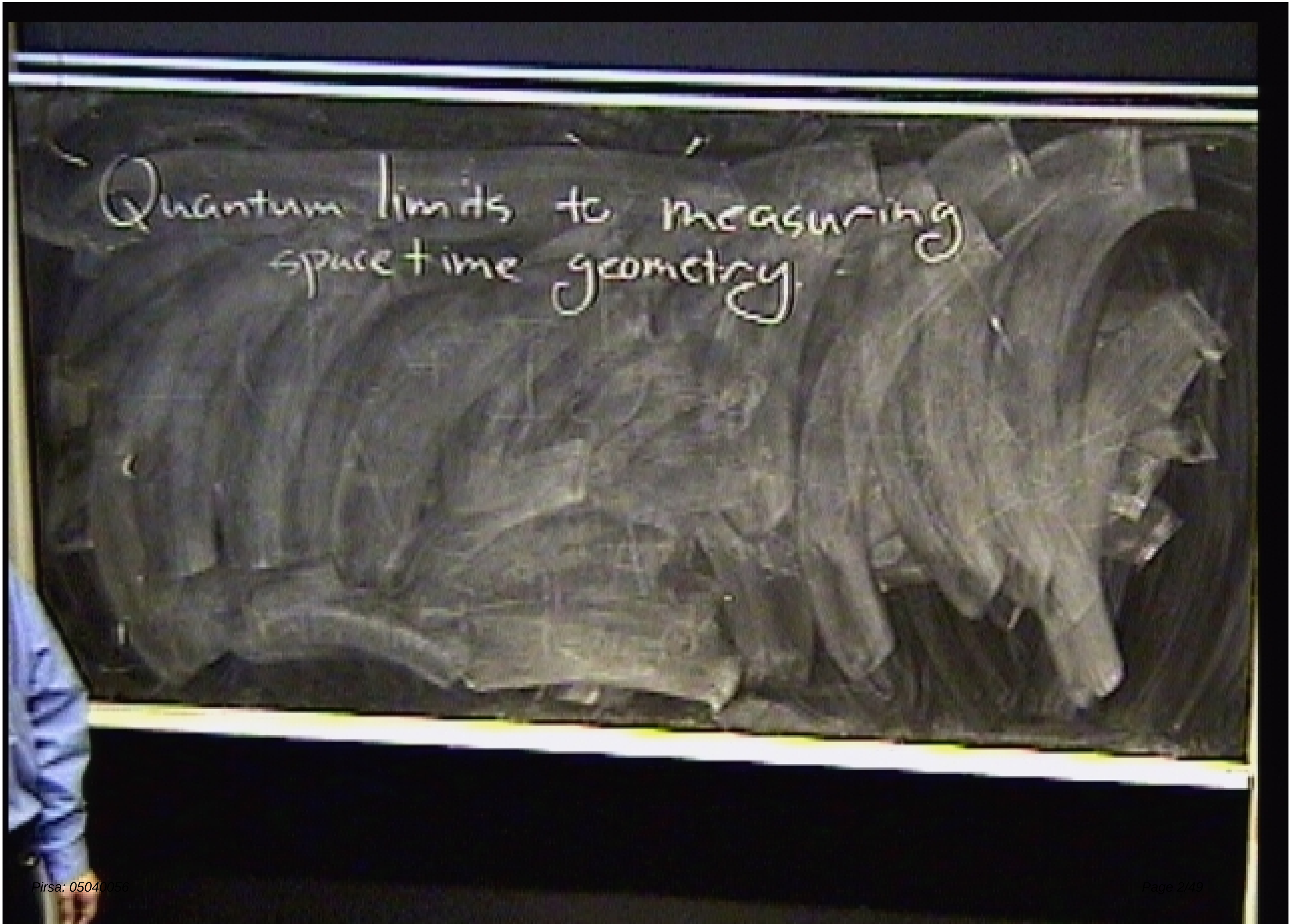
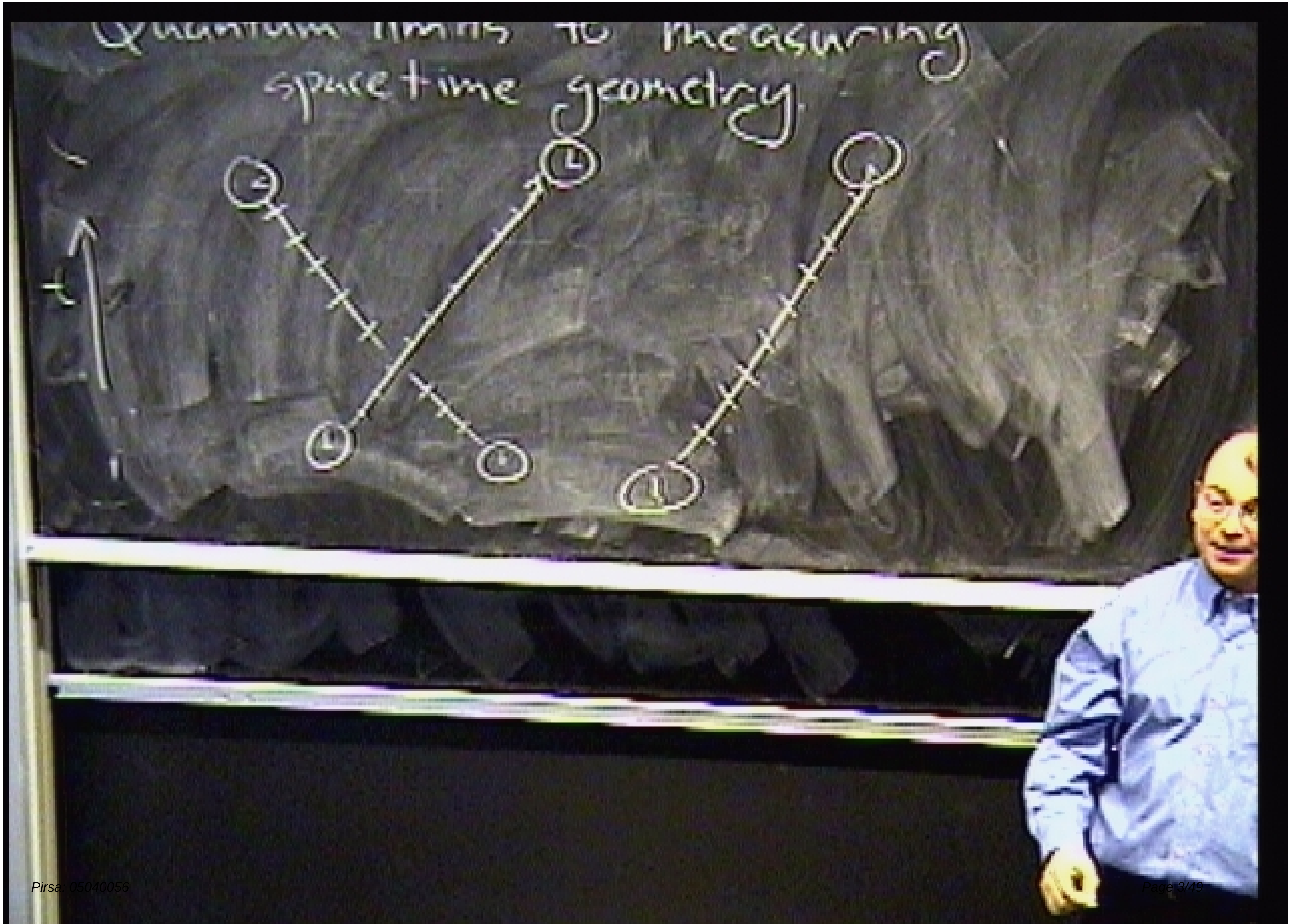
Title: Quantum limits to measuring spacetime geometry

Date: Apr 13, 2005 02:00 AM

URL: http://pirsa.org/05040056

Abstract:


Quantum limits to measuring
spacetime geometry


Quantum limits to measuring
spacetime geometry
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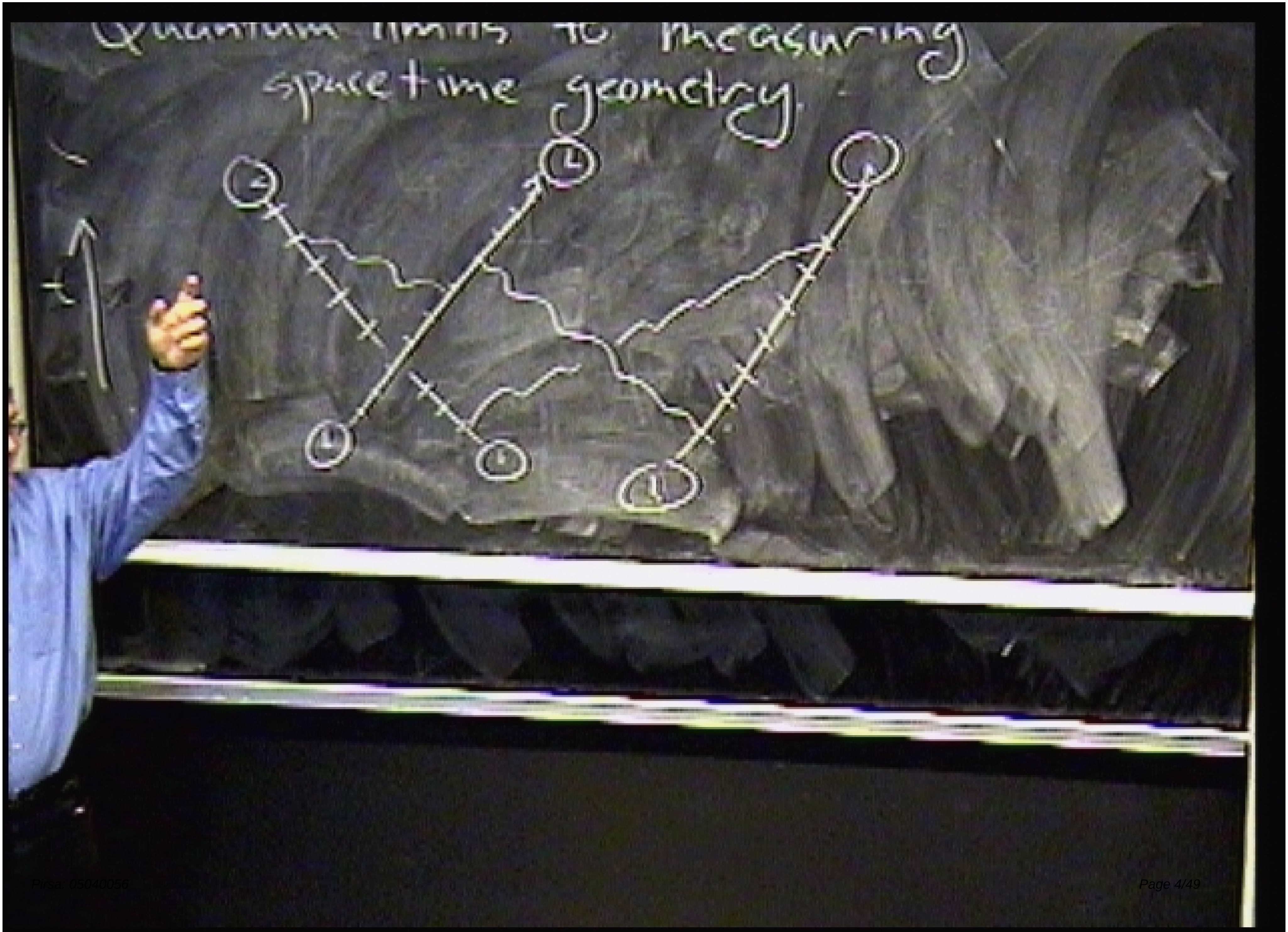
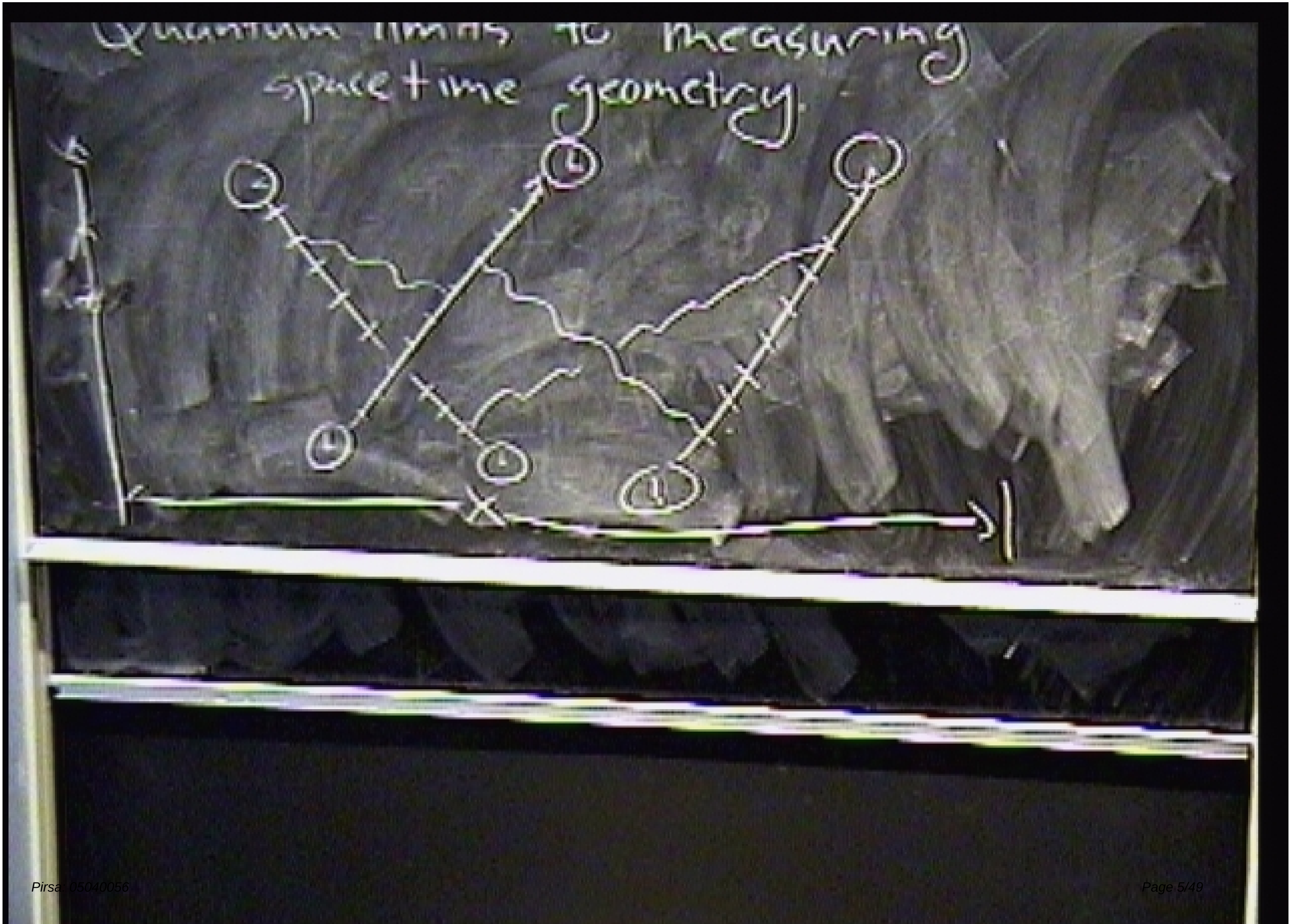
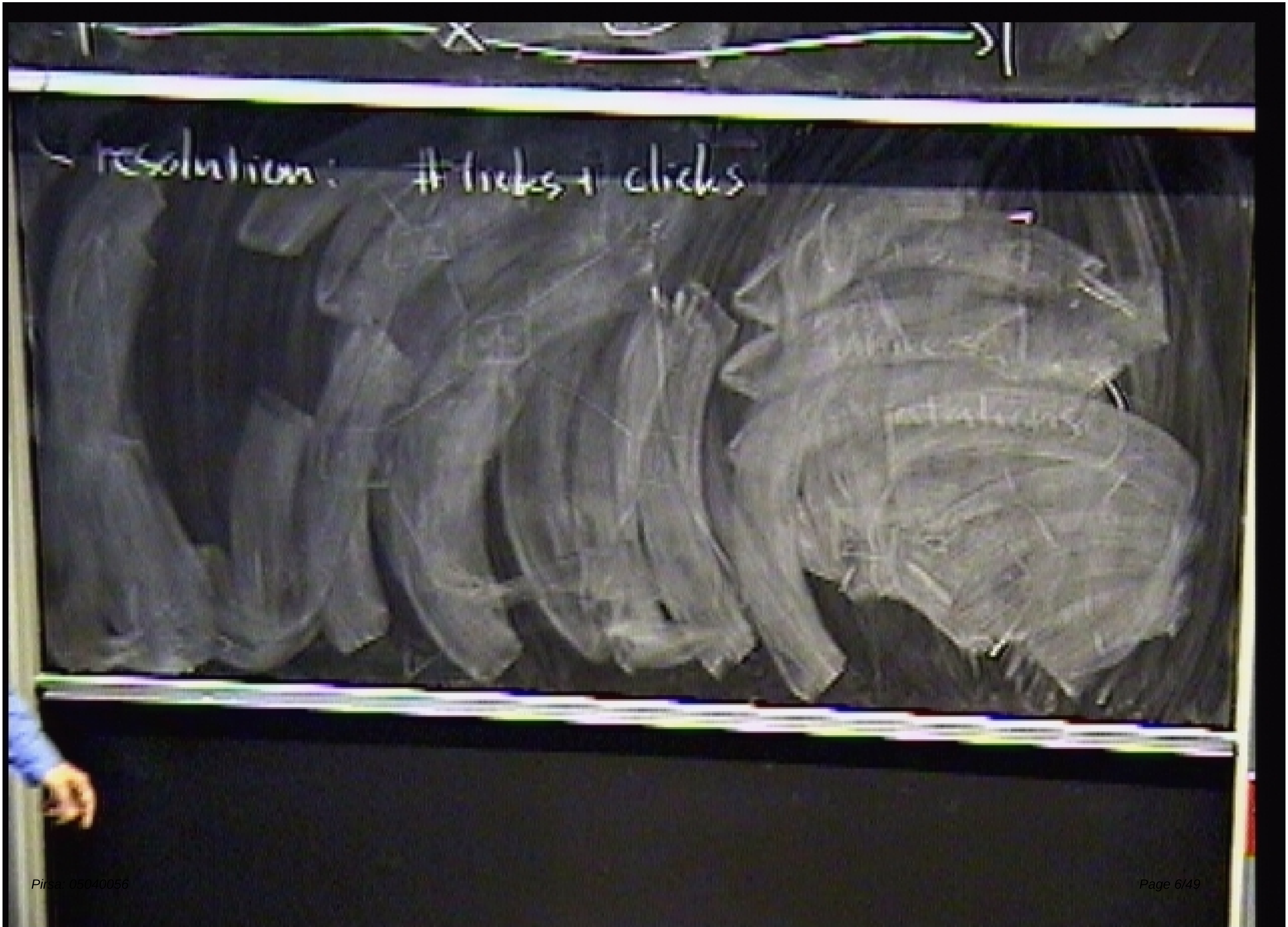
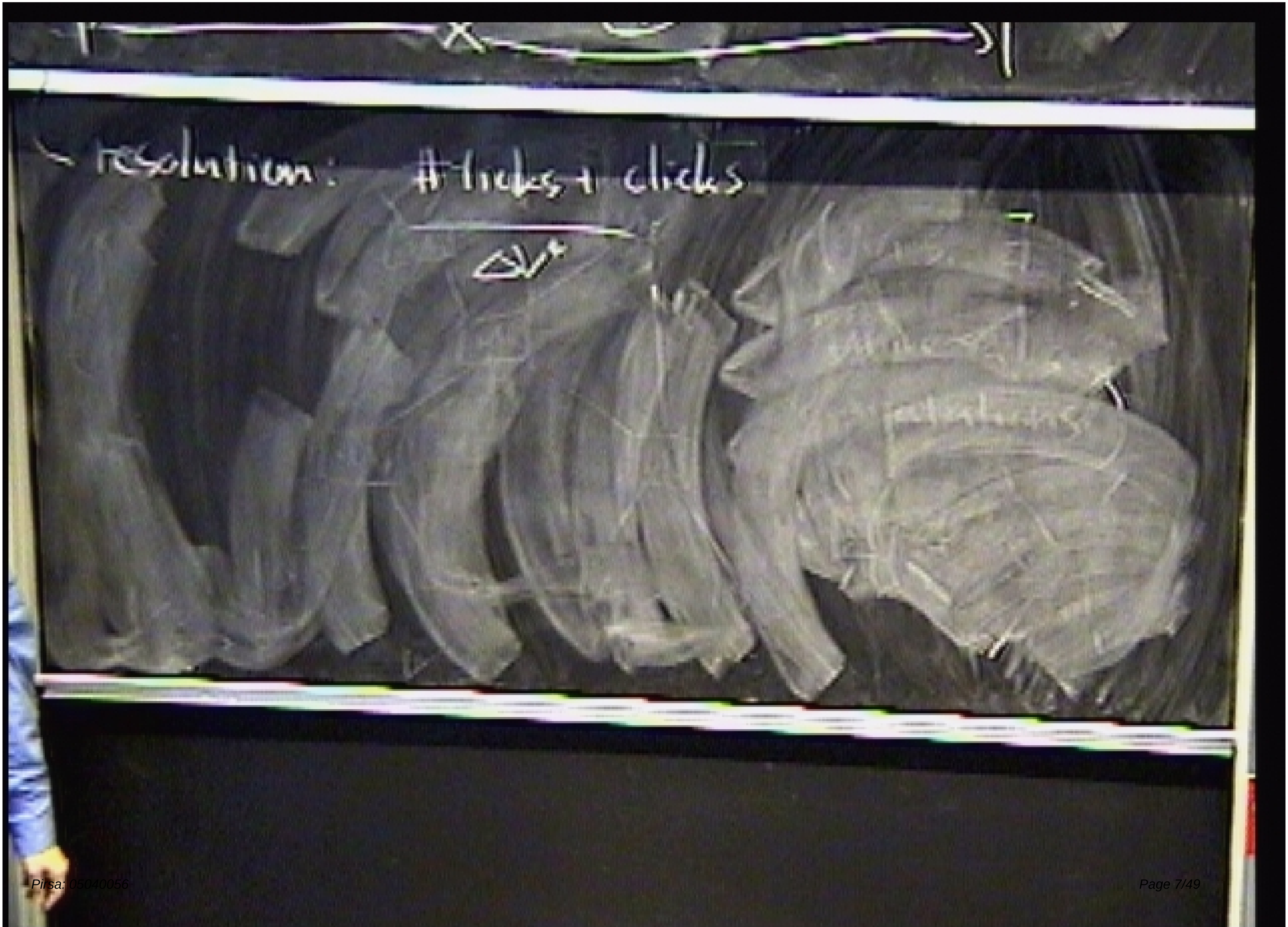

Quantum limits to measuring
spacetime geometry


Quantum limits to measuring
spacetime geometry


resolution: # ticks + clicks


resolution: # ticks + clicks
ΔV

- resolution: $\dfrac{\#\text{ ticks + clicks}}{\Delta V}$

- Two fundamental limits to measurement:

$$\Delta x \Delta p \geq \hbar/2$$

resolution: $\dfrac{\#\text{ticks} + \text{clicks}}{\Delta\nu}$

Two fundamental limits to measurement:

$\Delta x \Delta p \geq \hbar/2 \implies$ Standard Quantum Limit

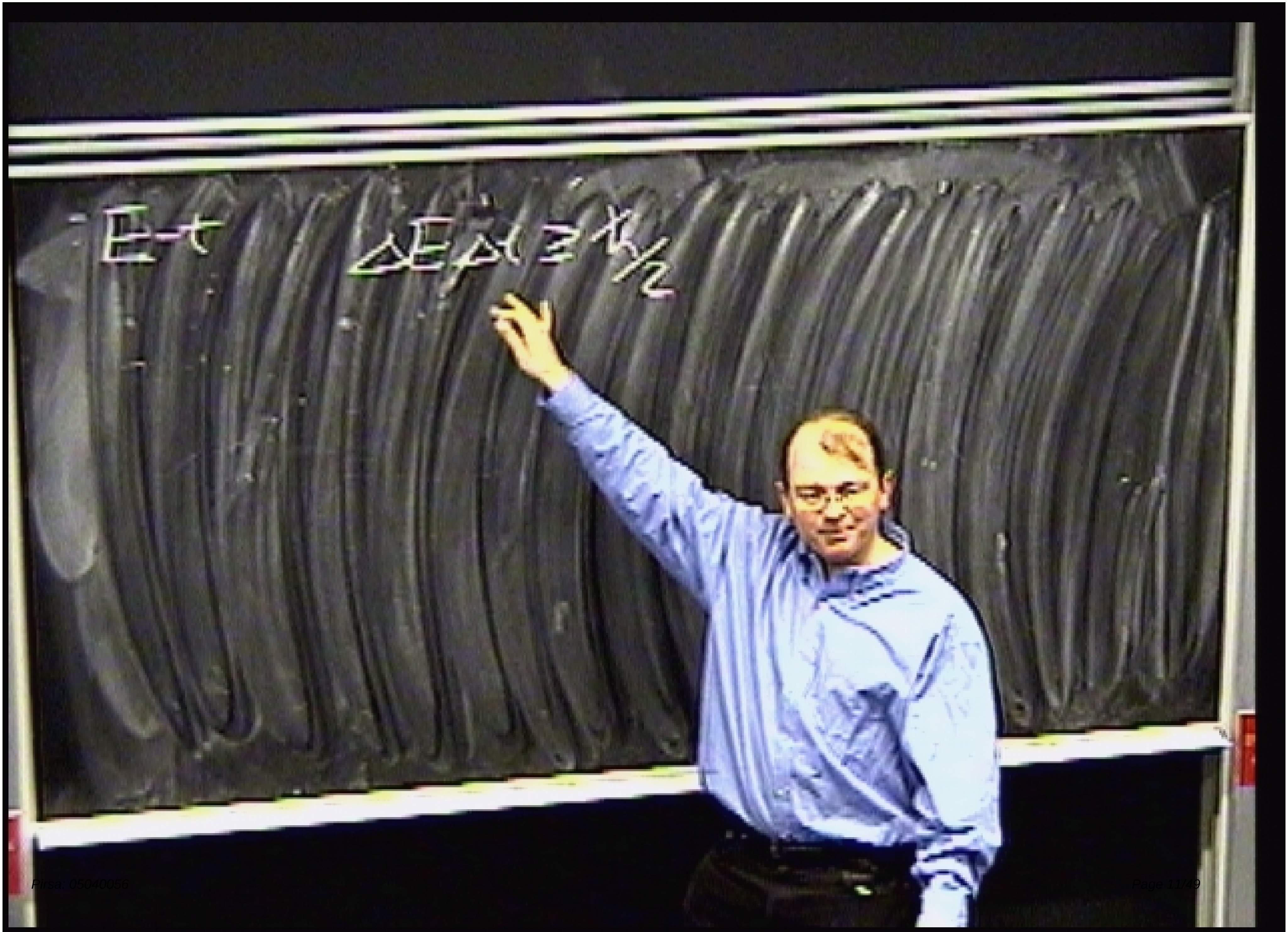
- resolution: $\dfrac{\#\text{flicks} + \text{clicks}}{\Delta\nu^{4}}$

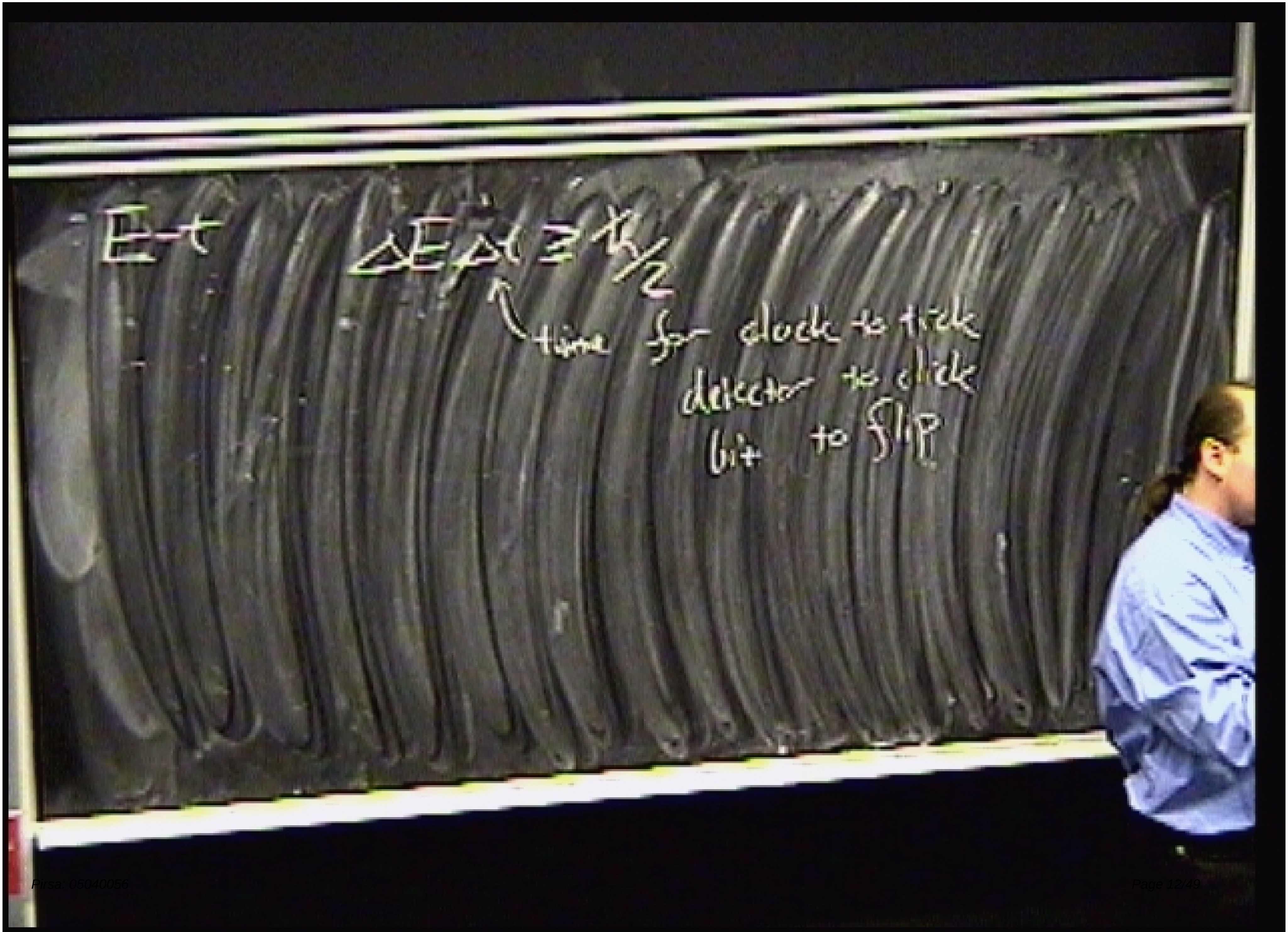
- Two fundamental limits to measurement:

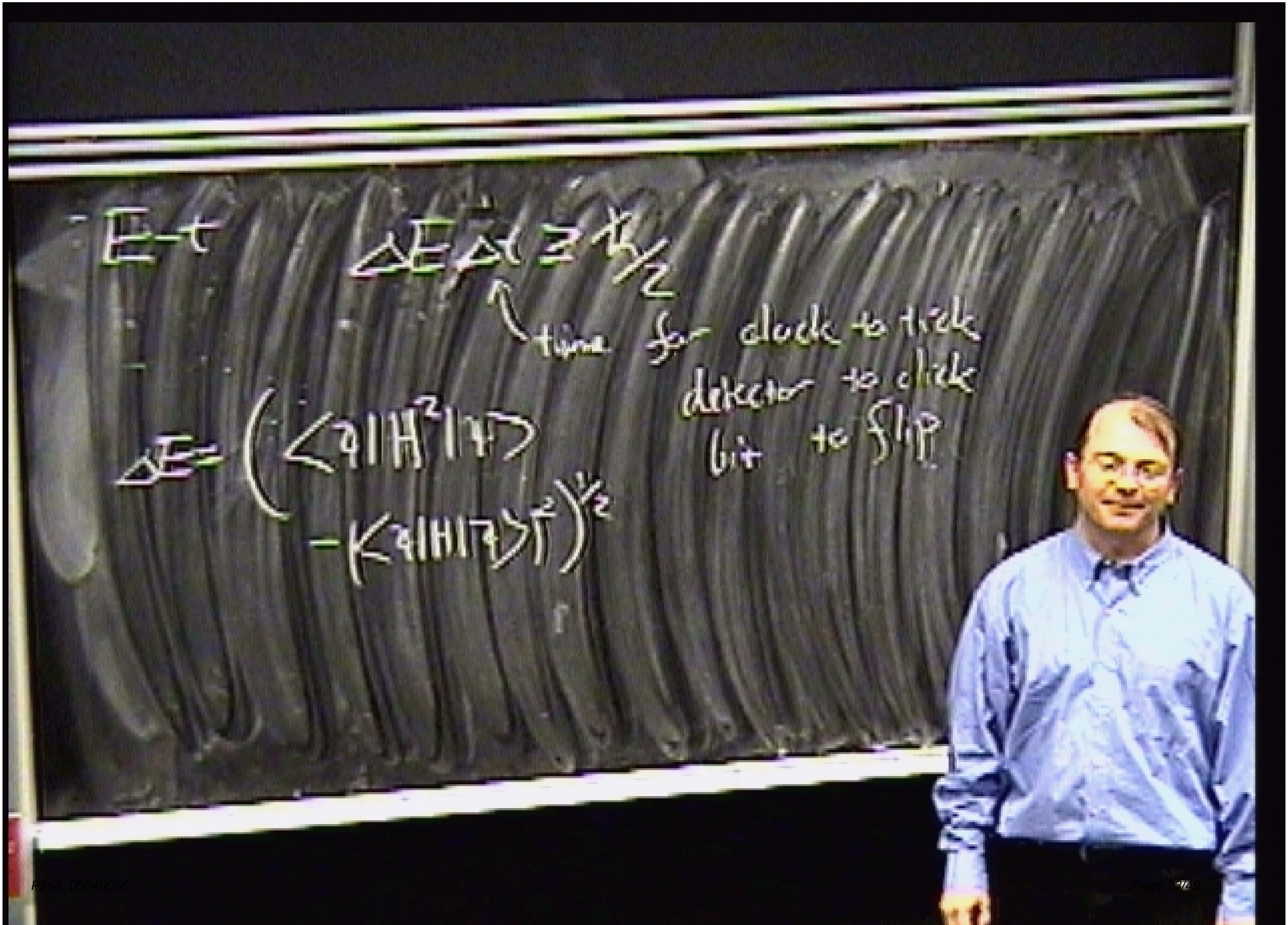
$\Delta x \Delta p \geq \hbar/2 \implies$ Standard Quantum Limit

not a limit

- Take care!


E-t
$\Delta E \Delta t \geq \hbar/2$


E-t
ΔE Δt ≥ ħ/2
time for clock to tick
detector to click
bit to flip


E–t
$\Delta E \Delta t \geq \hbar/2$
time for clock to tick
detector to click
bit to flip
$\Delta E = \left(\langle \psi | H^2 | \psi \rangle - \left(\langle \psi | H | \psi \rangle \right)^2 \right)^{1/2}$

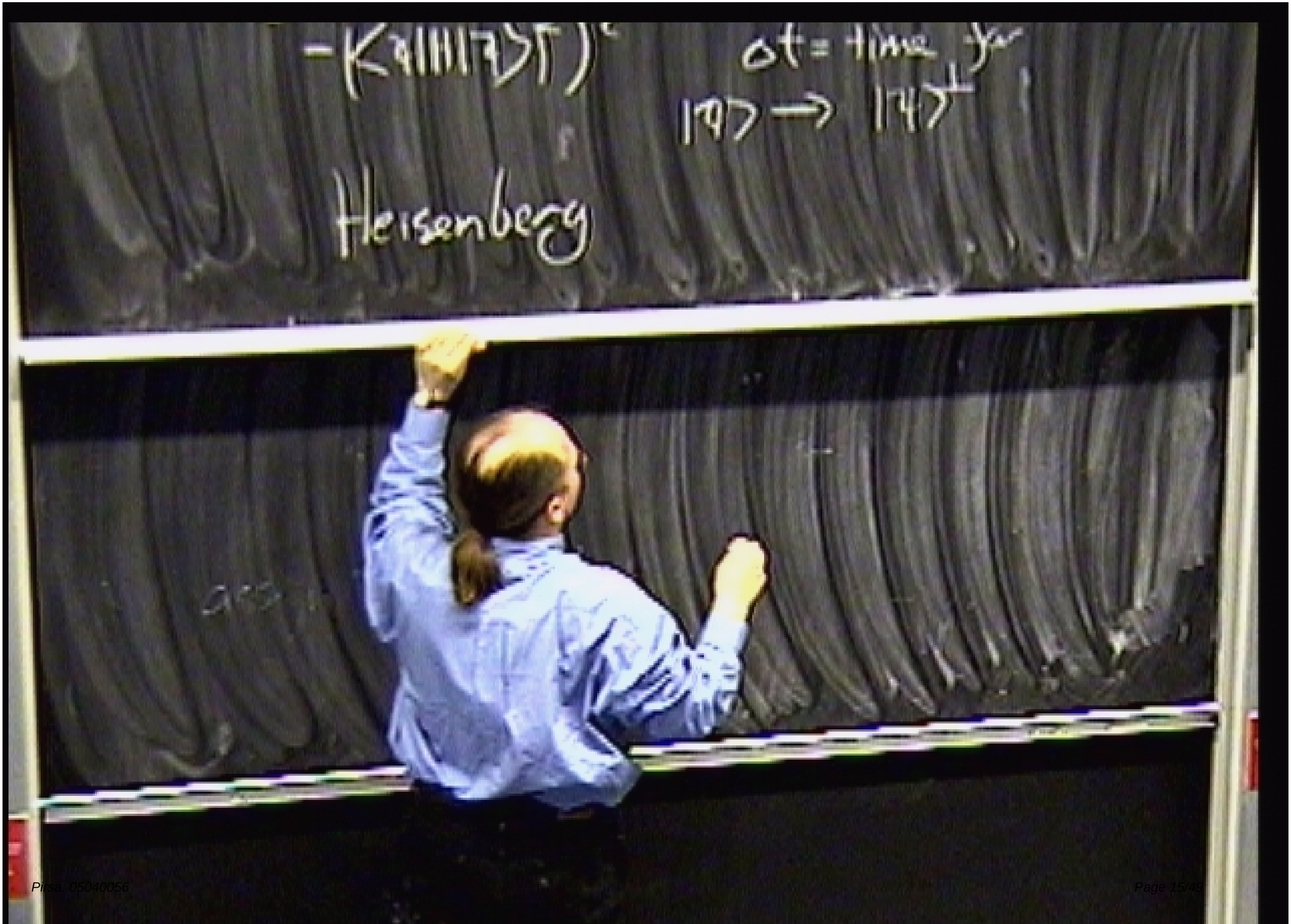
E–t

$$\Delta E \Delta t \geq \hbar/2$$

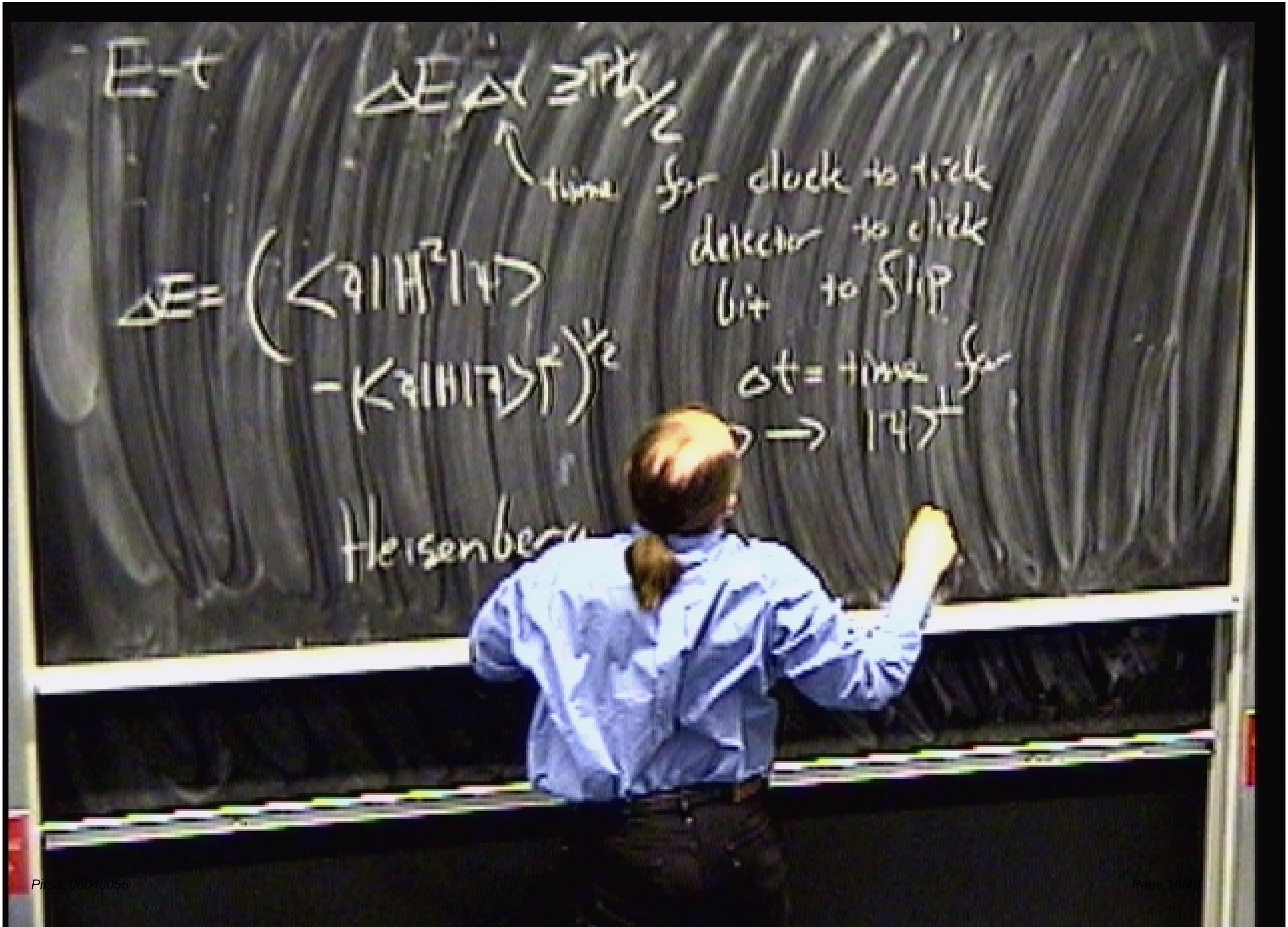
time for clock to tick
detector to click
bit to flip

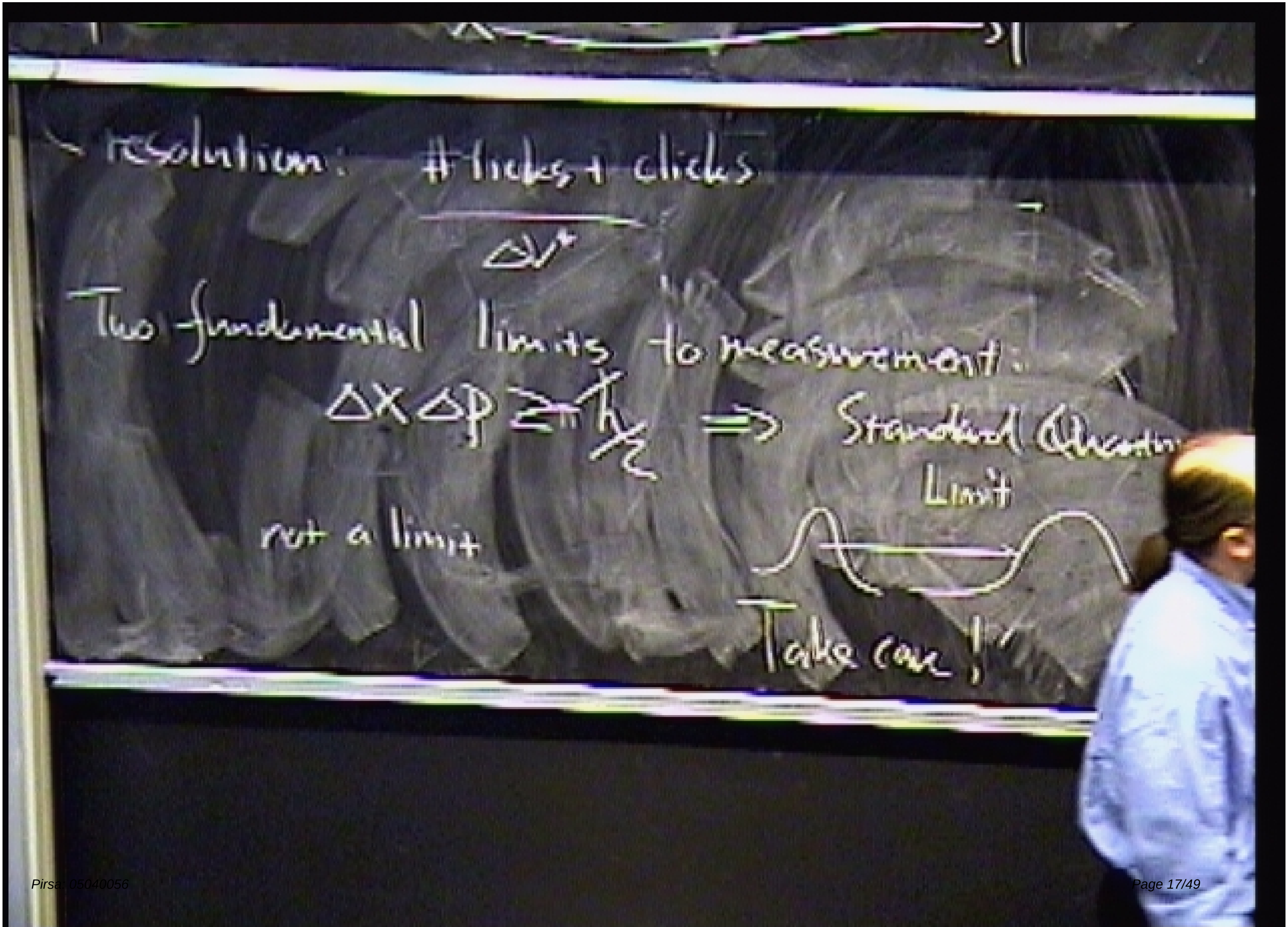
$$\Delta E = \left(\langle \psi | H^2 | \psi \rangle - \langle \psi | H | \psi \rangle^2 \right)^{1/2}$$

$\Delta t =$ time for $|\psi\rangle \rightarrow |\psi^{\perp}\rangle$



Δt = time for
|ψ⟩ → |ψ⟩⊥
Heisenberg


E-t
ΔE Δt ≥ πħ/2
time for clock to tick
detector to click
bit to flip
ΔE = (⟨ψ|H²|ψ⟩ − ⟨ψ|H|ψ⟩²)^½
Δt = time for
|ψ⟩ → |ψ⊥⟩
Heisenberg

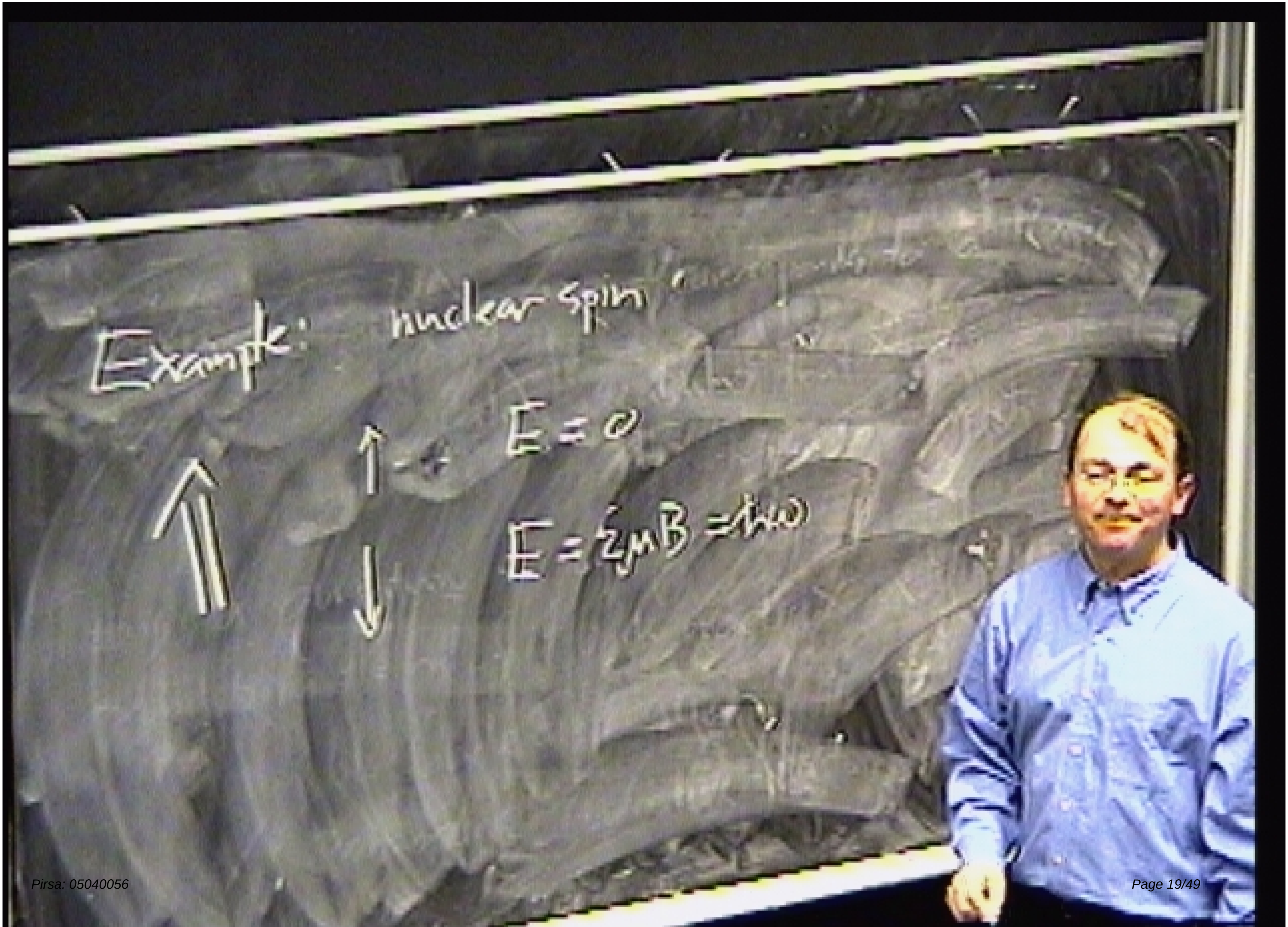

resolution: # flicks + clicks
Two fundamental limits to measurement:
ΔxΔp ≥ ħ/2 ⇒ Standard Quantum Limit
not a limit
Take care!

Heisenberg $E = \hbar\omega$

Margolus-Levitin thm. $E \Delta t \geq \pi\hbar/2$

$$E = \langle\psi|H|\psi\rangle - E_0$$

energy of ground state


Example: nuclear spin
E = 0
E = 2μB = ħω

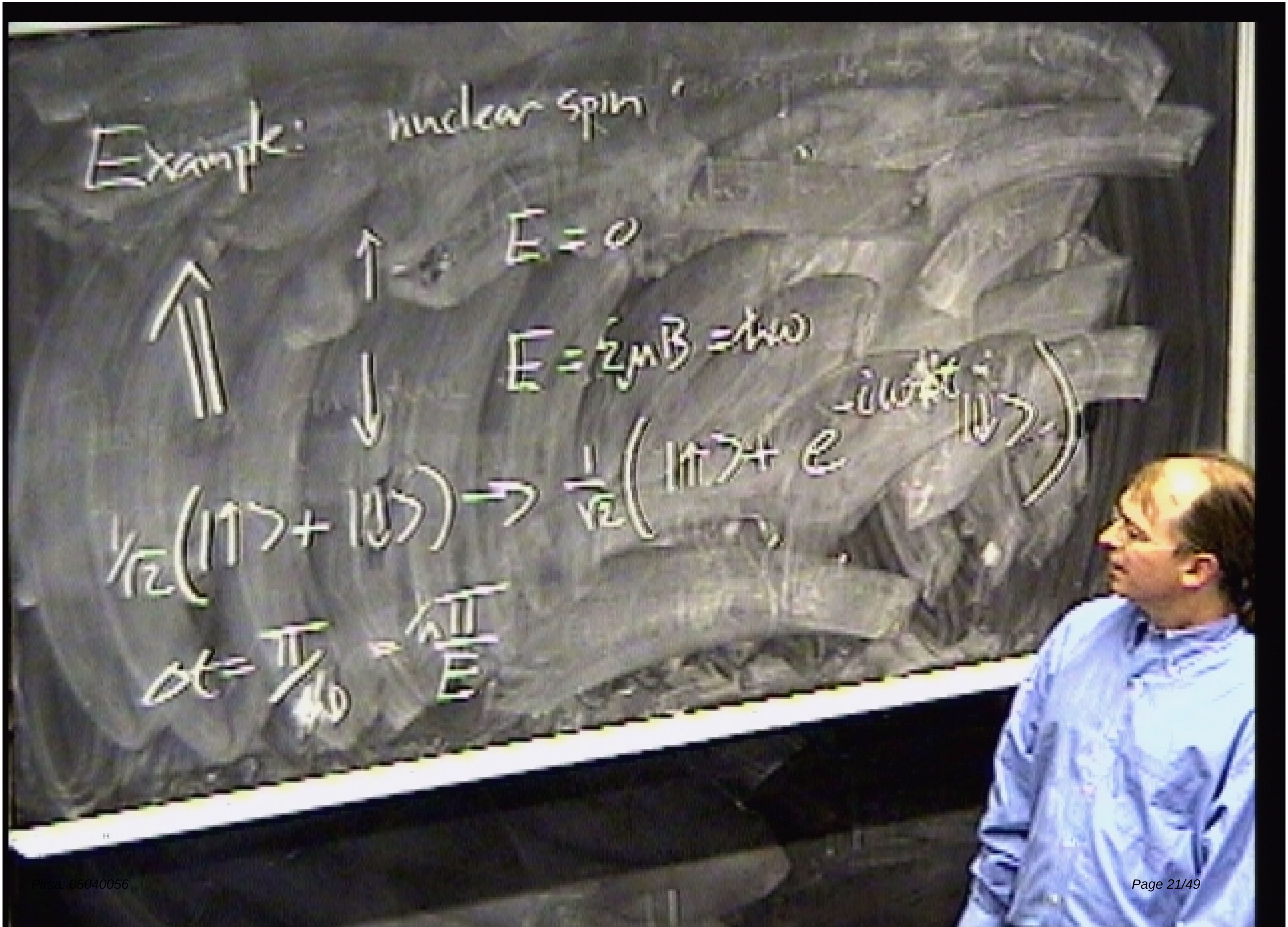
Example: nuclear spin

$\Uparrow$

$\uparrow \quad E = 0$

$\downarrow \quad E = 2\mu B = \hbar\omega$

$$\frac{1}{\sqrt{2}}\left(|\Uparrow\uparrow\rangle + |\Uparrow\downarrow\rangle\right) \rightarrow \frac{1}{\sqrt{2}}\left(|\Uparrow\uparrow\rangle + e^{-i\omega t}|\Uparrow\downarrow\rangle\right)$$


Example: nuclear spin
⇑ ↑ E = 0
↓ E = 2μB = ħω
1/√2 (|⇑⟩ + |⇓⟩) → 1/√2 (|⇑⟩ + e^{-iωt} |⇓⟩)
Δt = π/ω = πħ/E

Example: nuclear spin

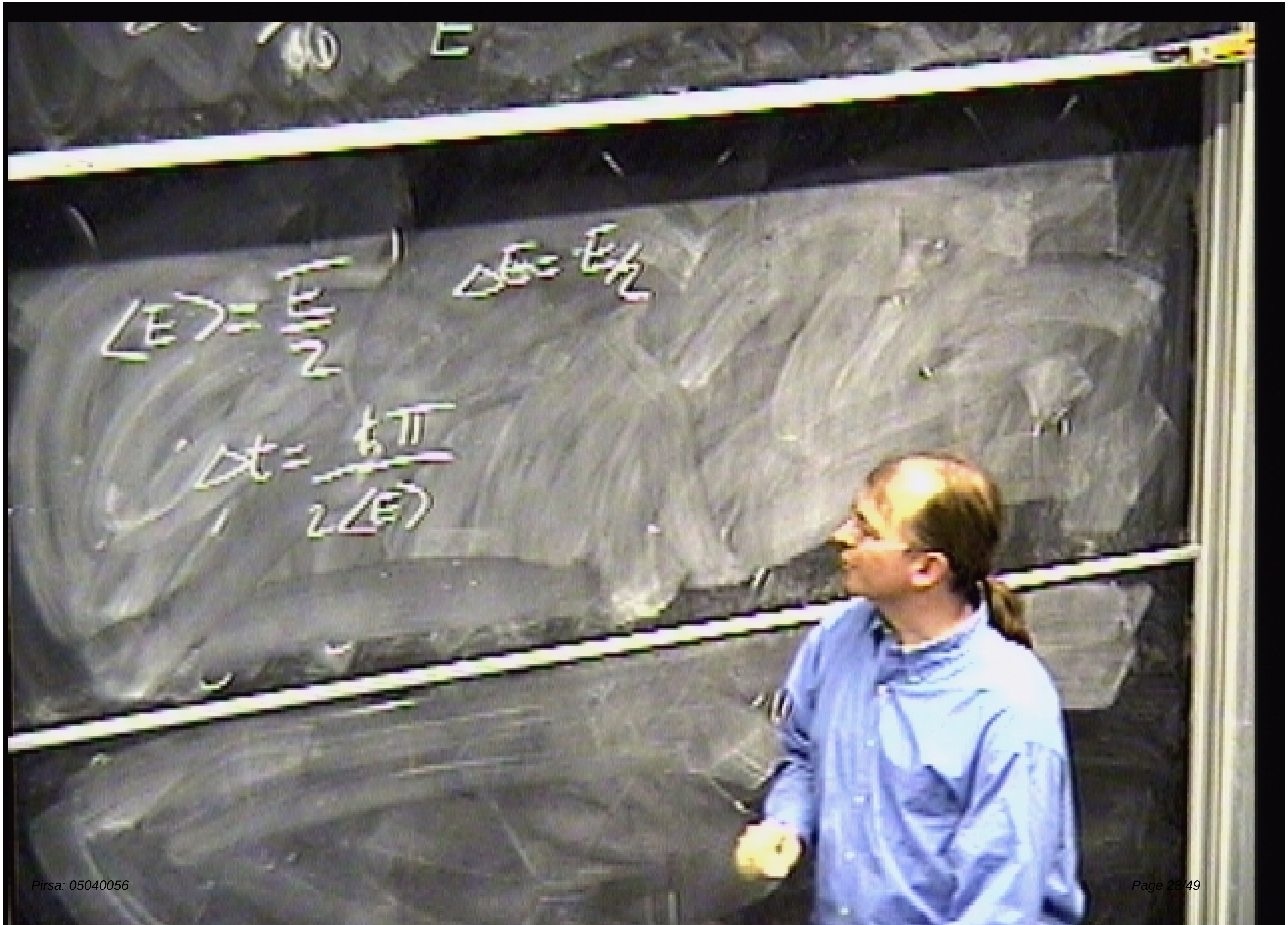
$\Uparrow$ $\quad \uparrow \;\; E = 0$

$\downarrow \;\; E = 2\mu B = \hbar\omega$

$$|\rightarrow\rangle = \tfrac{1}{\sqrt{2}}\left(|\uparrow\rangle + |\downarrow\rangle\right) \rightarrow \tfrac{1}{\sqrt{2}}\left(|\uparrow\rangle + e^{-i\omega t}|\downarrow\rangle\right)$$

$$= \tfrac{1}{\sqrt{2}}\left(|\uparrow\rangle - |\downarrow\rangle\right) = |\Leftarrow\rangle$$

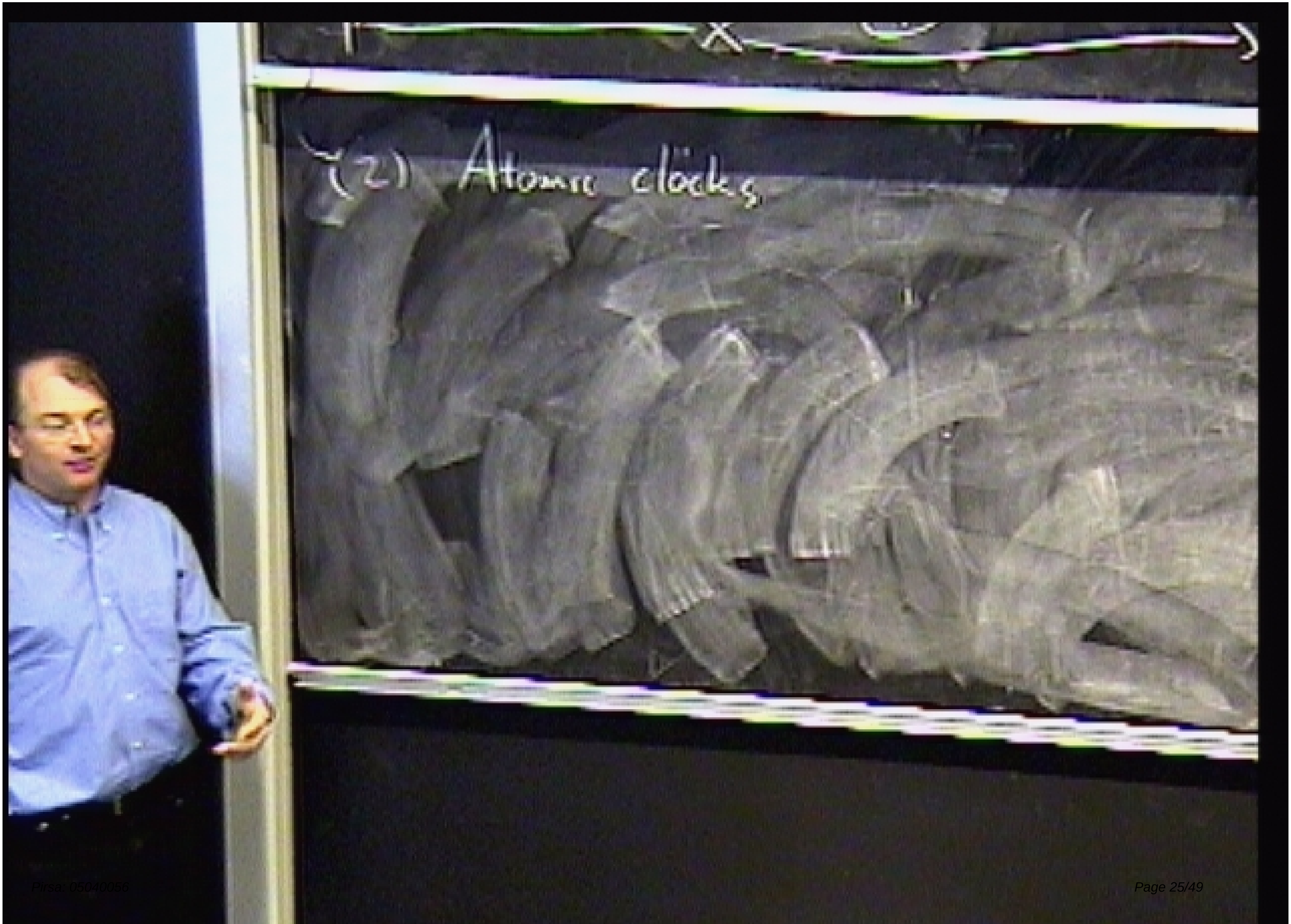
$$\Delta t = \pi/\omega = \frac{\hbar\pi}{E}$$

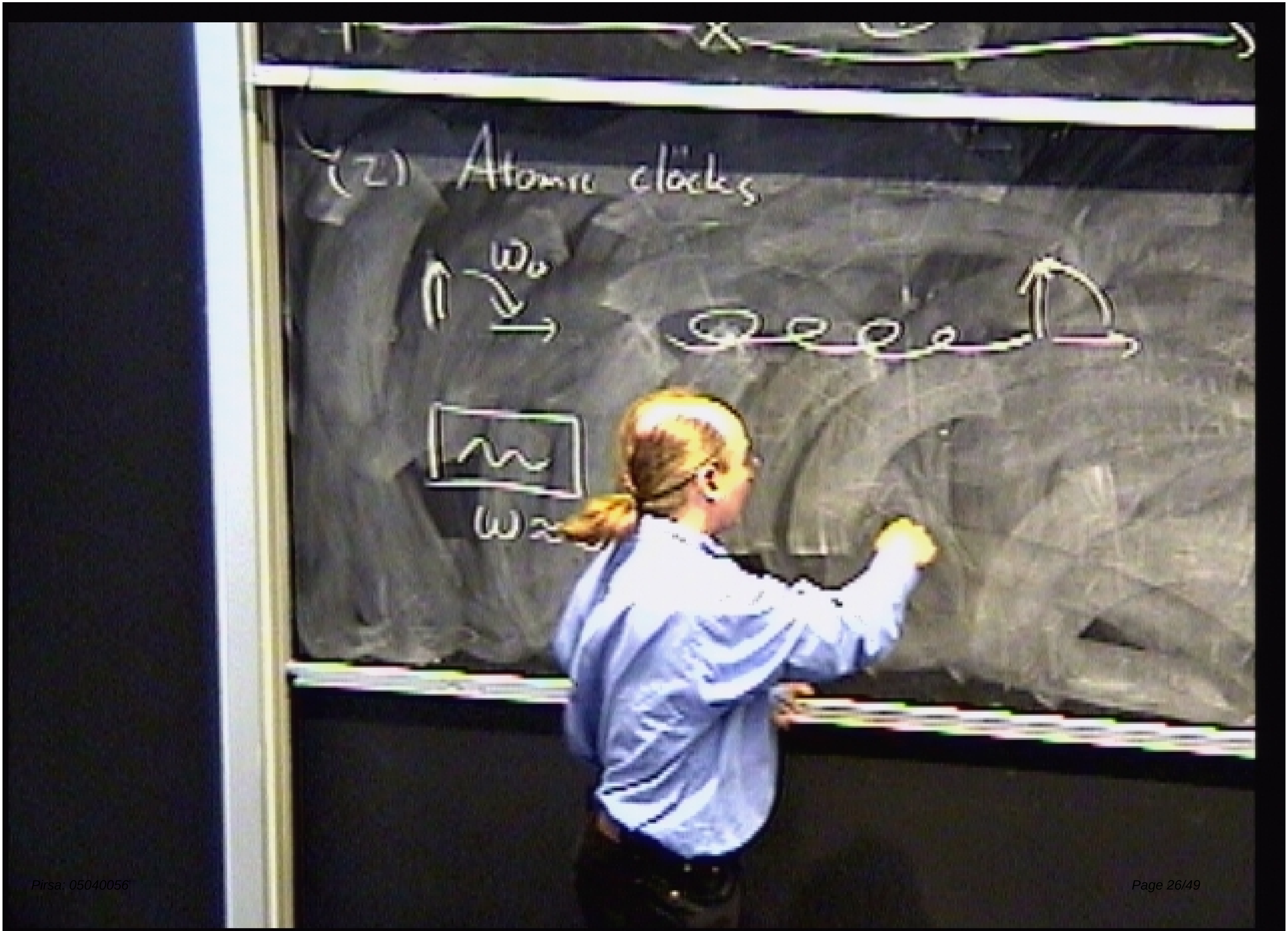

$\langle E \rangle = \frac{E}{2}$
$\Delta E = E/2$
$\Delta t = \frac{h\pi}{2\langle E \rangle}$

$\langle E \rangle = \frac{E}{2}$ $\Delta E = E/2$

$\Delta t = \frac{h\pi}{2\langle E \rangle} = \frac{h\pi}{2\Delta E}$

M-L H


(2) Atomic clocks

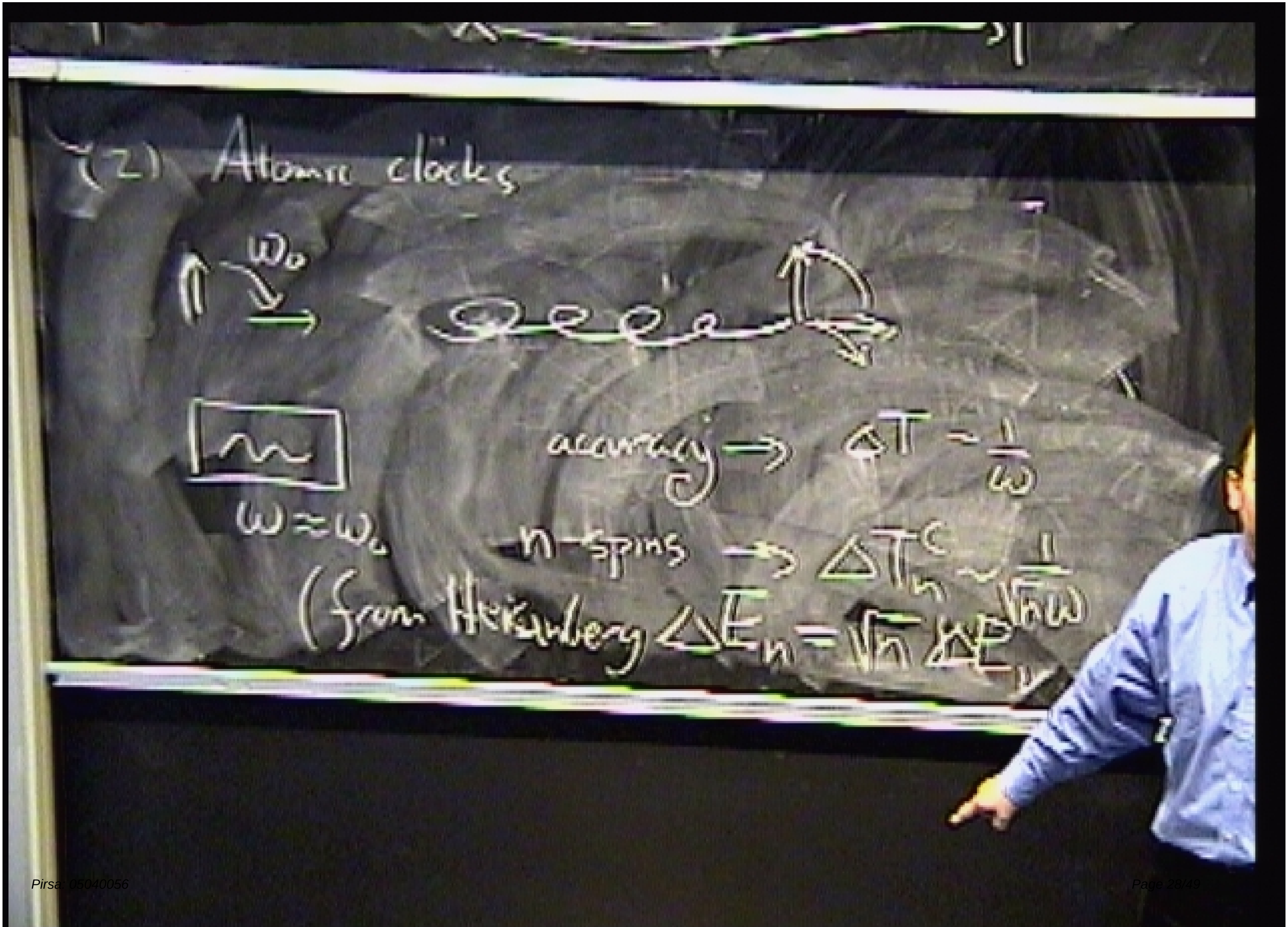

(2) Atomic clocks
ω_0
$\omega \approx$

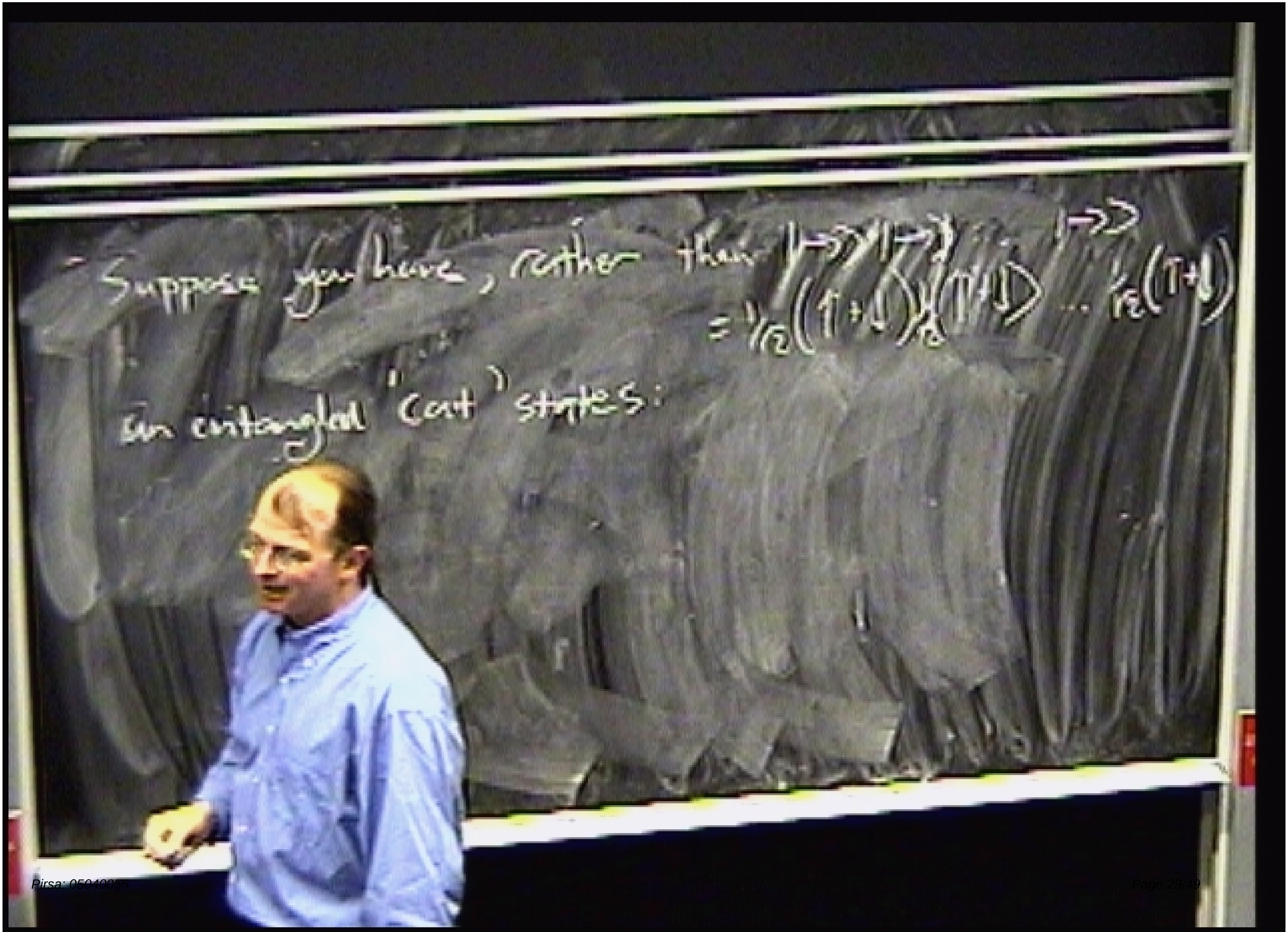
(2) Atomic clocks

$\omega \approx \omega_0$

accuracy $\rightarrow \Delta T \sim \frac{1}{\omega}$

n-spins $\rightarrow \Delta T_n^c \sim \frac{1}{\sqrt{n}\,\omega}$


(2) Atomic clocks
ω_0
accuracy $\to$ $\Delta T \sim \frac{1}{\omega}$
$\omega \approx \omega_0$
n-spins $\to$ $\Delta T_n^c \sim \frac{1}{\sqrt{n}\,\omega}$
(from Heisenberg $\Delta E_n = \sqrt{n}\,\Delta E_1$)


- Suppose you have, rather than |→>|→>... |→>
= 1/√2(↑+↓) 1/√2(↑+↓) ... 1/√2(↑+↓)
an entangled 'cat' states:

- Suppose you have, rather than $|\rightarrow\rangle|\rightarrow\rangle \dots |\rightarrow\rangle$

$$= \tfrac{1}{\sqrt{2}}(\uparrow+\downarrow)\tfrac{1}{\sqrt{2}}(\uparrow+\downarrow) \dots \tfrac{1}{\sqrt{2}}(\uparrow+\downarrow)$$

an entangled 'cat' states:

$$\tfrac{1}{\sqrt{2}}\left(|\uparrow\uparrow\dots\uparrow\rangle + |\downarrow\downarrow\dots\downarrow\rangle\right)$$

$$\longrightarrow \tfrac{1}{\sqrt{2}}\left(|\uparrow\uparrow\dots\uparrow\rangle + e^{-iN\omega t}|\downarrow\downarrow\dots\downarrow\rangle\right)$$

Suppose you have, rather than $|\rightarrow\rangle|\rightarrow\rangle \cdots |\rightarrow\rangle$

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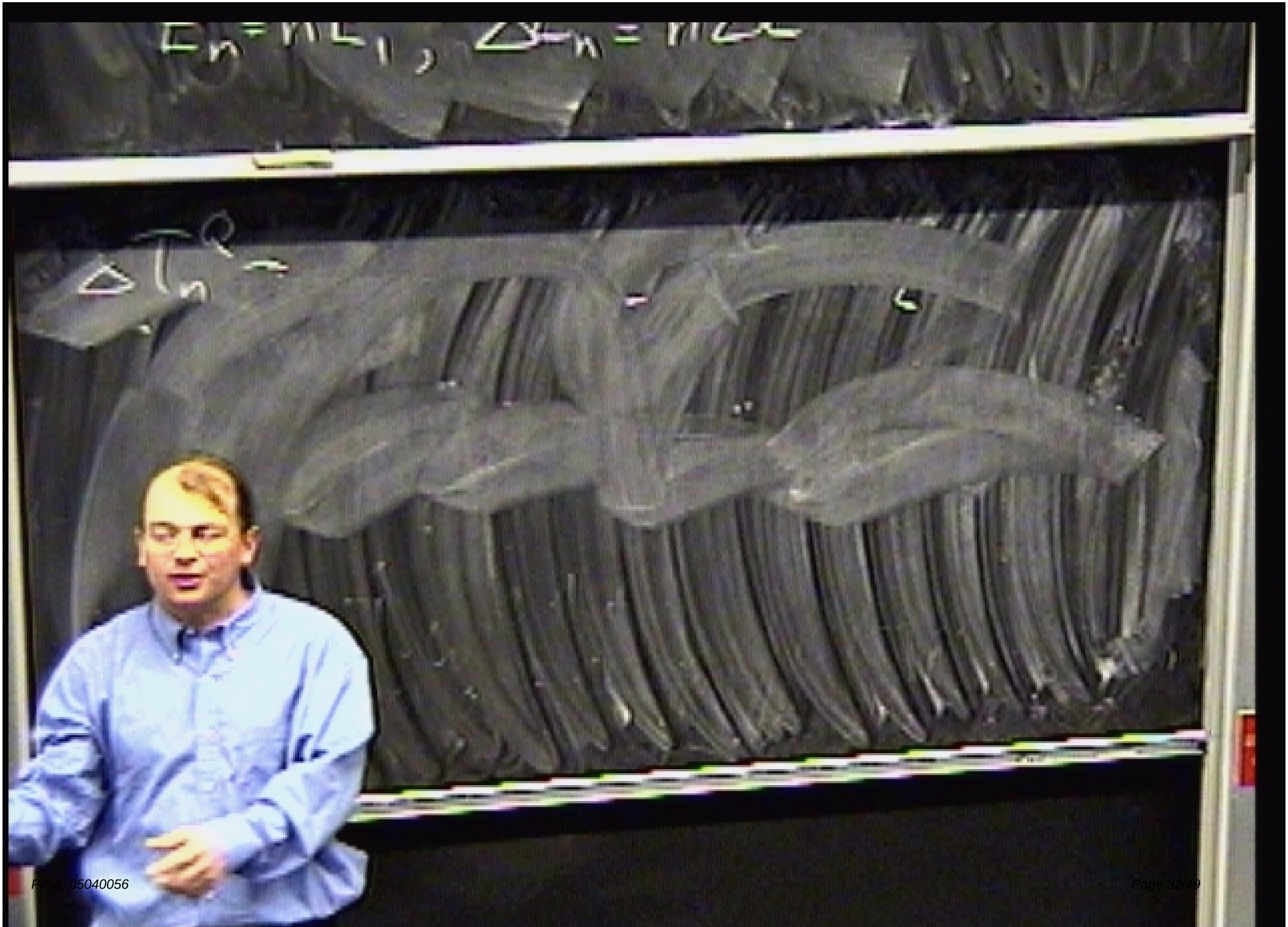
$$E_n = nE_1, \quad \Delta E_n = n\Delta E$$



$E_n = nE_1 , \quad \Delta E_n = n\Delta E$

$\Delta T_n^Q \sim \frac{1}{n\omega}$, $\sqrt{n}$ enhancement in accuracy

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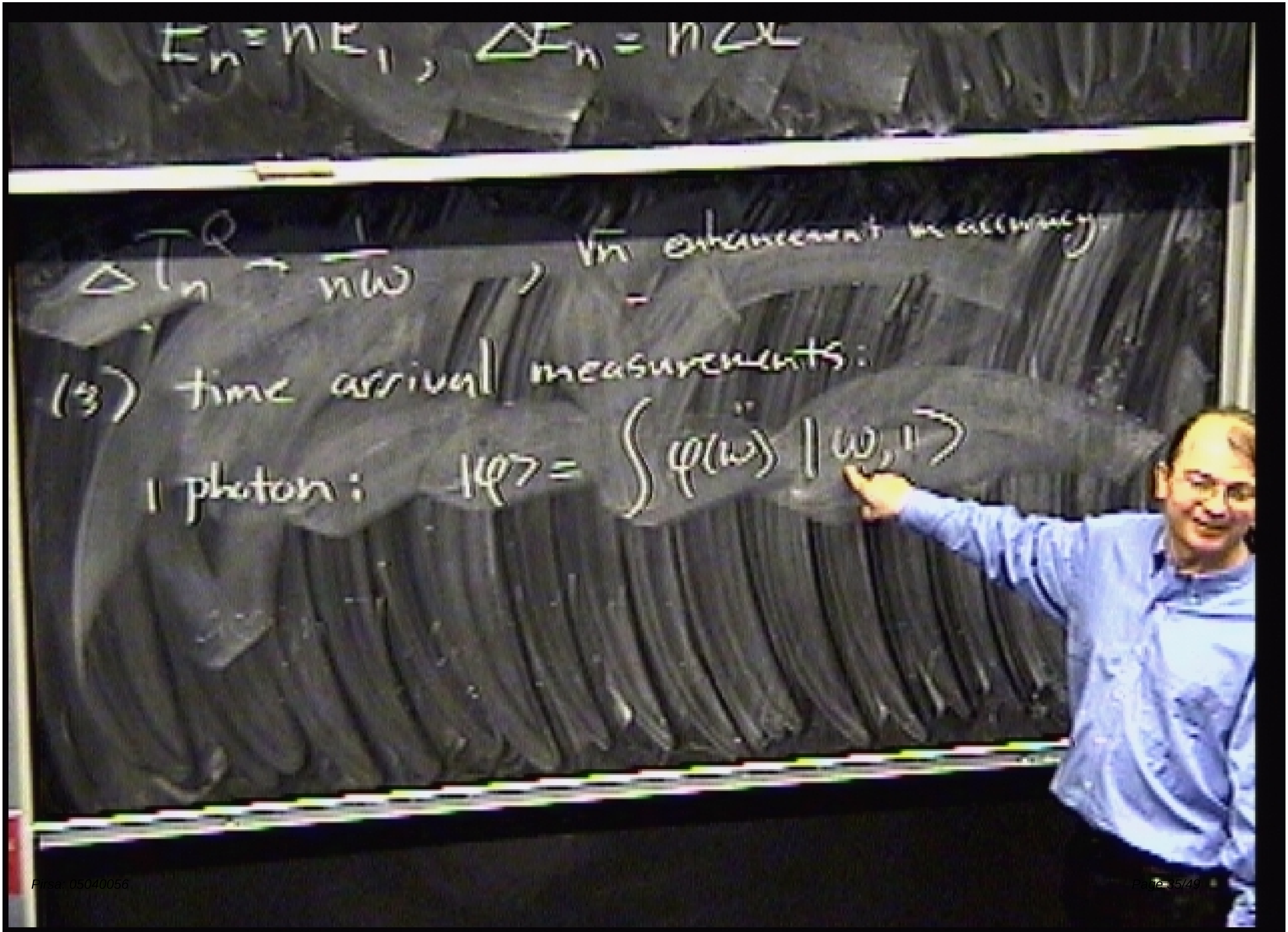
$\Delta T_n^Q \sim \frac{1}{n\omega}$, $\sqrt{n}$ enhancement in accuracy

(3) time arrival measurements:


E_n = nE_1 , ΔE_n = nΔE
ΔT_n^Q ~ 1/(nω) , √n enhancement in accuracy
(3) time arrival measurements:
1 photon: |φ> = ∫ φ(ω) |ω,1>

$E_n = nE_1$, $\Delta E_n = n\Delta E$

$\Delta t_n \sim \frac{1}{n\omega}$, $\sqrt{n}$ enhancement in accuracy.

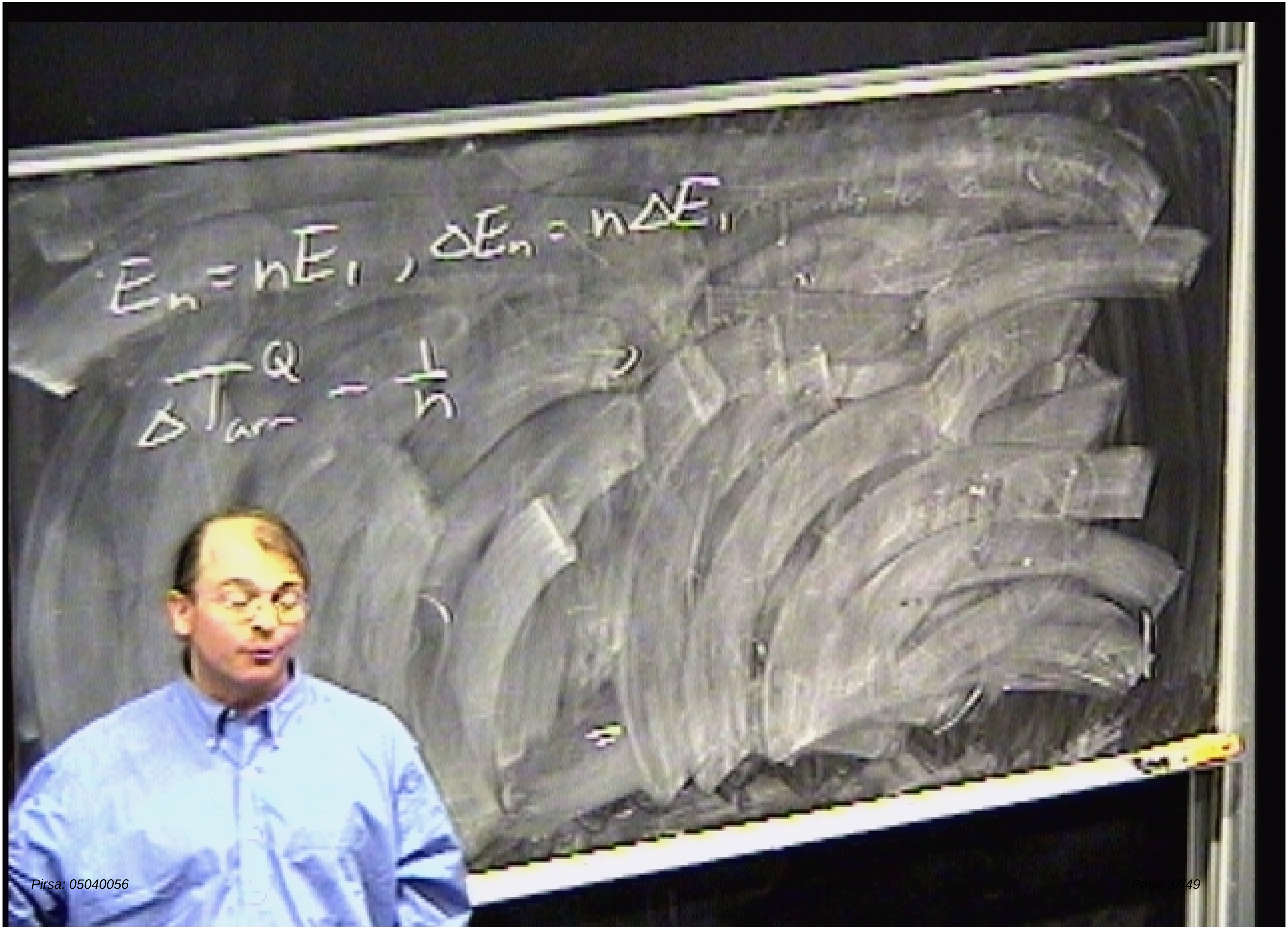
(3) time arrival measurements:

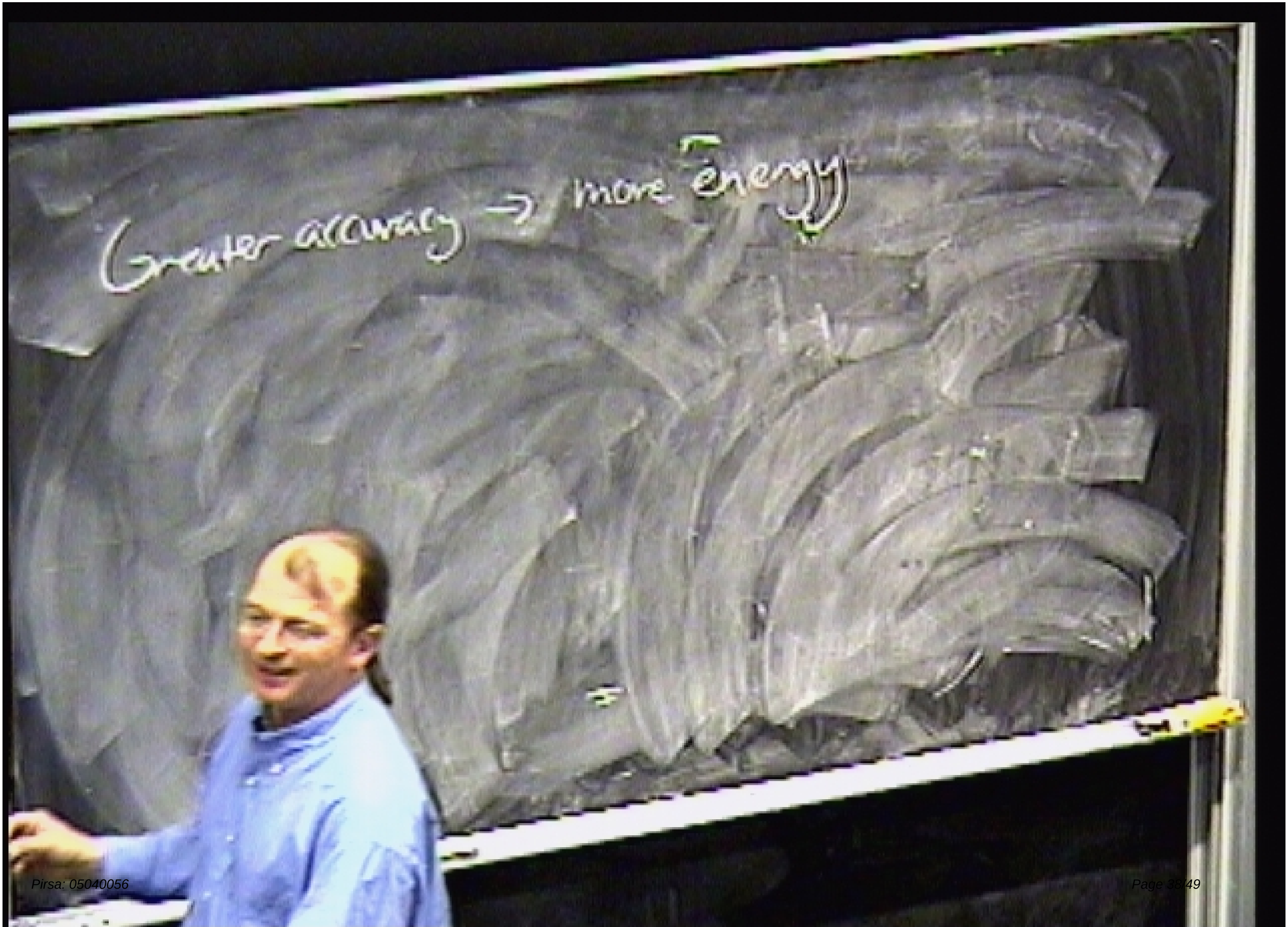
1 photon: $|\varphi\rangle = \int \varphi(\omega)\, |\omega, 1\rangle\, d\omega$

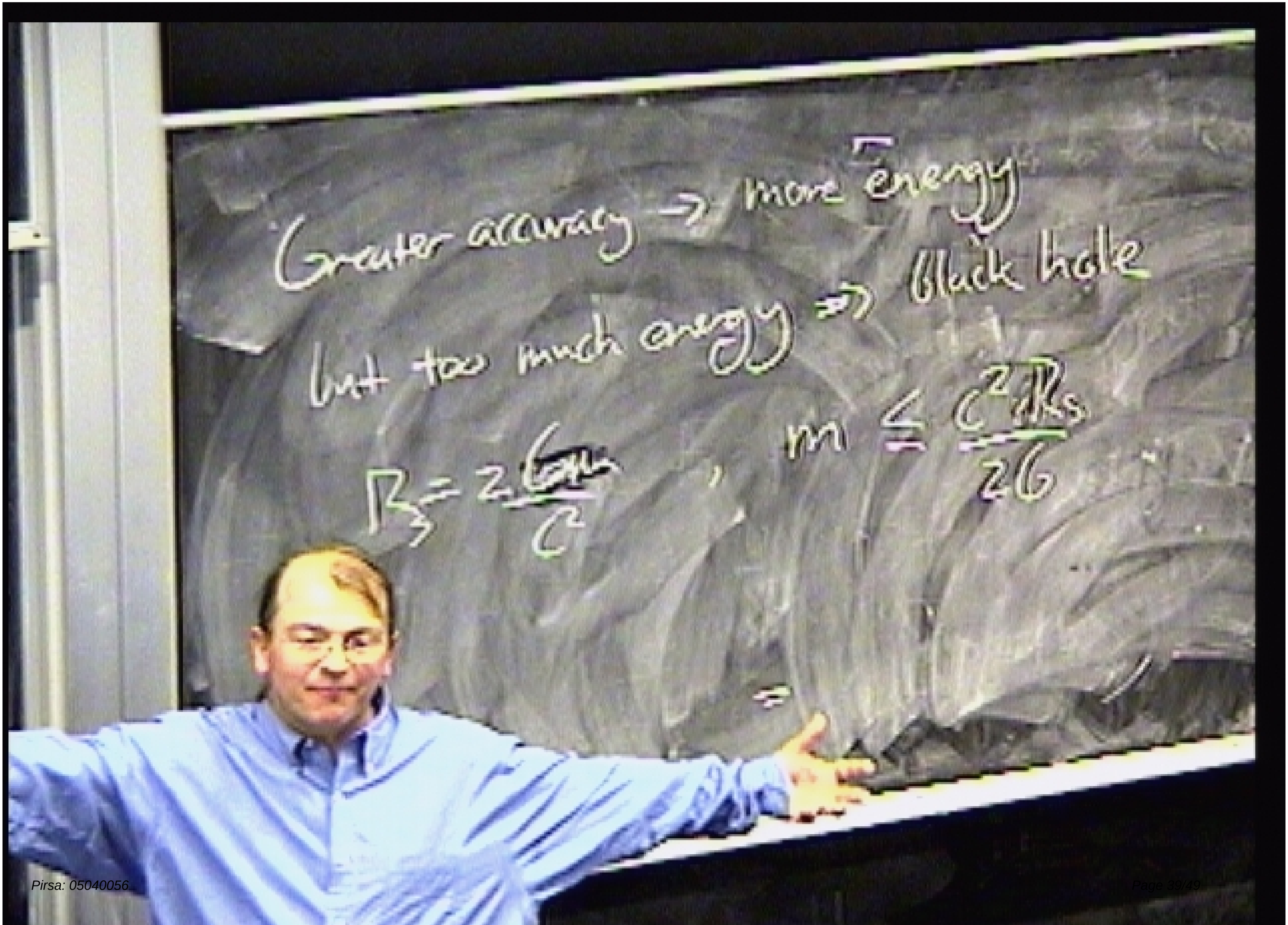
n uncorrelated: $|\varphi\rangle|\varphi\rangle \cdots |\varphi\rangle \longrightarrow \Delta E_n \sim \sqrt{n}\Delta E$

$\Delta T_n \sim \frac{1}{\sqrt{n}}$ shot noise limit

n entangled photons $\int \varphi(\omega)\, |\omega,1\rangle_1 |\omega,1\rangle_2 \cdots |\omega,1\rangle_n\, d\omega$


$E_n = nE_1 \,,\ \Delta E_n = n\Delta E_1$
$\Delta T^Q_{arr} \sim \frac{1}{n}$


Greater accuracy → more energy


Greater accuracy → more energy
but too much energy ⇒ black hole
$R_s = \frac{2Gm}{c^2}$, $m \le \frac{c^2 R_s}{2G}$

Greater accuracy $\Rightarrow$ more energy

but too much energy $\Rightarrow$ black hole

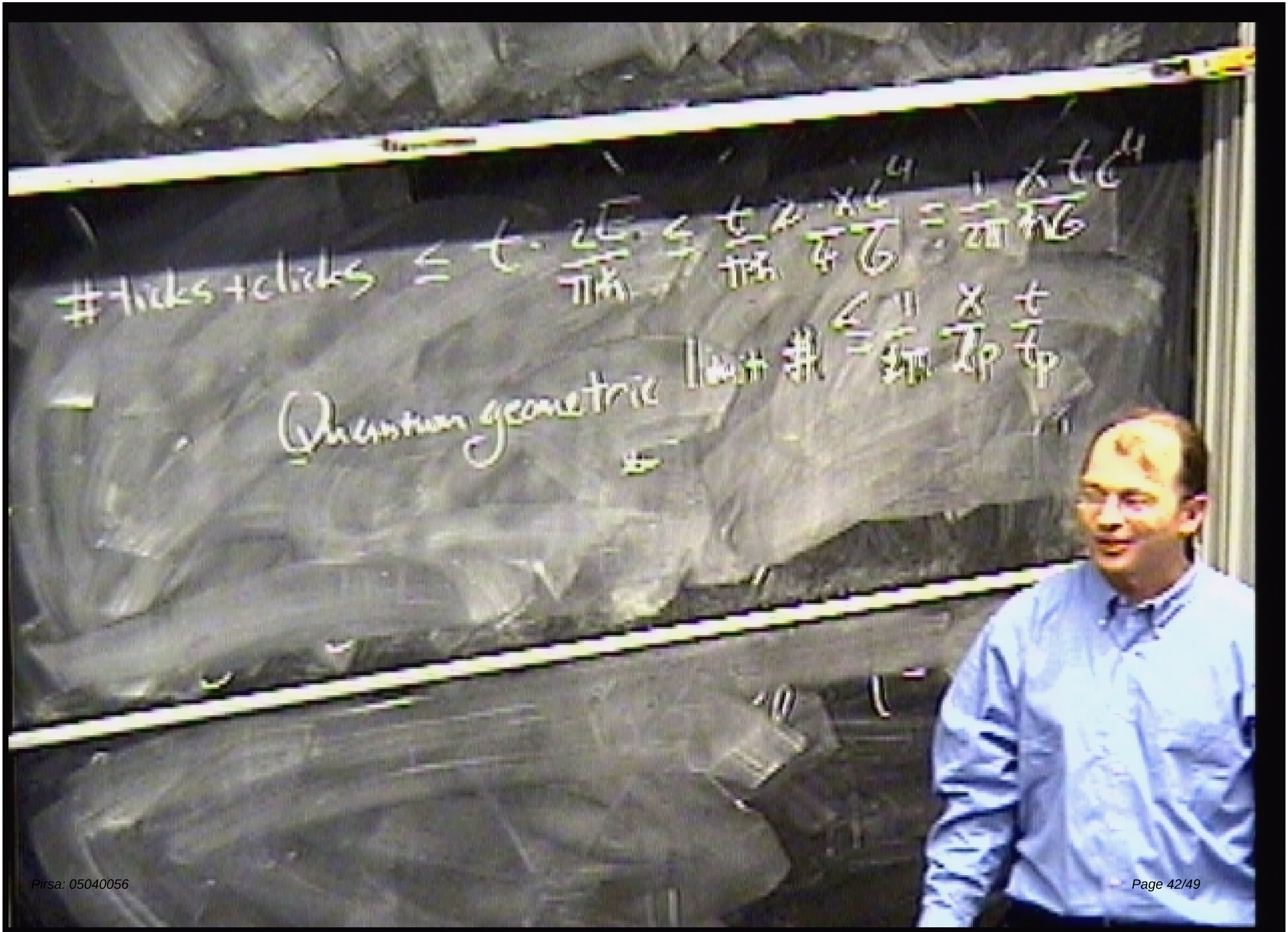
$$R_s = \frac{2Gm}{c^2}, \quad m \le \frac{c^2 R_s}{2G} = \frac{c^2 x}{4G}$$

$$E = mc^2 \le \frac{x c^4}{4G}$$

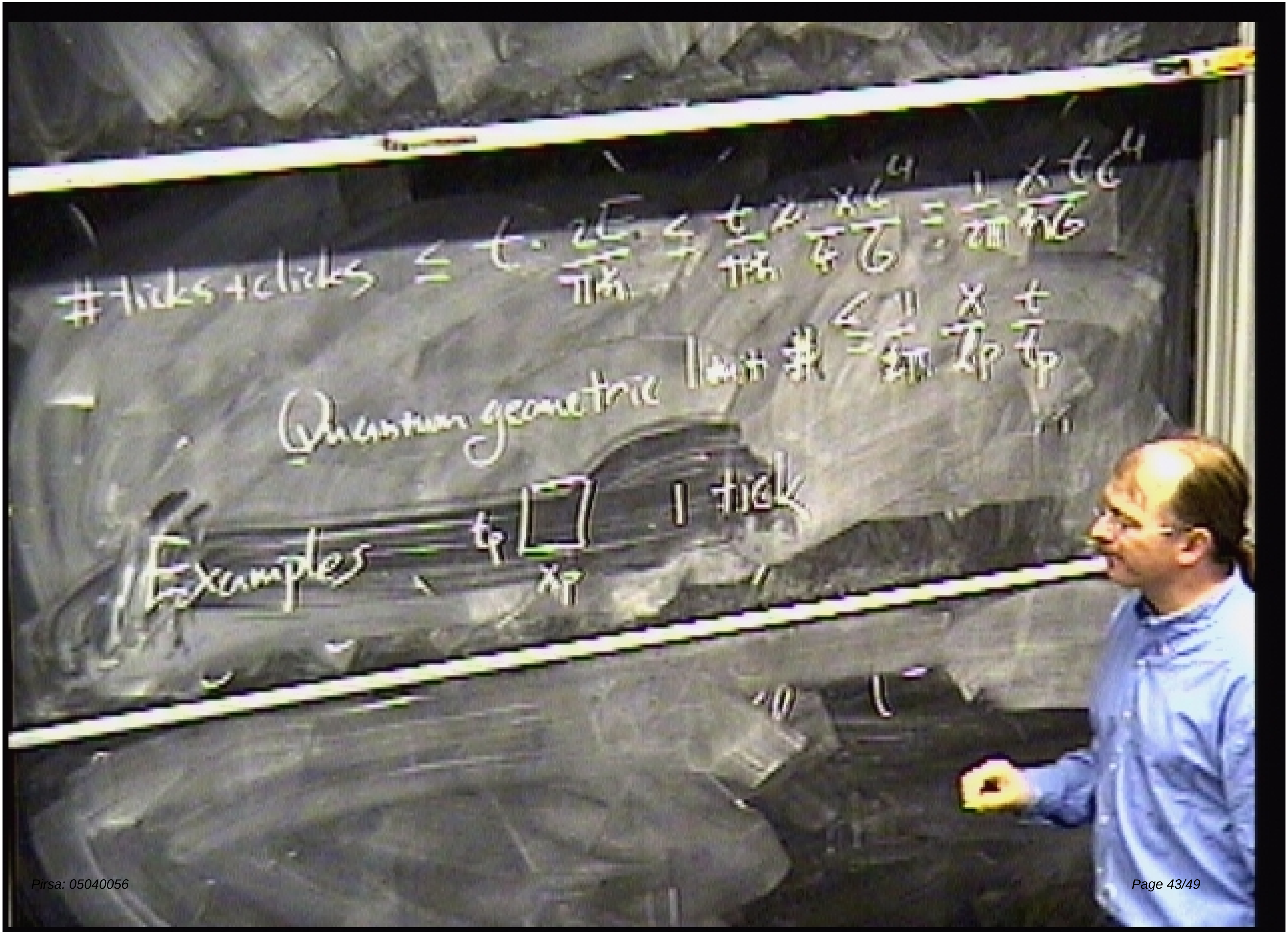
$\Rightarrow$

#ticks+clicks $\leq t \cdot \frac{2L}{[illegible]} \leq \frac{t}{[illegible]} \cdot \frac{x c^4}{[illegible]} = \frac{1}{[illegible]} \frac{x t c^4}{[illegible]}$

$= \frac{1}{[illegible]} \frac{x}{l_P} \frac{t}{t_P}$



#ticks+clicks
Quantum geometric limit


#ticks + clicks
Quantum geometric limit
Examples
1 tick

$$\#\,\text{ticks} + \text{clicks} \le t \cdot \frac{2E}{\pi\hbar} \le \frac{t}{\pi\hbar}\,\frac{x c^4}{G} = \frac{1}{2\pi}\,\frac{x\, t\, c^4}{\hbar G}$$

$$\le \frac{1}{2\pi}\,\frac{x}{x_P}\,\frac{t}{t_P}$$

Quantum geometric limit

Examples $t_P \; \boxed{} \; x_P$ 1 tick

$$\left(\frac{t}{t_P}\right)^2 \sim 10^{120}$$

Universe $x \sim ct$, $t \approx 10^{10}$ years $\sim 10^{17}$ sec.

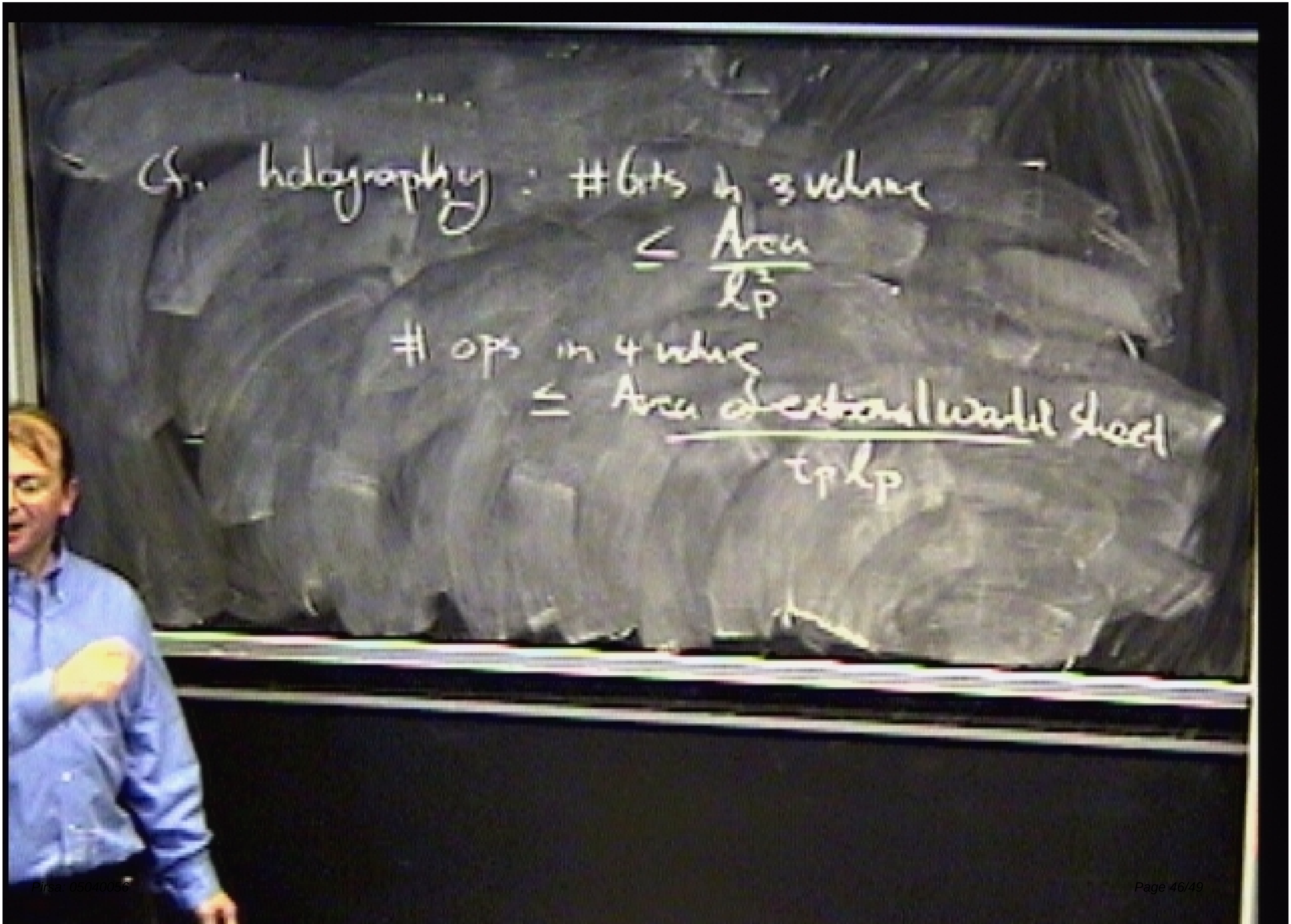
$$\#\,\text{ticks + clicks} \leq t \cdot \frac{2E}{\pi\hbar} \leq \frac{t}{\pi\hbar} \cdot \frac{x c^4}{4G} = \frac{1}{4\pi} \frac{x\, t\, c^4}{\hbar G}$$

$$\#\text{ops/sec} \longleftarrow$$

Quantum geometric limit $\# \leq \frac{1}{4\pi} \frac{x}{x_p} \frac{t}{t_p}$

Examples t_p $\square$ x_p 1 tick $\frac{t^2}{t_p^2} \sim 10^{120}$

Universe $x \sim ct$, $t \approx 10^{10}$ years $\sim 10^{17}$ sec.


Cf. holography : # bits in 3 volume
≤ Area / l_p^2
ops in 4 volume
≤ Area of extremal world sheet / $t_p l_p$

Cf. holography: # bits in 3 volume

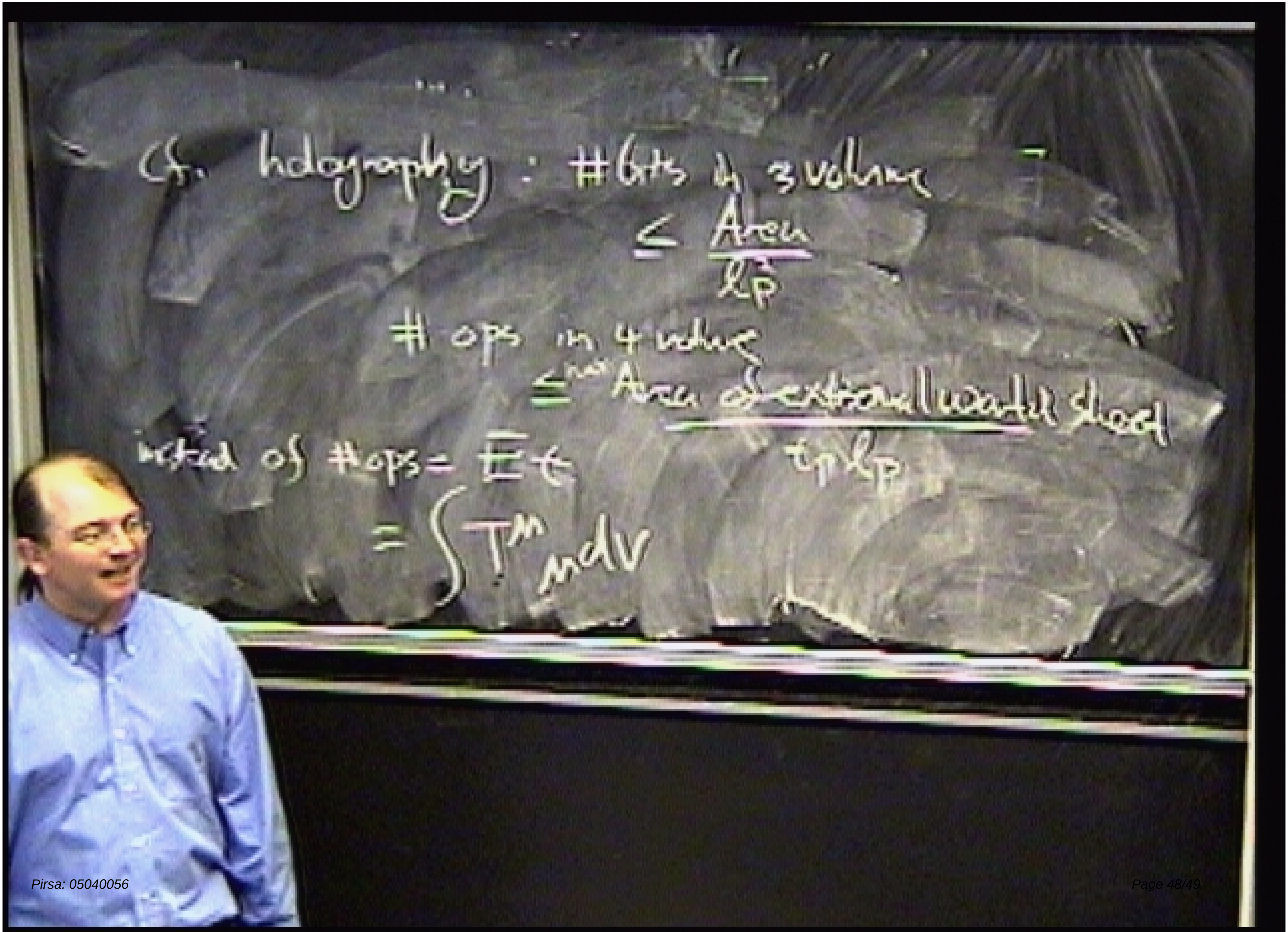
$$\leq \frac{\text{Area}}{l_P^2}$$

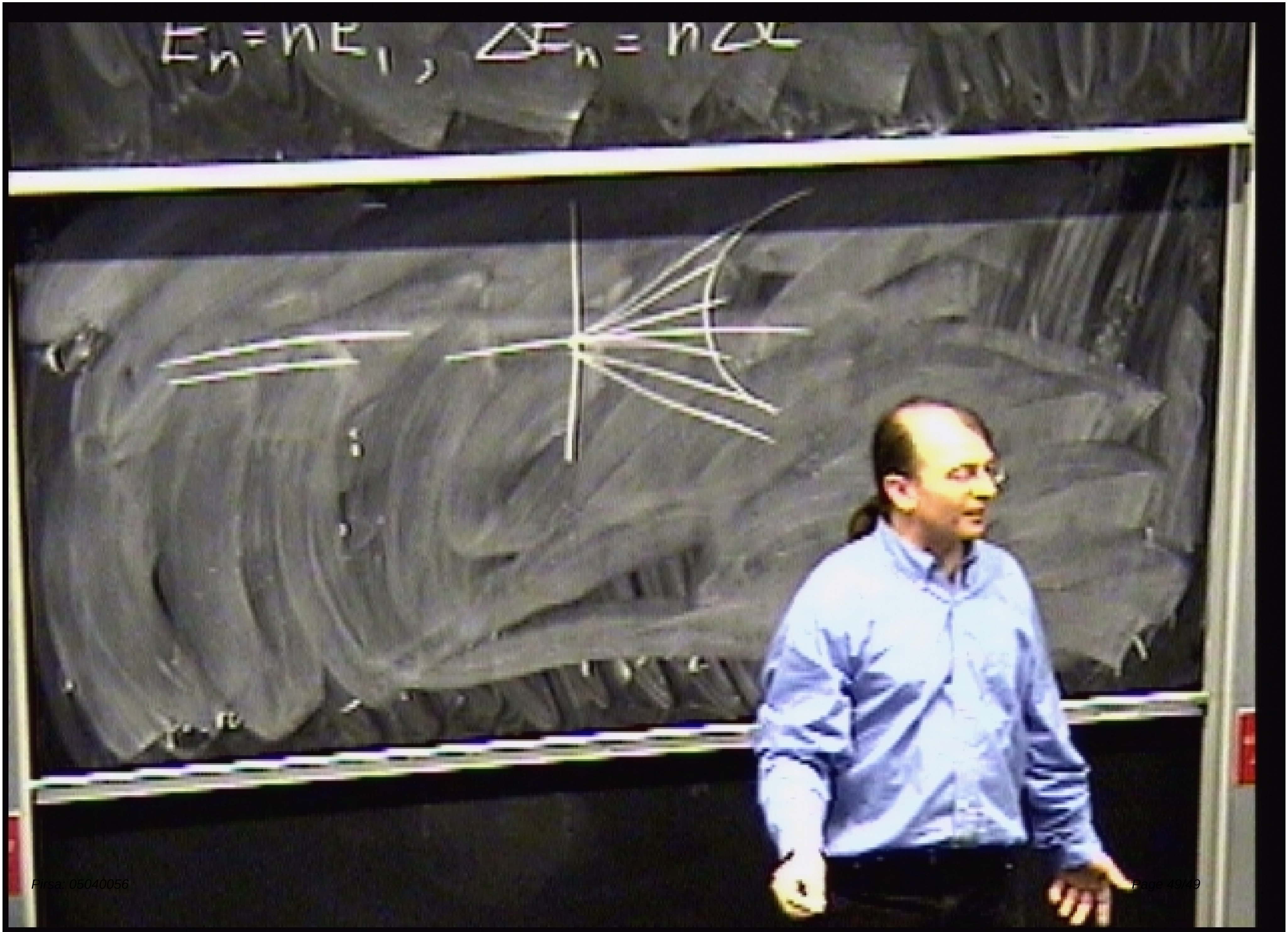
ops in 4 volume

$$\leq \frac{\text{Area of extremal world sheet}}{t_P\, l_P}$$

instead of # ops $= Et$

$$= \int T^{\mu\nu}\, dV_{\mu}$$


Cf. holography : # bits in 3 volume
≤ Area
ops in 4 volume
instead of # ops = E t


$E_n = nE_1$, $\Delta E_n = n$