Title: Moduli Stabilization by Magnetic Fluxes and Split Supersymmetry

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Abstract:

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Moduli stabilization
by
magnetic fluxes

with T. Maillard hep-th/0412008
also with A. Kumar and T. Maillard to appear

Outline

- Motivations

- Framework

  Type I string theory with magnetized D9 branes

- T^6 example
  - SUSY conditions
  - tadpole cancellation
  - general solution
  - explicit example

- T^6/Z_2 orientifold combined with 3-form fluxes

  fix also the dilaton

Moduli stabilization with 3-form fluxes:
significant progress but

- no exact string description
  low energy SUGRA approximation

- fix only complex structure

Type I with internal magnetic fluxes:
alternative/complementary approach

- exact string description

- Kähler class stabilization
  T^6: all geometric moduli fixed

- natural implementation in intersecting D-brane models

General framework

- Type I string theory compactified in 4d on 6d Calabi-Yau

$\Rightarrow$ $N = 2$ SUSY in the bulk, $N = 1$ on branes

- Magnetic fluxes on 2-cycles

$\Rightarrow$ SUSY breaking

Dirac quantization: $H = \frac{m}{nA} \equiv \frac{p}{A}$

H: constant magnetic field

m: units of magnetic flux

n: brane wrapping

A: area of the 2-cycle

Spin-dependent mass shifts for all charged states

$[p_i, p_j] = iqH\epsilon_{ij}$ $\quad$ q: charge

$\Rightarrow$ Landau spectrum

6d $\rightarrow$ 4d on T^2 with abelian magnetic field H

$\delta M^2 = (2k+1)|qH| + 2qH \cdot \Sigma$ — spin operator

$k = 0, 1, 2, \ldots$: Landau level

Landau multiplicity: mn

- spin-0: $\Sigma = 0 \Rightarrow$ mass gap

- spin-1/2: $\Sigma = \pm 1/2 \Rightarrow$ chiral 0-mode

$k = 0 \quad : \quad \delta M^2 = |qH| \pm qH$

$\Rightarrow \quad \delta M^2 = 0$ for $\Sigma = -1/2 \; (qH > 0)$

- spin-1: $\Sigma = \pm 1 \Rightarrow$ tachyon

Nielsen-Olesen instability

$k = 0 \quad : \quad \delta M^2 = |qH| \pm 2qH$

$\Rightarrow \quad \delta M^2 = -qH$ for $\Sigma = -1 \; (qH > 0)$

$(T^2)^3$ generalization: H_I with $I = 1, 2, 3$

$$\delta M^2 = \sum_I \{(2k_I + 1)|qH_I| + 2qH_I\Sigma_I\}$$

- spin-1/2: one chiral 0-mode

$\delta M^2 = 0$ for $k_I = 0$ and $\Sigma_I = -1/2$ $(qH_I > 0)$

- spin-1: tachyon can be avoided Bachas 95

$$\begin{array}{rcrcrl} & |H_1| & + & |H_2| & - & |H_3| > 0 \\ & |H_1| & - & |H_2| & + & |H_3| > 0 \\ - & |H_1| & + & |H_2| & + & |H_3| > 0 \end{array}$$

massless scalar ⇔ partial brane susy restoration

Angelantonj-I.A.-Dudas-Sagnotti 00

$$\theta_1 + \theta_2 + \theta_3 = 0$$

Magnetic fluxes can be used to stabilize moduli

I.A.-Maillard

e.g. T^6: 36 moduli (geometric deformations)

internal metric: $6 \times 7/2 = 21$

type IIB RR 2-form: $6 \times 5/2 = 15$

complexification $\Rightarrow \begin{cases} \text{Kähler class} & J \\ \text{complex structure} & \tau \end{cases}$

9 complex moduli for each

magnetic flux: 6×6 antisymmetric matrix F

complexification $\Rightarrow$

$F_{(2,0)}$ on holomorphic 2-cycles: potential for τ

$F_{(1,1)}$ on mixed (1,1)-cycles: potential for J

$N = 1$ susy conditions:

(1) $F_{(2,0)} = 0 \quad \Rightarrow \tau$

(2) $J \wedge J \wedge F_{(1,1)} = F_{(1,1)} \wedge F_{(1,1)} \wedge F_{(1,1)} \quad \Rightarrow J$

Appropriate choice of magnetic fluxes F^a
in several abelian directions $U(1)_a \Rightarrow$

all moduli vanish except the 6 radii of T^6
which are fixed in terms of the quantized fluxes

$T^6 = \prod_{I=1}^{3} T_I^2$ ← orthogonal 2-torus

$\tau_I = \frac{R_I}{R_I'} \qquad J_I = R_I R_I' \qquad H_I^a = \frac{F_I^a}{R_I R_I'}$

(1) fixes the ratios τ_I

(2) fixes the sizes J_I

$H_1 + H_2 + H_3 = H_1 H_2 H_3 \Leftrightarrow \theta_1 + \theta_2 + \theta_3 = 0$

DBI action:

$$\{[J\wedge J\wedge J - J\wedge F\wedge F]^2 + [J\wedge J\wedge F - F\wedge F\wedge F]^2\}^{1/2}$$

becomes polynomial

$$S_{\mathrm{DBI}} = -T_9 \int \sqrt{g}\,(1 - |H_1 H_2| + |H_2 H_3| + |H_1 H_3|)$$

or permutation of the minus sign

I.A.-Maillard, Bianchi-Trevigne

Tadpole conditions

$$Q_9 = \sum_a N_a \sum_{rst} \mathcal{K}_{rst} n^a_r n^a_s n^a_t = 16$$ — O9 charge

sic due to orientation choice

$$Q_5 = \sum_a N_a \sum_{st} \mathcal{K}_{rst} n^a_r m^a_s m^a_t = 0$$ $\forall$ 2-cycle r

- volume positivity: $J\wedge J\wedge J - J\wedge F^a \wedge F^a > 0$
- no antibranes: $\mathcal{K}_{rst} n^a_r > 0 \quad \forall s, t$

Main ingredients for moduli stabilization

1. "oblique" magnetic fields ⇒ fix off-diagonal components of the metric

2. magnetized D9's ⇒ -ve 5-brane tension

3. Non linear DBI action ⇒ fix overall volume

(1)+(2) : necessary for Q_5 cancellation

(2)+(3) : not valid in six dimensions

Stabilization of RR moduli

- Kähler class: absorbed by massive $U(1)$'s

$dC_2 \wedge \star(A^a \wedge F^a) \quad \Rightarrow$

4d G-S kinetic mixing with magnetized $U(1)$'s

$\Rightarrow$ need at least 9 brane stacks

- Complex structure: get potential through mixing with NS moduli

Bianchi-Trevigne

Stack $\sharp$	Fluxes	Fixed complex structure	5 − brane local.	Fixed Kähler class
$\sharp 1$	$(F^1_{x_1y_2}, F^1_{x_2y_1})$	$\tau_{31} = \tau_{32} = 0$ $p^1_{x_1y_2}\tau_{11} = \tau_{22}p^1_{x_2y_1}$	$[x_3, y_3]$	$V_{12} - V_{21} = 0$
$\sharp 2$	$(F^2_{x_1y_3}, F^2_{x_3y_1})$	$\tau_{21} = \tau_{23} = 0$ $p^2_{x_1y_3}\tau_{11} = \tau_{33}p^2_{x_3y_1}$	$[x_2, y_2]$	$V_{13} - V_{31} = 0$
$\sharp 3$	$(F^3_{x_1x_2}, F^3_{y_1y_2})$	$\tau_{13} = 0$ $\tau_{11}\tau_{22} = -\frac{p^3_{y_1y_2}}{p^3_{x_1x_2}}$	$[x_3, y_3]$	$V_{12} + V_{21} = 0$
$\sharp 4$	$(F^4_{x_2x_3}, F^4_{y_2y_3})$	$\tau_{12} = 0$	$[x_1, y_1]$	$V_{23} + V_{32} = 0$
$\sharp 5$	$(F^5_{x_1x_3}, F^5_{y_1y_3})$		$[x_2, y_2]$	$V_{13} + V_{31} = 0$
$\sharp 6$	$(F^6_{x_2y_3}, F^6_{x_3y_2})$		$[x_1, y_1]$	$V_{23} - V_{32} = 0$

Fixed complex structure and Kähler class moduli for each magnetized stack $\sharp$ of $D9$ branes depending on the quantized fluxes. The previous to last column gives the localization on the 2-cycles $C_r^{(2)}$, with $r = [x_i, y_i]$, of the induced 5-brane charges.

$V_{i\bar{j}} = (J \wedge J)_{i\bar{j}} = 0 \Rightarrow J_{i\bar{j}} = 0 \;\; \forall i \neq j$

Compatibility condition: p^4, p^5, p^6 constrained by p^1, p^2, p^3, so that the solution for τ_{ii} remains unchanged

Fix areas of the 3 T^2's $\Rightarrow$ add 3 more stacks:

Stack ♯	Fluxes	5 – brane localization
♯7	$(F^7_{x_1y_1}, F^7_{x_2y_2}, F^7_{x_3y_3})$	$[x_1, y_1]$ $[x_2, y_2]$ $[x_3, y_3]$
♯8	$(F^8_{x_1y_1}, F^7_{x_2y_2}, F^8_{x_3y_3})$	$[x_1, y_1]$ $[x_2, y_2]$ $[x_3, y_3]$
♯9	$(F^9_{x_1y_1}, F^9_{x_2y_2}, F^9_{x_3y_3})$	$[x_1, y_1]$ $[x_2, y_2]$ $[x_3, y_3]$

$$\Rightarrow \begin{pmatrix} \mathcal{F}_1^7 & \mathcal{F}_2^7 & \mathcal{F}_3^7 \\ \mathcal{F}_1^8 & \mathcal{F}_2^8 & \mathcal{F}_3^8 \\ \mathcal{F}_1^9 & \mathcal{F}_2^9 & \mathcal{F}_3^9 \end{pmatrix} \begin{pmatrix} J_2 J_3 \\ J_1 J_3 \\ J_1 J_2 \end{pmatrix} = \begin{pmatrix} \mathcal{F}_1^7 \mathcal{F}_2^7 \mathcal{F}_3^7 \\ \mathcal{F}_1^8 \mathcal{F}_2^8 \mathcal{F}_3^8 \\ \mathcal{F}_1^9 \mathcal{F}_2^9 \mathcal{F}_3^9 \end{pmatrix}$$

Consistency conditions and tadpole cancellation can be easily satisfied

- large volume: rescale $\mathcal{F}_1^{7,8,9} \to \Lambda \mathcal{F}_1^{7,8,9} \Rightarrow J_1 \to \Lambda J_1$

tadpoles can be satisfied by an appropriate rescaling of the first stacks

Stack $\sharp$	Fluxes	Fixed complex structure	$5-$brane local.	Fixed Kähler class
$\sharp 1$	$(F^1_{x_1y_2}, F^1_{x_2y_1})$	$\tau_{31} = \tau_{32} = 0$ $p^1_{x_1y_2}\tau_{11} = \tau_{22}p^1_{x_2y_1}$	$[x_3, y_3]$	$V_{12} - V_{21} = 0$
$\sharp 2$	$(F^2_{x_1y_3}, F^2_{x_3y_1})$	$\tau_{21} = \tau_{23} = 0$ $p^2_{x_1y_3}\tau_{11} = \tau_{33}p^2_{x_3y_1}$	$[x_2, y_2]$	$V_{13} - V_{31} = 0$
$\sharp 3$	$(F^3_{x_1x_2}, F^3_{y_1y_2})$	$\tau_{13} = 0$ $\tau_{11}\tau_{22} = -\frac{p^3_{y_1y_2}}{p^3_{x_1x_2}}$	$[x_3, y_3]$	$V_{12} + V_{21} = 0$
$\sharp 4$	$(F^4_{x_2x_3}, F^4_{y_2y_3})$	$\tau_{12} = 0$	$[x_1, y_1]$	$V_{23} + V_{32} = 0$
$\sharp 5$	$(F^5_{x_1x_3}, F^5_{y_1y_3})$		$[x_2, y_2]$	$V_{13} + V_{31} = 0$
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$$\Rightarrow \begin{pmatrix} \mathcal{F}_1^7 & \mathcal{F}_2^7 & \mathcal{F}_3^7 \\ \mathcal{F}_1^8 & \mathcal{F}_2^8 & \mathcal{F}_3^8 \\ \mathcal{F}_1^9 & \mathcal{F}_2^9 & \mathcal{F}_3^9 \end{pmatrix} \begin{pmatrix} J_2 J_3 \\ J_1 J_3 \\ J_1 J_2 \end{pmatrix} = \begin{pmatrix} \mathcal{F}_1^7 \mathcal{F}_2^7 \mathcal{F}_3^7 \\ \mathcal{F}_1^8 \mathcal{F}_2^8 \mathcal{F}_3^8 \\ \mathcal{F}_1^9 \mathcal{F}_2^9 \mathcal{F}_3^9 \end{pmatrix}$$

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$$\{[J\wedge J\wedge J - J\wedge F\wedge F]^2 + [J\wedge J\wedge F - F\wedge F\wedge F]^2\}^{1/2}$$

becomes polynomial

$$S_{\mathrm{DBI}} = -T_9 \int \sqrt{g}\,(1 - |H_1 H_2| + |H_2 H_3| + |H_1 H_3|)$$

or permutation of the minus sign

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sign due to orientation choice

$Q_5 = \sum_a N_a \sum_{st} \mathcal{K}_{rst} n_r^a m_s^a m_t^a = 0 \quad \forall$ 2-cycle r

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♯4	$(F^4_{x_2x_3}, F^4_{y_2y_3})$	$\tau_{12}=0$	$[x_1, y_1]$	$V_{23}+V_{32}=0$
♯5	$(F^5_{x_1x_3}, F^5_{y_1y_3})$		$[x_2, y_2]$	$V_{13}+V_{31}=0$
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- volume positivity: $J\wedge J\wedge J - J\wedge F^a \wedge F^a > 0$
- no antibranes: $\mathcal{K}_{rst} n_r^a > 0 \quad \forall s,t$

Live with the hierarchy

still unknown explanation perhaps related to the cosmological constant problem

Split Supersymmetry: raise SUSY breaking scale

but keep SUSY main predictions:

unification + dark matter candidate ⇒

keep all MSSM fermions light

but let squarks and sleptons become heavy

TeV physics: SM with a 'fine tuned' light Higgs

+ gauginos + a pair of higgsinos

All MSSM 'problems' solved:

FCNC, B/L violation, CP, nb of parameters,...

Gauge couplings

$$SU(N_a): \quad \frac{1}{\alpha_{N_a}} = \frac{V}{g_s} \prod_I |n_I^a| \sqrt{1 + (H_I^a \alpha')^2}$$

g_s: string coupling

V: compactification volume in string units

$$U(1)_a: \quad \alpha_{U(1)_a} = \frac{\alpha_{N_a}}{2N_a}$$

Non abelian unification conditions:

(i) $\prod_I |n_I^a|$ independent of a

follows from absence of chiral symmetric reps

no color sextets and weak triplets $\Rightarrow \prod_I n_I^a = 1$

(ii) $|H_I^a| \begin{cases} \text{independent of } a \\ << M_s^2 = \alpha'^{-1} \end{cases}$

$\Rightarrow$ more quantitative analysis

$$\frac{1}{\alpha_{N_a}} = \frac{V}{g_s} \prod_I \sqrt{1 + (H_I^a \alpha')^2}$$

1% error in $\alpha_3 = \alpha_2 \quad \Rightarrow \quad H_I^a \alpha' \lesssim 0.1$

$\Rightarrow V = \prod_I V_I \gtrsim 10^3$

too high to keep strings weakly coupled?

$\alpha_{\mathrm{GUT}} \simeq 1/25 \to g_s \gtrsim \mathcal{O}(10)$

can be partly relaxed if $H_I^3 = H_I^2$ for some I:

it follows from the absence of chiral $(\bar{3}, 2)$

no antiquark doublets

$\Rightarrow$ keep $g_s \lesssim \mathcal{O}(1)$

same stack: antisymmetric or symmetric

$$\text{multiplicities}: \begin{cases} A: \ \frac{1}{2}\left(\prod_I 2m_I^a\right)\left(\prod_J n_J^a + 1\right) \\ S: \ \frac{1}{2}\left(\prod_I 2m_I^a\right)\left(\prod_J n_J^a - 1\right) \end{cases}$$

different stacks: bifundamentals

$$\text{multiplicities}: \begin{cases} (N_a, N_b): \ \prod_I (m_I^a n_I^b + n_I^a m_I^b) \\ (N_a, \bar{N}_b): \ \prod_I (m_I^a n_I^b - n_I^a m_I^b) \end{cases}$$

⇒ Generic spectrum of split SUSY:

- massless gauginos
- massive squarks and sleptons
- massless Higgs ⇔ non chiral susy intersection
  two Higgs multiplets

Mass scales

- $M_{\text{GUT}} \simeq$ smallest compactification scale
  $\simeq 10^{16}$ GeV

- smallest $H_I^a \alpha' \sim 0.1 \Rightarrow$
  $M_s \simeq 3 \times M_{\text{GUT}}$

- $m_{\text{susy}} \sim$ largest scalar mass m_S
  : free parameter

branes: $m_S^2 \sim \delta H^a \equiv \epsilon_1 H_1^a + \epsilon_2 H_2^a + \epsilon_3 H_3^a$

brane intersections: $m_S^2 \sim \delta H^{ab} \equiv \delta H^a - \delta H^b$

"natural" scale: $m_S \sim M_{\text{GUT}}$

but can be much smaller stable due to SUSY

Minimal Standard Model embedding

New possibilities using intersecting branes

- no large dimensions for low string scale
- no need for B or L conservation
- but need $\sin^2\theta_W = \frac{3}{8}$

General analysis using 3 brane stacks

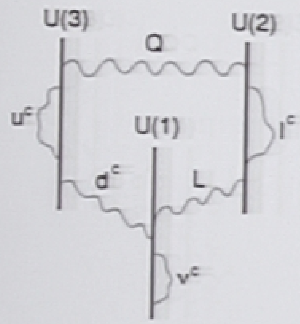
$\Rightarrow U(3) \times U(2) \times U(1)$

antiquarks u^c, d^c $(\bar{3}, 1)$:

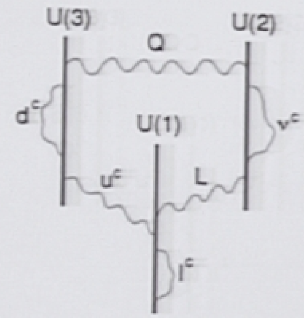
antisymmetric of $U(3)$ or

bifundamental $U(3) \leftrightarrow U(1)$

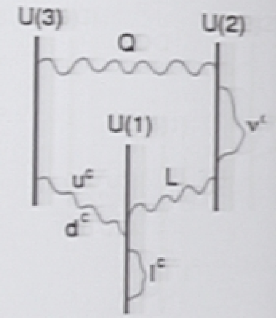
$\Rightarrow$ 3 models: antisymmetric is u^c, d^c or none



Model A



Model B



Model C

	Model A	Model B	Model C
Q	$(3,2;1,1,0)_{1/6}$	$(3,2;1,\varepsilon_Q,0)_{1/6}$	$(3,2;1,\varepsilon_Q,0)_{1/6}$
u^c	$(\bar{3},1;2,0,0)_{-2/3}$	$(\bar{3},1;-1,0,1)_{-2/3}$	$(\bar{3},1;-1,0,1)_{-2/3}$
d^c	$(\bar{3},1;-1,0,\varepsilon_d)_{1/3}$	$(\bar{3},1;2,0,0)_{1/3}$	$(\bar{3},1;-1,0,-1)_{1/3}$
L	$(1,2;0,-1,\varepsilon_L)_{-1/2}$	$(1,2;0,\varepsilon_L,1)_{-1/2}$	$(1,2;0,\varepsilon_L,1)_{-1/2}$
l^c	$(1,1;0,2,0)_1$	$(1,1;0,0,-2)_1$	$(1,1;0,0,-2)_1$
ν^c	$(1,1;0,0,2\varepsilon_\nu)_0$	$(1,1;0,2\varepsilon_\nu,0)_0$	$(1,1;0,2\varepsilon_\nu,0)_0$

$$Y_A = -\frac{1}{3}Q_3 + \frac{1}{2}Q_2 \qquad Y_{B,C} = \frac{1}{6}Q_3 - \frac{1}{2}Q_1$$

Model A : $\sin^2\theta_W = \frac{1}{2+2\alpha_2/3\alpha_3}\Big|_{\alpha_2=\alpha_3} = \frac{3}{8}$

Model B, C : $\sin^2\theta_W = \frac{1}{1+\alpha_2/2\alpha_1+\alpha_2/6\alpha_3}\Big|_{\alpha_2=\alpha_3} = \frac{6}{7+3\alpha_2/\alpha_1}$