Title: Reconnection of Colliding Cosmic Strings

Date: Mar 29, 2005 10:35 AM

URL: http://pirsa.org/05030136

Abstract:

29 March. 2005

Talk at Perimeter Institute,

String phenomenology workshop

Reconnection of Colliding Cosmic Strings

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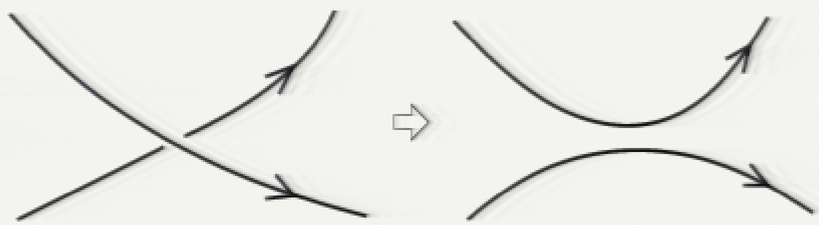
Center for Theoretical Physics, MIT

`hep-th/0501031` **w/ A. Hanany (MIT)**

work in progress **w/ D. Tong (Cambridge)**

How can we distinguish several possible origins of cosmic strings?

Ans.
Reconnection (or Recombination, Intercommutation) probability P in collision process



- $P = 1$ (deterministic reconnection) for field theory vortex strings

  [Shellard][Matzner][Moriarty-Myers-Rebbi]

  – Numerical simulations show that they always reconnect.
  – There seems to be an upperbound of velocity for reconnection.

- $P \neq 1$ (probabilistic reconnection) for cosmic super(/D-)strings

  [Copeland-Myers-Polchinski][Jackson-Jones-Polchinski]

  – Worldsheet calculations show $10^{-3} \lesssim P \lesssim 1$ for some compactification scenario.
  – Fundamental strings should be of this type, $P \sim g_s^2$.

In this talk, the following questions will be answered, from the view-point of effective field theory on the cosmic strings:

- What is the mechanism of the reconnection of the vortex strings and of the D-strings?
- Why are they different in reconnection property?
- How can one compute the reconnection probability for D-strings?

——————— Plan of this talk ———————

- Introduction
- Vortex strings and D-strings: the difference
- Reconnection of vortex strings
- Reconnection of D-strings
- Summary and discussions

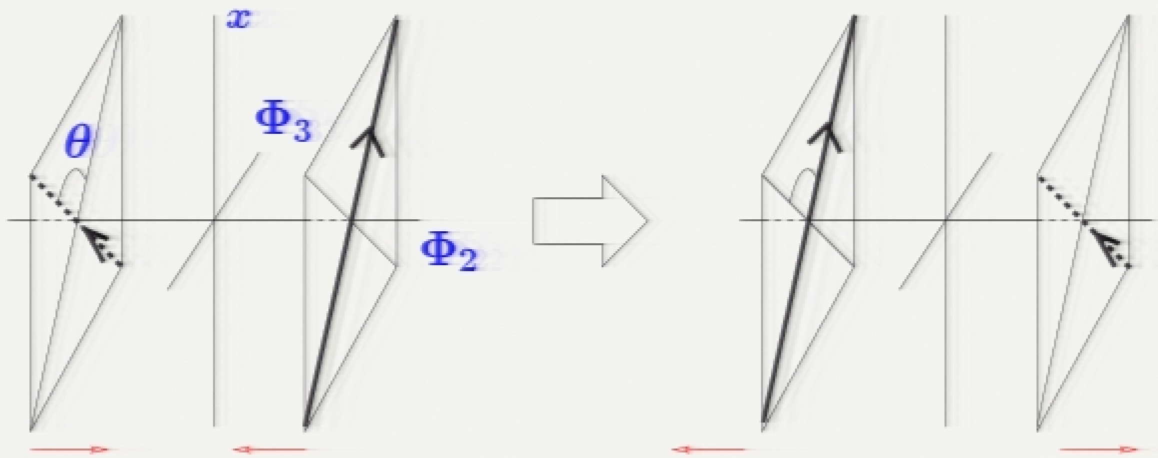
D-strings can pass through each other

$$S=\frac{2\pi l_s^2}{g_s}\int dtdx\ \mathrm{Tr}\left[-\frac{1}{4}F_{\mu\nu}F^{\mu\nu}-\frac{1}{2}D_\mu\Phi_i D^\mu\Phi^i+\frac{1}{4}[\Phi_i,\Phi_j]^2\right]$$

In this D-string action, there is a classical solution representing them passing through each other without reconnection:

$$2\pi l_s^2\Phi_2=\begin{pmatrix}\bar{v}t & 0\\ 0 & -\bar{v}t\end{pmatrix},\quad 2\pi l_s^2\Phi_3=\begin{pmatrix}\tan(\theta/2)x & 0\\ 0 & -\tan(\theta/2)x\end{pmatrix}$$

$\theta\ll 1,\ \bar{v}\ll 1,\ g_s\to 0$: Action (low energy approx.) is valid.



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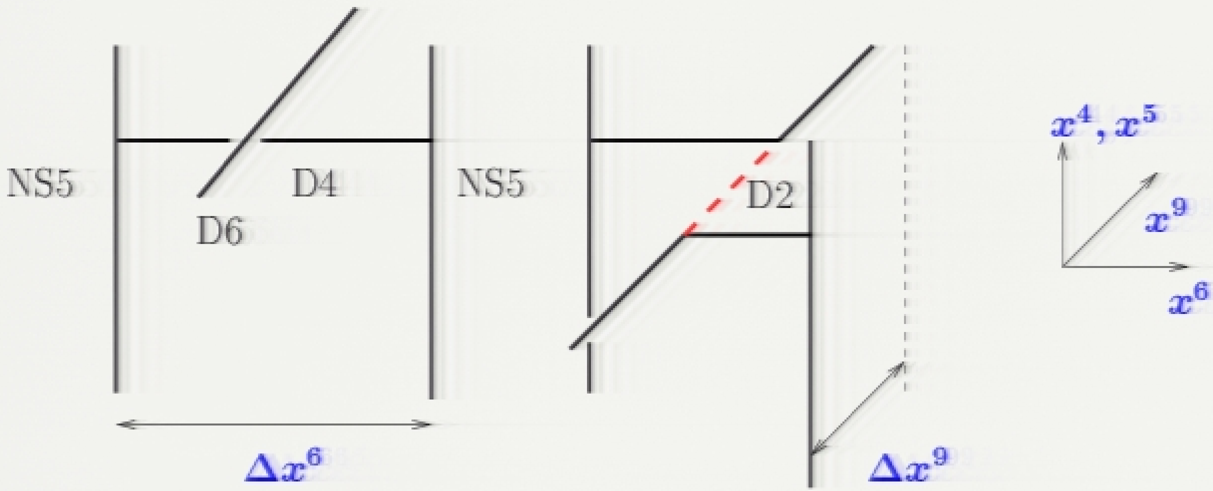

NS5: 0123-45
D4: 0123-6
D6: 0123-789

[Hanany-Witten]

Brane realization of 4d $\mathcal{N}=2$ Abelian Higgs model w/ $(A_\mu, \phi, q, \tilde{q})$

Gauge coupling: $\frac{1}{e_{\mathrm{AH}}^2} = \frac{\Delta x^6}{(2\pi)^2 g_s l_s}$, FI parameter : $\zeta_{\mathrm{AH}} = \frac{\Delta x^9}{(2\pi)^3 g_s l_s^3}$

Decoupling limit: $\Delta x^6 \sim \epsilon l_s, \quad \Delta x^9 \sim \epsilon^2 l_s, \quad g_s \sim \epsilon, \quad \epsilon \to 0$

Vortex strings = D2-branes suspending between D4s on D6

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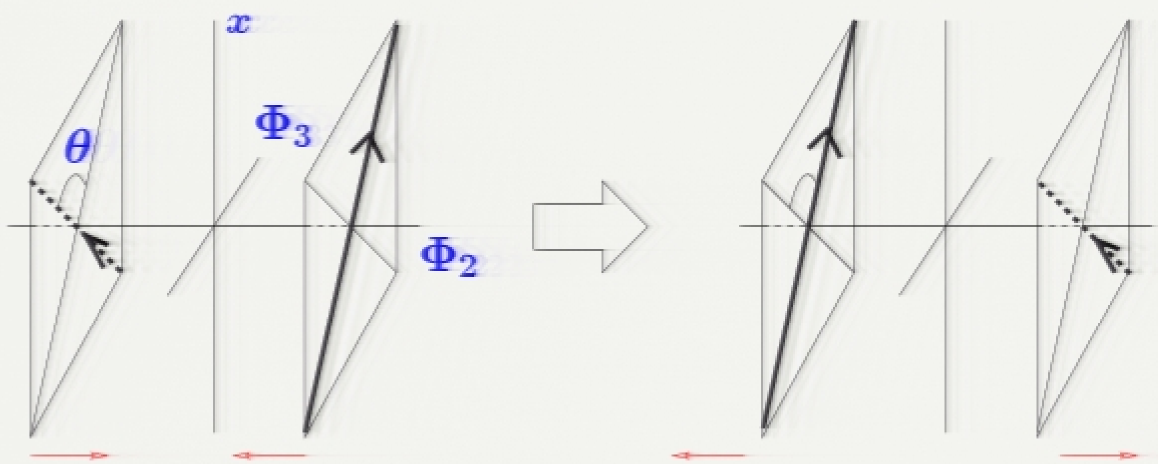

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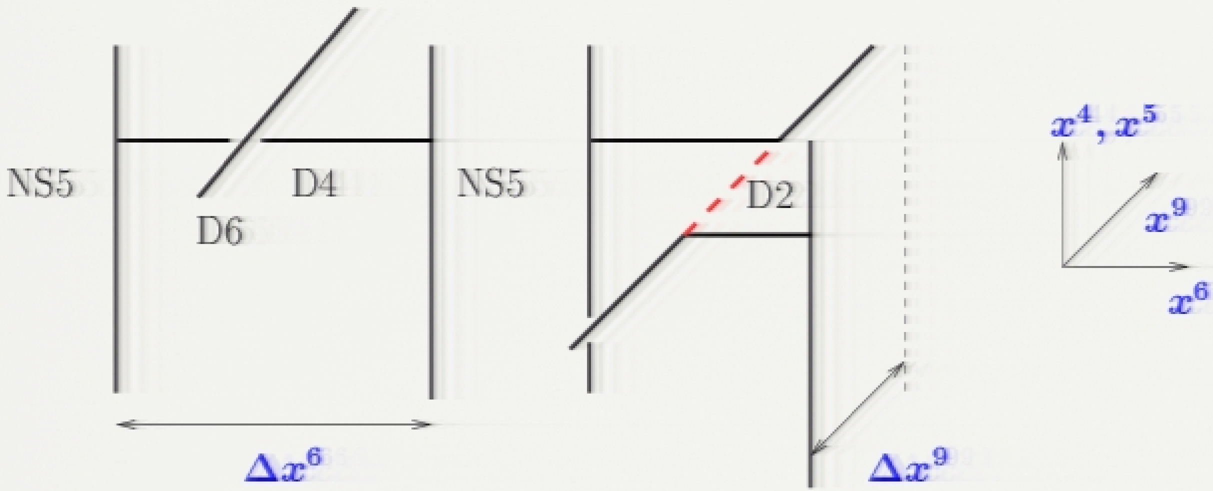

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Decoupling limit $\Rightarrow g \to \infty \Rightarrow$ Only the potential bottom survives!

> The effective theory of the vortex strings is a sigma model whose target space is the D-term equation $\psi\psi^\dagger - [Z, Z^\dagger] - r\mathbf{1}_{2\times 2} = 0$.

The important fact is that the D-term equation does not allow the passing-through solution due to the FI parameter r.

$\Rightarrow$ There is no naive classical solution of vortex strings passing through each other.

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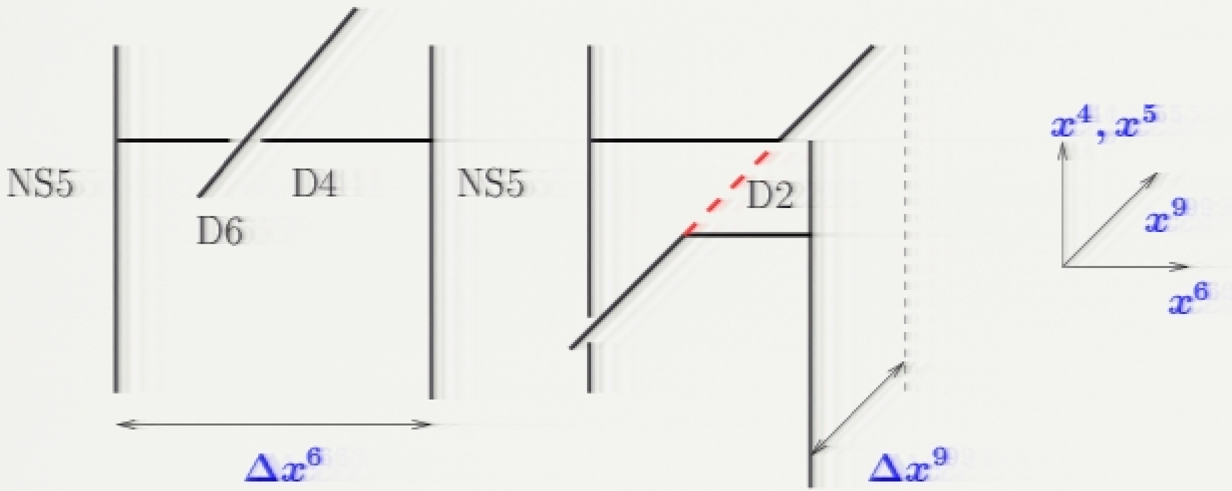

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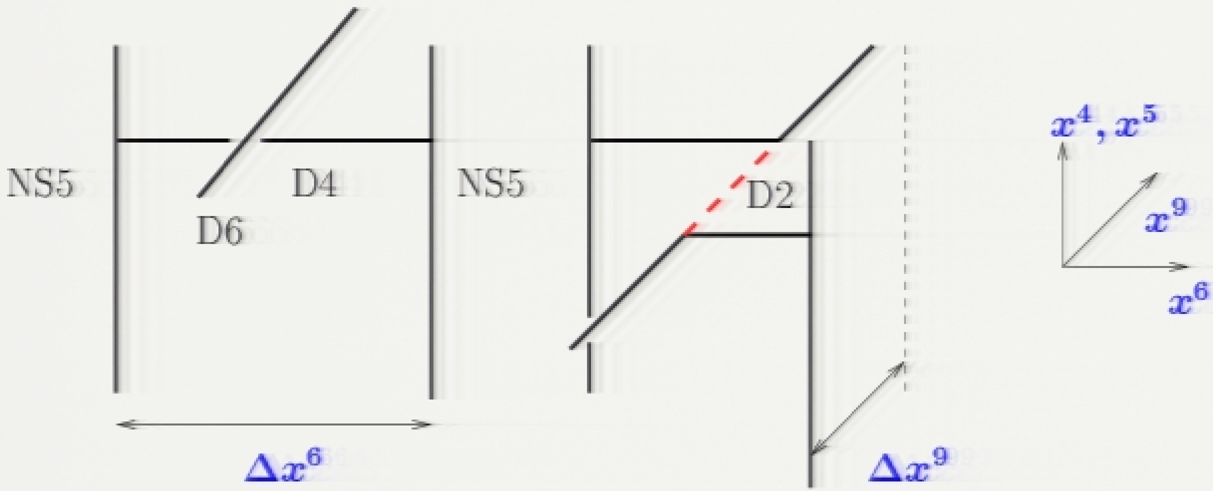

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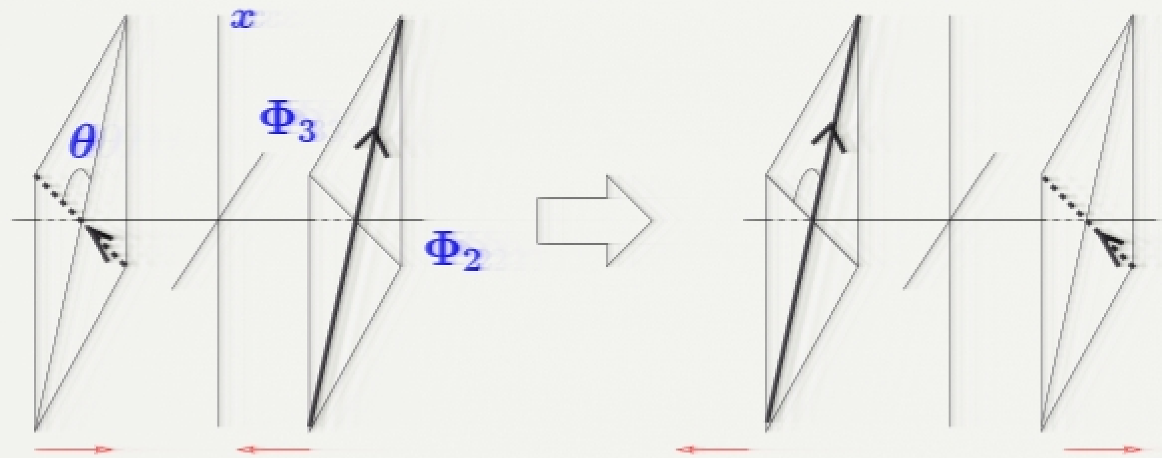

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3 Reconnection of colliding vortex strings

Solving the D-term condition by [Kim-Lee-Yi]

$$Z = w 1_{2\times 2} + z \begin{pmatrix} 1 & \sqrt{\frac{2b}{a}} \\ 0 & -1 \end{pmatrix}, \ \psi = \sqrt{r} \begin{pmatrix} \sqrt{1-b} \\ \sqrt{1+b} \end{pmatrix}, \ a \equiv \frac{2|z|^2}{r}, b \equiv \frac{1}{a+\sqrt{1+a^2}}$$

w : center-of-mass for the two vortex strings, $2z$: relative position

$$\Downarrow$$

Using Manton's method, we obtain the effective action of the relative motion of the two vortex strings:

$$S = \mathcal{T} \int dt dx \, g(|z|) \partial_\mu z(t,x) \partial^\mu \bar{z}(t,x), \quad g(|z|) \equiv \frac{|z|^2}{\sqrt{|z|^4 + r^2/4}}$$

- Behavior of the metric : $g(|z|) \sim \begin{cases} 2|z|^2/r & \text{for} \quad |z| < \sqrt{r/2} \\ 1 & \text{for} \quad |z| > \sqrt{r/2} \end{cases}$

  When strings are close ($z \sim 0$), a coordinate transformation $\tilde{z} \sim z^2/\sqrt{2r}$ gives a flat metric $ds^2 \sim |d\tilde{z}|^2$.

  $\Rightarrow$ Antipodal points in z space are identified

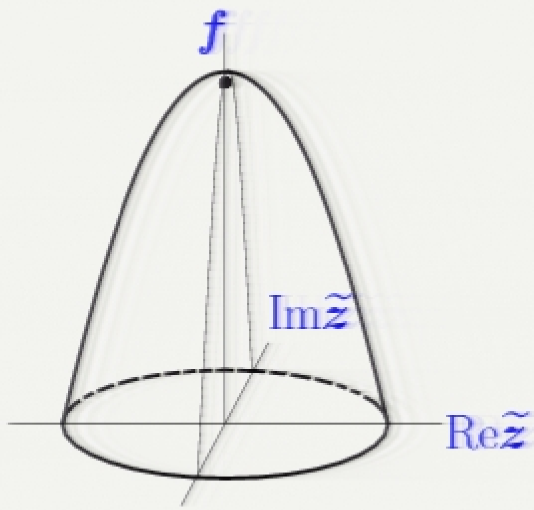
[Vilenkin-Shellard][Ruback]

We make the field redefinition

$$\tilde{z} \equiv \frac{z^2}{2(|z|^4 + r^2/4)^{1/4}}$$

Then in terms of $\tilde{z} \equiv \rho e^{i\varphi}$,

$$ds^2 = \left(1 + \left(\frac{df(\rho)}{d\rho}\right)^2\right) d\rho^2 + \rho^2 d\varphi^2$$



where $f(\rho)$ is a smooth function with

$$f(\rho) \sim -\sqrt{3}\rho \quad (\rho \sim \infty)\,, \quad f(\rho) \sim -\sqrt{\frac{2}{r}}\rho^2 \quad (\rho \sim 0)$$

$\Downarrow$

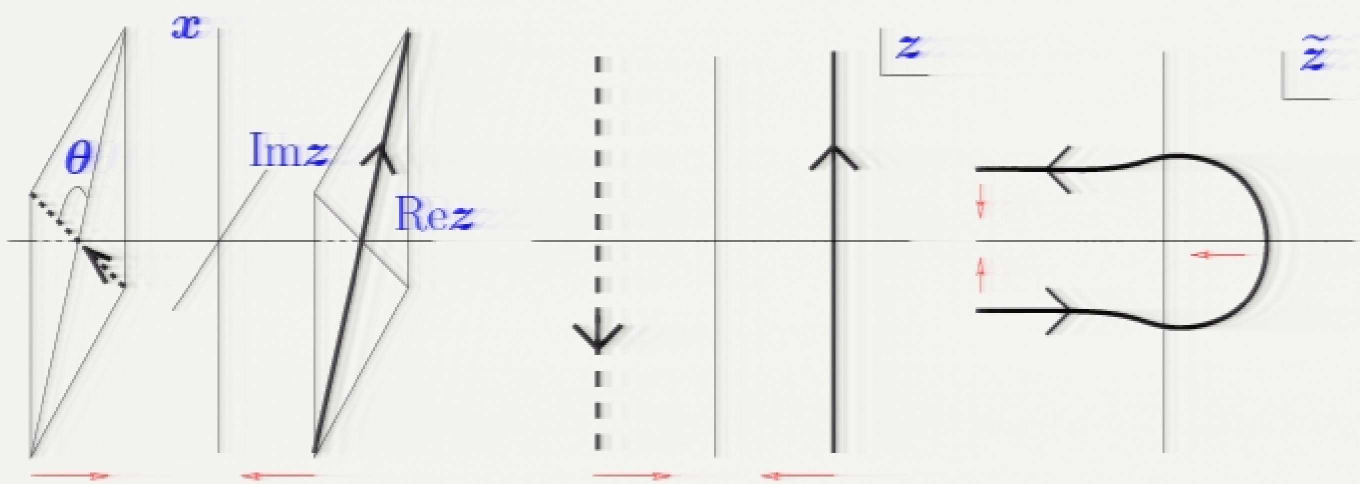
A smeared surface of a cone with a deficit angle π

$\Downarrow$

Effective sigma model describes

a Polyakov string moving on the surface of the smeared cone.

Proof of reconnection of colliding vortex strings

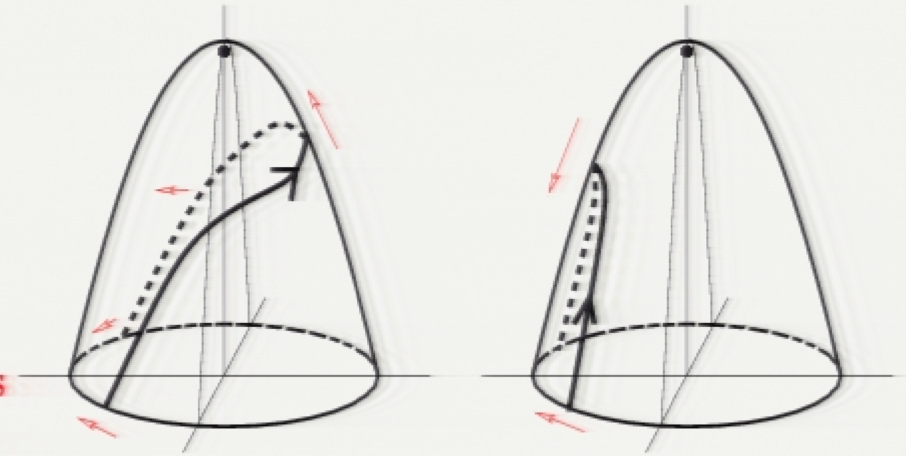


Initial condition at some time for the vortex strings :

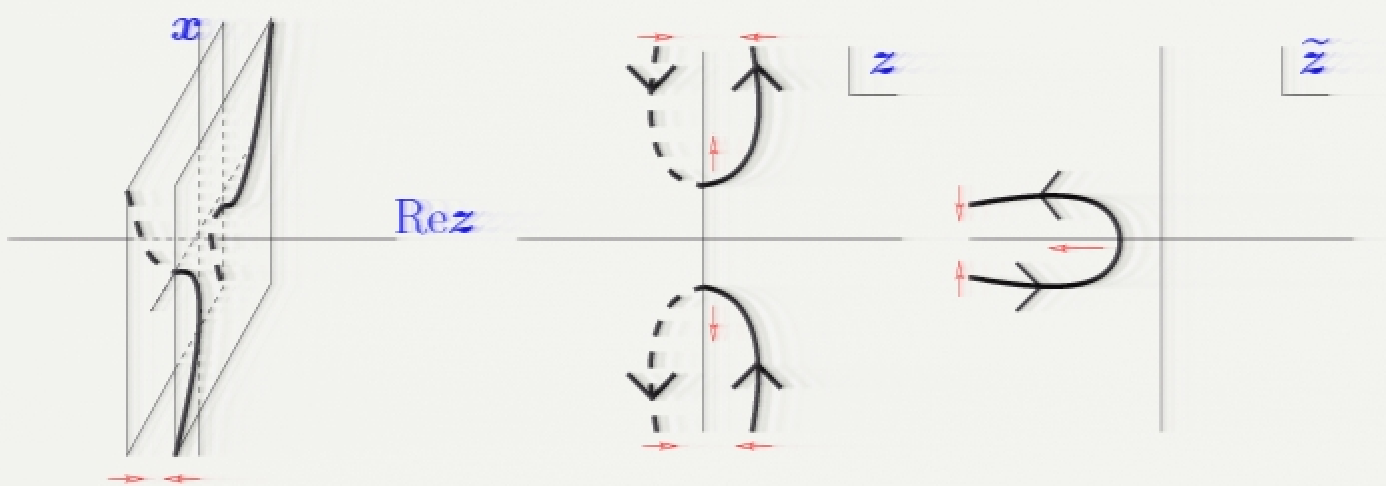
$$z = z_0 + i\tan(\theta/2)x, \quad \dot{z} = \frac{v}{2} \quad (\theta,\ z_0 \text{ and } v \text{ are real})$$

- When $\theta, v \ll 1$, Moduli space approximation is valid.
- Initial strings are straight, but in a natural metric they are equivalent to a single curved Polyakov string.

  → It moves in $\tilde{z}$ space.



The Polyakov string slips
off the top of the cone.



Going back to the original space,
the vortex strings have been reconnected!

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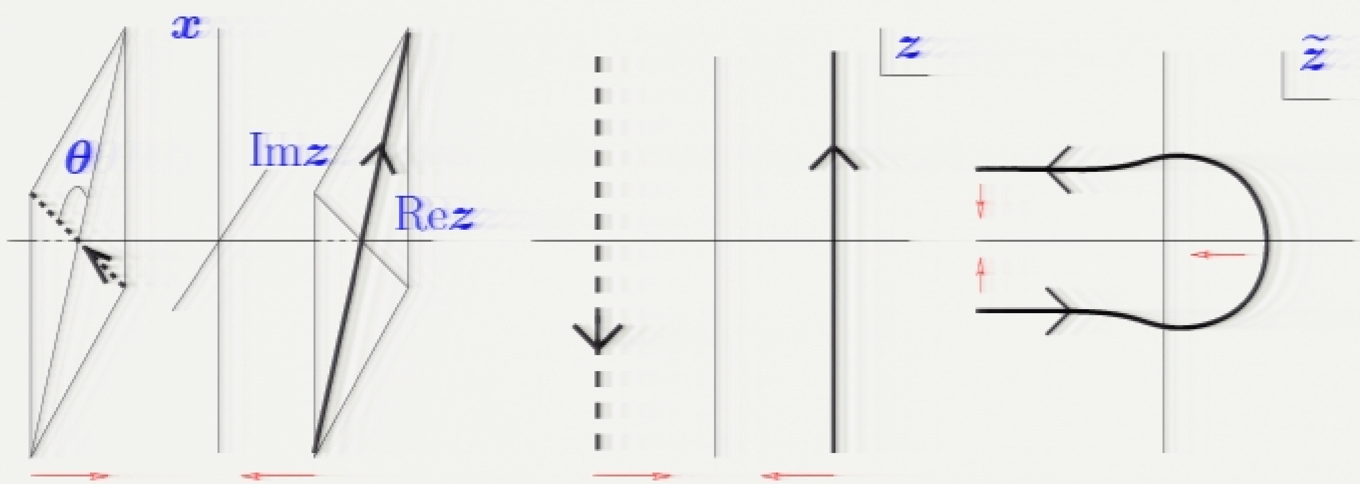

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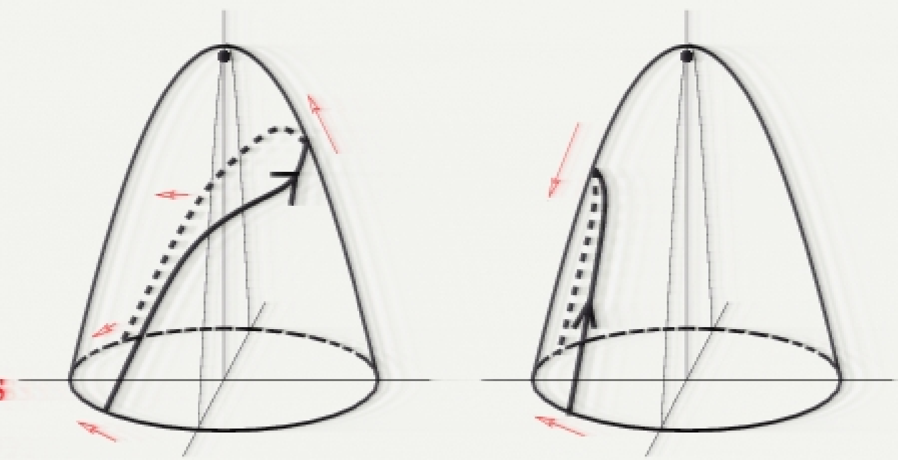

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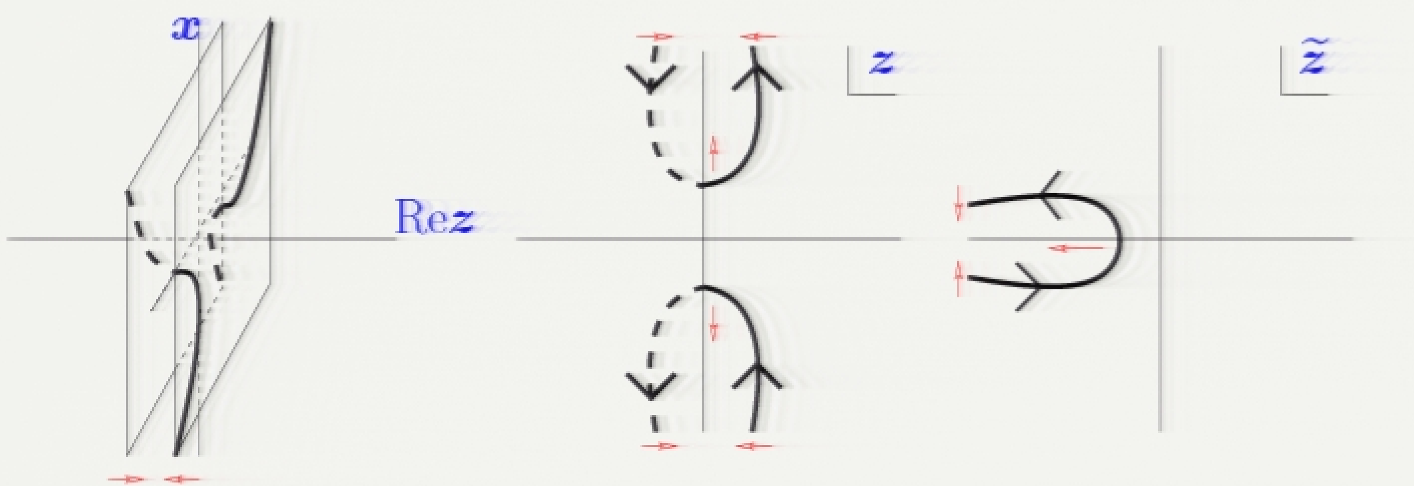

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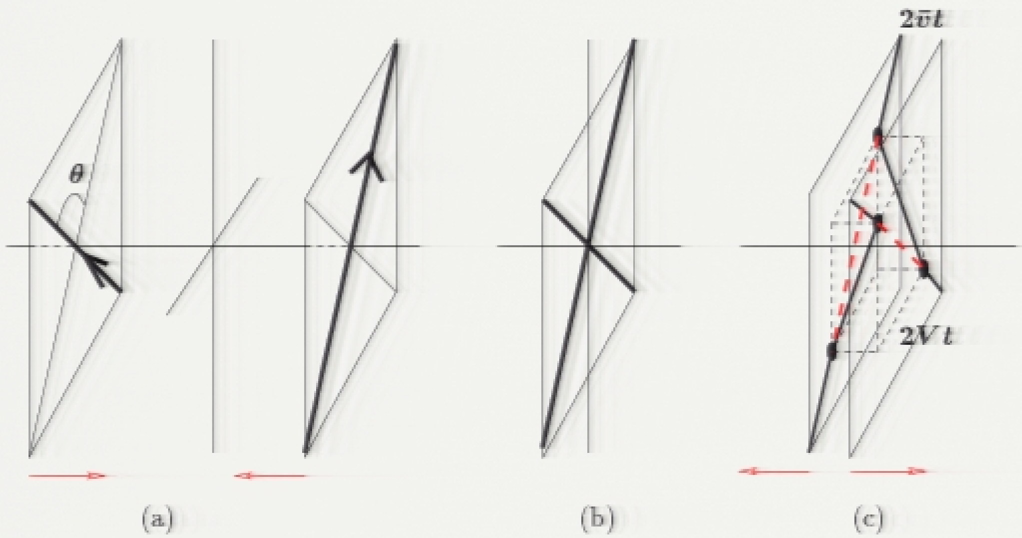
Velocity upperbound for the reconnection



$\bar{v}$: velocity of the original strings in the center-of-mass frame
V : velocity of the reconnected region
$t = 0$: collision incidence

Kink points are at $(Vt\cot(\theta/2), Vt, \bar{v}t), (-Vt\cot(\theta/2), Vt, -\bar{v}t)$.

Consider the energy gain, $\delta E = E_+ - E_-$.

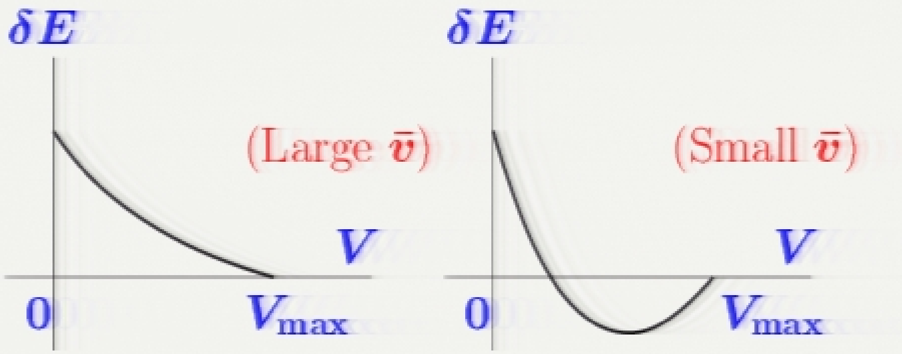
- E_+ : energy produced by reconnection = energy between kinks. (solid lines along the box surface)
- E_- : energy of the original strings which disappeared after the reconnection. (dashed lines)

Given $\bar{v}$, if $\delta E < 0$ for some velocity V, then the reconnection should occur.

$$\delta E = 4\mathcal{T}t\left(\frac{\sqrt{\bar{v}^2 + V^2\cot^2(\theta/2)}}{\sqrt{1-V^2}} - \frac{V}{\sqrt{1-\bar{v}^2}\sin(\theta/2)}\right)$$

Kinks should not travel faster than the speed of light

$$\Rightarrow \quad 0 \le V \le \sqrt{1-\bar{v}^2}\sin(\theta/2)\ (\equiv V_{\max})$$



$$\text{Velocity upperbound}: \quad \bar{v} < \frac{\sin(\theta/2)}{\sqrt{1+\sin^2(\theta/2)}}$$

This coincides with a field theory result by [Copeland-Turok],

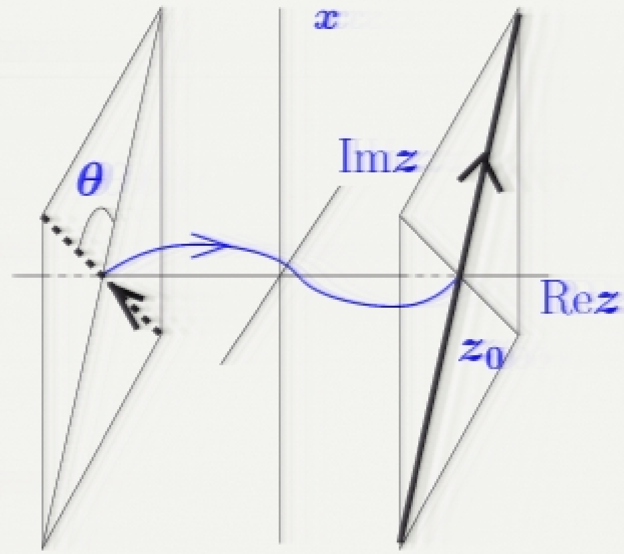
$$\bar{v} < \sqrt{\frac{4\alpha(1-\cos\theta)}{1+4\alpha(1-\cos\theta)}}$$ with the ansatz parameter $\alpha = 1/8$.

Reconnection of colliding D-strings

Reconnection = Tachyon condensation

Worldsheet string theory result: spectrum of a string connecting the two D-strings is [Berkooz-Douglas-Leigh]

$$m^2 = \left(n - \frac{1}{2}\right)\frac{\theta}{\pi l_s^2} + \frac{(2z_0)^2}{(2\pi l_s^2)^2}$$



The lowest mode $n = 0$ becomes tachyonic for sufficiently close D-strings.

It was proved that the condensation of the tachyonic mode corresponds to the reconnection. The proof is [Nagaoka-K.H.]

(1) Describe the tilted D-strings by 2d Yang-Mills.
(2) Fluctuation around the classical solution reproduces the spectrum, and the tachyon mode comes from an off-diagonal entry.
(3) Condensation of it leads to the reconnection by diagonalization.

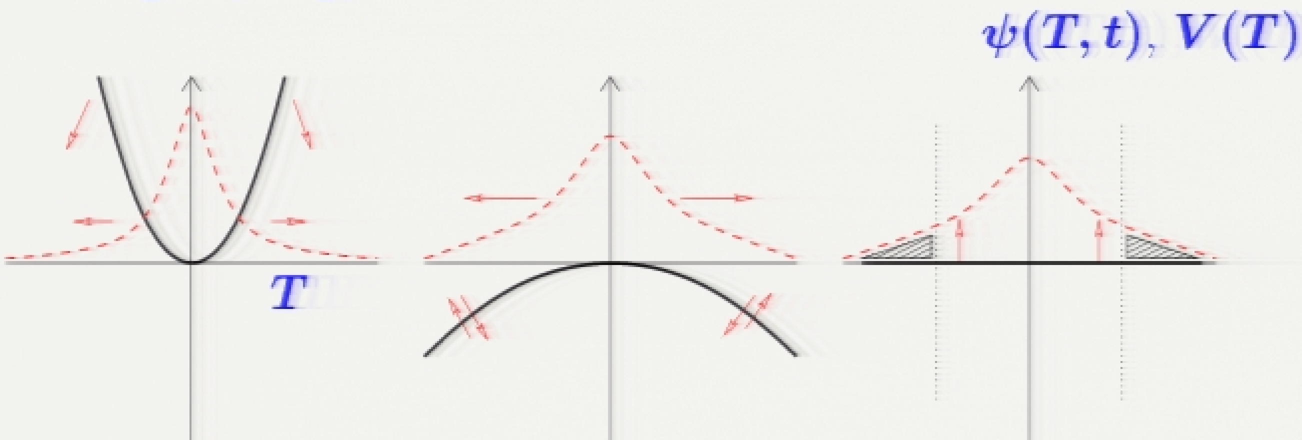
$$\Phi \sim \begin{pmatrix} \tan(\theta/2)x & T(t)e^{-\theta x^2} \\ T(t)e^{-\theta x^2} & -\tan(\theta/2)x \end{pmatrix} \to \pm\sqrt{(x\tan(\theta/2))^2 + T(t)^2 e^{-2\theta x^2}}$$

Effective action of the off-diagonal tachyonic mode:

$$S = \frac{1}{g_T}\int dt \left[\frac{1}{2}(\partial_t T(t))^2 - \frac{1}{2}m^2 T^2\right]$$

Tachyon dynamics and the reconnection is a quantum mechanical problem. A particle interpretation :

- $\frac{1}{g_T} = \frac{2\sqrt{2}\pi^2 l_s^3}{g_s\sqrt{\theta}}$ serves as a mass of the particle located at $T(t)$
- $m^2 = -\frac{\theta}{2\pi l_s^2} + \frac{(2z_0)^2}{(2\pi l_s^2)^2}$ gives a frequency of the harmonic potential



Moving D-strings $2z_0 = vt$ show a tachionic period $-t_0 < t < t_0$ ($t_0 \equiv \frac{l_s\sqrt{2\pi\theta}}{v}$) during which the potential becomes upside down.

Reconnection = Tachyon condensation

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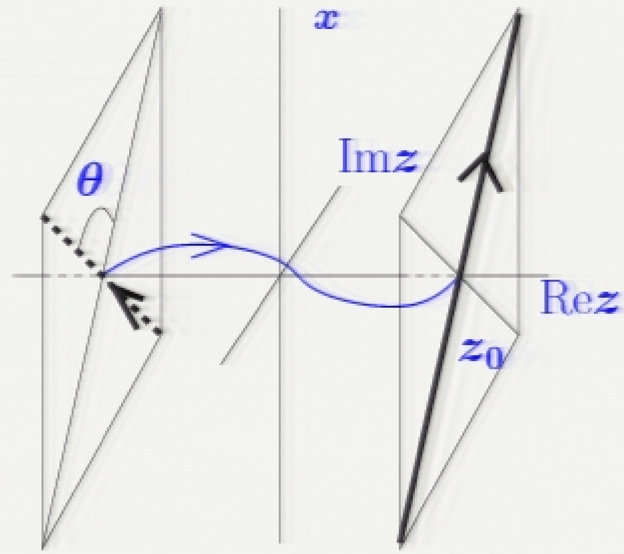

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$$S = \frac{1}{g_T}\int dt\left[\frac{1}{2}(\partial_t T(t))^2 - \frac{1}{2}m^2T^2\right]$$

Tachyon dynamics and the reconnection is a quantum mechanical problem. A particle interpretation :

- $\frac{1}{g_T} = \frac{2\sqrt{2}\pi^2 l_s^3}{g_s\sqrt{\theta}}$ serves as a mass of the particle located at $T(t)$
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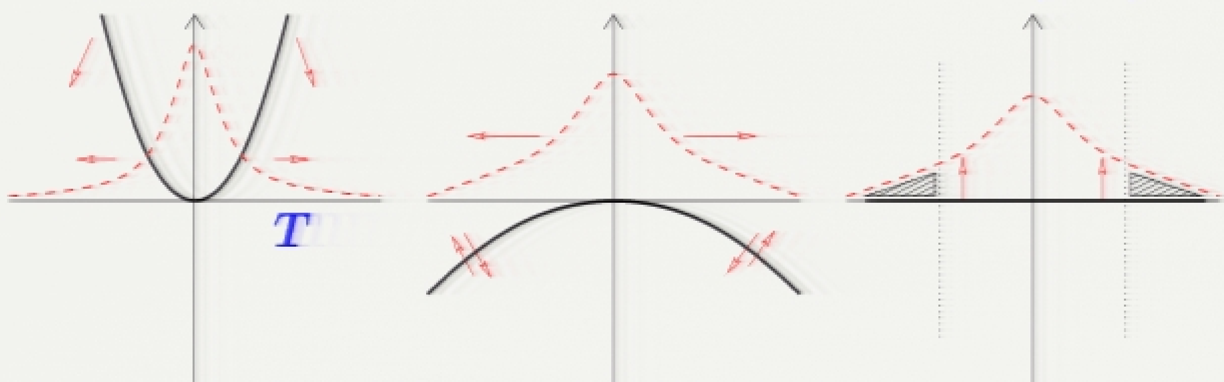
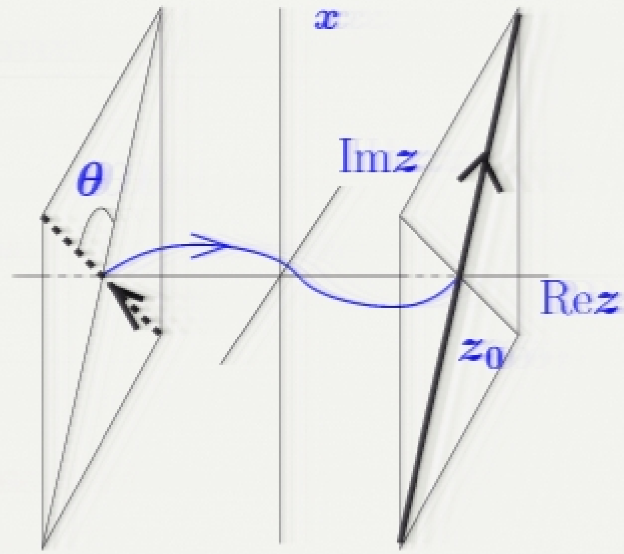
$\psi(T,t)$, $V(T)$



Moving D-strings $2z_0 = vt$ show a tachionic period $-t_0 < t < t_0$ ($t_0 \equiv \frac{l_s\sqrt{2\pi\theta}}{v}$) during which the potential becomes upside down.

Reconnection of colliding D-strings

Reconnection = Tachyon condensation



Worldsheet string theory result: spectrum of a string connecting the two D-strings is [Berkooz-Douglas-Leigh]

$$m^2 = \left(n - \frac{1}{2}\right)\frac{\theta}{\pi l_s^2} + \frac{(2z_0)^2}{(2\pi l_s^2)^2}$$

The lowest mode $n = 0$ becomes tachyonic for sufficiently close D-strings.

It was proved that the condensation of the tachyonic mode corresponds to the reconnection. The proof is [Nagaoka-K.H.]

(1) Describe the tilted D-strings by 2d Yang-Mills.
(2) Fluctuation around the classical solution reproduces the spectrum, and the tachyon mode comes from an off-diagonal entry.
(3) Condensation of it leads to the reconnection by diagonalization.

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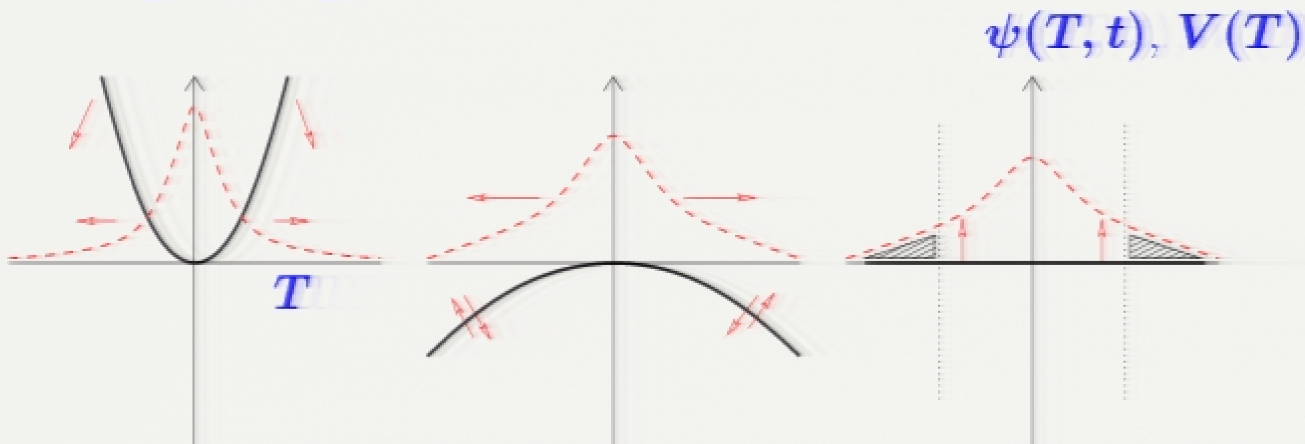

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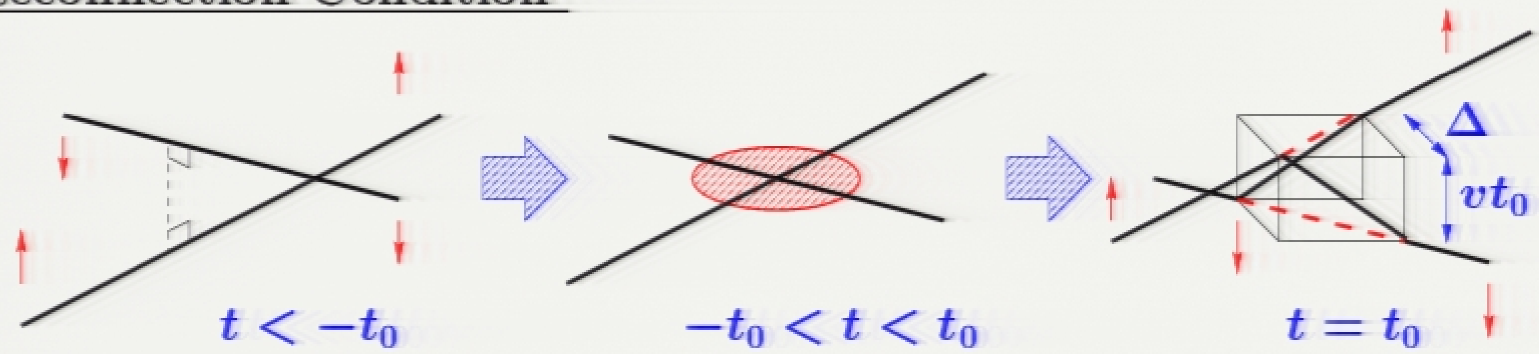
Reconnection Condition



If the tachyon value is large enough at the end of the tachyonic period, the reconnection occurs.

$$vt_0 < \Delta \quad \Leftrightarrow \quad T > \frac{\sqrt{\theta}}{\sqrt{2\pi} l_s} \quad : \text{Reconnection Condition}$$

Reconnection probability

$$P = 2\int_{\sqrt{\theta}/\sqrt{2\pi}l_s}^{\infty} dT\, |\psi(T, t = t_0)|^2 \simeq \frac{\sqrt{g_s}}{2\pi^{3/4}\theta^{3/4}} e^{2\sqrt{s}\theta/v} \exp\left[-\frac{4\sqrt{\pi}\theta^{3/2}}{g_s} e^{-4\sqrt{s}\theta/v}\right]$$

Our result is close to that of [Jackson-Jones-Polchinski], a string worldsheet calculation,

$$P = \exp\left[\left(4 - \frac{v}{2g_s}\right) e^{-\pi\theta/v}\right]$$

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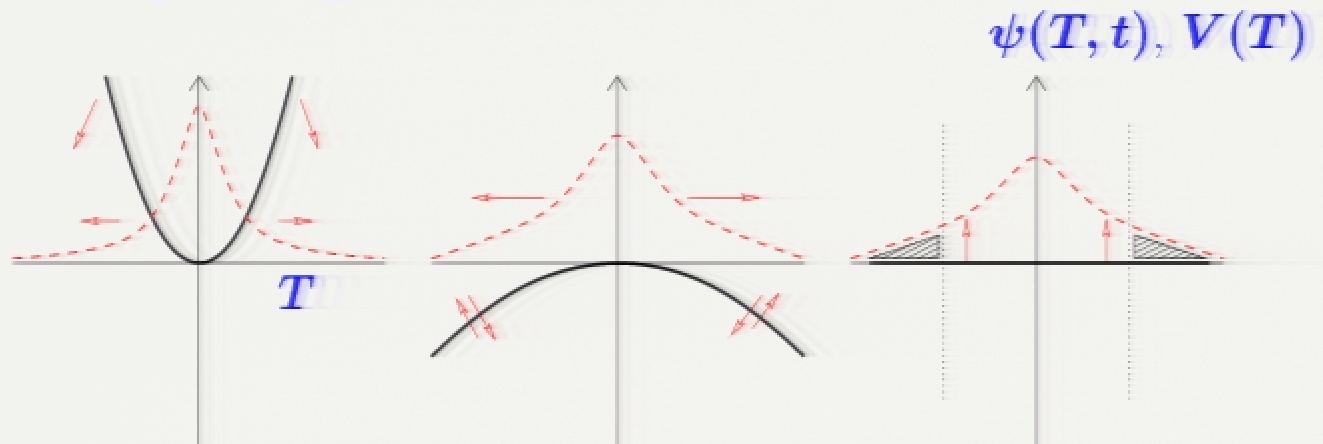

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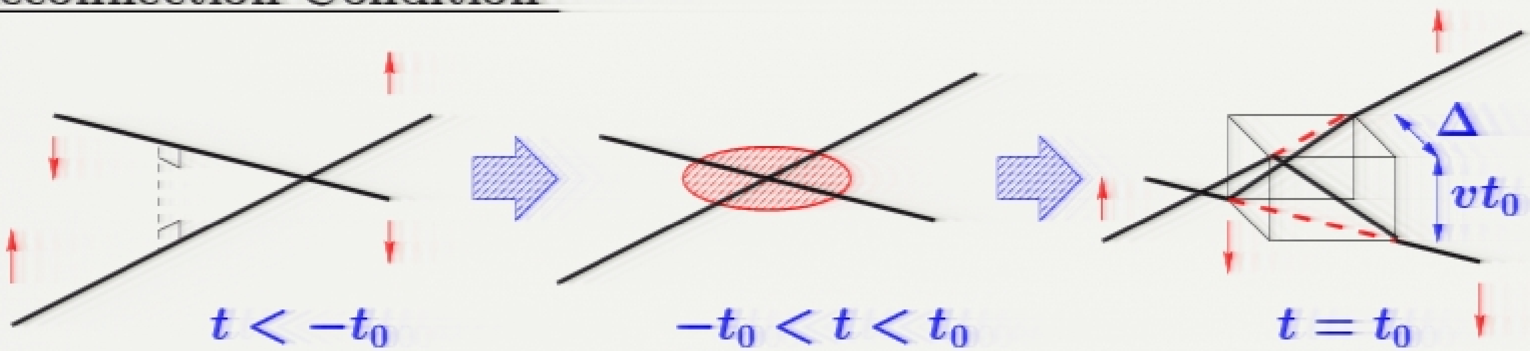

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5 Summary and Discussions

Summary

For vortex strings,

- Inevitable reconnection of colliding strings was shown classically.
- Velocity upperbound for it was derived.

For D-strings,

- Reconnection is a quantum phenomenon.
- Tachyon condensation leads to the reconnection.
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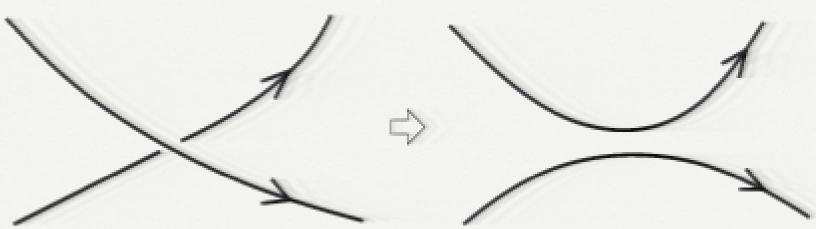
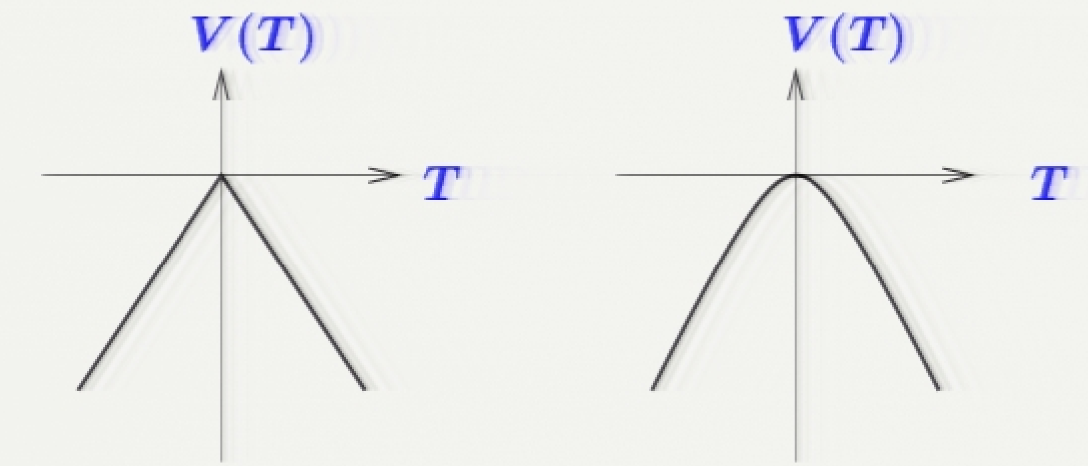


Origin of the difference : an energetic consideration

Why is there a classical difference for the strings?



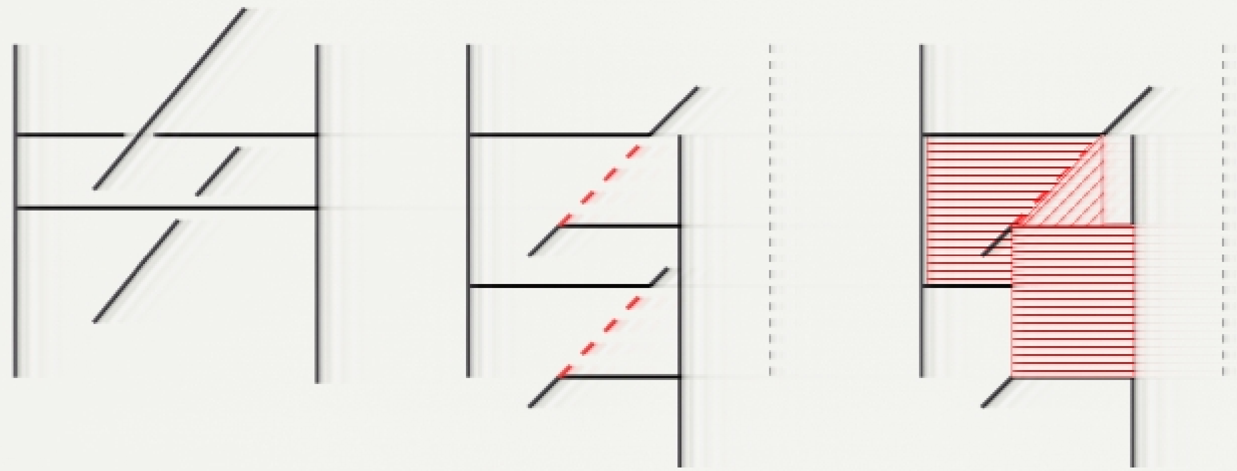
D-string reconnection is accompanied by bond production

[Taylor-K.H.][Sato]

Nonabelian vortex strings

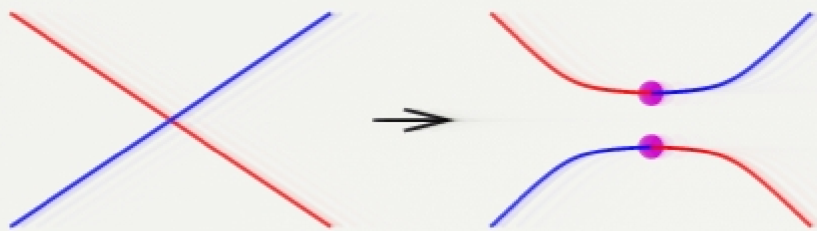
[Tong-K.H.]

In nonabelian Higgs models, several kinds of vortex strings appear.



Different kinds of strings can pass through each other.

$$P = 1/N + \mathcal{O}(e^{m^2/T}) \text{ for } N = N_c = N_f$$



Generation of Tong's Monopoles!

How can we distinguish field theory vortices from superstrings?!

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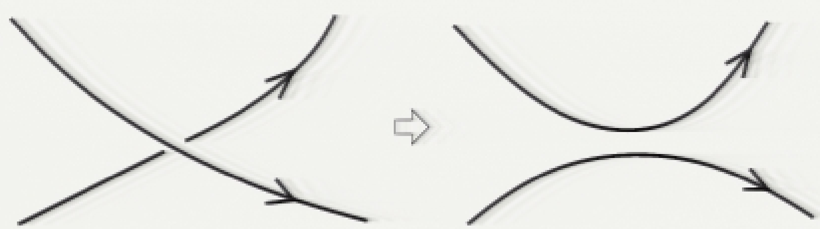


Vortex string effective action comes from a D2-brane action.

$$S_{\text{vortex}} = \int dt dx \, \text{Tr} \left[-\frac{1}{4g^2} F_{\mu\nu} F^{\mu\nu} - \mathcal{D}_\mu Z^\dagger \mathcal{D}^\mu Z - \mathcal{D}_\mu \psi^\dagger \mathcal{D}^\mu \psi \right.$$
$$\left. - \frac{g^2}{2} \left(\psi\psi^\dagger - [Z, Z^\dagger] - r \mathbf{1}_{2\times 2} \right)^2 \right]$$

$Z \propto \Phi^2 + i\Phi^3$: complex adjoint field, ψ : new fundamental field

$$\frac{1}{g^2} = \frac{l_s \Delta x^9}{g_s} = (2\pi)^3 l_s^4 \zeta_{\text{AH}} \,, \quad \text{FI parameter } r = \frac{\Delta x^6}{2\pi g_s l_s} = \frac{2\pi}{e_{\text{AH}}^2}$$

Decoupling limit $\Rightarrow g \to \infty \Rightarrow$ Only the potential bottom survives!

> The effective theory of the vortex strings is a sigma model whose target space is the D-term equation $\psi\psi^\dagger - [Z, Z^\dagger] - r \mathbf{1}_{2\times 2} = 0$.

The important fact is that the D-term equation does not allow the passing-through solution due to the FI parameter r.

$\Rightarrow$ There is no naive classical solution of vortex strings passing through each other.

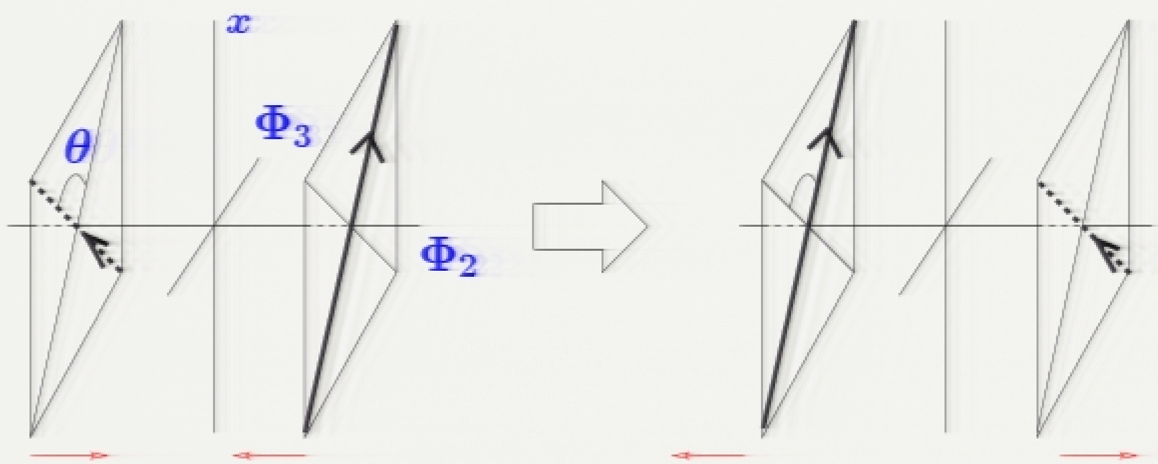
D-strings can pass through each other

$$S = \frac{2\pi l_s^2}{g_s}\int dt dx \,\mathrm{Tr}\left[-\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{2}D_\mu\Phi_i D^\mu\Phi^i + \frac{1}{4}[\Phi_i, \Phi_j]^2\right]$$

In this D-string action, there is a classical solution representing them passing through each other without reconnection:

$$2\pi l_s^2\Phi_2 = \begin{pmatrix} \bar{v}t & 0 \\ 0 & -\bar{v}t \end{pmatrix}, \quad 2\pi l_s^2\Phi_3 = \begin{pmatrix} \tan(\theta/2)x & 0 \\ 0 & -\tan(\theta/2)x \end{pmatrix}$$

$\theta \ll 1,\ \bar{v} \ll 1,\ g_s \to 0$: Action (low energy approx.) is valid.



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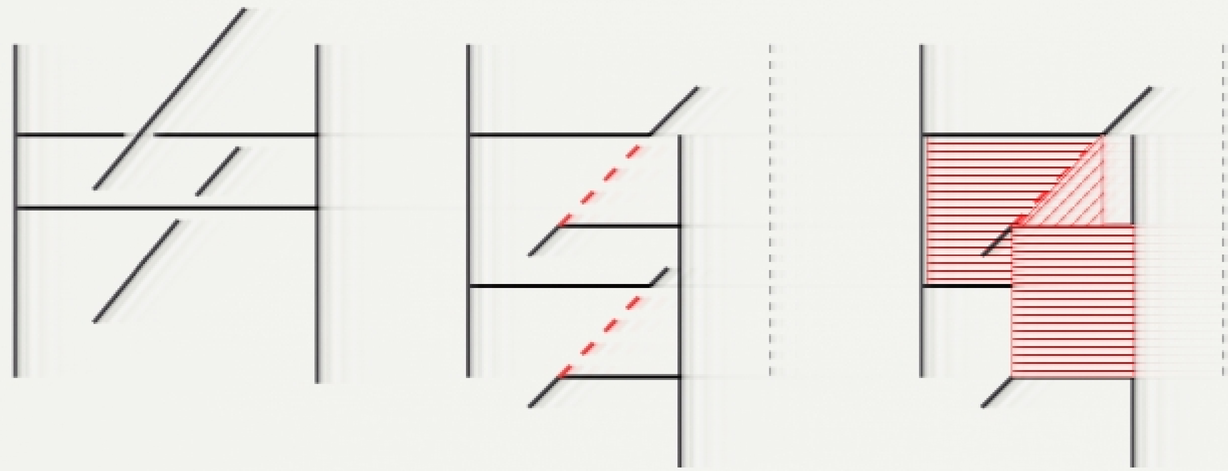

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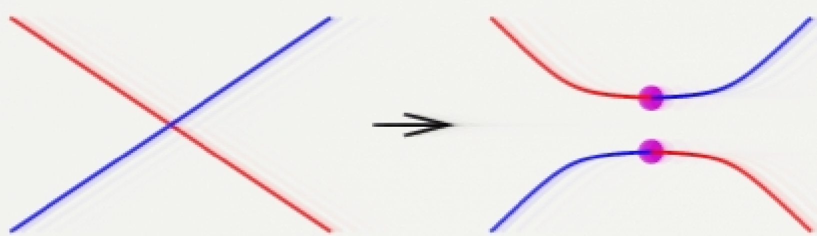

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