Title: Recent results in statistics of vacua

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Abstract:

1. Introductory comments

The stringy landscape is obviously interesting, as evidenced by the number of talks here discussing it. But what do we mean by it, and "what good is it?" Why does the idea still bother people?

And what do we mean by the "landscape" anyways?

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In general, that there are "lots" of vacua, meaning enough to solve the cosmological constant problem a la Weinberg and Bousso-Polchinski. This also includes many unrealistic and quasi-realistic vacua, with no simple way to find the realistic ones.

Five points of uncertainty:

1. The effective potential hypothesis
2. The "inelegant universe"
3. The unknown unknowns
4. How many vacua are there?
5. Are all vacua created equal?

1. The effective potential hypothesis – we can make controlled computations of an effective potential on moduli spaces of string vacua, and its minima are metastable vacua. Not totally obvious in a theory of quantum gravity.

2. With 100's of fields and discrete choices in these constructions, we can expect 10^{100}'s of vacua. Important features of our world, such as the small c.c., can emerge from random and meaningless combinations of effects, perhaps only selected by observational (anthopic) considerations.

3. At present we can only compute in a small subset of vacua, and there may even be huge undiscovered classes of vacua – non-geometric, with high scale susy breaking, etc.

5. Perhaps the vacua are weighted by a wave function or other early cosmological factor.

All are important and not well understood. But the best posed in many ways is (4), to characterize the number and distribution of string vacua defined according to the present state of the art.

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To give a very simplified example, one could study the fraction of vacua coming out of any given construction which realize the SM gauge group, by keeping track of both the numbers of vacua which do and do not do so. To the extent that this fraction agrees between different constructions, we have evidence that they are representative (at least for purposes of this question). We might instead eventually decide that the underlying distributions of gauge groups are different and that the landscape has different regions; for example this seems likely for brane constructions and the perturbative heterotic string. It is less clear non-perturbatively; doing these comparisons should help enlarge the sample set, as was true for duality in models with $N > 1$ susy.

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Given a large enough sample set, if the basic assumptions (effective potential, duality, extrapolations) are wrong, this should eventually show up in contradictions. For example, estimates of numbers of vacua coming out of different (putatively) dual constructions should agree. The same would be true for weighted distributions, if we are sure the weights agree. One should also be careful to actually count isolated vacua, and not special points in moduli space or other related constructs which are probably not duality invariant.

Barring direct evidence for or against string theory, statistics are essential data for understanding the predictivity and falsifiability of the theory. One can distinguish various regimes:

- $N_{vac} << 10^{60}$ – cannot solve the c.c. problem anthropically; need other solutions.

- $10^{60} << N_{vac} << 10^{500}$ – can expect interesting predictions, even without additional selection principles. Knowledge of the distributions could confirm or drastically modify present-day intuitions about physical "naturalness."

- $N_{vac} >> 10^{1000}$ – unless the distribution is very sharply peaked, or we bring in other selection principles, cannot expect predictions. Selection principles become very interesting.

- $N_{vac} \to \infty$ – even with selection principles (short of consistency), it would be hard to claim that indirect evidence (*i.e.* modelled within $d = 4$ EFT) bears on string theory. Anthropic arguments become central.

The most basic question along these lines is simply whether the number of quasi-realistic string/M theory vacua is even finite. There is some scope for definition here, and it is needed. For example, there are infinite series of "compactifications" such as $AdS_5 \times S^5$ with N units of flux. However, the radius of the S^5 $R \sim N^{1/4}$ goes to infinity for large N, so the number is finite if we put a lower bound on the gap to the Kaluza-Klein spectrum,

$$M_{KK} > M_{low}.$$

All the infinite series I know about are removed this way, and for this and other reasons I would conjecture that the total number of string/M theory vacua with this cut is finite. This conjecture is beyond proof at present, for example we cannot show that the number of Calabi-Yau threefolds is finite.

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Why is this important? Once we see an infinite-valued choice anywhere, we face the likelihood that such choices are possible in the hidden sector (and at yet unobserved energies). These choices affect parameters in the observable sector, and would allow arbitrary tunings. On the other hand, they are sufficiently unrelated to the observable sector to make the claim that infinite numbers of vacua concentrate on physically observed values implausible. Perhaps falsifiability could be saved by a weight factor whose sum over all vacua converges, but this brings problems as

2. Flux vacua

These have been studied in type IIb on CY orientifolds by several groups (Rutgers, Stanford, Cambridge), and in M theory on G_2 (Acharya et al).

In IIb, we start from $W = \int \Omega \wedge H$ on complex structure moduli space, with its classical Kähler potential. Similar formulae apply to M theory, F theory on fourfolds, heterotic on CY, and even the distribution of black hole attractor points (FKS, Moore, DD etc.).

The problems fall into two classes. In M on G_2, heterotic on CY and the BH problem, one set of fluxes is being scanned. Their number, b_3, is comparable to the number of (real) moduli, $2b^{2,1}$. This turns out not to be enough to tune the cosmological constant.

In IIb, there are two sets of fluxes (NS and RR three-forms) leading to $K = 2b_3$ adjustable choices for $b_3 - 2$ real moduli. This leads to many more vacua and uniform distributions for the c.c.

F and M theory on fourfolds have one set of four-form fluxes, but for many fourfolds one finds $K = b^4 >> 2b^{3,1}$ (the number of complex structure moduli) so this is more like IIb.

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The simplest result characterizing this distribution is the index of supersymmetric (AdS and Minkowski) vacua,

$$dI[z, |W_0|^2] = \sum_W \delta_z(DW(z))(\text{sgn } \det D^2W)\ \delta(|W_0|^2 - e^K|W|^2)$$

where $\delta_z(DW)$ counts solutions of $DW = 0$ with unit weight, and the sign is the Morse sign (for AdS) and $+1$ (for Minkowski). We also control the AdS cosmological constant $\Lambda = -3|W_0|^2$.

In the "one flux" problems, the equations $DW(z) = 0$ essentially determine the flux given z, and the cosmological constant is proportional to the flux. Thus one has

$$dI_1 \sim |W_0|^{K/2-1} \frac{\omega^n}{n!}$$

The two distributions are not really comparable because they have different control parameters. But clearly if two sets of vacua had statistics dI_1 and dI_2, they would not be dual to each other. On the other hand, duality between IIb on CY, heterotic on CY, and M on G_2 is fairly well motivated.

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In IIb theory, one finds

$$dI_2[z] = \frac{(2\pi L_*)^{K/2}}{(K/2)!\pi^{n+1}} \det(R + \omega \cdot 1)$$

where ω is the Kähler form on configuration space, R is its matrix of curvature two-forms, and

$$L_* \geq L = \int F \wedge H$$

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Thus, duality strongly suggests that the constructions with one flux have more free choices which enter into the superpotential. For example, the IIa mirror of a general IIb compactification is non-geometric, as discussed here by Hellerman, so one must count these choices.

In heterotic on CY, there is a wide choice of gauge bundles, and the superpotential contains the holomorphic Chern-Simons term,

$$W = \int \Omega \wedge \left(H + A\bar{\partial}A + \frac{2}{3}A^3 \right),$$

suggesting that the role of the "second flux" might be played by the gauge field. On the other hand, general IIb constructions have some choice of gauge field as well (on the 7-branes) which is dual to that choice.

Heterotic string and IIb orientifolds are both special cases of F theory on fourfolds; in particular the heterotic string comes from wrapping a D3 on the $\mathbb{P}^1$ base of a K3 fiber. Many F theory choices are non-perturbative in heterotic language, and one must count these choices as well to get the numbers to agree. Non-geometric choices might also be required.

Once we bring in more choices, we should ask whether the distribution is still dI_2, or if we expect the distribution to change again. A general (but heuristic) argument for dI_2 is the following. Suppose we draw the superpotential out of a larger ensemble,

$$W = \sum_\alpha N^\alpha \Pi_\alpha(z)$$

in which the "generalized fluxes" N^α vary, while the "generalized periods" Π_α do not. The periods are still sections of the same line bundle $\mathcal{L}$, since this is forced by supergravity.

Then, consider a region R of moduli space which is so large that we wash out all a priori structure in a typical superpotential. We can then ignore correlations with possible vacua outside of the region, and think of the region as if it were compact. Then, we would have the topological formula for the index counting vacua,

$$I \propto \int_R \det(R + \omega \cdot 1).$$

So, adding more fluxes or other choices would preserve this form. This could be checked on F and M theory on fourfolds (work in progress).

$$dI \propto \det(R + \omega \cdot 1).$$

Some general properties:

- Enhanced number of vacua near conifold points, going as

$$d\mu_{vac} \sim \frac{d^2 S}{|S \log S|^2}; \qquad \mathcal{N}_{vac}|_{S<S_*} \sim \frac{1}{|\log S_*|}$$

  where S is the distance to the conifold point, dual to the expectation value of the gaugino condensate in the gauge theory dual. Thus hierarchically small scales are common but a minority of vacua.

- Near large volume and weak coupling, and in other "generic" regimes, the distribution goes roughly as the volume form. Thus, we have

$$d\mu_{vac} \sim \frac{d^2\tau}{(\text{Im } \tau)^2}$$

  for the dilaton-axion $\tau = a/2\pi + 4\pi i/g_s$. This is equivalent to a uniform distribution $dg_s da$, so weak couplings, large volumes are also common.

- On the other hand, flux stabilized large extra dimensions are rare, $N \sim V^{-n/3}$ for n Kähler moduli.

In generic regions of moduli space, $R \sim \omega$ and this density is similar to the volume form on moduli space. The volume of Calabi-Yau complex structure moduli space for a general hypersurface in a toric variety has been shown to be finite (Z. Lu), a non-trivial result which implicity takes duality into account.

This argument may extend to $\det(R + \omega)$ but this is not literally proven yet. The idea is to show that while R can diverge, it cannot do so strongly enough to get contributions from singularities in codimension 2 and higher. On the other hand, the codimension 1 singularity can be analyzed and reduces to the conifold and large volume behaviors, which are integrable.

The formulas just discussed ignore flux quantization. One can show (work with Shiffman and Zelditch) that they are asymptotic to the finite L answer, with corrections falling faster than $1/\sqrt{L}$, and appear good for $L > b_3$. On the other hand, flux quantization is important if $L < b_3$, and more generally for the structure on scales $r < \sqrt{L/b_2}$, which shows concentrations at points where the region containing vacua lines up with the lattice of fluxes, and voids around these points. Recently DeWolfe et al 0411061 have identified (many of) these special points as points of enhanced discrete symmetry, for example Gepner points.

3. Supersymmetry breaking

Fine tuning can solve the hierarchy problem in a fraction $M_H^2/M_0^2 = M_H^2 M_{pl}^2/M_{susy}^4$ of vacua (assuming gravity mediation).

Thus, if the number of vacua grows with susy breaking scale at least as fast as $dN \sim M_{susy}^3 dM_{susy}$, high scale breaking is numerically favored.

Various mechanisms of susy breaking have been proposed, leading to different distributions. But if we can find a class of constructions which include two or more mechanisms, we can decide which among these is favored. Some proposals:

1. Nonperturbative effects in gauge theory.
2. Spontaneous (F term) breaking in the flux potential.
3. D term breaking, say through wrapping branes with incompatible susy.
4. Brane-antibrane breaking.

(3) and (4) are closely related. Similarly, extrapolating from known gauge-flux dualities, one can hypothesize that (1) and

With Denef, we have developed a formalism for counting F breaking flux vacua, and found that the generic distribution of breaking scales is

$$dN \sim |F|^5 dF \sim M_{susy}^{11} dM_{susy}.$$

A simple argument for this (Dine, O' Neil and Sun) is that in the typical susy breaking flux vacuum, all but one of the moduli have string scale masses, leaving an effective superpotential

$$W(\phi) = W_0 + \alpha\phi + \beta\phi^2 + \gamma\phi^3 + \ldots$$

near the minimum $V' = 0$. Then $F = DW = \alpha$ and small susy breaking requires tuning both real and imaginary parts of α. Getting a light goldstino requires tuning $|\beta| < |F|$, while eliminating tachyons turns out to require tuning $|\gamma| \le |F|$. Tuning the c.c. can be accomplished by tuning W_0, which decouples from the rest of the problem.

Finally, IIb vacua have enough fluxes to make the generic distribution for these parameters

$$d\mu \sim d^2\alpha d^2\beta d^2\gamma d|W_0|^2$$

leading to the result.

From another point of view, this result is surprising. We know that flux vacua can naturally produce the hierarchically small scales, from physics (gauge-flux duality, warp factor) and from our previous conifold counting result. Why can't these small scales control the supersymmetry breaking?

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The answer seems to be that the actual flux superpotentials contain, in addition to the terms $Mz + Nz \log z$ which lead to hierarchically small scales, additional $O(1)$ terms which contribute to the parameters discussed earlier, and must be tuned away to get stable low scale susy breaking.

This leads to two alternatives: either

- Gauge-flux duality is generally valid. While in principle gauge theory can break susy at a low scale, the actual effective Lagrangians coming out of string theory contain terms which generically spoil this breaking. In this case high scale breaking would be favored.

- Gauge-flux duality is valid in the cases which have been understood in some detail (non-chiral theories, or perhaps more general theories in non-compact CY) but not more generally. In more general cases, nonperturbative effects can generate a superpotential in which all parameters W_0, α, β, γ are controlled by the same low scale, allowing low

Either possibility seems reasonable and this question remains to be answered. If gauge theory breaking is different from flux breaking, then for low scale breaking to be favored at the end, we would also need

- A large fraction of flux vacua with $W = 0$ – not obvious in theories with many moduli.
- A sizable fraction of hidden sector gauge theories break susy dynamically.
- The μ problem must be generically solved.

More precisely, assuming that the same group of models can lead to either outcome, the product of these fractions should be greater than 10^{-30}.

If we can get a believable argument for the scale, we can then go on to ask which mediation mechanisms are favored, whether the "split" scenario is favored, etc.

4. Matter distributions

To decide what fraction of gauge theories break susy, we need a model for the ensemble of gauge theories coming out of string compactification. This could also be useful for understanding the possibilities for the hidden sector and for new strongly coupled physics at higher energies.

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Recently Blumenhagen et al 0411173 have studied the distribution of gauge and matter sectors on D-branes cancelling tadpoles in IIa compactification on tori. (see also Kumar and Wells 0409218.)

Their basic claim is that this problem is essentially that of partitioning the total tadpole charge among the branes, leading to results such as

$$N \sim \exp \sqrt{L \log L}$$

for the total number of brane constructions with tadpole L, and

$$d\mu[SU(M)] \sim \exp -M\sqrt{\log L/L}.$$

for the fraction of models containing an $SU(M)$ gauge theory factor. Typically there are $\sqrt{L}$ different gauge factors of average rank $\sqrt{L/\log L}$.

The distribution of intersection numbers I goes as $\exp -\sqrt{I}/L$ (as per the rough relation $I \sim M^2$).

Compare MRD 0303194 distribution dI/I (enforcing no adjoints, and only using the tadpole constraint implicitly). While this looks very different at first, this applied only up to a cutoff $I \sim L$. This cutoff is the scale of exponential falloff for Blumenhagen's results, and in this sense the two results are roughly compatible – most vacua have $I \leq L$.

In any case, most theories with several gauge group factors would be expected to have chiral matter, and are candidates for susy breaking. However we probably need to know the superpotential to judge this. Is the ansatz that superpotential terms allowed by symmetry are non-zero and independent (eg as in Nelson's talk here) reasonable?

Other issues: vacuum multiplicity within gauge sectors (e.g. pure $SU(M)$ gauge theory has M vacua, etc.), flux couplings, ...

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5. Conclusions and open questions

Basic aspects of the IIb flux vacua distribution are understood. We should be able to show that these vacua are finite in number (or not) in the near term.

There are interesting enhancements of number of vacua with discrete symmetry and $W = 0$ (DeWolfe et al) but it remains to be seen how important this is with many moduli (their arguments require periods in extension fields of low degree, which seems very rare).

Progress on the expected scale of susy breaking, but no definite claim. The usual low scale picture seems incompatible with claiming that all gauge theories which stabilize moduli can be dualized to flux.

One general question yet to address, is to what extent different cycles couple to each other; *e.g.* does varying one flux vary all the moduli significantly. This bears on susy breaking – if it is not true one can get significant enhancement of high scales – and on mediation.

Recent work (Kallosh et al, Trivedi et al) suggests that a large fraction of IIb vacua can stabilize Kähler moduli – how large?

The exploration of distributions coming from other constructions is just beginning. The main goal here has to be to decide whether or not we can get results consistent with the expected dualities, and if not why.

For most other constructions (M on G_2, heterotic on CY) the issue is to find more choices, which allow tuning the c.c.. This may require including non-geometric compactifications.

For F theory, taking the existing formulae at face value and plugging in $L = \chi/24 \sim 1000$ and $b_4 \sim 10000$ leads to extremely large numbers of vacua, even by present standards. In IIb language, most of these choices appear to correspond to the choice of gauge bundle in the D7-branes, and might be removed by the primitivity conditions or other susy constraints.

A particularly important confirmation of the landscape would be to understand AdS/CFT duals of supersymmetric flux vacua, along lines of Silverstein or possibly other gauge theory duals. IIb vacua with small Λ and large M_{KK} correspond to CFT's with a large gap in the spectrum; do these really exist?