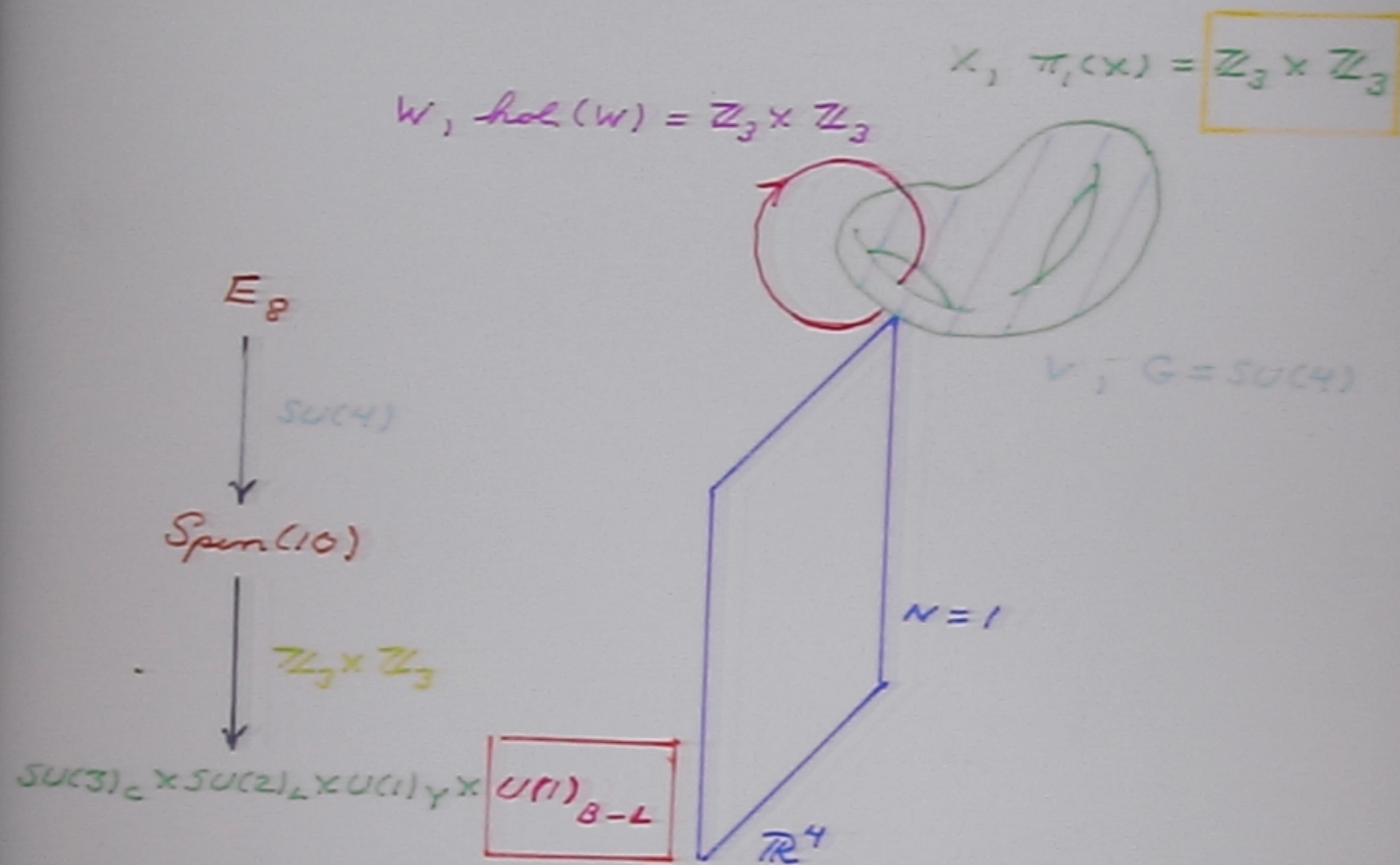


Title: A Heterotic Standard Model

Date: Mar 28, 2005 10:35 AM

URL: <http://pirsa.org/05030129>

Abstract:

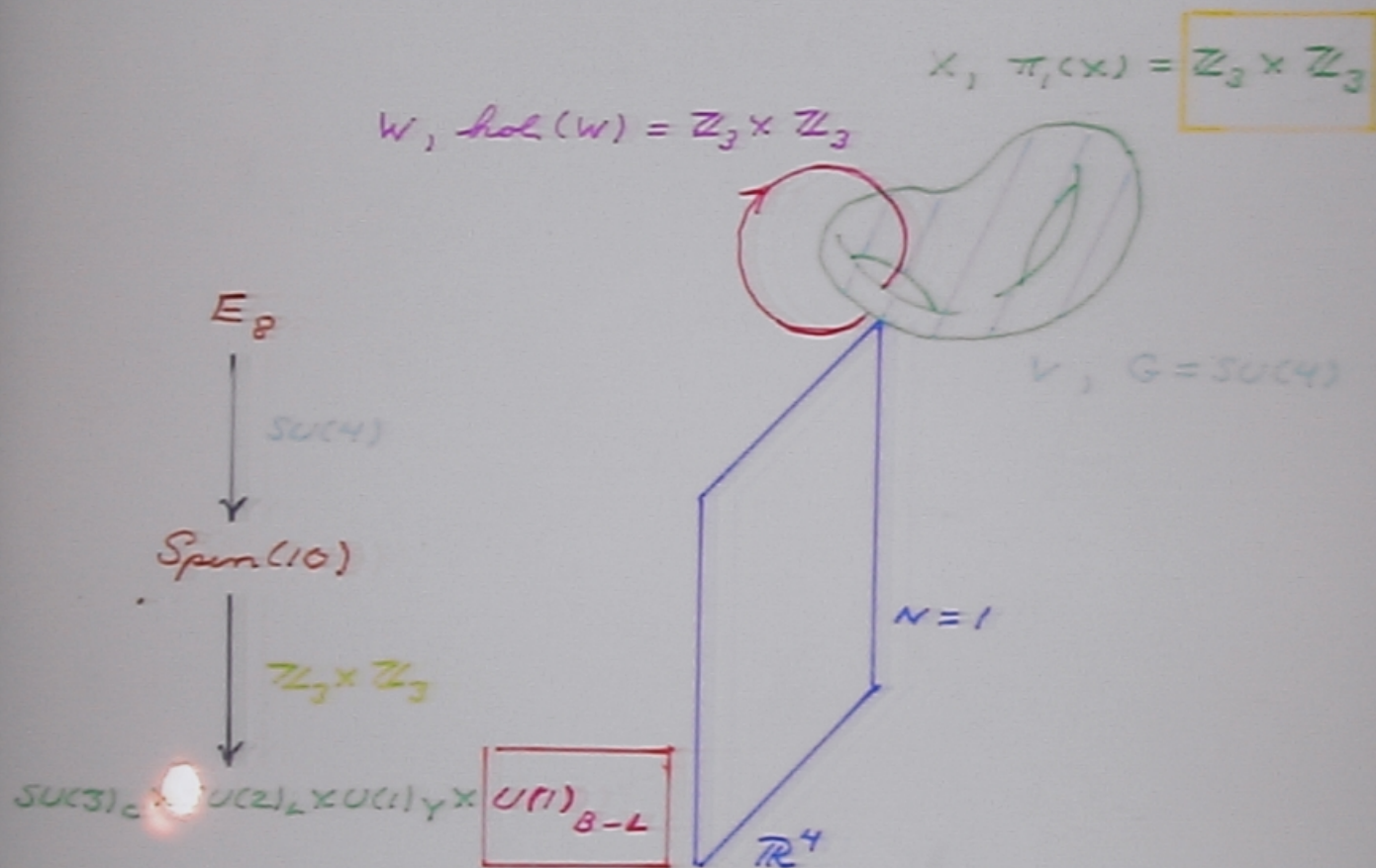


Spectrum:

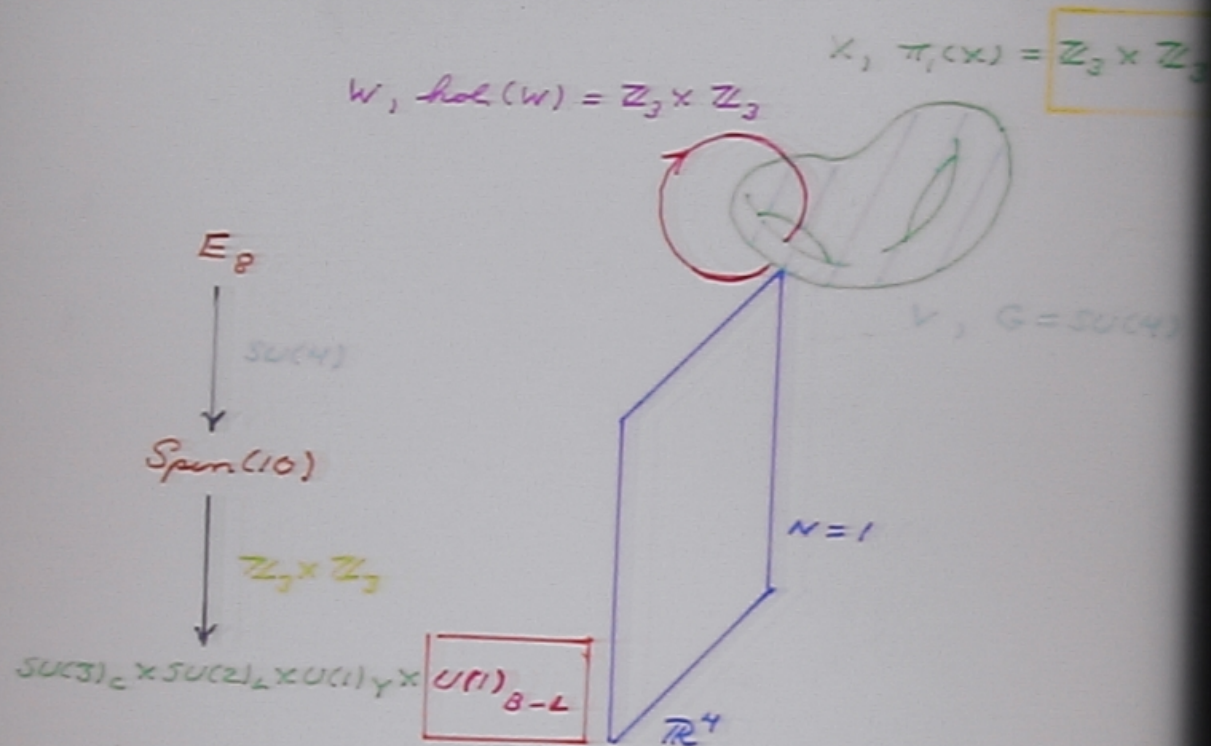
a) 3 families of quark/leptons. Each family is

$$(\underline{3}, 2, 1, 1), (\bar{\underline{3}}, 1, -4, -1), (\bar{\underline{3}}, 1, 2, -1)$$

①

Observable Sector:Spectrum:

Observable Sector:

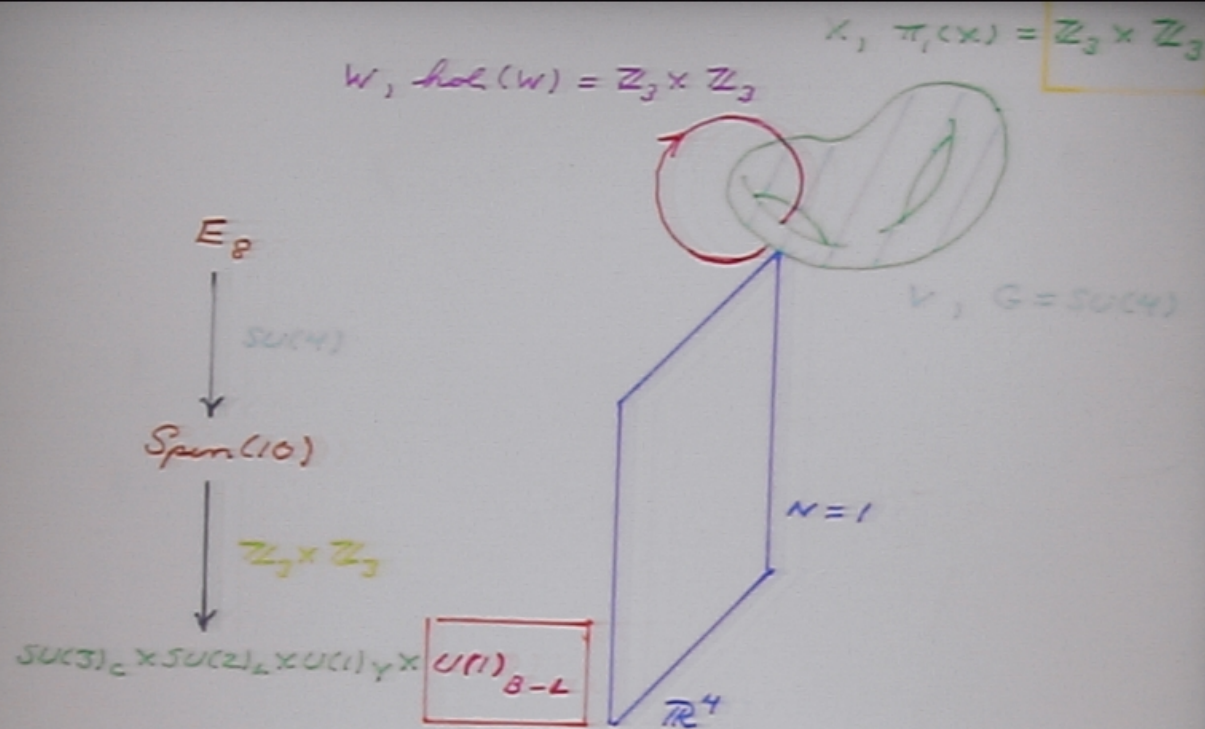


Spectrum:

a) 3 families of quark/leptons. Each family is

$$(\mathbf{3}, 2, 1, 1), (\bar{\mathbf{3}}, 1, -4, -1), (\bar{\mathbf{3}}, 1, 2, -1)$$

$$(1, 2, -3, -3), (1, 1, 6, -3)$$



Spectrum:

2) 3 families of quark/leptons. Each family is

$$(\mathbf{3}, 2, 1, 1), (\bar{\mathbf{3}}, 1, -4, -1), (\bar{\mathbf{3}}, 1, 2, -1)$$

$$(1, 2, -3, -3), (1, 1, 6, -3)$$

+

$$(\mathbf{1}, \mathbf{1}, \mathbf{0}, \mathbf{3}) \leftarrow \text{RH neutrino}$$

(2)

b) 2 pairs of Higgs / Higgs fields. Each pair is

$$(1, 2, 3, 0), (1, \bar{2}, -3, 0)$$

c) 6 geometric moduli and a small number
of vector bundle moduli.

d)

NO

EXOTIC MATTER!

How does one construct this observable sector?

X:

Consider a simply connected Calabi-Yau 3-fold

X elliptically fibred as

X

$\pi^{-1}(x)$

(2)

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Consider a simply connected Calabi-Yau 3-fold

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$\tilde{X}$

$\tilde{\tau}^*(x)$

(2)

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$\tilde{X}$ elliptically fibered as



③

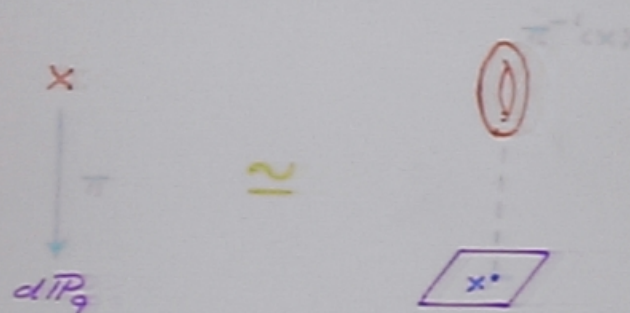
One can show there is a free action

$$\mathbb{Z}_3 \times \mathbb{Z}_3 : \tilde{X} \longrightarrow \tilde{X}$$

Then

$$X = \tilde{X} / \mathbb{Z}_3 \times \mathbb{Z}_3$$

is a smooth CY 3-fold which is torus fibred



and has non-trivial fundamental group

$$\pi_1(X) = \mathbb{Z}_3 \times \mathbb{Z}_3$$

(4)

$\Rightarrow$ 3 Kähler and 3 complex structure moduli.

V:

Consider a stable, holomorphic vector bundle

V_2 over X with structure group

$$G = SU(4)$$

V_2 is constructed by "extension"

$$0 \rightarrow V_2 \rightarrow \tilde{V} \rightarrow V_2' \rightarrow 0$$

where V_2, V_2' have rank 2. The space of extensions, $\text{Ext}^1(V_2', V_2)$, carries a representation

(4)

$\Rightarrow$ 3 Kähler and 3 complex structure moduli.

V:

Consider a stable, holomorphic vector bundle

$\tilde{V}$ over $\tilde{X}$ with structure group

$$G = SU(4)$$

$\tilde{V}$ is constructed by "extension"

$$0 \rightarrow V_2 \rightarrow \tilde{V} \rightarrow V_2' \rightarrow 0$$

where V_2, V_2' have rank 2. The space of extensions, $Ext^1(V_2', V_2)$, carries a representation of $U_3 \times U_3$. $\tilde{V}$ is equivariant if

$\tilde{V}$

$\in \Gamma(X, \tilde{V})$

$U_3 \times U_3$

⑤

Choose $\tilde{V}$ to be equivariant under $U_3 \times U_3$. Then

$$V = \tilde{V} / U_3 \times U_3$$

is a stable, holomorphic vector bundle on X

with structure group $G = SU(4)$.

Spin(10) Spectrum:

With respect to $SU(4) \times Spin(10)$

$$24\theta = (1, 45) \oplus (15, 1) \oplus (4, 16) \oplus (\bar{4}, \bar{16}) \oplus (6, 10)$$

The zero mode spectrum is

$$h_{\tilde{V}} = H^0(\tilde{X}, \mathcal{O}_{\tilde{X}}) \otimes 45 \oplus H^1(\tilde{X}, \mathcal{O}_{\tilde{X}} \otimes \tilde{V}) \otimes 1 \oplus$$

6

The number of 45 multiplets is

$$n_{45} = h^0(\tilde{X}, \mathcal{O}_{\tilde{X}}) = 1$$

For any other representation R

$$n_R = h^0(\tilde{X}, \mathcal{U}_R(\tilde{V}))$$

where $\mathcal{U}_R(\tilde{V})$ is the corresponding bundle. For example

$$n_{10} = h^0(\tilde{X}, \Lambda^2 \tilde{V})$$

Physical Constraints:

Realistic particle physics $\Rightarrow$

1. 3 generations

⑦

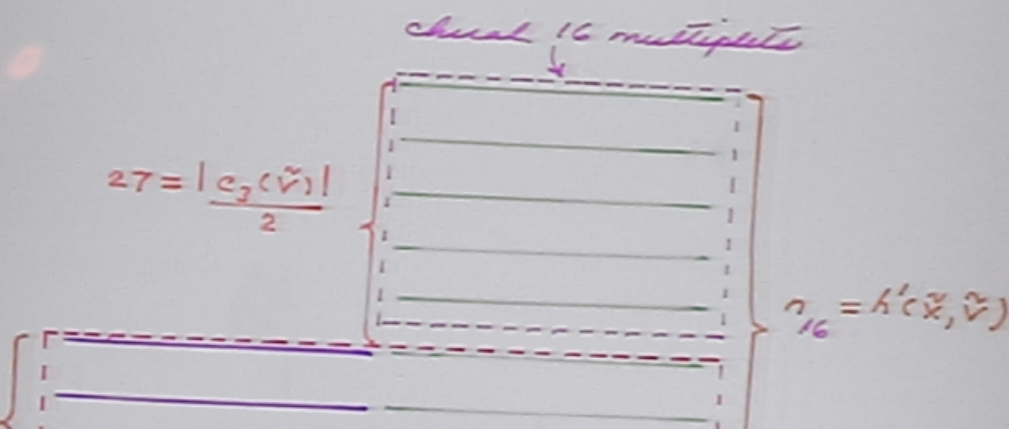
The Atiyah-Singer index Theorem and Serre
duality $\Rightarrow$

$$h'(x, v) - h'(x, v^*) = -\frac{1}{2} \int_X c_3(v)$$

Comparing $\Rightarrow$ must restrict $c_3(v) = -6 \Rightarrow$ choose

$$c_3(\tilde{v}) = -6 \times |\mathbb{Z}_3 \times \mathbb{Z}_3| = -54$$

Pictorially



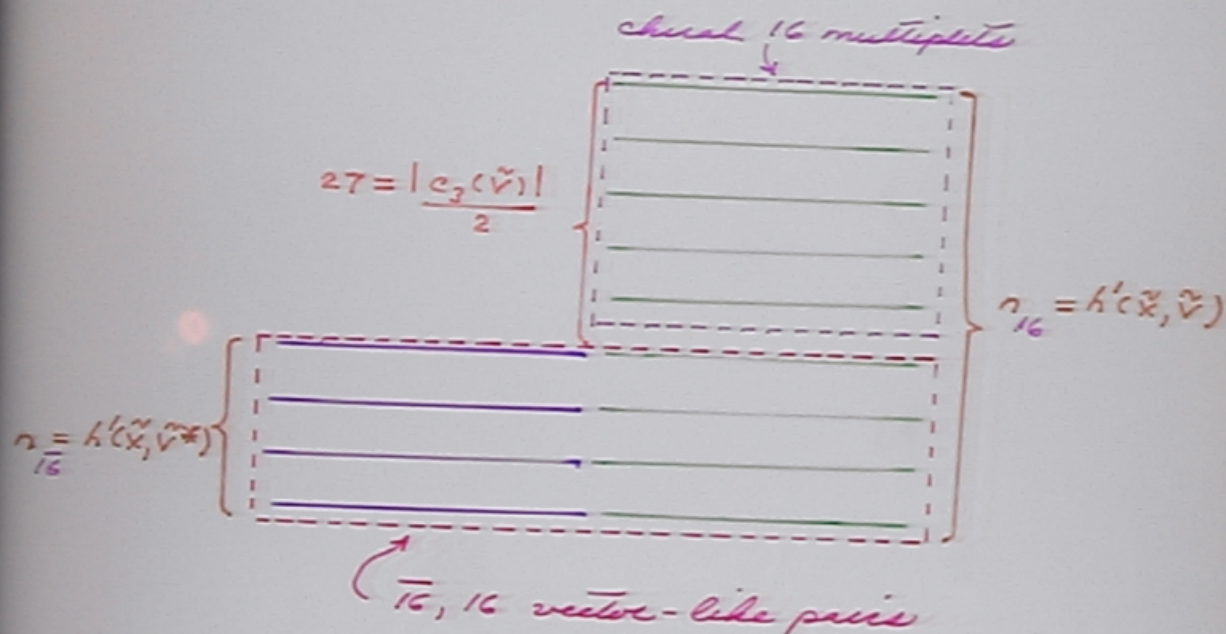
duality $\Rightarrow$

$$h'(x, v) - h'(x, v^*) = -\frac{1}{2} \int_X c_3(v)$$

Comparing $\Rightarrow$ must subtract $c_3(v) = -6 \Rightarrow$ choose

$$c_3(\tilde{v}) = -6 \times |\mathbb{Z}_3 \times \mathbb{Z}_3| = -54$$

Pictorially



2. No exotic matter

$\Rightarrow$ choose

$$n_{\overline{16}} = h'(\tilde{X}, \tilde{V}^*) = 0$$

3. Small number of Higgs

The Higgs arise from the decomposition of 10.

$\Rightarrow$ choose

$$n_{10} = h'(\tilde{X}, \wedge^2 \tilde{V}) \text{ minimal } (\neq 0)$$

Result:

Can construct bundles $\tilde{V}$ satisfying properties

1. and 2. with

(9)

$SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_{B-L}$ Spectrum:

The low energy spectrum is given by

$$\text{see } \phi_V = (H^0(\tilde{X}, \mathcal{O}_{\tilde{X}}) \otimes 45)^{\mathbb{Z}_3 \times \mathbb{Z}_3} \oplus (H^1(\tilde{X}, \omega \tilde{V}) \otimes 1)^{\mathbb{Z}_3 \times \mathbb{Z}_3} \oplus \\ (H^1(\tilde{X}, \tilde{V}) \otimes 16)^{\mathbb{Z}_3 \times \mathbb{Z}_3} \oplus (H^1(\tilde{X}, \tilde{V}) \otimes \overline{16})^{\mathbb{Z}_3 \times \mathbb{Z}_3} \oplus (H^1(\tilde{X}, \lambda^2 \tilde{V}) \otimes 10)^{\mathbb{Z}_3 \times \mathbb{Z}_3}$$

$\Rightarrow$ must find the representation of $\mathbb{Z}_3 \times \mathbb{Z}_3$ on both $H^1(\tilde{X}, \mathcal{O}_R(\tilde{V}))$ and R , take the $\otimes$ product and choose the invariant subspace.

To proceed, note that two 1-dim representations of $\mathbb{Z}_3 \times \mathbb{Z}_3$ are

$$\chi_1(g_1) = \omega, \chi_1(g_2) = 1; \chi_2(g_1) = 1, \chi_2(g_2) = \omega$$

The low energy spectrum is given by

$$\text{see } \mathcal{H}_V = (H^0(\tilde{X}, \mathcal{O}_{\tilde{X}}) \otimes 45)^{\mathbb{Z}_3 \times \mathbb{Z}_3} \oplus (H^1(\tilde{X}, \mathcal{O}_{\tilde{X}}) \otimes 1)^{\mathbb{Z}_3 \times \mathbb{Z}_3} \oplus \\ (H^1(\tilde{X}, \tilde{V}) \otimes 16)^{\mathbb{Z}_3 \times \mathbb{Z}_3} \oplus (H^1(\tilde{X}, \tilde{V}) \otimes \overline{16})^{\mathbb{Z}_3 \times \mathbb{Z}_3} \oplus (H^1(\tilde{X}, \wedge^2 \tilde{V}) \otimes 10)^{\mathbb{Z}_3 \times \mathbb{Z}_3}$$

$\Rightarrow$ must find the representation of $\mathbb{Z}_3 \times \mathbb{Z}_3$ on both $H^1(\tilde{X}, U_R(\tilde{V}))$ and R , take the $\otimes$ product and choose the invariant subspaces.

To proceed, note that two 1-dim representations of $\mathbb{Z}_3 \times \mathbb{Z}_3$ are

$$\chi_1(g_1) = \omega, \chi_1(g_2) = 1; \chi_2(g_1) = 1, \chi_2(g_2) = \omega$$

where g_1, g_2 are the generators of the two $\mathbb{Z}_3$ factors

$$\text{and } \omega = e^{\frac{2\pi i}{3}}.$$

16 representation:

(10)

Using Leray spectral sequences we find that
the $\mathbb{Z}_3 \times \mathbb{Z}_3$ action on $H^1(\tilde{X}, \tilde{V})$ is

$$H^1(\tilde{X}, \tilde{V}) = \text{Reg}(\mathbb{Z}_3 \times \mathbb{Z}_3)^{\oplus 3}$$

where

$$\text{Reg}(\mathbb{Z}_3 \times \mathbb{Z}_3) = 1 \oplus \chi_1 \oplus \chi_2 \oplus \chi_1^2 \oplus \chi_1 \chi_2 \oplus \chi_2^2 \oplus \chi_1^2 \chi_2 \oplus \chi_1 \chi_2^2 \oplus \chi_1^2 \chi_2^2$$

Note this is consistent with $H^1(\tilde{X}, \tilde{V}) = 27$. The Wilson
line acts on 16 as

$$16 = (\chi_1^2 \chi_2(3, 2) \oplus \chi_1^2 \chi_2^2(\bar{3}, 1) \oplus \chi_1^2(1, 1)) \oplus$$
$$(\chi_2^2(\bar{3}, 1) \oplus (1, 2)) \oplus (1, 1)$$

Tensoring these actions $\Rightarrow$ the invariant subspace

11

$$q \rightarrow (\bar{3}, 2, 1, 1), (\bar{3}, 1, -4, -1), (\bar{3}, 1, 2, -1)$$

$$L \rightarrow (1, 2, -3, -3), (1, 1, 6, 3)$$

$$(1, 1, 0, 3) \leftarrow r_h n$$

as desired.

10 representation

Using Leray spectral sequences we find that the

$\mathbb{Z}_3 \times \mathbb{Z}_3$ action on $H^1(\tilde{X}, \Lambda^2 \tilde{V})$ is

$$H^1(\tilde{X}, \Lambda^2 \tilde{V}) = 2 \oplus 2\chi_1 \oplus 2\chi_2 \oplus 2\chi_1^2 \oplus 2\chi_2^2 \oplus 2\chi_1\chi_2^2 \oplus 2\chi_1^2\chi_2$$

Note that this is consistent with $h^1(\tilde{X}, \Lambda^2 \tilde{V}) = 14$.

The Wilson line acts on 10 as

$$(1, 1, 0, 3) \leftarrow r_h n$$

as desired.

10 representation

Using Leray spectral sequences we find that the

$\mathbb{Z}_3 \times \mathbb{Z}_3$ action on $H^1(\tilde{X}, \Lambda^2 \tilde{V})$ is

$$H^1(\tilde{X}, \Lambda^2 \tilde{V}) = 2 \oplus 2\chi_1 \oplus 2\chi_2 \oplus 2\chi_1^2 \oplus 2\chi_2^2 \oplus 2\chi_1\chi_2^2 \oplus 2\chi_1^2\chi_2$$

Note that this is consistent with $H^1(\tilde{X}, \Lambda^2 \tilde{V}) = 14$.

The Wilson line acts on 10 as

$$10 = (\chi_1^2(1, 2) \oplus \chi_1^2\chi_2^2(3, 1)) \oplus (\chi_1(1, \bar{2}) \oplus \chi_1\chi_2(\bar{3}, 1))$$

(12)

Tensoring these actions $\Rightarrow$ the invariant subspace

$(H'(\tilde{X}, \Lambda^2 \tilde{V}) \otimes 10)^{\mathbb{Z}_3 \times \mathbb{Z}_3}$ is spanned by 2 copies of

$$(1, 2, 3, 0), (1, \bar{2}, -3, 0)$$

$\Rightarrow$ 2 copies of $H, \bar{H}$ pairs. Note that the

color triplets have been projected out!

This is an explicit mechanism for

Doublet - Triplet

Splitting

!

Finally,

Tensoring these actions $\Rightarrow$ the invariant subspace

$(H^1(\tilde{X}, \wedge^2 \tilde{V}) \otimes 10)^{\mathbb{Z}_3 \times \mathbb{Z}_3}$ is spanned by 2 copies of

$$(1, 2, 3, 0), (1, \bar{2}, -3, 0)$$

$\Rightarrow$ 2 copies of $H, \bar{H}$ pairs. Note that the color triplets have been projected out!

This is an explicit mechanism for

Doublet - Triplet

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!

Finally,

$$H^1(\tilde{X}, \wedge^2 \tilde{V}) \sim \mathcal{O}(10^2)$$

and $\mathbb{Z}_3 \times \mathbb{Z}_3$ acts trivially on 1. Therefore

$$\dim(H^1(\tilde{X}, \wedge^2 \tilde{V}) \otimes 1)^{\mathbb{Z}_3 \times \mathbb{Z}_3} \sim \mathcal{O}(10^2)$$

③

$\Rightarrow$ small number of vector bundle moduli.

Hidden Sector:

The condition for anomaly freedom is that

$$[W_5] = c_2(TX) - c_2(\tilde{V}) - c_2(\tilde{V}')$$

be an effective class. This $\Rightarrow$ a strong constraint

on $\tilde{V}'$. The simplest possibility is that $\tilde{V}'$ is

trivial. However, in this case

$$[W_5] \text{ not effective}$$

$\Rightarrow$ must choose $\tilde{V}'$ non-trivial. However, we
always take

(14)

Strong Coupling:Choose $\tilde{V}'$ with structure group

$$G' \approx SU(2)$$

This breaks

$$E_8' \longrightarrow E_7$$

and under $SU(2) \times E_7$

$$248' = \boxed{(1, 133)} \oplus \boxed{(3, 1)} \oplus (2, 56)$$

 $\Rightarrow$ for no exotic matter choose

(15)

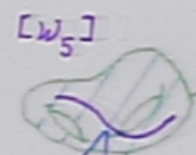
$$[w_5] \neq 0$$

$\Rightarrow$ strong coupling solution with 5-branes.

The 5-brane gauge group is

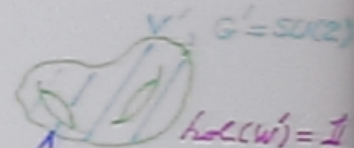
$$G'_5 = U(6) \longrightarrow U(1)^6$$

Pictorially



$U(6)$

moduli



E_7

moduli

(16)

Weak Coupling:

Want to find vacua with

$$[W_5] = 0$$

Choose $\tilde{V}' = \tilde{V}'_1 \oplus \tilde{V}'_2$ with structure group

$$G' = SU(2) \times SU(2)$$

This breaks

$$E'_0 \longrightarrow Spin(12)$$

and under $SU(2) \times SU(2) \times Spin(12)$

$$24e' = (\bar{3}, 1, 1) \oplus (1, \bar{3}, 1) \oplus (1, 1, 66) \oplus (1, 2, 32) \oplus (2, 1, 32) \oplus (2, 2, 12)$$

(17)

$$n_{32} = h'(\tilde{x}, \tilde{v}_1') = 0, \quad n'_{32} = h'(\tilde{x}, \tilde{v}_2') = 0$$

and

$$n_{12} = h'(\tilde{x}, \tilde{v}_1' \otimes \tilde{v}_2') = 0$$

However, we can only satisfy $h'(\tilde{x}, \tilde{v}_1') = 0$.

Then, minimally

$$h'(\tilde{x}, \tilde{v}_2') = 4, \quad h'(\tilde{x}, \tilde{v}_1' \otimes \tilde{v}_2') = 18$$

Be that as it may, we find

$$\dim(H'(\tilde{x}, \tilde{v}_2') \otimes 32)^{\mathbb{Z}_3 \times \mathbb{Z}_3} = \underline{\underline{0}}$$

$\Rightarrow$ 32 multiplets are projected out. Unfortunately,

$$\dim(H'(\tilde{x}, \tilde{v}_1' \otimes \tilde{v}_2') \otimes 12)^{\mathbb{Z}_3 \times \mathbb{Z}_3} = \underline{\underline{2}}$$

(19)

For such $\tilde{V}'$

$$[W_5] = 0$$

$\Rightarrow$ weak coupling solution (also a strong coupling solution) with no 5-branes.

Pictorially

