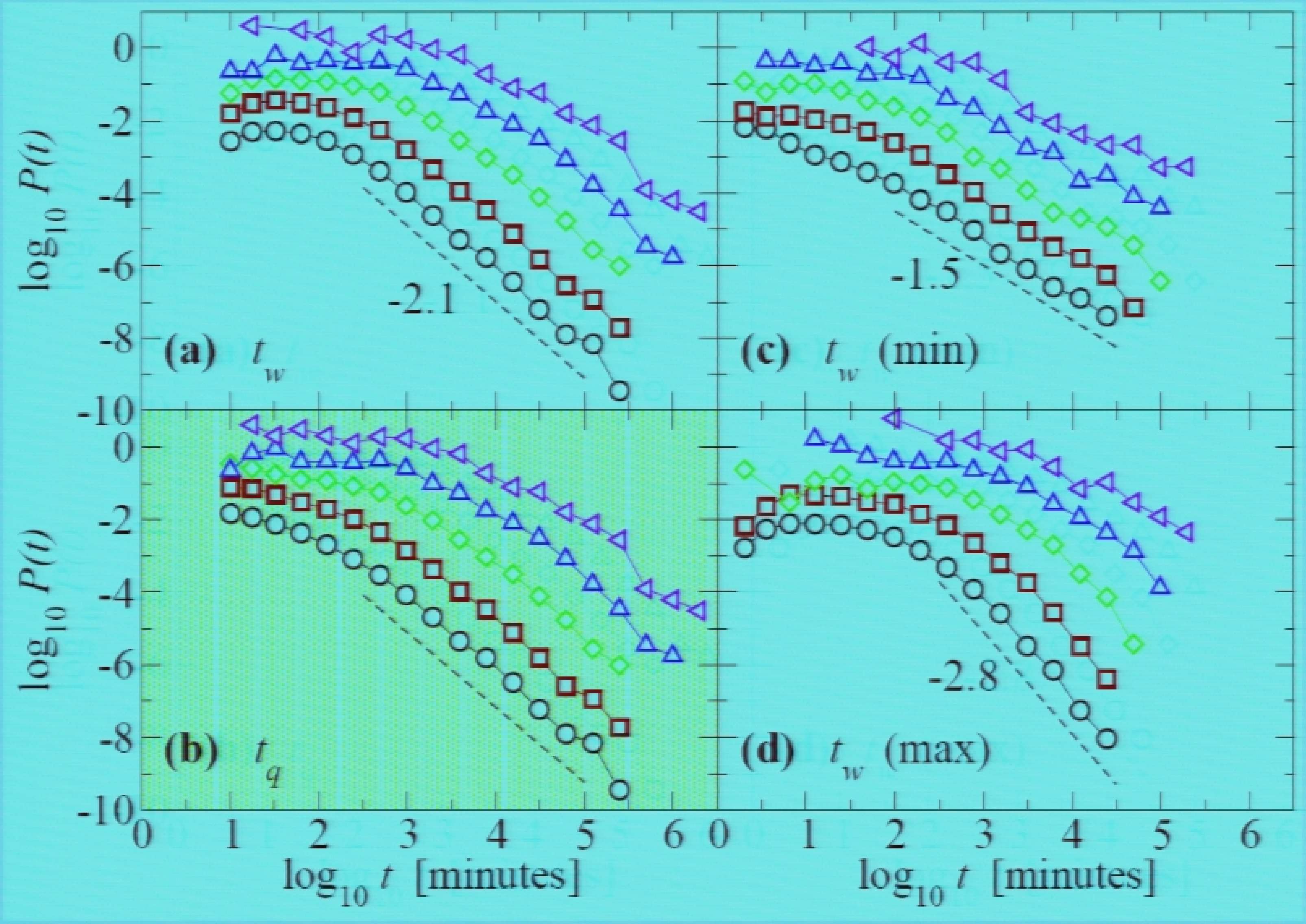

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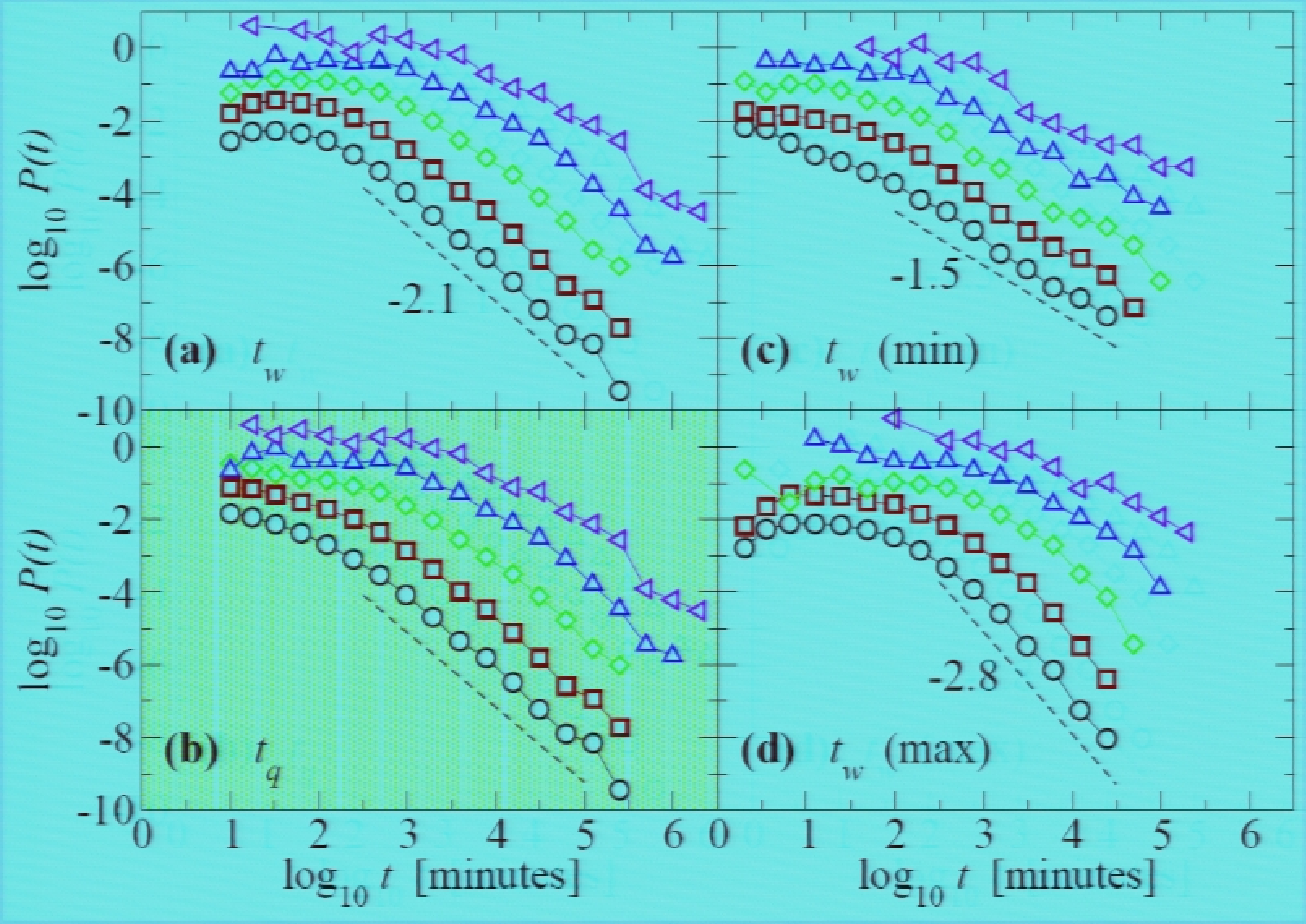

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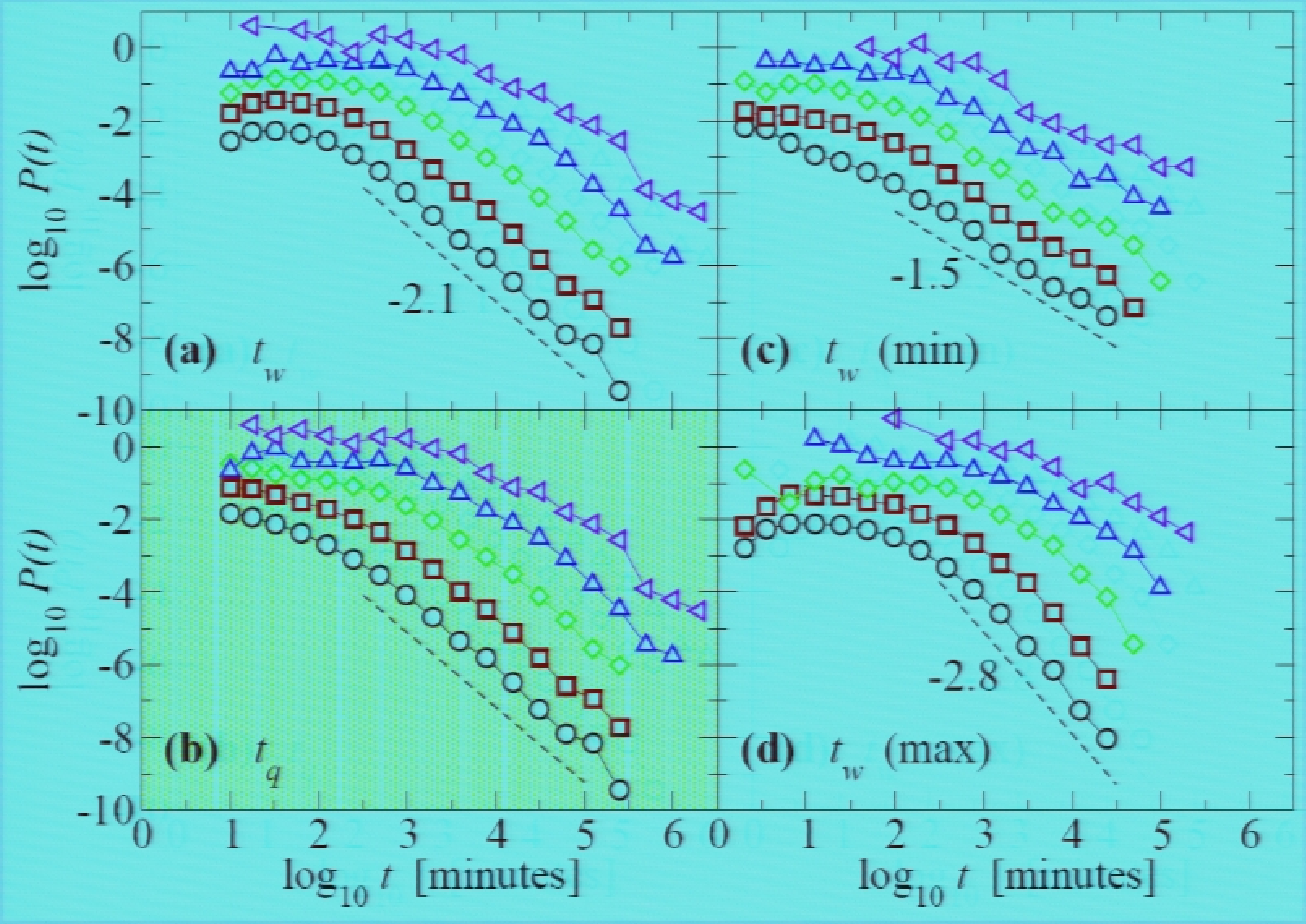

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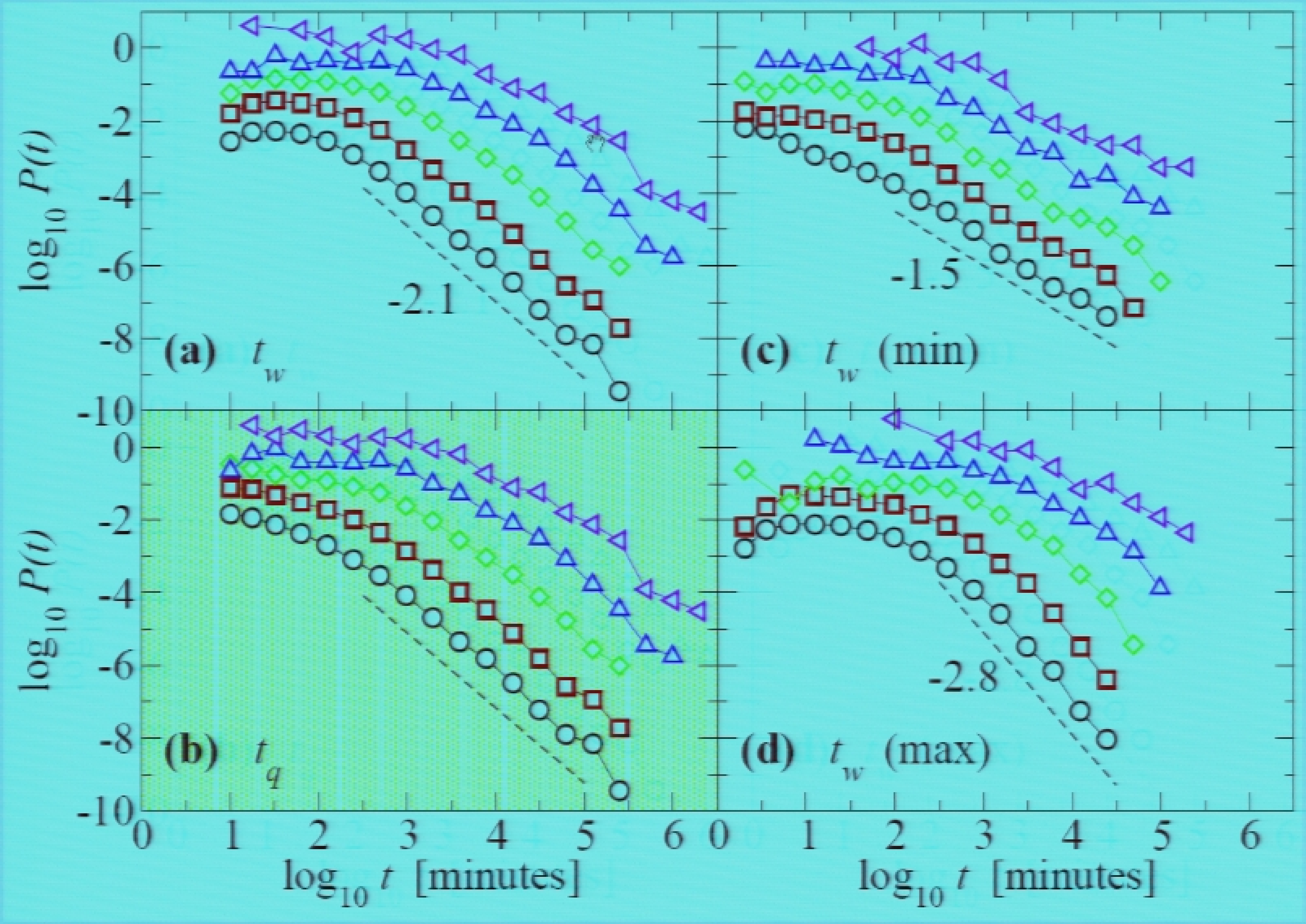

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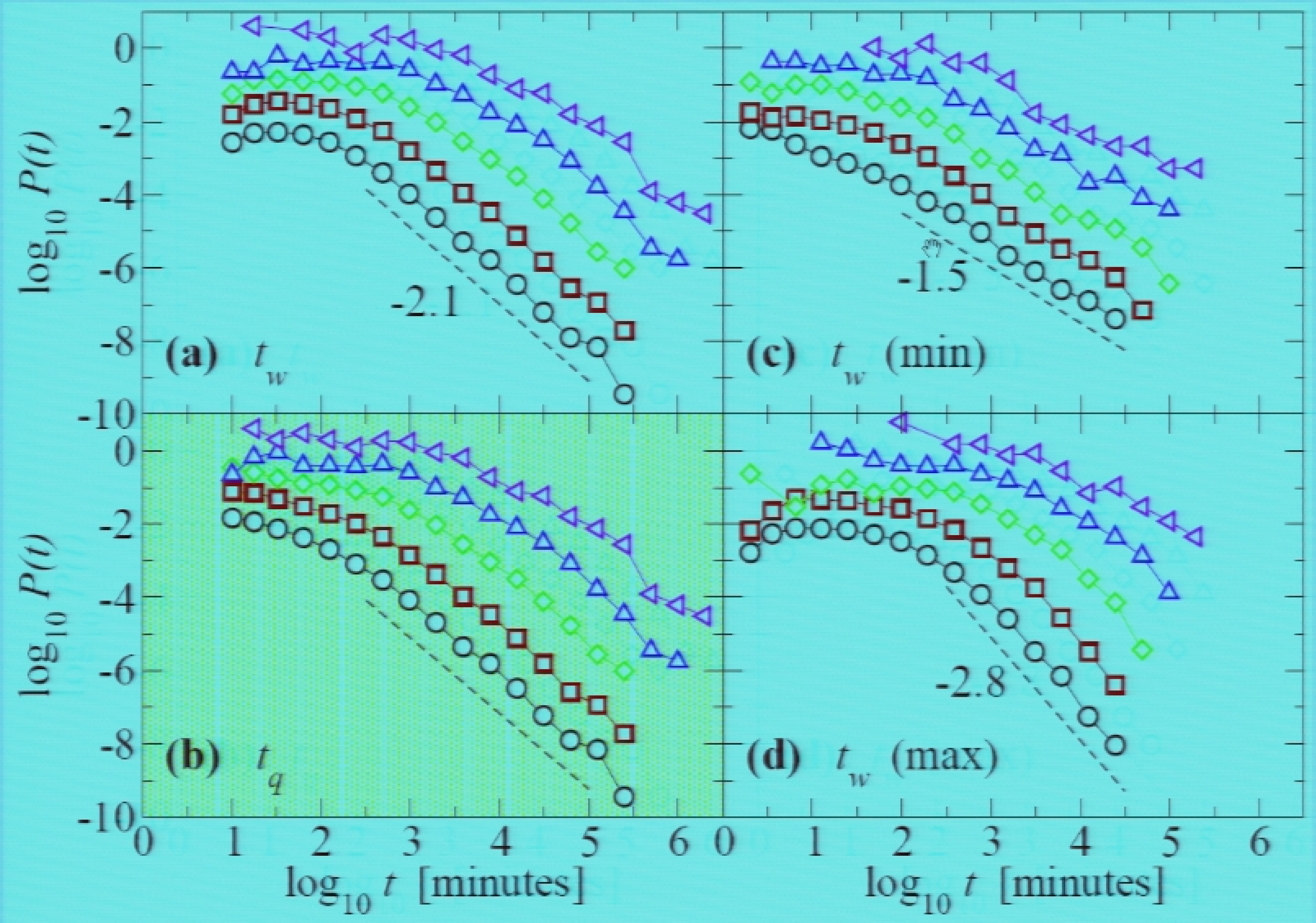

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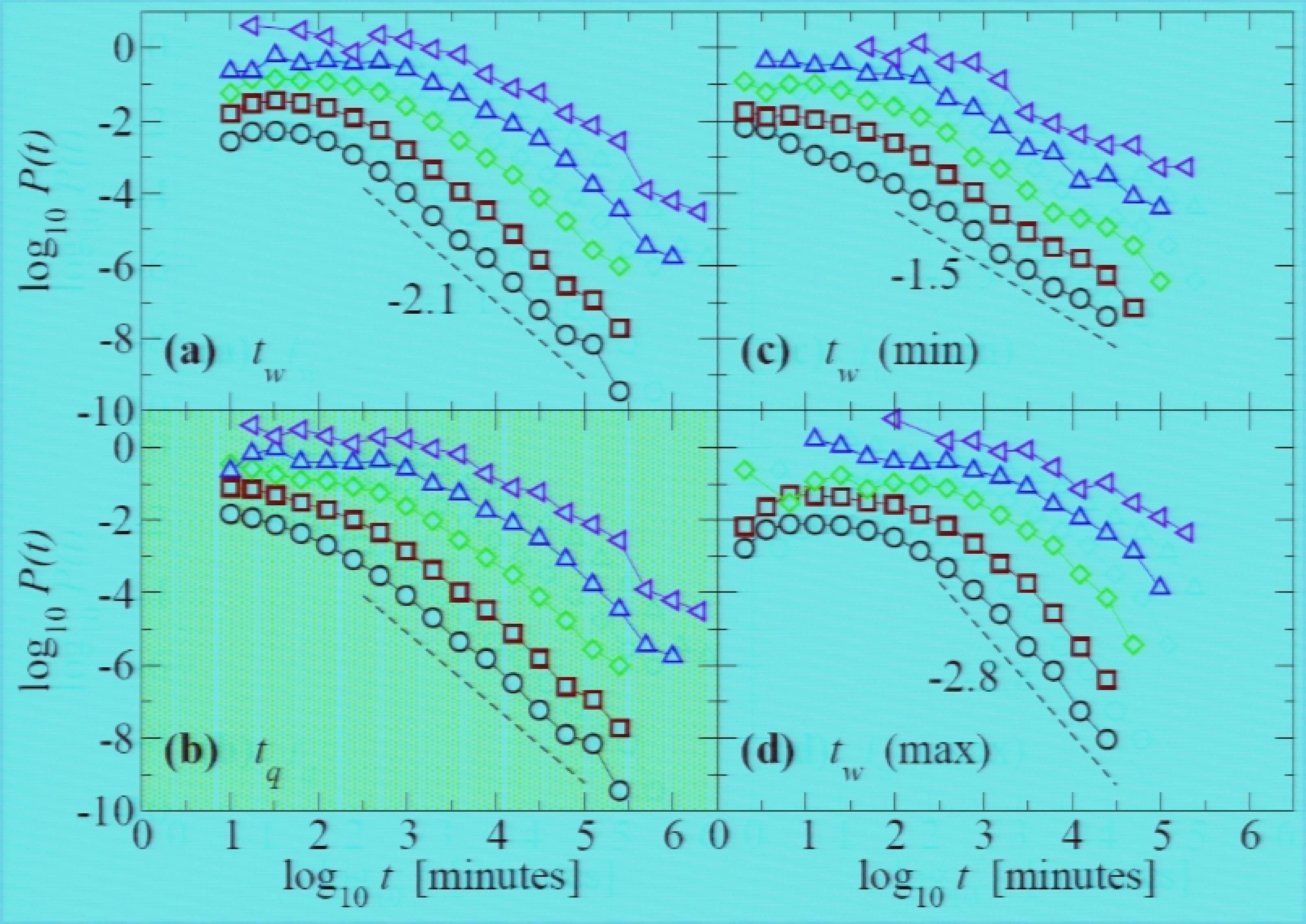

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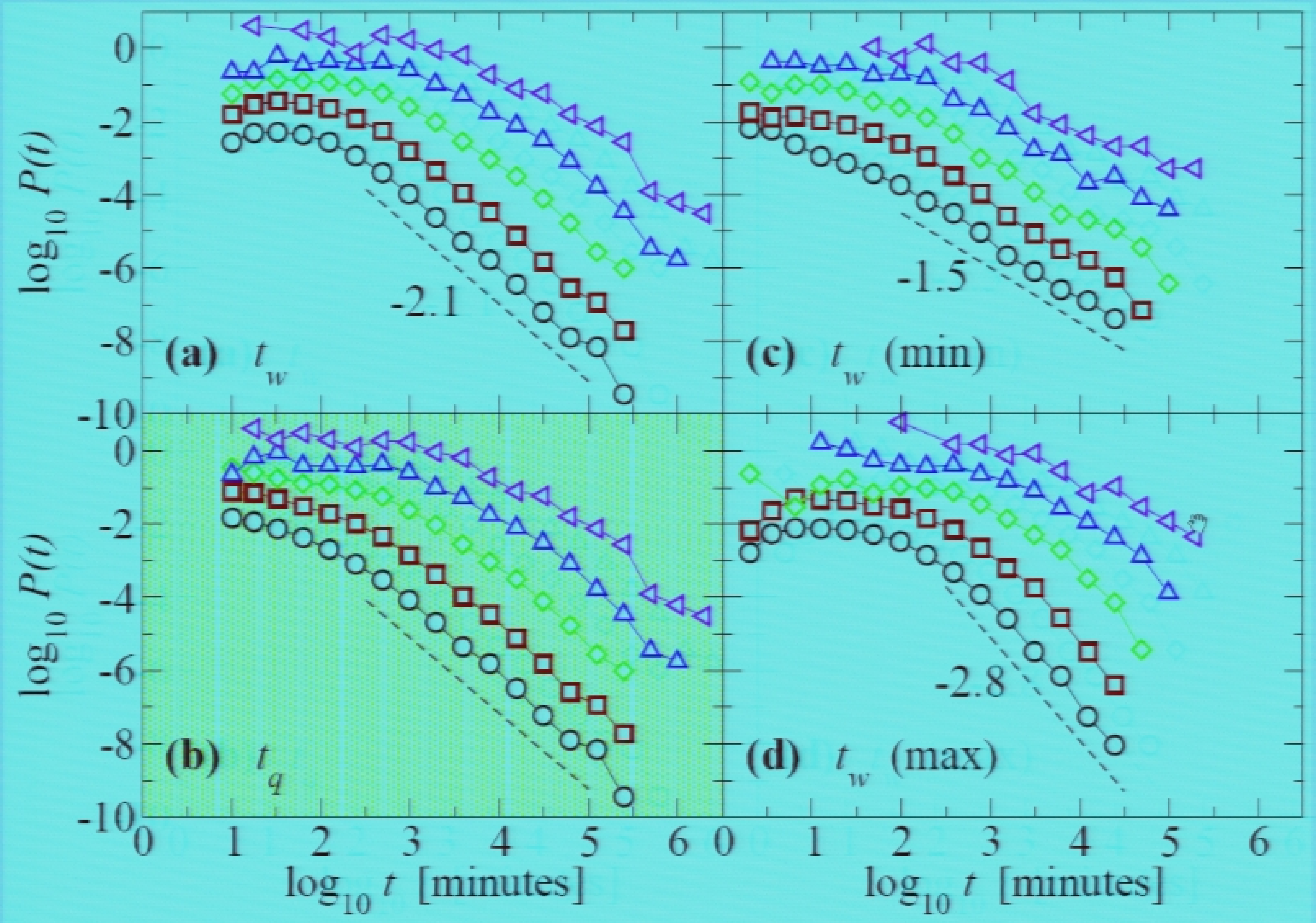
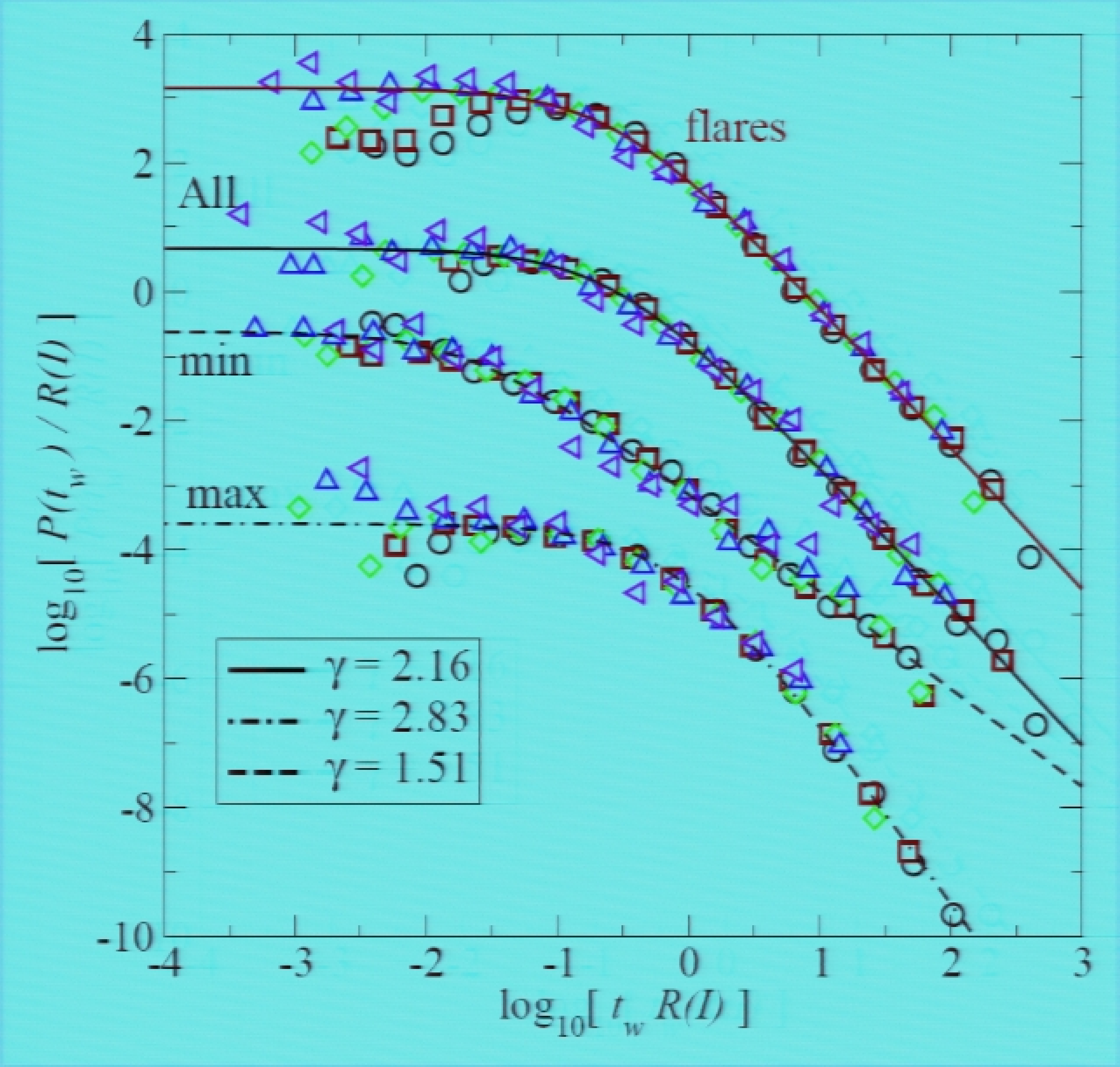


Complex correlations in self-organized critical phenomena

Distributions of waiting times rescaled by the flaring rates $R(I)$

$$\text{fit} \sim \frac{1}{(1+x/x^*)^\gamma}$$



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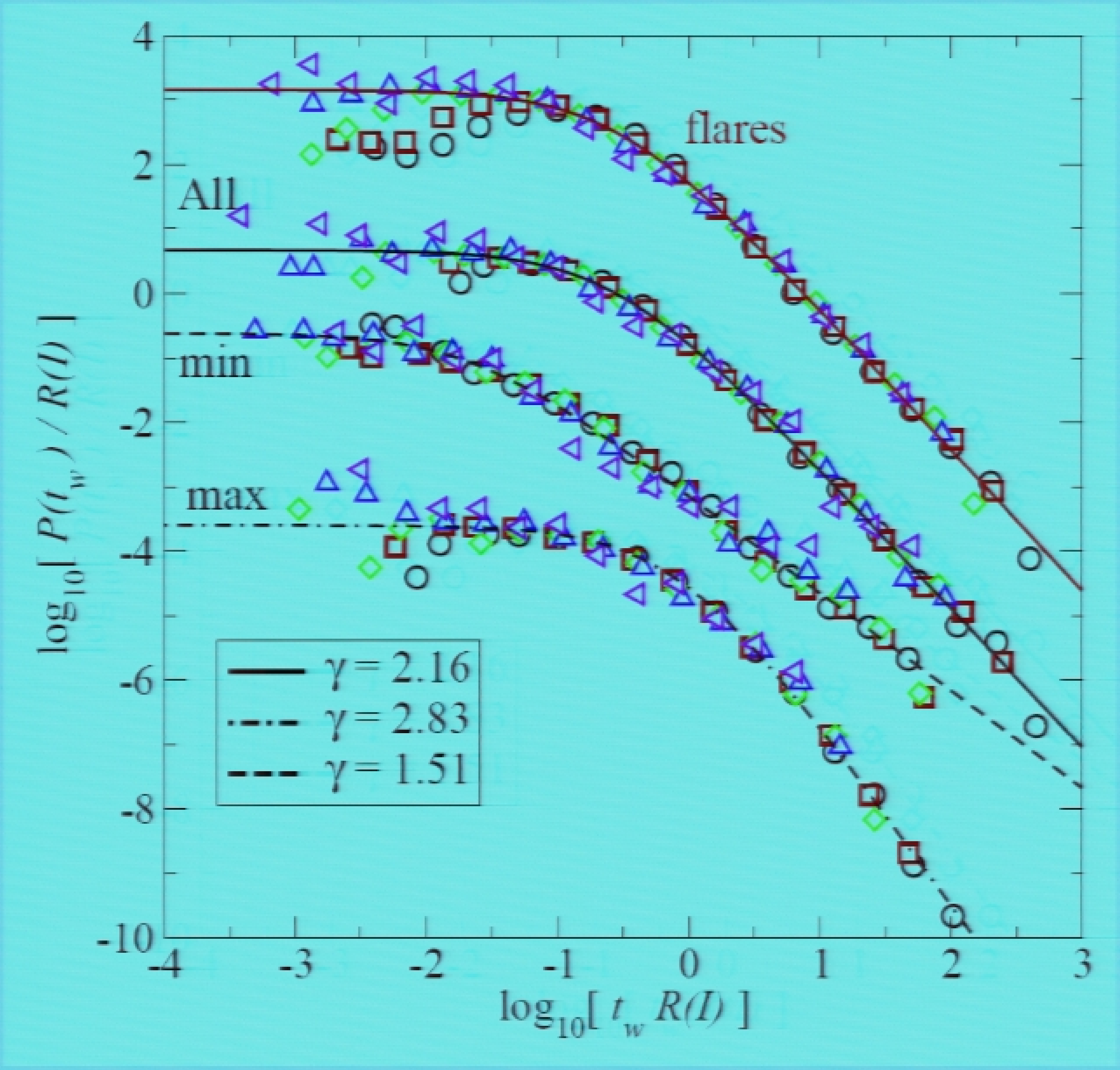

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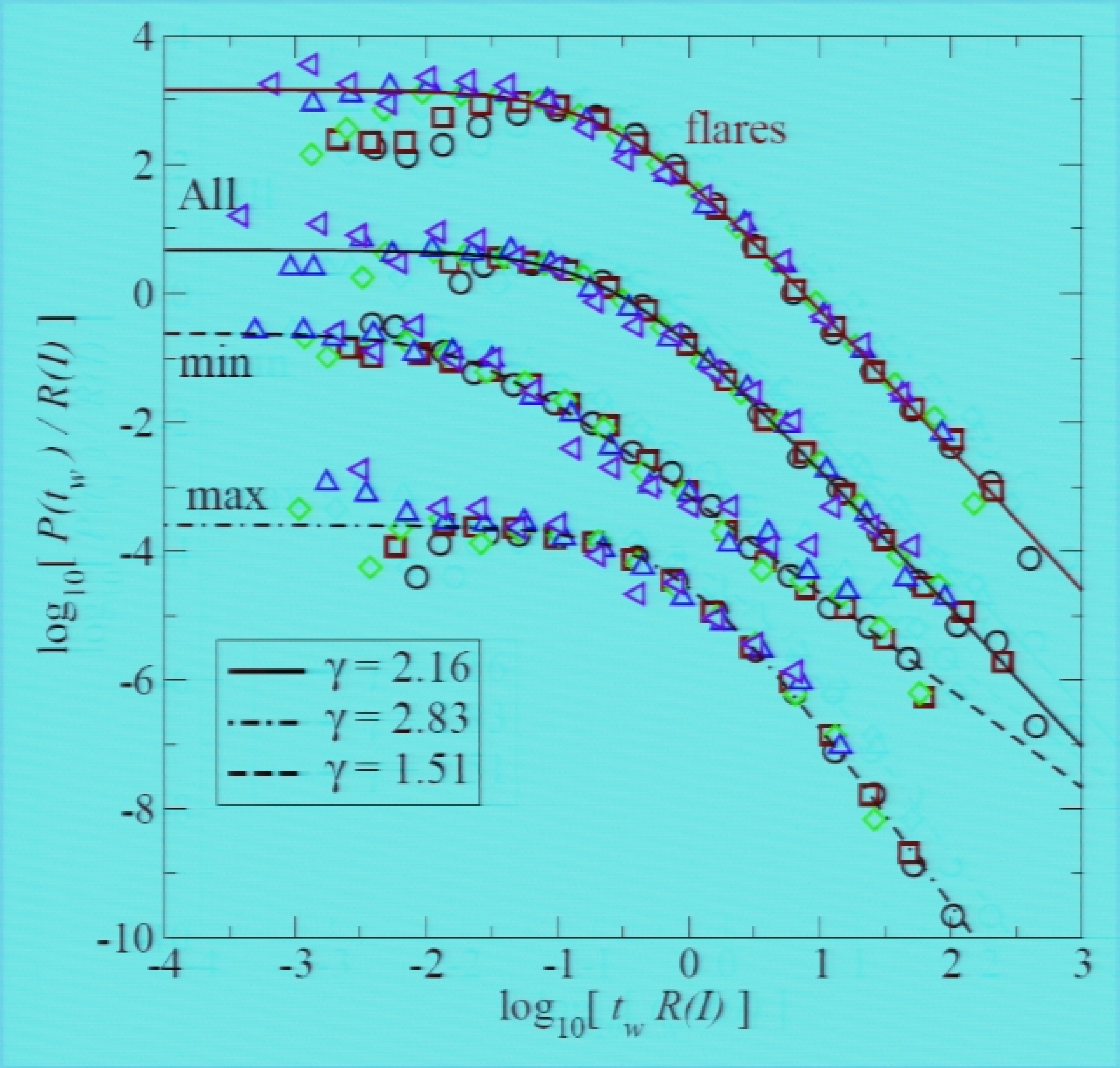

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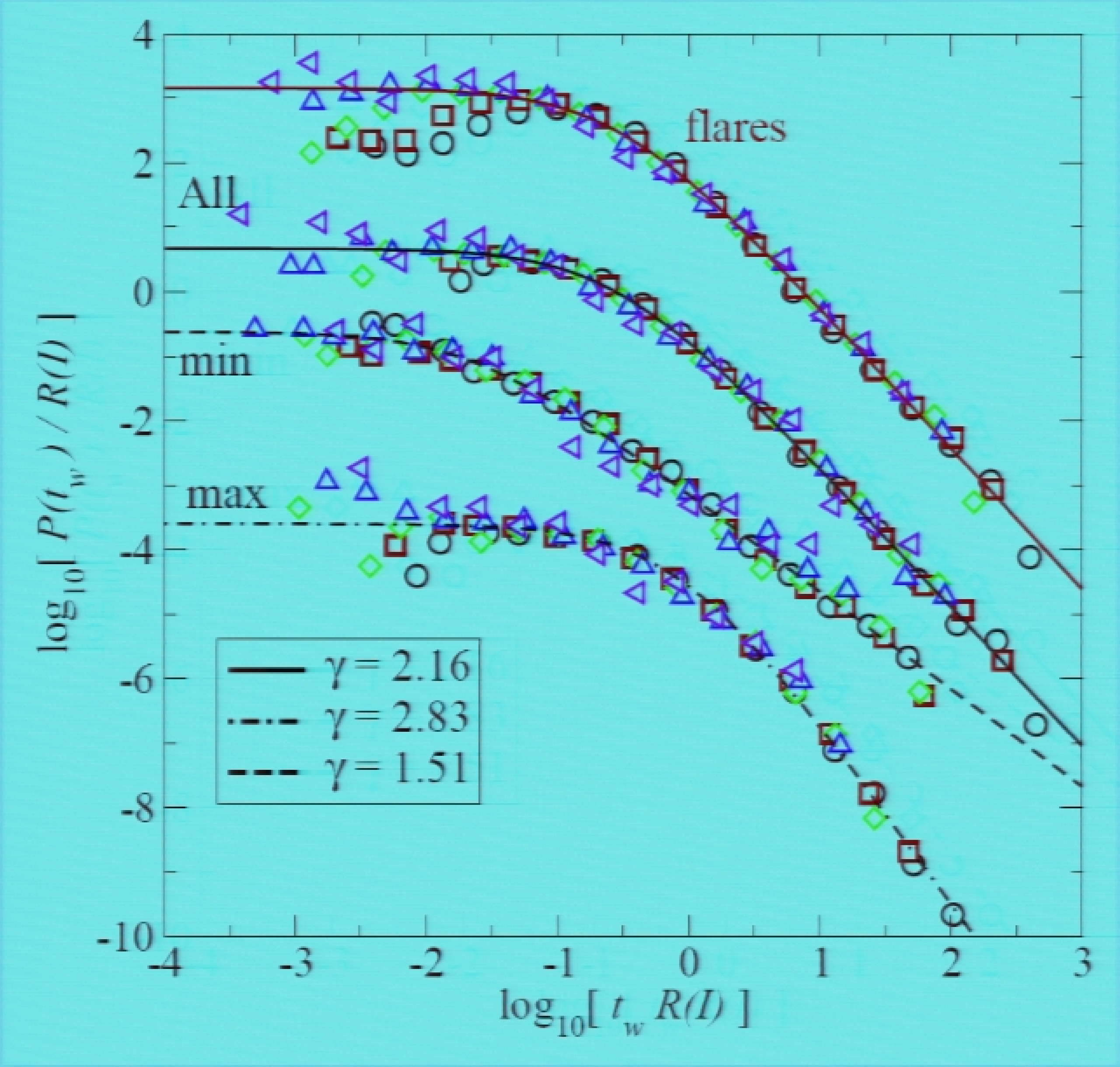

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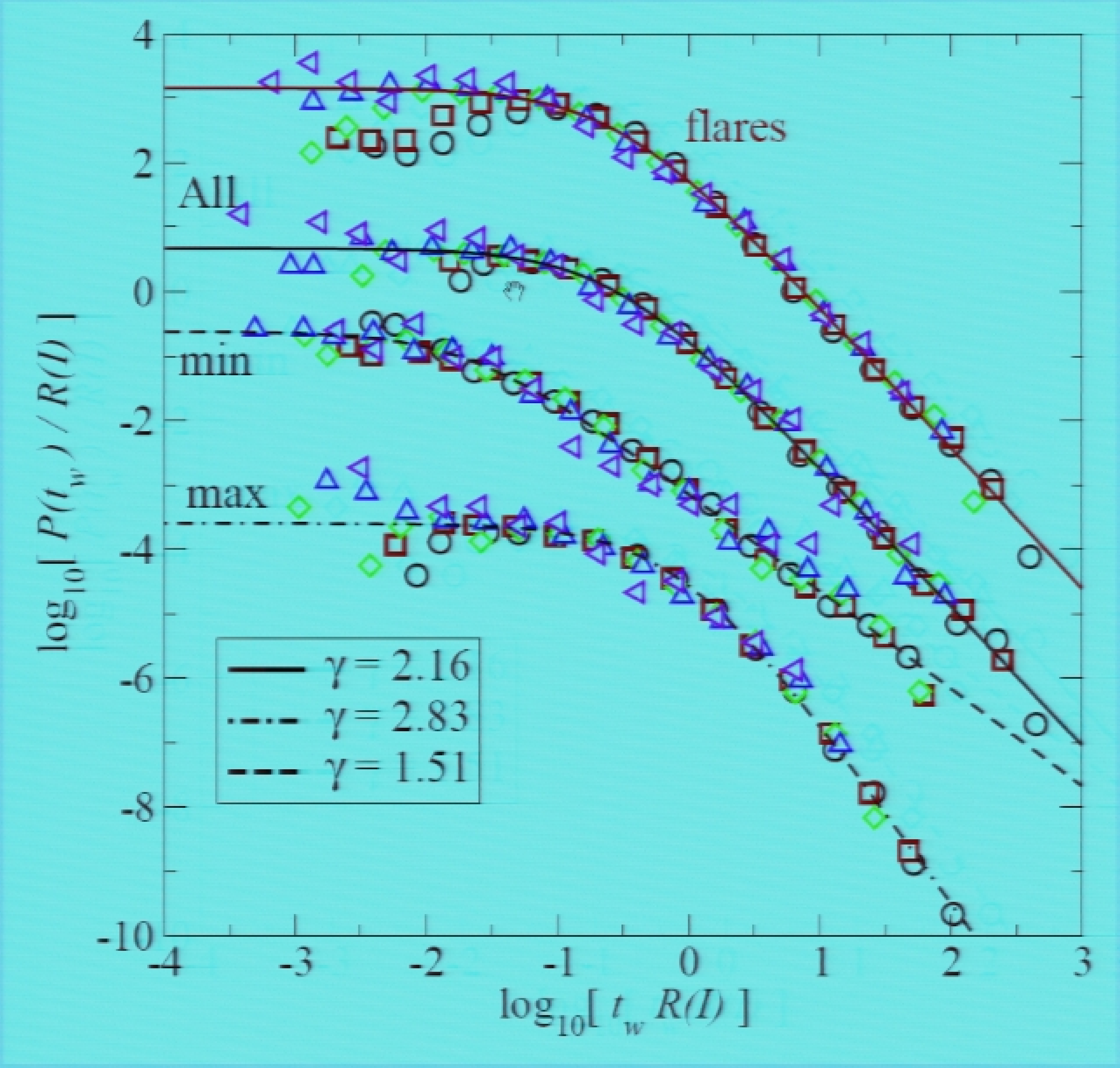

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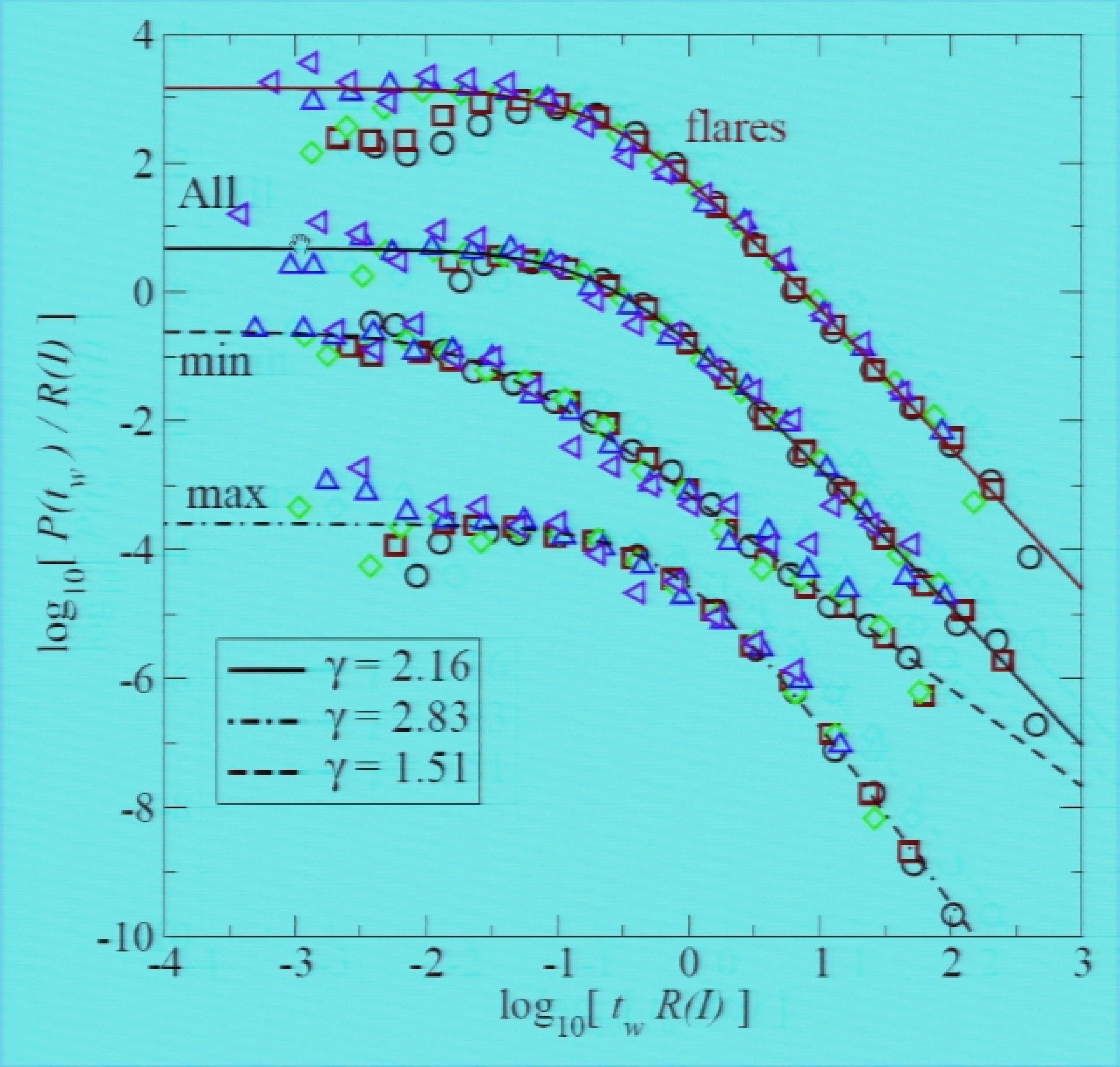

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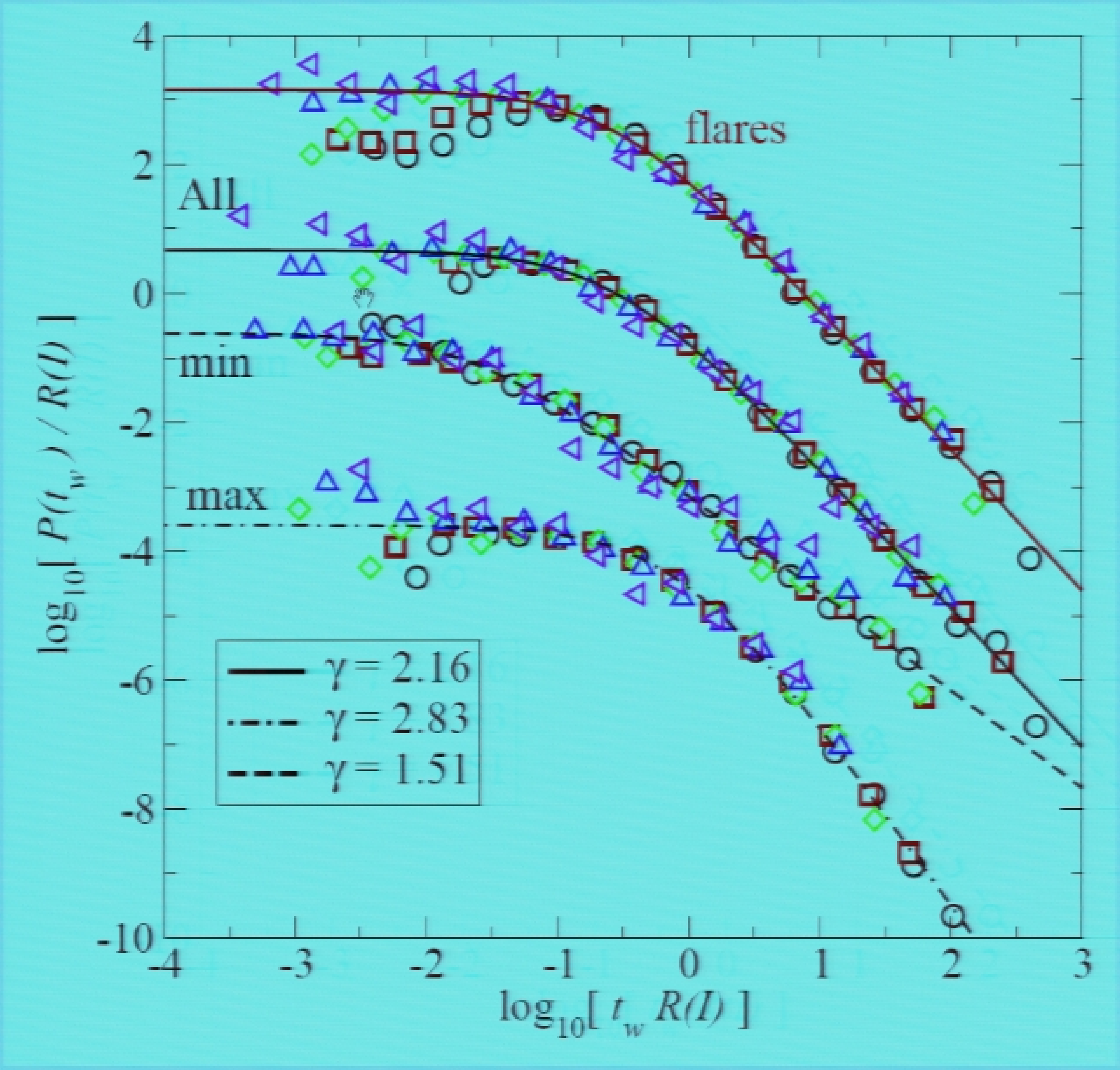

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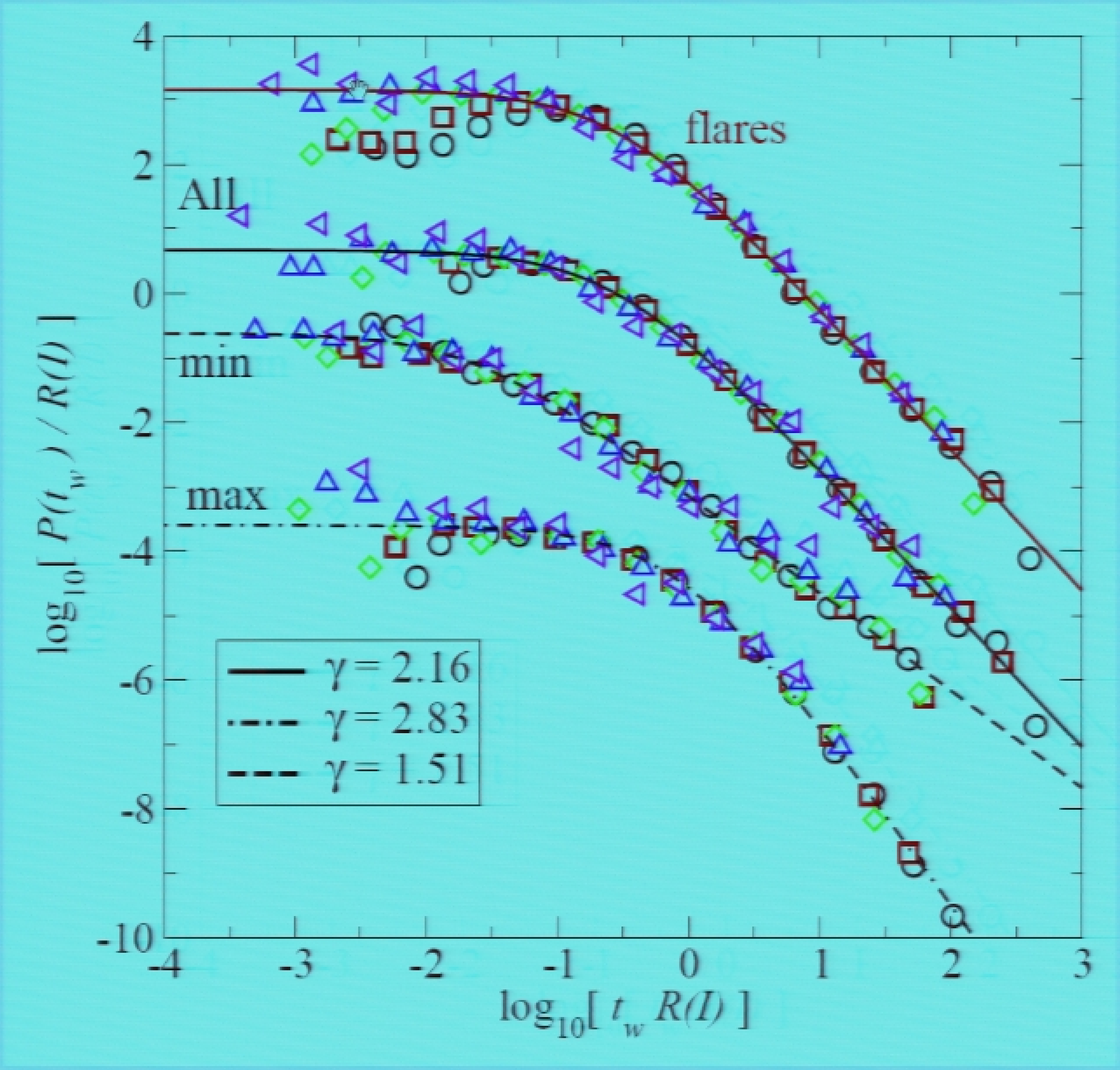

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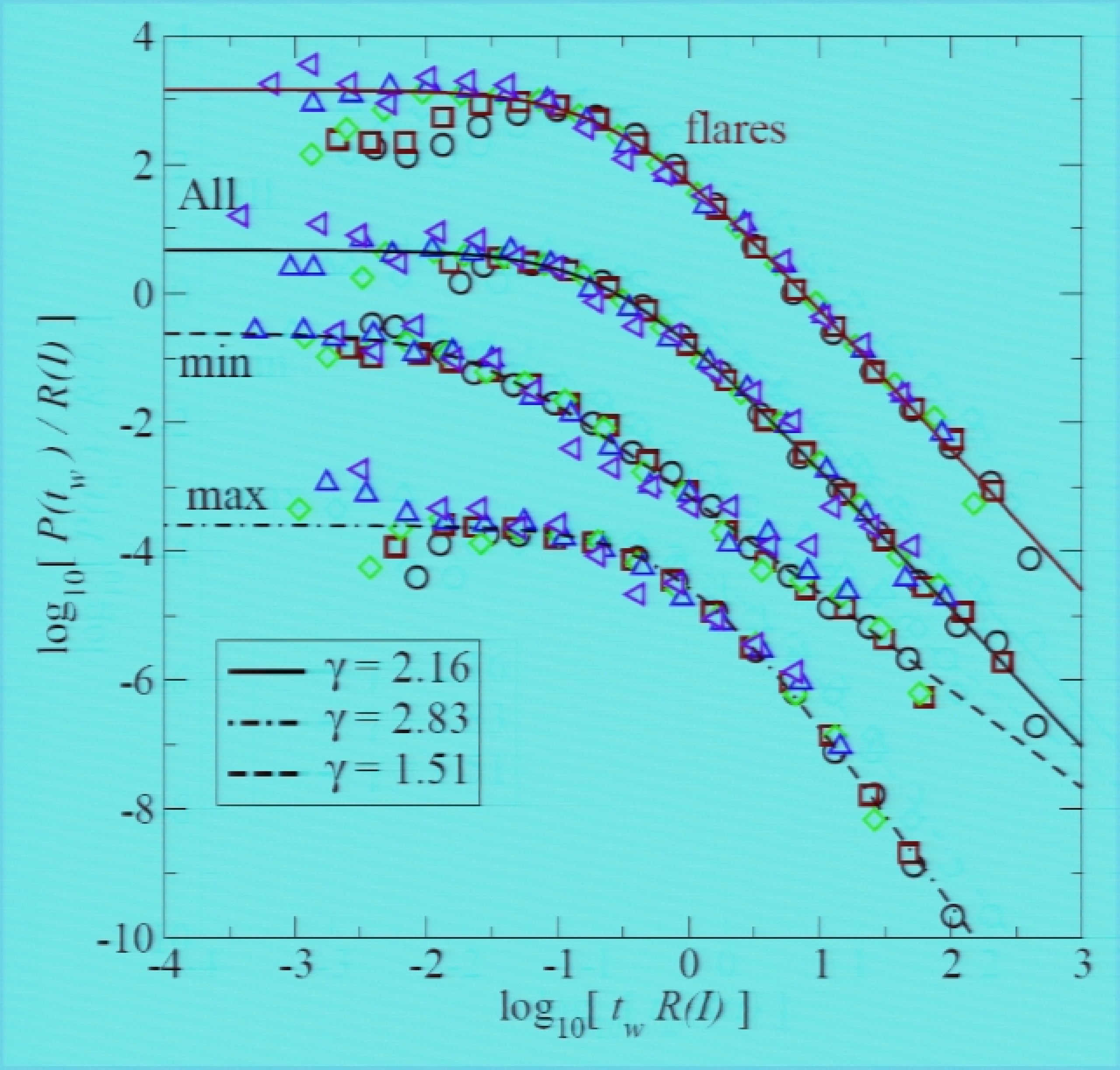

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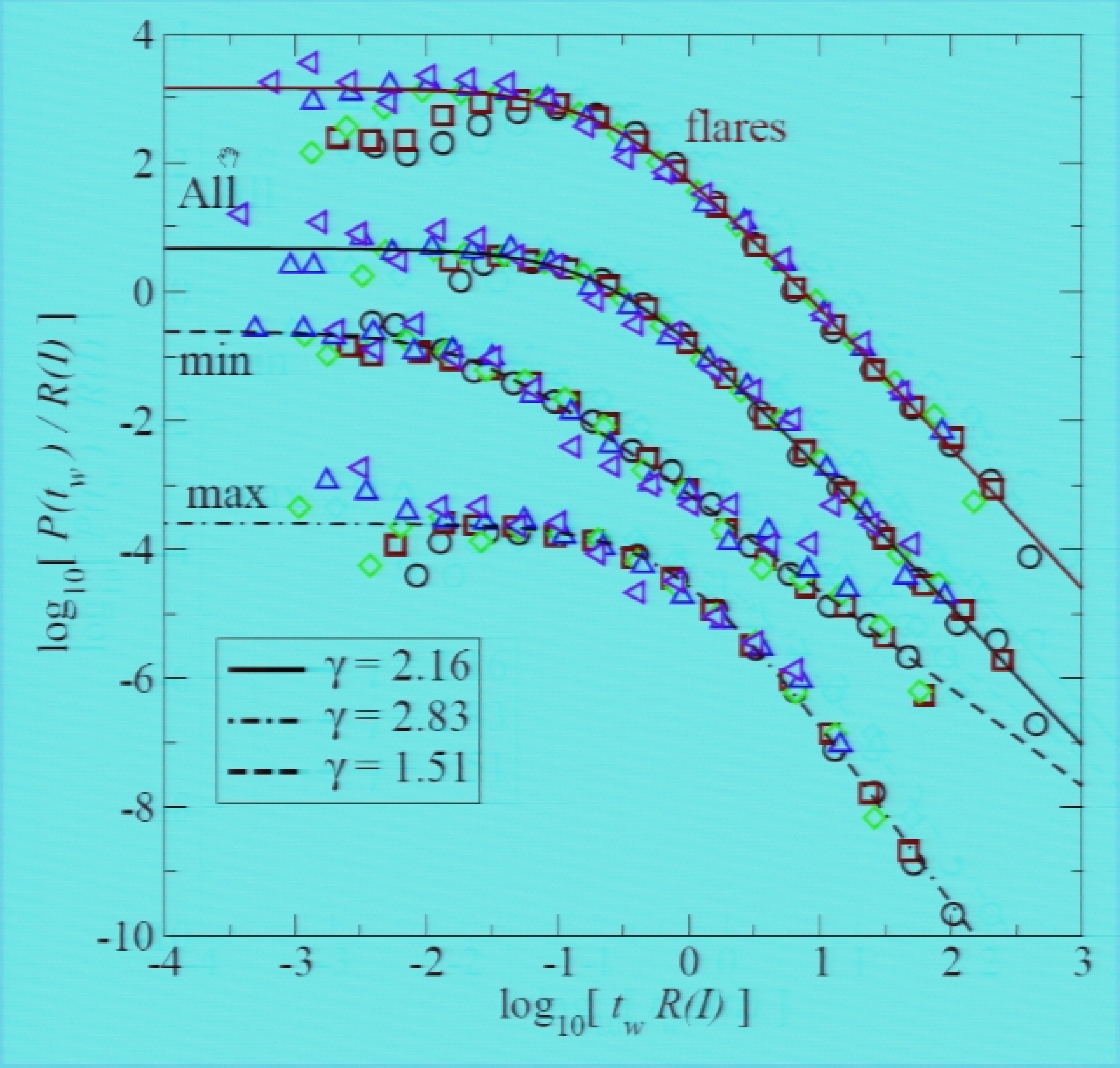

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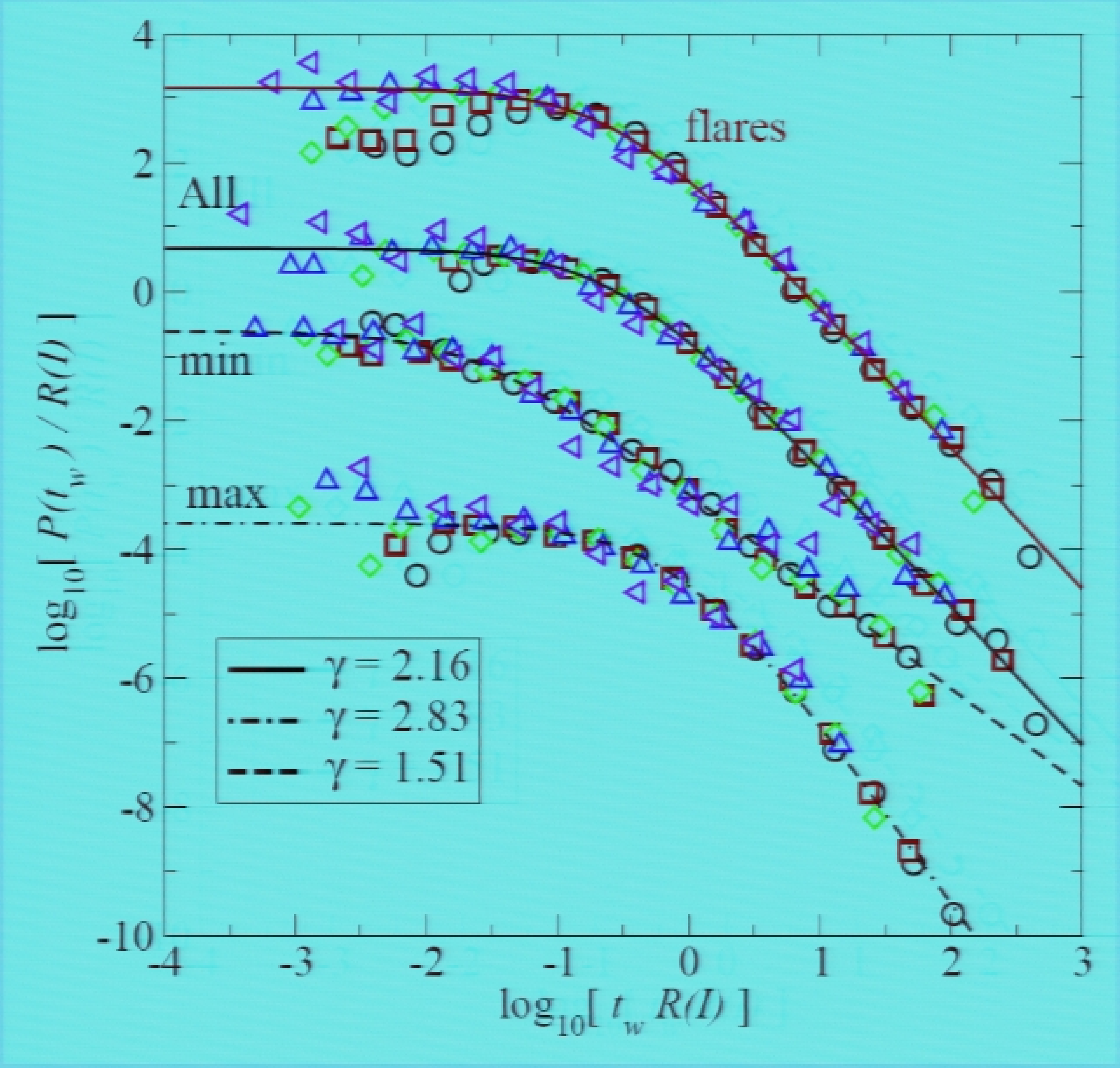

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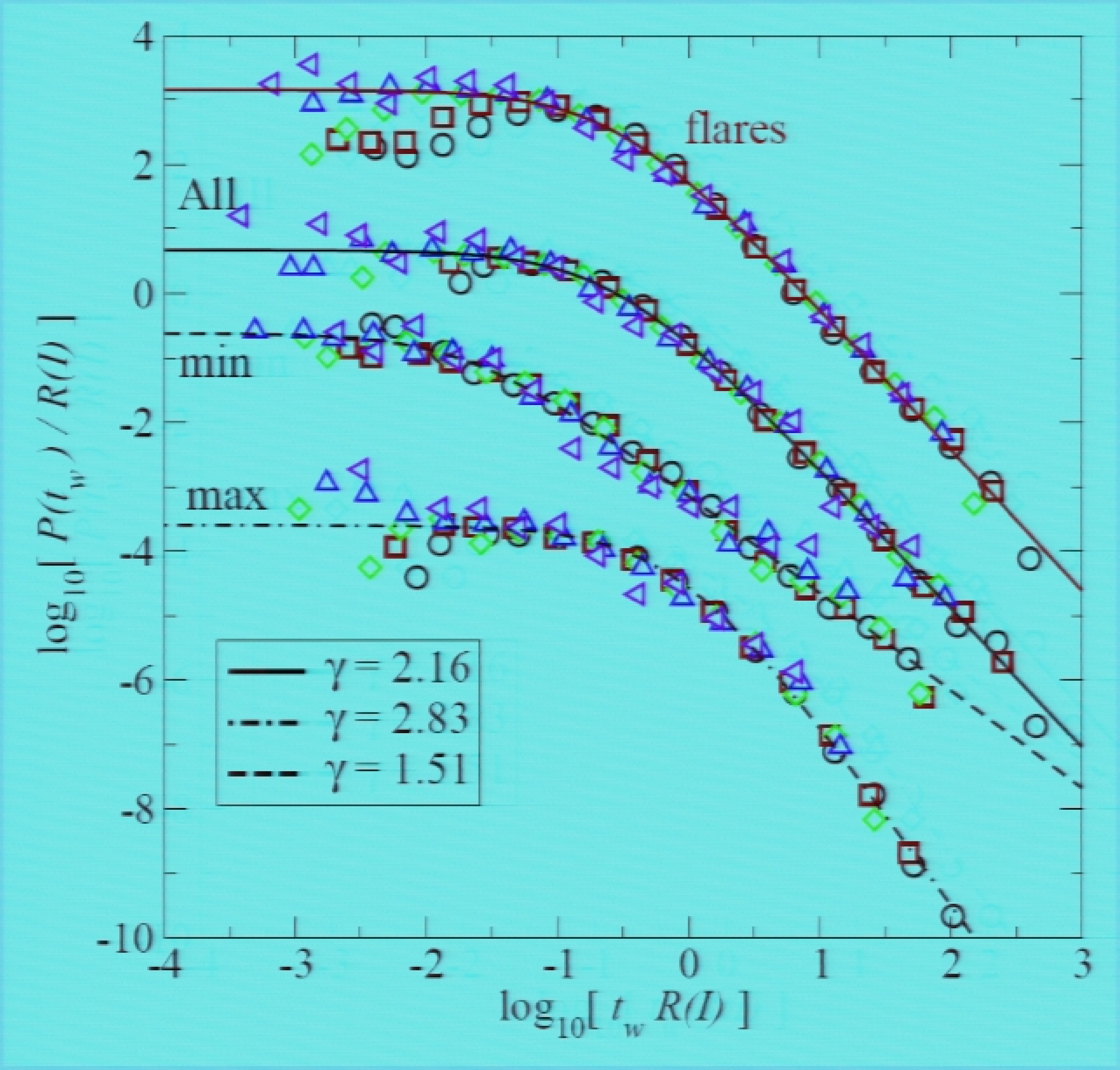

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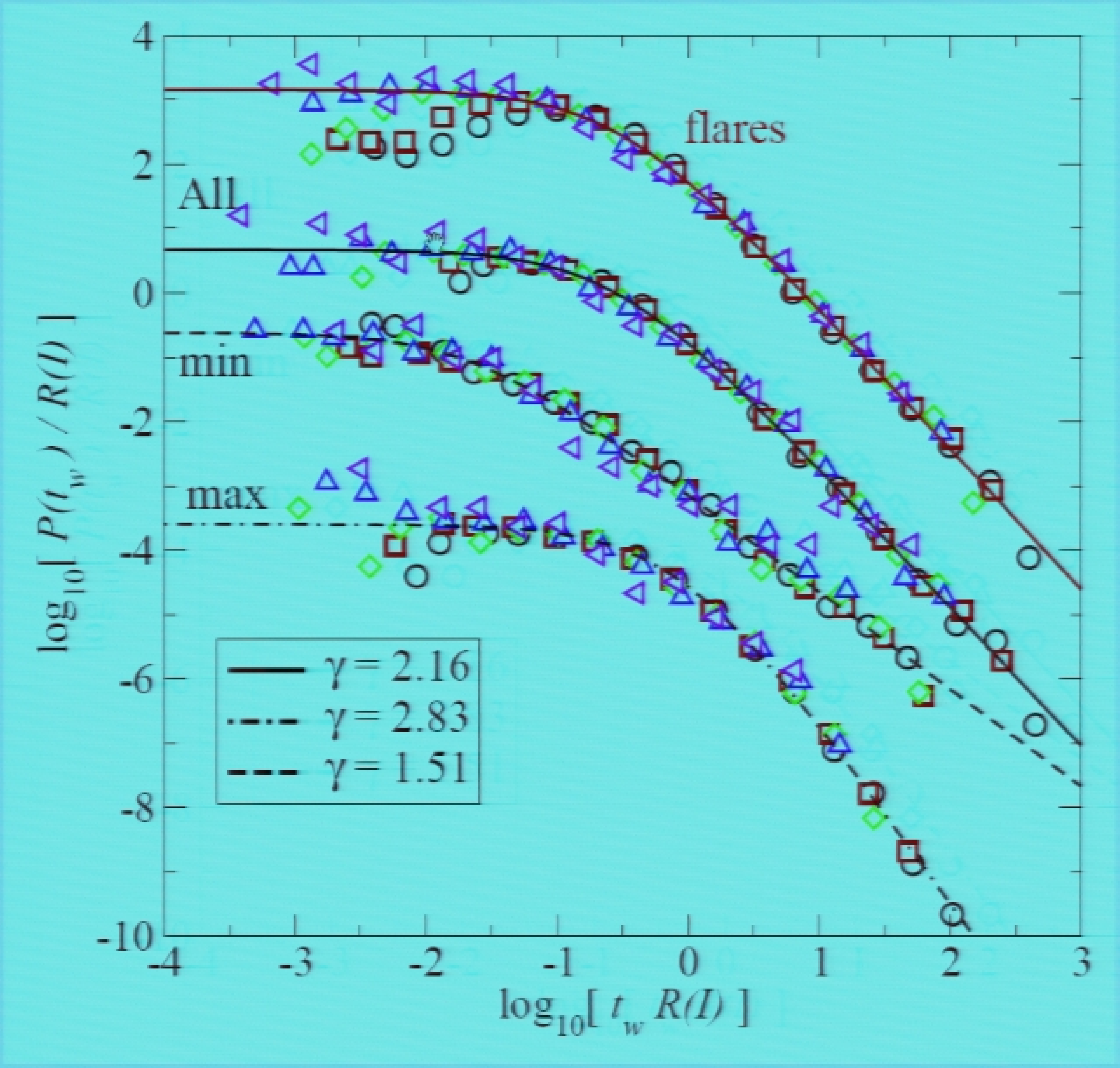

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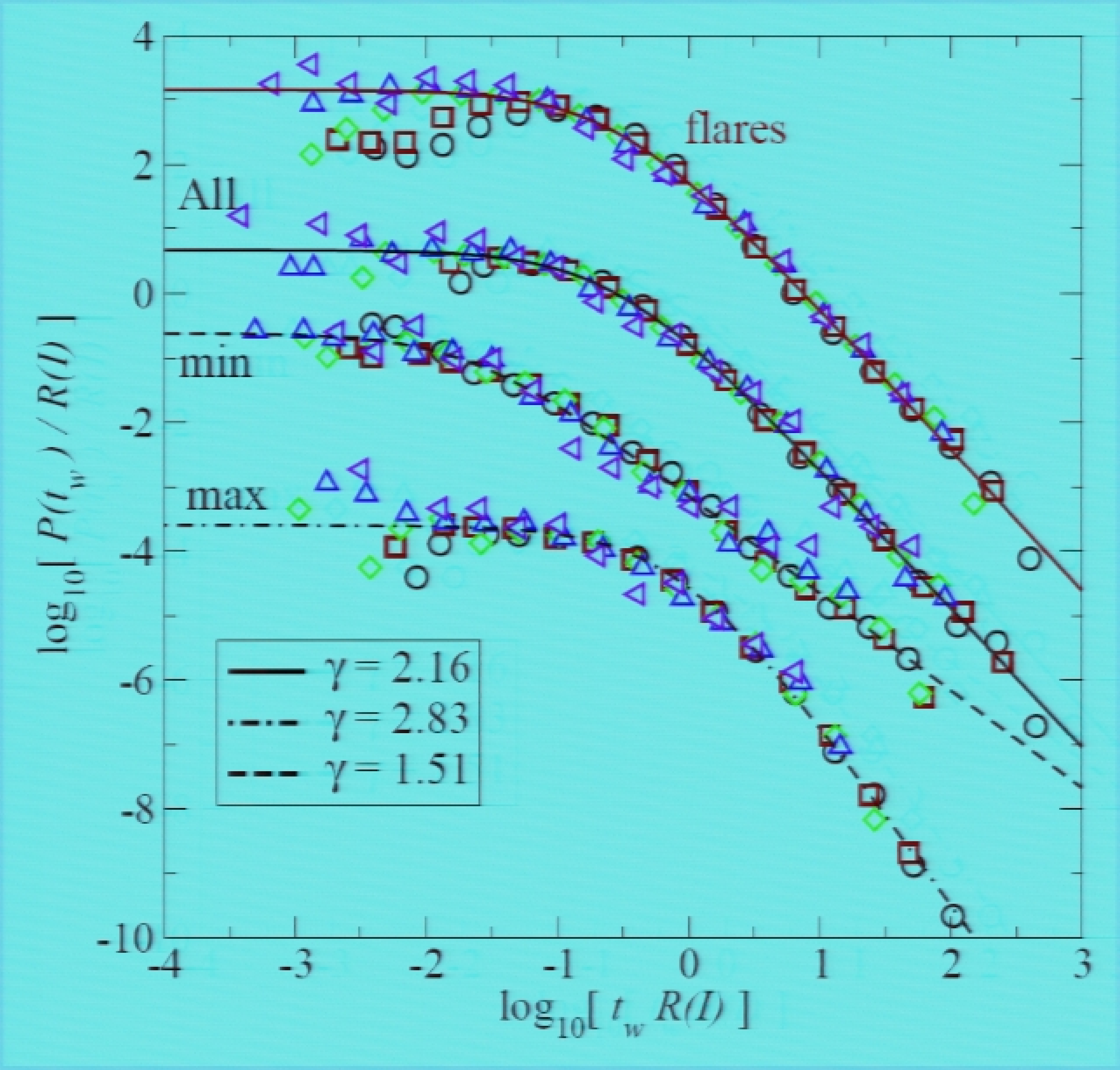
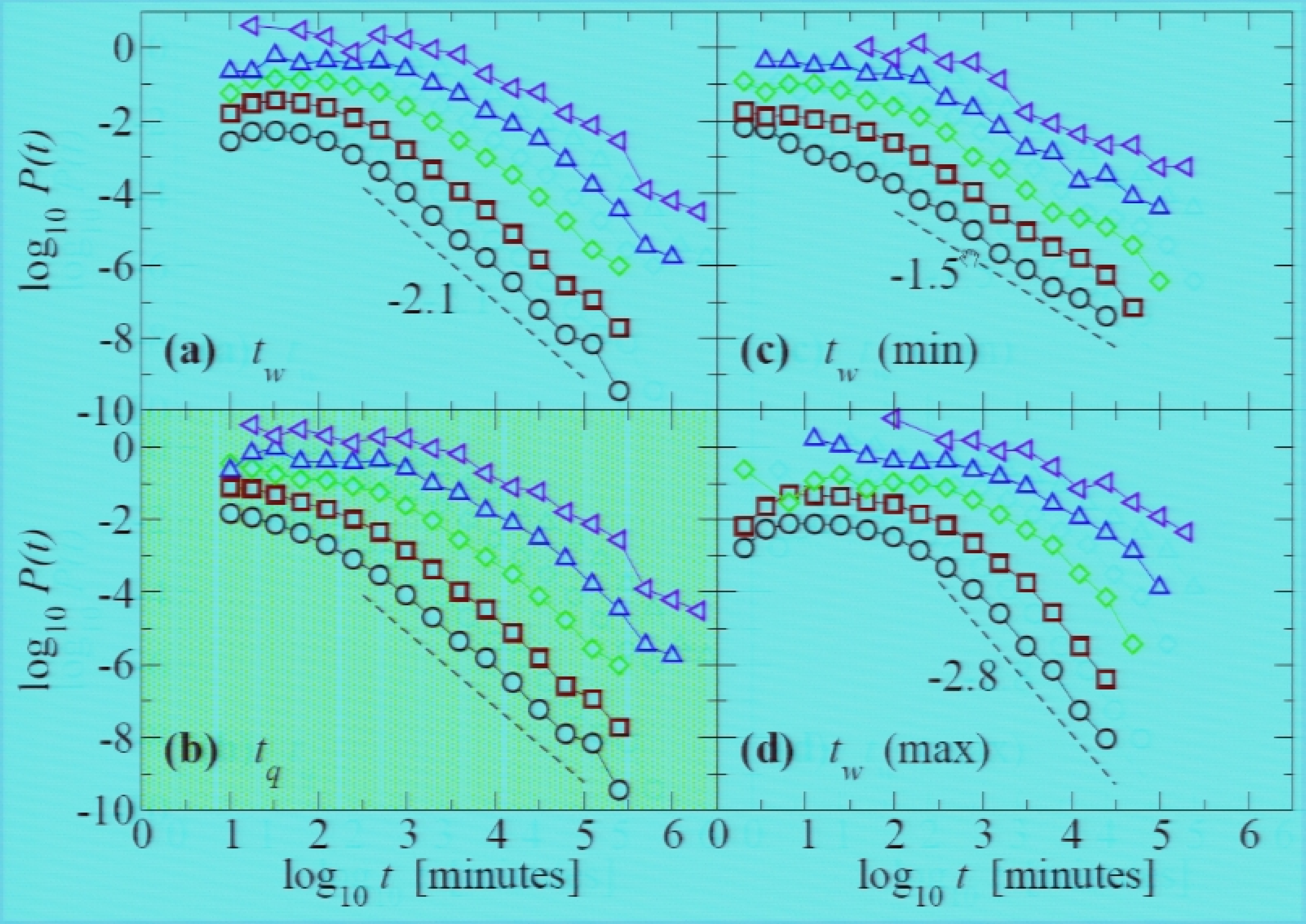


Complex correlations in self-organized critical phenomena

Waiting and quiet times for different thresholds



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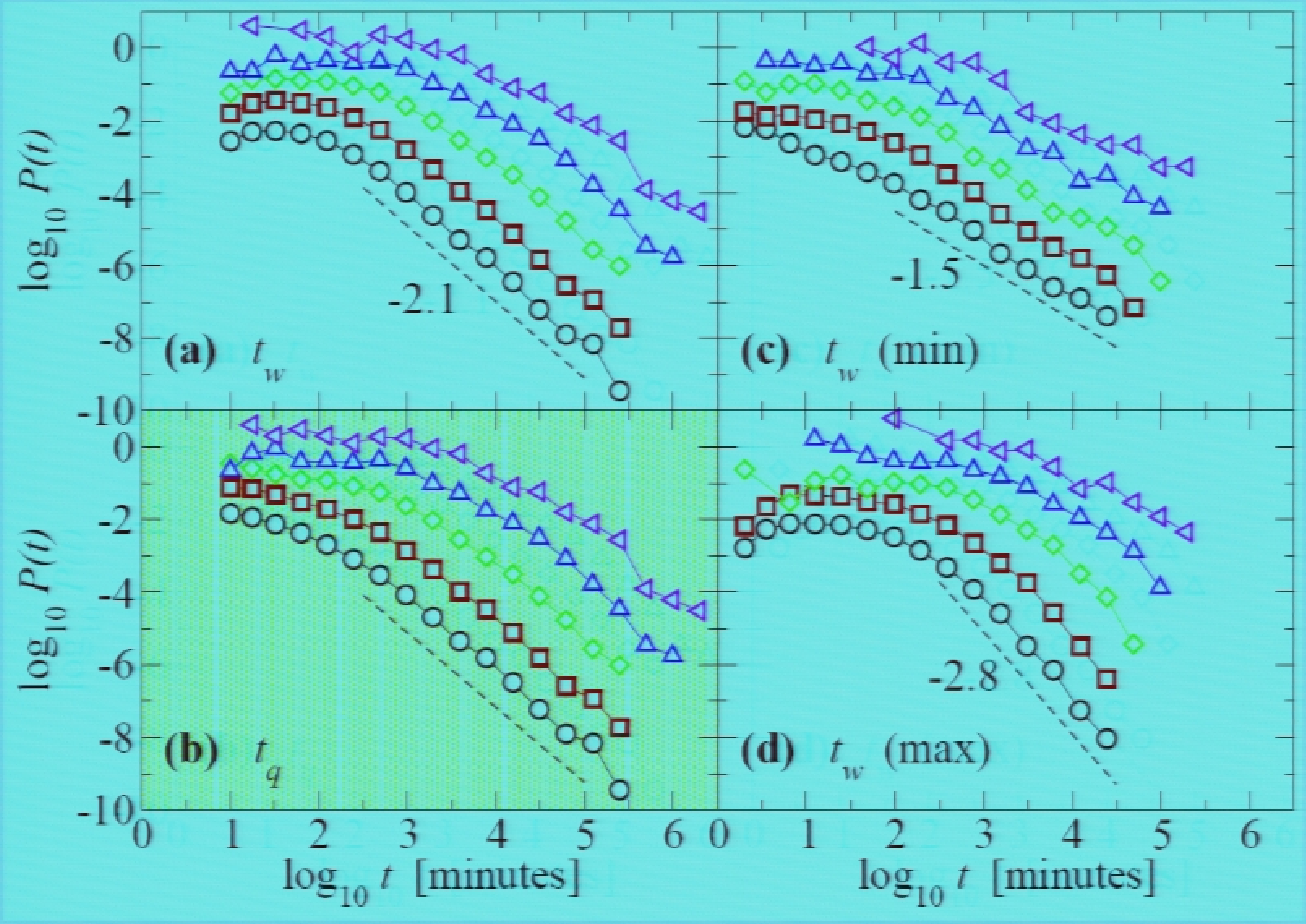

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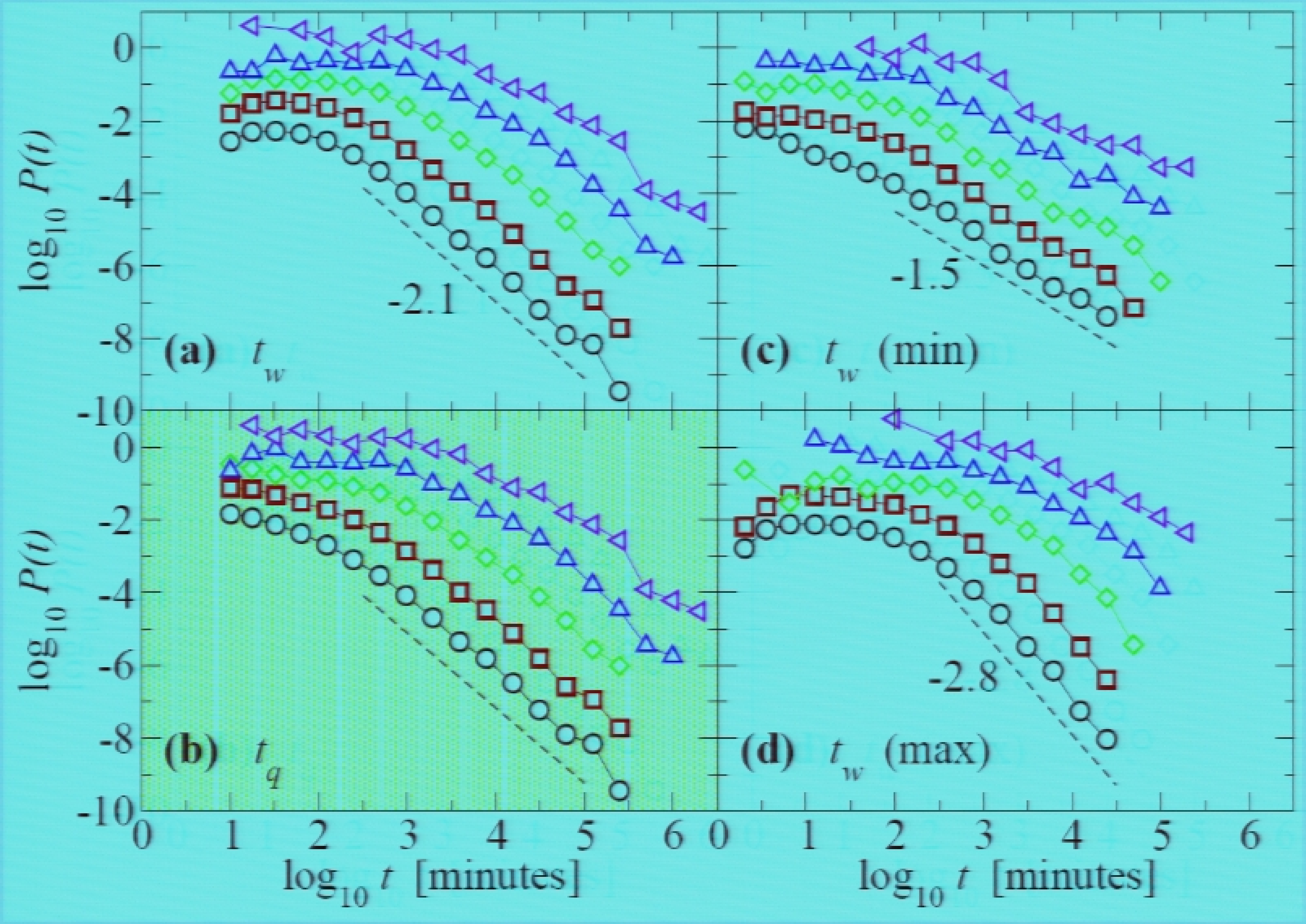

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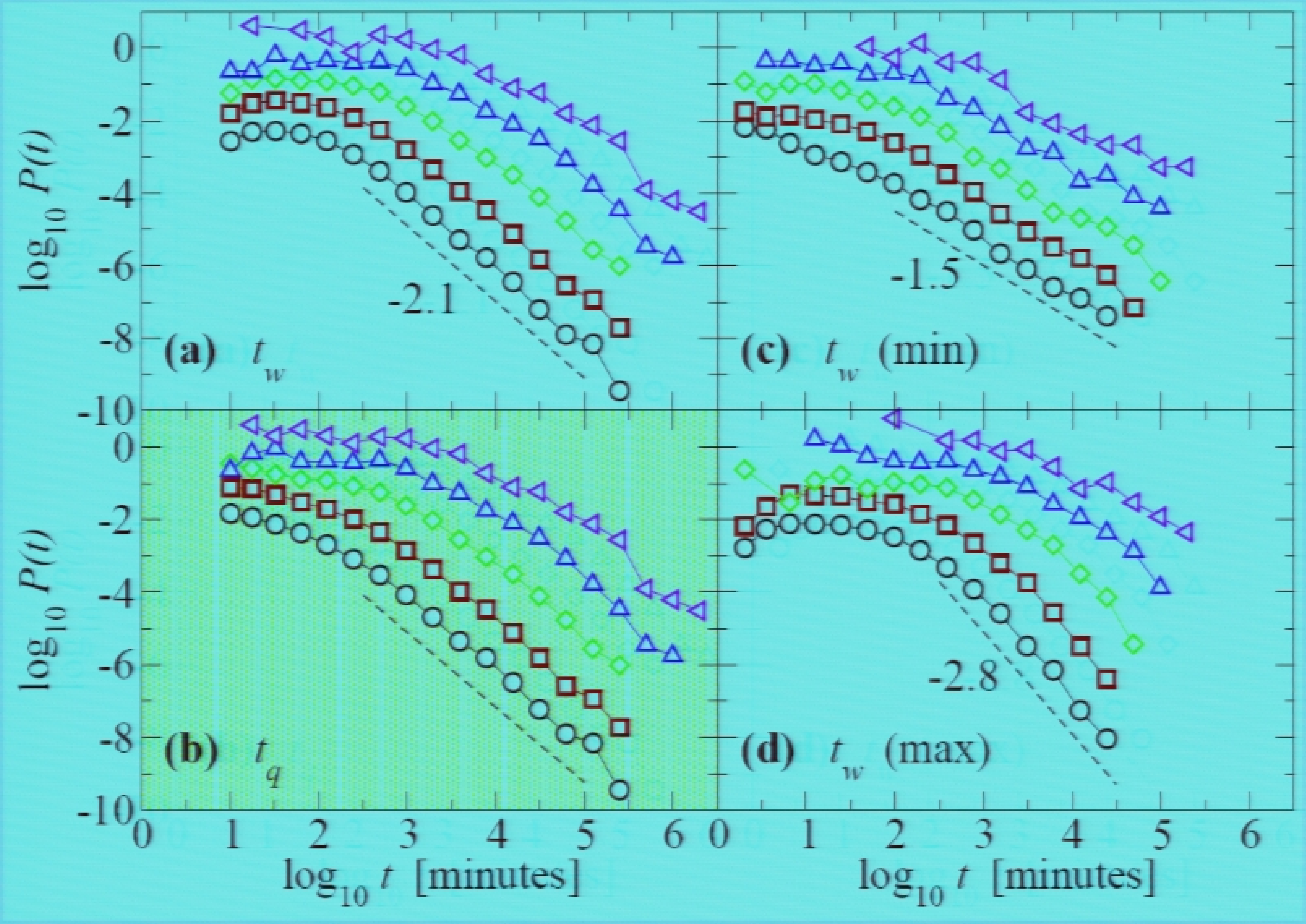

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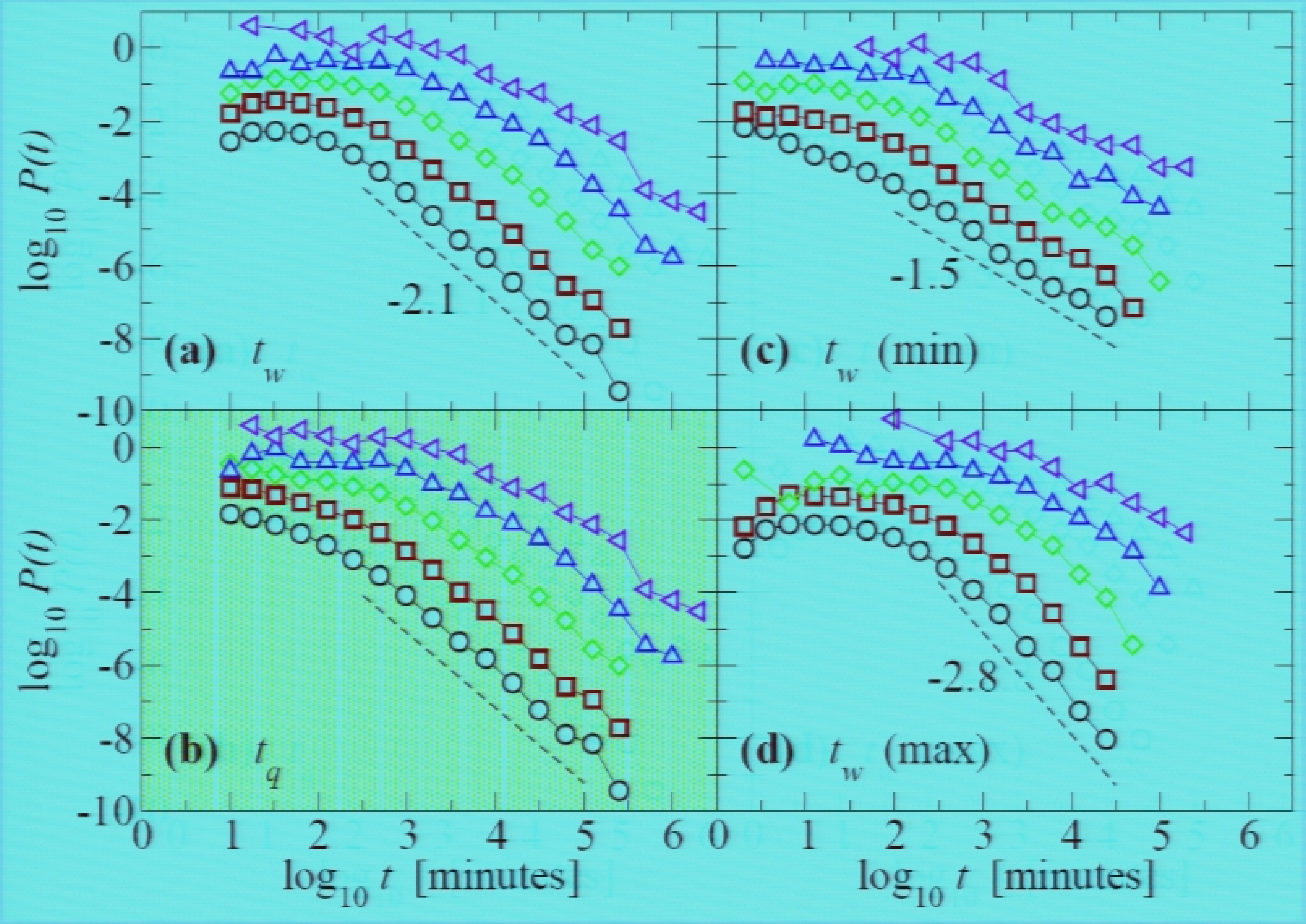

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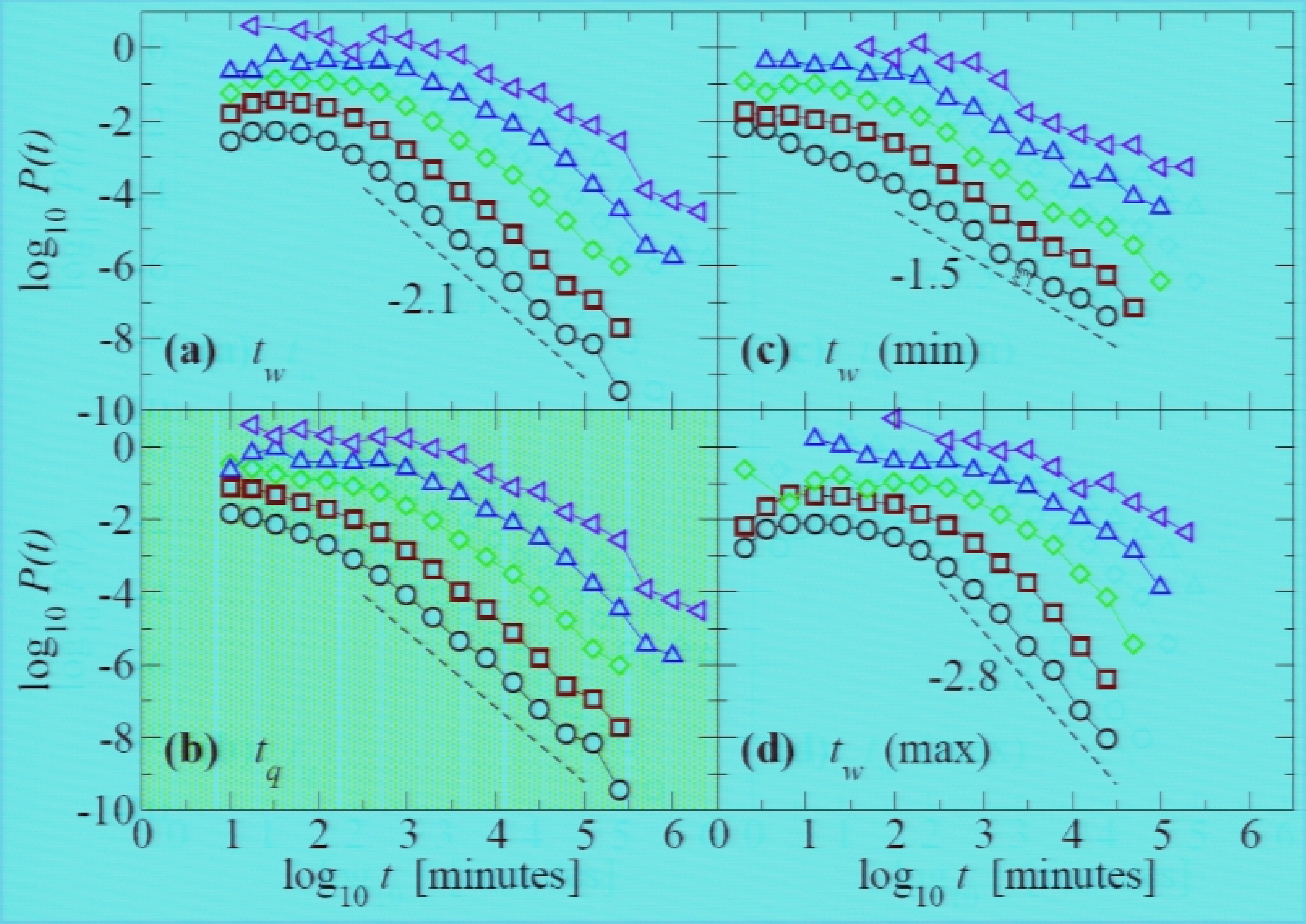

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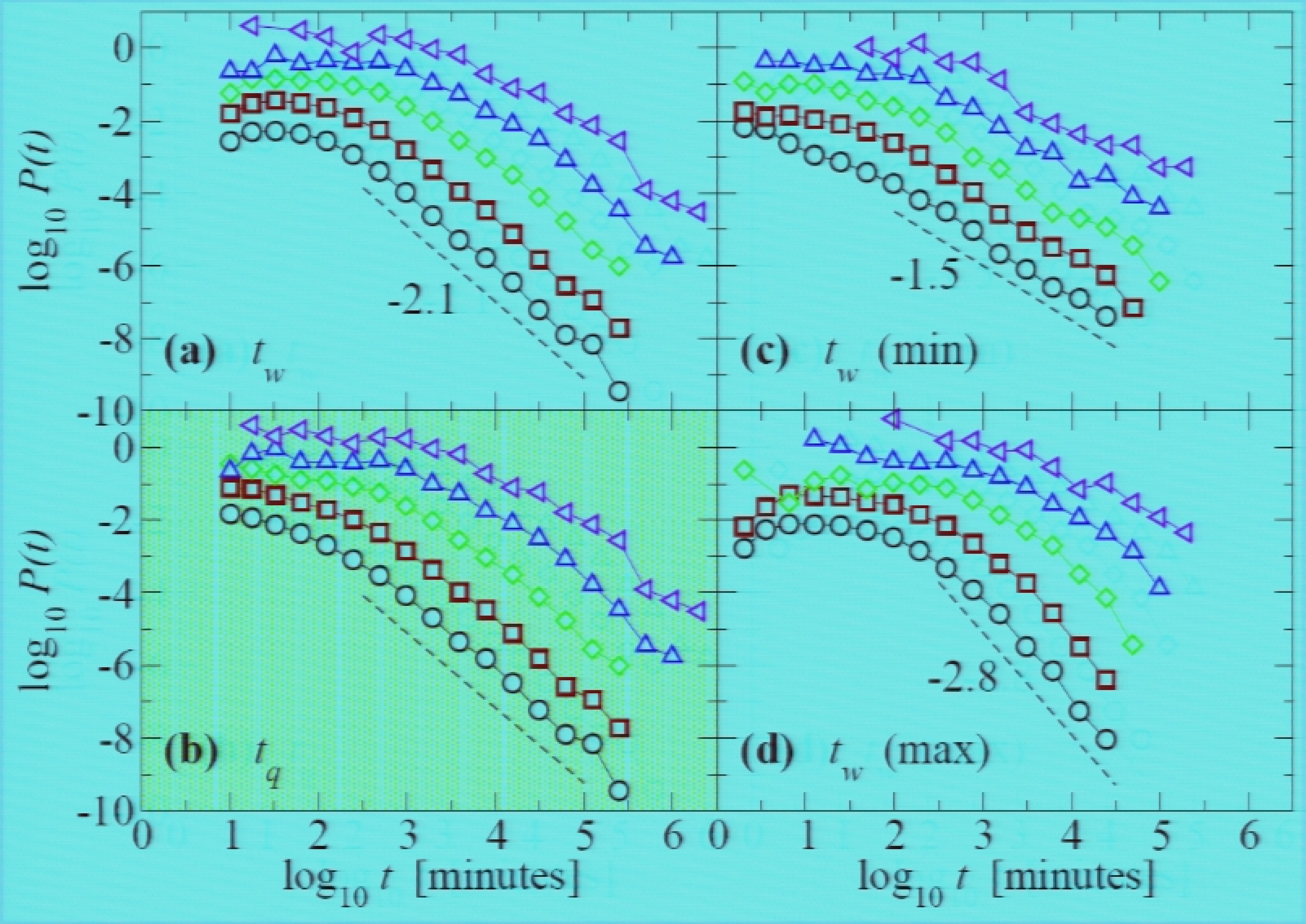

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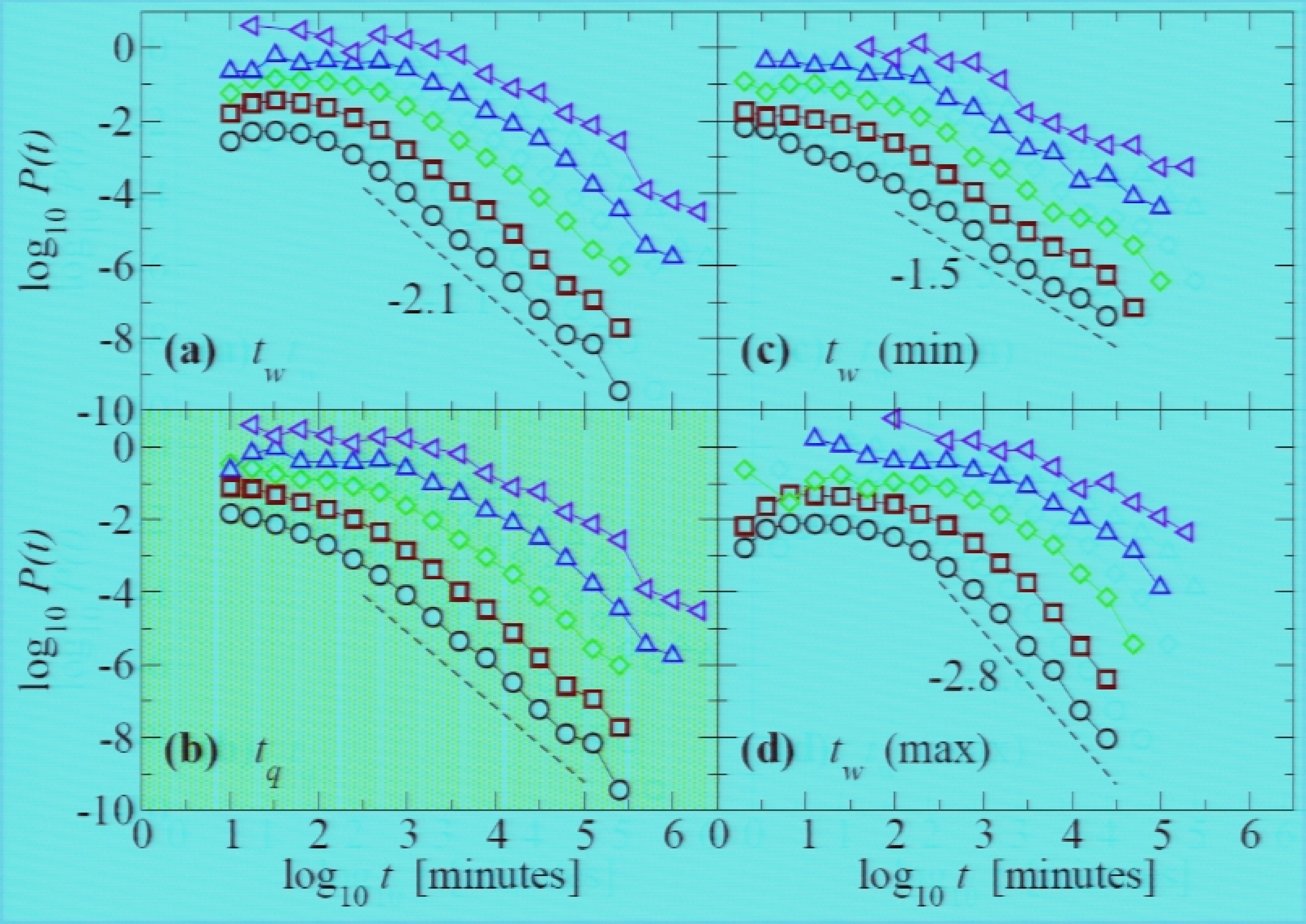

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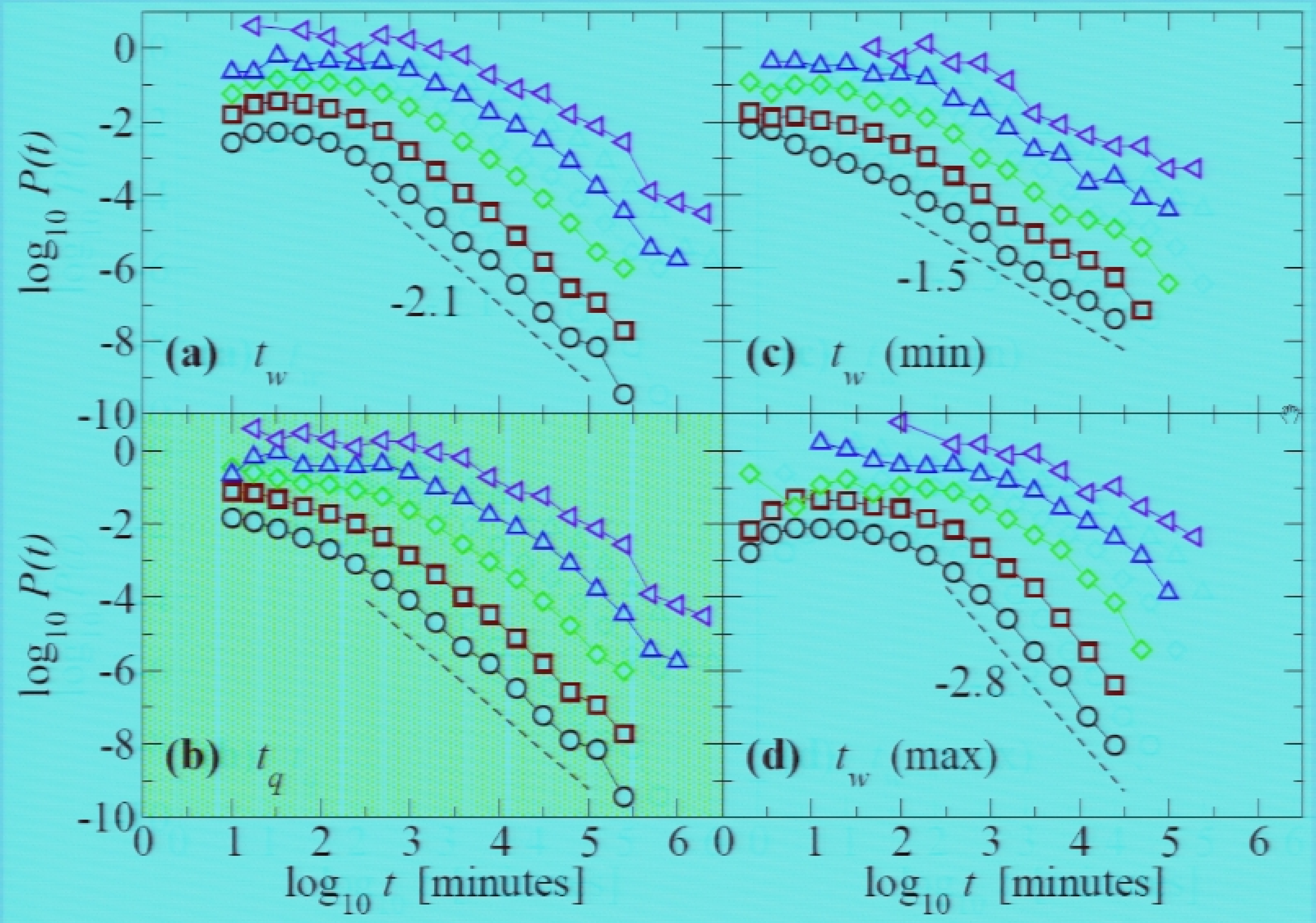

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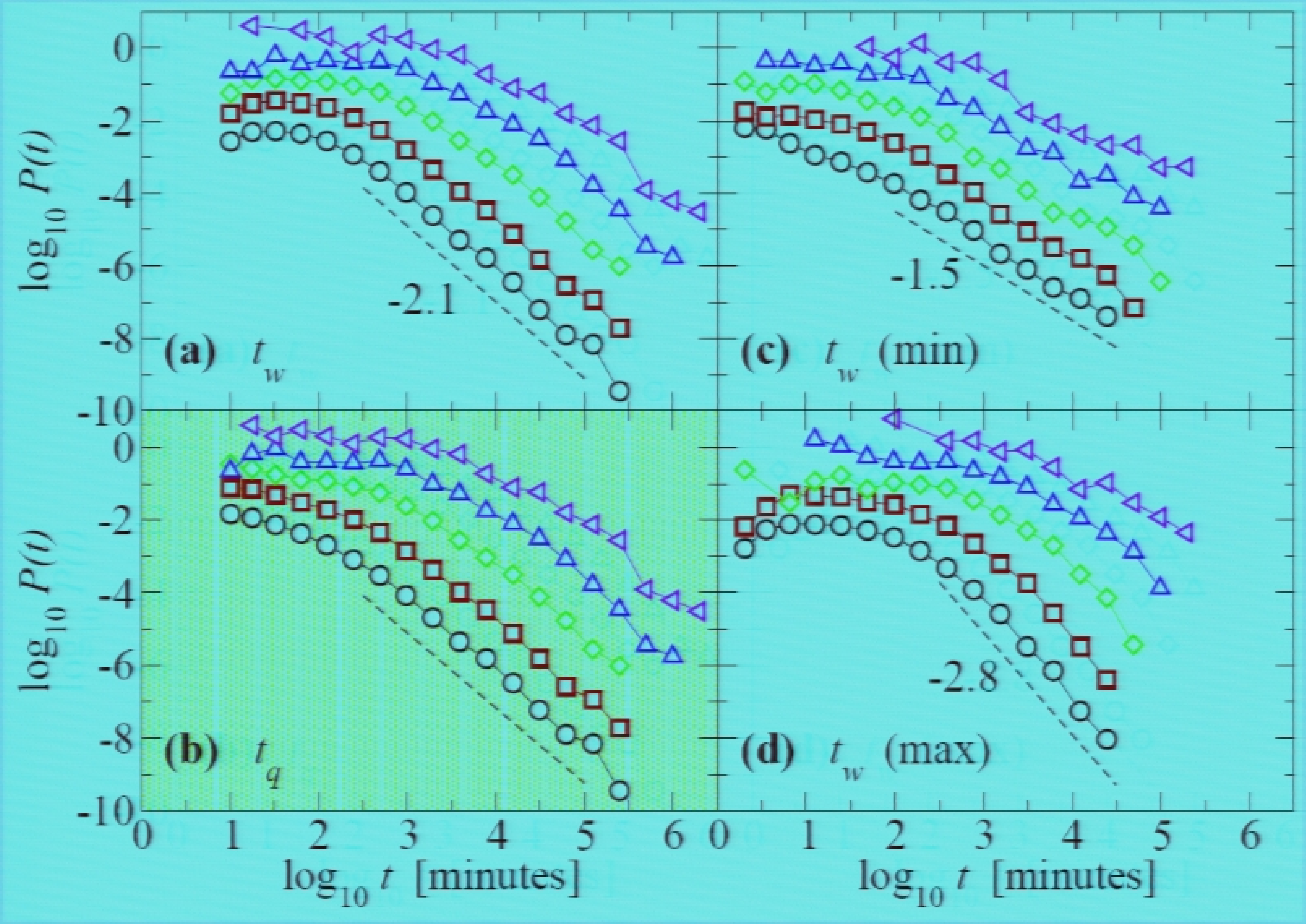

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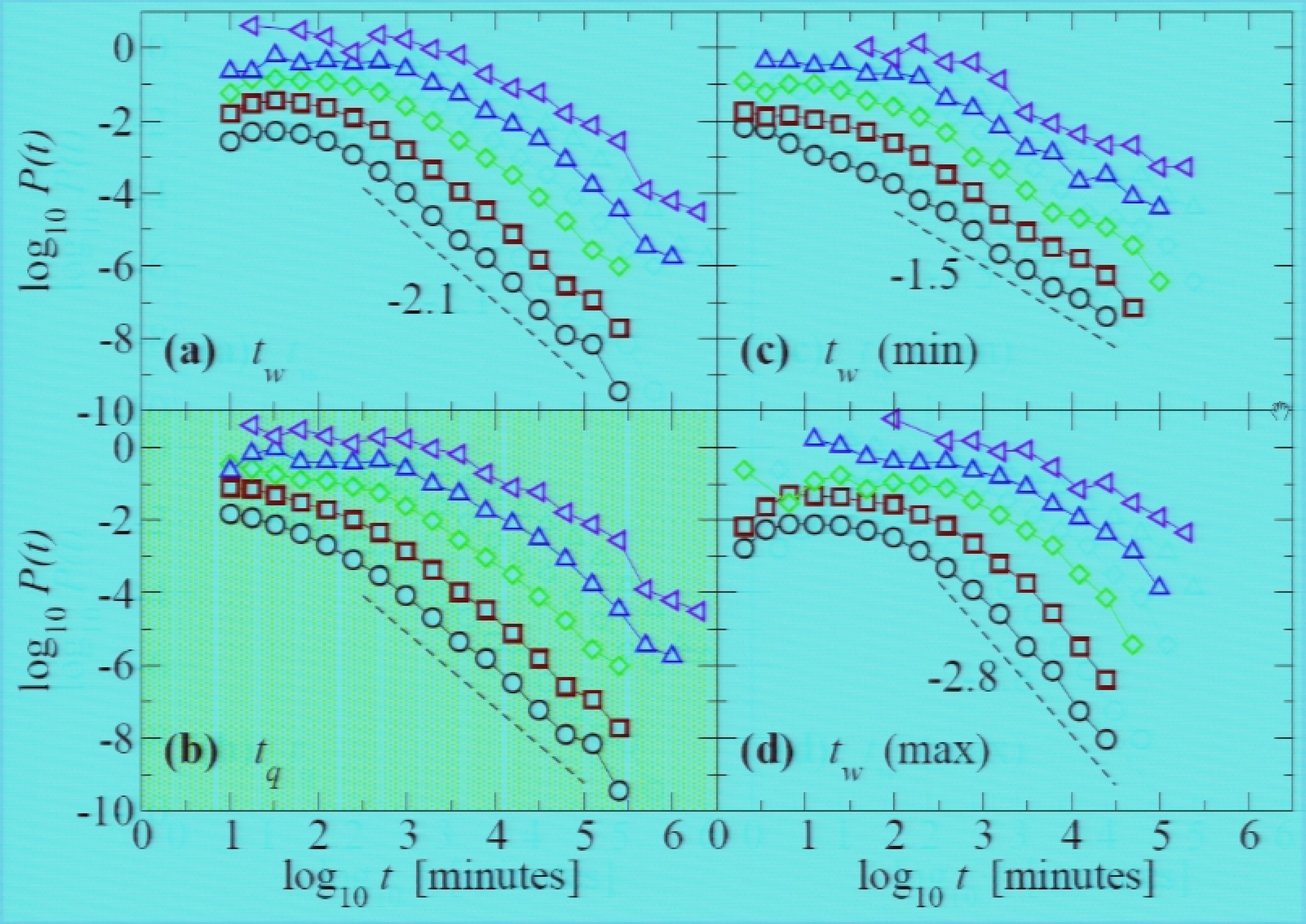

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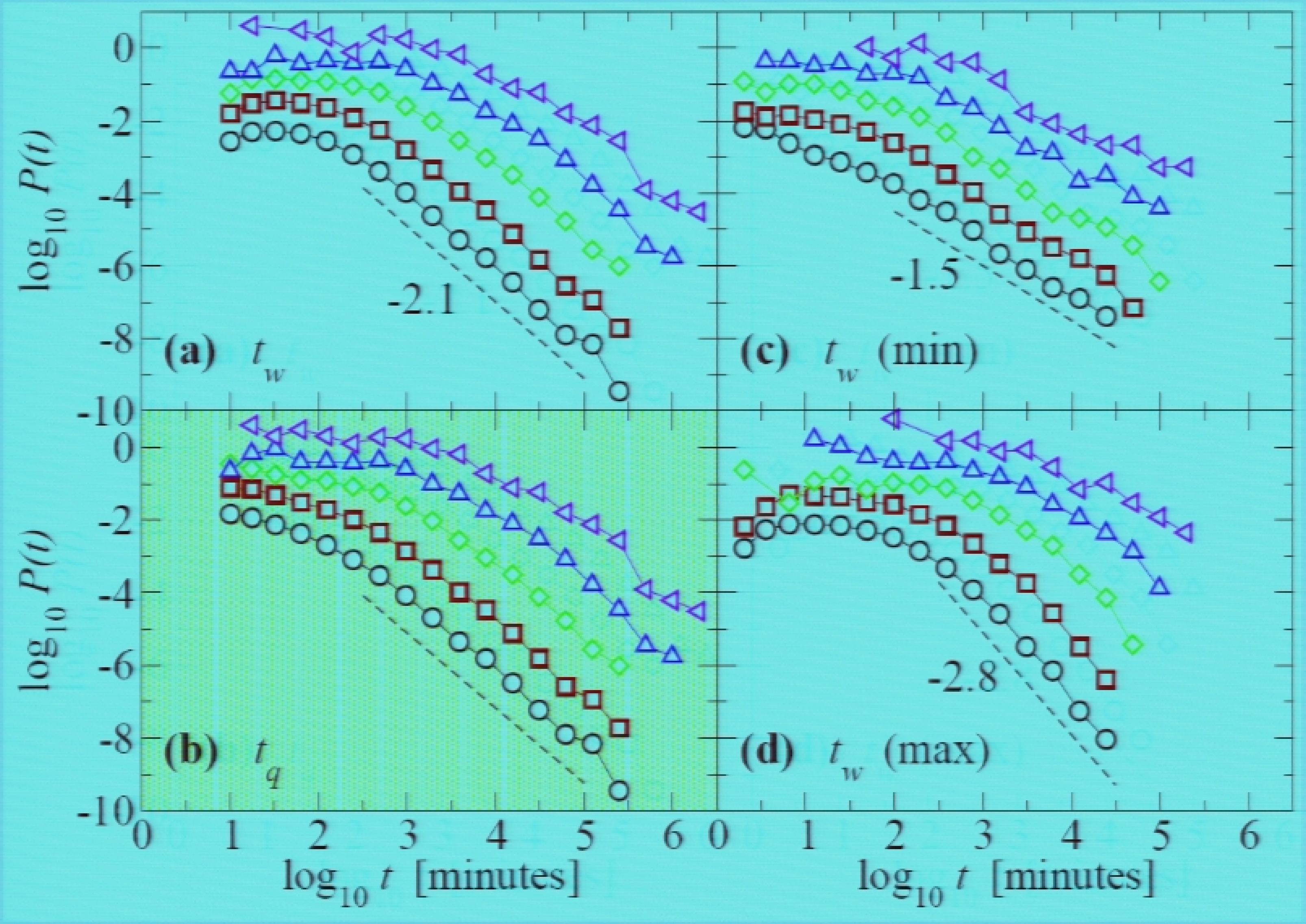
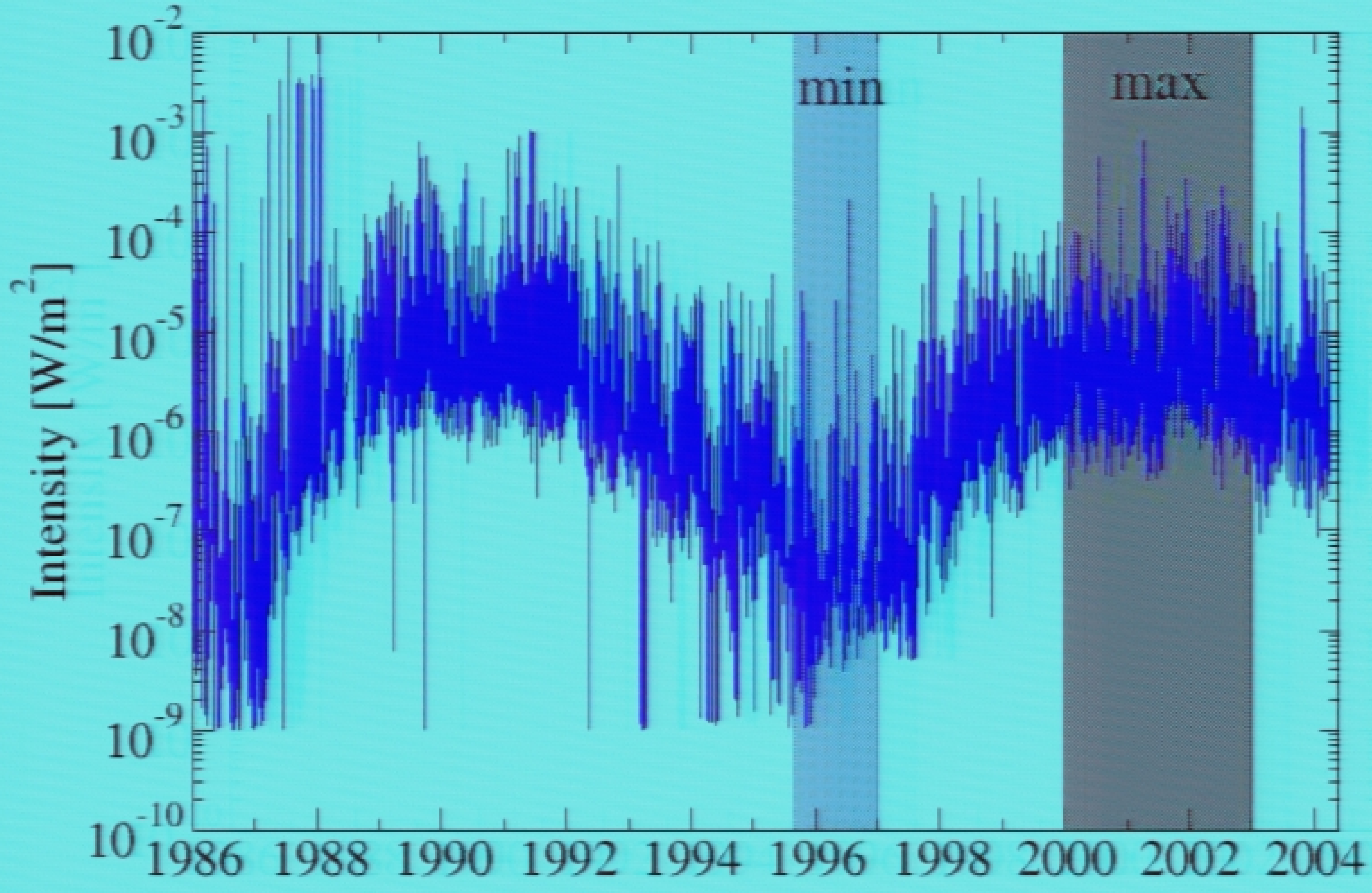


Complex correlations in self-organized critical phenomena

Solar flares: X-ray emissions, recorded by GOES satellites



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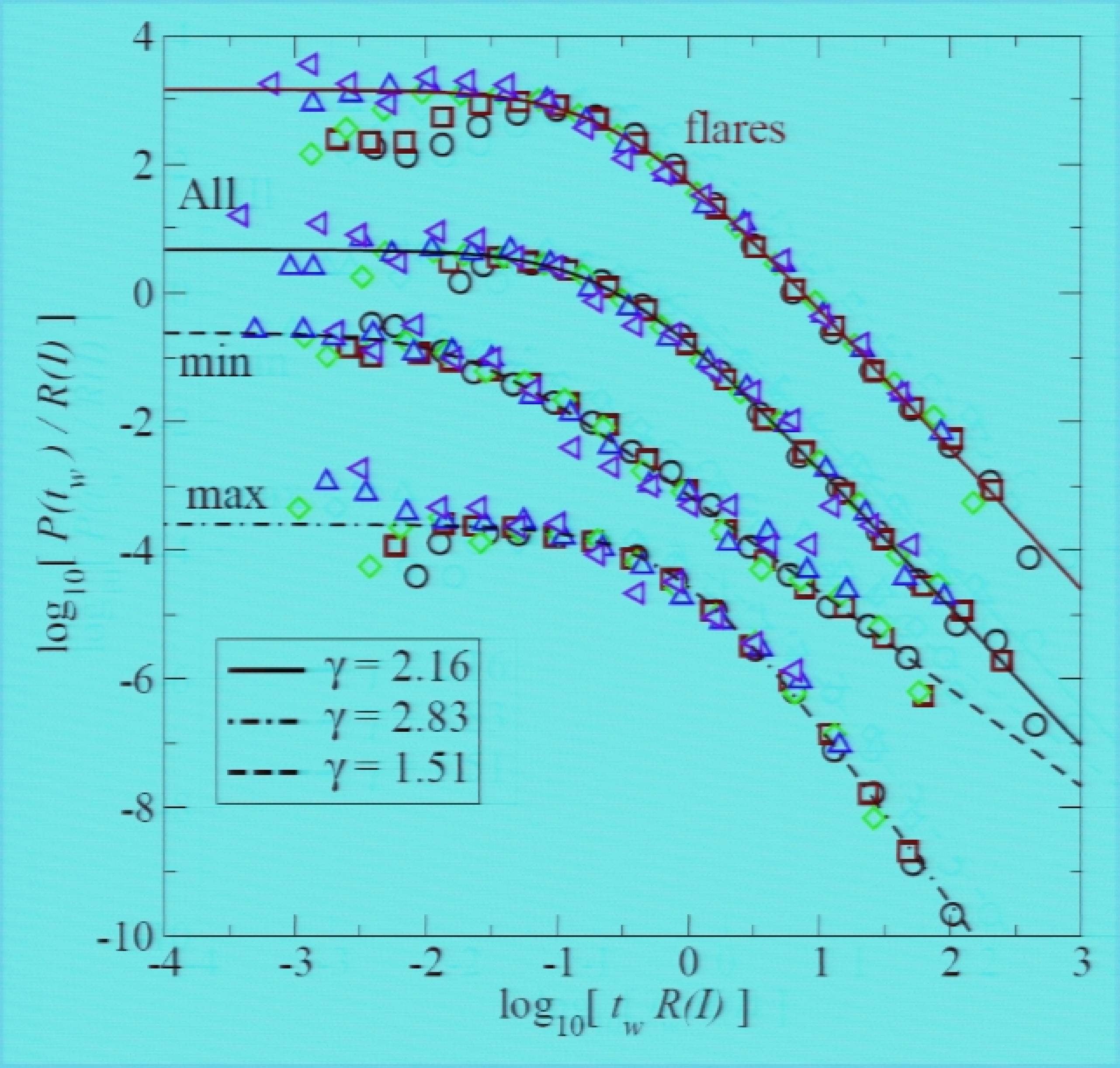

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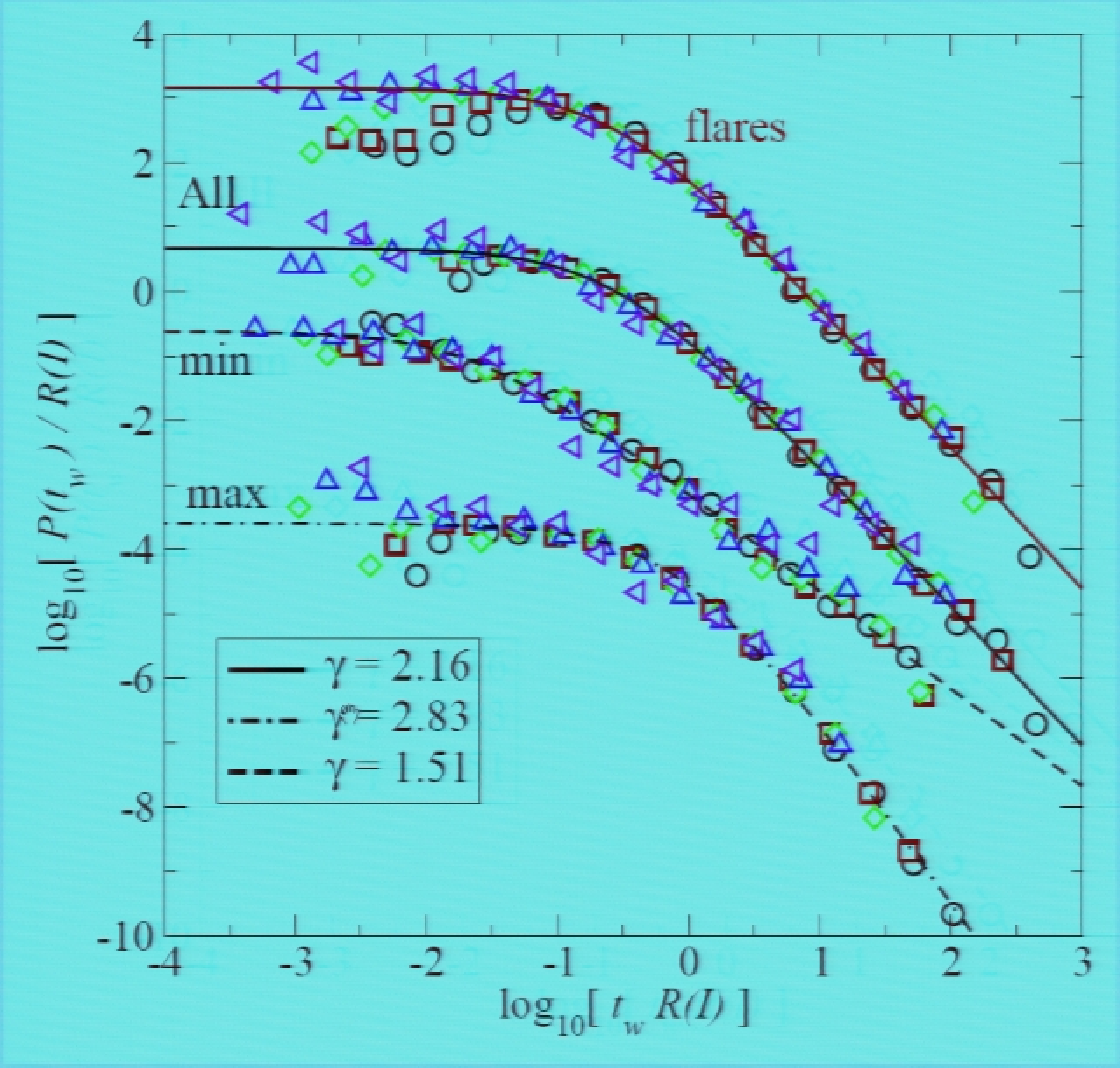

Complex correlations in self-organized critical phenomena

$$P(t_w, I) = \frac{1}{\bar{\lambda}_I} \int_0^\infty F_I(\lambda) \exp(-\lambda t_w) \lambda^2 d\lambda \quad ,$$

average flaring rate $\bar{\lambda}_I = \int_0^\infty F_I(\lambda) \lambda d\lambda$

$f_I(\lambda) = \lambda^2 F_I(\lambda)/\bar{\lambda}$

With $\lambda \to \beta, \quad t_w \to E, \quad f(\lambda) \to f(\beta):$

equivalent to χ^2-superstatistics with $f_I(\lambda) \to$ Gamma distribution:

$$f_I(\lambda) \sim \left(\frac{\lambda x^*}{R(I)}\right)^{\gamma-1} \exp\left(-\frac{\lambda x^*}{R(I)}\right) \quad .$$

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With $\lambda \to \beta, \quad t_w \to E, \quad f(\lambda) \to f(\beta)$:

equivalent to χ^2-superstatistics with $f_I(\lambda) \to$ Gamma distribution:

$$f_I(\lambda) \sim \left(\frac{\lambda x^*}{R(I)}\right)^{\gamma-1} \exp\left(-\frac{\lambda x^*}{R(I)}\right) \quad .$$

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Complex correlations in self-organized critical phenomena

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Solar flares: summary

Distribution of interoccurrence times:

- Power law tail
- Depends on the phase of the solar cycle
- Shape does not depend on the threshold
- Definition of flare/event not crucial

Scaling picture: self similar character

[Baiesi, Paczuski, and Stella, cond-mat/0411243]

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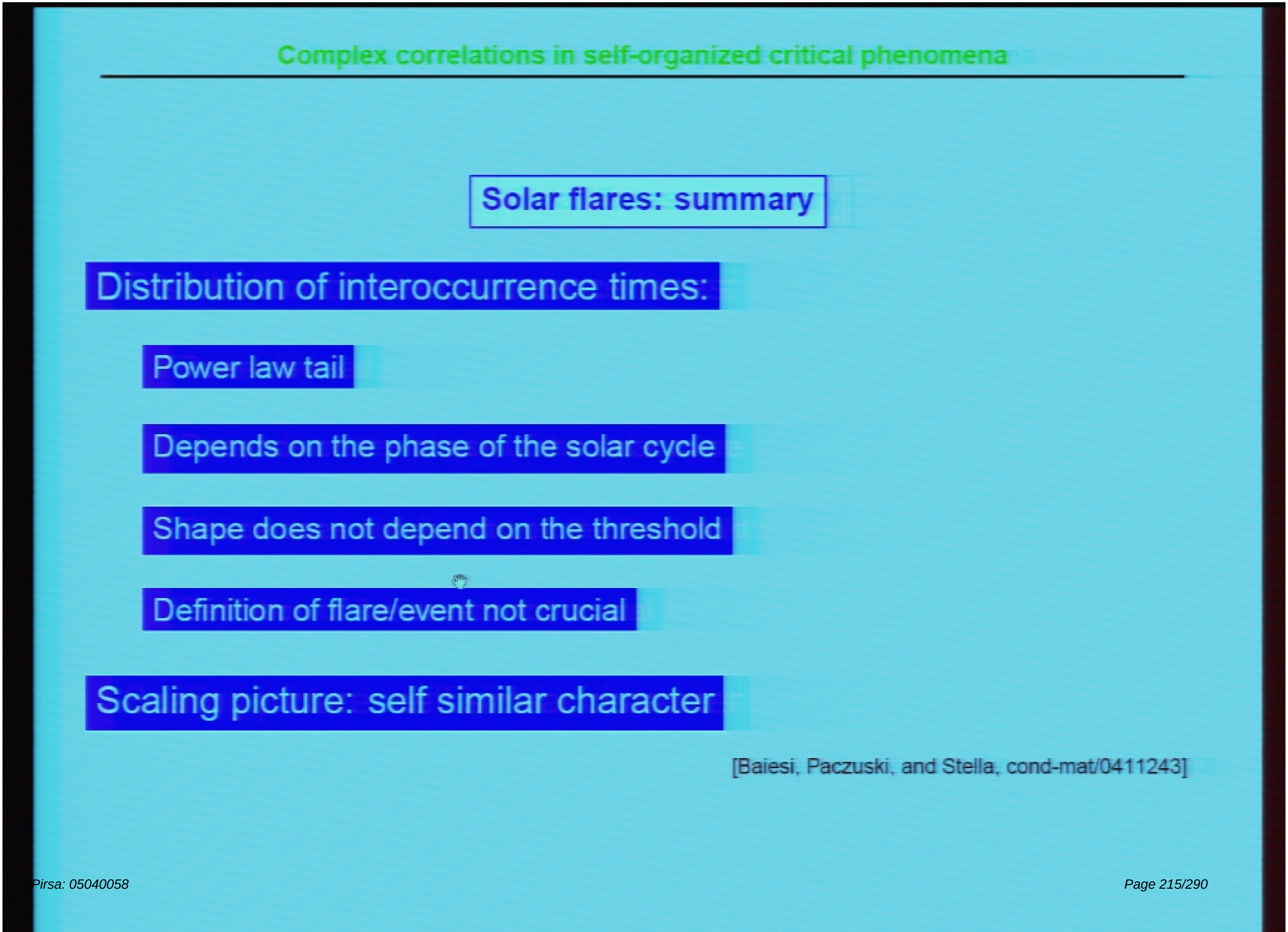

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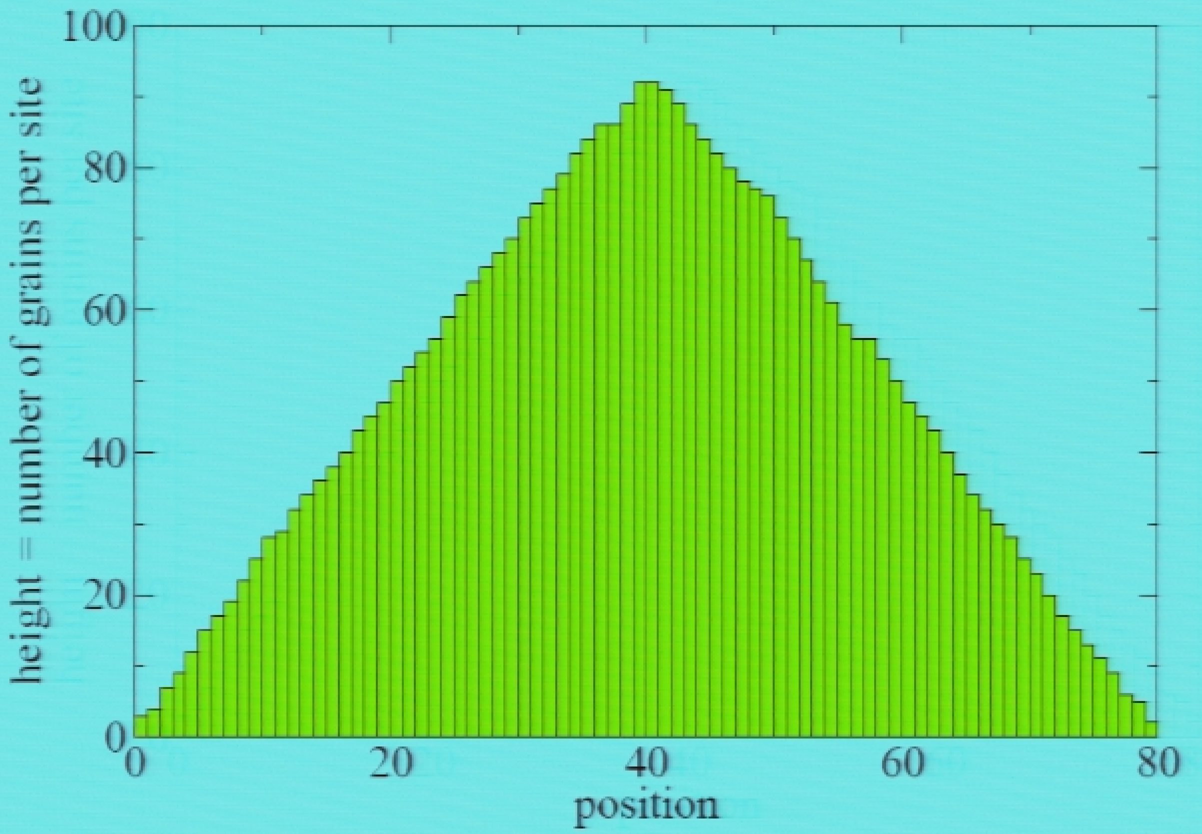
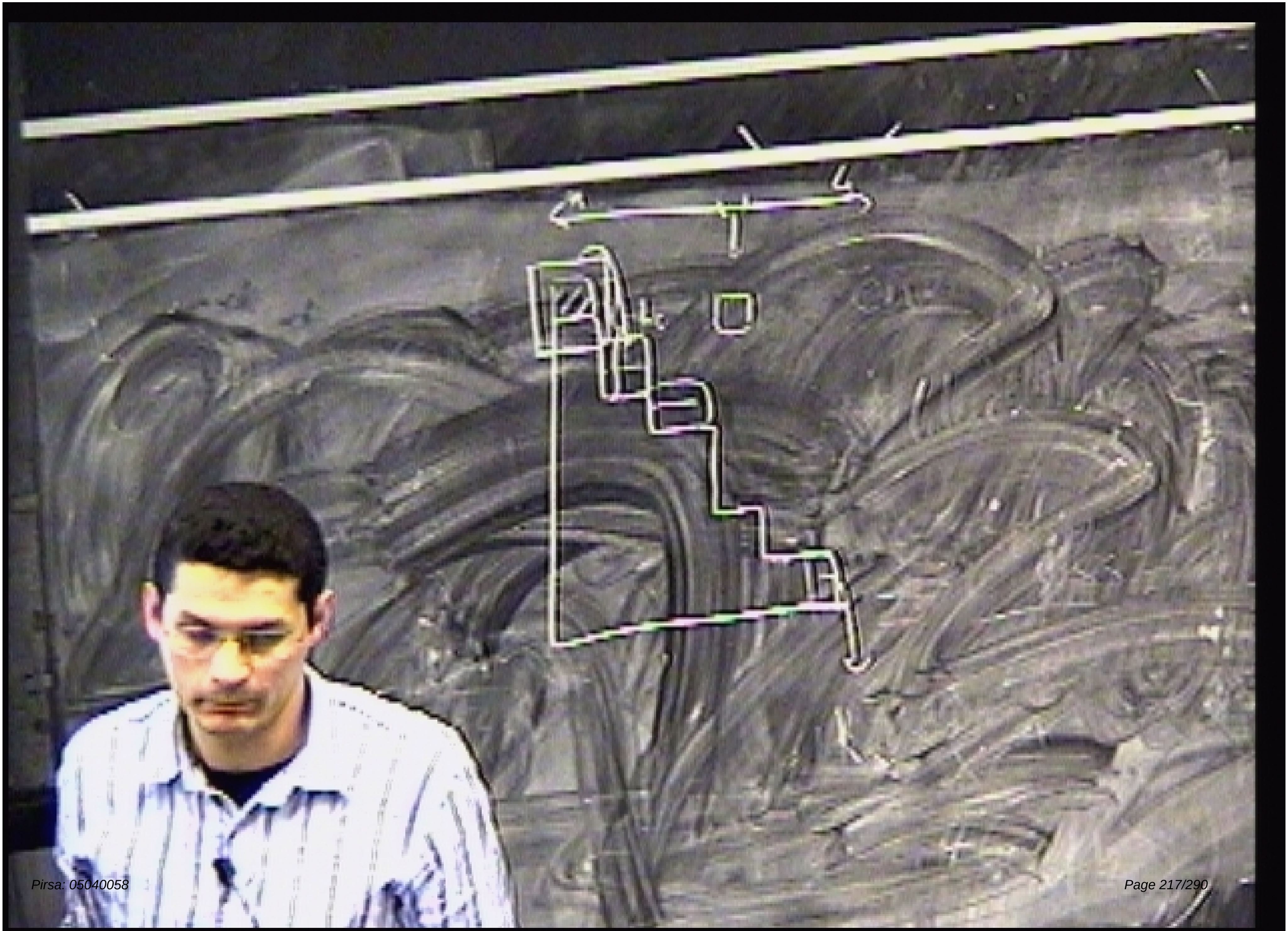
cellular
automaton



Criticality

1) scale-free avalanches

2) correlation between events?



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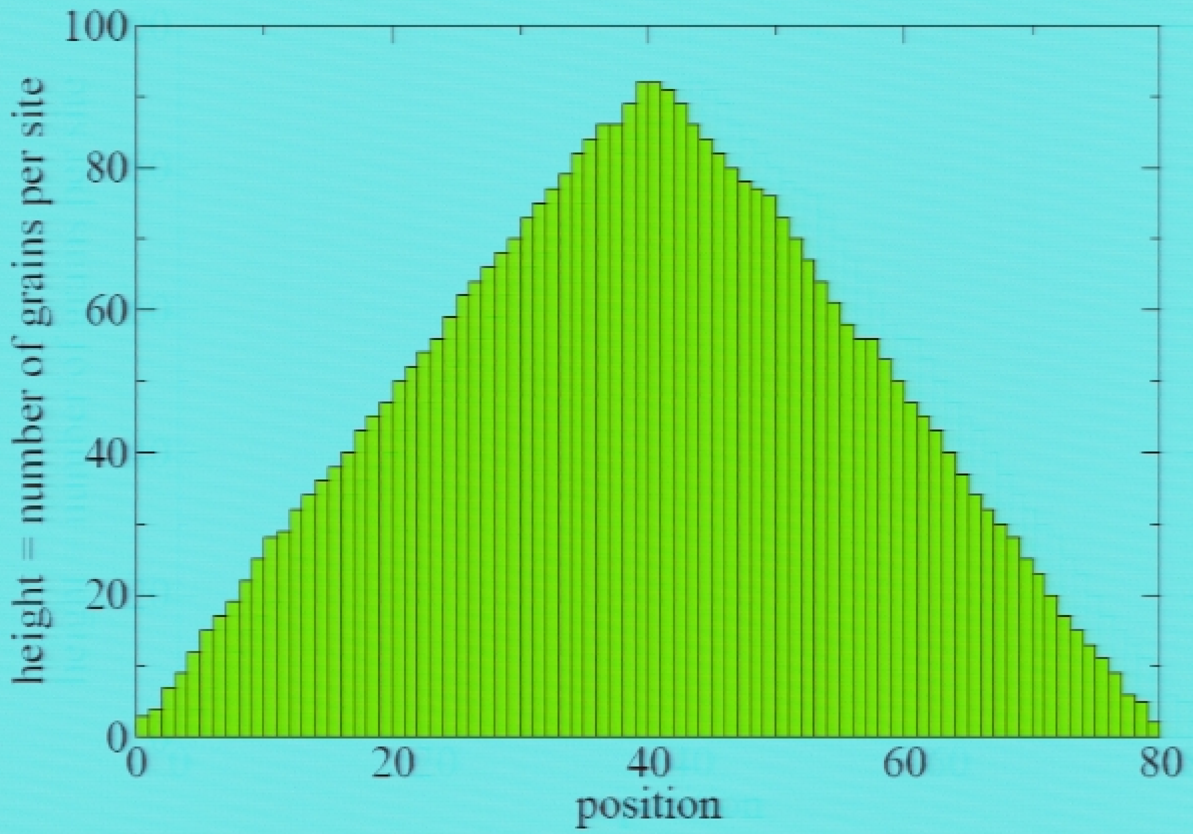

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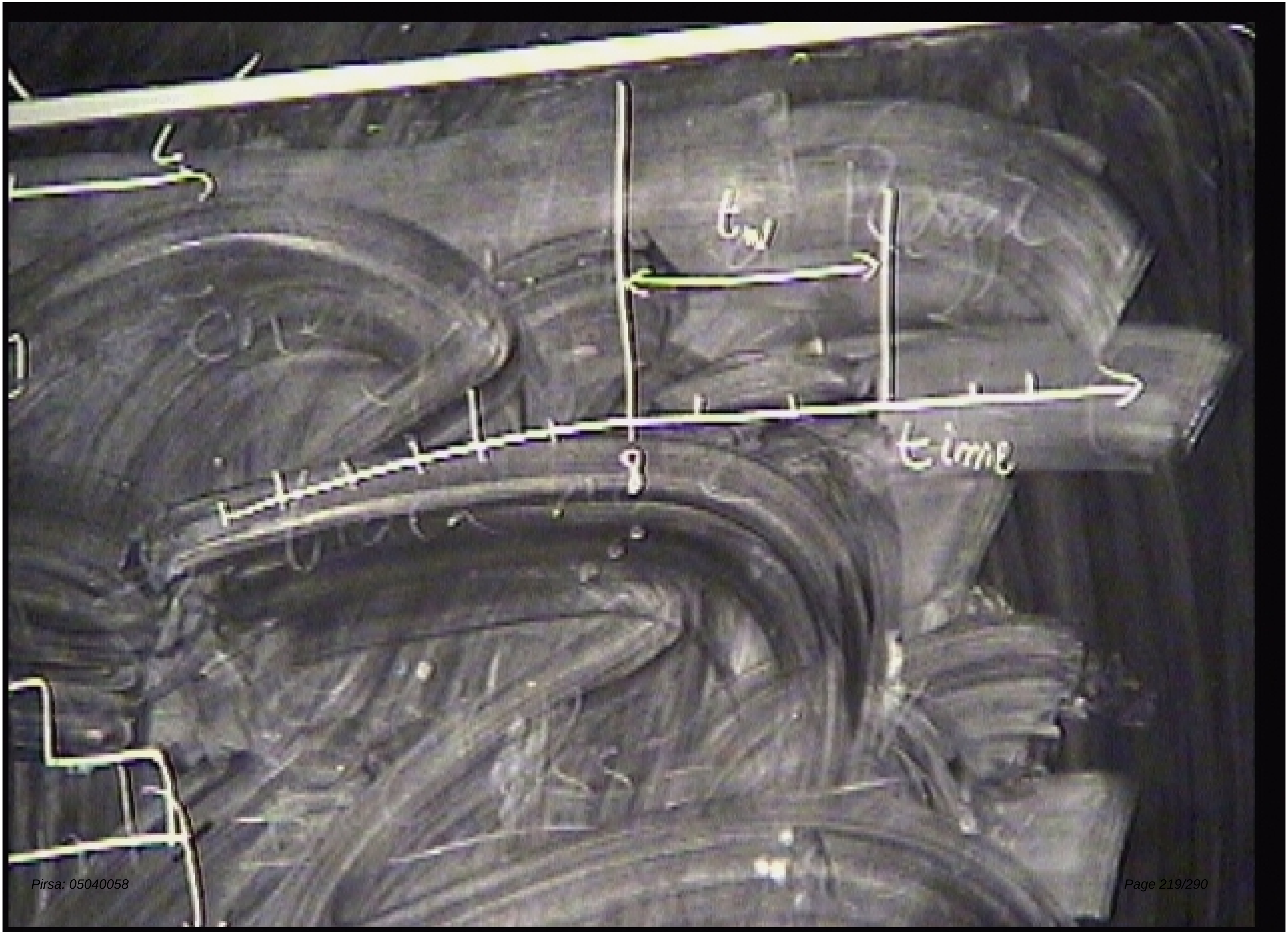
t_w
8
time

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Temporal correlations in SOC: $1/f$ noise

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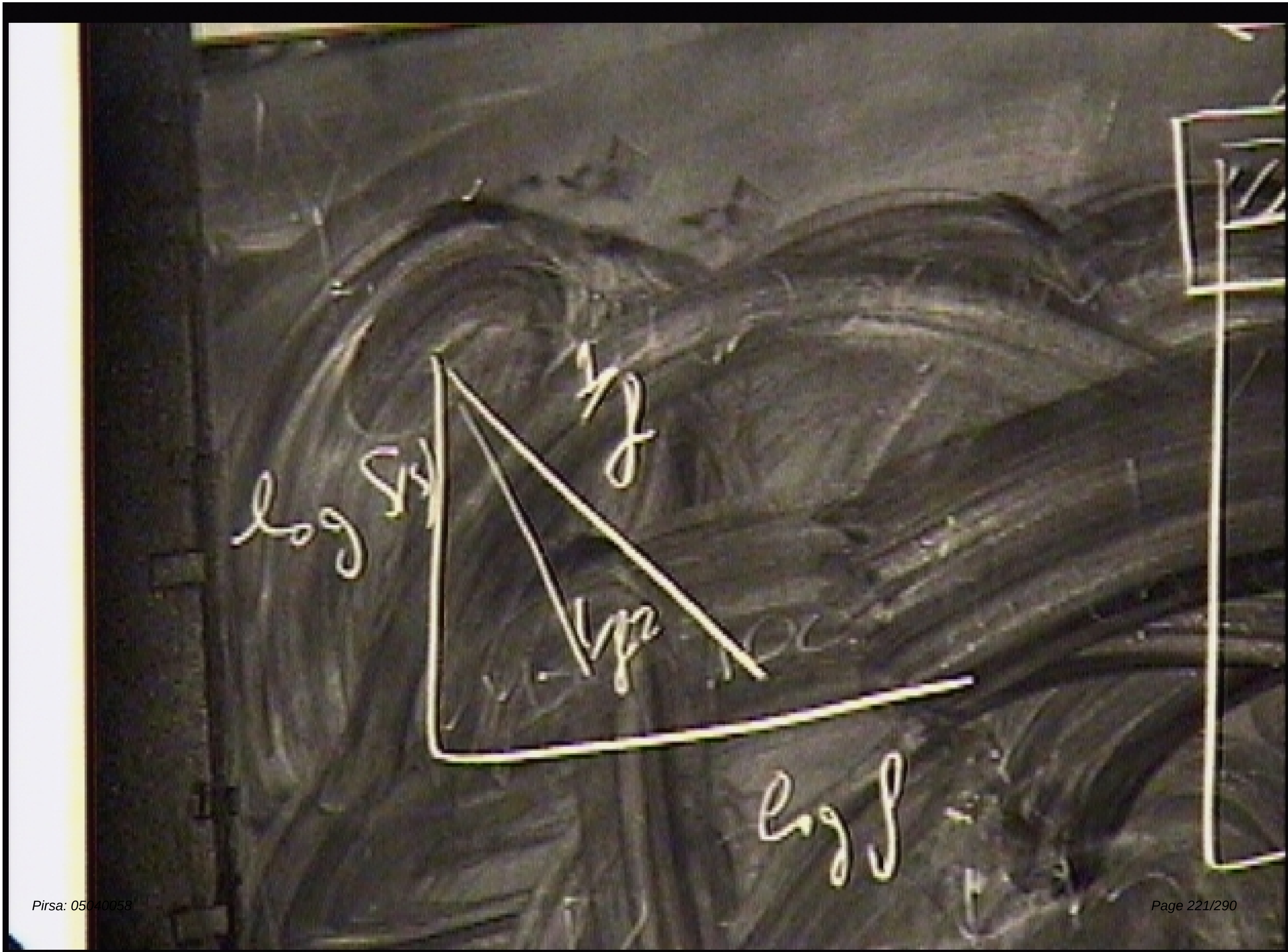
log S_V
$1/g$
$1/g^2$
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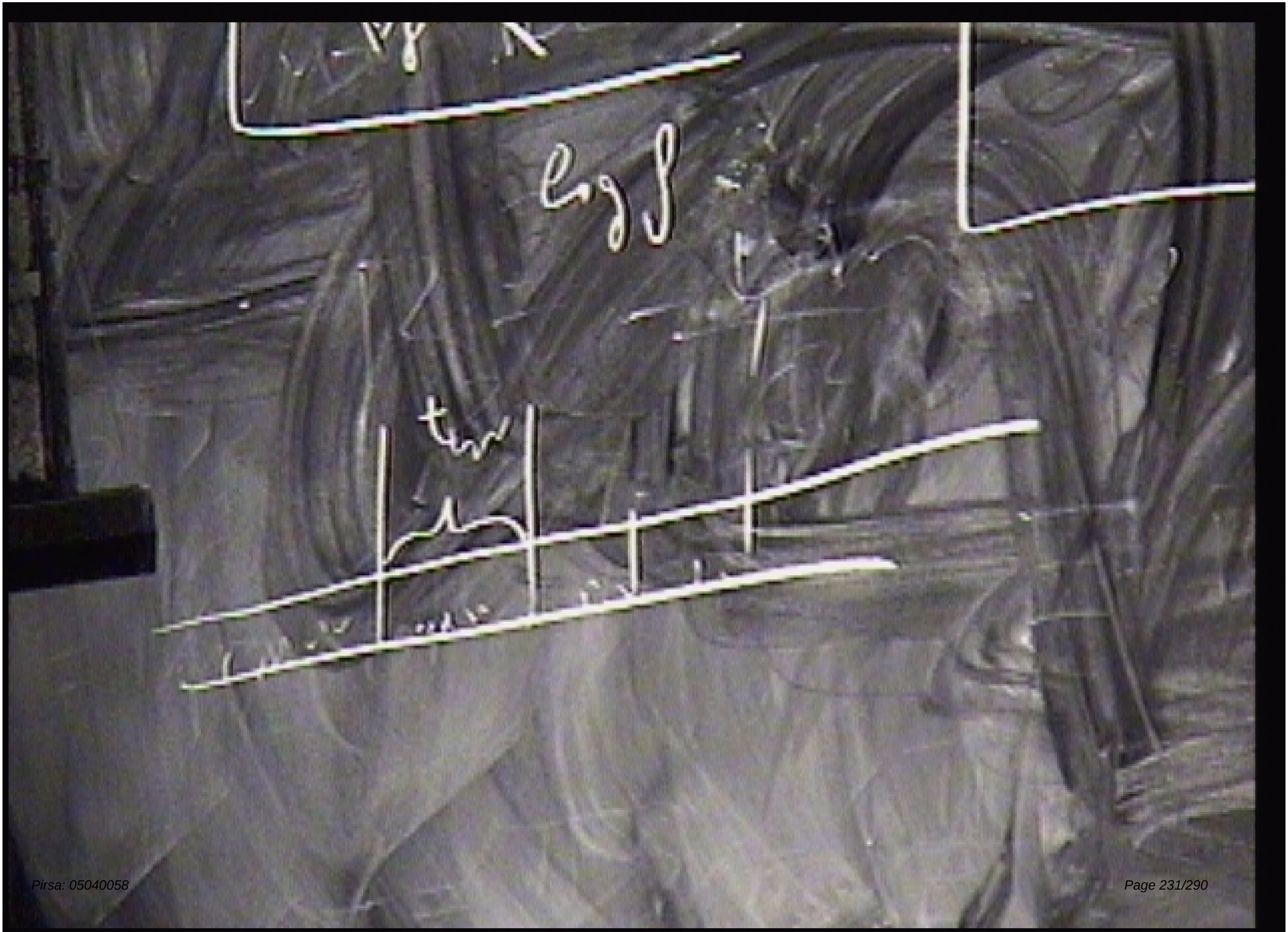
e.g. ρ
tw

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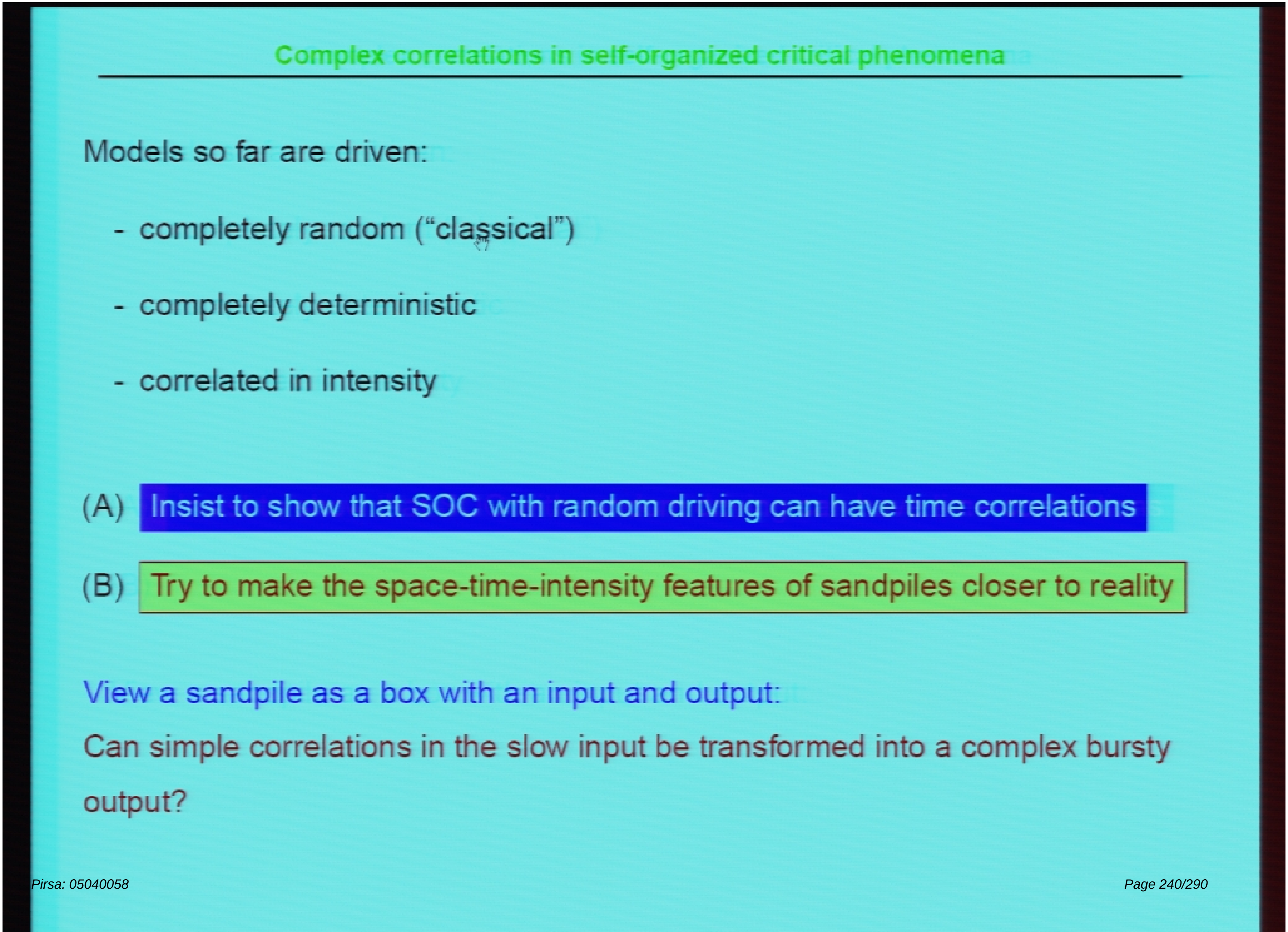
- completely random ("classical")
- completely deterministic
- correlated in intensity

(A) Insist to show that SOC with random driving can have time correlations

(B) Try to make the space-time-intensity features of sandpiles closer to reality

View a sandpile as a box with an input and output:

Can simple correlations in the slow input be transformed into a complex bursty output?



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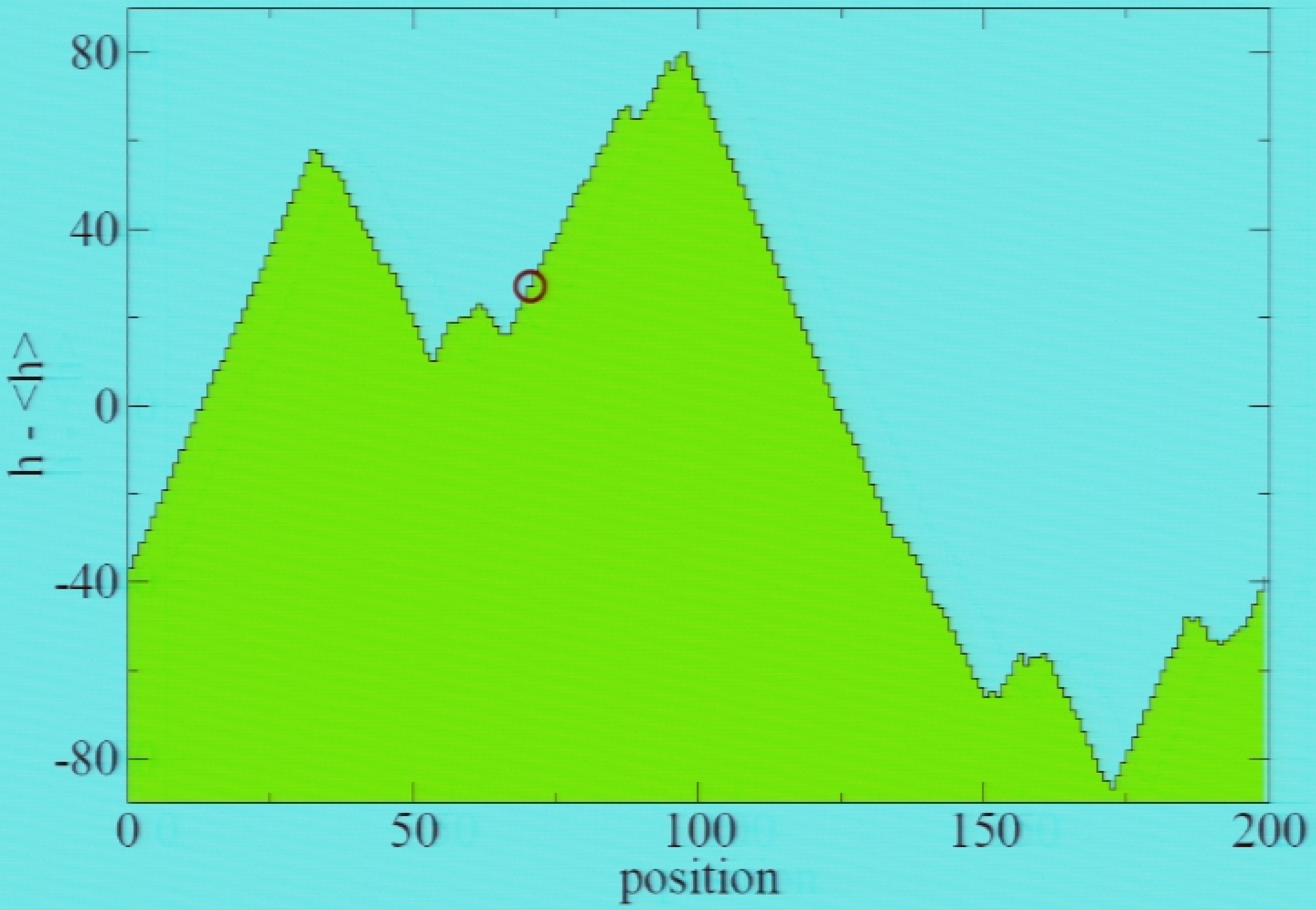
My new model

Place where grain is added (○) moves as a random walk



Complex correlations in self-organized critical phenomena

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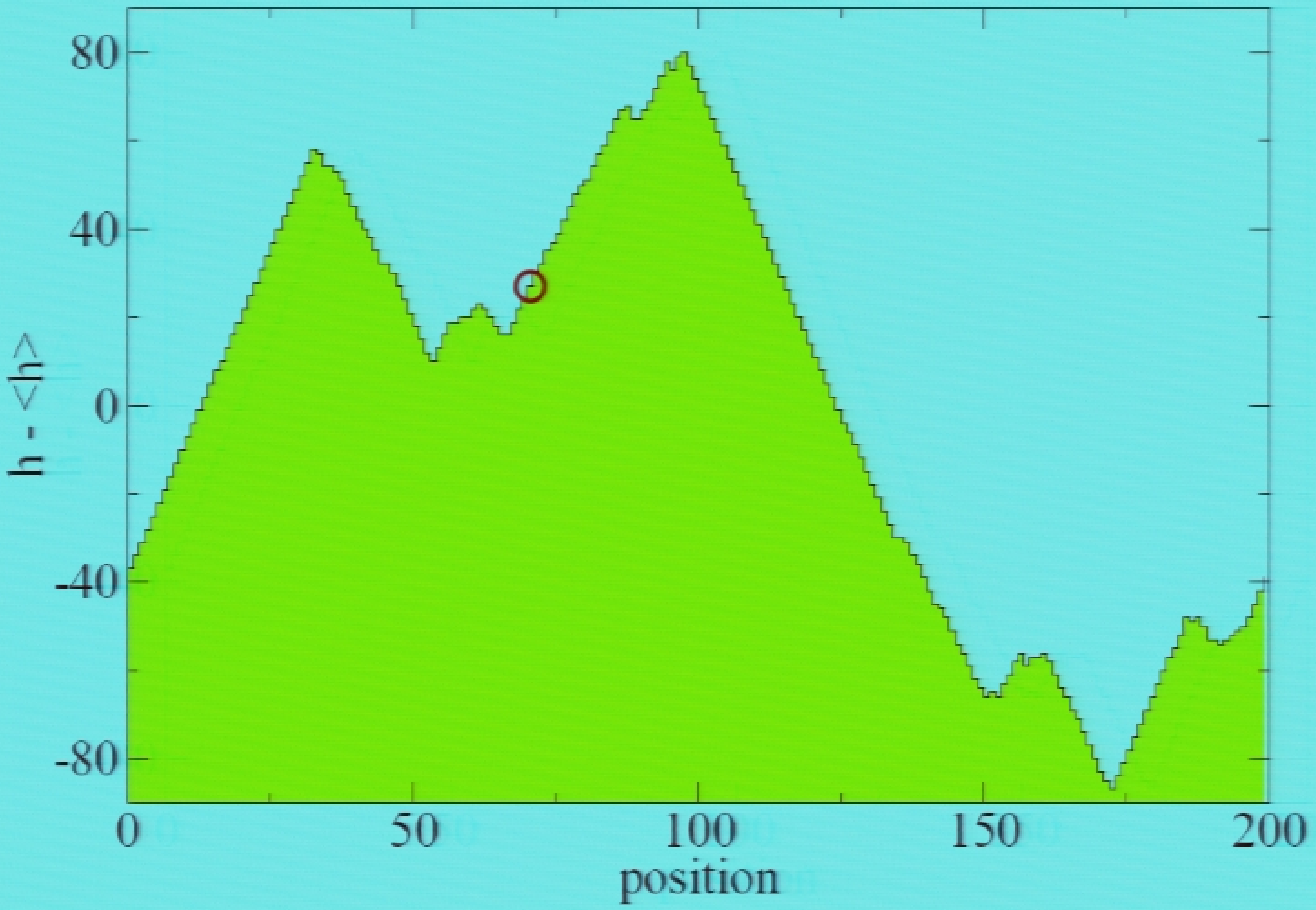
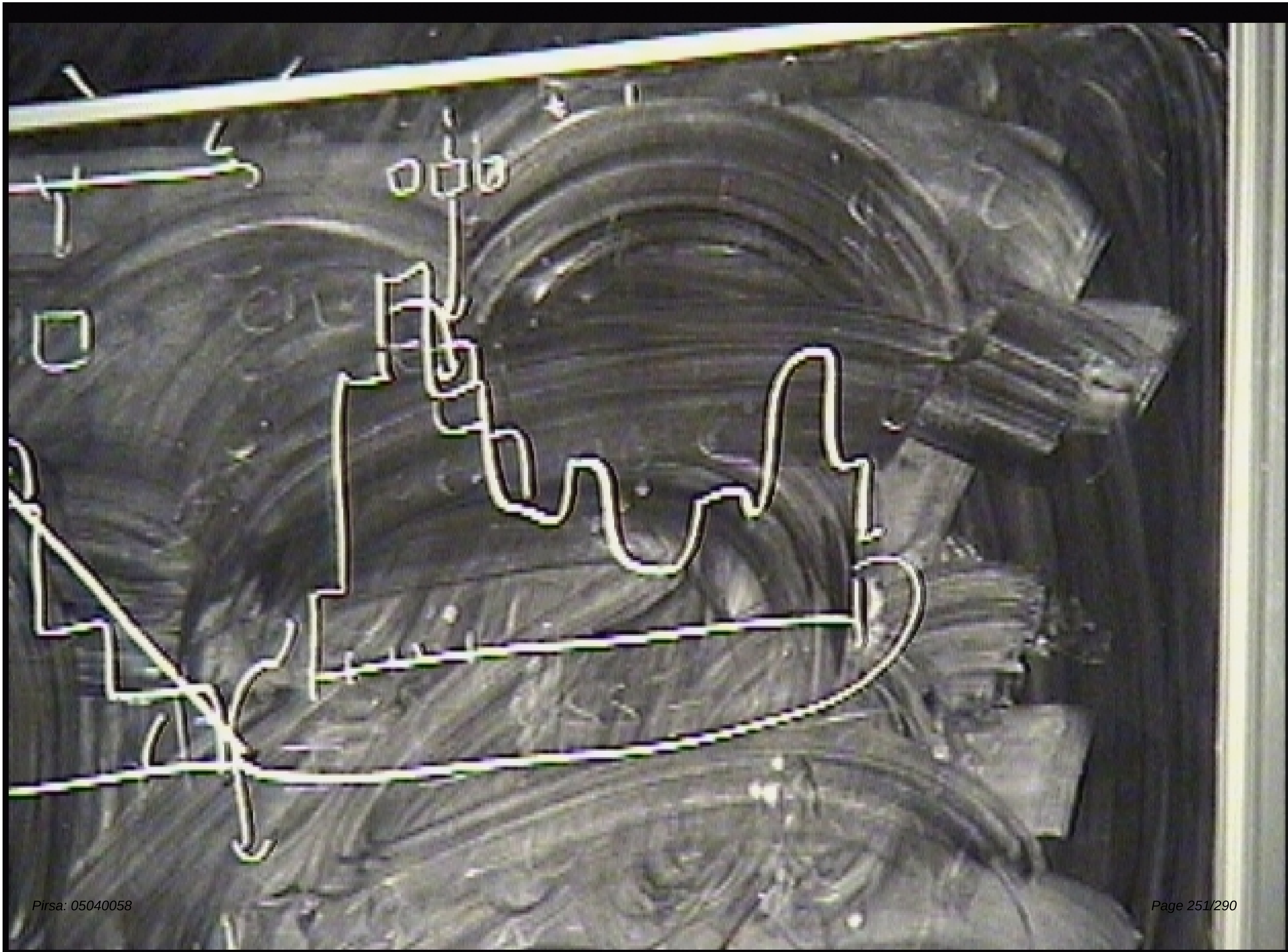
Place where grain is added (○) moves as a random walk





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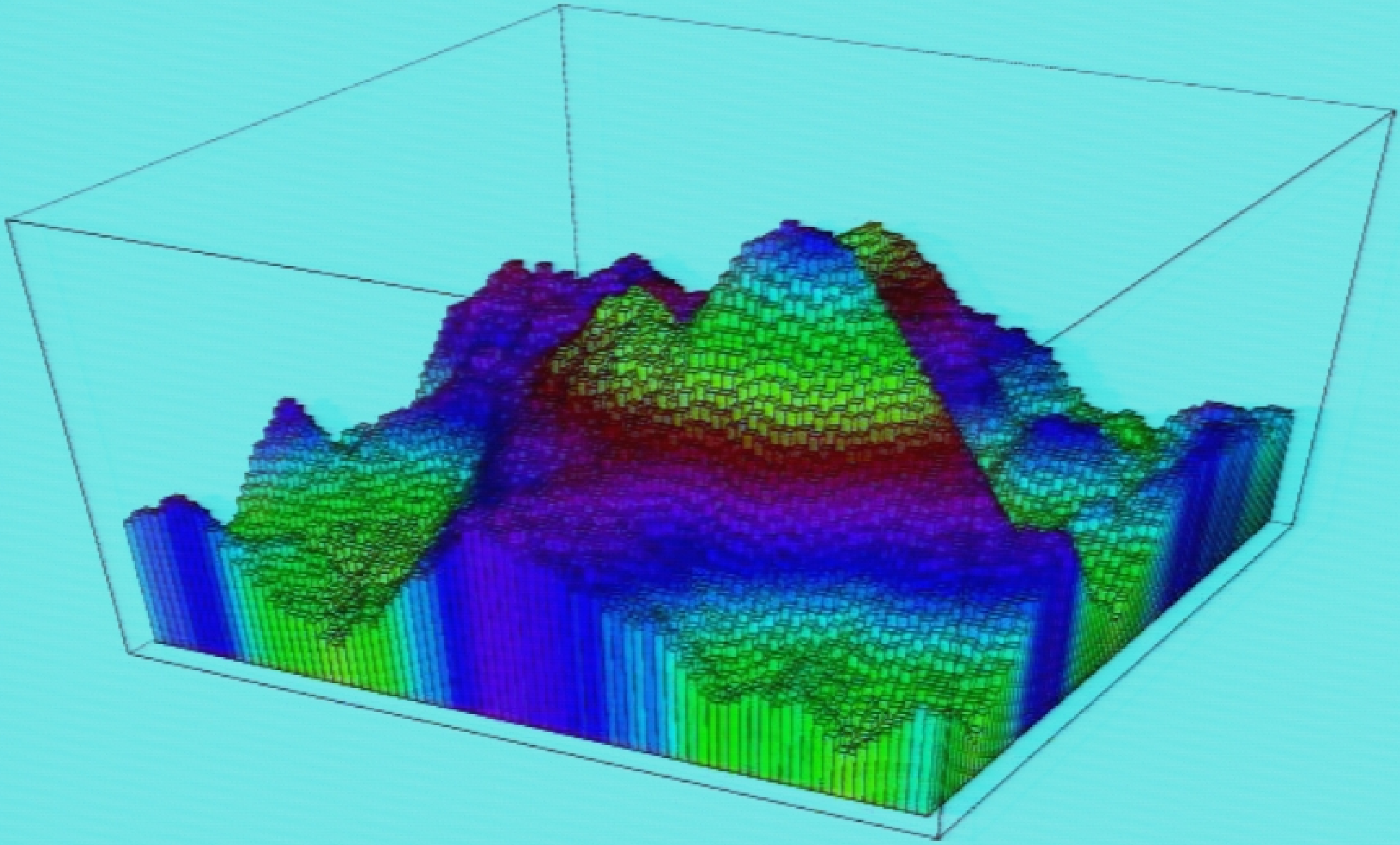
Model in 2d



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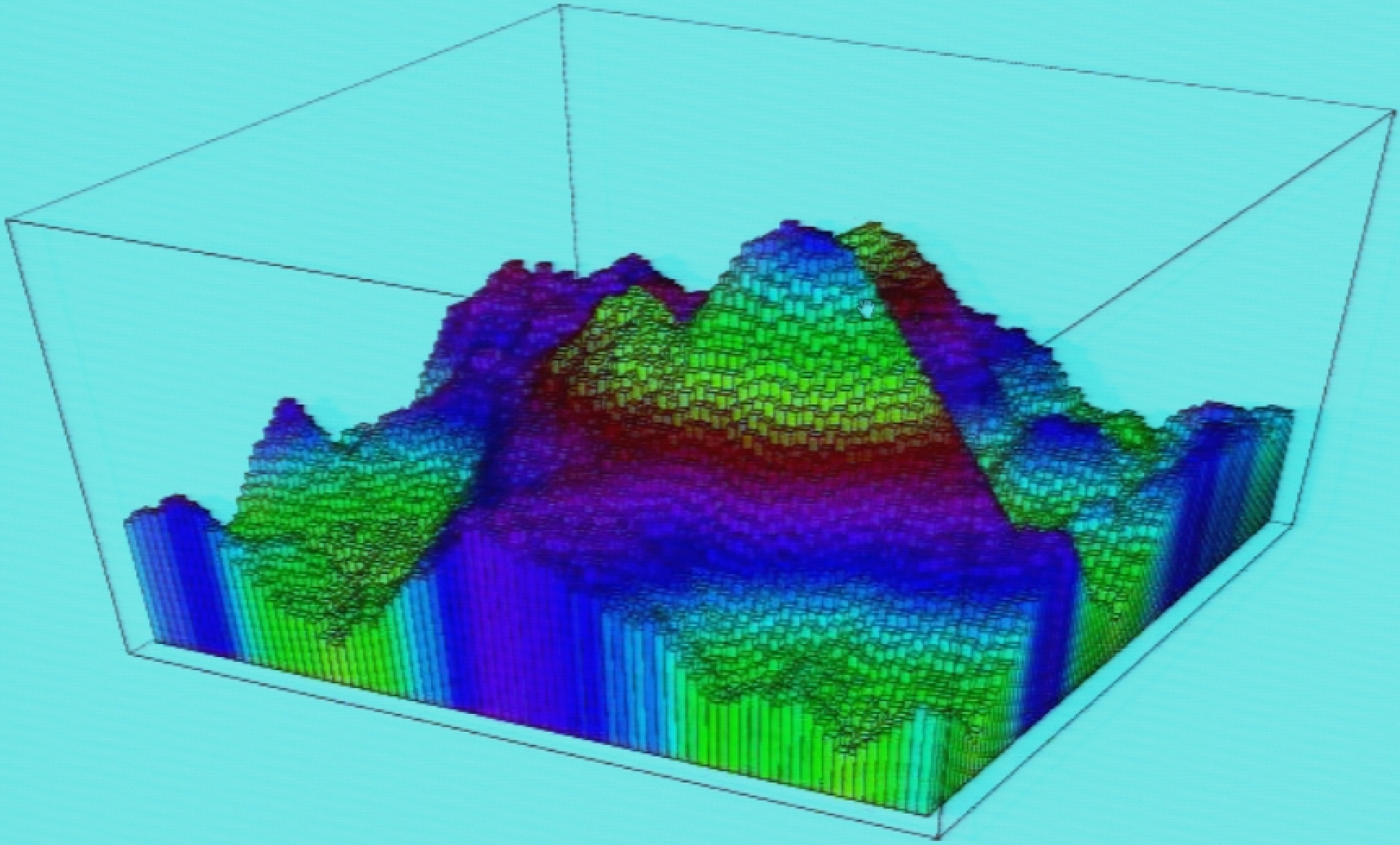

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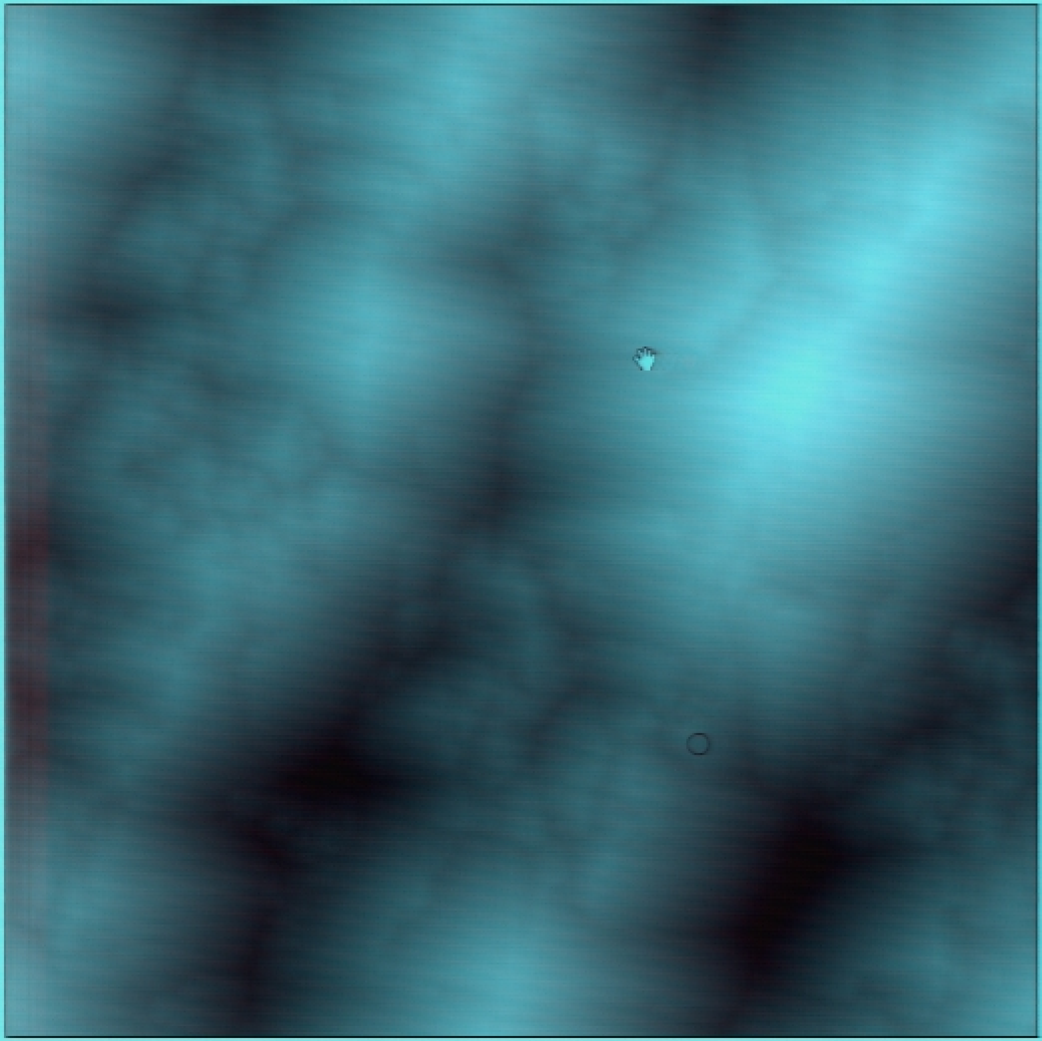

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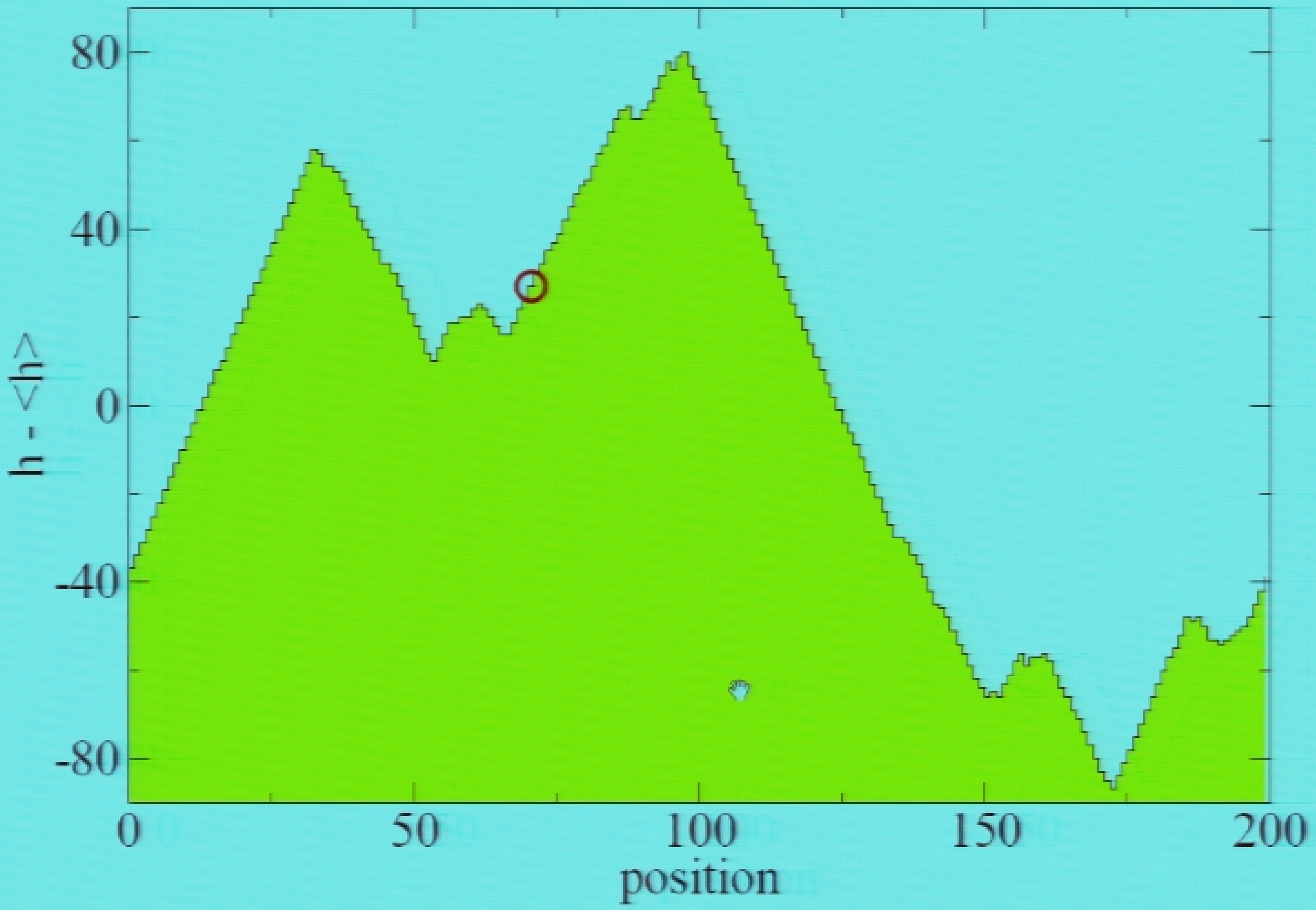

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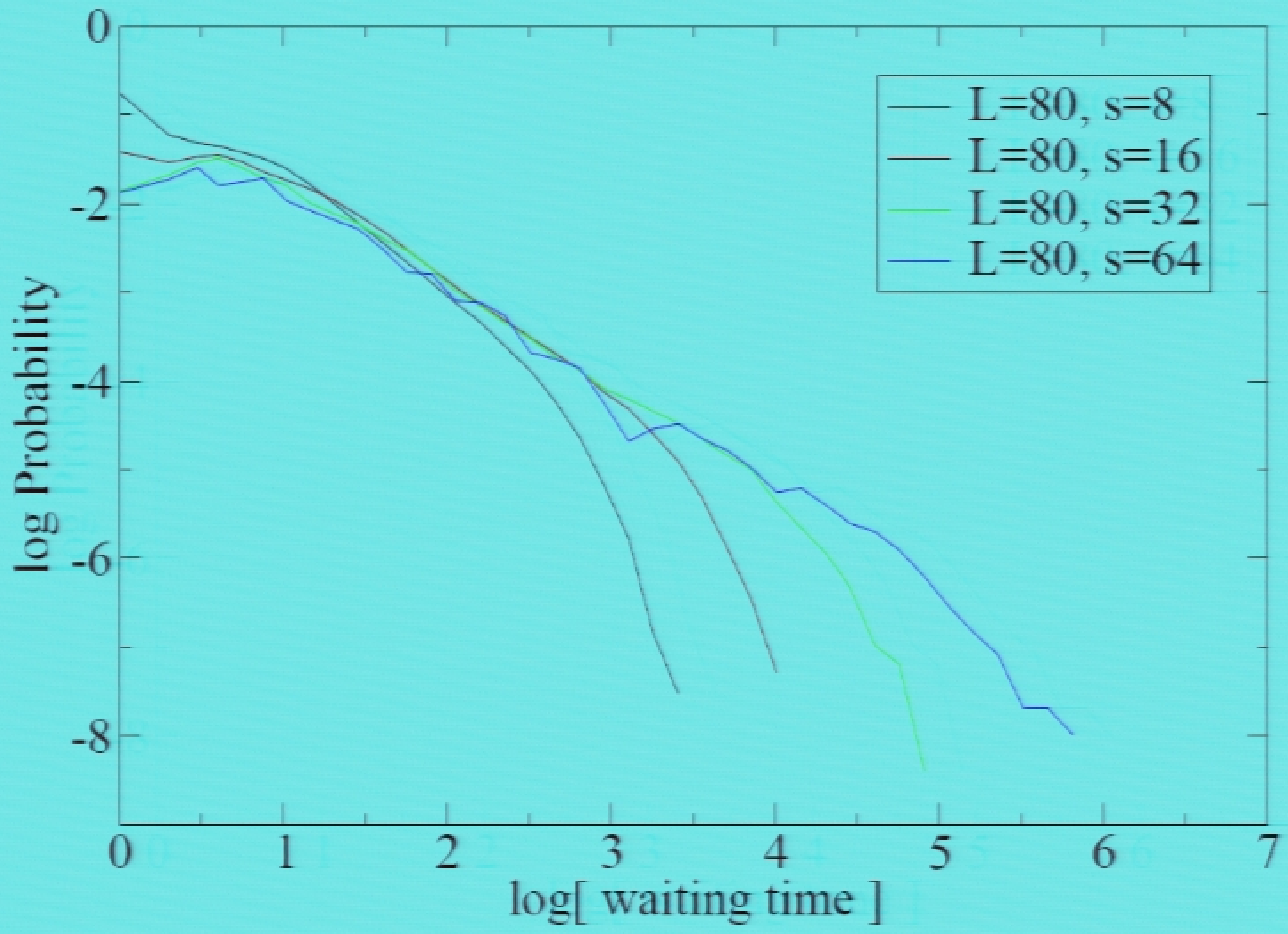

Complex correlations in self-organized critical phenomena

Conclusions

To further understand critical phenomena:

- Tools and ideas from statistical mechanics
- Novel approaches
- Simple models, but with features closer to reality
- Self-organized criticality

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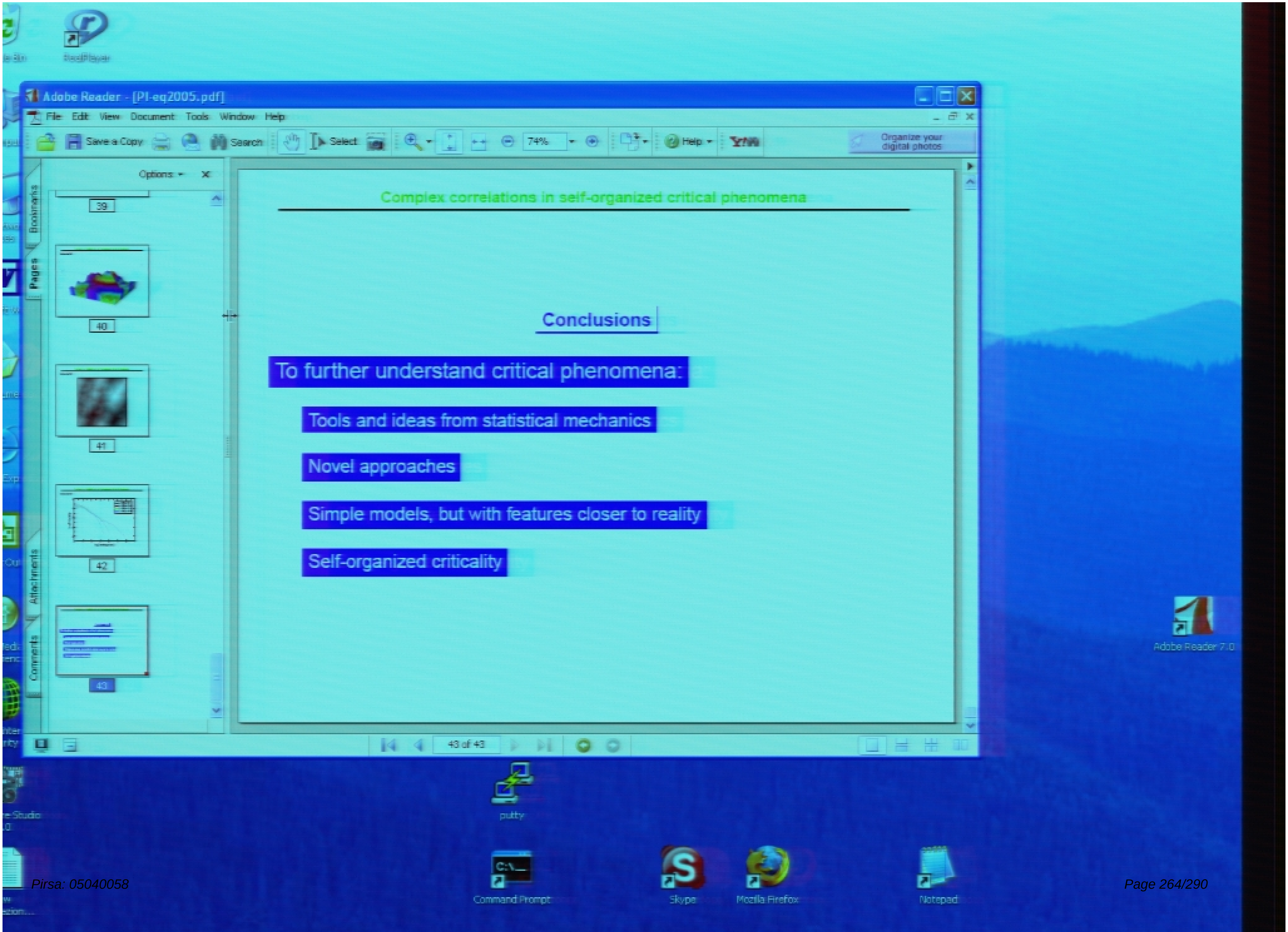

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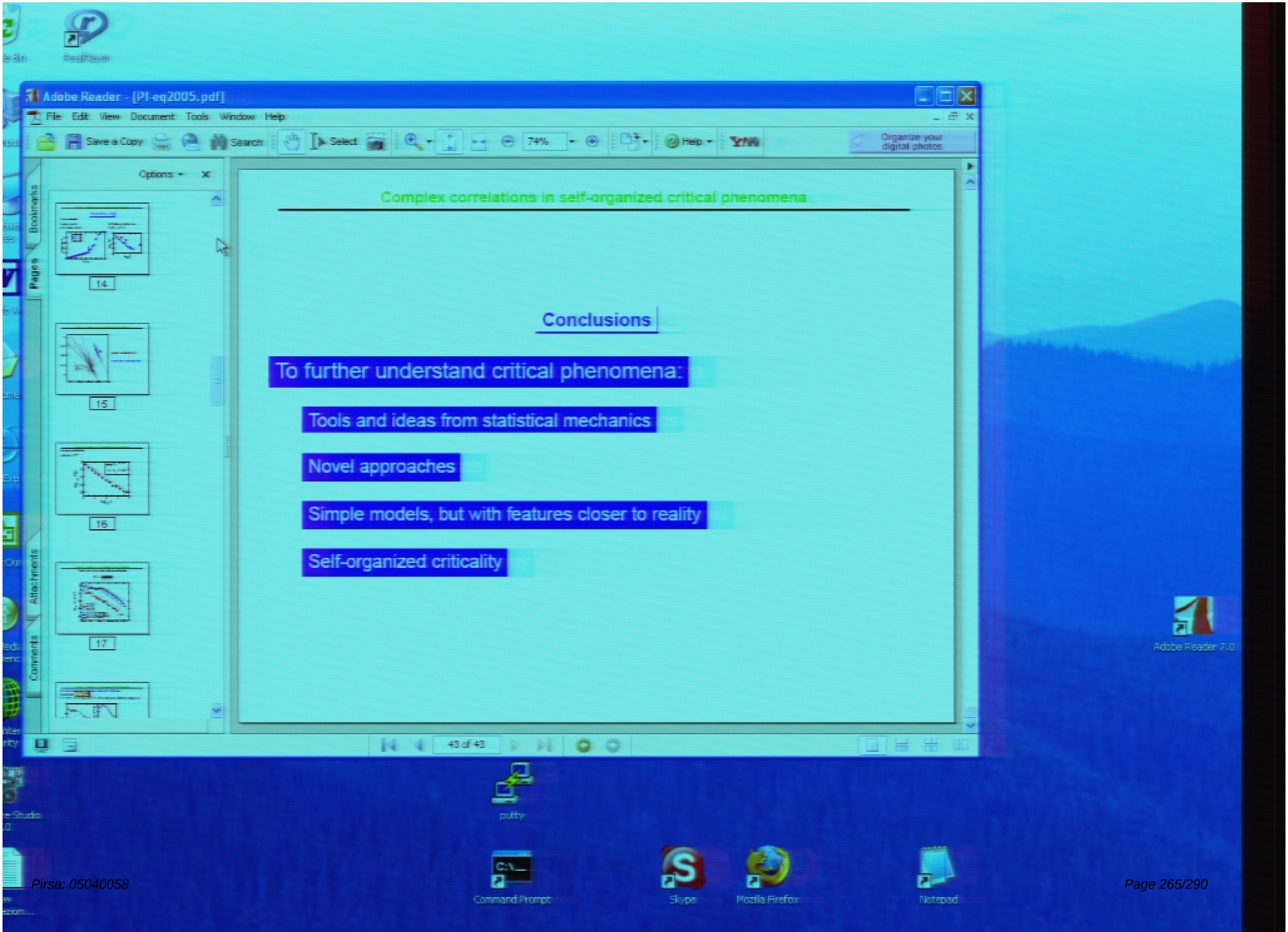

Complex correlations in self-organized critical phenomena

Conclusions

- To further understand critical phenomena:
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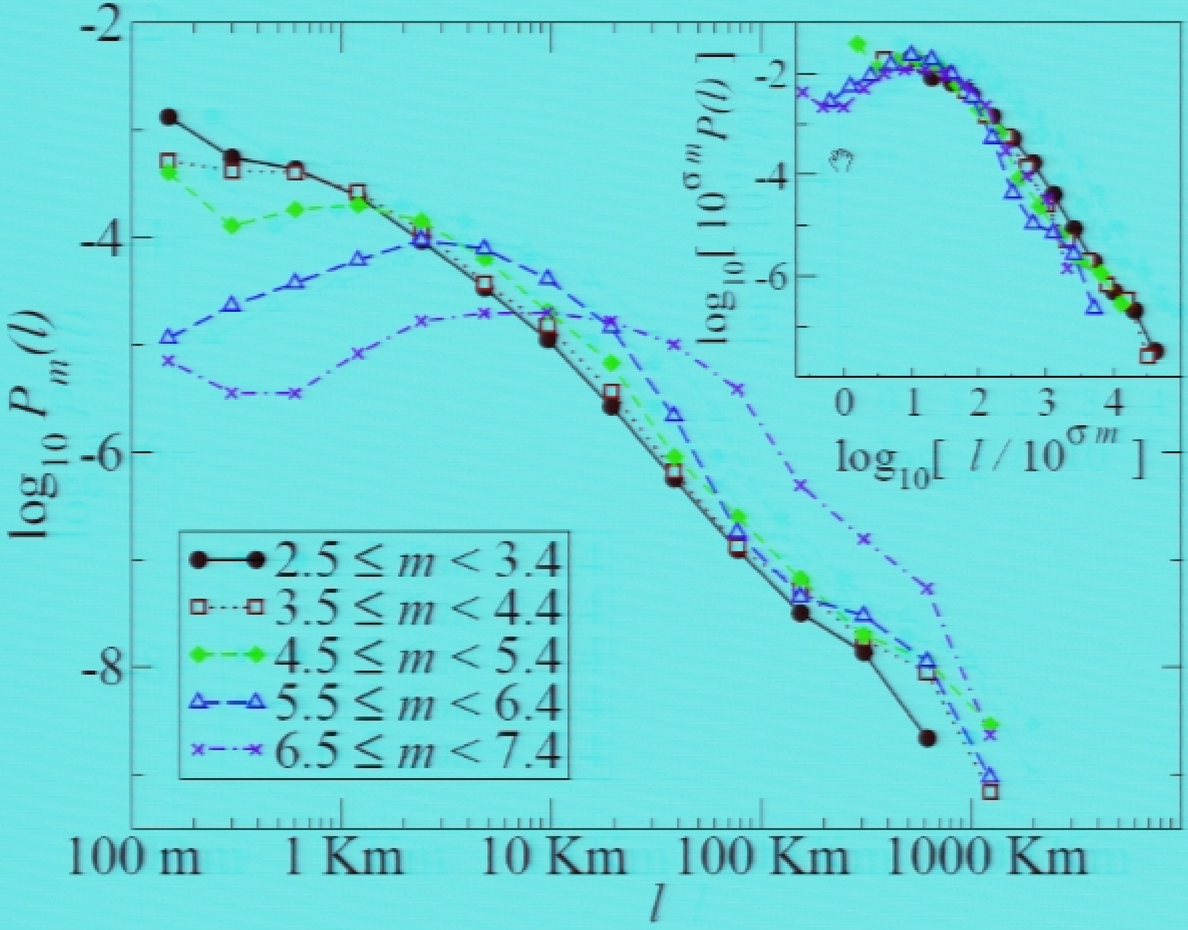
Complex correlations in self-organized critical phenomena

New phenomenology: Distribution of aftershock distances

typical length $\boxed{l_m \sim 10^{\sigma m}}$

$\sigma = 0.3 \div 0.4 \neq$ "classical" $\sigma = 1/2$ of aftershocks within the rupture area



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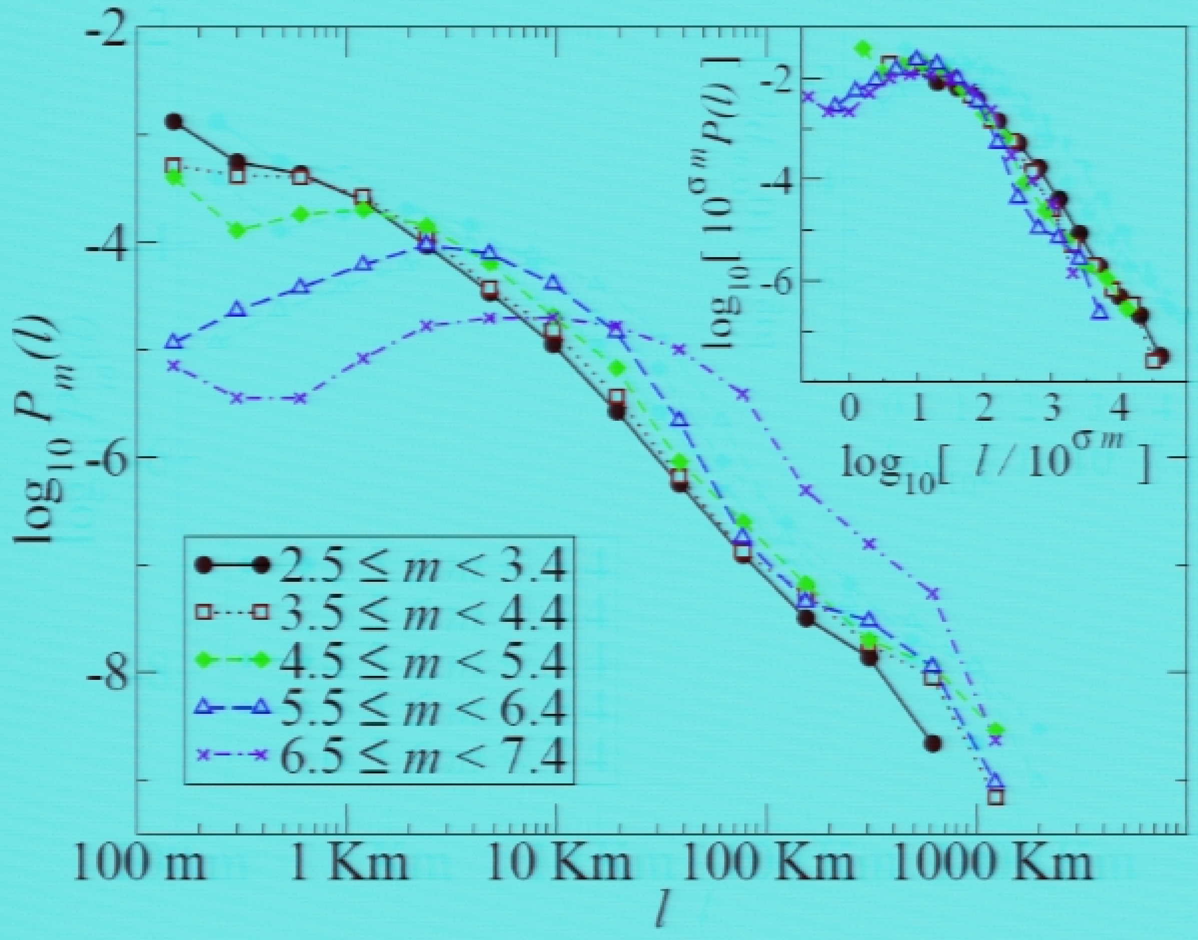

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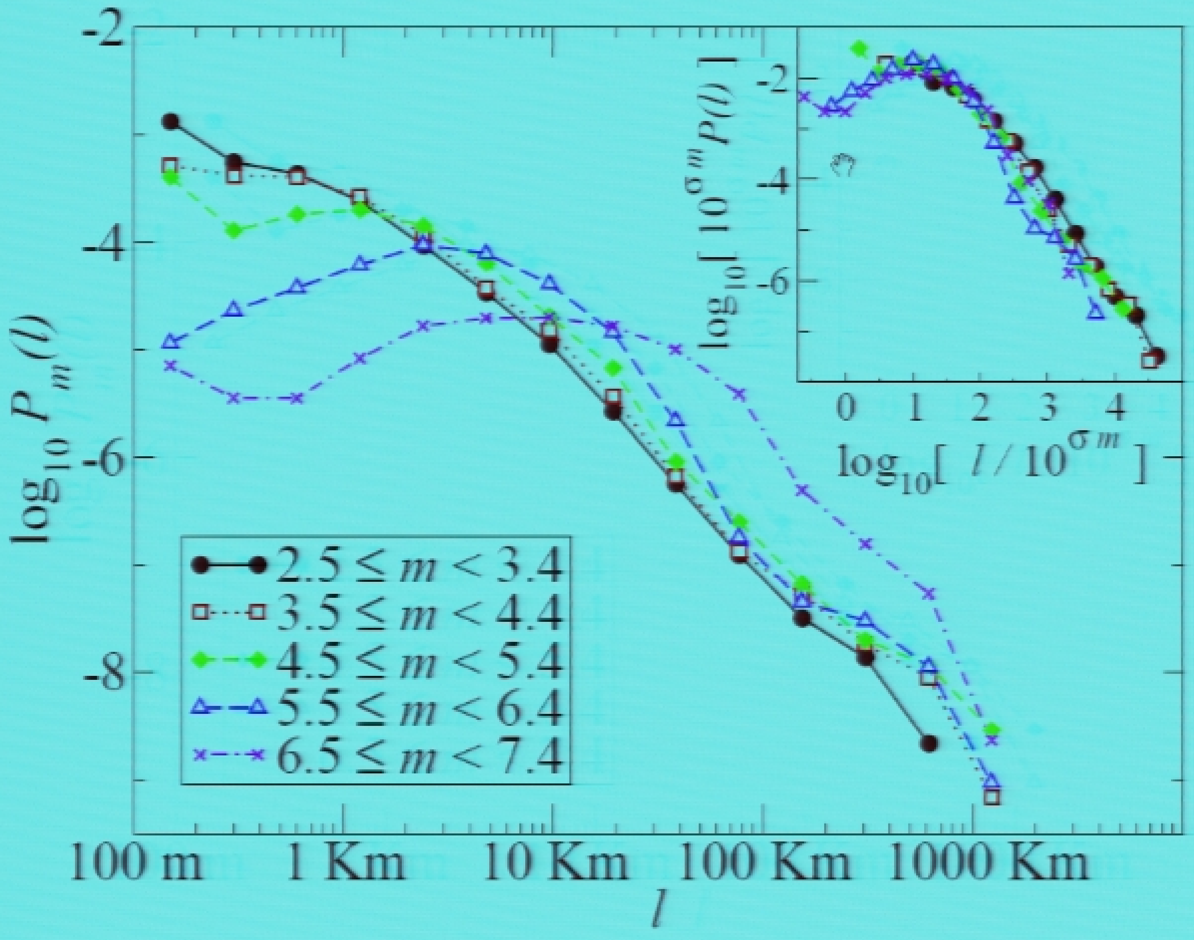

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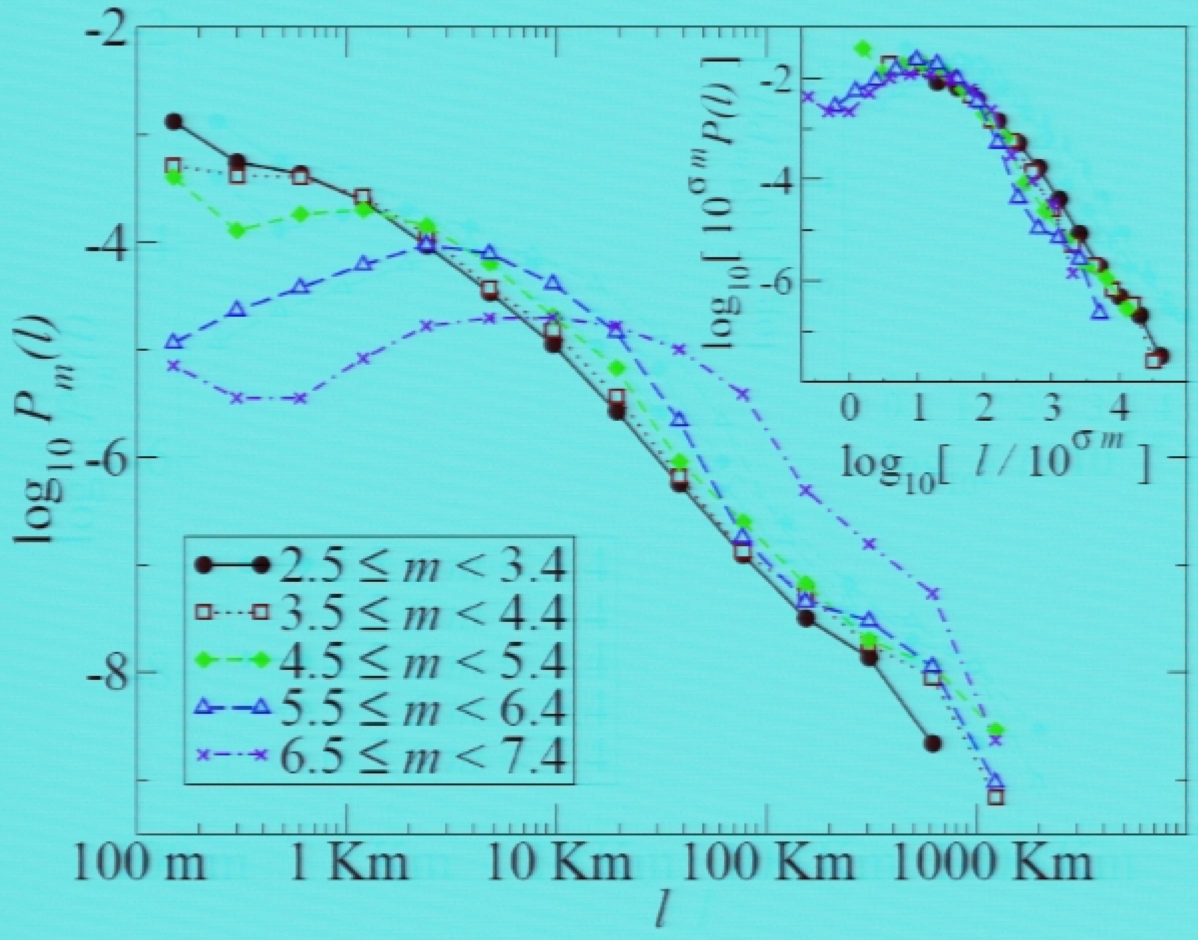

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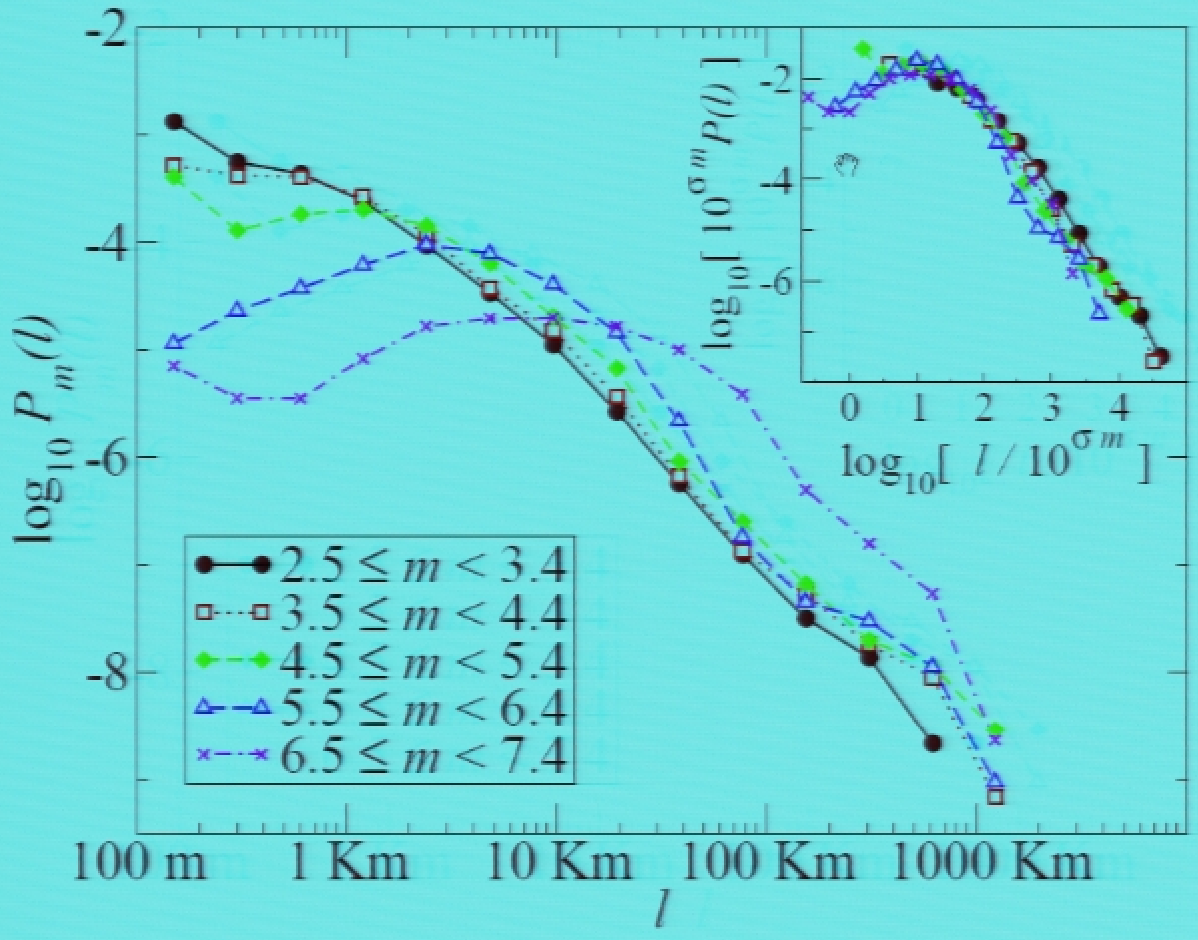

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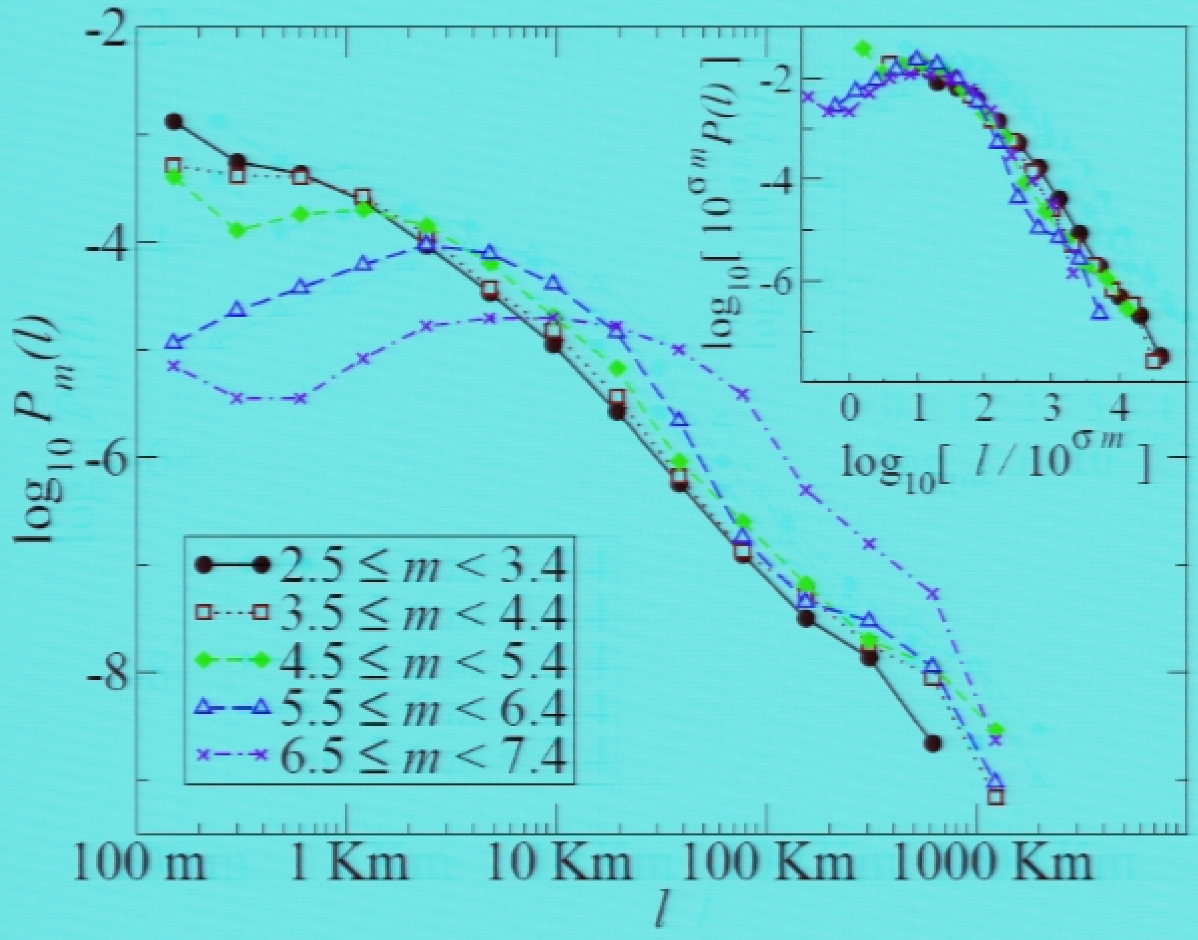

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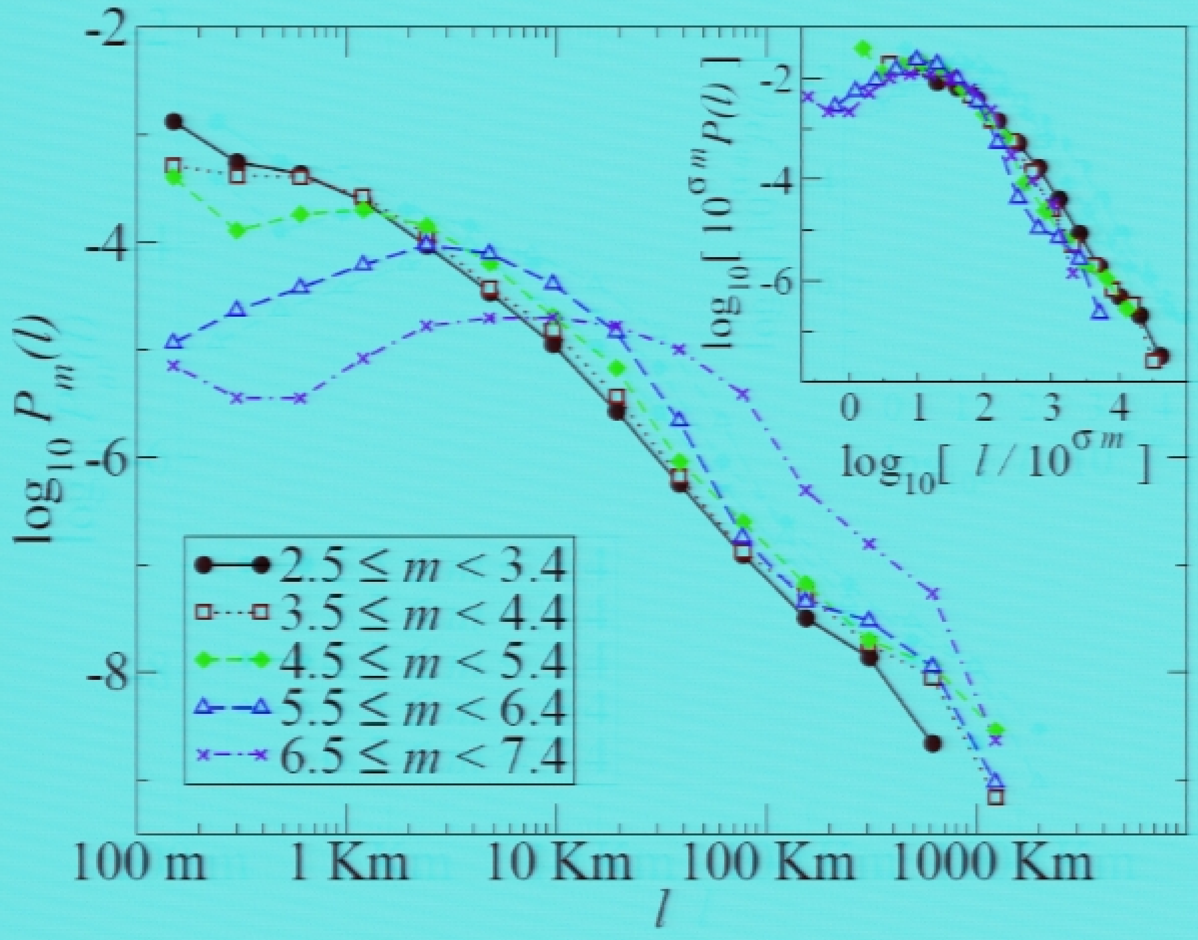

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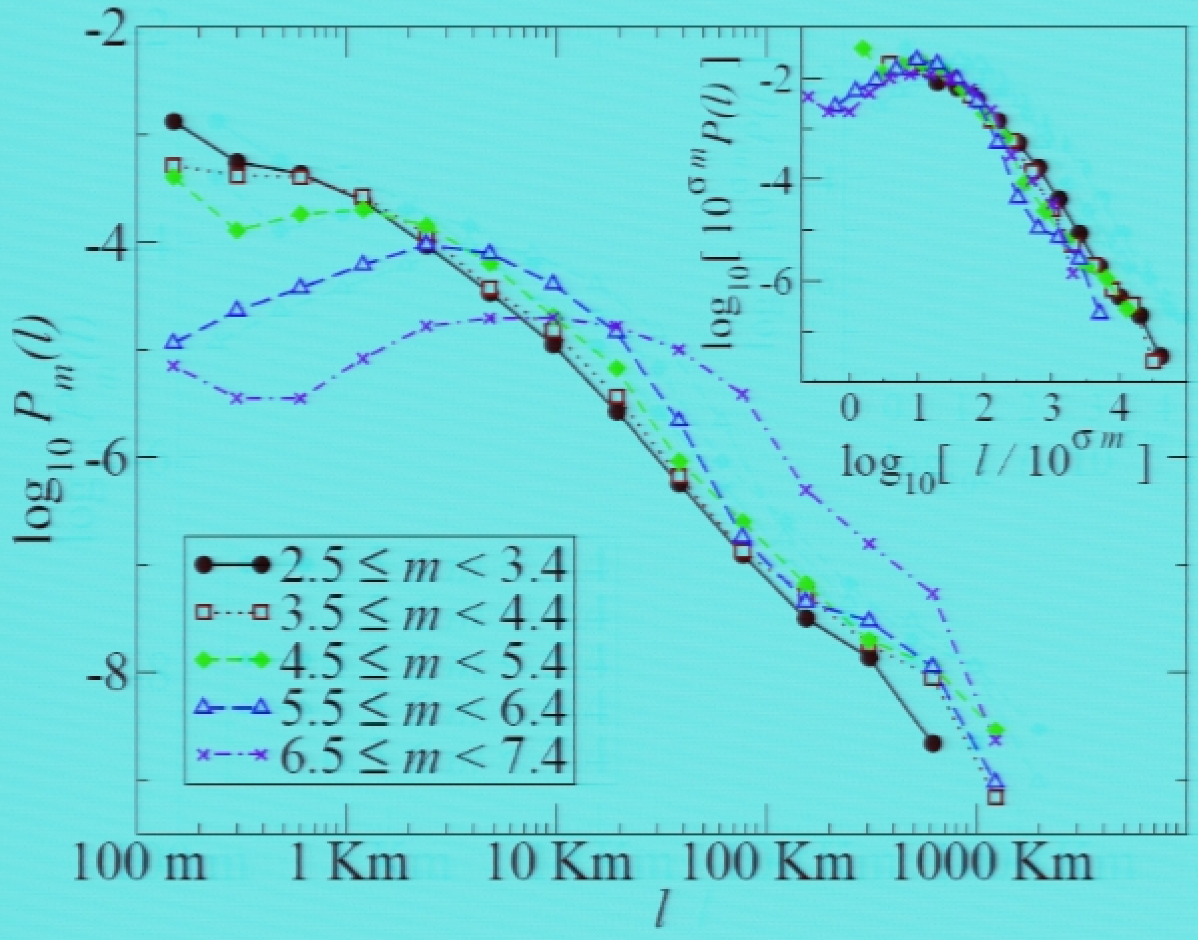

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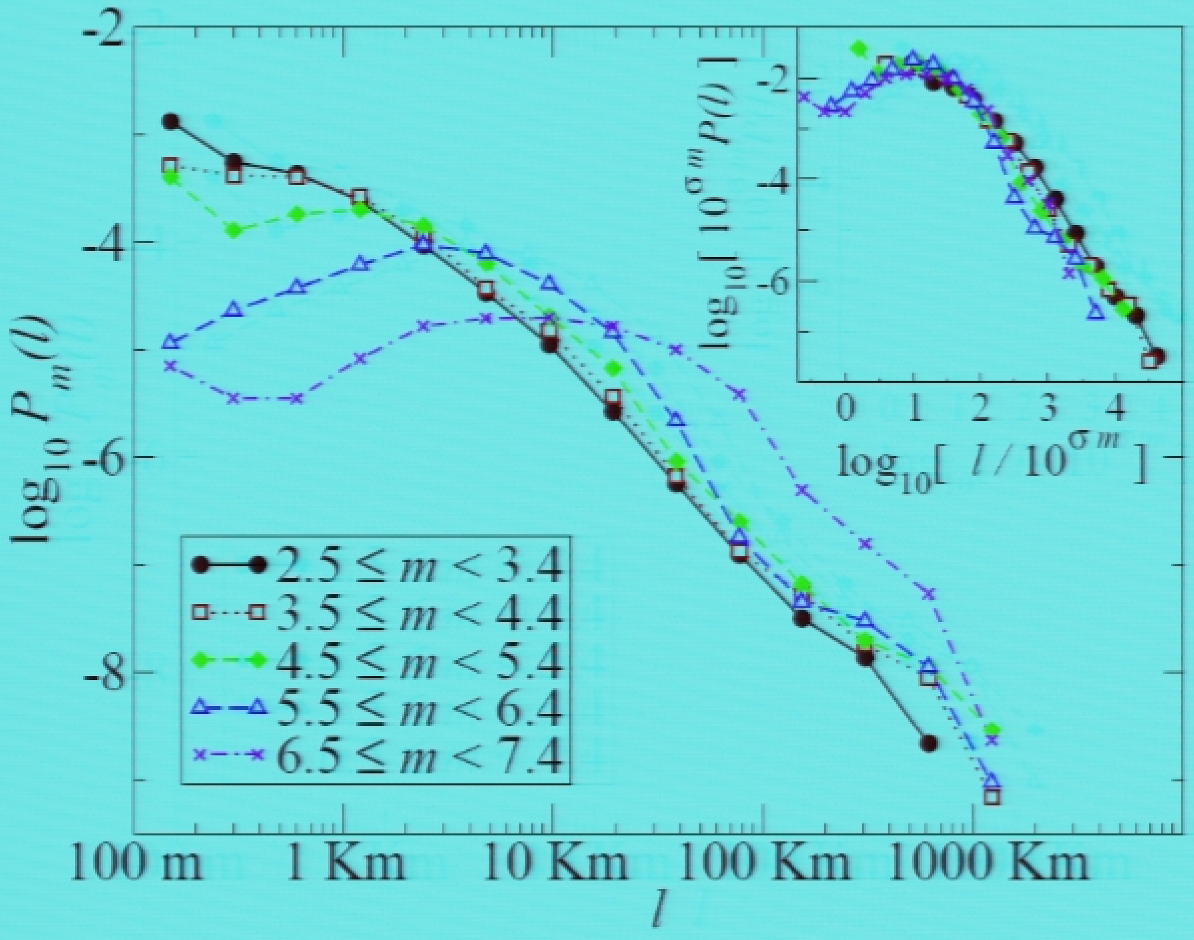

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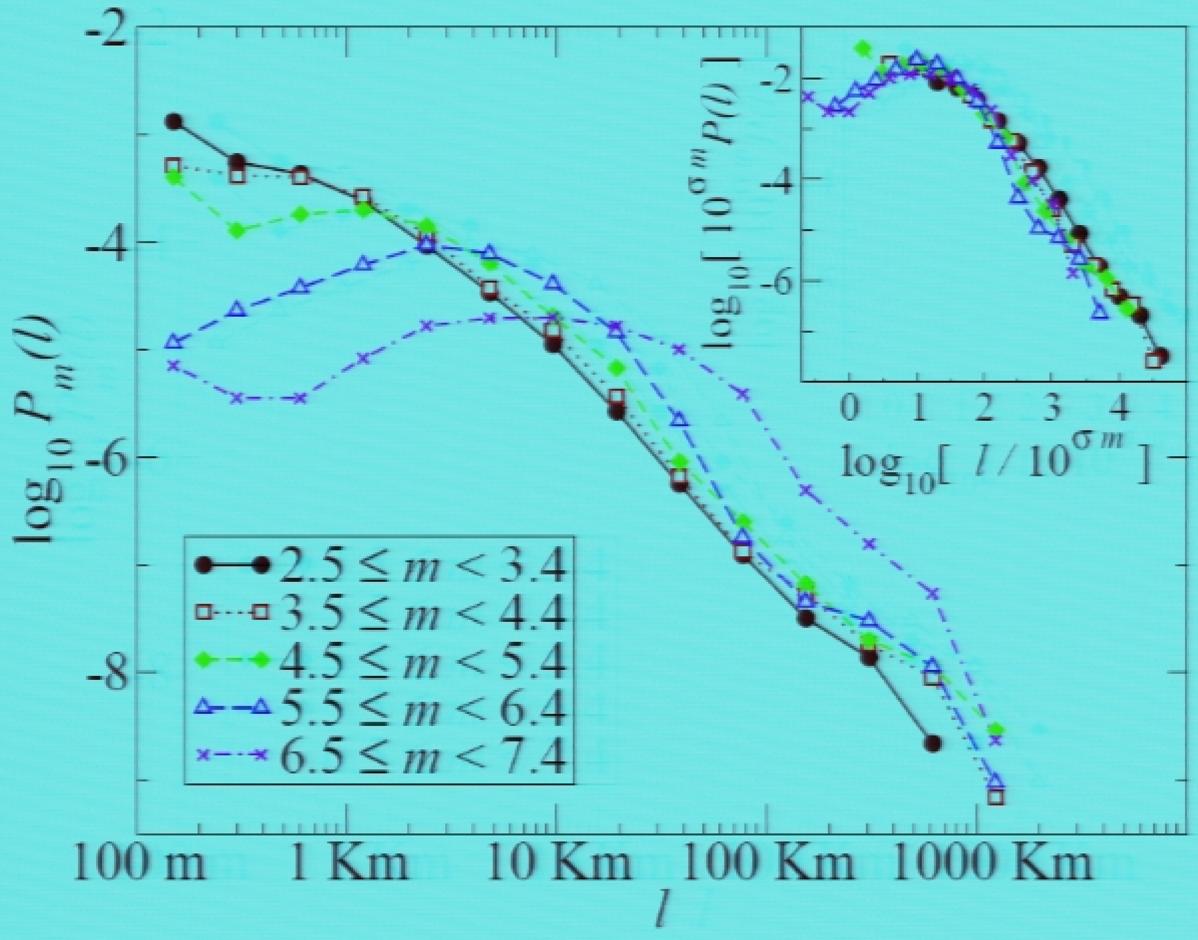

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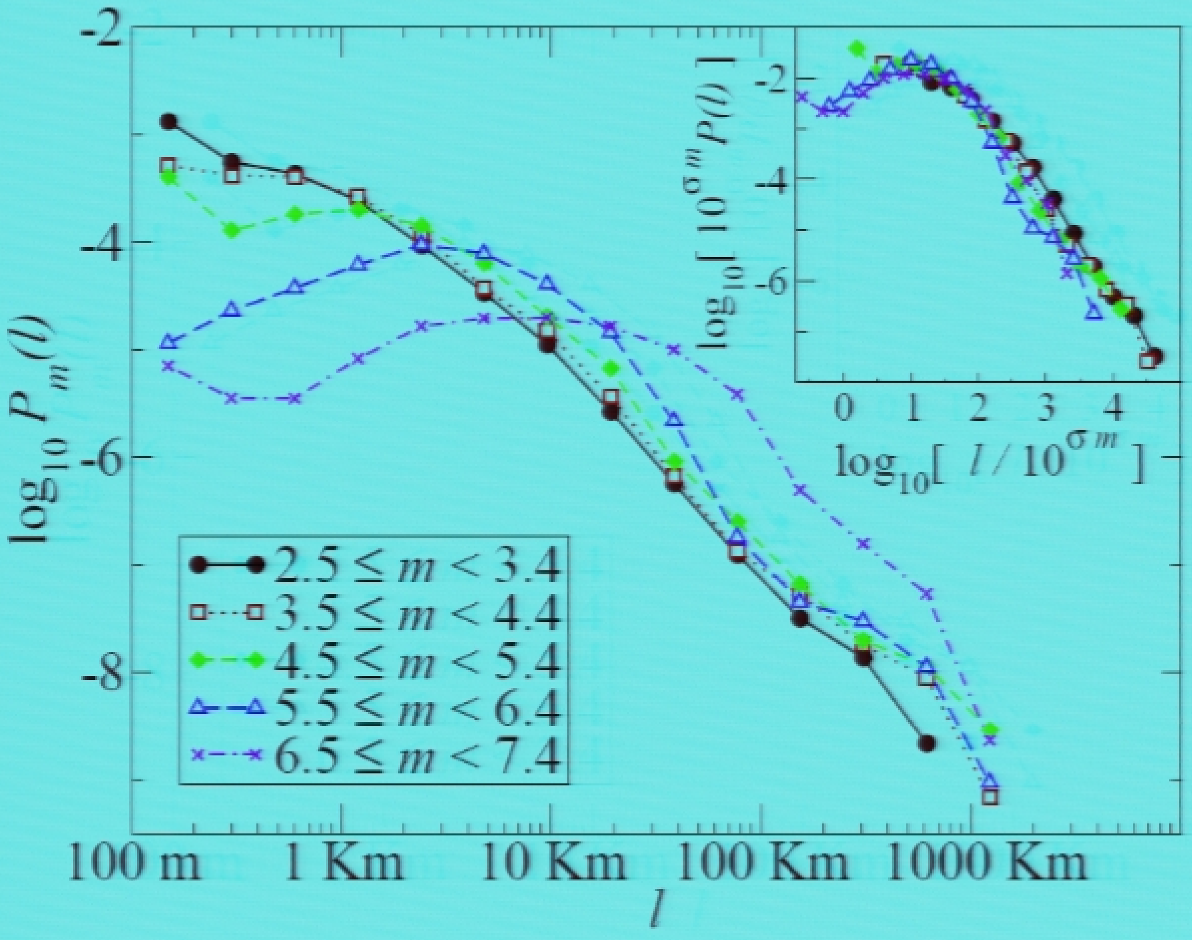

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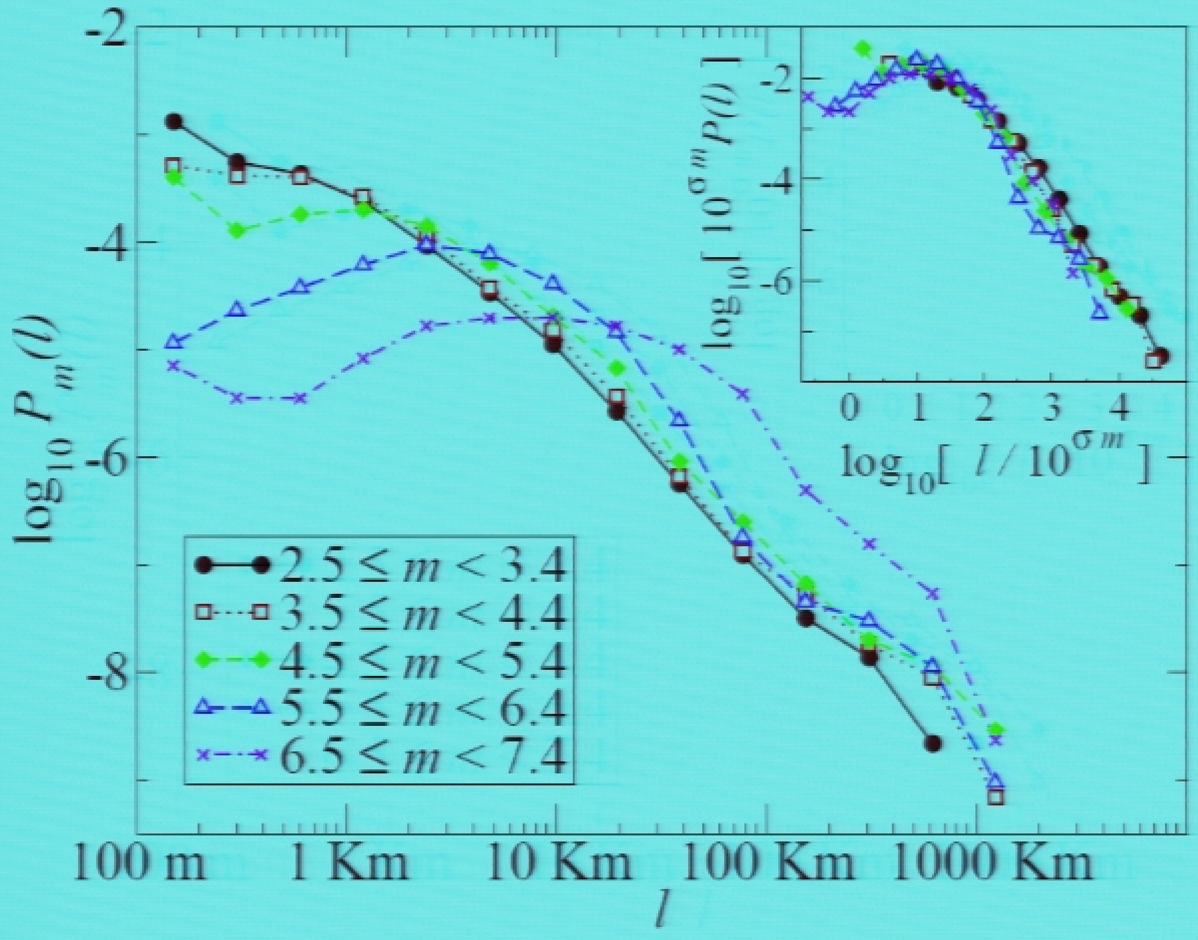

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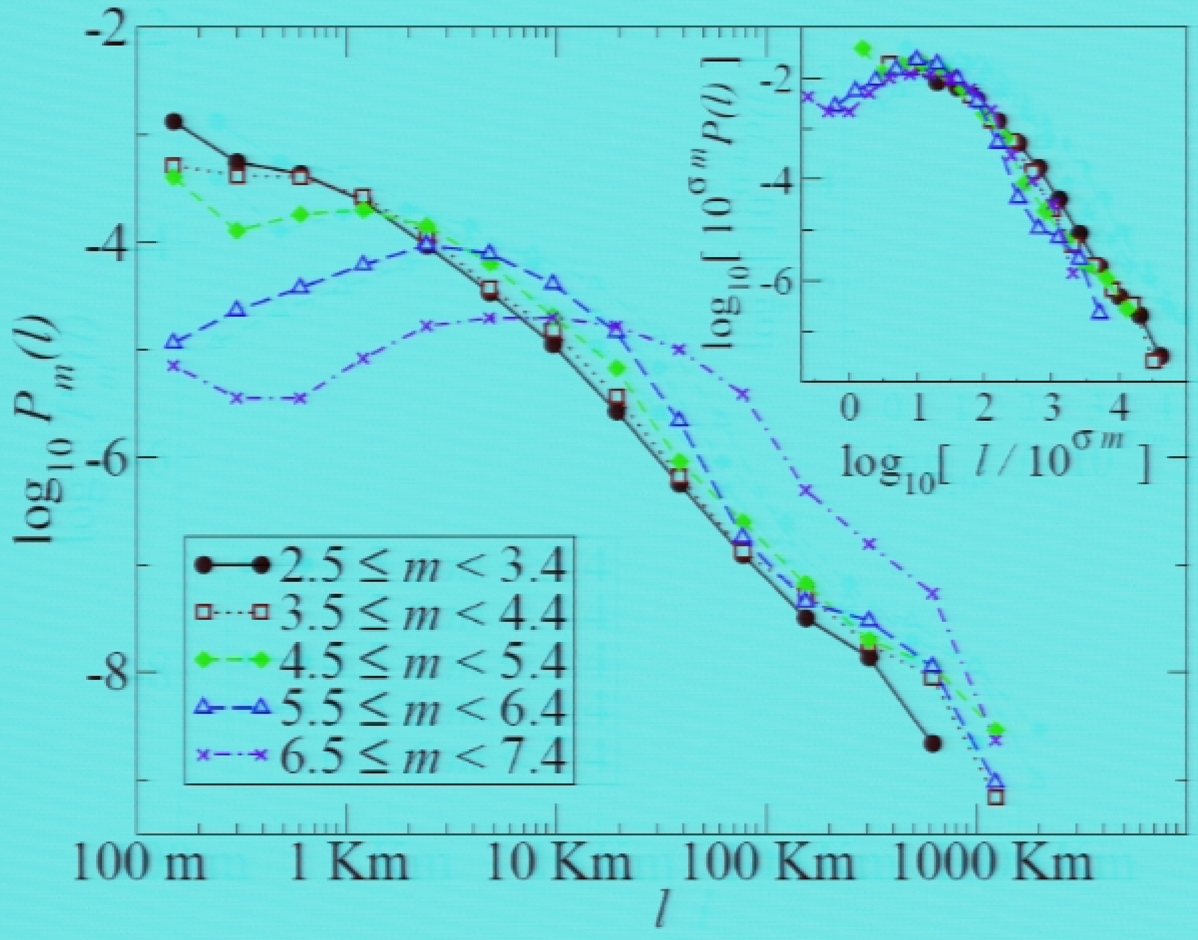

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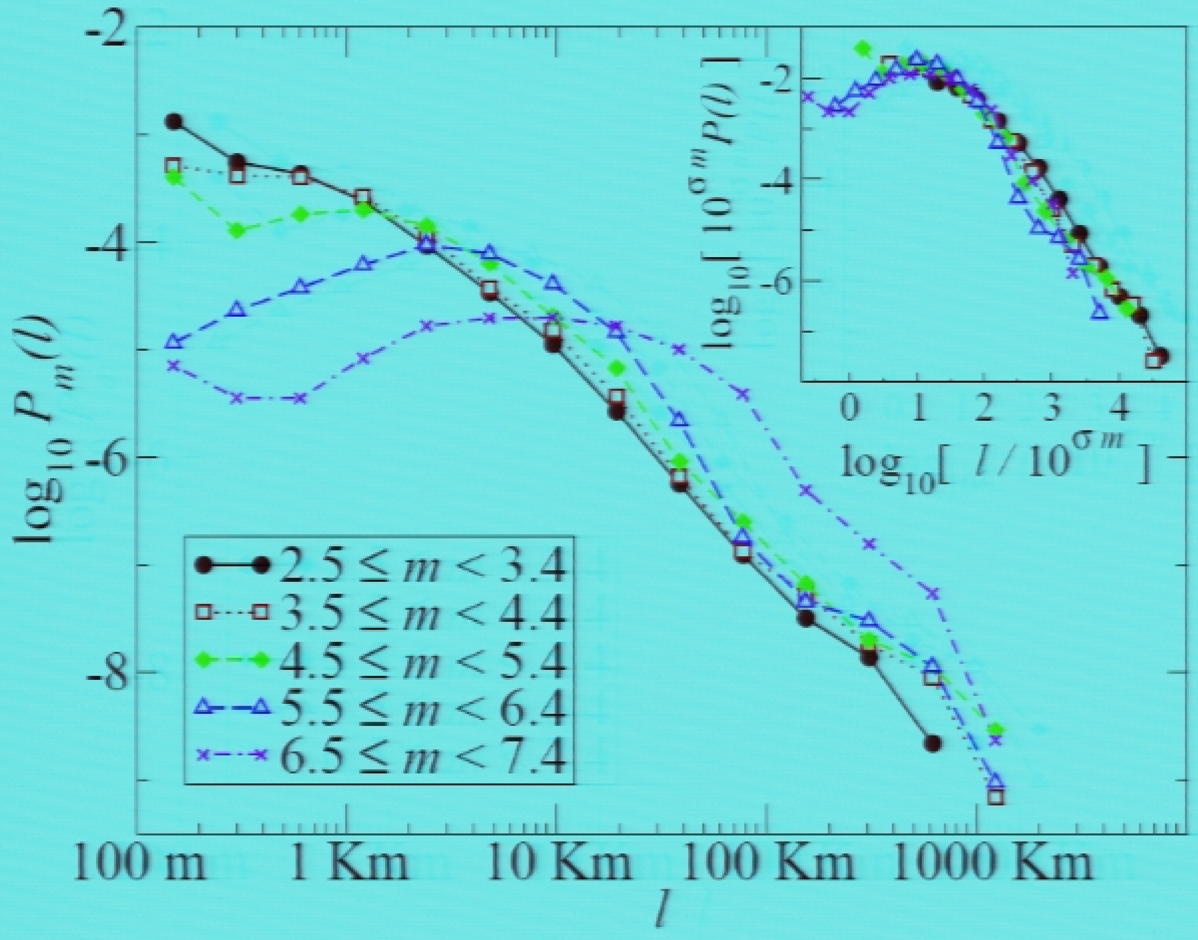

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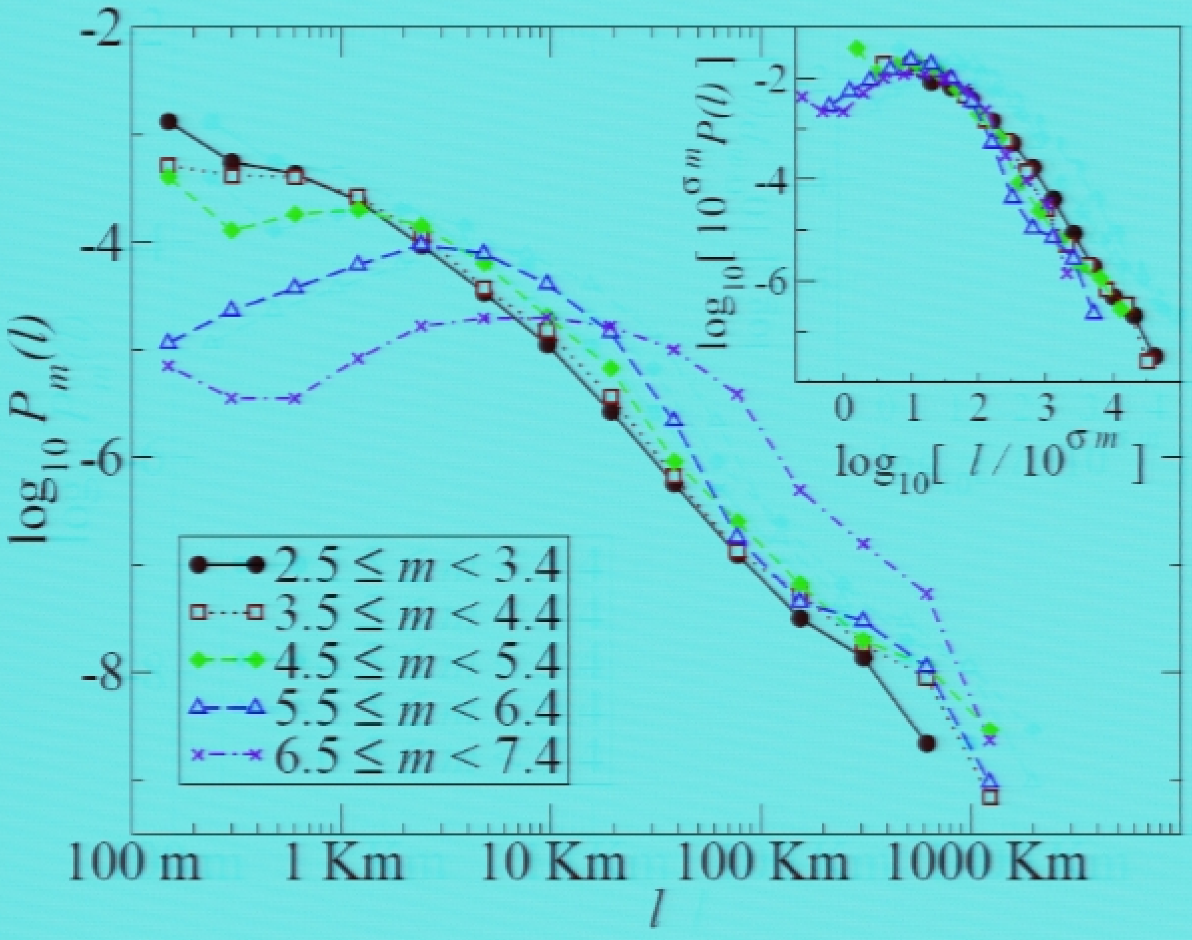

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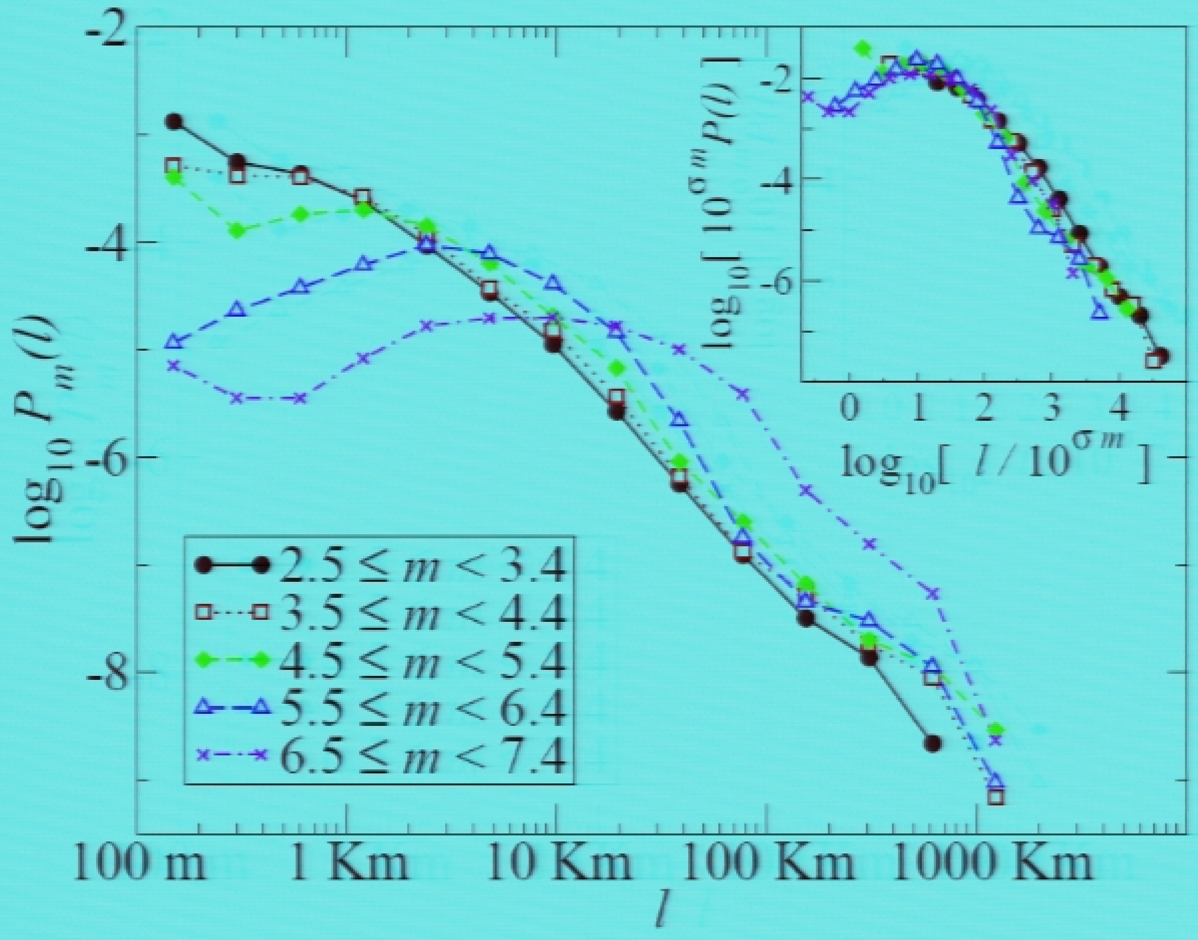

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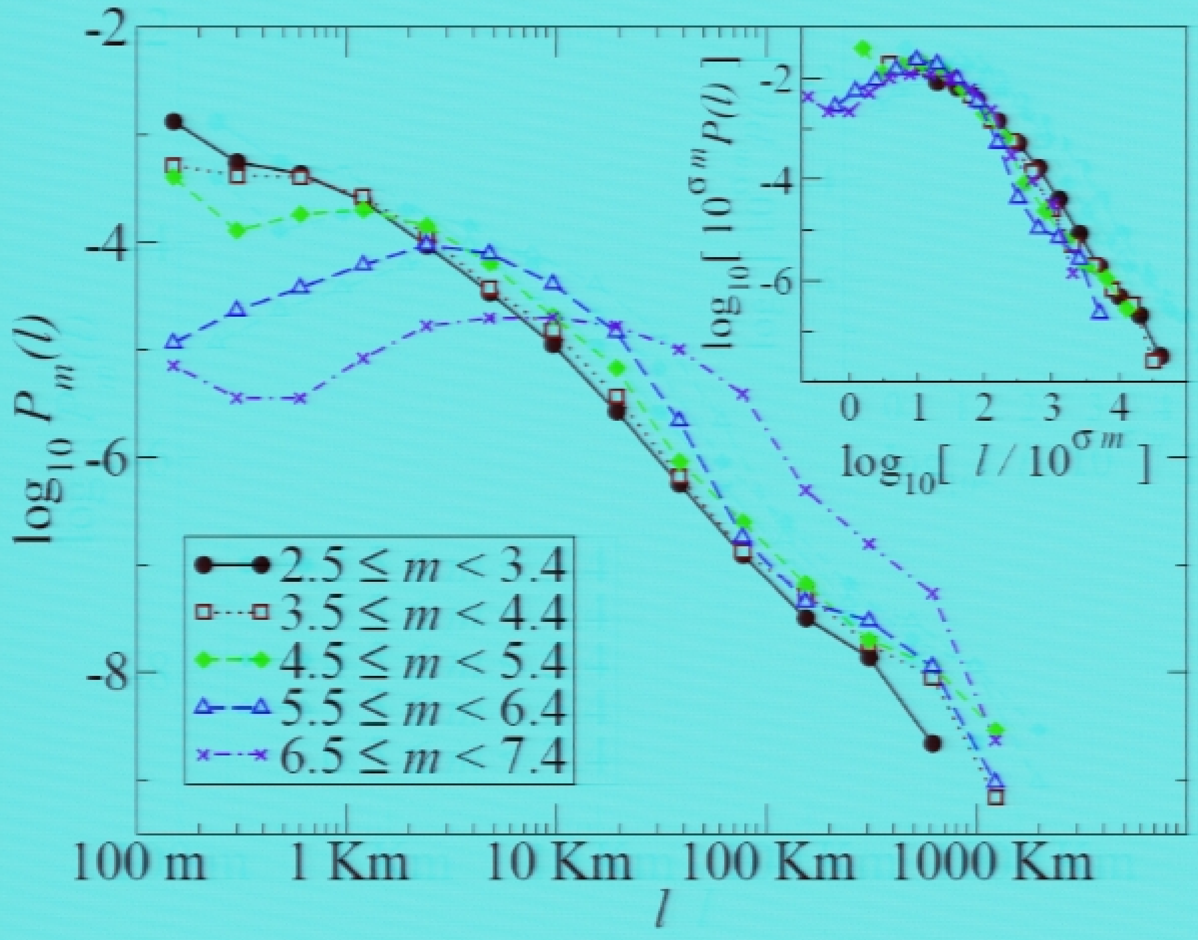

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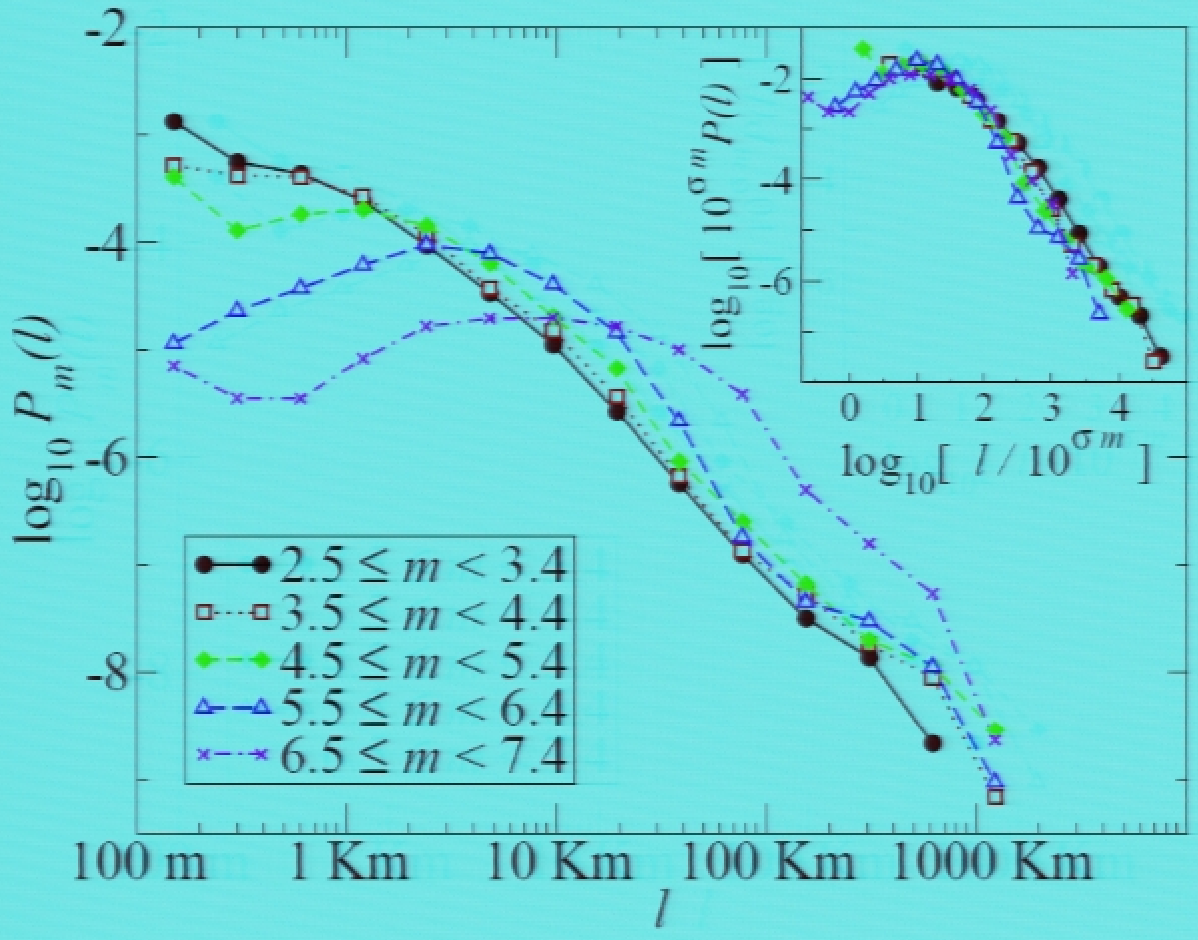

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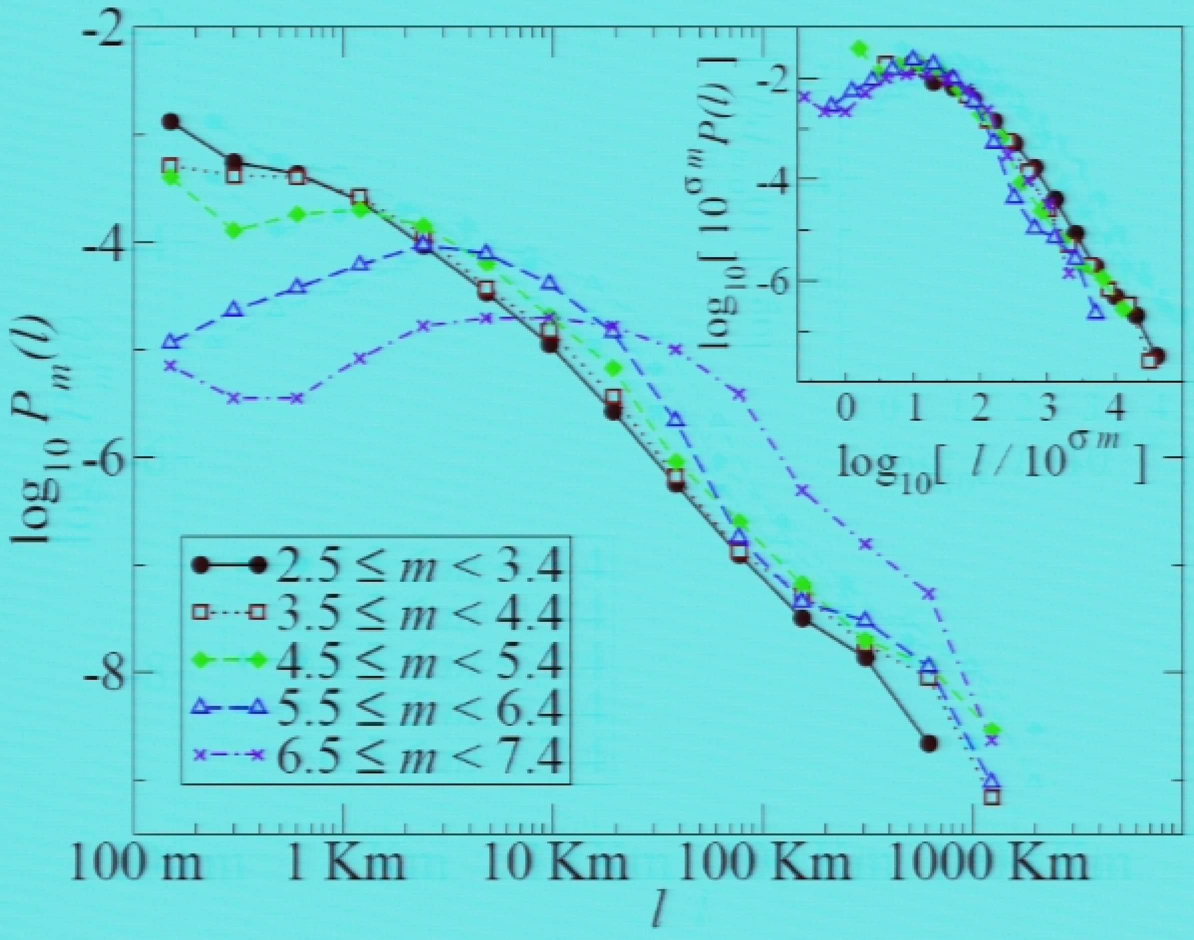

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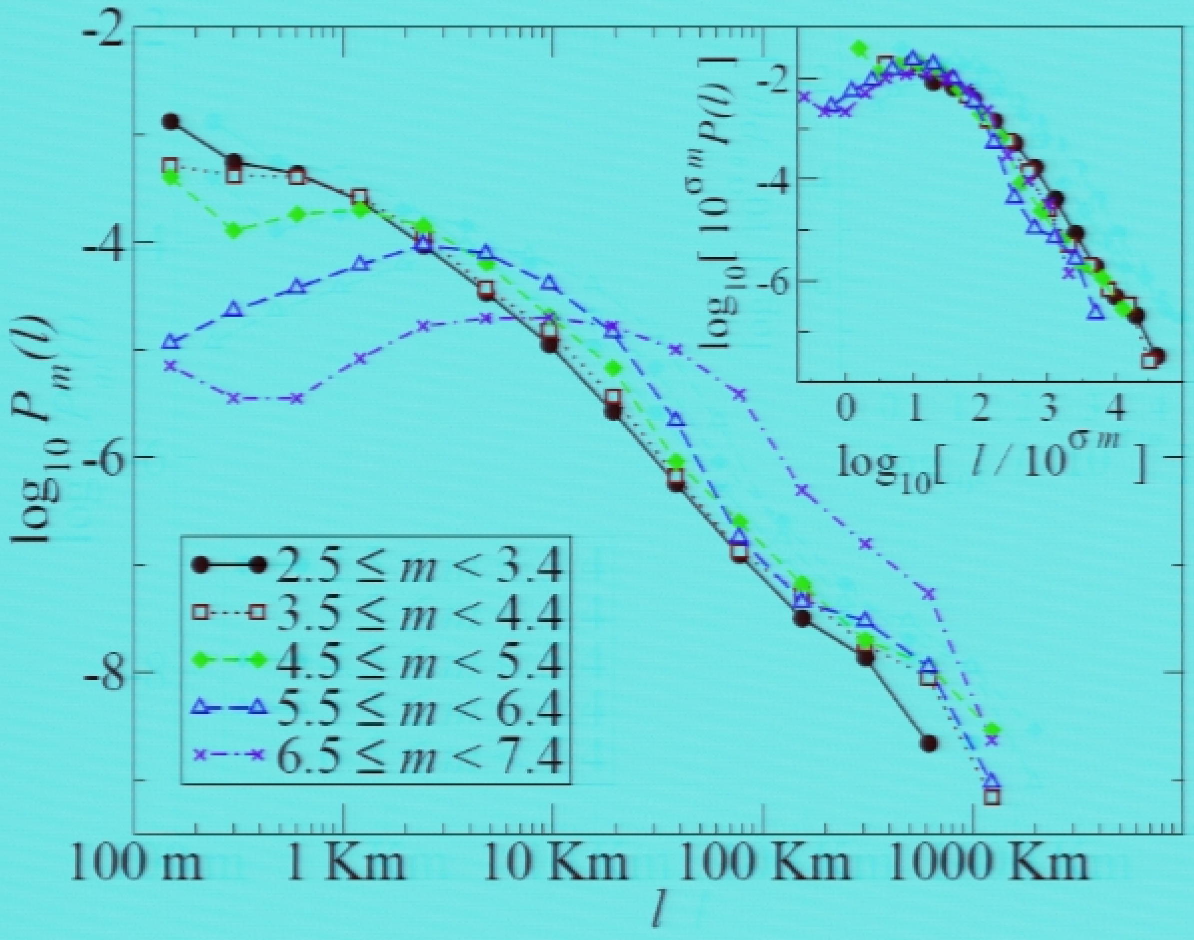

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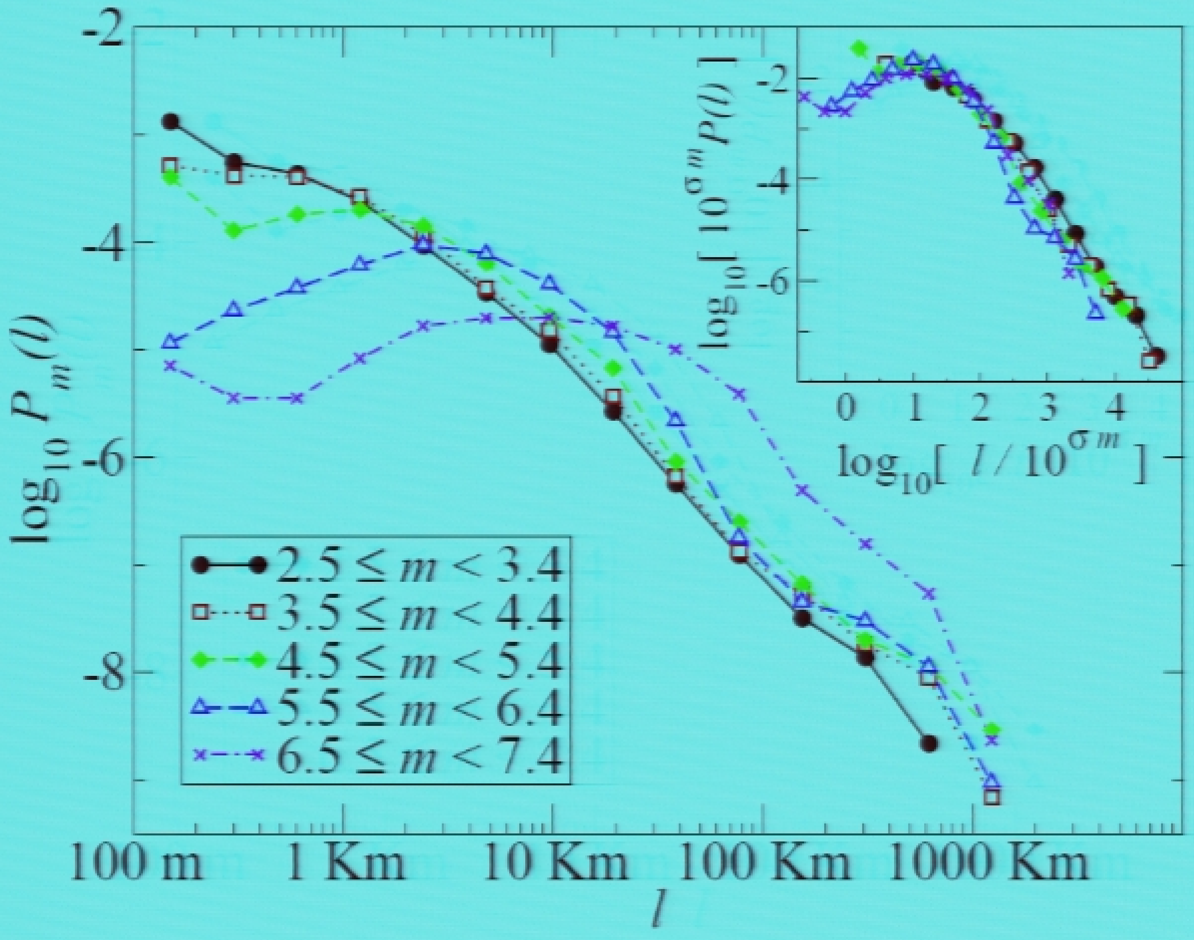

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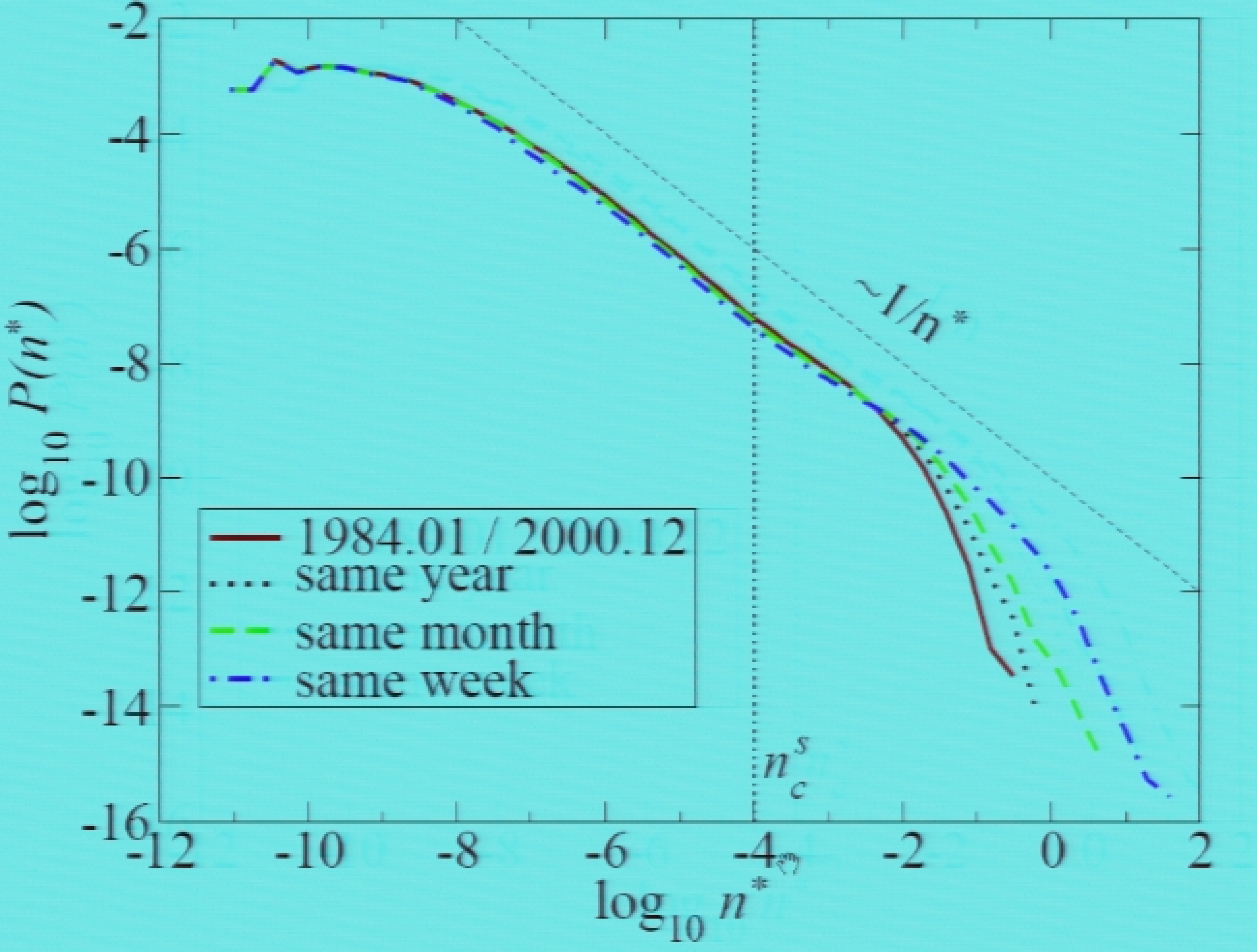
Draw only one link to a new event j: largest c_{ij} ($=$ or smallest $n_{ij} \equiv n_j^*$)



$P(n^*) \sim (n^*)^{-1}$

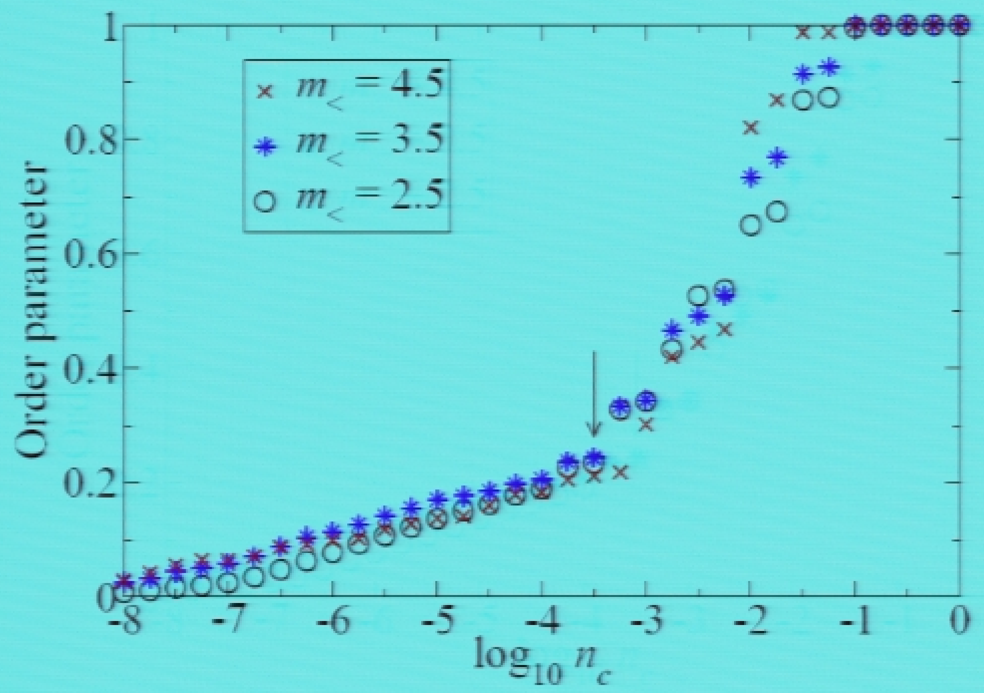
Complex correlations in self-organized critical phenomena

Put a link if $n_{ij} < n_c$

Order parameter:

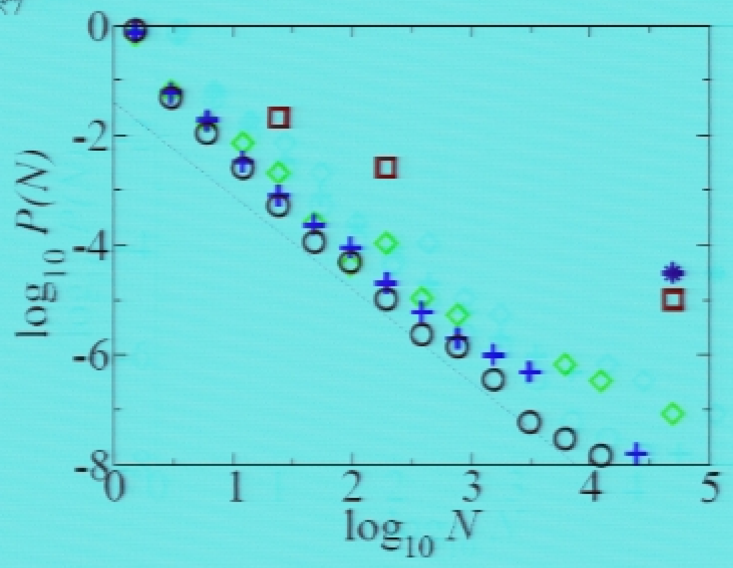
fraction of nodes

in the biggest cluster



Distribution of cluster sizes

$P(N) \sim N^{-1.7}$



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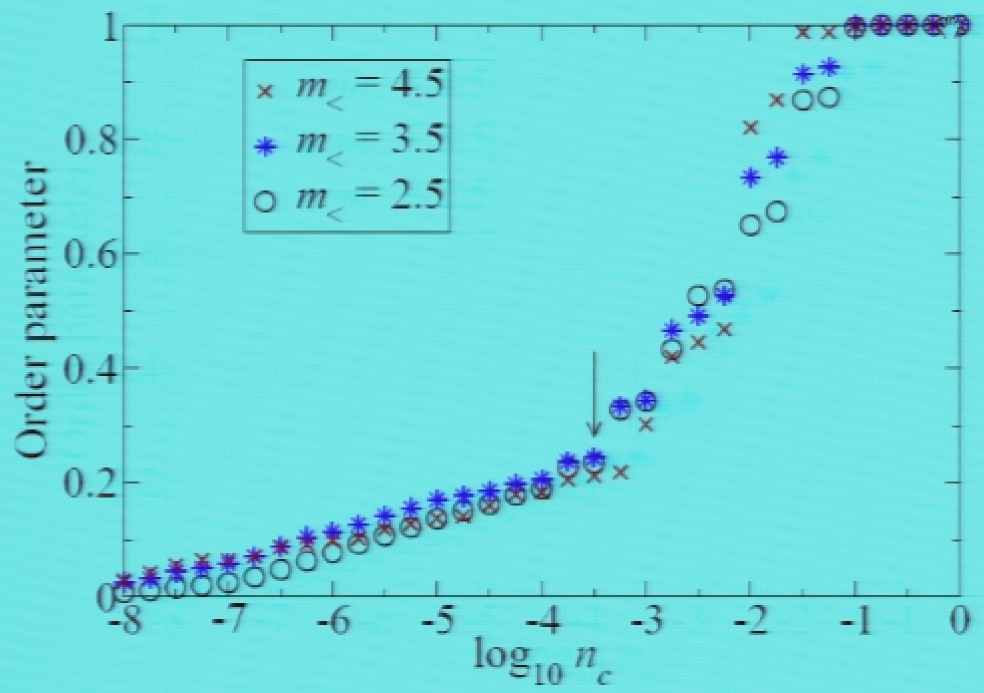

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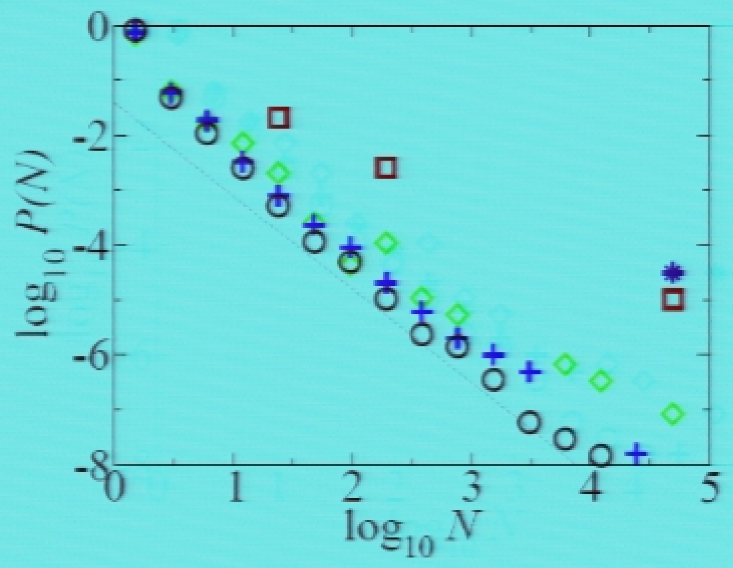

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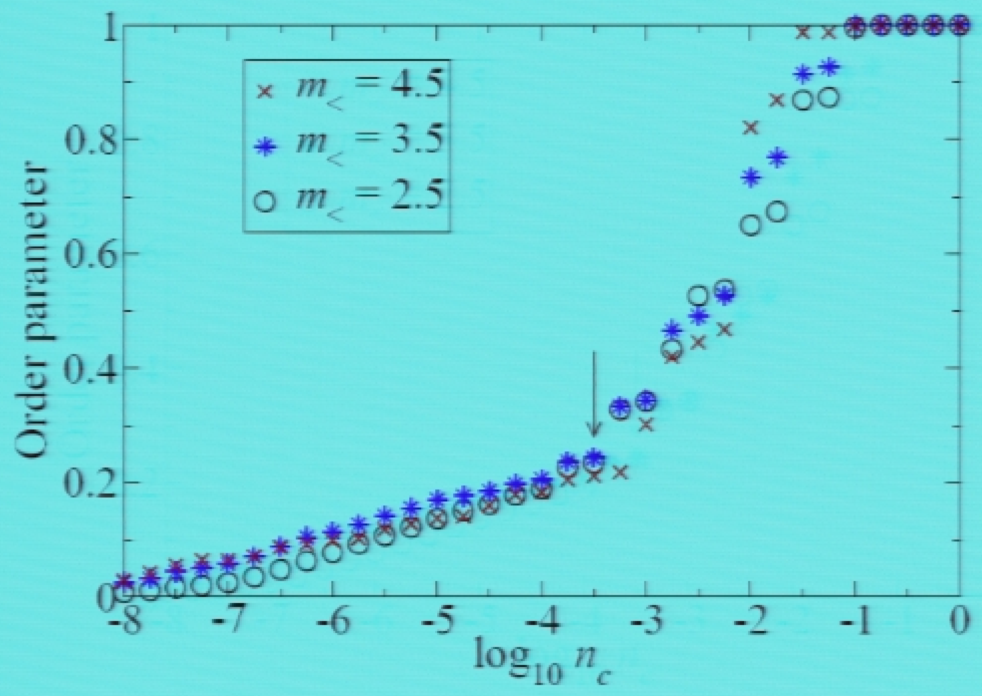

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