Title: A Caculable Toy Model of the Landscape

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Abstract:

A Calculable Toy Model of the Landscape

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hep-th/0412185

1. Introduction / Motivation
2. The FI model $(n=1)$
3. The Toy Model: detailed analysis $(n=2)$
   - vacuum structure
   - landscape probabilities
   - RG flow, phase transitions, RG fixed points
   - soft masses
4. Beyond the Toy Model (general n)
   - $n=3$, arbitrary n
   - flux interpretations
5. Extension to SUGRA

6. Λ for heterotic strings: a statistical study

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6. Λ for heterotic strings: a statistical study

Recent developments in the study of string-theory compactifications suggest the existence of huge numbers of string vacua ("landscape"), each with different low-energy phenomenologies

- susy-breaking scale
- gauge group and particle content
- cosmological constant Λ, ...

How to deal with this situation?

- vacuum selection principle?
- statistical approach?
- anthropic arguments?

Main difficulty:

DIFFICULT TO SURVEY LANDSCAPE CONCRETELY!

HIGHLY NON-TRIVIAL STRING CONSTRUCTIONS!
(e.g., flux compactifications, etc.)

approach:

Examine explicit field-theory counterparts of such constructions

→ field theory models which naturally give rise to large numbers of vacua

↳ can then explicitly calculate physical properties such as

- scale of SUSY breaking
- presence or absence of R-symmetries
- stability of vacua...

OUR FT models : SUSY models with

→ multiple abelian gauge groups
→ multiple charged scalar fields

↳

- straightforward to analyze explicitly
- have a rich vacuum structure
- actual component of full string landscape
- can be viewed as deconstructions of actual flux compactifications

⇒ Should be directly relevant to studies

Start small : a trivial example

The Fayet–Iliopoulos Model of SUSY-breaking

- one $U(1)$ gauge group, gauge coupling g
- FI D-term with coefficient ξ
- two charged chiral superfields $\Phi^{(\pm)}$
- superpotential $W = m\,\Phi^{(+)}\Phi^{(-)}$

Extremize scalar potential $V(\phi^+, \phi^-)$, classify resulting extrema according to vev's

$v_\pm \equiv \langle \phi^\pm \rangle$, find:

$\{\Phi\}$: $v_\pm = 0$: such extrema exist for all ξ, are stable only if $m^2 \pm g^2 \xi \geq 0$

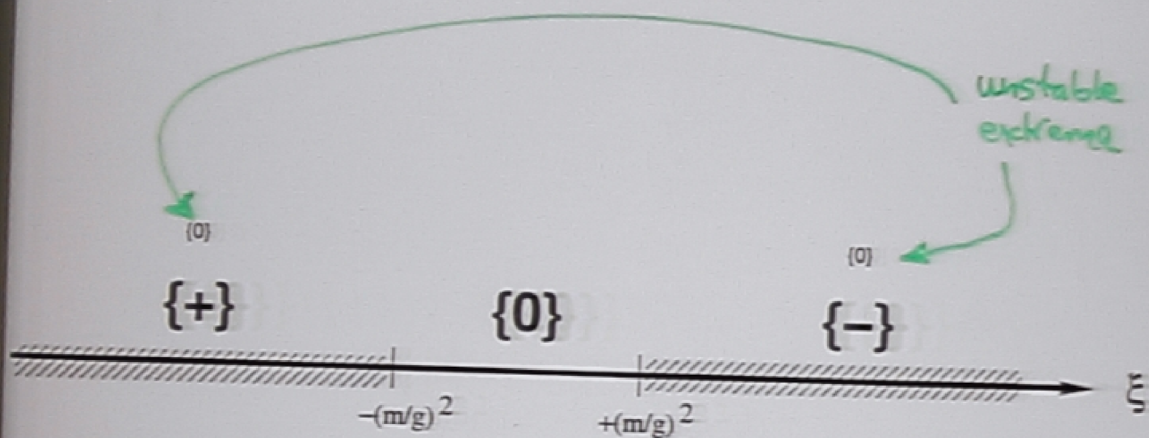
[$U(1)$ unbroken, R symm preserved]

$\{+\}$: $v_+ \neq 0$, $v_- = 0$: exist if $m^2 + g^2 \xi < 0$, always stable

$\{-\}$: $v_+ = 0$, $v_- \neq 0$: exist if $m^2 - g^2 \xi < 0$, always stable

Consider ξ as a parameter defining the model:

$\Rightarrow$ as a function of ξ,
vacuum structure of FI model is thus:

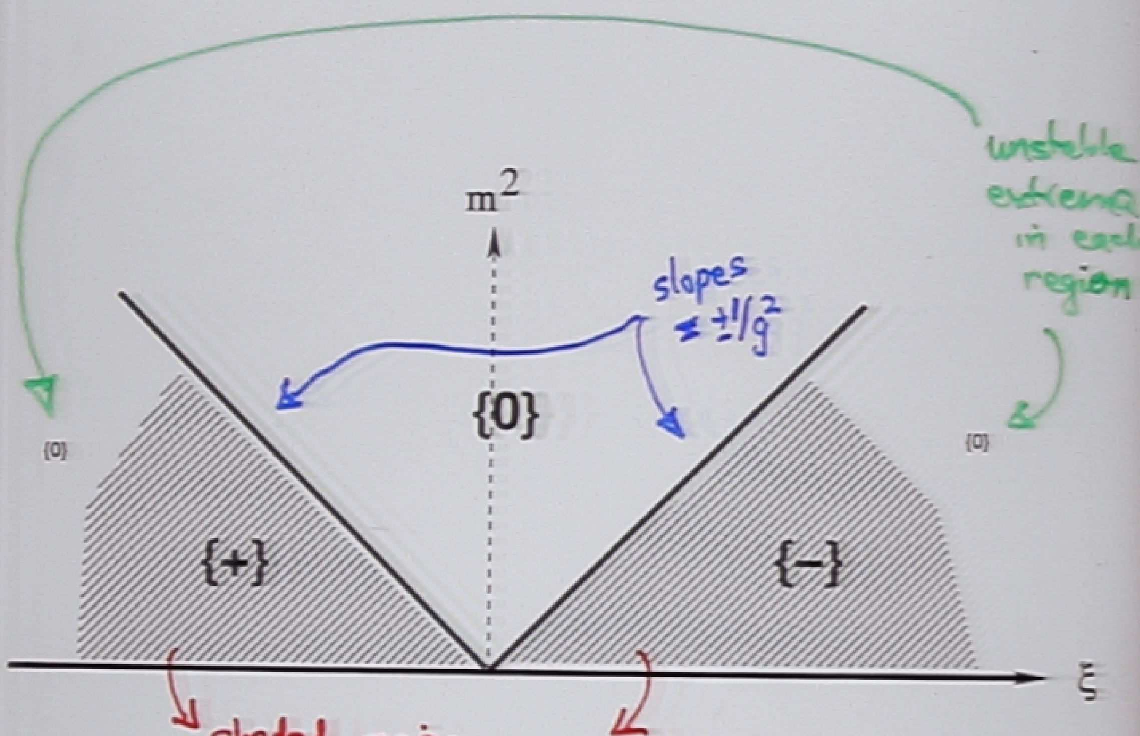


shaded regions indicate potential R-symm. breaking

Each point on this line corresponds to different ξ
$\Rightarrow$ a different theory with a unique vacuum
$\Rightarrow$ this is a field-Theory <u>landscape</u>
for the FI model!

Of course, we can also consider m^2
as a parameter defining the model

$\Rightarrow$ "landscape" is two-dimensional :



shaded region
indicates vacua
which potentially break R-symmetry

Indeed, g can also be considered to be
a landscape coordinate $\Rightarrow$ 3D landscape

Landscape probabilities?

– e.g., probability that R-symm. is preserved?

→ Depends on landscape coordinates & integration measures!

- **1D landscape:** ξ only, measure $d\xi$ 

  $\Rightarrow \underline{P(R)=0}$ (large ξ dominates)

- **2D landscape:** (ξ, m^2), measure $d\xi\, dm^2$ 

  $\Rightarrow P(R) = 1 - \frac{2}{\pi}\tan^{-1} g^2$

- **3D landscape** (ξ, m^2, g)

  If measure $d\xi\, dm^2\, dg \Rightarrow P(R) = 0$ (large g dominates)

  Maybe define angular variable

  $$\theta_g = \tan^{-1} g^2$$

  ↓ (suppresses large-g)

  Use measure $\int_0^{\pi/2} d\theta_g = \int_0^{\infty} dg^2 \frac{1}{1+g^4}$

  Then

  $$P(R) = \frac{2}{\pi}\int_0^{\pi/2} d\theta_g \left(1 - \frac{2}{\pi}\theta_g\right) = \frac{2}{\pi}\left(\frac{\pi}{4}\right) = \frac{1}{2}$$

Are these kinds of calculations meaningful?
Comments:

① Should impose cutoffs $\xi < M_p^2$, $g \lesssim \sqrt{4\pi}$, ...

Results still qualitatively OK.

② Choices of measure?

Underlying landscape parameters are fluxes, etc.
... quantized
... bounded by tadpole constraints

$\{\text{fluxes}\} \longrightarrow \{\xi, m^2, g\}$

measure HERE $\xrightarrow{\text{determines}}$ measure HERE

"effective landscapes"

③ Mapping is ENERGY-DEPENDENT :

$\{\xi, m^2, g\}$ are subject to RG flow!

$\Rightarrow$ these "effective" landscapes and our location in them are subject to

Constructing a TOY MODEL

- $n=2$ $U(1)$'s, gauge couplings g_1, g_2
- FI D-term coefficients ξ_1, ξ_2
- three chiral superfields $\Phi_{i=1,2,3}$

	$U(1)_1$	$U(1)_2$
Φ_1	-1	0
Φ_2	$+1$	-1
Φ_3	0	$+1$

motivated by deconstruction

- Wilson-line superpotential $W = \lambda \Phi_1 \Phi_2 \Phi_3$

We can simplify $g_1 = g_2 = 1$

$\Rightarrow$ model defined by 3 parameters: (ξ_1, ξ_2, λ)
(restrict to $\lambda \geq 0$ without loss of generality)

$\Rightarrow$ MODEL GIVES RISE TO A

RICH VACUUM STRUCTURE ...

To analyze vacuum structure,

perform standard D-term, F-term analysis:

scalar potential
$$V(\phi_i) = \frac{1}{2}\sum_{a=1}^{2} g_a D_a^2 + \sum_{i=1}^{3} |F_i|^2$$

where
$$D_a = \sum_{i=1}^{3} q_i^{(a)} |\phi_i|^2 + \zeta_a \; ; \quad F_i = \frac{\partial W}{\partial \phi_i} .$$

<u>Extrema of $V(\phi_i)$:</u>
$$\frac{\partial V}{\partial \phi_i} = \frac{\partial V}{\partial \phi_i^*} = 0$$

<u>Stability of extrema:</u>

$$\mathcal{M}^2 \equiv \begin{pmatrix} \frac{\partial^2 V}{\partial \phi_i^* \partial \phi_j} & \frac{\partial^2 V}{\partial \phi_i^* \partial \phi_j^*} \\ \frac{\partial^2 V}{\partial \phi_i \partial \phi_j} & \frac{\partial^2 V}{\partial \phi_i \partial \phi_j^*} \end{pmatrix} \quad \ldots \text{ 6×6 mass matrix}$$

→ calculate eigenvalues

$$= \begin{cases} 0 & \text{for each broken } U(1) \\ \text{others} \rightarrow & \underline{\text{STABLE}} \text{ if all } > 0 \\ & \underline{\text{FLAT DIR.}} \text{ if } = 0 \end{cases}$$

classes of vacua according to non-zero $v_i \equiv \langle \phi_i \rangle$

– e.g., $\{\phi\}$: $v_1 = v_2 = v_3 = 0$

$\{1\}$: $v_1 \neq 0$, $v_2 = v_3 = 0$

$\{12\}$: $v_1 \neq 0$, $v_2 \neq 0$, $v_3 = 0$

$\{123\}$: $v_1 \neq 0$, $v_2 \neq 0$, $v_3 \neq 0$

$\Rightarrow$ 8 possible classes of vacua!

SO WHAT DO WE FIND IN OUR TOY MODEL?

$\Rightarrow$ A surprisingly rich vacuum structure!

Start with $\lambda = 1$ $\Rightarrow$ physics depends on (ξ_1, ξ_2) location

Find:

$\{\phi\}$: exists everywhere, always unstable 

$\{1\}$: exists for $\xi_1 > 0$, stable if $|\xi_2| < \xi_1$

$\{2\}$: $\xi_2 > \xi_1$ $\quad$ $\xi_1 < 0$, $\xi_2 > 0$

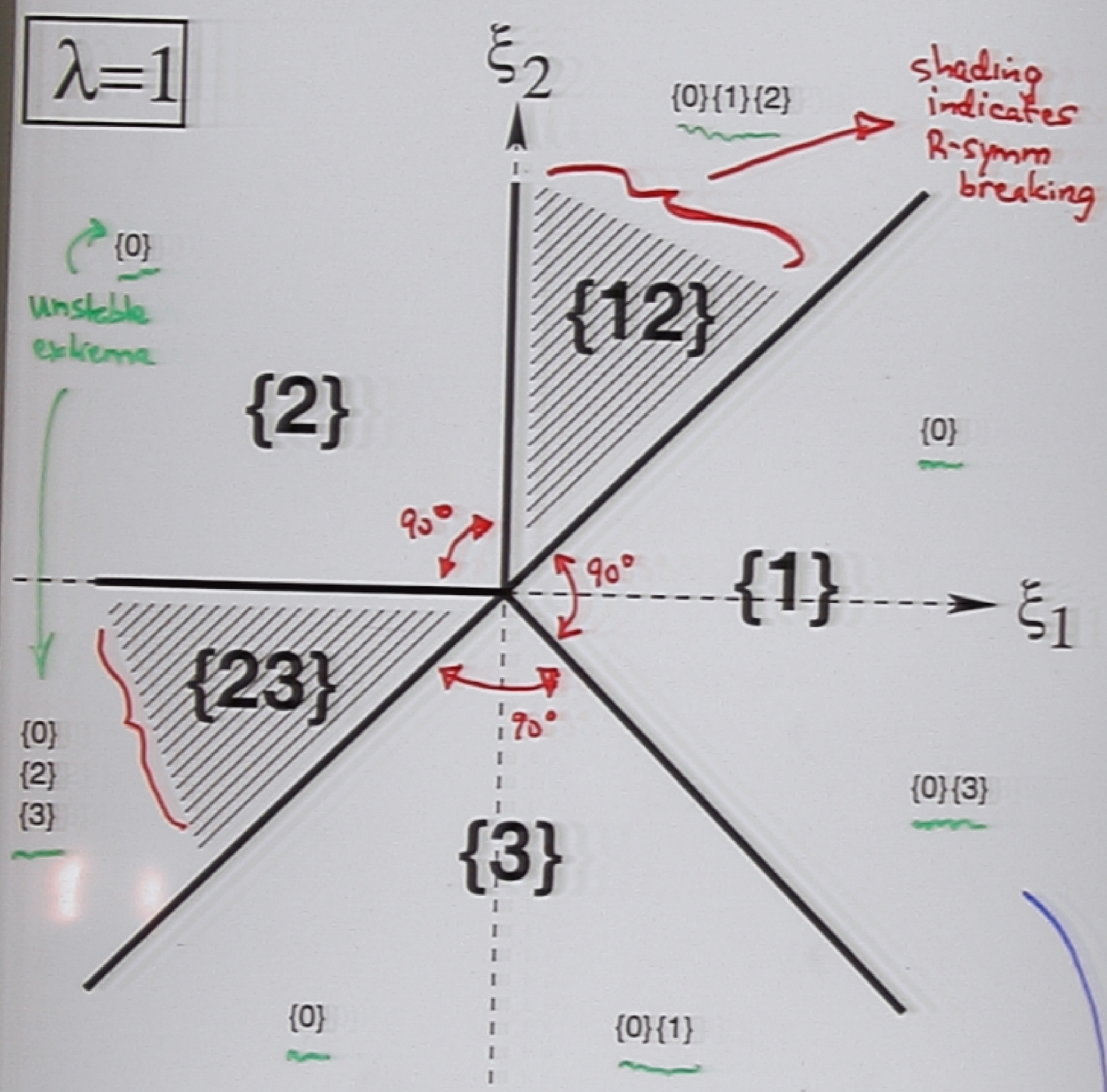
$\{3\}$: $\xi_2 < 0$ $\quad$ $|\xi_1| < |\xi_2|$

$\{12\}$: $0 < \xi_1 < \xi_2$ $\quad$ always

$\{23\}$: $\xi_1 < \xi_2 < 0$ $\quad$ always

$\{13\}$:
$\{123\}$: $\Big\}$ do not exist

- non-overlapping regions of stability!
- no region with overlapping stable vacua!



- each point in this "landscape" has a unique vacuum energy

- Probability that a random extremum is stable:

$$P = \frac{4}{11} \approx 0.36 \qquad \gg \left(\tfrac{1}{2}\right)^6$$

six eigenvalues for M^2

probability of <u>stable</u> extrema is significantly higher than might have been expected.

- Probability that R-symmetry is preserved:

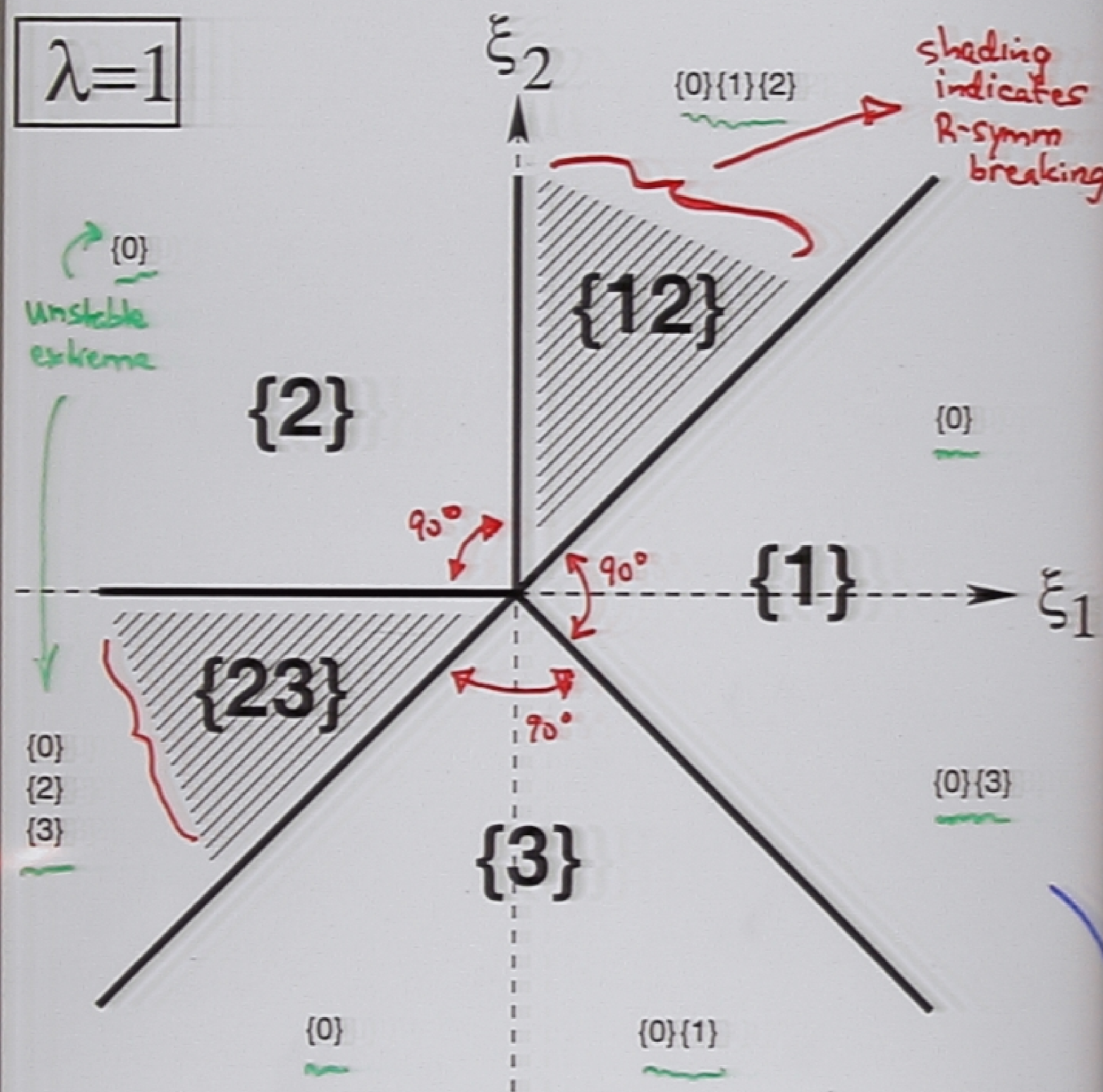
$$P = 75\% \text{ exactly} \quad (\text{for } \lambda = 1).$$

- Probability that SUSY is unbroken:

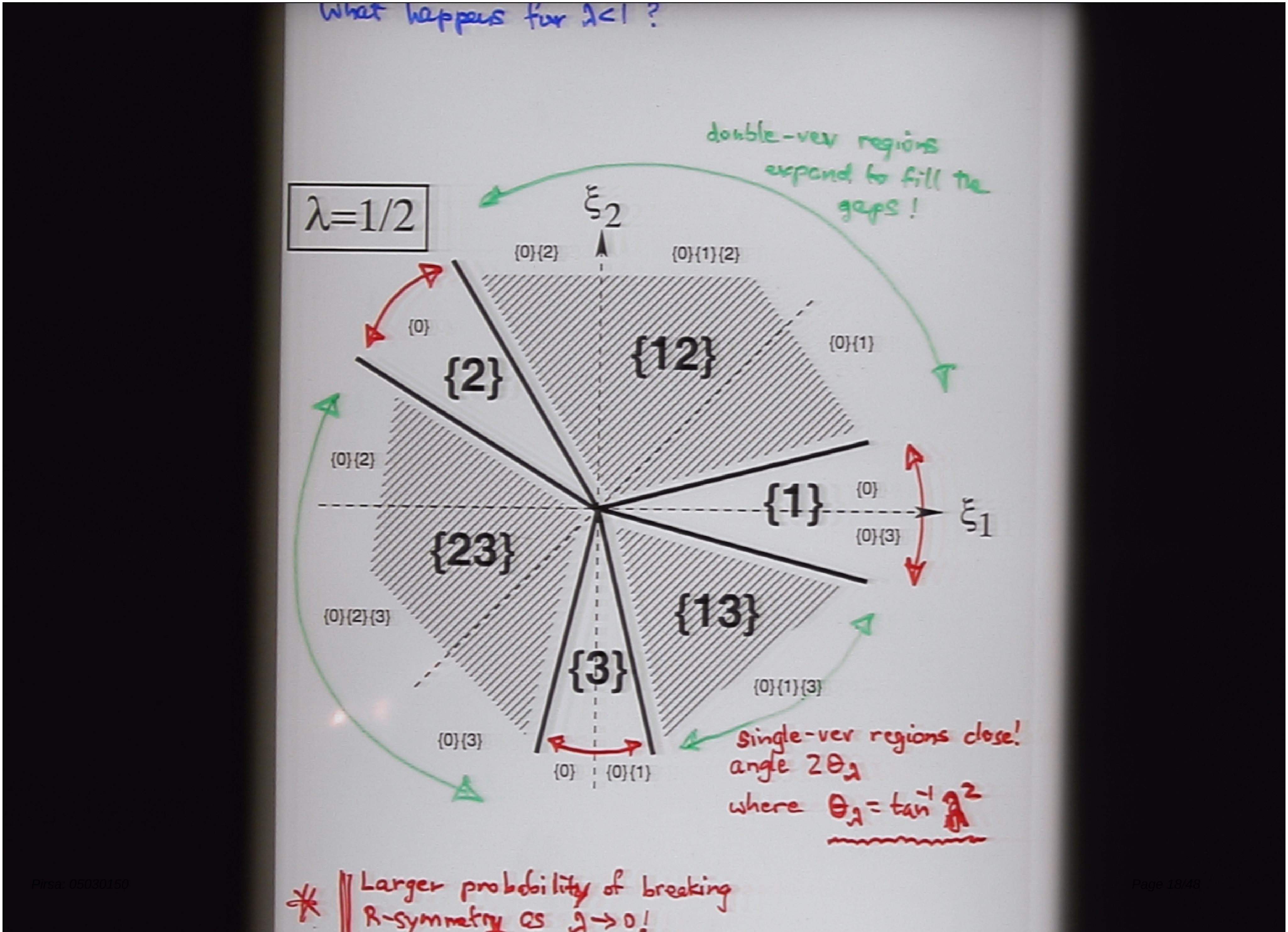
$$P = 0$$

[SUSY-preserving vacua exist only along one-dimensional lines in each single-vev region ... set of measure zero.]

• non-overlapping regions of stability!
• no region with overlapping stable vacua!

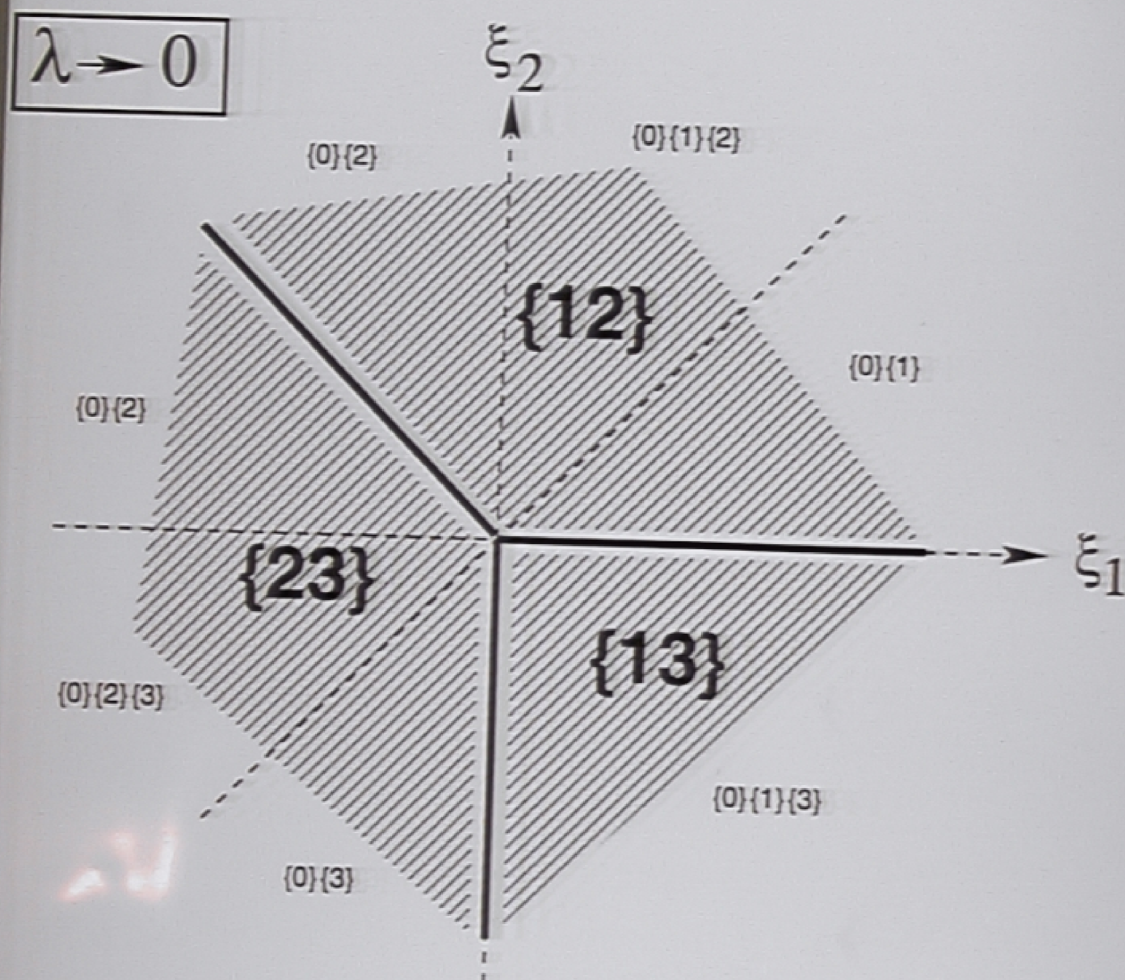


• each point in this "landscape" has a unique vacuum


What happens for λ<1 ?
λ=1/2
ξ_2
ξ_1
double-vev regions expand to fill the gaps!
{0}{2}
{0}{1}{2}
{0}
{12}
{0}{1}
{2}
{0}{2}
{1}
{0}
{0}{3}
{23}
{13}
{0}{2}{3}
{3}
{0}{1}{3}
{0}{3}
{0}
{0}{1}
single-vev regions close!
angle $2\theta_\lambda$
where $\theta_\lambda = \tan^{-1}\lambda^2$
* Larger probability of breaking R-symmetry as $\lambda \to 0$!

As $\lambda \to 0$, single-vev regions disappear entirely!

$\Rightarrow$ entire landscape breaks R-symmetry!



$\lambda = 0$ limit is special: $\Longrightarrow$ flat directions!

- SUSY unbroken for any (ξ_1, ξ_2)
- R-symmetry always preserved

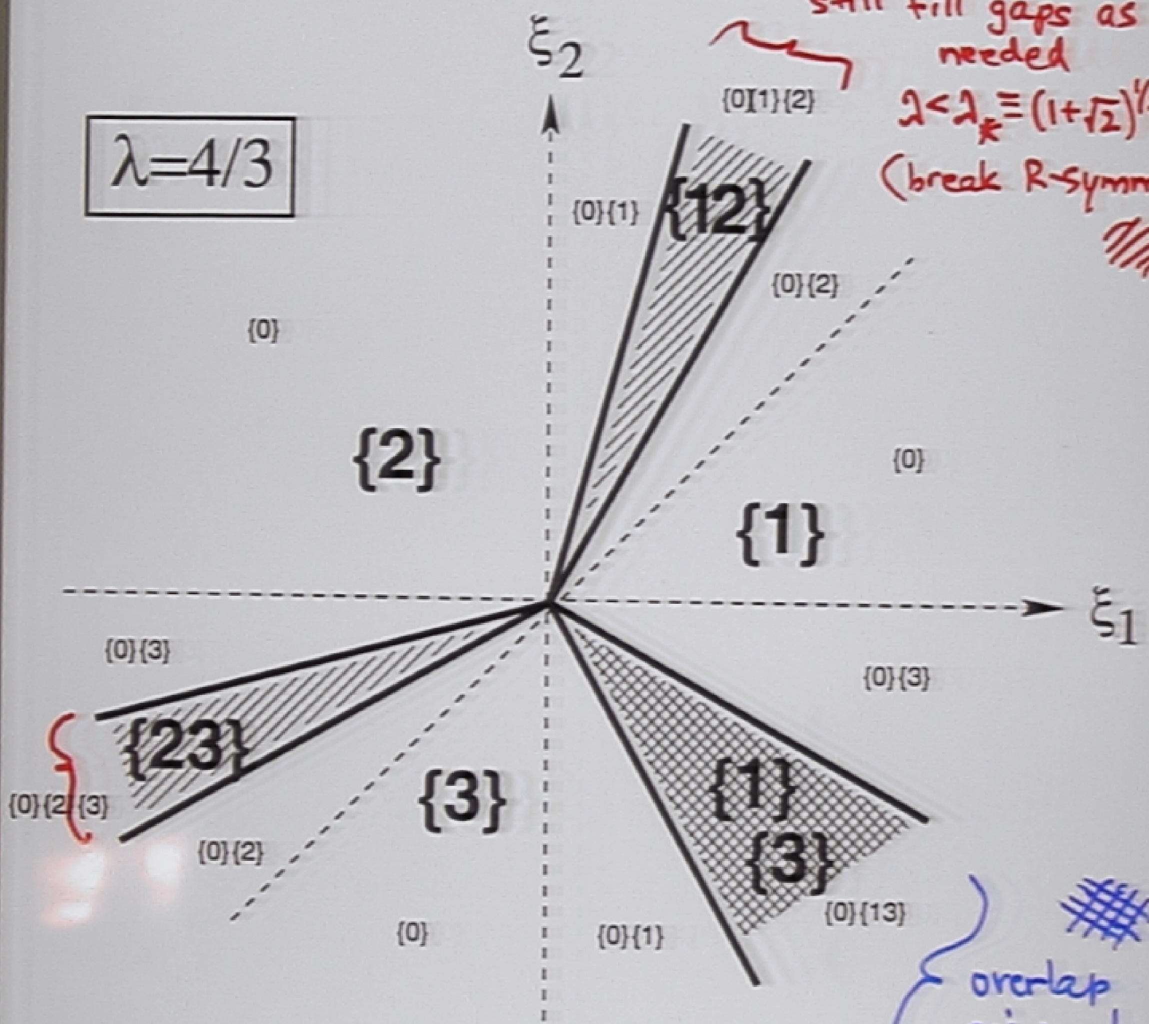
For $\lambda > 1$, single-vev regions grow, start overlapping!

e.g.,

double-vev regions still fill gaps as needed

$\lambda < \lambda_* \equiv (1+\sqrt{2})^{1/2}$

(break R-symm)



overlap regions! (for $\lambda > 1$)

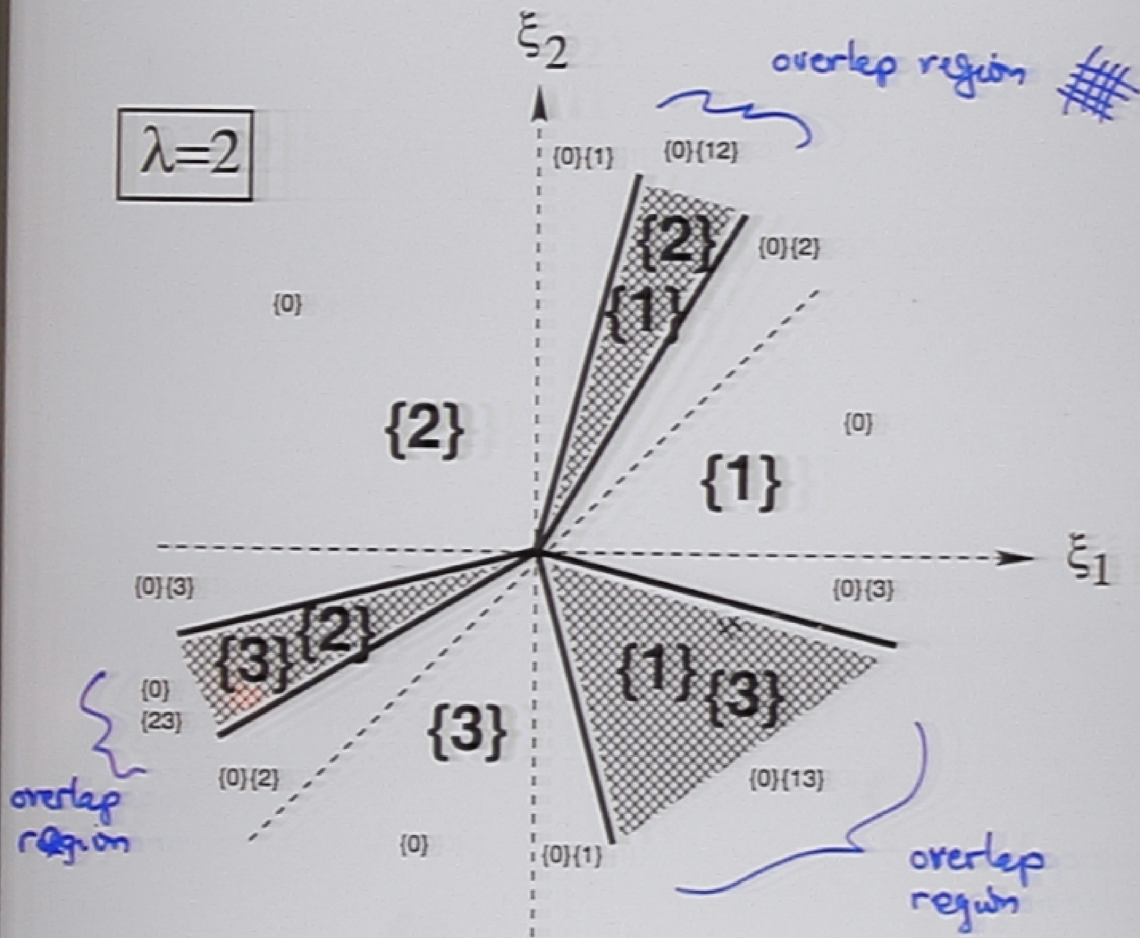
In such regions, each point in landscape has multiple (meta-) stable extrema!

For $\lambda \geq \lambda_* \equiv (1+\sqrt{2})^{1/2}$ (i.e., $\theta_\lambda \geq \theta_\lambda^* = \frac{3\pi}{8}$)

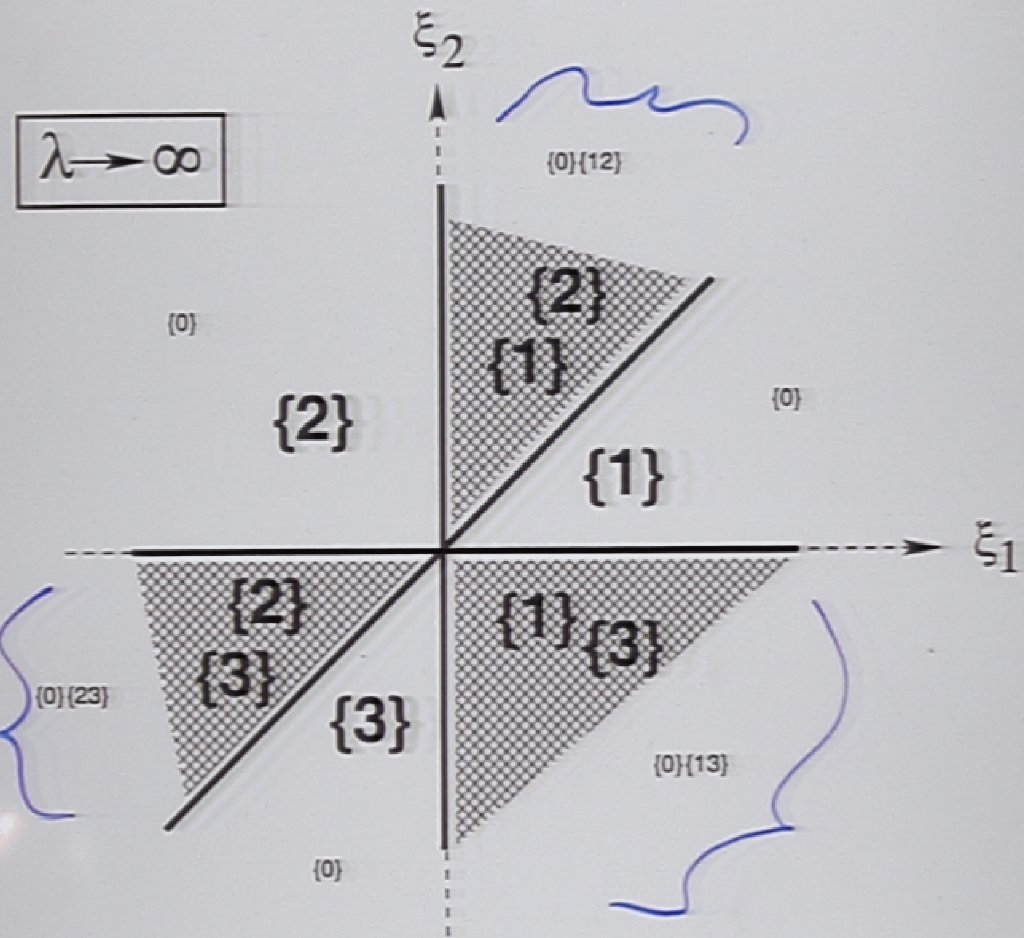
all double-vev regions vanish (become unstable)

$\Rightarrow$ all gaps have become overlap regions!

e.g.,



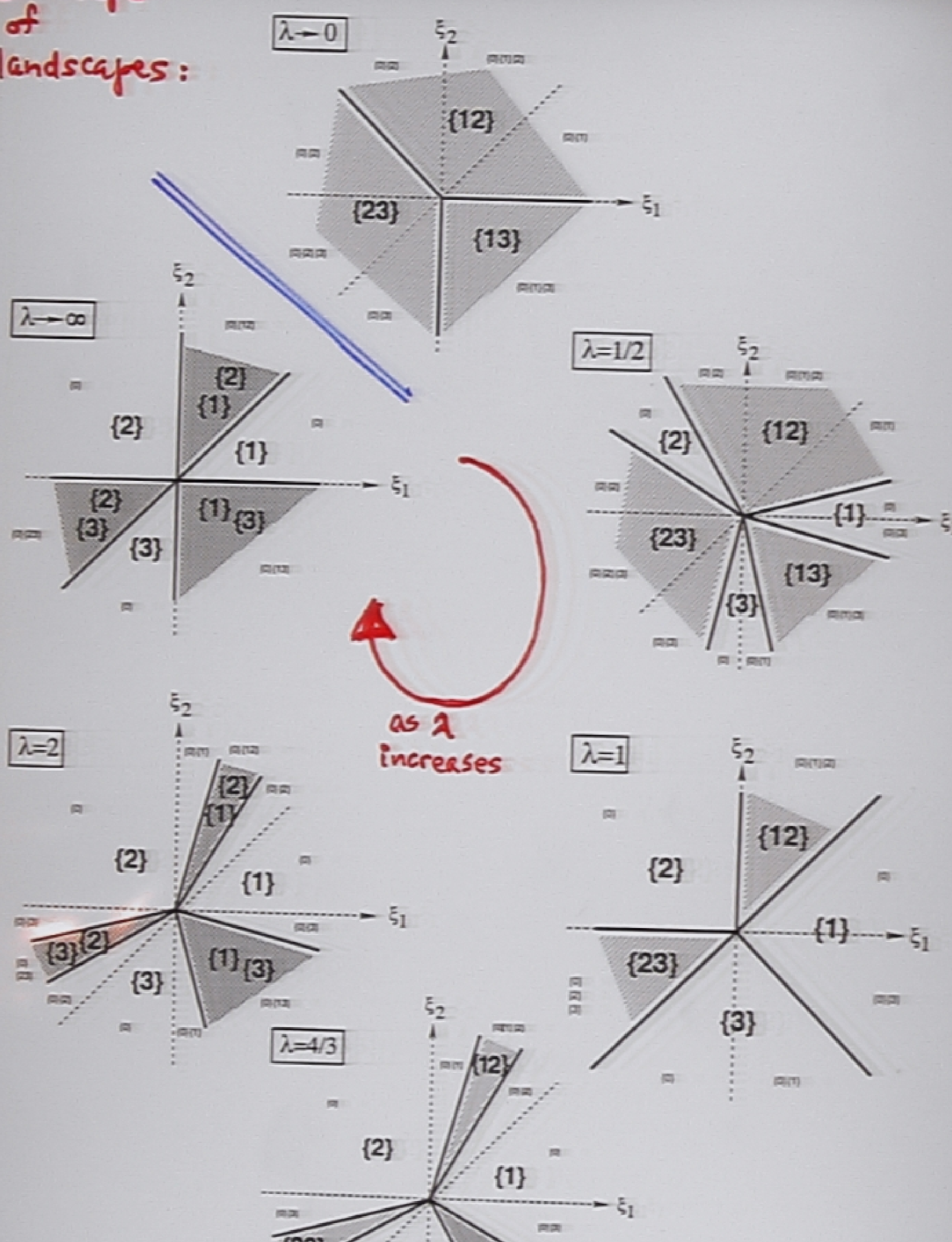
Finally, as $\lambda \to \infty$,

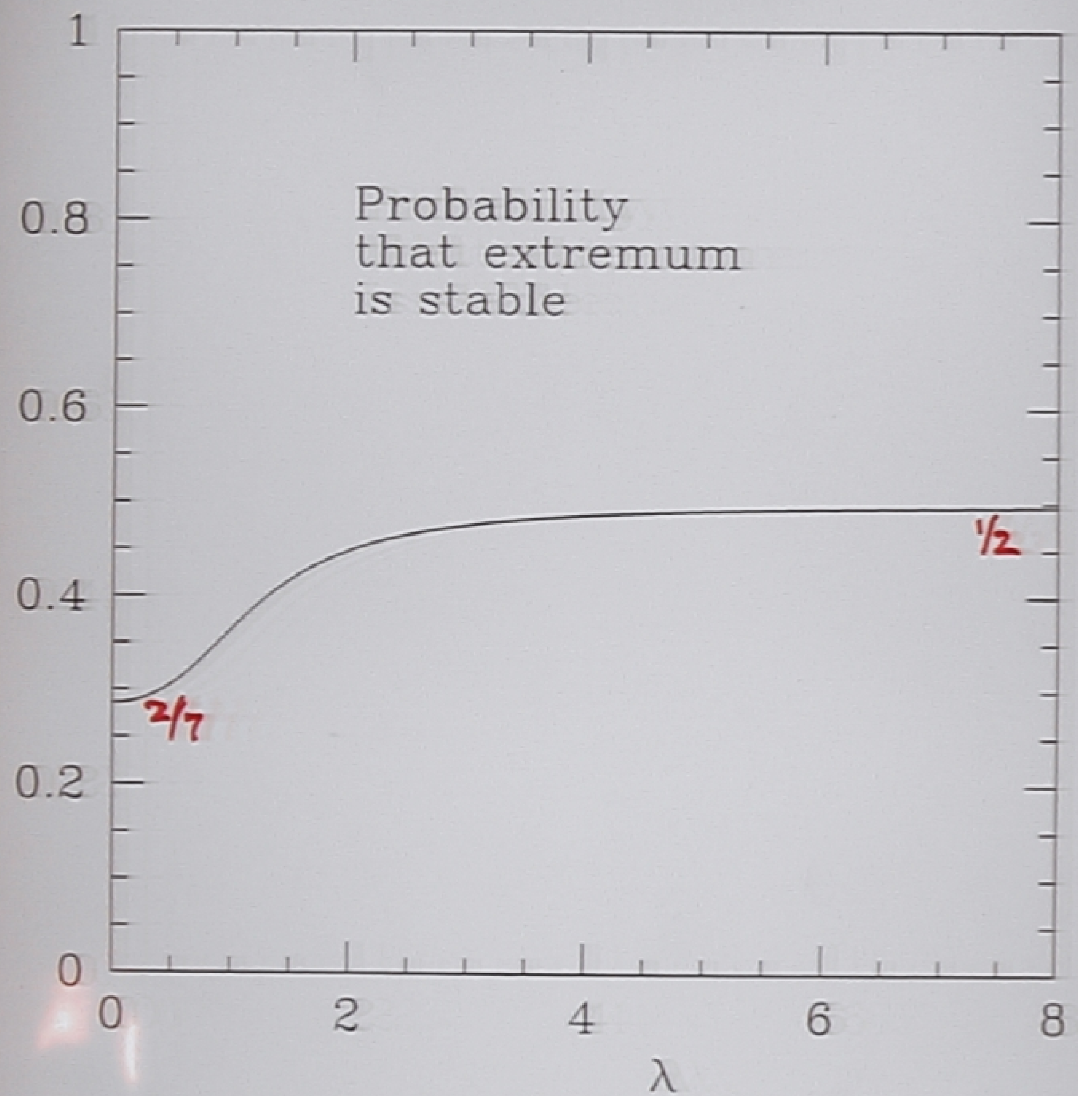


regions with <u>intra-model</u> metastability

- each point in these regions has <u>two stable</u> vacua (one metastable)

Landscape of landscapes:





$$P(\text{stable}) = \frac{2\pi}{7\pi - 6\tan^{-1}\lambda^2} \quad \text{for all } \lambda$$

<u>Note:</u> always $\gg \left(\frac{1}{2}\right)^6$ or even $\left(\frac{1}{2}\right)^4$!

But we should really integrate over λ as well!

MEASURE ?

- If use $\int_0^\infty d\lambda$ $\Rightarrow$ get $P = 1/2$ from dominance of large λ.
- Previous plots suggest defining an angular Yukawa variable

$$\theta_\lambda \equiv \tan^{-1} \lambda^2$$

Then use

$$\frac{2}{\pi}\int_0^{\pi/2} d\theta_\lambda = \frac{2}{\pi}\int_0^\infty d\lambda^2 \frac{1}{1+\lambda^4}$$

suppresses contributions from large λ

$\Rightarrow$ obtain

$$P(\text{stable}) = \frac{2}{\pi}\int_0^{\pi/2} \frac{2\pi \, d\theta_\lambda}{7\pi - 6\theta_\lambda} = \frac{2}{3}\ln\left(\frac{7}{4}\right)$$

$$\approx 0.373$$

Likewise, can calculate probability that R-symmetry is preserved

$\Longleftrightarrow$ probability of single-vev region

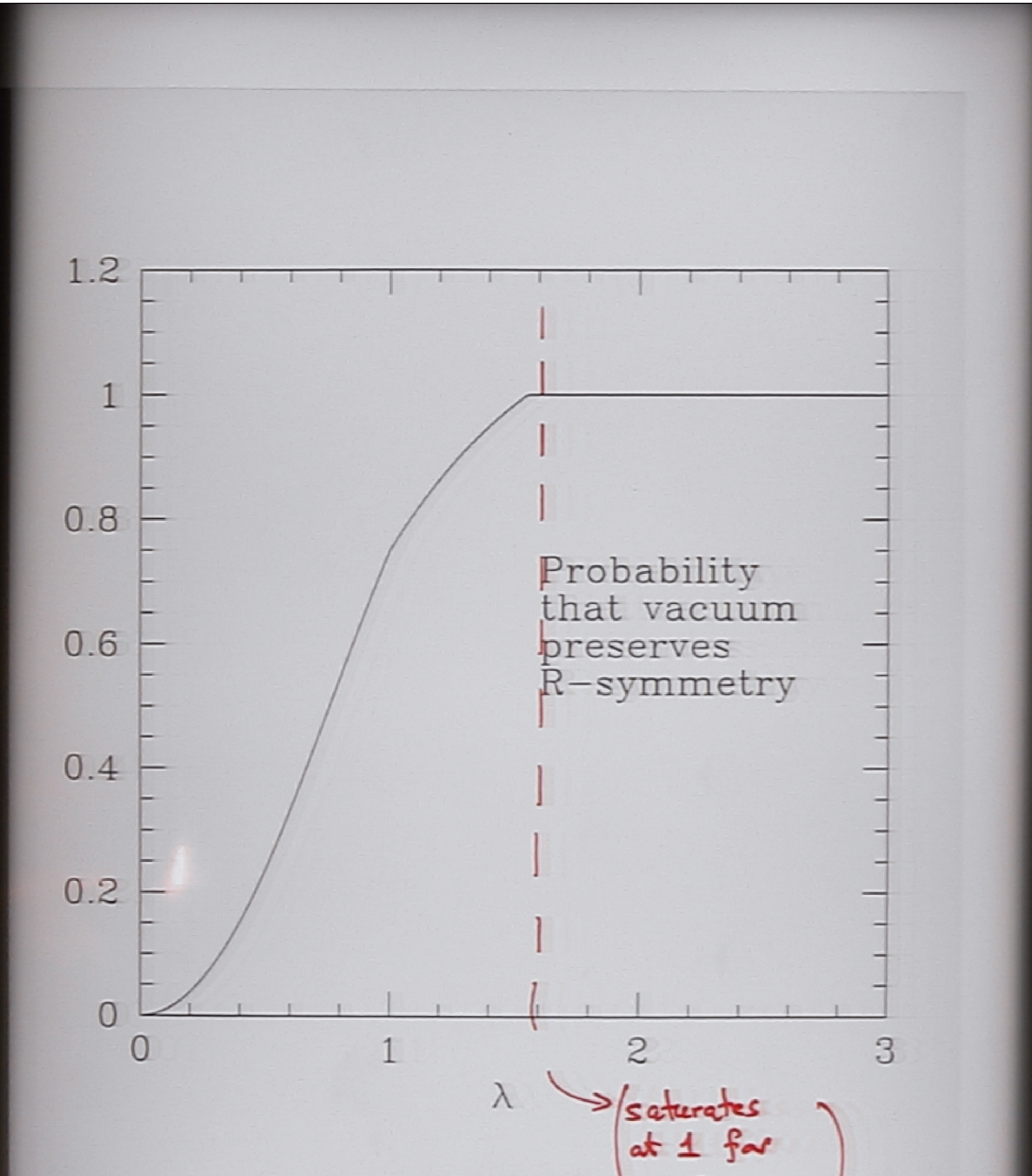
$$P(R\ \text{symm}) = \begin{cases} \frac{3}{\pi}\tan^{-1}\lambda^2 & \text{for } \lambda \le 1 \\ \frac{1}{4} + \frac{2}{\pi}\tan^{-1}\lambda^2 & \text{for } 1 \le \lambda \le \lambda_* \\ 1 & \text{for } \lambda \ge \lambda_* \end{cases}$$

↳ plot is continuous, saturates at 1 for $\lambda \geq \lambda_* \approx 1.55$

↳

Integrate over λ as well:

$$P(R\ \text{symm}) = \underbrace{\frac{2}{\pi}\int_0^{\pi/2} d\theta_\lambda}_{\text{using "angular" measure}}\, P(R,\lambda) = \frac{21}{32} \approx 0.656$$


1.2
1
0.8
0.6
0.4
0.2
0
0
1
2
3
λ
Probability
that vacuum
preserves
R−symmetry
saturates
at 1 for

$\Leftrightarrow$ probability of single-vev region

$$P(R\ \text{symm}) = \begin{cases} \frac{3}{\pi} \tan^{-1} \lambda^2 & \text{for } \lambda \leq 1 \\ \frac{1}{4} + \frac{2}{\pi} \tan^{-1} \lambda^2 & \text{for } 1 \leq \lambda \leq \lambda_* \\ 1 & \text{for } \lambda \geq \lambda_* \end{cases}$$

$\hookrightarrow$ plot is continuous, saturates at $\underline{1}$ for $\lambda \geq \lambda_* \approx 1.55$

$\hookrightarrow$

Integrate over λ as well:

$$P(R\,\text{symm}) = \underbrace{\frac{2}{\pi} \int_0^{\pi/2} d\theta_\lambda}_{\text{using "angular" measure}} P(R,\lambda) = \frac{21}{32} \approx 0.656$$

RG flow, phase transitions, and IR fixed points:

Our landscape coordinates (ξ_1, ξ_2, λ)
are all renormalized under RG flow
(energy-dependent, or T-dependent in early universe).

$\Rightarrow$ in principle, vacuum state can cross
a boundary between regions
because either

- landscape location of vacuum changes
- location of boundary changes

or both simultaneously.

NEED TO CALCULATE RGE's
for landscape parameters!

For proper treatment, must restore gauge couplings!

Find

$$\begin{cases} \mu \dfrac{d}{d\mu} g_a = \dfrac{g_a^3}{16\pi^2}(2) \\[2ex] \mu \dfrac{d}{d\mu} \lambda = \dfrac{\lambda}{16\pi^2}(3\lambda^2 - 4g_1^2 - 4g_2^2) \\[2ex] \mu \dfrac{d}{d\mu} \xi_a = 0 \end{cases}$$

$\mathrm{Tr}\, Q_a^2 = \sum_{i=1}^{3} q_i^{(a)2} = 2$

$\mathrm{Tr}\, Q_a = 0 \Rightarrow$ set at tree-level; one-loop not induced

$\Rightarrow$ Location in landscape is fixed ~~but~~ landscape itself may evolve.

Assume $(g_a^{(0)}, \lambda^{(0)})$ at some fixed UV scale

If $g_1^{(0)} = g_2^{(0)} \Rightarrow g_1(\mu) = g_2(\mu)$ at all scales

Restore g_a to previous equations $\Rightarrow$ only rescales

$$\lambda(\mu) \to Y(\mu) \equiv \frac{\lambda(\mu)}{g(\mu)}$$

This quantity $Y(\mu)$ is what labels/parametrizes the different landscape snapshots.

Thus, RG flow simply corresponds to moving between landscape pictures

($\Leftrightarrow$ evolution of $Y(\mu) \equiv \dfrac{\lambda(\mu)}{g(\mu)}$)

while holding our original landscape location $(\xi_1^{(0)}, \xi_2^{(0)})$ fixed!

RGE for $Y(\mu)$: find

$$3 - \frac{10}{Y^2(\mu)} = \left(3 - \frac{10}{Y_0^2}\right) \underbrace{\left[\frac{g(\mu)}{g_0}\right]^{10}}_{\to 0 \text{ in IR}}$$

$\Rightarrow$ regardless of Y_0,
theory flows to fixed point in IR at

$$\overline{Y} \equiv \sqrt{10/3} \approx 1.826$$

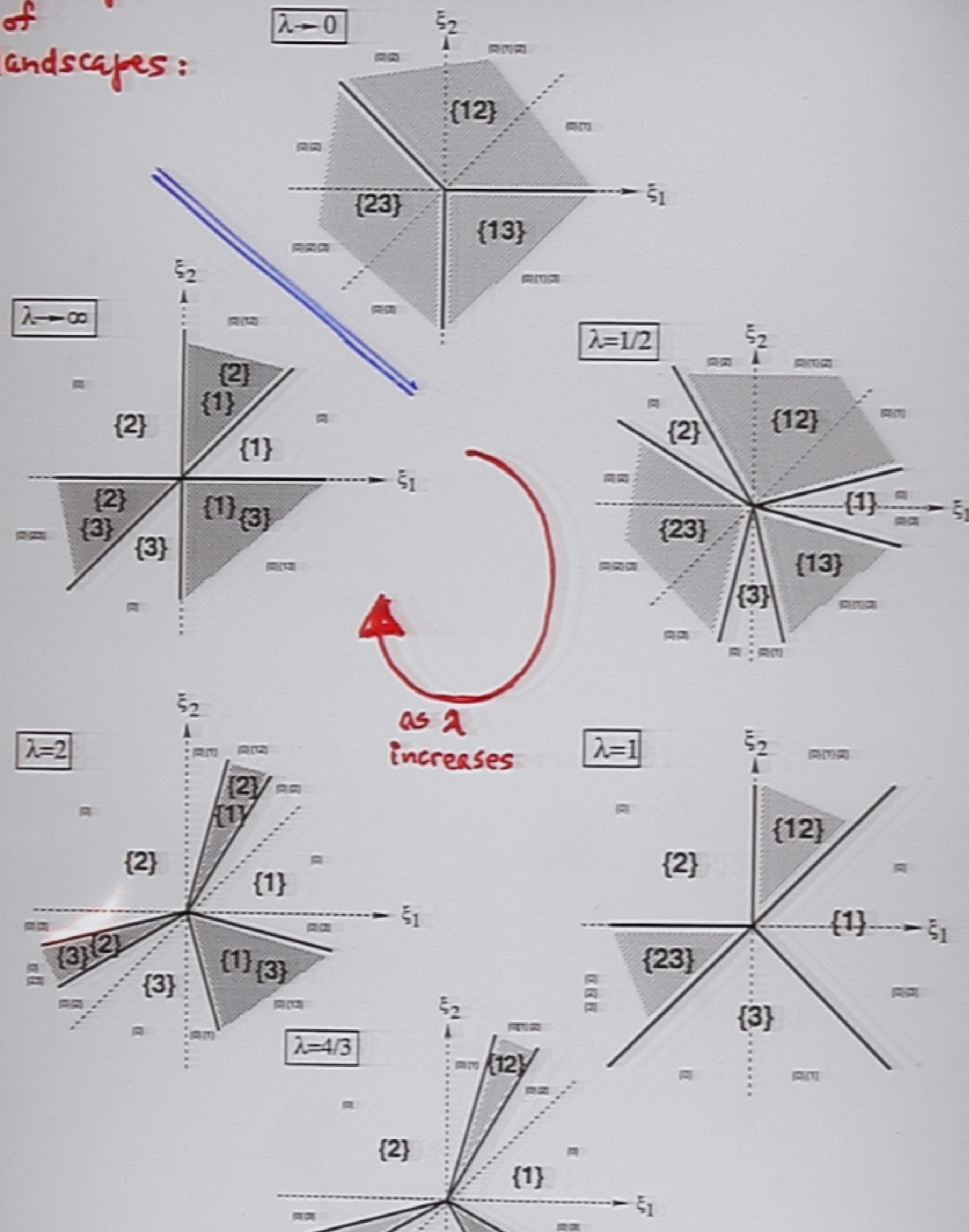
from either above or below!

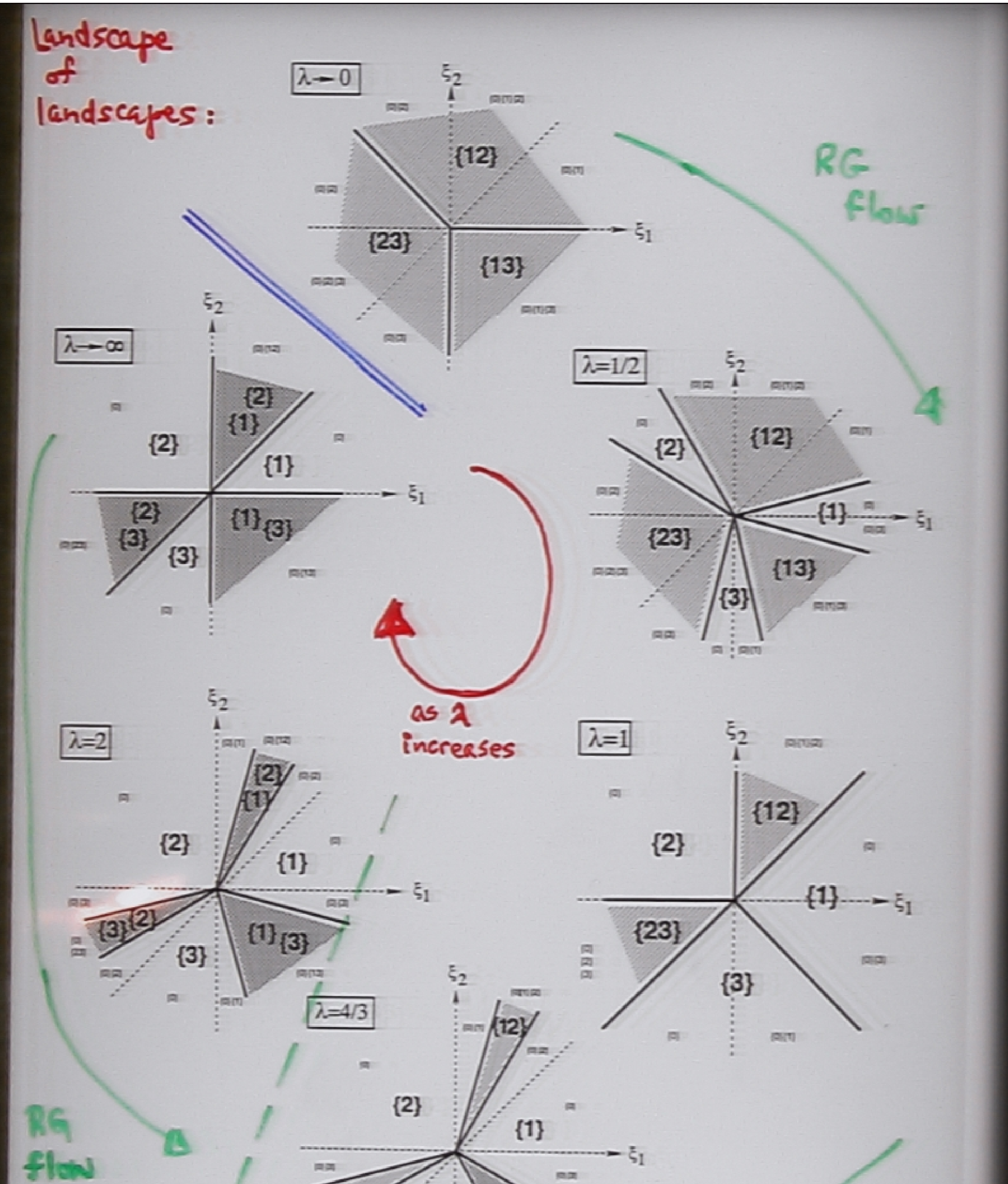
If $Y_0 < \overline{Y} \Rightarrow$ theory flows UPWARDS through previous diagrams

$Y_0 > \overline{Y} \Rightarrow$ DOWNWARDS

ALWAYS EVOLVES TOWARDS UNIVERSAL INFRARED LIMIT $\Rightarrow$ a fixed landscape $\overline{Y} = \sqrt{10/3}$

Landscape of landscapes:




Landscape of landscapes:
λ→0
{12}
{23}
{13}
λ→∞
{2}
{1}
{3}
λ=1/2
λ=2
λ=1
λ=4/3
as λ increases
RG flow
RG flow

In this RG flow, phase transitions can occur as boundaries are crossed.

$\boxed{\text{If } Y_0 > \bar{Y}}$ $\Rightarrow$ only boundaries are between single-vev and overlap regions

e.g., $\{1\}$ $\qquad$ $\{1\}+\{3\}$

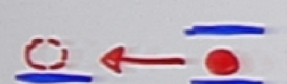
Since $Y(\mu)$ decreases,

$\Downarrow$ region gets larger $\qquad$ $\Downarrow$ region gets smaller

state falls between regions

$\{1\}$ —— $\Leftarrow$ —— $\{3\}$, —— $\{1\}$

Phase transition depends on initial state:

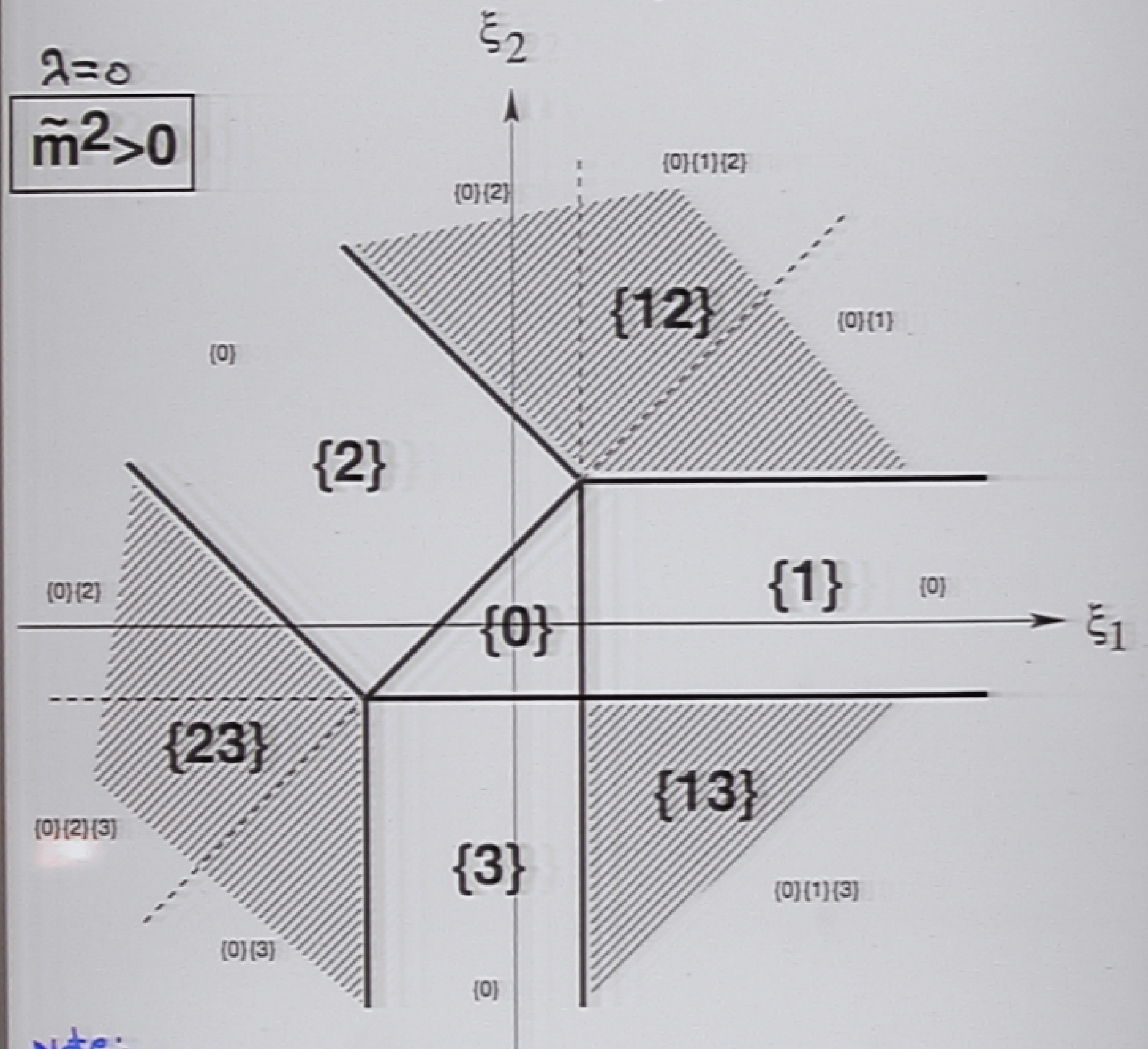
If  $\Rightarrow$ no transition

If  $\Rightarrow$ metastable state becomes unstable, theory drops discontinuously [1^{st} order]

Many possible extensions – e.g., add soft masses $\tilde{m}$:

$$V_{soft} = \tilde{m} \sum_{i=1}^{3} |\phi_i|^2$$

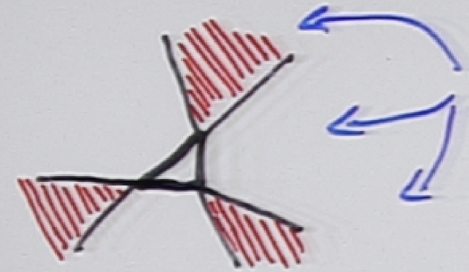
For $\tilde{m}^2 > 0$, increases stability of vacua:



Note:

Far from origin, only those regions with non-zero opening angles survive $\Rightarrow$ reproduce previous results

For $\lambda > 0$, similar behavior...



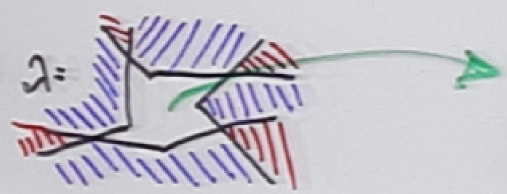
Same opening angles as before, but with new stable central region

Above pictures assume $\tilde{m}^2 > 0$.

$\boxed{\text{If } \tilde{m}^2 < 0}$ $\Rightarrow$ adds pressure towards destabilization:

- For $\lambda = 0$, entire landscape (formerly flat) destabilizes.
- As λ increases, stable regions move in from ∞

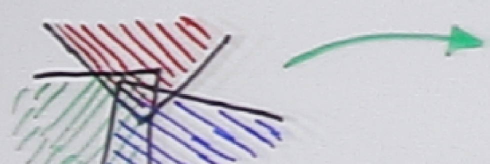
small λ:



HOLE IN THE LANDSCAPE!

unstable regions completely surrounded by regions of stability

larger λ:



CENTRAL MULTIPLE OVERLAP REGION!

region with multiple metastable

BEYOND THE TOY MODEL: $n \geq 3$ $U(1)$ factors

$$
\begin{array}{c|cccccc}
 & U(1)_1 & U(1)_2 & U(1)_3 & \cdots & & U(1)_n \\
\hline
\Phi_1 & -1 & & & & & \\
\Phi_2 & +1 & -1 & & & & \\
\Phi_3 & & +1 & -1 & & & \\
 & & & & \ddots & & \\
\Phi_n & & & & & +1 & -1 \\
\Phi_{n+1} & & & & & & +1
\end{array}
\left.\vphantom{\begin{array}{c}1\\1\\1\\1\\1\\1\end{array}}\right\} \text{motivated by deconstruction}
$$

- Assume ξ_1, ξ_n for "endpoint" $U(1)$'s only.
- soft scalar masses $\quad V_{soft} = \tilde{m}^2 \sum_i |\phi_i|^2$
- superpotential $\quad W = \lambda\, \Phi_1 \cdots \Phi_{n+1}$ $\quad$ non-renormalizable

$\Rightarrow$ λ suppressed by dimensional analysis

$\Rightarrow$ henceforth consider only $\lambda \to 0$

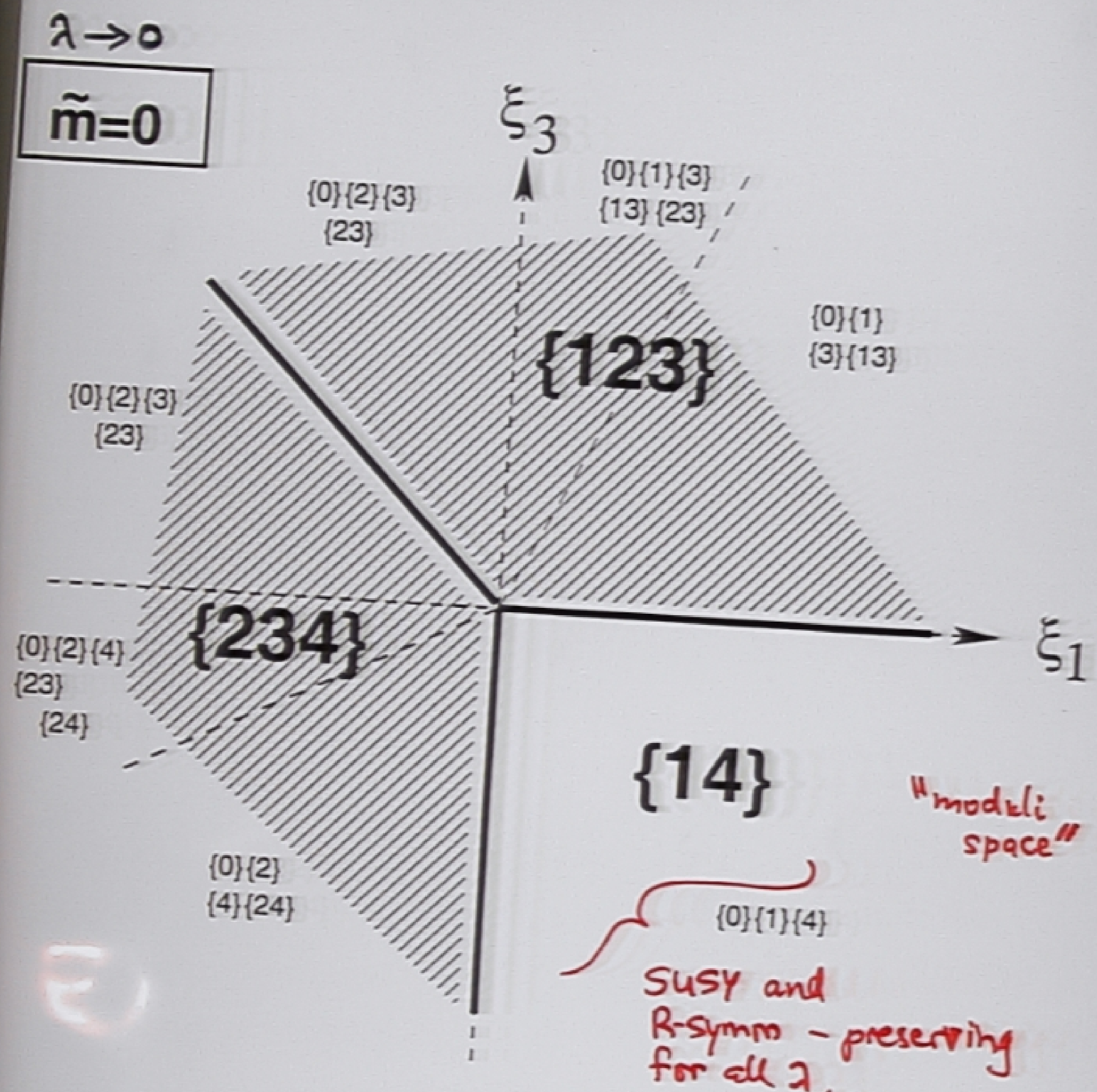
$\Rightarrow$ 2^{n+1} classes of vacua:

$\{\phi\}$

$\{1\}, \{2\}, \ldots, \{n+1\}$

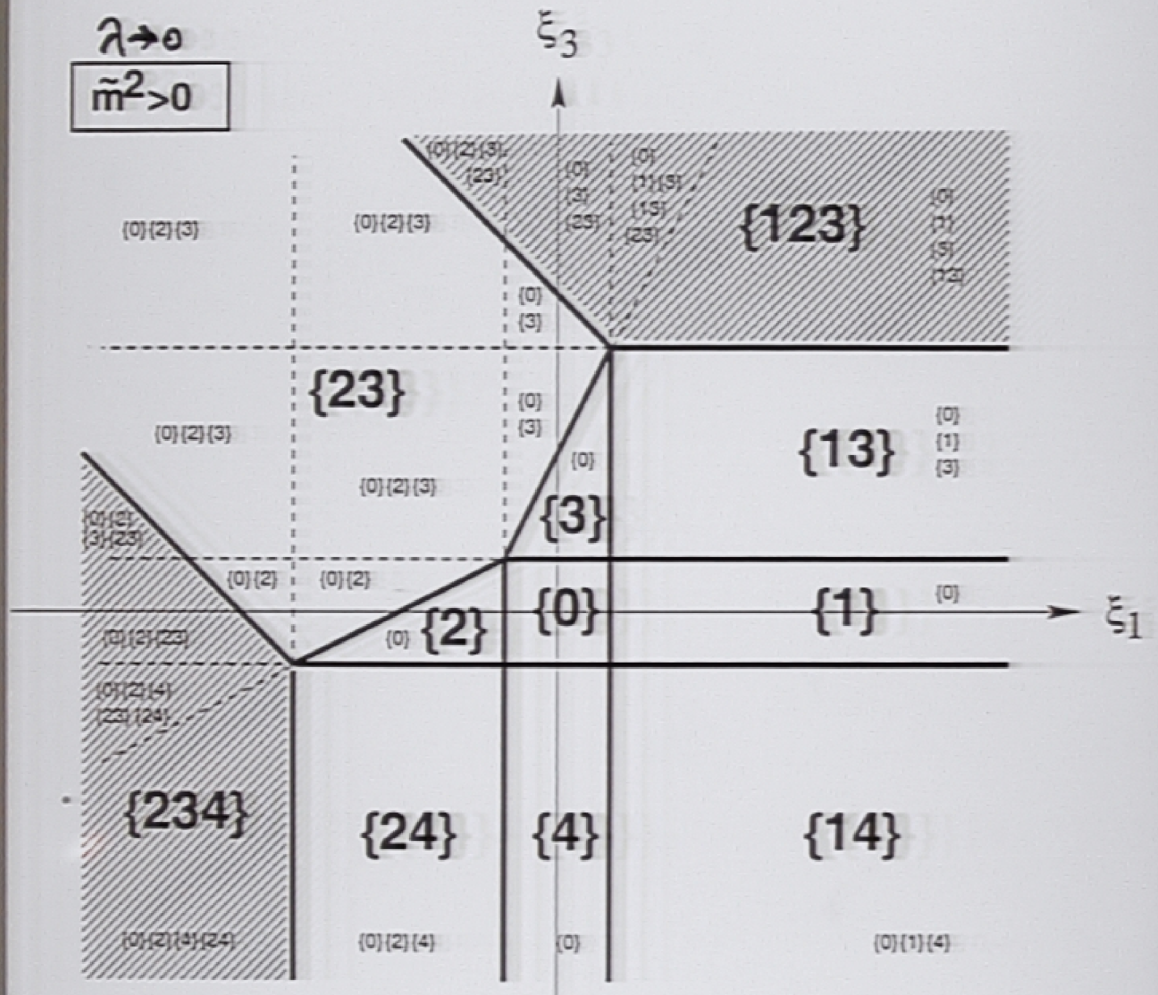
$\{1,2\}$ $\{1,3\} \ldots \{1,n+1\}$, $\{2,3\}$, $\{2,4\} \ldots \{2,n+1\}$, $\{3,1\} \ldots$

etc...



Similar to $\underline{n=2}$ case except ↑ (this region),
⇒ example of landscape with both
SUSY and ~~SUSY~~ regions of $\underline{\text{same}}$ dimensionality

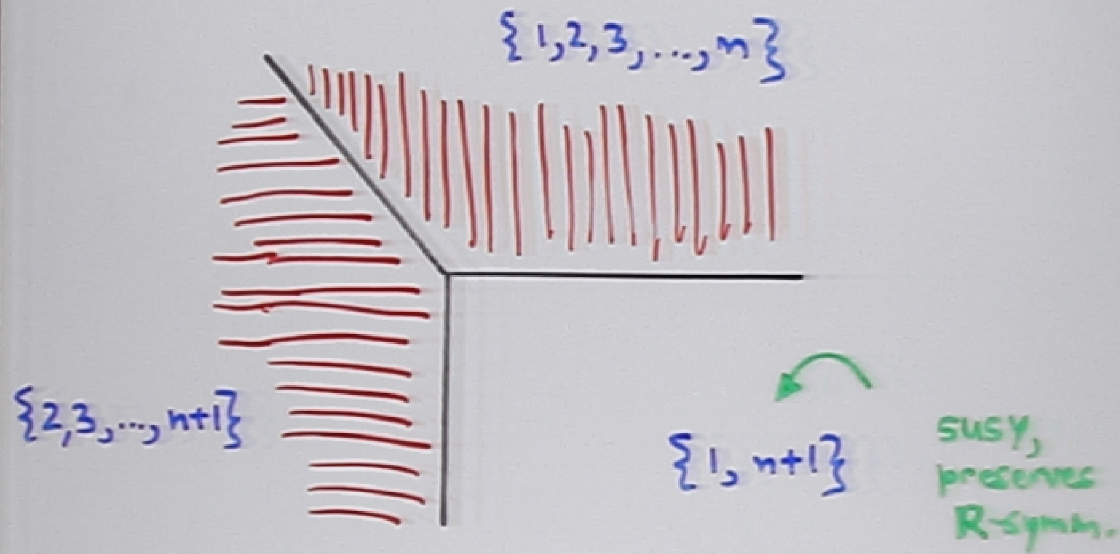
Now let $\tilde{m}^2 > 0$ $\Rightarrow$ opens up new central stability regions



⇒ obviously leads to a very rich

For general $\underline{n}$, $\{ \ldots \tilde{m}^2 = 0 \}$,

the three dominating vacua generalise to



these solutions are explicitly given as

$\{1,2,3,\ldots,n\}$: $|v_1|^2 = \xi_1 + \xi_n \; ; \; |v_2|^2 = \ldots = |v_n|^2 = \xi_n \; ; \; |v_{n+1}|^2 = 0$

$\{2,3,\ldots,n+1\}$: $|v_1|^2 = 0 \; ; \; |v_2|^2 = \ldots = |v_n|^2 = -\xi_1 \; ; \; |v_{n+1}|^2 = -(\xi_1 + \xi_n)$

both solutions have "flat" profiles $|v_k|^2$ $(2 \leq k \leq n)$

Other solutions correspond to deconstructed
<u>MAGNETIC FLUX</u> COMPACTIFICATIONS!

eg $\{2,3,\ldots,n\}$ vacuum has

$$|v_1|^2=0, \quad \underbrace{|v_k|^2 = -\xi_1 + \frac{k-1}{n}(\xi_1+\xi_n)}_{} \quad , |v_{n+1}|^2=0$$

linear profile $\Rightarrow$ smoking gun for flux interpretation!

To see this, recall 5D susy $U(1)$ theory on $\mathbb{R}^4 \times S^1/\mathbb{Z}_2$
Contains gauge field A and $\mathbb{Z}_2$-odd real scalar Σ.

Solve field equations for Σ, obtain solution

$$\langle \Sigma \rangle = \frac{\xi_1+\xi_n}{2\pi R}\, y - \frac{\xi_1}{2}\,\epsilon(y) \Rightarrow \text{linear profile!}$$

Identify $R \sim n \ldots$

To see flux,
consider Σ as sixth component of gauge field A_6

$$\Rightarrow \quad F_{56} = \langle \partial_5 A_6 \rangle = \underbrace{\frac{\xi_1+\xi_n}{2\pi R}}_{\text{magnetic flux}} - \underbrace{\xi_1 \delta(y) - \xi_n \delta(y-\pi R)}_{\text{fluxes localized at orb. fixed pts.}}$$

π/R

General comments:

- Solutions with many unbroken U(1)'s actually have these U(1)'s broken by mixed gauge anomalies
  - ↪ can be cancelled by Green-Schwarz mechanism, extra axionic fields, etc...
- In SUGRA context, heavy supergravity moduli fields can be integrated out
  - ↪ can be a source of large soft breaking terms and Wilson-line contributions
  - ⇒ increases stability of vacuum solutions
- large gravitino mass & large soft masses
  - ⇒ new methods of cancelling cosmological constant Λ.

Together, high-scale SUSY breaking and large numbers of stable vacua } preconditions for SPLIT

And now for something completely different —
for recreational purposes only —

"PREHISTORIC LANDSCAPING" [ca. 1990 AD]

- "landscape" in 1990 → heterotic strings, weakly coupled, gauge group ranks ≤ 22, ...
- huge classes of vacua → orbifolds w/ Wilson lines, Narain lattices, free WS fermions, ...

In 1990, explicitly constructed $> 10^5$ vacua with
- high-scale SUSY-breaking (Scherk-Schwarz, non-SUSY orbifolds)
- no physical tachyons

e.g., 4D analogues of $D=10$ $SO(16) \times SO(16)$ het. string.

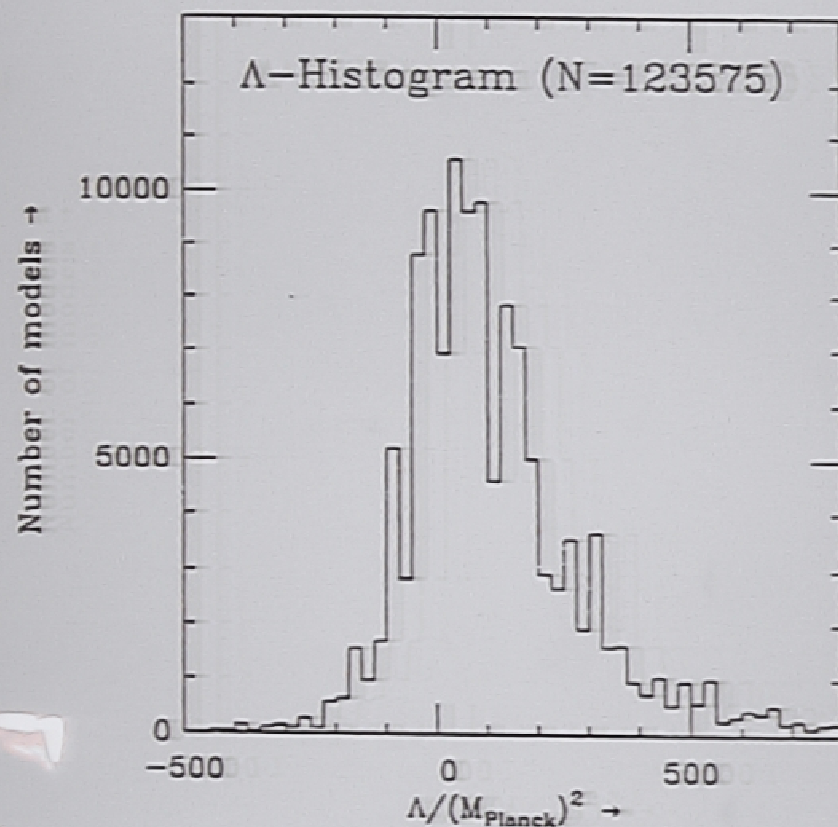
Each model was distinct → different spectrum, etc.

SUCH MODELS ARE NOT STABLE BEYOND TREE LEVEL

Still, can calculate one-loop cosmological constant Λ over this landscape...

132 → PhD thesis!

- both $\Lambda > 0$ and $\Lambda < 0$ found
- contributions from infinite towers of string states are <u>significant</u>!

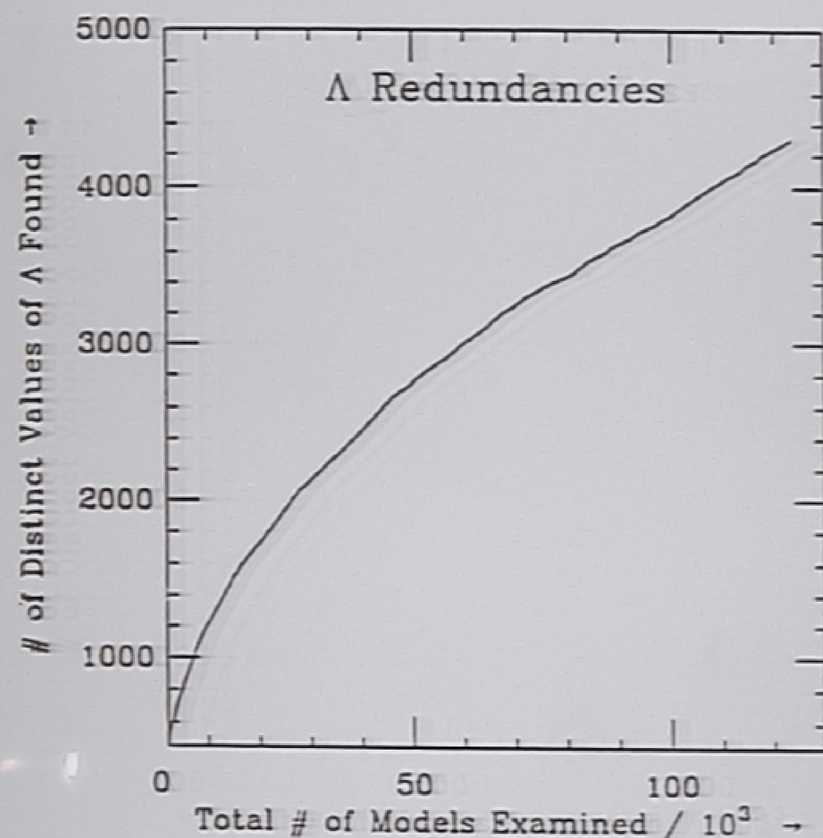


<u>N = 123,575 models</u>

↳ minimum value found
<u>$\Lambda \simeq 0.0187\ M_P^2$</u>

Figure 7. Histogram of values of Λ obtained in computer search.

found a cosmological constant redundancy :



More models $\not\Rightarrow$ More values of Λ !

Figure 8. Model/cosmological constant redundancy.

* || consequence of tight string consistency constraints, modular inv. on towers of string states

Generic feature:

Many different string models with entirely different phenomenologies (gauge groups, matter reps, hidden sectors, ...) have IDENTICAL values of Λ!

$\Longrightarrow$ there exist flat directions even in non-SUSY landscape!

Consequence of tight string consistency constraints acting on infinite towers of string states, even when SUSY is broken!

In Figure 7, we provide a histogram illustrating the distribution of cosmological constants obtained for the models in our search. As can be seen, both positive and negative values of Λ were obtained, and indeed the bulk of these models had values of Λ centered near zero. (Recall that it was for this reason that we restricted our search to those non-supersymmetric models containing the spin-structure vector $\mathbf{W}_1$.) However, we obtained no model with $\Lambda = 0$ (within numerical error); indeed, the closest value obtained was $\Lambda \cong 0.0187\,(M_{\text{Planck}})^2$.

In Figure 8, we plot the total number of different matrices $\hat{a}_{mn}^{(\text{tot})}$ obtained at different points during our computer search versus the total number of models then examined. It is clear that this cosmological-constant redundancy is quite severe, and in fact the shape of this curve might lead one to conclude that there may be a finite and relatively small number of self-consistent matrices $\hat{a}_{mn}^{(\text{tot})}$ which our fermionic spin-structure models can reach. If this were the case, then we would expect the number of such matrices already seen, Σ, to have a dependence on the total number of models examined, t, of the form:

saturates at $N \sim 5500$

$t_0 \sim 70{,}000$!

$$\boxed{\Sigma(t) \;=\; N\left(1 - e^{-t/t_0}\right)} \tag{F.2}$$

where N is this total number of matrices and t_0, the "time constant", is a parameter characterizing the scale of the redundancy. Fitting the curve in Figure 8 to (F.2), we find that values of $N \sim 5500$ and $t_0 \sim 70\,000$ seem to be indicated. (One cannot be more precise, since we have clearly *not* examined a sufficient number of models to observe saturation.) However, this approach assumes that our models uniformly span the space of $\hat{a}$ matrices (and also that our search uniformly spans the space of models). Both of these assumptions are more likely than not to be invalid. We note that a similar saturation plot and fit was attempted in [24], with similar conclusions as well.

These models are not stable
and represent only a small, limited class
of heterotic strings,

but may hold an important lesson
for the final string landscape
and its resulting architecture.