Title: Dynamics of Extra Dimensions

Date: Apr 06, 2005 02:00 PM

URL: http://pirsa.org/05040044

Abstract: A key issue in the context of (compact) extra dimensions is the one of their stability. Any stabilization mechanism is effective only up to some given energy scale; if they can approach this energy, 4$d observers can excite the fluctuations of the internal space, and probe its existence. Stabilization mechanisms introduce fields in the internal space; perturbations of these fields are mixed with perturbations of the metric, so that their study requires a complete GR treatment. After presenting the general framework, I will then discuss some relevant applications. I will present the exact coupling of the radion to codimension one branes, extending the regime of validity of the results in the literature. I will then focus on de Sitter compactifications, showing that the cosmological expansion has typically the effect of destabilizing the internal space. The final part of the talk will be devoted to related work in progress in less conventional areas of brane models: the localization of gravity towards the IR brane (corresponding to a dual description of emerging gravity from the CFT), and the inclusion of ghost fields in the bulk.

Dynamics of extra dimensions

Marco Peloso, Minnesota

Extra dim. are dynamical: possibility of detection. General formalism + applications:

- Coupling of the perturbatios to SM fields
- Gravitational instability, $m^2 \propto -H^2$, in the early universe (inflation).
- Study of ghost / tachyon configurations
- Changing the shape of gravity

Kofman, Martin, M. P., PRD 70:085015, 2004

Contaldi, Kofman and M.P., JCAP 0408:007, 2004

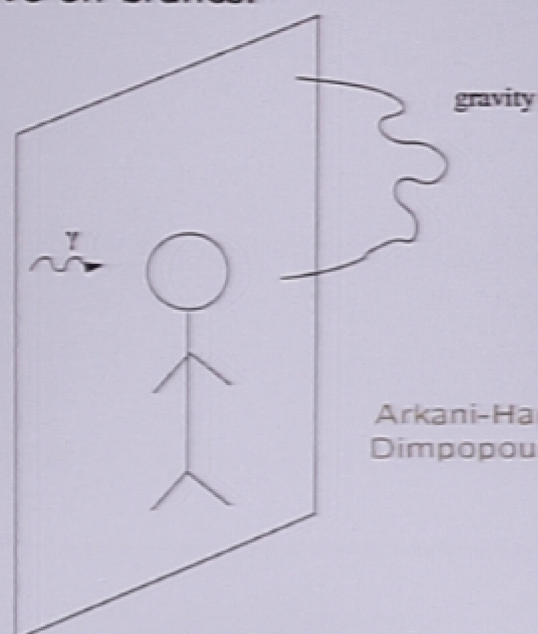
Nunes, M.P., in progress

Gherghetta, M.P., Poppitz, in progress

Many theoretical extensions of the SM have (compact) extra dimensions. However, the nature we see is 4 dim.

- KK compactification: they are too small, $R \sim 10^{-32}$ m, to be excited in lab

- Large, but we (= fermions and gauge fields) live on branes.



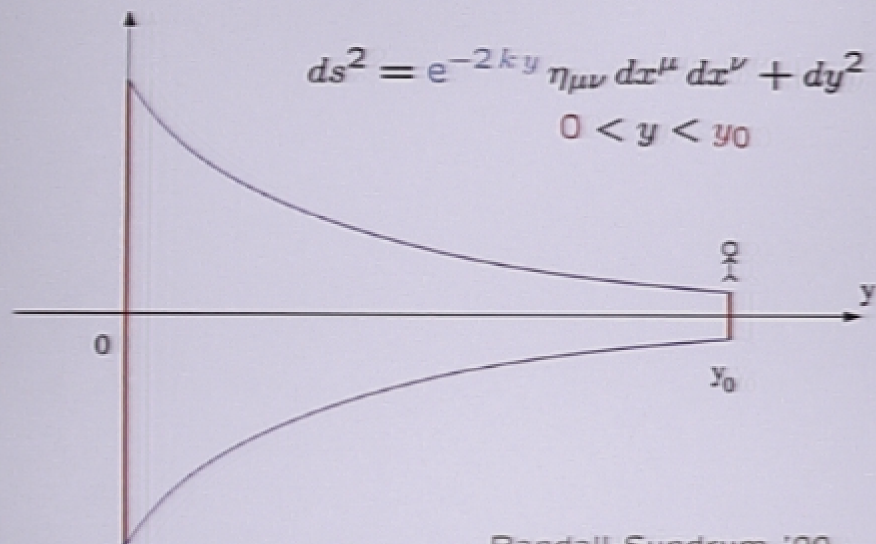
Arkani-Hamed, Dimpopoulos, Dvali, '98

Gravity is weaker because it spreads on a large volume (Gauss law). $R \lesssim 0.1$ mm

Warped extra-dimension

$$ds^2 = e^{-2ky}\,\eta_{\mu\nu}\,dx^\mu\,dx^\nu + dy^2$$

$$0 < y < y_0$$



Randall-Sundrum '99

Physical scales on our brane rescaled by the warped conformal factor

$$m_{\text{phys}}(y_0) = e^{-k y_0}\, m_0$$

$$k\, y_0 \sim 35 \;\Rightarrow\; e^{-k y_0} = \frac{\text{TeV}}{M_p}$$

y_0 is a modulus.

Need for a stabilization

Bulk volume / distance between branes not fixed by gravity.

- "Theoretical" issues

Huge hierarchy from a small one $10^{16} \sim \exp(35)$. Need to set y_0 with extreme accuracy.

- "Phenomenological" issues:

1. Electroweak coupling bulk fluctuations $\leftrightarrow$ SM field.
2. Expansion of the Universe 4d from BBN on; need to "freeze" the extra space, Kanti *et al.* '99; Csáki *et al.* '99

Stabilization mechanism: solitonic configuration in the bulk, stable due to topology and/or boundary conditions. Bulk stabilization from the interplay of gradient ($V \to \infty$) and potential energy ($V \to 0$).

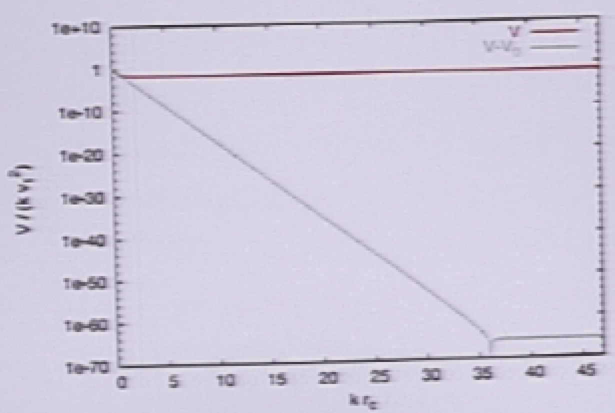
Ex: Scalar field with bulk + brane potentials

Goldberger, Wise '99

$$S = \int d^5x\sqrt{|g|}\left\{\frac{1}{2}R - \Lambda - \frac{1}{2}(\partial\phi)^2 - V(\phi)\right\}$$
$$- \sum_{i=1}^{2}\int d^4\xi\sqrt{|\gamma|}\left\{[K]_i + \Lambda_i + U_i(\phi)\right\}$$

Set $V = \frac{1}{2}m^2\phi^2$, $U_i = \lambda_i\left(\phi^2 - v_i^2\right)^2$

Effective potential: integrate (over y) action for ϕ in the unperturbed RS geometry.



Tiny minimum, $\Delta V \sim 10^{-83}$. Initial conditions ? (cf. Brustein, Steinhardt '93)

Quantization of perturbations in warped geometries

Kofman, Martin, M.P. '04

- Background of the form

$$ds^2 = A^2(y)\left[\eta_{\mu\nu}\, dx^\mu\, dx^\nu + dy^2\right]$$

$$A^2(y) = \frac{1}{1+k^2y^2} \quad \text{for RS}$$

- GW scalar field ϕ gives mass to the radion. It also modifies the AdS bulk geometry.

- Linearized perturbations on a given background (semiclassical calculation)

- Physical perturbations are not purely gravitational modes, $\delta g_{\mu\nu} \leftrightarrow \delta\phi$. Need to identify the physical modes (not at all obvious in the starting action)

Analogy with scalar field inflation in $3+1$ d

3+1	4+1
t−dep. background	y−dep. background
$\phi(t)$ for inflation	$\phi(y)$ for stabilization
$a(t)$ scale factor	$A(y)$ warp factor
$\delta g_{\mu\nu} \leftrightarrow \delta\phi \Rightarrow$ CMB	$\delta g_{\mu\nu} \leftrightarrow \delta\phi \Rightarrow$ LHC

Linearized Einstein eqs. for perturbations:

Initial-value pbm	Boundary-value pbm

Action perturbations for the normalization:

→ initial conditions	→ coupling to branes

Perturbations in 4d cosmology

Bardeen '80

- Classify perturbations into irreducible SO(3) representations (do not mix at linear level)

$$ds^2 = a^2(t)\left[(1+2\Phi)\,dt^2 - 2\left(E_{;i}+\bar{E}_i\right)dt\,dx^i + \right.$$
$$\left. -\left(\delta_{ij} - 2\psi\delta_{ij} - B_{;ij} - \bar{B}_{(i;j)} - h_{ij}\right)dx^i\,dx^j\right]$$

$\bar{E}, \bar{B}$ transverse; h transverse traceless:

4 scalar, 2×2 vector, 2 tensor modes ($=10$)

- Gauge invariance:

$x^\mu \to x^\mu + \delta x^\mu$ removes 2 scalars and 1 vector

Remaining degrees of freedom combined into physical (gauge invariant) modes

Extend this calculation to 5d. Scalar sector nontrivial: several modes. Constraint equations $\Rightarrow$ only one mode is dynamical

$$\left[\Box_4 + \frac{d^2}{dy^2} + \mathcal{F}[A,\phi]\right]\Psi = 0$$

Separate $\Psi = \sum_n \bar{\Psi}_n(y)\, Q_n(x)$

$$\begin{cases} \left(\Box_4 - m_n^2\right) Q_n = 0 \\ \left[\frac{d^2}{dy^2} + \mathcal{F} + m_n^2\right] \bar{\Psi}_n = 0 \end{cases}$$

Boundary conditions determine m_n, and the shape of $\bar{\Psi}_n$. However these are all linear eqs. Normalization undetermined. Needed to get the coupling to brane fields

$$\mathcal{L}_{\rm int} \sim \frac{\Psi}{\Lambda} T_{\rm br}{}^{\mu}_{\mu} \sim \frac{\bar{\Psi}_n(y_{\rm br})}{\Lambda} Q_n T_{\rm br}$$

First, no scalar field. In this case, KK tower absent

$$ds^2 = A(y)^2 \left[(1-\Phi)\, \eta_{\mu\nu} dx^\mu dx^\nu + (1+2\Phi)\, dy^2 \right]$$

$$\Phi = \bar{\Phi}(y)\, Q(x) \quad \Rightarrow \quad \bar{\Phi}(y) = \bar{\Phi}_0 \left[A_0 / A(y) \right]^2$$

Action for the perturbation:

$$\begin{aligned} S &= -\frac{3}{4} \int d^5x\, A^3\, \eta^{\mu\nu}\, \partial_\mu \Phi\, \partial_\nu \Phi \\ &= -\frac{3\, M^3\, A_0^4\, \bar{\Phi}_0^2}{4} \int \frac{dy}{A} \int d^4x\, \eta^{\mu\nu}\, \partial_\mu Q\, \partial_\nu Q \\ &\equiv -\frac{1}{2} \int d^4x\, \eta^{\mu\nu}\, \partial_\mu Q\, \partial_\nu Q \end{aligned}$$

$\equiv$ determines the normalization of Φ

$\Box Q = 0$, radion is a massless gravi-scalar.

- If $\phi \ll 1$ (units of 5d Planck mass) the above analysis is a good approximation, since the mixing $\Phi \leftrightarrow \delta\phi$ can be neglected. The coupling of the radion to SM $\sim$ electroweak scale, Csàki, Graesser, Kribs '01.

- At small ϕ, $m_{\rm radion} \sim \phi$ TeV , $m_{\rm KK} \sim$ TeV . $\phi \sim 1/50$ is probably acceptable, but $\phi \sim 1$ looks more natural. Moreover, it is compatible with a strong warping (solution hierarchy problem).

- Discontinuity in the number of modes. When ϕ is present (no matter how small the amplitude) the whole KK tower appears

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$$S_2 = M^3 \int d^5x\, A^3 \Big\{ \frac{\partial^\mu \delta\phi\, \partial_\nu \delta\phi}{-M^3} + 6\partial^\mu (\Phi + \Psi)\, \partial_\nu \Psi$$
$$- \frac{1}{M^3} \delta\phi'^2 + 12\Psi'^2 + \frac{2}{M^3} \phi' \delta\phi' (\Phi - 4\Psi)$$
$$+ 48 \frac{A'}{A} \Psi' \left(\Psi - \frac{\Phi}{2} \right) - \left(\frac{\phi'^2}{M^3} - 12 \frac{A'^2}{A^2} \right) (\Phi^2 + 8\Psi^2)$$
$$- \left[\frac{1}{M^3} A^2 V'' \delta\phi^2 - \frac{2}{M^3} A^2 V' \delta\phi (\Phi + 4\Psi) \right] \Big\}$$
$$- \sum_i \int d^4x A^4 \left[4U\Psi^2 + 4U'\delta\phi\Psi + \frac{1}{2} U'' \delta\phi^2 \right]$$

$$\Downarrow \qquad \Phi = \textstyle\sum_n \bar{\Phi}_n(y)\, Q_n(x) \ldots$$

$$S = \sum_n C_n \int d^4x\, Q_n \left[\Box - m_n^2 \right] Q_n$$

$$C_n = \frac{3\,M^3}{2} \int_0^{y_0} \frac{dy}{A} \left[\frac{3\,M^3}{2\,\phi'^2} \left(A^2 \bar{\Phi}_n \right)'^2 + \left(A^2 \bar{\Phi}_n \right)^2 \right]$$
$$\equiv \frac{1}{2}$$

Example: perturbations in the model by De Wolfe, Freedman, Gubser, Karch '99

$$V = -6\,k^2 + \left(2\,k\,u + \frac{u^2}{2}\right)\phi^2 - \frac{u^2}{6}\phi^4$$

Background solutions ($dw = A\,dy$)

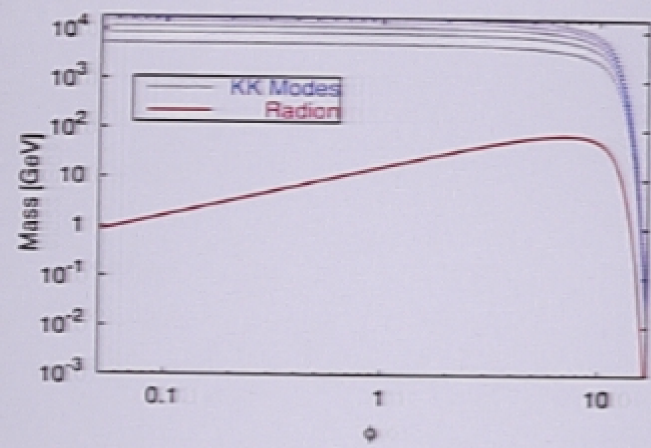
$$\phi = \phi_0\, e^{-u\,w}$$

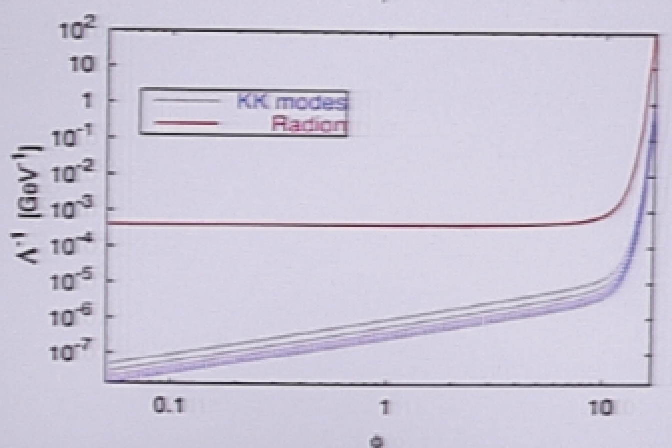
$$A = \exp\left[-k\,w + \frac{\phi_0^2}{12}\left(1 - e^{-2\,u\,w}\right)\right]$$

Place the second brane at w_b, and choose brane potentials to satisfy the junction conditions.

For any given ϕ_0, choose k and w_b to reproduce M_p and TeV at the second brane. In particular, the interbrane distande has to be increased when ϕ_0 increases.

Parameter chosen to preserve M_p and TeV at the second brane (interbrane distance increases when ϕ increses)





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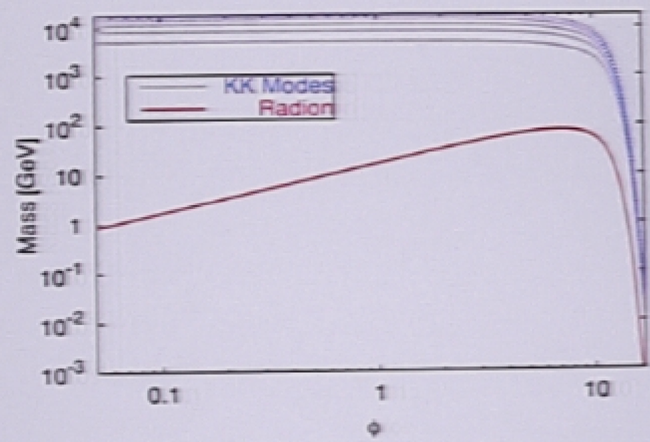

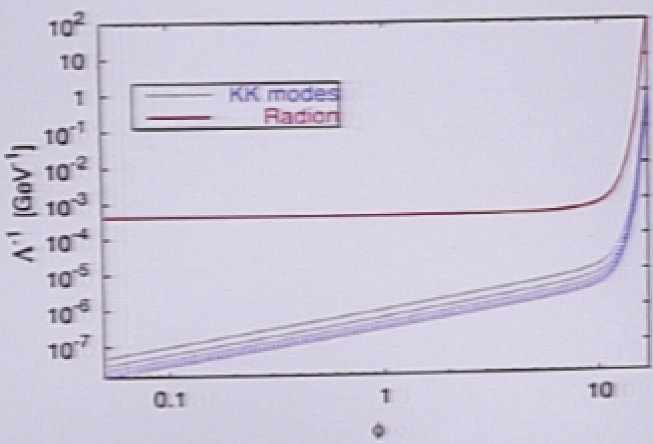

de Sitter compactifications $dS_4 \times \mathcal{M}_d$

Contaldi, Kofman, M.P. '04

- Consider (dS) expansion of the noncompact coordinates. Analysis as before. Solve the eigenvalue equation for $\bar{\Psi}_n$, and look at the equations for Q_n. $\Box_4$ always enters in the combination

$$-m_0^2 Q_n = -\left[\Box_4 + \frac{12\,d}{d+2} H^2\right] Q_n(x) =$$

$$\left[\partial_t^2 + 3\,H\,\partial_t - e^{-2Ht}\partial_i^2 - \frac{12\,d\,H^2}{d+2}\right] Q_n(x)$$

Hence

$$m_{\mathrm{phys}}^2 = m_0^2 - \frac{12\,d\,H^2}{d+2}$$

- General effect, not limited to codimension 1 braneworlds

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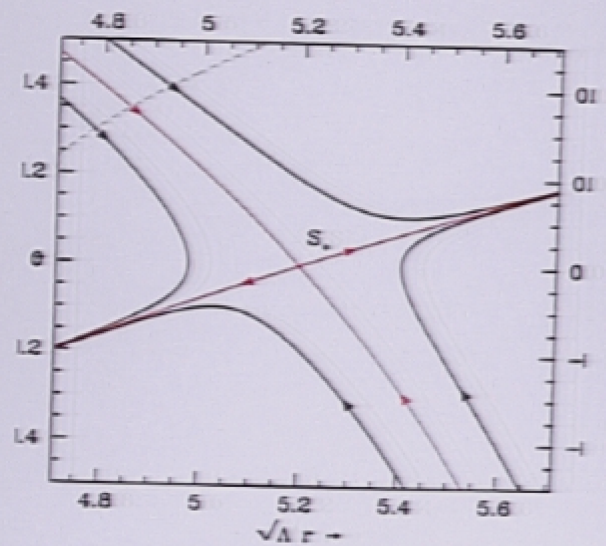
$dS_4 \times S_d$; $\Lambda > 0$, no stabilization mechanism;

(flux compactification, Bousso, DeWolfe, Myers, '02)

$$-\Box_4 - \frac{12\,d}{d+2}H^2 + \frac{4\,(d-1)}{d\,(d+2)}R_d = -\Box_4 - 6\,H^2$$



- Up-right: instability towards dS_{4+d}
- Down-left: S_d crunches with Kasner asymp.

- A possible causality argument

RS1 Cosmology with stabilization mechanism and brane fields. Radion of infinite mass (rigid extra dimension),

Lesgourgues, Pastor, M.P., Sorbo '00

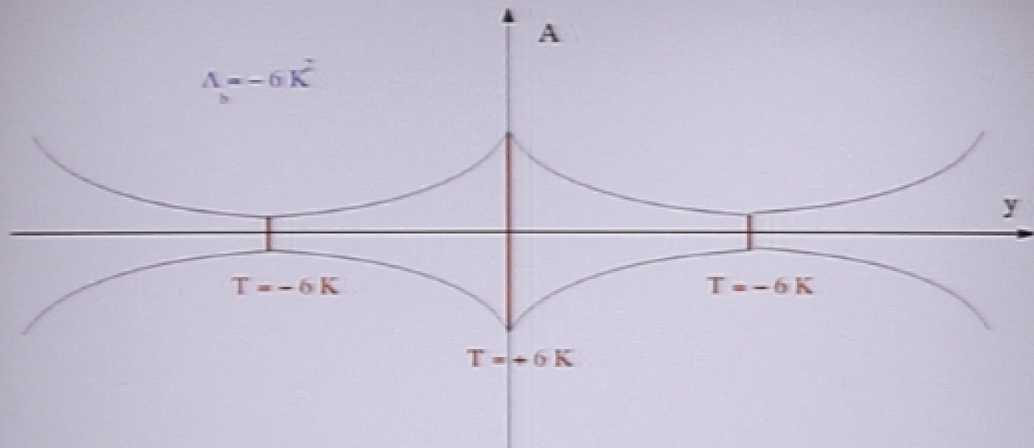
$$H_1^2 \simeq \frac{1}{3M_p^2} \frac{\rho_1 + \rho_0 - \rho_0\rho_1/\left(10\, M_p^2\, \mathrm{TeV}^2\right)}{1 - \rho_0/\left(10\, M_p^2\, \mathrm{TeV}^2\right)}$$

(phisical quantities; $H_1 = H e^{-A(y_1)}, \ldots$)

ρ_0 (hidden brane) dark matter/energy/radiation....
Intermediate scale $\bar{\rho} \sim \sqrt{M_p TeV}$?

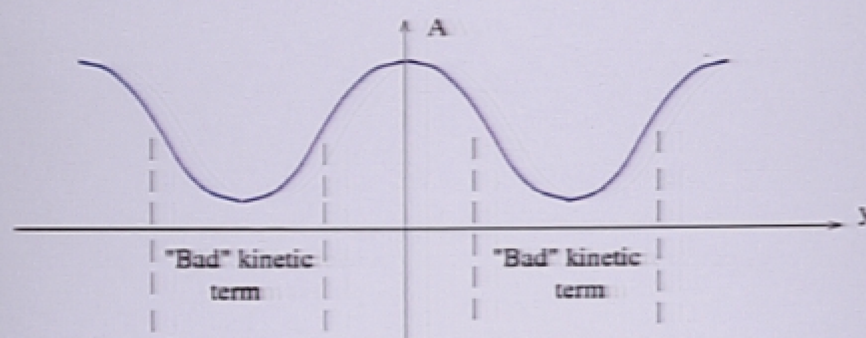
$$\rho_0 > \bar{\rho} \quad - \quad H > D^{-1}$$

No matter what the stabilization mechanism is, the bulk is not static when the size of the extra space is greater than H^{-1} ("different edges do not talk to each other")



Field-theory resolution

$$V(\phi) = -\frac{1}{2}(\partial\phi)^2\frac{\phi}{1-\phi^2} + |\Lambda_b|\left(1 - \frac{\omega}{2|\Lambda_b|}\phi - \phi^2\right)$$



$\phi = \phi_0 \cos\omega y$, $A = \exp[\cos\omega y]$

Ghost corresponding to $T < 0$

Pospelov '04

- $4-d$ ghost is bad, e.g. vacuum decay

$$E = \sqrt{m^2 - p^2} \quad \longleftarrow \qquad E = \sqrt{m^2 - p^2} \quad \longrightarrow$$

Solutions ? Possibly, cure it with higher derivatives (e.g. ghost condensation)

- Less clear in higher dimensions, when the sign of the kinetic term is not univoquely defined

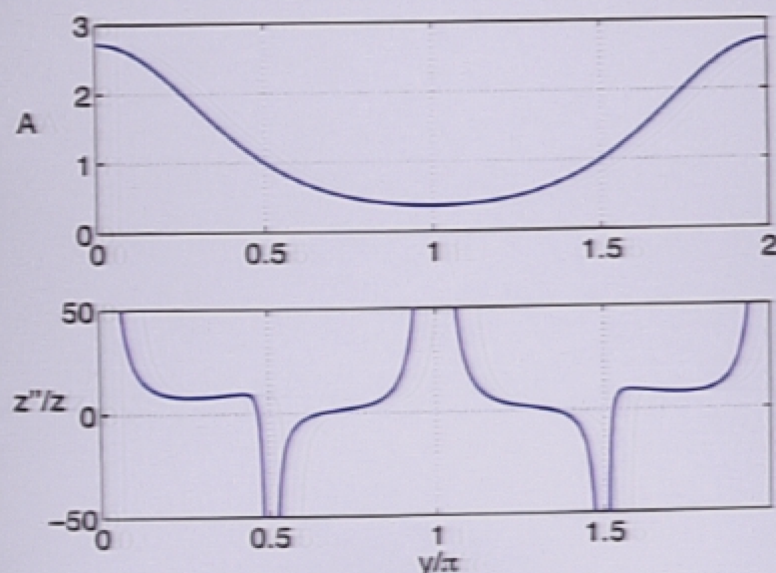
Need to compute the $4-d$ action, and check the resulting sign for the kinetic term. Semi-classical calculation in progress (with Nunes)

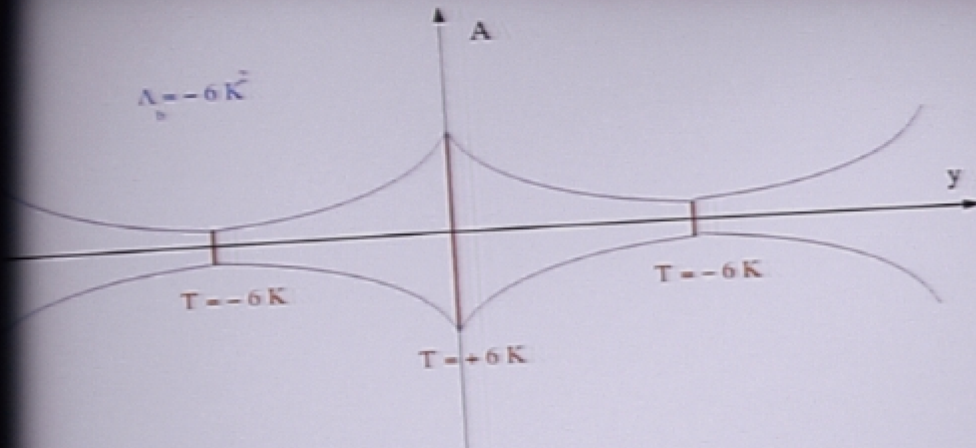
$$\mathcal{L} = -\frac{F(\phi)}{2}(\partial\phi)^2 - V(\phi)$$

$$S = \sum_n C_n \int d^4x\, Q_n \left[\Box - m_n^2\right] Q_n$$

$$C_n = \frac{1}{2}\int dy\, \sigma(F)\, \bar{\Psi}_n^2$$

$$\left(\frac{d^2}{dy^2} - \frac{z''}{z} + m_n^2\right)\bar{\Psi}_n = 0 \quad , \quad z = \sqrt{2|F|}\,\frac{A^{5/2}\phi'}{A'}$$





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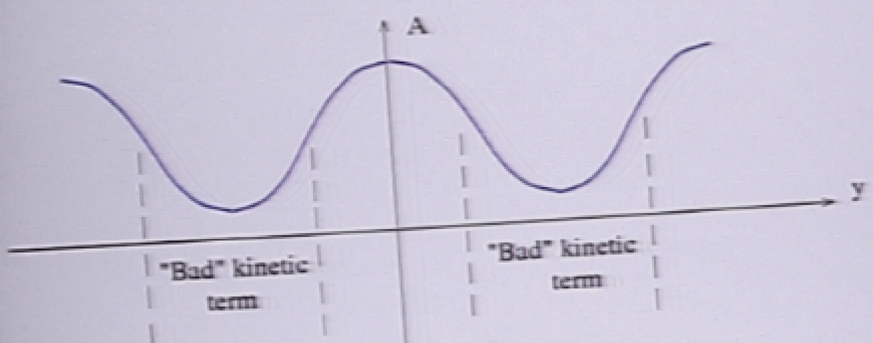

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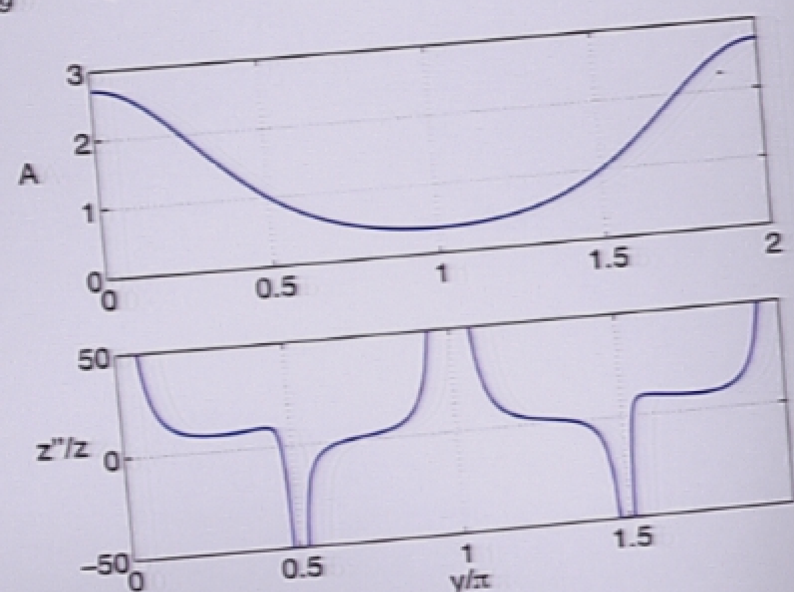

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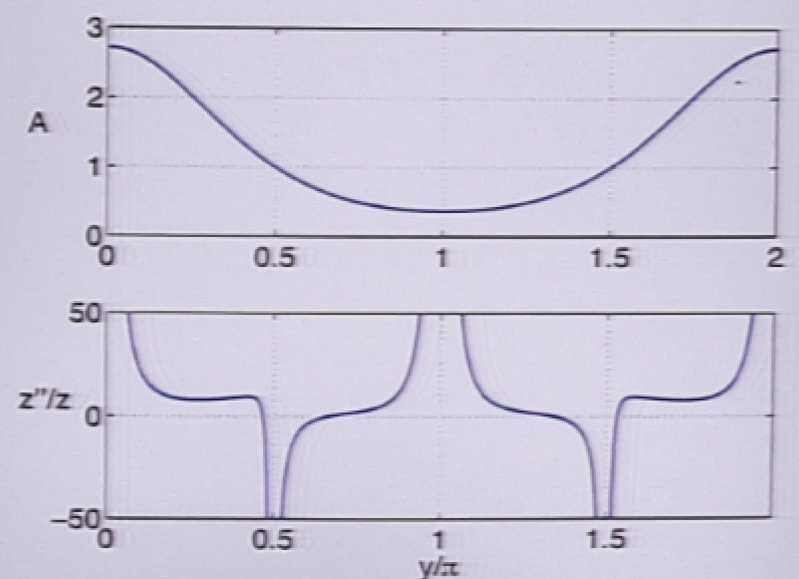


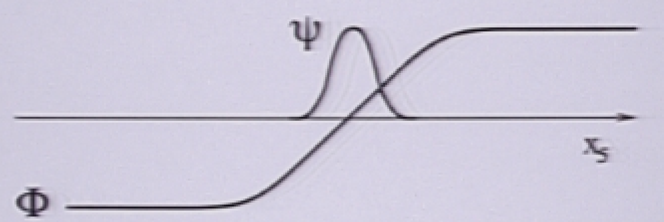
Changing the shape of gravity

Fermions on a kink, Rubakov, Shaposhnikov, '83

$$\mathcal{L} = \bar{\Psi}\,[i\partial\!\!\!/ - (m + g\,\Phi)]\,\Psi$$

Fermions localized where $m + g\phi = 0$

5d "mass" parameter → localization



- Fermion / scalar fields on AdS + branes. Also in this case, bulk profile determined by bulk / brane masses, Pomarol, Gherghetta '00
- Gauge fields, Ghoroku, Nakamura, '01
- Gravity in progress (with Gherghetta, Pop-pitz)

Gravity calculations less straightforward (cf. massive gauge bosons vs. massive gravitons in 4d: vDVZ discontinuity); possible ghosts (Chacko, Graesser, Grojean, Pilo, '03)

Suitable choice (tuning) of bulk-brane mass for the graviton: linearized spectrum contains a massless $4d$ graviton with arbitrary localization

$$g_{\mu\nu} = A(y)^2 \left[\eta_{\mu\nu} + A(y)^{-\alpha} h_{\mu\nu}(x)\right]$$

$$A = \frac{1}{1+ky} \quad , \quad A(y_0) = 10^{-16}$$

RS $\quad \alpha = 0$ graviton at UV brane

$\alpha > 2$ graviton at IR brane

Dual picture: emerging gravity from the CFT

Conclusions

- Extra dimensions are dynamical, which gives the chance to detect them. Need for a stabilization mechanism.

- Perturbations can be treated from well known techniques in cosmology. Same formalism for collider phenomenology as well as the early time dynamics

- General instability effect for dS compactifications. To be accounted for in model of inflation with extra dimensions.

- General bulk scalar fields under consideration. Check stability/instability in this case.

- Gravity towards the IR brane. Emerging gravity in the CFT (?)