Title: The Wavefunction of the Universe

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Abstract:

# The Wavefunction of the Universe

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Cornell University

- with Hassan Firouzjahi and Henry Tye. hep-th/0406107
- soon to appear, hep-th/0504... Sash Sarangi and Henry Tye

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## Key Questions for String Theorists

- Let us assume that string theory gives a landscape of supersymmetric and nonsupersymmetric vacua.
- Of all string theory vacua, why does the universe end up in a particular vacuum?
- Why only 3 large space dimensions? Why not a 10−D supersymmetric vacuum?
- Do we need some version of the Anthropic Principle?
- A framework to answer some of these questions
: Spontaneous Creation of the Universe
: Creation of a deSitter spacetime from nothing (no classical spacetime).

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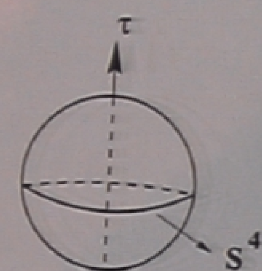
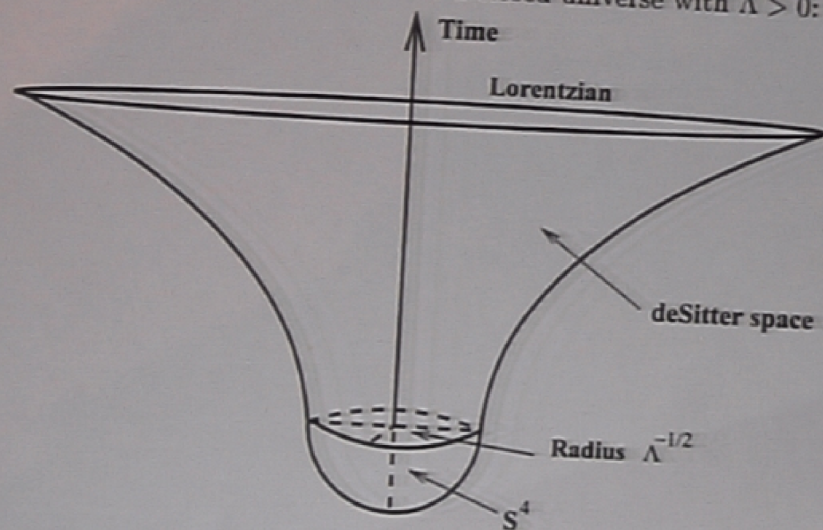
# The $S^4$ Instanton




The $S^4$ Instanton.

The $S^4$ can be cut and joined to a closed universe with $\Lambda > 0$:




The $S^4$ can be analytically continued to deSitter in the Lorentzian plane

## Hartle-Hawking Wavefunction

- $\Psi_{HH} = \int_{\phi}^{h_{ij}} D[g] e^{-S_E[g]}$ ; $P = |\Psi_{HH}|^2$
- The $4-D$ Euclidean Action:

$$S_E = -\frac{1}{16\pi G}\int d^4x\sqrt{|g|}\,(R-2\Lambda)$$

- Metric ansatz (minisuperspace):

$$ds^2 = \left(d\tau^2 + a^2(\tau)d\Omega_3^2\right)$$

- Euclidean Einstein's Equations (closed universe):

$$-2\frac{\ddot{a}}{a} - \frac{\dot{a}^2}{a^2} + \frac{1}{a^2} = 3\Lambda$$

which gives the $S^4$ instanton solution:

$$a(\tau) = \frac{1}{\sqrt{\Lambda}}\cos(\sqrt{\Lambda}\tau)$$

- continued to Lorentzian signature :

$$a(t) = \frac{1}{\sqrt{\Lambda}}\cosh(\sqrt{\Lambda}t)$$

-

$$S_E = -3\pi/2G\Lambda \quad ; \quad P \sim e^{3\pi/G\Lambda}$$

## Problems with Hartle-Hawking

- An infrared problem in scenarios with dynamical $\Lambda$ - as in scenarios with four-form flux, Bousso-Polchinski type scenarios.
- $\Psi_{HH} \sim e^{3\pi/2G_N\Lambda}$
  Such a universe prefers $\Lambda \to 0$ and Size $\to \infty$.
  $(a \sim \frac{1}{\sqrt{\Lambda}})$
- This means the Euclidean action does not have a minimum. $S_E \to -\infty$.
- This renders $\Psi_{HH}$ unnormalizable.
- This infrared divergence is related to the lack of a lower bound to the Euclidean action in theories with dynamical $\Lambda$.
- Problem with topology change : the $S^4$ is unstable to the formation of other topologies.
  Fischler, Morgan, Polchinski, 1990
- Loop corrections, and string corrections, not helpful

## Decoherence : Solution to the Infrared Problem of $\Psi_{HH}$

"Environment" induced tunneling suppression.

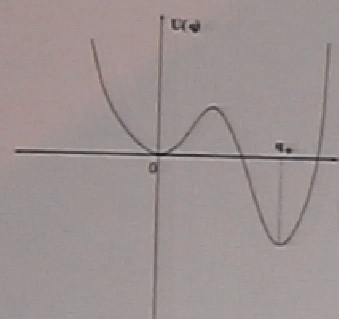
Example: Caldeira, Leggett, 1983

- $L = L_{system} + L_{environ} + L_{coupling}$

$$L_{system} = \frac{M}{2}\dot{q}^2 + U(q)$$

$$L_{environ} = \frac{1}{2}\sum_\alpha m_\alpha \left(\dot{x}_\alpha^2 - \omega_\alpha^2 x_\alpha^2\right)$$

$$L_{coupling} = q \sum_\alpha C_\alpha x_\alpha$$



- Calculation of the effective bounce by tracing out the environment

$$Z = \int D[q] e^{-S_o[q]} \prod_\alpha \int dx_\alpha e^{-S_{env}[q, x_\alpha]}$$

$$\simeq e^{-S_o[q] - \chi[q]}$$

$$\chi[q] > 0$$

**Comments**

- $$S_E = \int \sqrt{2MU(q, x_\alpha)}ds$$
$$ds^2 = dq^2 + \sum \frac{m_\alpha}{M} dx_\alpha$$
Increase in $S_E$ is due to the longer path length in the many dimensional $(q, x_\alpha)$ space.

- Interaction of $q$ with $x_\alpha$ interferes with its attempt to tunnel. This interaction can be seen as attempts to observe $q$. Repeated measurements of $q$ suppresses the tunneling rate.

- Interaction of $q$ with the environment introduces decoherence, .The system behaves more like a classical system than like a quantum system.

- For bounded case, there is a mere correction.

- For gravity, picture changes qualitatively.

- Not a 1−loop correction.

## Why an Environment for deSitter?

- Universe should be a superposition of different linearly independent wavefunctions.
- Environment offers as a source of decoherence to explain the classical evolution of the universe.
- Classical evolution crucial for the calculation of density perturbations.
- The environment from the Euclidean region can be continued to the Lorentzian region to get density perturbations in inflation.

## Metric and Non-Metric Perturbations

- Metric perturbation

$$h_{ij} = a^2 \left(\Omega_{ij} + \epsilon_{ij}\right)$$

- Expansion in spherical harmonics of $S^3$

$$\epsilon_{ij} = \sum_{n,l,m} \Big[ \sqrt{6} a_{nlm} \frac{1}{3} \Omega_{ij} Q^n_{lm} + \sqrt{6} b_{nlm} (P_{ij})^n_{lm} +$$

$$\sqrt{2} c^0_{nlm} (S^0_{ij})^n_{lm} + \sqrt{2} c^e_{nlm} (S^e_{ij})^n_{lm} + 2 d^0_{nlm} (G^0_{ij})^n_{lm} + 2 d^e_{nlm} (G^e_{ij})^n_{lm} \Big]$$

-

$$N = N_0 \left[ 1 + \frac{1}{\sqrt{6}} \sum_{n,l,m} g_{nlm} Q^n_{lm} \right]$$

$$N_i = a(t) \sum_{n,l,m} \left[ \frac{1}{\sqrt{6}} k_{nlm} (P_i)^n_{lm} + \sqrt{2} j_{nlm} (S_i)^n_{lm} \right]$$

$$\Phi = \sigma^{-1} \left[ \frac{1}{\sqrt{2\pi}} \phi(t) + \sum_{n,l,m} f_{nlm} Q^n_{lm} \right]$$

- Perturbed Action

$$I = I_0(a, \phi, N_0) + \sum_n I_n$$

"Environment" induced tunneling suppression for $\Psi_{HH}$

- Gravitational perturbations $ds^2 = d\tau^2 + a^2(\Omega_{ij} + \epsilon_{ij})dx^i dx^j$
- System : The scale factor $a(\tau)$

$$S_E^o = \frac{1}{2}\int d\tau \left(-a\dot{a}^2 - a + \Lambda a^3\right)$$

- Environment : Gravitational metric perturbations

$$S_E^n = \frac{1}{2}\int d\tau a^3 \left(\dot{d_n}^2 + \frac{(n^2-1)}{a^2} d_n^2\right)$$

- The path integral is

$$\int D[a] \prod_n \int_{d_n^i}^{d_n^f} D[d_n] \;\; e^{-S_E^o[a] - \sum_n S_E^n[a,d_n]}$$

- To find the effect of the environment, trace over the $d_n$'s with $(d_n^i \neq, d_n^f \neq 0)$

$$\Psi[a] = TrZ = \prod_n \int dd_n^i \int dd_n^f \; \delta(d_n^i - d_n^f) \; Z$$

## Calculation

- A new time variable

$$du = \frac{d\tau}{a(\tau)^3}$$

- The equation of motion

$$d_n'' - \omega_n(a)^2 d_n = 0$$

- Relevant modes $\dot{\omega}_n << \omega_n^2$.
- WKB approximation

$$d_n^{\pm} = \frac{1}{\sqrt{\omega_n}} \exp\left(\pm \int^u du' \omega_n(u')\right)$$

- Classical Action

$$S_E(d_{cl}) = \frac{1}{2(\exp(D_n) - \exp(-D_n))} \Big[ ((d_n^f)^2 \omega_n(u_f) + (d_n^i)^2 \omega_n(u_i)) \\ (\exp(D_n) - \exp(-D_n)) - 4 d_n^i d_n^f \sqrt{\omega_n(u_i)\omega_n(u_f)} \Big]$$

- Trace over $d_n$

$$\prod_n \int df_n^i \int_{f_n^i}^{f_n^i} D[f_n] \exp(S_E) \\ \simeq \prod_n \frac{1}{\sqrt{(\exp(D_n) - \exp(-D_n))}}$$

## The Modified Bounce

- Tracing out the environment leads to

$$S_E^{eff} = S_E^o[a] + D[a] = \frac{1}{2}\int d\tau \left(-a\dot{a}^2 - a + \Lambda a^3 + \frac{\nu}{\Lambda^3 a}\right)$$

where $\nu \simeq M_s^6$.

- $\nu$ related to the large wavelength (Hubble) and small wavelength (*string*) cutoffs.
- Equation of Motion

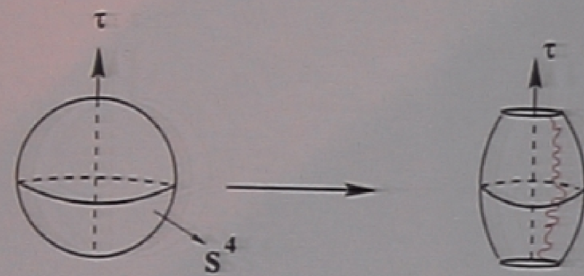
$$-2\frac{\ddot{a}}{a} - \frac{\dot{a}^2}{a^2} + \frac{1}{a^2} = 3\Lambda - \frac{\nu}{\Lambda^3 a^4}$$

- Modified Bounce Solution

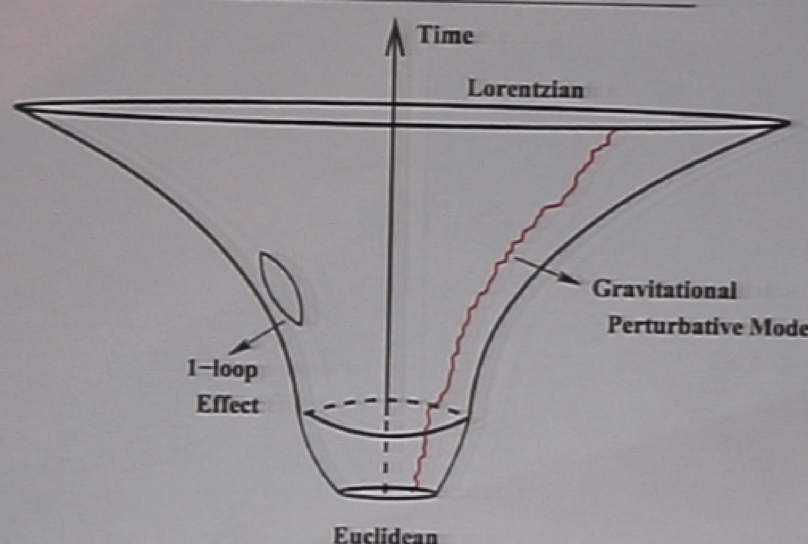
$$a(\tau) = \frac{1}{\sqrt{2\Lambda}}\sqrt{\left(1 + \sqrt{\left(1 - \frac{4\nu}{\Lambda^2}\right)}\cos(2\sqrt{\Lambda}\tau)\right)}$$

- $S^4$ recovered when $\nu = 0$.

# The Modified Bounce contn'd

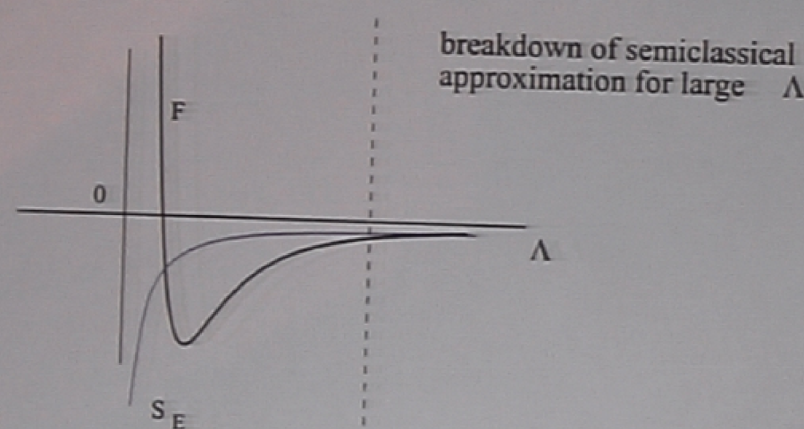



The enviroment deforms the $S^4$ bounce solution.



## The Improved Wavefunction

- $P \simeq e^F$
- Where $F$ is twice the modified Euclidean action
  $F = \frac{3\pi}{G\Lambda} - \frac{243\nu\pi^3}{G^3\Lambda^3}$




- $S_E$ is unbounded from below. But the interaction with the environment has made $F$ bounded from below.

## The Gravitational Potential

- Modified Lorentzian Action

$$S = \frac{1}{2}\int d\tau \left(-a\dot{a}^2 + a - \lambda a^3 - \frac{\nu}{\lambda^3 a}\right)$$

- Modified Hamiltonian constraint

$$H = \frac{1}{2a}\left(-\Pi_a^2 - a^2 + \lambda a^4 + \frac{\nu}{\lambda^3}\right) = 0$$

$\Pi_a = -a\dot{a}$ is the conjugate momentum.

- The gravitational potential is given by

$$U(a) = -a + \lambda a^3 + \frac{\nu}{\lambda^3 a}$$

-

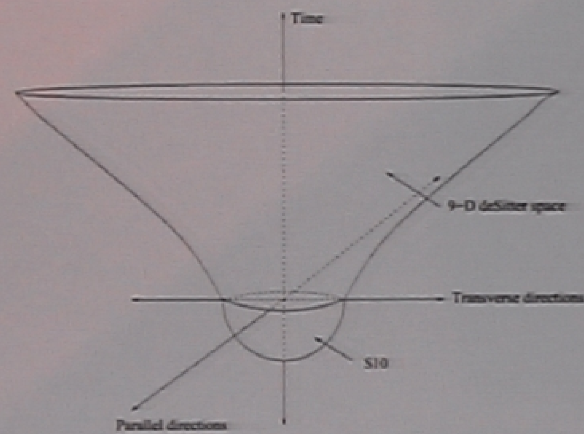
$$U_{min} = 2\sqrt{\frac{\lambda}{3}}\left(\frac{3\nu}{2\lambda^3} - \frac{1}{3\lambda} + 0(\frac{1}{\lambda^4})\right)$$

- Gravitational Potential bounded now.

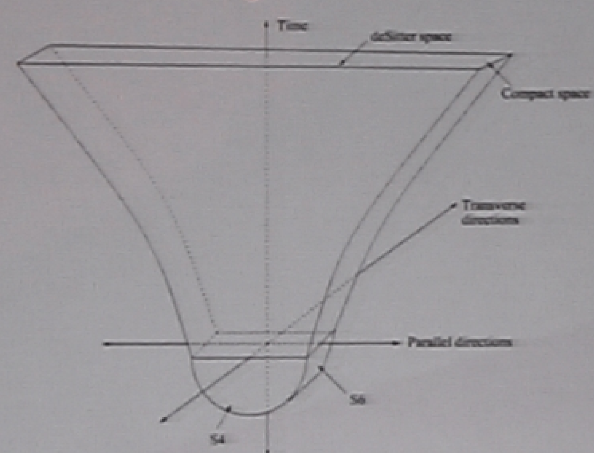
## Applying the Wavefunction to the Landscape

- The wavefunction can be applied as a selection principle on the cosmic landscape. Firouzjahi, Sarangi, Tye 2004 ; Ooguri, Vafa, Verlinde 2005
- Hartle-Hawking wavefunction, however, is problematic. One has to use the imroved wavefunction.
- There are 10-D generalizations of the $S^4$ instanton : $S^{10}$, $S^4 \times S^6$, $S^5 \times S^5$, $S^4 \times$ Calabi-Yau .... etc.
- String scale lower than the Planck scale : Semiclassical treatment still valid.
- Each instanton corresponds to certain number of inflating dimensions and some static dimensions whose size is fixed.

## $10 - D$ Gravitational Instantons




$S^{10}$ Instanton : 9 Inflating Dimensions + 1 Time Direction




$S^4 \times S^6$ Instanton : 3 Inflating Dimensions + 1 Time Direction + 6 Static Dimensions

## Improved Wavefunction in Higher Dimensions

- Modified wavefunction leads to a tunneling probability $P \sim e^F$.
- $F = -S_E - D$.
- In 10-D, $P \sim \exp(-S_E - cV_9^2 + ...)$.
- In effective 4-D theory, $F = \frac{3\pi}{G\Lambda} - \frac{cV_6^2}{G^3\Lambda^3}$
- The constant $c$ has to be calculated for each vacuum.

## Tunneling Probabilities

**The Improved Wavefunction can be used to find the probability of every single vacuum in the landscape.**

Since $S_E$ is bounded, $F = -S_E = -(S_E^o + D)$.

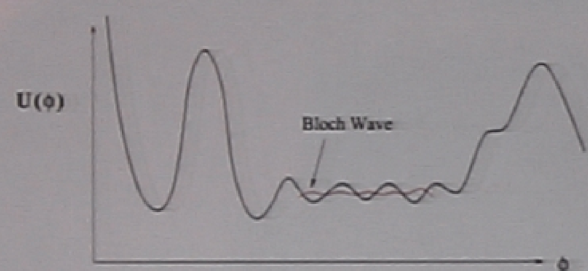
- Tunneling to an inflationary universe (KKLMMT model with fluxes M and K fixed to maximize $F$)
  $F \sim 10^{18}$
- To 10-D deSitter space $S^{10}$
  $F \sim 10^{9}$
  Similarly, for $S^4 \times S^6$, $S^5 \times S^5$, etc.
- To KKLT vacuum
  $F < 0$
- To a vacuum with today's cosmological constant
  $F < -10^{170}$

## Selection of the Original Universe Principle

- The improved wavefunction can give the probability for each individual vacuum in the landscape.
- Decoherence term to be calculated for each individual vacuum.
- Comparing the probabilities will tell which vacua are preferred.
- Once the inflationary universe gets created, it can follow a path that leads to todays vacuum with a low cosmological constant.

## Other Implications of Decoherence

- At the end of inflation, a Bloch wave covering a neighborhood of degenerate vacua Kane, Perry, Zytkow, hep-th/0311152
- Universe, after tunneling, will span the neighborhood of vacua.
- Decoherence will help the universe settle down to a single vacuum.




## Eternal Inflation?

- Decoherence suppressed tunneling may modify the criteria for eternal inflation.
- Decoherence may modify Coleman-DeLuccia instanton.

## Comments

- At large values of $\Lambda$ the semiclassical treatment breaksdown.
- At small values of $\Lambda$ decoherence effects become very large.
- In the region of interest (inflationary scale) we may have a good control over the behavior of the wavefunction.
- We have calculated the decoherence effect of a pure gravitational field. One can include matter fields as well. Qualitative features do not change.
- The parameter $\nu$ deforms the $S^4$ instanton. For large values of $\nu$, $S^4$ becomes $R^1 \times S^3$ and tunneling is totally suppressed. Such vacua will not allow the spontaneous creation of universe.

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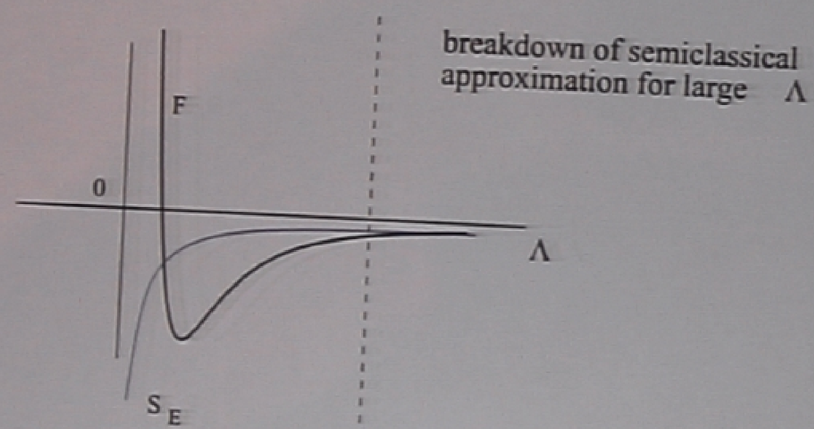


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## Summary and Conclusion

- The modified Hartle-Hawking wavefunction with the inclusion of decoherence can be used as a selection principle on the cosmic landscape.
- Decoherence effect can provide a lower bound to the gravitational action.
- A better understanding of decoherence and its determination important.
- Inflationary vacua seem to be favored over supersymmetric, KKLT vacua.
- Find other vacua, especially vacua with other large spatial dimensions; and determine the tunneling probability from nothing to any one of them. Find out whether 4D is selected or not.

## Tunneling Probabilities

**The Improved Wavefunction can be used to find the probability of every single vacuum in the landscape.**

Since $S_E$ is bounded, $F = -S_E = -(S_E^o + D)$.

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